Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/39-1.2.2.2-d-x-^m-a+b-x^2+c-x^4-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [1126]. This is test number [39].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1126)	0.00 (0)
Mathematica	100.00 (1126)	0.00 (0)
Maple	94.32 (1062)	5.68 (64)
Fricas	88.90 (1001)	11.10 (125)
Giac	72.91 (821)	27.09 (305)
Mupad	61.72 (695)	38.28 (431)
Maxima	61.10 (688)	38.90 (438)
Sympy	41.92 (472)	58.08 (654)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

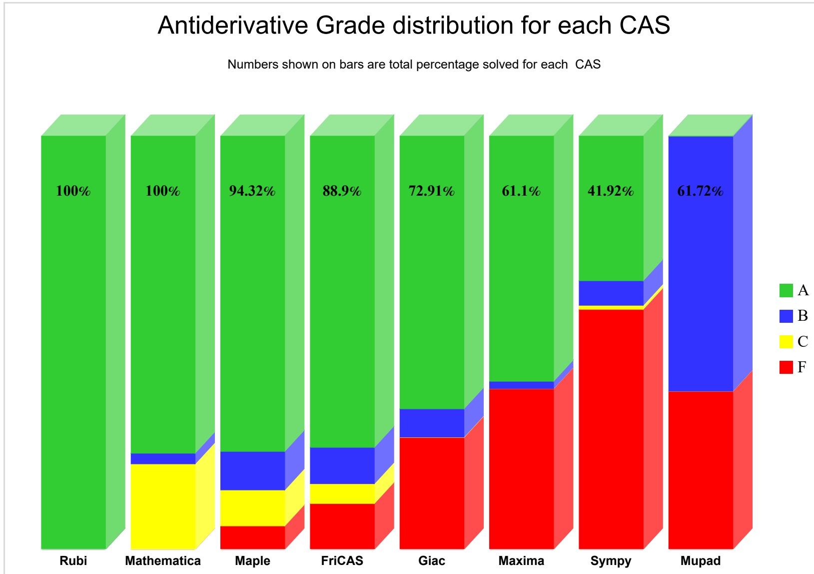
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

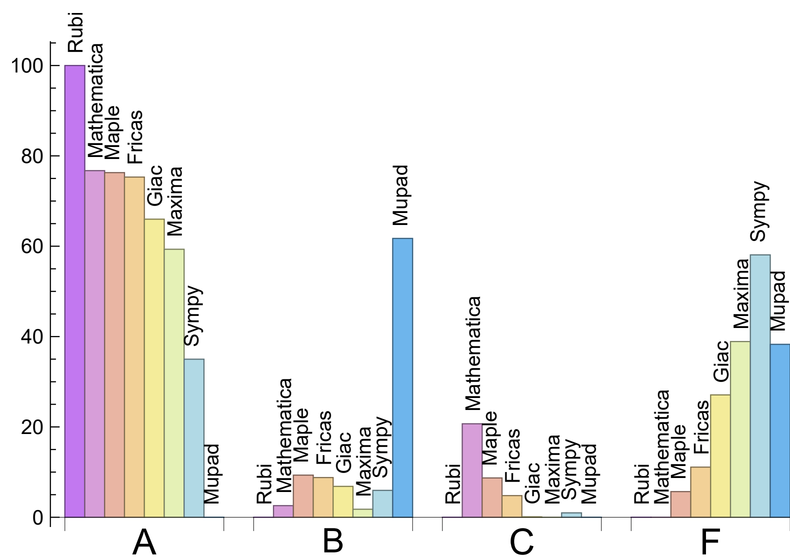
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	76.73	2.58	20.69	0.00
Maple	76.29	9.33	8.70	5.68
Fricas	75.31	8.79	4.80	11.10
Giac	65.99	6.84	0.09	27.09
Maxima	59.33	1.78	0.00	38.90
Sympy	34.99	5.95	0.98	58.08
Mupad	N/A	61.72	0.00	38.28

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	64	100.00 %	0.00 %	0.00 %
Fricas	125	77.60 %	3.20 %	19.20 %
Giac	305	98.69 %	0.33 %	0.98 %
Maxima	438	85.39 %	0.00 %	14.61 %
Sympy	654	83.03 %	14.37 %	2.60 %
Mupad	431	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

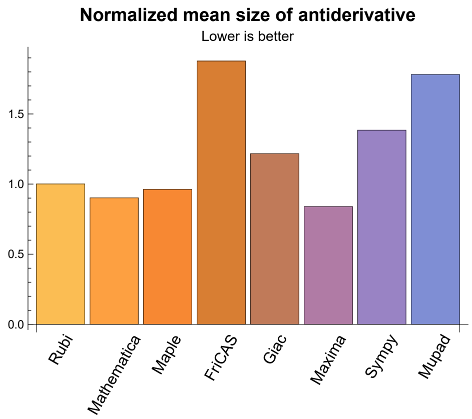
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

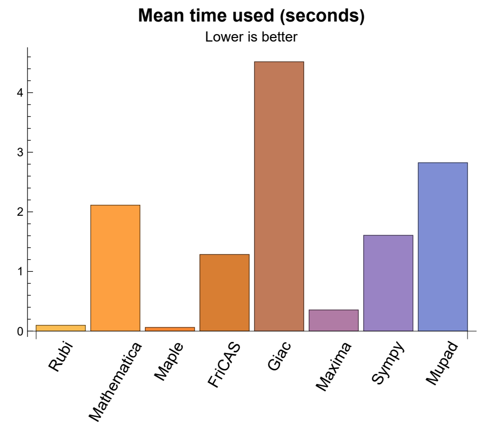
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.10	144.69	1.00	90.00	1.00
Mathematica	2.11	99.01	0.90	70.50	0.88
Maple	0.06	128.86	0.96	77.00	0.85
Maxima	0.35	90.16	0.84	55.00	0.88
Fricas	1.28	428.13	1.88	68.00	0.90
Sympy	1.61	102.71	1.38	51.00	0.96
Giac	4.52	179.04	1.22	70.00	0.88
Mupad	2.82	294.80	1.78	66.00	0.88

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {17, 18, 19, 23, 24, 25, 27, 28, 30, 31, 32, 36, 37, 38, 40, 41, 43, 44, 45, 49, 50, 51, 53, 54, 56, 57, 58, 62, 63, 64, 78, 79, 81, 124, 125, 126, 128, 129, 130, 132, 133, 1097, 1098, 1099, 1108, 1114, 1115, 1116, 1117, 1122, 1123, 1124, 1125, 1126}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

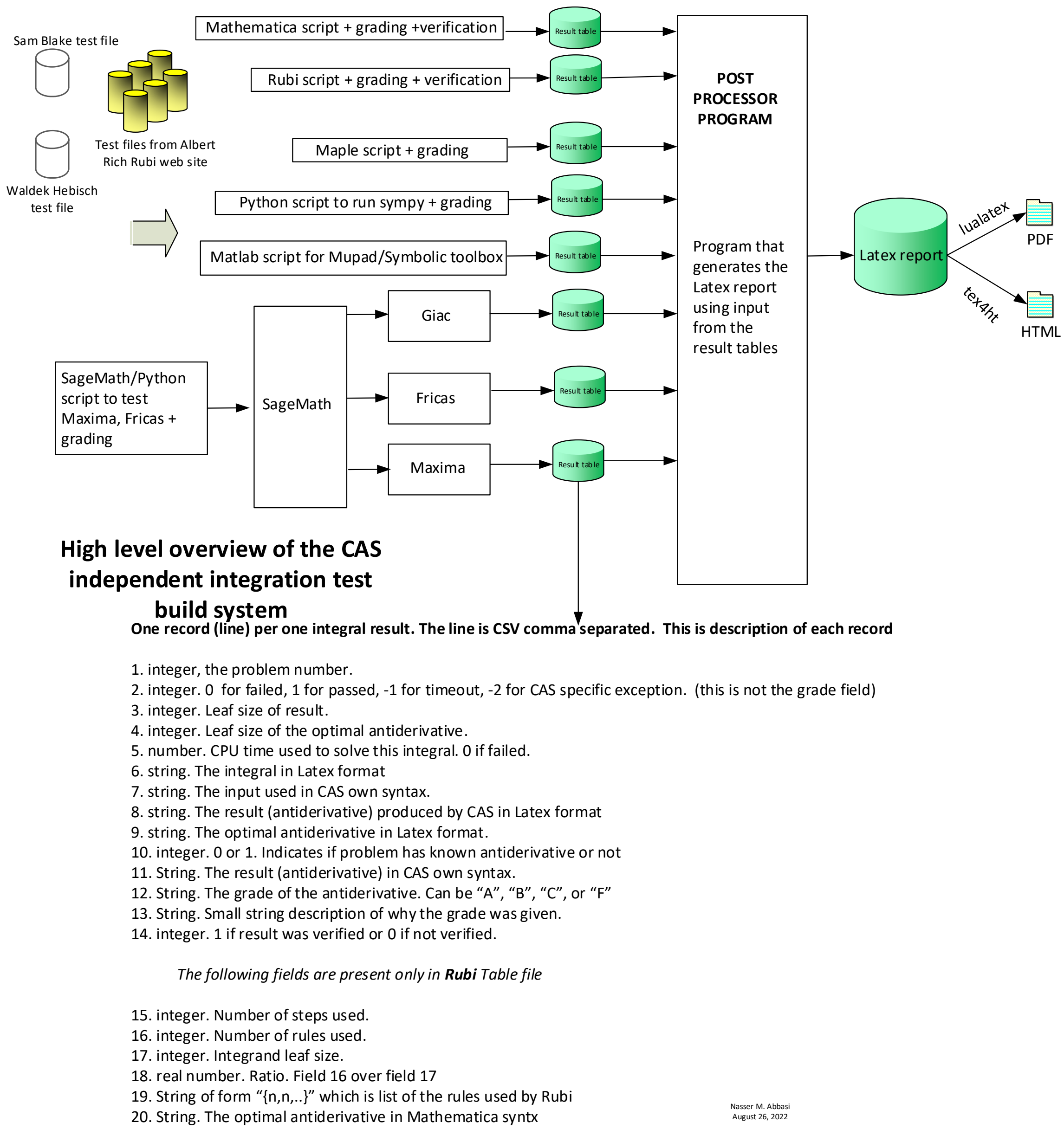
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 12, 21, 22, 33, 34, 42, 46, 47, 55, 60, 61, 76, 77, 92, 93, 94, 96, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 915, 916, 917, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 955, 956, 957, 958, 959, 960, 961, 962, 968, 969, 972, 973, 974, 975, 981, 982, 983, 984, 985, 986, 987, 988, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1097, 1098, 1099, 1105, 1106, 1107, 1111, 1112, 1113, 1114, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

B grade: { 11, 26, 39, 95, 97, 108, 109, 171, 438, 449, 467, 515, 970, 971, 1040, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1100, 1101, 1102, 1103, 1104, 1110 }

C grade: { 8, 10, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 40, 41, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 659, 660, 661, 662, 663, 664, 909, 910, 911, 912, 913, 914, 918, 919, 930, 931, 932, 933, 934, 935, 947, 948, 949, 950, 951, 952, 953, 954, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 989, 990, 991, 992, 993, 1003, 1005, 1007, 1009, 1011, 1039, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1108, 1109, 1115, 1116, 1117 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 4, 5, 6, 7, 9, 12, 13, 14, 66, 76, 77, 78, 79, 81, 92, 93, 94, 95, 96, 97, 108, 109, 119, 122, 124, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 766, 768, 787, 791, 792, 793, 798, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 876, 877, 878, 879, 880, 881, 882, 883, 884, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 912, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 930, 931, 932, 933, 934, 935, 940, 941, 942, 943, 947, 948, 949, 950, 951, 952, 953, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 999, 1000, 1001, 1002, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1107 }

B grade: { 8, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 125, 126, 127, 128, 129, 130, 131, 132, 133, 171, 243, 270, 287, 403, 426, 438, 449, 451, 467, 496, 515, 598, 757, 758, 759, 760, 761, 762, 763, 764, 765, 767, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 874, 875, 885, 886, 911, 913, 914, 926, 927, 928, 929, 936, 937, 938, 939, 944, 945, 946, 954, 981, 982, 998, 1039, 1105, 1106 }

C grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 123, 1003, 1005, 1007, 1009, 1011, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088 }

F grade: { 3, 406, 407, 408, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.4 Maxima

A grade: { 12, 13, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 403, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 517, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 761, 762, 763, 764, 773, 774, 775, 776, 777, 778, 779, 780, 785, 786, 787, 791, 792, 793, 798, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 897, 898, 899, 900, 901, 906, 907, 908, 909, 910, 915, 916, 917, 968, 969, 970, 971, 972, 973, 974, 975, 994, 995, 996, 997, 999, 1001, 1004, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1105, 1106, 1107 }

B grade: { 11, 161, 171, 210, 244, 245, 426, 438, 449, 451, 467, 496, 498, 515, 516, 518, 519, 598, 647, 1006 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 234, 235, 236, 237, 254, 255, 256, 257, 258, 259, 270, 271, 272, 283, 284, 285, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 659, 660, 661, 662, 663, 664, 757, 758, 759, 760, 765, 766, 767, 768, 769, 770, 771, 772, 781, 782, 783, 784, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 902, 903, 904, 905, 911, 912, 913, 914, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 998, 1000, 1002, 1003, 1005, 1007, 1009, 1011, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 517, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 787, 791, 792, 793, 798, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 888, 889, 890, 891, 892, 897, 898, 899, 900, 901, 906, 907, 908, 909, 910, 911, 912, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 954, 955, 956, 957, 958, 959, 960, 961, 962, 965, 968, 969, 970, 971, 972, 973, 974, 975, 978, 981, 982, 983, 984, 985, 987, 988, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1006, 1007, 1008, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1107 }

B grade: { 8, 10, 11, 14, 34, 161, 171, 210, 403, 426, 438, 449, 451, 467, 496, 498, 499, 500, 515, 516, 518, 519, 520, 521, 522, 598, 647, 785, 786, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 893, 894, 895, 896, 902, 903, 904, 905, 913, 914, 918, 986, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1082, 1083, 1084, 1086, 1105, 1106 }

C grade: { 80, 123, 287, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402 }

F grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 95, 96, 97, 108, 109, 406, 407, 408, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 930, 931, 932, 933, 934, 935, 947, 948, 949, 950, 951, 952, 953, 963, 964, 966, 967, 976, 977, 979, 980, 989, 990, 991, 992, 993, 1005, 1009, 1040, 1081, 1085, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.6 Sympy

A grade: { 8, 10, 12, 13, 14, 21, 34, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 179, 181, 183, 185, 187, 189, 190, 191, 192, 193, 194, 195, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 212, 213, 214, 215, 216, 217, 218, 219, 220, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 490, 491, 492, 493, 494, 495, 497, 499, 500, 501, 502, 503, 504, 505, 508, 509, 510, 511, 512, 513, 514, 517, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 691, 705, 723, 730, 752, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 855, 856, 857, 858, 859, 860, 869, 893, 894, 895, 896, 899, 902, 903, 904, 905, 911, 912, 913, 914, 915, 916, 917, 918, 919, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061 }

B grade: { 9, 11, 15, 148, 161, 171, 178, 180, 182, 184, 186, 188, 196, 198, 210, 211, 403, 404, 405, 426, 438, 449, 451, 467, 469, 488, 489, 496, 498, 506, 507, 515, 516, 518, 519, 785, 786, 787, 848, 849, 850, 851, 852, 862, 863, 864, 874, 875, 876, 877, 878, 888, 889, 890, 891, 897, 898, 900, 901, 906, 907, 908, 909, 910, 1105, 1106, 1107 }

C grade: { 47, 60, 71, 87, 102, 114, 1003, 1005, 1007, 1009, 1011 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 316, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 686, 687, 688, 689, 690, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 724, 725, 726, 727, 728, 729, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 853, 854, 861, 865, 866, 867, 868, 870, 871, 872, 873, 879, 880, 881, 882, 883, 884, 885, 886, 887, 892, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1024, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.7 Giac

A grade: { 1, 8, 9, 12, 13, 14, 15, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 793, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 861, 862, 863, 864, 865, 866, 873, 874, 875, 876, 877, 878, 879, 880, 888, 889, 890, 891, 892, 897, 898, 899, 900, 901, 906, 907, 908, 909, 910, 911, 912, 913, 915, 916, 917, 918, 919, 920, 921, 922, 923, 925, 955, 956, 957, 958, 959, 960, 968, 969, 970, 971, 972, 973, 981, 982, 983, 984, 985, 986, 987, 988, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1004, 1006, 1008, 1010, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061 }

B grade: { 11, 161, 171, 226, 227, 228, 244, 245, 246, 247, 248, 277, 403, 404, 426, 438, 449, 451, 467, 515, 567, 591, 598, 785, 786, 787, 791, 792, 798, 855, 856, 857, 858, 859, 860, 867, 868, 869, 870, 871, 872, 881, 882, 883, 884, 885, 886, 887, 893, 894, 895, 896, 902, 903, 904, 905, 914, 926, 927, 928, 929, 936, 937, 938, 939, 942, 943, 944, 945, 946, 961, 962, 974, 975, 1105, 1106, 1107 }

C grade: { 287 }

F grade: { 2, 3, 4, 5, 6, 7, 10, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 924, 930, 931, 932, 933, 934, 935, 940, 941, 947, 948, 949, 950, 951, 952, 953, 954, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 989, 990, 991, 992, 993, 1003, 1005, 1007, 1009, 1011, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.1.8 Mupad

A grade: { }

B grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 21, 34, 47, 60, 71, 87, 102, 114, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 262, 263, 264, 265, 266, 267, 268, 269, 271, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 403, 404, 405, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 554, 555, 556, 557, 561, 562, 567, 568, 569, 570, 571, 580, 581, 582, 583, 584, 591, 598, 599, 600, 601, 602, 603, 604, 617, 618, 619, 620, 621, 622, 623, 625, 626, 627, 628, 636, 637, 647, 648, 649, 650, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 731, 732, 733, 734, 738, 739, 740, 741, 745, 746, 747, 748, 785, 786, 787, 798, 799, 800, 801, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 938, 939, 957, 958, 959, 960, 970, 971, 972, 973, 983, 984, 985, 994, 995, 996, 997, 999, 1000, 1001, 1004, 1007, 1008, 1009, 1010, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1022, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1105, 1106, 1107 }

C grade: { }

F grade: { 1, 2, 3, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 225, 235, 236, 237, 240, 241, 242, 243, 254, 255, 256, 257, 258, 259, 260, 261, 270, 272, 273, 274, 283, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 550, 551, 552, 553, 558, 559, 560, 563, 564, 565, 566, 572, 573, 574, 575, 576, 577, 578, 579, 585, 586, 587, 588, 589, 590, 592, 593, 594, 595, 596, 597, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 624, 629, 630, 631, 632, 633, 634, 635, 638, 639, 640, 641, 642, 643, 644, 645, 646, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 728, 729, 730, 735, 736, 737, 742, 743, 744, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 961, 962, 963, 964, 965, 966, 967, 968, 969, 974, 975, 976, 977, 978, 979, 980, 981, 982, 986, 987, 988, 989, 990, 991, 992, 993, 998, 1002, 1003, 1005, 1006, 1011, 1012, 1021, 1023, 1024, 1039, 1040, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	88	77	0	177	0	59	-1
N.S.	1	1.00	0.69	0.60	0.00	1.38	0.00	0.46	-0.01
time (sec)	N/A	0.018	0.082	0.022	0.000	0.374	0.000	4.385	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	59	58	0	147	0	0	-1
N.S.	1	1.00	0.65	0.64	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.048	0.017	0.000	0.348	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	0	0	90	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	1.50	0.00	0.00	-0.02
time (sec)	N/A	0.008	0.010	0.003	0.000	0.365	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	25	33	0	34	0	0	34
N.S.	1	1.00	0.74	0.97	0.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.006	0.033	0.018	0.000	0.344	0.000	0.000	4.139

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	38	44	0	58	0	0	45
N.S.	1	1.03	0.56	0.65	0.00	0.85	0.00	0.00	0.66
time (sec)	N/A	0.010	0.049	0.016	0.000	0.429	0.000	0.000	4.205

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	49	55	0	80	0	0	56
N.S.	1	1.02	0.47	0.52	0.00	0.76	0.00	0.00	0.53
time (sec)	N/A	0.015	0.060	0.016	0.000	0.372	0.000	0.000	4.210

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	148	60	66	0	102	0	0	141
N.S.	1	1.10	0.44	0.49	0.00	0.76	0.00	0.00	1.04
time (sec)	N/A	0.022	0.013	0.016	0.000	0.376	0.000	0.000	4.129

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	81	610	0	583	63	75	872
N.S.	1	1.00	0.27	2.04	0.00	1.95	0.21	0.25	2.92
time (sec)	N/A	0.195	0.031	0.086	0.000	0.421	0.475	3.730	4.375

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	32	0	269	257	31	85
N.S.	1	1.00	0.91	0.68	0.00	5.72	5.47	0.66	1.81
time (sec)	N/A	0.015	0.016	0.029	0.000	0.360	0.331	5.997	0.102

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	52	586	0	613	48	0	469
N.S.	1	1.00	0.17	1.96	0.00	2.05	0.16	0.00	1.57
time (sec)	N/A	0.176	0.020	0.079	0.000	0.401	0.305	0.000	4.356

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	26	25	25	26	29	11
N.S.	1	1.00	2.18	1.53	1.47	1.47	1.53	1.71	0.65
time (sec)	N/A	0.005	0.005	0.020	0.270	0.361	0.075	4.833	0.036

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	18	17	17	20	17	17
N.S.	1	1.00	1.00	0.75	0.71	0.71	0.83	0.71	0.71
time (sec)	N/A	0.005	0.008	0.018	0.496	0.351	0.064	4.760	4.116

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	91	54	53	53	70	53	83
N.S.	1	1.00	1.36	0.81	0.79	0.79	1.04	0.79	1.24
time (sec)	N/A	0.029	0.048	0.026	0.497	0.336	0.093	5.157	4.147

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	77	57	0	163	63	56	47
N.S.	1	1.00	1.04	0.77	0.00	2.20	0.85	0.76	0.64
time (sec)	N/A	0.027	0.049	0.035	0.000	0.340	0.092	3.577	4.186

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	41	253	0	249	899	143	210
N.S.	1	1.00	0.23	1.44	0.00	1.41	5.11	0.81	1.19
time (sec)	N/A	0.109	0.028	0.059	0.000	0.372	0.620	3.159	4.205

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	8	0	0	-1
N.S.	1	1.00	6.50	5.10	0.00	0.80	0.00	0.00	-0.10
time (sec)	N/A	0.007	10.019	0.042	0.000	0.072	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	84	0	35	0	0	-1
N.S.	1	1.00	1.02	1.75	0.00	0.73	0.00	0.00	-0.02
time (sec)	N/A	0.082	10.051	0.068	0.000	0.092	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	80	0	40	0	0	-1
N.S.	1	1.00	1.10	1.67	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.066	10.056	0.073	0.000	0.090	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	49	84	0	35	0	0	-1
N.S.	1	1.00	1.11	1.91	0.00	0.80	0.00	0.00	-0.02
time (sec)	N/A	0.045	10.046	0.065	0.000	0.105	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	63	41	0	8	0	0	-1
N.S.	1	1.00	5.25	3.42	0.00	0.67	0.00	0.00	-0.08
time (sec)	N/A	0.007	10.029	0.027	0.000	0.084	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	54	0	16	37	0	16
N.S.	1	1.00	1.00	3.00	0.00	0.89	2.06	0.00	0.89
time (sec)	N/A	0.006	10.039	0.119	0.000	0.073	0.357	0.000	4.200

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	49	0	16	0	0	-1
N.S.	1	1.00	1.00	2.45	0.00	0.80	0.00	0.00	-0.05
time (sec)	N/A	0.008	10.033	0.043	0.000	0.082	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	84	0	35	0	0	-1
N.S.	1	1.00	1.21	2.00	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.043	10.045	0.066	0.000	0.081	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	80	0	40	0	0	-1
N.S.	1	1.00	1.20	1.74	0.00	0.87	0.00	0.00	-0.02
time (sec)	N/A	0.061	10.045	0.066	0.000	0.073	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	84	0	35	0	0	-1
N.S.	1	1.00	1.02	1.75	0.00	0.73	0.00	0.00	-0.02
time (sec)	N/A	0.064	10.054	0.067	0.000	0.081	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	54	50	0	15	0	0	-1
N.S.	1	1.00	3.00	2.78	0.00	0.83	0.00	0.00	-0.06
time (sec)	N/A	0.008	10.032	0.040	0.000	0.080	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	84	0	49	0	0	-1
N.S.	1	1.00	1.16	1.87	0.00	1.09	0.00	0.00	-0.02
time (sec)	N/A	0.042	10.039	0.069	0.000	0.092	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	84	0	50	0	0	-1
N.S.	1	1.00	0.98	1.91	0.00	1.14	0.00	0.00	-0.02
time (sec)	N/A	0.089	10.064	0.072	0.000	0.081	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	8	0	0	-1
N.S.	1	1.00	6.50	5.10	0.00	0.80	0.00	0.00	-0.10
time (sec)	N/A	0.007	10.044	0.054	0.000	0.072	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	84	0	35	0	0	-1
N.S.	1	1.00	1.16	1.91	0.00	0.80	0.00	0.00	-0.02
time (sec)	N/A	0.052	10.041	0.060	0.000	0.076	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	84	0	56	0	0	-1
N.S.	1	1.00	1.11	1.87	0.00	1.24	0.00	0.00	-0.02
time (sec)	N/A	0.042	10.046	0.074	0.000	0.093	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	49	84	0	35	0	0	-1
N.S.	1	1.00	1.11	1.91	0.00	0.80	0.00	0.00	-0.02
time (sec)	N/A	0.041	10.033	0.067	0.000	0.081	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	47	0	16	0	0	-1
N.S.	1	1.00	1.00	2.35	0.00	0.80	0.00	0.00	-0.05
time (sec)	N/A	0.007	10.036	0.039	0.000	0.073	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	54	0	34	37	0	16
N.S.	1	1.00	1.00	3.00	0.00	1.89	2.06	0.00	0.89
time (sec)	N/A	0.003	10.033	0.096	0.000	0.074	0.354	0.000	4.197

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	65	43	0	8	0	0	-1
N.S.	1	1.00	5.42	3.58	0.00	0.67	0.00	0.00	-0.08
time (sec)	N/A	0.007	10.023	0.028	0.000	0.083	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	84	0	35	0	0	-1
N.S.	1	1.00	1.21	2.00	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.033	10.045	0.056	0.000	0.090	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	52	84	0	57	0	0	-1
N.S.	1	1.00	1.21	1.95	0.00	1.33	0.00	0.00	-0.02
time (sec)	N/A	0.022	10.036	0.053	0.000	0.080	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	84	0	35	0	0	-1
N.S.	1	1.00	1.16	1.91	0.00	0.80	0.00	0.00	-0.02
time (sec)	N/A	0.046	10.053	0.051	0.000	0.097	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	54	50	0	15	0	0	-1
N.S.	1	1.00	3.00	2.78	0.00	0.83	0.00	0.00	-0.06
time (sec)	N/A	0.008	10.020	0.043	0.000	0.087	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	45	84	0	49	0	0	-1
N.S.	1	1.00	1.07	2.00	0.00	1.17	0.00	0.00	-0.02
time (sec)	N/A	0.062	10.044	0.074	0.000	0.083	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	84	0	49	0	0	-1
N.S.	1	1.00	1.16	1.87	0.00	1.09	0.00	0.00	-0.02
time (sec)	N/A	0.032	10.036	0.075	0.000	0.073	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	53	0	0	0	0	-1
N.S.	1	1.00	0.81	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.006	10.028	0.024	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	81	84	0	0	0	0	-1
N.S.	1	1.00	0.57	0.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	10.051	0.039	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	83	84	0	0	0	0	-1
N.S.	1	1.00	0.57	0.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	10.049	0.040	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	83	84	0	0	0	0	-1
N.S.	1	1.00	0.59	0.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	10.049	0.039	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	43	0	0	0	0	-1
N.S.	1	1.00	0.76	0.68	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.006	10.028	0.027	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	40	56	0	0	34	0	31
N.S.	1	1.00	0.35	0.49	0.00	0.00	0.30	0.00	0.27
time (sec)	N/A	0.010	10.035	0.099	0.000	0.000	0.338	0.000	0.080

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	53	0	0	0	0	-1
N.S.	1	1.00	0.92	0.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.005	10.033	0.039	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	81	84	0	0	0	0	-1
N.S.	1	1.00	0.55	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	10.044	0.042	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	81	84	0	0	0	0	-1
N.S.	1	1.00	0.53	0.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	10.053	0.039	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	81	84	0	0	0	0	-1
N.S.	1	1.00	0.55	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	10.051	0.043	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	53	0	0	0	0	-1
N.S.	1	1.00	1.03	0.84	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.005	10.033	0.030	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	80	84	0	0	0	0	-1
N.S.	1	1.00	0.54	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	10.043	0.045	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	77	84	0	0	0	0	-1
N.S.	1	1.00	0.52	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	10.058	0.039	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	53	0	0	0	0	-1
N.S.	1	1.00	0.81	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.006	10.034	0.023	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	83	84	0	0	0	0	-1
N.S.	1	1.00	0.56	0.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	10.054	0.050	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	80	84	0	0	0	0	-1
N.S.	1	1.00	0.55	0.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	10.052	0.041	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	83	84	0	0	0	0	-1
N.S.	1	1.00	0.58	0.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	10.047	0.040	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	51	0	0	0	0	-1
N.S.	1	1.00	1.00	0.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.005	10.030	0.045	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	40	56	0	0	34	0	31
N.S.	1	1.00	0.36	0.50	0.00	0.00	0.30	0.00	0.28
time (sec)	N/A	0.011	10.036	0.119	0.000	0.000	0.337	0.000	0.082

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	45	0	0	0	0	-1
N.S.	1	1.00	0.78	0.69	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.005	10.031	0.033	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	81	84	0	0	0	0	-1
N.S.	1	1.00	0.54	0.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.043	0.039	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	78	84	0	0	0	0	-1
N.S.	1	1.00	0.51	0.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	10.043	0.041	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	83	84	0	0	0	0	-1
N.S.	1	1.00	0.54	0.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	10.046	0.038	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	53	0	0	0	0	-1
N.S.	1	1.00	1.03	0.84	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.005	10.032	0.030	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	44	0	10	0	0	-1
N.S.	1	1.00	1.12	0.85	0.00	0.19	0.00	0.00	-0.02
time (sec)	N/A	0.005	10.024	0.030	0.000	0.083	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	35	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.059	0.058	0.000	0.083	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	35	0	0	-1
N.S.	1	1.00	1.57	0.95	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.012	10.104	0.056	0.000	0.095	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	35	0	0	-1
N.S.	1	1.00	1.57	0.95	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.013	10.057	0.058	0.000	0.076	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	142	85	0	35	0	0	-1
N.S.	1	1.00	1.61	0.97	0.00	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.010	10.067	0.058	0.000	0.085	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	25	66	0	16	36	0	16
N.S.	1	1.00	0.35	0.92	0.00	0.22	0.50	0.00	0.22
time (sec)	N/A	0.005	10.031	0.090	0.000	0.077	0.330	0.000	0.089

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	35	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.011	10.064	0.053	0.000	0.085	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	35	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.011	10.062	0.054	0.000	0.076	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	35	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.011	10.084	0.077	0.000	0.082	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	35	0	0	-1
N.S.	1	1.00	1.64	0.99	0.00	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.008	10.064	0.055	0.000	0.074	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	42	0	8	0	0	-1
N.S.	1	1.00	0.58	0.46	0.00	0.09	0.00	0.00	-0.01
time (sec)	N/A	0.010	10.028	0.024	0.000	0.098	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	82	0	35	0	0	-1
N.S.	1	1.00	0.94	0.91	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.010	10.064	0.058	0.000	0.080	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	82	0	57	0	0	-1
N.S.	1	1.00	0.88	0.75	0.00	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.061	10.058	0.066	0.000	0.081	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	82	0	35	0	0	-1
N.S.	1	1.00	0.89	0.75	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.053	10.071	0.067	0.000	0.088	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	61	50	0	10	0	0	-1
N.S.	1	1.00	1.02	0.83	0.00	0.17	0.00	0.00	-0.02
time (sec)	N/A	0.005	10.035	0.030	0.000	0.081	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	90	82	0	33	0	0	-1
N.S.	1	1.00	0.87	0.79	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.042	10.043	0.062	0.000	0.082	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	50	0	16	0	0	-1
N.S.	1	1.00	1.12	0.96	0.00	0.31	0.00	0.00	-0.02
time (sec)	N/A	0.005	10.035	0.027	0.000	0.080	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	35	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.012	10.071	0.052	0.000	0.079	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	142	87	0	51	0	0	-1
N.S.	1	1.00	1.54	0.95	0.00	0.55	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.079	0.051	0.000	0.078	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	35	0	0	-1
N.S.	1	1.00	1.57	0.95	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.060	0.060	0.000	0.077	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	140	85	0	41	0	0	-1
N.S.	1	1.00	1.59	0.97	0.00	0.47	0.00	0.00	-0.01
time (sec)	N/A	0.009	10.049	0.051	0.000	0.077	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	25	66	0	34	36	0	16
N.S.	1	1.00	0.35	0.92	0.00	0.47	0.50	0.00	0.22
time (sec)	N/A	0.005	10.037	0.089	0.000	0.075	0.331	0.000	0.088

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	41	0	0	-1
N.S.	1	1.00	1.58	0.97	0.00	0.46	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.062	0.070	0.000	0.085	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	35	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.060	0.052	0.000	0.090	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	53	0	0	-1
N.S.	1	1.00	1.58	0.97	0.00	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.070	0.052	0.000	0.078	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	35	0	0	-1
N.S.	1	1.00	1.64	0.99	0.00	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.061	0.064	0.000	0.074	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	42	0	8	0	0	-1
N.S.	1	1.00	0.58	0.46	0.00	0.09	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.023	0.025	0.000	0.080	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	81	82	0	35	0	0	-1
N.S.	1	1.00	0.90	0.91	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.053	0.049	0.000	0.081	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	58	49	0	15	0	0	-1
N.S.	1	1.00	0.63	0.53	0.00	0.16	0.00	0.00	-0.01
time (sec)	N/A	0.010	10.029	0.026	0.000	0.071	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	58	48	0	0	0	0	-1
N.S.	1	1.00	3.05	2.53	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.008	10.022	0.024	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	81	82	0	0	0	0	-1
N.S.	1	1.00	1.84	1.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	10.039	0.038	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	53	50	0	0	0	0	-1
N.S.	1	1.00	3.79	3.57	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.007	10.025	0.025	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	46	0	0	-1
N.S.	1	1.00	1.64	0.99	0.00	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.039	0.039	0.000	0.091	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	60	0	0	-1
N.S.	1	1.00	1.58	0.97	0.00	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.066	0.038	0.000	0.074	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	41	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.46	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.049	0.038	0.000	0.069	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	140	85	0	41	0	0	-1
N.S.	1	1.00	1.59	0.97	0.00	0.47	0.00	0.00	-0.01
time (sec)	N/A	0.006	10.039	0.042	0.000	0.080	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	47	66	0	34	39	0	31
N.S.	1	1.00	0.65	0.92	0.00	0.47	0.54	0.00	0.43
time (sec)	N/A	0.005	10.040	0.081	0.000	0.074	0.337	0.000	4.290

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	41	0	0	-1
N.S.	1	1.00	1.58	0.97	0.00	0.46	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.046	0.037	0.000	0.071	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	41	0	0	-1
N.S.	1	1.00	1.57	0.95	0.00	0.45	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.060	0.038	0.000	0.071	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	142	87	0	60	0	0	-1
N.S.	1	1.00	1.54	0.95	0.00	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.069	0.041	0.000	0.087	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	46	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.047	0.044	0.000	0.082	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	63	44	0	22	0	0	-1
N.S.	1	1.00	1.19	0.83	0.00	0.42	0.00	0.00	-0.02
time (sec)	N/A	0.012	10.018	0.049	0.000	0.067	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	85	82	0	0	0	0	-1
N.S.	1	1.00	2.02	1.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	10.058	0.036	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	53	42	0	0	0	0	-1
N.S.	1	1.00	8.83	7.00	0.00	0.00	0.00	0.00	-0.17
time (sec)	N/A	0.007	10.021	0.024	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	38	0	0	-1
N.S.	1	1.00	1.64	0.99	0.00	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.056	0.041	0.000	0.072	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	35	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.100	0.038	0.000	0.078	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	41	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.46	0.00	0.00	-0.01
time (sec)	N/A	0.010	10.060	0.041	0.000	0.071	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	142	85	0	35	0	0	-1
N.S.	1	1.00	1.61	0.97	0.00	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.006	10.055	0.059	0.000	0.083	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	47	66	0	22	39	0	31
N.S.	1	1.00	0.65	0.92	0.00	0.31	0.54	0.00	0.43
time (sec)	N/A	0.005	10.042	0.082	0.000	0.077	0.331	0.000	4.176

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	35	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.059	0.038	0.000	0.068	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	41	0	0	-1
N.S.	1	1.00	1.57	0.95	0.00	0.45	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.053	0.037	0.000	0.071	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	35	0	0	-1
N.S.	1	1.00	1.57	0.95	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.007	10.110	0.042	0.000	0.067	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	38	0	0	-1
N.S.	1	1.00	1.60	0.97	0.00	0.42	0.00	0.00	-0.01
time (sec)	N/A	0.008	10.067	0.039	0.000	0.066	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	63	50	0	10	0	0	-1
N.S.	1	1.00	1.21	0.96	0.00	0.19	0.00	0.00	-0.02
time (sec)	N/A	0.011	10.032	0.027	0.000	0.076	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	35	0	0	-1
N.S.	1	1.00	1.57	0.95	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.019	10.087	0.056	0.000	0.089	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	147	87	0	35	0	0	-1
N.S.	1	1.00	1.63	0.97	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.069	0.081	0.000	0.083	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	44	0	10	0	0	-1
N.S.	1	1.00	1.12	0.85	0.00	0.19	0.00	0.00	-0.02
time (sec)	N/A	0.005	10.023	0.013	0.000	0.081	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	48	0	10	0	0	-1
N.S.	1	1.00	1.00	0.83	0.00	0.17	0.00	0.00	-0.02
time (sec)	N/A	0.005	10.025	0.028	0.000	0.086	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	103	76	0	40	0	0	-1
N.S.	1	1.00	0.95	0.70	0.00	0.37	0.00	0.00	-0.01
time (sec)	N/A	0.031	10.060	0.064	0.000	0.103	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	55	80	0	40	0	0	-1
N.S.	1	1.00	1.15	1.67	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.070	10.061	0.064	0.000	0.084	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	76	0	40	0	0	-1
N.S.	1	1.00	1.16	1.69	0.00	0.89	0.00	0.00	-0.02
time (sec)	N/A	0.046	10.041	0.059	0.000	0.098	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	8	0	0	-1
N.S.	1	1.00	6.50	5.10	0.00	0.80	0.00	0.00	-0.10
time (sec)	N/A	0.007	10.018	0.015	0.000	0.072	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	40	0	0	-1
N.S.	1	1.00	1.14	1.63	0.00	0.82	0.00	0.00	-0.02
time (sec)	N/A	0.072	10.056	0.063	0.000	0.081	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	52	80	0	40	0	0	-1
N.S.	1	1.00	1.08	1.67	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.069	10.044	0.067	0.000	0.084	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	40	0	0	-1
N.S.	1	1.00	1.14	1.63	0.00	0.82	0.00	0.00	-0.02
time (sec)	N/A	0.054	10.046	0.064	0.000	0.083	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	65	43	0	8	0	0	-1
N.S.	1	1.00	5.42	3.58	0.00	0.67	0.00	0.00	-0.08
time (sec)	N/A	0.008	10.034	0.027	0.000	0.073	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	76	0	40	0	0	-1
N.S.	1	1.00	1.16	1.69	0.00	0.89	0.00	0.00	-0.02
time (sec)	N/A	0.045	10.047	0.082	0.000	0.076	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	40	0	0	-1
N.S.	1	1.00	1.14	1.63	0.00	0.82	0.00	0.00	-0.02
time (sec)	N/A	0.053	10.036	0.066	0.000	0.085	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.004	0.002	0.046	0.270	0.314	0.008	5.355	0.023

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.002	0.036	0.290	0.337	0.010	4.037	0.021

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.002	0.000	0.023	0.276	0.335	0.006	3.973	0.020

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	13	13	12	13	13
N.S.	1	1.00	1.00	0.88	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.001	0.028	0.279	0.315	0.007	5.371	0.020

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.003	0.000	0.007	0.269	0.321	0.005	4.611	0.017

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	14	11
N.S.	1	1.00	1.00	0.92	1.08	0.85	0.77	1.08	0.85
time (sec)	N/A	0.005	0.001	0.009	0.270	0.345	0.021	3.068	0.022

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	13	5	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.30	0.50	1.00	1.00
time (sec)	N/A	0.003	0.001	0.015	0.268	0.320	0.020	3.852	0.024

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	20	11
N.S.	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	0.85
time (sec)	N/A	0.003	0.002	0.010	0.287	0.345	0.034	4.841	0.039

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.004	0.002	0.010	0.289	0.329	0.039	6.833	0.027

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.004	0.003	0.029	0.292	0.330	0.043	6.036	0.027

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.004	0.002	0.010	0.307	0.325	0.053	5.930	0.028

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.008	0.001	0.084	0.279	0.331	0.009	4.722	0.037

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.016	0.001	0.092	0.279	0.324	0.008	4.278	0.036

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.010	0.001	0.079	0.276	0.315	0.015	3.742	0.035

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	24	24	24	24	24
N.S.	1	1.00	1.00	0.94	1.50	1.50	1.50	1.50	1.50
time (sec)	N/A	0.006	0.001	0.083	0.268	0.324	0.010	4.633	0.032

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.009	0.001	0.073	0.300	0.330	0.009	4.471	0.033

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	21	20	24	21
N.S.	1	1.00	1.00	0.96	1.04	0.91	0.87	1.04	0.91
time (sec)	N/A	0.013	0.001	0.086	0.272	0.346	0.027	5.336	0.029

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92
time (sec)	N/A	0.011	0.001	0.089	0.271	0.316	0.026	4.315	0.035

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	24	27	24	32	23
N.S.	1	1.00	1.00	0.89	0.89	1.00	0.89	1.19	0.85
time (sec)	N/A	0.013	0.001	0.074	0.287	0.369	0.044	5.860	0.032

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	24
N.S.	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.04
time (sec)	N/A	0.010	0.001	0.067	0.276	0.335	0.053	3.721	0.027

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	26	28	24	34	24
N.S.	1	1.00	1.00	0.96	1.08	1.17	1.00	1.42	1.00
time (sec)	N/A	0.013	0.001	0.073	0.276	0.327	0.074	4.626	0.045

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	25
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.89
time (sec)	N/A	0.011	0.001	0.073	0.278	0.330	0.078	4.640	0.036

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	24	24	26	24	26
N.S.	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37
time (sec)	N/A	0.006	0.001	0.088	0.277	0.334	0.078	4.485	0.036

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.011	0.001	0.081	0.282	0.332	0.082	3.824	0.037

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.017	0.002	0.083	0.278	0.322	0.010	4.175	0.044

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	36	35	35	37	35	35
N.S.	1	1.00	1.26	1.06	1.03	1.03	1.09	1.03	1.03
time (sec)	N/A	0.025	0.002	0.088	0.267	0.331	0.012	3.478	0.043

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.014	0.002	0.083	0.288	0.315	0.011	4.585	0.043

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	35	35	37	35	35
N.S.	1	1.00	1.00	0.94	2.19	2.19	2.31	2.19	2.19
time (sec)	N/A	0.006	0.002	0.079	0.298	0.316	0.012	5.037	0.042

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.011	0.001	0.083	0.284	0.311	0.013	3.485	0.040

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	36	33	37	36	33
N.S.	1	1.00	1.00	0.87	0.92	0.85	0.95	0.92	0.85
time (sec)	N/A	0.017	0.003	0.083	0.286	0.342	0.035	4.817	0.036

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	29	32	32
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.85	0.94	0.94
time (sec)	N/A	0.014	0.003	0.079	0.322	0.333	0.034	5.688	0.042

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	36	38	37	46	34
N.S.	1	1.00	1.00	0.88	0.90	0.95	0.92	1.15	0.85
time (sec)	N/A	0.019	0.005	0.100	0.270	0.342	0.050	4.628	0.037

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	36	36	34	36
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.97	0.92	0.97
time (sec)	N/A	0.013	0.003	0.086	0.295	0.364	0.056	3.714	0.038

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	37	39	37	46	37
N.S.	1	1.00	1.00	0.88	0.92	0.98	0.92	1.15	0.92
time (sec)	N/A	0.017	0.004	0.080	0.278	0.335	0.092	3.894	0.035

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	33	37	34	33	34
N.S.	1	1.00	1.00	0.97	0.97	1.09	1.00	0.97	1.00
time (sec)	N/A	0.013	0.004	0.069	0.275	0.321	0.081	4.186	0.032

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	39	39	37	47	36
N.S.	1	1.00	1.00	0.87	1.00	1.00	0.95	1.21	0.92
time (sec)	N/A	0.017	0.004	0.091	0.277	0.339	0.107	3.348	0.047

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	37	37	39	37	35
N.S.	1	1.00	1.00	0.92	0.95	0.95	1.00	0.95	0.90
time (sec)	N/A	0.014	0.003	0.086	0.296	0.323	0.107	3.349	0.029

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	35	35	37	35	37
N.S.	1	1.00	2.26	1.89	1.84	1.84	1.95	1.84	1.95
time (sec)	N/A	0.006	0.006	0.086	0.282	0.335	0.118	4.612	0.030

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.013	0.003	0.092	0.276	0.321	0.124	3.679	0.032

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	36	37	37	39	37	37
N.S.	1	1.00	1.08	0.90	0.92	0.92	0.98	0.92	0.92
time (sec)	N/A	0.016	0.003	0.070	0.274	0.326	0.138	5.736	0.033

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	60	148	107	65	54
N.S.	1	1.00	1.00	0.88	0.88	2.18	1.57	0.96	0.79
time (sec)	N/A	0.024	0.019	0.112	0.510	0.373	0.077	4.690	0.032

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	46	45	44	47	45
N.S.	1	1.00	1.00	0.87	0.87	0.85	0.83	0.89	0.85
time (sec)	N/A	0.031	0.005	0.095	0.273	0.382	0.064	5.543	0.047

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	50	126	95	55	43
N.S.	1	1.00	1.00	0.89	0.91	2.29	1.73	1.00	0.78
time (sec)	N/A	0.021	0.018	0.083	0.491	0.365	0.073	4.640	0.052

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	34	33	32	35	33
N.S.	1	1.00	1.00	0.88	0.85	0.82	0.80	0.88	0.82
time (sec)	N/A	0.022	0.005	0.103	0.281	0.389	0.057	3.660	0.046

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	37	99	80	40	32
N.S.	1	1.00	1.00	0.90	0.88	2.36	1.90	0.95	0.76
time (sec)	N/A	0.018	0.014	0.087	0.493	0.330	0.067	4.611	0.047

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
N.S.	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.017	0.004	0.095	0.302	0.349	0.053	4.783	0.036

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	23
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74
time (sec)	N/A	0.012	0.006	0.095	0.496	0.349	0.061	4.246	0.036

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.006	0.002	0.093	0.277	0.326	0.038	5.182	0.029

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.007	0.004	0.088	0.501	0.366	0.050	2.946	4.198

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	23	18	15	22	18
N.S.	1	1.00	1.00	0.95	1.05	0.82	0.68	1.00	0.82
time (sec)	N/A	0.010	0.004	0.094	0.285	0.347	0.086	4.675	0.064

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	26
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76
time (sec)	N/A	0.009	0.009	0.117	0.499	0.340	0.069	5.719	4.273

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	33	33	31	43	31
N.S.	1	1.00	1.00	0.91	0.94	0.94	0.89	1.23	0.89
time (sec)	N/A	0.019	0.005	0.099	0.278	0.361	0.115	4.589	0.059

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	40	106	87	40	37
N.S.	1	1.00	1.00	0.91	0.93	2.47	2.02	0.93	0.86
time (sec)	N/A	0.017	0.015	0.099	0.508	0.334	0.096	4.807	4.144

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	47	45	42	57	46
N.S.	1	1.00	1.00	0.90	0.96	0.92	0.86	1.16	0.94
time (sec)	N/A	0.023	0.006	0.095	0.283	0.335	0.145	3.214	0.060

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	52	52	132	100	52	48
N.S.	1	1.00	1.00	0.90	0.90	2.28	1.72	0.90	0.83
time (sec)	N/A	0.021	0.019	0.095	0.490	0.375	0.132	2.529	0.052

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	58	58	56	70	58
N.S.	1	1.00	1.00	0.89	0.92	0.92	0.89	1.11	0.92
time (sec)	N/A	0.029	0.006	0.080	0.278	0.349	0.166	3.273	0.068

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	65	71	190	124	73	66
N.S.	1	1.00	0.90	0.82	0.90	2.41	1.57	0.92	0.84
time (sec)	N/A	0.025	0.036	0.102	0.502	0.368	0.144	7.825	0.042

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	55	54	70	53	67	57
N.S.	1	1.00	0.86	0.96	0.95	1.23	0.93	1.18	1.00
time (sec)	N/A	0.036	0.013	0.107	0.271	0.383	0.117	4.748	4.144

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	54	59	164	107	61	56
N.S.	1	1.00	0.91	0.82	0.89	2.48	1.62	0.92	0.85
time (sec)	N/A	0.022	0.031	0.082	0.499	0.356	0.130	5.143	0.058

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	44	43	56	39	49	45
N.S.	1	1.00	0.86	1.00	0.98	1.27	0.89	1.11	1.02
time (sec)	N/A	0.026	0.013	0.111	0.290	0.331	0.106	6.070	0.042

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	42	45	136	83	42	43
N.S.	1	1.00	0.93	0.76	0.82	2.47	1.51	0.76	0.78
time (sec)	N/A	0.016	0.023	0.094	0.527	0.374	0.120	5.357	4.174

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	32	35	29	32	29
N.S.	1	1.00	0.82	0.91	0.97	1.06	0.88	0.97	0.88
time (sec)	N/A	0.021	0.007	0.091	0.276	0.344	0.081	5.488	4.178

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	36	120	78	35	33
N.S.	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	0.73
time (sec)	N/A	0.012	0.015	0.114	0.482	0.343	0.092	4.949	4.149

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	15	15	14	14
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.94	0.88	0.88
time (sec)	N/A	0.006	0.002	0.081	0.286	0.413	0.062	5.057	0.024

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.011	0.018	0.093	0.505	0.404	0.093	5.925	0.037

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	42	37	47	34	36	34
N.S.	1	1.00	0.87	1.11	0.97	1.24	0.89	0.95	0.89
time (sec)	N/A	0.022	0.012	0.094	0.274	0.369	0.136	6.605	4.177

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	45	49	136	92	47	44
N.S.	1	1.00	0.95	0.79	0.86	2.39	1.61	0.82	0.77
time (sec)	N/A	0.016	0.026	0.085	0.525	0.354	0.150	6.704	0.064

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	55	52	73	51	50	51
N.S.	1	1.00	0.84	1.12	1.06	1.49	1.04	1.02	1.04
time (sec)	N/A	0.027	0.026	0.085	0.281	0.353	0.190	5.010	4.211

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	55	64	172	114	59	58
N.S.	1	1.00	0.99	0.81	0.94	2.53	1.68	0.87	0.85
time (sec)	N/A	0.018	0.027	0.079	0.526	0.337	0.179	3.557	4.171

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	65	70	90	68	86	67
N.S.	1	1.00	0.86	0.98	1.06	1.36	1.03	1.30	1.02
time (sec)	N/A	0.035	0.039	0.092	0.301	0.351	0.215	3.299	4.170

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	67	75	198	126	70	70
N.S.	1	1.00	0.99	0.83	0.93	2.44	1.56	0.86	0.86
time (sec)	N/A	0.025	0.033	0.106	0.499	0.392	0.199	4.553	4.285

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	63	82	230	133	73	77
N.S.	1	1.00	0.91	0.74	0.96	2.71	1.56	0.86	0.91
time (sec)	N/A	0.027	0.034	0.102	0.508	0.342	0.222	5.548	4.211

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	62	66	91	68	62	68
N.S.	1	1.00	0.74	0.95	1.02	1.40	1.05	0.95	1.05
time (sec)	N/A	0.035	0.041	0.090	0.282	0.358	0.179	4.836	4.257

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	51	68	202	107	54	64
N.S.	1	1.00	0.89	0.69	0.92	2.73	1.45	0.73	0.86
time (sec)	N/A	0.021	0.032	0.098	0.492	0.352	0.203	4.606	4.249

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	46	55	69	53	42	52
N.S.	1	1.00	0.80	0.94	1.12	1.41	1.08	0.86	1.06
time (sec)	N/A	0.031	0.012	0.099	0.286	0.328	0.147	4.838	4.181

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	47	59	188	110	45	56
N.S.	1	1.00	0.86	0.73	0.92	2.94	1.72	0.70	0.88
time (sec)	N/A	0.017	0.031	0.100	0.496	0.372	0.157	3.352	4.226

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
N.S.	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.006	0.005	0.079	0.303	0.356	0.129	5.829	4.179

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	49	62	190	110	50	55
N.S.	1	1.00	0.89	0.75	0.95	2.92	1.69	0.77	0.85
time (sec)	N/A	0.017	0.021	0.095	0.501	0.331	0.157	5.293	4.226

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	26	26	27	14	28
N.S.	1	1.00	1.00	0.94	1.62	1.62	1.69	0.88	1.75
time (sec)	N/A	0.006	0.002	0.093	0.274	0.345	0.105	6.101	0.028

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	57	58	188	105	45	55
N.S.	1	1.00	0.89	0.92	0.94	3.03	1.69	0.73	0.89
time (sec)	N/A	0.014	0.024	0.103	0.496	0.348	0.152	4.799	4.207

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	59	60	90	56	59	56
N.S.	1	1.00	0.80	1.09	1.11	1.67	1.04	1.09	1.04
time (sec)	N/A	0.029	0.023	0.098	0.289	0.368	0.210	4.545	0.055

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	54	71	202	116	57	66
N.S.	1	1.00	0.89	0.71	0.93	2.66	1.53	0.75	0.87
time (sec)	N/A	0.022	0.028	0.100	0.499	0.334	0.218	4.029	4.256

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	72	77	119	80	66	75
N.S.	1	1.00	0.88	1.07	1.15	1.78	1.19	0.99	1.12
time (sec)	N/A	0.037	0.041	0.106	0.295	0.340	0.251	3.098	0.064

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	79	64	86	238	138	71	80
N.S.	1	1.00	0.91	0.74	0.99	2.74	1.59	0.82	0.92
time (sec)	N/A	0.026	0.030	0.102	0.493	0.343	0.241	3.198	4.261

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	74	83	92	134	90	79	88
N.S.	1	1.00	0.86	0.97	1.07	1.56	1.05	0.92	1.02
time (sec)	N/A	0.046	0.034	0.127	0.291	0.334	0.274	3.670	4.248

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	75	97	264	150	80	92
N.S.	1	1.00	0.90	0.75	0.97	2.64	1.50	0.80	0.92
time (sec)	N/A	0.031	0.038	0.099	0.490	0.426	0.266	5.102	4.235

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	94	103	145	104	110	101
N.S.	1	1.00	0.89	0.99	1.08	1.53	1.09	1.16	1.06
time (sec)	N/A	0.055	0.053	0.089	0.276	0.356	0.299	5.426	0.101

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	113	124	121	188	0	101	105
N.S.	1	1.00	0.95	1.04	1.02	1.58	0.00	0.85	0.88
time (sec)	N/A	0.083	0.101	0.105	0.292	0.393	0.000	3.407	4.685

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	102	104	97	167	0	85	77
N.S.	1	1.00	1.12	1.14	1.07	1.84	0.00	0.93	0.85
time (sec)	N/A	0.065	0.083	0.093	0.304	0.368	0.000	5.566	4.364

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	88	84	73	140	0	69	64
N.S.	1	1.00	1.29	1.24	1.07	2.06	0.00	1.01	0.94
time (sec)	N/A	0.042	0.060	0.081	0.284	0.377	0.000	6.001	4.366

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	64	64	49	115	0	52	50
N.S.	1	1.00	1.16	1.16	0.89	2.09	0.00	0.95	0.91
time (sec)	N/A	0.044	0.041	0.089	0.278	0.342	0.000	7.724	4.210

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	68	84	51	115	0	61	-1
N.S.	1	1.00	1.31	1.62	0.98	2.21	0.00	1.17	-0.02
time (sec)	N/A	0.047	0.048	0.107	0.281	0.389	0.000	7.380	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	41	28	0	63	28
N.S.	1	1.00	1.00	1.16	1.64	1.12	0.00	2.52	1.12
time (sec)	N/A	0.024	0.044	0.093	0.285	0.325	0.000	20.003	4.148

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	39	65	42	0	120	41
N.S.	1	1.00	0.88	0.75	1.25	0.81	0.00	2.31	0.79
time (sec)	N/A	0.047	0.051	0.097	0.294	0.384	0.000	7.445	4.263

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	57	50	89	53	0	148	89
N.S.	1	1.00	0.71	0.62	1.11	0.66	0.00	1.85	1.11
time (sec)	N/A	0.074	0.059	0.082	0.299	0.366	0.000	7.596	4.338

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	68	61	113	64	0	178	113
N.S.	1	1.00	0.63	0.56	1.05	0.59	0.00	1.65	1.05
time (sec)	N/A	0.103	0.065	0.118	0.287	0.370	0.000	5.318	4.505

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	79	72	137	75	0	206	137
N.S.	1	1.00	0.58	0.53	1.01	0.55	0.00	1.51	1.01
time (sec)	N/A	0.129	0.074	0.107	0.302	0.382	0.000	5.216	4.619

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	57	50	46	53	0	60	53
N.S.	1	1.00	0.73	0.64	0.59	0.68	0.00	0.77	0.68
time (sec)	N/A	0.062	0.025	0.098	0.295	0.362	0.000	5.901	4.233

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	39	34	41	0	44	41
N.S.	1	1.00	0.87	0.75	0.65	0.79	0.00	0.85	0.79
time (sec)	N/A	0.033	0.022	0.097	0.312	0.348	0.000	5.825	4.135

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	29
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	1.16
time (sec)	N/A	0.004	0.004	0.082	0.284	0.321	0.000	7.830	4.136

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	60	65	0	117	0	69	68
N.S.	1	1.00	1.20	1.30	0.00	2.34	0.00	1.38	1.36
time (sec)	N/A	0.030	0.042	0.092	0.000	0.364	0.000	7.388	4.308

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	63	85	0	134	0	50	-1
N.S.	1	1.00	1.12	1.52	0.00	2.39	0.00	0.89	-0.02
time (sec)	N/A	0.030	0.071	0.101	0.000	0.368	0.000	8.095	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	91	106	0	159	0	78	-1
N.S.	1	1.00	1.08	1.26	0.00	1.89	0.00	0.93	-0.01
time (sec)	N/A	0.065	0.091	0.085	0.000	0.373	0.000	6.574	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	103	128	0	185	0	100	-1
N.S.	1	1.00	0.92	1.14	0.00	1.65	0.00	0.89	-0.01
time (sec)	N/A	0.091	0.108	0.125	0.000	0.347	0.000	4.610	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	124	142	142	210	0	115	134
N.S.	1	1.00	1.00	1.15	1.15	1.69	0.00	0.93	1.08
time (sec)	N/A	0.085	0.124	0.100	0.311	0.354	0.000	4.839	4.351

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	113	122	118	189	0	99	99
N.S.	1	1.00	1.12	1.21	1.17	1.87	0.00	0.98	0.98
time (sec)	N/A	0.061	0.092	0.101	0.296	0.370	0.000	3.042	4.445

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	102	102	91	166	0	84	-1
N.S.	1	1.00	1.16	1.16	1.03	1.89	0.00	0.95	-0.01
time (sec)	N/A	0.061	0.090	0.101	0.307	0.381	0.000	3.714	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	74	84	70	145	0	68	-1
N.S.	1	1.00	0.92	1.05	0.88	1.81	0.00	0.85	-0.01
time (sec)	N/A	0.061	0.070	0.097	0.292	0.336	0.000	4.343	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	80	107	71	139	0	79	-1
N.S.	1	1.00	1.05	1.41	0.93	1.83	0.00	1.04	-0.01
time (sec)	N/A	0.062	0.093	0.098	0.302	0.321	0.000	3.521	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	85	129	89	135	0	122	-1
N.S.	1	1.00	1.13	1.72	1.19	1.80	0.00	1.63	-0.01
time (sec)	N/A	0.060	0.082	0.103	0.287	0.368	0.000	4.354	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	81	39	0	92	30
N.S.	1	1.00	1.00	1.16	3.24	1.56	0.00	3.68	1.20
time (sec)	N/A	0.026	0.062	0.102	0.292	0.324	0.000	4.342	4.380

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	105	53	0	178	87
N.S.	1	1.00	0.67	0.75	2.02	1.02	0.00	3.42	1.67
time (sec)	N/A	0.052	0.072	0.090	0.293	0.325	0.000	4.197	4.579

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	50	129	64	0	206	111
N.S.	1	1.00	0.58	0.62	1.61	0.80	0.00	2.58	1.39
time (sec)	N/A	0.088	0.081	0.101	0.292	0.349	0.000	3.795	4.722

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	57	61	153	75	0	236	135
N.S.	1	1.00	0.53	0.56	1.42	0.69	0.00	2.19	1.25
time (sec)	N/A	0.119	0.087	0.108	0.280	0.359	0.000	3.945	4.990

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	68	72	177	86	0	264	159
N.S.	1	1.00	0.50	0.53	1.30	0.63	0.00	1.94	1.17
time (sec)	N/A	0.150	0.097	0.112	0.297	0.405	0.000	4.114	5.174

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	68	72	79	86	0	92	73
N.S.	1	1.00	0.51	0.54	0.59	0.64	0.00	0.69	0.54
time (sec)	N/A	0.137	0.036	0.096	0.295	0.365	0.000	4.741	4.478

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	57	61	68	75	0	76	62
N.S.	1	1.00	0.54	0.58	0.64	0.71	0.00	0.72	0.58
time (sec)	N/A	0.104	0.035	0.082	0.300	0.334	0.000	3.944	4.304

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	50	57	64	0	60	51
N.S.	1	1.00	0.58	0.62	0.71	0.80	0.00	0.75	0.64
time (sec)	N/A	0.066	0.031	0.084	0.306	0.368	0.000	3.622	4.187

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	45	52	0	44	40
N.S.	1	1.00	0.67	0.75	0.87	1.00	0.00	0.85	0.77
time (sec)	N/A	0.032	0.027	0.117	0.280	0.375	0.000	3.368	4.160

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	32	39	0	27	30
N.S.	1	1.00	1.00	1.16	1.28	1.56	0.00	1.08	1.20
time (sec)	N/A	0.029	0.006	0.088	0.279	0.431	0.000	3.993	4.149

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	76	78	0	140	0	89	-1
N.S.	1	1.00	1.04	1.07	0.00	1.92	0.00	1.22	-0.01
time (sec)	N/A	0.061	0.059	0.095	0.000	0.402	0.000	3.648	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	83	102	0	147	0	69	-1
N.S.	1	1.00	1.05	1.29	0.00	1.86	0.00	0.87	-0.01
time (sec)	N/A	0.061	0.078	0.094	0.000	0.417	0.000	3.506	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	125	0	164	0	76	-1
N.S.	1	1.00	1.14	1.54	0.00	2.02	0.00	0.94	-0.01
time (sec)	N/A	0.063	0.102	0.109	0.000	0.391	0.000	4.278	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	104	145	0	185	0	100	-1
N.S.	1	1.00	0.95	1.33	0.00	1.70	0.00	0.92	-0.01
time (sec)	N/A	0.094	0.113	0.108	0.000	0.408	0.000	3.044	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	114	165	0	207	0	119	-1
N.S.	1	1.00	0.83	1.20	0.00	1.51	0.00	0.87	-0.01
time (sec)	N/A	0.123	0.136	0.105	0.000	0.352	0.000	4.009	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	126	186	0	229	0	138	-1
N.S.	1	1.00	0.76	1.13	0.00	1.39	0.00	0.84	-0.01
time (sec)	N/A	0.155	0.156	0.102	0.000	0.420	0.000	3.649	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	102	105	100	166	0	97	-1
N.S.	1	1.00	0.89	0.92	0.88	1.46	0.00	0.85	-0.01
time (sec)	N/A	0.083	0.070	0.094	0.285	0.348	0.000	4.378	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	91	85	76	145	0	79	-1
N.S.	1	1.00	1.06	0.99	0.88	1.69	0.00	0.92	-0.01
time (sec)	N/A	0.064	0.058	0.099	0.286	0.398	0.000	3.200	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	74	64	52	114	0	59	53
N.S.	1	1.00	1.28	1.10	0.90	1.97	0.00	1.02	0.91
time (sec)	N/A	0.050	0.044	0.097	0.287	0.421	0.000	3.044	4.302

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	52	44	32	74	0	38	33
N.S.	1	1.00	1.68	1.42	1.03	2.39	0.00	1.23	1.06
time (sec)	N/A	0.030	0.010	0.093	0.286	0.363	0.000	2.658	4.360

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	21	21	0	34	21
N.S.	1	1.00	1.00	1.13	0.91	0.91	0.00	1.48	0.91
time (sec)	N/A	0.024	0.031	0.082	0.300	0.335	0.000	4.596	4.213

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	37	44	31	0	59	29
N.S.	1	1.00	0.67	0.71	0.85	0.60	0.00	1.13	0.56
time (sec)	N/A	0.050	0.045	0.106	0.299	0.389	0.000	3.773	4.259

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	50	68	42	0	81	42
N.S.	1	1.00	0.58	0.62	0.85	0.52	0.00	1.01	0.52
time (sec)	N/A	0.075	0.053	0.096	0.293	0.368	0.000	3.291	4.326

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	57	61	92	53	0	107	92
N.S.	1	1.00	0.53	0.56	0.85	0.49	0.00	0.99	0.85
time (sec)	N/A	0.106	0.058	0.092	0.281	0.350	0.000	4.278	4.273

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	37	34	30	0	48	33
N.S.	1	1.00	0.68	0.74	0.68	0.60	0.00	0.96	0.66
time (sec)	N/A	0.040	0.023	0.092	0.303	0.377	0.000	4.015	4.248

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	13	20	0	28	20
N.S.	1	1.00	1.00	1.18	0.59	0.91	0.00	1.27	0.91
time (sec)	N/A	0.010	0.004	0.089	0.293	0.368	0.000	4.279	4.258

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	52	50	0	80	0	46	-1
N.S.	1	1.00	1.73	1.67	0.00	2.67	0.00	1.53	-0.03
time (sec)	N/A	0.006	0.023	0.086	0.000	0.391	0.000	2.856	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	76	73	0	133	0	55	76
N.S.	1	1.00	1.29	1.24	0.00	2.25	0.00	0.93	1.29
time (sec)	N/A	0.032	0.059	0.099	0.000	0.367	0.000	5.471	4.472

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	91	94	0	163	0	79	-1
N.S.	1	1.00	1.05	1.08	0.00	1.87	0.00	0.91	-0.01
time (sec)	N/A	0.059	0.080	0.122	0.000	0.384	0.000	3.440	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	91	87	103	209	0	96	-1
N.S.	1	1.00	0.83	0.80	0.94	1.92	0.00	0.88	-0.01
time (sec)	N/A	0.079	0.087	0.097	0.307	0.408	0.000	3.302	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	73	77	180	0	74	-1
N.S.	1	1.00	0.95	0.90	0.95	2.22	0.00	0.91	-0.01
time (sec)	N/A	0.065	0.071	0.090	0.287	0.356	0.000	5.479	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	64	63	54	150	0	57	55
N.S.	1	1.00	1.16	1.15	0.98	2.73	0.00	1.04	1.00
time (sec)	N/A	0.055	0.052	0.087	0.342	0.384	0.000	4.200	4.328

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	28	20	26	0	18	26
N.S.	1	1.00	1.00	1.27	0.91	1.18	0.00	0.82	1.18
time (sec)	N/A	0.031	0.030	0.096	0.317	0.331	0.000	5.616	4.131

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	29	37	41	41	0	58	26
N.S.	1	1.00	1.04	1.32	1.46	1.46	0.00	2.07	0.93
time (sec)	N/A	0.028	0.045	0.104	0.287	0.423	0.000	6.106	4.125

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	46	45	65	54	0	114	51
N.S.	1	1.00	0.62	0.61	0.88	0.73	0.00	1.54	0.69
time (sec)	N/A	0.081	0.059	0.104	0.288	0.336	0.000	5.828	4.236

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	57	59	89	63	0	169	60
N.S.	1	1.00	0.56	0.58	0.87	0.62	0.00	1.66	0.59
time (sec)	N/A	0.108	0.072	0.103	0.292	0.355	0.000	5.353	4.305

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	68	72	113	76	0	222	114
N.S.	1	1.00	0.52	0.55	0.87	0.58	0.00	1.71	0.88
time (sec)	N/A	0.137	0.081	0.105	0.297	0.383	0.000	5.861	4.407

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	29	37	22	39	0	51	38
N.S.	1	1.00	0.62	0.79	0.47	0.83	0.00	1.09	0.81
time (sec)	N/A	0.042	0.024	0.106	0.296	0.382	0.000	4.133	4.225

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	29	14	29	0	28	30
N.S.	1	1.00	1.00	1.38	0.67	1.38	0.00	1.33	1.43
time (sec)	N/A	0.012	0.004	0.094	0.290	0.344	0.000	6.460	4.152

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	59	65	0	162	0	79	-1
N.S.	1	1.00	1.16	1.27	0.00	3.18	0.00	1.55	-0.02
time (sec)	N/A	0.037	0.039	0.089	0.000	0.372	0.000	4.765	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	77	0	199	0	80	42
N.S.	1	1.00	0.96	0.95	0.00	2.46	0.00	0.99	0.52
time (sec)	N/A	0.039	0.072	0.126	0.000	0.357	0.000	5.280	4.341

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	92	94	0	229	0	99	44
N.S.	1	1.00	0.84	0.86	0.00	2.10	0.00	0.91	0.40
time (sec)	N/A	0.094	0.093	0.113	0.000	0.366	0.000	5.781	4.637

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	58	48	26	37	0	30	42
N.S.	1	1.00	1.71	1.41	0.76	1.09	0.00	0.88	1.24
time (sec)	N/A	0.033	0.037	0.125	0.515	0.352	0.000	7.485	4.332

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	59	54	26	59	0	43	41
N.S.	1	1.00	1.74	1.59	0.76	1.74	0.00	1.26	1.21
time (sec)	N/A	0.035	0.033	0.119	0.532	0.350	0.000	4.621	4.363

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	58	48	41	45	0	43	40
N.S.	1	1.00	1.29	1.07	0.91	1.00	0.00	0.96	0.89
time (sec)	N/A	0.036	0.006	0.088	0.503	0.382	0.000	4.914	4.397

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	58	60	41	45	0	44	40
N.S.	1	1.00	1.29	1.33	0.91	1.00	0.00	0.98	0.89
time (sec)	N/A	0.034	0.007	0.118	0.501	0.343	0.000	3.760	4.456

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	74	64	52	114	0	59	53
N.S.	1	1.00	1.28	1.10	0.90	1.97	0.00	1.02	0.91
time (sec)	N/A	0.054	0.050	0.089	0.306	0.362	0.000	3.847	4.707

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	86	67	42	120	0	73	60
N.S.	1	1.00	1.43	1.12	0.70	2.00	0.00	1.22	1.00
time (sec)	N/A	0.056	0.076	0.124	0.537	0.343	0.000	4.474	4.621

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.004	0.012	0.046	0.314	0.337	0.774	4.686	0.035

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.003	0.012	0.036	0.294	0.333	0.496	3.926	0.029

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.003	0.012	0.036	0.297	0.331	0.306	5.714	0.027

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	14	13	18	19	13	15
N.S.	1	1.00	0.90	0.67	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.003	0.011	0.033	0.282	0.342	0.802	4.044	0.028

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.71
time (sec)	N/A	0.003	0.011	0.049	0.325	0.344	0.147	5.728	0.026

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	16	19	13	15
N.S.	1	1.00	1.00	0.67	0.62	0.76	0.90	0.62	0.71
time (sec)	N/A	0.003	0.011	0.020	0.281	0.339	0.185	3.718	0.028

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	20	14	13	14	17	13	14
N.S.	1	1.00	1.05	0.74	0.68	0.74	0.89	0.68	0.74
time (sec)	N/A	0.003	0.011	0.018	0.330	0.336	0.228	3.712	0.030

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	14	17	13	15
N.S.	1	1.00	1.00	0.74	0.68	0.74	0.89	0.68	0.79
time (sec)	N/A	0.003	0.013	0.025	0.283	0.362	0.366	3.778	0.031

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.010	0.016	0.085	0.267	0.350	1.677	2.700	0.049

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.009	0.016	0.095	0.291	0.352	1.178	3.193	0.042

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.010	0.016	0.081	0.265	0.343	0.807	3.952	4.271

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.010	0.016	0.095	0.280	0.350	1.311	4.321	0.041

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.69
time (sec)	N/A	0.009	0.016	0.078	0.276	0.343	0.492	4.558	0.040

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	26
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.72
time (sec)	N/A	0.010	0.016	0.092	0.276	0.340	0.606	4.227	4.438

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	26
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.72
time (sec)	N/A	0.011	0.015	0.107	0.273	0.353	0.674	3.774	0.043

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	27	34	24	26
N.S.	1	1.00	0.83	0.69	0.67	0.75	0.94	0.67	0.72
time (sec)	N/A	0.010	0.015	0.111	0.282	0.320	0.864	5.183	0.047

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	36	35	40	49	35	35
N.S.	1	1.00	1.00	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.014	0.029	0.111	0.288	0.343	3.154	4.083	0.050

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	36	35	40	49	35	35
N.S.	1	1.00	1.00	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.013	0.028	0.102	0.282	0.380	2.317	3.542	0.051

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	36	35	40	49	35	35
N.S.	1	1.00	0.92	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.013	0.018	0.106	0.275	0.321	1.697	4.311	0.050

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.012	0.017	0.106	0.283	0.357	2.046	4.182	0.051

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.013	0.017	0.109	0.303	0.350	1.214	3.695	0.047

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.013	0.018	0.106	0.289	0.401	1.428	3.923	0.049

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.013	0.017	0.113	0.299	0.353	1.625	4.000	0.052

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.014	0.016	0.114	0.279	0.337	1.826	4.074	0.050

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	129	128	198	182	0	197	66
N.S.	1	1.00	0.59	0.59	0.91	0.84	0.00	0.91	0.30
time (sec)	N/A	0.144	0.161	0.130	0.501	0.383	0.000	4.288	0.114

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	128	125	194	170	136	196	67
N.S.	1	1.00	0.60	0.58	0.90	0.79	0.63	0.91	0.31
time (sec)	N/A	0.132	0.140	0.118	0.518	0.350	109.381	4.995	4.494

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	119	116	186	165	124	178	54
N.S.	1	1.00	0.58	0.57	0.91	0.81	0.61	0.87	0.26
time (sec)	N/A	0.109	0.126	0.123	0.512	0.363	53.361	3.709	4.359

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	118	115	185	124	110	178	55
N.S.	1	1.00	0.58	0.57	0.92	0.61	0.54	0.88	0.27
time (sec)	N/A	0.110	0.126	0.121	0.525	0.345	26.401	4.117	4.360

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	91	106	172	126	104	182	38
N.S.	1	1.00	0.47	0.55	0.90	0.66	0.54	0.95	0.20
time (sec)	N/A	0.093	0.103	0.106	0.500	0.346	16.627	3.807	0.079

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	92	106	172	126	104	182	37
N.S.	1	1.00	0.48	0.55	0.90	0.66	0.54	0.95	0.19
time (sec)	N/A	0.094	0.100	0.120	0.501	0.373	9.162	3.951	4.434

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	117	115	186	142	114	190	54
N.S.	1	1.00	0.58	0.57	0.92	0.70	0.56	0.94	0.27
time (sec)	N/A	0.108	0.139	0.132	0.545	0.357	5.890	4.008	4.530

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	119	116	187	167	128	178	53
N.S.	1	1.00	0.58	0.57	0.92	0.82	0.63	0.87	0.26
time (sec)	N/A	0.107	0.136	0.139	0.500	0.402	7.762	4.121	0.099

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	127	125	198	193	139	200	66
N.S.	1	1.00	0.59	0.58	0.92	0.90	0.65	0.93	0.31
time (sec)	N/A	0.122	0.162	0.132	0.515	0.359	13.995	4.905	0.093

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	129	127	201	189	146	192	65
N.S.	1	1.00	0.59	0.59	0.93	0.87	0.67	0.88	0.30
time (sec)	N/A	0.124	0.172	0.124	0.498	0.386	31.332	5.045	4.386

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	140	138	209	204	160	199	77
N.S.	1	1.00	0.61	0.60	0.91	0.89	0.70	0.87	0.33
time (sec)	N/A	0.136	0.178	0.125	0.527	0.387	76.134	4.857	4.466

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	149	148	217	227	0	216	92
N.S.	1	1.00	0.61	0.61	0.89	0.93	0.00	0.89	0.38
time (sec)	N/A	0.138	0.273	0.140	0.500	0.375	0.000	2.601	0.097

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	138	136	207	229	0	196	80
N.S.	1	1.00	0.60	0.59	0.90	1.00	0.00	0.85	0.35
time (sec)	N/A	0.120	0.263	0.129	0.500	0.380	0.000	4.399	0.107

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	138	136	206	192	0	196	80
N.S.	1	1.00	0.60	0.59	0.90	0.83	0.00	0.85	0.35
time (sec)	N/A	0.119	0.262	0.130	0.522	0.382	0.000	3.736	4.320

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	128	124	195	185	0	199	64
N.S.	1	1.00	0.59	0.57	0.89	0.85	0.00	0.91	0.29
time (sec)	N/A	0.106	0.247	0.116	0.519	0.374	0.000	4.997	0.089

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	127	127	195	187	0	199	64
N.S.	1	1.00	0.58	0.58	0.89	0.86	0.00	0.91	0.29
time (sec)	N/A	0.108	0.246	0.119	0.523	0.373	0.000	5.118	4.311

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	128	127	194	182	0	199	64
N.S.	1	1.00	0.59	0.58	0.89	0.83	0.00	0.91	0.29
time (sec)	N/A	0.108	0.236	0.086	0.505	0.362	0.000	4.880	4.340

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	128	124	194	179	0	199	64
N.S.	1	1.00	0.59	0.57	0.89	0.82	0.00	0.91	0.29
time (sec)	N/A	0.108	0.227	0.117	0.510	0.347	0.000	5.577	0.095

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	138	136	208	208	0	210	77
N.S.	1	1.00	0.60	0.59	0.90	0.90	0.00	0.91	0.33
time (sec)	N/A	0.118	0.281	0.135	0.510	0.366	0.000	4.450	0.085

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	138	136	209	228	0	196	77
N.S.	1	1.00	0.60	0.59	0.91	0.99	0.00	0.85	0.33
time (sec)	N/A	0.120	0.288	0.139	0.490	0.373	0.000	3.614	0.107

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	149	147	221	251	0	220	87
N.S.	1	1.00	0.61	0.60	0.91	1.03	0.00	0.91	0.36
time (sec)	N/A	0.134	0.312	0.135	0.510	0.374	0.000	4.185	4.369

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	149	147	224	245	0	212	87
N.S.	1	1.00	0.61	0.60	0.92	1.01	0.00	0.87	0.36
time (sec)	N/A	0.136	0.307	0.132	0.540	0.396	0.000	4.152	0.106

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	160	158	232	262	0	219	99
N.S.	1	1.00	0.62	0.61	0.90	1.02	0.00	0.85	0.38
time (sec)	N/A	0.150	0.329	0.135	0.511	0.377	0.000	3.922	4.369

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	149	145	229	247	0	208	101
N.S.	1	1.00	0.59	0.58	0.91	0.98	0.00	0.83	0.40
time (sec)	N/A	0.137	0.349	0.138	0.556	0.353	0.000	4.293	4.389

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	138	136	218	248	0	209	87
N.S.	1	1.00	0.58	0.57	0.91	1.04	0.00	0.87	0.36
time (sec)	N/A	0.122	0.342	0.115	0.520	0.362	0.000	3.814	4.281

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	138	139	218	254	0	209	87
N.S.	1	1.00	0.58	0.58	0.91	1.06	0.00	0.87	0.36
time (sec)	N/A	0.126	0.339	0.107	0.527	0.372	0.000	4.577	0.097

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	136	138	222	260	0	212	85
N.S.	1	1.00	0.56	0.57	0.92	1.07	0.00	0.88	0.35
time (sec)	N/A	0.123	0.327	0.108	0.492	0.376	0.000	3.648	0.087

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	137	138	221	257	0	211	85
N.S.	1	1.00	0.57	0.57	0.91	1.06	0.00	0.87	0.35
time (sec)	N/A	0.122	0.317	0.107	0.500	0.354	0.000	3.647	0.102

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	138	150	217	250	0	209	86
N.S.	1	1.00	0.58	0.63	0.91	1.05	0.00	0.87	0.36
time (sec)	N/A	0.125	0.222	0.116	0.504	0.344	0.000	4.488	0.089

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	138	147	217	241	0	209	86
N.S.	1	1.00	0.58	0.62	0.91	1.01	0.00	0.87	0.36
time (sec)	N/A	0.122	0.220	0.112	0.550	0.368	0.000	3.539	4.292

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	149	145	230	263	0	220	99
N.S.	1	1.00	0.59	0.58	0.92	1.05	0.00	0.88	0.39
time (sec)	N/A	0.139	0.359	0.144	0.503	0.381	0.000	3.638	4.366

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	149	145	231	283	0	208	99
N.S.	1	1.00	0.59	0.58	0.92	1.13	0.00	0.83	0.39
time (sec)	N/A	0.133	0.366	0.128	0.527	0.382	0.000	4.063	0.128

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	160	156	243	306	0	232	109
N.S.	1	1.00	0.61	0.59	0.92	1.16	0.00	0.88	0.41
time (sec)	N/A	0.146	0.377	0.151	0.504	0.376	0.000	4.947	0.118

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	160	156	246	300	0	224	109
N.S.	1	1.00	0.61	0.59	0.93	1.14	0.00	0.85	0.41
time (sec)	N/A	0.153	0.372	0.142	0.533	0.376	0.000	6.131	4.348

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	171	167	254	317	0	231	121
N.S.	1	1.00	0.61	0.60	0.91	1.14	0.00	0.83	0.43
time (sec)	N/A	0.167	0.388	0.133	0.535	0.370	0.000	6.455	0.139

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	171	167	257	311	0	243	121
N.S.	1	1.00	0.61	0.60	0.92	1.11	0.00	0.87	0.43
time (sec)	N/A	0.172	0.385	0.120	0.525	0.378	0.000	5.323	4.389

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	102	237	0	73	0	0	-1
N.S.	1	1.00	0.32	0.73	0.00	0.23	0.00	0.00	-0.00
time (sec)	N/A	0.254	10.049	0.131	0.000	0.100	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	102	157	0	68	0	0	-1
N.S.	1	1.00	0.58	0.89	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.153	10.039	0.138	0.000	0.085	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	86	226	0	61	0	0	-1
N.S.	1	1.00	0.29	0.77	0.00	0.21	0.00	0.00	-0.00
time (sec)	N/A	0.193	10.036	0.097	0.000	0.083	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	86	145	0	58	0	0	-1
N.S.	1	1.00	0.59	0.99	0.00	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.118	9.542	0.128	0.000	0.101	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	57	213	0	48	0	0	-1
N.S.	1	1.00	0.22	0.81	0.00	0.18	0.00	0.00	-0.00
time (sec)	N/A	0.150	7.825	0.098	0.000	0.114	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	55	130	0	43	0	0	-1
N.S.	1	1.00	0.47	1.10	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.085	6.854	0.094	0.000	0.079	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	55	202	0	48	0	0	-1
N.S.	1	1.00	0.22	0.80	0.00	0.19	0.00	0.00	-0.00
time (sec)	N/A	0.149	10.011	0.120	0.000	0.109	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	57	125	0	41	0	0	-1
N.S.	1	1.00	0.48	1.06	0.00	0.35	0.00	0.00	-0.01
time (sec)	N/A	0.084	10.023	0.108	0.000	0.074	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	57	224	0	59	0	0	-1
N.S.	1	1.00	0.19	0.76	0.00	0.20	0.00	0.00	-0.00
time (sec)	N/A	0.191	10.013	0.106	0.000	0.086	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	57	142	0	53	0	0	-1
N.S.	1	1.00	0.39	0.97	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.118	10.015	0.109	0.000	0.088	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	57	239	0	72	0	0	-1
N.S.	1	1.00	0.18	0.74	0.00	0.22	0.00	0.00	-0.00
time (sec)	N/A	0.234	10.014	0.120	0.000	0.083	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	57	156	0	64	0	0	-1
N.S.	1	1.00	0.32	0.89	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.152	10.025	0.115	0.000	0.089	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	101	248	0	84	0	0	-1
N.S.	1	1.00	0.29	0.71	0.00	0.24	0.00	0.00	-0.00
time (sec)	N/A	0.277	10.074	0.100	0.000	0.083	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	101	168	0	79	0	0	-1
N.S.	1	1.00	0.50	0.83	0.00	0.39	0.00	0.00	-0.00
time (sec)	N/A	0.184	10.046	0.115	0.000	0.101	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	90	237	0	72	0	0	-1
N.S.	1	1.00	0.28	0.74	0.00	0.22	0.00	0.00	-0.00
time (sec)	N/A	0.237	10.043	0.121	0.000	0.086	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	90	157	0	69	0	0	-1
N.S.	1	1.00	0.52	0.91	0.00	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.148	10.031	0.095	0.000	0.103	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	58	226	0	62	0	0	-1
N.S.	1	1.00	0.20	0.78	0.00	0.21	0.00	0.00	-0.00
time (sec)	N/A	0.193	10.023	0.105	0.000	0.109	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	56	145	0	56	0	0	-1
N.S.	1	1.00	0.39	1.01	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.116	9.024	0.097	0.000	0.106	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	56	216	0	59	0	0	-1
N.S.	1	1.00	0.20	0.76	0.00	0.21	0.00	0.00	-0.00
time (sec)	N/A	0.188	10.024	0.115	0.000	0.102	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	58	130	0	50	0	0	-1
N.S.	1	1.00	0.41	0.91	0.00	0.35	0.00	0.00	-0.01
time (sec)	N/A	0.115	10.012	0.122	0.000	0.106	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	58	221	0	56	0	0	-1
N.S.	1	1.00	0.20	0.77	0.00	0.20	0.00	0.00	-0.00
time (sec)	N/A	0.197	10.015	0.117	0.000	0.095	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	58	140	0	49	0	0	-1
N.S.	1	1.00	0.41	0.98	0.00	0.34	0.00	0.00	-0.01
time (sec)	N/A	0.120	10.016	0.109	0.000	0.077	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	58	239	0	72	0	0	-1
N.S.	1	1.00	0.18	0.75	0.00	0.22	0.00	0.00	-0.00
time (sec)	N/A	0.231	10.015	0.126	0.000	0.084	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	58	156	0	64	0	0	-1
N.S.	1	1.00	0.34	0.90	0.00	0.37	0.00	0.00	-0.01
time (sec)	N/A	0.153	10.020	0.121	0.000	0.097	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	58	250	0	83	0	0	-1
N.S.	1	1.00	0.17	0.71	0.00	0.24	0.00	0.00	-0.00
time (sec)	N/A	0.278	10.020	0.124	0.000	0.079	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	58	167	0	75	0	0	-1
N.S.	1	1.00	0.29	0.82	0.00	0.37	0.00	0.00	-0.00
time (sec)	N/A	0.182	10.026	0.111	0.000	0.085	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	97	148	0	69	0	0	-1
N.S.	1	1.00	0.54	0.83	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.159	10.043	0.107	0.000	0.110	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	86	217	0	62	0	0	-1
N.S.	1	1.00	0.29	0.73	0.00	0.21	0.00	0.00	-0.00
time (sec)	N/A	0.192	10.038	0.114	0.000	0.079	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	86	137	0	57	0	0	-1
N.S.	1	1.00	0.58	0.92	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.114	10.032	0.117	0.000	0.096	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	70	206	0	47	0	0	-1
N.S.	1	1.00	0.26	0.77	0.00	0.18	0.00	0.00	-0.00
time (sec)	N/A	0.152	10.024	0.115	0.000	0.082	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	70	123	0	43	0	0	-1
N.S.	1	1.00	0.58	1.02	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.083	10.042	0.116	0.000	0.078	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	57	131	0	22	0	0	-1
N.S.	1	1.00	0.25	0.57	0.00	0.10	0.00	0.00	-0.00
time (sec)	N/A	0.116	10.023	0.097	0.000	0.094	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	55	106	0	14	0	0	-1
N.S.	1	1.00	0.61	1.18	0.00	0.16	0.00	0.00	-0.01
time (sec)	N/A	0.055	10.019	0.082	0.000	0.085	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	55	195	0	50	0	0	-1
N.S.	1	1.00	0.21	0.75	0.00	0.19	0.00	0.00	-0.00
time (sec)	N/A	0.153	10.017	0.125	0.000	0.082	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	57	119	0	42	0	0	-1
N.S.	1	1.00	0.47	0.98	0.00	0.35	0.00	0.00	-0.01
time (sec)	N/A	0.084	10.016	0.086	0.000	0.083	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	57	215	0	61	0	0	-1
N.S.	1	1.00	0.19	0.73	0.00	0.21	0.00	0.00	-0.00
time (sec)	N/A	0.190	10.016	0.106	0.000	0.076	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	57	134	0	53	0	0	-1
N.S.	1	1.00	0.38	0.90	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.115	10.019	0.099	0.000	0.081	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	57	230	0	72	0	0	-1
N.S.	1	1.00	0.17	0.71	0.00	0.22	0.00	0.00	-0.00
time (sec)	N/A	0.231	10.024	0.123	0.000	0.076	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	57	147	0	64	0	0	-1
N.S.	1	1.00	0.32	0.82	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.149	10.041	0.126	0.000	0.096	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	86	144	0	88	0	0	-1
N.S.	1	1.00	0.49	0.83	0.00	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.163	10.041	0.135	0.000	0.092	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	72	213	0	80	0	0	-1
N.S.	1	1.00	0.25	0.73	0.00	0.27	0.00	0.00	-0.00
time (sec)	N/A	0.197	10.023	0.138	0.000	0.094	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	73	131	0	76	0	0	-1
N.S.	1	1.00	0.50	0.90	0.00	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.122	10.022	0.126	0.000	0.102	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	61	200	0	65	0	0	-1
N.S.	1	1.00	0.24	0.77	0.00	0.25	0.00	0.00	-0.00
time (sec)	N/A	0.155	10.030	0.111	0.000	0.083	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	60	120	0	59	0	0	-1
N.S.	1	1.00	0.50	1.01	0.00	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.091	10.026	0.130	0.000	0.092	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	60	203	0	64	0	0	-1
N.S.	1	1.00	0.23	0.78	0.00	0.25	0.00	0.00	-0.00
time (sec)	N/A	0.159	10.021	0.090	0.000	0.086	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	60	123	0	59	0	0	-1
N.S.	1	1.00	0.51	1.04	0.00	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.091	9.884	0.112	0.000	0.086	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	58	203	0	81	0	0	-1
N.S.	1	1.00	0.20	0.71	0.00	0.28	0.00	0.00	-0.00
time (sec)	N/A	0.201	10.019	0.115	0.000	0.079	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	60	127	0	73	0	0	-1
N.S.	1	1.00	0.41	0.88	0.00	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.120	10.017	0.125	0.000	0.094	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	60	222	0	95	0	0	-1
N.S.	1	1.00	0.19	0.69	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.237	10.019	0.140	0.000	0.085	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	60	141	0	87	0	0	-1
N.S.	1	1.00	0.35	0.82	0.00	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.150	10.023	0.129	0.000	0.088	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	60	237	0	108	0	0	-1
N.S.	1	1.00	0.17	0.68	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.277	10.031	0.132	0.000	0.088	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	59	181	76	161	731	264	171
N.S.	1	1.00	0.81	2.48	1.04	2.21	10.01	3.62	2.34
time (sec)	N/A	0.035	0.058	0.105	0.284	0.354	0.737	5.690	4.290

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	96	55	89	337	141	97
N.S.	1	1.00	0.83	1.85	1.06	1.71	6.48	2.71	1.87
time (sec)	N/A	0.026	0.050	0.103	0.290	0.370	0.427	5.015	4.195

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	36	34	39	112	56	38
N.S.	1	1.00	0.79	1.06	1.00	1.15	3.29	1.65	1.12
time (sec)	N/A	0.010	0.026	0.013	0.280	0.363	0.192	7.171	4.148

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.027	0.023	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.030	0.015	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.032	0.015	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.007	0.001	0.063	0.288	0.306	0.007	5.224	0.038

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.006	0.001	0.011	0.305	0.338	0.006	3.585	0.033

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	16	25	24	24	24	24	24
N.S.	1	1.00	0.53	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.007	0.002	0.008	0.281	0.336	0.007	4.905	0.031

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.003	0.000	0.010	0.276	0.313	0.006	15.230	0.028

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	21	20	24	21
N.S.	1	1.00	1.00	0.96	1.04	0.91	0.87	1.04	0.91
time (sec)	N/A	0.005	0.001	0.025	0.284	0.368	0.023	12.316	4.097

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92
time (sec)	N/A	0.006	0.001	0.011	0.299	0.329	0.022	14.257	0.034

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	24	27	24	32	23
N.S.	1	1.00	1.00	0.89	0.89	1.00	0.89	1.19	0.85
time (sec)	N/A	0.007	0.001	0.014	0.289	0.333	0.039	7.866	0.032

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	24
N.S.	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.04
time (sec)	N/A	0.006	0.001	0.013	0.287	0.340	0.045	4.971	4.106

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	26	28	24	34	24
N.S.	1	1.00	1.00	0.96	1.08	1.17	1.00	1.42	1.00
time (sec)	N/A	0.006	0.001	0.013	0.282	0.338	0.063	3.490	0.044

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	25
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.89
time (sec)	N/A	0.006	0.001	0.011	0.286	0.333	0.064	4.300	0.036

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87
time (sec)	N/A	0.007	0.001	0.013	0.302	0.345	0.067	3.655	0.035

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87
time (sec)	N/A	0.006	0.001	0.015	0.306	0.326	0.075	3.947	0.034

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	53	46	46
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.82
time (sec)	N/A	0.020	0.002	0.053	0.296	0.320	0.010	2.978	0.025

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	56	47	46	46	49	46	46
N.S.	1	1.00	1.06	0.89	0.87	0.87	0.92	0.87	0.87
time (sec)	N/A	0.044	0.002	0.052	0.280	0.324	0.013	3.638	0.022

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	53	46	46
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.82
time (sec)	N/A	0.017	0.002	0.048	0.301	0.341	0.010	3.392	0.023

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	56	47	46	46	53	46	46
N.S.	1	1.00	1.65	1.38	1.35	1.35	1.56	1.35	1.35
time (sec)	N/A	0.027	0.002	0.049	0.289	0.352	0.012	4.201	0.022

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	53	46	46
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.82
time (sec)	N/A	0.017	0.002	0.046	0.281	0.337	0.010	4.282	0.022

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	45	44	44	44	44	44
N.S.	1	1.00	1.00	2.81	2.75	2.75	2.75	2.75	2.75
time (sec)	N/A	0.003	0.002	0.045	0.281	0.335	0.012	4.203	0.022

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	55	43	49	43	43
N.S.	1	1.00	1.00	0.86	1.08	0.84	0.96	0.84	0.84
time (sec)	N/A	0.015	0.001	0.010	0.276	0.331	0.010	5.405	0.021

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	47	44	49	47	44
N.S.	1	1.00	1.00	0.90	0.94	0.88	0.98	0.94	0.88
time (sec)	N/A	0.024	0.003	0.045	0.285	0.372	0.034	6.359	0.027

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	44	48	44	44	44
N.S.	1	1.00	1.00	0.94	0.92	1.00	0.92	0.92	0.92
time (sec)	N/A	0.017	0.006	0.039	0.292	0.352	0.031	7.395	0.025

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	46	49	46	56	44
N.S.	1	1.00	1.00	0.94	0.96	1.02	0.96	1.17	0.92
time (sec)	N/A	0.026	0.004	0.020	0.304	0.349	0.050	3.819	0.027

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	45	48	49	45	47
N.S.	1	1.00	1.00	0.90	0.90	0.96	0.98	0.90	0.94
time (sec)	N/A	0.017	0.005	0.017	0.287	0.369	0.055	5.958	0.045

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	48	49	49	59	48
N.S.	1	1.00	1.00	0.94	0.98	1.00	1.00	1.20	0.98
time (sec)	N/A	0.024	0.004	0.031	0.300	0.392	0.078	3.190	0.038

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	47	48	49	47	47
N.S.	1	1.00	1.00	0.90	0.94	0.96	0.98	0.94	0.94
time (sec)	N/A	0.017	0.008	0.018	0.310	0.336	0.082	3.333	0.045

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	48	50	49	57	47
N.S.	1	1.00	1.00	0.94	0.98	1.02	1.00	1.16	0.96
time (sec)	N/A	0.023	0.004	0.017	0.296	0.334	0.107	3.444	0.038

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	46	48	48	46	46
N.S.	1	1.00	1.00	0.94	0.98	1.02	1.02	0.98	0.98
time (sec)	N/A	0.017	0.005	0.015	0.321	0.324	0.116	3.739	4.190

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	50	50	49	58	47
N.S.	1	1.00	1.00	0.90	1.00	1.00	0.98	1.16	0.94
time (sec)	N/A	0.022	0.004	0.017	0.322	0.367	0.146	3.890	0.051

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	47	48	48	51	48	47
N.S.	1	1.00	1.00	0.87	0.89	0.89	0.94	0.89	0.87
time (sec)	N/A	0.018	0.006	0.014	0.280	0.333	0.143	3.385	0.034

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	52	47	46	46	49	46	46
N.S.	1	1.00	2.74	2.47	2.42	2.42	2.58	2.42	2.42
time (sec)	N/A	0.004	0.003	0.014	0.381	0.367	0.157	3.622	0.034

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	48	48	51	48	48
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.017	0.005	0.015	0.306	0.343	0.161	4.028	4.840

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	56	47	48	48	51	48	48
N.S.	1	1.00	1.40	1.18	1.20	1.20	1.28	1.20	1.20
time (sec)	N/A	0.017	0.003	0.016	0.337	0.311	0.173	4.025	4.218

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	48	48	51	48	48
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.017	0.006	0.017	0.300	0.319	0.173	4.022	0.037

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	48	48	51	48	48
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.023	0.003	0.017	0.300	0.320	0.189	4.183	4.329

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	48	48	51	48	48
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86
time (sec)	N/A	0.017	0.005	0.018	0.426	0.326	0.190	3.902	4.351

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	68	68	80	68	68
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83
time (sec)	N/A	0.030	0.003	0.053	0.541	0.358	0.013	3.319	0.033

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	82	69	68	68	78	68	68
N.S.	1	1.00	1.14	0.96	0.94	0.94	1.08	0.94	0.94
time (sec)	N/A	0.078	0.002	0.053	0.313	0.323	0.016	3.353	0.031

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	67	67	76	67	67
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.96	0.85	0.85
time (sec)	N/A	0.025	0.002	0.053	0.348	0.341	0.013	4.275	0.031

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	82	69	68	68	80	68	68
N.S.	1	1.00	1.55	1.30	1.28	1.28	1.51	1.28	1.28
time (sec)	N/A	0.052	0.002	0.052	0.314	0.346	0.016	4.581	0.033

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	68	68	80	68	68
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83
time (sec)	N/A	0.025	0.002	0.052	0.298	0.333	0.012	4.126	0.033

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	77	68	67	67	75	67	67
N.S.	1	1.00	2.26	2.00	1.97	1.97	2.21	1.97	1.97
time (sec)	N/A	0.029	0.002	0.054	0.289	0.338	0.016	3.952	0.032

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	68	68	80	68	68
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.83
time (sec)	N/A	0.026	0.002	0.053	0.292	0.359	0.012	5.430	0.032

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	69	68	68	78	68	68
N.S.	1	1.00	1.00	4.31	4.25	4.25	4.88	4.25	4.25
time (sec)	N/A	0.003	0.002	0.050	0.299	0.331	0.016	4.928	0.031

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	66	100	65	73	65	65
N.S.	1	1.00	1.00	0.90	1.37	0.89	1.00	0.89	0.89
time (sec)	N/A	0.023	0.001	0.010	0.353	0.358	0.012	6.163	0.030

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	67	69	66	76	69	66
N.S.	1	1.00	1.00	0.88	0.91	0.87	1.00	0.91	0.87
time (sec)	N/A	0.034	0.004	0.060	0.356	0.333	0.042	4.006	0.036

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	66	70	70	66	66
N.S.	1	1.00	1.00	0.93	0.92	0.97	0.97	0.92	0.92
time (sec)	N/A	0.027	0.006	0.093	0.292	0.314	0.040	7.552	0.034

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	68	69	72	76	79	67
N.S.	1	1.00	1.00	0.88	0.90	0.94	0.99	1.03	0.87
time (sec)	N/A	0.038	0.006	0.022	0.303	0.342	0.058	11.147	0.039

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	67	67	70	75	67	69
N.S.	1	1.00	1.00	0.91	0.91	0.95	1.01	0.91	0.93
time (sec)	N/A	0.025	0.009	0.020	0.357	0.326	0.064	4.087	0.032

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	69	71	73	80	69
N.S.	1	1.00	1.00	0.93	0.96	0.99	1.01	1.11	0.96
time (sec)	N/A	0.036	0.004	0.039	0.290	0.346	0.087	3.289	0.036

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	67	70	73	67	69
N.S.	1	1.00	1.00	0.93	0.93	0.97	1.01	0.93	0.96
time (sec)	N/A	0.026	0.005	0.027	0.295	0.331	0.090	3.566	0.033

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	70	71	76	81	70
N.S.	1	1.00	1.00	0.86	0.89	0.90	0.96	1.03	0.89
time (sec)	N/A	0.036	0.004	0.023	0.326	0.331	0.122	4.319	4.344

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	69	70	73	69	69
N.S.	1	1.00	1.00	0.93	0.96	0.97	1.01	0.96	0.96
time (sec)	N/A	0.025	0.007	0.018	0.291	0.362	0.128	3.460	0.055

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	68	70	72	73	81	69
N.S.	1	1.00	1.00	0.93	0.96	0.99	1.00	1.11	0.95
time (sec)	N/A	0.035	0.006	0.031	0.301	0.406	0.163	3.563	0.047

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	67	69	70	73	69	70
N.S.	1	1.00	1.00	0.91	0.93	0.95	0.99	0.93	0.95
time (sec)	N/A	0.026	0.007	0.019	0.301	0.336	0.164	4.199	0.053

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	68	72	72	75	81	70
N.S.	1	1.00	1.00	0.88	0.94	0.94	0.97	1.05	0.91
time (sec)	N/A	0.033	0.004	0.025	0.298	0.333	0.208	3.896	4.403

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	66	68	70	71	68	68
N.S.	1	1.00	1.00	0.93	0.96	0.99	1.00	0.96	0.96
time (sec)	N/A	0.025	0.004	0.016	0.280	0.347	0.206	4.035	4.300

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	67	72	72	73	80	69
N.S.	1	1.00	1.00	0.88	0.95	0.95	0.96	1.05	0.91
time (sec)	N/A	0.032	0.004	0.018	0.325	0.346	0.256	5.019	0.065

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	69	70	70	75	70	69
N.S.	1	1.00	1.00	0.91	0.92	0.92	0.99	0.92	0.91
time (sec)	N/A	0.026	0.007	0.020	0.376	0.320	0.241	4.093	0.053

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	82	69	68	68	73	68	70
N.S.	1	1.00	4.32	3.63	3.58	3.58	3.84	3.58	3.68
time (sec)	N/A	0.004	0.006	0.019	0.367	0.361	0.268	4.037	4.357

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	70	70	75	70	70
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.91	0.85	0.85
time (sec)	N/A	0.026	0.007	0.039	0.277	0.351	0.258	3.733	0.051

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	78	69	70	70	75	70	69
N.S.	1	1.00	1.95	1.72	1.75	1.75	1.88	1.75	1.72
time (sec)	N/A	0.017	0.004	0.019	0.294	0.344	0.297	4.023	4.366

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	70	70	75	70	70
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.91	0.85	0.85
time (sec)	N/A	0.026	0.005	0.021	0.294	0.348	0.284	4.494	0.047

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	82	69	70	70	75	70	70
N.S.	1	1.00	1.32	1.11	1.13	1.13	1.21	1.13	1.13
time (sec)	N/A	0.027	0.004	0.021	0.295	0.318	0.309	4.671	4.313

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	69	70	70	75	70	69
N.S.	1	1.00	1.00	0.86	0.88	0.88	0.94	0.88	0.86
time (sec)	N/A	0.026	0.007	0.020	0.281	0.317	0.300	4.250	0.053

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	69	70	70	75	70	70
N.S.	1	1.00	0.98	0.82	0.83	0.83	0.89	0.83	0.83
time (sec)	N/A	0.037	0.006	0.030	0.294	0.346	0.327	3.870	0.053

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	70	70	75	70	70
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.91	0.85	0.85
time (sec)	N/A	0.025	0.007	0.022	0.276	0.342	0.314	3.876	0.053

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	75	77	93	80	92	79
N.S.	1	1.00	0.87	0.90	0.93	1.12	0.96	1.11	0.95
time (sec)	N/A	0.054	0.016	0.027	0.282	0.336	0.126	5.359	4.361

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	64	65	81	66	80	68
N.S.	1	1.00	0.86	0.91	0.93	1.16	0.94	1.14	0.97
time (sec)	N/A	0.044	0.016	0.025	0.271	0.344	0.116	4.423	0.044

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	53	54	70	53	67	57
N.S.	1	1.00	0.86	0.93	0.95	1.23	0.93	1.18	1.00
time (sec)	N/A	0.035	0.012	0.033	0.277	0.357	0.108	5.229	0.055

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	41	43	56	39	49	45
N.S.	1	1.00	0.86	0.93	0.98	1.27	0.89	1.11	1.02
time (sec)	N/A	0.027	0.013	0.020	0.283	0.348	0.094	6.365	0.052

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	32	35	29	30	29
N.S.	1	1.00	0.82	0.91	0.97	1.06	0.88	0.91	0.88
time (sec)	N/A	0.020	0.007	0.037	0.290	0.335	0.073	2.601	0.051

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	15	15	14	14
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.94	0.88	0.88
time (sec)	N/A	0.004	0.002	0.013	0.275	0.340	0.055	2.987	4.325

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	42	37	47	34	47	34
N.S.	1	1.00	0.87	1.11	0.97	1.24	0.89	1.24	0.89
time (sec)	N/A	0.027	0.011	0.027	0.274	0.352	0.130	3.558	4.388

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	55	52	73	51	51	51
N.S.	1	1.00	0.84	1.12	1.06	1.49	1.04	1.04	1.04
time (sec)	N/A	0.031	0.027	0.030	0.285	0.325	0.165	3.391	0.077

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	65	70	90	68	86	67
N.S.	1	1.00	0.86	0.98	1.06	1.36	1.03	1.30	1.02
time (sec)	N/A	0.036	0.039	0.031	0.291	0.331	0.199	4.051	0.072

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	76	82	212	134	84	77
N.S.	1	1.00	0.89	0.83	0.89	2.30	1.46	0.91	0.84
time (sec)	N/A	0.035	0.038	0.036	0.493	0.367	0.144	4.678	0.044

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	65	71	190	124	73	66
N.S.	1	1.00	0.90	0.82	0.90	2.41	1.57	0.92	0.84
time (sec)	N/A	0.030	0.033	0.055	0.491	0.332	0.133	4.652	4.274

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	54	59	164	107	61	56
N.S.	1	1.00	0.91	0.82	0.89	2.48	1.62	0.92	0.85
time (sec)	N/A	0.025	0.031	0.030	0.502	0.365	0.124	4.475	0.064

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	42	45	136	83	42	43
N.S.	1	1.00	0.93	0.76	0.82	2.47	1.51	0.76	0.78
time (sec)	N/A	0.016	0.023	0.027	0.521	0.367	0.110	4.457	4.286

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	36	120	78	35	33
N.S.	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	0.73
time (sec)	N/A	0.011	0.015	0.029	0.512	0.342	0.084	7.087	0.046

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.012	0.017	0.024	0.505	0.355	0.088	4.115	0.043

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	45	49	136	92	47	44
N.S.	1	1.00	0.95	0.79	0.86	2.39	1.61	0.82	0.77
time (sec)	N/A	0.018	0.027	0.033	0.542	0.350	0.128	3.950	4.489

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	55	64	172	114	59	58
N.S.	1	1.00	0.99	0.81	0.94	2.53	1.68	0.87	0.85
time (sec)	N/A	0.024	0.026	0.037	0.518	0.377	0.158	3.972	4.431

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	67	75	198	126	70	70
N.S.	1	1.00	0.99	0.83	0.93	2.44	1.56	0.86	0.86
time (sec)	N/A	0.031	0.033	0.040	0.508	0.361	0.183	3.422	4.713

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	78	90	99	137	100	91	98
N.S.	1	1.00	0.86	0.99	1.09	1.51	1.10	1.00	1.08
time (sec)	N/A	0.063	0.022	0.056	0.285	0.321	0.275	4.543	4.483

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	59	79	88	124	90	73	88
N.S.	1	1.00	0.77	1.03	1.14	1.61	1.17	0.95	1.14
time (sec)	N/A	0.050	0.035	0.034	0.286	0.328	0.253	3.589	4.506

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	50	64	77	102	76	53	75
N.S.	1	1.00	0.70	0.90	1.08	1.44	1.07	0.75	1.06
time (sec)	N/A	0.043	0.014	0.049	0.286	0.357	0.208	3.935	4.327

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	35	48	58	58	60	33	60
N.S.	1	1.00	1.84	2.53	3.05	3.05	3.16	1.74	3.16
time (sec)	N/A	0.005	0.010	0.027	0.331	0.338	0.175	4.019	4.287

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	47	47	48	22	48
N.S.	1	1.00	0.71	0.91	1.38	1.38	1.41	0.65	1.41
time (sec)	N/A	0.021	0.006	0.027	0.298	0.348	0.156	3.751	4.229

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	37	37	39	14	39
N.S.	1	1.00	1.00	0.94	2.31	2.31	2.44	0.88	2.44
time (sec)	N/A	0.004	0.002	0.023	0.324	0.316	0.142	2.815	4.284

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	76	82	134	80	70	78
N.S.	1	1.00	0.77	1.09	1.17	1.91	1.14	1.00	1.11
time (sec)	N/A	0.047	0.029	0.039	0.297	0.352	0.246	2.789	4.467

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	89	99	163	102	93	97
N.S.	1	1.00	0.83	1.06	1.18	1.94	1.21	1.11	1.15
time (sec)	N/A	0.058	0.047	0.043	0.295	0.346	0.295	3.945	0.152

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	85	100	114	178	116	108	111
N.S.	1	1.00	0.84	0.99	1.13	1.76	1.15	1.07	1.10
time (sec)	N/A	0.069	0.040	0.048	0.301	0.338	0.332	5.888	4.664

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	99	85	116	322	172	96	109
N.S.	1	1.00	0.85	0.73	0.99	2.75	1.47	0.82	0.93
time (sec)	N/A	0.048	0.041	0.046	0.520	0.353	0.302	4.521	0.062

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	89	74	104	296	156	84	99
N.S.	1	1.00	0.86	0.71	1.00	2.85	1.50	0.81	0.95
time (sec)	N/A	0.042	0.032	0.039	0.519	0.372	0.287	4.020	4.359

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	77	62	90	268	131	65	86
N.S.	1	1.00	0.83	0.67	0.97	2.88	1.41	0.70	0.92
time (sec)	N/A	0.032	0.031	0.039	0.537	0.331	0.263	4.373	0.103

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	66	58	81	254	134	56	78
N.S.	1	1.00	0.80	0.70	0.98	3.06	1.61	0.67	0.94
time (sec)	N/A	0.026	0.027	0.040	0.505	0.345	0.220	3.978	4.388

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	69	58	87	258	143	62	75
N.S.	1	1.00	0.82	0.69	1.04	3.07	1.70	0.74	0.89
time (sec)	N/A	0.026	0.031	0.056	0.543	0.367	0.194	4.058	4.349

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	69	58	87	258	139	62	74
N.S.	1	1.00	0.81	0.68	1.02	3.04	1.64	0.73	0.87
time (sec)	N/A	0.026	0.027	0.040	0.490	0.338	0.188	3.856	4.313

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	78	80	254	129	56	77
N.S.	1	1.00	0.84	0.99	1.01	3.22	1.63	0.71	0.97
time (sec)	N/A	0.024	0.025	0.035	0.488	0.343	0.193	4.098	4.356

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	65	93	268	139	68	88
N.S.	1	1.00	0.83	0.68	0.98	2.82	1.46	0.72	0.93
time (sec)	N/A	0.036	0.030	0.044	0.541	0.361	0.261	3.719	4.438

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	91	75	108	304	162	82	102
N.S.	1	1.00	0.86	0.71	1.02	2.87	1.53	0.77	0.96
time (sec)	N/A	0.042	0.034	0.046	0.530	0.358	0.327	3.657	4.449

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	101	87	119	330	173	93	114
N.S.	1	1.00	0.85	0.73	1.00	2.77	1.45	0.78	0.96
time (sec)	N/A	0.050	0.037	0.051	0.512	0.362	0.325	3.971	4.465

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	124	143	203	150	113	142
N.S.	1	1.00	0.86	0.93	1.08	1.53	1.13	0.85	1.07
time (sec)	N/A	0.097	0.018	0.075	0.284	0.355	0.477	4.133	0.130

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	101	113	132	190	138	95	132
N.S.	1	1.00	0.86	0.96	1.12	1.61	1.17	0.81	1.12
time (sec)	N/A	0.079	0.020	0.052	0.291	0.378	0.453	3.239	4.600

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	72	98	121	168	124	75	119
N.S.	1	1.00	0.66	0.90	1.11	1.54	1.14	0.69	1.09
time (sec)	N/A	0.068	0.017	0.063	0.279	0.327	0.374	3.051	4.370

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	57	81	102	102	107	55	104
N.S.	1	1.00	3.00	4.26	5.37	5.37	5.63	2.89	5.47
time (sec)	N/A	0.005	0.012	0.044	0.285	0.346	0.324	3.866	4.446

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	46	65	91	91	95	44	93
N.S.	1	1.00	1.18	1.67	2.33	2.33	2.44	1.13	2.38
time (sec)	N/A	0.018	0.011	0.045	0.320	0.343	0.291	3.834	0.054

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	48	80	80	83	33	81
N.S.	1	1.00	0.66	0.91	1.51	1.51	1.57	0.62	1.53
time (sec)	N/A	0.032	0.010	0.041	0.278	0.323	0.264	4.110	4.618

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	69	69	71	22	70
N.S.	1	1.00	0.71	0.91	2.03	2.03	2.09	0.65	2.06
time (sec)	N/A	0.021	0.007	0.043	0.283	0.320	0.248	3.791	4.478

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	59	59	63	14	61
N.S.	1	1.00	1.00	0.94	3.69	3.69	3.94	0.88	3.81
time (sec)	N/A	0.004	0.002	0.039	0.290	0.360	0.232	5.521	0.058

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	76	110	126	222	128	92	122
N.S.	1	1.00	0.75	1.08	1.24	2.18	1.25	0.90	1.20
time (sec)	N/A	0.069	0.042	0.059	0.283	0.339	0.365	3.585	0.238

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	92	123	143	251	150	115	141
N.S.	1	1.00	0.79	1.06	1.23	2.16	1.29	0.99	1.22
time (sec)	N/A	0.086	0.059	0.066	0.295	0.380	0.427	3.571	4.677

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	107	134	158	266	165	130	155
N.S.	1	1.00	0.76	0.96	1.13	1.90	1.18	0.93	1.11
time (sec)	N/A	0.099	0.042	0.072	0.289	0.380	0.469	3.314	4.912

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	122	107	159	454	218	117	153
N.S.	1	1.00	0.79	0.69	1.03	2.93	1.41	0.75	0.99
time (sec)	N/A	0.069	0.044	0.060	0.501	0.347	0.491	2.856	0.106

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	111	96	148	428	204	106	143
N.S.	1	1.00	0.78	0.68	1.04	3.01	1.44	0.75	1.01
time (sec)	N/A	0.061	0.040	0.056	0.496	0.338	0.477	3.000	4.520

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	100	84	134	400	178	87	130
N.S.	1	1.00	0.76	0.64	1.02	3.05	1.36	0.66	0.99
time (sec)	N/A	0.050	0.035	0.055	0.492	0.360	0.434	3.589	0.158

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	88	80	125	386	182	78	122
N.S.	1	1.00	0.73	0.66	1.03	3.19	1.50	0.64	1.01
time (sec)	N/A	0.047	0.035	0.056	0.494	0.429	0.363	4.698	4.522

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	91	80	131	390	194	84	119
N.S.	1	1.00	0.75	0.66	1.07	3.20	1.59	0.69	0.98
time (sec)	N/A	0.046	0.040	0.053	0.487	0.354	0.337	5.479	4.421

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	91	78	133	390	196	84	117
N.S.	1	1.00	0.74	0.63	1.08	3.17	1.59	0.68	0.95
time (sec)	N/A	0.048	0.041	0.056	0.488	0.345	0.320	3.460	4.504

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	91	78	133	390	196	84	116
N.S.	1	1.00	0.73	0.63	1.07	3.15	1.58	0.68	0.94
time (sec)	N/A	0.047	0.035	0.052	0.498	0.335	0.297	3.473	4.468

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	91	80	131	390	190	84	118
N.S.	1	1.00	0.73	0.64	1.05	3.12	1.52	0.67	0.94
time (sec)	N/A	0.046	0.036	0.052	0.501	0.357	0.290	3.986	4.482

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	89	120	124	386	177	78	121
N.S.	1	1.00	0.79	1.06	1.10	3.42	1.57	0.69	1.07
time (sec)	N/A	0.042	0.032	0.066	0.509	0.351	0.297	3.834	4.707

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	101	87	137	400	187	90	132
N.S.	1	1.00	0.76	0.65	1.03	3.01	1.41	0.68	0.99
time (sec)	N/A	0.056	0.039	0.059	0.501	0.351	0.391	3.773	4.581

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	113	97	152	436	209	104	146
N.S.	1	1.00	0.78	0.67	1.06	3.03	1.45	0.72	1.01
time (sec)	N/A	0.068	0.042	0.065	0.507	0.361	0.414	4.100	4.623

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	123	109	163	462	221	115	158
N.S.	1	1.00	0.78	0.69	1.04	2.94	1.41	0.73	1.01
time (sec)	N/A	0.076	0.043	0.072	0.519	0.437	0.450	4.221	4.648

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	15	19	12	15	16
N.S.	1	1.00	0.84	0.84	0.79	1.00	0.63	0.79	0.84
time (sec)	N/A	0.002	0.005	0.011	0.487	0.322	0.031	4.098	0.034

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	11
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	1.00
time (sec)	N/A	0.002	0.001	0.021	0.296	0.365	0.018	3.882	0.018

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89
time (sec)	N/A	0.003	0.006	0.015	0.523	0.379	0.029	3.691	0.027

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	18	23	15	18	18
N.S.	1	1.00	0.82	0.86	0.82	1.05	0.68	0.82	0.82
time (sec)	N/A	0.007	0.005	0.012	0.283	0.362	0.024	3.862	0.035

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	9	8	9	11
N.S.	1	1.00	0.85	0.77	0.69	0.69	0.62	0.69	0.85
time (sec)	N/A	0.001	0.001	0.029	0.281	0.330	0.021	4.103	0.046

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	19	18	23	14	19	18
N.S.	1	1.00	0.83	0.79	0.75	0.96	0.58	0.79	0.75
time (sec)	N/A	0.011	0.005	0.013	0.291	0.324	0.024	3.882	4.228

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	71
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	0.90
time (sec)	N/A	0.042	0.010	0.049	0.285	0.344	0.010	3.598	4.455

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	39	36	13	13	12	23	59
N.S.	1	1.00	0.58	0.54	0.19	0.19	0.18	0.34	0.88
time (sec)	N/A	0.034	0.005	0.054	0.304	0.346	0.010	3.115	4.310

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	24	14	13	12	22	33
N.S.	1	1.00	1.06	0.67	0.39	0.36	0.33	0.61	0.92
time (sec)	N/A	0.018	0.005	0.036	0.296	0.358	0.011	3.131	4.352

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	37	34	14	11	10	30	109
N.S.	1	1.00	0.49	0.45	0.19	0.15	0.13	0.40	1.45
time (sec)	N/A	0.014	0.007	0.020	0.288	0.334	0.023	3.307	4.394

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	39	38	14	17	10	45	112
N.S.	1	1.00	0.52	0.51	0.19	0.23	0.13	0.60	1.49
time (sec)	N/A	0.015	0.008	0.018	0.284	0.346	0.036	3.603	4.450

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	34	13	13	14	30	33
N.S.	1	1.00	0.95	0.87	0.33	0.33	0.36	0.77	0.85
time (sec)	N/A	0.025	0.006	0.020	0.286	0.336	0.040	4.262	4.212

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	39	36	15	15	15	31	35
N.S.	1	1.00	0.54	0.50	0.21	0.21	0.21	0.43	0.49
time (sec)	N/A	0.010	0.006	0.019	0.281	0.317	0.046	4.319	4.238

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.039	0.006	0.020	0.309	0.318	0.055	4.311	4.242

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.039	0.006	0.020	0.281	0.365	0.063	4.524	4.215

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	-1
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01
time (sec)	N/A	0.016	0.005	0.047	0.273	0.355	0.010	5.978	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	-1
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01
time (sec)	N/A	0.015	0.005	0.046	0.276	0.349	0.011	3.657	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	36	33	10	10	8	20	-1
N.S.	1	1.00	0.49	0.45	0.14	0.14	0.11	0.27	-0.01
time (sec)	N/A	0.009	0.005	0.014	0.282	0.391	0.009	6.428	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	35	34	10	13	5	26	-1
N.S.	1	1.00	0.49	0.47	0.14	0.18	0.07	0.36	-0.01
time (sec)	N/A	0.014	0.007	0.033	0.282	0.345	0.022	4.023	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	37	34	13	13	14	30	33
N.S.	1	1.00	0.48	0.44	0.17	0.17	0.18	0.39	0.43
time (sec)	N/A	0.015	0.006	0.018	0.289	0.344	0.039	3.963	4.244

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.015	0.005	0.019	0.288	0.407	0.046	4.223	4.211

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.016	0.006	0.018	0.272	0.336	0.054	4.213	4.184

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.015	0.006	0.020	0.284	0.371	0.062	4.025	4.195

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.077	0.013	0.047	0.294	0.339	0.000	3.899	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.077	0.011	0.046	0.275	0.375	0.000	3.760	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	119	61	58	35	35	0	67	-1
N.S.	1	1.12	0.58	0.55	0.33	0.33	0.00	0.63	-0.01
time (sec)	N/A	0.056	0.011	0.060	0.281	0.345	0.000	3.893	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	61	58	35	35	0	45	46
N.S.	1	1.00	0.91	0.87	0.52	0.52	0.00	0.67	0.69
time (sec)	N/A	0.035	0.011	0.038	0.322	0.353	0.000	4.209	4.286

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	24	35	35	0	44	36
N.S.	1	1.00	0.75	0.67	0.97	0.97	0.00	1.22	1.00
time (sec)	N/A	0.018	0.008	0.045	0.305	0.323	0.000	3.646	4.253

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	60	57	33	33	0	68	-1
N.S.	1	1.00	0.37	0.35	0.20	0.20	0.00	0.42	-0.01
time (sec)	N/A	0.031	0.014	0.021	0.288	0.332	0.000	3.437	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	62	59	34	38	0	87	-1
N.S.	1	1.00	0.38	0.36	0.21	0.23	0.00	0.53	-0.01
time (sec)	N/A	0.033	0.015	0.035	0.288	0.381	0.000	4.037	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	61	60	34	39	0	87	-1
N.S.	1	1.00	0.37	0.37	0.21	0.24	0.00	0.53	-0.01
time (sec)	N/A	0.032	0.011	0.024	0.286	0.357	0.000	3.604	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	63	60	33	39	0	87	-1
N.S.	1	1.00	0.39	0.37	0.20	0.24	0.00	0.53	-0.01
time (sec)	N/A	0.033	0.015	0.023	0.297	0.355	0.000	4.336	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	59	56	35	35	0	68	151
N.S.	1	1.00	1.44	1.37	0.85	0.85	0.00	1.66	3.68
time (sec)	N/A	0.026	0.011	0.023	0.283	0.370	0.000	4.012	4.243

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	61	58	35	37	0	69	151
N.S.	1	1.00	0.85	0.81	0.49	0.51	0.00	0.96	2.10
time (sec)	N/A	0.011	0.010	0.023	0.300	0.349	0.000	4.247	4.203

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.073	0.010	0.023	0.292	0.364	0.000	3.752	4.207

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.074	0.012	0.025	0.279	0.415	0.000	4.186	4.212

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.071	0.010	0.025	0.290	0.363	0.000	3.937	4.234

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.029	0.011	0.046	0.293	0.424	0.000	4.036	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.028	0.011	0.047	0.287	0.365	0.000	4.372	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.029	0.011	0.047	0.280	0.379	0.000	3.136	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.028	0.008	0.047	0.286	0.342	0.000	2.884	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	59	56	31	31	0	63	-1
N.S.	1	1.00	0.37	0.35	0.19	0.19	0.00	0.40	-0.01
time (sec)	N/A	0.022	0.009	0.016	0.304	0.371	0.000	4.071	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	60	58	32	36	0	64	-1
N.S.	1	1.00	0.38	0.37	0.20	0.23	0.00	0.41	-0.01
time (sec)	N/A	0.027	0.011	0.022	0.281	0.407	0.000	4.973	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	59	56	33	36	0	67	-1
N.S.	1	1.00	0.37	0.35	0.20	0.22	0.00	0.42	-0.01
time (sec)	N/A	0.027	0.010	0.023	0.271	0.356	0.000	3.096	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	59	56	32	37	0	66	-1
N.S.	1	1.00	0.37	0.35	0.20	0.23	0.00	0.42	-0.01
time (sec)	N/A	0.027	0.010	0.022	0.294	0.368	0.000	3.118	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	35	37	0	69	151
N.S.	1	1.00	0.37	0.36	0.21	0.23	0.00	0.42	0.93
time (sec)	N/A	0.029	0.009	0.021	0.277	0.350	0.000	4.840	4.255

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.032	0.009	0.021	0.280	0.325	0.000	6.164	4.264

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.028	0.012	0.021	0.279	0.335	0.000	4.103	4.549

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.029	0.010	0.023	0.324	0.331	0.000	5.692	4.630

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.21	0.22	0.00	0.41	0.90
time (sec)	N/A	0.029	0.010	0.023	0.299	0.327	0.000	4.959	4.297

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.113	0.017	0.048	0.279	0.332	0.000	4.410	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.108	0.014	0.047	0.312	0.324	0.000	4.051	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	83	80	57	57	0	105	-1
N.S.	1	1.00	0.41	0.40	0.28	0.28	0.00	0.52	-0.00
time (sec)	N/A	0.091	0.014	0.048	0.331	0.328	0.000	4.596	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	83	80	57	57	0	105	-1
N.S.	1	1.00	0.52	0.50	0.36	0.36	0.00	0.66	-0.01
time (sec)	N/A	0.081	0.014	0.049	0.297	0.350	0.000	4.103	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	83	80	56	56	0	104	-1
N.S.	1	1.00	0.70	0.67	0.47	0.47	0.00	0.87	-0.01
time (sec)	N/A	0.065	0.014	0.046	0.287	0.340	0.000	4.214	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	83	80	56	56	0	67	-1
N.S.	1	1.00	1.24	1.19	0.84	0.84	0.00	1.00	-0.01
time (sec)	N/A	0.034	0.014	0.037	0.281	0.344	0.000	3.324	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	24	57	57	0	66	36
N.S.	1	1.00	0.75	0.67	1.58	1.58	0.00	1.83	1.00
time (sec)	N/A	0.018	0.009	0.037	0.294	0.346	0.000	3.458	4.404

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	82	79	55	55	0	106	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.42	-0.00
time (sec)	N/A	0.044	0.017	0.023	0.286	0.342	0.000	3.709	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	56	61	0	125	-1
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.050	0.017	0.026	0.276	0.352	0.000	3.916	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	56	61	0	127	-1
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.51	-0.00
time (sec)	N/A	0.048	0.018	0.026	0.300	0.334	0.000	4.319	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	56	61	0	128	-1
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.51	-0.00
time (sec)	N/A	0.050	0.017	0.037	0.289	0.327	0.000	2.908	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	56	61	0	126	-1
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.046	0.014	0.026	0.298	0.327	0.000	3.978	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	85	82	55	61	0	125	-1
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.046	0.019	0.029	0.282	0.336	0.000	4.186	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	81	78	57	57	0	106	231
N.S.	1	1.00	1.98	1.90	1.39	1.39	0.00	2.59	5.63
time (sec)	N/A	0.027	0.012	0.036	0.275	0.363	0.000	4.481	4.178

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	80	57	59	0	107	231
N.S.	1	1.00	1.15	1.11	0.79	0.82	0.00	1.49	3.21
time (sec)	N/A	0.011	0.013	0.023	0.280	0.324	0.000	4.476	4.219

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	83	80	57	59	0	107	231
N.S.	1	1.00	0.65	0.62	0.45	0.46	0.00	0.84	1.80
time (sec)	N/A	0.062	0.012	0.025	0.274	0.370	0.000	4.048	4.235

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.104	0.012	0.025	0.274	0.332	0.000	4.973	4.266

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.109	0.012	0.028	0.277	0.336	0.000	4.811	4.225

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.107	0.013	0.026	0.277	0.355	0.000	3.437	4.233

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.107	0.014	0.028	0.270	0.358	0.000	3.475	4.223

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.040	0.016	0.048	0.286	0.326	0.000	4.011	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.041	0.014	0.048	0.286	0.353	0.000	4.027	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.039	0.014	0.046	0.284	0.323	0.000	3.314	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.041	0.014	0.046	0.275	0.343	0.000	3.002	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.041	0.014	0.046	0.276	0.327	0.000	3.753	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	56	56	0	104	-1
N.S.	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.041	0.015	0.045	0.280	0.340	0.000	3.915	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	81	78	54	54	0	102	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.034	0.012	0.017	0.279	0.320	0.000	3.974	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	55	59	0	103	-1
N.S.	1	1.00	0.34	0.32	0.22	0.24	0.00	0.42	-0.00
time (sec)	N/A	0.040	0.014	0.023	0.310	0.314	0.000	3.490	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	83	80	54	59	0	104	-1
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.42	-0.00
time (sec)	N/A	0.039	0.016	0.026	0.274	0.332	0.000	5.899	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	83	80	55	59	0	106	-1
N.S.	1	1.00	0.33	0.32	0.22	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.039	0.014	0.030	0.277	0.346	0.000	4.615	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	55	59	0	106	-1
N.S.	1	1.00	0.34	0.32	0.22	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.039	0.012	0.023	0.290	0.337	0.000	5.293	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	83	80	54	59	0	105	-1
N.S.	1	1.00	0.34	0.33	0.22	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.039	0.013	0.023	0.289	0.359	0.000	6.077	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	57	59	0	107	231
N.S.	1	1.00	0.33	0.32	0.23	0.24	0.00	0.43	0.92
time (sec)	N/A	0.040	0.012	0.024	0.279	0.349	0.000	5.071	4.221

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	57	59	0	107	231
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.040	0.012	0.023	0.290	0.335	0.000	3.931	4.350

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.040	0.012	0.026	0.275	0.333	0.000	3.289	4.208

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.040	0.012	0.024	0.288	0.358	0.000	4.424	4.315

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.041	0.014	0.025	0.282	0.324	0.000	3.199	4.267

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.040	0.012	0.025	0.271	0.349	0.000	3.962	4.337

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.22	0.23	0.00	0.42	0.91
time (sec)	N/A	0.040	0.013	0.029	0.281	0.352	0.000	5.979	4.311

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	55	52	34	33	32	59	-1
N.S.	1	1.00	0.43	0.41	0.27	0.26	0.25	0.46	-0.01
time (sec)	N/A	0.071	0.018	0.124	0.285	0.328	0.055	4.357	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	44	41	23	22	20	33	64
N.S.	1	1.00	0.59	0.55	0.31	0.29	0.27	0.44	0.85
time (sec)	N/A	0.038	0.009	0.122	0.271	0.342	0.048	5.992	4.519

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	35	32	13	13	10	22	33
N.S.	1	1.00	0.80	0.73	0.30	0.30	0.23	0.50	0.75
time (sec)	N/A	0.022	0.006	0.121	0.274	0.322	0.035	5.400	4.415

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	42	37	23	18	15	33	40
N.S.	1	1.00	0.52	0.46	0.29	0.22	0.19	0.41	0.50
time (sec)	N/A	0.023	0.008	0.128	0.291	0.357	0.089	4.732	4.451

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	122	54	52	33	33	31	52	75
N.S.	1	0.98	0.43	0.42	0.26	0.26	0.25	0.42	0.60
time (sec)	N/A	0.034	0.011	0.138	0.281	0.361	0.124	4.033	4.454

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	66	64	37	99	80	64	-1
N.S.	1	1.00	0.51	0.50	0.29	0.77	0.62	0.50	-0.01
time (sec)	N/A	0.031	0.018	0.141	0.498	0.343	0.066	6.241	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	54	48	26	82	56	42	-1
N.S.	1	1.00	0.61	0.54	0.29	0.92	0.63	0.47	-0.01
time (sec)	N/A	0.021	0.010	0.126	0.504	0.357	0.056	4.955	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	34	15	67	53	23	-1
N.S.	1	1.00	0.83	0.64	0.28	1.26	1.00	0.43	-0.02
time (sec)	N/A	0.010	0.009	0.149	0.495	0.355	0.050	3.687	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	56	50	29	82	65	37	-1
N.S.	1	1.00	0.61	0.54	0.32	0.89	0.71	0.40	-0.01
time (sec)	N/A	0.021	0.010	0.132	0.494	0.352	0.074	3.341	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	130	70	68	40	106	87	50	-1
N.S.	1	0.98	0.53	0.51	0.30	0.80	0.65	0.38	-0.01
time (sec)	N/A	0.030	0.018	0.129	0.532	0.370	0.097	4.670	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	81	103	66	91	0	92	-1
N.S.	1	1.00	0.51	0.65	0.42	0.58	0.00	0.58	-0.01
time (sec)	N/A	0.088	0.022	0.029	0.287	0.364	0.000	3.679	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	61	81	55	69	0	62	-1
N.S.	1	1.00	0.54	0.72	0.49	0.61	0.00	0.55	-0.01
time (sec)	N/A	0.068	0.016	0.048	0.292	0.316	0.000	4.287	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	69	39	32	36	36	0	32	42
N.S.	1	1.68	0.95	0.78	0.88	0.88	0.00	0.78	1.02
time (sec)	N/A	0.034	0.009	0.024	0.291	0.349	0.000	4.633	4.244

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	26	26	0	24	34
N.S.	1	1.00	0.71	0.63	0.68	0.68	0.00	0.63	0.89
time (sec)	N/A	0.017	0.006	0.017	0.284	0.331	0.000	4.249	4.343

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	74	107	57	90	0	89	-1
N.S.	1	1.00	0.50	0.73	0.39	0.61	0.00	0.61	-0.01
time (sec)	N/A	0.054	0.020	0.030	0.292	0.342	0.000	4.629	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	97	133	75	119	0	122	-1
N.S.	1	1.00	0.51	0.70	0.40	0.63	0.00	0.65	-0.01
time (sec)	N/A	0.063	0.027	0.054	0.280	0.341	0.000	4.490	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	84	97	59	188	0	65	-1
N.S.	1	1.00	0.66	0.76	0.46	1.47	0.00	0.51	-0.01
time (sec)	N/A	0.033	0.022	0.038	0.521	0.341	0.000	3.884	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	81	99	62	190	0	70	-1
N.S.	1	1.00	0.63	0.77	0.48	1.47	0.00	0.54	-0.01
time (sec)	N/A	0.033	0.020	0.053	0.496	0.339	0.000	4.490	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	83	97	58	188	0	65	-1
N.S.	1	1.00	0.61	0.72	0.43	1.39	0.00	0.48	-0.01
time (sec)	N/A	0.024	0.017	0.042	0.494	0.345	0.000	4.184	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	93	119	71	202	0	87	-1
N.S.	1	1.00	0.55	0.70	0.42	1.20	0.00	0.51	-0.01
time (sec)	N/A	0.041	0.023	0.034	0.502	0.379	0.000	6.020	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	105	139	86	238	0	101	-1
N.S.	1	1.00	0.50	0.67	0.41	1.14	0.00	0.48	-0.00
time (sec)	N/A	0.053	0.027	0.036	0.489	0.331	0.000	5.125	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	103	163	110	157	0	114	-1
N.S.	1	1.00	0.43	0.68	0.46	0.66	0.00	0.48	-0.00
time (sec)	N/A	0.129	0.027	0.031	0.290	0.313	0.000	3.434	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	83	141	99	135	0	84	-1
N.S.	1	1.00	0.42	0.72	0.51	0.69	0.00	0.43	-0.01
time (sec)	N/A	0.111	0.021	0.027	0.279	0.331	0.000	4.596	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	61	54	80	80	0	54	144
N.S.	1	1.00	1.49	1.32	1.95	1.95	0.00	1.32	3.51
time (sec)	N/A	0.027	0.012	0.030	0.283	0.339	0.000	3.940	4.291

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	50	43	69	69	0	43	53
N.S.	1	1.00	0.68	0.58	0.93	0.93	0.00	0.58	0.72
time (sec)	N/A	0.010	0.013	0.026	0.284	0.334	0.000	3.840	4.235

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	39	32	58	58	0	32	42
N.S.	1	1.00	0.57	0.46	0.84	0.84	0.00	0.46	0.61
time (sec)	N/A	0.034	0.010	0.025	0.279	0.350	0.000	3.604	4.260

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	48	48	0	24	34
N.S.	1	1.00	0.71	0.63	1.26	1.26	0.00	0.63	0.89
time (sec)	N/A	0.017	0.007	0.021	0.290	0.327	0.000	3.588	4.273

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	96	193	101	178	0	111	-1
N.S.	1	1.00	0.43	0.87	0.45	0.80	0.00	0.50	-0.00
time (sec)	N/A	0.081	0.030	0.033	0.283	0.370	0.000	4.127	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	119	219	119	207	0	144	-1
N.S.	1	1.00	0.45	0.82	0.45	0.78	0.00	0.54	-0.00
time (sec)	N/A	0.095	0.032	0.038	0.285	0.376	0.000	3.549	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	105	172	109	324	0	93	-1
N.S.	1	1.00	0.50	0.82	0.52	1.54	0.00	0.44	-0.00
time (sec)	N/A	0.060	0.028	0.039	0.497	0.377	0.000	3.395	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	105	172	111	324	0	93	-1
N.S.	1	1.00	0.50	0.81	0.52	1.53	0.00	0.44	-0.00
time (sec)	N/A	0.056	0.028	0.040	0.495	0.356	0.000	4.174	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	105	172	109	324	0	93	-1
N.S.	1	1.00	0.49	0.81	0.51	1.52	0.00	0.44	-0.00
time (sec)	N/A	0.056	0.024	0.037	0.493	0.350	0.000	4.262	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	105	169	102	320	0	87	-1
N.S.	1	1.00	0.49	0.79	0.48	1.50	0.00	0.41	-0.00
time (sec)	N/A	0.047	0.027	0.035	0.498	0.330	0.000	3.944	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	115	191	115	334	0	109	-1
N.S.	1	1.00	0.46	0.76	0.46	1.33	0.00	0.43	-0.00
time (sec)	N/A	0.069	0.031	0.037	0.497	0.351	0.000	3.941	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	127	211	130	370	0	123	-1
N.S.	1	1.00	0.44	0.73	0.45	1.27	0.00	0.42	-0.00
time (sec)	N/A	0.083	0.033	0.042	0.510	0.342	0.000	4.052	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	64	0	0	0	0	0	-1
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.158	4.785	0.005	0.000	0.000	0.000	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	48	0	0	0	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.086	4.647	0.003	0.000	0.000	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	51	0	0	0	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.112	5.270	0.003	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	618	618	64	0	0	0	0	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	5.753	0.005	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	64	0	0	0	0	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	5.350	0.003	0.000	0.000	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	61	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	6.045	0.003	0.000	0.000	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	42	41	40	48	48	40
N.S.	1	1.00	0.65	0.82	0.80	0.78	0.94	0.94	0.78
time (sec)	N/A	0.010	0.020	0.030	0.280	0.361	0.422	3.237	0.074

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	42	41	34	48	42	41
N.S.	1	1.00	0.65	0.82	0.80	0.67	0.94	0.82	0.80
time (sec)	N/A	0.010	0.019	0.025	0.280	0.340	0.233	3.686	4.224

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	42	41	29	48	37	41
N.S.	1	1.00	0.65	0.82	0.80	0.57	0.94	0.73	0.80
time (sec)	N/A	0.010	0.015	0.023	0.323	0.342	0.120	4.309	0.050

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	41	41	31	46	41	41
N.S.	1	1.00	0.67	0.84	0.84	0.63	0.94	0.84	0.84
time (sec)	N/A	0.009	0.016	0.024	0.337	0.347	0.220	4.517	0.045

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	42	44	34	46	51	31
N.S.	1	1.00	0.67	0.86	0.90	0.69	0.94	1.04	0.63
time (sec)	N/A	0.010	0.019	0.028	0.279	0.344	0.230	3.342	0.052

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	42	43	34	46	53	34
N.S.	1	1.00	0.67	0.86	0.88	0.69	0.94	1.08	0.69
time (sec)	N/A	0.010	0.019	0.030	0.285	0.319	0.286	3.609	4.231

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	38	42	47	34	46	48	34
N.S.	1	1.00	0.78	0.86	0.96	0.69	0.94	0.98	0.69
time (sec)	N/A	0.010	0.022	0.033	0.306	0.360	0.430	4.479	0.050

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	74	73	68	88	86	71
N.S.	1	1.00	0.60	0.81	0.80	0.75	0.97	0.95	0.78
time (sec)	N/A	0.031	0.024	0.055	0.298	0.360	0.699	3.845	4.199

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	74	73	58	88	74	71
N.S.	1	1.00	0.60	0.81	0.80	0.64	0.97	0.81	0.78
time (sec)	N/A	0.028	0.022	0.059	0.289	0.326	0.430	4.193	0.030

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	74	73	51	88	69	71
N.S.	1	1.00	0.60	0.81	0.80	0.56	0.97	0.76	0.78
time (sec)	N/A	0.029	0.020	0.055	0.268	0.329	0.238	3.446	0.029

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	73	90	53	87	73	71
N.S.	1	1.00	0.62	0.82	1.01	0.60	0.98	0.82	0.80
time (sec)	N/A	0.027	0.020	0.030	0.279	0.355	0.338	6.404	0.031

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	74	76	56	87	89	71
N.S.	1	1.00	0.62	0.83	0.85	0.63	0.98	1.00	0.80
time (sec)	N/A	0.028	0.023	0.035	0.274	0.329	0.353	2.888	0.033

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	74	76	56	87	92	71
N.S.	1	1.00	0.62	0.83	0.85	0.63	0.98	1.03	0.80
time (sec)	N/A	0.028	0.023	0.056	0.277	0.349	0.389	3.341	0.030

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	60	74	82	56	85	95	75
N.S.	1	1.00	0.69	0.85	0.94	0.64	0.98	1.09	0.86
time (sec)	N/A	0.028	0.026	0.040	0.281	0.338	0.482	3.883	0.058

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	77	105	105	96	128	124	103
N.S.	1	1.00	0.60	0.81	0.81	0.74	0.99	0.96	0.80
time (sec)	N/A	0.045	0.029	0.059	0.303	0.342	1.065	4.107	0.039

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	77	106	105	82	129	106	103
N.S.	1	1.00	0.59	0.81	0.80	0.63	0.98	0.81	0.79
time (sec)	N/A	0.042	0.026	0.059	0.299	0.328	0.694	4.291	0.037

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	77	106	105	73	129	101	103
N.S.	1	1.00	0.59	0.81	0.80	0.56	0.98	0.77	0.79
time (sec)	N/A	0.041	0.022	0.078	0.304	0.340	0.432	3.343	0.038

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	77	105	155	75	128	105	103
N.S.	1	1.00	0.60	0.81	1.20	0.58	0.99	0.81	0.80
time (sec)	N/A	0.041	0.024	0.035	0.288	0.336	0.517	3.877	0.037

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	77	105	108	78	124	127	103
N.S.	1	1.00	0.62	0.84	0.86	0.62	0.99	1.02	0.82
time (sec)	N/A	0.044	0.028	0.041	0.309	0.341	0.550	3.846	0.039

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	77	106	108	78	126	130	103
N.S.	1	1.00	0.61	0.83	0.85	0.61	0.99	1.02	0.81
time (sec)	N/A	0.041	0.028	0.041	0.287	0.361	0.612	3.749	0.038

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	82	106	114	78	126	133	107
N.S.	1	1.00	0.65	0.83	0.90	0.61	0.99	1.05	0.84
time (sec)	N/A	0.041	0.030	0.046	0.275	0.342	0.769	3.802	0.039

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	178	207	300	283	0	297	129
N.S.	1	1.00	0.56	0.66	0.95	0.90	0.00	0.94	0.41
time (sec)	N/A	0.265	0.298	0.072	0.509	0.356	0.000	4.277	4.273

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	167	188	273	283	0	277	112
N.S.	1	1.00	0.56	0.63	0.92	0.95	0.00	0.93	0.38
time (sec)	N/A	0.211	0.282	0.086	0.514	0.356	0.000	3.836	0.124

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	167	190	282	247	0	263	112
N.S.	1	1.00	0.56	0.64	0.95	0.83	0.00	0.88	0.38
time (sec)	N/A	0.202	0.270	0.065	0.491	0.344	0.000	3.522	0.120

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	154	171	256	247	0	277	92
N.S.	1	1.00	0.55	0.61	0.91	0.88	0.00	0.99	0.33
time (sec)	N/A	0.182	0.262	0.047	0.501	0.354	0.000	3.464	4.252

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	153	177	265	234	0	261	92
N.S.	1	1.00	0.54	0.63	0.94	0.83	0.00	0.93	0.33
time (sec)	N/A	0.174	0.261	0.046	0.510	0.359	0.000	4.071	4.355

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	152	180	255	232	78	264	90
N.S.	1	1.00	0.54	0.64	0.90	0.82	0.28	0.93	0.32
time (sec)	N/A	0.180	0.237	0.043	0.499	0.398	2.540	3.300	0.112

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	154	177	261	232	0	269	90
N.S.	1	1.00	0.54	0.63	0.92	0.82	0.00	0.95	0.32
time (sec)	N/A	0.172	0.245	0.046	0.508	0.368	0.000	3.292	0.105

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	165	191	268	276	0	294	102
N.S.	1	1.00	0.55	0.64	0.89	0.92	0.00	0.98	0.34
time (sec)	N/A	0.203	0.275	0.069	0.508	0.341	0.000	3.077	0.122

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	165	194	275	300	0	276	102
N.S.	1	1.00	0.55	0.65	0.92	1.00	0.00	0.92	0.34
time (sec)	N/A	0.195	0.277	0.069	0.505	0.377	0.000	3.208	4.396

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	181	206	290	323	0	307	113
N.S.	1	1.00	0.57	0.65	0.91	1.02	0.00	0.97	0.36
time (sec)	N/A	0.223	0.298	0.073	0.497	0.349	0.000	3.953	4.350

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	205	237	361	399	0	336	188
N.S.	1	1.00	0.56	0.64	0.98	1.08	0.00	0.91	0.51
time (sec)	N/A	0.277	0.568	0.105	0.523	0.398	0.000	4.017	0.129

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	194	218	334	399	0	316	171
N.S.	1	1.00	0.55	0.62	0.95	1.14	0.00	0.90	0.49
time (sec)	N/A	0.251	0.547	0.105	0.520	0.392	0.000	4.149	4.333

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	175	220	343	363	0	302	171
N.S.	1	1.00	0.50	0.63	0.98	1.04	0.00	0.86	0.49
time (sec)	N/A	0.250	0.472	0.093	0.502	0.372	0.000	3.174	4.300

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	164	203	317	370	0	314	153
N.S.	1	1.00	0.49	0.61	0.95	1.11	0.00	0.94	0.46
time (sec)	N/A	0.227	0.470	0.065	0.500	0.361	0.000	3.418	0.110

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	164	209	326	373	0	301	153
N.S.	1	1.00	0.49	0.63	0.98	1.12	0.00	0.90	0.46
time (sec)	N/A	0.222	0.460	0.062	0.506	0.350	0.000	3.462	4.289

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	164	206	323	390	0	317	150
N.S.	1	1.00	0.49	0.61	0.96	1.16	0.00	0.94	0.45
time (sec)	N/A	0.226	0.442	0.062	0.522	0.347	0.000	3.957	4.260

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	164	206	332	389	0	304	150
N.S.	1	1.00	0.49	0.61	0.99	1.16	0.00	0.90	0.45
time (sec)	N/A	0.226	0.428	0.063	0.500	0.353	0.000	3.286	4.264

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	161	206	323	396	0	317	149
N.S.	1	1.00	0.48	0.61	0.96	1.18	0.00	0.95	0.44
time (sec)	N/A	0.230	0.393	0.080	0.510	0.360	0.000	3.672	4.232

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	161	206	332	373	0	302	149
N.S.	1	1.00	0.48	0.61	0.99	1.11	0.00	0.90	0.44
time (sec)	N/A	0.228	0.378	0.085	0.527	0.350	0.000	4.210	4.272

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	161	212	317	359	252	302	150
N.S.	1	1.00	0.48	0.63	0.95	1.07	0.75	0.90	0.45
time (sec)	N/A	0.228	0.273	0.062	0.505	0.362	11.487	3.226	0.099

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	161	209	322	357	0	308	150
N.S.	1	1.00	0.48	0.62	0.96	1.07	0.00	0.92	0.45
time (sec)	N/A	0.223	0.264	0.063	0.512	0.365	0.000	3.954	4.284

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	173	221	328	410	0	327	166
N.S.	1	1.00	0.49	0.63	0.93	1.16	0.00	0.93	0.47
time (sec)	N/A	0.260	0.486	0.099	0.510	0.385	0.000	4.814	0.138

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	173	224	335	434	0	308	166
N.S.	1	1.00	0.49	0.64	0.95	1.23	0.00	0.88	0.47
time (sec)	N/A	0.255	0.464	0.109	0.513	0.378	0.000	6.983	4.253

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	189	236	350	457	0	349	179
N.S.	1	1.00	0.51	0.64	0.95	1.24	0.00	0.94	0.48
time (sec)	N/A	0.286	0.495	0.114	0.511	0.369	0.000	5.542	4.327

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	227	269	421	515	0	374	248
N.S.	1	1.00	0.54	0.64	1.00	1.23	0.00	0.89	0.59
time (sec)	N/A	0.346	0.935	0.148	0.533	0.348	0.000	6.562	4.402

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	216	250	394	515	0	354	231
N.S.	1	1.00	0.54	0.62	0.98	1.28	0.00	0.88	0.57
time (sec)	N/A	0.318	0.893	0.142	0.509	0.371	0.000	5.718	0.236

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	216	252	403	479	0	340	231
N.S.	1	1.00	0.54	0.63	1.00	1.19	0.00	0.85	0.57
time (sec)	N/A	0.308	0.835	0.123	0.510	0.365	0.000	6.358	4.358

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	205	235	377	486	0	352	213
N.S.	1	1.00	0.53	0.61	0.98	1.26	0.00	0.91	0.55
time (sec)	N/A	0.288	0.829	0.084	0.513	0.375	0.000	6.297	0.214

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	205	241	386	489	0	339	213
N.S.	1	1.00	0.53	0.63	1.00	1.27	0.00	0.88	0.55
time (sec)	N/A	0.278	0.660	0.089	0.508	0.389	0.000	6.361	4.272

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	205	238	383	506	0	355	210
N.S.	1	1.00	0.53	0.61	0.99	1.30	0.00	0.91	0.54
time (sec)	N/A	0.289	0.741	0.096	0.516	0.349	0.000	11.854	4.289

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	186	238	392	505	0	342	210
N.S.	1	1.00	0.48	0.61	1.01	1.30	0.00	0.88	0.54
time (sec)	N/A	0.286	0.696	0.082	0.506	0.345	0.000	14.295	0.127

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	186	236	385	518	0	355	208
N.S.	1	1.00	0.48	0.60	0.98	1.32	0.00	0.91	0.53
time (sec)	N/A	0.293	0.628	0.086	0.503	0.368	0.000	10.905	4.322

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	186	236	394	513	0	342	208
N.S.	1	1.00	0.48	0.60	1.01	1.31	0.00	0.87	0.53
time (sec)	N/A	0.284	0.618	0.080	0.509	0.373	0.000	18.071	4.230

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	186	236	385	520	0	355	207
N.S.	1	1.00	0.47	0.60	0.98	1.32	0.00	0.90	0.53
time (sec)	N/A	0.290	0.566	0.081	0.511	0.367	0.000	15.021	0.116

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	186	236	394	513	0	342	207
N.S.	1	1.00	0.47	0.60	1.00	1.30	0.00	0.87	0.53
time (sec)	N/A	0.292	0.552	0.079	0.514	0.370	0.000	16.389	4.265

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	183	238	383	512	0	355	209
N.S.	1	1.00	0.47	0.61	0.98	1.32	0.00	0.91	0.54
time (sec)	N/A	0.296	0.468	0.082	0.515	0.348	0.000	7.904	4.280

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	183	238	392	485	0	340	209
N.S.	1	1.00	0.47	0.61	1.01	1.25	0.00	0.87	0.54
time (sec)	N/A	0.291	0.457	0.079	0.501	0.370	0.000	7.478	0.131

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	183	244	377	469	547	340	210
N.S.	1	1.00	0.47	0.63	0.97	1.21	1.41	0.88	0.54
time (sec)	N/A	0.296	0.339	0.079	0.522	0.376	33.308	7.660	4.247

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	183	241	382	475	0	346	210
N.S.	1	1.00	0.47	0.62	0.99	1.23	0.00	0.89	0.54
time (sec)	N/A	0.287	0.334	0.080	0.515	0.377	0.000	4.898	4.289

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	195	253	388	544	0	365	226
N.S.	1	1.00	0.48	0.63	0.96	1.35	0.00	0.90	0.56
time (sec)	N/A	0.326	0.751	0.156	0.514	0.369	0.000	3.829	0.208

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	195	256	395	568	0	356	226
N.S.	1	1.00	0.48	0.63	0.98	1.41	0.00	0.88	0.56
time (sec)	N/A	0.330	0.733	0.136	0.520	0.372	0.000	3.198	4.463

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	211	268	410	591	0	362	239
N.S.	1	1.00	0.50	0.64	0.97	1.40	0.00	0.86	0.57
time (sec)	N/A	0.360	0.741	0.156	0.526	0.368	0.000	3.423	0.273

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	44	41	25	26	0	45	-1
N.S.	1	1.00	0.47	0.44	0.27	0.28	0.00	0.48	-0.01
time (sec)	N/A	0.019	0.020	0.048	0.288	0.323	0.000	4.809	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	44	41	25	22	0	42	-1
N.S.	1	1.00	0.47	0.44	0.27	0.24	0.00	0.45	-0.01
time (sec)	N/A	0.019	0.017	0.039	0.293	0.345	0.000	4.559	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	44	41	25	18	27	37	-1
N.S.	1	1.00	0.47	0.44	0.27	0.19	0.29	0.40	-0.01
time (sec)	N/A	0.019	0.017	0.013	0.283	0.329	39.238	4.583	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	43	40	24	19	0	40	47
N.S.	1	1.00	0.47	0.44	0.26	0.21	0.00	0.44	0.52
time (sec)	N/A	0.019	0.016	0.020	0.280	0.347	0.000	4.272	4.358

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	44	41	25	22	0	41	52
N.S.	1	1.00	0.48	0.45	0.27	0.24	0.00	0.45	0.57
time (sec)	N/A	0.019	0.017	0.018	0.284	0.347	0.000	4.272	4.345

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	42	39	24	23	0	42	53
N.S.	1	1.00	0.46	0.43	0.26	0.25	0.00	0.46	0.58
time (sec)	N/A	0.020	0.018	0.019	0.283	0.338	0.000	3.463	4.377

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	42	39	25	21	0	44	56
N.S.	1	1.00	0.46	0.43	0.27	0.23	0.00	0.48	0.62
time (sec)	N/A	0.019	0.018	0.020	0.280	0.366	0.000	3.624	4.319

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	66	63	83	54	0	99	-1
N.S.	1	1.00	0.34	0.32	0.43	0.28	0.00	0.51	-0.01
time (sec)	N/A	0.039	0.029	0.055	0.284	0.329	0.000	3.359	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	66	63	83	46	0	90	-1
N.S.	1	1.00	0.34	0.32	0.43	0.24	0.00	0.46	-0.01
time (sec)	N/A	0.037	0.025	0.041	0.297	0.346	0.000	2.949	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	66	63	83	40	0	85	-1
N.S.	1	1.00	0.34	0.32	0.43	0.21	0.00	0.44	-0.01
time (sec)	N/A	0.035	0.023	0.041	0.292	0.335	0.000	2.846	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	66	63	87	42	0	89	76
N.S.	1	1.00	0.34	0.33	0.45	0.22	0.00	0.46	0.39
time (sec)	N/A	0.037	0.023	0.014	0.295	0.352	0.000	3.076	4.496

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	66	63	87	45	0	102	87
N.S.	1	1.00	0.35	0.33	0.46	0.24	0.00	0.53	0.46
time (sec)	N/A	0.036	0.028	0.022	0.284	0.336	0.000	3.736	4.535

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	66	63	86	45	0	105	88
N.S.	1	1.00	0.34	0.33	0.45	0.23	0.00	0.54	0.46
time (sec)	N/A	0.036	0.029	0.035	0.295	0.356	0.000	3.387	4.490

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	66	63	86	45	0	107	91
N.S.	1	1.00	0.35	0.33	0.45	0.24	0.00	0.56	0.48
time (sec)	N/A	0.035	0.028	0.023	0.296	0.334	0.000	3.780	4.533

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	88	85	147	82	0	153	-1
N.S.	1	1.00	0.30	0.29	0.49	0.28	0.00	0.52	-0.00
time (sec)	N/A	0.056	0.035	0.041	0.279	0.346	0.000	4.007	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	88	85	147	70	0	138	-1
N.S.	1	1.00	0.30	0.29	0.49	0.24	0.00	0.46	-0.00
time (sec)	N/A	0.050	0.031	0.043	0.297	0.332	0.000	3.958	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	88	85	147	62	0	133	-1
N.S.	1	1.00	0.30	0.29	0.49	0.21	0.00	0.45	-0.00
time (sec)	N/A	0.053	0.027	0.044	0.293	0.328	0.000	4.275	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	88	85	151	64	0	137	112
N.S.	1	1.00	0.30	0.29	0.52	0.22	0.00	0.47	0.38
time (sec)	N/A	0.052	0.028	0.015	0.290	0.365	0.000	4.004	4.565

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	88	85	151	67	0	156	116
N.S.	1	1.00	0.30	0.29	0.51	0.23	0.00	0.53	0.39
time (sec)	N/A	0.051	0.031	0.022	0.288	0.336	0.000	3.977	4.536

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	88	85	151	67	0	159	116
N.S.	1	1.00	0.30	0.29	0.52	0.23	0.00	0.54	0.40
time (sec)	N/A	0.057	0.033	0.021	0.289	0.359	0.000	4.091	4.564

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	88	85	150	67	0	162	118
N.S.	1	1.00	0.30	0.29	0.51	0.23	0.00	0.55	0.40
time (sec)	N/A	0.052	0.039	0.043	0.297	0.357	0.000	4.534	4.716

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	163	239	266	223	0	273	-1
N.S.	1	1.00	0.36	0.52	0.58	0.49	0.00	0.60	-0.00
time (sec)	N/A	0.209	0.173	0.152	0.497	0.363	0.000	5.096	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	151	221	241	219	0	254	-1
N.S.	1	1.00	0.37	0.54	0.58	0.53	0.00	0.62	-0.00
time (sec)	N/A	0.192	0.145	0.150	0.506	0.363	0.000	2.971	0.000

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	150	214	250	170	0	238	-1
N.S.	1	1.00	0.37	0.52	0.61	0.41	0.00	0.58	-0.00
time (sec)	N/A	0.186	0.143	0.136	0.511	0.366	0.000	3.270	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	123	183	216	173	41	242	-1
N.S.	1	1.00	0.33	0.50	0.59	0.47	0.11	0.66	-0.00
time (sec)	N/A	0.166	0.112	0.132	0.509	0.355	15.625	4.319	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	125	182	226	165	0	251	-1
N.S.	1	1.00	0.34	0.49	0.61	0.45	0.00	0.68	-0.00
time (sec)	N/A	0.158	0.111	0.127	0.496	0.376	0.000	3.688	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	150	224	234	198	0	264	-1
N.S.	1	1.00	0.36	0.54	0.57	0.48	0.00	0.64	-0.00
time (sec)	N/A	0.191	0.147	0.137	0.495	0.360	0.000	3.078	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	152	239	242	227	0	256	-1
N.S.	1	1.00	0.37	0.58	0.58	0.55	0.00	0.62	-0.00
time (sec)	N/A	0.189	0.155	0.135	0.506	0.359	0.000	3.911	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	160	251	259	253	0	284	-1
N.S.	1	1.00	0.35	0.55	0.56	0.55	0.00	0.62	-0.00
time (sec)	N/A	0.217	0.167	0.130	0.505	0.347	0.000	4.049	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	206	737	0	341	0	377	-1
N.S.	1	1.00	0.37	1.34	0.00	0.62	0.00	0.68	-0.00
time (sec)	N/A	0.269	0.368	0.067	0.000	0.345	0.000	3.567	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	504	195	679	0	341	0	357	-1
N.S.	1	1.00	0.39	1.35	0.00	0.68	0.00	0.71	-0.00
time (sec)	N/A	0.247	0.377	0.069	0.000	0.372	0.000	3.130	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	504	195	696	0	305	0	343	-1
N.S.	1	1.00	0.39	1.38	0.00	0.61	0.00	0.68	-0.00
time (sec)	N/A	0.241	0.364	0.070	0.000	0.375	0.000	3.618	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	184	612	0	312	0	345	-1
N.S.	1	1.00	0.40	1.34	0.00	0.68	0.00	0.75	-0.00
time (sec)	N/A	0.224	0.354	0.043	0.000	0.398	0.000	4.263	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	184	666	279	315	0	332	-1
N.S.	1	1.00	0.40	1.45	0.61	0.69	0.00	0.72	-0.00
time (sec)	N/A	0.211	0.348	0.043	0.529	0.389	0.000	4.781	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	179	617	272	326	0	348	-1
N.S.	1	1.00	0.39	1.34	0.59	0.71	0.00	0.76	-0.00
time (sec)	N/A	0.220	0.343	0.043	0.528	0.346	0.000	3.550	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	180	668	281	308	0	332	-1
N.S.	1	1.00	0.39	1.46	0.61	0.67	0.00	0.72	-0.00
time (sec)	N/A	0.212	0.333	0.044	0.532	0.354	0.000	4.221	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	181	617	265	304	0	333	-1
N.S.	1	1.00	0.39	1.34	0.58	0.66	0.00	0.72	-0.00
time (sec)	N/A	0.225	0.240	0.040	0.523	0.341	0.000	4.584	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	181	638	0	298	0	339	-1
N.S.	1	1.00	0.39	1.39	0.00	0.65	0.00	0.74	-0.00
time (sec)	N/A	0.219	0.228	0.039	0.000	0.355	0.000	3.823	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	193	645	0	343	0	368	-1
N.S.	1	1.00	0.38	1.27	0.00	0.68	0.00	0.73	-0.00
time (sec)	N/A	0.252	0.378	0.069	0.000	0.367	0.000	3.928	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	193	707	0	367	0	359	-1
N.S.	1	1.00	0.38	1.40	0.00	0.73	0.00	0.71	-0.00
time (sec)	N/A	0.248	0.375	0.074	0.000	0.371	0.000	3.710	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	204	687	0	390	0	390	-1
N.S.	1	1.00	0.37	1.24	0.00	0.71	0.00	0.71	-0.00
time (sec)	N/A	0.274	0.383	0.069	0.000	0.366	0.000	4.077	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	647	647	229	1287	0	457	0	415	-1
N.S.	1	1.00	0.35	1.99	0.00	0.71	0.00	0.64	-0.00
time (sec)	N/A	0.331	0.730	0.086	0.000	0.369	0.000	4.425	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	218	1171	0	457	0	395	-1
N.S.	1	1.00	0.36	1.95	0.00	0.76	0.00	0.66	-0.00
time (sec)	N/A	0.311	0.670	0.080	0.000	0.363	0.000	4.139	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	218	1202	0	421	0	381	-1
N.S.	1	1.00	0.36	2.00	0.00	0.70	0.00	0.64	-0.00
time (sec)	N/A	0.310	0.626	0.091	0.000	0.377	0.000	4.541	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	207	1046	0	428	0	383	-1
N.S.	1	1.00	0.37	1.89	0.00	0.77	0.00	0.69	-0.00
time (sec)	N/A	0.281	0.551	0.048	0.000	0.371	0.000	3.089	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	201	1134	583	431	0	370	-1
N.S.	1	1.00	0.36	2.05	1.05	0.78	0.00	0.67	-0.00
time (sec)	N/A	0.279	0.577	0.051	0.560	0.364	0.000	3.781	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	201	1051	577	448	0	386	-1
N.S.	1	1.00	0.36	1.89	1.04	0.80	0.00	0.69	-0.00
time (sec)	N/A	0.283	0.569	0.048	0.560	0.395	0.000	3.759	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	202	1136	595	447	0	373	-1
N.S.	1	1.00	0.36	2.04	1.07	0.80	0.00	0.67	-0.00
time (sec)	N/A	0.278	0.576	0.048	0.551	0.355	0.000	2.676	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	201	1051	584	462	0	386	-1
N.S.	1	1.00	0.36	1.88	1.04	0.82	0.00	0.69	-0.00
time (sec)	N/A	0.289	0.488	0.048	0.556	0.354	0.000	2.847	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	201	1136	597	455	0	373	-1
N.S.	1	1.00	0.36	2.03	1.07	0.81	0.00	0.67	-0.00
time (sec)	N/A	0.284	0.470	0.048	0.555	0.342	0.000	3.568	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	201	1051	582	454	0	386	-1
N.S.	1	1.00	0.36	1.89	1.04	0.82	0.00	0.69	-0.00
time (sec)	N/A	0.290	0.461	0.047	0.551	0.361	0.000	4.518	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	201	1136	586	429	0	371	-1
N.S.	1	1.00	0.36	2.04	1.05	0.77	0.00	0.67	-0.00
time (sec)	N/A	0.283	0.439	0.047	0.550	0.361	0.000	2.914	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	201	1051	569	414	0	371	-1
N.S.	1	1.00	0.36	1.89	1.02	0.74	0.00	0.67	-0.00
time (sec)	N/A	0.287	0.304	0.046	0.540	0.363	0.000	3.879	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	201	1133	0	416	0	377	-1
N.S.	1	1.00	0.36	2.04	0.00	0.75	0.00	0.68	-0.00
time (sec)	N/A	0.281	0.294	0.048	0.000	0.353	0.000	4.114	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	602	602	213	1081	0	477	0	406	-1
N.S.	1	1.00	0.35	1.80	0.00	0.79	0.00	0.67	-0.00
time (sec)	N/A	0.310	0.605	0.084	0.000	0.364	0.000	3.901	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	602	602	213	1183	0	501	0	397	-1
N.S.	1	1.00	0.35	1.97	0.00	0.83	0.00	0.66	-0.00
time (sec)	N/A	0.317	0.578	0.076	0.000	0.364	0.000	4.467	0.000

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	224	1129	0	524	0	428	-1
N.S.	1	1.00	0.35	1.74	0.00	0.81	0.00	0.66	-0.00
time (sec)	N/A	0.355	0.594	0.085	0.000	0.359	0.000	4.395	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	105	602	144	507	3104	847	540
N.S.	1	1.00	0.70	4.01	0.96	3.38	20.69	5.65	3.60
time (sec)	N/A	0.082	0.182	0.018	0.293	0.342	0.950	4.430	4.579

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	73	292	100	253	1278	415	263
N.S.	1	1.00	0.70	2.81	0.96	2.43	12.29	3.99	2.53
time (sec)	N/A	0.052	0.059	0.014	0.286	0.353	0.514	2.578	4.513

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	57	56	87	330	135	95
N.S.	1	1.00	0.71	0.98	0.97	1.50	5.69	2.33	1.64
time (sec)	N/A	0.016	0.041	0.015	0.289	0.363	0.239	3.031	4.270

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.025	0.020	0.000	0.000	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.026	0.005	0.000	0.000	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.030	0.006	0.000	0.000	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	111	453	243	369	0	900	-1
N.S.	1	1.00	0.35	1.45	0.78	1.18	0.00	2.88	-0.00
time (sec)	N/A	0.082	0.119	0.033	0.302	0.383	0.000	4.447	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	131	199	119	159	0	384	-1
N.S.	1	1.00	0.64	0.97	0.58	0.78	0.00	1.87	-0.00
time (sec)	N/A	0.054	0.071	0.014	0.284	0.358	0.000	2.734	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	53	56	35	35	0	83	-1
N.S.	1	1.00	0.55	0.58	0.36	0.36	0.00	0.86	-0.01
time (sec)	N/A	0.025	0.029	0.010	0.281	0.348	0.000	3.568	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.032	0.010	0.000	0.000	0.000	0.000	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.034	0.006	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.035	0.006	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	66	0	0	0	0	0	-1
N.S.	1	1.04	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.029	0.025	0.000	0.000	0.000	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	110	150	115	163	0	375	206
N.S.	1	1.00	0.63	0.86	0.66	0.94	0.00	2.16	1.18
time (sec)	N/A	0.078	0.068	0.039	0.304	0.348	0.000	3.773	4.403

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	77	96	79	108	0	235	137
N.S.	1	1.00	0.59	0.74	0.61	0.83	0.00	1.81	1.05
time (sec)	N/A	0.055	0.052	0.030	0.271	0.356	0.000	2.567	4.267

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	51	58	54	70	0	132	85
N.S.	1	1.00	0.61	0.69	0.64	0.83	0.00	1.57	1.01
time (sec)	N/A	0.039	0.039	0.023	0.286	0.336	0.000	3.211	4.264

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	31	30	37	0	58	46
N.S.	1	1.00	0.71	0.76	0.73	0.90	0.00	1.41	1.12
time (sec)	N/A	0.018	0.003	0.015	0.278	0.351	0.000	3.504	4.669

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.031	0.005	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.033	0.012	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.037	0.016	0.000	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.031	0.010	0.000	0.000	0.000	0.000	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	46	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.008	0.027	0.004	0.000	0.000	0.000	0.000	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.038	0.012	0.000	0.000	0.000	0.000	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.040	0.015	0.000	0.000	0.000	0.000	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.112	0.005	0.000	0.000	0.000	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.088	0.005	0.000	0.000	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.099	0.006	0.000	0.000	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.133	0.007	0.000	0.000	0.000	0.000	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.141	0.005	0.000	0.000	0.000	0.000	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.005	0.002	0.009	0.278	0.340	0.006	3.899	0.030

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.005	0.001	0.007	0.277	0.360	0.007	3.546	0.028

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.002	0.000	0.006	0.294	0.338	0.006	3.438	0.024

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	20	17	17	20	17
N.S.	1	1.00	1.00	0.86	0.95	0.81	0.81	0.95	0.81
time (sec)	N/A	0.004	0.002	0.023	0.299	0.349	0.021	3.993	0.025

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	12	16	16
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.67	0.89	0.89
time (sec)	N/A	0.004	0.001	0.011	0.281	0.349	0.020	4.173	0.029

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	20	22	17	26	17
N.S.	1	1.00	1.00	0.86	0.95	1.05	0.81	1.24	0.81
time (sec)	N/A	0.005	0.002	0.013	0.280	0.384	0.033	6.094	0.029

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	17	21	17	17	18
N.S.	1	1.00	1.00	0.94	0.94	1.17	0.94	0.94	1.00
time (sec)	N/A	0.004	0.004	0.010	0.282	0.360	0.038	3.636	0.023

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	21	23	19	27	20
N.S.	1	1.00	1.00	0.86	1.00	1.10	0.90	1.29	0.95
time (sec)	N/A	0.005	0.003	0.016	0.281	0.363	0.094	3.052	0.043

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	21	21	22	21	20
N.S.	1	1.00	1.00	0.87	0.91	0.91	0.96	0.91	0.87
time (sec)	N/A	0.005	0.002	0.010	0.277	0.323	0.108	4.178	0.031

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	21	21	22	21	21
N.S.	1	1.00	1.00	0.80	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.005	0.002	0.025	0.282	0.357	0.152	3.898	0.031

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	21	21	22	21	21
N.S.	1	1.00	1.00	0.80	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.005	0.002	0.010	0.285	0.347	0.145	3.880	0.033

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	51	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.94	0.85	0.83
time (sec)	N/A	0.021	0.005	0.040	0.278	0.362	0.010	3.095	0.027

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	44	44	46	46	45
N.S.	1	1.00	0.89	0.83	0.81	0.81	0.85	0.85	0.83
time (sec)	N/A	0.027	0.006	0.040	0.286	0.347	0.011	4.159	0.021

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	45	41	48	43	42
N.S.	1	1.00	1.00	0.86	0.92	0.84	0.98	0.88	0.86
time (sec)	N/A	0.014	0.004	0.010	0.294	0.348	0.009	4.997	0.020

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	44	41	42	46	42
N.S.	1	1.00	1.00	0.94	0.94	0.87	0.89	0.98	0.89
time (sec)	N/A	0.028	0.009	0.054	0.278	0.366	0.037	3.841	0.024

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	42	46	44	44	43
N.S.	1	1.00	1.00	0.94	0.88	0.96	0.92	0.92	0.90
time (sec)	N/A	0.015	0.012	0.030	0.295	0.354	0.035	3.060	0.023

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	45	44	47	44	53	43
N.S.	1	1.00	0.90	0.88	0.86	0.92	0.86	1.04	0.84
time (sec)	N/A	0.027	0.011	0.016	0.278	0.334	0.052	3.873	0.026

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	42	42	46	46	42	44
N.S.	1	1.00	1.00	0.89	0.89	0.98	0.98	0.89	0.94
time (sec)	N/A	0.015	0.014	0.015	0.278	0.329	0.058	3.743	0.041

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	42	45	47	44	60	43
N.S.	1	1.00	0.91	0.93	1.00	1.04	0.98	1.33	0.96
time (sec)	N/A	0.027	0.014	0.030	0.284	0.376	0.171	3.713	0.037

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	43	45	46	48	47	44
N.S.	1	1.00	1.02	0.90	0.94	0.96	1.00	0.98	0.92
time (sec)	N/A	0.017	0.015	0.015	0.275	0.358	0.196	3.129	0.041

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	44	45	48	48	54	46
N.S.	1	1.00	0.98	0.86	0.88	0.94	0.94	1.06	0.90
time (sec)	N/A	0.025	0.012	0.026	0.280	0.347	0.370	3.506	4.137

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	42	44	46	46	46	45
N.S.	1	1.00	1.04	0.89	0.94	0.98	0.98	0.98	0.96
time (sec)	N/A	0.016	0.016	0.016	0.284	0.348	0.364	4.386	4.173

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	43	48	48	48	58	45
N.S.	1	1.00	1.04	0.90	1.00	1.00	1.00	1.21	0.94
time (sec)	N/A	0.024	0.018	0.016	0.297	0.366	0.692	4.176	4.182

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	45	46	46	49	48	46
N.S.	1	1.00	0.96	0.87	0.88	0.88	0.94	0.92	0.88
time (sec)	N/A	0.017	0.014	0.014	0.290	0.364	0.751	4.619	0.035

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	45	46	46	49	48	47
N.S.	1	1.00	0.98	0.83	0.85	0.85	0.91	0.89	0.87
time (sec)	N/A	0.024	0.011	0.016	0.283	0.321	1.050	3.851	4.119

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	56	45	46	46	49	48	47
N.S.	1	1.00	1.04	0.83	0.85	0.85	0.91	0.89	0.87
time (sec)	N/A	0.016	0.017	0.019	0.306	0.356	1.208	4.942	4.160

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	45	46	46	49	48	47
N.S.	1	1.00	0.93	0.83	0.85	0.85	0.91	0.89	0.87
time (sec)	N/A	0.025	0.012	0.016	0.272	0.328	1.482	4.011	4.158

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	111	81	81	97	87	76
N.S.	1	1.00	1.00	1.25	0.91	0.91	1.09	0.98	0.85
time (sec)	N/A	0.041	0.010	0.046	0.280	0.321	0.015	5.106	0.034

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	111	81	81	92	87	76
N.S.	1	1.00	0.89	1.25	0.91	0.91	1.03	0.98	0.85
time (sec)	N/A	0.055	0.012	0.044	0.273	0.351	0.015	3.448	0.030

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	107	85	77	87	83	72
N.S.	1	1.00	1.00	1.32	1.05	0.95	1.07	1.02	0.89
time (sec)	N/A	0.031	0.007	0.013	0.272	0.336	0.014	5.069	0.030

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	85	82	79	92	87	73
N.S.	1	1.00	1.00	1.00	0.96	0.93	1.08	1.02	0.86
time (sec)	N/A	0.050	0.015	0.038	0.278	0.333	0.063	5.007	0.034

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	84	78	83	82	83	73
N.S.	1	1.00	1.00	1.05	0.98	1.04	1.02	1.04	0.91
time (sec)	N/A	0.027	0.017	0.037	0.273	0.330	0.062	3.873	0.033

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	87	82	85	92	98	75
N.S.	1	1.00	0.91	1.01	0.95	0.99	1.07	1.14	0.87
time (sec)	N/A	0.052	0.025	0.020	0.274	0.370	0.081	4.142	0.036

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	84	80	83	90	84	77
N.S.	1	1.00	1.00	1.01	0.96	1.00	1.08	1.01	0.93
time (sec)	N/A	0.028	0.018	0.020	0.268	0.330	0.083	4.170	0.031

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	93	105	0	313	391	92	842
N.S.	1	1.00	0.93	1.05	0.00	3.13	3.91	0.92	8.42
time (sec)	N/A	0.089	0.061	0.060	0.000	0.371	1.806	4.957	4.396

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	83	0	254	316	75	655
N.S.	1	1.00	0.96	1.02	0.00	3.14	3.90	0.93	8.09
time (sec)	N/A	0.057	0.031	0.042	0.000	0.336	1.147	4.699	4.750

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	197	223	59	118
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	1.87
time (sec)	N/A	0.041	0.016	0.033	0.000	0.361	0.520	4.107	4.263

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	129	131	35	41
N.S.	1	1.00	1.08	1.00	0.00	3.58	3.64	0.97	1.14
time (sec)	N/A	0.023	0.007	0.018	0.000	0.353	0.258	3.551	4.270

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	113	65	0	223	253	68	1014
N.S.	1	1.00	1.64	0.94	0.00	3.23	3.67	0.99	14.70
time (sec)	N/A	0.048	0.049	0.029	0.000	0.365	8.324	3.708	4.936

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	135	85	0	293	0	94	2033
N.S.	1	1.00	1.52	0.96	0.00	3.29	0.00	1.06	22.84
time (sec)	N/A	0.092	0.084	0.035	0.000	0.385	0.000	3.733	5.892

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	188	134	0	374	0	126	2451
N.S.	1	1.00	1.65	1.18	0.00	3.28	0.00	1.11	21.50
time (sec)	N/A	0.128	0.148	0.039	0.000	0.385	0.000	5.127	6.367

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	250	217	0	1564	194	2457	2500
N.S.	1	1.00	1.23	1.07	0.00	7.70	0.96	12.10	12.32
time (sec)	N/A	0.465	0.105	0.043	0.000	0.409	11.345	4.840	5.014

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	202	169	0	1059	129	2109	3026
N.S.	1	1.00	1.13	0.94	0.00	5.92	0.72	11.78	16.91
time (sec)	N/A	0.175	0.071	0.033	0.000	0.366	1.222	5.980	0.653

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	165	149	0	559	75	503	416
N.S.	1	1.00	1.10	0.99	0.00	3.73	0.50	3.35	2.77
time (sec)	N/A	0.075	0.056	0.031	0.000	0.352	0.433	4.763	4.457

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	117	0	613	87	1024	763
N.S.	1	1.00	0.86	0.78	0.00	4.09	0.58	6.83	5.09
time (sec)	N/A	0.059	0.052	0.029	0.000	0.361	0.631	4.226	4.612

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	191	159	0	1116	148	1839	2997
N.S.	1	1.00	1.10	0.91	0.00	6.41	0.85	10.57	17.22
time (sec)	N/A	0.155	0.268	0.042	0.000	0.368	1.899	5.040	4.854

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	216	179	0	1622	211	1640	2500
N.S.	1	1.00	1.10	0.91	0.00	8.28	1.08	8.37	12.76
time (sec)	N/A	0.260	0.094	0.044	0.000	0.383	71.427	4.651	0.788

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	121	179	0	663	0	152	1336
N.S.	1	1.00	0.92	1.36	0.00	5.02	0.00	1.15	10.12
time (sec)	N/A	0.111	0.118	0.073	0.000	0.352	0.000	4.993	5.098

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	93	104	0	407	282	96	187
N.S.	1	1.00	1.19	1.33	0.00	5.22	3.62	1.23	2.40
time (sec)	N/A	0.042	0.059	0.039	0.000	0.352	0.830	4.185	0.177

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	77	0	360	269	82	178
N.S.	1	1.00	1.05	1.03	0.00	4.80	3.59	1.09	2.37
time (sec)	N/A	0.040	0.044	0.039	0.000	0.358	0.724	3.896	4.566

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	79	75	0	361	267	82	172
N.S.	1	1.00	1.07	1.01	0.00	4.88	3.61	1.11	2.32
time (sec)	N/A	0.037	0.051	0.038	0.000	0.357	0.694	4.933	4.311

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	207	185	0	813	0	166	2500
N.S.	1	1.00	1.70	1.52	0.00	6.66	0.00	1.36	20.49
time (sec)	N/A	0.127	0.217	0.056	0.000	0.421	0.000	3.560	8.292

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	248	213	0	1007	0	182	2500
N.S.	1	1.00	1.53	1.31	0.00	6.22	0.00	1.12	15.43
time (sec)	N/A	0.165	0.176	0.058	0.000	0.477	0.000	4.717	8.812

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	327	319	0	2856	0	3339	2500
N.S.	1	1.00	0.99	0.96	0.00	8.63	0.00	10.09	7.55
time (sec)	N/A	0.518	0.423	0.067	0.000	0.591	0.000	5.295	1.566

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	282	279	0	2257	0	2736	2500
N.S.	1	1.00	1.04	1.03	0.00	8.33	0.00	10.10	9.23
time (sec)	N/A	0.377	0.330	0.048	0.000	0.419	0.000	7.369	5.999

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	235	230	0	1668	296	2132	2500
N.S.	1	1.00	0.99	0.97	0.00	7.04	1.25	9.00	10.55
time (sec)	N/A	0.265	0.267	0.041	0.000	0.396	7.352	4.148	5.910

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	222	271	0	1680	0	1969	2500
N.S.	1	1.00	1.00	1.23	0.00	7.60	0.00	8.91	11.31
time (sec)	N/A	0.171	0.290	0.074	0.000	0.407	0.000	5.339	1.348

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	243	320	0	2309	0	2682	2500
N.S.	1	1.00	0.96	1.27	0.00	9.16	0.00	10.64	9.92
time (sec)	N/A	0.330	0.278	0.065	0.000	0.461	0.000	4.277	5.996

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	302	294	0	2912	0	3087	2500
N.S.	1	1.00	0.98	0.95	0.00	9.45	0.00	10.02	8.12
time (sec)	N/A	0.915	0.392	0.063	0.000	0.564	0.000	5.697	6.716

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	244	367	0	1631	0	306	2588
N.S.	1	1.00	1.17	1.76	0.00	7.80	0.00	1.46	12.38
time (sec)	N/A	0.253	0.214	0.102	0.000	0.406	0.000	5.803	7.296

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	194	267	0	973	554	212	444
N.S.	1	1.00	1.60	2.21	0.00	8.04	4.58	1.75	3.67
time (sec)	N/A	0.075	0.116	0.077	0.000	0.359	2.399	5.761	4.532

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	137	230	0	892	524	171	423
N.S.	1	1.00	1.15	1.93	0.00	7.50	4.40	1.44	3.55
time (sec)	N/A	0.067	0.130	0.056	0.000	0.385	1.800	4.476	4.444

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	145	216	0	907	580	161	460
N.S.	1	1.00	1.12	1.66	0.00	6.98	4.46	1.24	3.54
time (sec)	N/A	0.083	0.091	0.067	0.000	0.379	2.537	6.144	4.457

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	128	0	808	491	143	400
N.S.	1	1.00	1.01	1.13	0.00	7.15	4.35	1.27	3.54
time (sec)	N/A	0.062	0.068	0.056	0.000	0.361	1.401	5.996	4.392

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	106	126	0	809	481	144	386
N.S.	1	1.00	0.94	1.12	0.00	7.16	4.26	1.27	3.42
time (sec)	N/A	0.060	0.068	0.051	0.000	0.398	1.372	5.277	4.337

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	342	360	0	2017	0	323	2500
N.S.	1	1.00	1.71	1.80	0.00	10.08	0.00	1.62	12.50
time (sec)	N/A	0.203	0.331	0.079	0.000	0.652	0.000	6.426	10.945

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	402	412	0	2312	0	382	2500
N.S.	1	1.00	1.58	1.62	0.00	9.07	0.00	1.50	9.80
time (sec)	N/A	0.258	0.388	0.102	0.000	0.814	0.000	6.181	11.756

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	455	496	0	4279	0	2432	2500
N.S.	1	1.00	1.14	1.24	0.00	10.70	0.00	6.08	6.25
time (sec)	N/A	1.196	0.736	0.068	0.000	0.832	0.000	9.219	9.036

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	381	420	0	3725	0	4558	2500
N.S.	1	1.00	1.09	1.21	0.00	10.70	0.00	13.10	7.18
time (sec)	N/A	0.593	0.611	0.059	0.000	0.589	0.000	6.823	8.537

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	343	374	0	3128	0	1750	2500
N.S.	1	1.00	1.15	1.26	0.00	10.50	0.00	5.87	8.39
time (sec)	N/A	0.476	0.545	0.058	0.000	0.426	0.000	6.537	8.179

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	285	338	0	3128	0	1863	2500
N.S.	1	1.00	0.99	1.17	0.00	10.82	0.00	6.45	8.65
time (sec)	N/A	0.462	0.465	0.056	0.000	0.446	0.000	8.358	7.590

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	334	836	0	3777	0	4270	2500
N.S.	1	1.00	1.07	2.69	0.00	12.14	0.00	13.73	8.04
time (sec)	N/A	0.492	0.547	0.158	0.000	0.572	0.000	5.259	8.367

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	372	920	0	4323	0	2707	2500
N.S.	1	1.00	1.05	2.59	0.00	12.18	0.00	7.63	7.04
time (sec)	N/A	1.199	0.650	0.145	0.000	0.802	0.000	6.245	8.997

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	454	517	0	4924	0	5273	2500
N.S.	1	1.00	1.07	1.22	0.00	11.59	0.00	12.41	5.88
time (sec)	N/A	0.607	1.124	0.087	0.000	1.382	0.000	7.082	9.370

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	86	0	259	311	78	656
N.S.	1	1.00	0.98	1.05	0.00	3.16	3.79	0.95	8.00
time (sec)	N/A	0.061	0.036	0.039	0.000	0.356	1.155	5.643	4.736

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	63	0	206	223	62	120
N.S.	1	1.00	1.02	0.98	0.00	3.22	3.48	0.97	1.88
time (sec)	N/A	0.042	0.017	0.033	0.000	0.373	0.512	3.131	4.398

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	41	38	0	134	131	37	42
N.S.	1	1.00	1.17	1.09	0.00	3.83	3.74	1.06	1.20
time (sec)	N/A	0.026	0.007	0.014	0.000	0.333	0.258	3.858	4.297

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	117	68	0	230	253	71	1015
N.S.	1	1.00	1.67	0.97	0.00	3.29	3.61	1.01	14.50
time (sec)	N/A	0.053	0.049	0.029	0.000	0.371	8.311	3.627	4.892

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	139	87	0	298	0	95	2032
N.S.	1	1.00	1.56	0.98	0.00	3.35	0.00	1.07	22.83
time (sec)	N/A	0.097	0.096	0.035	0.000	0.364	0.000	3.724	5.844

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	208	167	0	1051	129	2153	3000
N.S.	1	1.00	1.16	0.93	0.00	5.87	0.72	12.03	16.76
time (sec)	N/A	0.230	0.082	0.039	0.000	0.372	1.223	4.784	0.673

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	137	149	0	551	75	513	416
N.S.	1	1.00	0.91	0.99	0.00	3.67	0.50	3.42	2.77
time (sec)	N/A	0.068	0.071	0.027	0.000	0.382	0.430	4.139	4.537

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	137	117	0	605	87	1050	763
N.S.	1	1.00	0.91	0.78	0.00	4.03	0.58	7.00	5.09
time (sec)	N/A	0.048	0.053	0.023	0.000	0.342	0.631	6.162	0.486

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	199	159	0	1108	148	1877	2979
N.S.	1	1.00	1.16	0.92	0.00	6.44	0.86	10.91	17.32
time (sec)	N/A	0.139	0.267	0.038	0.000	0.382	1.884	4.716	4.932

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	62	63	74	156	138	60	166
N.S.	1	1.00	0.90	0.91	1.07	2.26	2.00	0.87	2.41
time (sec)	N/A	0.066	0.027	0.033	0.496	0.361	0.645	3.305	0.391

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	49	60	134	110	46	153
N.S.	1	1.00	0.91	0.88	1.07	2.39	1.96	0.82	2.73
time (sec)	N/A	0.040	0.013	0.023	0.509	0.354	0.317	3.476	0.170

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	37	91	53	23	31
N.S.	1	1.00	1.00	0.84	1.19	2.94	1.71	0.74	1.00
time (sec)	N/A	0.020	0.007	0.019	0.491	0.362	0.152	3.419	4.341

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	90	70	85	151	184	71	183
N.S.	1	1.00	1.17	0.91	1.10	1.96	2.39	0.92	2.38
time (sec)	N/A	0.056	0.034	0.025	0.515	0.352	2.587	4.752	4.563

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	146	82	123	209	372	126	389
N.S.	1	1.00	1.51	0.85	1.27	2.15	3.84	1.30	4.01
time (sec)	N/A	0.117	0.065	0.041	0.522	0.369	20.615	4.374	4.870

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	144	101	0	603	105	512	1097
N.S.	1	1.00	1.26	0.89	0.00	5.29	0.92	4.49	9.62
time (sec)	N/A	0.125	0.060	0.035	0.000	0.366	0.781	6.132	4.791

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	128	96	0	267	44	199	216
N.S.	1	1.00	1.17	0.88	0.00	2.45	0.40	1.83	1.98
time (sec)	N/A	0.035	0.069	0.037	0.000	0.361	0.269	4.289	0.297

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	105	74	0	553	63	299	322
N.S.	1	1.00	0.96	0.68	0.00	5.07	0.58	2.74	2.95
time (sec)	N/A	0.036	0.041	0.023	0.000	0.352	0.468	5.744	5.779

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	143	119	0	1612	134	698	2774
N.S.	1	1.00	1.18	0.98	0.00	13.32	1.11	5.77	22.93
time (sec)	N/A	0.079	0.100	0.033	0.000	0.363	2.173	7.882	5.116

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	62	62	58	157	144	58	302
N.S.	1	1.00	0.90	0.90	0.84	2.28	2.09	0.84	4.38
time (sec)	N/A	0.052	0.028	0.034	0.498	0.364	0.641	6.550	0.180

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	47	42	131	117	42	85
N.S.	1	1.00	0.91	0.87	0.78	2.43	2.17	0.78	1.57
time (sec)	N/A	0.033	0.013	0.024	0.489	0.378	0.309	5.185	0.086

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	21	91	60	21	24
N.S.	1	1.00	1.00	0.84	0.68	2.94	1.94	0.68	0.77
time (sec)	N/A	0.019	0.006	0.018	0.509	0.348	0.146	2.557	0.051

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	105	63	61	147	194	61	71
N.S.	1	1.00	1.52	0.91	0.88	2.13	2.81	0.88	1.03
time (sec)	N/A	0.047	0.039	0.025	0.480	0.363	2.517	2.552	4.641

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	163	75	104	208	386	125	2500
N.S.	1	1.00	1.83	0.84	1.17	2.34	4.34	1.40	28.09
time (sec)	N/A	0.093	0.066	0.034	0.495	0.370	20.467	3.325	7.390

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	164	777	0	615	105	533	1147
N.S.	1	1.00	0.38	1.80	0.00	1.42	0.24	1.23	2.66
time (sec)	N/A	0.591	0.067	0.098	0.000	0.352	0.776	3.705	4.650

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	143	339	0	279	44	203	222
N.S.	1	1.00	0.43	1.02	0.00	0.84	0.13	0.61	0.67
time (sec)	N/A	0.179	0.077	0.054	0.000	0.346	0.257	3.443	0.283

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	119	499	0	567	63	307	986
N.S.	1	1.00	0.33	1.39	0.00	1.58	0.18	0.86	2.75
time (sec)	N/A	0.183	0.044	0.064	0.000	0.338	0.449	3.860	5.157

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	174	1262	0	1582	134	742	2848
N.S.	1	1.00	0.40	2.91	0.00	3.65	0.31	1.71	6.58
time (sec)	N/A	0.369	0.104	0.071	0.000	0.373	2.107	3.388	5.274

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	26	18	20
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.30	0.90	1.00
time (sec)	N/A	0.015	0.005	0.016	0.486	0.345	0.031	2.774	0.057

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.011	0.005	0.013	0.502	0.368	0.028	4.165	0.059

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	20	18	18
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.87	0.78	0.78
time (sec)	N/A	0.008	0.010	0.023	0.485	0.367	0.050	4.092	4.368

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	94	69	0	163	63	56	44
N.S.	1	1.00	1.27	0.93	0.00	2.20	0.85	0.76	0.59
time (sec)	N/A	0.036	0.091	0.026	0.000	0.386	0.073	3.701	0.082

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	39	168	0	240	24	147	101
N.S.	1	1.00	0.21	0.89	0.00	1.28	0.13	0.78	0.54
time (sec)	N/A	0.124	0.021	0.050	0.000	0.387	0.280	5.024	4.371

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	166	296	0	367	0	172	315
N.S.	1	1.00	0.97	1.73	0.00	2.15	0.00	1.01	1.84
time (sec)	N/A	0.107	0.303	0.072	0.000	0.386	0.000	3.913	5.313

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	132	247	0	303	0	134	193
N.S.	1	1.00	0.86	1.61	0.00	1.98	0.00	0.88	1.26
time (sec)	N/A	0.094	0.232	0.045	0.000	0.399	0.000	3.912	4.639

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	103	139	0	237	0	98	87
N.S.	1	1.00	0.95	1.29	0.00	2.19	0.00	0.91	0.81
time (sec)	N/A	0.056	0.199	0.039	0.000	0.366	0.000	3.953	4.520

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	85	101	0	197	0	76	72
N.S.	1	1.00	1.02	1.22	0.00	2.37	0.00	0.92	0.87
time (sec)	N/A	0.038	0.142	0.035	0.000	0.367	0.000	3.708	4.622

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	104	91	0	566	0	0	88
N.S.	1	1.00	0.95	0.83	0.00	5.19	0.00	0.00	0.81
time (sec)	N/A	0.073	0.168	0.030	0.000	0.378	0.000	0.000	4.423

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	107	140	0	601	0	148	91
N.S.	1	1.00	0.96	1.25	0.00	5.37	0.00	1.32	0.81
time (sec)	N/A	0.074	0.167	0.046	0.000	0.398	0.000	4.001	4.553

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	193	0	215	0	241	-1
N.S.	1	1.00	1.00	2.19	0.00	2.44	0.00	2.74	-0.01
time (sec)	N/A	0.050	0.235	0.045	0.000	0.387	0.000	4.517	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	222	0	261	0	359	-1
N.S.	1	1.00	0.93	1.91	0.00	2.25	0.00	3.09	-0.01
time (sec)	N/A	0.067	0.413	0.052	0.000	0.381	0.000	4.362	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	141	387	0	325	0	617	-1
N.S.	1	1.00	0.88	2.40	0.00	2.02	0.00	3.83	-0.01
time (sec)	N/A	0.102	0.501	0.058	0.000	0.421	0.000	3.620	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	176	442	0	389	0	842	-1
N.S.	1	1.00	0.88	2.22	0.00	1.95	0.00	4.23	-0.01
time (sec)	N/A	0.159	0.676	0.076	0.000	0.446	0.000	5.502	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	538	476	0	0	0	0	-1
N.S.	1	1.00	1.36	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.173	8.175	0.047	0.000	0.000	0.000	0.000	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	479	417	0	0	0	0	-1
N.S.	1	1.00	1.40	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.095	7.016	0.036	0.000	0.000	0.000	0.000	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	445	379	0	0	0	0	-1
N.S.	1	1.00	1.44	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.067	6.380	0.047	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	435	381	0	0	0	0	-1
N.S.	1	1.00	1.44	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.059	10.539	0.037	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	459	404	0	0	0	0	-1
N.S.	1	1.00	1.35	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.100	10.588	0.040	0.000	0.000	0.000	0.000	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	530	452	0	0	0	0	-1
N.S.	1	1.00	1.34	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	10.877	0.046	0.000	0.000	0.000	0.000	0.000

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	220	534	0	535	0	669	-1
N.S.	1	1.00	0.99	2.39	0.00	2.40	0.00	3.00	-0.00
time (sec)	N/A	0.140	0.657	0.064	0.000	0.399	0.000	2.818	0.000

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	194	432	0	451	0	535	-1
N.S.	1	1.00	0.95	2.12	0.00	2.21	0.00	2.62	-0.00
time (sec)	N/A	0.122	0.531	0.059	0.000	0.363	0.000	6.515	0.000

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	140	316	0	361	0	414	223
N.S.	1	1.00	0.93	2.11	0.00	2.41	0.00	2.76	1.49
time (sec)	N/A	0.083	0.371	0.052	0.000	0.386	0.000	5.469	4.880

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	110	242	0	297	0	317	115
N.S.	1	1.00	0.89	1.95	0.00	2.40	0.00	2.56	0.93
time (sec)	N/A	0.057	0.307	0.045	0.000	0.352	0.000	4.695	4.965

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	142	192	0	727	0	0	-1
N.S.	1	1.00	0.92	1.24	0.00	4.69	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.464	0.039	0.000	0.460	0.000	0.000	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	132	170	0	713	0	0	-1
N.S.	1	1.00	0.88	1.13	0.00	4.75	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.491	0.091	0.000	0.423	0.000	0.000	0.000

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	131	174	0	713	0	302	-1
N.S.	1	1.00	0.87	1.15	0.00	4.72	0.00	2.00	-0.01
time (sec)	N/A	0.110	0.508	0.066	0.000	0.428	0.000	4.865	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	148	202	0	771	0	412	-1
N.S.	1	1.00	0.91	1.24	0.00	4.73	0.00	2.53	-0.01
time (sec)	N/A	0.122	0.683	0.067	0.000	0.409	0.000	4.867	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	120	260	0	319	0	606	-1
N.S.	1	1.00	0.90	1.95	0.00	2.40	0.00	4.56	-0.01
time (sec)	N/A	0.077	0.683	0.065	0.000	0.416	0.000	4.253	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	160	337	0	383	0	832	-1
N.S.	1	1.00	0.99	2.08	0.00	2.36	0.00	5.14	-0.01
time (sec)	N/A	0.098	0.973	0.075	0.000	0.467	0.000	3.181	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	201	457	0	473	0	1235	-1
N.S.	1	1.00	0.93	2.12	0.00	2.19	0.00	5.72	-0.00
time (sec)	N/A	0.141	1.355	0.088	0.000	0.543	0.000	3.415	0.000

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	657	674	0	0	0	0	-1
N.S.	1	1.00	1.33	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	11.461	0.048	0.000	0.000	0.000	0.000	0.000

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	602	545	0	0	0	0	-1
N.S.	1	1.00	1.36	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	11.229	0.045	0.000	0.000	0.000	0.000	0.000

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	533	471	0	0	0	0	-1
N.S.	1	1.00	1.40	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.140	10.063	0.041	0.000	0.000	0.000	0.000	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	505	430	0	0	0	0	-1
N.S.	1	1.00	1.40	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.118	10.825	0.044	0.000	0.000	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	473	428	0	0	0	0	-1
N.S.	1	1.00	1.34	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.100	10.589	0.049	0.000	0.000	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	527	450	0	0	0	0	-1
N.S.	1	1.00	1.32	1.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.151	10.877	0.052	0.000	0.000	0.000	0.000	0.000

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	572	495	0	0	0	0	-1
N.S.	1	1.00	1.28	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	11.037	0.056	0.000	0.000	0.000	0.000	0.000

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	59	114	0	24	0	0	-1
N.S.	1	1.00	1.23	2.38	0.00	0.50	0.00	0.00	-0.02
time (sec)	N/A	0.026	4.037	0.038	0.000	0.078	0.000	0.000	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	101	162	0	241	0	103	-1
N.S.	1	1.00	0.83	1.34	0.00	1.99	0.00	0.85	-0.01
time (sec)	N/A	0.071	0.222	0.043	0.000	0.363	0.000	3.169	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	116	0	203	0	82	-1
N.S.	1	1.00	0.88	1.12	0.00	1.95	0.00	0.79	-0.01
time (sec)	N/A	0.057	0.182	0.039	0.000	0.374	0.000	4.135	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	70	56	0	161	0	61	55
N.S.	1	1.00	1.03	0.82	0.00	2.37	0.00	0.90	0.81
time (sec)	N/A	0.034	0.115	0.039	0.000	0.369	0.000	4.792	4.428

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	35	0	118	0	40	34
N.S.	1	1.00	0.95	0.81	0.00	2.74	0.00	0.93	0.79
time (sec)	N/A	0.020	0.073	0.036	0.000	0.347	0.000	4.956	4.690

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	0	124	0	38	44
N.S.	1	1.00	0.93	0.89	0.00	2.82	0.00	0.86	1.00
time (sec)	N/A	0.024	0.071	0.024	0.000	0.359	0.000	5.652	4.441

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	63	0	179	0	114	56
N.S.	1	1.00	1.00	0.88	0.00	2.49	0.00	1.58	0.78
time (sec)	N/A	0.039	0.138	0.043	0.000	0.348	0.000	5.532	4.484

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	91	127	0	221	0	221	-1
N.S.	1	1.00	0.84	1.18	0.00	2.05	0.00	2.05	-0.01
time (sec)	N/A	0.067	0.221	0.049	0.000	0.394	0.000	5.580	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	110	176	0	265	0	335	-1
N.S.	1	1.00	0.76	1.21	0.00	1.83	0.00	2.31	-0.01
time (sec)	N/A	0.106	0.358	0.050	0.000	0.418	0.000	4.319	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	444	388	0	0	0	0	-1
N.S.	1	1.00	1.42	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.072	10.610	0.034	0.000	0.000	0.000	0.000	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	278	216	0	0	0	0	-1
N.S.	1	1.00	1.04	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.044	10.085	0.021	0.000	0.000	0.000	0.000	0.000

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	186	144	0	120	0	0	-1
N.S.	1	1.00	1.63	1.26	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.009	10.063	0.020	0.000	0.081	0.000	0.000	0.000

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	298	239	0	0	0	0	-1
N.S.	1	1.00	1.01	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.077	10.336	0.033	0.000	0.000	0.000	0.000	0.000

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	459	413	0	0	0	0	-1
N.S.	1	1.00	1.33	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.105	10.593	0.038	0.000	0.000	0.000	0.000	0.000

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	107	168	153	249	0	112	-1
N.S.	1	1.00	0.86	1.35	1.23	2.01	0.00	0.90	-0.01
time (sec)	N/A	0.072	10.068	0.067	0.501	0.367	0.000	4.313	0.000

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	89	120	105	211	0	91	-1
N.S.	1	1.00	0.83	1.12	0.98	1.97	0.00	0.85	-0.01
time (sec)	N/A	0.064	10.041	0.043	0.500	0.378	0.000	3.835	0.000

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	394	58	50	169	0	70	62
N.S.	1	1.00	5.63	0.83	0.71	2.41	0.00	1.00	0.89
time (sec)	N/A	0.036	2.115	0.036	0.503	0.354	0.000	3.736	4.593

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	124	36	28	124	0	45	40
N.S.	1	1.00	2.82	0.82	0.64	2.82	0.00	1.02	0.91
time (sec)	N/A	0.021	0.148	0.022	0.494	0.365	0.000	3.218	4.789

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	45	36	129	0	36	52
N.S.	1	1.00	0.94	0.96	0.77	2.74	0.00	0.77	1.11
time (sec)	N/A	0.028	0.079	0.031	0.493	0.352	0.000	3.535	4.520

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	74	62	188	0	111	64
N.S.	1	1.00	0.99	0.96	0.81	2.44	0.00	1.44	0.83
time (sec)	N/A	0.041	0.128	0.045	0.505	0.356	0.000	7.402	4.546

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	149	126	230	0	224	-1
N.S.	1	1.00	0.83	1.30	1.10	2.00	0.00	1.95	-0.01
time (sec)	N/A	0.071	0.208	0.049	0.523	0.398	0.000	9.121	0.000

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	114	202	179	272	0	344	-1
N.S.	1	1.00	0.74	1.31	1.16	1.77	0.00	2.23	-0.01
time (sec)	N/A	0.115	0.319	0.052	0.484	0.407	0.000	7.702	0.000

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	459	391	0	0	0	0	-1
N.S.	1	1.00	1.12	0.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	10.522	0.043	0.000	0.000	0.000	0.000	0.000

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	271	217	0	0	0	0	-1
N.S.	1	1.00	0.72	0.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.173	10.090	0.025	0.000	0.000	0.000	0.000	0.000

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	177	145	0	126	0	0	-1
N.S.	1	1.00	1.05	0.86	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.043	10.072	0.020	0.000	0.084	0.000	0.000	0.000

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	283	241	0	0	0	0	-1
N.S.	1	1.00	0.69	0.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.228	10.283	0.037	0.000	0.000	0.000	0.000	0.000

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	472	417	0	0	0	0	-1
N.S.	1	1.00	1.06	0.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	10.488	0.041	0.000	0.000	0.000	0.000	0.000

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	166	354	0	591	0	215	-1
N.S.	1	1.00	0.87	1.86	0.00	3.11	0.00	1.13	-0.01
time (sec)	N/A	0.161	0.621	0.072	0.000	0.443	0.000	5.021	0.000

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	131	264	0	459	0	154	-1
N.S.	1	1.00	0.98	1.97	0.00	3.43	0.00	1.15	-0.01
time (sec)	N/A	0.077	0.449	0.063	0.000	0.406	0.000	8.476	0.000

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	96	149	0	387	0	101	84
N.S.	1	1.00	0.83	1.30	0.00	3.37	0.00	0.88	0.73
time (sec)	N/A	0.062	0.369	0.033	0.000	0.405	0.000	10.241	4.765

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	38	0	67	0	44	37
N.S.	1	1.00	1.00	1.06	0.00	1.86	0.00	1.22	1.03
time (sec)	N/A	0.020	0.242	0.029	0.000	0.368	0.000	7.038	4.474

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	36	0	67	0	45	35
N.S.	1	1.00	1.03	1.00	0.00	1.86	0.00	1.25	0.97
time (sec)	N/A	0.015	0.244	0.032	0.000	0.397	0.000	3.078	4.362

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	124	99	0	389	0	110	-1
N.S.	1	1.00	1.39	1.11	0.00	4.37	0.00	1.24	-0.01
time (sec)	N/A	0.054	0.384	0.042	0.000	0.404	0.000	3.236	0.000

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	127	195	0	485	0	200	-1
N.S.	1	1.00	0.91	1.40	0.00	3.49	0.00	1.44	-0.01
time (sec)	N/A	0.090	0.513	0.079	0.000	0.398	0.000	5.175	0.000

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	166	314	0	615	0	350	-1
N.S.	1	1.00	0.85	1.61	0.00	3.15	0.00	1.79	-0.01
time (sec)	N/A	0.147	0.695	0.093	0.000	0.455	0.000	5.005	0.000

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	489	482	0	0	0	0	-1
N.S.	1	1.00	1.20	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	10.902	0.038	0.000	0.000	0.000	0.000	0.000

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	452	450	0	0	0	0	-1
N.S.	1	1.00	1.32	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.095	10.543	0.033	0.000	0.000	0.000	0.000	0.000

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	437	446	0	0	0	0	-1
N.S.	1	1.00	1.28	1.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.090	10.503	0.047	0.000	0.000	0.000	0.000	0.000

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	456	481	0	0	0	0	-1
N.S.	1	1.00	1.29	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.083	10.549	0.031	0.000	0.000	0.000	0.000	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	428	515	536	0	0	0	0	-1
N.S.	1	1.00	1.20	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.138	10.864	0.085	0.000	0.000	0.000	0.000	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	37	34	30	0	48	33
N.S.	1	1.00	0.68	0.74	0.68	0.60	0.00	0.96	0.66
time (sec)	N/A	0.041	0.008	0.115	0.304	0.324	0.000	3.760	4.604

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	74	64	52	114	0	59	53
N.S.	1	1.00	1.28	1.10	0.90	1.97	0.00	1.02	0.91
time (sec)	N/A	0.052	0.017	0.130	0.278	0.395	0.000	4.113	4.613

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	13	20	0	28	20
N.S.	1	1.00	1.00	1.18	0.59	0.91	0.00	1.27	0.91
time (sec)	N/A	0.011	0.003	0.139	0.295	0.391	0.000	4.480	4.369

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	52	44	32	74	0	38	33
N.S.	1	1.00	1.68	1.42	1.03	2.39	0.00	1.23	1.06
time (sec)	N/A	0.031	0.009	0.128	0.312	0.346	0.000	4.776	4.563

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	52	50	0	80	0	46	-1
N.S.	1	1.00	1.73	1.67	0.00	2.67	0.00	1.53	-0.03
time (sec)	N/A	0.006	0.007	0.126	0.000	0.340	0.000	4.189	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	21	21	0	34	21
N.S.	1	1.00	1.00	1.13	0.91	0.91	0.00	1.48	0.91
time (sec)	N/A	0.024	0.006	0.128	0.283	0.347	0.000	4.140	4.315

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	76	73	0	133	0	55	76
N.S.	1	1.00	1.29	1.24	0.00	2.25	0.00	0.93	1.29
time (sec)	N/A	0.034	0.016	0.133	0.000	0.366	0.000	4.894	4.637

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	37	44	31	0	59	29
N.S.	1	1.00	0.67	0.71	0.85	0.60	0.00	1.13	0.56
time (sec)	N/A	0.052	0.010	0.135	0.308	0.358	0.000	4.552	4.471

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	91	94	0	163	0	79	-1
N.S.	1	1.00	1.05	1.08	0.00	1.87	0.00	0.91	-0.01
time (sec)	N/A	0.063	0.019	0.138	0.000	0.374	0.000	4.514	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	62	91	0	44	37	0	-1
N.S.	1	1.00	0.57	0.84	0.00	0.41	0.34	0.00	-0.01
time (sec)	N/A	0.015	10.031	0.137	0.000	0.086	0.388	0.000	0.000

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	22	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.22	0.78	0.78
time (sec)	N/A	0.003	0.013	0.143	0.276	0.348	0.198	3.949	4.663

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	51	97	0	0	37	0	-1
N.S.	1	1.00	0.24	0.46	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.033	10.025	0.123	0.000	0.000	0.362	0.000	0.000

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	45	63	20	25	-1
N.S.	1	1.00	1.00	0.80	1.50	2.10	0.67	0.83	-0.03
time (sec)	N/A	0.010	0.098	0.131	0.494	0.363	0.442	3.952	0.000

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	28	36	0	37
N.S.	1	1.00	0.84	0.80	0.00	0.32	0.41	0.00	0.42
time (sec)	N/A	0.007	10.034	0.125	0.000	0.084	0.353	0.000	4.347

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	37	63	22	23	19
N.S.	1	1.00	1.00	1.07	1.37	2.33	0.81	0.85	0.70
time (sec)	N/A	0.013	0.021	0.115	0.494	0.360	0.454	4.687	4.549

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	49	115	0	0	39	0	40
N.S.	1	1.00	0.21	0.50	0.00	0.00	0.17	0.00	0.17
time (sec)	N/A	0.046	10.024	0.122	0.000	0.000	0.401	0.000	4.562

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	31	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	1.48	0.81
time (sec)	N/A	0.003	0.102	0.129	0.274	0.340	0.333	4.539	4.514

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	51	93	0	47	41	0	-1
N.S.	1	1.00	0.46	0.85	0.00	0.43	0.37	0.00	-0.01
time (sec)	N/A	0.016	10.020	0.138	0.000	0.076	0.440	0.000	0.000

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	63	51	124	95	54	-1
N.S.	1	1.00	0.86	0.86	0.70	1.70	1.30	0.74	-0.01
time (sec)	N/A	0.015	0.064	0.135	0.288	0.390	2.461	3.169	0.000

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	34	33	23	44	30	24
N.S.	1	1.00	0.75	0.94	0.92	0.64	1.22	0.83	0.67
time (sec)	N/A	0.017	0.019	0.140	0.278	0.347	0.207	4.606	4.599

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	51	39	31	93	42	40	56
N.S.	1	1.00	1.04	0.80	0.63	1.90	0.86	0.82	1.14
time (sec)	N/A	0.009	0.006	0.138	0.273	0.355	1.119	4.664	4.640

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87
time (sec)	N/A	0.002	0.002	0.129	0.282	0.358	0.172	5.698	4.331

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	0.80
time (sec)	N/A	0.004	0.005	0.133	0.277	0.351	0.432	6.601	0.124

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	17	60	19	22	19
N.S.	1	1.00	1.00	1.16	0.68	2.40	0.76	0.88	0.76
time (sec)	N/A	0.012	0.003	0.127	0.279	0.374	0.447	4.892	4.573

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	19	30	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	1.00	1.58	0.89
time (sec)	N/A	0.004	0.027	0.138	0.279	0.350	0.311	7.802	0.040

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	48	36	105	42	51	38
N.S.	1	1.00	1.00	0.96	0.72	2.10	0.84	1.02	0.76
time (sec)	N/A	0.019	0.056	0.154	0.277	0.367	1.136	6.497	4.535

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	37	36	27	46	55	25
N.S.	1	1.00	0.70	0.84	0.82	0.61	1.05	1.25	0.57
time (sec)	N/A	0.008	0.041	0.138	0.277	0.340	0.396	7.685	4.552

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	13	12	8	-1
N.S.	1	1.00	1.00	0.81	0.75	0.81	0.75	0.50	-0.06
time (sec)	N/A	0.001	0.002	0.020	0.294	0.356	0.201	5.108	0.000

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	8	10
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.50	0.62
time (sec)	N/A	0.001	0.002	0.024	0.271	0.328	0.185	7.787	4.505

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	14	10	5	-1
N.S.	1	1.00	1.00	0.92	0.85	1.08	0.77	0.38	-0.08
time (sec)	N/A	0.001	0.001	0.026	0.286	0.328	0.171	5.200	0.000

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	16	0	14	-1
N.S.	1	1.00	1.00	0.93	0.87	1.07	0.00	0.93	-0.07
time (sec)	N/A	0.001	0.001	0.015	0.272	0.332	0.000	4.581	0.000

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	10	8	13
N.S.	1	1.00	1.00	0.92	0.83	1.25	0.83	0.67	1.08
time (sec)	N/A	0.001	0.001	0.016	0.282	0.345	0.165	3.719	4.300

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	15	12	8	10
N.S.	1	1.00	1.00	0.77	0.69	1.15	0.92	0.62	0.77
time (sec)	N/A	0.001	0.001	0.019	0.275	0.338	0.176	3.883	4.338

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	14	8	13
N.S.	1	1.00	1.00	0.81	0.75	0.94	0.88	0.50	0.81
time (sec)	N/A	0.001	0.001	0.019	0.277	0.337	0.190	3.957	4.305

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	12	15	15	8	13
N.S.	1	1.00	1.06	0.81	0.75	0.94	0.94	0.50	0.81
time (sec)	N/A	0.001	0.002	0.020	0.277	0.340	0.210	4.042	4.272

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	13	12	15	15	8	13
N.S.	1	1.00	0.94	0.81	0.75	0.94	0.94	0.50	0.81
time (sec)	N/A	0.001	0.001	0.020	0.275	0.316	0.233	3.582	4.331

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.001	0.018	0.294	0.330	0.006	3.475	0.016

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.000	0.017	0.285	0.326	0.006	4.112	0.027

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.000	0.016	0.281	0.335	0.006	4.222	0.013

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.000	0.016	0.297	0.343	0.006	4.208	0.016

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.001	0.000	0.013	0.281	0.327	0.020	4.458	0.002

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	7	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.88	0.75
time (sec)	N/A	0.001	0.000	0.016	0.281	0.352	0.008	4.731	4.241

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.001	0.000	0.016	0.276	0.355	0.009	3.733	0.028

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	12	8	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	1.00	0.67	0.67
time (sec)	N/A	0.001	0.000	0.016	0.293	0.329	0.009	5.089	4.400

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	12	8	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	1.00	0.67	0.67
time (sec)	N/A	0.001	0.000	0.017	0.295	0.340	0.008	4.497	4.329

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	8	0	0	-1
N.S.	1	1.00	1.50	3.58	0.00	0.67	0.00	0.00	-0.08
time (sec)	N/A	0.006	10.025	0.020	0.000	0.074	0.000	0.000	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	82	0	0	0	0	-1
N.S.	1	1.00	2.23	2.10	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	10.089	0.057	0.000	0.000	0.000	0.000	0.000

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	20	19	24	29	19	21
N.S.	1	1.00	0.81	0.65	0.61	0.77	0.94	0.61	0.68
time (sec)	N/A	0.005	0.016	0.023	0.300	0.327	0.475	3.878	4.291

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	20	19	24	29	19	21
N.S.	1	1.00	0.81	0.65	0.61	0.77	0.94	0.61	0.68
time (sec)	N/A	0.004	0.015	0.025	0.292	0.334	0.285	3.229	0.036

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	20	19	22	29	19	21
N.S.	1	1.00	0.81	0.65	0.61	0.71	0.94	0.61	0.68
time (sec)	N/A	0.005	0.015	0.022	0.305	0.317	0.885	3.249	0.034

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	19	21	27	19	21
N.S.	1	1.00	0.86	0.69	0.66	0.72	0.93	0.66	0.72
time (sec)	N/A	0.004	0.015	0.023	0.283	0.333	0.139	3.733	0.031

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	19	21	27	19	21
N.S.	1	1.00	0.86	0.69	0.66	0.72	0.93	0.66	0.72
time (sec)	N/A	0.004	0.018	0.026	0.296	0.373	0.237	3.666	0.039

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	19	21	27	19	21
N.S.	1	1.00	0.86	0.69	0.66	0.72	0.93	0.66	0.72
time (sec)	N/A	0.004	0.019	0.027	0.335	0.334	0.306	4.106	0.034

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	20	21	27	20	21
N.S.	1	1.00	0.86	0.69	0.69	0.72	0.93	0.69	0.72
time (sec)	N/A	0.004	0.022	0.031	0.294	0.319	0.340	3.554	4.326

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	45	44	49	70	46	45
N.S.	1	1.00	0.97	0.70	0.69	0.77	1.09	0.72	0.70
time (sec)	N/A	0.017	0.033	0.046	0.296	0.344	1.149	3.452	4.415

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	45	44	49	70	46	45
N.S.	1	1.00	0.89	0.70	0.69	0.77	1.09	0.72	0.70
time (sec)	N/A	0.015	0.032	0.046	0.302	0.323	0.744	3.920	0.026

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	45	44	47	63	46	45
N.S.	1	1.00	0.89	0.70	0.69	0.73	0.98	0.72	0.70
time (sec)	N/A	0.016	0.032	0.058	0.299	0.319	1.564	3.282	0.028

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	45	48	46	68	46	45
N.S.	1	1.00	0.92	0.73	0.77	0.74	1.10	0.74	0.73
time (sec)	N/A	0.016	0.031	0.029	0.329	0.351	0.471	3.483	0.025

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	52	47	44	46	68	46	45
N.S.	1	1.00	0.84	0.76	0.71	0.74	1.10	0.74	0.73
time (sec)	N/A	0.015	0.033	0.036	0.279	0.334	0.630	4.509	0.026

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	52	47	44	46	68	46	45
N.S.	1	1.00	0.84	0.76	0.71	0.74	1.10	0.74	0.73
time (sec)	N/A	0.015	0.033	0.040	0.310	0.344	0.699	3.661	0.027

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	52	47	45	46	68	47	48
N.S.	1	1.00	0.84	0.76	0.73	0.74	1.10	0.76	0.77
time (sec)	N/A	0.015	0.041	0.040	0.305	0.336	0.867	3.703	0.046

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	111	111	81	86	129	87	76
N.S.	1	1.00	1.08	1.08	0.79	0.83	1.25	0.84	0.74
time (sec)	N/A	0.032	0.053	0.056	0.302	0.366	2.147	3.283	0.041

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	111	111	81	86	129	87	76
N.S.	1	1.00	1.08	1.08	0.79	0.83	1.25	0.84	0.74
time (sec)	N/A	0.029	0.054	0.073	0.288	0.396	1.648	3.854	0.037

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	107	111	81	84	112	87	76
N.S.	1	1.00	1.04	1.08	0.79	0.82	1.09	0.84	0.74
time (sec)	N/A	0.029	0.053	0.059	0.288	0.323	2.452	4.005	0.035

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	111	88	83	128	87	76
N.S.	1	1.00	0.96	1.10	0.87	0.82	1.27	0.86	0.75
time (sec)	N/A	0.030	0.056	0.036	0.283	0.392	1.155	3.901	0.035

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	88	81	83	126	87	76
N.S.	1	1.00	0.94	0.89	0.82	0.84	1.27	0.88	0.77
time (sec)	N/A	0.030	0.062	0.043	0.357	0.324	1.392	4.026	0.038

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	93	88	81	83	128	87	76
N.S.	1	1.00	0.92	0.87	0.80	0.82	1.27	0.86	0.75
time (sec)	N/A	0.030	0.057	0.043	0.283	0.334	1.522	3.978	0.037

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	88	82	83	124	88	79
N.S.	1	1.00	0.94	0.89	0.83	0.84	1.25	0.89	0.80
time (sec)	N/A	0.031	0.063	0.047	0.286	0.350	1.851	4.243	0.036

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	80	65	0	6649	0	0	2500
N.S.	1	1.00	0.21	0.17	0.00	17.09	0.00	0.00	6.43
time (sec)	N/A	0.602	0.092	0.064	0.000	3.694	0.000	0.000	5.820

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	80	64	0	5319	0	0	2500
N.S.	1	1.00	0.21	0.17	0.00	13.82	0.00	0.00	6.49
time (sec)	N/A	0.574	0.055	0.038	0.000	1.171	0.000	0.000	6.858

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	48	45	0	4058	0	0	2500
N.S.	1	1.00	0.15	0.14	0.00	12.26	0.00	0.00	7.55
time (sec)	N/A	0.310	0.058	0.046	0.000	0.642	0.000	0.000	6.512

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	46	45	0	2482	0	0	2500
N.S.	1	1.00	0.14	0.14	0.00	7.50	0.00	0.00	7.55
time (sec)	N/A	0.281	0.050	0.040	0.000	0.443	0.000	0.000	6.024

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	47	45	0	2769	0	0	2500
N.S.	1	1.00	0.14	0.14	0.00	8.37	0.00	0.00	7.55
time (sec)	N/A	0.251	0.054	0.038	0.000	0.450	0.000	0.000	5.308

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	49	42	0	4045	0	0	2500
N.S.	1	1.00	0.15	0.13	0.00	12.22	0.00	0.00	7.55
time (sec)	N/A	0.280	0.039	0.050	0.000	0.596	0.000	0.000	6.258

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	78	65	0	5384	0	0	2500
N.S.	1	1.00	0.21	0.18	0.00	14.51	0.00	0.00	6.74
time (sec)	N/A	0.399	0.083	0.040	0.000	1.876	0.000	0.000	5.737

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	82	64	0	6671	0	0	2500
N.S.	1	1.00	0.22	0.17	0.00	17.98	0.00	0.00	6.74
time (sec)	N/A	0.362	0.059	0.046	0.000	2.541	0.000	0.000	8.637

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	106	84	0	7995	0	0	2500
N.S.	1	1.00	0.26	0.20	0.00	19.41	0.00	0.00	6.07
time (sec)	N/A	0.611	0.109	0.048	0.000	11.597	0.000	0.000	6.479

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	232	149	0	13791	0	0	2500
N.S.	1	1.00	0.43	0.27	0.00	25.35	0.00	0.00	4.60
time (sec)	N/A	1.766	0.484	0.059	0.000	101.297	0.000	0.000	7.012

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	237	146	0	11906	0	0	2500
N.S.	1	1.00	0.46	0.28	0.00	22.90	0.00	0.00	4.81
time (sec)	N/A	1.071	0.393	0.059	0.000	17.985	0.000	0.000	11.849

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	189	121	0	11032	0	0	2500
N.S.	1	1.00	0.40	0.26	0.00	23.42	0.00	0.00	5.31
time (sec)	N/A	0.623	0.320	0.067	0.000	22.262	0.000	0.000	6.445

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	483	194	118	0	9245	0	0	2500
N.S.	1	1.00	0.40	0.24	0.00	19.14	0.00	0.00	5.18
time (sec)	N/A	0.712	0.267	0.072	0.000	3.002	0.000	0.000	10.885

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	109	121	0	9757	0	0	2500
N.S.	1	1.00	0.24	0.27	0.00	21.68	0.00	0.00	5.56
time (sec)	N/A	0.471	0.209	0.055	0.000	8.379	0.000	0.000	6.058

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	110	118	0	10570	0	0	2500
N.S.	1	1.00	0.25	0.27	0.00	23.91	0.00	0.00	5.66
time (sec)	N/A	0.467	0.179	0.056	0.000	6.565	0.000	0.000	10.634

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	136	149	0	12411	0	0	2500
N.S.	1	1.00	0.28	0.30	0.00	25.38	0.00	0.00	5.11
time (sec)	N/A	0.655	0.256	0.056	0.000	42.364	0.000	0.000	6.560

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	140	144	0	13351	0	0	2500
N.S.	1	1.00	0.28	0.29	0.00	26.54	0.00	0.00	4.97
time (sec)	N/A	0.891	0.213	0.055	0.000	32.052	0.000	0.000	7.286

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	573	265	172	0	15175	0	0	2500
N.S.	1	1.00	0.46	0.30	0.00	26.48	0.00	0.00	4.36
time (sec)	N/A	1.670	0.437	0.097	0.000	203.766	0.000	0.000	11.419

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	506	275	0	18827	0	0	2500
N.S.	1	1.00	0.81	0.44	0.00	30.32	0.00	0.00	4.03
time (sec)	N/A	1.299	0.843	0.097	0.000	115.547	0.000	0.000	9.350

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	392	242	0	0	0	0	2500
N.S.	1	1.00	0.69	0.43	0.00	0.00	0.00	0.00	4.39
time (sec)	N/A	1.269	0.760	0.095	0.000	0.000	0.000	0.000	8.015

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	397	241	0	16141	0	0	2500
N.S.	1	1.00	0.70	0.42	0.00	28.37	0.00	0.00	4.39
time (sec)	N/A	1.443	0.646	0.091	0.000	37.227	0.000	0.000	8.524

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	339	244	0	17098	0	0	2500
N.S.	1	1.00	0.64	0.46	0.00	32.08	0.00	0.00	4.69
time (sec)	N/A	0.963	0.674	0.102	0.000	140.915	0.000	0.000	7.659

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	345	237	0	17577	0	0	2500
N.S.	1	1.00	0.65	0.44	0.00	32.98	0.00	0.00	4.69
time (sec)	N/A	0.970	0.567	0.092	0.000	62.016	0.000	0.000	8.384

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	214	281	0	0	0	0	2500
N.S.	1	1.00	0.36	0.47	0.00	0.00	0.00	0.00	4.21
time (sec)	N/A	1.593	0.385	0.103	0.000	0.000	0.000	0.000	8.019

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	215	270	0	20151	0	0	2500
N.S.	1	1.00	0.36	0.45	0.00	33.92	0.00	0.00	4.21
time (sec)	N/A	1.765	0.333	0.092	0.000	157.947	0.000	0.000	9.347

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	658	255	321	0	0	0	0	2500
N.S.	1	1.00	0.39	0.49	0.00	0.00	0.00	0.00	3.80
time (sec)	N/A	3.623	0.476	0.096	0.000	0.000	0.000	0.000	8.746

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	658	259	316	0	0	0	0	2500
N.S.	1	1.00	0.39	0.48	0.00	0.00	0.00	0.00	3.80
time (sec)	N/A	3.809	0.432	0.097	0.000	0.000	0.000	0.000	9.847

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	365	0	0	0	0	0	-1
N.S.	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	10.381	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	342	0	0	0	0	0	-1
N.S.	1	1.00	2.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	10.291	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	342	0	0	0	0	0	-1
N.S.	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	10.263	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	345	0	0	0	0	0	-1
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	10.258	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	459	0	0	0	0	0	-1
N.S.	1	1.00	3.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	10.582	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	417	0	0	0	0	0	-1
N.S.	1	1.00	2.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	10.543	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	415	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	10.515	0.011	0.000	0.000	0.000	0.000	0.000

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	384	0	0	0	0	0	-1
N.S.	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	10.504	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	173	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	10.097	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	173	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	10.077	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	171	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	10.066	0.013	0.000	0.000	0.000	0.000	0.000

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	348	0	0	0	0	0	-1
N.S.	1	1.00	2.40	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	10.363	0.011	0.000	0.000	0.000	0.000	0.000

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	348	0	0	0	0	0	-1
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	10.277	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	367	0	0	0	0	0	-1
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	10.397	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	366	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	10.373	0.017	0.000	0.000	0.000	0.000	0.000

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	409	0	0	0	0	0	-1
N.S.	1	1.00	2.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	10.512	0.016	0.000	0.000	0.000	0.000	0.000

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	111	782	195	594	4332	1132	546
N.S.	1	1.00	0.71	5.01	1.25	3.81	27.77	7.26	3.50
time (sec)	N/A	0.065	0.393	0.017	0.318	0.350	1.050	3.212	4.834

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	110	241	1435	449	260
N.S.	1	1.00	0.69	2.98	1.09	2.39	14.21	4.45	2.57
time (sec)	N/A	0.035	0.106	0.013	0.298	0.330	0.533	3.741	4.584

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	51	50	71	299	119	89
N.S.	1	1.00	0.67	0.98	0.96	1.37	5.75	2.29	1.71
time (sec)	N/A	0.013	0.043	0.012	0.299	0.362	0.237	3.965	4.395

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	82	0	0	0	0	0	-1
N.S.	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.076	0.015	0.000	0.000	0.000	0.000	0.000

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	78	0	0	0	0	0	-1
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.459	0.102	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	357	0	0	0	0	0	-1
N.S.	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.692	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.503	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.742	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	221	0	0	0	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	8.679	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	179	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.190	0.025	0.000	0.000	0.000	0.000	0.000

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	162	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.378	0.025	0.000	0.000	0.000	0.000	0.000

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	162	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.148	0.336	0.021	0.000	0.000	0.000	0.000	0.000

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	162	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.254	0.014	0.000	0.000	0.000	0.000	0.000

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	135	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.104	0.009	0.000	0.000	0.000	0.000	0.000

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	152	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.220	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	163	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.245	0.013	0.000	0.000	0.000	0.000	0.000

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	159	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.270	0.017	0.000	0.000	0.000	0.000	0.000

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.394	0.014	0.000	0.000	0.000	0.000	0.000

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.392	0.012	0.000	0.000	0.000	0.000	0.000

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.120	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	164	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.398	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.394	0.013	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [728] had the largest ratio of [30]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	22	0.136
2	A	3	3	1.00	22	0.136
3	A	2	2	1.00	22	0.091
4	A	2	2	1.00	22	0.091
5	A	3	3	1.03	22	0.136
6	A	4	3	1.02	22	0.136
7	A	5	3	1.10	22	0.136
8	A	9	5	1.00	16	0.312
9	A	3	3	1.00	16	0.188
10	A	9	5	1.00	16	0.312
11	A	3	2	1.00	12	0.167
12	A	3	2	1.00	12	0.167
13	A	9	5	1.00	12	0.417
14	A	9	5	1.00	12	0.417
15	A	9	5	1.00	12	0.417
16	A	2	2	1.00	16	0.125
17	A	2	2	1.00	16	0.125
18	A	2	2	1.00	16	0.125
19	A	2	2	1.00	16	0.125
20	A	2	2	1.00	14	0.143
21	A	1	1	1.00	11	0.091
22	A	2	2	1.00	16	0.125
23	A	2	2	1.00	16	0.125
24	A	2	2	1.00	16	0.125
25	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	16	0.125
27	A	2	2	1.00	16	0.125
28	A	2	2	1.00	16	0.125
29	A	2	2	1.00	16	0.125
30	A	2	2	1.00	16	0.125
31	A	2	2	1.00	16	0.125
32	A	2	2	1.00	16	0.125
33	A	2	2	1.00	14	0.143
34	A	1	1	1.00	11	0.091
35	A	2	2	1.00	16	0.125
36	A	2	2	1.00	16	0.125
37	A	2	2	1.00	16	0.125
38	A	2	2	1.00	16	0.125
39	A	2	2	1.00	16	0.125
40	A	2	2	1.00	16	0.125
41	A	2	2	1.00	16	0.125
42	A	1	1	1.00	16	0.062
43	A	1	1	1.00	16	0.062
44	A	1	1	1.00	16	0.062
45	A	1	1	1.00	16	0.062
46	A	1	1	1.00	14	0.071
47	A	1	1	1.00	11	0.091
48	A	1	1	1.00	16	0.062
49	A	1	1	1.00	16	0.062
50	A	1	1	1.00	16	0.062
51	A	1	1	1.00	16	0.062
52	A	1	1	1.00	16	0.062
53	A	1	1	1.00	16	0.062
54	A	1	1	1.00	16	0.062
55	A	1	1	1.00	16	0.062
56	A	1	1	1.00	16	0.062
57	A	1	1	1.00	16	0.062
58	A	1	1	1.00	16	0.062
59	A	1	1	1.00	14	0.071
60	A	1	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	1	1	1.00	16	0.062
62	A	1	1	1.00	16	0.062
63	A	1	1	1.00	16	0.062
64	A	1	1	1.00	16	0.062
65	A	1	1	1.00	16	0.062
66	A	1	1	1.00	16	0.062
67	A	1	1	1.00	16	0.062
68	A	1	1	1.00	16	0.062
69	A	1	1	1.00	16	0.062
70	A	1	1	1.00	14	0.071
71	A	1	1	1.00	11	0.091
72	A	1	1	1.00	16	0.062
73	A	1	1	1.00	16	0.062
74	A	1	1	1.00	16	0.062
75	A	1	1	1.00	16	0.062
76	A	1	1	1.00	16	0.062
77	A	1	1	1.00	16	0.062
78	A	1	1	1.00	16	0.062
79	A	1	1	1.00	16	0.062
80	A	1	1	1.00	16	0.062
81	A	1	1	1.00	16	0.062
82	A	1	1	1.00	16	0.062
83	A	1	1	1.00	16	0.062
84	A	1	1	1.00	16	0.062
85	A	1	1	1.00	16	0.062
86	A	1	1	1.00	14	0.071
87	A	1	1	1.00	11	0.091
88	A	1	1	1.00	16	0.062
89	A	1	1	1.00	16	0.062
90	A	1	1	1.00	16	0.062
91	A	1	1	1.00	16	0.062
92	A	1	1	1.00	16	0.062
93	A	1	1	1.00	16	0.062
94	A	1	1	1.00	16	0.062
95	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	2	1.00	16	0.125
97	A	2	2	1.00	16	0.125
98	A	1	1	1.00	16	0.062
99	A	1	1	1.00	16	0.062
100	A	1	1	1.00	16	0.062
101	A	1	1	1.00	14	0.071
102	A	1	1	1.00	11	0.091
103	A	1	1	1.00	16	0.062
104	A	1	1	1.00	16	0.062
105	A	1	1	1.00	16	0.062
106	A	1	1	1.00	16	0.062
107	A	2	2	1.00	16	0.125
108	A	2	2	1.00	16	0.125
109	A	2	2	1.00	16	0.125
110	A	1	1	1.00	16	0.062
111	A	1	1	1.00	16	0.062
112	A	1	1	1.00	16	0.062
113	A	1	1	1.00	14	0.071
114	A	1	1	1.00	11	0.091
115	A	1	1	1.00	16	0.062
116	A	1	1	1.00	16	0.062
117	A	1	1	1.00	16	0.062
118	A	1	1	1.00	16	0.062
119	A	2	2	1.00	16	0.125
120	A	1	1	1.00	16	0.062
121	A	1	1	1.00	16	0.062
122	A	1	1	1.00	16	0.062
123	A	1	1	1.00	16	0.062
124	A	1	1	1.00	14	0.071
125	A	2	2	1.00	16	0.125
126	A	2	2	1.00	16	0.125
127	A	2	2	1.00	16	0.125
128	A	2	2	1.00	16	0.125
129	A	2	2	1.00	16	0.125
130	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.00	16	0.125
132	A	2	2	1.00	16	0.125
133	A	2	2	1.00	16	0.125
134	A	2	1	1.00	15	0.067
135	A	2	1	1.00	13	0.077
136	A	1	0	1.00	11	0.000
137	A	2	1	1.00	15	0.067
138	A	2	1	1.00	15	0.067
139	A	2	1	1.00	15	0.067
140	A	2	1	1.00	15	0.067
141	A	2	1	1.00	15	0.067
142	A	2	1	1.00	15	0.067
143	A	2	1	1.00	15	0.067
144	A	2	1	1.00	15	0.067
145	A	3	2	1.00	13	0.154
146	A	4	3	1.00	17	0.176
147	A	3	2	1.00	17	0.118
148	A	2	2	1.00	17	0.118
149	A	3	2	1.00	17	0.118
150	A	4	3	1.00	17	0.176
151	A	3	2	1.00	17	0.118
152	A	4	3	1.00	17	0.176
153	A	3	2	1.00	17	0.118
154	A	4	3	1.00	17	0.176
155	A	3	2	1.00	17	0.118
156	A	2	2	1.00	17	0.118
157	A	3	2	1.00	17	0.118
158	A	3	2	1.00	17	0.118
159	A	4	3	1.00	17	0.176
160	A	3	2	1.00	17	0.118
161	A	2	2	1.00	17	0.118
162	A	3	2	1.00	17	0.118
163	A	4	3	1.00	17	0.176
164	A	3	2	1.00	17	0.118
165	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	2	1.00	17	0.118
167	A	4	3	1.00	17	0.176
168	A	3	2	1.00	17	0.118
169	A	4	3	1.00	17	0.176
170	A	3	2	1.00	17	0.118
171	A	2	2	1.00	17	0.118
172	A	3	2	1.00	17	0.118
173	A	4	4	1.00	17	0.235
174	A	4	3	1.00	17	0.176
175	A	4	3	1.00	17	0.176
176	A	4	3	1.00	17	0.176
177	A	4	3	1.00	17	0.176
178	A	4	3	1.00	17	0.176
179	A	4	3	1.00	17	0.176
180	A	3	3	1.00	17	0.176
181	A	2	2	1.00	17	0.118
182	A	2	2	1.00	17	0.118
183	A	5	5	1.00	15	0.333
184	A	3	3	1.00	13	0.231
185	A	4	3	1.00	17	0.176
186	A	4	3	1.00	17	0.176
187	A	4	3	1.00	17	0.176
188	A	5	3	1.00	17	0.176
189	A	4	3	1.00	17	0.176
190	A	5	4	1.00	17	0.235
191	A	4	3	1.00	17	0.176
192	A	5	4	1.00	17	0.235
193	A	4	3	1.00	17	0.176
194	A	4	4	1.00	17	0.235
195	A	4	3	1.00	17	0.176
196	A	3	3	1.00	17	0.176
197	A	2	2	1.00	17	0.118
198	A	3	3	1.00	17	0.176
199	A	4	3	1.00	17	0.176
200	A	4	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	3	1.00	15	0.200
202	A	5	4	1.00	13	0.308
203	A	4	3	1.00	17	0.176
204	A	6	4	1.00	17	0.235
205	A	6	4	1.00	17	0.235
206	A	4	3	1.00	17	0.176
207	A	5	4	1.00	17	0.235
208	A	4	3	1.00	17	0.176
209	A	4	3	1.00	17	0.176
210	A	2	2	1.00	17	0.118
211	A	4	4	1.00	17	0.235
212	A	2	2	1.00	17	0.118
213	A	4	3	1.00	17	0.176
214	A	4	3	1.00	17	0.176
215	A	5	4	1.00	17	0.235
216	A	4	3	1.00	17	0.176
217	A	6	4	1.00	17	0.235
218	A	4	3	1.00	15	0.200
219	A	7	4	1.00	13	0.308
220	A	4	3	1.00	17	0.176
221	A	6	6	1.00	19	0.316
222	A	5	5	1.00	19	0.263
223	A	4	4	1.00	17	0.235
224	A	4	4	1.00	19	0.210
225	A	4	4	1.00	19	0.210
226	A	1	1	1.00	19	0.053
227	A	2	2	1.00	19	0.105
228	A	3	2	1.00	19	0.105
229	A	4	2	1.00	19	0.105
230	A	5	2	1.00	19	0.105
231	A	3	2	1.00	19	0.105
232	A	2	2	1.00	19	0.105
233	A	1	1	1.00	15	0.067
234	A	3	3	1.00	19	0.158
235	A	3	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	4	4	1.00	19	0.210
237	A	5	4	1.00	19	0.210
238	A	6	5	1.00	19	0.263
239	A	5	4	1.00	17	0.235
240	A	5	5	1.00	19	0.263
241	A	5	4	1.00	19	0.210
242	A	5	5	1.00	19	0.263
243	A	5	4	1.00	19	0.210
244	A	1	1	1.00	19	0.053
245	A	2	2	1.00	19	0.105
246	A	3	2	1.00	19	0.105
247	A	4	2	1.00	19	0.105
248	A	5	2	1.00	19	0.105
249	A	5	3	1.00	19	0.158
250	A	4	3	1.00	19	0.158
251	A	3	3	1.00	19	0.158
252	A	2	2	1.00	15	0.133
253	A	1	1	1.00	19	0.053
254	A	4	3	1.00	19	0.158
255	A	4	4	1.00	19	0.210
256	A	4	3	1.00	19	0.158
257	A	5	4	1.00	19	0.210
258	A	6	4	1.00	19	0.210
259	A	7	4	1.00	19	0.210
260	A	6	5	1.00	19	0.263
261	A	5	5	1.00	19	0.263
262	A	4	4	1.00	19	0.210
263	A	3	3	1.00	17	0.176
264	A	1	1	1.00	19	0.053
265	A	2	2	1.00	19	0.105
266	A	3	2	1.00	19	0.105
267	A	4	2	1.00	19	0.105
268	A	2	2	1.00	19	0.105
269	A	1	1	1.00	19	0.053
270	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	3	3	1.00	19	0.158
272	A	4	3	1.00	19	0.158
273	A	6	6	1.00	19	0.316
274	A	5	5	1.00	19	0.263
275	A	4	4	1.00	19	0.210
276	A	1	1	1.00	19	0.053
277	A	2	2	1.00	17	0.118
278	A	3	3	1.00	19	0.158
279	A	4	3	1.00	19	0.158
280	A	5	3	1.00	19	0.158
281	A	2	2	1.00	19	0.105
282	A	1	1	1.00	19	0.053
283	A	3	3	1.00	19	0.158
284	A	4	4	1.00	15	0.267
285	A	5	4	1.00	19	0.210
286	A	4	4	1.00	19	0.210
287	A	4	4	1.00	19	0.210
288	A	4	4	1.00	19	0.210
289	A	4	4	1.00	19	0.210
290	A	4	4	1.00	19	0.210
291	A	4	4	1.00	20	0.200
292	A	2	1	1.00	17	0.059
293	A	2	1	1.00	17	0.059
294	A	2	1	1.00	17	0.059
295	A	2	1	1.00	17	0.059
296	A	2	1	1.00	17	0.059
297	A	2	1	1.00	17	0.059
298	A	2	1	1.00	17	0.059
299	A	2	1	1.00	17	0.059
300	A	3	2	1.00	19	0.105
301	A	3	2	1.00	19	0.105
302	A	3	2	1.00	19	0.105
303	A	3	2	1.00	19	0.105
304	A	3	2	1.00	19	0.105
305	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	3	2	1.00	19	0.105
307	A	3	2	1.00	19	0.105
308	A	3	2	1.00	19	0.105
309	A	3	2	1.00	19	0.105
310	A	3	2	1.00	19	0.105
311	A	3	2	1.00	19	0.105
312	A	3	2	1.00	19	0.105
313	A	3	2	1.00	19	0.105
314	A	3	2	1.00	19	0.105
315	A	3	2	1.00	19	0.105
316	A	13	9	1.00	19	0.474
317	A	13	9	1.00	19	0.474
318	A	12	9	1.00	19	0.474
319	A	12	9	1.00	19	0.474
320	A	11	8	1.00	19	0.421
321	A	11	8	1.00	19	0.421
322	A	12	9	1.00	19	0.474
323	A	12	9	1.00	19	0.474
324	A	13	9	1.00	19	0.474
325	A	13	9	1.00	19	0.474
326	A	14	9	1.00	19	0.474
327	A	14	10	1.00	19	0.526
328	A	13	10	1.00	19	0.526
329	A	13	10	1.00	19	0.526
330	A	12	9	1.00	19	0.474
331	A	12	9	1.00	19	0.474
332	A	12	9	1.00	19	0.474
333	A	12	9	1.00	19	0.474
334	A	13	10	1.00	19	0.526
335	A	13	10	1.00	19	0.526
336	A	14	10	1.00	19	0.526
337	A	14	10	1.00	19	0.526
338	A	15	10	1.00	19	0.526
339	A	14	10	1.00	19	0.526
340	A	13	9	1.00	19	0.474

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
341	A	13	9	1.00	19	0.474
342	A	13	10	1.00	19	0.526
343	A	13	10	1.00	19	0.526
344	A	13	9	1.00	19	0.474
345	A	13	9	1.00	19	0.474
346	A	14	10	1.00	19	0.526
347	A	14	10	1.00	19	0.526
348	A	15	10	1.00	19	0.526
349	A	15	10	1.00	19	0.526
350	A	16	10	1.00	19	0.526
351	A	16	10	1.00	19	0.526
352	A	8	7	1.00	21	0.333
353	A	6	5	1.00	21	0.238
354	A	7	7	1.00	21	0.333
355	A	5	5	1.00	21	0.238
356	A	6	6	1.00	21	0.286
357	A	4	4	1.00	21	0.190
358	A	6	6	1.00	21	0.286
359	A	4	4	1.00	21	0.190
360	A	7	7	1.00	21	0.333
361	A	5	5	1.00	21	0.238
362	A	8	7	1.00	21	0.333
363	A	6	5	1.00	21	0.238
364	A	9	7	1.00	21	0.333
365	A	7	5	1.00	21	0.238
366	A	8	7	1.00	21	0.333
367	A	6	5	1.00	21	0.238
368	A	7	6	1.00	21	0.286
369	A	5	4	1.00	21	0.190
370	A	7	7	1.00	21	0.333
371	A	5	5	1.00	21	0.238
372	A	7	6	1.00	21	0.286
373	A	5	4	1.00	21	0.190
374	A	8	7	1.00	21	0.333
375	A	6	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	9	7	1.00	21	0.333
377	A	7	5	1.00	21	0.238
378	A	6	4	1.00	21	0.190
379	A	7	6	1.00	21	0.286
380	A	5	4	1.00	21	0.190
381	A	6	6	1.00	21	0.286
382	A	4	4	1.00	21	0.190
383	A	5	5	1.00	21	0.238
384	A	3	3	1.00	21	0.143
385	A	6	6	1.00	21	0.286
386	A	4	4	1.00	21	0.190
387	A	7	6	1.00	21	0.286
388	A	5	4	1.00	21	0.190
389	A	8	6	1.00	21	0.286
390	A	6	4	1.00	21	0.190
391	A	6	5	1.00	21	0.238
392	A	7	7	1.00	21	0.333
393	A	5	5	1.00	21	0.238
394	A	6	6	1.00	21	0.286
395	A	4	4	1.00	21	0.190
396	A	6	6	1.00	21	0.286
397	A	4	4	1.00	21	0.190
398	A	7	7	1.00	21	0.333
399	A	5	5	1.00	21	0.238
400	A	8	7	1.00	21	0.333
401	A	6	5	1.00	21	0.238
402	A	9	7	1.00	21	0.333
403	A	4	3	1.00	19	0.158
404	A	4	3	1.00	19	0.158
405	A	2	1	1.00	17	0.059
406	A	3	3	1.00	19	0.158
407	A	3	3	1.00	19	0.158
408	A	3	3	1.00	19	0.158
409	A	2	1	1.00	22	0.045
410	A	2	1	1.00	22	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	2	1	1.00	20	0.050
412	A	1	0	1.00	18	0.000
413	A	2	1	1.00	22	0.045
414	A	2	1	1.00	22	0.045
415	A	2	1	1.00	22	0.045
416	A	2	1	1.00	22	0.045
417	A	2	1	1.00	22	0.045
418	A	2	1	1.00	22	0.045
419	A	2	1	1.00	22	0.045
420	A	2	1	1.00	22	0.045
421	A	3	2	1.00	24	0.083
422	A	4	3	1.00	24	0.125
423	A	3	2	1.00	24	0.083
424	A	4	3	1.00	24	0.125
425	A	3	2	1.00	24	0.083
426	A	2	2	1.00	22	0.091
427	A	3	2	1.00	20	0.100
428	A	4	3	1.00	24	0.125
429	A	3	2	1.00	24	0.083
430	A	4	3	1.00	24	0.125
431	A	3	2	1.00	24	0.083
432	A	4	3	1.00	24	0.125
433	A	3	2	1.00	24	0.083
434	A	4	3	1.00	24	0.125
435	A	3	2	1.00	24	0.083
436	A	4	3	1.00	24	0.125
437	A	3	2	1.00	24	0.083
438	A	2	2	1.00	24	0.083
439	A	3	2	1.00	24	0.083
440	A	4	4	1.00	24	0.167
441	A	3	2	1.00	24	0.083
442	A	4	3	1.00	24	0.125
443	A	3	2	1.00	24	0.083
444	A	3	2	1.00	24	0.083
445	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	3	2	1.00	24	0.083
447	A	4	3	1.00	24	0.125
448	A	3	2	1.00	24	0.083
449	A	4	3	1.00	24	0.125
450	A	3	2	1.00	24	0.083
451	A	2	2	1.00	22	0.091
452	A	3	2	1.00	20	0.100
453	A	4	3	1.00	24	0.125
454	A	3	2	1.00	24	0.083
455	A	4	3	1.00	24	0.125
456	A	3	2	1.00	24	0.083
457	A	4	3	1.00	24	0.125
458	A	3	2	1.00	24	0.083
459	A	4	3	1.00	24	0.125
460	A	3	2	1.00	24	0.083
461	A	4	3	1.00	24	0.125
462	A	3	2	1.00	24	0.083
463	A	4	3	1.00	24	0.125
464	A	3	2	1.00	24	0.083
465	A	4	3	1.00	24	0.125
466	A	3	2	1.00	24	0.083
467	A	2	2	1.00	24	0.083
468	A	3	2	1.00	24	0.083
469	A	4	4	1.00	24	0.167
470	A	3	2	1.00	24	0.083
471	A	5	4	1.00	24	0.167
472	A	3	2	1.00	24	0.083
473	A	6	4	1.00	24	0.167
474	A	3	2	1.00	24	0.083
475	A	4	3	1.00	24	0.125
476	A	4	3	1.00	24	0.125
477	A	4	3	1.00	24	0.125
478	A	4	3	1.00	24	0.125
479	A	4	3	1.00	24	0.125
480	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	4	3	1.00	24	0.125
482	A	4	3	1.00	24	0.125
483	A	4	3	1.00	24	0.125
484	A	5	4	1.00	24	0.167
485	A	5	4	1.00	24	0.167
486	A	5	4	1.00	24	0.167
487	A	4	4	1.00	24	0.167
488	A	3	3	1.00	24	0.125
489	A	3	3	1.00	20	0.150
490	A	4	4	1.00	24	0.167
491	A	5	4	1.00	24	0.167
492	A	6	4	1.00	24	0.167
493	A	4	3	1.00	24	0.125
494	A	4	3	1.00	24	0.125
495	A	4	3	1.00	24	0.125
496	A	2	2	1.00	24	0.083
497	A	4	3	1.00	24	0.125
498	A	2	2	1.00	22	0.091
499	A	4	3	1.00	24	0.125
500	A	4	3	1.00	24	0.125
501	A	4	3	1.00	24	0.125
502	A	7	4	1.00	24	0.167
503	A	7	4	1.00	24	0.167
504	A	6	4	1.00	24	0.167
505	A	5	3	1.00	24	0.125
506	A	5	4	1.00	24	0.167
507	A	5	4	1.00	24	0.167
508	A	5	3	1.00	20	0.150
509	A	6	4	1.00	24	0.167
510	A	7	4	1.00	24	0.167
511	A	8	4	1.00	24	0.167
512	A	4	3	1.00	24	0.125
513	A	4	3	1.00	24	0.125
514	A	4	3	1.00	24	0.125
515	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	4	4	1.00	24	0.167
517	A	4	3	1.00	24	0.125
518	A	4	3	1.00	24	0.125
519	A	2	2	1.00	22	0.091
520	A	4	3	1.00	24	0.125
521	A	4	3	1.00	24	0.125
522	A	4	3	1.00	24	0.125
523	A	9	4	1.00	24	0.167
524	A	9	4	1.00	24	0.167
525	A	8	4	1.00	24	0.167
526	A	7	3	1.00	24	0.125
527	A	7	4	1.00	24	0.167
528	A	7	4	1.00	24	0.167
529	A	7	4	1.00	24	0.167
530	A	7	4	1.00	24	0.167
531	A	7	3	1.00	20	0.150
532	A	8	4	1.00	24	0.167
533	A	9	4	1.00	24	0.167
534	A	10	4	1.00	24	0.167
535	A	3	3	1.00	12	0.250
536	A	2	2	1.00	14	0.143
537	A	3	3	1.00	16	0.188
538	A	4	3	1.00	16	0.188
539	A	2	2	1.00	14	0.143
540	A	4	3	1.00	16	0.188
541	A	4	3	1.00	26	0.115
542	A	3	3	1.00	26	0.115
543	A	2	2	1.00	24	0.083
544	A	3	2	1.00	26	0.077
545	A	3	2	1.00	26	0.077
546	A	3	3	1.00	26	0.115
547	A	1	1	1.00	26	0.038
548	A	4	3	1.00	26	0.115
549	A	4	3	1.00	26	0.115
550	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	3	2	1.00	26	0.077
552	A	2	1	1.00	22	0.045
553	A	3	2	1.00	26	0.077
554	A	3	2	1.00	26	0.077
555	A	3	2	1.00	26	0.077
556	A	3	2	1.00	26	0.077
557	A	3	2	1.00	26	0.077
558	A	4	3	1.00	26	0.115
559	A	4	3	1.00	26	0.115
560	A	3	2	1.12	26	0.077
561	A	3	3	1.00	26	0.115
562	A	2	2	1.00	24	0.083
563	A	4	3	1.00	26	0.115
564	A	4	3	1.00	26	0.115
565	A	4	3	1.00	26	0.115
566	A	4	3	1.00	26	0.115
567	A	3	3	1.00	26	0.115
568	A	1	1	1.00	26	0.038
569	A	4	3	1.00	26	0.115
570	A	4	3	1.00	26	0.115
571	A	4	3	1.00	26	0.115
572	A	3	2	1.00	26	0.077
573	A	3	2	1.00	26	0.077
574	A	3	2	1.00	26	0.077
575	A	3	2	1.00	26	0.077
576	A	3	2	1.00	22	0.091
577	A	3	2	1.00	26	0.077
578	A	3	2	1.00	26	0.077
579	A	3	2	1.00	26	0.077
580	A	3	2	1.00	26	0.077
581	A	3	2	1.00	26	0.077
582	A	3	2	1.00	26	0.077
583	A	3	2	1.00	26	0.077
584	A	3	2	1.00	26	0.077
585	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	4	3	1.00	26	0.115
587	A	3	2	1.00	26	0.077
588	A	4	3	1.00	26	0.115
589	A	4	3	1.00	26	0.115
590	A	3	3	1.00	26	0.115
591	A	2	2	1.00	24	0.083
592	A	4	3	1.00	26	0.115
593	A	4	3	1.00	26	0.115
594	A	4	3	1.00	26	0.115
595	A	4	3	1.00	26	0.115
596	A	4	3	1.00	26	0.115
597	A	4	3	1.00	26	0.115
598	A	3	3	1.00	26	0.115
599	A	1	1	1.00	26	0.038
600	A	5	4	1.00	26	0.154
601	A	4	3	1.00	26	0.115
602	A	4	3	1.00	26	0.115
603	A	4	3	1.00	26	0.115
604	A	4	3	1.00	26	0.115
605	A	3	2	1.00	26	0.077
606	A	3	2	1.00	26	0.077
607	A	3	2	1.00	26	0.077
608	A	3	2	1.00	26	0.077
609	A	3	2	1.00	26	0.077
610	A	3	2	1.00	26	0.077
611	A	3	2	1.00	22	0.091
612	A	3	2	1.00	26	0.077
613	A	3	2	1.00	26	0.077
614	A	3	2	1.00	26	0.077
615	A	3	2	1.00	26	0.077
616	A	3	2	1.00	26	0.077
617	A	3	2	1.00	26	0.077
618	A	3	2	1.00	26	0.077
619	A	3	2	1.00	26	0.077
620	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
621	A	3	2	1.00	26	0.077
622	A	3	2	1.00	26	0.077
623	A	3	2	1.00	26	0.077
624	A	4	3	1.00	26	0.115
625	A	4	4	1.00	26	0.154
626	A	3	3	1.00	24	0.125
627	A	5	5	1.00	26	0.192
628	A	4	3	0.98	26	0.115
629	A	4	3	1.00	26	0.115
630	A	3	3	1.00	26	0.115
631	A	2	2	1.00	22	0.091
632	A	3	3	1.00	26	0.115
633	A	4	3	0.98	26	0.115
634	A	4	3	1.00	26	0.115
635	A	4	3	1.00	26	0.115
636	A	3	3	1.68	26	0.115
637	A	2	2	1.00	24	0.083
638	A	4	3	1.00	26	0.115
639	A	4	3	1.00	26	0.115
640	A	4	3	1.00	26	0.115
641	A	4	4	1.00	26	0.154
642	A	4	3	1.00	22	0.136
643	A	5	4	1.00	26	0.154
644	A	6	4	1.00	26	0.154
645	A	4	3	1.00	26	0.115
646	A	4	3	1.00	26	0.115
647	A	3	3	1.00	26	0.115
648	A	1	1	1.00	26	0.038
649	A	3	3	1.00	26	0.115
650	A	2	2	1.00	24	0.083
651	A	4	3	1.00	26	0.115
652	A	4	3	1.00	26	0.115
653	A	6	4	1.00	26	0.154
654	A	6	4	1.00	26	0.154
655	A	6	4	1.00	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
656	A	6	3	1.00	22	0.136
657	A	7	4	1.00	26	0.154
658	A	8	4	1.00	26	0.154
659	A	4	4	1.00	26	0.154
660	A	3	3	1.00	22	0.136
661	A	4	4	1.00	26	0.154
662	A	6	6	1.00	26	0.231
663	A	6	6	1.00	22	0.273
664	A	7	7	1.00	26	0.269
665	A	2	1	1.00	26	0.038
666	A	2	1	1.00	26	0.038
667	A	2	1	1.00	26	0.038
668	A	2	1	1.00	26	0.038
669	A	2	1	1.00	26	0.038
670	A	2	1	1.00	26	0.038
671	A	2	1	1.00	26	0.038
672	A	3	2	1.00	28	0.071
673	A	3	2	1.00	28	0.071
674	A	3	2	1.00	28	0.071
675	A	3	2	1.00	28	0.071
676	A	3	2	1.00	28	0.071
677	A	3	2	1.00	28	0.071
678	A	3	2	1.00	28	0.071
679	A	3	2	1.00	28	0.071
680	A	3	2	1.00	28	0.071
681	A	3	2	1.00	28	0.071
682	A	3	2	1.00	28	0.071
683	A	3	2	1.00	28	0.071
684	A	3	2	1.00	28	0.071
685	A	3	2	1.00	28	0.071
686	A	14	10	1.00	28	0.357
687	A	13	10	1.00	28	0.357
688	A	13	10	1.00	28	0.357
689	A	12	9	1.00	28	0.321
690	A	12	9	1.00	28	0.321

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	12	9	1.00	28	0.321
692	A	12	9	1.00	28	0.321
693	A	13	10	1.00	28	0.357
694	A	13	10	1.00	28	0.357
695	A	14	10	1.00	28	0.357
696	A	16	10	1.00	28	0.357
697	A	15	10	1.00	28	0.357
698	A	15	10	1.00	28	0.357
699	A	14	9	1.00	28	0.321
700	A	14	9	1.00	28	0.321
701	A	14	10	1.00	28	0.357
702	A	14	10	1.00	28	0.357
703	A	14	10	1.00	28	0.357
704	A	14	10	1.00	28	0.357
705	A	14	9	1.00	28	0.321
706	A	14	9	1.00	28	0.321
707	A	15	10	1.00	28	0.357
708	A	15	10	1.00	28	0.357
709	A	16	10	1.00	28	0.357
710	A	18	10	1.00	28	0.357
711	A	17	10	1.00	28	0.357
712	A	17	10	1.00	28	0.357
713	A	16	9	1.00	28	0.321
714	A	16	9	1.00	28	0.321
715	A	16	10	1.00	28	0.357
716	A	16	10	1.00	28	0.357
717	A	16	10	1.00	28	0.357
718	A	16	10	1.00	28	0.357
719	A	16	10	1.00	28	0.357
720	A	16	10	1.00	28	0.357
721	A	16	10	1.00	28	0.357
722	A	16	10	1.00	28	0.357
723	A	16	9	1.00	28	0.321
724	A	16	9	1.00	28	0.321
725	A	17	10	1.00	28	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	17	10	1.00	28	0.357
727	A	18	10	1.00	28	0.357
728	A	3	2	1.00	30	0.067
729	A	3	2	1.00	30	0.067
730	A	3	2	1.00	30	0.067
731	A	3	2	1.00	30	0.067
732	A	3	2	1.00	30	0.067
733	A	3	2	1.00	30	0.067
734	A	3	2	1.00	30	0.067
735	A	3	2	1.00	30	0.067
736	A	3	2	1.00	30	0.067
737	A	3	2	1.00	30	0.067
738	A	3	2	1.00	30	0.067
739	A	3	2	1.00	30	0.067
740	A	3	2	1.00	30	0.067
741	A	3	2	1.00	30	0.067
742	A	3	2	1.00	30	0.067
743	A	3	2	1.00	30	0.067
744	A	3	2	1.00	30	0.067
745	A	3	2	1.00	30	0.067
746	A	3	2	1.00	30	0.067
747	A	3	2	1.00	30	0.067
748	A	3	2	1.00	30	0.067
749	A	13	9	1.00	30	0.300
750	A	12	9	1.00	30	0.300
751	A	12	9	1.00	30	0.300
752	A	11	8	1.00	30	0.267
753	A	11	8	1.00	30	0.267
754	A	12	9	1.00	30	0.300
755	A	12	9	1.00	30	0.300
756	A	13	9	1.00	30	0.300
757	A	15	10	1.00	30	0.333
758	A	14	10	1.00	30	0.333
759	A	14	10	1.00	30	0.333
760	A	13	9	1.00	30	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	13	9	1.00	30	0.300
762	A	13	10	1.00	30	0.333
763	A	13	10	1.00	30	0.333
764	A	13	9	1.00	30	0.300
765	A	13	9	1.00	30	0.300
766	A	14	10	1.00	30	0.333
767	A	14	10	1.00	30	0.333
768	A	15	10	1.00	30	0.333
769	A	17	10	1.00	30	0.333
770	A	16	10	1.00	30	0.333
771	A	16	10	1.00	30	0.333
772	A	15	9	1.00	30	0.300
773	A	15	9	1.00	30	0.300
774	A	15	10	1.00	30	0.333
775	A	15	10	1.00	30	0.333
776	A	15	10	1.00	30	0.333
777	A	15	10	1.00	30	0.333
778	A	15	10	1.00	30	0.333
779	A	15	10	1.00	30	0.333
780	A	15	9	1.00	30	0.300
781	A	15	9	1.00	30	0.300
782	A	16	10	1.00	30	0.333
783	A	16	10	1.00	30	0.333
784	A	17	10	1.00	30	0.333
785	A	3	2	1.00	26	0.077
786	A	3	2	1.00	26	0.077
787	A	2	1	1.00	24	0.042
788	A	2	2	1.00	26	0.077
789	A	2	2	1.00	26	0.077
790	A	2	2	1.00	26	0.077
791	A	3	2	1.00	28	0.071
792	A	3	2	1.00	28	0.071
793	A	3	2	1.00	28	0.071
794	A	2	2	1.00	28	0.071
795	A	2	2	1.00	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	2	2	1.00	28	0.071
797	A	2	2	1.04	26	0.077
798	A	4	3	1.00	24	0.125
799	A	4	3	1.00	24	0.125
800	A	4	3	1.00	24	0.125
801	A	2	2	1.00	22	0.091
802	A	3	3	1.00	24	0.125
803	A	3	3	1.00	24	0.125
804	A	2	2	1.00	24	0.083
805	A	2	2	1.00	24	0.083
806	A	2	2	1.00	20	0.100
807	A	2	2	1.00	24	0.083
808	A	2	2	1.00	24	0.083
809	A	2	2	1.00	28	0.071
810	A	2	2	1.00	28	0.071
811	A	2	2	1.00	28	0.071
812	A	2	2	1.00	28	0.071
813	A	2	2	1.00	28	0.071
814	A	2	1	1.00	16	0.062
815	A	2	1	1.00	14	0.071
816	A	1	0	1.00	12	0.000
817	A	2	1	1.00	16	0.062
818	A	2	1	1.00	16	0.062
819	A	2	1	1.00	16	0.062
820	A	2	1	1.00	16	0.062
821	A	2	1	1.00	16	0.062
822	A	2	1	1.00	16	0.062
823	A	2	1	1.00	16	0.062
824	A	2	1	1.00	16	0.062
825	A	2	1	1.00	18	0.056
826	A	3	2	1.00	16	0.125
827	A	2	1	1.00	14	0.071
828	A	3	2	1.00	18	0.111
829	A	2	1	1.00	18	0.056
830	A	3	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
831	A	2	1	1.00	18	0.056
832	A	3	2	1.00	18	0.111
833	A	2	1	1.00	18	0.056
834	A	3	2	1.00	18	0.111
835	A	2	1	1.00	18	0.056
836	A	3	2	1.00	18	0.111
837	A	2	1	1.00	18	0.056
838	A	3	2	1.00	18	0.111
839	A	2	1	1.00	18	0.056
840	A	3	2	1.00	18	0.111
841	A	2	1	1.00	18	0.056
842	A	3	2	1.00	16	0.125
843	A	2	1	1.00	14	0.071
844	A	3	2	1.00	18	0.111
845	A	2	1	1.00	18	0.056
846	A	3	2	1.00	18	0.111
847	A	2	1	1.00	18	0.056
848	A	7	6	1.00	18	0.333
849	A	6	6	1.00	18	0.333
850	A	5	5	1.00	18	0.278
851	A	3	3	1.00	16	0.188
852	A	7	7	1.00	18	0.389
853	A	8	7	1.00	18	0.389
854	A	8	7	1.00	18	0.389
855	A	5	4	1.00	18	0.222
856	A	4	3	1.00	18	0.167
857	A	3	2	1.00	18	0.111
858	A	3	2	1.00	14	0.143
859	A	4	3	1.00	18	0.167
860	A	5	4	1.00	18	0.222
861	A	7	7	1.00	18	0.389
862	A	4	4	1.00	18	0.222
863	A	4	4	1.00	18	0.222
864	A	4	4	1.00	16	0.250
865	A	8	7	1.00	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	8	7	1.00	18	0.389
867	A	6	4	1.00	18	0.222
868	A	5	4	1.00	18	0.222
869	A	4	3	1.00	18	0.167
870	A	4	3	1.00	18	0.167
871	A	4	3	1.00	14	0.214
872	A	5	4	1.00	18	0.222
873	A	8	8	1.00	18	0.444
874	A	5	4	1.00	18	0.222
875	A	5	5	1.00	18	0.278
876	A	5	5	1.00	18	0.278
877	A	5	5	1.00	18	0.278
878	A	5	4	1.00	16	0.250
879	A	9	8	1.00	18	0.444
880	A	9	8	1.00	18	0.444
881	A	7	5	1.00	18	0.278
882	A	6	5	1.00	18	0.278
883	A	5	4	1.00	18	0.222
884	A	5	4	1.00	18	0.222
885	A	5	4	1.00	18	0.222
886	A	5	4	1.00	14	0.286
887	A	6	5	1.00	18	0.278
888	A	6	6	1.00	19	0.316
889	A	5	5	1.00	19	0.263
890	A	3	3	1.00	17	0.176
891	A	7	7	1.00	19	0.368
892	A	8	7	1.00	19	0.368
893	A	4	3	1.00	19	0.158
894	A	3	2	1.00	19	0.105
895	A	3	2	1.00	15	0.133
896	A	4	3	1.00	19	0.158
897	A	6	6	1.00	22	0.273
898	A	5	5	1.00	22	0.227
899	A	3	3	1.00	20	0.150
900	A	7	7	1.00	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
901	A	8	7	1.00	22	0.318
902	A	4	3	1.00	22	0.136
903	A	3	2	1.00	22	0.091
904	A	3	2	1.00	18	0.111
905	A	4	3	1.00	22	0.136
906	A	6	6	1.00	20	0.300
907	A	5	5	1.00	20	0.250
908	A	3	3	1.00	18	0.167
909	A	7	7	1.00	20	0.350
910	A	8	7	1.00	20	0.350
911	A	10	6	1.00	20	0.300
912	A	9	5	1.00	20	0.250
913	A	9	5	1.00	16	0.312
914	A	10	6	1.00	20	0.300
915	A	3	3	1.00	12	0.250
916	A	3	3	1.00	14	0.214
917	A	3	2	1.00	16	0.125
918	A	9	6	1.00	16	0.375
919	A	9	6	1.00	16	0.375
920	A	6	6	1.00	20	0.300
921	A	6	6	1.00	20	0.300
922	A	5	5	1.00	20	0.250
923	A	4	4	1.00	18	0.222
924	A	7	6	1.00	20	0.300
925	A	7	6	1.00	20	0.300
926	A	4	4	1.00	20	0.200
927	A	5	5	1.00	20	0.250
928	A	6	6	1.00	20	0.300
929	A	7	7	1.00	20	0.350
930	A	5	5	1.00	20	0.250
931	A	4	4	1.00	20	0.200
932	A	4	4	1.00	16	0.250
933	A	4	4	1.00	20	0.200
934	A	5	5	1.00	20	0.250
935	A	6	5	1.00	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	7	6	1.00	20	0.300
937	A	7	6	1.00	20	0.300
938	A	6	5	1.00	20	0.250
939	A	5	4	1.00	18	0.222
940	A	8	7	1.00	20	0.350
941	A	8	7	1.00	20	0.350
942	A	8	7	1.00	20	0.350
943	A	8	7	1.00	20	0.350
944	A	5	4	1.00	20	0.200
945	A	6	5	1.00	20	0.250
946	A	7	6	1.00	20	0.300
947	A	6	6	1.00	20	0.300
948	A	5	5	1.00	20	0.250
949	A	5	5	1.00	16	0.312
950	A	5	5	1.00	20	0.250
951	A	5	5	1.00	20	0.250
952	A	6	6	1.00	20	0.300
953	A	7	6	1.00	20	0.300
954	A	5	5	1.00	16	0.312
955	A	5	5	1.00	20	0.250
956	A	5	5	1.00	20	0.250
957	A	4	4	1.00	20	0.200
958	A	3	3	1.00	18	0.167
959	A	3	3	1.00	20	0.150
960	A	4	4	1.00	20	0.200
961	A	5	5	1.00	20	0.250
962	A	6	6	1.00	20	0.300
963	A	4	4	1.00	20	0.200
964	A	3	3	1.00	20	0.150
965	A	1	1	1.00	16	0.062
966	A	5	5	1.00	20	0.250
967	A	5	5	1.00	20	0.250
968	A	5	5	1.00	21	0.238
969	A	5	5	1.00	21	0.238
970	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
971	A	3	3	1.00	19	0.158
972	A	3	3	1.00	22	0.136
973	A	4	4	1.00	22	0.182
974	A	5	5	1.00	22	0.227
975	A	6	6	1.00	22	0.273
976	A	5	5	1.00	21	0.238
977	A	4	4	1.00	21	0.190
978	A	2	2	1.00	17	0.118
979	A	6	6	1.00	21	0.286
980	A	6	6	1.00	21	0.286
981	A	6	6	1.00	20	0.300
982	A	5	5	1.00	20	0.250
983	A	5	5	1.00	20	0.250
984	A	2	2	1.00	20	0.100
985	A	2	2	1.00	18	0.111
986	A	5	5	1.00	20	0.250
987	A	5	5	1.00	20	0.250
988	A	6	6	1.00	20	0.300
989	A	5	5	1.00	20	0.250
990	A	4	4	1.00	20	0.200
991	A	4	4	1.00	20	0.200
992	A	4	4	1.00	16	0.250
993	A	5	5	1.00	20	0.250
994	A	3	3	1.00	28	0.107
995	A	5	5	1.00	28	0.179
996	A	2	2	1.00	28	0.071
997	A	4	4	1.00	26	0.154
998	A	3	3	1.00	24	0.125
999	A	2	2	1.00	28	0.071
1000	A	4	4	1.00	28	0.143
1001	A	3	3	1.00	28	0.107
1002	A	5	4	1.00	28	0.143
1003	A	3	3	1.00	29	0.103
1004	A	2	2	1.00	29	0.069
1005	A	4	4	1.00	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1006	A	4	4	1.00	27	0.148
1007	A	2	2	1.00	25	0.080
1008	A	4	4	1.00	29	0.138
1009	A	5	5	1.00	29	0.172
1010	A	2	2	1.00	29	0.069
1011	A	3	3	1.00	29	0.103
1012	A	5	4	1.00	29	0.138
1013	A	4	3	1.00	29	0.103
1014	A	4	4	1.00	29	0.138
1015	A	2	2	1.00	27	0.074
1016	A	3	3	1.00	25	0.120
1017	A	4	4	1.00	29	0.138
1018	A	2	2	1.00	29	0.069
1019	A	5	5	1.00	29	0.172
1020	A	3	3	1.00	29	0.103
1021	A	3	3	1.00	23	0.130
1022	A	3	3	1.00	23	0.130
1023	A	3	3	1.00	23	0.130
1024	A	3	3	1.00	21	0.143
1025	A	3	3	1.00	19	0.158
1026	A	3	3	1.00	23	0.130
1027	A	3	3	1.00	23	0.130
1028	A	3	3	1.00	23	0.130
1029	A	3	3	1.00	23	0.130
1030	A	3	3	1.00	24	0.125
1031	A	3	3	1.00	24	0.125
1032	A	3	3	1.00	24	0.125
1033	A	3	3	1.00	22	0.136
1034	A	2	2	1.00	20	0.100
1035	A	3	3	1.00	24	0.125
1036	A	3	3	1.00	24	0.125
1037	A	3	3	1.00	24	0.125
1038	A	3	3	1.00	24	0.125
1039	A	2	2	1.00	16	0.125
1040	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1041	A	2	1	1.00	18	0.056
1042	A	2	1	1.00	18	0.056
1043	A	2	1	1.00	18	0.056
1044	A	2	1	1.00	18	0.056
1045	A	2	1	1.00	18	0.056
1046	A	2	1	1.00	18	0.056
1047	A	2	1	1.00	18	0.056
1048	A	2	1	1.00	20	0.050
1049	A	2	1	1.00	20	0.050
1050	A	2	1	1.00	20	0.050
1051	A	2	1	1.00	20	0.050
1052	A	2	1	1.00	20	0.050
1053	A	2	1	1.00	20	0.050
1054	A	2	1	1.00	20	0.050
1055	A	2	1	1.00	20	0.050
1056	A	2	1	1.00	20	0.050
1057	A	2	1	1.00	20	0.050
1058	A	2	1	1.00	20	0.050
1059	A	2	1	1.00	20	0.050
1060	A	2	1	1.00	20	0.050
1061	A	2	1	1.00	20	0.050
1062	A	9	6	1.00	20	0.300
1063	A	9	6	1.00	20	0.300
1064	A	8	5	1.00	20	0.250
1065	A	8	5	1.00	20	0.250
1066	A	8	5	1.00	20	0.250
1067	A	8	5	1.00	20	0.250
1068	A	9	6	1.00	20	0.300
1069	A	9	6	1.00	20	0.300
1070	A	10	7	1.00	20	0.350
1071	A	10	7	1.00	20	0.350
1072	A	10	7	1.00	20	0.350
1073	A	9	6	1.00	20	0.300
1074	A	9	6	1.00	20	0.300
1075	A	9	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1076	A	9	6	1.00	20	0.300
1077	A	9	6	1.00	20	0.300
1078	A	9	6	1.00	20	0.300
1079	A	10	7	1.00	20	0.350
1080	A	11	8	1.00	20	0.400
1081	A	10	7	1.00	20	0.350
1082	A	10	7	1.00	20	0.350
1083	A	10	7	1.00	20	0.350
1084	A	10	7	1.00	20	0.350
1085	A	10	7	1.00	20	0.350
1086	A	10	7	1.00	20	0.350
1087	A	10	7	1.00	20	0.350
1088	A	10	7	1.00	20	0.350
1089	A	2	2	1.00	24	0.083
1090	A	2	2	1.00	24	0.083
1091	A	2	2	1.00	24	0.083
1092	A	2	2	1.00	24	0.083
1093	A	2	2	1.00	24	0.083
1094	A	2	2	1.00	24	0.083
1095	A	2	2	1.00	24	0.083
1096	A	2	2	1.00	24	0.083
1097	A	2	2	1.00	24	0.083
1098	A	2	2	1.00	24	0.083
1099	A	2	2	1.00	24	0.083
1100	A	2	2	1.00	24	0.083
1101	A	2	2	1.00	24	0.083
1102	A	2	2	1.00	24	0.083
1103	A	2	2	1.00	24	0.083
1104	A	2	2	1.00	24	0.083
1105	A	2	1	1.00	20	0.050
1106	A	2	1	1.00	20	0.050
1107	A	2	1	1.00	18	0.056
1108	A	3	2	1.00	20	0.100
1109	A	4	3	1.00	20	0.150
1110	A	2	2	1.00	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1111	A	2	2	1.00	22	0.091
1112	A	2	2	1.00	22	0.091
1113	A	2	2	1.00	22	0.091
1114	A	2	2	1.00	20	0.100
1115	A	4	4	1.00	18	0.222
1116	A	4	4	1.00	18	0.222
1117	A	3	3	1.00	18	0.167
1118	A	2	2	1.00	16	0.125
1119	A	3	3	1.00	18	0.167
1120	A	3	3	1.00	18	0.167
1121	A	3	3	1.00	18	0.167
1122	A	2	2	1.00	18	0.111
1123	A	2	2	1.00	18	0.111
1124	A	2	2	1.00	14	0.143
1125	A	2	2	1.00	18	0.111
1126	A	2	2	1.00	18	0.111

Chapter 3

Listing of integrals

Local contents

3.1 $\int (a^2+2abx^2+b^2x^4)^{3/4}\,dx$

Optimal. Leaf size=128

$$\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3ax(a^2+2abx^2+b^2x^4)^{3/4}}{8\,(a+bx^2)}+\frac{3\sqrt{a}\,(a^2+2abx^2+b^2x^4)^{3/4}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}\,\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

```
[Out] 1/4*x*(b^2*x^4+2*a*b*x^2+a^2)^(3/4)+3/8*a*x*(b^2*x^4+2*a*b*x^2+a^2)^(3/4)/(
b*x^2+a)+3/8*(b^2*x^4+2*a*b*x^2+a^2)^(3/4)*arcsinh(x*b^(1/2)/a^(1/2))*a^(1/
2)/(1+b*x^2/a)^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1103, 201, 221}

$$\frac{3ax(a^2+2abx^2+b^2x^4)^{3/4}}{8\,(a+bx^2)}+\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3\sqrt{a}\,(a^2+2abx^2+b^2x^4)^{3/4}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}\,\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))/4 + (3*a*x*(a^2 + 2*a*b*x^2 + b^2*x^4
)^(3/4))/(8*(a + b*x^2)) + (3*Sqrt[a]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)*Arc
Sinh[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[b]*(1 + (b*x^2)/a)^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1103

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[a^IntPart[
p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2*FracPart[p])), Int
[(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
```

```
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}\,dx=\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/4}\int \left(1+\frac{bx^2}{a}\right)^{3/2}\,dx}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$=\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{\left(3\left(a^2+2abx^2+b^2x^4\right)^{3/4}\right)\int \sqrt{1+\frac{bx^2}{a}}\,dx}{4\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

$$=\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)}+\frac{\left(3\left(a^2+2abx^2+\right.\right.}{8}$$

$$=\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)}+\frac{3\sqrt{a}\left(a^2+2abx^2\right.}{8\sqrt{}}$$

Mathematica [A]
time = 0.08, size = 88, normalized size = 0.69

$$\frac{\left(\left(a+bx^2\right)^2\right)^{3/4}\left(\sqrt{b}\,x\sqrt{a+bx^2}\left(5a+2bx^2\right)-3a^2\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)\right)}{8\sqrt{b}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
[Out] (((a + b*x^2)^2)^(3/4)*(Sqrt[b]*x*Sqrt[a + b*x^2]*(5*a + 2*b*x^2) - 3*a^2*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]))/(8*Sqrt[b]*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.02, size = 77, normalized size = 0.60

method	result	size
risch	$\frac{x\left(2b\,x^2+5a\right)\left(b\,x^2+a\right)}{8\left(\left(b\,x^2+a\right)^2\right)^{\frac{1}{4}}}+\frac{3a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)\sqrt{b\,x^2+a}}{8\sqrt{b}\left(\left(b\,x^2+a\right)^2\right)^{\frac{1}{4}}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(2*b*x^2+5*a)*(b*x^2+a)/((b*x^2+a)^2)^(1/4)+3/8*a^2*ln(x*b^(1/2)+(b*x
^2+a)^(1/2))/b^(1/2)/((b*x^2+a)^2)^(1/4)*(b*x^2+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/4), x)
```

Fricas [A]
time = 0.37, size = 177, normalized size = 1.38

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,bx^2-2\,(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{b}\,x-a\right)+2\,(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}(2\,b^2x^3+5\,abx)}{16\,b},-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{-b}\,x}{bx^2+a}\right)-(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}(2\,b^2x^3+5\,abx)}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqr
t(b)*x - a) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b^2*x^3 + 5*a*b*x))/b,
 -1/8*(3*a^2*sqrt(-b)*arctan((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(
b*x^2 + a)) - (b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b^2*x^3 + 5*a*b*x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4), x)
```

Giac [A]
time = 4.38, size = 59, normalized size = 0.46

$$-\frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{-bx^2-a}\,x-\frac{3\,a^2\log\left(\left|-\sqrt{-b}\,x+\sqrt{-bx^2-a}\right|\right)}{8\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="giac")
```

```
[Out] -1/8*(2*b*x^2 + 5*a)*sqrt(-b*x^2 - a)*x - 3/8*a^2*log(abs(-sqrt(-b)*x + sqr
t(-b*x^2 - a)))/sqrt(-b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4), x)
```

3.2 $\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=91

$$\frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4}+\frac{\sqrt{a}\,\sqrt[4]{a^2+2abx^2+b^2x^4}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}\,\sqrt{1+\frac{bx^2}{a}}}$$

```
[Out] 1/2*x*(b^2*x^4+2*a*b*x^2+a^2)^(1/4)+1/2*(b^2*x^4+2*a*b*x^2+a^2)^(1/4)*arcsi
nh(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)/(1+b*x^2/a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1103, 201, 221}

$$\frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4}+\frac{\sqrt{a}\,\sqrt[4]{a^2+2abx^2+b^2x^4}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}\,\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))/2 + (Sqrt[a]*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(1/4)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[b]*Sqrt[1 + (b*x^2)/a])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1103

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[a^IntPart[
p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2*FracPart[p])), Int
[(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
```

```
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt[4]{a^2+2abx^2+b^2x^4}\int \sqrt{1+\frac{bx^2}{a}}\,dx}{\sqrt{1+\frac{bx^2}{a}}}$$

$$= \frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4} + \frac{\sqrt[4]{a^2+2abx^2+b^2x^4}\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{2\sqrt{1+\frac{bx^2}{a}}}$$

$$= \frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4} + \frac{\sqrt{a}\sqrt[4]{a^2+2abx^2+b^2x^4}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{1+\frac{bx^2}{a}}}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.65

$$\frac{1}{2}\sqrt[4]{(a+bx^2)^2}\left(x-\frac{a\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{\sqrt{b}\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
[Out] (((a + b*x^2)^2)^(1/4)*(x - (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(Sqrt[b
]*Sqrt[a + b*x^2])))/2
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.64

method	result	size
risch	$\frac{x\left((b\,x^2+a)^2\right)^{\frac{1}{4}}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)\left((b\,x^2+a)^2\right)^{\frac{1}{4}}}{2\sqrt{b}\,\sqrt{b\,x^2+a}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*((b*x^2+a)^2)^(1/4)+1/2*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)*((b*x
^2+a)^2)^(1/4)/(b*x^2+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4), x)
```

Fricas [A]
time = 0.35, size = 147, normalized size = 1.62

$$\left[\frac{a\sqrt{b}\log\left(-2\,bx^2-2\,(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{b}\,x-a\right)+2\,(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}bx}{4\,b}, -\frac{a\sqrt{-b}\,\arctan\left(\frac{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{-b}\,x}{bx^2+a}\right)-(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}bx}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(b)*
x - a) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*b*x)/b, -1/2*(a*sqrt(-b)*arcta
n((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(b*x^2 + a)) - (b^2*x^4 + 2*
a*b*x^2 + a^2)^(1/4)*b*x)/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt[4]{a^2+2abx^2+b^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4),x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4), x)
```

3.3 $\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

```
[Out] arcsinh(x*b^(1/2)/a^(1/2))*a^(1/2)*(1+b*x^2/a)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2
)^(1/4)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1103, 221}

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(a^2 +
2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1103

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[a^IntPart[
p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2*FracPart[p])), Int
[(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt[4]{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\,\sqrt[4]{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
[Out] (Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*((a + b*x^2
)^2)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^2x^4+2ab\,x^2+a^2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/4), x)
```

Fricas [A]
time = 0.36, size = 90, normalized size = 1.50

$$\left[\frac{\log\left(-2\,bx^2-2\,(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{b}\,x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}\sqrt{-b}\,x}{bx^2+a}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(b)*x - a)/sqrt
(b), -sqrt(-b)*arctan((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*sqrt(-b)*x/(b*x^2 +
 a))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4),x)
```

```
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4), x)
```

3.4 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/4}}\,dx$

Optimal. Leaf size=34

$$\frac{x(a+bx^2)}{a\,(a^2+2abx^2+b^2x^4)^{3/4}}$$

[Out] x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1103, 197}

$$\frac{x(a+bx^2)}{a\,(a^2+2abx^2+b^2x^4)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]

[Out] (x*(a + b*x^2))/(a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1103

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[a^IntPart[
p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2*FracPart[p])), Int
[(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/4}}\,dx &= \frac{\left(1+\frac{bx^2}{a}\right)^{3/2}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{(a^2+2abx^2+b^2x^4)^{3/4}}\\ &= \frac{x(a+bx^2)}{a\,(a^2+2abx^2+b^2x^4)^{3/4}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.74

$$\frac{x(a+bx^2)}{a\left((a+bx^2)^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
[Out] (x*(a + b*x^2))/(a*((a + b*x^2)^2)^(3/4))
```

Maple [A]
time = 0.02, size = 33, normalized size = 0.97

method	result	size
gosper	$\frac{x\left(b\,x^{2}+a\right)}{a\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{4}}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x,method=_RETURNVERBOSE)
[Out] x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/4), x)
```

Fricas [A]
time = 0.34, size = 34, normalized size = 1.00

$$\frac{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{4}}x}{abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="fricas")
[Out] (b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*x/(a*b*x^2 + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/4), x)
```

Mupad [B]
time = 4.14, size = 34, normalized size = 1.00

$$\frac{x\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{1/4}}{a\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4),x)
```

```
[Out] (x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4))/(a*(a + b*x^2))
```

3.5 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx$

Optimal. Leaf size=68

$$\frac{x(a+bx^2)}{3a\left(a^2+2abx^2+b^2x^4\right)^{5/4}}+\frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/3*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)+2/3*x/a^2/(b^2*x^4+2*a*b*x^
2+a^2)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1103, 198, 197}

$$\frac{x}{3a\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
```

```
[Out] (2*x)/(3*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + x/(3*a*(a + b*x^2)*(a^2 +
 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1103

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[a^IntPart[
p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2*FracPart[p])), Int
[(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/2}}\,dx}{a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{x}{3a\,(a+bx^2)\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{\left(2\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/2}}\,dx}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{x}{3a\,(a+bx^2)\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 0.56

$$\frac{(a+bx^2)\,(3ax+2bx^3)}{3a^2\left((a+bx^2)^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
```

```
[Out] ((a + b*x^2)*(3*a*x + 2*b*x^3))/(3*a^2*((a + b*x^2)^2)^(5/4))
```

Maple [A]
time = 0.02, size = 44, normalized size = 0.65

method	result	size
gosper	$\frac{(b\,x^2+a)\,x\,(2b\,x^2+3a)}{3a^2\,(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{4}}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)*x*(2*b*x^2+3*a)/a^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

Fricas [A]
time = 0.43, size = 58, normalized size = 0.85

$$\frac{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{1}{4}}(2\,bx^3 + 3\,ax)}{3\,(a^2b^2x^4 + 2\,a^3bx^2 + a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b*x^3 + 3*a*x)/(a^2*b^2*x^4 + 2*a^
3*b*x^2 + a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

Mupad [B]
time = 4.20, size = 45, normalized size = 0.66

$$\frac{x\,(2\,b\,x^2 + 3\,a)\,\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/4}}{3\,a^2\,\left(b\,x^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/4),x)
```

```
[Out] (x*(3*a + 2*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4))/(3*a^2*(a + b*x^2)^3)
```

3.6 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{7/4}}\,dx$

Optimal. Leaf size=105

$$\frac{x(a+bx^2)}{5a\left(a^2+2abx^2+b^2x^4\right)^{7/4}}+\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{8x(a+bx^2)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

```
[Out] 1/5*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(7/4)+4/15*x/a^2/(b^2*x^4+2*a*b*x
^2+a^2)^(3/4)+8/15*x*(b*x^2+a)/a^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1103, 198, 197}

$$\frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{8x(a+bx^2)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

```
[Out] (4*x)/(15*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)) + x/(5*a*(a + b*x^2)*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(3/4)) + (8*x*(a + b*x^2))/(15*a^3*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(3/4))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1103

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[a^IntPart[
p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2*FracPart[p])), Int
[(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{7/4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{3/2}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{7/2}}\,dx}{a^2\,(a^2+2abx^2+b^2x^4)^{3/4}}$$
$$= \frac{x}{5a\,(a+bx^2)\,(a^2+2abx^2+b^2x^4)^{3/4}} + \frac{\left(4\left(1+\frac{bx^2}{a}\right)^{3/2}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/2}}\,dx}{5a^2\,(a^2+2abx^2+b^2x^4)^{3/4}}$$
$$= \frac{4x}{15a^2\,(a^2+2abx^2+b^2x^4)^{3/4}} + \frac{x}{5a\,(a+bx^2)\,(a^2+2abx^2+b^2x^4)^{3/4}} + \frac{\left(8\left(1+\right.\right.}{15a^2}$$
$$= \frac{4x}{15a^2\,(a^2+2abx^2+b^2x^4)^{3/4}} + \frac{x}{5a\,(a+bx^2)\,(a^2+2abx^2+b^2x^4)^{3/4}} + \frac{}{15a^3\,(a^2}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.47

$$\frac{(a+bx^2)\,(15a^2x+20abx^3+8b^2x^5)}{15a^3\,\left((a+bx^2)^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

```
[Out] ((a + b*x^2)*(15*a^2*x + 20*a*b*x^3 + 8*b^2*x^5))/(15*a^3*((a + b*x^2)^2)^(
7/4))
```

Maple [A]
time = 0.02, size = 55, normalized size = 0.52

method	result	size
gosper	$\frac{(b\,x^2+a)\,x\,(8b^2x^4+20ab\,x^2+15a^2)}{15a^3(b^2x^4+2ab\,x^2+a^2)^{\frac{7}{4}}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(b*x^2+a)*x*(8*b^2*x^4+20*a*b*x^2+15*a^2)/a^3/(b^2*x^4+2*a*b*x^2+a^2)^
(7/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

Fricas [A]
time = 0.37, size = 80, normalized size = 0.76

$$\frac{(8\,b^2x^5 + 20\,abx^3 + 15\,a^2x)(b^2x^4 + 2\,abx^2 + a^2)^{\frac{1}{4}}}{15\,(a^3b^3x^6 + 3\,a^4b^2x^4 + 3\,a^5bx^2 + a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/15*(8*b^2*x^5 + 20*a*b*x^3 + 15*a^2*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)/
(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(7/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-7/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

Mupad [B]
time = 4.21, size = 56, normalized size = 0.53

$$\frac{x\,(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{1/4}\,(15\,a^2 + 20\,a\,b\,x^2 + 8\,b^2\,x^4)}{15\,a^3\,(b\,x^2 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(7/4),x)
```

```
[Out] (x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/4)*(15*a^2 + 8*b^2*x^4 + 20*a*b*x^2))/(15
*a^3*(a + b*x^2)^3)
```

3.7 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{9/4}}\,dx$

Optimal. Leaf size=135

$$\frac{x(a+bx^2)}{7a\,(a^2+2abx^2+b^2x^4)^{9/4}}+\frac{6x}{35a^2\,(a^2+2abx^2+b^2x^4)^{5/4}}+\frac{8x(a+bx^2)}{35a^3\,(a^2+2abx^2+b^2x^4)^{5/4}}+\frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/7*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(9/4)+6/35*x/a^2/(b^2*x^4+2*a*b*x
^2+a^2)^(5/4)+8/35*x*(b*x^2+a)/a^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)+16/35*x/a^
4/(b^2*x^4+2*a*b*x^2+a^2)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1103, 198, 197}

$$\frac{6x}{35a^2\,(a+bx^2)^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{x}{7a\,(a+bx^2)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{8x}{35a^3\,(a+bx^2)\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

```
[Out] (16*x)/(35*a^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + x/(7*a*(a + b*x^2)^3*(a
^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + (6*x)/(35*a^2*(a + b*x^2)^2*(a^2 + 2*a*b
*x^2 + b^2*x^4)^(1/4)) + (8*x)/(35*a^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(1/4))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1103

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[a^IntPart[
p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2*FracPart[p])), Int
[(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{9/4}}\,dx = \frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{9/2}}\,dx}{a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{x}{7a\,(a+bx^2)^3\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{\left(6\sqrt{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{7/2}}\,dx}{7a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{x}{7a\,(a+bx^2)^3\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{6x}{35a^2\,(a+bx^2)^2\sqrt[4]{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{x}{7a\,(a+bx^2)^3\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{6x}{35a^2\,(a+bx^2)^2\sqrt[4]{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{x}{7a\,(a+bx^2)^3\sqrt[4]{a^2+2abx^2+b^2x^4}} + \frac{}{35a^2\,(a+}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 0.44

$$\frac{(a+bx^2)\,(35a^3x+70a^2bx^3+56ab^2x^5+16b^3x^7)}{35a^4\left((a+bx^2)^2\right)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
[Out] ((a + b*x^2)*(35*a^3*x + 70*a^2*b*x^3 + 56*a*b^2*x^5 + 16*b^3*x^7))/(35*a^4
*((a + b*x^2)^2)^(9/4))
```

Maple [A]
time = 0.02, size = 66, normalized size = 0.49

method	result	size
gosper	$\frac{(b\,x^2+a)\,x\,(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3)}{35a^4(b^2x^4+2ab\,x^2+a^2)^{\frac{9}{4}}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(b*x^2+a)*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/(b^2*x^4
+2*a*b*x^2+a^2)^(9/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

Fricas [A]
time = 0.38, size = 102, normalized size = 0.76

$$\frac{(16\,b^3x^7 + 56\,ab^2x^5 + 70\,a^2bx^3 + 35\,a^3x)(b^2x^4 + 2\,abx^2 + a^2)^{\frac{1}{4}}}{35\,(a^4b^4x^8 + 4\,a^5b^3x^6 + 6\,a^6b^2x^4 + 4\,a^7bx^2 + a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*(b^2*x^4 + 2*a*b
*x^2 + a^2)^(1/4)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^
2 + a^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(9/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-9/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

Mupad [B]
time = 4.13, size = 141, normalized size = 1.04

$$\frac{x\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/4}}{7\,a\,{\left(b\,x^2+a\right)}^5}+\frac{6\,x\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/4}}{35\,a^2\,{\left(b\,x^2+a\right)}^4}+\frac{8\,x\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/4}}{35\,a^3\,{\left(b\,x^2+a\right)}^3}+\frac{16\,x\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/4}}{35\,a^4\,{\left(b\,x^2+a\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(9/4),x)
```

```
[Out] (x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/4))/(7*a*(a + b*x^2)^5) + (6*x*(a^2 + b^2
*x^4 + 2*a*b*x^2)^(3/4))/(35*a^2*(a + b*x^2)^4) + (8*x*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(3/4))/(35*a^3*(a + b*x^2)^3) + (16*x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
3/4))/(35*a^4*(a + b*x^2)^2)
```

3.8 $\int \frac{1}{a^2+b+2ax^2+x^4}\,dx$

Optimal. Leaf size=299

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{a^2+b}}-\sqrt{2}\,x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{a+\sqrt{a^2+b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{a^2+b}}+\sqrt{2}\,x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{a+\sqrt{a^2+b}}}-\frac{\log\left(\sqrt{a^2+b}-\sqrt{2}\,\sqrt{-a+\cdots}\right.}{4\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{-a+\cdots}}$$

```
[Out] -1/8*ln(x^2+(a^2+b)^(1/2)-x*2^(1/2)*(-a+(a^2+b)^(1/2))^(1/2))*2^(1/2)/(a^2+
b)^(1/2)/(-a+(a^2+b)^(1/2))^(1/2)+1/8*ln(x^2+(a^2+b)^(1/2)+x*2^(1/2)*(-a+(a
^2+b)^(1/2))^(1/2))*2^(1/2)/(a^2+b)^(1/2)/(-a+(a^2+b)^(1/2))^(1/2)-1/4*arct
an((-x*2^(1/2)+(-a+(a^2+b)^(1/2))^(1/2))/(a+(a^2+b)^(1/2))^(1/2))*2^(1/2)/(
a^2+b)^(1/2)/(a+(a^2+b)^(1/2))^(1/2)+1/4*arctan((x*2^(1/2)+(-a+(a^2+b)^(1/2
))^(1/2))/(a+(a^2+b)^(1/2))^(1/2))*2^(1/2)/(a^2+b)^(1/2)/(a+(a^2+b)^(1/2))^
(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a^2+b}-a}-\sqrt{2}\,x}{\sqrt{\sqrt{a^2+b}+a}}\right)}{2\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{\sqrt{a^2+b}+a}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a^2+b}-a}+\sqrt{2}\,x}{\sqrt{\sqrt{a^2+b}+a}}\right)}{2\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{\sqrt{a^2+b}+a}}-\frac{\log\left(-\sqrt{2}\,x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{\sqrt{a^2+b}-a}}+\frac{\log\left(\sqrt{2}\,x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\,\sqrt{a^2+b}\,\sqrt{\sqrt{a^2+b}-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[-a + Sqrt[a^2 + b]] - Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]
/(Sqrt[2]*Sqrt[a^2 + b]*Sqrt[a + Sqrt[a^2 + b]]) + ArcTan[(Sqrt[-a + Sqrt[a
^2 + b]] + Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]/(2*Sqrt[2]*Sqrt[a^2 + b]*Sqr
t[a + Sqrt[a^2 + b]]) - Log[Sqrt[a^2 + b] - Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]
]*x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[-a + Sqrt[a^2 + b]]) + Log[Sqrt[a^
2 + b] + Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]
*Sqrt[-a + Sqrt[a^2 + b]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a^2+b+2ax^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}-x}{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2}\,dx}{2\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}} + \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}+x \cdots}{\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+b \cdots}}}}{2\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2 \cdots}}}$$

$$= \frac{\int \frac{1}{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2}\,dx}{4\sqrt{a^2+b}} + \frac{\int \frac{1}{\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+b \cdots}}}}{4\sqrt{a^2+b}}$$

$$= -\frac{\log\left(\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}\,x+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}} + \frac{\log\left(\sqrt{a^2+b}+\sqrt{2}\sqrt{- \cdots}\right.}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\cdots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{a^2+b}}-\sqrt{2}\,x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{a^2+b}}+\sqrt{2}\,x}{\sqrt{a+\sqrt{a^2+b}}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 81, normalized size = 0.27

$$-\frac{i\left(\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-i\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+i\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{b}}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] ((-1/2*I)*(ArcTan[x/Sqrt[a - I*Sqrt[b]]]/Sqrt[a - I*Sqrt[b]] - ArcTan[x/Sqr
t[a + I*Sqrt[b]]]/Sqrt[a + I*Sqrt[b]]))/Sqrt[b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 609 vs. 2(223) = 446.
time = 0.09, size = 610, normalized size = 2.04

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+2a\_Z^{2}+a^{2}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}+\_Ra}\right)}{4}$
default	$\frac{\left(\sqrt{2\sqrt{a^{2}+b}-2a}\sqrt{a^{2}+b}\,a^{2}+\sqrt{2\sqrt{a^{2}+b}-2a}\,a^{3}+\sqrt{2\sqrt{a^{2}+b}-2a}\sqrt{a^{2}+b}\,b+\sqrt{2\sqrt{a^{2}+b}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/(a^2+b)^(3/2)*(1/2*((2*(a^2+b)^(1/2)-2*a)^(1/2)*(a^2+b)^(1/2)*a^2+(2*
(a^2+b)^(1/2)-2*a)^(1/2)*a^3+(2*(a^2+b)^(1/2)-2*a)^(1/2)*(a^2+b)^(1/2)*b+(2
*(a^2+b)^(1/2)-2*a)^(1/2)*a*b)*ln(x^2+x*(2*(a^2+b)^(1/2)-2*a)^(1/2)+(a^2+b)
^(1/2))+2*(2*a^2*b+2*b^2-1/2*((2*(a^2+b)^(1/2)-2*a)^(1/2)*(a^2+b)^(1/2)*a^2
+(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^3+(2*(a^2+b)^(1/2)-2*a)^(1/2)*(a^2+b)^(1/2)*
b+(2*(a^2+b)^(1/2)-2*a)^(1/2)*a*b)*(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^
(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+
2*a)^(1/2)))+1/4/b/(a^2+b)^(3/2)*(-1/2*((2*(a^2+b)^(1/2)-2*a)^(1/2)*(a^2+b)
^(1/2)*a^2+(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^3+(2*(a^2+b)^(1/2)-2*a)^(1/2)*(a^2
+b)^(1/2)*b+(2*(a^2+b)^(1/2)-2*a)^(1/2)*a*b)*ln(x*(2*(a^2+b)^(1/2)-2*a)^(1/
```

```
2)-x^2-(a^2+b)^(1/2))+2*(-2*a^2*b-2*b^2+1/2*((2*(a^2+b)^(1/2)-2*a)^(1/2)*(a
^2+b)^(1/2)*a^2+(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^3+(2*(a^2+b)^(1/2)-2*a)^(1/2)
*(a^2+b)^(1/2)*b+(2*(a^2+b)^(1/2)-2*a)^(1/2)*a*b)*(2*(a^2+b)^(1/2)-2*a)^(1/
2))/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2
*(a^2+b)^(1/2)+2*a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(225) = 450.
time = 0.42, size = 583, normalized size = 1.95

$$\frac{1}{4}\sqrt{\frac{\left(a^{2}b+b^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}+a}{a^{2}b+b^{2}}}\log\left(\left(\left(a^{3}b+ab^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}+b\right)\sqrt{\frac{\left(a^{2}b+b^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}+a}{a^{2}b+b^{2}}}+x\right)-\frac{1}{4}\sqrt{\frac{\left(a^{2}b+b^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}+a}{a^{2}b+b^{2}}}\log\left(-\left(\left(a^{3}b+ab^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}+b\right)\sqrt{\frac{\left(a^{2}b+b^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}+a}{a^{2}b+b^{2}}}+x\right)-\frac{1}{4}\sqrt{-\frac{\left(a^{2}b+b^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}-a}{a^{2}b+b^{2}}}\log\left(\left(\left(a^{3}b+ab^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}-b\right)\sqrt{-\frac{\left(a^{2}b+b^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}-a}{a^{2}b+b^{2}}}+x\right)+\frac{1}{4}\sqrt{-\frac{\left(a^{2}b+b^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}-a}{a^{2}b+b^{2}}}\log\left(-\left(\left(a^{3}b+ab^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}-b\right)\sqrt{-\frac{\left(a^{2}b+b^{2}\right)\sqrt{-\frac{1}{a^{4}b+2\,a^{2}b^{2}+b^{3}}}-a}{a^{2}b+b^{2}}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^
2))*log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + b)*sqrt(((a^2
*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2)) + x) - 1/4
*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2))
*log(-((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + b)*sqrt(((a^2*b
 + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2)) + x) - 1/4*s
qrt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2))*
log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - b)*sqrt(-((a^2*b
+ b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2)) + x) + 1/4*sq
rt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2))*l
og(-((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - b)*sqrt(-((a^2*b
+ b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2)) + x)
```

Sympy [A]
time = 0.47, size = 63, normalized size = 0.21

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{2}b^{2}+256b^{3}\right)-32t^{2}ab+1,\left(t\mapsto t\log\left(64t^{3}a^{3}b+64t^{3}ab^{2}-4ta^{2}+4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2+b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 + 256*b**3) - 32*_t**2*a*b + 1, Lambda(_t, _t*
log(64*_t**3*a**3*b + 64*_t**3*a*b**2 - 4*_t*a**2 + 4*_t*b + x)))
```

Giac [A]
time = 3.73, size = 75, normalized size = 0.25

$$-\frac{\sqrt{a+\sqrt{-b}}\arctan\left(\frac{x}{\sqrt{a+\sqrt{-b}}}\right)}{2\left(a\sqrt{-b}-b\right)}+\frac{\sqrt{a-\sqrt{-b}}\arctan\left(\frac{x}{\sqrt{a-\sqrt{-b}}}\right)}{2\left(a\sqrt{-b}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+b),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + sqrt(-b))*arctan(x/sqrt(a + sqrt(-b)))/(a*sqrt(-b) - b) + 1/2
*sqrt(a - sqrt(-b))*arctan(x/sqrt(a - sqrt(-b)))/(a*sqrt(-b) + b)
```

Mupad [B]
time = 4.38, size = 872, normalized size = 2.92

$$-2\,\mathrm{atanh}\left(\frac{8\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}-\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}}{\frac{2\,b\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a\,b^2}{a^2\,b^2+b^3}}-\frac{8\,a^2\,b^2\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}-\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}-\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}}+\frac{8\,a\,b\,x\,\sqrt{\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}-\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}\,\sqrt{-b^3}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}-\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}-\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}}\right)\,\sqrt{\frac{a\,b-\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}-2\,\mathrm{atanh}\left(\frac{8\,a^2\,b^2\,x\,\sqrt{\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}+\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}+\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}+\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}}-\frac{8\,x\,\sqrt{\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}+\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}}}{\frac{2\,b\,\sqrt{-b^3}}{a^2\,b^2+b^3}+\frac{2\,a\,b^2}{a^2\,b^2+b^3}}+\frac{8\,a\,b\,x\,\sqrt{\frac{\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}+\frac{a\,b}{16\,\left(a^2\,b^2+b^3\right)}}\,\sqrt{-b^3}}{\frac{2\,b^4\,\sqrt{-b^3}}{a^2\,b^2+b^3}+\frac{2\,a^3\,b^4}{a^2\,b^2+b^3}+\frac{2\,a\,b^5}{a^2\,b^2+b^3}+\frac{2\,a^2\,b^3\,\sqrt{-b^3}}{a^2\,b^2+b^3}}\right)\,\sqrt{\frac{a\,b+\sqrt{-b^3}}{16\,\left(a^2\,b^2+b^3\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b + 2*a*x^2 + a^2 + x^4),x)
```

```
[Out] - 2*atanh((8*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 + a^2*b^
2)))^(1/2))/((2*b*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*a*b^2)/(b^3 + a^2*b^2)
) - (8*a^2*b^2*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 + a^2*
b^2)))^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*a^3*b^4)/(b^3 + a^
2*b^2) - (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b^3 + a^2*b^
2)) + (8*a*b*x*((a*b)/(16*(b^3 + a^2*b^2)) - (-b^3)^(1/2)/(16*(b^3 + a^2*b^
2)))^(1/2)*(-b^3)^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) - (2*a^3*b^4
)/(b^3 + a^2*b^2) - (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b
^3 + a^2*b^2)))*((a*b - (-b^3)^(1/2))/(16*(b^3 + a^2*b^2)))^(1/2) - 2*atanh
((8*a^2*b^2*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3 + a^2*b^2
)))^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*a^3*b^4)/(b^3 + a^2*b
^2) + (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b^3 + a^2*b^2))
 - (8*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3 + a^2*b^2)))^(1
/2))/((2*b*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*a*b^2)/(b^3 + a^2*b^2)) + (8*
a*b*x*((-b^3)^(1/2)/(16*(b^3 + a^2*b^2)) + (a*b)/(16*(b^3 + a^2*b^2)))^(1/2
)*(-b^3)^(1/2))/((2*b^4*(-b^3)^(1/2))/(b^3 + a^2*b^2) + (2*a^3*b^4)/(b^3 +
a^2*b^2) + (2*a*b^5)/(b^3 + a^2*b^2) + (2*a^2*b^3*(-b^3)^(1/2))/(b^3 + a^2*
b^2)))*((a*b + (-b^3)^(1/2))/(16*(b^3 + a^2*b^2)))^(1/2)
```

3.9 $\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+a}}\right)}{2\sqrt{1+a}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

```
[Out] -1/2*arctanh(x/(1-a)^(1/2))/(1-a)^(1/2)-1/2*arctan(x/(1+a)^(1/2))/(1+a)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1107, 213, 209}

$$-\frac{\text{ArcTan}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[x/Sqrt[1 + a]]/Sqrt[1 + a] - ArcTanh[x/Sqrt[1 - a]]/(2*Sqrt[1 -
 a])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx = \frac{1}{2}\int \frac{1}{-1+a+x^2}\,dx - \frac{1}{2}\int \frac{1}{1+a+x^2}\,dx$$
$$= -\frac{\tan^{-1}\left(\frac{x}{\sqrt{1+a}}\right)}{2\sqrt{1+a}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

Mathematica [**A**]
time = 0.02, size = 43, normalized size = 0.91

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+a}}\right)}{2\sqrt{-1+a}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{1+a}}\right)}{2\sqrt{1+a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + a]]/(2*Sqrt[-1 + a]) - ArcTan[x/Sqrt[1 + a]]/(2*Sqrt[1 +
 a])
```

Maple [**A**]
time = 0.03, size = 32, normalized size = 0.68

method	result	size
default	$\frac{\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}} - \frac{\arctan\left(\frac{x}{\sqrt{1+a}}\right)}{2\sqrt{1+a}}$	32
risch	$-\frac{\ln\left(x\sqrt{1-a}-a+1\right)}{4\sqrt{1-a}} + \frac{\ln\left(x\sqrt{1-a}+a-1\right)}{4\sqrt{1-a}} - \frac{\ln\left(-x\sqrt{-1-a}-a-1\right)}{4\sqrt{-1-a}} + \frac{\ln\left(-x\sqrt{-1-a}+a+1\right)}{4\sqrt{-1-a}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a-1)^(1/2)*arctan(x/(a-1)^(1/2))-1/2*arctan(x/(1+a)^(1/2))/(1+a)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1.0>0)', see `assume?` for more d
etails)
```

<u>Fricas</u> [**A**]
time = 0.36, size = 269, normalized size = 5.72

$$\left[-\frac{(a-1)\sqrt{-a-1}\log\left(\frac{x^2+2\sqrt{-a-1}\,x-a-1}{x^2+a+1}\right)+(a+1)\sqrt{-a+1}\log\left(\frac{x^2-2\sqrt{-a+1}\,x-a+1}{x^2+a-1}\right)}{4\,(a^2-1)},\ \frac{2\,(a+1)\sqrt{a-1}\arctan\left(\frac{x}{\sqrt{a-1}}\right)-(a-1)\sqrt{-a-1}\log\left(\frac{x^2+2\sqrt{-a-1}\,x-a-1}{x^2+a+1}\right)}{4\,(a^2-1)},\ -\frac{2\sqrt{a+1}\,(a-1)\arctan\left(\frac{x}{\sqrt{a+1}}\right)+(a+1)\sqrt{-a+1}\log\left(\frac{x^2-2\sqrt{-a+1}\,x-a+1}{x^2+a-1}\right)}{4\,(a^2-1)},\ -\frac{\sqrt{a+1}\,(a-1)\arctan\left(\frac{x}{\sqrt{a+1}}\right)-(a+1)\sqrt{a-1}\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\,(a^2-1)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - 1)*sqrt(-a - 1)*log((x^2 + 2*sqrt(-a - 1)*x - a - 1)/(x^2 + a +
 1)) + (a + 1)*sqrt(-a + 1)*log((x^2 - 2*sqrt(-a + 1)*x - a + 1)/(x^2 + a -
 1)))/(a^2 - 1), 1/4*(2*(a + 1)*sqrt(a - 1)*arctan(x/sqrt(a - 1)) - (a - 1)
*sqrt(-a - 1)*log((x^2 + 2*sqrt(-a - 1)*x - a - 1)/(x^2 + a + 1)))/(a^2 - 1
), -1/4*(2*sqrt(a + 1)*(a - 1)*arctan(x/sqrt(a + 1)) + (a + 1)*sqrt(-a + 1)
*log((x^2 - 2*sqrt(-a + 1)*x - a + 1)/(x^2 + a - 1)))/(a^2 - 1), -1/2*(sqrt
(a + 1)*(a - 1)*arctan(x/sqrt(a + 1)) - (a + 1)*sqrt(a - 1)*arctan(x/sqrt(a
 - 1)))/(a^2 - 1)]
```

<u>Sympy</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(37) = 74$.
time = 0.33, size = 257, normalized size = 5.47

$$\frac{\sqrt{-\frac{1}{a-1}}\log\left(-a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}-a^2\sqrt{-\frac{1}{a-1}}+a\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+x-\sqrt{-\frac{1}{a-1}}\right)}{4}-\frac{\sqrt{-\frac{1}{a-1}}\log\left(a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+a^2\sqrt{-\frac{1}{a-1}}-a\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+x+\sqrt{-\frac{1}{a-1}}\right)}{4}+\frac{\sqrt{-\frac{1}{a+1}}\log\left(-a^3\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}-a^2\sqrt{-\frac{1}{a+1}}+a\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+x-\sqrt{-\frac{1}{a+1}}\right)}{4}-\frac{\sqrt{-\frac{1}{a+1}}\log\left(a^3\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+a^2\sqrt{-\frac{1}{a+1}}-a\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+x+\sqrt{-\frac{1}{a+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2-1),x)
```

```
[Out] sqrt(-1/(a - 1))*log(-a**3*(-1/(a - 1))**(3/2) - a**2*sqrt(-1/(a - 1)) + a*
(-1/(a - 1))**(3/2) + x - sqrt(-1/(a - 1)))/4 - sqrt(-1/(a - 1))*log(a**3*(
-1/(a - 1))**(3/2) + a**2*sqrt(-1/(a - 1)) - a*(-1/(a - 1))**(3/2) + x + sq
rt(-1/(a - 1)))/4 + sqrt(-1/(a + 1))*log(-a**3*(-1/(a + 1))**(3/2) - a**2*s
qrt(-1/(a + 1)) + a*(-1/(a + 1))**(3/2) + x - sqrt(-1/(a + 1)))/4 - sqrt(-1
/(a + 1))*log(a**3*(-1/(a + 1))**(3/2) + a**2*sqrt(-1/(a + 1)) - a*(-1/(a +
 1))**(3/2) + x + sqrt(-1/(a + 1)))/4
```

<u>Giac</u> [**A**]
time = 6.00, size = 31, normalized size = 0.66

$$-\frac{\arctan\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}+\frac{\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2-1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x/sqrt(a + 1))/sqrt(a + 1) + 1/2*arctan(x/sqrt(a - 1))/sqrt(a -
 1)
```

Mupad [B]
time = 0.10, size = 85, normalized size = 1.81

$$\frac{\operatorname{atanh}\left(\frac{2x\left(\frac{a}{2}-\frac{1}{2}\right)}{\sqrt{1-a}}+\frac{2ax\left(\frac{a}{2}-\frac{1}{2}\right)}{(1-a)^{3/2}}\right)}{2\sqrt{1-a}}+\frac{\operatorname{atanh}\left(\frac{2x\left(\frac{a}{2}+\frac{1}{2}\right)}{\sqrt{-a-1}}+\frac{2ax\left(\frac{a}{2}+\frac{1}{2}\right)}{(-a-1)^{3/2}}\right)}{2\sqrt{-a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a*x^2 + a^2 + x^4 - 1),x)
```

```
[Out] atanh((2*x*(a/2 - 1/2))/(1 - a)^(1/2) + (2*a*x*(a/2 - 1/2))/(1 - a)^(3/2))/
(2*(1 - a)^(1/2)) + atanh((2*x*(a/2 + 1/2))/(- a - 1)^(1/2) + (2*a*x*(a/2 +
 1/2))/(- a - 1)^(3/2))/(2*(- a - 1)^(1/2))
```

3.10 $\int \frac{1}{1+a^2+2ax^2+x^4} \, dx$

Optimal. Leaf size=299

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{1+a^2}}-\sqrt{2}x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{1+a^2}}+\sqrt{2}x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}}-\frac{\log\left(\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\right.}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a-}}$$

```
[Out] -1/8*ln(x^2+(a^2+1)^(1/2)-x*2^(1/2)*(-a+(a^2+1)^(1/2))^(1/2))*2^(1/2)/(a^2+
1)^(1/2)/(-a+(a^2+1)^(1/2))^(1/2)+1/8*ln(x^2+(a^2+1)^(1/2)+x*2^(1/2)*(-a+(a
^2+1)^(1/2))^(1/2))*2^(1/2)/(a^2+1)^(1/2)/(-a+(a^2+1)^(1/2))^(1/2)-1/4*arct
an((-x*2^(1/2)+(-a+(a^2+1)^(1/2))^(1/2))/(a+(a^2+1)^(1/2))^(1/2))*2^(1/2)/(
a^2+1)^(1/2)/(a+(a^2+1)^(1/2))^(1/2)+1/4*arctan((x*2^(1/2)+(-a+(a^2+1)^(1/2
))^(1/2))/(a+(a^2+1)^(1/2))^(1/2))*2^(1/2)/(a^2+1)^(1/2)/(a+(a^2+1)^(1/2))^
(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a^2+1}-a}-\sqrt{2}x}{\sqrt{\sqrt{a^2+1}+a}}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}+a}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a^2+1}-a}+\sqrt{2}x}{\sqrt{\sqrt{a^2+1}+a}}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}+a}}-\frac{\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\log\left(\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[-a + Sqrt[1 + a^2]] - Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]
/(Sqrt[2]*Sqrt[1 + a^2]*Sqrt[a + Sqrt[1 + a^2]]) + ArcTan[(Sqrt[-a + Sqrt[1
 + a^2]] + Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]/(2*Sqrt[2]*Sqrt[1 + a^2]*Sqr
t[a + Sqrt[1 + a^2]]) - Log[Sqrt[1 + a^2] - Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]
]*x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[-a + Sqrt[1 + a^2]]) + Log[Sqrt[1
+ a^2] + Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]
*Sqrt[-a + Sqrt[1 + a^2]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+a^2+2ax^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}-x}{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2}\,dx}{2\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}} + \frac{\int \frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}+x}{\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}} \cdots}}{2\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+} \cdots}}$$

$$= \frac{\int \frac{1}{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2}\,dx}{4\sqrt{1+a^2}} + \frac{\int \frac{1}{\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}} \cdots}}{4\sqrt{1+a^2}}$$

$$= -\frac{\log\left(\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,x+x^2\right)}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}} + \frac{\log\left(\sqrt{1+a^2}+\sqrt{2}\sqrt{- \cdots}\right.}{4\sqrt{2}\sqrt{1+a^2}\sqrt{- \cdots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{1+a^2}}-\sqrt{2}x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-a+\sqrt{1+a^2}}+\sqrt{2}x}{\sqrt{a+\sqrt{1+a^2}}}\right)}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 52, normalized size = 0.17

$$-\frac{1}{2}i\left(\frac{\tan^{-1}\left(\frac{x}{\sqrt{-i+a}}\right)}{\sqrt{-i+a}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{i+a}}\right)}{\sqrt{i+a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] (-1/2*I)*(ArcTan[x/Sqrt[-I + a]]/Sqrt[-I + a] - ArcTan[x/Sqrt[I + a]]/Sqrt[
I + a])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(223) = 446.
time = 0.08, size = 586, normalized size = 1.96

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2a\_Z^2+a^2+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3+\_Ra}\right)}{4}$
default	$\frac{\frac{\left(\sqrt{2\sqrt{a^2+1}-2a}\,\sqrt{a^2+1}\,a^2+\sqrt{2\sqrt{a^2+1}-2a}\,a^3+\sqrt{2\sqrt{a^2+1}-2a}\,\sqrt{a^2+1}+\sqrt{2\sqrt{a^2+}\right.}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*a*x^2+a^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(a^2+1)^(3/2)*(1/2*((2*(a^2+1)^(1/2)-2*a)^(1/2)*(a^2+1)^(1/2)*a^2+(2*(a
^2+1)^(1/2)-2*a)^(1/2)*a^3+(2*(a^2+1)^(1/2)-2*a)^(1/2)*(a^2+1)^(1/2)+(2*(a^
2+1)^(1/2)-2*a)^(1/2)*a)*ln(x^2+x*(2*(a^2+1)^(1/2)-2*a)^(1/2)+(a^2+1)^(1/2)
)+2*(2*a^2+2-1/2*((2*(a^2+1)^(1/2)-2*a)^(1/2)*(a^2+1)^(1/2)*a^2+(2*(a^2+1)^
(1/2)-2*a)^(1/2)*a^3+(2*(a^2+1)^(1/2)-2*a)^(1/2)*(a^2+1)^(1/2)+(2*(a^2+1)^(
1/2)-2*a)^(1/2)*a)*(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2)
*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2)))+1/4
/(a^2+1)^(3/2)*(-1/2*((2*(a^2+1)^(1/2)-2*a)^(1/2)*(a^2+1)^(1/2)*a^2+(2*(a^2
+1)^(1/2)-2*a)^(1/2)*a^3+(2*(a^2+1)^(1/2)-2*a)^(1/2)*(a^2+1)^(1/2)+(2*(a^2+
1)^(1/2)-2*a)^(1/2)*a)*ln(x*(2*(a^2+1)^(1/2)-2*a)^(1/2)-x^2-(a^2+1)^(1/2))+
2*(-2*a^2-2+1/2*((2*(a^2+1)^(1/2)-2*a)^(1/2)*(a^2+1)^(1/2)*a^2+(2*(a^2+1)^(
1/2)-2*a)^(1/2)*a^3+(2*(a^2+1)^(1/2)-2*a)^(1/2)*(a^2+1)^(1/2)+(2*(a^2+1)^(1
/2)-2*a)^(1/2)*a)*(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2)*
arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(225) = 450.
time = 0.40, size = 613, normalized size = 2.05

$$\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\left(\frac{a}{\sqrt{a^2+1}}+1\right)\log\left(x^2+\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{1}{4}}}+\sqrt{a^2+1}\right)}{8(a^2+1)^{\frac{1}{4}}}-\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\left(\frac{a}{\sqrt{a^2+1}}+1\right)\log\left(x^2-\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{1}{4}}}+\sqrt{a^2+1}\right)}{8(a^2+1)^{\frac{1}{4}}}-\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,(a^2+1)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{a^4+2a^2+1}\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{5}{4}}}+\frac{a^3+a}{\sqrt{a^4+2a^2+1}}+\frac{\sqrt{a^4+2a^2+1}\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\sqrt{x^2+\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{1}{4}}}+\sqrt{a^2+1}}}{(a^2+1)^{\frac{5}{4}}}-\frac{\sqrt{a^4+2a^2+1}}{\sqrt{a^2+1}}\right)}{2\sqrt{a^4+2a^2+1}}-\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,(a^2+1)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{a^4+2a^2+1}\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{5}{4}}}-\frac{a^3+a}{\sqrt{a^4+2a^2+1}}+\frac{\sqrt{a^4+2a^2+1}\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\sqrt{x^2-\frac{\sqrt{2a^2-\frac{2(a^3+a)}{\sqrt{a^2+1}}+2}\,x}{(a^2+1)^{\frac{1}{4}}}+\sqrt{a^2+1}}}{(a^2+1)^{\frac{5}{4}}}+\frac{\sqrt{a^4+2a^2+1}}{\sqrt{a^2+1}}\right)}{2\sqrt{a^4+2a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*(a/sqrt(a^2 + 1) + 1)*log(x
^2 + sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(1/4) + sqrt(a
^2 + 1))/(a^2 + 1)^(1/4) - 1/8*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*
(a/sqrt(a^2 + 1) + 1)*log(x^2 - sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)
*x/(a^2 + 1)^(1/4) + sqrt(a^2 + 1))/(a^2 + 1)^(1/4) - 1/2*sqrt(2*a^2 - 2*(a
^3 + a)/sqrt(a^2 + 1) + 2)*(a^2 + 1)^(1/4)*arctan(-sqrt(a^4 + 2*a^2 + 1)*sq
rt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(5/4) + (a^3 + a)/sqr
t(a^4 + 2*a^2 + 1) + sqrt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^
2 + 1) + 2)*sqrt(x^2 + sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 +
 1)^(1/4) + sqrt(a^2 + 1))/(a^2 + 1)^(5/4) - sqrt(a^4 + 2*a^2 + 1)/sqrt(a^2
 + 1))/sqrt(a^4 + 2*a^2 + 1) - 1/2*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) +
 2)*(a^2 + 1)^(1/4)*arctan(-sqrt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/
sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(5/4) - (a^3 + a)/sqrt(a^4 + 2*a^2 + 1) + sq
rt(a^4 + 2*a^2 + 1)*sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*sqrt(x^2 -
sqrt(2*a^2 - 2*(a^3 + a)/sqrt(a^2 + 1) + 2)*x/(a^2 + 1)^(1/4) + sqrt(a^2 +
1))/(a^2 + 1)^(5/4) + sqrt(a^4 + 2*a^2 + 1)/sqrt(a^2 + 1))/sqrt(a^4 + 2*a^2
 + 1)
```

Sympy [A]
time = 0.31, size = 48, normalized size = 0.16

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^2+256\right)-32t^2a+1,\left(t\mapsto t\log\left(64t^3a^3+64t^3a-4ta^2+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*a*x**2+a**2+1),x)
```

```
[Out] RootSum(_t**4*(256*a**2 + 256) - 32*_t**2*a + 1, Lambda(_t, _t*log(64*_t**3
*a**3 + 64*_t**3*a - 4*_t*a**2 + 4*_t + x)))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*a*x^2+a^2+1),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.36, size = 469, normalized size = 1.57

$$-\frac{\operatorname{atanh}\left(-\frac{2\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1\mathrm{i}}{a^2+1}}}{\frac{2\,a}{a^2+1}+\frac{2\mathrm{i}}{a^2+1}}+\frac{a\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1\mathrm{i}}{a^2+1}}\,2\mathrm{i}}{\frac{2\,a}{a^2+1}+\frac{2\,a^3}{a^2+1}+\frac{2\mathrm{i}}{a^2+1}+\frac{a^2\,2\mathrm{i}}{a^2+1}}+\frac{2\,a^2\,x\,\sqrt{\frac{a}{a^2+1}+\frac{1\mathrm{i}}{a^2+1}}}{\frac{2\,a}{a^2+1}+\frac{2\,a^3}{a^2+1}+\frac{2\mathrm{i}}{a^2+1}+\frac{a^2\,2\mathrm{i}}{a^2+1}}\right)\,\sqrt{\frac{a+1\mathrm{i}}{a^2+1}}}{2}+2\,\operatorname{atanh}\left(\frac{8\,x\,\sqrt{\frac{a}{16\,a^2+16}-\frac{1\mathrm{i}}{16\,a^2+16}}}{\frac{32\,a}{16\,a^2+16}-\frac{32\mathrm{i}}{16\,a^2+16}}+\frac{a\,x\,\sqrt{\frac{a}{16\,a^2+16}-\frac{1\mathrm{i}}{16\,a^2+16}}\,128\mathrm{i}}{\frac{512\,a}{16\,a^2+16}+\frac{512\,a^3}{16\,a^2+16}-\frac{512\mathrm{i}}{16\,a^2+16}-\frac{a^2\,512\mathrm{i}}{16\,a^2+16}}-\frac{128\,a^2\,x\,\sqrt{\frac{a}{16\,a^2+16}-\frac{1\mathrm{i}}{16\,a^2+16}}}{\frac{512\,a}{16\,a^2+16}+\frac{512\,a^3}{16\,a^2+16}-\frac{512\mathrm{i}}{16\,a^2+16}-\frac{a^2\,512\mathrm{i}}{16\,a^2+16}}\right)\,\sqrt{\frac{a-\mathrm{i}}{16\,a^2+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a*x^2 + a^2 + x^4 + 1),x)
```

```
[Out] 2*atanh((8*x*(a/(16*a^2 + 16) - 1i/(16*a^2 + 16))^(1/2))/((32*a)/(16*a^2 +
16) - 32i/(16*a^2 + 16)) + (a*x*(a/(16*a^2 + 16) - 1i/(16*a^2 + 16))^(1/2)*
128i)/((512*a)/(16*a^2 + 16) - 512i/(16*a^2 + 16) - (a^2*512i)/(16*a^2 + 16
) + (512*a^3)/(16*a^2 + 16)) - (128*a^2*x*(a/(16*a^2 + 16) - 1i/(16*a^2 + 1
6))^(1/2))/((512*a)/(16*a^2 + 16) - 512i/(16*a^2 + 16) - (a^2*512i)/(16*a^2
 + 16) + (512*a^3)/(16*a^2 + 16)))*((a - 1i)/(16*a^2 + 16))^(1/2) - (atanh(
(a*x*(a/(a^2 + 1) + 1i/(a^2 + 1))^(1/2)*2i)/((2*a)/(a^2 + 1) + 2i/(a^2 + 1)
 + (a^2*2i)/(a^2 + 1) + (2*a^3)/(a^2 + 1)) - (2*x*(a/(a^2 + 1) + 1i/(a^2 +
1))^(1/2))/((2*a)/(a^2 + 1) + 2i/(a^2 + 1)) + (2*a^2*x*(a/(a^2 + 1) + 1i/(a
^2 + 1))^(1/2))/((2*a)/(a^2 + 1) + 2i/(a^2 + 1) + (a^2*2i)/(a^2 + 1) + (2*a
^3)/(a^2 + 1)))*((a + 1i)/(a^2 + 1))^(1/2))/2
```

3.11 $\int \frac{1}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] -1/6*arctanh(1/2*x)+1/3*arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1107, 213}

$$\frac{1}{3}\tanh^{-1}(x)-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 5*x^2 + x^4)^(-1),x]
```

```
[Out] -1/6*ArcTanh[x/2] + ArcTanh[x]/3
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-5x^2+x^4}\,dx &= \frac{1}{3}\int \frac{1}{-4+x^2}\,dx-\frac{1}{3}\int \frac{1}{-1+x^2}\,dx\\ &= -\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.
time = 0.00, size = 37, normalized size = 2.18

$$-\frac{1}{6}\log(1-x)+\frac{1}{12}\log(2-x)+\frac{1}{6}\log(1+x)-\frac{1}{12}\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 5*x^2 + x^4)^(-1),x]
```

```
[Out] -1/6*Log[1 - x] + Log[2 - x]/12 + Log[1 + x]/6 - Log[2 + x]/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.
time = 0.02, size = 26, normalized size = 1.53

method	result	size
default	$-\frac{\ln(x+2)}{12}+\frac{\ln(x-2)}{12}-\frac{\ln(-1+x)}{6}+\frac{\ln(1+x)}{6}$	26
norman	$-\frac{\ln(x+2)}{12}+\frac{\ln(x-2)}{12}-\frac{\ln(-1+x)}{6}+\frac{\ln(1+x)}{6}$	26
risch	$-\frac{\ln(x+2)}{12}+\frac{\ln(x-2)}{12}-\frac{\ln(-1+x)}{6}+\frac{\ln(1+x)}{6}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*ln(x+2)+1/12*ln(x-2)-1/6*ln(-1+x)+1/6*ln(1+x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.
time = 0.27, size = 25, normalized size = 1.47

$$-\frac{1}{12}\log(x+2)+\frac{1}{6}\log(x+1)-\frac{1}{6}\log(x-1)+\frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x - 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.
time = 0.36, size = 25, normalized size = 1.47

$$-\frac{1}{12}\log(x+2)+\frac{1}{6}\log(x+1)-\frac{1}{6}\log(x-1)+\frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x - 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.07, size = 26, normalized size = 1.53

$$\frac{\log(x-2)}{12}-\frac{\log(x-1)}{6}+\frac{\log(x+1)}{6}-\frac{\log(x+2)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-5*x**2+4),x)
```

```
[Out] log(x - 2)/12 - log(x - 1)/6 + log(x + 1)/6 - log(x + 2)/12
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.
time = 4.83, size = 29, normalized size = 1.71

$$-\frac{1}{12}\log\left(|x+2|\right)+\frac{1}{6}\log\left(|x+1|\right)-\frac{1}{6}\log\left(|x-1|\right)+\frac{1}{12}\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*log(abs(x + 2)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1)) + 1/12*lo
g(abs(x - 2))
```

Mupad [B]
time = 0.04, size = 11, normalized size = 0.65

$$\frac{\operatorname{atanh}(x)}{3}-\frac{\operatorname{atanh}\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 5*x^2 + 4),x)
```

```
[Out] atanh(x)/3 - atanh(x/2)/6
```

3.12 $\int \frac{1}{3+4x^2+x^4}\,dx$

Optimal. Leaf size=24

$$\frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/2*arctan(x)-1/6*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {1107, 209}

$$\frac{\text{ArcTan}(x)}{2}-\frac{\text{ArcTan}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+4x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{1}{2}\int \frac{1}{3+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.75

method	result	size
default	$\frac{\arctan(x)}{2}-\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$	18
risch	$\frac{\arctan(x)}{2}-\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(x)-1/6*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.50, size = 17, normalized size = 0.71

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

Fricas [A]
time = 0.35, size = 17, normalized size = 0.71

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="fricas")
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.83

$$\frac{\operatorname{atan}(x)}{2} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**2+3),x)
[Out] atan(x)/2 - sqrt(3)*atan(sqrt(3)*x/3)/6
```

Giac [A]
time = 4.76, size = 17, normalized size = 0.71

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+3),x, algorithm="giac")
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

Mupad [B]
time = 4.12, size = 17, normalized size = 0.71

$$\frac{\operatorname{atan}(x)}{2} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + x^4 + 3),x)
[Out] atan(x)/2 - (3^(1/2)*atan((3^(1/2)*x)/3))/6
```

3.13 $\int \frac{1}{9+5x^2+x^4} \, dx$

Optimal. Leaf size=67

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{11}}\right)}{6\sqrt{11}}-\frac{1}{12}\log\left(3-x+x^2\right)+\frac{1}{12}\log\left(3+x+x^2\right)$$

```
[Out] -1/12*ln(x^2-x+3)+1/12*ln(x^2+x+3)-1/66*arctan(1/11*(1-2*x)*11^(1/2))*11^(1
/2)+1/66*arctan(1/11*(1+2*x)*11^(1/2))*11^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{11}}\right)}{6\sqrt{11}}-\frac{1}{12}\log\left(x^2-x+3\right)+\frac{1}{12}\log\left(x^2+x+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 5*x^2 + x^4)^(-1),x]
```

```
[Out] -1/6*ArcTan[(1 - 2*x)/Sqrt[11]]/Sqrt[11] + ArcTan[(1 + 2*x)/Sqrt[11]]/(6*Sq
rt[11]) - Log[3 - x + x^2]/12 + Log[3 + x + x^2]/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{9+5x^2+x^4}\,dx &= \frac{1}{6}\int \frac{1-x}{3-x+x^2}\,dx+\frac{1}{6}\int \frac{1+x}{3+x+x^2}\,dx\\ &= \frac{1}{12}\int \frac{1}{3-x+x^2}\,dx-\frac{1}{12}\int \frac{-1+2x}{3-x+x^2}\,dx+\frac{1}{12}\int \frac{1}{3+x+x^2}\,dx+\frac{1}{12}\int \frac{1+2x}{3+x+x}\\ &= -\frac{1}{12}\log\left(3-x+x^2\right)+\frac{1}{12}\log\left(3+x+x^2\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx,x,-1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{11}}\right)}{6\sqrt{11}}-\frac{1}{12}\log\left(3-x+x^2\right)+\frac{1}{12}\log\left(3+x+x^2\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 91, normalized size = 1.36

$$-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(5-i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5-i\sqrt{11}\right)}}+\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(5+i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5+i\sqrt{11}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 5*x^2 + x^4)^(-1),x]
[Out] ((-I)*ArcTan[x/Sqrt[(5 - I*Sqrt[11])/2]])/Sqrt[(11*(5 - I*Sqrt[11]))/2] + (
I*ArcTan[x/Sqrt[(5 + I*Sqrt[11])/2]])/Sqrt[(11*(5 + I*Sqrt[11]))/2]
```

Maple [A]
time = 0.03, size = 54, normalized size = 0.81

method	result	size
default	$-\frac{\ln(x^2-x+3)}{12}+\frac{\sqrt{11}\,\arctan\left(\frac{(2x-1)\sqrt{11}}{11}\right)}{66}+\frac{\ln(x^2+x+3)}{12}+\frac{\arctan\left(\frac{(2x+1)\sqrt{11}}{11}\right)\sqrt{11}}{66}$	54
risch	$-\frac{\ln(4x^2-4x+12)}{12}+\frac{\sqrt{11}\,\arctan\left(\frac{(2x-1)\sqrt{11}}{11}\right)}{66}+\frac{\ln(4x^2+4x+12)}{12}+\frac{\arctan\left(\frac{(2x+1)\sqrt{11}}{11}\right)\sqrt{11}}{66}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+5*x^2+9),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*ln(x^2-x+3)+1/66*11^(1/2)*arctan(1/11*(2*x-1)*11^(1/2))+1/12*ln(x^2+x
+3)+1/66*arctan(1/11*(2*x+1)*11^(1/2))*11^(1/2)
```

Maxima [A]
time = 0.50, size = 53, normalized size = 0.79

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\,(2\,x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\,(2\,x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="maxima")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

Fricas [A]
time = 0.34, size = 53, normalized size = 0.79

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\,(2\,x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\,(2\,x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="fricas")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

Sympy [A]
time = 0.09, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^2-x+3\right)}{12}+\frac{\log\left(x^2+x+3\right)}{12}+\frac{\sqrt{11}\,\operatorname{atan}\left(\frac{2\sqrt{11}\,x}{11}-\frac{\sqrt{11}}{11}\right)}{66}+\frac{\sqrt{11}\,\operatorname{atan}\left(\frac{2\sqrt{11}\,x}{11}+\frac{\sqrt{11}}{11}\right)}{66}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+5*x**2+9),x)
```

```
[Out] -log(x**2 - x + 3)/12 + log(x**2 + x + 3)/12 + sqrt(11)*atan(2*sqrt(11)*x/1
1 - sqrt(11)/11)/66 + sqrt(11)*atan(2*sqrt(11)*x/11 + sqrt(11)/11)/66
```

Giac [A]
time = 5.16, size = 53, normalized size = 0.79

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+9),x, algorithm="giac")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arctan(1/11*s
qrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x^2 - x + 3)
```

Mupad [B]
time = 4.15, size = 83, normalized size = 1.24

$$\operatorname{atan}\left(\frac{x\,8\mathrm{i}}{27\left(-\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}-\frac{2\sqrt{11}\,x}{27\left(-\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}\right)\left(\frac{\sqrt{11}}{66}+\frac{1}{6}\mathrm{i}\right)+\operatorname{atan}\left(\frac{x\,8\mathrm{i}}{27\left(\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}+\frac{2\sqrt{11}\,x}{27\left(\frac{5}{9}+\frac{\sqrt{11}\,1\mathrm{i}}{9}\right)}\right)\left(\frac{\sqrt{11}}{66}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + x^4 + 9),x)
```

```
[Out] atan((x*8i)/(27*((11^(1/2)*1i)/9 - 5/9)) - (2*11^(1/2)*x)/(27*((11^(1/2)*1i
)/9 - 5/9)))*(11^(1/2)/66 + 1i/6) + atan((x*8i)/(27*((11^(1/2)*1i)/9 + 5/9)
) + (2*11^(1/2)*x)/(27*((11^(1/2)*1i)/9 + 5/9)))*(11^(1/2)/66 - 1i/6)
```

3.14 $\int \frac{1}{1-x^2+x^4}\,dx$

Optimal. Leaf size=74

$$-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}$$

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))-1/12*ln(1+x^2-x*3^(1/2))*3^
(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{1}{2}\text{ArcTan}\left(\sqrt{3}-2x\right)+\frac{1}{2}\text{ArcTan}\left(2x+\sqrt{3}\right)-\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]/2 - Log[1 - Sqrt[3]*x +
x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1-x^2+x^4}\,dx = \frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}\,x+x^2}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}\,x+x^2}\,dx}{2\sqrt{3}}$$

$$= \frac{1}{4}\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx - \frac{\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}\,x+x^2}\,dx}{4\sqrt{3}} + \frac{\int \frac{\sqrt{3}+2x}{1+\sqrt{3}\,x+x^2}\,d}{4\sqrt{3}}$$

$$= -\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}} - \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}\,+\right.$$

$$= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right) + \frac{1}{2}\tan^{-1}\left(\sqrt{3}+2x\right) - \frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}\right.}{4\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 77, normalized size = 1.04

$$\frac{i\left(\sqrt{-1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-\sqrt{-1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - Sqrt[-1 + I*Sqrt[3
]]*ArcTan[((1 + I*Sqrt[3])*x)/2]))/Sqrt[6]
```

Maple [A]
time = 0.04, size = 57, normalized size = 0.77

method	result	size

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3-\_R}\right)}{2}$	35
default	$\frac{\arctan\left(2x-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x+\sqrt{3}\right)}{2}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))-1/12*ln(1+x^2-x*3^(1/2))*3^
(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="maxima")
[Out] integrate(1/(x^4 - x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(56) = 112$.
time = 0.34, size = 163, normalized size = 2.20

$$-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{36}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-72\sqrt{6}\sqrt{2}x+144x^2+144}+\sqrt{3}\right)-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+\frac{1}{24}\sqrt{6}\sqrt{2}\log\left(72\sqrt{6}\sqrt{2}x+144x^2+144\right)-\frac{1}{24}\sqrt{6}\sqrt{2}\log\left(-72\sqrt{6}\sqrt{2}x+144x^2+144\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="fricas")
[Out] -1/6*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/36*s
qrt(6)*sqrt(3)*sqrt(2)*sqrt(-72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) + sqrt(3
)) - 1/6*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/
3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 1/24*sqr
t(6)*sqrt(2)*log(72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) - 1/24*sqrt(6)*sqrt(
2)*log(-72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144)
```

Sympy [A]
time = 0.09, size = 63, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

Giac [A]
time = 3.58, size = 56, normalized size = 0.76

$$\frac{1}{12}\,\sqrt{3}\,\log\left(x^2+\sqrt{3}\,x+1\right)-\frac{1}{12}\,\sqrt{3}\,\log\left(x^2-\sqrt{3}\,x+1\right)+\frac{1}{2}\,\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{2}\,\arctan\left(2\,x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
1) + 1/2*arctan(2*x + sqrt(3)) + 1/2*arctan(2*x - sqrt(3))
```

Mupad [B]
time = 4.19, size = 47, normalized size = 0.64

$$\mathrm{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\mathrm{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - x^2 + 1),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atan((2*x)/(3^(1/2)*1
i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

3.15 $\int \frac{1}{2+2x^2+x^4}\,dx$

Optimal. Leaf size=176

$$-\frac{1}{4}\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{4}\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}+2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)-\frac{\log}{}$$

```
[Out] -1/8*ln(x^2+2^(1/2)-x*(-2+2*2^(1/2))^(1/2))/(2^(1/2)-1)^(1/2)+1/8*ln(x^2+2^
(1/2)+x*(-2+2*2^(1/2))^(1/2))/(2^(1/2)-1)^(1/2)-1/4*arctan((-2*x+(-2+2*2^(1
/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)+1/4*arctan((2*x+(-2+2*2^
(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{1}{4}\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{4}\sqrt{\sqrt{2}-1}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(\sqrt{2}-1\right)}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)-\frac{\log\left(x^2-\sqrt{2\left(\sqrt{2}-1\right)}\,x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\log\left(x^2+\sqrt{2\left(\sqrt{2}-1\right)}\,x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] -1/4*(Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] - 2*x)/Sqrt[2*(1 +
Sqrt[2])]]) + (Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] + 2*x)/Sqr
t[2*(1 + Sqrt[2])]])/4 - Log[Sqrt[2] - Sqrt[2*(-1 + Sqrt[2])]*x + x^2]/(8*S
qrt[-1 + Sqrt[2]]) + Log[Sqrt[2] + Sqrt[2*(-1 + Sqrt[2])]*x + x^2]/(8*Sqrt[
-1 + Sqrt[2]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{2+2x^2+x^4}\,dx = \frac{\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{-1+\sqrt{2}}} + \frac{\int \frac{\sqrt{2\left(-1+\sqrt{2}\right)}+x}{\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{-1+\sqrt{2}}}$$

$$= \frac{\int \frac{1}{\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{2}} + \frac{\int \frac{1}{\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{-\sqrt{2\left(-\right.}}{\sqrt{2}-\sqrt{2\left(\right.}}}{8\sqrt{}}$$

$$= -\frac{\log\left(\sqrt{2}-\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2\right)}{8\sqrt{-1+\sqrt{2}}} + \frac{\log\left(\sqrt{2}+\sqrt{2\left(-1+\sqrt{2}\right)}x+x^2\right)}{8\sqrt{-1+\sqrt{2}}} -$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)}{4\sqrt{1+\sqrt{2}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}+2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)}{4\sqrt{1+\sqrt{2}}} - \frac{\log\left(\sqrt{2}-\right.}{8}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 41, normalized size = 0.23

$$\frac{1}{4}\left((1-i)^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{1+i}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] ((1 - I)^(3/2)*ArcTan[x/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTan[x/Sqrt[1 + I]])
/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(124) = 248$.
time = 0.06, size = 253, normalized size = 1.44

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+2\right)}\frac{\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{4}$
default	$\frac{\left(\sqrt{-2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{-2+2\sqrt{2}}\right)\ln\left(x^2+\sqrt{2}+x\sqrt{-2+2\sqrt{2}}\right)}{16}+\frac{\left(2\sqrt{2}-\frac{\left(\sqrt{-2+2\sqrt{2}}\,\sqrt{2}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((-2+2*2^(1/2))^(1/2)*2^(1/2)+2*(-2+2*2^(1/2))^(1/2))*ln(x^2+2^(1/2)+x
*(-2+2*2^(1/2))^(1/2))+1/4*(2*2^(1/2)-1/2*((-2+2*2^(1/2))^(1/2)*2^(1/2)+2*(
-2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)*arctan((2*x+
(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))+1/16*(-(-2+2*2^(1/2))^(1/2)*2^(1
/2)-2*(-2+2*2^(1/2))^(1/2))*ln(x^2+2^(1/2)-x*(-2+2*2^(1/2))^(1/2))+1/4*(2*2
^(1/2)+1/2*(-(-2+2*2^(1/2))^(1/2)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2))*(-2+2*2^(
1/2))^(1/2))/(2+2*2^(1/2))^(1/2)*arctan((2*x-(-2+2*2^(1/2))^(1/2))/(2+2*2^(
1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*x^2 + 2), x)
```

Fricas [A]
time = 0.37, size = 249, normalized size = 1.41

$$\frac{1}{16} \cdot 2^{\frac{1}{4}} \left(\sqrt{2}+1\right) \sqrt{-2\sqrt{2}+4} \log\left(2 \cdot 2^{\frac{3}{4}} x \sqrt{-2\sqrt{2}+4} + 4x^2 + 4\sqrt{2}\right) - \frac{1}{16} \cdot 2^{\frac{1}{4}} \left(\sqrt{2}+1\right) \sqrt{-2\sqrt{2}+4} \log\left(-2 \cdot 2^{\frac{3}{4}} x \sqrt{-2\sqrt{2}+4} + 4x^2 + 4\sqrt{2}\right) - \frac{1}{4} \cdot 2^{\frac{1}{4}} \sqrt{-2\sqrt{2}+4} \arctan\left(-\frac{1}{2} \cdot 2^{\frac{3}{4}} x \sqrt{-2\sqrt{2}+4} + \frac{1}{4} \cdot 2^{\frac{3}{4}} \sqrt{2 \cdot 2^{\frac{3}{4}} x \sqrt{-2\sqrt{2}+4} + 4x^2 + 4\sqrt{2}} \sqrt{-2\sqrt{2}+4} - \sqrt{2} + 1\right) - \frac{1}{4} \cdot 2^{\frac{1}{4}} \sqrt{-2\sqrt{2}+4} \arctan\left(-\frac{1}{2} \cdot 2^{\frac{3}{4}} x \sqrt{-2\sqrt{2}+4} + \frac{1}{4} \cdot 2^{\frac{3}{4}} \sqrt{-2 \cdot 2^{\frac{3}{4}} x \sqrt{-2\sqrt{2}+4} + 4x^2 + 4\sqrt{2}} \sqrt{-2\sqrt{2}+4} + \sqrt{2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log(2*2^(3/4)*x*sqrt(-2*sqr
t(2) + 4) + 4*x^2 + 4*sqrt(2)) - 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2)
 + 4)*log(-2*2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 4*x^2 + 4*sqrt(2)) - 1/4*2^(1
/4)*sqrt(-2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 1/4*2
^(3/4)*sqrt(2*2^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 4*x^2 + 4*sqrt(2))*sqrt(-2*s
qrt(2) + 4) - sqrt(2) + 1) - 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(-1/2*2
^(3/4)*x*sqrt(-2*sqrt(2) + 4) + 1/4*2^(3/4)*sqrt(-2*2^(3/4)*x*sqrt(-2*sqrt(
2) + 4) + 4*x^2 + 4*sqrt(2))*sqrt(-2*sqrt(2) + 4) + sqrt(2) - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(151) = 302$.
time = 0.62, size = 899, normalized size = 5.11

$$\sqrt{\frac{1}{64}+\frac{\sqrt{2}}{64}} \log\left(x^2 + x\left(-4\sqrt{2}\sqrt{1+\sqrt{2}} - \sqrt{1+\sqrt{2}} + 3\sqrt{1+\sqrt{2}}\sqrt{2\sqrt{2}+3}\right) - 15\sqrt{2\sqrt{2}+3} - 7\sqrt{2}\sqrt{2\sqrt{2}+3} + 29 + 23\sqrt{2}\right) - \sqrt{\frac{1}{64}+\frac{\sqrt{2}}{64}} \log\left(x^2 + x\left(-3\sqrt{1+\sqrt{2}}\sqrt{2\sqrt{2}+3} + \sqrt{1+\sqrt{2}} + 4\sqrt{2}\sqrt{1+\sqrt{2}}\right) - 15\sqrt{2\sqrt{2}+3} - 7\sqrt{2}\sqrt{2\sqrt{2}+3} + 29 + 23\sqrt{2}\right) + 2\sqrt{-\frac{\sqrt{2\sqrt{2}+3}}{32} + \frac{1}{64} + \frac{3\sqrt{2}}{64}} \operatorname{atan}\left(\ldots \right)$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*x**2+2),x)
```

```
[Out] sqrt(1/64 + sqrt(2)/64)*log(x**2 + x*(-4*sqrt(2)*sqrt(1 + sqrt(2)) - sqrt(1
 + sqrt(2)) + 3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3)) - 15*sqrt(2*sqrt(2)
+ 3) - 7*sqrt(2)*sqrt(2*sqrt(2) + 3) + 29 + 23*sqrt(2)) - sqrt(1/64 + sqrt(
2)/64)*log(x**2 + x*(-3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3) + sqrt(1 + sq
rt(2)) + 4*sqrt(2)*sqrt(1 + sqrt(2))) - 15*sqrt(2*sqrt(2) + 3) - 7*sqrt(2)*
sqrt(2*sqrt(2) + 3) + 29 + 23*sqrt(2)) + 2*sqrt(-sqrt(2*sqrt(2) + 3)/32 + 1
/64 + 3*sqrt(2)/64)*atan(2*x/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))
+ sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) - 4*sqr
t(2)*sqrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt
(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) - sqrt(1 + sq
rt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*
sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) + 3*sqrt(1 + sqrt(2))*sqrt(2*
sqrt(2) + 3)/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2)
 + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)))) + 2*sqrt(-sqrt(2*sqrt(
2) + 3)/32 + 1/64 + 3*sqrt(2)/64)*atan(2*x/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1
 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqr
t(2))) - 3*sqrt(1 + sqrt(2))*sqrt(2*sqrt(2) + 3)/(sqrt(-2*sqrt(2*sqrt(2) +
3) + 1 + 3*sqrt(2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 +
```

```
 3*sqrt(2))) + sqrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(
2)) + sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))) + 4
*sqrt(2)*sqrt(1 + sqrt(2))/(sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2)) +
sqrt(2*sqrt(2) + 3)*sqrt(-2*sqrt(2*sqrt(2) + 3) + 1 + 3*sqrt(2))))
```

Giac [A]
time = 3.16, size = 143, normalized size = 0.81

$$\frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x+2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\right)}{2\sqrt{\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{\sqrt{2}-1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x-2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\right)}{2\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(x^2+2^{\frac{1}{4}}x\sqrt{-\sqrt{2}+2}+\sqrt{2}\right)-\frac{1}{8}\sqrt{\sqrt{2}+1}\log\left(x^2-2^{\frac{1}{4}}x\sqrt{-\sqrt{2}+2}+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(-sqrt(2) + 2))
/sqrt(sqrt(2) + 2)) + 1/4*sqrt(sqrt(2) - 1)*arctan(1/2*2^(3/4)*(2*x - 2^(1/
4)*sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) + 1/8*sqrt(sqrt(2) + 1)*log(x^2 +
 2^(1/4)*x*sqrt(-sqrt(2) + 2) + sqrt(2)) - 1/8*sqrt(sqrt(2) + 1)*log(x^2 -
2^(1/4)*x*sqrt(-sqrt(2) + 2) + sqrt(2))
```

Mupad [B]
time = 4.21, size = 210, normalized size = 1.19

$$\operatorname{atanh}\left(\frac{4\sqrt{2}\,x\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}+\frac{4\sqrt{2}\,x\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}-2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)-\operatorname{atanh}\left(\frac{4\sqrt{2}\,x\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1}-\frac{4\sqrt{2}\,x\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}+2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + x^4 + 2),x)
```

```
[Out] atanh((4*2^(1/2)*x*(1/64 - 2^(1/2)/64)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)
*(2^(1/2)/64 + 1/64)^(1/2) - 1) + (4*2^(1/2)*x*(2^(1/2)/64 + 1/64)^(1/2))/(
64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2) - 1))*(2*(1/64 - 2^(
1/2)/64)^(1/2) - 2*(2^(1/2)/64 + 1/64)^(1/2)) - atanh((4*2^(1/2)*x*(1/64 -
2^(1/2)/64)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2)
+ 1) - (4*2^(1/2)*x*(2^(1/2)/64 + 1/64)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2
)*(2^(1/2)/64 + 1/64)^(1/2) + 1))*(2*(1/64 - 2^(1/2)/64)^(1/2) + 2*(2^(1/2)
/64 + 1/64)^(1/2))
```

3.16 $\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

```
[Out] EllipticF(1/2*x*2^(1/2),I*6^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{12-6x^2}\,\sqrt{2+6x^2}}\,dx\\ &= F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.02, size = 65, normalized size = 6.50

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{1+3x^2}\,F\left(i\sinh^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}\sqrt{2+5x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6]
)/(Sqrt[3]*Sqrt[2 + 5*x^2 - 3*x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(13) = 26.
time = 0.04, size = 51, normalized size = 5.10

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{6}\right)}{2\sqrt{-3x^4+5x^2+2}}$	51
elliptic	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{6}\right)}{2\sqrt{-3x^4+5x^2+2}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*Ellipti
cF(1/2*2^(1/2)*x,I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.07, size = 8, normalized size = 0.80

$$\mathrm{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x,-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] ellipticF(1/2*sqrt(2)*x, -6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 3*x^4 + 2)^(1/2), x)
```

3.17 $\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}\,x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

```
[Out] 1/6*EllipticF(1/2*x*(-4+2*10^(1/2))^(1/2),1/3*I*6^(1/2)+1/3*I*15^(1/2))*(12
+6*10^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}\,F\left(\text{ArcSin}\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}\,x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[(2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(-2 + Sqrt[10])/2]*x], (-7 - 2
*Sqrt[10])/3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{4+2\sqrt{10}-6x^2}\,\sqrt{-4+2\sqrt{10}+6x^2}}\,dx\\ &= \sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}\,x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 49, normalized size = 1.02

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{2}}}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x], (-7 + 2*Sqrt[10])/3])/Sqr
t[2 + Sqrt[10]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.
time = 0.07, size = 84, normalized size = 1.75

method	result	s
default	$\frac{2\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-4+2\sqrt{10}}}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{-4+2\sqrt{10}}\,\sqrt{-3x^4+4x^2+2}}$	8
elliptic	$\frac{2\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-4+2\sqrt{10}}}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{-4+2\sqrt{10}}\,\sqrt{-3x^4+4x^2+2}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-4+2*10^(1/2))^(1/2)*(1-(-1+1/2*10^(1/2))*x^2)^(1/2)*(1-(-1-1/2*10^(1/2)
)*x^2)^(1/2)/(-3*x^4+4*x^2+2)^(1/2)*EllipticF(1/2*x*(-4+2*10^(1/2))^(1/2),1
/3*I*6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 + 2), x)
```

Fricas [A]
time = 0.09, size = 35, normalized size = 0.73

$$\frac{1}{6}\left(\sqrt{10}+2\right)\sqrt{\sqrt{10}-2}\,\text{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{10}-2},-\frac{2}{3}\sqrt{10}-\frac{7}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/6*(sqrt(10) + 2)*sqrt(sqrt(10) - 2)*ellipticF(1/2*sqrt(2)*x*sqrt(sqrt(10)
 - 2), -2/3*sqrt(10) - 7/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-3x^4+4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+4*x**2+2)**(1/2),x)
[Out] Integral(1/sqrt(-3*x**4 + 4*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 3*x^4 + 2)^(1/2),x)
[Out] int(1/(4*x^2 - 3*x^4 + 2)^(1/2), x)
```

3.18 $\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{-3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

```
[Out] EllipticF(x*6^(1/2)/(3+33^(1/2))^(1/2),1/4*I*6^(1/2)+1/4*I*22^(1/2))*2^(1/2
)/(-3+33^(1/2))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{\sqrt{33}-3}}\,F\left(\text{ArcSin}\left(\sqrt{\frac{6}{3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(3 + Sqrt[33])]*x], (-7 - S
qrt[33])/4]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{3+\sqrt{33}-6x^2}\,\sqrt{-3+\sqrt{33}+6x^2}}\,dx\\ &= \sqrt{\frac{2}{-3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 53, normalized size = 1.10

$$-i\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 3*x^2 - 3*x^4],x]
[Out] (-I)*Sqrt[2/(3 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x],
 (-7 + Sqrt[33])/4]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(37) = 74.
time = 0.07, size = 80, normalized size = 1.67

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3+\sqrt{33}}}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{-3+\sqrt{33}}\,\sqrt{-3x^4+3x^2+2}}$	80
elliptic	$\frac{2\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3+\sqrt{33}}}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{-3+\sqrt{33}}\,\sqrt{-3x^4+3x^2+2}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(-3+33^(1/2))^(1/2)*(1-(-3/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*33^(1/
2))*x^2)^(1/2)/(-3*x^4+3*x^2+2)^(1/2)*EllipticF(1/2*x*(-3+33^(1/2))^(1/2),1
/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 + 2), x)
```

Fricas [A]
time = 0.09, size = 40, normalized size = 0.83

$$\frac{1}{24}\left(\sqrt{33}\,\sqrt{2}+3\sqrt{2}\right)\sqrt{\sqrt{33}-3}\,\mathrm{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{33}-3}\,,-\frac{1}{4}\sqrt{33}-\frac{7}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(33)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(33) - 3)*ellipticF(1/2*x*sqrt
(sqrt(33) - 3), -1/4*sqrt(33) - 7/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 3*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^2 - 3*x^4 + 2)^(1/2), x)
```

3.19 $\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{-1+\sqrt{7}}}$$

```
[Out] EllipticF(x*3^(1/2)/(1+7^(1/2))^(1/2),1/6*I*6^(1/2)+1/6*I*42^(1/2))/(-1+7^(
1/2))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{3}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt[-1 + Sqrt
[7]]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{2+2\sqrt{7}-6x^2}\,\sqrt{-2+2\sqrt{7}+6x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{-1+\sqrt{7}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 49, normalized size = 1.11

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3])/Sqr
t[1 + Sqrt[7]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(34) = 68.
time = 0.06, size = 84, normalized size = 1.91

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\sqrt{7}}}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{-2+2\sqrt{7}}\,\sqrt{-3x^4+2x^2+2}}$	84
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2\sqrt{7}}}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{-2+2\sqrt{7}}\,\sqrt{-3x^4+2x^2+2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-2+2*7^(1/2))^(1/2)*(1-(-1/2+1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*7^(1/2
))*x^2)^(1/2)/(-3*x^4+2*x^2+2)^(1/2)*EllipticF(1/2*x*(-2+2*7^(1/2))^(1/2),1
/6*I*6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 + 2), x)
```

Fricas [A]
time = 0.11, size = 35, normalized size = 0.80

$$\frac{1}{6}\left(\sqrt{7}+1\right)\sqrt{\sqrt{7}-1}\,\text{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{7}-1},-\frac{1}{3}\sqrt{7}-\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/6*(sqrt(7) + 1)*sqrt(sqrt(7) - 1)*ellipticF(1/2*sqrt(2)*x*sqrt(sqrt(7) -
1), -1/3*sqrt(7) - 4/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-3x^4+2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+2*x**2+2)**(1/2),x)
[Out] Integral(1/sqrt(-3*x**4 + 2*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 - 3*x^4 + 2)^(1/2),x)
[Out] int(1/(2*x^2 - 3*x^4 + 2)^(1/2), x)
```

3.20 $\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}(x)\,\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{6-6x^2}\,\sqrt{4+6x^2}}\,dx\\ &= \frac{F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 63, normalized size = 5.25

$$-\frac{i\sqrt{1-x^2}\,\sqrt{2+3x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{3}\,\sqrt{2+x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], -2/3]
)/(Sqrt[3]*Sqrt[2 + x^2 - 3*x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(13) = 26.
time = 0.03, size = 41, normalized size = 3.42

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{-3x^4+x^2+2}}$	41
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(x,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{-3x^4+x^2+2}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(-3*x^4+x^2+2)^(1/2)*EllipticF(x,1/2*I*6
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 8, normalized size = 0.67

$$\frac{1}{2}\,\sqrt{2}\,\text{ellipticF}\left(x,-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*ellipticF(x, -3/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4 + x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{-3\,x^4 + x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(x^2 - 3*x^4 + 2)^(1/2), x)
```

3.21 $\int \frac{1}{\sqrt{2-3x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

```
[Out] 1/6*EllipticF(1/2*3^(1/4)*2^(3/4)*x,I)*6^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {227}

$$\frac{F\left(\text{ArcSin}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b, 4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^4}}\,dx = \frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Mathematica [A]
time = 10.04, size = 18, normalized size = 1.00

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(17) = 34$.
time = 0.12, size = 54, normalized size = 3.00

method	result	size
meijerg	$\frac{\sqrt{2}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], \frac{3 x^{4}}{2}\right)}{2}$	18
default	$\frac{\sqrt{2}\, 6^{\frac{3}{4}} \sqrt{4-2 x^{2} \sqrt{6}}\, \sqrt{4+2 x^{2} \sqrt{6}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}\, 6^{\frac{1}{4}}}{2}, i\right)}{24 \sqrt{-3 x^{4}+2}}$	54
elliptic	$\frac{\sqrt{2}\, 6^{\frac{3}{4}} \sqrt{4-2 x^{2} \sqrt{6}}\, \sqrt{4+2 x^{2} \sqrt{6}}\, \mathrm{EllipticF}\left(\frac{x \sqrt{2}\, 6^{\frac{1}{4}}}{2}, i\right)}{24 \sqrt{-3 x^{4}+2}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*2^(1/2)*6^(3/4)*(4-2*x^2*6^(1/2))^(1/2)*(4+2*x^2*6^(1/2))^(1/2)/(-3*x^
4+2)^(1/2)*EllipticF(1/2*x*2^(1/2)*6^(1/4),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2), x)
```

Fricas [A]
time = 0.07, size = 16, normalized size = 0.89

$$\frac{1}{6} \cdot 6^{\frac{3}{4}} \mathrm{ellipticF}\left(\frac{1}{2} \cdot 6^{\frac{1}{4}} \sqrt{2}\, x, -1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*6^(3/4)*ellipticF(1/2*6^(1/4)*sqrt(2)*x, -1)
```

Sympy [**A**]
time = 0.36, size = 37, normalized size = 2.06

$$\frac{\sqrt{2}\, x\Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{3x^{4}e^{2i\pi}}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+2)**(1/2),x)
```

```
[Out] sqrt(2)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(2*I*pi)/2)/
(8*gamma(5/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2), x)
```

Mupad [**B**]
time = 4.20, size = 16, normalized size = 0.89

$$\frac{\sqrt{2}\, x\, {}_2\mathrm{F}_1\left(\frac{1}{4}, \frac{1}{2};\ \frac{5}{4};\ \frac{3\,x^4}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4)^(1/2),x)
```

```
[Out] (2^(1/2)*x*hypergeom([1/4, 1/2], 5/4, (3*x^4)/2))/2
```

3.22 $\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx$

Optimal. Leaf size=20

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{4-6x^2}\,\sqrt{6+6x^2}}\,dx\\ &= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 10.03, size = 20, normalized size = 1.00

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(18) = 36.
time = 0.04, size = 49, normalized size = 2.45

method	result	size
default	$\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-x^2+2}}$	49
elliptic	$\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-x^2+2}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-x^2+2)^(1/2)*EllipticF(1
/2*x*6^(1/2),1/3*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 16, normalized size = 0.80

$$\frac{1}{3}\,\sqrt{3}\,\mathrm{ellipticF}\left(\frac{1}{2}\,\sqrt{3}\,\sqrt{2}\,x,-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*ellipticF(1/2*sqrt(3)*sqrt(2)*x, -2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4 - x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{-3\,x^4 - x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4 - x^2)^(1/2),x)
```

```
[Out] int(1/(2 - 3*x^4 - x^2)^(1/2), x)
```

3.23 $\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx$

Optimal. Leaf size=42

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

```
[Out] EllipticF(x*3^(1/2)/(-1+7^(1/2))^(1/2),1/6*I*42^(1/2)-1/6*I*6^(1/2))/(1+7^(
1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 2*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqrt[1 + Sqrt
[7]]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-2+2\sqrt{7}-6x^2}\,\sqrt{2+2\sqrt{7}+6x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 51, normalized size = 1.21

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}\,x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{-1+\sqrt{7}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 - 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3])/Sqrt
[-1 + Sqrt[7]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(34) = 68.
time = 0.07, size = 84, normalized size = 2.00

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2+2\sqrt{7}}}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{2+2\sqrt{7}}\,\sqrt{-3x^4-2x^2+2}}$	84
elliptic	$\frac{2\sqrt{1-\left(\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2+2\sqrt{7}}}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{2+2\sqrt{7}}\,\sqrt{-3x^4-2x^2+2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(2+2*7^(1/2))^(1/2)*(1-(1/2*7^(1/2)+1/2)*x^2)^(1/2)*(1-(1/2-1/2*7^(1/2))*
x^2)^(1/2)/(-3*x^4-2*x^2+2)^(1/2)*EllipticF(1/2*x*(2+2*7^(1/2))^(1/2),1/6*I
*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.83

$$\frac{1}{6}\sqrt{\sqrt{7}+1}\left(\sqrt{7}-1\right)\text{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{7}+1},\frac{1}{3}\sqrt{7}-\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(sqrt(7) + 1)*(sqrt(7) - 1)*ellipticF(1/2*sqrt(2)*x*sqrt(sqrt(7) +
1), 1/3*sqrt(7) - 4/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-3x^4-2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-2*x**2+2)**(1/2),x)
[Out] Integral(1/sqrt(-3*x**4 - 2*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-3\,x^4-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4 - 2*x^2)^(1/2),x)
[Out] int(1/(2 - 3*x^4 - 2*x^2)^(1/2), x)
```

3.24 $\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx$

Optimal. Leaf size=46

$$\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

[Out] EllipticF(x*6^(1/2)/(-3+33^(1/2))^(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/2))*2^(1/2)/(3+33^(1/2))^(1/2)

Rubi [A]

time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\text{ArcSin}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 3*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(-3 + Sqrt[33])]*x], (-7 + Sqrt[33])/4]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-3+\sqrt{33}-6x^2}\,\sqrt{3+\sqrt{33}+6x^2}}\,dx\\ &= \sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 55, normalized size = 1.20

$$-i\sqrt{\frac{2}{-3+\sqrt{33}}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}\,x\right)\middle|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 - 3*x^2 - 3*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-3 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqrt[33])]*x],
 -7/4 - Sqrt[33]/4]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(37) = 74.
time = 0.07, size = 80, normalized size = 1.74

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{\sqrt{33}}{4}+\frac{3}{4}\right)x^{2}}\,\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+\sqrt{33}}}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{3+\sqrt{33}}\,\sqrt{-3x^{4}-3x^{2}+2}}$	80
elliptic	$\frac{2\sqrt{1-\left(\frac{\sqrt{33}}{4}+\frac{3}{4}\right)x^{2}}\,\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+\sqrt{33}}}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{3+\sqrt{33}}\,\sqrt{-3x^{4}-3x^{2}+2}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(3+33^(1/2))^(1/2)*(1-(1/4*33^(1/2)+3/4)*x^2)^(1/2)*(1-(3/4-1/4*33^(1/2))
*x^2)^(1/2)/(-3*x^4-3*x^2+2)^(1/2)*EllipticF(1/2*x*(3+33^(1/2))^(1/2),1/4*I
*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 + 2), x)
```

Fricas [A]
time = 0.07, size = 40, normalized size = 0.87

$$\frac{1}{24}\left(\sqrt{33}\,\sqrt{2}-3\,\sqrt{2}\right)\sqrt{\sqrt{33}+3}\;\mathrm{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{33}+3}\,,\frac{1}{4}\,\sqrt{33}-\frac{7}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(33)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(33) + 3)*ellipticF(1/2*x*sqrt
(sqrt(33) + 3), 1/4*sqrt(33) - 7/4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4 - 3x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 3*x**2 + 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^4 - 3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4 - 3*x^2)^(1/2),x)
```

```
[Out] int(1/(2 - 3*x^4 - 3*x^2)^(1/2), x)
```

3.25 $\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{1}{6}\left(-2+\sqrt{10}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

```
[Out] 1/6*EllipticF(1/2*x*(4+2*10^(1/2))^(1/2),1/3*I*15^(1/2)-1/3*I*6^(1/2))*(-12
+6*10^(1/2))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{1}{6}\left(\sqrt{10}-2\right)}\,F\left(\text{ArcSin}\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 4*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[(-2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(2 + Sqrt[10])/2]*x], (-7 + 2
*Sqrt[10])/3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-4+2\sqrt{10}-6x^2}\,\sqrt{4+2\sqrt{10}+6x^2}}\,dx\\ &= \sqrt{\frac{1}{6}\left(-2+\sqrt{10}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 49, normalized size = 1.02

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{2}}}\ x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{-2+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 - 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x], (-7 - 2*Sqrt[10])/3])/Sq
rt[-2 + Sqrt[10]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.
time = 0.07, size = 84, normalized size = 1.75

method	result	size
default	$\frac{2\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{4+2\sqrt{10}}}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{4+2\sqrt{10}}\,\sqrt{-3x^4-4x^2+2}}$	84
elliptic	$\frac{2\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{4+2\sqrt{10}}}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{4+2\sqrt{10}}\,\sqrt{-3x^4-4x^2+2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(4+2*10^(1/2))^(1/2)*(1-(1+1/2*10^(1/2))*x^2)^(1/2)*(1-(1-1/2*10^(1/2))*x
^2)^(1/2)/(-3*x^4-4*x^2+2)^(1/2)*EllipticF(1/2*x*(4+2*10^(1/2))^(1/2),1/3*I
*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.73

$$\frac{1}{6}\sqrt{\sqrt{10}+2}\left(\sqrt{10}-2\right)\mathrm{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{10}+2}\,,\frac{2}{3}\sqrt{10}-\frac{7}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(sqrt(10) + 2)*(sqrt(10) - 2)*ellipticF(1/2*sqrt(2)*x*sqrt(sqrt(10)
 + 2), 2/3*sqrt(10) - 7/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4-4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 4*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4 - 4*x^2)^(1/2),x)
```

```
[Out] int(1/(2 - 3*x^4 - 4*x^2)^(1/2), x)
```

3.26 $\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

```
[Out] 1/6*EllipticF(x*3^(1/2),1/6*I*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 5*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3]*x], -1/6]/Sqrt[6]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-5x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{2-6x^2}\,\sqrt{12+6x^2}}\,dx\\ &= \frac{F\left(\sin^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(18) = 36.
time = 10.03, size = 54, normalized size = 3.00

$$\frac{\sqrt{1-3x^2}\sqrt{2+x^2}\,F\left(\sin^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}\sqrt{2-5x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 5*x^2 - 3*x^4],x]
```

```
[Out] (Sqrt[1 - 3*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x], -1/6])/(Sqrt[6]
*Sqrt[2 - 5*x^2 - 3*x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(17) = 34.
time = 0.04, size = 50, normalized size = 2.78

method	result	size
default	$\frac{\sqrt{3}\sqrt{-3x^2+1}\sqrt{2x^2+4}\,\text{EllipticF}\left(x\sqrt{3},\frac{i\sqrt{6}}{6}\right)}{6\sqrt{-3x^4-5x^2+2}}$	50
elliptic	$\frac{\sqrt{3}\sqrt{-3x^2+1}\sqrt{2x^2+4}\,\text{EllipticF}\left(x\sqrt{3},\frac{i\sqrt{6}}{6}\right)}{6\sqrt{-3x^4-5x^2+2}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*3^(1/2)*(-3*x^2+1)^(1/2)*(2*x^2+4)^(1/2)/(-3*x^4-5*x^2+2)^(1/2)*Ellipti
cF(x*3^(1/2),1/6*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 15, normalized size = 0.83

$$\frac{1}{6}\sqrt{3}\sqrt{2}\,\text{ellipticF}\left(\sqrt{3}\,x,-\frac{1}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*sqrt(2)*ellipticF(sqrt(3)*x, -1/6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4 - 5x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-5*x**2+2)**(1/2),x)
[Out] Integral(1/sqrt(-3*x**4 - 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{-3\,x^4 - 5\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2 - 3*x^4 - 5*x^2)^(1/2),x)
[Out] int(1/(2 - 3*x^4 - 5*x^2)^(1/2), x)
```

3.27 $\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{-7+\sqrt{73}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

```
[Out] EllipticF(2*x/(7+73^(1/2))^(1/2),7/12*I*6^(1/2)+1/12*I*438^(1/2))*2^(1/2)/(
-7+73^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{\sqrt{73}-7}}\,F\left(\text{ArcSin}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 7*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[7 + Sqrt[73]]], (-61 -
7*Sqrt[73])/12]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{7+\sqrt{73}-4x^2}\,\sqrt{-7+\sqrt{73}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{-7+\sqrt{73}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{7+\sqrt{73}}}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 7*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(7 + Sqrt[73])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-7 + Sqrt[73]]],
 (-61 + 7*Sqrt[73])/12]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(35) = 70.
time = 0.07, size = 84, normalized size = 1.87

method	result
default	$\frac{6\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-42+6\sqrt{73}}}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{\sqrt{-42+6\sqrt{73}}\,\sqrt{-2x^4+7x^2+3}}$
elliptic	$\frac{6\sqrt{1-\left(-\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-42+6\sqrt{73}}}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{\sqrt{-42+6\sqrt{73}}\,\sqrt{-2x^4+7x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+7*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(-42+6*73^(1/2))^(1/2)*(1-(-7/6+1/6*73^(1/2))*x^2)^(1/2)*(1-(-7/6-1/6*73^
(1/2))*x^2)^(1/2)/(-2*x^4+7*x^2+3)^(1/2)*EllipticF(1/6*x*(-42+6*73^(1/2))^(
1/2),7/12*I*6^(1/2)+1/12*I*438^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 + 3), x)
```

Fricas [A]
time = 0.09, size = 49, normalized size = 1.09

$$\frac{1}{72}\left(\sqrt{73}\sqrt{6}\sqrt{3}+7\sqrt{6}\sqrt{3}\right)\sqrt{\sqrt{73}-7}\,\mathrm{ellipticF}\left(\frac{1}{6}\sqrt{6}\,x\sqrt{\sqrt{73}-7},-\frac{7}{12}\sqrt{73}-\frac{61}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/72*(sqrt(73)*sqrt(6)*sqrt(3) + 7*sqrt(6)*sqrt(3))*sqrt(sqrt(73) - 7)*elli
pticF(1/6*sqrt(6)*x*sqrt(sqrt(73) - 7), -7/12*sqrt(73) - 61/12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4+7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+7*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + 7*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-2\,x^4+7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x^2 - 2*x^4 + 3)^(1/2),x)
[Out] int(1/(7*x^2 - 2*x^4 + 3)^(1/2), x)
```

3.28 $\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}\,x\right)\middle|-4-\sqrt{15}\right)$$

```
[Out] 1/6*EllipticF(1/3*x*(-9+3*15^(1/2))^(1/2),1/2*I*6^(1/2)+1/2*I*10^(1/2))*(18
+6*15^(1/2))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}\,F\left(\text{ArcSin}\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}\,x\right)\middle|-4-\sqrt{15}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 6*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[(3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(-3 + Sqrt[15])/3]*x], -4 - Sq
rt[15]]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx &= \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6+2\sqrt{15}-4x^2}\,\sqrt{-6+2\sqrt{15}+4x^2}}\,dx\\ &= \sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}\,x\right)\middle|-4-\sqrt{15}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 43, normalized size = 0.98

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{3}}}\,x\right)\middle|-4+\sqrt{15}\right)}{\sqrt{3+\sqrt{15}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 6*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/3]]*x], -4 + Sqrt[15]])/Sqrt[3 +
Sqrt[15]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.
time = 0.07, size = 84, normalized size = 1.91

method	result	siz
default	$\frac{3\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-9+3\sqrt{15}}}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{\sqrt{-9+3\sqrt{15}}\,\sqrt{-2x^4+6x^2+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(-1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(-1-\frac{\sqrt{15}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-9+3\sqrt{15}}}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{\sqrt{-9+3\sqrt{15}}\,\sqrt{-2x^4+6x^2+3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+6*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(-9+3*15^(1/2))^(1/2)*(1-(-1+1/3*15^(1/2))*x^2)^(1/2)*(1-(-1-1/3*15^(1/2)
)*x^2)^(1/2)/(-2*x^4+6*x^2+3)^(1/2)*EllipticF(1/3*x*(-9+3*15^(1/2))^(1/2),1
/2*I*6^(1/2)+1/2*I*10^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 50, normalized size = 1.14

$$\frac{1}{6}\left(\sqrt{5}\,\sqrt{3}+3\right)\sqrt{\sqrt{5}\,\sqrt{3}-3}\,\mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\,\sqrt{\sqrt{5}\,\sqrt{3}-3}\,x,-\sqrt{5}\,\sqrt{3}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(5)*sqrt(3) + 3)*sqrt(sqrt(5)*sqrt(3) - 3)*ellipticF(1/3*sqrt(3)*s
qrt(sqrt(5)*sqrt(3) - 3)*x, -sqrt(5)*sqrt(3) - 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4+6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 6*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x^2 - 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(6*x^2 - 2*x^4 + 3)^(1/2), x)
```

3.29 $\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-6\right)$$

```
[Out] EllipticF(1/3*x*3^(1/2),I*6^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$F\left(\text{ArcSin}\left(\frac{x}{\sqrt{3}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 5*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[3]], -6]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx &= \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{12-4x^2}\,\sqrt{2+4x^2}}\,dx\\ &= F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-6\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 65, normalized size = 6.50

$$-\frac{i\sqrt{1-\frac{x^2}{3}}\sqrt{1+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{2}\sqrt{3+5x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 5*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/3]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1/6]
)/(Sqrt[2]*Sqrt[3 + 5*x^2 - 2*x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(13) = 26.
time = 0.05, size = 51, normalized size = 5.10

method	result	size
default	$\frac{\sqrt{3}\sqrt{-3x^2+9}\sqrt{2x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}}{3},i\sqrt{6}\right)}{3\sqrt{-2x^4+5x^2+3}}$	51
elliptic	$\frac{\sqrt{3}\sqrt{-3x^2+9}\sqrt{2x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}}{3},i\sqrt{6}\right)}{3\sqrt{-2x^4+5x^2+3}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(-2*x^4+5*x^2+3)^(1/2)*Ellipti
cF(1/3*x*3^(1/2),I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 3), x)
```

Fricas [A]
time = 0.07, size = 8, normalized size = 0.80

$$\mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x,-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] ellipticF(1/3*sqrt(3)*x, -6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+5*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{-2\,x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 2*x^4 + 3)^(1/2),x)
[Out] int(1/(5*x^2 - 2*x^4 + 3)^(1/2), x)
```

3.30 $\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{-2+\sqrt{10}}}$$

```
[Out] EllipticF(x*2^(1/2)/(2+10^(1/2))^(1/2),1/3*I*6^(1/2)+1/3*I*15^(1/2))/(-2+10
^(1/2))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 4*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(2 + Sqrt[10])]*x], (-7 - 2*Sqrt[10])/3]/Sqrt[-2 +
Sqrt[10]]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+4x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{4+2\sqrt{10}-4x^2}\,\sqrt{-4+2\sqrt{10}+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right)\middle|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{-2+\sqrt{10}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 51, normalized size = 1.16

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}\,x\right)\middle|-\frac{7}{3}+\frac{2\sqrt{10}}{3}\right)}{\sqrt{2+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*x], -7/3 + (2*Sqrt[10])/3
])/Sqrt[2 + Sqrt[10]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(34) = 68.
time = 0.06, size = 84, normalized size = 1.91

method	result	si
default	$\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+3\sqrt{10}}}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{-6+3\sqrt{10}}\,\sqrt{-2x^4+4x^2+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+3\sqrt{10}}}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{-6+3\sqrt{10}}\,\sqrt{-2x^4+4x^2+3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+4*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(-6+3*10^(1/2))^(1/2)*(1-(-2/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/3-1/3*10^(
1/2))*x^2)^(1/2)/(-2*x^4+4*x^2+3)^(1/2)*EllipticF(1/3*x*(-6+3*10^(1/2))^(1/
2),1/3*I*6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.80

$$\frac{1}{6}\left(\sqrt{10}+2\right)\sqrt{\sqrt{10}-2}\,\text{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{10}-2}\,,-\frac{2}{3}\sqrt{10}-\frac{7}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/6*(sqrt(10) + 2)*sqrt(sqrt(10) - 2)*ellipticF(1/3*sqrt(3)*x*sqrt(sqrt(10)
 - 2), -2/3*sqrt(10) - 7/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+4*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + 4*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 2*x^4 + 3)^(1/2),x)
[Out] int(1/(4*x^2 - 2*x^4 + 3)^(1/2), x)
```

3.31 $\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{-3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

```
[Out] EllipticF(2*x/(3+33^(1/2))^(1/2),1/4*I*6^(1/2)+1/4*I*22^(1/2))*2^(1/2)/(-3+
33^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{\sqrt{33}-3}}\,F\left(\text{ArcSin}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 3*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[3 + Sqrt[33]]], (-7 - S
qrt[33])/4]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{3+\sqrt{33}-4x^2}\,\sqrt{-3+\sqrt{33}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{-3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 50, normalized size = 1.11

$$-i\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 3*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(3 + Sqrt[33])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 + Sqrt[33]]],
 (-7 + Sqrt[33])/4]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(35) = 70.
time = 0.07, size = 84, normalized size = 1.87

method	result
default	$\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-18+6\sqrt{33}}}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{-18+6\sqrt{33}}\,\sqrt{-2x^4+3x^2+3}}$
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-18+6\sqrt{33}}}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{-18+6\sqrt{33}}\,\sqrt{-2x^4+3x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(-18+6*33^(1/2))^(1/2)*(1-(-1/2+1/6*33^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*33^
(1/2))*x^2)^(1/2)/(-2*x^4+3*x^2+3)^(1/2)*EllipticF(1/6*x*(-18+6*33^(1/2))^(
1/2),1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 + 3), x)
```

Fricas [A]
time = 0.09, size = 56, normalized size = 1.24

$$\frac{1}{24}\left(\sqrt{11}\,\sqrt{6}+\sqrt{6}\,\sqrt{3}\right)\sqrt{\sqrt{11}\,\sqrt{3}-3}\,\mathrm{ellipticF}\left(\frac{1}{6}\,\sqrt{6}\,\sqrt{\sqrt{11}\,\sqrt{3}-3}\,x,-\frac{1}{4}\,\sqrt{11}\,\sqrt{3}-\frac{7}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/24*(sqrt(11)*sqrt(6) + sqrt(6)*sqrt(3))*sqrt(sqrt(11)*sqrt(3) - 3)*ellipt
icF(1/6*sqrt(6)*sqrt(sqrt(11)*sqrt(3) - 3)*x, -1/4*sqrt(11)*sqrt(3) - 7/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4+3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+3*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + 3*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 2*x^4 + 3)^(1/2),x)
[Out] int(1/(3*x^2 - 2*x^4 + 3)^(1/2), x)
```

3.32 $\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{-1+\sqrt{7}}}$$

```
[Out] EllipticF(x*2^(1/2)/(1+7^(1/2))^(1/2),1/6*I*6^(1/2)+1/6*I*42^(1/2))/(-1+7^(
1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{2}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 2*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt[-1 + Sqrt
[7]]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{2+2\sqrt{7}-4x^2}\,\sqrt{-2+2\sqrt{7}+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{-1+\sqrt{7}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 49, normalized size = 1.11

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3])/Sqr
t[1 + Sqrt[7]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(34) = 68.
time = 0.07, size = 84, normalized size = 1.91

method	result	size
default	$\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3+3\sqrt{7}}}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{-3+3\sqrt{7}}\,\sqrt{-2x^4+2x^2+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3+3\sqrt{7}}}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{-3+3\sqrt{7}}\,\sqrt{-2x^4+2x^2+3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(-3+3*7^(1/2))^(1/2)*(1-(-1/3+1/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3-1/3*7^(1/2
))*x^2)^(1/2)/(-2*x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x*(-3+3*7^(1/2))^(1/2),1
/6*I*6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.80

$$\frac{1}{6}\left(\sqrt{7}+1\right)\sqrt{\sqrt{7}-1}\,\mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{7}-1},-\frac{1}{3}\sqrt{7}-\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/6*(sqrt(7) + 1)*sqrt(sqrt(7) - 1)*ellipticF(1/3*sqrt(3)*x*sqrt(sqrt(7) -
1), -1/3*sqrt(7) - 4/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4+2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+2*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + 2*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 - 2*x^4 + 3)^(1/2),x)
[Out] int(1/(2*x^2 - 2*x^4 + 3)^(1/2), x)
```

3.33 $\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx$

Optimal. Leaf size=20

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(1/3*x*6^(1/2),1/2*I*6^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3+x^2-2x^4}}\,dx &= \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6-4x^2}\,\sqrt{4+4x^2}}\,dx\\ &= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|-\frac{3}{2}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [**A**]
time = 10.04, size = 20, normalized size = 1.00

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.
time = 0.04, size = 47, normalized size = 2.35

method	result	size
default	$\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{6\sqrt{-2x^4+x^2+3}}$	47
elliptic	$\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{6\sqrt{-2x^4+x^2+3}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-2*x^4+x^2+3)^(1/2)*EllipticF(1
/3*x*6^(1/2),1/2*I*6^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 + 3), x)
```

Fricas [**A**]
time = 0.07, size = 16, normalized size = 0.80

$$\frac{1}{2}\,\sqrt{2}\,\mathrm{ellipticF}\left(\frac{1}{3}\,\sqrt{3}\,\sqrt{2}\,x,-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*ellipticF(1/3*sqrt(3)*sqrt(2)*x, -3/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 + x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{-2\,x^4 + x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*x^4 + 3)^(1/2),x)
[Out] int(1/(x^2 - 2*x^4 + 3)^(1/2), x)
```

3.34 $\int \frac{1}{\sqrt{3-2x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

```
[Out] 1/6*EllipticF(1/3*2^(1/4)*3^(3/4)*x,I)*6^(3/4)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {227}

$$\frac{F\left(\text{ArcSin}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2x^4}}\,dx = \frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Mathematica [A]
time = 10.03, size = 18, normalized size = 1.00

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(17) = 34$.
time = 0.10, size = 54, normalized size = 3.00

method	result	size
meijerg	$\frac{\sqrt{3}\, x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],\frac{2x^4}{3}\right)}{3}$	18
default	$\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{9-3x^2\sqrt{6}}\, \sqrt{9+3x^2\sqrt{6}}\, \operatorname{EllipticF}\left(\frac{x\sqrt{3}\, 6^{\frac{1}{4}}}{3}, i\right)}{54\sqrt{-2x^4+3}}$	54
elliptic	$\frac{\sqrt{3}\, 6^{\frac{3}{4}} \sqrt{9-3x^2\sqrt{6}}\, \sqrt{9+3x^2\sqrt{6}}\, \operatorname{EllipticF}\left(\frac{x\sqrt{3}\, 6^{\frac{1}{4}}}{3}, i\right)}{54\sqrt{-2x^4+3}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*3^(1/2)*6^(3/4)*(9-3*x^2*6^(1/2))^(1/2)*(9+3*x^2*6^(1/2))^(1/2)/(-2*x^
4+3)^(1/2)*EllipticF(1/3*x*3^(1/2)*6^(1/4),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.
time = 0.07, size = 34, normalized size = 1.89

$$\frac{1}{6}\sqrt{3}\,\sqrt{2}\,\sqrt{\sqrt{3}\,\sqrt{2}}\,\operatorname{ellipticF}\left(\frac{1}{3}\sqrt{3}\,\sqrt{\sqrt{3}\,\sqrt{2}}\,x, -1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2)*sqrt(sqrt(3)*sqrt(2))*ellipticF(1/3*sqrt(3)*sqrt(sqrt(3
)*sqrt(2))*x, -1)
```

Sympy [A]
time = 0.35, size = 37, normalized size = 2.06

$$\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{2i\pi}}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+3)**(1/2),x)
[Out] sqrt(3)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(2*I*pi)/3)/
(12*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + 3), x)
```

Mupad [B]
time = 4.20, size = 16, normalized size = 0.89

$$\frac{\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{2\,x^4}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4)^(1/2),x)
[Out] (3^(1/2)*x*hypergeom([1/4, 1/2], 5/4, (2*x^4)/3))/3
```

3.35 $\int \frac{1}{\sqrt{3-x^2-2x^4}} \, dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(x,1/3*I*6^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}(x)\,\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -2/3]/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-x^2-2x^4}} \, dx &= \left(2\sqrt{2}\right) \int \frac{1}{\sqrt{4-4x^2}\,\sqrt{6+4x^2}} \, dx \\ &= \frac{F\left(\sin^{-1}(x)\middle|-\frac{2}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.02, size = 65, normalized size = 5.42

$$-\frac{i\sqrt{1-x^2}\,\sqrt{3+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{3-x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], -3/2]
)/(Sqrt[2]*Sqrt[3 - x^2 - 2*x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.
time = 0.03, size = 43, normalized size = 3.58

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{6}}{3}\right)}{3\sqrt{-2x^4-x^2+3}}$	43
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{6}}{3}\right)}{3\sqrt{-2x^4-x^2+3}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(6*x^2+9)^(1/2)/(-2*x^4-x^2+3)^(1/2)*EllipticF(x,1/3*I*6
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 8, normalized size = 0.67

$$\frac{1}{3}\,\sqrt{3}\,\mathrm{ellipticF}\left(x,-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*ellipticF(x, -2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 - x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{-2\,x^4 - x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - x^2)^(1/2),x)
```

```
[Out] int(1/(3 - 2*x^4 - x^2)^(1/2), x)
```

3.36 $\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx$

Optimal. Leaf size=42

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

```
[Out] EllipticF(x*2^(1/2)/(-1+7^(1/2))^(1/2),1/6*I*42^(1/2)-1/6*I*6^(1/2))/(1+7^(
1/2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 2*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqrt[1 + Sqrt
[7]]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-2+2\sqrt{7}-4x^2}\,\sqrt{2+2\sqrt{7}+4x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 51, normalized size = 1.21

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}\,x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{-1+\sqrt{7}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7]/3])/Sqrt
[-1 + Sqrt[7]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(34) = 68.
time = 0.06, size = 84, normalized size = 2.00

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{\sqrt{7}}{3}+\frac{1}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+3\sqrt{7}}}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{3+3\sqrt{7}}\,\sqrt{-2x^4-2x^2+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(\frac{\sqrt{7}}{3}+\frac{1}{3}\right)x^2}\,\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+3\sqrt{7}}}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{3+3\sqrt{7}}\,\sqrt{-2x^4-2x^2+3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(3+3*7^(1/2))^(1/2)*(1-(1/3*7^(1/2)+1/3)*x^2)^(1/2)*(1-(1/3-1/3*7^(1/2))*
x^2)^(1/2)/(-2*x^4-2*x^2+3)^(1/2)*EllipticF(1/3*x*(3+3*7^(1/2))^(1/2),1/6*I
*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 + 3), x)
```

Fricas [A]
time = 0.09, size = 35, normalized size = 0.83

$$\frac{1}{6}\sqrt{\sqrt{7}+1}\left(\sqrt{7}-1\right)\text{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{7}+1},\frac{1}{3}\sqrt{7}-\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(sqrt(7) + 1)*(sqrt(7) - 1)*ellipticF(1/3*sqrt(3)*x*sqrt(sqrt(7) +
1), 1/3*sqrt(7) - 4/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4-2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-2*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 - 2*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 2*x^2)^(1/2),x)
[Out] int(1/(3 - 2*x^4 - 2*x^2)^(1/2), x)
```

3.37 $\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx$

Optimal. Leaf size=43

$$\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

```
[Out] EllipticF(2*x/(-3+33^(1/2))^(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/2))*2^(1/2)/(3+
33^(1/2))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\text{ArcSin}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 3*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[-3 + Sqrt[33]]], (-7 + S
qrt[33])/4]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-3+\sqrt{33}-4x^2}\,\sqrt{3+\sqrt{33}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{3+\sqrt{33}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 52, normalized size = 1.21

$$-i\sqrt{\frac{2}{-3+\sqrt{33}}}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 3*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-3 + Sqrt[33])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[33]]],
 -7/4 - Sqrt[33]/4]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(35) = 70.
time = 0.05, size = 84, normalized size = 1.95

method	result	size
default	$\frac{6\sqrt{1-\left(\frac{\sqrt{33}}{6}+\frac{1}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{18+6\sqrt{33}}}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{18+6\sqrt{33}}\,\sqrt{-2x^4-3x^2+3}}$	84
elliptic	$\frac{6\sqrt{1-\left(\frac{\sqrt{33}}{6}+\frac{1}{2}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{18+6\sqrt{33}}}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{18+6\sqrt{33}}\,\sqrt{-2x^4-3x^2+3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(18+6*33^(1/2))^(1/2)*(1-(1/6*33^(1/2)+1/2)*x^2)^(1/2)*(1-(1/2-1/6*33^(1/
2))*x^2)^(1/2)/(-2*x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*(18+6*33^(1/2))^(1/2)
,1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 57, normalized size = 1.33

$$\frac{1}{24}\left(\sqrt{11}\,\sqrt{6}-\sqrt{6}\,\sqrt{3}\right)\sqrt{\sqrt{11}\,\sqrt{3}+3}\,\mathrm{ellipticF}\left(\frac{1}{6}\,\sqrt{6}\,\sqrt{\sqrt{11}\,\sqrt{3}+3}\,x,\frac{1}{4}\,\sqrt{11}\,\sqrt{3}-\frac{7}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/24*(sqrt(11)*sqrt(6) - sqrt(6)*sqrt(3))*sqrt(sqrt(11)*sqrt(3) + 3)*ellipt
icF(1/6*sqrt(6)*sqrt(sqrt(11)*sqrt(3) + 3)*x, 1/4*sqrt(11)*sqrt(3) - 7/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-3*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 - 3*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 3*x^2)^(1/2),x)
[Out] int(1/(3 - 2*x^4 - 3*x^2)^(1/2), x)
```

3.38 $\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

```
[Out] EllipticF(x*2^(1/2)/(-2+10^(1/2))^(1/2),1/3*I*15^(1/2)-1/3*I*6^(1/2))/(2+10
^(1/2))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 4*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(-2 + Sqrt[10])]*x], (-7 + 2*Sqrt[10])/3]/Sqrt[2 +
Sqrt[10]]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-4+2\sqrt{10}-4x^2}\,\sqrt{4+2\sqrt{10}+4x^2}}\,dx$$

$$= \frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 51, normalized size = 1.16

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right)|-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{-2+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x], -7/3 - (2*Sqrt[10])/3]
)/Sqrt[-2 + Sqrt[10]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(34) = 68.
time = 0.05, size = 84, normalized size = 1.91

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6+3\sqrt{10}}}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{6+3\sqrt{10}}\,\sqrt{-2x^4-4x^2+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6+3\sqrt{10}}}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{6+3\sqrt{10}}\,\sqrt{-2x^4-4x^2+3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-4*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(6+3*10^(1/2))^(1/2)*(1-(2/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3-1/3*10^(1/2
))*x^2)^(1/2)/(-2*x^4-4*x^2+3)^(1/2)*EllipticF(1/3*x*(6+3*10^(1/2))^(1/2),1
/3*I*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 + 3), x)
```

Fricas [A]
time = 0.10, size = 35, normalized size = 0.80

$$\frac{1}{6}\sqrt{\sqrt{10}+2}\left(\sqrt{10}-2\right)\mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{10}+2},\frac{2}{3}\sqrt{10}-\frac{7}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(sqrt(10) + 2)*(sqrt(10) - 2)*ellipticF(1/3*sqrt(3)*x*sqrt(sqrt(10)
 + 2), 2/3*sqrt(10) - 7/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4-4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-4*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 - 4*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 4*x^2)^(1/2),x)
[Out] int(1/(3 - 2*x^4 - 4*x^2)^(1/2), x)
```

3.39 $\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

```
[Out] 1/6*EllipticF(x*2^(1/2),1/6*I*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 5*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/6]/Sqrt[6]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-5x^2-2x^4}}\,dx &= \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{2-4x^2}\,\sqrt{12+4x^2}}\,dx\\ &= \frac{F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(18) = 36.
time = 10.02, size = 54, normalized size = 3.00

$$\frac{\sqrt{1-2x^2}\sqrt{3+x^2}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}\sqrt{3-5x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 5*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/6])/(Sqrt[6]
*Sqrt[3 - 5*x^2 - 2*x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(17) = 34.
time = 0.04, size = 50, normalized size = 2.78

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(\sqrt{2}\,x,\frac{i\sqrt{6}}{6}\right)}{6\sqrt{-2x^4-5x^2+3}}$	50
elliptic	$\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(\sqrt{2}\,x,\frac{i\sqrt{6}}{6}\right)}{6\sqrt{-2x^4-5x^2+3}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(1/2)*(-2*x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-2*x^4-5*x^2+3)^(1/2)*Ellipti
cF(2^(1/2)*x,1/6*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 + 3), x)
```

Fricas [A]
time = 0.09, size = 15, normalized size = 0.83

$$\frac{1}{6}\sqrt{3}\sqrt{2}\,\mathrm{ellipticF}\left(\sqrt{2}\,x,-\frac{1}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2)*ellipticF(sqrt(2)*x, -1/6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 - 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 5*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{-2\,x^4 - 5\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 5*x^2)^(1/2),x)
```

```
[Out] int(1/(3 - 2*x^4 - 5*x^2)^(1/2), x)
```

3.40 $\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx$

Optimal. Leaf size=42

$$\sqrt{\frac{1}{6}\left(-3+\sqrt{15}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}\,x\right)|-4+\sqrt{15}\right)$$

```
[Out] 1/6*EllipticF(1/3*x*(9+3*15^(1/2))^(1/2),1/2*I*10^(1/2)-1/2*I*6^(1/2))*(-18
+6*15^(1/2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{1}{6}\left(\sqrt{15}-3\right)}\,F\left(\text{ArcSin}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}\,x\right)|-4+\sqrt{15}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 6*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[(-3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(3 + Sqrt[15])/3]*x], -4 + Sq
rt[15]]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx &= \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-6+2\sqrt{15}-4x^2}\,\sqrt{6+2\sqrt{15}+4x^2}}\,dx\\ &= \sqrt{\frac{1}{6}\left(-3+\sqrt{15}\right)}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}\,x\right)|-4+\sqrt{15}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 45, normalized size = 1.07

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{3}}}\,x\right)\middle|-4-\sqrt{15}\right)}{\sqrt{-3+\sqrt{15}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 6*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x], -4 - Sqrt[15]])/Sqrt[-3
+ Sqrt[15]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.
time = 0.07, size = 84, normalized size = 2.00

method	result	size
default	$\frac{3\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{9+3\sqrt{15}}}{3},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{\sqrt{9+3\sqrt{15}}\,\sqrt{-2x^4-6x^2+3}}$	84
elliptic	$\frac{3\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{9+3\sqrt{15}}}{3},\frac{i\sqrt{10}}{2}-\frac{i\sqrt{6}}{2}\right)}{\sqrt{9+3\sqrt{15}}\,\sqrt{-2x^4-6x^2+3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-6*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(9+3*15^(1/2))^(1/2)*(1-(1+1/3*15^(1/2))*x^2)^(1/2)*(1-(1-1/3*15^(1/2))*x
^2)^(1/2)/(-2*x^4-6*x^2+3)^(1/2)*EllipticF(1/3*x*(9+3*15^(1/2))^(1/2),1/2*I
*10^(1/2)-1/2*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 6*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 49, normalized size = 1.17

$$\frac{1}{6}\sqrt{\sqrt{5}\sqrt{3}+3}\left(\sqrt{5}\sqrt{3}-3\right)\mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\sqrt{\sqrt{5}\sqrt{3}+3}\,x,\sqrt{5}\sqrt{3}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(sqrt(5)*sqrt(3) + 3)*(sqrt(5)*sqrt(3) - 3)*ellipticF(1/3*sqrt(3)*s
qrt(sqrt(5)*sqrt(3) + 3)*x, sqrt(5)*sqrt(3) - 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4-6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 6*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-6*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 6*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 6*x^2)^(1/2),x)
```

```
[Out] int(1/(3 - 2*x^4 - 6*x^2)^(1/2), x)
```

3.41 $\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{7+\sqrt{73}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)\middle|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

```
[Out] EllipticF(2*x/(-7+73^(1/2))^(1/2),1/12*I*438^(1/2)-7/12*I*6^(1/2))*2^(1/2)/
(7+73^(1/2))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{7+\sqrt{73}}}\,F\left(\text{ArcSin}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)\middle|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 7*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[-7 + Sqrt[73]]], (-61 +
7*Sqrt[73])/12]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-7+\sqrt{73}-4x^2}\,\sqrt{7+\sqrt{73}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{7+\sqrt{73}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{-7+\sqrt{73}}}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 - 7*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-7 + Sqrt[73])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[7 + Sqrt[73]]],
 (-61 - 7*Sqrt[73])/12]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(35) = 70.
time = 0.08, size = 84, normalized size = 1.87

method	result	siz
default	$\frac{6\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{42+6\sqrt{73}}}{6},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{\sqrt{42+6\sqrt{73}}\,\sqrt{-2x^4-7x^2+3}}$	84
elliptic	$\frac{6\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\sqrt{1-\left(\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{42+6\sqrt{73}}}{6},\frac{i\sqrt{438}}{12}-\frac{7i\sqrt{6}}{12}\right)}{\sqrt{42+6\sqrt{73}}\,\sqrt{-2x^4-7x^2+3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-7*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(42+6*73^(1/2))^(1/2)*(1-(7/6+1/6*73^(1/2))*x^2)^(1/2)*(1-(7/6-1/6*73^(1/
2))*x^2)^(1/2)/(-2*x^4-7*x^2+3)^(1/2)*EllipticF(1/6*x*(42+6*73^(1/2))^(1/2)
,1/12*I*438^(1/2)-7/12*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 - 7*x^2 + 3), x)
```

Fricas [A]
time = 0.07, size = 49, normalized size = 1.09

$$\frac{1}{72}\left(\sqrt{73}\,\sqrt{6}\,\sqrt{3}-7\,\sqrt{6}\,\sqrt{3}\right)\sqrt{\sqrt{73}+7}\,\mathrm{ellipticF}\left(\frac{1}{6}\,\sqrt{6}\,x\sqrt{\sqrt{73}+7},\frac{7}{12}\,\sqrt{73}-\frac{61}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/72*(sqrt(73)*sqrt(6)*sqrt(3) - 7*sqrt(6)*sqrt(3))*sqrt(sqrt(73) + 7)*elli
pticF(1/6*sqrt(6)*x*sqrt(sqrt(73) + 7), 7/12*sqrt(73) - 61/12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4-7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-7*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 - 7*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-7*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 - 7*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - 2*x^4 - 7*x^2)^(1/2),x)
[Out] int(1/(3 - 2*x^4 - 7*x^2)^(1/2), x)
```

3.42 $\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{2+x^2}\,\sqrt{-1+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{2}}\,x}{\sqrt{-1+3x^2}}\right)|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{-2+5x^2+3x^4}}$$

```
[Out] 1/7*EllipticF(1/2*x*14^(1/2)/(3*x^2-1)^(1/2),1/7*42^(1/2))*(x^2+2)^(1/2)*(3
*x^2-1)^(1/2)*7^(1/2)/(3*x^4+5*x^2-2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1111}

$$\frac{\sqrt{x^2+2}\,\sqrt{3x^2-1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{7}{2}}\,x}{\sqrt{3x^2-1}}\right)|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{3x^4+5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[2 + x^2]*Sqrt[-1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7/2]*x)/Sqrt[-1 + 3*
x^2]], 6/7])/(Sqrt[7]*Sqrt[-2 + 5*x^2 + 3*x^4])
```

Rule 1111

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx = \frac{\sqrt{2+x^2}\,\sqrt{-1+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{2}}\,x}{\sqrt{-1+3x^2}}\right)|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{-2+5x^2+3x^4}}$$

Mathematica [A]
time = 10.03, size = 54, normalized size = 0.81

$$\frac{\sqrt{1-3x^2}\,\sqrt{2+x^2}\,F\left(\sin^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}\,\sqrt{-2+5x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[1 - 3*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x], -1/6])/(Sqrt[6]
*Sqrt[-2 + 5*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.02, size = 53, normalized size = 0.79

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-3x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},i\sqrt{6}\right)}{2\sqrt{3x^4+5x^2-2}}$	53
elliptic	$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-3x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},i\sqrt{6}\right)}{2\sqrt{3x^4+5x^2-2}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+5*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)/(3*x^4+5*x^2-2)^(1/2)*Ellip
ticF(1/2*I*2^(1/2)*x,I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 - 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 3*x^4 - 2)^(1/2), x)
```

3.43 $\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{-2+\left(2+\sqrt{10}\right)x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-2+\left(2+\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{-2+4x^2+3x^4}}$$

```
[Out] 1/20*EllipticF(2^(3/4)*5^(1/4)*x/(-2+x^2*(2+10^(1/2)))^(1/2),1/10*(50+10*10
^(1/2))^(1/2))*((2-x^2*(2-10^(1/2)))/(2-x^2*(2+10^(1/2))))^(1/2)*(-2+x^2*(2
+10^(1/2)))^(1/2)*10^(3/4)/(3*x^4+4*x^2-2)^(1/2)/(1/(2-x^2*(2+10^(1/2))))^(
1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-2}\,F\left(\text{ArcSin}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{\left(2+\sqrt{10}\right)x^2-2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{3x^4+4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2 - (2 - Sqrt[10])*x^2)/(2 - (2 + Sqrt[10])*x^2)]*Sqrt[-2 + (2 + Sqr
t[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 + (2 + Sqrt[10])*x
^2]], (5 + Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2 - (2 + Sqrt[10])*x^2)^(-1)]*S
qrt[-2 + 4*x^2 + 3*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx = \frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{-2+\left(2+\sqrt{10}\right)x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-2+\left(2+\sqrt{10}\right.}}\right.\right.}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{-2+4x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 81, normalized size = 0.57

$$-\frac{i\sqrt{2-4x^2-3x^4}\,F\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{2}}}\,x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{-2+\sqrt{10}}\sqrt{-2+4x^2+3x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 + 4*x^2 + 3*x^4],x]
[Out] ((-I)*Sqrt[2 - 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x],
(-7 - 2*Sqrt[10])/3])/(Sqrt[-2 + Sqrt[10]]*Sqrt[-2 + 4*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.60

method	result	size
default	$\frac{2\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{4-2\sqrt{10}}\sqrt{3x^4+4x^2-2}}$	84
elliptic	$\frac{2\sqrt{1-\left(1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-\left(1+\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{4-2\sqrt{10}}\sqrt{3x^4+4x^2-2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+4*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(4-2*10^(1/2))^(1/2)*(1-(1-1/2*10^(1/2))*x^2)^(1/2)*(1-(1+1/2*10^(1/2))*x
^2)^(1/2)/(3*x^4+4*x^2-2)^(1/2)*EllipticF(1/2*(4-2*10^(1/2))^(1/2)*x,1/3*I*
6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 + 4x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+4*x**2-2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**4 + 4*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2-2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 + 4\,x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 3*x^4 - 2)^(1/2),x)
[Out] int(1/(4*x^2 + 3*x^4 - 2)^(1/2), x)
```

3.44 $\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{-4+\left(3+\sqrt{33}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-4+\left(3+\sqrt{33}\right)x^2}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{-2+3x^2+3x^4}}$$

```
[Out] 1/132*EllipticF(33^(1/4)*x*2^(1/2)/(-4+x^2*(3+33^(1/2)))^(1/2),1/22*(242+22
*33^(1/2))^(1/2))*((4-x^2*(3-33^(1/2)))/(4-x^2*(3+33^(1/2))))^(1/2)*(-4+x^2
*(3+33^(1/2)))^(1/2)*33^(3/4)*2^(1/2)/(3*x^4+3*x^2-2)^(1/2)/(1/(4-x^2*(3+33
^(1/2))))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-4}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{\left(3+\sqrt{33}\right)x^2-4}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{3x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 3*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(4 - (3 - Sqrt[33])*x^2)/(4 - (3 + Sqrt[33])*x^2)]*Sqrt[-4 + (3 + Sqr
t[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 + (3 + Sqrt[33])*
x^2]], (11 + Sqrt[33])/22])/(2*Sqrt[2]*33^(1/4)*Sqrt[(4 - (3 + Sqrt[33])*x^
2)^(-1)]*Sqrt[-2 + 3*x^2 + 3*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx = \frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{-4+\left(3+\sqrt{33}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-4+\left(3+\sqrt{33}\right)}}\right.\right.}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{-2+3x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 83, normalized size = 0.57

$$-\frac{i\sqrt{4-6x^2-6x^4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}\,x\right)|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{-3+\sqrt{33}}\sqrt{-2+3x^2+3x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 + 3*x^2 + 3*x^4],x]
[Out] ((-I)*Sqrt[4 - 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqrt[33])]*x]
, -7/4 - Sqrt[33]/4])/(Sqrt[-3 + Sqrt[33]]*Sqrt[-2 + 3*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.58

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\sqrt{1-\left(\frac{\sqrt{33}}{4}+\frac{3}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3-\sqrt{33}}}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{3-\sqrt{33}}\sqrt{3x^4+3x^2-2}}$	84
elliptic	$\frac{2\sqrt{1-\left(\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\sqrt{1-\left(\frac{\sqrt{33}}{4}+\frac{3}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3-\sqrt{33}}}{2},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{3-\sqrt{33}}\sqrt{3x^4+3x^2-2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+3*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(3-33^(1/2))^(1/2)*(1-(3/4-1/4*33^(1/2))*x^2)^(1/2)*(1-(1/4*33^(1/2)+3/4)
*x^2)^(1/2)/(3*x^4+3*x^2-2)^(1/2)*EllipticF(1/2*x*(3-33^(1/2))^(1/2),1/4*I*
6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 + 3x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+3*x**2-2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**4 + 3*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2-2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 + 3\,x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 3*x^4 - 2)^(1/2),x)
[Out] int(1/(3*x^2 + 3*x^4 - 2)^(1/2), x)
```

3.45 $\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{-2+\left(1+\sqrt{7}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}\,x}{\sqrt{-2+\left(1+\sqrt{7}\right)x^2}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{-2+2x^2+3x^4}}$$

```
[Out] 1/14*EllipticF(7^(1/4)*x*2^(1/2)/(-2+x^2*(1+7^(1/2)))^(1/2),1/14*(98+14*7^(
1/2))^(1/2))*((2-x^2*(1-7^(1/2)))/(2-x^2*(1+7^(1/2))))^(1/2)*(-2+x^2*(1+7^(
1/2)))^(1/2)*7^(3/4)/(3*x^4+2*x^2-2)^(1/2)/(1/(2-x^2*(1+7^(1/2))))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-2}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt[4]{7}\,x}{\sqrt{\left(1+\sqrt{7}\right)x^2-2}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{3x^4+2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 2*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2 - (1 - Sqrt[7])*x^2)/(2 - (1 + Sqrt[7])*x^2)]*Sqrt[-2 + (1 + Sqrt[
7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 + (1 + Sqrt[7])*x^2]]
, (7 + Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 - (1 + Sqrt[7])*x^2)^(-1)]*Sqrt[-2
+ 2*x^2 + 3*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx = \frac{\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\,\sqrt{-2+\left(1+\sqrt{7}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{7}\,x}{\sqrt{-2+\left(1+\sqrt{7}\right)x^2}}\right.\right.}{2\sqrt[4]{7}\,\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\,\sqrt{-2+2x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 83, normalized size = 0.59

$$-\frac{i\sqrt{2-2x^2-3x^4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}\,x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{-1+\sqrt{7}}\,\sqrt{-2+2x^2+3x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 + 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 - 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x],
 -4/3 - Sqrt[7]/3])/(Sqrt[-1 + Sqrt[7]]*Sqrt[-2 + 2*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.60

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-\left(\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{2-2\sqrt{7}}\,\sqrt{3x^4+2x^2-2}}$	84
elliptic	$\frac{2\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\,\sqrt{1-\left(\frac{\sqrt{7}}{2}+\frac{1}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{2-2\sqrt{7}}\,\sqrt{3x^4+2x^2-2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+2*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(2-2*7^(1/2))^(1/2)*(1-(1/2-1/2*7^(1/2))*x^2)^(1/2)*(1-(1/2*7^(1/2)+1/2)*
x^2)^(1/2)/(3*x^4+2*x^2-2)^(1/2)*EllipticF(1/2*(2-2*7^(1/2))^(1/2)*x,1/6*I*
6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 + 2x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+2*x**2-2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**4 + 2*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2-2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 + 2\,x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + 3*x^4 - 2)^(1/2),x)
[Out] int(1/(2*x^2 + 3*x^4 - 2)^(1/2), x)
```

3.46 $\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{1+x^2}\,\sqrt{-2+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{5}\,x}{\sqrt{-2+3x^2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{-2+x^2+3x^4}}$$

```
[Out] 1/5*EllipticF(x*5^(1/2)/(3*x^2-2)^(1/2),1/5*15^(1/2))*(x^2+1)^(1/2)*(3*x^2-
2)^(1/2)*5^(1/2)/(3*x^4+x^2-2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1111}

$$\frac{\sqrt{x^2+1}\,\sqrt{3x^2-2}\,F\left(\text{ArcSin}\left(\frac{\sqrt{5}\,x}{\sqrt{3x^2-2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{3x^4+x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[-2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-2 + 3*x^
2]], 3/5])/(Sqrt[5]*Sqrt[-2 + x^2 + 3*x^4])
```

Rule 1111

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx = \frac{\sqrt{1+x^2}\,\sqrt{-2+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{5}\,x}{\sqrt{-2+3x^2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{-2+x^2+3x^4}}$$

Mathematica [A]
time = 10.03, size = 48, normalized size = 0.76

$$\frac{\sqrt{\left(\frac{2}{3}-x^2\right)(1+x^2)}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|-\frac{2}{3}\right)}{\sqrt{-2+x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2/3 - x^2)*(1 + x^2)]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/Sqrt[-2
+ x^2 + 3*x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 43, normalized size = 0.68

method	result	size
default	$-\frac{i\sqrt{x^{2}+1}\,\sqrt{-6x^{2}+4}\,\mathrm{EllipticF}\left(ix, \frac{i\sqrt{6}}{2}\right)}{2\sqrt{3x^{4}+x^{2}-2}}$	43
elliptic	$-\frac{i\sqrt{x^{2}+1}\,\sqrt{-6x^{2}+4}\,\mathrm{EllipticF}\left(ix, \frac{i\sqrt{6}}{2}\right)}{2\sqrt{3x^{4}+x^{2}-2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(3*x^4+x^2-2)^(1/2)*EllipticF(I*x,1/2
*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^{4}+x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(x^2 + 3*x^4 - 2)^(1/2), x)
```

3.47 $\int \frac{1}{\sqrt{-2+3x^4}}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{-2+\sqrt{6}\,x^2}\sqrt{\frac{2+\sqrt{6}\,x^2}{2-\sqrt{6}\,x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}\,x}{\sqrt{-2+\sqrt{6}\,x^2}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}\,x^2}}\sqrt{-2+3x^4}}$$

```
[Out] 1/12*EllipticF(2^(3/4)*3^(1/4)*x/(-2+x^2*6^(1/2))^(1/2),1/2*2^(1/2))*(-2+x^
2*6^(1/2))^(1/2)*((2+x^2*6^(1/2))/(2-x^2*6^(1/2)))^(1/2)*6^(3/4)/(3*x^4-2)^
(1/2)/(1/(2-x^2*6^(1/2)))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {229}

$$\frac{\sqrt{\sqrt{6}\,x^2-2}\sqrt{\frac{\sqrt{6}\,x^2+2}{2-\sqrt{6}\,x^2}}\,F\left(\text{ArcSin}\left(\frac{2^{3/4}\sqrt[4]{3}\,x}{\sqrt{\sqrt{6}\,x^2-2}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}\,x^2}}\sqrt{3x^4-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 + Sqrt[6]*x^2]*Sqrt[(2 + Sqrt[6]*x^2)/(2 - Sqrt[6]*x^2)]*EllipticF
[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-2 + Sqrt[6]*x^2]], 1/2])/(2*6^(1/4)*Sqrt[
(2 - Sqrt[6]*x^2)^(-1)]*Sqrt[-2 + 3*x^4])
```

Rule 229

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[(a - q*x^2)/(a + q*x^2)]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[a + b*x^
4]*Sqrt[a/(a + q*x^2)]))*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2],
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+3x^4}}\,dx = \frac{\sqrt{-2+\sqrt{6}\,x^2}\,\sqrt{\frac{2+\sqrt{6}\,x^2}{2-\sqrt{6}\,x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}\,x}{\sqrt{-2+\sqrt{6}\,x^2}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}\,x^2}}\,\sqrt{-2+3x^4}}$$

Mathematica [A]
time = 10.03, size = 40, normalized size = 0.35

$$\frac{\sqrt{2-3x^4}\,F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}\,\sqrt{-2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 3*x^4],x]
```

```
[Out] (Sqrt[2 - 3*x^4]*EllipticF[ArcSin[(3/2)^(1/4)*x], -1])/(6^(1/4)*Sqrt[-2 + 3
*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.10, size = 56, normalized size = 0.49

method	result	size
meijerg	$\frac{\sqrt{2}\,\sqrt{-\mathrm{signum}\left(-1+\frac{3x^4}{2}\right)}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],\frac{3x^4}{2}\right)}{2\sqrt{\mathrm{signum}\left(-1+\frac{3x^4}{2}\right)}}$	40
default	$\frac{\sqrt{4+2x^2\sqrt{6}}\,\sqrt{4-2x^2\sqrt{6}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2\sqrt{6}}}{2},i\right)}{2\sqrt{-2\sqrt{6}}\,\sqrt{3x^4-2}}$	56
elliptic	$\frac{\sqrt{4+2x^2\sqrt{6}}\,\sqrt{4-2x^2\sqrt{6}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2\sqrt{6}}}{2},i\right)}{2\sqrt{-2\sqrt{6}}\,\sqrt{3x^4-2}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-2*6^(1/2))^(1/2)*(4+2*x^2*6^(1/2))^(1/2)*(4-2*x^2*6^(1/2))^(1/2)/(3*x
^4-2)^(1/2)*EllipticF(1/2*x*(-2*6^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [C] Result contains complex when optimal does not.
time = 0.34, size = 34, normalized size = 0.30

$$-\frac{\sqrt{2}\, ix\Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| {\frac{3x^{4}}{2}} \right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-2)**(1/2),x)
[Out] -sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4/2)/(8*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(3*x^4 - 2), x)
```

Mupad [B]
time = 0.08, size = 31, normalized size = 0.27

$$\frac{x\,\sqrt{4-6\,x^4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{3\,x^4}{2}\right)}{2\,\sqrt{3\,x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 2)^(1/2),x)
```

```
[Out] (x*(4 - 6*x^4)^(1/2)*hypergeom([1/4, 1/2], 5/4, (3*x^4)/2))/(2*(3*x^4 - 2)^
(1/2))
```

3.48 $\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{-1+x^2}\,\sqrt{2+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{2}}\,x}{\sqrt{-1+x^2}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{-2-x^2+3x^4}}$$

```
[Out] 1/5*EllipticF(1/2*x*10^(1/2)/(x^2-1)^(1/2),1/5*10^(1/2))*(x^2-1)^(1/2)*(3*x
^2+2)^(1/2)*5^(1/2)/(3*x^4-x^2-2)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1111\}$

$$\frac{\sqrt{x^2-1}\,\sqrt{3x^2+2}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{5}{2}}\,x}{\sqrt{x^2-1}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{3x^4-x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5/2]*x)/Sqrt[-1 + x^
2]], 2/5])/(Sqrt[5]*Sqrt[-2 - x^2 + 3*x^4])
```

Rule 1111

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx = \frac{\sqrt{-1+x^2}\,\sqrt{2+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{2}}\,x}{\sqrt{-1+x^2}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{-2-x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 60, normalized size = 0.92

$$-\frac{i\sqrt{1-x^2}\sqrt{2+3x^2}F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{-6-3x^2+9x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], -2/3]
)/Sqrt[-6 - 3*x^2 + 9*x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 53, normalized size = 0.82

method	result	size
default	$-\frac{i\sqrt{6}\sqrt{6x^2+4}\sqrt{-x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{3x^4-x^2-2}}$	53
elliptic	$-\frac{i\sqrt{6}\sqrt{6x^2+4}\sqrt{-x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{3x^4-x^2-2}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+4)^(1/2)*(-x^2+1)^(1/2)/(3*x^4-x^2-2)^(1/2)*EllipticF
(1/2*I*x*6^(1/2),1/3*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 - x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\,x^4 - x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - x^2 - 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - x^2 - 2)^(1/2), x)
```

3.49 $\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{-2-\left(1-\sqrt{7}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{7}\right)x^2}{2+\left(1-\sqrt{7}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{7}\,x}{\sqrt{-2-\left(1-\sqrt{7}\right)x^2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2+\left(1-\sqrt{7}\right)x^2}}\sqrt{-2-2x^2+3x^4}}$$

```
[Out] 1/14*EllipticF(7^(1/4)*x*2^(1/2)/(-2-x^2*(1-7^(1/2)))^(1/2),1/14*(98-14*7^(
1/2))^(1/2))*(-2-x^2*(1-7^(1/2)))^(1/2)*((2+x^2*(1+7^(1/2)))/(2+x^2*(1-7^(1
/2))))^(1/2)*7^(3/4)/(3*x^4-2*x^2-2)^(1/2)/(1/(2+x^2*(1-7^(1/2))))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-2}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+2}{\left(1-\sqrt{7}\right)x^2+2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\,\sqrt[4]{7}\,x}{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+2}}\sqrt{3x^4-2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 2*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 - (1 - Sqrt[7])*x^2]*Sqrt[(2 + (1 + Sqrt[7])*x^2)/(2 + (1 - Sqrt[7
])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 - (1 - Sqrt[7])*x^2]]
, (7 - Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 + (1 - Sqrt[7])*x^2)^(-1)]*Sqrt[-2
- 2*x^2 + 3*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx = \frac{\sqrt{-2-\left(1-\sqrt{7}\right)x^2}\sqrt{\frac{2+\left(1+\sqrt{7}\right)x^2}{2+\left(1-\sqrt{7}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}\,x}{\sqrt{-2-\left(1-\sqrt{7}\right)x^2}}\right.\right.}{2\sqrt[4]{7}\sqrt{\frac{1}{2+\left(1-\sqrt{7}\right)x^2}}\sqrt{-2-2x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 81, normalized size = 0.55

$$-\frac{i\sqrt{2+2x^2-3x^4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}\,x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}\sqrt{-2-2x^2+3x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 - 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 + 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x]
, (-4 + Sqrt[7])/3])/(Sqrt[1 + Sqrt[7]]*Sqrt[-2 - 2*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.57

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{-2-2\sqrt{7}}\sqrt{3x^4-2x^2-2}}$	84
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{7}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-2-2\sqrt{7}}\,x}{2},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{-2-2\sqrt{7}}\sqrt{3x^4-2x^2-2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-2*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-2-2*7^(1/2))^(1/2)*(1-(-1/2-1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2+1/2*7^(1/2
))*x^2)^(1/2)/(3*x^4-2*x^2-2)^(1/2)*EllipticF(1/2*(-2-2*7^(1/2))^(1/2)*x,1/
6*I*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 - 2x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-2*x**2-2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**4 - 2*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2-2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 - 2\,x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 2*x^2 - 2)^(1/2),x)
[Out] int(1/(3*x^4 - 2*x^2 - 2)^(1/2), x)
```

3.50 $\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{-4-\left(3-\sqrt{33}\right)x^2}\sqrt{\frac{4+\left(3+\sqrt{33}\right)x^2}{4+\left(3-\sqrt{33}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-4-\left(3-\sqrt{33}\right)x^2}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4+\left(3-\sqrt{33}\right)x^2}}\sqrt{-2-3x^2+3x^4}}$$

```
[Out] 1/132*EllipticF(33^(1/4)*x*2^(1/2)/(-4-x^2*(3-33^(1/2)))^(1/2),1/22*(242-22
*33^(1/2))^(1/2))*(-4-x^2*(3-33^(1/2)))^(1/2)*((4+x^2*(3+33^(1/2)))/(4+x^2*
(3-33^(1/2))))^(1/2)*33^(3/4)*2^(1/2)/(3*x^4-3*x^2-2)^(1/2)/(1/(4+x^2*(3-33
^(1/2))))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-4}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+4}{\left(3-\sqrt{33}\right)x^2+4}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-4}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+4}}\sqrt{3x^4-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 3*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-4 - (3 - Sqrt[33])*x^2]*Sqrt[(4 + (3 + Sqrt[33])*x^2)/(4 + (3 - Sqrt
[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-4 - (3 - Sqrt[33])*
x^2]], (11 - Sqrt[33])/22])/(2*Sqrt[2]*33^(1/4)*Sqrt[(4 + (3 - Sqrt[33])*x^
2)^(-1)]*Sqrt[-2 - 3*x^2 + 3*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx = \frac{\sqrt{-4-\left(3-\sqrt{33}\right)x^2}\,\sqrt{\frac{4+\left(3+\sqrt{33}\right)x^2}{4+\left(3-\sqrt{33}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{33}\,x}{\sqrt{-4-\left(3-\sqrt{33}\right.}}\right.\right.}{2\sqrt{2}\,\sqrt[4]{33}\,\sqrt{\frac{1}{4+\left(3-\sqrt{33}\right)x^2}}\,\sqrt{-2-3x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 81, normalized size = 0.53

$$-\frac{i\sqrt{4+6x^2-6x^4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{\sqrt{3+\sqrt{33}}\,\sqrt{-2-3x^2+3x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 - 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[4 + 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sqrt[33])]*x
], (-7 + Sqrt[33])/4])/(Sqrt[3 + Sqrt[33]]*Sqrt[-2 - 3*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.55

method	result	si
default	$\frac{2\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3-\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{-3-\sqrt{33}}\,\sqrt{3x^4-3x^2-2}}$	8
elliptic	$\frac{2\sqrt{1-\left(-\frac{3}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{3}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3-\sqrt{33}}\,x}{2},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{-3-\sqrt{33}}\,\sqrt{3x^4-3x^2-2}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-3*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-3-33^(1/2))^(1/2)*(1-(-3/4-1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4+1/4*33^(1/
2))*x^2)^(1/2)/(3*x^4-3*x^2-2)^(1/2)*EllipticF(1/2*(-3-33^(1/2))^(1/2)*x,1/
4*I*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 - 3x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-3*x**2-2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**4 - 3*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2-2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 - 3\,x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 3*x^2 - 2)^(1/2),x)
[Out] int(1/(3*x^4 - 3*x^2 - 2)^(1/2), x)
```

3.51 $\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{-2-\left(2-\sqrt{10}\right)x^2}\sqrt{\frac{2+\left(2+\sqrt{10}\right)x^2}{2+\left(2-\sqrt{10}\right)x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-2-\left(2-\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2+\left(2-\sqrt{10}\right)x^2}}\sqrt{-2-4x^2+3x^4}}$$

```
[Out] 1/20*EllipticF(2^(3/4)*5^(1/4)*x/(-2-x^2*(2-10^(1/2)))^(1/2),1/10*(50-10*10
^(1/2))^(1/2))*(-2-x^2*(2-10^(1/2)))^(1/2)*((2+x^2*(2+10^(1/2)))/(2+x^2*(2-
10^(1/2))))^(1/2)*10^(3/4)/(3*x^4-4*x^2-2)^(1/2)/(1/(2+x^2*(2-10^(1/2))))^(
1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-2}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+2}{\left(2-\sqrt{10}\right)x^2+2}}F\left(\text{ArcSin}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+2}}\sqrt{3x^4-4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 4*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 - (2 - Sqrt[10])*x^2]*Sqrt[(2 + (2 + Sqrt[10])*x^2)/(2 + (2 - Sqrt
[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-2 - (2 - Sqrt[10])*x
^2]], (5 - Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2 + (2 - Sqrt[10])*x^2)^(-1)]*S
qrt[-2 - 4*x^2 + 3*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx = \frac{\sqrt{-2-\left(2-\sqrt{10}\right)x^2}\sqrt{\frac{2+\left(2+\sqrt{10}\right)x^2}{2+\left(2-\sqrt{10}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-2-\left(2-\sqrt{10}\right)}\right.\right.}{2\sqrt[4]{10}\sqrt{\frac{1}{2+\left(2-\sqrt{10}\right)x^2}}\sqrt{-2-4x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 81, normalized size = 0.55

$$-\frac{i\sqrt{2+4x^2-3x^4}\,F\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{2}}}\,x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}\sqrt{-2-4x^2+3x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-2 - 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 + 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x], (
-7 + 2*Sqrt[10])/3])/(Sqrt[2 + Sqrt[10]]*Sqrt[-2 - 4*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.57

method	result	si
default	$\frac{2\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{-4-2\sqrt{10}}\sqrt{3x^4-4x^2-2}}$	84
elliptic	$\frac{2\sqrt{1-\left(-1-\frac{\sqrt{10}}{2}\right)x^2}\sqrt{1-\left(-1+\frac{\sqrt{10}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-4-2\sqrt{10}}\,x}{2},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{-4-2\sqrt{10}}\sqrt{3x^4-4x^2-2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-4*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-4-2*10^(1/2))^(1/2)*(1-(-1-1/2*10^(1/2))*x^2)^(1/2)*(1-(-1+1/2*10^(1/2)
)*x^2)^(1/2)/(3*x^4-4*x^2-2)^(1/2)*EllipticF(1/2*(-4-2*10^(1/2))^(1/2)*x,1/
3*I*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 - 4x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-4*x**2-2)**(1/2),x)
[Out] Integral(1/sqrt(3*x**4 - 4*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2-2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 - 4\,x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 4*x^2 - 2)^(1/2),x)
[Out] int(1/(3*x^4 - 4*x^2 - 2)^(1/2), x)
```

3.52 $\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{-2+x^2}\,\sqrt{1+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{7}\,x}{\sqrt{-2+x^2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{-2-5x^2+3x^4}}$$

```
[Out] 1/7*EllipticF(x*7^(1/2)/(x^2-2)^(1/2),1/7*7^(1/2))*(x^2-2)^(1/2)*(3*x^2+1)^
(1/2)*7^(1/2)/(3*x^4-5*x^2-2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1111\}$

$$\frac{\sqrt{x^2-2}\,\sqrt{3x^2+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{7}\,x}{\sqrt{x^2-2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{3x^4-5x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 + x^2]*Sqrt[1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-2 + x^2]
], 1/7])/(Sqrt[7]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Rule 1111

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx = \frac{\sqrt{-2+x^2}\,\sqrt{1+3x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{7}\,x}{\sqrt{-2+x^2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{-2-5x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 65, normalized size = 1.03

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\,\sqrt{1+3x^2}\,F\left(i\sinh^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}\,\sqrt{-2-5x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6]
)/(Sqrt[3]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 53, normalized size = 0.84

method	result	size
default	$-\frac{i\sqrt{3}\,\sqrt{3x^2+1}\,\sqrt{-2x^2+4}\,\text{EllipticF}\left(i\sqrt{3}\,x,\frac{i\sqrt{6}}{6}\right)}{6\sqrt{3x^4-5x^2-2}}$	53
elliptic	$-\frac{i\sqrt{3}\,\sqrt{3x^2+1}\,\sqrt{-2x^2+4}\,\text{EllipticF}\left(i\sqrt{3}\,x,\frac{i\sqrt{6}}{6}\right)}{6\sqrt{3x^4-5x^2-2}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-5*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*3^(1/2)*(3*x^2+1)^(1/2)*(-2*x^2+4)^(1/2)/(3*x^4-5*x^2-2)^(1/2)*Ellip
ticF(I*3^(1/2)*x,1/6*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4-5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 5*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 5*x^2 - 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 5*x^2 - 2)^(1/2), x)
```

3.53 $\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{-6+\left(7+\sqrt{73}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{73}\,x}{\sqrt{-6+\left(7+\sqrt{73}\right)x^2}}\right)|\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{-3+7x^2+2x^4}}$$

```
[Out] 1/438*EllipticF(73^(1/4)*x*2^(1/2)/(-6+x^2*(7+73^(1/2)))^(1/2),1/146*(10658
+1022*73^(1/2))^(1/2))*((6-x^2*(7-73^(1/2)))/(6-x^2*(7+73^(1/2))))^(1/2)*(-
6+x^2*(7+73^(1/2)))^(1/2)*73^(3/4)*3^(1/2)/(2*x^4+7*x^2-3)^(1/2)/(1/(6-x^2*
(7+73^(1/2))))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-6}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt[4]{73}\,x}{\sqrt{\left(7+\sqrt{73}\right)x^2-6}}\right)|\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{2x^4+7x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(6 - (7 - Sqrt[73])*x^2)/(6 - (7 + Sqrt[73])*x^2)]*Sqrt[-6 + (7 + Sqr
t[73])*x^2]*EllipticF[ArcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[-6 + (7 + Sqrt[73])*
x^2]], (73 + 7*Sqrt[73])/146])/(2*Sqrt[3]*73^(1/4)*Sqrt[(6 - (7 + Sqrt[73])
*x^2)^(-1)]*Sqrt[-3 + 7*x^2 + 2*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx = \frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{-6+\left(7+\sqrt{73}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{73}\,x}{\sqrt{-6+\left(7+\sqrt{73}\right)}}\right.\right.}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{-3+7x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 80, normalized size = 0.54

$$-\frac{i\sqrt{6-14x^2-4x^4}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)}{\sqrt{-7+\sqrt{73}}\sqrt{-3+7x^2+2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 7*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[6 - 14*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[7 + Sqrt[73]]
], (-61 - 7*Sqrt[73])/12])/(Sqrt[-7 + Sqrt[73]]*Sqrt[-3 + 7*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.57

method	result	size
default	$\frac{6\sqrt{1-\left(\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{42-6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{\sqrt{42-6\sqrt{73}}\sqrt{2x^4+7x^2-3}}$	84
elliptic	$\frac{6\sqrt{1-\left(\frac{7}{6}-\frac{\sqrt{73}}{6}\right)x^2}\sqrt{1-\left(\frac{7}{6}+\frac{\sqrt{73}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{42-6\sqrt{73}}\,x}{6},\frac{7i\sqrt{6}}{12}+\frac{i\sqrt{438}}{12}\right)}{\sqrt{42-6\sqrt{73}}\sqrt{2x^4+7x^2-3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+7*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(42-6*73^(1/2))^(1/2)*(1-(7/6-1/6*73^(1/2))*x^2)^(1/2)*(1-(7/6+1/6*73^(1/
2))*x^2)^(1/2)/(2*x^4+7*x^2-3)^(1/2)*EllipticF(1/6*(42-6*73^(1/2))^(1/2)*x,
7/12*I*6^(1/2)+1/12*I*438^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + 7x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+7*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 + 7*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 + 7\,x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x^2 + 2*x^4 - 3)^(1/2),x)
[Out] int(1/(7*x^2 + 2*x^4 - 3)^(1/2), x)
```

3.54 $\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{-3+\left(3+\sqrt{15}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{15}\,x}{\sqrt{-3+\left(3+\sqrt{15}\right)x^2}}\right)|\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{\sqrt{2}\,3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{-3+6x^2+2x^4}}$$

```
[Out] 1/30*EllipticF(15^(1/4)*x*2^(1/2)/(-3+x^2*(3+15^(1/2)))^(1/2),1/10*(50+10*1
5^(1/2))^(1/2))*((3-x^2*(3-15^(1/2)))/(3-x^2*(3+15^(1/2))))^(1/2)*(-3+x^2*(
3+15^(1/2)))^(1/2)*3^(1/4)*5^(3/4)*2^(1/2)/(2*x^4+6*x^2-3)^(1/2)/(1/(3-x^2*
(3+15^(1/2))))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-3}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt[4]{15}\,x}{\sqrt{\left(3+\sqrt{15}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{\sqrt{2}\,3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{2x^4+6x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (3 - Sqrt[15])*x^2)/(3 - (3 + Sqrt[15])*x^2)]*Sqrt[-3 + (3 + Sqr
t[15])*x^2]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[-3 + (3 + Sqrt[15])*
x^2]], (5 + Sqrt[15])/10])/(Sqrt[2]*3^(3/4)*5^(1/4)*Sqrt[(3 - (3 + Sqrt[15]
)*x^2)^(-1)]*Sqrt[-3 + 6*x^2 + 2*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\,\sqrt{-3+\left(3+\sqrt{15}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{15}\,x}{\sqrt{-3+\left(3+\sqrt{15}\right.}}\right.\right.}{\sqrt{2}\,3^{3/4}\sqrt[4]{5}\,\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\,\sqrt{-3+6x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 77, normalized size = 0.52

$$-\frac{i\sqrt{3-6x^2-2x^4}\,F\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{3}}}\,x\right)|-4-\sqrt{15}\right)}{\sqrt{-3+\sqrt{15}}\,\sqrt{-3+6x^2+2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 6*x^2 + 2*x^4],x]
[Out] ((-I)*Sqrt[3 - 6*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x],
-4 - Sqrt[15]])/(Sqrt[-3 + Sqrt[15]]*Sqrt[-3 + 6*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.57

method	result	size
default	$\frac{3\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{\sqrt{9-3\sqrt{15}}\,\sqrt{2x^4+6x^2-3}}$	84
elliptic	$\frac{3\sqrt{1-\left(1-\frac{\sqrt{15}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{15}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{9-3\sqrt{15}}\,x}{3},\frac{i\sqrt{6}}{2}+\frac{i\sqrt{10}}{2}\right)}{\sqrt{9-3\sqrt{15}}\,\sqrt{2x^4+6x^2-3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+6*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 3/(9-3*15^(1/2))^(1/2)*(1-(1-1/3*15^(1/2))*x^2)^(1/2)*(1-(1+1/3*15^(1/2))*x
^2)^(1/2)/(2*x^4+6*x^2-3)^(1/2)*EllipticF(1/3*(9-3*15^(1/2))^(1/2)*x,1/2*I*
6^(1/2)+1/2*I*10^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + 6x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+6*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 + 6*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 + 6\,x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x^2 + 2*x^4 - 3)^(1/2),x)
[Out] int(1/(6*x^2 + 2*x^4 - 3)^(1/2), x)
```

3.55 $\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{3+x^2}\,\sqrt{-1+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{3}}\,x}{\sqrt{-1+2x^2}}\right)\middle|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{-3+5x^2+2x^4}}$$

```
[Out] 1/7*EllipticF(1/3*x*21^(1/2)/(2*x^2-1)^(1/2),1/7*42^(1/2))*(x^2+3)^(1/2)*(2
*x^2-1)^(1/2)*7^(1/2)/(2*x^4+5*x^2-3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1111\}$

$$\frac{\sqrt{x^2+3}\,\sqrt{2x^2-1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{7}{3}}\,x}{\sqrt{2x^2-1}}\right)\middle|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{2x^4+5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 + x^2]*Sqrt[-1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7/3]*x)/Sqrt[-1 + 2*
x^2]], 6/7])/(Sqrt[7]*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Rule 1111

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx = \frac{\sqrt{3+x^2}\,\sqrt{-1+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{3}}\,x}{\sqrt{-1+2x^2}}\right)\middle|\frac{6}{7}\right)}{\sqrt{7}\,\sqrt{-3+5x^2+2x^4}}$$

Mathematica [A]
time = 10.03, size = 54, normalized size = 0.81

$$\frac{\sqrt{1-2x^2}\,\sqrt{3+x^2}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{6}\,\sqrt{-3+5x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/6])/(Sqrt[6]
*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.02, size = 53, normalized size = 0.79

method	result	size
default	$-\frac{i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{-2x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{3}\,x}{3},i\sqrt{6}\right)}{3\sqrt{2x^4+5x^2-3}}$	53
elliptic	$-\frac{i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{-2x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{3}\,x}{3},i\sqrt{6}\right)}{3\sqrt{2x^4+5x^2-3}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+5*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*3^(1/2)*(3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(2*x^4+5*x^2-3)^(1/2)*Ellip
ticF(1/3*I*3^(1/2)*x,I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + 5x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 5*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 + 5\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 2*x^4 - 3)^(1/2), x)
```

3.56 $\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{-3+\left(2+\sqrt{10}\right)x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-3+\left(2+\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\,\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{-3+4x^2+2x^4}}$$

```
[Out] 1/30*EllipticF(2^(3/4)*5^(1/4)*x/(-3+x^2*(2+10^(1/2)))^(1/2),1/10*(50+10*10
^(1/2))^(1/2))*((3-x^2*(2-10^(1/2)))/(3-x^2*(2+10^(1/2))))^(1/2)*(-3+x^2*(2
+10^(1/2)))^(1/2)*2^(1/4)*5^(3/4)*3^(1/2)/(2*x^4+4*x^2-3)^(1/2)/(1/(3-x^2*(
2+10^(1/2))))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-3}\,F\left(\text{ArcSin}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{\left(2+\sqrt{10}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\,\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{2x^4+4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 4*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (2 - Sqrt[10])*x^2)/(3 - (2 + Sqrt[10])*x^2)]*Sqrt[-3 + (2 + Sqr
t[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 + (2 + Sqrt[10])*x
^2]], (5 + Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3 - (2 + Sqrt[10])
*x^2)^(-1)]*Sqrt[-3 + 4*x^2 + 2*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{-3+\left(2+\sqrt{10}\right)x^2}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-3+\left(2+\sqrt{10}\right.}}\right.\right.}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{-3+4x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 83, normalized size = 0.56

$$-\frac{i\sqrt{3-4x^2-2x^4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right)|-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{-2+\sqrt{10}}\sqrt{-3+4x^2+2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 - 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x]
, -7/3 - (2*Sqrt[10])/3])/(Sqrt[-2 + Sqrt[10]]*Sqrt[-3 + 4*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 84, normalized size = 0.57

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{6-3\sqrt{10}}\sqrt{2x^4+4x^2-3}}$	84
elliptic	$\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{6}}{3}+\frac{i\sqrt{15}}{3}\right)}{\sqrt{6-3\sqrt{10}}\sqrt{2x^4+4x^2-3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+4*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(6-3*10^(1/2))^(1/2)*(1-(2/3-1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3+1/3*10^(1/2
))*x^2)^(1/2)/(2*x^4+4*x^2-3)^(1/2)*EllipticF(1/3*(6-3*10^(1/2))^(1/2)*x,1/
3*I*6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + 4x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+4*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 + 4*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 + 4\,x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 2*x^4 - 3)^(1/2),x)
[Out] int(1/(4*x^2 + 2*x^4 - 3)^(1/2), x)
```

3.57 $\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{\frac{6-\left(3-\sqrt{33}\right)x^2}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{-6+\left(3+\sqrt{33}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-6+\left(3+\sqrt{33}\right)x^2}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{-3+3x^2+2x^4}}$$

```
[Out] 1/66*EllipticF(33^(1/4)*x*2^(1/2)/(-6+x^2*(3+33^(1/2)))^(1/2),1/22*(242+22*
33^(1/2))^(1/2))*((6-x^2*(3-33^(1/2)))/(6-x^2*(3+33^(1/2))))^(1/2)*(-6+x^2*
(3+33^(1/2)))^(1/2)*3^(1/4)*11^(3/4)/(2*x^4+3*x^2-3)^(1/2)/(1/(6-x^2*(3+33^
(1/2))))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{\frac{6-\left(3-\sqrt{33}\right)x^2}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-6}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{\left(3+\sqrt{33}\right)x^2-6}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{2x^4+3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 3*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(6 - (3 - Sqrt[33])*x^2)/(6 - (3 + Sqrt[33])*x^2)]*Sqrt[-6 + (3 + Sqr
t[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 + (3 + Sqrt[33])*
x^2]], (11 + Sqrt[33])/22])/(2*3^(3/4)*11^(1/4)*Sqrt[(6 - (3 + Sqrt[33])*x^
2)^(-1)]*Sqrt[-3 + 3*x^2 + 2*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx = \frac{\sqrt{\frac{6-\left(3-\sqrt{33}\right)x^2}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{-6+\left(3+\sqrt{33}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-6+\left(3+\sqrt{33}\right)}\right.\right.}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{-3+3x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 80, normalized size = 0.55

$$-\frac{i\sqrt{6-6x^2-4x^4}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)\middle|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{-3+\sqrt{33}}\sqrt{-3+3x^2+2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 3*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[6 - 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + Sqrt[33]]]
, -7/4 - Sqrt[33]/4])/(Sqrt[-3 + Sqrt[33]]*Sqrt[-3 + 3*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.58

method	result	size
default	$\frac{6\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-\left(\frac{\sqrt{33}}{6}+\frac{1}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{18-6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{18-6\sqrt{33}}\sqrt{2x^4+3x^2-3}}$	84
elliptic	$\frac{6\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-\left(\frac{\sqrt{33}}{6}+\frac{1}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{18-6\sqrt{33}}\,x}{6},\frac{i\sqrt{6}}{4}+\frac{i\sqrt{22}}{4}\right)}{\sqrt{18-6\sqrt{33}}\sqrt{2x^4+3x^2-3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+3*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(18-6*33^(1/2))^(1/2)*(1-(1/2-1/6*33^(1/2))*x^2)^(1/2)*(1-(1/6*33^(1/2)+1
/2)*x^2)^(1/2)/(2*x^4+3*x^2-3)^(1/2)*EllipticF(1/6*(18-6*33^(1/2))^(1/2)*x,
1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + 3x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+3*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 + 3*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 + 3\,x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 2*x^4 - 3)^(1/2),x)
[Out] int(1/(3*x^2 + 2*x^4 - 3)^(1/2), x)
```

3.58 $\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{-3+\left(1+\sqrt{7}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-3+\left(1+\sqrt{7}\right)x^2}}\right)\middle|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{-3+2x^2+2x^4}}$$

```
[Out] 1/42*EllipticF(7^(1/4)*x*2^(1/2)/(-3+x^2*(1+7^(1/2)))^(1/2),1/14*(98+14*7^(
1/2))^(1/2))*((3-x^2*(1-7^(1/2)))/(3-x^2*(1+7^(1/2))))^(1/2)*(-3+x^2*(1+7^(
1/2)))^(1/2)*7^(3/4)*6^(1/2)/(2*x^4+2*x^2-3)^(1/2)/(1/(3-x^2*(1+7^(1/2))))^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-3}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-3}}\right)\middle|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{2x^4+2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 2*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (1 - Sqrt[7])*x^2)/(3 - (1 + Sqrt[7])*x^2)]*Sqrt[-3 + (1 + Sqrt[
7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 + (1 + Sqrt[7])*x^2]]
, (7 + Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3 - (1 + Sqrt[7])*x^2)^(-1)]*Sq
rt[-3 + 2*x^2 + 2*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{-3+\left(1+\sqrt{7}\right)x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}\,x}{\sqrt{-3+\left(1+\sqrt{7}\right)x^2}}\right.\right.}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{-3+2x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 83, normalized size = 0.58

$$-\frac{i\sqrt{3-2x^2-2x^4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}\,x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{-1+\sqrt{7}}\sqrt{-3+2x^2+2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 + 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 - 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x],
 -4/3 - Sqrt[7]/3])/(Sqrt[-1 + Sqrt[7]]*Sqrt[-3 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.59

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\sqrt{1-\left(\frac{\sqrt{7}}{3}+\frac{1}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{3-3\sqrt{7}}\sqrt{2x^4+2x^2-3}}$	84
elliptic	$\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\sqrt{1-\left(\frac{\sqrt{7}}{3}+\frac{1}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{6}}{6}+\frac{i\sqrt{42}}{6}\right)}{\sqrt{3-3\sqrt{7}}\sqrt{2x^4+2x^2-3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+2*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(3-3*7^(1/2))^(1/2)*(1-(1/3-1/3*7^(1/2))*x^2)^(1/2)*(1-(1/3*7^(1/2)+1/3)*
x^2)^(1/2)/(2*x^4+2*x^2-3)^(1/2)*EllipticF(1/3*(3-3*7^(1/2))^(1/2)*x,1/6*I*
6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + 2x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+2*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 + 2*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 + 2\,x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + 2*x^4 - 3)^(1/2),x)
[Out] int(1/(2*x^2 + 2*x^4 - 3)^(1/2), x)
```

3.59 $\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{-1+x^2}\,\sqrt{3+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{-1+x^2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{-3+x^2+2x^4}}$$

```
[Out] 1/5*EllipticF(1/3*x*15^(1/2)/(x^2-1)^(1/2),1/5*15^(1/2))*(x^2-1)^(1/2)*(2*x
^2+3)^(1/2)*5^(1/2)/(2*x^4+x^2-3)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1111\}$

$$\frac{\sqrt{x^2-1}\,\sqrt{2x^2+3}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{x^2-1}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5/3]*x)/Sqrt[-1 + x^
2]], 3/5])/(Sqrt[5]*Sqrt[-3 + x^2 + 2*x^4])
```

Rule 1111

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx = \frac{\sqrt{-1+x^2}\,\sqrt{3+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{-1+x^2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\,\sqrt{-3+x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 63, normalized size = 1.00

$$-\frac{i\sqrt{1-x^2}\,\sqrt{3+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|-\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-3+x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], -3/2]
)/(Sqrt[2]*Sqrt[-3 + x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 51, normalized size = 0.81

method	result	size
default	$-\frac{i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{6\sqrt{2x^4+x^2-3}}$	51
elliptic	$-\frac{i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{i\sqrt{6}}{2}\right)}{6\sqrt{2x^4+x^2-3}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(2*x^4+x^2-3)^(1/2)*EllipticF
(1/3*I*x*6^(1/2),1/2*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + x**2 - 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 - 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4 + x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(x^2 + 2*x^4 - 3)^(1/2), x)
```

3.60 $\int \frac{1}{\sqrt{-3+2x^4}}\,dx$

Optimal. Leaf size=112

$$\frac{\sqrt{-3+\sqrt{6}\,x^2}\sqrt{\frac{3+\sqrt{6}\,x^2}{3-\sqrt{6}\,x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}\,x}{\sqrt{-3+\sqrt{6}\,x^2}}\right)|\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}\,x^2}}\sqrt{-3+2x^4}}$$

```
[Out] 1/6*EllipticF(2^(3/4)*3^(1/4)*x/(-3+x^2*6^(1/2))^(1/2),1/2*2^(1/2))*(-3+x^2
*6^(1/2))^(1/2)*((3+x^2*6^(1/2))/(3-x^2*6^(1/2)))^(1/2)*6^(1/4)/(2*x^4-3)^(
1/2)/(1/(3-x^2*6^(1/2)))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {229}

$$\frac{\sqrt{\sqrt{6}\,x^2-3}\sqrt{\frac{\sqrt{6}\,x^2+3}{3-\sqrt{6}\,x^2}}\,F\left(\text{ArcSin}\left(\frac{2^{3/4}\sqrt[4]{3}\,x}{\sqrt{\sqrt{6}\,x^2-3}}\right)|\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}\,x^2}}\sqrt{2x^4-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 + Sqrt[6]*x^2]*Sqrt[(3 + Sqrt[6]*x^2)/(3 - Sqrt[6]*x^2)]*EllipticF
[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-3 + Sqrt[6]*x^2]], 1/2])/(6^(3/4)*Sqrt[(3
 - Sqrt[6]*x^2)^(-1)]*Sqrt[-3 + 2*x^4])
```

Rule 229

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[(a - q*x^2)/(a + q*x^2)]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[a + b*x^
4]*Sqrt[a/(a + q*x^2)]))*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2],
 x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+2x^4}}\,dx = \frac{\sqrt{-3+\sqrt{6}\,x^2}\sqrt{\frac{3+\sqrt{6}\,x^2}{3-\sqrt{6}\,x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}\,x}{\sqrt{-3+\sqrt{6}\,x^2}}\right)|\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}\,x^2}}\,\sqrt{-3+2x^4}}$$

Mathematica [A]
time = 10.04, size = 40, normalized size = 0.36

$$\frac{\sqrt{3-2x^4}\,F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|-1\right)}{\sqrt[4]{6}\,\sqrt{-3+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^4]*EllipticF[ArcSin[(2/3)^(1/4)*x], -1])/(6^(1/4)*Sqrt[-3 + 2
*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 56, normalized size = 0.50

method	result	size
meijerg	$\frac{\sqrt{3}\,\sqrt{-\mathrm{signum}\left(-1+\frac{2x^4}{3}\right)}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],\frac{2x^4}{3}\right)}{3\sqrt{\mathrm{signum}\left(-1+\frac{2x^4}{3}\right)}}$	40
default	$\frac{\sqrt{9+3x^2\sqrt{6}}\,\sqrt{9-3x^2\sqrt{6}}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3\sqrt{6}}\,x}{3},i\right)}{3\sqrt{-3\sqrt{6}}\,\sqrt{2x^4-3}}$	56
elliptic	$\frac{\sqrt{9+3x^2\sqrt{6}}\,\sqrt{9-3x^2\sqrt{6}}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3\sqrt{6}}\,x}{3},i\right)}{3\sqrt{-3\sqrt{6}}\,\sqrt{2x^4-3}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(-3*6^(1/2))^(1/2)*(9+3*x^2*6^(1/2))^(1/2)*(9-3*x^2*6^(1/2))^(1/2)/(2*x
^4-3)^(1/2)*EllipticF(1/3*(-3*6^(1/2))^(1/2)*x,I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [C] Result contains complex when optimal does not.
time = 0.34, size = 34, normalized size = 0.30

$$-\frac{\sqrt{3}\,ix\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-3)**(1/2),x)
[Out] -sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4/3)/(12*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 - 3), x)
```

Mupad [B]
time = 0.08, size = 31, normalized size = 0.28

$$\frac{x\sqrt{9-6x^4}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\;\frac{5}{4};\;\frac{2x^4}{3}\right)}{3\sqrt{2x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 3)^(1/2),x)
```

```
[Out] (x*(9 - 6*x^4)^(1/2)*hypergeom([1/4, 1/2], 5/4, (2*x^4)/3))/(3*(2*x^4 - 3)^
(1/2))
```

3.61 $\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{1+x^2}\,\sqrt{-3+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{5}\,x}{\sqrt{-3+2x^2}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{-3-x^2+2x^4}}$$

```
[Out] 1/5*EllipticF(x*5^(1/2)/(2*x^2-3)^(1/2),1/5*10^(1/2))*(x^2+1)^(1/2)*(2*x^2-
3)^(1/2)*5^(1/2)/(2*x^4-x^2-3)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1111}

$$\frac{\sqrt{x^2+1}\,\sqrt{2x^2-3}\,F\left(\text{ArcSin}\left(\frac{\sqrt{5}\,x}{\sqrt{2x^2-3}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{2x^4-x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[-3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt[-3 + 2*x^
2]], 2/5])/(Sqrt[5]*Sqrt[-3 - x^2 + 2*x^4])
```

Rule 1111

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx = \frac{\sqrt{1+x^2}\,\sqrt{-3+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{5}\,x}{\sqrt{-3+2x^2}}\right)|\frac{2}{5}\right)}{\sqrt{5}\,\sqrt{-3-x^2+2x^4}}$$

Mathematica [A]
time = 10.03, size = 51, normalized size = 0.78

$$\frac{\sqrt{3-2x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|-\frac{3}{2}\right)}{\sqrt{-6-2x^2+4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], -3/2])/Sqrt[-
6 - 2*x^2 + 4*x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 45, normalized size = 0.69

method	result	size
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+9}\,\mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{3}\right)}{3\sqrt{2x^4-x^2-3}}$	45
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{-6x^2+9}\,\mathrm{EllipticF}\left(ix,\frac{i\sqrt{6}}{3}\right)}{3\sqrt{2x^4-x^2-3}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*(x^2+1)^(1/2)*(-6*x^2+9)^(1/2)/(2*x^4-x^2-3)^(1/2)*EllipticF(I*x,1/3
*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - x**2 - 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 - 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - x^2 - 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - x^2 - 3)^(1/2), x)
```

3.62 $\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{-3-\left(1-\sqrt{7}\right)x^2}\sqrt{\frac{3+\left(1+\sqrt{7}\right)x^2}{3+\left(1-\sqrt{7}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{7}\,x}{\sqrt{-3-\left(1-\sqrt{7}\right)x^2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{6}\,\sqrt[4]{7}\sqrt{\frac{1}{3+\left(1-\sqrt{7}\right)x^2}}\sqrt{-3-2x^2+2x^4}}$$

```
[Out] 1/42*EllipticF(7^(1/4)*x*2^(1/2)/(-3-x^2*(1-7^(1/2)))^(1/2),1/14*(98-14*7^(
1/2))^(1/2))*(-3-x^2*(1-7^(1/2)))^(1/2)*((3+x^2*(1+7^(1/2)))/(3+x^2*(1-7^(1
/2))))^(1/2)*7^(3/4)*6^(1/2)/(2*x^4-2*x^2-3)^(1/2)/(1/(3+x^2*(1-7^(1/2))))^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-3}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+3}{\left(1-\sqrt{7}\right)x^2+3}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\,\sqrt[4]{7}\,x}{\sqrt{-\left(\left(1-\sqrt{7}\right)x^2\right)-3}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{6}\,\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+3}}\sqrt{2x^4-2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 2*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 - (1 - Sqrt[7])*x^2]*Sqrt[(3 + (1 + Sqrt[7])*x^2)/(3 + (1 - Sqrt[7
])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 - (1 - Sqrt[7])*x^2]]
, (7 - Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3 + (1 - Sqrt[7])*x^2)^(-1)]*Sq
rt[-3 - 2*x^2 + 2*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx = \frac{\sqrt{-3-\left(1-\sqrt{7}\right)x^2}\sqrt{\frac{3+\left(1+\sqrt{7}\right)x^2}{3+\left(1-\sqrt{7}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-3-\left(1-\sqrt{7}\right)x^2}}\right.\right.}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3+\left(1-\sqrt{7}\right)x^2}}\sqrt{-3-2x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 81, normalized size = 0.54

$$-\frac{i\sqrt{3+2x^2-2x^4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}\,x\right)\middle|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}\sqrt{-3-2x^2+2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 - 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x]
, (-4 + Sqrt[7])/3])/(Sqrt[1 + Sqrt[7]]*Sqrt[-3 - 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.56

method	result	size
default	$\frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{-3-3\sqrt{7}}\sqrt{2x^4-2x^2-3}}$	84
elliptic	$\frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{\sqrt{7}}{3}\right)x^2}\sqrt{1-\left(-\frac{1}{3}+\frac{\sqrt{7}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3-3\sqrt{7}}\,x}{3},\frac{i\sqrt{42}}{6}-\frac{i\sqrt{6}}{6}\right)}{\sqrt{-3-3\sqrt{7}}\sqrt{2x^4-2x^2-3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-2*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(-3-3*7^(1/2))^(1/2)*(1-(-1/3-1/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3+1/3*7^(1/2
))*x^2)^(1/2)/(2*x^4-2*x^2-3)^(1/2)*EllipticF(1/3*(-3-3*7^(1/2))^(1/2)*x,1/
6*I*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 - 2x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-2*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 - 2*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 - 2\,x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 2*x^2 - 3)^(1/2),x)
[Out] int(1/(2*x^4 - 2*x^2 - 3)^(1/2), x)
```

3.63 $\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{-6-\left(3-\sqrt{33}\right)x^2}\sqrt{\frac{6+\left(3+\sqrt{33}\right)x^2}{6+\left(3-\sqrt{33}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-6-\left(3-\sqrt{33}\right)x^2}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6+\left(3-\sqrt{33}\right)x^2}}\sqrt{-3-3x^2+2x^4}}$$

```
[Out] 1/66*EllipticF(33^(1/4)*x*2^(1/2)/(-6-x^2*(3-33^(1/2)))^(1/2),1/22*(242-22*
33^(1/2))^(1/2))*(-6-x^2*(3-33^(1/2)))^(1/2)*((6+x^2*(3+33^(1/2)))/(6+x^2*(
3-33^(1/2))))^(1/2)*3^(1/4)*11^(3/4)/(2*x^4-3*x^2-3)^(1/2)/(1/(6+x^2*(3-33^
(1/2))))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-6}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+6}{\left(3-\sqrt{33}\right)x^2+6}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-\left(\left(3-\sqrt{33}\right)x^2\right)-6}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+6}}\sqrt{2x^4-3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 3*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-6 - (3 - Sqrt[33])*x^2]*Sqrt[(6 + (3 + Sqrt[33])*x^2)/(6 + (3 - Sqrt
[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[-6 - (3 - Sqrt[33])*
x^2]], (11 - Sqrt[33])/22])/(2*3^(3/4)*11^(1/4)*Sqrt[(6 + (3 - Sqrt[33])*x^
2)^(-1)]*Sqrt[-3 - 3*x^2 + 2*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx = \frac{\sqrt{-6-\left(3-\sqrt{33}\right)x^2}\sqrt{\frac{6+\left(3+\sqrt{33}\right)x^2}{6+\left(3-\sqrt{33}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}\,x}{\sqrt{-6-\left(3-\sqrt{33}\right.}}\right.\right.}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6+\left(3-\sqrt{33}\right)x^2}}\sqrt{-3-3x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 78, normalized size = 0.51

$$-\frac{i\sqrt{6+6x^2-4x^4}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{\sqrt{3+\sqrt{33}}\sqrt{-3-3x^2+2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 - 3*x^2 + 2*x^4],x]
[Out] ((-I)*Sqrt[6 + 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 + Sqrt[33]]
], (-7 + Sqrt[33])/4])/(Sqrt[3 + Sqrt[33]]*Sqrt[-3 - 3*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.55

method	result
default	$\frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-18-6\sqrt{33}}\,x}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{-18-6\sqrt{33}}\sqrt{2x^4-3x^2-3}}$
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{33}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{33}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-18-6\sqrt{33}}\,x}{6},\frac{i\sqrt{22}}{4}-\frac{i\sqrt{6}}{4}\right)}{\sqrt{-18-6\sqrt{33}}\sqrt{2x^4-3x^2-3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-3*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 6/(-18-6*33^(1/2))^(1/2)*(1-(-1/2-1/6*33^(1/2))*x^2)^(1/2)*(1-(-1/2+1/6*33^
(1/2))*x^2)^(1/2)/(2*x^4-3*x^2-3)^(1/2)*EllipticF(1/6*(-18-6*33^(1/2))^(1/2
)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 - 3x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-3*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 - 3*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 - 3\,x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 3*x^2 - 3)^(1/2),x)
[Out] int(1/(2*x^4 - 3*x^2 - 3)^(1/2), x)
```

3.64 $\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx$

Optimal. Leaf size=155

$$\frac{\sqrt{-3-\left(2-\sqrt{10}\right)x^2}\sqrt{\frac{3+\left(2+\sqrt{10}\right)x^2}{3+\left(2-\sqrt{10}\right)x^2}}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-3-\left(2-\sqrt{10}\right)x^2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3+\left(2-\sqrt{10}\right)x^2}}\sqrt{-3-4x^2+2x^4}}$$

```
[Out] 1/30*EllipticF(2^(3/4)*5^(1/4)*x/(-3-x^2*(2-10^(1/2)))^(1/2),1/10*(50-10*10
^(1/2))^(1/2))*(-3-x^2*(2-10^(1/2)))^(1/2)*((3+x^2*(2+10^(1/2)))/(3+x^2*(2-
10^(1/2))))^(1/2)*2^(1/4)*5^(3/4)*3^(1/2)/(2*x^4-4*x^2-3)^(1/2)/(1/(3+x^2*(
2-10^(1/2))))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1112}

$$\frac{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-3}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+3}{\left(2-\sqrt{10}\right)x^2+3}}\,F\left(\text{ArcSin}\left(\frac{2^{3/4}\sqrt[4]{5}\,x}{\sqrt{-\left(\left(2-\sqrt{10}\right)x^2\right)-3}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+3}}\sqrt{2x^4-4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 4*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 - (2 - Sqrt[10])*x^2]*Sqrt[(3 + (2 + Sqrt[10])*x^2)/(3 + (2 - Sqrt
[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-3 - (2 - Sqrt[10])*x
^2]], (5 - Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^(1/4)*Sqrt[(3 + (2 - Sqrt[10])
*x^2)^(-1)]*Sqrt[-3 - 4*x^2 + 2*x^4])
```

Rule 1112

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[(2*a + (b - q)*x^2)/(2*a + (b + q)*x^2)]*(Sqrt[(
2*a + (b + q)*x^2)/q]/(2*Sqrt[a + b*x^2 + c*x^4]*Sqrt[a/(2*a + (b + q)*x^2)
]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b + q)*x^2)/(2*q)]], (b + q)/(2*q)], x]
] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx = \frac{\sqrt{-3-\left(2-\sqrt{10}\right)x^2}\sqrt{\frac{3+\left(2+\sqrt{10}\right)x^2}{3+\left(2-\sqrt{10}\right)x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-3-\left(2-\sqrt{10}\right)}}\right.\right.}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3+\left(2-\sqrt{10}\right)x^2}}\sqrt{-3-4x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 83, normalized size = 0.54

$$-\frac{i\sqrt{3+4x^2-2x^4}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}\,x\right)\middle|-\frac{7}{3}+\frac{2\sqrt{10}}{3}\right)}{\sqrt{2+\sqrt{10}}\sqrt{-3-4x^2+2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-3 - 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*x
], -7/3 + (2*Sqrt[10])/3])/(Sqrt[2 + Sqrt[10]]*Sqrt[-3 - 4*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 84, normalized size = 0.54

method	result	si
default	$\frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{-6-3\sqrt{10}}\sqrt{2x^4-4x^2-3}}$	84
elliptic	$\frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)x^2}\sqrt{1-\left(-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6-3\sqrt{10}}\,x}{3},\frac{i\sqrt{15}}{3}-\frac{i\sqrt{6}}{3}\right)}{\sqrt{-6-3\sqrt{10}}\sqrt{2x^4-4x^2-3}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-4*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(-6-3*10^(1/2))^(1/2)*(1-(-2/3-1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/3+1/3*10^(
1/2))*x^2)^(1/2)/(2*x^4-4*x^2-3)^(1/2)*EllipticF(1/3*(-6-3*10^(1/2))^(1/2)*
x,1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 - 4x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-4*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 - 4*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 - 4\,x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 4*x^2 - 3)^(1/2),x)
[Out] int(1/(2*x^4 - 4*x^2 - 3)^(1/2), x)
```

3.65 $\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{-3+x^2}\,\sqrt{1+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{7}\,x}{\sqrt{-3+x^2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{-3-5x^2+2x^4}}$$

```
[Out] 1/7*EllipticF(x*7^(1/2)/(x^2-3)^(1/2),1/7*7^(1/2))*(x^2-3)^(1/2)*(2*x^2+1)^
(1/2)*7^(1/2)/(2*x^4-5*x^2-3)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1111}

$$\frac{\sqrt{x^2-3}\,\sqrt{2x^2+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{7}\,x}{\sqrt{x^2-3}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{2x^4-5x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt[-3 + x^2]
], 1/7])/(Sqrt[7]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Rule 1111

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[-2*a - (b - q)*x^2]*(Sqrt[(2*a + (b + q)*x^2)/q]
/(2*Sqrt[-a]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[ArcSin[x/Sqrt[(2*a + (b +
q)*x^2)/(2*q)]], (b + q)/(2*q)], x] /; IntegerQ[q]] /; FreeQ[{a, b, c}, x]
&& GtQ[b^2 - 4*a*c, 0] && LtQ[a, 0] && GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx = \frac{\sqrt{-3+x^2}\,\sqrt{1+2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{7}\,x}{\sqrt{-3+x^2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\,\sqrt{-3-5x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 65, normalized size = 1.03

$$-\frac{i\sqrt{1-\frac{x^2}{3}}\,\sqrt{1+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{2}\,x\right)|-\frac{1}{6}\right)}{\sqrt{2}\,\sqrt{-3-5x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 5*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/3]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1/6]
)/(Sqrt[2]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 53, normalized size = 0.84

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{-3x^2+9}\,\mathrm{EllipticF}\left(i\sqrt{2}\,x,\frac{i\sqrt{6}}{6}\right)}{6\sqrt{2x^4-5x^2-3}}$	53
elliptic	$-\frac{i\sqrt{2}\,\sqrt{2x^2+1}\,\sqrt{-3x^2+9}\,\mathrm{EllipticF}\left(i\sqrt{2}\,x,\frac{i\sqrt{6}}{6}\right)}{6\sqrt{2x^4-5x^2-3}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-5*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*2^(1/2)*(2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-5*x^2-3)^(1/2)*Ellip
ticF(I*2^(1/2)*x,1/6*I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-5x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 5*x**2 - 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 5*x^2 - 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 5*x^2 - 3)^(1/2), x)
```

3.66 $\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

```
[Out] 1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2)
)*((3*x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1114}

$$\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]
*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rule 1114

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.02, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{1+x^2}\sqrt{2+3x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|\frac{2}{3}\right)}{\sqrt{6+15x^2+9x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])
/Sqrt[6 + 15*x^2 + 9*x^4]
```

Maple [A]
time = 0.03, size = 44, normalized size = 0.85

method	result	size
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$	44
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/
2*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 10, normalized size = 0.19

$$-\frac{1}{2}i\,\sqrt{2}\,\mathrm{ellipticF}\left(i\,x,\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(2)*ellipticF(I*x, 3/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 + 5x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\,x^4 + 5\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

3.67 $\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2+4x^2+3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-6*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+4*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(3*x^4+4*x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1117\}$

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+4x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 4*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[2 + 4*x^2 + 3*x^
4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2+4x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{3x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{-2+i\sqrt{2}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-2-i\sqrt{2}}}\,x\right)|\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{2+4x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[3]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[2 + 4*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 87, normalized size = 0.97

method	result	s
default	$\frac{2\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-4+2i\sqrt{2}}}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{-4+2i\sqrt{2}}\sqrt{3x^4+4x^2+2}}$	8
elliptic	$\frac{2\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-4+2i\sqrt{2}}}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{-4+2i\sqrt{2}}\sqrt{3x^4+4x^2+2}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-4+2*I*2^(1/2))^(1/2)*(1-(-1+1/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1-1/2*I*2^(1
/2))*x^2)^(1/2)/(3*x^4+4*x^2+2)^(1/2)*EllipticF(1/2*x*(-4+2*I*2^(1/2))^(1/2
),1/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.39

$$-\frac{1}{6}\left(\sqrt{-2}+2\right)\sqrt{\sqrt{-2}-2}\,\mathrm{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{-2}-2},\,\frac{2}{3}\sqrt{-2}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(-2) + 2)*sqrt(sqrt(-2) - 2)*ellipticF(1/2*sqrt(2)*x*sqrt(sqrt(-2
) - 2), 2/3*sqrt(-2) + 1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 4*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(4*x^2 + 3*x^4 + 2)^(1/2), x)
```

3.68 $\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2+3x^2+3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8-2*6^(1/
2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+3*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(3*x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 + 3*x^2 + 3*x^
4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2+3x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{6x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{-3+i\sqrt{15}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-3-i\sqrt{15}}}\,x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{2+3x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(-3 + I*Sqrt[15]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-3 - I*Sqrt[15])]*x], (-3 - I*Sqrt[15])/(-3
 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[2 + 3*x^2 + 3*
x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 87, normalized size = 0.95

method	result
default	$\frac{2\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-3+i\sqrt{15}}}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{-3+i\sqrt{15}}\sqrt{3x^4+3x^2+2}}$
elliptic	$\frac{2\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-3+i\sqrt{15}}}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{-3+i\sqrt{15}}\sqrt{3x^4+3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-3+I*15^(1/2))^(1/2)*(1-(-3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-3/4-1/4*I*
15^(1/2))*x^2)^(1/2)/(3*x^4+3*x^2+2)^(1/2)*EllipticF(1/2*x*(-3+I*15^(1/2))^
(1/2),1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

Fricas [A]
time = 0.10, size = 35, normalized size = 0.38

$$-\frac{1}{24}\sqrt{2}\left(\sqrt{-15}+3\right)\sqrt{\sqrt{-15}-3}\,\mathrm{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{-15}-3},\frac{1}{4}\sqrt{-15}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(2)*(sqrt(-15) + 3)*sqrt(sqrt(-15) - 3)*ellipticF(1/2*x*sqrt(sqrt
(-15) - 3), 1/4*sqrt(-15) - 1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 3*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{3\,x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^2 + 3*x^4 + 2)^(1/2), x)
```

3.69 $\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2+2x^2+3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-3*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+2*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(3*x^4+2*x^2+2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+2x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[2 + 2*x^2 + 3*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2+2x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{3x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{-1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-1-i\sqrt{5}}}x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{2+2x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[3]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[2 + 2*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 87, normalized size = 0.95

method	result
default	$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{5}}}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{-2+2i\sqrt{5}}\sqrt{3x^4+2x^2+2}}$
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{5}}}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{-2+2i\sqrt{5}}\sqrt{3x^4+2x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-2+2*I*5^(1/2))^(1/2)*(1-(-1/2+1/2*I*5^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
5^(1/2))*x^2)^(1/2)/(3*x^4+2*x^2+2)^(1/2)*EllipticF(1/2*x*(-2+2*I*5^(1/2))^
(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.38

$$-\frac{1}{6}\left(\sqrt{-5}+1\right)\sqrt{\sqrt{-5}-1}\,\mathrm{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{-5}-1},\frac{1}{3}\sqrt{-5}-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(-5) + 1)*sqrt(sqrt(-5) - 1)*ellipticF(1/2*sqrt(2)*x*sqrt(sqrt(-5
) - 1), 1/3*sqrt(-5) - 2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3x^4+2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 2*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(2*x^2 + 3*x^4 + 2)^(1/2), x)
```

3.70 $\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2+x^2+3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72-6*6^(
1/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(3*x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 + x^2 + 3*x^4]
)
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2+x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 142, normalized size = 1.61

$$-\frac{i\sqrt{1-\frac{6x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{-1+i\sqrt{23}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-1-i\sqrt{23}}}\,x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{2+x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(-1 + I*Sqrt[23]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-1 - I*Sqrt[23])]*x], (-1 - I*Sqrt[23])/(-1
 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[2 + x^2 + 3*x^
4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 85, normalized size = 0.97

method	result
default	$\frac{2\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-1+i\sqrt{23}}}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{-1+i\sqrt{23}}\,\sqrt{3x^4+x^2+2}}$
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-1+i\sqrt{23}}}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{-1+i\sqrt{23}}\,\sqrt{3x^4+x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-1+I*23^(1/2))^(1/2)*(1-(-1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-1/4-1/4*I*
23^(1/2))*x^2)^(1/2)/(3*x^4+x^2+2)^(1/2)*EllipticF(1/2*x*(-1+I*23^(1/2))^(1
/2),1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

[Out] integrate(1/sqrt(3*x^4 + x^2 + 2), x)

Fricas [A]
time = 0.09, size = 35, normalized size = 0.40

$$-\frac{1}{24}\sqrt{2}\left(\sqrt{-23}+1\right)\sqrt{\sqrt{-23}-1}\,\text{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{-23}-1},\frac{1}{12}\sqrt{-23}-\frac{11}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="fricas")

[Out] -1/24*sqrt(2)*(sqrt(-23) + 1)*sqrt(sqrt(-23) - 1)*ellipticF(1/2*x*sqrt(sqrt(-23) - 1), 1/12*sqrt(-23) - 11/12)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x**4+x**2+2)**(1/2),x)

[Out] Integral(1/sqrt(3*x**4 + x**2 + 2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3*x^4+x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(3*x^4 + x^2 + 2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{3\,x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 + 3*x^4 + 2)^(1/2),x)

[Out] int(1/(x^2 + 3*x^4 + 2)^(1/2), x)

3.71 $\int \frac{1}{\sqrt{2+3x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{2+3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/2*2^(1/2))*(
2+x^2*6^(1/2))*((3*x^4+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/(3*x^4+2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {226}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{3x^4+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[2 + 3*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 25, normalized size = 0.35

$$-\sqrt[4]{-\frac{1}{6}}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{3}{2}}\,x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 3*x^4],x]
[Out] -((-1/6)^(1/4)*EllipticF[I*ArcSinh[(-3/2)^(1/4)*x], -1])
```

Maple [C] Result contains complex when optimal does not.
time = 0.09, size = 66, normalized size = 0.92

method	result	size
meijerg	$\frac{\sqrt{2}\,x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-\frac{3x^4}{2}\right)}{2}$	18
default	$\frac{\sqrt{2}\,\sqrt{4-2i\sqrt{6}\,x^2}\,\sqrt{4+2i\sqrt{6}\,x^2}\,\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{i\sqrt{6}}}{2},i\right)}{4\sqrt{i\sqrt{6}}\,\sqrt{3x^4+2}}$	66
elliptic	$\frac{\sqrt{2}\,\sqrt{4-2i\sqrt{6}\,x^2}\,\sqrt{4+2i\sqrt{6}\,x^2}\,\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{i\sqrt{6}}}{2},i\right)}{4\sqrt{i\sqrt{6}}\,\sqrt{3x^4+2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*2^(1/2)/(I*6^(1/2))^(1/2)*(4-2*I*6^(1/2)*x^2)^(1/2)*(4+2*I*6^(1/2)*x^2)
^(1/2)/(3*x^4+2)^(1/2)*EllipticF(1/2*x*2^(1/2)*(I*6^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 + 2), x)
```

Fricas [A]
time = 0.08, size = 16, normalized size = 0.22

$$-\frac{1}{6}\left(-6\right)^{\frac{3}{4}}\,\text{ellipticF}\left(\frac{1}{2}\,\sqrt{2}\,\left(-6\right)^{\frac{1}{4}}\,x,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(-6)^(3/4)*ellipticF(1/2*sqrt(2)*(-6)^(1/4)*x, -1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.33, size = 36, normalized size = 0.50

$$\frac{\sqrt{2}\,x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{3x^4e^{i\pi}}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+2)**(1/2),x)
```

```
[Out] sqrt(2)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(I*pi)/2)/(8
*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2), x)
```

Mupad [B]
time = 0.09, size = 16, normalized size = 0.22

$$\frac{\sqrt{2}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{3\,x^4}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 + 2)^(1/2),x)
```

```
[Out] (2^(1/2)*x*hypergeom([1/4, 1/2], 5/4, -(3*x^4)/2))/2
```

3.72 $\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-x^2+3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72+6*6^(
1/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(3*x^4-x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 - x^2 + 3*x^4]
)
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{6x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{1-i\sqrt{23}}}x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{2-x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(1 + I*Sqrt[23])]
*EllipticF[I*ArcSinh[Sqrt[-6/(1 - I*Sqrt[23])]*x], (1 - I*Sqrt[23])/(1 + I*
Sqrt[23])])/(Sqrt[6]*Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[2 - x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 87, normalized size = 0.97

method	result	s
default	$\frac{2\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{1+i\sqrt{23}}}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{1+i\sqrt{23}}\sqrt{3x^4-x^2+2}}$	8
elliptic	$\frac{2\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{1+i\sqrt{23}}}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{1+i\sqrt{23}}\sqrt{3x^4-x^2+2}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(1+I*23^(1/2))^(1/2)*(1-(1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4-1/4*I*23^
(1/2))*x^2)^(1/2)/(3*x^4-x^2+2)^(1/2)*EllipticF(1/2*x*(1+I*23^(1/2))^(1/2),
1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 + 2), x)
```

Fricas [A]
time = 0.09, size = 35, normalized size = 0.39

$$-\frac{1}{24}\sqrt{2}\sqrt{\sqrt{-23}+1}\left(\sqrt{-23}-1\right)\text{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{-23}+1},-\frac{1}{12}\sqrt{-23}-\frac{11}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(2)*sqrt(sqrt(-23) + 1)*(sqrt(-23) - 1)*ellipticF(1/2*x*sqrt(sqrt
(-23) + 1), -1/12*sqrt(-23) - 11/12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 - x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 - x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - x^2 + 2)^(1/2), x)
```

3.73 $\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-2x^2+3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+3*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-2*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(3*x^4-2*x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1117\}$

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-2x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 2*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[2 - 2*x^2 + 3*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-2x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{3x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{1+i\sqrt{5}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{1-i\sqrt{5}}}\,x\right)|\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{2-2x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-3/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[3]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[2 - 2*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 87, normalized size = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{i\sqrt{5}}{2}+\frac{1}{2}\right)x^2}\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2+2i\sqrt{5}}}{2},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{2+2i\sqrt{5}}\sqrt{3x^4-2x^2+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{i\sqrt{5}}{2}+\frac{1}{2}\right)x^2}\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2+2i\sqrt{5}}}{2},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{2+2i\sqrt{5}}\sqrt{3x^4-2x^2+2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(2+2*I*5^(1/2))^(1/2)*(1-(1/2*I*5^(1/2)+1/2)*x^2)^(1/2)*(1-(1/2-1/2*I*5^(
1/2))*x^2)^(1/2)/(3*x^4-2*x^2+2)^(1/2)*EllipticF(1/2*x*(2+2*I*5^(1/2))^(1/2
),1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.39

$$-\frac{1}{6}\sqrt{\sqrt{-5}+1}\,(\sqrt{-5}-1)\text{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{-5}+1},-\frac{1}{3}\sqrt{-5}-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(sqrt(-5) + 1)*(sqrt(-5) - 1)*ellipticF(1/2*sqrt(2)*x*sqrt(sqrt(-5
) + 1), -1/3*sqrt(-5) - 2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4-2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 2*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 2*x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 2*x^2 + 2)^(1/2), x)
```

3.74 $\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-3x^2+3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8+2*6^(1/
2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-3*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(3*x^4-3*x^2+2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 3*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[2 - 3*x^2 + 3*x^
4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-3x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.08, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{6x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{3+i\sqrt{15}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{3-i\sqrt{15}}}\,x\right)|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{2-3x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(3 + I*Sqrt[15])]
*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[15])]*x], (3 - I*Sqrt[15])/(3 + I*
Sqrt[15])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[2 - 3*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 87, normalized size = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+i\sqrt{15}}}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{3+i\sqrt{15}}\sqrt{3x^4-3x^2+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+i\sqrt{15}}}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{3+i\sqrt{15}}\sqrt{3x^4-3x^2+2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(3+I*15^(1/2))^(1/2)*(1-(3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4-1/4*I*15^
(1/2))*x^2)^(1/2)/(3*x^4-3*x^2+2)^(1/2)*EllipticF(1/2*x*(3+I*15^(1/2))^(1/2
),1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.39

$$-\frac{1}{24}\sqrt{2}\sqrt{\sqrt{-15}+3}\left(\sqrt{-15}-3\right)\text{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{-15}+3},-\frac{1}{4}\sqrt{-15}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(2)*sqrt(sqrt(-15) + 3)*(sqrt(-15) - 3)*ellipticF(1/2*x*sqrt(sqrt
(-15) + 3), -1/4*sqrt(-15) - 1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 3*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 3*x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 3*x^2 + 2)^(1/2), x)
```

3.75 $\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2-4x^2+3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+6*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-4*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(3*x^4-4*x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-4x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 4*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[2 - 4*x^2 + 3*x^
4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2-4x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{3x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{2+i\sqrt{2}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{2-i\sqrt{2}}}\,x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{2-4x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-3/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[3]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[2 - 4*x^2 + 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 87, normalized size = 0.99

method	result	size
default	$\frac{2\sqrt{1-\left(1+\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{4+2i\sqrt{2}}}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{4+2i\sqrt{2}}\,\sqrt{3x^4-4x^2+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(1+\frac{i\sqrt{2}}{2}\right)x^2}\,\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{4+2i\sqrt{2}}}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{4+2i\sqrt{2}}\,\sqrt{3x^4-4x^2+2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(4+2*I*2^(1/2))^(1/2)*(1-(1+1/2*I*2^(1/2))*x^2)^(1/2)*(1-(1-1/2*I*2^(1/2)
)*x^2)^(1/2)/(3*x^4-4*x^2+2)^(1/2)*EllipticF(1/2*x*(4+2*I*2^(1/2))^(1/2),1/
3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

Fricas [A]
time = 0.07, size = 35, normalized size = 0.40

$$-\frac{1}{6}\sqrt{\sqrt{-2}+2}\,(\sqrt{-2}-2)\text{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{-2}+2}\,,-\frac{2}{3}\sqrt{-2}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(sqrt(-2) + 2)*(sqrt(-2) - 2)*ellipticF(1/2*sqrt(2)*x*sqrt(sqrt(-2
) + 2), -2/3*sqrt(-2) + 1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4-4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 4*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-4*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 4*x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 4*x^2 + 2)^(1/2), x)
```

3.76 $\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-5x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-5x^2+3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72+30*6^
(1/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-5*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(
3/4)/(3*x^4-5*x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1110}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-5x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 5*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 5*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[2 - 5*x^2 +
3*x^4])
```

Rule 1110

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-5x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-5x^2+3x^4}}$$

Mathematica [A]
time = 10.03, size = 53, normalized size = 0.58

$$\frac{\sqrt{2-3x^2}\,\sqrt{1-x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)|\frac{2}{3}\right)}{\sqrt{6-15x^2+9x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 5*x^2 + 3*x^4],x]
[Out] (Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/Sqrt[6
- 15*x^2 + 9*x^4]
```

Maple [A]
time = 0.02, size = 42, normalized size = 0.46

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4-5x^2+2}}$	42
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\text{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4-5x^2+2}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(3*x^4-5*x^2+2)^(1/2)*EllipticF(x,1/2*6
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

Fricas [A]
time = 0.10, size = 8, normalized size = 0.09

$$\frac{1}{2}\sqrt{2}\,\text{ellipticF}\left(x,\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*ellipticF(x, 3/2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 - 5x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 5*x**2 + 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 - 5\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 5*x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 5*x^2 + 2)^(1/2), x)
```

3.77 $\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-6x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-6x^2+3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/2*(2+6^(1/2)
)^(1/2))*(2+x^2*6^(1/2))*((3*x^4-6*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/
(3*x^4-6*x^2+2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1110}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-6x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3x^4-6x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 - 6*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 6*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*Sqrt[2 - 6*x^2 + 3*x^
4])
```

Rule 1110

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-6x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2-6x^2+3x^4}}$$

Mathematica [**A**]
time = 10.06, size = 85, normalized size = 0.94

$$\frac{\sqrt{3-\sqrt{3}-3x^2}\,\sqrt{2+\left(-3+\sqrt{3}\right)x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}\,x\right)|2-\sqrt{3}\right)}{\sqrt{6}\,\sqrt{2-6x^2+3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 - 6*x^2 + 3*x^4],x]
[Out] (Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[2 - 6*x^2 + 3*x^4])
```

Maple [**A**]
time = 0.06, size = 82, normalized size = 0.91

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2}\,\sqrt{1-\left(-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6+2\sqrt{3}}}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{6+2\sqrt{3}}\,\sqrt{3x^4-6x^2+2}}$	82
elliptic	$\frac{2\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2}\,\sqrt{1-\left(-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6+2\sqrt{3}}}{2},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{6+2\sqrt{3}}\,\sqrt{3x^4-6x^2+2}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4-6*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(6+2*3^(1/2))^(1/2)*(1-(1/2*3^(1/2)+3/2)*x^2)^(1/2)*(1-(-1/2*3^(1/2)+3/2)
*x^2)^(1/2)/(3*x^4-6*x^2+2)^(1/2)*EllipticF(1/2*x*(6+2*3^(1/2))^(1/2),1/2*6
^(1/2)-1/2*2^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

Fricas [**A**]
time = 0.08, size = 35, normalized size = 0.39

$$-\frac{1}{6}\sqrt{\sqrt{3}+3}\left(\sqrt{3}-3\right)\mathrm{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{3}+3},-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(sqrt(3) + 3)*(sqrt(3) - 3)*ellipticF(1/2*sqrt(2)*x*sqrt(sqrt(3) +
 3), -sqrt(3) + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 - 6x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-6*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 6*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4-6*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4 - 6\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 - 6*x^2 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^4 - 6*x^2 + 2)^(1/2), x)
```

3.78 $\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{\frac{6+\left(9-\sqrt{57}\right)x^2}{6+\left(9+\sqrt{57}\right)x^2}}\left(6+\left(9+\sqrt{57}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}\,x\right)|\frac{1}{4}\left(-19+3\sqrt{57}\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{3+9x^2+2x^4}}$$

```
[Out] (1/(36+x^2*(54+6*57^(1/2))))^(1/2)*(36+x^2*(54+6*57^(1/2)))^(1/2)*EllipticF
(x*(54+6*57^(1/2))^(1/2)/(36+x^2*(54+6*57^(1/2)))^(1/2),1/2*(-19+3*57^(1/2)
)^(1/2))*(6+x^2*(57^(1/2)+9))*((6+x^2*(9-57^(1/2)))/(6+x^2*(57^(1/2)+9)))^(
1/2)/(2*x^4+9*x^2+3)^(1/2)/(54+6*57^(1/2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1113}

$$\frac{\sqrt{\frac{\left(9-\sqrt{57}\right)x^2+6}{\left(9+\sqrt{57}\right)x^2+6}}\left(\left(9+\sqrt{57}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}\,x\right)|\frac{1}{4}\left(-19+3\sqrt{57}\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{2x^4+9x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 9*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(6 + (9 - Sqrt[57])*x^2)/(6 + (9 + Sqrt[57])*x^2)]*(6 + (9 + Sqrt[57]
)*x^2)*EllipticF[ArcTan[Sqrt[(9 + Sqrt[57])/6]*x], (-19 + 3*Sqrt[57])/4])/(
Sqrt[6*(9 + Sqrt[57])]*Sqrt[3 + 9*x^2 + 2*x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx = \frac{\sqrt{\frac{6+\left(9-\sqrt{57}\right)x^2}{6+\left(9+\sqrt{57}\right)x^2}}\left(6+\left(9+\sqrt{57}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}\,x\right)|\frac{1}{4}\left(\right.\right.}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{3+9x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 97, normalized size = 0.88

$$-\frac{i\sqrt{\frac{-9+\sqrt{57}-4x^2}{-9+\sqrt{57}}}\sqrt{9+\sqrt{57}+4x^2}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{9+\sqrt{57}}}\right)|\frac{23}{4}+\frac{3\sqrt{57}}{4}\right)}{2\sqrt{3+9x^2+2x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 9*x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[(-9 + Sqrt[57] - 4*x^2)/(-9 + Sqrt[57])]*Sqrt[9 + Sqrt[57] +
 4*x^2]*EllipticF[I*ArcSinh[(2*x)/Sqrt[9 + Sqrt[57]]], 23/4 + (3*Sqrt[57])/
4])/Sqrt[3 + 9*x^2 + 2*x^4]
```

Maple [A]
time = 0.07, size = 82, normalized size = 0.75

method	result	s
default	$\frac{6\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{57}}{6}\right)x^2}\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{57}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-54+6\sqrt{57}}}{6},\frac{3\sqrt{6}}{4}+\frac{\sqrt{38}}{4}\right)}{\sqrt{-54+6\sqrt{57}}\sqrt{2x^4+9x^2+3}}$	8
elliptic	$\frac{6\sqrt{1-\left(-\frac{3}{2}+\frac{\sqrt{57}}{6}\right)x^2}\sqrt{1-\left(-\frac{3}{2}-\frac{\sqrt{57}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-54+6\sqrt{57}}}{6},\frac{3\sqrt{6}}{4}+\frac{\sqrt{38}}{4}\right)}{\sqrt{-54+6\sqrt{57}}\sqrt{2x^4+9x^2+3}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+9*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(-54+6*57^(1/2))^(1/2)*(1-(-3/2+1/6*57^(1/2))*x^2)^(1/2)*(1-(-3/2-1/6*57^
(1/2))*x^2)^(1/2)/(2*x^4+9*x^2+3)^(1/2)*EllipticF(1/6*x*(-54+6*57^(1/2))^(1
/2),3/4*6^(1/2)+1/4*38^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 57, normalized size = 0.52

$$-\frac{1}{24}\left(\sqrt{19}\,\sqrt{6}+3\,\sqrt{6}\,\sqrt{3}\right)\sqrt{\sqrt{19}\,\sqrt{3}-9}\;\text{ellipticF}\left(\frac{1}{6}\,\sqrt{6}\,\sqrt{\sqrt{19}\,\sqrt{3}-9}\,x,\frac{3}{4}\,\sqrt{19}\,\sqrt{3}+\frac{23}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(sqrt(19)*sqrt(6) + 3*sqrt(6)*sqrt(3))*sqrt(sqrt(19)*sqrt(3) - 9)*ell
ipticF(1/6*sqrt(6)*sqrt(sqrt(19)*sqrt(3) - 9)*x, 3/4*sqrt(19)*sqrt(3) + 23/
4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{2x^4+9x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+9*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 9*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+9*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{2\,x^4+9\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2 + 2*x^4 + 3)^(1/2),x)
[Out] int(1/(9*x^2 + 2*x^4 + 3)^(1/2), x)
```

3.79 $\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{\frac{3+\left(4-\sqrt{10}\right)x^2}{3+\left(4+\sqrt{10}\right)x^2}}\left(3+\left(4+\sqrt{10}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}\,x\right)|-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{3+8x^2+2x^4}}$$

```
[Out] (1/(9+x^2*(12+3*10^(1/2))))^(1/2)*(9+x^2*(12+3*10^(1/2)))^(1/2)*EllipticF(x
*(12+3*10^(1/2))^(1/2)/(9+x^2*(12+3*10^(1/2)))^(1/2),1/3*(-30+12*10^(1/2))^
(1/2))*(3+x^2*(10^(1/2)+4))*((3+x^2*(4-10^(1/2)))/(3+x^2*(10^(1/2)+4)))^(1/
2)/(2*x^4+8*x^2+3)^(1/2)/(12+3*10^(1/2))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1113}

$$\frac{\sqrt{\frac{\left(4-\sqrt{10}\right)x^2+3}{\left(4+\sqrt{10}\right)x^2+3}}\left(\left(4+\sqrt{10}\right)x^2+3\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}\,x\right)|-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{2x^4+8x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 8*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 + (4 - Sqrt[10])*x^2)/(3 + (4 + Sqrt[10])*x^2)]*(3 + (4 + Sqrt[10]
)*x^2)*EllipticF[ArcTan[Sqrt[(4 + Sqrt[10])/3]*x], (-2*(5 - 2*Sqrt[10]))/3]
)/(Sqrt[3*(4 + Sqrt[10])]*Sqrt[3 + 8*x^2 + 2*x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3+\left(4-\sqrt{10}\right)x^2}{3+\left(4+\sqrt{10}\right)x^2}}\left(3+\left(4+\sqrt{10}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}\,x\right)|-\frac{2}{3}\right.}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{3+8x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 98, normalized size = 0.89

$$-\frac{i\sqrt{\frac{-4+\sqrt{10}-2x^2}{-4+\sqrt{10}}}\sqrt{4+\sqrt{10}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{4+\sqrt{10}}}\,x\right)|\frac{13}{3}+\frac{4\sqrt{10}}{3}\right)}{\sqrt{6+16x^2+4x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 8*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[(-4 + Sqrt[10] - 2*x^2)/(-4 + Sqrt[10])]*Sqrt[4 + Sqrt[10] + 2*x
^2]*EllipticF[I*ArcSinh[Sqrt[2/(4 + Sqrt[10])]*x], 13/3 + (4*Sqrt[10])/3])/
Sqrt[6 + 16*x^2 + 4*x^4]
```

Maple [A]
time = 0.07, size = 82, normalized size = 0.75

method	result	s
default	$\frac{3\sqrt{1-\left(-\frac{4}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{4}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-12+3\sqrt{10}}}{3},\frac{2\sqrt{6}}{3}+\frac{\sqrt{15}}{3}\right)}{\sqrt{-12+3\sqrt{10}}\,\sqrt{2x^4+8x^2+3}}$	8
elliptic	$\frac{3\sqrt{1-\left(-\frac{4}{3}+\frac{\sqrt{10}}{3}\right)x^2}\,\sqrt{1-\left(-\frac{4}{3}-\frac{\sqrt{10}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-12+3\sqrt{10}}}{3},\frac{2\sqrt{6}}{3}+\frac{\sqrt{15}}{3}\right)}{\sqrt{-12+3\sqrt{10}}\,\sqrt{2x^4+8x^2+3}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+8*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(-12+3*10^(1/2))^(1/2)*(1-(-4/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-4/3-1/3*10^
(1/2))*x^2)^(1/2)/(2*x^4+8*x^2+3)^(1/2)*EllipticF(1/3*x*(-12+3*10^(1/2))^(1
/2),2/3*6^(1/2)+1/3*15^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

Fricas [A]
time = 0.09, size = 35, normalized size = 0.32

$$-\frac{1}{6}\left(\sqrt{10}+4\right)\sqrt{\sqrt{10}-4}\,\text{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{10}-4}\,,\frac{4}{3}\sqrt{10}+\frac{13}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="fricas")
[Out] -1/6*(sqrt(10) + 4)*sqrt(sqrt(10) - 4)*ellipticF(1/3*sqrt(3)*x*sqrt(sqrt(10
) - 4), 4/3*sqrt(10) + 13/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{2x^4+8x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+8*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 + 8*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+8*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{2\,x^4+8\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^2 + 2*x^4 + 3)^(1/2),x)
[Out] int(1/(8*x^2 + 2*x^4 + 3)^(1/2), x)
```

3.80 $\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{\frac{3+x^2}{1+2x^2}}\,(1+2x^2)\,F\left(\tan^{-1}\left(\sqrt{2}\,x\right)|\frac{5}{6}\right)}{\sqrt{6}\,\sqrt{3+7x^2+2x^4}}$$

```
[Out] 1/6*(2*x^2+1)^(3/2)*(1/(2*x^2+1))^(1/2)*EllipticF(x*2^(1/2)/(2*x^2+1)^(1/2)
,1/6*30^(1/2))*((x^2+3)/(2*x^2+1))^(1/2)*6^(1/2)/(2*x^4+7*x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1113}

$$\frac{\sqrt{\frac{x^2+3}{2x^2+1}}\,(2x^2+1)\,F\left(\text{ArcTan}\left(\sqrt{2}\,x\right)|\frac{5}{6}\right)}{\sqrt{6}\,\sqrt{2x^4+7x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 5/6])
/(Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3+x^2}{1+2x^2}}\,(1+2x^2)\,F\left(\tan^{-1}\left(\sqrt{2}\,x\right)|\frac{5}{6}\right)}{\sqrt{6}\,\sqrt{3+7x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 61, normalized size = 1.02

$$-\frac{i\sqrt{3+x^2}\,\sqrt{1+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{2}\,x\right)|\frac{1}{6}\right)}{\sqrt{6}\,\sqrt{3+7x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 7*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], 1/6])/(
Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 50, normalized size = 0.83

method	result	size
default	$-\frac{i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{2x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{3}\,x}{3},\sqrt{6}\right)}{3\sqrt{2x^4+7x^2+3}}$	50
elliptic	$-\frac{i\sqrt{3}\,\sqrt{3x^2+9}\,\sqrt{2x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{3}\,x}{3},\sqrt{6}\right)}{3\sqrt{2x^4+7x^2+3}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+7*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*3^(1/2)*(3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+7*x^2+3)^(1/2)*Ellipt
icF(1/3*I*3^(1/2)*x,6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.08, size = 10, normalized size = 0.17

$$-i\,\mathrm{ellipticF}\left(\frac{1}{3}i\,\sqrt{3}\,x,6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -I*ellipticF(1/3*I*sqrt(3)*x, 6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + 7x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 7*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+7*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4 + 7\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(7*x^2 + 2*x^4 + 3)^(1/2), x)
```

3.81 $\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt{\frac{3+\left(3-\sqrt{3}\right)x^2}{3+\left(3+\sqrt{3}\right)x^2}}\left(3+\left(3+\sqrt{3}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}\,x\right)\middle|-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{3+6x^2+2x^4}}$$

```
[Out] (1/(9+x^2*(9+3*3^(1/2))))^(1/2)*(9+x^2*(9+3*3^(1/2)))^(1/2)*EllipticF(x*(9+
3*3^(1/2))^(1/2)/(9+x^2*(9+3*3^(1/2)))^(1/2),(3^(1/2)-1)^(1/2))*(3+x^2*(3^(
1/2)+3))*((3+x^2*(3-3^(1/2)))/(3+x^2*(3^(1/2)+3)))^(1/2)/(2*x^4+6*x^2+3)^(1
/2)/(9+3*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1113}

$$\frac{\sqrt{\frac{\left(3-\sqrt{3}\right)x^2+3}{\left(3+\sqrt{3}\right)x^2+3}}\left(\left(3+\sqrt{3}\right)x^2+3\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}\,x\right)\middle|-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{2x^4+6x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 + (3 - Sqrt[3])*x^2)/(3 + (3 + Sqrt[3])*x^2)]*(3 + (3 + Sqrt[3])*x
^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[3])/3]*x], -1 + Sqrt[3]])/(Sqrt[3*(3 +
Sqrt[3])]*Sqrt[3 + 6*x^2 + 2*x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx = \frac{\sqrt{\frac{3+\left(3-\sqrt{3}\right)x^2}{3+\left(3+\sqrt{3}\right)x^2}}\left(3+\left(3+\sqrt{3}\right)x^2\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}\,x\right)|-1+\right.}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{3+6x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 90, normalized size = 0.87

$$-\frac{i\sqrt{\frac{-3+\sqrt{3}-2x^2}{-3+\sqrt{3}}}\sqrt{3+\sqrt{3}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-\frac{1}{\sqrt{3}}}\,x\right)|2+\sqrt{3}\right)}{\sqrt{6+12x^2+4x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[3 + 6*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[(-3 + Sqrt[3] - 2*x^2)/(-3 + Sqrt[3])]*Sqrt[3 + Sqrt[3] + 2*x^2]
*EllipticF[I*ArcSinh[Sqrt[1 - 1/Sqrt[3]]*x], 2 + Sqrt[3]])/Sqrt[6 + 12*x^2
+ 4*x^4]
```

Maple [A]
time = 0.06, size = 82, normalized size = 0.79

method	result	size
default	$\frac{3\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-9+3\sqrt{3}}}{3},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-9+3\sqrt{3}}\sqrt{2x^4+6x^2+3}}$	82
elliptic	$\frac{3\sqrt{1-\left(-1+\frac{\sqrt{3}}{3}\right)x^2}\sqrt{1-\left(-1-\frac{\sqrt{3}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-9+3\sqrt{3}}}{3},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{-9+3\sqrt{3}}\sqrt{2x^4+6x^2+3}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+6*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(-9+3*3^(1/2))^(1/2)*(1-(-1+1/3*3^(1/2))*x^2)^(1/2)*(1-(-1-1/3*3^(1/2))*x
^2)^(1/2)/(2*x^4+6*x^2+3)^(1/2)*EllipticF(1/3*x*(-9+3*3^(1/2))^(1/2),1/2*6^
(1/2)+1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 33, normalized size = 0.32

$$-\frac{1}{6}\left(\sqrt{3}+3\right)\sqrt{\sqrt{3}-3}\,\mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{3}-3},\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="fricas")
[Out] -1/6*(sqrt(3) + 3)*sqrt(sqrt(3) - 3)*ellipticF(1/3*sqrt(3)*x*sqrt(sqrt(3) -
 3), sqrt(3) + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+6*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 + 6*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+6*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x^2 + 2*x^4 + 3)^(1/2),x)
[Out] int(1/(6*x^2 + 2*x^4 + 3)^(1/2), x)
```

3.82 $\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{(1+x^2)\sqrt{\frac{3+2x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{3}\right)}{\sqrt{3}\,\sqrt{3+5x^2+2x^4}}$$

```
[Out] 1/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/3*3^(1/2))*
((2*x^2+3)/(x^2+1))^(1/2)*3^(1/2)/(2*x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1113}

$$\frac{(x^2+1)\sqrt{\frac{2x^2+3}{x^2+1}}\,F\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{3}\right)}{\sqrt{3}\,\sqrt{2x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(3 + 2*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]*
Sqrt[3 + 5*x^2 + 2*x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx = \frac{(1+x^2)\sqrt{\frac{3+2x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{3}\right)}{\sqrt{3}\,\sqrt{3+5x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{1+x^2}\,\sqrt{3+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|\frac{3}{2}\right)}{\sqrt{6+10x^2+4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], 3/2])
/Sqrt[6 + 10*x^2 + 4*x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 50, normalized size = 0.96

method	result	size
default	$-\frac{i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{6\sqrt{2x^4+5x^2+3}}$	50
elliptic	$-\frac{i\sqrt{6}\,\sqrt{6x^2+9}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{6\sqrt{2x^4+5x^2+3}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(2*x^4+5*x^2+3)^(1/2)*Elliptic
F(1/3*I*x*6^(1/2),1/2*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 16, normalized size = 0.31

$$-\frac{1}{2}\,\sqrt{-2}\,\mathrm{ellipticF}\left(\frac{1}{3}\,\sqrt{3}\,\sqrt{-2}\,x,\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-2)*ellipticF(1/3*sqrt(3)*sqrt(-2)*x, 3/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+5*x**2+3)**(1/2),x)
[Out] Integral(1/sqrt(2*x**4 + 5*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4 + 5\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 2*x^4 + 3)^(1/2),x)
[Out] int(1/(5*x^2 + 2*x^4 + 3)^(1/2), x)
```

3.83 $\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3+4x^2+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-6*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+4*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(2*x^4+4*x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+4x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 + 4*x^2 + 2*x^
4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3+4x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-2-i\sqrt{2}}}\,x\right)|\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{3+4x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[2]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[3 + 4*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 87, normalized size = 0.97

method	result	siz
default	$\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+3i\sqrt{2}}}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{-6+3i\sqrt{2}}\sqrt{2x^4+4x^2+3}}$	87
elliptic	$\frac{3\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+3i\sqrt{2}}}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{-6+3i\sqrt{2}}\sqrt{2x^4+4x^2+3}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+4*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(-6+3*I*2^(1/2))^(1/2)*(1-(-2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-2/3-1/3*I*
2^(1/2))*x^2)^(1/2)/(2*x^4+4*x^2+3)^(1/2)*EllipticF(1/3*x*(-6+3*I*2^(1/2))^
(1/2),1/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.39

$$-\frac{1}{6}\left(\sqrt{-2}+2\right)\sqrt{\sqrt{-2}-2}\ \mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{-2}-2},\frac{2}{3}\sqrt{-2}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(-2) + 2)*sqrt(sqrt(-2) - 2)*ellipticF(1/3*sqrt(3)*x*sqrt(sqrt(-2
) - 2), 2/3*sqrt(-2) + 1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 4*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+4*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(4*x^2 + 2*x^4 + 3)^(1/2), x)
```

3.84 $\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3+3x^2+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8-2*6^(1/
2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+3*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(2*x^4+3*x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 3*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[3 + 3*x^2 + 2*x^
4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3+3x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.08, size = 142, normalized size = 1.54

$$-\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\,x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{3+3x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 3*x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(-3 + I*Sqrt
[15])]*EllipticF[I*ArcSinh[2*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*x], (-3 - I*Sqrt
[15])/(-3 + I*Sqrt[15])]])/(Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[3 + 3*x^2 + 2
*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 87, normalized size = 0.95

method	result
default	$\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-18+6i\sqrt{15}}}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right.}{\sqrt{-18+6i\sqrt{15}}\sqrt{2x^4+3x^2+3}}$
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-18+6i\sqrt{15}}}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right.}{\sqrt{-18+6i\sqrt{15}}\sqrt{2x^4+3x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(-18+6*I*15^(1/2))^(1/2)*(1-(-1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6
*I*15^(1/2))*x^2)^(1/2)/(2*x^4+3*x^2+3)^(1/2)*EllipticF(1/6*x*(-18+6*I*15^(
1/2))^(1/2),1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 51, normalized size = 0.55

$$-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{3}\sqrt{-5}-3}\left(\sqrt{3}+\sqrt{-5}\right)\text{ellipticF}\left(\frac{1}{6}\sqrt{6}\sqrt{\sqrt{3}\sqrt{-5}-3}\,x,\frac{1}{4}\sqrt{3}\sqrt{-5}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*sqrt(sqrt(3)*sqrt(-5) - 3)*(sqrt(3) + sqrt(-5))*ellipticF(1/6
*sqrt(6)*sqrt(sqrt(3)*sqrt(-5) - 3)*x, 1/4*sqrt(3)*sqrt(-5) - 1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{2x^4+3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 3*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(3*x^2 + 2*x^4 + 3)^(1/2), x)
```

3.85 $\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3+2x^2+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-3*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+2*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(2*x^4+2*x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+2x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 2*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[3 + 2*x^2 + 2*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3+2x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{2x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+i\sqrt{5}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-1-i\sqrt{5}}}\,x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{3+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[2]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[3 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 87, normalized size = 0.95

method	result
default	$\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3+3i\sqrt{5}}}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{-3+3i\sqrt{5}}\sqrt{2x^4+2x^2+3}}$
elliptic	$\frac{3\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-3+3i\sqrt{5}}}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{-3+3i\sqrt{5}}\sqrt{2x^4+2x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(-3+3*I*5^(1/2))^(1/2)*(1-(-1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-1/3-1/3*I*
5^(1/2))*x^2)^(1/2)/(2*x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x*(-3+3*I*5^(1/2))^
(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.38

$$-\frac{1}{6}\left(\sqrt{-5}+1\right)\sqrt{\sqrt{-5}-1}\,\mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{-5}-1},\frac{1}{3}\sqrt{-5}-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(-5) + 1)*sqrt(sqrt(-5) - 1)*ellipticF(1/3*sqrt(3)*x*sqrt(sqrt(-5
) - 1), 1/3*sqrt(-5) - 2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4+2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 2*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^2 + 2*x^4 + 3)^(1/2), x)
```

3.86 $\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3+x^2+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72-6*6^(
1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(2*x^4+x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 + x^2 + 2*x^4]
)
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3+x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 140, normalized size = 1.59

$$-\frac{i\sqrt{1-\frac{4x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+i\sqrt{23}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\,x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{3+x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(-1 + I*Sqrt
[23])]*EllipticF[I*ArcSinh[2*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*x], (-1 - I*Sqrt
[23])/(-1 + I*Sqrt[23])])/(Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[3 + x^2 + 2*x
^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 85, normalized size = 0.97

method	result
default	$\frac{6\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+6i\sqrt{23}}}{6},\frac{\sqrt{-33+3i\sqrt{2}}}{6}\right.}{\sqrt{-6+6i\sqrt{23}}\sqrt{2x^4+x^2+3}}$
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+6i\sqrt{23}}}{6},\frac{\sqrt{-33+3i\sqrt{2}}}{6}\right.}{\sqrt{-6+6i\sqrt{23}}\sqrt{2x^4+x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(-6+6*I*23^(1/2))^(1/2)*(1-(-1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*
I*23^(1/2))*x^2)^(1/2)/(2*x^4+x^2+3)^(1/2)*EllipticF(1/6*x*(-6+6*I*23^(1/2)
)^(1/2),1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 41, normalized size = 0.47

$$-\frac{1}{72}\sqrt{6}\sqrt{3}\left(\sqrt{-23}+1\right)\sqrt{\sqrt{-23}-1}\,\mathrm{ellipticF}\left(\frac{1}{6}\sqrt{6}\,x\sqrt{\sqrt{-23}-1},\frac{1}{12}\sqrt{-23}-\frac{11}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/72*sqrt(6)*sqrt(3)*(sqrt(-23) + 1)*sqrt(sqrt(-23) - 1)*ellipticF(1/6*sqr
t(6)*x*sqrt(sqrt(-23) - 1), 1/12*sqrt(-23) - 11/12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{2x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 2*x^4 + 3)^(1/2),x)
```

```
[Out] int(1/(x^2 + 2*x^4 + 3)^(1/2), x)
```

3.87 $\int \frac{1}{\sqrt{3+2x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{3+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/2*2^(1/2))*(
3+x^2*6^(1/2))*((2*x^4+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/(2*x^4+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {226}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{2x^4+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[3 + 2*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{3+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 25, normalized size = 0.35

$$-\sqrt[4]{-\frac{1}{6}}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{2}{3}}\,x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 2*x^4],x]
```

```
[Out] -((-1/6)^(1/4)*EllipticF[I*ArcSinh[(-2/3)^(1/4)*x], -1])
```

Maple [C] Result contains complex when optimal does not.
time = 0.09, size = 66, normalized size = 0.92

method	result	size
meijerg	$\frac{\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-\frac{2x^4}{3}\right)}{3}$	18
default	$\frac{\sqrt{3}\,\sqrt{9-3i\sqrt{6}\,x^2}\,\sqrt{9+3i\sqrt{6}\,x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{i\sqrt{6}}}{3},i\right)}{9\sqrt{i\sqrt{6}}\,\sqrt{2x^4+3}}$	66
elliptic	$\frac{\sqrt{3}\,\sqrt{9-3i\sqrt{6}\,x^2}\,\sqrt{9+3i\sqrt{6}\,x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{i\sqrt{6}}}{3},i\right)}{9\sqrt{i\sqrt{6}}\,\sqrt{2x^4+3}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*3^(1/2)/(I*6^(1/2))^(1/2)*(9-3*I*6^(1/2)*x^2)^(1/2)*(9+3*I*6^(1/2)*x^2)
^(1/2)/(2*x^4+3)^(1/2)*EllipticF(1/3*x*3^(1/2)*(I*6^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3), x)
```

Fricas [A]
time = 0.07, size = 34, normalized size = 0.47

$$-\frac{1}{6}\,\sqrt{3}\,\sqrt{-2}\,\sqrt{\sqrt{3}\,\sqrt{-2}}\,\mathrm{ellipticF}\left(\frac{1}{3}\,\sqrt{3}\,\sqrt{\sqrt{3}\,\sqrt{-2}}\,x,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(-2)*sqrt(sqrt(3)*sqrt(-2))*ellipticF(1/3*sqrt(3)*sqrt(sqr
t(3)*sqrt(-2))*x, -1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.33, size = 36, normalized size = 0.50

$$\frac{\sqrt{3}\,x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^{4}e^{i\pi}}{3}\right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+3)**(1/2),x)
```

```
[Out] sqrt(3)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(I*pi)/3)/(1
2*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3), x)
```

Mupad [B]
time = 0.09, size = 16, normalized size = 0.22

$$\frac{\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{2\,x^4}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 + 3)^(1/2),x)
```

```
[Out] (3^(1/2)*x*hypergeom([1/4, 1/2], 5/4, -(2*x^4)/3))/3
```

3.88 $\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-x^2+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72+6*6^(
1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(2*x^4-x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - x^2 + 2*x^4]
)
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{1+i\sqrt{23}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{1-i\sqrt{23}}}\,x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{3-x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(1 + I*Sqrt[2
3])]*EllipticF[I*ArcSinh[2*Sqrt[-(1 - I*Sqrt[23])^(-1)]*x], (1 - I*Sqrt[23]
)/(1 + I*Sqrt[23])])/(Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[3 - x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.07, size = 87, normalized size = 0.97

method	result
default	$\frac{6\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6+6i\sqrt{23}}}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{6+6i\sqrt{23}}\sqrt{2x^4-x^2+3}}$
elliptic	$\frac{6\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6+6i\sqrt{23}}}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{6+6i\sqrt{23}}\sqrt{2x^4-x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(6+6*I*23^(1/2))^(1/2)*(1-(1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*I*2
3^(1/2))*x^2)^(1/2)/(2*x^4-x^2+3)^(1/2)*EllipticF(1/6*x*(6+6*I*23^(1/2))^(1
/2),1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 41, normalized size = 0.46

$$-\frac{1}{72}\sqrt{6}\sqrt{3}\sqrt{\sqrt{-23}+1}\left(\sqrt{-23}-1\right)\text{ellipticF}\left(\frac{1}{6}\sqrt{6}\,x\sqrt{\sqrt{-23}+1},-\frac{1}{12}\sqrt{-23}-\frac{11}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/72*sqrt(6)*sqrt(3)*sqrt(sqrt(-23) + 1)*(sqrt(-23) - 1)*ellipticF(1/6*sqr
t(6)*x*sqrt(sqrt(-23) + 1), -1/12*sqrt(-23) - 11/12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - x^2 + 3)^(1/2), x)
```

3.89 $\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-2x^2+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+3*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-2*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(2*x^4-2*x^2+3)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-2x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 2*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[3 - 2*x^2 + 2*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-2x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{1-i\sqrt{5}}}x\right)|\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{3-2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-2/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[2]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[3 - 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 87, normalized size = 0.97

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+3i\sqrt{5}}}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{3+3i\sqrt{5}}\sqrt{2x^4-2x^2+3}}$	87
elliptic	$\frac{3\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3+3i\sqrt{5}}}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{3+3i\sqrt{5}}\sqrt{2x^4-2x^2+3}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(3+3*I*5^(1/2))^(1/2)*(1-(1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3-1/3*I*5^(
1/2))*x^2)^(1/2)/(2*x^4-2*x^2+3)^(1/2)*EllipticF(1/3*x*(3+3*I*5^(1/2))^(1/2
),1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

Fricas [A]
time = 0.09, size = 35, normalized size = 0.39

$$-\frac{1}{6}\sqrt{\sqrt{-5}+1}\left(\sqrt{-5}-1\right)\text{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{-5}+1},-\frac{1}{3}\sqrt{-5}-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(sqrt(-5) + 1)*(sqrt(-5) - 1)*ellipticF(1/3*sqrt(3)*x*sqrt(sqrt(-5
) + 1), -1/3*sqrt(-5) - 2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 2*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 2*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 2*x^2 + 3)^(1/2), x)
```

3.90 $\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-3x^2+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8+2*6^(1/
2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-3*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(2*x^4-3*x^2+3)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 3*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[3 - 3*x^2 + 2*x^
4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-3x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{3-i\sqrt{15}}}\,x\right)|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{3-3x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 3*x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(3 + I*Sqrt[1
5])]*EllipticF[I*ArcSinh[2*Sqrt[-(3 - I*Sqrt[15])^(-1)]*x], (3 - I*Sqrt[15]
)/(3 + I*Sqrt[15])])/(Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[3 - 3*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 87, normalized size = 0.97

method	result
default	$\frac{6\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{18+6i\sqrt{15}}}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{18+6i\sqrt{15}}\sqrt{2x^4-3x^2+3}}$
elliptic	$\frac{6\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{18+6i\sqrt{15}}}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{18+6i\sqrt{15}}\sqrt{2x^4-3x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(18+6*I*15^(1/2))^(1/2)*(1-(1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*I*
15^(1/2))*x^2)^(1/2)/(2*x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*(18+6*I*15^(1/2)
)^(1/2),1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 53, normalized size = 0.59

$$\frac{1}{24}\sqrt{6}\sqrt{\sqrt{3}\sqrt{-5}+3}\left(\sqrt{3}-\sqrt{-5}\right)\text{ellipticF}\left(\frac{1}{6}\sqrt{6}\sqrt{\sqrt{3}\sqrt{-5}+3}\,x,-\frac{1}{4}\sqrt{3}\sqrt{-5}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(6)*sqrt(sqrt(3)*sqrt(-5) + 3)*(sqrt(3) - sqrt(-5))*ellipticF(1/6*
sqrt(6)*sqrt(sqrt(3)*sqrt(-5) + 3)*x, -1/4*sqrt(3)*sqrt(-5) - 1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 3*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 3*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 3*x^2 + 3)^(1/2), x)
```

3.91 $\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3-4x^2+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+6*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-4*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(2*x^4-4*x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-4x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 4*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[3 - 4*x^2 + 2*x^
4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{3-4x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{2x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{2-i\sqrt{2}}}x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{3-4x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-2/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[2]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[3 - 4*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 87, normalized size = 0.99

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6+3i\sqrt{2}}}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{6+3i\sqrt{2}}\sqrt{2x^4-4x^2+3}}$	87
elliptic	$\frac{3\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6+3i\sqrt{2}}}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{6+3i\sqrt{2}}\sqrt{2x^4-4x^2+3}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-4*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(6+3*I*2^(1/2))^(1/2)*(1-(2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3-1/3*I*2^(
1/2))*x^2)^(1/2)/(2*x^4-4*x^2+3)^(1/2)*EllipticF(1/3*x*(6+3*I*2^(1/2))^(1/2
),1/3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

Fricas [A]
time = 0.07, size = 35, normalized size = 0.40

$$-\frac{1}{6}\sqrt{\sqrt{-2}+2}\,\left(\sqrt{-2}-2\right)\text{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{-2}+2}\,,-\frac{2}{3}\sqrt{-2}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(sqrt(-2) + 2)*(sqrt(-2) - 2)*ellipticF(1/3*sqrt(3)*x*sqrt(sqrt(-2
) + 2), -2/3*sqrt(-2) + 1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{2x^4-4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 4*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-4*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 4*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 4*x^2 + 3)^(1/2), x)
```

3.92 $\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-5x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-5x^2+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72+30*6^
(1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-5*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(
3/4)/(2*x^4-5*x^2+3)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1110}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-5x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 5*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 5*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - 5*x^2 +
2*x^4])
```

Rule 1110

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-5x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-5x^2+2x^4}}$$

Mathematica [A]
time = 10.02, size = 53, normalized size = 0.58

$$\frac{\sqrt{3-2x^2}\,\sqrt{1-x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|\frac{3}{2}\right)}{\sqrt{6-10x^2+4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 5*x^2 + 2*x^4],x]
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], 3/2])/Sqrt[6
- 10*x^2 + 4*x^4]
```

Maple [A]
time = 0.02, size = 42, normalized size = 0.46

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+9}\,\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)}{3\sqrt{2x^4-5x^2+3}}$	42
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+9}\,\mathrm{EllipticF}\left(x,\frac{\sqrt{6}}{3}\right)}{3\sqrt{2x^4-5x^2+3}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(-x^2+1)^(1/2)*(-6*x^2+9)^(1/2)/(2*x^4-5*x^2+3)^(1/2)*EllipticF(x,1/3*6
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 8, normalized size = 0.09

$$\frac{1}{3}\,\sqrt{3}\,\mathrm{ellipticF}\left(x,\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*ellipticF(x, 2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 - 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 5*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 - 5\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 5*x^2 + 3)^(1/2), x)
```

3.93 $\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-6x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-6x^2+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/2*(2+6^(1/2)
)^(1/2))*(3+x^2*6^(1/2))*((2*x^4-6*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/
(2*x^4-6*x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1110}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-6x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-6x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 6*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 6*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*Sqrt[3 - 6*x^2 + 2*x^
4])
```

Rule 1110

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-6x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)\middle|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-6x^2+2x^4}}$$

Mathematica [A]
time = 10.05, size = 81, normalized size = 0.90

$$\frac{\sqrt{3-\sqrt{3}-2x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}\,F\left(\sin^{-1}\left(\sqrt{1+\frac{1}{\sqrt{3}}}\,x\right)|2-\sqrt{3}\right)}{\sqrt{6}\sqrt{3-6x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[3 - 6*x^2 + 2*x^4])
```

Maple [A]
time = 0.05, size = 82, normalized size = 0.91

method	result	size
default	$\frac{3\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{9+3\sqrt{3}}}{3},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{9+3\sqrt{3}}\sqrt{2x^4-6x^2+3}}$	82
elliptic	$\frac{3\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{9+3\sqrt{3}}}{3},\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}\right)}{\sqrt{9+3\sqrt{3}}\sqrt{2x^4-6x^2+3}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-6*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(9+3*3^(1/2))^(1/2)*(1-(1+1/3*3^(1/2))*x^2)^(1/2)*(1-(1-1/3*3^(1/2))*x^2)
^(1/2)/(2*x^4-6*x^2+3)^(1/2)*EllipticF(1/3*x*(9+3*3^(1/2))^(1/2),1/2*6^(1/2
)-1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.39

$$-\frac{1}{6}\sqrt{\sqrt{3}+3}\left(\sqrt{3}-3\right)\mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{3}+3},-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(sqrt(3) + 3)*(sqrt(3) - 3)*ellipticF(1/3*sqrt(3)*x*sqrt(sqrt(3) +
 3), -sqrt(3) + 2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 - 6x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 6*x**2 + 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-6*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 - 6\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 6*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 6*x^2 + 3)^(1/2), x)
```

3.94 $\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-7x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-7x^2+2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72+42*6^
(1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-7*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(
3/4)/(2*x^4-7*x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1110}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-7x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{2x^4-7x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 7*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 7*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (12 + 7*Sqrt[6])/24])/(2*6^(1/4)*Sqrt[3 - 7*x^2 +
2*x^4])
```

Rule 1110

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-7x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{3-7x^2+2x^4}}$$

Mathematica [A]
time = 10.03, size = 58, normalized size = 0.63

$$\frac{\sqrt{1-2x^2}\sqrt{1-\frac{x^2}{3}}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|\frac{1}{6}\right)}{\sqrt{2}\sqrt{3-7x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2/3]*EllipticF[ArcSin[Sqrt[2]*x], 1/6])/(Sqrt[2
]*Sqrt[3 - 7*x^2 + 2*x^4])
```

Maple [A]
time = 0.03, size = 49, normalized size = 0.53

method	result	size
default	$\frac{\sqrt{2}\sqrt{-2x^2+1}\sqrt{-3x^2+9}\,\mathrm{EllipticF}\left(\sqrt{2}\,x,\frac{\sqrt{6}}{6}\right)}{6\sqrt{2x^4-7x^2+3}}$	49
elliptic	$\frac{\sqrt{2}\sqrt{-2x^2+1}\sqrt{-3x^2+9}\,\mathrm{EllipticF}\left(\sqrt{2}\,x,\frac{\sqrt{6}}{6}\right)}{6\sqrt{2x^4-7x^2+3}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4-7*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(1/2)*(-2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-7*x^2+3)^(1/2)*Ellipti
cF(2^(1/2)*x,1/6*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

Fricas [A]
time = 0.07, size = 15, normalized size = 0.16

$$\frac{1}{6}\sqrt{3}\sqrt{2}\,\mathrm{ellipticF}\left(\sqrt{2}\,x,\frac{1}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2)*ellipticF(sqrt(2)*x, 1/6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 - 7x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4-7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 7*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4-7*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2\,x^4 - 7\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 - 7*x^2 + 3)^(1/2),x)
```

```
[Out] int(1/(2*x^4 - 7*x^2 + 3)^(1/2), x)
```

3.95 $\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx$

Optimal. Leaf size=19

$$-\frac{F\left(\cos^{-1}\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}}$$

```
[Out] -1/5*(x^2)^(1/2)/x*EllipticF(1/3*(-3*x^2+9)^(1/2),1/5*30^(1/2))*5^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 431}

$$-\frac{F\left(\text{ArcCos}\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 7*x^2 - 2*x^4],x]
```

```
[Out] -(EllipticF[ArcCos[x/Sqrt[3]], 6/5]/Sqrt[5])
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx &= \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{12-4x^2}\sqrt{-2+4x^2}}\,dx\\ &= -\frac{F\left(\cos^{-1}\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 58 vs. 2(19) = 38.
time = 10.02, size = 58, normalized size = 3.05

$$\frac{\sqrt{1-2x^2}\sqrt{1-\frac{x^2}{3}}\,F\left(\sin^{-1}\left(\sqrt{2}\,x\right)|\frac{1}{6}\right)}{\sqrt{2}\sqrt{-3+7x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 7*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2/3]*EllipticF[ArcSin[Sqrt[2]*x], 1/6])/(Sqrt[2
]*Sqrt[-3 + 7*x^2 - 2*x^4])
```

Maple [A]
time = 0.02, size = 48, normalized size = 2.53

method	result	size
default	$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{-2x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}}{3},\sqrt{6}\right)}{3\sqrt{-2x^4+7x^2-3}}$	48
elliptic	$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{-2x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{3}}{3},\sqrt{6}\right)}{3\sqrt{-2x^4+7x^2-3}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+7*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(-2*x^4+7*x^2-3)^(1/2)*Ellipt
icF(1/3*x*3^(1/2),6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+7x^2-3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+7*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 7*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+7*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(7*x^2 - 2*x^4 - 3)^(1/2), x)
```

3.96 $\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

```
[Out] -1/6*(x^2*(9-3*3^(1/2)))^(1/2)/x/(9-3*3^(1/2))^(1/2)*EllipticF(1/3*(9-x^2*(
9-3*3^(1/2)))^(1/2),1/2*(2+2*3^(1/2))^(1/2))*3^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1109, 431}

$$-\frac{F\left(\text{ArcCos}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 6*x^2 - 2*x^4],x]
```

```
[Out] -(EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqrt[3])/2]/(Sqrt[2]*3^(1
/4)))
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6+2\sqrt{3}-4x^2}\,\sqrt{-6+2\sqrt{3}+4x^2}}\,dx$$
$$= -\frac{F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\,\sqrt[4]{3}}$$

Mathematica [**A**]
time = 10.04, size = 81, normalized size = 1.84

$$\frac{\sqrt{3-\sqrt{3}-2x^2}\,\sqrt{3+\left(-3+\sqrt{3}\right)x^2}\,F\left(\sin^{-1}\left(\sqrt{1+\frac{1}{\sqrt{3}}}\,x\right)|2-\sqrt{3}\right)}{\sqrt{6}\,\sqrt{-3+6x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 6*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[-3 + 6*x^2 - 2*x^4])
```

Maple [**A**]
time = 0.04, size = 82, normalized size = 1.86

method	result	size
default	$\frac{3\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{9-3\sqrt{3}}}{3},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{9-3\sqrt{3}}\,\sqrt{-2x^4+6x^2-3}}$	82
elliptic	$\frac{3\sqrt{1-\left(1-\frac{\sqrt{3}}{3}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{9-3\sqrt{3}}}{3},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{9-3\sqrt{3}}\,\sqrt{-2x^4+6x^2-3}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+6*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(9-3*3^(1/2))^(1/2)*(1-(1-1/3*3^(1/2))*x^2)^(1/2)*(1-(1+1/3*3^(1/2))*x^2)
^(1/2)/(-2*x^4+6*x^2-3)^(1/2)*EllipticF(1/3*x*(9-3*3^(1/2))^(1/2),1/2*6^(1/
2)+1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+6x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+6*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + 6*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+6*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x^2 - 2*x^4 - 3)^(1/2),x)
[Out] int(1/(6*x^2 - 2*x^4 - 3)^(1/2), x)
```

3.97 $\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx$

Optimal. Leaf size=14

$$-F\left(\cos^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|3\right)$$

```
[Out] -(x^2)^(1/2)/x*EllipticF(1/3*(-6*x^2+9)^(1/2),3^(1/2))
```

Rubi [A]

time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 431}

$$-F\left(\text{ArcCos}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|3\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 5*x^2 - 2*x^4],x]
```

```
[Out] -EllipticF[ArcCos[Sqrt[2/3]*x], 3]
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx &= \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{6-4x^2}\,\sqrt{-4+4x^2}}\,dx\\ &= -F\left(\cos^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|3\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.
time = 10.03, size = 53, normalized size = 3.79

$$\frac{\sqrt{3-2x^2}\,\sqrt{1-x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)|\frac{3}{2}\right)}{\sqrt{-6+10x^2-4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 5*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], 3/2])/Sqrt[-6
 + 10*x^2 - 4*x^4]
```

Maple [A]
time = 0.02, size = 50, normalized size = 3.57

method	result	size
default	$\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{6\sqrt{-2x^4+5x^2-3}}$	50
elliptic	$\frac{\sqrt{6}\,\sqrt{-6x^2+9}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)}{6\sqrt{-2x^4+5x^2-3}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+5*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(-2*x^4+5*x^2-3)^(1/2)*Elliptic
F(1/3*x*6^(1/2),1/2*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 + 5x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\sqrt{-2\,x^4 + 5\,x^2 - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 2*x^4 - 3)^(1/2), x)
```

3.98 $\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3+4x^2-2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+6*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-4*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-2*x^4+4*x^2-3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4+4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 4*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 + 4*x^2 - 2*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3+4x^2-2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{2x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{2+i\sqrt{2}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{2-i\sqrt{2}}}\,x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{-3+4x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-2/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[2]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[-3 + 4*x^2 - 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.99

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{6-3i\sqrt{2}}\sqrt{-2x^4+4x^2-3}}$	87
elliptic	$\frac{3\sqrt{1-\left(\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{6-3i\sqrt{2}}\sqrt{-2x^4+4x^2-3}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+4*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(6-3*I*2^(1/2))^(1/2)*(1-(2/3-1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3+1/3*I*2^(
1/2))*x^2)^(1/2)/(-2*x^4+4*x^2-3)^(1/2)*EllipticF(1/3*(6-3*I*2^(1/2))^(1/2)
*x,1/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 - 3), x)
```

Fricas [A]
time = 0.09, size = 46, normalized size = 0.52

$$\frac{1}{18}\sqrt{3}\left(\sqrt{-2}\sqrt{-3}-2\sqrt{-3}\right)\sqrt{\sqrt{-2}+2}\,\mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{-2}+2},-\frac{2}{3}\sqrt{-2}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(-2)*sqrt(-3) - 2*sqrt(-3))*sqrt(sqrt(-2) + 2)*ellipticF(
1/3*sqrt(3)*x*sqrt(sqrt(-2) + 2), -2/3*sqrt(-2) + 1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4+4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 4*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+4*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{-2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(4*x^2 - 2*x^4 - 3)^(1/2), x)
```

3.99 $\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3+3x^2-2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8+2*6^(1/
2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-3*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-2*x^4+3*x^2-3)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4+3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 3*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-3 + 3*x^2 - 2*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3+3x^2-2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{3-i\sqrt{15}}}\,x\right)\middle|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{-3+3x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 3*x^2 - 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(3 + I*Sqrt[1
5])]*EllipticF[I*ArcSinh[2*Sqrt[-(3 - I*Sqrt[15])^(-1)]*x], (3 - I*Sqrt[15]
)/(3 + I*Sqrt[15])]])/(Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[-3 + 3*x^2 - 2*x^4]
)
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.97

method	result	s
default	$\frac{6\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{18-6i\sqrt{15}}\sqrt{-2x^4+3x^2-3}}$	8
elliptic	$\frac{6\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{18-6i\sqrt{15}}\sqrt{-2x^4+3x^2-3}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+3*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(18-6*I*15^(1/2))^(1/2)*(1-(1/2-1/6*I*15^(1/2))*x^2)^(1/2)*(1-(1/2+1/6*I*
15^(1/2))*x^2)^(1/2)/(-2*x^4+3*x^2-3)^(1/2)*EllipticF(1/6*(18-6*I*15^(1/2))
^(1/2)*x,1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 - 3), x)
```

Fricas [A]
time = 0.07, size = 60, normalized size = 0.67

$$\frac{1}{72}\sqrt{6}\left(\sqrt{3}\sqrt{-3}\sqrt{-5}-3\sqrt{-3}\right)\sqrt{\sqrt{3}\sqrt{-5}+3}\,\text{ellipticF}\left(\frac{1}{6}\sqrt{6}\sqrt{\sqrt{3}\sqrt{-5}+3}\,x,-\frac{1}{4}\sqrt{3}\sqrt{-5}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="fricas")
[Out] 1/72*sqrt(6)*(sqrt(3)*sqrt(-3)*sqrt(-5) - 3*sqrt(-3))*sqrt(sqrt(3)*sqrt(-5)
 + 3)*ellipticF(1/6*sqrt(6)*sqrt(sqrt(3)*sqrt(-5) + 3)*x, -1/4*sqrt(3)*sqrt
(-5) - 1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+3*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + 3*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+3*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 2*x^4 - 3)^(1/2),x)
[Out] int(1/(3*x^2 - 2*x^4 - 3)^(1/2), x)
```

3.100 $\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3+2x^2-2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18+3*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-2*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-2*x^4+2*x^2-3)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-3 + 2*x^2 - 2*
x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3+2x^2-2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{1-i\sqrt{5}}}\,x\right)|\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-3+2x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-2/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[2]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[-3 + 2*x^2 - 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.97

method	result	size
default	$\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{3-3i\sqrt{5}}\sqrt{-2x^4+2x^2-3}}$	87
elliptic	$\frac{3\sqrt{1-\left(\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{3-3i\sqrt{5}}\sqrt{-2x^4+2x^2-3}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+2*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(3-3*I*5^(1/2))^(1/2)*(1-(1/3-1/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3+1/3*I*5^(
1/2))*x^2)^(1/2)/(-2*x^4+2*x^2-3)^(1/2)*EllipticF(1/3*(3-3*I*5^(1/2))^(1/2)
*x,1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 - 3), x)
```

Fricas [A]
time = 0.07, size = 41, normalized size = 0.46

$$\frac{1}{18}\sqrt{3}\sqrt{-3}\sqrt{\sqrt{-5}+1}\left(\sqrt{-5}-1\right)\text{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{-5}+1},-\frac{1}{3}\sqrt{-5}-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*sqrt(-3)*sqrt(sqrt(-5) + 1)*(sqrt(-5) - 1)*ellipticF(1/3*sqrt(
3)*x*sqrt(sqrt(-5) + 1), -1/3*sqrt(-5) - 2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+2x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 2*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+2*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(2*x^2 - 2*x^4 - 3)^(1/2), x)
```

3.101 $\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3+x^2-2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72+6*6^(
1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4-x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-2*x^4+x^2-3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-3 + x^2 - 2*x^4
])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3-x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3+x^2-2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 140, normalized size = 1.59

$$-\frac{i\sqrt{1-\frac{4x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{1+i\sqrt{23}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{1-i\sqrt{23}}}\,x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-3+x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + x^2 - 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(1 + I*Sqrt[2
3])]*EllipticF[I*ArcSinh[2*Sqrt[-(1 - I*Sqrt[23])^(-1)]*x], (1 - I*Sqrt[23]
)/(1 + I*Sqrt[23])])/(Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[-3 + x^2 - 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 85, normalized size = 0.97

method	result
default	$\frac{6\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{6-6i\sqrt{23}}\sqrt{-2x^4+x^2-3}}$
elliptic	$\frac{6\sqrt{1-\left(\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1-\left(\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{6-6i\sqrt{23}}\sqrt{-2x^4+x^2-3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(6-6*I*23^(1/2))^(1/2)*(1-(1/6-1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1/6+1/6*I*2
3^(1/2))*x^2)^(1/2)/(-2*x^4+x^2-3)^(1/2)*EllipticF(1/6*(6-6*I*23^(1/2))^(1/
2)*x,1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 - 3), x)
```

Fricas [A]
time = 0.08, size = 41, normalized size = 0.47

$$\frac{1}{72}\sqrt{6}\sqrt{-3}\sqrt{\sqrt{-23}+1}\left(\sqrt{-23}-1\right)\text{ellipticF}\left(\frac{1}{6}\sqrt{6}\,x\sqrt{\sqrt{-23}+1},-\frac{1}{12}\sqrt{-23}-\frac{11}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(6)*sqrt(-3)*sqrt(sqrt(-23) + 1)*(sqrt(-23) - 1)*ellipticF(1/6*sqr
t(6)*x*sqrt(sqrt(-23) + 1), -1/12*sqrt(-23) - 11/12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(x^2 - 2*x^4 - 3)^(1/2), x)
```

3.102 $\int \frac{1}{\sqrt{-3-2x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-3-2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/2*2^(1/2))*(
3+x^2*6^(1/2))*((2*x^4+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/(-2*x^4-3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {226}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-3 - 2*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-3-2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 47, normalized size = 0.65

$$-\frac{\sqrt[4]{-\frac{1}{6}}\sqrt{3+2x^4}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{2}{3}}\,x\right)\middle|-1\right)}{\sqrt{-3-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 2*x^4],x]
```

```
[Out] -(((-1/6)^(1/4)*Sqrt[3 + 2*x^4]*EllipticF[I*ArcSinh[(-2/3)^(1/4)*x], -1])/S
qrt[-3 - 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 66, normalized size = 0.92

method	result	size
meijerg	$-\frac{i\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-\frac{2x^4}{3}\right)}{3}$	19
default	$\frac{\sqrt{3}\,\sqrt{9+3i\sqrt{6}\,x^2}\,\sqrt{9-3i\sqrt{6}\,x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{-i\sqrt{6}}\,x}{3},i\right)}{9\sqrt{-i\sqrt{6}}\,\sqrt{-2x^4-3}}$	66
elliptic	$\frac{\sqrt{3}\,\sqrt{9+3i\sqrt{6}\,x^2}\,\sqrt{9-3i\sqrt{6}\,x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{-i\sqrt{6}}\,x}{3},i\right)}{9\sqrt{-i\sqrt{6}}\,\sqrt{-2x^4-3}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*3^(1/2)/(-I*6^(1/2))^(1/2)*(9+3*I*6^(1/2)*x^2)^(1/2)*(9-3*I*6^(1/2)*x^2
)^(1/2)/(-2*x^4-3)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*6^(1/2))^(1/2)*x,I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3), x)
```

Fricas [A]
time = 0.07, size = 34, normalized size = 0.47

$$\frac{1}{6}\sqrt{-2}\,\sqrt{-3}\,\sqrt{\sqrt{3}\,\sqrt{-2}}\,\mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\,\sqrt{\sqrt{3}\,\sqrt{-2}}\,x,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(-2)*sqrt(-3)*sqrt(sqrt(3)*sqrt(-2))*ellipticF(1/3*sqrt(3)*sqrt(sqr
t(3)*sqrt(-2))*x, -1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.34, size = 39, normalized size = 0.54

$$-\frac{\sqrt{3}\, ix\Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| {\frac{2x^{4}e^{i\pi}}{3}} \right)}{12\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-3)**(1/2),x)
```

```
[Out] -sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(I*pi)/3)
/(12*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3), x)
```

Mupad [B]
time = 4.29, size = 31, normalized size = 0.43

$$\frac{x\,\sqrt{6\,x^4+9}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{2\,x^4}{3}\right)}{3\,\sqrt{-2\,x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 2*x^4 - 3)^(1/2),x)
```

```
[Out] (x*(6*x^4 + 9)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(2*x^4)/3))/(3*(- 2*x^4 -
3)^(1/2))
```

3.103 $\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3-x^2-2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/12*(72-6*6^(
1/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-2*x^4-x^2-3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-3 - x^2 - 2*x^4
])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3-x^2-2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+i\sqrt{23}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\,x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{-3-x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - x^2 - 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(-1 + I*Sqrt
[23])]*EllipticF[I*ArcSinh[2*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*x], (-1 - I*Sqrt
[23])/(-1 + I*Sqrt[23])]])/(Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[-3 - x^2 - 2*
x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.97

method	result
default	$\frac{6\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{-6-6i\sqrt{23}}\sqrt{-2x^4-x^2-3}}$
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{6}-\frac{i\sqrt{23}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{6}+\frac{i\sqrt{23}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6-6i\sqrt{23}}\,x}{6},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{-6-6i\sqrt{23}}\sqrt{-2x^4-x^2-3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(-6-6*I*23^(1/2))^(1/2)*(1-(-1/6-1/6*I*23^(1/2))*x^2)^(1/2)*(1-(-1/6+1/6*
I*23^(1/2))*x^2)^(1/2)/(-2*x^4-x^2-3)^(1/2)*EllipticF(1/6*(-6-6*I*23^(1/2))
^(1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 - 3), x)
```

Fricas [A]
time = 0.07, size = 41, normalized size = 0.46

$$\frac{1}{72}\,\sqrt{6}\,\sqrt{-3}\,\left(\sqrt{-23}\,+1\right)\sqrt{\sqrt{-23}\,-1}\,\mathrm{ellipticF}\left(\frac{1}{6}\,\sqrt{6}\,x\sqrt{\sqrt{-23}\,-1}\,,\frac{1}{12}\,\sqrt{-23}\,-\frac{11}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(6)*sqrt(-3)*(sqrt(-23) + 1)*sqrt(sqrt(-23) - 1)*ellipticF(1/6*sqr
t(6)*x*sqrt(sqrt(-23) - 1), 1/12*sqrt(-23) - 11/12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{-2\,x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(- x^2 - 2*x^4 - 3)^(1/2), x)
```

3.104 $\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3-2x^2-2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-3*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+2*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-2*x^4-2*x^2-3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-2x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-3 - 2*x^2 - 2*
x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+2x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3-2x^2-2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{2x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-1-i\sqrt{5}}}x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{-3-2x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[2]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[-3 - 2*x^2 - 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.95

method	result
default	$\frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{-3-3i\sqrt{5}}\sqrt{-2x^4-2x^2-3}}$
elliptic	$\frac{3\sqrt{1-\left(-\frac{1}{3}-\frac{i\sqrt{5}}{3}\right)x^2}\sqrt{1-\left(-\frac{1}{3}+\frac{i\sqrt{5}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3-3i\sqrt{5}}\,x}{3},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{-3-3i\sqrt{5}}\sqrt{-2x^4-2x^2-3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-2*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(-3-3*I*5^(1/2))^(1/2)*(1-(-1/3-1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-1/3+1/3*I*
5^(1/2))*x^2)^(1/2)/(-2*x^4-2*x^2-3)^(1/2)*EllipticF(1/3*(-3-3*I*5^(1/2))^(
1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 - 3), x)
```

Fricas [A]
time = 0.07, size = 41, normalized size = 0.45

$$\frac{1}{18}\sqrt{3}\sqrt{-3}\left(\sqrt{-5}+1\right)\sqrt{\sqrt{-5}-1}\,\mathrm{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{-5}-1},\frac{1}{3}\sqrt{-5}-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*sqrt(-3)*(sqrt(-5) + 1)*sqrt(sqrt(-5) - 1)*ellipticF(1/3*sqrt(
3)*x*sqrt(sqrt(-5) - 1), 1/3*sqrt(-5) - 2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4-2x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 2*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-2*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{-2\,x^4-2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 2*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(- 2*x^2 - 2*x^4 - 3)^(1/2), x)
```

3.105 $\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3-3x^2-2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/4*(8-2*6^(1/
2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+3*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-2*x^4-3*x^2-3)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-3x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 3*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-3 - 3*x^2 - 2*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx = \frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+3x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3-3x^2-2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 142, normalized size = 1.54

$$-\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\,x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{-3-3x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 3*x^2 - 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (4*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(-3 + I*Sqrt
[15])]*EllipticF[I*ArcSinh[2*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*x], (-3 - I*Sqrt
[15])/(-3 + I*Sqrt[15])])/(Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[-3 - 3*x^2 -
2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.95

method	result
default	$\frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right.}{\sqrt{-18-6i\sqrt{15}}\sqrt{-2x^4-3x^2-3}}$
elliptic	$\frac{6\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{15}}{6}\right)x^2}\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{15}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-18-6i\sqrt{15}}\,x}{6},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right.}{\sqrt{-18-6i\sqrt{15}}\sqrt{-2x^4-3x^2-3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-3*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6/(-18-6*I*15^(1/2))^(1/2)*(1-(-1/2-1/6*I*15^(1/2))*x^2)^(1/2)*(1-(-1/2+1/6
*I*15^(1/2))*x^2)^(1/2)/(-2*x^4-3*x^2-3)^(1/2)*EllipticF(1/6*(-18-6*I*15^(1
/2))^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 - 3), x)
```

Fricas [A]
time = 0.09, size = 60, normalized size = 0.65

$$\frac{1}{72}\sqrt{6}\left(\sqrt{3}\sqrt{-3}\sqrt{-5}+3\sqrt{-3}\right)\sqrt{\sqrt{3}\sqrt{-5}-3}\,\text{ellipticF}\left(\frac{1}{6}\sqrt{6}\sqrt{\sqrt{3}\sqrt{-5}-3}\,x,\frac{1}{4}\sqrt{3}\sqrt{-5}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(6)*(sqrt(3)*sqrt(-3)*sqrt(-5) + 3*sqrt(-3))*sqrt(sqrt(3)*sqrt(-5)
 - 3)*ellipticF(1/6*sqrt(6)*sqrt(sqrt(3)*sqrt(-5) - 3)*x, 1/4*sqrt(3)*sqrt(
-5) - 1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4-3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 3*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-3*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{-2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 3*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(- 3*x^2 - 2*x^4 - 3)^(1/2), x)
```

3.106 $\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3-4x^2-2x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/3*2^(1/4)*3^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/3*2^(1/4
)*3^(3/4)*x))*EllipticF(sin(2*arctan(1/3*2^(1/4)*3^(3/4)*x)),1/6*(18-6*6^(1
/2))^(1/2))*(3+x^2*6^(1/2))*((2*x^4+4*x^2+3)/(3+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-2*x^4-4*x^2-3)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}\,x^2+3\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2x^4-4x^2-3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-3 - 4*x^2 - 2*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx=\frac{\left(3+\sqrt{6}\,x^2\right)\sqrt{\frac{3+4x^2+2x^4}{\left(3+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3-4x^2-2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-2-i\sqrt{2}}}\,x\right)|\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{-3-4x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[2]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[-3 - 4*x^2 - 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.97

method	result
default	$\frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{-6-3i\sqrt{2}}\sqrt{-2x^4-4x^2-3}}$
elliptic	$\frac{3\sqrt{1-\left(-\frac{2}{3}-\frac{i\sqrt{2}}{3}\right)x^2}\sqrt{1-\left(-\frac{2}{3}+\frac{i\sqrt{2}}{3}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-6-3i\sqrt{2}}\,x}{3},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{-6-3i\sqrt{2}}\sqrt{-2x^4-4x^2-3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-4*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/(-6-3*I*2^(1/2))^(1/2)*(1-(-2/3-1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-2/3+1/3*I*
2^(1/2))*x^2)^(1/2)/(-2*x^4-4*x^2-3)^(1/2)*EllipticF(1/3*(-6-3*I*2^(1/2))^(
1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 - 3), x)
```

Fricas [A]
time = 0.08, size = 46, normalized size = 0.51

$$\frac{1}{18}\sqrt{3}\left(\sqrt{-2}\sqrt{-3}+2\sqrt{-3}\right)\sqrt{\sqrt{-2}-2}\,\text{ellipticF}\left(\frac{1}{3}\sqrt{3}\,x\sqrt{\sqrt{-2}-2},\frac{2}{3}\sqrt{-2}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(-2)*sqrt(-3) + 2*sqrt(-3))*sqrt(sqrt(-2) - 2)*ellipticF(
1/3*sqrt(3)*x*sqrt(sqrt(-2) - 2), 2/3*sqrt(-2) + 1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4-4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 4*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-4*x^2-3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 4*x^2 - 2*x^4 - 3)^(1/2),x)
```

```
[Out] int(1/(- 4*x^2 - 2*x^4 - 3)^(1/2), x)
```

3.107 $\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{3+2x^2}\,F\left(\tan^{-1}(x)\middle|\frac{1}{3}\right)}{\sqrt{3}\,\sqrt{-1-x^2}\,\sqrt{\frac{3+2x^2}{1+x^2}}}$$

```
[Out] 1/3*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/3*3^(1/2))*
(2*x^2+3)^(1/2)*3^(1/2)/(-x^2-1)^(1/2)/((2*x^2+3)/(x^2+1))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 429}

$$\frac{\sqrt{2x^2+3}\,F\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{3}\right)}{\sqrt{3}\,\sqrt{-x^2-1}\,\sqrt{\frac{2x^2+3}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 - 5*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 + 2*x^2]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]*Sqrt[-1 - x^2]*Sqrt[(3
 + 2*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{-4-4x^2}\,\sqrt{6+4x^2}}\,dx$$
$$= \frac{\sqrt{3+2x^2}\,F\left(\tan^{-1}(x)\middle|\frac{1}{3}\right)}{\sqrt{3}\,\sqrt{-1-x^2}\,\sqrt{\frac{3+2x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.02, size = 63, normalized size = 1.19

$$-\frac{i\sqrt{1+x^2}\,\sqrt{3+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\middle|\frac{3}{2}\right)}{\sqrt{2}\,\sqrt{-3-5x^2-2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 - 5*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]*x], 3/2])
/(Sqrt[2]*Sqrt[-3 - 5*x^2 - 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 44, normalized size = 0.83

method	result	size
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+9}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)}{3\sqrt{-2x^4-5x^2-3}}$	44
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+9}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{3}\right)}{3\sqrt{-2x^4-5x^2-3}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4-5*x^2-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I*(x^2+1)^(1/2)*(6*x^2+9)^(1/2)/(-2*x^4-5*x^2-3)^(1/2)*EllipticF(I*x,1
/3*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 - 3), x)
```

Fricas [A]
time = 0.07, size = 22, normalized size = 0.42

$$\frac{1}{6}\sqrt{3}\sqrt{-2}\sqrt{-3}\,\text{ellipticF}\left(\frac{1}{3}\sqrt{3}\sqrt{-2}\,x, \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*sqrt(-2)*sqrt(-3)*ellipticF(1/3*sqrt(3)*sqrt(-2)*x, 3/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2x^4 - 5x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4-5*x**2-3)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 - 5*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4-5*x^2-3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-2\,x^4 - 5\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 5*x^2 - 2*x^4 - 3)^(1/2),x)
[Out] int(1/(- 5*x^2 - 2*x^4 - 3)^(1/2), x)
```

3.108 $\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx$

Optimal. Leaf size=42

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{3+\sqrt{3}}}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\,\sqrt[4]{3}}$$

```
[Out] -1/6*3^(3/4)*(x^2/(3^(1/2)+3))^(1/2)/x*(3^(1/2)+3)^(1/2)*EllipticF((1-3*x^2
/(3^(1/2)+3))^(1/2),1/2*(2+2*3^(1/2))^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 431}

$$-\frac{F\left(\text{ArcCos}\left(\sqrt{\frac{3}{3+\sqrt{3}}}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\,\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 6*x^2 - 3*x^4],x]
```

```
[Out] -(EllipticF[ArcCos[Sqrt[3/(3 + Sqrt[3])]*x], (1 + Sqrt[3])/2]/(Sqrt[2]*3^(1
/4)))
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{6+2\sqrt{3}-6x^2}\,\sqrt{-6+2\sqrt{3}+6x^2}}\,dx$$
$$= -\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{3+\sqrt{3}}}\,x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\,\sqrt[4]{3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 85 vs. 2(42) = 84.
time = 10.06, size = 85, normalized size = 2.02

$$\frac{\sqrt{3-\sqrt{3}-3x^2}\,\sqrt{2+\left(-3+\sqrt{3}\right)x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}\,x\right)|2-\sqrt{3}\right)}{\sqrt{6}\,\sqrt{-2+6x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 6*x^2 - 3*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sqrt[3])*x^2]*EllipticF[ArcSin[Sq
rt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[-2 + 6*x^2 - 3*x^4])
```

Maple [A]
time = 0.04, size = 82, normalized size = 1.95

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2}\,\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{6-2\sqrt{3}}\,\sqrt{-3x^4+6x^2-2}}$	82
elliptic	$\frac{2\sqrt{1-\left(-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2}\,\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{3}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{6-2\sqrt{3}}\,x}{2},\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{\sqrt{6-2\sqrt{3}}\,\sqrt{-3x^4+6x^2-2}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+6*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(6-2*3^(1/2))^(1/2)*(1-(-1/2*3^(1/2)+3/2)*x^2)^(1/2)*(1-(1/2*3^(1/2)+3/2)
*x^2)^(1/2)/(-3*x^4+6*x^2-2)^(1/2)*EllipticF(1/2*(6-2*3^(1/2))^(1/2)*x,1/2*
6^(1/2)+1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-3*x^4 + 6*x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+6x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+6*x**2-2)**(1/2),x)
[Out] Integral(1/sqrt(-3*x**4 + 6*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+6*x^2-2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 + 6*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-3\,x^4+6\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6*x^2 - 3*x^4 - 2)^(1/2),x)
[Out] int(1/(6*x^2 - 3*x^4 - 2)^(1/2), x)
```

3.109 $\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx$

Optimal. Leaf size=6

$$-F\left(\cos^{-1}(x)\middle|3\right)$$

```
[Out] -(x^2)^(1/2)/x*EllipticF((-x^2+1)^(1/2),3^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 431}

$$-F(\text{ArcCos}(x)|3)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 5*x^2 - 3*x^4],x]
```

```
[Out] -EllipticF[ArcCos[x], 3]
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{6-6x^2}\,\sqrt{-4+6x^2}}\,dx\\ &= -F\left(\cos^{-1}(x)\middle|3\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 53 vs. 2(6) = 12.
time = 10.02, size = 53, normalized size = 8.83

$$\frac{\sqrt{2-3x^2}\,\sqrt{1-x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|\frac{2}{3}\right)}{\sqrt{-6+15x^2-9x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 5*x^2 - 3*x^4],x]
```

```
[Out] (Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/Sqrt[-6
 + 15*x^2 - 9*x^4]
```

Maple [A]
time = 0.02, size = 42, normalized size = 7.00

method	result	size
default	$\frac{\sqrt{-x^{2}+1}\,\sqrt{-6x^{2}+4}\,\operatorname{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{-3x^{4}+5x^{2}-2}}$	42
elliptic	$\frac{\sqrt{-x^{2}+1}\,\sqrt{-6x^{2}+4}\,\operatorname{EllipticF}\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{-3x^{4}+5x^{2}-2}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+5*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(-3*x^4+5*x^2-2)^(1/2)*EllipticF(x,1/2*
6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^{4}+5x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 - 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 - 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.17

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 3*x^4 - 2)^(1/2), x)
```

3.110 $\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2+4x^2-3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+6*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-4*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-3*x^4+4*x^2-2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-2 + 4*x^2 - 3*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2+4x^2-3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{3x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{2+i\sqrt{2}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{2-i\sqrt{2}}}\,x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{-2+4x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(2 + I*Sqrt[2])]*E
llipticF[I*ArcSinh[Sqrt[-3/(2 - I*Sqrt[2])]*x], (2 - I*Sqrt[2])/(2 + I*Sqrt
[2])])/(Sqrt[3]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*Sqrt[-2 + 4*x^2 - 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.99

method	result	size
default	$\frac{2\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-\left(1+\frac{i\sqrt{2}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{4-2i\sqrt{2}}\sqrt{-3x^4+4x^2-2}}$	87
elliptic	$\frac{2\sqrt{1-\left(1-\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-\left(1+\frac{i\sqrt{2}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3+6i\sqrt{2}}}{3}\right)}{\sqrt{4-2i\sqrt{2}}\sqrt{-3x^4+4x^2-2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+4*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(4-2*I*2^(1/2))^(1/2)*(1-(1-1/2*I*2^(1/2))*x^2)^(1/2)*(1-(1+1/2*I*2^(1/2)
)*x^2)^(1/2)/(-3*x^4+4*x^2-2)^(1/2)*EllipticF(1/2*(4-2*I*2^(1/2))^(1/2)*x,1
/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 - 2), x)
```

Fricas [A]
time = 0.07, size = 38, normalized size = 0.43

$$-\frac{1}{6}\sqrt{2}\sqrt{\sqrt{-2}+2}\,(\sqrt{-2}+1)\mathrm{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{-2}+2}\,,-\frac{2}{3}\sqrt{-2}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*sqrt(sqrt(-2) + 2)*(sqrt(-2) + 1)*ellipticF(1/2*sqrt(2)*x*sqrt
(sqrt(-2) + 2), -2/3*sqrt(-2) + 1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-3x^4+4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 4*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+4*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{-3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(4*x^2 - 3*x^4 - 2)^(1/2), x)
```

3.111 $\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2+3x^2-3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8+2*6^(1/
2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-3*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-3*x^4+3*x^2-2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 3*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 + 3*x^2 - 3*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2+3x^2-3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{6x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{3+i\sqrt{15}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{3-i\sqrt{15}}}\,x\right)|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{-2+3x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 3*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(3 + I*Sqrt[15])]
*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[15])]*x], (3 - I*Sqrt[15])/(3 + I*
Sqrt[15])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[15])^(-1)]*Sqrt[-2 + 3*x^2 - 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{3-i\sqrt{15}}\sqrt{-3x^4+3x^2-2}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\sqrt{1-\left(\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1+i\sqrt{15}}}{2}\right)}{\sqrt{3-i\sqrt{15}}\sqrt{-3x^4+3x^2-2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+3*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(3-I*15^(1/2))^(1/2)*(1-(3/4-1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4+1/4*I*15^
(1/2))*x^2)^(1/2)/(-3*x^4+3*x^2-2)^(1/2)*EllipticF(1/2*(3-I*15^(1/2))^(1/2)
*x,1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 - 2), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.39

$$\frac{1}{24}\sqrt{-2}\sqrt{\sqrt{-15}+3}\left(\sqrt{-15}-3\right)\text{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{-15}+3},-\frac{1}{4}\sqrt{-15}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(-2)*sqrt(sqrt(-15) + 3)*(sqrt(-15) - 3)*ellipticF(1/2*x*sqrt(sqrt
(-15) + 3), -1/4*sqrt(-15) - 1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-3x^4+3x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 3*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+3*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{-3\,x^4+3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(3*x^2 - 3*x^4 - 2)^(1/2), x)
```

3.112 $\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2+2x^2-3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18+3*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-2*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-3*x^4+2*x^2-2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + 2*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-2 + 2*x^2 - 3*
x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2+2x^2-3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{3x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{1+i\sqrt{5}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{1-i\sqrt{5}}}\,x\right)|\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-2+2x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(1 + I*Sqrt[5])]*E
llipticF[I*ArcSinh[Sqrt[-3/(1 - I*Sqrt[5])]*x], (1 - I*Sqrt[5])/(1 + I*Sqrt
[5])])/(Sqrt[3]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*Sqrt[-2 + 2*x^2 - 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{5}}{2}+\frac{1}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{2-2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{2-2i\sqrt{5}}\,\sqrt{-3x^4+2x^2-2}}$	87
elliptic	$\frac{2\sqrt{1-\left(\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\,\sqrt{1-\left(\frac{i\sqrt{5}}{2}+\frac{1}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{2-2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6+3i\sqrt{5}}}{3}\right)}{\sqrt{2-2i\sqrt{5}}\,\sqrt{-3x^4+2x^2-2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+2*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(2-2*I*5^(1/2))^(1/2)*(1-(1/2-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2*I*5^(1/2)
+1/2)*x^2)^(1/2)/(-3*x^4+2*x^2-2)^(1/2)*EllipticF(1/2*(2-2*I*5^(1/2))^(1/2)
*x,1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 - 2), x)
```

Fricas [A]
time = 0.07, size = 41, normalized size = 0.46

$$\frac{1}{12}\sqrt{2}\sqrt{-2}\sqrt{\sqrt{-5}+1}\left(\sqrt{-5}-1\right)\text{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{-5}+1},-\frac{1}{3}\sqrt{-5}-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*sqrt(-2)*sqrt(sqrt(-5) + 1)*(sqrt(-5) - 1)*ellipticF(1/2*sqrt(
2)*x*sqrt(sqrt(-5) + 1), -1/3*sqrt(-5) - 2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+2x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 2*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+2*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(2*x^2 - 3*x^4 - 2)^(1/2), x)
```

3.113 $\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2+x^2-3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72+6*6^(
1/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4-x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-3*x^4+x^2-2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4+x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 + x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-2 + x^2 - 3*x^4
])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2-x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2+x^2-3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 142, normalized size = 1.61

$$-\frac{i\sqrt{1-\frac{6x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{1+i\sqrt{23}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{1-i\sqrt{23}}}\,x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-2+x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 + x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(1 + I*Sqrt[23])]
*EllipticF[I*ArcSinh[Sqrt[-6/(1 - I*Sqrt[23])]*x], (1 - I*Sqrt[23])/(1 + I*
Sqrt[23])])/(Sqrt[6]*Sqrt[-(1 - I*Sqrt[23])^(-1)]*Sqrt[-2 + x^2 - 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 85, normalized size = 0.97

method	result	s
default	$\frac{2\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{1-i\sqrt{23}}\,\sqrt{-3x^4+x^2-2}}$	8
elliptic	$\frac{2\sqrt{1-\left(\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\,\sqrt{1-\left(\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33+3i\sqrt{23}}}{6}\right)}{\sqrt{1-i\sqrt{23}}\,\sqrt{-3x^4+x^2-2}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(1-I*23^(1/2))^(1/2)*(1-(1/4-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4+1/4*I*23^
(1/2))*x^2)^(1/2)/(-3*x^4+x^2-2)^(1/2)*EllipticF(1/2*(1-I*23^(1/2))^(1/2)*x
,1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 - 2), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.40

$$\frac{1}{24}\sqrt{-2}\sqrt{\sqrt{-23}+1}\left(\sqrt{-23}-1\right)\text{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{-23}+1},-\frac{1}{12}\sqrt{-23}-\frac{11}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="fricas")
[Out] 1/24*sqrt(-2)*sqrt(sqrt(-23) + 1)*(sqrt(-23) - 1)*ellipticF(1/2*x*sqrt(sqrt
(-23) + 1), -1/12*sqrt(-23) - 11/12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+x**2-2)**(1/2),x)
[Out] Integral(1/sqrt(-3*x**4 + x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+x^2-2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 + x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 3*x^4 - 2)^(1/2),x)
[Out] int(1/(x^2 - 3*x^4 - 2)^(1/2), x)
```

3.114 $\int \frac{1}{\sqrt{-2-3x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-2-3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/2*2^(1/2))*(
2+x^2*6^(1/2))*((3*x^4+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4)/(-3*x^4-2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {226}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*ArcTan
[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-2 - 3*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\,\sqrt{-2-3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 47, normalized size = 0.65

$$-\frac{\sqrt[4]{-\frac{1}{6}}\sqrt{2+3x^4}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{3}{2}}\,x\right)\middle|-1\right)}{\sqrt{-2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 3*x^4],x]
```

```
[Out] -(((-1/6)^(1/4)*Sqrt[2 + 3*x^4]*EllipticF[I*ArcSinh[(-3/2)^(1/4)*x], -1])/S
qrt[-2 - 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 66, normalized size = 0.92

method	result	size
meijerg	$-\frac{i\sqrt{2}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-\frac{3x^4}{2}\right)}{2}$	19
default	$\frac{\sqrt{2}\,\sqrt{4+2i\sqrt{6}\,x^2}\,\sqrt{4-2i\sqrt{6}\,x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{-i\sqrt{6}}\,x}{2},i\right)}{4\sqrt{-i\sqrt{6}}\,\sqrt{-3x^4-2}}$	66
elliptic	$\frac{\sqrt{2}\,\sqrt{4+2i\sqrt{6}\,x^2}\,\sqrt{4-2i\sqrt{6}\,x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{-i\sqrt{6}}\,x}{2},i\right)}{4\sqrt{-i\sqrt{6}}\,\sqrt{-3x^4-2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)/(-I*6^(1/2))^(1/2)*(4+2*I*6^(1/2)*x^2)^(1/2)*(4-2*I*6^(1/2)*x^2
)^(1/2)/(-3*x^4-2)^(1/2)*EllipticF(1/2*2^(1/2)*(-I*6^(1/2))^(1/2)*x,I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2), x)
```

Fricas [A]
time = 0.08, size = 22, normalized size = 0.31

$$\frac{1}{12}\sqrt{2}\,\sqrt{-2}\,(-6)^{\frac{3}{4}}\,\mathrm{ellipticF}\left(\frac{1}{2}\sqrt{2}\,(-6)^{\frac{1}{4}}\,x,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*sqrt(-2)*(-6)^(3/4)*ellipticF(1/2*sqrt(2)*(-6)^(1/4)*x, -1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.33, size = 39, normalized size = 0.54

$$-\frac{\sqrt{2}\, ix\Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{3x^{4}e^{i\pi}}{2}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-2)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(I*pi)/2)
/(8*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2), x)
```

Mupad [B]
time = 4.18, size = 31, normalized size = 0.43

$$\frac{x\,\sqrt{6\,x^4+4}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{3\,x^4}{2}\right)}{2\,\sqrt{-3\,x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 3*x^4 - 2)^(1/2),x)
```

```
[Out] (x*(6*x^4 + 4)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(3*x^4)/2))/(2*(- 3*x^4 -
2)^(1/2))
```

3.115 $\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2-x^2-3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/12*(72-6*6^(
1/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-3*x^4-x^2-2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[2*
ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*Sqrt[-2 - x^2 - 3*x^4
])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx=\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)\middle|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2-x^2-3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{6x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{-1+i\sqrt{23}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-1-i\sqrt{23}}}\,x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{-2-x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(-1 + I*Sqrt[23]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-1 - I*Sqrt[23])]*x], (-1 - I*Sqrt[23])/(-1
 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(-1 - I*Sqrt[23])^(-1)]*Sqrt[-2 - x^2 - 3*x
^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.97

method	result
default	$\frac{2\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{-1-i\sqrt{23}}\sqrt{-3x^4-x^2-2}}$
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)x^2}\sqrt{1-\left(-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-1-i\sqrt{23}}\,x}{2},\frac{\sqrt{-33-3i\sqrt{23}}}{6}\right)}{\sqrt{-1-i\sqrt{23}}\sqrt{-3x^4-x^2-2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-1-I*23^(1/2))^(1/2)*(1-(-1/4-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-1/4+1/4*I*
23^(1/2))*x^2)^(1/2)/(-3*x^4-x^2-2)^(1/2)*EllipticF(1/2*(-1-I*23^(1/2))^(1/
2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 - 2), x)
```

Fricas [A]
time = 0.07, size = 35, normalized size = 0.39

$$\frac{1}{24}\sqrt{-2}\left(\sqrt{-23}+1\right)\sqrt{\sqrt{-23}-1}\,\text{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{-23}-1},\frac{1}{12}\sqrt{-23}-\frac{11}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(-2)*(sqrt(-23) + 1)*sqrt(sqrt(-23) - 1)*ellipticF(1/2*x*sqrt(sqrt
(-23) - 1), 1/12*sqrt(-23) - 11/12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(- x^2 - 3*x^4 - 2)^(1/2), x)
```

3.116 $\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2-2x^2-3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-3*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+2*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-3*x^4-2*x^2-2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-2x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 2*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*Sqrt[-2 - 2*x^2 - 3*
x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+2x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2-2x^2-3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{3x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{-1+i\sqrt{5}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-1-i\sqrt{5}}}\,x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{-2-2x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(-1 + I*Sqrt[5])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-1 - I*Sqrt[5])]*x], (-1 - I*Sqrt[5])/(-1 + I
*Sqrt[5])])/(Sqrt[3]*Sqrt[-(-1 - I*Sqrt[5])^(-1)]*Sqrt[-2 - 2*x^2 - 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.95

method	result
default	$\frac{2\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-2-2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{-2-2i\sqrt{5}}\sqrt{-3x^4-2x^2-2}}$
elliptic	$\frac{2\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{5}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{5}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-2-2i\sqrt{5}}\,x}{2},\frac{\sqrt{-6-3i\sqrt{5}}}{3}\right)}{\sqrt{-2-2i\sqrt{5}}\sqrt{-3x^4-2x^2-2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-2*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-2-2*I*5^(1/2))^(1/2)*(1-(-1/2-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(-1/2+1/2*I*
5^(1/2))*x^2)^(1/2)/(-3*x^4-2*x^2-2)^(1/2)*EllipticF(1/2*(-2-2*I*5^(1/2))^(
1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 - 2), x)
```

Fricas [A]
time = 0.07, size = 41, normalized size = 0.45

$$\frac{1}{12}\sqrt{2}\sqrt{-2}\left(\sqrt{-5}+1\right)\sqrt{\sqrt{-5}-1}\,\mathrm{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{-5}-1},\frac{1}{3}\sqrt{-5}-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*sqrt(-2)*(sqrt(-5) + 1)*sqrt(sqrt(-5) - 1)*ellipticF(1/2*sqrt(
2)*x*sqrt(sqrt(-5) - 1), 1/3*sqrt(-5) - 2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4-2x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 2*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-2*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4-2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 2*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(- 2*x^2 - 3*x^4 - 2)^(1/2), x)
```

3.117 $\int \frac{1}{\sqrt{-2-3x^2-3x^4}} \, dx$

Optimal. Leaf size=92

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2-3x^2-3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/4*(8-2*6^(1/
2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+3*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/4
)/(-3*x^4-3*x^2-2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 3*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*Sqrt[-2 - 3*x^2 - 3*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}} \, dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+3x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\,\sqrt{-2-3x^2-3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.11, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{6x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{-3+i\sqrt{15}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-3-i\sqrt{15}}}\,x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{-2-3x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 3*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(-3 + I*Sqrt[15]
)]*EllipticF[I*ArcSinh[Sqrt[-6/(-3 - I*Sqrt[15])]*x], (-3 - I*Sqrt[15])/(-3
 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(-3 - I*Sqrt[15])^(-1)]*Sqrt[-2 - 3*x^2 - 3
*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.95

method	result
default	$\frac{2\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{-3-i\sqrt{15}}\sqrt{-3x^4-3x^2-2}}$
elliptic	$\frac{2\sqrt{1-\left(-\frac{3}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\sqrt{1-\left(-\frac{3}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-3-i\sqrt{15}}\,x}{2},\frac{\sqrt{-1-i\sqrt{15}}}{2}\right)}{\sqrt{-3-i\sqrt{15}}\sqrt{-3x^4-3x^2-2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-3*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-3-I*15^(1/2))^(1/2)*(1-(-3/4-1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-3/4+1/4*I*
15^(1/2))*x^2)^(1/2)/(-3*x^4-3*x^2-2)^(1/2)*EllipticF(1/2*(-3-I*15^(1/2))^(
1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="maxima")
```

[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 - 2), x)

Fricas [A]
time = 0.07, size = 35, normalized size = 0.38

$$\frac{1}{24}\sqrt{-2}\left(\sqrt{-15}+3\right)\sqrt{\sqrt{-15}-3}\,\text{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{-15}-3}\,,\frac{1}{4}\,\sqrt{-15}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="fricas")

[Out] 1/24*sqrt(-2)*(sqrt(-15) + 3)*sqrt(sqrt(-15) - 3)*ellipticF(1/2*x*sqrt(sqrt(-15) - 3), 1/4*sqrt(-15) - 1/4)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-3x^4-3x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x**4-3*x**2-2)**(1/2),x)

[Out] Integral(1/sqrt(-3*x**4 - 3*x**2 - 2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^4-3*x^2-2)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 - 2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{-3\,x^4-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(- 3*x^2 - 3*x^4 - 2)^(1/2),x)

[Out] int(1/(- 3*x^2 - 3*x^4 - 2)^(1/2), x)

3.118 $\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2-4x^2-3x^4}}$$

```
[Out] 1/12*(cos(2*arctan(1/2*3^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*3^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*3^(1/4)*2^(3/4)*x)),1/6*(18-6*6^(1
/2))^(1/2))*(2+x^2*6^(1/2))*((3*x^4+4*x^2+2)/(2+x^2*6^(1/2))^2)^(1/2)*6^(3/
4)/(-3*x^4-4*x^2-2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{6}\,x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-3x^4-4x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 4*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*EllipticF[
2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*Sqrt[-2 - 4*x^2 - 3*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx = \frac{\left(2+\sqrt{6}\,x^2\right)\sqrt{\frac{2+4x^2+3x^4}{\left(2+\sqrt{6}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}\,x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\,\sqrt{-2-4x^2-3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 144, normalized size = 1.60

$$-\frac{i\sqrt{1-\frac{3x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{-2+i\sqrt{2}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-2-i\sqrt{2}}}\,x\right)|\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{-2-4x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(-2 + I*Sqrt[2])]
*EllipticF[I*ArcSinh[Sqrt[-3/(-2 - I*Sqrt[2])]*x], (-2 - I*Sqrt[2])/(-2 + I
*Sqrt[2])])/(Sqrt[3]*Sqrt[-(-2 - I*Sqrt[2])^(-1)]*Sqrt[-2 - 4*x^2 - 3*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 87, normalized size = 0.97

method	result	s
default	$\frac{2\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{-4-2i\sqrt{2}}\sqrt{-3x^4-4x^2-2}}$	8
elliptic	$\frac{2\sqrt{1-\left(-1-\frac{i\sqrt{2}}{2}\right)x^2}\sqrt{1-\left(-1+\frac{i\sqrt{2}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{\sqrt{-4-2i\sqrt{2}}\,x}{2},\frac{\sqrt{3-6i\sqrt{2}}}{3}\right)}{\sqrt{-4-2i\sqrt{2}}\sqrt{-3x^4-4x^2-2}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-4*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-4-2*I*2^(1/2))^(1/2)*(1-(-1-1/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1+1/2*I*2^(1
/2))*x^2)^(1/2)/(-3*x^4-4*x^2-2)^(1/2)*EllipticF(1/2*(-4-2*I*2^(1/2))^(1/2)
*x,1/3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 - 2), x)
```

Fricas [A]
time = 0.07, size = 38, normalized size = 0.42

$$\frac{1}{6}\sqrt{2}\left(\sqrt{-2}-1\right)\sqrt{\sqrt{-2}-2}\,\text{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{-2}-2},\frac{2}{3}\sqrt{-2}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*(sqrt(-2) - 1)*sqrt(sqrt(-2) - 2)*ellipticF(1/2*sqrt(2)*x*sqrt(
sqrt(-2) - 2), 2/3*sqrt(-2) + 1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4-4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 4*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-4*x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 4*x^2 - 3*x^4 - 2)^(1/2),x)
```

```
[Out] int(1/(- 4*x^2 - 3*x^4 - 2)^(1/2), x)
```

3.119 $\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{-2-3x^2}\,F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{1+x^2}\,\sqrt{\frac{2+3x^2}{1+x^2}}}$$

```
[Out] -1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*(-3*x^2-2)^
(1/2)*2^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 429}

$$-\frac{\sqrt{-3x^2-2}\,F\left(\text{ArcTan}(x)\,\middle|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 5*x^2 - 3*x^4],x]
```

```
[Out] -((Sqrt[-2 - 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt
[(2 + 3*x^2)/(1 + x^2)]))
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx = \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{-4-6x^2}\sqrt{6+6x^2}}\,dx$$
$$= -\frac{\sqrt{-2-3x^2}\,F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 63, normalized size = 1.21

$$-\frac{i\sqrt{1+x^2}\sqrt{2+3x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|\frac{2}{3}\right)}{\sqrt{3}\sqrt{-2-5x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])
/(Sqrt[3]*Sqrt[-2 - 5*x^2 - 3*x^4])
```

Maple [A]
time = 0.03, size = 50, normalized size = 0.96

method	result	size
default	$-\frac{i\sqrt{6}\sqrt{6x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-5x^2-2}}$	50
elliptic	$-\frac{i\sqrt{6}\sqrt{6x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-5x^2-2}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4-5*x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-5*x^2-2)^(1/2)*Ellipti
cF(1/2*I*x*6^(1/2),1/3*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 - 2), x)
```

Fricas [A]
time = 0.08, size = 10, normalized size = 0.19

$$\frac{1}{2}i\sqrt{-2}\,\mathrm{ellipticF}\left(i\,x,\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="fricas")
[Out] 1/2*I*sqrt(-2)*ellipticF(I*x, 3/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-3x^4-5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4-5*x**2-2)**(1/2),x)
[Out] Integral(1/sqrt(-3*x**4 - 5*x**2 - 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4-5*x^2-2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 5*x^2 - 3*x^4 - 2)^(1/2),x)
[Out] int(1/(- 5*x^2 - 3*x^4 - 2)^(1/2), x)
```

3.120 $\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(2+\sqrt{10}\,x^2\right)\sqrt{\frac{2+5x^2+5x^4}{\left(2+\sqrt{10}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{5}{2}}\,x\right)\,|\,\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\,\sqrt{2+5x^2+5x^4}}$$

```
[Out] 1/20*(cos(2*arctan(1/2*5^(1/4)*2^(3/4)*x))^2)^(1/2)/cos(2*arctan(1/2*5^(1/4
)*2^(3/4)*x))*EllipticF(sin(2*arctan(1/2*5^(1/4)*2^(3/4)*x)),1/4*(8-2*10^(1
/2))^(1/2))*(2+x^2*10^(1/2))*((5*x^4+5*x^2+2)/(2+x^2*10^(1/2))^2)^(1/2)*10^
(3/4)/(5*x^4+5*x^2+2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{10}\,x^2+2\right)\sqrt{\frac{5x^4+5x^2+2}{\left(\sqrt{10}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{\frac{5}{2}}\,x\right)\,|\,\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\,\sqrt{5x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 5*x^4],x]
```

```
[Out] ((2 + Sqrt[10]*x^2)*Sqrt[(2 + 5*x^2 + 5*x^4)/(2 + Sqrt[10]*x^2)^2]*Elliptic
F[2*ArcTan[(5/2)^(1/4)*x], (4 - Sqrt[10])/8])/(2*10^(1/4)*Sqrt[2 + 5*x^2 +
5*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx=\frac{\left(2+\sqrt{10}\,x^2\right)\sqrt{\frac{2+5x^2+5x^4}{\left(2+\sqrt{10}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{\frac{5}{2}}\,x\right)\,|\,\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\,\sqrt{2+5x^2+5x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{10x^2}{-5-i\sqrt{15}}}\sqrt{1-\frac{10x^2}{-5+i\sqrt{15}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{10}{-5-i\sqrt{15}}}x\right)|\frac{-5-i\sqrt{15}}{-5+i\sqrt{15}}\right)}{\sqrt{10}\sqrt{-\frac{1}{-5-i\sqrt{15}}}\sqrt{2+5x^2+5x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 5*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (10*x^2)/(-5 - I*Sqrt[15])]*Sqrt[1 - (10*x^2)/(-5 + I*Sqrt[1
5])]*EllipticF[I*ArcSinh[Sqrt[-10/(-5 - I*Sqrt[15])]*x], (-5 - I*Sqrt[15])/
(-5 + I*Sqrt[15])])/(Sqrt[10]*Sqrt[-(-5 - I*Sqrt[15])^(-1)]*Sqrt[2 + 5*x^2
+ 5*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 87, normalized size = 0.95

method	result
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{15}}}{2},\frac{\sqrt{1+i\sqrt{15}}}{2}\right)}{\sqrt{-5+i\sqrt{15}}\sqrt{5x^4+5x^2+2}}$
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{15}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{15}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{15}}}{2},\frac{\sqrt{1+i\sqrt{15}}}{2}\right)}{\sqrt{-5+i\sqrt{15}}\sqrt{5x^4+5x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-5+I*15^(1/2))^(1/2)*(1-(-5/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*I*
15^(1/2))*x^2)^(1/2)/(5*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+I*15^(1/2))^
(1/2),1/2*(1+I*15^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.09, size = 35, normalized size = 0.38

$$-\frac{1}{40}\sqrt{2}\left(\sqrt{-15}+5\right)\sqrt{\sqrt{-15}-5}\,\mathrm{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{-15}-5},\frac{1}{4}\sqrt{-15}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/40*sqrt(2)*(sqrt(-15) + 5)*sqrt(sqrt(-15) - 5)*ellipticF(1/2*x*sqrt(sqrt
(-15) - 5), 1/4*sqrt(-15) + 1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{5x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**4+5*x**2+2)**(1/2),x)
[Out] Integral(1/sqrt(5*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{5\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 5*x^4 + 2)^(1/2),x)
[Out] int(1/(5*x^2 + 5*x^4 + 2)^(1/2), x)
```

3.121 $\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{2+5x^2+4x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right)|\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{2+5x^2+4x^4}}$$

```
[Out] 1/4*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(s
in(2*arctan(2^(1/4)*x)),1/4*(8-5*2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((4*x^4+5*
x^2+2)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(4*x^4+5*x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{4x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 4*x^4],x]
```

```
[Out] ((1 + Sqrt[2]*x^2)*Sqrt[(2 + 5*x^2 + 4*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[
2*ArcTan[2^(1/4)*x], (8 - 5*Sqrt[2])/16])/(2*2^(3/4)*Sqrt[2 + 5*x^2 + 4*x^4
])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx=\frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{2+5x^2+4x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right)|\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{2+5x^2+4x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 147, normalized size = 1.63

$$-\frac{i\sqrt{1-\frac{8x^2}{-5-i\sqrt{7}}}\sqrt{1-\frac{8x^2}{-5+i\sqrt{7}}}\,F\left(i\sinh^{-1}\left(2\sqrt{-\frac{2}{-5-i\sqrt{7}}}\,x\right)|\frac{-5-i\sqrt{7}}{-5+i\sqrt{7}}\right)}{2\sqrt{2}\sqrt{-\frac{1}{-5-i\sqrt{7}}}\sqrt{2+5x^2+4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 4*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 - (8*x^2)/(-5 - I*Sqrt[7])]*Sqrt[1 - (8*x^2)/(-5 + I*Sqrt[
7])]*EllipticF[I*ArcSinh[2*Sqrt[-2/(-5 - I*Sqrt[7])]*x], (-5 - I*Sqrt[7])/(
-5 + I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-5 - I*Sqrt[7])^(-1)]*Sqrt[2 + 5*x^2 + 4*
x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 87, normalized size = 0.97

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{7}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{7}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{7}}}{2},\frac{\sqrt{9+5i\sqrt{7}}}{4}\right)}{\sqrt{-5+i\sqrt{7}}\sqrt{4x^4+5x^2+2}}$	87
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{i\sqrt{7}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{i\sqrt{7}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+i\sqrt{7}}}{2},\frac{\sqrt{9+5i\sqrt{7}}}{4}\right)}{\sqrt{-5+i\sqrt{7}}\sqrt{4x^4+5x^2+2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-5+I*7^(1/2))^(1/2)*(1-(-5/4+1/4*I*7^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*I*7^
(1/2))*x^2)^(1/2)/(4*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+I*7^(1/2))^(1/2
),1/4*(9+5*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 35, normalized size = 0.39

$$-\frac{1}{32}\,\sqrt{2}\,\left(\sqrt{-7}+5\right)\sqrt{\sqrt{-7}-5}\,\mathrm{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{-7}-5}\,,\,\frac{5}{16}\,\sqrt{-7}+\frac{9}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*sqrt(2)*(sqrt(-7) + 5)*sqrt(sqrt(-7) - 5)*ellipticF(1/2*x*sqrt(sqrt(-
7) - 5), 5/16*sqrt(-7) + 9/16)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{4x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 4*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 4*x^4 + 2)^(1/2), x)
```

3.122 $\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{2+5x^2+3x^4}}$$

```
[Out] 1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2)
)*((3*x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1114}

$$\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,\middle|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]
*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rule 1114

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx=\frac{(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{2+5x^2+3x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.02, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{1+x^2}\,\sqrt{2+3x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\middle|\frac{2}{3}\right)}{\sqrt{6+15x^2+9x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])
/Sqrt[6 + 15*x^2 + 9*x^4]
```

Maple [A]
time = 0.01, size = 44, normalized size = 0.85

method	result	size
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$	44
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/
2*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 10, normalized size = 0.19

$$-\frac{1}{2}i\sqrt{2}\,\mathrm{ellipticF}\left(i\,x,\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(2)*ellipticF(I*x, 3/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

3.123 $\int \frac{1}{\sqrt{2+5x^2+2x^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{\frac{2+x^2}{1+2x^2}}\,(1+2x^2)\,F\left(\tan^{-1}\left(\sqrt{2}\,x\right)|\frac{3}{4}\right)}{2\sqrt{2+5x^2+2x^4}}$$

```
[Out] 1/2*(2*x^2+1)^(3/2)*(1/(2*x^2+1))^(1/2)*EllipticF(x*2^(1/2)/(2*x^2+1)^(1/2)
,1/2*3^(1/2))*((x^2+2)/(2*x^2+1))^(1/2)/(2*x^4+5*x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1113}

$$\frac{\sqrt{\frac{x^2+2}{2x^2+1}}\,(2x^2+1)\,F\left(\text{ArcTan}\left(\sqrt{2}\,x\right)|\frac{3}{4}\right)}{2\sqrt{2x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(2 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]*x], 3/4])
/(2*Sqrt[2 + 5*x^2 + 2*x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+2x^4}}\,dx = \frac{\sqrt{\frac{2+x^2}{1+2x^2}}\,(1+2x^2)\,F\left(\tan^{-1}\left(\sqrt{2}\,x\right)|\frac{3}{4}\right)}{2\sqrt{2+5x^2+2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.02, size = 58, normalized size = 1.00

$$-\frac{i\sqrt{2+x^2}\,\sqrt{1+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{2}\,x\right)|\frac{1}{4}\right)}{2\sqrt{2+5x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[2 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], 1/4
])/Sqrt[2 + 5*x^2 + 2*x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 48, normalized size = 0.83

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{2x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},2\right)}{2\sqrt{2x^4+5x^2+2}}$	48
elliptic	$-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{2x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},2\right)}{2\sqrt{2x^4+5x^2+2}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+5*x^2+2)^(1/2)*Ellipt
icF(1/2*I*2^(1/2)*x,2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.09, size = 10, normalized size = 0.17

$$-i\,\mathrm{ellipticF}\left(\frac{1}{2}i\,\sqrt{2}\,x,4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -I*ellipticF(1/2*I*sqrt(2)*x, 4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2x^4 + 5x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 5*x**2 + 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2\,x^4 + 5\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + 2*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 + 2*x^4 + 2)^(1/2), x)
```

3.124 $\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{\frac{4+\left(5-\sqrt{17}\right)x^2}{4+\left(5+\sqrt{17}\right)x^2}}\left(4+\left(5+\sqrt{17}\right)x^2\right)F\left(\tan^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{17}}\,x\right)|\frac{1}{4}\left(-17+5\sqrt{17}\right)\right)}{2\sqrt{5+\sqrt{17}}\sqrt{2+5x^2+x^4}}$$

```
[Out] 1/2*(1/(4+x^2*(5+17^(1/2))))^(1/2)*(4+x^2*(5+17^(1/2)))^(3/2)*EllipticF(x*(
5+17^(1/2))^(1/2)/(4+x^2*(5+17^(1/2)))^(1/2),1/2*(-17+5*17^(1/2))^(1/2))*((
4+x^2*(5-17^(1/2)))/(4+x^2*(5+17^(1/2))))^(1/2)/(x^4+5*x^2+2)^(1/2)/(5+17^(
1/2))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1113}

$$\frac{\sqrt{\frac{\left(5-\sqrt{17}\right)x^2+4}{\left(5+\sqrt{17}\right)x^2+4}}\left(\left(5+\sqrt{17}\right)x^2+4\right)F\left(\text{ArcTan}\left(\frac{1}{2}\sqrt{5+\sqrt{17}}\,x\right)|\frac{1}{4}\left(-17+5\sqrt{17}\right)\right)}{2\sqrt{5+\sqrt{17}}\sqrt{x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[(4 + (5 - Sqrt[17])*x^2)/(4 + (5 + Sqrt[17])*x^2)]*(4 + (5 + Sqrt[17]
)*x^2)*EllipticF[ArcTan[(Sqrt[5 + Sqrt[17]]*x)/2], (-17 + 5*Sqrt[17])/4])/(
2*Sqrt[5 + Sqrt[17]]*Sqrt[2 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx = \frac{\sqrt{\frac{4+\left(5-\sqrt{17}\right)x^2}{4+\left(5+\sqrt{17}\right)x^2}}\left(4+\left(5+\sqrt{17}\right)x^2\right)F\left(\tan^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{17}}\,x\right)|\frac{1}{4}\left(-17\right.\right.}{2\sqrt{5+\sqrt{17}}\sqrt{2+5x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 103, normalized size = 0.95

$$-\frac{i\sqrt{5-\sqrt{17}+2x^2}\sqrt{5+\sqrt{17}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{17}}}\,x\right)|\frac{21}{4}+\frac{5\sqrt{17}}{4}\right)}{\sqrt{2\left(5-\sqrt{17}\right)}\sqrt{2+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[5 - Sqrt[17] + 2*x^2]*Sqrt[5 + Sqrt[17] + 2*x^2]*EllipticF[I*Arc
Sinh[Sqrt[2/(5 + Sqrt[17])]*x], 21/4 + (5*Sqrt[17])/4])/(Sqrt[2*(5 - Sqrt[1
7])]*Sqrt[2 + 5*x^2 + x^4])
```

Maple [A]
time = 0.06, size = 76, normalized size = 0.70

method	result	si
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{17}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{17}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{17}}}{2},\frac{5\sqrt{2}}{4}+\frac{\sqrt{34}}{4}\right)}{\sqrt{-5+\sqrt{17}}\sqrt{x^4+5x^2+2}}$	76
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{17}}{4}\right)x^2}\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{17}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{17}}}{2},\frac{5\sqrt{2}}{4}+\frac{\sqrt{34}}{4}\right)}{\sqrt{-5+\sqrt{17}}\sqrt{x^4+5x^2+2}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-5+17^(1/2))^(1/2)*(1-(-5/4+1/4*17^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*17^(1/
2))*x^2)^(1/2)/(x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+17^(1/2))^(1/2),5/4*
2^(1/2)+1/4*34^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.10, size = 40, normalized size = 0.37

$$-\frac{1}{8}\left(\sqrt{17}\sqrt{2}+5\sqrt{2}\right)\sqrt{\sqrt{17}-5}\,\text{ellipticF}\left(\frac{1}{2}x\sqrt{\sqrt{17}-5},\frac{5}{4}\sqrt{17}+\frac{21}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/8*(sqrt(17)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(17) - 5)*ellipticF(1/2*x*sqrt
(sqrt(17) - 5), 5/4*sqrt(17) + 21/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+5*x**2+2)**(1/2),x)
[Out] Integral(1/sqrt(x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 + x^4 + 2)^(1/2),x)
[Out] int(1/(5*x^2 + x^4 + 2)^(1/2), x)
```

3.125 $\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{-5+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{5+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

```
[Out] EllipticF(x*2^(1/2)/(5+33^(1/2))^(1/2),5/4*I*2^(1/2)+1/4*I*66^(1/2))*2^(1/2
)/(-5+33^(1/2))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{\sqrt{33}-5}}\,F\left(\text{ArcSin}\left(\sqrt{\frac{2}{5+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[2/(5 + Sqrt[33])]*x], (-29 -
5*Sqrt[33])/4]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx &= 2\int \frac{1}{\sqrt{5+\sqrt{33}-2x^2}\,\sqrt{-5+\sqrt{33}+2x^2}}\,dx\\ &= \sqrt{\frac{2}{-5+\sqrt{33}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{5+\sqrt{33}}}\,x\right)|\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 55, normalized size = 1.15

$$-i\sqrt{\frac{2}{5+\sqrt{33}}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-5+\sqrt{33}}}\,x\right)|-\frac{29}{4}+\frac{5\sqrt{33}}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - x^4],x]
[Out] (-I)*Sqrt[2/(5 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[2/(-5 + Sqrt[33])]*x],
 -29/4 + (5*Sqrt[33])/4]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(37) = 74.
time = 0.06, size = 80, normalized size = 1.67

method	result	siz
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{33}}}{2},\frac{5i\sqrt{2}}{4}+\frac{i\sqrt{66}}{4}\right)}{\sqrt{-5+\sqrt{33}}\,\sqrt{-x^4+5x^2+2}}$	80
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{33}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{33}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{33}}}{2},\frac{5i\sqrt{2}}{4}+\frac{i\sqrt{66}}{4}\right)}{\sqrt{-5+\sqrt{33}}\,\sqrt{-x^4+5x^2+2}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(-5+33^(1/2))^(1/2)*(1-(-5/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*33^(1/
2))*x^2)^(1/2)/(-x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+33^(1/2))^(1/2),5/4
*I*2^(1/2)+1/4*I*66^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 40, normalized size = 0.83

$$\frac{1}{8}\left(\sqrt{33}\,\sqrt{2}+5\,\sqrt{2}\right)\sqrt{\sqrt{33}-5}\;\mathrm{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{33}-5},-\frac{5}{4}\,\sqrt{33}-\frac{29}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(33)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(33) - 5)*ellipticF(1/2*x*sqrt(
sqrt(33) - 5), -5/4*sqrt(33) - 29/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - x^4 + 2)^(1/2), x)
```

3.126 $\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{-5+\sqrt{41}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)|\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

```
[Out] EllipticF(2*x/(5+41^(1/2))^(1/2),5/4*I+1/4*I*41^(1/2))*2^(1/2)/(-5+41^(1/2)
)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{\sqrt{41}-5}}\,F\left(\text{ArcSin}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)|\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[41])]*EllipticF[ArcSin[(2*x)/Sqrt[5 + Sqrt[41]]], (-33 -
5*Sqrt[41])/8]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx = \left(2\sqrt{2}\right)\int \frac{1}{\sqrt{5+\sqrt{41}-4x^2}\,\sqrt{-5+\sqrt{41}+4x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{41}}}\,F\left(\sin^{-1}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)|\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{5+\sqrt{41}}}\,F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-5+\sqrt{41}}}\right)|-\frac{33}{8}+\frac{5\sqrt{41}}{8}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[41])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-5 + Sqrt[41]]],
 -33/8 + (5*Sqrt[41])/8]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(31) = 62.
time = 0.06, size = 76, normalized size = 1.69

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{41}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{41}}}{2},\frac{5i}{4}+\frac{i\sqrt{41}}{4}\right)}{\sqrt{-5+\sqrt{41}}\,\sqrt{-2x^4+5x^2+2}}$	76
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{41}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{41}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{41}}}{2},\frac{5i}{4}+\frac{i\sqrt{41}}{4}\right)}{\sqrt{-5+\sqrt{41}}\,\sqrt{-2x^4+5x^2+2}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-5+41^(1/2))^(1/2)*(1-(-5/4+1/4*41^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*41^(1/
2))*x^2)^(1/2)/(-2*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+41^(1/2))^(1/2),5
/4*I+1/4*I*41^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.10, size = 40, normalized size = 0.89

$$\frac{1}{16}\left(\sqrt{41}\,\sqrt{2}+5\,\sqrt{2}\right)\sqrt{\sqrt{41}-5}\,\mathrm{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{41}-5}\,,-\frac{5}{8}\,\sqrt{41}-\frac{33}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/16*(sqrt(41)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(41) - 5)*ellipticF(1/2*x*sqrt
(sqrt(41) - 5), -5/8*sqrt(41) - 33/8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-2x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+5*x**2+2)**(1/2),x)
[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{-2\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 2*x^4 + 2)^(1/2),x)
[Out] int(1/(5*x^2 - 2*x^4 + 2)^(1/2), x)
```

3.127 $\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

[Out] EllipticF(1/2*x*2^(1/2),I*6^(1/2))

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + 5*x^2 - 3*x^4],x]

[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx &= \left(2\sqrt{3}\right)\int \frac{1}{\sqrt{12-6x^2}\sqrt{2+6x^2}}\,dx\\ &= F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.02, size = 65, normalized size = 6.50

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{1+3x^2}\,F\left(i\sinh^{-1}\left(\sqrt{3}\,x\right)|-\frac{1}{6}\right)}{\sqrt{3}\sqrt{2+5x^2-3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]*x], -1/6]
)/(Sqrt[3]*Sqrt[2 + 5*x^2 - 3*x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(13) = 26.
time = 0.02, size = 51, normalized size = 5.10

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{6}\right)}{2\sqrt{-3x^4+5x^2+2}}$	51
elliptic	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{3x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{6}\right)}{2\sqrt{-3x^4+5x^2+2}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/2)*Ellipti
cF(1/2*2^(1/2)*x,I*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.07, size = 8, normalized size = 0.80

$$\mathrm{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x,-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] ellipticF(1/2*sqrt(2)*x, -6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 3*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 3*x^4 + 2)^(1/2), x)
```

3.128 $\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{-5+\sqrt{57}}}\,F\left(\sin^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{57}}}\,x\right)|\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

```
[Out] EllipticF(2*x*2^(1/2)/(5+57^(1/2))^(1/2),5/8*I*2^(1/2)+1/8*I*114^(1/2))*2^(
1/2)/(-5+57^(1/2))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{\sqrt{57}-5}}\,F\left(\text{ArcSin}\left(2\sqrt{\frac{2}{5+\sqrt{57}}}\,x\right)|\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 4*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[57])]*EllipticF[ArcSin[2*Sqrt[2/(5 + Sqrt[57])]*x], (-41
- 5*Sqrt[57])/16]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx &= 4\int \frac{1}{\sqrt{5+\sqrt{57}-8x^2}\,\sqrt{-5+\sqrt{57}+8x^2}}\,dx\\ &= \sqrt{\frac{2}{-5+\sqrt{57}}}\,F\left(\sin^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{57}}}\,x\right)|\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{57}}}\,F\left(i\sinh^{-1}\left(2\sqrt{\frac{2}{-5+\sqrt{57}}}\,x\right)|\frac{1}{16}\left(-41+5\sqrt{57}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 4*x^4],x]
[Out] (-I)*Sqrt[2/(5 + Sqrt[57])]*EllipticF[I*ArcSinh[2*Sqrt[2/(-5 + Sqrt[57])]*x
], (-41 + 5*Sqrt[57])/16]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(38) = 76.
time = 0.06, size = 80, normalized size = 1.63

method	result
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{57}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{57}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{57}}}{2},\frac{5i\sqrt{2}}{8}+\frac{i\sqrt{114}}{8}\right)}{\sqrt{-5+\sqrt{57}}\,\sqrt{-4x^4+5x^2+2}}$
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{57}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{57}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{57}}}{2},\frac{5i\sqrt{2}}{8}+\frac{i\sqrt{114}}{8}\right)}{\sqrt{-5+\sqrt{57}}\,\sqrt{-4x^4+5x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(-5+57^(1/2))^(1/2)*(1-(-5/4+1/4*57^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*57^(1/
2))*x^2)^(1/2)/(-4*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+57^(1/2))^(1/2),5
/8*I*2^(1/2)+1/8*I*114^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-4*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 40, normalized size = 0.82

$$\frac{1}{32}\left(\sqrt{57}\,\sqrt{2}+5\,\sqrt{2}\right)\sqrt{\sqrt{57}-5}\,\mathrm{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{57}-5},-\frac{5}{16}\,\sqrt{57}-\frac{41}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(57)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(57) - 5)*ellipticF(1/2*x*sqrt
(sqrt(57) - 5), -5/16*sqrt(57) - 41/16)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-4x^4 + 5x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-4*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-4*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-4\,x^4 + 5\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 4*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 4*x^4 + 2)^(1/2), x)
```

3.129 $\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{-5+\sqrt{65}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{10}{5+\sqrt{65}}}\,x\right)|\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

```
[Out] EllipticF(x*10^(1/2)/(5+65^(1/2))^(1/2),1/4*I*10^(1/2)+1/4*I*26^(1/2))*2^(1
/2)/(-5+65^(1/2))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{\sqrt{65}-5}}\,F\left(\text{ArcSin}\left(\sqrt{\frac{10}{5+\sqrt{65}}}\,x\right)|\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 5*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[65])]*EllipticF[ArcSin[Sqrt[10/(5 + Sqrt[65])]*x], (-9 -
Sqrt[65])/4]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx &= \left(2\sqrt{5}\right)\int \frac{1}{\sqrt{5+\sqrt{65}-10x^2}\,\sqrt{-5+\sqrt{65}+10x^2}}\,dx\\ &= \sqrt{\frac{2}{-5+\sqrt{65}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{10}{5+\sqrt{65}}}\,x\right)|\frac{1}{4}\left(-9-\sqrt{65}\right)\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 52, normalized size = 1.08

$$-i\sqrt{\frac{2}{5+\sqrt{65}}}\,F\left(i\sinh^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{65}}\,x\right)|\frac{1}{4}\left(-9+\sqrt{65}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 5*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[65])]*EllipticF[I*ArcSinh[(Sqrt[5 + Sqrt[65]]*x)/2],
(-9 + Sqrt[65])/4]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(37) = 74.
time = 0.07, size = 80, normalized size = 1.67

method	result	si
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{65}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{65}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{65}}}{2},\frac{i\sqrt{10}}{4}+\frac{i\sqrt{26}}{4}\right)}{\sqrt{-5+\sqrt{65}}\,\sqrt{-5x^4+5x^2+2}}$	80
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{65}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{65}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{65}}}{2},\frac{i\sqrt{10}}{4}+\frac{i\sqrt{26}}{4}\right)}{\sqrt{-5+\sqrt{65}}\,\sqrt{-5x^4+5x^2+2}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-5+65^(1/2))^(1/2)*(1-(-5/4+1/4*65^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*65^(1/
2))*x^2)^(1/2)/(-5*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+65^(1/2))^(1/2),1
/4*I*10^(1/2)+1/4*I*26^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-5*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 40, normalized size = 0.83

$$\frac{1}{40}\left(\sqrt{65}\,\sqrt{2}+5\,\sqrt{2}\right)\sqrt{\sqrt{65}-5}\,\mathrm{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{65}-5},-\frac{1}{4}\,\sqrt{65}-\frac{9}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(sqrt(65)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(65) - 5)*ellipticF(1/2*x*sqrt
(sqrt(65) - 5), -1/4*sqrt(65) - 9/4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-5x^4 + 5x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-5*x**4 + 5*x**2 + 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-5*x^4 + 5*x^2 + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-5\,x^4 + 5\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 5*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 5*x^4 + 2)^(1/2), x)
```

3.130 $\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{-5+\sqrt{73}}}\,F\left(\sin^{-1}\left(2\sqrt{\frac{3}{5+\sqrt{73}}}\,x\right)|\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

```
[Out] EllipticF(2*x*3^(1/2)/(5+73^(1/2))^(1/2),5/12*I*3^(1/2)+1/12*I*219^(1/2))*2
^(1/2)/(-5+73^(1/2))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{\sqrt{73}-5}}\,F\left(\text{ArcSin}\left(2\sqrt{\frac{3}{5+\sqrt{73}}}\,x\right)|\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 6*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[73])]*EllipticF[ArcSin[2*Sqrt[3/(5 + Sqrt[73])]*x], (-49
- 5*Sqrt[73])/24]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx &= \left(2\sqrt{6}\right)\int \frac{1}{\sqrt{5+\sqrt{73}-12x^2}\,\sqrt{-5+\sqrt{73}+12x^2}}\,dx\\ &= \sqrt{\frac{2}{-5+\sqrt{73}}}\,F\left(\sin^{-1}\left(2\sqrt{\frac{3}{5+\sqrt{73}}}\,x\right)|\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{73}}}\,F\left(i\sinh^{-1}\left(2\sqrt{\frac{3}{-5+\sqrt{73}}}\,x\right)|\frac{1}{24}\left(-49+5\sqrt{73}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 6*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[73])]*EllipticF[I*ArcSinh[2*Sqrt[3/(-5 + Sqrt[73])]*x
], (-49 + 5*Sqrt[73])/24]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(38) = 76.
time = 0.06, size = 80, normalized size = 1.63

method	result
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{73}}}{2},\frac{5i\sqrt{3}}{12}+\frac{i\sqrt{219}}{12}\right)}{\sqrt{-5+\sqrt{73}}\,\sqrt{-6x^4+5x^2+2}}$
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{73}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{73}}{4}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{73}}}{2},\frac{5i\sqrt{3}}{12}+\frac{i\sqrt{219}}{12}\right)}{\sqrt{-5+\sqrt{73}}\,\sqrt{-6x^4+5x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-6*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-5+73^(1/2))^(1/2)*(1-(-5/4+1/4*73^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*73^(1/
2))*x^2)^(1/2)/(-6*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+73^(1/2))^(1/2),5
/12*I*3^(1/2)+1/12*I*219^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-6*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 40, normalized size = 0.82

$$\frac{1}{48}\left(\sqrt{73}\,\sqrt{2}+5\,\sqrt{2}\right)\sqrt{\sqrt{73}-5}\,\mathrm{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{73}-5}\,,-\frac{5}{24}\,\sqrt{73}-\frac{49}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(sqrt(73)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(73) - 5)*ellipticF(1/2*x*sqrt
(sqrt(73) - 5), -5/24*sqrt(73) - 49/24)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-6x^4 + 5x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-6*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-6*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-6*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-6\,x^4 + 5\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 6*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 6*x^4 + 2)^(1/2), x)
```

3.131 $\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{7}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*14^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}(x)\,\middle|-\frac{7}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 7*x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -7/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx &= \left(2\sqrt{7}\right)\int \frac{1}{\sqrt{14-14x^2}\,\sqrt{4+14x^2}}\,dx\\ &= \frac{F\left(\sin^{-1}(x)\middle|-\frac{7}{2}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 65, normalized size = 5.42

$$-\frac{i\sqrt{1-x^2}\,\sqrt{2+7x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{7}{2}}\,x\right)|-\frac{2}{7}\right)}{\sqrt{7}\,\sqrt{2+5x^2-7x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 7*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 7*x^2]*EllipticF[I*ArcSinh[Sqrt[7/2]*x], -2/7]
)/(Sqrt[7]*Sqrt[2 + 5*x^2 - 7*x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(13) = 26.
time = 0.03, size = 43, normalized size = 3.58

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{14x^2+4}\,\text{EllipticF}\left(x,\frac{i\sqrt{14}}{2}\right)}{2\sqrt{-7x^4+5x^2+2}}$	43
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{14x^2+4}\,\text{EllipticF}\left(x,\frac{i\sqrt{14}}{2}\right)}{2\sqrt{-7x^4+5x^2+2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-7*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(14*x^2+4)^(1/2)/(-7*x^4+5*x^2+2)^(1/2)*EllipticF(x,1/2*
I*14^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-7*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.07, size = 8, normalized size = 0.67

$$\frac{1}{2}\,\sqrt{2}\,\text{ellipticF}\left(x,-\frac{7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*ellipticF(x, -7/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-7x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-7*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-7*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-7*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 7*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 7*x^4 + 2)^(1/2), x)
```

3.132 $\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{-5+\sqrt{89}}}\,F\left(\sin^{-1}\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)|\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

```
[Out] EllipticF(4*x/(5+89^(1/2))^(1/2),5/8*I+1/8*I*89^(1/2))*2^(1/2)/(-5+89^(1/2)
)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{\sqrt{89}-5}}\,F\left(\text{ArcSin}\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)|\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 8*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[89])]*EllipticF[ArcSin[(4*x)/Sqrt[5 + Sqrt[89]]], (-57 -
5*Sqrt[89])/32]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx = \left(4\sqrt{2}\right)\int \frac{1}{\sqrt{5+\sqrt{89}-16x^2}\,\sqrt{-5+\sqrt{89}+16x^2}}\,dx$$
$$= \sqrt{\frac{2}{-5+\sqrt{89}}}\,F\left(\sin^{-1}\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)|\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{5+\sqrt{89}}}\,F\left(i\sinh^{-1}\left(\frac{4x}{\sqrt{-5+\sqrt{89}}}\right)|\frac{1}{32}\left(-57+5\sqrt{89}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 8*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[89])]*EllipticF[I*ArcSinh[(4*x)/Sqrt[-5 + Sqrt[89]]],
 (-57 + 5*Sqrt[89])/32]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(31) = 62.
time = 0.08, size = 76, normalized size = 1.69

method	result	size
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{89}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{89}}{4}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{89}}}{2},\frac{5i}{8}+\frac{i\sqrt{89}}{8}\right)}{\sqrt{-5+\sqrt{89}}\,\sqrt{-8x^4+5x^2+2}}$	76
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{89}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{89}}{4}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{89}}}{2},\frac{5i}{8}+\frac{i\sqrt{89}}{8}\right)}{\sqrt{-5+\sqrt{89}}\,\sqrt{-8x^4+5x^2+2}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-8*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-5+89^(1/2))^(1/2)*(1-(-5/4+1/4*89^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*89^(1/
2))*x^2)^(1/2)/(-8*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+89^(1/2))^(1/2),5
/8*I+1/8*I*89^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-8*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 40, normalized size = 0.89

$$\frac{1}{64}\left(\sqrt{89}\,\sqrt{2}+5\,\sqrt{2}\right)\sqrt{\sqrt{89}-5}\ \text{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{89}-5},\,-\frac{5}{32}\,\sqrt{89}-\frac{57}{32}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/64*(sqrt(89)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(89) - 5)*ellipticF(1/2*x*sqrt
(sqrt(89) - 5), -5/32*sqrt(89) - 57/32)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-8x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x**4+5*x**2+2)**(1/2),x)
[Out] Integral(1/sqrt(-8*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-8*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-8*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-8\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 8*x^4 + 2)^(1/2),x)
[Out] int(1/(5*x^2 - 8*x^4 + 2)^(1/2), x)
```

3.133 $\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{-5+\sqrt{97}}}\,F\left(\sin^{-1}\left(3\sqrt{\frac{2}{5+\sqrt{97}}}\,x\right)|\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

```
[Out] EllipticF(3*x*2^(1/2)/(5+97^(1/2))^(1/2),5/12*I*2^(1/2)+1/12*I*194^(1/2))*2
^(1/2)/(-5+97^(1/2))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\sqrt{\frac{2}{\sqrt{97}-5}}\,F\left(\text{ArcSin}\left(3\sqrt{\frac{2}{5+\sqrt{97}}}\,x\right)|\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 5*x^2 - 9*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[97])]*EllipticF[ArcSin[3*Sqrt[2/(5 + Sqrt[97])]*x], (-61
- 5*Sqrt[97])/36]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx &= 6\int \frac{1}{\sqrt{5+\sqrt{97}-18x^2}\,\sqrt{-5+\sqrt{97}+18x^2}}\,dx\\ &= \sqrt{\frac{2}{-5+\sqrt{97}}}\,F\left(\sin^{-1}\left(3\sqrt{\frac{2}{5+\sqrt{97}}}\,x\right)|\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{97}}}\,F\left(i\sinh^{-1}\left(3\sqrt{\frac{2}{-5+\sqrt{97}}}\,x\right)|\frac{1}{36}\left(-61+5\sqrt{97}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2 + 5*x^2 - 9*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[97])]*EllipticF[I*ArcSinh[3*Sqrt[2/(-5 + Sqrt[97])]*x
], (-61 + 5*Sqrt[97])/36]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(38) = 76.
time = 0.07, size = 80, normalized size = 1.63

method	result	s
default	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{97}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{97}}{4}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{97}}}{2},\frac{5i\sqrt{2}}{12}+\frac{i\sqrt{194}}{12}\right)}{\sqrt{-5+\sqrt{97}}\,\sqrt{-9x^4+5x^2+2}}$	8
elliptic	$\frac{2\sqrt{1-\left(-\frac{5}{4}+\frac{\sqrt{97}}{4}\right)x^2}\,\sqrt{1-\left(-\frac{5}{4}-\frac{\sqrt{97}}{4}\right)x^2}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-5+\sqrt{97}}}{2},\frac{5i\sqrt{2}}{12}+\frac{i\sqrt{194}}{12}\right)}{\sqrt{-5+\sqrt{97}}\,\sqrt{-9x^4+5x^2+2}}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-9*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-5+97^(1/2))^(1/2)*(1-(-5/4+1/4*97^(1/2))*x^2)^(1/2)*(1-(-5/4-1/4*97^(1/
2))*x^2)^(1/2)/(-9*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+97^(1/2))^(1/2),5
/12*I*2^(1/2)+1/12*I*194^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-9*x^4 + 5*x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 40, normalized size = 0.82

$$\frac{1}{72}\left(\sqrt{97}\,\sqrt{2}+5\,\sqrt{2}\right)\sqrt{\sqrt{97}-5}\;\mathrm{ellipticF}\left(\frac{1}{2}\,x\sqrt{\sqrt{97}-5}\,,-\frac{5}{36}\,\sqrt{97}-\frac{61}{36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(97)*sqrt(2) + 5*sqrt(2))*sqrt(sqrt(97) - 5)*ellipticF(1/2*x*sqrt
(sqrt(97) - 5), -5/36*sqrt(97) - 61/36)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-9x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-9*x**4 + 5*x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-9*x^4 + 5*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-9\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - 9*x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(5*x^2 - 9*x^4 + 2)^(1/2), x)
```

3.134 $\int x^2(bx^2+cx^4)\,dx$

Optimal. Leaf size=17

$$\frac{bx^5}{5}+\frac{cx^7}{7}$$

```
[Out] 1/5*b*x^5+1/7*c*x^7
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {14}

$$\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^2 + c*x^4),x]
[Out] (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(bx^2+cx^4\right)\,dx&=\int\left(bx^4+cx^6\right)\,dx\\&=\frac{bx^5}{5}+\frac{cx^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^2 + c*x^4),x]
[Out] (b*x^5)/5 + (c*x^7)/7
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.82

method	result	size
default	$\frac{1}{5}b\,x^5+\frac{1}{7}c\,x^7$	14
norman	$\frac{1}{5}b\,x^5+\frac{1}{7}c\,x^7$	14
risch	$\frac{1}{5}b\,x^5+\frac{1}{7}c\,x^7$	14
gosper	$\frac{x^5(5c\,x^2+7b)}{35}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/5*b*x^5+1/7*c*x^7
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.76

$$\frac{1}{7}\,cx^7+\frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/7*c*x^7 + 1/5*b*x^5
```

Fricas [A]
time = 0.31, size = 13, normalized size = 0.76

$$\frac{1}{7}\,cx^7+\frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/7*c*x^7 + 1/5*b*x^5
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{bx^5}{5}+\frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2),x)
```

```
[Out] b*x**5/5 + c*x**7/7
```

Giac [A]
time = 5.36, size = 13, normalized size = 0.76

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^7}{7} + \frac{b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2 + c*x^4),x)
```

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

3.135 $\int x(bx^2+cx^4)\,dx$

Optimal. Leaf size=17

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

[Out] 1/4*b*x^4+1/6*c*x^6

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {14}

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

[In] Int[x*(b*x^2 + c*x^4),x]

[Out] (b*x^4)/4 + (c*x^6)/6

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(bx^2+cx^4\right)\,dx&=\int\left(bx^3+cx^5\right)\,dx\\&=\frac{bx^4}{4}+\frac{cx^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[x*(b*x^2 + c*x^4),x]

[Out] (b*x^4)/4 + (c*x^6)/6

Maple [A]
time = 0.04, size = 14, normalized size = 0.82

method	result	size
default	$\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	14
norman	$\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	14
risch	$\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	14
gosper	$\frac{x^{4}\left(2c\,x^{2}+3b\right)}{12}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*x^4+1/6*c*x^6
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.76

$$\frac{1}{6}\,cx^{6}+\frac{1}{4}\,bx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.76

$$\frac{1}{6}\,cx^{6}+\frac{1}{4}\,bx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{bx^{4}}{4}+\frac{cx^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2),x)
```

```
[Out] b*x**4/4 + c*x**6/6
```

Giac [A]
time = 4.04, size = 13, normalized size = 0.76

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^6}{6} + \frac{b\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2 + c*x^4),x)
```

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

3.136 $\int (bx^2 + cx^4) \, dx$

Optimal. Leaf size=17

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

```
[Out] 1/3*b*x^3+1/5*c*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[b*x^2 + c*x^4,x]
```

```
[Out] (b*x^3)/3 + (c*x^5)/5
```

Rubi steps

$$\int (bx^2 + cx^4) \, dx = \frac{bx^3}{3} + \frac{cx^5}{5}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x^2 + c*x^4,x]
```

```
[Out] (b*x^3)/3 + (c*x^5)/5
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.82

method	result	size
default	$\frac{1}{3}b\,x^3 + \frac{1}{5}c\,x^5$	14
norman	$\frac{1}{3}b\,x^3 + \frac{1}{5}c\,x^5$	14

risch	$\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	14
gosper	$\frac{x^{3}\left(3c\,x^{2}+5b\right)}{15}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^4+b*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*x^3+1/5*c*x^5
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$\frac{1}{5}\,cx^{5}+\frac{1}{3}\,bx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="maxima")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.76

$$\frac{1}{5}\,cx^{5}+\frac{1}{3}\,bx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="fricas")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{bx^{3}}{3}+\frac{cx^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**2,x)
```

```
[Out] b*x**3/3 + c*x**5/5
```

Giac [A]
time = 3.97, size = 13, normalized size = 0.76

$$\frac{1}{5}\,cx^{5}+\frac{1}{3}\,bx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^5}{5}+\frac{b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^2 + c*x^4,x)
```

```
[Out] (b*x^3)/3 + (c*x^5)/5
```

3.137 $\int \frac{bx^2+cx^4}{x}\,dx$

Optimal. Leaf size=17

$$\frac{bx^2}{2}+\frac{cx^4}{4}$$

[Out] 1/2*b*x^2+1/4*c*x^4

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)/x,x]

[Out] (b*x^2)/2 + (c*x^4)/4

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x}\,dx &= \int \left(bx+cx^3\right)\,dx\\ &= \frac{bx^2}{2}+\frac{cx^4}{4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^2 + c*x^4)/x,x]

[Out] (b*x^2)/2 + (c*x^4)/4

Maple [A]
time = 0.03, size = 15, normalized size = 0.88

method	result	size
norman	$\frac{1}{2}b\,x^2+\frac{1}{4}c\,x^4$	14
gosper	$\frac{x^2(c\,x^2+2b)}{4}$	15
default	$\frac{(c\,x^2+b)^2}{4c}$	15
risch	$\frac{c\,x^4}{4}+\frac{b\,x^2}{2}+\frac{b^2}{4c}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(c*x^2+b)^2/c
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

Fricas [A]
time = 0.31, size = 13, normalized size = 0.76

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{bx^2}{2}+\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x,x)
```

```
[Out] b*x**2/2 + c*x**4/4
```

Giac [A]
time = 5.37, size = 13, normalized size = 0.76

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{c\,x^4}{4} + \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x,x)
```

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

3.138 $\int \frac{bx^2+cx^4}{x^2} \, dx$

Optimal. Leaf size=12

$$bx + \frac{cx^3}{3}$$

```
[Out] b*x+1/3*c*x^3
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^2,x]
```

```
[Out] b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2 + cx^4}{x^2} \, dx &= \int \left(b + cx^2\right) \, dx \\ &= bx + \frac{cx^3}{3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^2,x]
```

```
[Out] b*x + (c*x^3)/3
```

Maple [A]
time = 0.01, size = 11, normalized size = 0.92

method	result	size
default	$bx+\frac{1}{3}c\,x^{3}$	11
risch	$bx+\frac{1}{3}c\,x^{3}$	11
gosper	$\frac{x\left(c\,x^{2}+3b\right)}{3}$	13
norman	$\frac{b\,x^{2}+\frac{1}{3}c\,x^{4}}{x}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*x+1/3*c*x^3
```

Maxima [A]
time = 0.27, size = 10, normalized size = 0.83

$$\frac{1}{3}\,cx^{3}+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + b*x
```

Fricas [A]
time = 0.32, size = 10, normalized size = 0.83

$$\frac{1}{3}\,cx^{3}+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*c*x^3 + b*x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$bx+\frac{cx^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**2,x)
```

```
[Out] b*x + c*x**3/3
```

Giac [A]
time = 4.61, size = 10, normalized size = 0.83

$$\frac{1}{3} cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + b*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{c\,x^3}{3} + b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^2,x)
```

```
[Out] b*x + (c*x^3)/3
```

3.139 $\int \frac{bx^2+cx^4}{x^3} \, dx$

Optimal. Leaf size=13

$$\frac{cx^2}{2} + b\log(x)$$

[Out] 1/2*c*x^2+b*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$b\log(x) + \frac{cx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)/x^3,x]

[Out] (c*x^2)/2 + b*Log[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2+cx^4}{x^3} \, dx &= \int \left(\frac{b}{x}+cx\right) \, dx \\ &= \frac{cx^2}{2} + b\log(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{cx^2}{2} + b\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^2 + c*x^4)/x^3,x]

[Out] (c*x^2)/2 + b*Log[x]

Maple [A]
time = 0.01, size = 12, normalized size = 0.92

method	result	size
default	$\frac{cx^2}{2} + b\ln(x)$	12
norman	$\frac{cx^2}{2} + b\ln(x)$	12
risch	$\frac{cx^2}{2} + b\ln(x)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2*c*x^2+b*ln(x)
```

Maxima [A]
time = 0.27, size = 14, normalized size = 1.08

$$\frac{1}{2}cx^2 + \frac{1}{2}b\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="maxima")
[Out] 1/2*c*x^2 + 1/2*b*log(x^2)
```

Fricas [A]
time = 0.35, size = 11, normalized size = 0.85

$$\frac{1}{2}cx^2 + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="fricas")
[Out] 1/2*c*x^2 + b*log(x)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.77

$$b\log(x) + \frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**3,x)
[Out] b*log(x) + c*x**2/2
```

Giac [A]
time = 3.07, size = 14, normalized size = 1.08

$$\frac{1}{2}\,cx^2 + \frac{1}{2}\,b\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*c*x^2 + 1/2*b*log(x^2)
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.85

$$\frac{c\,x^2}{2} + b\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^3,x)
```

```
[Out] (c*x^2)/2 + b*log(x)
```

3.140 $\int \frac{bx^2+cx^4}{x^4}\,dx$

Optimal. Leaf size=10

$$-\frac{b}{x}+cx$$

[Out] -b/x+c*x

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{14\}$

$$cx-\frac{b}{x}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)/x^4,x]

[Out] -(b/x) + c*x

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^4}\,dx &= \int \left(c+\frac{b}{x^2}\right)dx\\ &= -\frac{b}{x}+cx\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{b}{x}+cx$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^2 + c*x^4)/x^4,x]

[Out] -(b/x) + c*x

Maple [A]
time = 0.02, size = 11, normalized size = 1.10

method	result	size
default	$-\frac{b}{x}+cx$	11
risch	$-\frac{b}{x}+cx$	11
gosper	$-\frac{-c\,x^{2}+b}{x}$	14
norman	$\frac{c\,x^{4}-b\,x^{2}}{x^{3}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^4,x,method=_RETURNVERBOSE)
[Out] -b/x+c*x
```

Maxima [A]
time = 0.27, size = 10, normalized size = 1.00

$$cx-\frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="maxima")
[Out] c*x - b/x
```

Fricas [A]
time = 0.32, size = 13, normalized size = 1.30

$$\frac{cx^2-b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="fricas")
[Out] (c*x^2 - b)/x
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.50

$$-\frac{b}{x}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**4,x)
```

```
[Out] -b/x + c*x
```

Giac [A]
time = 3.85, size = 10, normalized size = 1.00

$$cx - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^4,x, algorithm="giac")
```

```
[Out] c*x - b/x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 1.00

$$c\,x - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^4,x)
```

```
[Out] c*x - b/x
```

3.141 $\int \frac{bx^2+cx^4}{x^5}\,dx$

Optimal. Leaf size=13

$$-\frac{b}{2x^2}+c\log(x)$$

```
[Out] -1/2*b/x^2+c*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {14}

$$c\log(x)-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -1/2*b/x^2 + c*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^5}\,dx &= \int \left(\frac{b}{x^3}+\frac{c}{x}\right)\,dx\\ &= -\frac{b}{2x^2}+c\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{b}{2x^2}+c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -1/2*b/x^2 + c*Log[x]
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.92

method	result	size
default	$-\frac{b}{2x^2}+c\ln\left(x\right)$	12
norman	$-\frac{b}{2x^2}+c\ln\left(x\right)$	12
risch	$-\frac{b}{2x^2}+c\ln\left(x\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/2*b/x^2+c*ln(x)
```

Maxima [A]
time = 0.29, size = 14, normalized size = 1.08

$$\frac{1}{2}\,c\log\left(x^2\right)-\frac{b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="maxima")
[Out] 1/2*c*log(x^2) - 1/2*b/x^2
```

Fricas [A]
time = 0.35, size = 17, normalized size = 1.31

$$\frac{2\,cx^2\log\left(x\right)-b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="fricas")
[Out] 1/2*(2*c*x^2*log(x) - b)/x^2
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.77

$$-\frac{b}{2x^2}+c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**5,x)
[Out] -b/(2*x**2) + c*log(x)
```

Giac [A]
time = 4.84, size = 20, normalized size = 1.54

$$\frac{1}{2}\,c\log\left(x^2\right)-\frac{cx^2+b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2) - 1/2*(c*x^2 + b)/x^2
```

Mupad [B]
time = 0.04, size = 11, normalized size = 0.85

$$c\,\ln\left(x\right)-\frac{b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^5,x)
```

```
[Out] c*log(x) - b/(2*x^2)
```

3.142 $\int \frac{bx^2+cx^4}{x^6} dx$

Optimal. Leaf size=15

$$-\frac{b}{3x^3} - \frac{c}{x}$$

[Out] -1/3*b/x^3-c/x

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$-\frac{b}{3x^3} - \frac{c}{x}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)/x^6,x]

[Out] -1/3*b/x^3 - c/x

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^6}\,dx &= \int \left(\frac{b}{x^4}+\frac{c}{x^2}\right) dx\\ &= -\frac{b}{3x^3} - \frac{c}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{b}{3x^3} - \frac{c}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^2 + c*x^4)/x^6,x]

[Out] -1/3*b/x^3 - c/x

Maple [A]
time = 0.01, size = 14, normalized size = 0.93

method	result	size
gosper	$-\frac{3c\,x^2+b}{3x^3}$	14
default	$-\frac{b}{3x^3}-\frac{c}{x}$	14
risch	$\frac{-c\,x^2-\frac{b}{3}}{x^3}$	15
norman	$\frac{-\frac{1}{3}b\,x^2-c\,x^4}{x^5}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/x^3-c/x
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.87

$$-\frac{3\,cx^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/3*(3*c*x^2 + b)/x^3
```

Fricas [A]
time = 0.33, size = 13, normalized size = 0.87

$$-\frac{3\,cx^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/3*(3*c*x^2 + b)/x^3
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.93

$$\frac{-b-3cx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**6,x)
```

```
[Out] (-b - 3*c*x**2)/(3*x**3)
```

Giac [A]
time = 6.83, size = 13, normalized size = 0.87

$$-\frac{3\,cx^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^6,x, algorithm="giac")
```

```
[Out] -1/3*(3*c*x^2 + b)/x^3
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.87

$$-\frac{3\,c\,x^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^6,x)
```

```
[Out] -(b + 3*c*x^2)/(3*x^3)
```

3.143 $\int \frac{bx^2+cx^4}{x^7}\,dx$

Optimal. Leaf size=17

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

[Out] -1/4*b/x^4-1/2*c/x^2

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)/x^7,x]

[Out] -1/4*b/x^4 - c/(2*x^2)

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^7}\,dx &= \int \left(\frac{b}{x^5}+\frac{c}{x^3}\right)dx\\ &= -\frac{b}{4x^4}-\frac{c}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^2 + c*x^4)/x^7,x]

[Out] -1/4*b/x^4 - c/(2*x^2)

Maple [A]
time = 0.03, size = 14, normalized size = 0.82

method	result	size
gosper	$-\frac{2cx^2+b}{4x^4}$	14
default	$-\frac{b}{4x^4}-\frac{c}{2x^2}$	14
risch	$\frac{-\frac{cx^2}{2}-\frac{b}{4}}{x^4}$	15
norman	$\frac{-\frac{1}{4}bx^2-\frac{1}{2}cx^4}{x^6}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/4*b/x^4-1/2*c/x^2
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.76

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="maxima")
[Out] -1/4*(2*c*x^2 + b)/x^4
```

Fricas [A]
time = 0.33, size = 13, normalized size = 0.76

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="fricas")
[Out] -1/4*(2*c*x^2 + b)/x^4
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.82

$$\frac{-b-2cx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**7,x)
```

```
[Out] (-b - 2*c*x**2)/(4*x**4)
```

Giac [A]
time = 6.04, size = 13, normalized size = 0.76

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^7,x, algorithm="giac")
```

```
[Out] -1/4*(2*c*x^2 + b)/x^4
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.76

$$-\frac{2\,c\,x^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^7,x)
```

```
[Out] -(b + 2*c*x^2)/(4*x^4)
```

3.144 $\int \frac{bx^2+cx^4}{x^8}\,dx$

Optimal. Leaf size=17

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

```
[Out] -1/5*b/x^5-1/3*c/x^3
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -1/5*b/x^5 - c/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^8}\,dx &= \int \left(\frac{b}{x^6}+\frac{c}{x^4}\right)dx\\ &= -\frac{b}{5x^5}-\frac{c}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -1/5*b/x^5 - c/(3*x^3)
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{b}{5x^5}-\frac{c}{3x^3}$	14
risch	$\frac{-\frac{c\,x^2}{3}-\frac{b}{5}}{x^5}$	15
gosper	$-\frac{5c\,x^2+3b}{15x^5}$	16
norman	$\frac{-\frac{1}{5}b\,x^2-\frac{1}{3}c\,x^4}{x^7}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/5*b/x^5-1/3*c/x^3
```

Maxima [A]
time = 0.31, size = 15, normalized size = 0.88

$$-\frac{5\,cx^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="maxima")
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

Fricas [A]
time = 0.32, size = 15, normalized size = 0.88

$$-\frac{5\,cx^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="fricas")
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.88

$$\frac{-3b-5cx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**8,x)
```

```
[Out] (-3*b - 5*c*x**2)/(15*x**5)
```

Giac [A]
time = 5.93, size = 15, normalized size = 0.88

$$-\frac{5\,cx^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^8,x, algorithm="giac")
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.88

$$-\frac{5\,c\,x^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^8,x)
[Out] -(3*b + 5*c*x^2)/(15*x^5)
```

3.145 $\int (bx^2 + cx^4)^2 \, dx$

Optimal. Leaf size=30

$$\frac{b^2x^5}{5} + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

```
[Out] 1/5*b^2*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1607, 276}

$$\frac{b^2x^5}{5} + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int (bx^2 + cx^4)^2 \, dx &= \int x^4 (b + cx^2)^2 \, dx \\
&= \int (b^2x^4 + 2bcx^6 + c^2x^8) \, dx \\
&= \frac{b^2x^5}{5} + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{b^2x^5}{5} + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.83

method	result	size
default	$\frac{1}{5}b^2x^5+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	25
norman	$\frac{1}{5}b^2x^5+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	25
risch	$\frac{1}{5}b^2x^5+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	25
gosper	$\frac{x^5\left(35c^2x^4+90bc\,x^2+63b^2\right)}{315}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^2*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

Fricas [A]
time = 0.33, size = 24, normalized size = 0.80

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.87

$$\frac{b^2x^5}{5}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2,x)
```

```
[Out] b**2*x**5/5 + 2*b*c*x**7/7 + c**2*x**9/9
```

Giac [A]
time = 4.72, size = 24, normalized size = 0.80

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.80

$$\frac{b^2\,x^5}{5}+\frac{2\,b\,c\,x^7}{7}+\frac{c^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2,x)
```

```
[Out] (b^2*x^5)/5 + (c^2*x^9)/9 + (2*b*c*x^7)/7
```

3.146 $\int \frac{(bx^2+cx^4)^2}{x} dx$

Optimal. Leaf size=30

$$\frac{b^2x^4}{4} + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

```
[Out] 1/4*b^2*x^4+1/3*b*c*x^6+1/8*c^2*x^8
```

Rubi [A]

time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 45}

$$\frac{b^2x^4}{4} + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x,x]
```

```
[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x}\,dx = \int x^3\left(b+cx^2\right)^2\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int x(b+cx)^2\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(b^2x+2bcx^2+c^2x^3\right)\,dx, x, x^2\right)$$
$$= \frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Mathematica [**A**]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{b^2x^4}{4}+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x,x]
```

```
[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

Maple [**A**]
time = 0.09, size = 25, normalized size = 0.83

method	result	size
default	$\frac{1}{4}b^2x^4+\frac{1}{3}bc\,x^6+\frac{1}{8}c^2x^8$	25
norman	$\frac{1}{4}b^2x^4+\frac{1}{3}bc\,x^6+\frac{1}{8}c^2x^8$	25
risch	$\frac{1}{4}b^2x^4+\frac{1}{3}bc\,x^6+\frac{1}{8}c^2x^8$	25
gosper	$\frac{x^4\left(3c^2x^4+8bc\,x^2+6b^2\right)}{24}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^2*x^4+1/3*b*c*x^6+1/8*c^2*x^8
```

Maxima [**A**]
time = 0.28, size = 24, normalized size = 0.80

$$\frac{1}{8}\,c^2x^8+\frac{1}{3}\,bcx^6+\frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="maxima")
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

Fricas [A]
time = 0.32, size = 24, normalized size = 0.80

$$\frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="fricas")
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.80

$$\frac{b^2x^4}{4} + \frac{bcx^6}{3} + \frac{c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x,x)
[Out] b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8
```

Giac [A]
time = 4.28, size = 24, normalized size = 0.80

$$\frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x,x, algorithm="giac")
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.80

$$\frac{b^2\,x^4}{4} + \frac{b\,c\,x^6}{3} + \frac{c^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x,x)
[Out] (b^2*x^4)/4 + (c^2*x^8)/8 + (b*c*x^6)/3
```

3.147 $\int \frac{(bx^2+cx^4)^2}{x^2} dx$

Optimal. Leaf size=30

$$\frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

```
[Out] 1/3*b^2*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 276}

$$\frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{x^2} dx &= \int x^2(b+cx^2)^2 \, dx \\
&= \int (b^2x^2 + 2bcx^4 + c^2x^6) \, dx \\
&= \frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{b^2x^3}{3}+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.83

method	result	size
default	$\frac{1}{3}b^2x^3+\frac{2}{5}bc\,x^5+\frac{1}{7}c^2x^7$	25
risch	$\frac{1}{3}b^2x^3+\frac{2}{5}bc\,x^5+\frac{1}{7}c^2x^7$	25
gosper	$\frac{x^3\left(15c^2x^4+42bc\,x^2+35b^2\right)}{105}$	27
norman	$\frac{\frac{1}{3}b^2x^4+\frac{1}{7}c^2x^8+\frac{2}{5}bc\,x^6}{x}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$\frac{1}{7}\,c^2x^7+\frac{2}{5}\,bcx^5+\frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

Fricas [A]
time = 0.31, size = 24, normalized size = 0.80

$$\frac{1}{7}\,c^2x^7+\frac{2}{5}\,bcx^5+\frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

Sympy [A]
time = 0.02, size = 26, normalized size = 0.87

$$\frac{b^2x^3}{3} + \frac{2bcx^5}{5} + \frac{c^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**2,x)
[Out] b**2*x**3/3 + 2*b*c*x**5/5 + c**2*x**7/7
```

Giac [A]
time = 3.74, size = 24, normalized size = 0.80

$$\frac{1}{7}c^2x^7 + \frac{2}{5}bcx^5 + \frac{1}{3}b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="giac")
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.80

$$\frac{b^2\,x^3}{3} + \frac{2\,b\,c\,x^5}{5} + \frac{c^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^2,x)
[Out] (b^2*x^3)/3 + (c^2*x^7)/7 + (2*b*c*x^5)/5
```

3.148 $\int \frac{(bx^2+cx^4)^2}{x^3}\,dx$

Optimal. Leaf size=16

$$\frac{(b+cx^2)^3}{6c}$$

```
[Out] 1/6*(c*x^2+b)^3/c
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 267}

$$\frac{(b+cx^2)^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] (b + c*x^2)^3/(6*c)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^3}\,dx &= \int x\left(b+cx^2\right)^2\,dx\\ &= \frac{(b+cx^2)^3}{6c}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{(b+cx^2)^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] (b + c*x^2)^3/(6*c)
```

Maple [A]
time = 0.08, size = 15, normalized size = 0.94

method	result	size
default	$\frac{(c\,x^2+b)^3}{6c}$	15
gosper	$\frac{x^2(c^2x^4+3bc\,x^2+3b^2)}{6}$	26
norman	$\frac{\frac{1}{2}b^2x^4+\frac{1}{6}c^2x^8+\frac{1}{2}bc\,x^6}{x^2}$	29
risch	$\frac{c^2x^6}{6}+\frac{bc\,x^4}{2}+\frac{b^2x^2}{2}+\frac{b^3}{6c}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(c*x^2+b)^3/c
```

Maxima [A]
time = 0.27, size = 24, normalized size = 1.50

$$\frac{1}{6}\,c^2x^6+\frac{1}{2}\,bcx^4+\frac{1}{2}\,b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

Fricas [A]
time = 0.32, size = 24, normalized size = 1.50

$$\frac{1}{6}\,c^2x^6+\frac{1}{2}\,bcx^4+\frac{1}{2}\,b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.
time = 0.01, size = 24, normalized size = 1.50

$$\frac{b^2x^2}{2}+\frac{bcx^4}{2}+\frac{c^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**3,x)
```

```
[Out] b**2*x**2/2 + b*c*x**4/2 + c**2*x**6/6
```

Giac [A]
time = 4.63, size = 24, normalized size = 1.50

$$\frac{1}{6}\,c^2x^6+\frac{1}{2}\,bcx^4+\frac{1}{2}\,b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

Mupad [B]
time = 0.03, size = 24, normalized size = 1.50

$$\frac{b^2\,x^2}{2}+\frac{b\,c\,x^4}{2}+\frac{c^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^3,x)
```

```
[Out] (b^2*x^2)/2 + (c^2*x^6)/6 + (b*c*x^4)/2
```

3.149 $\int \frac{(bx^2+cx^4)^2}{x^4} dx$

Optimal. Leaf size=25

$$b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

```
[Out] b^2*x+2/3*b*c*x^3+1/5*c^2*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 200}

$$b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{x^4} dx &= \int (b+cx^2)^2 dx \\
&= \int (b^2+2bcx^2+c^2x^4) dx \\
&= b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^4,x]
[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Maple [A]
time = 0.07, size = 22, normalized size = 0.88

method	result	size
default	$b^{2} x+\frac{2}{3} b c \,x^{3}+\frac{1}{5} c^{2} x^{5}$	22
risch	$b^{2} x+\frac{2}{3} b c \,x^{3}+\frac{1}{5} c^{2} x^{5}$	22
gosper	$\frac{x \left(3 c^{2} x^{4}+10 b c \,x^{2}+15 b^{2}\right)}{15}$	25
norman	$\frac{b^{2} x^{4}+\frac{1}{5} c^{2} x^{8}+\frac{2}{3} b c \,x^{6}}{x^{3}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^4,x,method=_RETURNVERBOSE)
[Out] b^2*x+2/3*b*c*x^3+1/5*c^2*x^5
```

Maxima [A]
time = 0.30, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5+\frac{2}{3}\,bcx^3+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="maxima")
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

Fricas [A]
time = 0.33, size = 21, normalized size = 0.84

$$\frac{1}{5}\,c^2x^5+\frac{2}{3}\,bcx^3+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="fricas")
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

Sympy [A]
time = 0.01, size = 22, normalized size = 0.88

$$b^2x+\frac{2bcx^3}{3}+\frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**4,x)
```

```
[Out] b**2*x + 2*b*c*x**3/3 + c**2*x**5/5
```

Giac [A]
time = 4.47, size = 21, normalized size = 0.84

$$\frac{1}{5}c^2x^5 + \frac{2}{3}bcx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.84

$$b^2\,x+\frac{2\,b\,c\,x^3}{3}+\frac{c^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^4,x)
```

```
[Out] b^2*x + (c^2*x^5)/5 + (2*b*c*x^3)/3
```

3.150 $\int \frac{(bx^2+cx^4)^2}{x^5} \, dx$

Optimal. Leaf size=23

$$bcx^2 + \frac{c^2x^4}{4} + b^2\log(x)$$

[Out] b*c*x^2+1/4*c^2*x^4+b^2*ln(x)

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 45}

$$b^2\log(x) + bcx^2 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)^2/x^5,x]

[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^5}\,dx = \int \frac{(b+cx^2)^2}{x}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(b+cx)^2}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(2bc+\frac{b^2}{x}+c^2x\right)\,dx, x, x^2\right)$$
$$= bcx^2+\frac{c^2x^4}{4}+b^2\log(x)$$

<u>**Mathematica**</u> [A]
time = 0.00, size = 23, normalized size = 1.00

$$bcx^2+\frac{c^2x^4}{4}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

<u>**Maple**</u> [A]
time = 0.09, size = 22, normalized size = 0.96

method	result	size
default	$bc\,x^2+\frac{c^2x^4}{4}+b^2\ln(x)$	22
risch	$\frac{c^2x^4}{4}+bc\,x^2+b^2+b^2\ln(x)$	25
norman	$\frac{bc\,x^6+\frac{1}{4}c^2x^8}{x^4}+b^2\ln(x)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] b*c*x^2+1/4*c^2*x^4+b^2*ln(x)
```

<u>**Maxima**</u> [A]
time = 0.27, size = 24, normalized size = 1.04

$$\frac{1}{4}c^2x^4+bcx^2+\frac{1}{2}b^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*log(x^2)
```

Fricas [A]
time = 0.35, size = 21, normalized size = 0.91

$$\frac{1}{4}c^2x^4 + bcx^2 + b^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + b^2*log(x)
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.87

$$b^2\log(x) + bcx^2 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**5,x)
```

```
[Out] b**2*log(x) + b*c*x**2 + c**2*x**4/4
```

Giac [A]
time = 5.34, size = 24, normalized size = 1.04

$$\frac{1}{4}c^2x^4 + bcx^2 + \frac{1}{2}b^2\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*log(x^2)
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.91

$$b^2\ln(x) + \frac{c^2x^4}{4} + b\,c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^5,x)
```

```
[Out] b^2*log(x) + (c^2*x^4)/4 + b*c*x^2
```

3.151 $\int \frac{(bx^2+cx^4)^2}{x^6} dx$

Optimal. Leaf size=24

$$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

```
[Out] -b^2/x+2*b*c*x+1/3*c^2*x^3
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 276}

$$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^6} dx &= \int \frac{(b+cx^2)^2}{x^2} dx\\ &= \int \left(2bc+\frac{b^2}{x^2}+c^2x^2\right) dx\\ &= -\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^6,x]
[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Maple [A]
time = 0.09, size = 23, normalized size = 0.96

method	result	size
default	$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$	23
risch	$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$	23
gosper	$-\frac{-c^2x^4-6bc\,x^2+3b^2}{3x}$	27
norman	$\frac{-b^2x^4+\frac{1}{3}c^2x^8+2bc\,x^6}{x^5}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^6,x,method=_RETURNVERBOSE)
[Out] -b^2/x+2*b*c*x+1/3*c^2*x^3
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.92

$$\frac{1}{3}\,c^2x^3+2\,bcx-\frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="maxima")
[Out] 1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

Fricas [A]
time = 0.32, size = 25, normalized size = 1.04

$$\frac{c^2x^4+6\,bcx^2-3\,b^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/3*(c^2*x^4 + 6*b*c*x^2 - 3*b^2)/x
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.79

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**6,x)
```

```
[Out] -b**2/x + 2*b*c*x + c**2*x**3/3
```

Giac [A]
time = 4.31, size = 22, normalized size = 0.92

$$\frac{1}{3}c^2x^3 + 2\,bcx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.92

$$\frac{c^2\,x^3}{3} - \frac{b^2}{x} + 2\,b\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^6,x)
```

```
[Out] (c^2*x^3)/3 - b^2/x + 2*b*c*x
```

3.152 $\int \frac{(bx^2+cx^4)^2}{x^7} dx$

Optimal. Leaf size=27

$$-\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)$$

```
[Out] -1/2*b^2/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1598, 272, 45}

$$-\frac{b^2}{2x^2}+2bc\log(x)+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -1/2*b^2/x^2 + (c^2*x^2)/2 + 2*b*c*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^7}\,dx = \int \frac{(b+cx^2)^2}{x^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(b+cx)^2}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(c^2+\frac{b^2}{x^2}+\frac{2bc}{x}\right)\,dx, x, x^2\right)$$
$$= -\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$-\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -1/2*b^2/x^2 + (c^2*x^2)/2 + 2*b*c*Log[x]
```

Maple [A]
time = 0.07, size = 24, normalized size = 0.89

method	result	size
default	$-\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\ln(x)$	24
risch	$-\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\ln(x)$	24
norman	$\frac{-\frac{1}{2}b^2x^4+\frac{1}{2}c^2x^8}{x^6}+2bc\ln(x)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.89

$$\frac{1}{2}\,c^2x^2+bc\log\left(x^2\right)-\frac{b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="maxima")
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/2*b^2/x^2
```

Fricas [A]
time = 0.37, size = 27, normalized size = 1.00

$$\frac{c^2x^4 + 4\,bcx^2\log\left(x\right) - b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="fricas")
[Out] 1/2*(c^2*x^4 + 4*b*c*x^2*log(x) - b^2)/x^2
```

Sympy [A]
time = 0.04, size = 24, normalized size = 0.89

$$-\frac{b^2}{2x^2} + 2bc\log\left(x\right) + \frac{c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**7,x)
[Out] -b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2
```

Giac [A]
time = 5.86, size = 32, normalized size = 1.19

$$\frac{1}{2}\,c^2x^2 + bc\log\left(x^2\right) - \frac{2\,bcx^2 + b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="giac")
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/2*(2*b*c*x^2 + b^2)/x^2
```

Mupad [B]
time = 0.03, size = 23, normalized size = 0.85

$$\frac{c^2\,x^2}{2} - \frac{b^2}{2\,x^2} + 2\,b\,c\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^7,x)
[Out] (c^2*x^2)/2 - b^2/(2*x^2) + 2*b*c*log(x)
```

3.153 $\int \frac{(bx^2+cx^4)^2}{x^8}\,dx$

Optimal. Leaf size=23

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

```
[Out] -1/3*b^2/x^3-2*b*c/x+c^2*x
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 276}

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -1/3*b^2/x^3 - (2*b*c)/x + c^2*x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{x^8}\,dx &= \int \frac{(b+cx^2)^2}{x^4}\,dx\\
&= \int \left(c^2+\frac{b^2}{x^4}+\frac{2bc}{x^2}\right)dx\\
&= -\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{b^2}{3x^3} - \frac{2bc}{x} + c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^8,x]
[Out] -1/3*b^2/x^3 - (2*b*c)/x + c^2*x
```

Maple [A]
time = 0.07, size = 22, normalized size = 0.96

method	result	size
default	$-\frac{b^2}{3x^3} - \frac{2bc}{x} + c^2x$	22
risch	$c^2x + \frac{-2bc\,x^2 - \frac{1}{3}b^2}{x^3}$	24
gosper	$-\frac{-3c^2x^4 + 6bc\,x^2 + b^2}{3x^3}$	25
norman	$\frac{c^2x^8 - \frac{1}{3}b^2x^4 - 2bc\,x^6}{x^7}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^8,x,method=_RETURNVERBOSE)
[Out] -1/3*b^2/x^3-2*b*c/x+c^2*x
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.96

$$c^2x - \frac{6\,bcx^2 + b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="maxima")
[Out] c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

Fricas [A]
time = 0.34, size = 26, normalized size = 1.13

$$\frac{3\,c^2x^4 - 6\,bcx^2 - b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^2*x^4 - 6*b*c*x^2 - b^2)/x^3
```

Sympy [A]
time = 0.05, size = 22, normalized size = 0.96

$$c^2x + \frac{-b^2 - 6bcx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**8,x)
```

```
[Out] c**2*x + (-b**2 - 6*b*c*x**2)/(3*x**3)
```

Giac [A]
time = 3.72, size = 22, normalized size = 0.96

$$c^2x - \frac{6\,bcx^2 + b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="giac")
```

```
[Out] c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

Mupad [B]
time = 0.03, size = 24, normalized size = 1.04

$$c^2\,x - \frac{\frac{b^2}{3} + 2\,c\,b\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^8,x)
```

```
[Out] c^2*x - (b^2/3 + 2*b*c*x^2)/x^3
```

3.154 $\int \frac{(bx^2+cx^4)^2}{x^9} dx$

Optimal. Leaf size=24

$$-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)$$

```
[Out] -1/4*b^2/x^4-b*c/x^2+c^2*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 45}

$$-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -1/4*b^2/x^4 - (b*c)/x^2 + c^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^2}{x^9}\,dx = \int \frac{(b+cx^2)^2}{x^5}\,dx$$
$$= \frac{1}{2}\mathrm{Subst}\left(\int \frac{(b+cx)^2}{x^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{b^2}{x^3}+\frac{2bc}{x^2}+\frac{c^2}{x}\right)dx, x, x^2\right)$$
$$= -\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$-\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^9,x]
[Out] -1/4*b^2/x^4 - (b*c)/x^2 + c^2*Log[x]
```

Maple [A]
time = 0.07, size = 23, normalized size = 0.96

method	result	size
default	$-\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\ln(x)$	23
risch	$\frac{-bc\,x^2-\frac{1}{4}b^2}{x^4}+c^2\ln(x)$	25
norman	$\frac{-\frac{1}{4}b^2x^4-bc\,x^6}{x^8}+c^2\ln(x)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^9,x,method=_RETURNVERBOSE)
[Out] -1/4*b^2/x^4-b*c/x^2+c^2*ln(x)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 1.08

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{4\,bcx^2+b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2) - 1/4*(4*b*c*x^2 + b^2)/x^4
```

Fricas [A]
time = 0.33, size = 28, normalized size = 1.17

$$\frac{4\,c^2x^4\log\left(x\right)-4\,bcx^2-b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/4*(4*c^2*x^4*log(x) - 4*b*c*x^2 - b^2)/x^4
```

Sympy [A]
time = 0.07, size = 24, normalized size = 1.00

$$c^2\log\left(x\right)+\frac{-b^2-4bcx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**9,x)
```

```
[Out] c**2*log(x) + (-b**2 - 4*b*c*x**2)/(4*x**4)
```

Giac [A]
time = 4.63, size = 34, normalized size = 1.42

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{3\,c^2x^4+4\,bcx^2+b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2) - 1/4*(3*c^2*x^4 + 4*b*c*x^2 + b^2)/x^4
```

Mupad [B]
time = 0.05, size = 24, normalized size = 1.00

$$c^2\,\ln\left(x\right)-\frac{\frac{b^2}{4}+c\,b\,x^2}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^9,x)
```

```
[Out] c^2*log(x) - (b^2/4 + b*c*x^2)/x^4
```

3.155 $\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx$

Optimal. Leaf size=28

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

```
[Out] -1/5*b^2/x^5-2/3*b*c/x^3-c^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 276}

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -1/5*b^2/x^5 - (2*b*c)/(3*x^3) - c^2/x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx &= \int \frac{(b+cx^2)^2}{x^6}\,dx\\ &= \int \left(\frac{b^2}{x^6}+\frac{2bc}{x^4}+\frac{c^2}{x^2}\right)dx\\ &= -\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^10,x]
[Out] -1/5*b^2/x^5 - (2*b*c)/(3*x^3) - c^2/x
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.89

method	result	size
default	$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$	25
risch	$\frac{-c^2x^4-\frac{2}{3}bc\,x^2-\frac{1}{5}b^2}{x^5}$	26
gosper	$-\frac{15c^2x^4+10bc\,x^2+3b^2}{15x^5}$	27
norman	$\frac{-\frac{1}{5}b^2x^4-c^2x^8-\frac{2}{3}bc\,x^6}{x^9}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^10,x,method=_RETURNVERBOSE)
[Out] -1/5*b^2/x^5-2/3*b*c/x^3-c^2/x
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.93

$$-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="maxima")
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

Fricas [A]
time = 0.33, size = 26, normalized size = 0.93

$$-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

Sympy [A]
time = 0.08, size = 27, normalized size = 0.96

$$\frac{-3b^2 - 10bcx^2 - 15c^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**10,x)
```

```
[Out] (-3*b**2 - 10*b*c*x**2 - 15*c**2*x**4)/(15*x**5)
```

Giac [A]
time = 4.64, size = 26, normalized size = 0.93

$$-\frac{15\,c^2x^4 + 10\,bcx^2 + 3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.89

$$-\frac{\frac{b^2}{5} + \frac{2\,b\,c\,x^2}{3} + c^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^10,x)
```

```
[Out] -(b^2/5 + c^2*x^4 + (2*b*c*x^2)/3)/x^5
```

3.156 $\int \frac{(bx^2+cx^4)^2}{x^{11}} dx$

Optimal. Leaf size=19

$$-\frac{(b+cx^2)^3}{6bx^6}$$

[Out] -1/6*(c*x^2+b)^3/b/x^6

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 270}

$$-\frac{(b+cx^2)^3}{6bx^6}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)^2/x^11,x]

[Out] -1/6*(b + c*x^2)^3/(b*x^6)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx &= \int \frac{(b+cx^2)^2}{x^7}\,dx\\ &= -\frac{(b+cx^2)^3}{6bx^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.58

$$-\frac{b^2}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -1/6*b^2/x^6 - (b*c)/(2*x^4) - c^2/(2*x^2)
```

Maple [A]
time = 0.09, size = 25, normalized size = 1.32

method	result	size
gosper	$-\frac{3c^2x^4+3bc\,x^2+b^2}{6x^6}$	25
default	$-\frac{bc}{2x^4}-\frac{c^2}{2x^2}-\frac{b^2}{6x^6}$	25
risch	$\frac{-\frac{1}{2}c^2x^4-\frac{1}{2}bc\,x^2-\frac{1}{6}b^2}{x^6}$	26
norman	$\frac{-\frac{1}{6}b^2x^4-\frac{1}{2}c^2x^8-\frac{1}{2}bc\,x^6}{x^{10}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*c/x^4-1/2*c^2/x^2-1/6*b^2/x^6
```

Maxima [A]
time = 0.28, size = 24, normalized size = 1.26

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

Fricas [A]
time = 0.33, size = 24, normalized size = 1.26

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

Sympy [A]
time = 0.08, size = 26, normalized size = 1.37

$$\frac{-b^2-3bcx^2-3c^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**11,x)
```

```
[Out] (-b**2 - 3*b*c*x**2 - 3*c**2*x**4)/(6*x**6)
```

Giac [A]
time = 4.49, size = 24, normalized size = 1.26

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

Mupad [B]
time = 0.04, size = 26, normalized size = 1.37

$$-\frac{\frac{b^2}{6}+\frac{b\,c\,x^2}{2}+\frac{c^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^11,x)
```

```
[Out] -(b^2/6 + (c^2*x^4)/2 + (b*c*x^2)/2)/x^6
```

3.157 $\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx$

Optimal. Leaf size=30

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

```
[Out] -1/7*b^2/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 276}

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -1/7*b^2/x^7 - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx &= \int \frac{(b+cx^2)^2}{x^8}\,dx\\ &= \int \left(\frac{b^2}{x^8}+\frac{2bc}{x^6}+\frac{c^2}{x^4}\right)dx\\ &= -\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^12,x]
[Out] -1/7*b^2/x^7 - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.83

method	result	size
default	$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$	25
risch	$\frac{-\frac{1}{3}c^2x^4-\frac{2}{5}bc\,x^2-\frac{1}{7}b^2}{x^7}$	26
gosper	$-\frac{35c^2x^4+42bc\,x^2+15b^2}{105x^7}$	27
norman	$\frac{-\frac{1}{7}b^2x^4-\frac{1}{3}c^2x^8-\frac{2}{5}bc\,x^6}{x^{11}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^12,x,method=_RETURNVERBOSE)
[Out] -1/7*b^2/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.87

$$-\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="maxima")
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

Fricas [A]
time = 0.33, size = 26, normalized size = 0.87

$$-\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

Sympy [A]
time = 0.08, size = 27, normalized size = 0.90

$$\frac{-15b^2 - 42bcx^2 - 35c^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**12,x)
```

```
[Out] (-15*b**2 - 42*b*c*x**2 - 35*c**2*x**4)/(105*x**7)
```

Giac [A]
time = 3.82, size = 26, normalized size = 0.87

$$-\frac{35\,c^2x^4 + 42\,bcx^2 + 15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.87

$$-\frac{\frac{b^2}{7} + \frac{2\,b\,c\,x^2}{5} + \frac{c^2\,x^4}{3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^12,x)
```

```
[Out] -(b^2/7 + (c^2*x^4)/3 + (2*b*c*x^2)/5)/x^7
```

3.158 $\int \frac{(bx^2+cx^4)^3}{x^2}\,dx$

Optimal. Leaf size=43

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

```
[Out] 1/5*b^3*x^5+3/7*b^2*c*x^7+1/3*b*c^2*x^9+1/11*c^3*x^11
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {1598, 276}

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^2}\,dx &= \int x^4\left(b+cx^2\right)^3\,dx\\
&= \int \left(b^3x^4+3b^2cx^6+3bc^2x^8+c^3x^{10}\right)\,dx\\
&= \frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^2,x]
[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Maple [A]
time = 0.08, size = 36, normalized size = 0.84

method	result	size
default	$\frac{1}{5}b^3x^5+\frac{3}{7}b^2c\,x^7+\frac{1}{3}b\,c^2x^9+\frac{1}{11}c^3x^{11}$	36
risch	$\frac{1}{5}b^3x^5+\frac{3}{7}b^2c\,x^7+\frac{1}{3}b\,c^2x^9+\frac{1}{11}c^3x^{11}$	36
gosper	$\frac{x^5\left(105c^3x^6+385b\,c^2x^4+495b^2c\,x^2+231b^3\right)}{1155}$	38
norman	$\frac{\frac{1}{5}b^3x^6+\frac{1}{11}c^3x^{12}+\frac{1}{3}b\,c^2x^{10}+\frac{3}{7}b^2c\,x^8}{x}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^2,x,method=_RETURNVERBOSE)
[Out] 1/5*b^3*x^5+3/7*b^2*c*x^7+1/3*b*c^2*x^9+1/11*c^3*x^11
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.81

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="maxima")
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

Fricas [A]
time = 0.32, size = 35, normalized size = 0.81

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

Sympy [A]
time = 0.01, size = 37, normalized size = 0.86

$$\frac{b^3x^5}{5} + \frac{3b^2cx^7}{7} + \frac{bc^2x^9}{3} + \frac{c^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**2,x)
```

```
[Out] b**3*x**5/5 + 3*b**2*c*x**7/7 + b*c**2*x**9/3 + c**3*x**11/11
```

Giac [A]
time = 4.18, size = 35, normalized size = 0.81

$$\frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,b^2cx^7 + \frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{b^3\,x^5}{5} + \frac{3\,b^2\,c\,x^7}{7} + \frac{b\,c^2\,x^9}{3} + \frac{c^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^2,x)
```

```
[Out] (b^3*x^5)/5 + (c^3*x^11)/11 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3
```

3.159 $\int \frac{(bx^2+cx^4)^3}{x^3} dx$

Optimal. Leaf size=34

$$-\frac{b(b+cx^2)^4}{8c^2}+\frac{(b+cx^2)^5}{10c^2}$$

```
[Out] -1/8*b*(c*x^2+b)^4/c^2+1/10*(c*x^2+b)^5/c^2
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 45}

$$\frac{(b+cx^2)^5}{10c^2}-\frac{b(b+cx^2)^4}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] -1/8*(b*(b + c*x^2)^4)/c^2 + (b + c*x^2)^5/(10*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^3}\,dx = \int x^3\left(b+cx^2\right)^3\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int x(b+cx)^3\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b(b+cx)^3}{c}+\frac{(b+cx)^4}{c}\right)dx, x, x^2\right)$$
$$= -\frac{b(b+cx^2)^4}{8c^2}+\frac{(b+cx^2)^5}{10c^2}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.26

$$\frac{b^3x^4}{4}+\frac{1}{2}b^2cx^6+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] (b^3*x^4)/4 + (b^2*c*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10
```

Maple [A]
time = 0.09, size = 36, normalized size = 1.06

method	result	size
default	$\frac{1}{10}c^3x^{10}+\frac{3}{8}b\,c^2x^8+\frac{1}{2}b^2c\,x^6+\frac{1}{4}b^3x^4$	36
risch	$\frac{1}{10}c^3x^{10}+\frac{3}{8}b\,c^2x^8+\frac{1}{2}b^2c\,x^6+\frac{1}{4}b^3x^4$	36
gosper	$\frac{x^4\left(4c^3x^6+15b\,c^2x^4+20b^2c\,x^2+10b^3\right)}{40}$	38
norman	$\frac{\frac{1}{4}b^3x^6+\frac{1}{10}c^3x^{12}+\frac{3}{8}b\,c^2x^{10}+\frac{1}{2}b^2c\,x^8}{x^2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c^3*x^10+3/8*b*c^2*x^8+1/2*b^2*c*x^6+1/4*b^3*x^4
```

Maxima [A]
time = 0.27, size = 35, normalized size = 1.03

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

Fricas [A]
time = 0.33, size = 35, normalized size = 1.03

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

Sympy [A]
time = 0.01, size = 37, normalized size = 1.09

$$\frac{b^3x^4}{4}+\frac{b^2cx^6}{2}+\frac{3bc^2x^8}{8}+\frac{c^3x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**3,x)
```

```
[Out] b**3*x**4/4 + b**2*c*x**6/2 + 3*b*c**2*x**8/8 + c**3*x**10/10
```

Giac [A]
time = 3.48, size = 35, normalized size = 1.03

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

Mupad [B]
time = 0.04, size = 35, normalized size = 1.03

$$\frac{b^3\,x^4}{4}+\frac{b^2\,c\,x^6}{2}+\frac{3\,b\,c^2\,x^8}{8}+\frac{c^3\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^3,x)
```

```
[Out] (b^3*x^4)/4 + (c^3*x^10)/10 + (b^2*c*x^6)/2 + (3*b*c^2*x^8)/8
```

3.160 $\int \frac{(bx^2+cx^4)^3}{x^4}\,dx$

Optimal. Leaf size=43

$$\frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

```
[Out] 1/3*b^3*x^3+3/5*b^2*c*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 276}

$$\frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^4}\,dx &= \int x^2\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^2+3b^2cx^4+3bc^2x^6+c^3x^8\right)dx\\ &= \frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^4,x]
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Maple [A]
time = 0.08, size = 36, normalized size = 0.84

method	result	size
default	$\frac{1}{3}b^3x^3+\frac{3}{5}b^2c\,x^5+\frac{3}{7}b\,c^2x^7+\frac{1}{9}c^3x^9$	36
risch	$\frac{1}{3}b^3x^3+\frac{3}{5}b^2c\,x^5+\frac{3}{7}b\,c^2x^7+\frac{1}{9}c^3x^9$	36
gosper	$\frac{x^3(35c^3x^6+135b\,c^2x^4+189b^2c\,x^2+105b^3)}{315}$	38
norman	$\frac{\frac{1}{3}b^3x^6+\frac{1}{9}c^3x^{12}+\frac{3}{7}b\,c^2x^{10}+\frac{3}{5}b^2c\,x^8}{x^3}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^4,x,method=_RETURNVERBOSE)
[Out] 1/3*b^3*x^3+3/5*b^2*c*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.81

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{1}{3}\,b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="maxima")
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

Fricas [A]
time = 0.31, size = 35, normalized size = 0.81

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{1}{3}\,b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

Sympy [**A**]
time = 0.01, size = 39, normalized size = 0.91

$$\frac{b^3x^3}{3} + \frac{3b^2cx^5}{5} + \frac{3bc^2x^7}{7} + \frac{c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**4,x)
```

```
[Out] b**3*x**3/3 + 3*b**2*c*x**5/5 + 3*b*c**2*x**7/7 + c**3*x**9/9
```

Giac [**A**]
time = 4.58, size = 35, normalized size = 0.81

$$\frac{1}{9}\,c^3x^9 + \frac{3}{7}\,bc^2x^7 + \frac{3}{5}\,b^2cx^5 + \frac{1}{3}\,b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

Mupad [**B**]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{b^3\,x^3}{3} + \frac{3\,b^2\,c\,x^5}{5} + \frac{3\,b\,c^2\,x^7}{7} + \frac{c^3\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^4,x)
```

```
[Out] (b^3*x^3)/3 + (c^3*x^9)/9 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7
```

3.161 $\int \frac{(bx^2+cx^4)^3}{x^5}\,dx$

Optimal. Leaf size=16

$$\frac{(b+cx^2)^4}{8c}$$

[Out] 1/8*(c*x^2+b)^4/c

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 267}

$$\frac{(b+cx^2)^4}{8c}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)^3/x^5,x]

[Out] (b + c*x^2)^4/(8*c)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^5}\,dx &= \int x\left(b+cx^2\right)^3\,dx\\ &= \frac{(b+cx^2)^4}{8c}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{(b+cx^2)^4}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^5,x]
```

```
[Out] (b + c*x^2)^4/(8*c)
```

Maple [A]
time = 0.08, size = 15, normalized size = 0.94

method	result	size
default	$\frac{\left(c\,x^{2}+b\right)^{4}}{8c}$	15
gosper	$\frac{x^{2}\left(c^{3}x^{6}+4b\,c^{2}x^{4}+6b^{2}c\,x^{2}+4b^{3}\right)}{8}$	37
norman	$\frac{\frac{1}{2}b^{3}x^{6}+\frac{1}{8}c^{3}x^{12}+\frac{1}{2}b\,c^{2}x^{10}+\frac{3}{4}b^{2}c\,x^{8}}{x^{4}}$	40
risch	$\frac{c^{3}x^{8}}{8}+\frac{b\,c^{2}x^{6}}{2}+\frac{3b^{2}c\,x^{4}}{4}+\frac{b^{3}x^{2}}{2}+\frac{b^{4}}{8c}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*x^2+b)^4/c
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.
time = 0.30, size = 35, normalized size = 2.19

$$\frac{1}{8}c^3x^8 + \frac{1}{2}bc^2x^6 + \frac{3}{4}b^2cx^4 + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.
time = 0.32, size = 35, normalized size = 2.19

$$\frac{1}{8}c^3x^8 + \frac{1}{2}bc^2x^6 + \frac{3}{4}b^2cx^4 + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.
time = 0.01, size = 37, normalized size = 2.31

$$\frac{b^3x^2}{2} + \frac{3b^2cx^4}{4} + \frac{bc^2x^6}{2} + \frac{c^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**5,x)
```

```
[Out] b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 + c**3*x**8/8
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.
time = 5.04, size = 35, normalized size = 2.19

$$\frac{1}{8}c^3x^8 + \frac{1}{2}bc^2x^6 + \frac{3}{4}b^2cx^4 + \frac{1}{2}b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

Mupad [B]
time = 0.04, size = 35, normalized size = 2.19

$$\frac{b^3\,x^2}{2} + \frac{3\,b^2\,c\,x^4}{4} + \frac{b\,c^2\,x^6}{2} + \frac{c^3\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^5,x)
```

```
[Out] (b^3*x^2)/2 + (c^3*x^8)/8 + (3*b^2*c*x^4)/4 + (b*c^2*x^6)/2
```

3.162 $\int \frac{(bx^2+cx^4)^3}{x^6} dx$

Optimal. Leaf size=35

$$b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

[Out] b^3*x+b^2*c*x^3+3/5*b*c^2*x^5+1/7*c^3*x^7

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 200}

$$b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)^3/x^6,x]

[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^6}\,dx &= \int \left(b+cx^2\right)^3\,dx \\
&= \int \left(b^3+3b^2cx^2+3bc^2x^4+c^3x^6\right)\,dx \\
&= b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^6,x]
[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

Maple [A]
time = 0.08, size = 32, normalized size = 0.91

method	result	size
default	$b^{3} x +b^{2} c \,x^{3}+\frac{3}{5} b \,c^{2} x^{5}+\frac{1}{7} c^{3} x^{7}$	32
risch	$b^{3} x +b^{2} c \,x^{3}+\frac{3}{5} b \,c^{2} x^{5}+\frac{1}{7} c^{3} x^{7}$	32
gosper	$\frac{x \left(5 c^{3} x^{6}+21 b \,c^{2} x^{4}+35 b^{2} c \,x^{2}+35 b^{3}\right)}{35}$	36
norman	$\frac{b^{3} x^{6}+b^{2} c \,x^{8}+\frac{1}{7} c^{3} x^{12}+\frac{3}{5} b \,c^{2} x^{10}}{x^{5}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^6,x,method=_RETURNVERBOSE)
[Out] b^3*x+b^2*c*x^3+3/5*b*c^2*x^5+1/7*c^3*x^7
```

Maxima [A]
time = 0.28, size = 31, normalized size = 0.89

$$\frac{1}{7} c^{3} x^{7} + \frac{3}{5} b c^{2} x^{5} + b^{2} c x^{3} + b^{3} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="maxima")
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

Fricas [A]
time = 0.31, size = 31, normalized size = 0.89

$$\frac{1}{7} c^{3} x^{7} + \frac{3}{5} b c^{2} x^{5} + b^{2} c x^{3} + b^{3} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="fricas")
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

Sympy [A]
time = 0.01, size = 32, normalized size = 0.91

$$b^{3} x + b^{2} c x^{3} + \frac{3 b c^{2} x^{5}}{5} + \frac{c^{3} x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**6,x)
```

```
[Out] b**3*x + b**2*c*x**3 + 3*b*c**2*x**5/5 + c**3*x**7/7
```

Giac [A]
time = 3.49, size = 31, normalized size = 0.89

$$\frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.89

$$b^3\,x + b^2\,c\,x^3 + \frac{3\,b\,c^2\,x^5}{5} + \frac{c^3\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^6,x)
```

```
[Out] b^3*x + (c^3*x^7)/7 + b^2*c*x^3 + (3*b*c^2*x^5)/5
```

3.163 $\int \frac{(bx^2+cx^4)^3}{x^7} dx$

Optimal. Leaf size=39

$$\frac{3}{2}b^2cx^2 + \frac{3}{4}bc^2x^4 + \frac{c^3x^6}{6} + b^3\log(x)$$

```
[Out] 3/2*b^2*c*x^2+3/4*b*c^2*x^4+1/6*c^3*x^6+b^3*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 45}

$$b^3\log(x) + \frac{3}{2}b^2cx^2 + \frac{3}{4}bc^2x^4 + \frac{c^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^7,x]
```

```
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^7}\,dx = \int \frac{(b+cx^2)^3}{x}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(b+cx)^3}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(3b^2c+\frac{b^3}{x}+3bc^2x+c^3x^2\right)\,dx, x, x^2\right)$$
$$= \frac{3}{2}b^2cx^2+\frac{3}{4}bc^2x^4+\frac{c^3x^6}{6}+b^3\log(x)$$

Mathematica [A]
time = 0.00, size = 39, normalized size = 1.00

$$\frac{3}{2}b^2cx^2+\frac{3}{4}bc^2x^4+\frac{c^3x^6}{6}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^7,x]
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

Maple [A]
time = 0.08, size = 34, normalized size = 0.87

method	result	size
default	$\frac{3b^2cx^2}{2}+\frac{3bc^2x^4}{4}+\frac{c^3x^6}{6}+b^3\ln(x)$	34
risch	$\frac{3b^2cx^2}{2}+\frac{3bc^2x^4}{4}+\frac{c^3x^6}{6}+b^3\ln(x)$	34
norman	$\frac{\frac{1}{6}c^3x^{12}+\frac{3}{4}bc^2x^{10}+\frac{3}{2}b^2cx^8}{x^6}+b^3\ln(x)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^7,x,method=_RETURNVERBOSE)
[Out] 3/2*b^2*c*x^2+3/4*b*c^2*x^4+1/6*c^3*x^6+b^3*ln(x)
```

Maxima [A]
time = 0.29, size = 36, normalized size = 0.92

$$\frac{1}{6}c^3x^6+\frac{3}{4}bc^2x^4+\frac{3}{2}b^2cx^2+\frac{1}{2}b^3\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*log(x^2)
```

Fricas [A]
time = 0.34, size = 33, normalized size = 0.85

$$\frac{1}{6}\,c^3x^6+\frac{3}{4}\,bc^2x^4+\frac{3}{2}\,b^2cx^2+b^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + b^3*log(x)
```

Sympy [A]
time = 0.04, size = 37, normalized size = 0.95

$$b^3\log\left(x\right)+\frac{3b^2cx^2}{2}+\frac{3bc^2x^4}{4}+\frac{c^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**7,x)
```

```
[Out] b**3*log(x) + 3*b**2*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6
```

Giac [A]
time = 4.82, size = 36, normalized size = 0.92

$$\frac{1}{6}\,c^3x^6+\frac{3}{4}\,bc^2x^4+\frac{3}{2}\,b^2cx^2+\frac{1}{2}\,b^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*log(x^2)
```

Mupad [B]
time = 0.04, size = 33, normalized size = 0.85

$$b^3\,\ln\left(x\right)+\frac{c^3\,x^6}{6}+\frac{3\,b^2\,c\,x^2}{2}+\frac{3\,b\,c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^7,x)
```

```
[Out] b^3*log(x) + (c^3*x^6)/6 + (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4
```

3.164 $\int \frac{(bx^2+cx^4)^3}{x^8} \, dx$

Optimal. Leaf size=34

$$-\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

[Out] -b^3/x+3*b^2*c*x+b*c^2*x^3+1/5*c^3*x^5

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 276}

$$-\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)^3/x^8,x]

[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^8} \, dx &= \int \frac{(b+cx^2)^3}{x^2} \, dx \\
&= \int \left(3b^2c + \frac{b^3}{x^2} + 3bc^2x^2 + c^3x^4\right) dx \\
&= -\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 1.00

$$-\frac{b^3}{x}+3b^2cx+bc^2x^3+\frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^8,x]
[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

Maple [A]
time = 0.08, size = 33, normalized size = 0.97

method	result	size
default	$-\frac{b^3}{x}+3b^2cx+b\,c^2x^3+\frac{c^3x^5}{5}$	33
risch	$-\frac{b^3}{x}+3b^2cx+b\,c^2x^3+\frac{c^3x^5}{5}$	33
gosper	$-\frac{-c^3x^6-5b\,c^2x^4-15b^2c\,x^2+5b^3}{5x}$	38
norman	$\frac{b\,c^2x^{10}-b^3x^6+\frac{1}{5}c^3x^{12}+3b^2c\,x^8}{x^7}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^8,x,method=_RETURNVERBOSE)
[Out] -b^3/x+3*b^2*c*x+b*c^2*x^3+1/5*c^3*x^5
```

Maxima [A]
time = 0.32, size = 32, normalized size = 0.94

$$\frac{1}{5}\,c^3x^5+bc^2x^3+3\,b^2cx-\frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="maxima")
[Out] 1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

Fricas [A]
time = 0.33, size = 36, normalized size = 1.06

$$\frac{c^3x^6+5\,bc^2x^4+15\,b^2cx^2-5\,b^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/5*(c^3*x^6 + 5*b*c^2*x^4 + 15*b^2*c*x^2 - 5*b^3)/x
```

Sympy [A]
time = 0.03, size = 29, normalized size = 0.85

$$-\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**8,x)
[Out] -b**3/x + 3*b**2*c*x + b*c**2*x**3 + c**3*x**5/5
```

Giac [A]
time = 5.69, size = 32, normalized size = 0.94

$$\frac{1}{5}c^3x^5 + bc^2x^3 + 3\,b^2cx - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="giac")
[Out] 1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

Mupad [B]
time = 0.04, size = 32, normalized size = 0.94

$$\frac{c^3\,x^5}{5} - \frac{b^3}{x} + b\,c^2\,x^3 + 3\,b^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^8,x)
[Out] (c^3*x^5)/5 - b^3/x + b*c^2*x^3 + 3*b^2*c*x
```

3.165 $\int \frac{(bx^2+cx^4)^3}{x^9} dx$

Optimal. Leaf size=40

$$-\frac{b^3}{2x^2} + \frac{3}{2}bc^2x^2 + \frac{c^3x^4}{4} + 3b^2c\log(x)$$

```
[Out] -1/2*b^3/x^2+3/2*b*c^2*x^2+1/4*c^3*x^4+3*b^2*c*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 45}

$$-\frac{b^3}{2x^2} + 3b^2c\log(x) + \frac{3}{2}bc^2x^2 + \frac{c^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^9,x]
```

```
[Out] -1/2*b^3/x^2 + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^9}\,dx=\int \frac{(b+cx^2)^3}{x^3}\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int \frac{(b+cx)^3}{x^2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int \left(3bc^2+\frac{b^3}{x^2}+\frac{3b^2c}{x}+c^3x\right)\,dx,x,x^2\right)$$
$$=-\frac{b^3}{2x^2}+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}+3b^2c\log(x)$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.00

$$-\frac{b^3}{2x^2}+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}+3b^2c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^9,x]
[Out] -1/2*b^3/x^2 + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

Maple [A]
time = 0.10, size = 35, normalized size = 0.88

method	result	size
default	$-\frac{b^3}{2x^2}+\frac{3b\,c^2x^2}{2}+\frac{c^3x^4}{4}+3b^2c\ln\left(x\right)$	35
norman	$\frac{-\frac{1}{2}b^3x^6+\frac{1}{4}c^3x^{12}+\frac{3}{2}b\,c^2x^{10}}{x^8}+3b^2c\ln\left(x\right)$	40
risch	$\frac{c^3x^4}{4}+\frac{3b\,c^2x^2}{2}+\frac{9b^2c}{4}-\frac{b^3}{2x^2}+3b^2c\ln\left(x\right)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^9,x,method=_RETURNVERBOSE)
[Out] -1/2*b^3/x^2+3/2*b*c^2*x^2+1/4*c^3*x^4+3*b^2*c*ln(x)
```

Maxima [A]
time = 0.27, size = 36, normalized size = 0.90

$$\frac{1}{4}\,c^3x^4+\frac{3}{2}\,bc^2x^2+\frac{3}{2}\,b^2c\log\left(x^2\right)-\frac{b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*log(x^2) - 1/2*b^3/x^2
```

Fricas [A]
time = 0.34, size = 38, normalized size = 0.95

$$\frac{c^3x^6 + 6\,bc^2x^4 + 12\,b^2cx^2\log\left(x\right) - 2\,b^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*x^6 + 6*b*c^2*x^4 + 12*b^2*c*x^2*log(x) - 2*b^3)/x^2
```

Sympy [A]
time = 0.05, size = 37, normalized size = 0.92

$$-\frac{b^3}{2x^2} + 3b^2c\log\left(x\right) + \frac{3bc^2x^2}{2} + \frac{c^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**9,x)
```

```
[Out] -b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4
```

Giac [A]
time = 4.63, size = 46, normalized size = 1.15

$$\frac{1}{4}\,c^3x^4 + \frac{3}{2}\,bc^2x^2 + \frac{3}{2}\,b^2c\log\left(x^2\right) - \frac{3\,b^2cx^2 + b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*log(x^2) - 1/2*(3*b^2*c*x^2 + b^3)/
x^2
```

Mupad [B]
time = 0.04, size = 34, normalized size = 0.85

$$\frac{c^3\,x^4}{4} - \frac{b^3}{2\,x^2} + \frac{3\,b\,c^2\,x^2}{2} + 3\,b^2\,c\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^9,x)
```

```
[Out] (c^3*x^4)/4 - b^3/(2*x^2) + (3*b*c^2*x^2)/2 + 3*b^2*c*log(x)
```

3.166 $\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx$

Optimal. Leaf size=37

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

```
[Out] -1/3*b^3/x^3-3*b^2*c/x+3*b*c^2*x+1/3*c^3*x^3
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 276}

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^10,x]
```

```
[Out] -1/3*b^3/x^3 - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx &= \int \frac{(b+cx^2)^3}{x^4}\,dx\\
&= \int \left(3bc^2+\frac{b^3}{x^4}+\frac{3b^2c}{x^2}+c^3x^2\right)dx\\
&= -\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.00

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^10,x]
[Out] -1/3*b^3/x^3 - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Maple [A]
time = 0.09, size = 34, normalized size = 0.92

method	result	size
default	$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3b\,c^2x+\frac{c^3x^3}{3}$	34
gosper	$-\frac{-c^3x^6-9b\,c^2x^4+9b^2c\,x^2+b^3}{3x^3}$	36
risch	$\frac{c^3x^3}{3}+3b\,c^2x+\frac{-3b^2c\,x^2-\frac{1}{3}b^3}{x^3}$	36
norman	$\frac{-\frac{1}{3}b^3x^6+\frac{1}{3}c^3x^{12}+3b\,c^2x^{10}-3b^2c\,x^8}{x^9}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^10,x,method=_RETURNVERBOSE)
[Out] -1/3*b^3/x^3-3*b^2*c/x+3*b*c^2*x+1/3*c^3*x^3
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.92

$$\frac{1}{3}\,c^3x^3+3\,bc^2x-\frac{9\,b^2cx^2+b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="maxima")
[Out] 1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

Fricas [A]
time = 0.36, size = 36, normalized size = 0.97

$$\frac{c^3x^6+9\,bc^2x^4-9\,b^2cx^2-b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/3*(c^3*x^6 + 9*b*c^2*x^4 - 9*b^2*c*x^2 - b^3)/x^3
```

Sympy [A]
time = 0.06, size = 36, normalized size = 0.97

$$3bc^2x + \frac{c^3x^3}{3} + \frac{-b^3 - 9b^2cx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**10,x)
[Out] 3*b*c**2*x + c**3*x**3/3 + (-b**3 - 9*b**2*c*x**2)/(3*x**3)
```

Giac [A]
time = 3.71, size = 34, normalized size = 0.92

$$\frac{1}{3}\,c^3x^3 + 3\,bc^2x - \frac{9\,b^2cx^2 + b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="giac")
[Out] 1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

Mupad [B]
time = 0.04, size = 36, normalized size = 0.97

$$\frac{c^3\,x^3}{3} - \frac{\frac{b^3}{3} + 3\,c\,b^2\,x^2}{x^3} + 3\,b\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^10,x)
[Out] (c^3*x^3)/3 - (b^3/3 + 3*b^2*c*x^2)/x^3 + 3*b*c^2*x
```

3.167 $\int \frac{(bx^2+cx^4)^3}{x^{11}} dx$

Optimal. Leaf size=40

$$-\frac{b^3}{4x^4} - \frac{3b^2c}{2x^2} + \frac{c^3x^2}{2} + 3bc^2\log(x)$$

```
[Out] -1/4*b^3/x^4-3/2*b^2*c/x^2+1/2*c^3*x^2+3*b*c^2*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 45}

$$-\frac{b^3}{4x^4} - \frac{3b^2c}{2x^2} + 3bc^2\log(x) + \frac{c^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^11,x]
```

```
[Out] -1/4*b^3/x^4 - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx &= \int \frac{(b+cx^2)^3}{x^5}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(b+cx)^3}{x^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(c^3+\frac{b^3}{x^3}+\frac{3b^2c}{x^2}+\frac{3bc^2}{x}\right)\,dx, x, x^2\right) \\
&= -\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+\frac{c^3x^2}{2}+3bc^2\log(x)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.00

$$-\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+\frac{c^3x^2}{2}+3bc^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^11,x]
[Out] -1/4*b^3/x^4 - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

Maple [A]
time = 0.08, size = 35, normalized size = 0.88

method	result	size
default	$-\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+\frac{c^3x^2}{2}+3b\,c^2\ln\left(x\right)$	35
risch	$\frac{c^3x^2}{2}+\frac{-\frac{3}{2}b^2c\,x^2-\frac{1}{4}b^3}{x^4}+3b\,c^2\ln\left(x\right)$	37
norman	$\frac{-\frac{1}{4}b^3x^6+\frac{1}{2}c^3x^{12}-\frac{3}{2}b^2c\,x^8}{x^{10}}+3b\,c^2\ln\left(x\right)$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^11,x,method=_RETURNVERBOSE)
[Out] -1/4*b^3/x^4-3/2*b^2*c/x^2+1/2*c^3*x^2+3*b*c^2*ln(x)
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.92

$$\frac{1}{2}\,c^3x^2+\frac{3}{2}\,bc^2\log\left(x^2\right)-\frac{6\,b^2cx^2+b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*x^2 + 3/2*b*c^2*log(x^2) - 1/4*(6*b^2*c*x^2 + b^3)/x^4
```

Fricas [A]
time = 0.34, size = 39, normalized size = 0.98

$$\frac{2\,c^3x^6 + 12\,bc^2x^4\log\left(x\right) - 6\,b^2cx^2 - b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*x^6 + 12*b*c^2*x^4*log(x) - 6*b^2*c*x^2 - b^3)/x^4
```

Sympy [A]
time = 0.09, size = 37, normalized size = 0.92

$$3bc^2\log\left(x\right) + \frac{c^3x^2}{2} + \frac{-b^3 - 6b^2cx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**11,x)
```

```
[Out] 3*b*c**2*log(x) + c**3*x**2/2 + (-b**3 - 6*b**2*c*x**2)/(4*x**4)
```

Giac [A]
time = 3.89, size = 46, normalized size = 1.15

$$\frac{1}{2}\,c^3x^2 + \frac{3}{2}\,bc^2\log\left(x^2\right) - \frac{9\,bc^2x^4 + 6\,b^2cx^2 + b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/2*c^3*x^2 + 3/2*b*c^2*log(x^2) - 1/4*(9*b*c^2*x^4 + 6*b^2*c*x^2 + b^3)/x^
4
```

Mupad [B]
time = 0.03, size = 37, normalized size = 0.92

$$\frac{c^3\,x^2}{2} - \frac{\frac{b^3}{4} + \frac{3\,c\,b^2\,x^2}{2}}{x^4} + 3\,b\,c^2\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^11,x)
```

```
[Out] (c^3*x^2)/2 - (b^3/4 + (3*b^2*c*x^2)/2)/x^4 + 3*b*c^2*log(x)
```

3.168 $\int \frac{(bx^2+cx^4)^3}{x^{12}} dx$

Optimal. Leaf size=34

$$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x$$

```
[Out] -1/5*b^3/x^5-b^2*c/x^3-3*b*c^2/x+c^3*x
```

Rubi [A]

time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 276}

$$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^12,x]
```

```
[Out] -1/5*b^3/x^5 - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx &= \int \frac{(b+cx^2)^3}{x^6}\,dx\\ &= \int \left(c^3+\frac{b^3}{x^6}+\frac{3b^2c}{x^4}+\frac{3bc^2}{x^2}\right)dx\\ &= -\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 1.00

$$-\frac{b^3}{5x^5} - \frac{b^2c}{x^3} - \frac{3bc^2}{x} + c^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^12,x]
[Out] -1/5*b^3/x^5 - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Maple [A]
time = 0.07, size = 33, normalized size = 0.97

method	result	size
default	$-\frac{b^3}{5x^5} - \frac{b^2c}{x^3} - \frac{3b\,c^2}{x} + c^3x$	33
risch	$c^3x + \frac{-3b\,c^2x^4 - b^2c\,x^2 - \frac{1}{5}b^3}{x^5}$	35
gosper	$-\frac{-5c^3x^6 + 15b\,c^2x^4 + 5b^2c\,x^2 + b^3}{5x^5}$	36
norman	$\frac{c^3x^{12} - \frac{1}{5}b^3x^6 - 3b\,c^2x^{10} - b^2c\,x^8}{x^{11}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^12,x,method=_RETURNVERBOSE)
[Out] -1/5*b^3/x^5-b^2*c/x^3-3*b*c^2/x+c^3*x
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.97

$$c^3x - \frac{15\,bc^2x^4 + 5\,b^2cx^2 + b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="maxima")
[Out] c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

Fricas [A]
time = 0.32, size = 37, normalized size = 1.09

$$\frac{5\,c^3x^6 - 15\,bc^2x^4 - 5\,b^2cx^2 - b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/5*(5*c^3*x^6 - 15*b*c^2*x^4 - 5*b^2*c*x^2 - b^3)/x^5
```

Sympy [A]
time = 0.08, size = 34, normalized size = 1.00

$$c^3x + \frac{-b^3 - 5b^2cx^2 - 15bc^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**12,x)
[Out] c**3*x + (-b**3 - 5*b**2*c*x**2 - 15*b*c**2*x**4)/(5*x**5)
```

Giac [A]
time = 4.19, size = 33, normalized size = 0.97

$$c^3x - \frac{15\,bc^2x^4 + 5\,b^2cx^2 + b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="giac")
[Out] c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

Mupad [B]
time = 0.03, size = 34, normalized size = 1.00

$$c^3\,x - \frac{\frac{b^3}{5} + b^2\,c\,x^2 + 3\,b\,c^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^12,x)
[Out] c^3*x - (b^3/5 + b^2*c*x^2 + 3*b*c^2*x^4)/x^5
```

3.169 $\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx$

Optimal. Leaf size=39

$$-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

```
[Out] -1/6*b^3/x^6-3/4*b^2*c/x^4-3/2*b*c^2/x^2+c^3*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1598, 272, 45}

$$-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^13,x]
```

```
[Out] -1/6*b^3/x^6 - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx = \int \frac{(b+cx^2)^3}{x^7}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(b+cx)^3}{x^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3b^2c}{x^3}+\frac{3bc^2}{x^2}+\frac{c^3}{x}\right)dx, x, x^2\right)$$
$$= -\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

Mathematica [A]
time = 0.00, size = 39, normalized size = 1.00

$$-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^13,x]
[Out] -1/6*b^3/x^6 - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

Maple [A]
time = 0.09, size = 34, normalized size = 0.87

method	result	size
default	$-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3b\,c^2}{2x^2}+c^3\ln\left(x\right)$	34
risch	$\frac{-\frac{3}{2}b\,c^2x^4-\frac{3}{4}b^2c\,x^2-\frac{1}{6}b^3}{x^6}+c^3\ln\left(x\right)$	36
norman	$\frac{-\frac{1}{6}b^3x^6-\frac{3}{2}b\,c^2x^{10}-\frac{3}{4}b^2c\,x^8}{x^{12}}+c^3\ln\left(x\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^13,x,method=_RETURNVERBOSE)
[Out] -1/6*b^3/x^6-3/4*b^2*c/x^4-3/2*b*c^2/x^2+c^3*ln(x)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 1.00

$$\frac{1}{2}\,c^3\log\left(x^2\right)-\frac{18\,bc^2x^4+9\,b^2cx^2+2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="maxima")
[Out] 1/2*c^3*log(x^2) - 1/12*(18*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3)/x^6
```

Fricas [A]
time = 0.34, size = 39, normalized size = 1.00

$$\frac{12\,c^3x^6\log\left(x\right)-18\,bc^2x^4-9\,b^2cx^2-2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="fricas")
[Out] 1/12*(12*c^3*x^6*log(x) - 18*b*c^2*x^4 - 9*b^2*c*x^2 - 2*b^3)/x^6
```

Sympy [A]
time = 0.11, size = 37, normalized size = 0.95

$$c^3\log\left(x\right)+\frac{-2b^3-9b^2cx^2-18bc^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**13,x)
[Out] c**3*log(x) + (-2*b**3 - 9*b**2*c*x**2 - 18*b*c**2*x**4)/(12*x**6)
```

Giac [A]
time = 3.35, size = 47, normalized size = 1.21

$$\frac{1}{2}\,c^3\log\left(x^2\right)-\frac{11\,c^3x^6+18\,bc^2x^4+9\,b^2cx^2+2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="giac")
[Out] 1/2*c^3*log(x^2) - 1/12*(11*c^3*x^6 + 18*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3)/x
^6
```

Mupad [B]
time = 0.05, size = 36, normalized size = 0.92

$$c^3\,\ln\left(x\right)-\frac{\frac{b^3}{6}+\frac{3\,b^2\,c\,x^2}{4}+\frac{3\,b\,c^2\,x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^13,x)
[Out] c^3*log(x) - (b^3/6 + (3*b^2*c*x^2)/4 + (3*b*c^2*x^4)/2)/x^6
```

3.170 $\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx$

Optimal. Leaf size=39

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

```
[Out] -1/7*b^3/x^7-3/5*b^2*c/x^5-b*c^2/x^3-c^3/x
```

Rubi [A]

time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 276}

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^14,x]
```

```
[Out] -1/7*b^3/x^7 - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx &= \int \frac{(b+cx^2)^3}{x^8}\,dx\\
&= \int \left(\frac{b^3}{x^8}+\frac{3b^2c}{x^6}+\frac{3bc^2}{x^4}+\frac{c^3}{x^2}\right)dx\\
&= -\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 39, normalized size = 1.00

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^14,x]
[Out] -1/7*b^3/x^7 - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.92

method	result	size
default	$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{b\,c^2}{x^3}-\frac{c^3}{x}$	36
risch	$\frac{-c^3x^6-b\,c^2x^4-\frac{3}{5}b^2c\,x^2-\frac{1}{7}b^3}{x^7}$	37
gosper	$-\frac{35c^3x^6+35b\,c^2x^4+21b^2c\,x^2+5b^3}{35x^7}$	38
norman	$\frac{-\frac{1}{7}b^3x^6-c^3x^{12}-b\,c^2x^{10}-\frac{3}{5}b^2c\,x^8}{x^{13}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^14,x,method=_RETURNVERBOSE)
[Out] -1/7*b^3/x^7-3/5*b^2*c/x^5-b*c^2/x^3-c^3/x
```

Maxima [A]
time = 0.30, size = 37, normalized size = 0.95

$$-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="maxima")
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

Fricas [A]
time = 0.32, size = 37, normalized size = 0.95

$$-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="fricas")
```

```
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

Sympy [A]
time = 0.11, size = 39, normalized size = 1.00

$$\frac{-5b^3 - 21b^2cx^2 - 35bc^2x^4 - 35c^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**14,x)
[Out] (-5*b**3 - 21*b**2*c*x**2 - 35*b*c**2*x**4 - 35*c**3*x**6)/(35*x**7)
```

Giac [A]
time = 3.35, size = 37, normalized size = 0.95

$$-\frac{35\,c^3x^6 + 35\,bc^2x^4 + 21\,b^2cx^2 + 5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="giac")
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

Mupad [B]
time = 0.03, size = 35, normalized size = 0.90

$$-\frac{\frac{b^3}{7} + \frac{3\,b^2\,c\,x^2}{5} + b\,c^2\,x^4 + c^3\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^14,x)
[Out] -(b^3/7 + c^3*x^6 + (3*b^2*c*x^2)/5 + b*c^2*x^4)/x^7
```

3.171 $\int \frac{(bx^2+cx^4)^3}{x^{15}} dx$

Optimal. Leaf size=19

$$-\frac{(b+cx^2)^4}{8bx^8}$$

[Out] -1/8*(c*x^2+b)^4/b/x^8

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 270}

$$-\frac{(b+cx^2)^4}{8bx^8}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)^3/x^15,x]

[Out] -1/8*(b + c*x^2)^4/(b*x^8)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{15}} dx &= \int \frac{(b+cx^2)^3}{x^9} dx \\ &= -\frac{(b+cx^2)^4}{8bx^8}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.
time = 0.01, size = 43, normalized size = 2.26

$$-\frac{b^3}{8x^8}-\frac{b^2c}{2x^6}-\frac{3bc^2}{4x^4}-\frac{c^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^15,x]
```

```
[Out] -1/8*b^3/x^8 - (b^2*c)/(2*x^6) - (3*b*c^2)/(4*x^4) - c^3/(2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 0.09, size = 36, normalized size = 1.89

method	result	size
gosper	$-\frac{4c^3x^6+6b\,c^2x^4+4b^2c\,x^2+b^3}{8x^8}$	36
default	$-\frac{3b\,c^2}{4x^4}-\frac{c^3}{2x^2}-\frac{b^3}{8x^8}-\frac{b^2c}{2x^6}$	36
risch	$\frac{-\frac{1}{2}c^3x^6-\frac{3}{4}b\,c^2x^4-\frac{1}{2}b^2c\,x^2-\frac{1}{8}b^3}{x^8}$	37
norman	$\frac{-\frac{1}{8}b^3x^6-\frac{1}{2}c^3x^{12}-\frac{3}{4}b\,c^2x^{10}-\frac{1}{2}b^2c\,x^8}{x^{14}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*b*c^2/x^4-1/2*c^3/x^2-1/8*b^3/x^8-1/2*b^2*c/x^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 0.28, size = 35, normalized size = 1.84

$$-\frac{4\,c^3x^6+6\,bc^2x^4+4\,b^2cx^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="maxima")
```

```
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 0.33, size = 35, normalized size = 1.84

$$-\frac{4\,c^3x^6+6\,bc^2x^4+4\,b^2cx^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="fricas")
```

```
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.
time = 0.12, size = 37, normalized size = 1.95

$$\frac{-b^3 - 4b^2cx^2 - 6bc^2x^4 - 4c^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**15,x)
```

```
[Out] (-b**3 - 4*b**2*c*x**2 - 6*b*c**2*x**4 - 4*c**3*x**6)/(8*x**8)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 4.61, size = 35, normalized size = 1.84

$$-\frac{4\,c^3x^6 + 6\,bc^2x^4 + 4\,b^2cx^2 + b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="giac")
```

```
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

Mupad [B]
time = 0.03, size = 37, normalized size = 1.95

$$-\frac{\frac{b^3}{8} + \frac{b^2\,c\,x^2}{2} + \frac{3\,b\,c^2\,x^4}{4} + \frac{c^3\,x^6}{2}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^15,x)
```

```
[Out] -(b^3/8 + (c^3*x^6)/2 + (b^2*c*x^2)/2 + (3*b*c^2*x^4)/4)/x^8
```

3.172 $\int \frac{(bx^2+cx^4)^3}{x^{16}} dx$

Optimal. Leaf size=43

$$-\frac{b^3}{9x^9} - \frac{3b^2c}{7x^7} - \frac{3bc^2}{5x^5} - \frac{c^3}{3x^3}$$

```
[Out] -1/9*b^3/x^9-3/7*b^2*c/x^7-3/5*b*c^2/x^5-1/3*c^3/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 276}

$$-\frac{b^3}{9x^9} - \frac{3b^2c}{7x^7} - \frac{3bc^2}{5x^5} - \frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^16,x]
```

```
[Out] -1/9*b^3/x^9 - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx &= \int \frac{(b+cx^2)^3}{x^{10}}\,dx \\
&= \int \left(\frac{b^3}{x^{10}} + \frac{3b^2c}{x^8} + \frac{3bc^2}{x^6} + \frac{c^3}{x^4}\right) dx \\
&= -\frac{b^3}{9x^9} - \frac{3b^2c}{7x^7} - \frac{3bc^2}{5x^5} - \frac{c^3}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$-\frac{b^3}{9x^9} - \frac{3b^2c}{7x^7} - \frac{3bc^2}{5x^5} - \frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^16,x]
```

```
[Out] -1/9*b^3/x^9 - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3)
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.84

method	result	size
default	$-\frac{b^3}{9x^9} - \frac{3b^2c}{7x^7} - \frac{3b\,c^2}{5x^5} - \frac{c^3}{3x^3}$	36
risch	$\frac{-\frac{1}{3}c^3x^6 - \frac{3}{5}b\,c^2x^4 - \frac{3}{7}b^2c\,x^2 - \frac{1}{9}b^3}{x^9}$	37
gosper	$-\frac{105c^3x^6 + 189b\,c^2x^4 + 135b^2c\,x^2 + 35b^3}{315x^9}$	38
norman	$\frac{-\frac{1}{9}b^3x^6 - \frac{1}{3}c^3x^{12} - \frac{3}{5}b\,c^2x^{10} - \frac{3}{7}b^2c\,x^8}{x^{15}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*b^3/x^9-3/7*b^2*c/x^7-3/5*b*c^2/x^5-1/3*c^3/x^3
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.86

$$-\frac{105\,c^3x^6 + 189\,bc^2x^4 + 135\,b^2cx^2 + 35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

Fricas [A]
time = 0.32, size = 37, normalized size = 0.86

$$-\frac{105\,c^3x^6 + 189\,bc^2x^4 + 135\,b^2cx^2 + 35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

Sympy [A]
time = 0.12, size = 39, normalized size = 0.91

$$\frac{-35b^3 - 135b^2cx^2 - 189bc^2x^4 - 105c^3x^6}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**16,x)
[Out] (-35*b**3 - 135*b**2*c*x**2 - 189*b*c**2*x**4 - 105*c**3*x**6)/(315*x**9)
```

Giac [A]
time = 3.68, size = 37, normalized size = 0.86

$$-\frac{105\,c^3x^6 + 189\,bc^2x^4 + 135\,b^2cx^2 + 35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="giac")
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

Mupad [B]
time = 0.03, size = 37, normalized size = 0.86

$$-\frac{\frac{b^3}{9} + \frac{3\,b^2\,c\,x^2}{7} + \frac{3\,b\,c^2\,x^4}{5} + \frac{c^3\,x^6}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^16,x)
[Out] -(b^3/9 + (c^3*x^6)/3 + (3*b^2*c*x^2)/7 + (3*b*c^2*x^4)/5)/x^9
```

3.173 $\int \frac{(bx^2+cx^4)^3}{x^{17}}\,dx$

Optimal. Leaf size=40

$$-\frac{(b+cx^2)^4}{10bx^{10}}+\frac{c(b+cx^2)^4}{40b^2x^8}$$

```
[Out] -1/10*(c*x^2+b)^4/b/x^10+1/40*c*(c*x^2+b)^4/b^2/x^8
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1598, 272, 47, 37}

$$\frac{c(b+cx^2)^4}{40b^2x^8}-\frac{(b+cx^2)^4}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^17,x]
```

```
[Out] -1/10*(b + c*x^2)^4/(b*x^10) + (c*(b + c*x^2)^4)/(40*b^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{x^{17}}\,dx &= \int \frac{(b+cx^2)^3}{x^{11}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(b+cx)^3}{x^6}\,dx,x,x^2\right)\\ &= -\frac{(b+cx^2)^4}{10bx^{10}}-\frac{c\text{Subst}\left(\int \frac{(b+cx)^3}{x^5}\,dx,x,x^2\right)}{10b}\\ &= -\frac{(b+cx^2)^4}{10bx^{10}}+\frac{c(b+cx^2)^4}{40b^2x^8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.08

$$-\frac{b^3}{10x^{10}}-\frac{3b^2c}{8x^8}-\frac{bc^2}{2x^6}-\frac{c^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^17,x]
[Out] -1/10*b^3/x^10 - (3*b^2*c)/(8*x^8) - (b*c^2)/(2*x^6) - c^3/(4*x^4)
```

Maple [A]
time = 0.07, size = 36, normalized size = 0.90

method	result	size
default	$-\frac{c^3}{4x^4}-\frac{3b^2c}{8x^8}-\frac{b^3}{10x^{10}}-\frac{b\,c^2}{2x^6}$	36
risch	$\frac{-\frac{1}{4}c^3x^6-\frac{1}{2}b\,c^2x^4-\frac{3}{8}b^2c\,x^2-\frac{1}{10}b^3}{x^{10}}$	37
gosper	$-\frac{10c^3x^6+20b\,c^2x^4+15b^2c\,x^2+4b^3}{40x^{10}}$	38
norman	$\frac{-\frac{1}{10}b^3x^6-\frac{1}{4}c^3x^{12}-\frac{1}{2}b\,c^2x^{10}-\frac{3}{8}b^2c\,x^8}{x^{16}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^17,x,method=_RETURNVERBOSE)
[Out] -1/4*c^3/x^4-3/8*b^2*c/x^8-1/10*b^3/x^10-1/2*b*c^2/x^6
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.92

$$-\frac{10\,c^3x^6+20\,bc^2x^4+15\,b^2cx^2+4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="maxima")
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

Fricas [A]
time = 0.33, size = 37, normalized size = 0.92

$$-\frac{10\,c^3x^6+20\,bc^2x^4+15\,b^2cx^2+4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="fricas")
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

Sympy [A]
time = 0.14, size = 39, normalized size = 0.98

$$\frac{-4b^3-15b^2cx^2-20bc^2x^4-10c^3x^6}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**17,x)
[Out] (-4*b**3 - 15*b**2*c*x**2 - 20*b*c**2*x**4 - 10*c**3*x**6)/(40*x**10)
```

Giac [A]
time = 5.74, size = 37, normalized size = 0.92

$$-\frac{10\,c^3x^6+20\,bc^2x^4+15\,b^2cx^2+4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="giac")
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

Mupad [B]
time = 0.03, size = 37, normalized size = 0.92

$$-\frac{\frac{b^3}{10}+\frac{3\,b^2\,c\,x^2}{8}+\frac{b\,c^2\,x^4}{2}+\frac{c^3\,x^6}{4}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^17,x)
[Out] -(b^3/10 + (c^3*x^6)/4 + (3*b^2*c*x^2)/8 + (b*c^2*x^4)/2)/x^10
```

3.174 $\int \frac{x^{10}}{bx^2+cx^4} \, dx$

Optimal. Leaf size=68

$$-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}$$

```
[Out] -b^3*x/c^4+1/3*b^2*x^3/c^3-1/5*b*x^5/c^2+1/7*x^7/c+b^(7/2)*arctan(x*c^(1/2)
/b^(1/2))/c^(9/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1598, 308, 211}

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c) + (b^(7/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{10}}{bx^2+cx^4}\,dx = \int \frac{x^8}{b+cx^2}\,dx$$
$$= \int \left(-\frac{b^3}{c^4}+\frac{b^2x^2}{c^3}-\frac{bx^4}{c^2}+\frac{x^6}{c}+\frac{b^4}{c^4(b+cx^2)}\right)dx$$
$$= -\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^4\int \frac{1}{b+cx^2}\,dx}{c^4}$$
$$= -\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 1.00

$$-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4),x]
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c) + (b^(7/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Maple [A]
time = 0.11, size = 60, normalized size = 0.88

method	result	size
default	$-\frac{-\frac{1}{7}c^3x^7+\frac{1}{5}b\,c^2x^5-\frac{1}{3}b^2c\,x^3+b^3x}{c^4}+\frac{b^4\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^4\sqrt{bc}}$	60
risch	$\frac{x^7}{7c}-\frac{b\,x^5}{5c^2}+\frac{b^2x^3}{3c^3}-\frac{b^3x}{c^4}+\frac{\sqrt{-bc}\,b^3\ln\left(-\sqrt{-bc}\,x+b\right)}{2c^5}-\frac{\sqrt{-bc}\,b^3\ln\left(\sqrt{-bc}\,x+b\right)}{2c^5}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] -1/c^4*(-1/7*c^3*x^7+1/5*b*c^2*x^5-1/3*b^2*c*x^3+b^3*x)+b^4/c^4/(b*c)^(1/2)
*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.51, size = 60, normalized size = 0.88

$$\frac{b^4\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^4}+\frac{15\,c^3x^7-21\,bc^2x^5+35\,b^2cx^3-105\,b^3x}{105\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] b^4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*c^3*x^7 - 21*b*c^2*x^
5 + 35*b^2*c*x^3 - 105*b^3*x)/c^4
```

Fricas [A]
time = 0.37, size = 148, normalized size = 2.18

$$\left[\frac{30\,c^3x^7-42\,bc^2x^5+70\,b^2cx^3+105\,b^3\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-210\,b^3x}{210\,c^4},\ \frac{15\,c^3x^7-21\,bc^2x^5+35\,b^2cx^3+105\,b^3\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-105\,b^3x}{105\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/210*(30*c^3*x^7 - 42*b*c^2*x^5 + 70*b^2*c*x^3 + 105*b^3*sqrt(-b/c)*log((
c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) - 210*b^3*x)/c^4, 1/105*(15*c^3*
x^7 - 21*b*c^2*x^5 + 35*b^2*c*x^3 + 105*b^3*sqrt(b/c)*arctan(c*x*sqrt(b/c)/
b) - 105*b^3*x)/c^4]
```

Sympy [A]
time = 0.08, size = 107, normalized size = 1.57

$$-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}-\frac{\sqrt{-\frac{b^7}{c^9}}\,\log\left(x-\frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2}+\frac{\sqrt{-\frac{b^7}{c^9}}\,\log\left(x+\frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2}+\frac{x^7}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2),x)
```

```
[Out] -b**3*x/c**4 + b**2*x**3/(3*c**3) - b*x**5/(5*c**2) - sqrt(-b**7/c**9)*log(
x - c**4*sqrt(-b**7/c**9)/b**3)/2 + sqrt(-b**7/c**9)*log(x + c**4*sqrt(-b**
7/c**9)/b**3)/2 + x**7/(7*c)
```

Giac [A]
time = 4.69, size = 65, normalized size = 0.96

$$\frac{b^4\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^4}+\frac{15\,c^6x^7-21\,bc^5x^5+35\,b^2c^4x^3-105\,b^3c^3x}{105\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] b^4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*c^6*x^7 - 21*b*c^5*x^
5 + 35*b^2*c^4*x^3 - 105*b^3*c^3*x)/c^7
```

Mupad [B]
time = 0.03, size = 54, normalized size = 0.79

$$\frac{x^7}{7\,c}-\frac{b\,x^5}{5\,c^2}-\frac{b^3\,x}{c^4}+\frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}+\frac{b^2\,x^3}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2 + c*x^4),x)
```

```
[Out] x^7/(7*c) - (b*x^5)/(5*c^2) - (b^3*x)/c^4 + (b^(7/2)*atan((c^(1/2)*x)/b^(1/
2)))/c^(9/2) + (b^2*x^3)/(3*c^3)
```

3.175 $\int \frac{x^9}{bx^2+cx^4}\,dx$

Optimal. Leaf size=53

$$\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}-\frac{b^3\log\left(b+cx^2\right)}{2c^4}$$

```
[Out] 1/2*b^2*x^2/c^3-1/4*b*x^4/c^2+1/6*x^6/c-1/2*b^3*ln(c*x^2+b)/c^4
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {1598, 272, 45}

$$-\frac{b^3\log\left(b+cx^2\right)}{2c^4}+\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*x^2])/(2*c
^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{bx^2+cx^4}\,dx &= \int \frac{x^7}{b+cx^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{b+cx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2}{c^3}-\frac{bx}{c^2}+\frac{x^2}{c}-\frac{b^3}{c^3(b+cx)}\right)\,dx,x,x^2\right)\\ &= \frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}-\frac{b^3\log(b+cx^2)}{2c^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 53, normalized size = 1.00

$$\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}-\frac{b^3\log(b+cx^2)}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*x^2])/(2*c
^4)
```

Maple [A]
time = 0.10, size = 46, normalized size = 0.87

method	result	size
default	$\frac{\frac{1}{3}c^2x^6-\frac{1}{2}bc\,x^4+b^2x^2}{2c^3}-\frac{b^3\ln(c\,x^2+b)}{2c^4}$	46
risch	$\frac{b^2x^2}{2c^3}-\frac{b\,x^4}{4c^2}+\frac{x^6}{6c}-\frac{b^3\ln(c\,x^2+b)}{2c^4}$	46
norman	$\frac{\frac{x^7}{6c}-\frac{b\,x^5}{4c^2}+\frac{b^2x^3}{2c^3}}{x}-\frac{b^3\ln(c\,x^2+b)}{2c^4}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^3*(1/3*c^2*x^6-1/2*b*c*x^4+b^2*x^2)-1/2*b^3*ln(c*x^2+b)/c^4
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.87

$$-\frac{b^3\log(cx^2+b)}{2\,c^4}+\frac{2\,c^2x^6-3\,bcx^4+6\,b^2x^2}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(c*x^2 + b)/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6*b^2*x^2)/c^3
```

Fricas [A]
time = 0.38, size = 45, normalized size = 0.85

$$\frac{2\,c^3x^6 - 3\,bc^2x^4 + 6\,b^2cx^2 - 6\,b^3\log\left(cx^2 + b\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*c^3*x^6 - 3*b*c^2*x^4 + 6*b^2*c*x^2 - 6*b^3*log(c*x^2 + b))/c^4
```

Sympy [A]
time = 0.06, size = 44, normalized size = 0.83

$$-\frac{b^3\log\left(b + cx^2\right)}{2c^4} + \frac{b^2x^2}{2c^3} - \frac{bx^4}{4c^2} + \frac{x^6}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2),x)
```

```
[Out] -b**3*log(b + c*x**2)/(2*c**4) + b**2*x**2/(2*c**3) - b*x**4/(4*c**2) + x**
6/(6*c)
```

Giac [A]
time = 5.54, size = 47, normalized size = 0.89

$$-\frac{b^3\log\left(|cx^2 + b|\right)}{2\,c^4} + \frac{2\,c^2x^6 - 3\,bcx^4 + 6\,b^2x^2}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(abs(c*x^2 + b))/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6*b^2*x^2)
/c^3
```

Mupad [B]
time = 0.05, size = 45, normalized size = 0.85

$$\frac{x^6}{6\,c} - \frac{b\,x^4}{4\,c^2} - \frac{b^3\,\ln\left(c\,x^2 + b\right)}{2\,c^4} + \frac{b^2\,x^2}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4),x)
```

```
[Out] x^6/(6*c) - (b*x^4)/(4*c^2) - (b^3*log(b + c*x^2))/(2*c^4) + (b^2*x^2)/(2*c
^3)
```

3.176 $\int \frac{x^8}{bx^2+cx^4} dx$

Optimal. Leaf size=55

$$\frac{b^2x}{c^3} - \frac{bx^3}{3c^2} + \frac{x^5}{5c} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}$$

[Out] b^2*x/c^3-1/3*b*x^3/c^2+1/5*x^5/c-b^(5/2)*arctan(x*c^(1/2)/b^(1/2))/c^(7/2)

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 308, 211}

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}} + \frac{b^2x}{c^3} - \frac{bx^3}{3c^2} + \frac{x^5}{5c}$$

Antiderivative was successfully verified.

[In] Int[x^8/(b*x^2 + c*x^4),x]

[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqr t[b]])/c^(7/2)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^8}{bx^2+cx^4}\,dx = \int \frac{x^6}{b+cx^2}\,dx$$
$$= \int \left(\frac{b^2}{c^3} - \frac{bx^2}{c^2} + \frac{x^4}{c} - \frac{b^3}{c^3\,(b+cx^2)}\right) dx$$
$$= \frac{b^2x}{c^3} - \frac{bx^3}{3c^2} + \frac{x^5}{5c} - \frac{b^3 \int \frac{1}{b+cx^2}\,dx}{c^3}$$
$$= \frac{b^2x}{c^3} - \frac{bx^3}{3c^2} + \frac{x^5}{5c} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 1.00

$$\frac{b^2x}{c^3} - \frac{bx^3}{3c^2} + \frac{x^5}{5c} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4),x]
[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqr
t[b]])/c^(7/2)
```

Maple [A]
time = 0.08, size = 49, normalized size = 0.89

method	result	size
default	$\frac{\frac{1}{5}c^2x^5-\frac{1}{3}bc\,x^3+b^2x}{c^3}-\frac{b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^3\sqrt{bc}}$	49
risch	$\frac{x^5}{5c}-\frac{b\,x^3}{3c^2}+\frac{b^2x}{c^3}+\frac{\sqrt{-bc}\,b^2\ln\left(-\sqrt{-bc}\,x-b\right)}{2c^4}-\frac{\sqrt{-bc}\,b^2\ln\left(\sqrt{-bc}\,x-b\right)}{2c^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/c^3*(1/5*c^2*x^5-1/3*b*c*x^3+b^2*x)-b^3/c^3/(b*c)^(1/2)*arctan(c*x/(b*c)^
(1/2))
```

Maxima [A]
time = 0.49, size = 50, normalized size = 0.91

$$-\frac{b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^3} + \frac{3\,c^2x^5 - 5\,bcx^3 + 15\,b^2x}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*c^2*x^5 - 5*b*c*x^3 +
15*b^2*x)/c^3
```

Fricas [A]
time = 0.36, size = 126, normalized size = 2.29

$$\left[\frac{6\,c^2x^5-10\,bcx^3+15\,b^2\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+30\,b^2x}{30\,c^3},\ \frac{3\,c^2x^5-5\,bcx^3-15\,b^2\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)+15\,b^2x}{15\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*c^2*x^5 - 10*b*c*x^3 + 15*b^2*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-
b/c) - b)/(c*x^2 + b)) + 30*b^2*x)/c^3, 1/15*(3*c^2*x^5 - 5*b*c*x^3 - 15*b^
2*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 15*b^2*x)/c^3]
```

Sympy [A]
time = 0.07, size = 95, normalized size = 1.73

$$\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{\sqrt{-\frac{b^5}{c^7}}\log\left(x-\frac{c^3\sqrt{-\frac{b^5}{c^7}}}{b^2}\right)}{2}-\frac{\sqrt{-\frac{b^5}{c^7}}\log\left(x+\frac{c^3\sqrt{-\frac{b^5}{c^7}}}{b^2}\right)}{2}+\frac{x^5}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2),x)
```

```
[Out] b**2*x/c**3 - b*x**3/(3*c**2) + sqrt(-b**5/c**7)*log(x - c**3*sqrt(-b**5/c*
*7)/b**2)/2 - sqrt(-b**5/c**7)*log(x + c**3*sqrt(-b**5/c**7)/b**2)/2 + x**5
/(5*c)
```

Giac [A]
time = 4.64, size = 55, normalized size = 1.00

$$-\frac{b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^3}+\frac{3\,c^4x^5-5\,bc^3x^3+15\,b^2c^2x}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*c^4*x^5 - 5*b*c^3*x^3
+ 15*b^2*c^2*x)/c^5
```

Mupad [B]
time = 0.05, size = 43, normalized size = 0.78

$$\frac{x^5}{5\,c}-\frac{b\,x^3}{3\,c^2}+\frac{b^2\,x}{c^3}-\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2 + c*x^4),x)
```

```
[Out] x^5/(5*c) - (b*x^3)/(3*c^2) + (b^2*x)/c^3 - (b^(5/2)*atan((c^(1/2)*x)/b^(1/
2)))/c^(7/2)
```

3.177 $\int \frac{x^7}{bx^2+cx^4}\,dx$

Optimal. Leaf size=40

$$-\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b^2\log\left(b+cx^2\right)}{2c^3}$$

```
[Out] -1/2*b*x^2/c^2+1/4*x^4/c+1/2*b^2*ln(c*x^2+b)/c^3
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1598, 272, 45}

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4),x]
```

```
[Out] -1/2*(b*x^2)/c^2 + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^7}{bx^2+cx^4}\,dx = \int \frac{x^5}{b+cx^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{b+cx}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{b^2}{c^2(b+cx)}\right)\,dx, x, x^2\right)$$
$$= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b^2\log\left(b+cx^2\right)}{2c^3}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.00

$$-\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b^2\log\left(b+cx^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4),x]
[Out] -1/2*(b*x^2)/c^2 + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

Maple [A]
time = 0.10, size = 35, normalized size = 0.88

method	result	size
default	$-\frac{-\frac{1}{2}c\,x^4+b\,x^2}{2c^2}+\frac{b^2\ln\left(c\,x^2+b\right)}{2c^3}$	35
norman	$\frac{\frac{x^5}{4c}-\frac{b\,x^3}{2c^2}}{x}+\frac{b^2\ln\left(c\,x^2+b\right)}{2c^3}$	40
risch	$\frac{x^4}{4c}-\frac{b\,x^2}{2c^2}+\frac{b^2}{4c^3}+\frac{b^2\ln\left(c\,x^2+b\right)}{2c^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] -1/2/c^2*(-1/2*c*x^4+b*x^2)+1/2*b^2*ln(c*x^2+b)/c^3
```

Maxima [A]
time = 0.28, size = 34, normalized size = 0.85

$$\frac{b^2\log\left(cx^2+b\right)}{2\,c^3}+\frac{cx^4-2\,bx^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(c*x^2 + b)/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

Fricas [A]
time = 0.39, size = 33, normalized size = 0.82

$$\frac{c^2x^4 - 2\,bcx^2 + 2\,b^2\log\left(cx^2 + b\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(c^2*x^4 - 2*b*c*x^2 + 2*b^2*log(c*x^2 + b))/c^3
```

Sympy [A]
time = 0.06, size = 32, normalized size = 0.80

$$\frac{b^2\log\left(b + cx^2\right)}{2c^3} - \frac{bx^2}{2c^2} + \frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2),x)
```

```
[Out] b**2*log(b + c*x**2)/(2*c**3) - b*x**2/(2*c**2) + x**4/(4*c)
```

Giac [A]
time = 3.66, size = 35, normalized size = 0.88

$$\frac{b^2\log\left(|cx^2 + b|\right)}{2\,c^3} + \frac{cx^4 - 2\,bx^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(abs(c*x^2 + b))/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

Mupad [B]
time = 0.05, size = 33, normalized size = 0.82

$$\frac{2\,b^2\,\ln\left(c\,x^2 + b\right) + c^2\,x^4 - 2\,b\,c\,x^2}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4),x)
```

```
[Out] (2*b^2*log(b + c*x^2) + c^2*x^4 - 2*b*c*x^2)/(4*c^3)
```

3.178 $\int \frac{x^6}{bx^2+cx^4}\,dx$

Optimal. Leaf size=42

$$-\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}$$

```
[Out] -b*x/c^2+1/3*x^3/c+b^(3/2)*arctan(x*c^(1/2)/b^(1/2))/c^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 308, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{bx^2+cx^4}\,dx &= \int \frac{x^4}{b+cx^2}\,dx\\ &= \int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2(b+cx^2)}\right)dx\\ &= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{b^2\int \frac{1}{b+cx^2}\,dx}{c^2}\\ &= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$-\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4),x]
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)
```

Maple [A]
time = 0.09, size = 38, normalized size = 0.90

method	result	size
default	$-\frac{-\frac{1}{3}c\,x^3+bx}{c^2}+\frac{b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^2\sqrt{bc}}$	38
risch	$\frac{x^3}{3c}-\frac{bx}{c^2}+\frac{\sqrt{-bc}\,b\ln\left(-\sqrt{-bc}\,x+b\right)}{2c^3}-\frac{\sqrt{-bc}\,b\ln\left(\sqrt{-bc}\,x+b\right)}{2c^3}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] -1/c^2*(-1/3*c*x^3+b*x)+b^2/c^2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.49, size = 37, normalized size = 0.88

$$\frac{b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^2}+\frac{cx^3-3\,bx}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(c*x^3 - 3*b*x)/c^2
```

Fricas [A]
time = 0.33, size = 99, normalized size = 2.36

$$\left[\frac{2\,cx^3+3\,b\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-6\,bx}{6\,c^2},\ \frac{cx^3+3\,b\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-3\,bx}{3\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c*x^3 + 3*b*sqrt(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 +
b)) - 6*b*x)/c^2, 1/3*(c*x^3 + 3*b*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 3*b*
x)/c^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.
time = 0.07, size = 80, normalized size = 1.90

$$-\frac{bx}{c^2}-\frac{\sqrt{-\frac{b^3}{c^5}}\,\log\left(x-\frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2}+\frac{\sqrt{-\frac{b^3}{c^5}}\,\log\left(x+\frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2}+\frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2),x)
```

```
[Out] -b*x/c**2 - sqrt(-b**3/c**5)*log(x - c**2*sqrt(-b**3/c**5)/b)/2 + sqrt(-b**
3/c**5)*log(x + c**2*sqrt(-b**3/c**5)/b)/2 + x**3/(3*c)
```

Giac [A]
time = 4.61, size = 40, normalized size = 0.95

$$\frac{b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c^2}+\frac{c^2x^3-3\,bcx}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2),x, algorithm="giac")
[Out] b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

Mupad [B]
time = 0.05, size = 32, normalized size = 0.76

$$\frac{x^3}{3\,c}+\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{b\,x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4),x)
[Out] x^3/(3*c) + (b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/c^(5/2) - (b*x)/c^2
```

3.179 $\int \frac{x^5}{bx^2+cx^4}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}$$

```
[Out] 1/2*x^2/c-1/2*b*ln(c*x^2+b)/c^2
```

Rubi [A]

time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1598, 272, 45}

$$\frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{bx^2+cx^4}\,dx &= \int \frac{x^3}{b+cx^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{b+cx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{c}-\frac{b}{c(b+cx)}\right)\,dx, x, x^2\right) \\
&= \frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$
\frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

Maple [A]
time = 0.10, size = 24, normalized size = 0.89

method	result	size
default	$\frac{x^2}{2c}-\frac{b\ln\left(c\,x^2+b\right)}{2c^2}$	24
norman	$\frac{x^2}{2c}-\frac{b\ln\left(c\,x^2+b\right)}{2c^2}$	24
risch	$\frac{x^2}{2c}-\frac{b\ln\left(c\,x^2+b\right)}{2c^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c-1/2*b*ln(c*x^2+b)/c^2
```

Maxima [A]
time = 0.30, size = 23, normalized size = 0.85

$$
\frac{x^2}{2\,c}-\frac{b\log\left(cx^2+b\right)}{2\,c^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - 1/2*b*log(c*x^2 + b)/c^2
```

Fricas [A]
time = 0.35, size = 22, normalized size = 0.81

$$\frac{cx^2 - b\log\left(cx^2 + b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^2 - b*log(c*x^2 + b))/c^2
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.74

$$-\frac{b\log\left(b + cx^2\right)}{2c^2} + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2),x)
```

```
[Out] -b*log(b + c*x**2)/(2*c**2) + x**2/(2*c)
```

Giac [A]
time = 4.78, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,c} - \frac{b\log\left(|cx^2 + b|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/2*b*log(abs(c*x^2 + b))/c^2
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.81

$$-\frac{b\,\ln\left(c\,x^2 + b\right) - c\,x^2}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4),x)
```

```
[Out] -(b*log(b + c*x^2) - c*x^2)/(2*c^2)
```

3.180 $\int \frac{x^4}{bx^2+cx^4}\,dx$

Optimal. Leaf size=31

$$\frac{x}{c}-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

```
[Out] x/c-arctan(x*c^(1/2)/b^(1/2))*b^(1/2)/c^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 327, 211}

$$\frac{x}{c}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4),x]
```

```
[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^4}{bx^2+cx^4}\,dx = \int \frac{x^2}{b+cx^2}\,dx$$
$$= \frac{x}{c} - \frac{b\int \frac{1}{b+cx^2}\,dx}{c}$$
$$= \frac{x}{c} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$\frac{x}{c} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4),x]
```

```
[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Maple [A]
time = 0.10, size = 27, normalized size = 0.87

method	result	size
default	$\frac{x}{c}-\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c\sqrt{bc}}$	27
risch	$\frac{x}{c}+\frac{\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)}{2c^2}-\frac{\sqrt{-bc}\,\ln\left(\sqrt{-bc}\,x-b\right)}{2c^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x/c-b/c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 26, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) + x/c
```

Fricas [A]
time = 0.35, size = 82, normalized size = 2.65

$$\left[\frac{\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+2\,x}{2\,c},\ -\frac{\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-x}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 2*x)/c,
-(sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - x)/c]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.
time = 0.06, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{b}{c^3}}\log\left(-c\sqrt{-\frac{b}{c^3}}+x\right)}{2}-\frac{\sqrt{-\frac{b}{c^3}}\log\left(c\sqrt{-\frac{b}{c^3}}+x\right)}{2}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-b/c**3)*log(-c*sqrt(-b/c**3) + x)/2 - sqrt(-b/c**3)*log(c*sqrt(-b/c**
3) + x)/2 + x/c
```

Giac [A]
time = 4.25, size = 26, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) + x/c
```

Mupad [B]
time = 0.04, size = 23, normalized size = 0.74

$$\frac{x}{c}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4),x)
```

```
[Out] x/c - (b^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/c^(3/2)
```

3.181 $\int \frac{x^3}{bx^2+cx^4}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^2)}{2c}$$

```
[Out] 1/2*ln(c*x^2+b)/c
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 266}

$$\frac{\log(b+cx^2)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4),x]
```

```
[Out] Log[b + c*x^2]/(2*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{bx^2+cx^4}\,dx &= \int \frac{x}{b+cx^2}\,dx\\ &= \frac{\log(b+cx^2)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^2)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4),x]
```

```
[Out] Log[b + c*x^2]/(2*c)
```

Maple [A]
time = 0.09, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\ln(c x^2+b)}{2c}$	14
norman	$\frac{\ln(c x^2+b)}{2c}$	14
risch	$\frac{\ln(c x^2+b)}{2c}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*x^2+b)/c
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$\frac{\log(cx^2+b)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b)/c
```

Fricas [A]
time = 0.33, size = 13, normalized size = 0.87

$$\frac{\log(cx^2+b)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b)/c
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.67

$$\frac{\log(b+cx^2)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2),x)
```

```
[Out] log(b + c*x**2)/(2*c)
```

Giac [A]
time = 5.18, size = 14, normalized size = 0.93

$$\frac{\log\left(|cx^2+b|\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^2 + b))/c
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^2+b\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4),x)
```

```
[Out] log(b + c*x^2)/(2*c)
```

3.182 $\int \frac{x^2}{bx^2+cx^4} dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

```
[Out] arctan(x*c^(1/2)/b^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{bx^2+cx^4}\,dx &= \int \frac{1}{b+cx^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4),x]
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Maple [A]
time = 0.09, size = 16, normalized size = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$	16
risch	$-\frac{\ln\left(cx+\sqrt{-bc}\right)}{2\sqrt{-bc}}+\frac{\ln\left(-cx+\sqrt{-bc}\right)}{2\sqrt{-bc}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] arctan(c*x/sqrt(b*c))/sqrt(b*c)
```

Fricas [A]
time = 0.37, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-bc}\,\log\left(\frac{cx^2-2\sqrt{-bc}\,x-b}{cx^2+b}\right)}{2\,bc},\,\frac{\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b))/(b*c), sqrt(
b*c)*arctan(sqrt(b*c)*x/b)/(b*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.
time = 0.05, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{bc}}\log\left(-b\sqrt{-\frac{1}{bc}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{bc}}\log\left(b\sqrt{-\frac{1}{bc}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-1/(b*c))*log(-b*sqrt(-1/(b*c)) + x)/2 + sqrt(-1/(b*c))*log(b*sqrt(-1
/(b*c)) + x)/2
```

Giac [A]
time = 2.95, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] arctan(c*x/sqrt(b*c))/sqrt(b*c)
```

Mupad [B]
time = 4.20, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4),x)
```

```
[Out] atan((c^(1/2)*x)/b^(1/2))/(b^(1/2)*c^(1/2))
```

3.183 $\int \frac{x}{bx^2+cx^4}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{b}-\frac{\log(b+cx^2)}{2b}$$

```
[Out] ln(x)/b-1/2*ln(c*x^2+b)/b
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1598, 272, 36, 29, 31}

$$\frac{\log(x)}{b}-\frac{\log(b+cx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{bx^2+cx^4}\,dx &= \int \frac{1}{x\left(b+cx^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(b+cx)}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2b} - \frac{c\text{Subst}\left(\int \frac{1}{b+cx}\,dx, x, x^2\right)}{2b}\\
&= \frac{\log(x)}{b} - \frac{\log\left(b+cx^2\right)}{2b}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{b} - \frac{\log\left(b+cx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4),x]
[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

Maple [A]
time = 0.09, size = 21, normalized size = 0.95

method	result	size
default	$\frac{\ln(x)}{b} - \frac{\ln\left(c\,x^2+b\right)}{2b}$	21
norman	$\frac{\ln(x)}{b} - \frac{\ln\left(c\,x^2+b\right)}{2b}$	21
risch	$\frac{\ln(x)}{b} - \frac{\ln\left(c\,x^2+b\right)}{2b}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] ln(x)/b-1/2*ln(c*x^2+b)/b
```

Maxima [A]
time = 0.28, size = 23, normalized size = 1.05

$$-\frac{\log\left(cx^2+b\right)}{2\,b} + \frac{\log\left(x^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] -1/2*log(c*x^2 + b)/b + 1/2*log(x^2)/b
```

Fricas [A]
time = 0.35, size = 18, normalized size = 0.82

$$-\frac{\log\left(cx^2+b\right)-2\,\log\left(x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] -1/2*(log(c*x^2 + b) - 2*log(x))/b
```

Sympy [A]
time = 0.09, size = 15, normalized size = 0.68

$$\frac{\log\left(x\right)}{b}-\frac{\log\left(\frac{b}{c}+x^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2),x)
[Out] log(x)/b - log(b/c + x**2)/(2*b)
```

Giac [A]
time = 4.68, size = 22, normalized size = 1.00

$$-\frac{\log\left(|cx^2+b|\right)}{2\,b}+\frac{\log\left(|x|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2),x, algorithm="giac")
[Out] -1/2*log(abs(c*x^2 + b))/b + log(abs(x))/b
```

Mupad [B]
time = 0.06, size = 18, normalized size = 0.82

$$-\frac{\ln\left(c\,x^2+b\right)-2\,\ln\left(x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4),x)
[Out] -(log(b + c*x^2) - 2*log(x))/(2*b)
```

3.184 $\int \frac{1}{bx^2+cx^4}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{bx}-\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}$$

```
[Out] -1/b/x-arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1607, 331, 211}

$$-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-1),x]
```

```
[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{bx^2 + cx^4}\, dx = \int \frac{1}{x^2(b + cx^2)}\, dx$$
$$= -\frac{1}{bx} - \frac{c\int \frac{1}{b+cx^2}\, dx}{b}$$
$$= -\frac{1}{bx} - \frac{\sqrt{c}\, \tan^{-1}\left(\frac{\sqrt{c}\, x}{\sqrt{b}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$-\frac{1}{bx} - \frac{\sqrt{c}\, \tan^{-1}\left(\frac{\sqrt{c}\, x}{\sqrt{b}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-1),x]
```

```
[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Maple [A]
time = 0.12, size = 30, normalized size = 0.88

method	result	size
default	$-\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b\sqrt{bc}} - \frac{1}{bx}$	30
risch	$-\frac{1}{bx} + \frac{\sqrt{-bc}\, \ln\left(-cx + \sqrt{-bc}\right)}{2b^2} - \frac{\sqrt{-bc}\, \ln\left(-cx - \sqrt{-bc}\right)}{2b^2}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -c/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-1/b/x
```

Maxima [A]
time = 0.50, size = 29, normalized size = 0.85

$$-\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, b} - \frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - 1/(b*x)
```

Fricas [A]
time = 0.34, size = 82, normalized size = 2.41

$$\left[\frac{x\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-2}{2\,bx}, -\frac{x\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+1}{bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) - 2)/(b*
x), -(x*sqrt(c/b)*arctan(x*sqrt(c/b)) + 1)/(b*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.
time = 0.07, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{c}{b^3}}\log\left(-\frac{b^2\sqrt{-\frac{c}{b^3}}}{c}+x\right)}{2}-\frac{\sqrt{-\frac{c}{b^3}}\log\left(\frac{b^2\sqrt{-\frac{c}{b^3}}}{c}+x\right)}{2}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c/b**3)*log(-b**2*sqrt(-c/b**3)/c + x)/2 - sqrt(-c/b**3)*log(b**2*sqr
t(-c/b**3)/c + x)/2 - 1/(b*x)
```

Giac [A]
time = 5.72, size = 29, normalized size = 0.85

$$-\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - 1/(b*x)
```

Mupad [B]
time = 4.27, size = 26, normalized size = 0.76

$$-\frac{1}{b\,x}-\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4),x)
```

```
[Out] - 1/(b*x) - (c^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/b^(3/2)
```

3.185 $\int \frac{1}{x(bx^2+cx^4)}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{2bx^2}-\frac{c\log(x)}{b^2}+\frac{c\log(b+cx^2)}{2b^2}$$

```
[Out] -1/2/b/x^2-c*ln(x)/b^2+1/2*c*ln(c*x^2+b)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1598, 272, 46}

$$\frac{c\log(b+cx^2)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*1/(b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x\left(bx^2+cx^4\right)}\,dx = \int \frac{1}{x^3\left(b+cx^2\right)}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(b+cx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{bx^2}-\frac{c}{b^2x}+\frac{c^2}{b^2(b+cx)}\right)\,dx, x, x^2\right)$$
$$= -\frac{1}{2bx^2}-\frac{c\log(x)}{b^2}+\frac{c\log\left(b+cx^2\right)}{2b^2}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.00

$$-\frac{1}{2bx^2}-\frac{c\log(x)}{b^2}+\frac{c\log\left(b+cx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*1/(b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.91

method	result	size
default	$-\frac{1}{2b\,x^2}-\frac{c\ln(x)}{b^2}+\frac{c\ln(c\,x^2+b)}{2b^2}$	32
norman	$-\frac{1}{2b\,x^2}-\frac{c\ln(x)}{b^2}+\frac{c\ln(c\,x^2+b)}{2b^2}$	32
risch	$-\frac{1}{2b\,x^2}-\frac{c\ln(x)}{b^2}+\frac{c\ln(-c\,x^2-b)}{2b^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/x^2-c*ln(x)/b^2+1/2*c*ln(c*x^2+b)/b^2
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.94

$$\frac{c\log\left(cx^2+b\right)}{2\,b^2}-\frac{c\log\left(x^2\right)}{2\,b^2}-\frac{1}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(c*x^2 + b)/b^2 - 1/2*c*log(x^2)/b^2 - 1/2/(b*x^2)
```

Fricas [A]
time = 0.36, size = 33, normalized size = 0.94

$$\frac{cx^2\log\left(cx^2+b\right)-2\,cx^2\log\left(x\right)-b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^2*log(c*x^2 + b) - 2*c*x^2*log(x) - b)/(b^2*x^2)
```

Sympy [A]
time = 0.11, size = 31, normalized size = 0.89

$$-\frac{1}{2bx^2}-\frac{c\log\left(x\right)}{b^2}+\frac{c\log\left(\frac{b}{c}+x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2),x)
```

```
[Out] -1/(2*b*x**2) - c*log(x)/b**2 + c*log(b/c + x**2)/(2*b**2)
```

Giac [A]
time = 4.59, size = 43, normalized size = 1.23

$$-\frac{c\log\left(x^2\right)}{2\,b^2}+\frac{c\log\left(\left|cx^2+b\right|\right)}{2\,b^2}+\frac{cx^2-b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*c*log(x^2)/b^2 + 1/2*c*log(abs(c*x^2 + b))/b^2 + 1/2*(c*x^2 - b)/(b^2*
x^2)
```

Mupad [B]
time = 0.06, size = 31, normalized size = 0.89

$$\frac{c\,\ln\left(c\,x^2+b\right)}{2\,b^2}-\frac{1}{2\,b\,x^2}-\frac{c\,\ln\left(x\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)),x)
```

```
[Out] (c*log(b + c*x^2))/(2*b^2) - 1/(2*b*x^2) - (c*log(x))/b^2
```

3.186 $\int \frac{1}{x^2(bx^2+cx^4)}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{3bx^3}+\frac{c}{b^2x}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}$$

```
[Out] -1/3/b/x^3+c/b^2/x+c^(3/2)*arctan(x*c^(1/2)/b^(1/2))/b^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1598, 331, 211}

$$\frac{c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{c}{b^2x}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*1/(b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(bx^2+cx^4)}\,dx &= \int \frac{1}{x^4(b+cx^2)}\,dx\\ &= -\frac{1}{3bx^3}-\frac{c\int \frac{1}{x^2(b+cx^2)}\,dx}{b}\\ &= -\frac{1}{3bx^3}+\frac{c}{b^2x}+\frac{c^2\int \frac{1}{b+cx^2}\,dx}{b^2}\\ &= -\frac{1}{3bx^3}+\frac{c}{b^2x}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.00

$$-\frac{1}{3bx^3}+\frac{c}{b^2x}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)),x]
[Out] -1/3*1/(b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(5/2)
```

Maple [A]
time = 0.10, size = 39, normalized size = 0.91

method	result	size
default	$\frac{c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b^2\sqrt{bc}}-\frac{1}{3b\,x^3}+\frac{c}{b^2x}$	39
risch	$\frac{\frac{c\,x^2}{b^2}-\frac{1}{3b}}{x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^5\_Z^2+c^3\right)}\_R\ln\left(\left(3\_R^2b^5+2c^3\right)x-b^3c\_R\right)\right)}{2}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] c^2/b^2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-1/3/b/x^3+c/b^2/x
```

Maxima [A]
time = 0.51, size = 40, normalized size = 0.93

$$\frac{c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^2}+\frac{3\,cx^2-b}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/3*(3*c*x^2 - b)/(b^2*x^3)
```

Fricas [**A**]
time = 0.33, size = 106, normalized size = 2.47

$$\left[\frac{3\,cx^3\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+6\,cx^2-2\,b}{6\,b^2x^3},\ \frac{3\,cx^3\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+3\,cx^2-b}{3\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*c*x^3*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) +
6*c*x^2 - 2*b)/(b^2*x^3), 1/3*(3*c*x^3*sqrt(c/b)*arctan(x*sqrt(c/b)) + 3*c*
x^2 - b)/(b^2*x^3)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(37) = 74$.
time = 0.10, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{c^3}{b^5}}\log\left(-\frac{b^3\sqrt{-\frac{c^3}{b^5}}}{c^2}+x\right)}{2}+\frac{\sqrt{-\frac{c^3}{b^5}}\log\left(\frac{b^3\sqrt{-\frac{c^3}{b^5}}}{c^2}+x\right)}{2}+\frac{-b+3cx^2}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-c**3/b**5)*log(-b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + sqrt(-c**3/b**5)
*log(b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + (-b + 3*c*x**2)/(3*b**2*x**3)
```

Giac [**A**]
time = 4.81, size = 40, normalized size = 0.93

$$\frac{c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^2}+\frac{3\,cx^2-b}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="giac")
[Out] c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/3*(3*c*x^2 - b)/(b^2*x^3)
```

Mupad [B]
time = 4.14, size = 37, normalized size = 0.86

$$\frac{c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}}-\frac{\frac{1}{3\,b}-\frac{c\,x^2}{b^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)),x)
[Out] (c^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/b^(5/2) - (1/(3*b) - (c*x^2)/b^2)/x^3
```

3.187 $\int \frac{1}{x^3(bx^2+cx^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{4bx^4}+\frac{c}{2b^2x^2}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log(b+cx^2)}{2b^3}$$

```
[Out] -1/4/b/x^4+1/2*c/b^2/x^2+c^2*ln(x)/b^3-1/2*c^2*ln(c*x^2+b)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1598, 272, 46}

$$-\frac{c^2\log(b+cx^2)}{2b^3}+\frac{c^2\log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*1/(b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^2])/(2
*b^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^3 (bx^2 + cx^4)}\, dx = \int \frac{1}{x^5 (b + cx^2)}\, dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(b+cx)}\, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{bx^3} - \frac{c}{b^2x^2} + \frac{c^2}{b^3x} - \frac{c^3}{b^3(b+cx)}\right) dx, x, x^2\right)$$
$$= -\frac{1}{4bx^4} + \frac{c}{2b^2x^2} + \frac{c^2\log(x)}{b^3} - \frac{c^2\log(b+cx^2)}{2b^3}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$-\frac{1}{4bx^4} + \frac{c}{2b^2x^2} + \frac{c^2\log(x)}{b^3} - \frac{c^2\log(b+cx^2)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*1/(b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^2])/(2
*b^3)
```

Maple [A]
time = 0.10, size = 44, normalized size = 0.90

method	result	size
default	$-\frac{1}{4b\,x^4}+\frac{c}{2b^2x^2}+\frac{c^2\ln(x)}{b^3}-\frac{c^2\ln(c\,x^2+b)}{2b^3}$	44
norman	$\frac{-\frac{1}{4b}+\frac{c\,x^2}{2b^2}}{x^4}+\frac{c^2\ln(x)}{b^3}-\frac{c^2\ln(c\,x^2+b)}{2b^3}$	46
risch	$\frac{-\frac{1}{4b}+\frac{c\,x^2}{2b^2}}{x^4}+\frac{c^2\ln(x)}{b^3}-\frac{c^2\ln(c\,x^2+b)}{2b^3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/x^4+1/2*c/b^2/x^2+c^2*ln(x)/b^3-1/2*c^2*ln(c*x^2+b)/b^3
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.96

$$-\frac{c^2\log(cx^2+b)}{2\,b^3} + \frac{c^2\log(x^2)}{2\,b^3} + \frac{2\,cx^2 - b}{4\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*c^2*log(c*x^2 + b)/b^3 + 1/2*c^2*log(x^2)/b^3 + 1/4*(2*c*x^2 - b)/(b^2
*x^4)
```

Fricas [A]
time = 0.33, size = 45, normalized size = 0.92

$$-\frac{2\,c^2x^4\log\left(cx^2+b\right)-4\,c^2x^4\log\left(x\right)-2\,bcx^2+b^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*c^2*x^4*log(c*x^2 + b) - 4*c^2*x^4*log(x) - 2*b*c*x^2 + b^2)/(b^3*x
^4)
```

Sympy [A]
time = 0.14, size = 42, normalized size = 0.86

$$\frac{-b+2cx^2}{4b^2x^4}+\frac{c^2\log\left(x\right)}{b^3}-\frac{c^2\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2),x)
```

```
[Out] (-b + 2*c*x**2)/(4*b**2*x**4) + c**2*log(x)/b**3 - c**2*log(b/c + x**2)/(2*
b**3)
```

Giac [A]
time = 3.21, size = 57, normalized size = 1.16

$$\frac{c^2\log\left(x^2\right)}{2\,b^3}-\frac{c^2\log\left(\left|cx^2+b\right|\right)}{2\,b^3}-\frac{3\,c^2x^4-2\,bcx^2+b^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2)/b^3 - 1/2*c^2*log(abs(c*x^2 + b))/b^3 - 1/4*(3*c^2*x^4 - 2
*b*c*x^2 + b^2)/(b^3*x^4)
```

Mupad [B]
time = 0.06, size = 46, normalized size = 0.94

$$\frac{c^2\,\ln\left(x\right)}{b^3}-\frac{c^2\,\ln\left(c\,x^2+b\right)}{2\,b^3}-\frac{\frac{1}{4\,b}-\frac{c\,x^2}{2\,b^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)),x)
```

```
[Out] (c^2*log(x))/b^3 - (c^2*log(b + c*x^2))/(2*b^3) - (1/(4*b) - (c*x^2)/(2*b^2
))/x^4
```

3.188 $\int \frac{1}{x^4(bx^2+cx^4)}\,dx$

Optimal. Leaf size=58

$$-\frac{1}{5bx^5}+\frac{c}{3b^2x^3}-\frac{c^2}{b^3x}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}$$

```
[Out] -1/5/b/x^5+1/3*c/b^2/x^3-c^2/b^3/x-c^(5/2)*arctan(x*c^(1/2)/b^(1/2))/b^(7/2
)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 331, 211}

$$-\frac{c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{c^2}{b^3x}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*1/(b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqrt[c]*x)/
Sqrt[b]])/b^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(bx^2+cx^4)}\,dx &= \int \frac{1}{x^6(b+cx^2)}\,dx\\ &= -\frac{1}{5bx^5} - \frac{c\int \frac{1}{x^4(b+cx^2)}\,dx}{b}\\ &= -\frac{1}{5bx^5} + \frac{c}{3b^2x^3} + \frac{c^2\int \frac{1}{x^2(b+cx^2)}\,dx}{b^2}\\ &= -\frac{1}{5bx^5} + \frac{c}{3b^2x^3} - \frac{c^2}{b^3x} - \frac{c^3\int \frac{1}{b+cx^2}\,dx}{b^3}\\ &= -\frac{1}{5bx^5} + \frac{c}{3b^2x^3} - \frac{c^2}{b^3x} - \frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 1.00

$$-\frac{1}{5bx^5} + \frac{c}{3b^2x^3} - \frac{c^2}{b^3x} - \frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*1/(b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqrt[c]*x)/
Sqrt[b]])/b^(7/2)
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.90

method	result	size
default	$-\frac{c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b^3\sqrt{bc}} - \frac{1}{5b\,x^5} - \frac{c^2}{b^3x} + \frac{c}{3b^2x^3}$	52
risch	$\frac{-\frac{c^2x^4}{b^3}+\frac{c\,x^2}{3b^2}-\frac{1}{5b}}{x^5} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^7\_Z^2+c^5\right)}\_R\ln\left(\left(3\_R^2b^7+2c^5\right)x+b^4c^2\_R\right)\right)}{2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -c^3/b^3/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-1/5/b/x^5-c^2/b^3/x+1/3*c/b^2/
x^3
```

Maxima [A]
time = 0.49, size = 52, normalized size = 0.90

$$-\frac{c^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\,b^{3}}-\frac{15\,c^{2}x^{4}-5\,bcx^{2}+3\,b^{2}}{15\,b^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/15*(15*c^2*x^4 - 5*b*c*x^2 +
 3*b^2)/(b^3*x^5)
```

Fricas [A]
time = 0.37, size = 132, normalized size = 2.28

$$\left[\frac{15\,c^{2}x^{5}\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^{2}-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)-30\,c^{2}x^{4}+10\,bcx^{2}-6\,b^{2}}{30\,b^{3}x^{5}},-\frac{15\,c^{2}x^{5}\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+15\,c^{2}x^{4}-5\,bcx^{2}+3\,b^{2}}{15\,b^{3}x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*c^2*x^5*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)
) - 30*c^2*x^4 + 10*b*c*x^2 - 6*b^2)/(b^3*x^5), -1/15*(15*c^2*x^5*sqrt(c/b)
*arctan(x*sqrt(c/b)) + 15*c^2*x^4 - 5*b*c*x^2 + 3*b^2)/(b^3*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(49) = 98$.
time = 0.13, size = 100, normalized size = 1.72

$$\frac{\sqrt{-\frac{c^{5}}{b^{7}}}\log\left(-\frac{b^{4}\sqrt{-\frac{c^{5}}{b^{7}}}}{c^{3}}+x\right)}{2}-\frac{\sqrt{-\frac{c^{5}}{b^{7}}}\log\left(\frac{b^{4}\sqrt{-\frac{c^{5}}{b^{7}}}}{c^{3}}+x\right)}{2}+\frac{-3b^{2}+5bcx^{2}-15c^{2}x^{4}}{15b^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c**5/b**7)*log(-b**4*sqrt(-c**5/b**7)/c**3 + x)/2 - sqrt(-c**5/b**7)*
log(b**4*sqrt(-c**5/b**7)/c**3 + x)/2 + (-3*b**2 + 5*b*c*x**2 - 15*c**2*x**
4)/(15*b**3*x**5)
```

Giac [A]
time = 2.53, size = 52, normalized size = 0.90

$$-\frac{c^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}\, b^3} - \frac{15\, c^2 x^4 - 5\, bcx^2 + 3\, b^2}{15\, b^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/15*(15*c^2*x^4 - 5*b*c*x^2 +
 3*b^2)/(b^3*x^5)
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.83

$$-\frac{\frac{1}{5\,b} - \frac{c\,x^2}{3\,b^2} + \frac{c^2\,x^4}{b^3}}{x^5} - \frac{c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b*x^2 + c*x^4)),x)
```

```
[Out] - (1/(5*b) - (c*x^2)/(3*b^2) + (c^2*x^4)/b^3)/x^5 - (c^(5/2)*atan((c^(1/2)*
x)/b^(1/2)))/b^(7/2)
```

3.189 $\int \frac{1}{x^5(bx^2+cx^4)}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{6bx^6}+\frac{c}{4b^2x^4}-\frac{c^2}{2b^3x^2}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log(b+cx^2)}{2b^4}$$

```
[Out] -1/6/b/x^6+1/4*c/b^2/x^4-1/2*c^2/b^3/x^2-c^3*ln(x)/b^4+1/2*c^3*ln(c*x^2+b)/
b^4
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1598, 272, 46}

$$\frac{c^3\log(b+cx^2)}{2b^4}-\frac{c^3\log(x)}{b^4}-\frac{c^2}{2b^3x^2}+\frac{c}{4b^2x^4}-\frac{1}{6bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*1/(b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4 + (c^3*
Log[b + c*x^2])/(2*b^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^5\,(bx^2+cx^4)}\,dx = \int \frac{1}{x^7\,(b+cx^2)}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(b+cx)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{bx^4}-\frac{c}{b^2x^3}+\frac{c^2}{b^3x^2}-\frac{c^3}{b^4x}+\frac{c^4}{b^4(b+cx)}\right)\,dx, x, x^2\right)$$
$$= -\frac{1}{6bx^6}+\frac{c}{4b^2x^4}-\frac{c^2}{2b^3x^2}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(b+cx^2\right)}{2b^4}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 1.00

$$-\frac{1}{6bx^6}+\frac{c}{4b^2x^4}-\frac{c^2}{2b^3x^2}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(b+cx^2\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*1/(b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4 + (c^3*
Log[b + c*x^2])/(2*b^4)
```

Maple [A]
time = 0.08, size = 56, normalized size = 0.89

method	result	size
default	$-\frac{1}{6b\,x^6}+\frac{c}{4b^2x^4}-\frac{c^2}{2b^3x^2}-\frac{c^3\ln(x)}{b^4}+\frac{c^3\ln(c\,x^2+b)}{2b^4}$	56
norman	$\frac{-\frac{1}{6b}+\frac{c\,x^2}{4b^2}-\frac{c^2x^4}{2b^3}}{x^6}-\frac{c^3\ln(x)}{b^4}+\frac{c^3\ln(c\,x^2+b)}{2b^4}$	58
risch	$\frac{-\frac{1}{6b}+\frac{c\,x^2}{4b^2}-\frac{c^2x^4}{2b^3}}{x^6}-\frac{c^3\ln(x)}{b^4}+\frac{c^3\ln(-c\,x^2-b)}{2b^4}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b/x^6+1/4*c/b^2/x^4-1/2*c^2/b^3/x^2-c^3*ln(x)/b^4+1/2*c^3*ln(c*x^2+b)/
b^4
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.92

$$\frac{c^3\log\left(cx^2+b\right)}{2\,b^4}-\frac{c^3\log\left(x^2\right)}{2\,b^4}-\frac{6\,c^2x^4-3\,bcx^2+2\,b^2}{12\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(c*x^2 + b)/b^4 - 1/2*c^3*log(x^2)/b^4 - 1/12*(6*c^2*x^4 - 3*b*c
*x^2 + 2*b^2)/(b^3*x^6)
```

Fricas [A]
time = 0.35, size = 58, normalized size = 0.92

$$\frac{6\,c^3x^6\log\left(cx^2+b\right)-12\,c^3x^6\log\left(x\right)-6\,bc^2x^4+3\,b^2cx^2-2\,b^3}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*c^3*x^6*log(c*x^2 + b) - 12*c^3*x^6*log(x) - 6*b*c^2*x^4 + 3*b^2*c*
x^2 - 2*b^3)/(b^4*x^6)
```

Sympy [A]
time = 0.17, size = 56, normalized size = 0.89

$$\frac{-2b^2+3bcx^2-6c^2x^4}{12b^3x^6}-\frac{c^3\log\left(x\right)}{b^4}+\frac{c^3\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2),x)
```

```
[Out] (-2*b**2 + 3*b*c*x**2 - 6*c**2*x**4)/(12*b**3*x**6) - c**3*log(x)/b**4 + c*
*3*log(b/c + x**2)/(2*b**4)
```

Giac [A]
time = 3.27, size = 70, normalized size = 1.11

$$-\frac{c^3\log\left(x^2\right)}{2\,b^4}+\frac{c^3\log\left(\left|cx^2+b\right|\right)}{2\,b^4}+\frac{11\,c^3x^6-6\,bc^2x^4+3\,b^2cx^2-2\,b^3}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*c^3*log(x^2)/b^4 + 1/2*c^3*log(abs(c*x^2 + b))/b^4 + 1/12*(11*c^3*x^6
- 6*b*c^2*x^4 + 3*b^2*c*x^2 - 2*b^3)/(b^4*x^6)
```

Mupad [B]
time = 0.07, size = 58, normalized size = 0.92

$$\frac{c^3\,\ln\left(c\,x^2+b\right)}{2\,b^4}-\frac{\frac{1}{6\,b}-\frac{c\,x^2}{4\,b^2}+\frac{c^2\,x^4}{2\,b^3}}{x^6}-\frac{c^3\,\ln\left(x\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 + c*x^4)),x)
```

```
[Out] (c^3*log(b + c*x^2))/(2*b^4) - (1/(6*b) - (c*x^2)/(4*b^2) + (c^2*x^4)/(2*b^
3))/x^6 - (c^3*log(x))/b^4
```

3.190 $\int \frac{x^{12}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=79

$$\frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}+\frac{7x^5}{10c^2}-\frac{x^7}{2c\,(b+cx^2)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}$$

```
[Out] 7/2*b^2*x/c^4-7/6*b*x^3/c^3+7/10*x^5/c^2-1/2*x^7/c/(c*x^2+b)-7/2*b^(5/2)*ar
ctan(x*c^(1/2)/b^(1/2))/c^(9/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1598, 294, 308, 211}

$$-\frac{7b^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}+\frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}-\frac{x^7}{2c\,(b+cx^2)}+\frac{7x^5}{10c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (7*b^2*x)/(2*c^4) - (7*b*x^3)/(6*c^3) + (7*x^5)/(10*c^2) - x^7/(2*c*(b + c*
x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{12}}{(bx^2+cx^4)^2}\,dx &= \int \frac{x^8}{(b+cx^2)^2}\,dx \\
&= -\frac{x^7}{2c(b+cx^2)} + \frac{7\int \frac{x^6}{b+cx^2}\,dx}{2c} \\
&= -\frac{x^7}{2c(b+cx^2)} + \frac{7\int \left(\frac{b^2}{c^3} - \frac{bx^2}{c^2} + \frac{x^4}{c} - \frac{b^3}{c^3(b+cx^2)}\right)dx}{2c} \\
&= \frac{7b^2x}{2c^4} - \frac{7bx^3}{6c^3} + \frac{7x^5}{10c^2} - \frac{x^7}{2c(b+cx^2)} - \frac{(7b^3)\int \frac{1}{b+cx^2}\,dx}{2c^4} \\
&= \frac{7b^2x}{2c^4} - \frac{7bx^3}{6c^3} + \frac{7x^5}{10c^2} - \frac{x^7}{2c(b+cx^2)} - \frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.90

$$\frac{x\left(90b^2-20bcx^2+6c^2x^4+\frac{15b^3}{b+cx^2}\right)}{30c^4}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(90*b^2 - 20*b*c*x^2 + 6*c^2*x^4 + (15*b^3)/(b + c*x^2)))/(30*c^4) - (7*
b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Maple [A]
time = 0.10, size = 65, normalized size = 0.82

method	result	size

default	$\frac{\frac{1}{5}c^2x^5-\frac{2}{3}bc\,x^3+3b^2x}{c^4}-\frac{b^3\left(-\frac{x}{2\left(c\,x^2+b\right)}+\frac{7\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^4}$	65
risch	$\frac{x^5}{5c^2}-\frac{2b\,x^3}{3c^3}+\frac{3b^2x}{c^4}+\frac{b^3x}{2c^4(c\,x^2+b)}+\frac{7\sqrt{-bc}\,b^2\ln\left(-\sqrt{-bc}\,x-b\right)}{4c^5}-\frac{7\sqrt{-bc}\,b^2\ln\left(\sqrt{-bc}\,x-b\right)}{4c^5}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/5*c^2*x^5-2/3*b*c*x^3+3*b^2*x)-b^3/c^4*(-1/2*x/(c*x^2+b)+7/2/(b*c)
^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 71, normalized size = 0.90

$$\frac{b^3x}{2\left(c^5x^2+bc^4\right)}-\frac{7\,b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,c^4}+\frac{3\,c^2x^5-10\,bcx^3+45\,b^2x}{15\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x/(c^5*x^2 + b*c^4) - 7/2*b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4)
 + 1/15*(3*c^2*x^5 - 10*b*c*x^3 + 45*b^2*x)/c^4
```

Fricas [A]
time = 0.37, size = 190, normalized size = 2.41

$$\left[\frac{12\,c^3x^7-28\,bc^2x^5+140\,b^2cx^3+210\,b^3x+105\,\left(b^2cx^2+b^3\right)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{60\,\left(c^5x^2+bc^4\right)},\ \frac{6\,c^3x^7-14\,bc^2x^5+70\,b^2cx^3+105\,b^3x-105\,\left(b^2cx^2+b^3\right)\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)}{30\,\left(c^5x^2+bc^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^3*x^7 - 28*b*c^2*x^5 + 140*b^2*c*x^3 + 210*b^3*x + 105*(b^2*c*x
^2 + b^3)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^5*
x^2 + b*c^4), 1/30*(6*c^3*x^7 - 14*b*c^2*x^5 + 70*b^2*c*x^3 + 105*b^3*x - 1
05*(b^2*c*x^2 + b^3)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^5*x^2 + b*c^4)]
```

Sympy [A]
time = 0.14, size = 124, normalized size = 1.57

$$\frac{b^3x}{2bc^4+2c^5x^2}+\frac{3b^2x}{c^4}-\frac{2bx^3}{3c^3}+\frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x-\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4}-\frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x+\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4}+\frac{x^5}{5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(c*x**4+b*x**2)**2,x)
```

```
[Out] b**3*x/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*x/c**4 - 2*b*x**3/(3*c**3) + 7*sqr
t(-b**5/c**9)*log(x - c**4*sqrt(-b**5/c**9)/b**2)/4 - 7*sqrt(-b**5/c**9)*lo
g(x + c**4*sqrt(-b**5/c**9)/b**2)/4 + x**5/(5*c**2)
```

Giac [A]
time = 7.83, size = 73, normalized size = 0.92

$$-\frac{7\,b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^4}+\frac{b^3x}{2\,(cx^2+b)c^4}+\frac{3\,c^8x^5-10\,bc^7x^3+45\,b^2c^6x}{15\,c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/2*b^3*x/((c*x^2 + b)*c^4
) + 1/15*(3*c^8*x^5 - 10*b*c^7*x^3 + 45*b^2*c^6*x)/c^10
```

Mupad [B]
time = 0.04, size = 66, normalized size = 0.84

$$\frac{x^5}{5\,c^2}-\frac{2\,b\,x^3}{3\,c^3}+\frac{3\,b^2\,x}{c^4}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{9/2}}+\frac{b^3\,x}{2\,\left(c^5\,x^2+b\,c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5/(5*c^2) - (2*b*x^3)/(3*c^3) + (3*b^2*x)/c^4 - (7*b^(5/2)*atan((c^(1/2)*
x)/b^(1/2)))/(2*c^(9/2)) + (b^3*x)/(2*(b*c^4 + c^5*x^2))
```

3.191 $\int \frac{x^{11}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{bx^2}{c^3}+\frac{x^4}{4c^2}+\frac{b^3}{2c^4\left(b+cx^2\right)}+\frac{3b^2\log\left(b+cx^2\right)}{2c^4}$$

```
[Out] -b*x^2/c^3+1/4*x^4/c^2+1/2*b^3/c^4/(c*x^2+b)+3/2*b^2*ln(c*x^2+b)/c^4
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 45}

$$\frac{b^3}{2c^4\left(b+cx^2\right)}+\frac{3b^2\log\left(b+cx^2\right)}{2c^4}-\frac{bx^2}{c^3}+\frac{x^4}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*x^2)/c^3) + x^4/(4*c^2) + b^3/(2*c^4*(b + c*x^2)) + (3*b^2*Log[b + c*x
^2])/(2*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11}}{(bx^2+cx^4)^2}\,dx = \int \frac{x^7}{(b+cx^2)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(b+cx)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{2b}{c^3}+\frac{x}{c^2}-\frac{b^3}{c^3(b+cx)^2}+\frac{3b^2}{c^3(b+cx)}\right)\,dx, x, x^2\right)$$
$$= -\frac{bx^2}{c^3}+\frac{x^4}{4c^2}+\frac{b^3}{2c^4(b+cx^2)}+\frac{3b^2\log(b+cx^2)}{2c^4}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.86

$$\frac{-4bcx^2+c^2x^4+\frac{2b^3}{b+cx^2}+6b^2\log(b+cx^2)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(b*x^2 + c*x^4)^2,x]
[Out] (-4*b*c*x^2 + c^2*x^4 + (2*b^3)/(b + c*x^2) + 6*b^2*Log[b + c*x^2])/(4*c^4)
```

Maple [A]
time = 0.11, size = 55, normalized size = 0.96

method	result	size
default	$\frac{(-c\,x^2+2b)^2}{4c^4}+\frac{b^2\left(\frac{3\ln(c\,x^2+b)}{c}+\frac{b}{c(c\,x^2+b)}\right)}{2c^3}$	55
risch	$\frac{x^4}{4c^2}-\frac{b\,x^2}{c^3}+\frac{b^2}{c^4}+\frac{b^3}{2c^4(c\,x^2+b)}+\frac{3b^2\ln(c\,x^2+b)}{2c^4}$	59
norman	$\frac{\frac{x^9}{4c}-\frac{3b\,x^7}{4c^2}+\frac{3b^3x^3}{2c^4}}{x^3(c\,x^2+b)}+\frac{3b^2\ln(c\,x^2+b)}{2c^4}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*(-c*x^2+2*b)^2/c^4+1/2*b^2/c^3*(3*ln(c*x^2+b)/c+b/c/(c*x^2+b))
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.95

$$\frac{b^3}{2(c^5x^2+bc^4)}+\frac{3\,b^2\log(cx^2+b)}{2\,c^4}+\frac{cx^4-4\,bx^2}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3/(c^5*x^2 + b*c^4) + 3/2*b^2*log(c*x^2 + b)/c^4 + 1/4*(c*x^4 - 4*b*x
^2)/c^3
```

Fricas [A]
time = 0.38, size = 70, normalized size = 1.23

$$\frac{c^3x^6 - 3\,bc^2x^4 - 4\,b^2cx^2 + 2\,b^3 + 6\,(b^2cx^2 + b^3)\log\left(cx^2 + b\right)}{4\,(c^5x^2 + bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*x^6 - 3*b*c^2*x^4 - 4*b^2*c*x^2 + 2*b^3 + 6*(b^2*c*x^2 + b^3)*log(
c*x^2 + b))/(c^5*x^2 + b*c^4)
```

Sympy [A]
time = 0.12, size = 53, normalized size = 0.93

$$\frac{b^3}{2bc^4 + 2c^5x^2} + \frac{3b^2\log\left(b + cx^2\right)}{2c^4} - \frac{bx^2}{c^3} + \frac{x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2)**2,x)
```

```
[Out] b**3/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*log(b + c*x**2)/(2*c**4) - b*x**2/c*
*3 + x**4/(4*c**2)
```

Giac [A]
time = 4.75, size = 67, normalized size = 1.18

$$\frac{3\,b^2\log\left(|cx^2 + b|\right)}{2\,c^4} + \frac{c^2x^4 - 4\,bcx^2}{4\,c^4} - \frac{3\,b^2cx^2 + 2\,b^3}{2\,(cx^2 + b)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(abs(c*x^2 + b))/c^4 + 1/4*(c^2*x^4 - 4*b*c*x^2)/c^4 - 1/2*(3*b^
2*c*x^2 + 2*b^3)/((c*x^2 + b)*c^4)
```

Mupad [B]
time = 4.14, size = 57, normalized size = 1.00

$$\frac{x^4}{4\,c^2} + \frac{b^3}{2\,c\,\left(c^4\,x^2 + b\,c^3\right)} - \frac{b\,x^2}{c^3} + \frac{3\,b^2\,\ln\left(c\,x^2 + b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^4/(4*c^2) + b^3/(2*c*(b*c^3 + c^4*x^2)) - (b*x^2)/c^3 + (3*b^2*log(b + c*
x^2))/(2*c^4)
```

3.192 $\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{5bx}{2c^3}+\frac{5x^3}{6c^2}-\frac{x^5}{2c\,(b+cx^2)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}$$

```
[Out] -5/2*b*x/c^3+5/6*x^3/c^2-1/2*x^5/c/(c*x^2+b)+5/2*b^(3/2)*arctan(x*c^(1/2)/b
^(1/2))/c^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1598, 294, 308, 211}

$$\frac{5b^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{5bx}{2c^3}-\frac{x^5}{2c\,(b+cx^2)}+\frac{5x^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-5*b*x)/(2*c^3) + (5*x^3)/(6*c^2) - x^5/(2*c*(b + c*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx &= \int \frac{x^6}{(b+cx^2)^2}\,dx\\ &= -\frac{x^5}{2c(b+cx^2)}+\frac{5\int \frac{x^4}{b+cx^2}\,dx}{2c}\\ &= -\frac{x^5}{2c(b+cx^2)}+\frac{5\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2(b+cx^2)}\right)dx}{2c}\\ &= -\frac{5bx}{2c^3}+\frac{5x^3}{6c^2}-\frac{x^5}{2c(b+cx^2)}+\frac{(5b^2)\int \frac{1}{b+cx^2}\,dx}{2c^3}\\ &= -\frac{5bx}{2c^3}+\frac{5x^3}{6c^2}-\frac{x^5}{2c(b+cx^2)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.91

$$\frac{x\left(-12b+2cx^2-\frac{3b^2}{b+cx^2}\right)}{6c^3}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4)^2,x]
[Out] (x*(-12*b + 2*c*x^2 - (3*b^2)/(b + c*x^2)))/(6*c^3) + (5*b^(3/2)*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Maple [A]
time = 0.08, size = 54, normalized size = 0.82

method	result	size

default	$-\frac{-\frac{1}{3}c\,x^3+2bx}{c^3}+\frac{b^2\left(-\frac{x}{2\left(c\,x^2+b\right)}+\frac{5\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^3}$	54
risch	$\frac{x^3}{3c^2}-\frac{2bx}{c^3}-\frac{b^2x}{2c^3(c\,x^2+b)}+\frac{5\sqrt{-bc}\,b\ln\left(-\sqrt{-bc}\,x+b\right)}{4c^4}-\frac{5\sqrt{-bc}\,b\ln\left(\sqrt{-bc}\,x+b\right)}{4c^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/3*c*x^3+2*b*x)+b^2/c^3*(-1/2*x/(c*x^2+b)+5/2/(b*c)^(1/2)*arctan(
c*x/(b*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 59, normalized size = 0.89

$$-\frac{b^2x}{2\,(c^4x^2+bc^3)}+\frac{5\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^3}+\frac{cx^3-6\,bx}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*x/(c^4*x^2 + b*c^3) + 5/2*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3
) + 1/3*(c*x^3 - 6*b*x)/c^3
```

Fricas [A]
time = 0.36, size = 164, normalized size = 2.48

$$\left[\frac{4\,c^2x^5-20\,bcx^3-30\,b^2x+15\,(bcx^2+b^2)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{12\,(c^4x^2+bc^3)},\ \frac{2\,c^2x^5-10\,bcx^3-15\,b^2x+15\,(bcx^2+b^2)\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)}{6\,(c^4x^2+bc^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*c^2*x^5 - 20*b*c*x^3 - 30*b^2*x + 15*(b*c*x^2 + b^2)*sqrt(-b/c)*lo
g((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^4*x^2 + b*c^3), 1/6*(2*c^
2*x^5 - 10*b*c*x^3 - 15*b^2*x + 15*(b*c*x^2 + b^2)*sqrt(b/c)*arctan(c*x*sqr
t(b/c)/b))/(c^4*x^2 + b*c^3)]
```

Sympy [A]
time = 0.13, size = 107, normalized size = 1.62

$$-\frac{b^2x}{2bc^3+2c^4x^2}-\frac{2bx}{c^3}-\frac{5\sqrt{-\frac{b^3}{c^7}}\log\left(x-\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4}+\frac{5\sqrt{-\frac{b^3}{c^7}}\log\left(x+\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4}+\frac{x^3}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2)**2,x)
```

```
[Out] -b**2*x/(2*b*c**3 + 2*c**4*x**2) - 2*b*x/c**3 - 5*sqrt(-b**3/c**7)*log(x -
c**3*sqrt(-b**3/c**7)/b)/4 + 5*sqrt(-b**3/c**7)*log(x + c**3*sqrt(-b**3/c**
7)/b)/4 + x**3/(3*c**2)
```

Giac [A]
time = 5.14, size = 61, normalized size = 0.92

$$\frac{5\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^3}-\frac{b^2x}{2\,(cx^2+b)c^3}+\frac{c^4x^3-6\,bc^3x}{3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) - 1/2*b^2*x/((c*x^2 + b)*c^3)
 + 1/3*(c^4*x^3 - 6*b*c^3*x)/c^6
```

Mupad [B]
time = 0.06, size = 56, normalized size = 0.85

$$\frac{x^3}{3\,c^2}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{7/2}}-\frac{b^2\,x}{2\,\left(c^4\,x^2+b\,c^3\right)}-\frac{2\,b\,x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^3/(3*c^2) + (5*b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*c^(7/2)) - (b^2*x)/(
2*(b*c^3 + c^4*x^2)) - (2*b*x)/c^3
```

3.193 $\int \frac{x^9}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=44

$$\frac{x^2}{2c^2}-\frac{b^2}{2c^3\,(b+cx^2)}-\frac{b\log\,(b+cx^2)}{c^3}$$

```
[Out] 1/2*x^2/c^2-1/2*b^2/c^3/(c*x^2+b)-b*ln(c*x^2+b)/c^3
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1598, 272, 45}

$$-\frac{b^2}{2c^3\,(b+cx^2)}-\frac{b\log\,(b+cx^2)}{c^3}+\frac{x^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^2/(2*c^2) - b^2/(2*c^3*(b + c*x^2)) - (b*Log[b + c*x^2])/c^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^9}{(bx^2+cx^4)^2}\,dx &= \int \frac{x^5}{(b+cx^2)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{c^2}+\frac{b^2}{c^2(b+cx)^2}-\frac{2b}{c^2(b+cx)}\right)\,dx, x, x^2\right) \\
&= \frac{x^2}{2c^2}-\frac{b^2}{2c^3\,(b+cx^2)}-\frac{b\log\left(b+cx^2\right)}{c^3}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.86

$$
\frac{cx^2-\frac{b^2}{b+cx^2}-2b\log\left(b+cx^2\right)}{2c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (c*x^2 - b^2/(b + c*x^2) - 2*b*Log[b + c*x^2])/(2*c^3)
```

Maple [A]
time = 0.11, size = 44, normalized size = 1.00

method	result	size
risch	$\frac{x^2}{2c^2}-\frac{b^2}{2c^3(c\,x^2+b)}-\frac{b\ln\left(c\,x^2+b\right)}{c^3}$	41
default	$\frac{x^2}{2c^2}-\frac{b\left(\frac{2\ln\left(c\,x^2+b\right)}{c}+\frac{b}{c\left(c\,x^2+b\right)}\right)}{2c^2}$	44
norman	$\frac{\frac{x^7}{2c}-\frac{b^2x^3}{c^3}}{x^3(c\,x^2+b)}-\frac{b\ln\left(c\,x^2+b\right)}{c^3}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c^2-1/2*b/c^2*(2*ln(c*x^2+b)/c+b/c/(c*x^2+b))
```

Maxima [A]
time = 0.29, size = 43, normalized size = 0.98

$$
-\frac{b^2}{2\left(c^4x^2+bc^3\right)}+\frac{x^2}{2\,c^2}-\frac{b\log\left(cx^2+b\right)}{c^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2/(c^4*x^2 + b*c^3) + 1/2*x^2/c^2 - b*log(c*x^2 + b)/c^3
```

Fricas [A]
time = 0.33, size = 56, normalized size = 1.27

$$\frac{c^2x^4 + bcx^2 - b^2 - 2\left(bcx^2 + b^2\right)\log\left(cx^2 + b\right)}{2\left(c^4x^2 + bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^4 + b*c*x^2 - b^2 - 2*(b*c*x^2 + b^2)*log(c*x^2 + b))/(c^4*x^2 +
 b*c^3)
```

Sympy [A]
time = 0.11, size = 39, normalized size = 0.89

$$-\frac{b^2}{2bc^3 + 2c^4x^2} - \frac{b\log\left(b + cx^2\right)}{c^3} + \frac{x^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**2,x)
```

```
[Out] -b**2/(2*b*c**3 + 2*c**4*x**2) - b*log(b + c*x**2)/c**3 + x**2/(2*c**2)
```

Giac [A]
time = 6.07, size = 49, normalized size = 1.11

$$\frac{x^2}{2\,c^2} - \frac{b\log\left(|cx^2 + b|\right)}{c^3} + \frac{2\,bcx^2 + b^2}{2\,(cx^2 + b)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/c^2 - b*log(abs(c*x^2 + b))/c^3 + 1/2*(2*b*c*x^2 + b^2)/((c*x^2 + b
)*c^3)
```

Mupad [B]
time = 0.04, size = 45, normalized size = 1.02

$$\frac{x^2}{2\,c^2} - \frac{b^2}{2\,\left(c^4\,x^2 + b\,c^3\right)} - \frac{b\,\ln\left(c\,x^2 + b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^2/(2*c^2) - b^2/(2*(b*c^3 + c^4*x^2)) - (b*log(b + c*x^2))/c^3
```

3.194 $\int \frac{x^8}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=55

$$\frac{3x}{2c^2} - \frac{x^3}{2c(b+cx^2)} - \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{5/2}}$$

```
[Out] 3/2*x/c^2-1/2*x^3/c/(c*x^2+b)-3/2*arctan(x*c^(1/2)/b^(1/2))*b^(1/2)/c^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1598, 294, 327, 211}

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{5/2}} - \frac{x^3}{2c(b+cx^2)} + \frac{3x}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (3*x)/(2*c^2) - x^3/(2*c*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[
b]])/(2*c^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(bx^2+cx^4)^2}\,dx &= \int \frac{x^4}{(b+cx^2)^2}\,dx \\
&= -\frac{x^3}{2c(b+cx^2)} + \frac{3\int \frac{x^2}{b+cx^2}\,dx}{2c} \\
&= \frac{3x}{2c^2} - \frac{x^3}{2c(b+cx^2)} - \frac{(3b)\int \frac{1}{b+cx^2}\,dx}{2c^2} \\
&= \frac{3x}{2c^2} - \frac{x^3}{2c(b+cx^2)} - \frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 0.93

$$\frac{x}{c^2} + \frac{bx}{2c^2(b+cx^2)} - \frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4)^2,x]
[Out] x/c^2 + (b*x)/(2*c^2*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])
/(2*c^(5/2))
```

Maple [A]
time = 0.09, size = 42, normalized size = 0.76

method	result	size
default	$\frac{x}{c^2} - \frac{b\left(-\frac{x}{2(c\,x^2+b)} + \frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^2}$	42

risch	$\frac{x}{c^2}+\frac{bx}{2c^2(c\,x^2+b)}+\frac{3\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)}{4c^3}-\frac{3\sqrt{-bc}\,\ln\left(\sqrt{-bc}\,x-b\right)}{4c^3}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^2-b/c^2*(-1/2*x/(c*x^2+b)+3/2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Maxima [A]
time = 0.53, size = 45, normalized size = 0.82

$$\frac{bx}{2\,(c^3x^2+bc^2)}-\frac{3\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^2}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x/(c^3*x^2 + b*c^2) - 3/2*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + x
/c^2
```

Fricas [A]
time = 0.37, size = 136, normalized size = 2.47

$$\left[\frac{4\,cx^3+3\,(cx^2+b)\sqrt{-\frac{b}{c}}\,\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+6\,bx}{4\,(c^3x^2+bc^2)},\ \frac{2\,cx^3-3\,(cx^2+b)\sqrt{\frac{b}{c}}\,\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)+3\,bx}{2\,(c^3x^2+bc^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*x^3 + 3*(c*x^2 + b)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)
/(c*x^2 + b)) + 6*b*x)/(c^3*x^2 + b*c^2), 1/2*(2*c*x^3 - 3*(c*x^2 + b)*sqrt
(b/c)*arctan(c*x*sqrt(b/c)/b) + 3*b*x)/(c^3*x^2 + b*c^2)]
```

Sympy [A]
time = 0.12, size = 83, normalized size = 1.51

$$\frac{bx}{2bc^2+2c^3x^2}+\frac{3\sqrt{-\frac{b}{c^5}}\,\log\left(-c^2\sqrt{-\frac{b}{c^5}}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{c^5}}\,\log\left(c^2\sqrt{-\frac{b}{c^5}}+x\right)}{4}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2)**2,x)
```

```
[Out] b*x/(2*b*c**2 + 2*c**3*x**2) + 3*sqrt(-b/c**5)*log(-c**2*sqrt(-b/c**5) + x)
/4 - 3*sqrt(-b/c**5)*log(c**2*sqrt(-b/c**5) + x)/4 + x/c**2
```

Giac [A]
time = 5.36, size = 42, normalized size = 0.76

$$-\frac{3\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c^2}+\frac{bx}{2\,(cx^2+b)c^2}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/2*b*x/((c*x^2 + b)*c^2) +
x/c^2
```

Mupad [B]
time = 4.17, size = 43, normalized size = 0.78

$$\frac{x}{c^2}+\frac{b\,x}{2\,\left(c^3\,x^2+b\,c^2\right)}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x/c^2 + (b*x)/(2*(b*c^2 + c^3*x^2)) - (3*b^(1/2)*atan((c^(1/2)*x)/b^(1/2)))
/(2*c^(5/2))
```

3.195 $\int \frac{x^7}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=33

$$\frac{b}{2c^2(b+cx^2)}+\frac{\log(b+cx^2)}{2c^2}$$

```
[Out] 1/2*b/c^2/(c*x^2+b)+1/2*ln(c*x^2+b)/c^2
```

Rubi [A]

time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1598, 272, 45}

$$\frac{b}{2c^2(b+cx^2)}+\frac{\log(b+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4)^2,x]
```

```
[Out] b/(2*c^2*(b + c*x^2)) + Log[b + c*x^2]/(2*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(bx^2+cx^4)^2}\,dx &= \int \frac{x^3}{(b+cx^2)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(b+cx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b}{c(b+cx)^2}+\frac{1}{c(b+cx)}\right)\,dx, x, x^2\right)\\ &= \frac{b}{2c^2(b+cx^2)}+\frac{\log(b+cx^2)}{2c^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{b}{b+cx^2}+\log(b+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b/(b + c*x^2) + Log[b + c*x^2])/(2*c^2)
```

Maple [A]
time = 0.09, size = 30, normalized size = 0.91

method	result	size
default	$\frac{b}{2c^2(cx^2+b)}+\frac{\ln(cx^2+b)}{2c^2}$	30
norman	$\frac{b}{2c^2(cx^2+b)}+\frac{\ln(cx^2+b)}{2c^2}$	30
risch	$\frac{b}{2c^2(cx^2+b)}+\frac{\ln(cx^2+b)}{2c^2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/c^2/(c*x^2+b)+1/2*ln(c*x^2+b)/c^2
```

Maxima [A]
time = 0.28, size = 32, normalized size = 0.97

$$\frac{b}{2(c^3x^2+bc^2)}+\frac{\log(cx^2+b)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b/(c^3*x^2 + b*c^2) + 1/2*log(c*x^2 + b)/c^2
```

Fricas [A]
time = 0.34, size = 35, normalized size = 1.06

$$\frac{(cx^2+b)\log(cx^2+b)+b}{2(c^3x^2+bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((c*x^2 + b)*log(c*x^2 + b) + b)/(c^3*x^2 + b*c^2)
```

Sympy [A]
time = 0.08, size = 29, normalized size = 0.88

$$\frac{b}{2bc^2+2c^3x^2}+\frac{\log(b+cx^2)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**2,x)
```

```
[Out] b/(2*b*c**2 + 2*c**3*x**2) + log(b + c*x**2)/(2*c**2)
```

Giac [A]
time = 5.49, size = 32, normalized size = 0.97

$$-\frac{x^2}{2(cx^2+b)c}+\frac{\log(|cx^2+b|)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^2/((c*x^2 + b)*c) + 1/2*log(abs(c*x^2 + b))/c^2
```

Mupad [B]
time = 4.18, size = 29, normalized size = 0.88

$$\frac{\ln(c\,x^2+b)}{2\,c^2}+\frac{b}{2\,c^2\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^2,x)
```

```
[Out] log(b + c*x^2)/(2*c^2) + b/(2*c^2*(b + c*x^2))
```

3.196 $\int \frac{x^6}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=45

$$-\frac{x}{2c(b+cx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}}$$

```
[Out] -1/2*x/c/(c*x^2+b)+1/2*arctan(x*c^(1/2)/b^(1/2))/c^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 294, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}}-\frac{x}{2c(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^2,x]
[Out] -1/2*x/(c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^6}{(bx^2+cx^4)^2}\,dx = \int \frac{x^2}{(b+cx^2)^2}\,dx$$
$$= -\frac{x}{2c(b+cx^2)} + \frac{\int \frac{1}{b+cx^2}\,dx}{2c}$$
$$= -\frac{x}{2c(b+cx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}}$$

Mathematica [A]
time = 0.01, size = 45, normalized size = 1.00

$$-\frac{x}{2c(b+cx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^2,x]
[Out] -1/2*x/(c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.80

method	result	size
default	$-\frac{x}{2c(c\,x^2+b)} + \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2c\sqrt{bc}}$	36
risch	$-\frac{x}{2c(c\,x^2+b)} - \frac{\ln\left(cx+\sqrt{-bc}\right)}{4\sqrt{-bc}\,c} + \frac{\ln\left(-cx+\sqrt{-bc}\right)}{4\sqrt{-bc}\,c}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*x/c/(c*x^2+b)+1/2/c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.48, size = 36, normalized size = 0.80

$$-\frac{x}{2(c^2x^2+bc)} + \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(c^2*x^2 + b*c) + 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

Fricas [A]
time = 0.34, size = 120, normalized size = 2.67

$$\left[-\frac{2\,bcx + (cx^2+b)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{4\,(bc^3x^2+b^2c^2)},\,-\frac{bcx-(cx^2+b)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{2\,(bc^3x^2+b^2c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b*c*x + (c*x^2 + b)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c
*x^2 + b)))/(b*c^3*x^2 + b^2*c^2), -1/2*(b*c*x - (c*x^2 + b)*sqrt(b*c)*arct
an(sqrt(b*c)*x/b))/(b*c^3*x^2 + b^2*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.
time = 0.09, size = 78, normalized size = 1.73

$$-\frac{x}{2bc+2c^2x^2}-\frac{\sqrt{-\frac{1}{bc^3}}\log\left(-bc\sqrt{-\frac{1}{bc^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{bc^3}}\log\left(bc\sqrt{-\frac{1}{bc^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**2,x)
```

```
[Out] -x/(2*b*c + 2*c**2*x**2) - sqrt(-1/(b*c**3))*log(-b*c*sqrt(-1/(b*c**3)) + x
)/4 + sqrt(-1/(b*c**3))*log(b*c*sqrt(-1/(b*c**3)) + x)/4
```

Giac [A]
time = 4.95, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,c}-\frac{x}{2\,(cx^2+b)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) - 1/2*x/((c*x^2 + b)*c)
```

Mupad [B]
time = 4.15, size = 33, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,\sqrt{b}\,c^{3/2}}-\frac{x}{2\,c\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4)^2,x)
```

```
[Out] atan((c^(1/2)*x)/b^(1/2))/(2*b^(1/2)*c^(3/2)) - x/(2*c*(b + c*x^2))
```

3.197 $\int \frac{x^5}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2c\,(b+cx^2)}$$

```
[Out] -1/2/c/(c*x^2+b)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 267}

$$-\frac{1}{2c\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*1/(c*(b + c*x^2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(bx^2+cx^4)^2} \, dx &= \int \frac{x}{(b+cx^2)^2} \, dx\\ &= -\frac{1}{2c\,(b+cx^2)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2c\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^2,x]
[Out] -1/2*1/(c*(b + c*x^2))
```

Maple [A]
time = 0.08, size = 15, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{2c(c\,x^2+b)}$	15
default	$-\frac{1}{2c(c\,x^2+b)}$	15
norman	$-\frac{1}{2c(c\,x^2+b)}$	15
risch	$-\frac{1}{2c(c\,x^2+b)}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/2/c/(c*x^2+b)
```

Maxima [A]
time = 0.29, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] -1/2/(c^2*x^2 + b*c)
```

Fricas [A]
time = 0.41, size = 15, normalized size = 0.94

$$-\frac{1}{2\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] -1/2/(c^2*x^2 + b*c)
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.94

$$-\frac{1}{2bc+2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**2,x)
```

```
[Out] -1/(2*b*c + 2*c**2*x**2)
```

Giac [A]
time = 5.06, size = 14, normalized size = 0.88

$$-\frac{1}{2\,(cx^2+b)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2/((c*x^2 + b)*c)
```

Mupad [B]
time = 0.02, size = 14, normalized size = 0.88

$$-\frac{1}{2\,c\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^2,x)
```

```
[Out] -1/(2*c*(b + c*x^2))
```

3.198 $\int \frac{x^4}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=45

$$\frac{x}{2b\,(b+cx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}$$

```
[Out] 1/2*x/b/(c*x^2+b)+1/2*arctan(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^4}{(bx^2+cx^4)^2}\,dx = \int \frac{1}{(b+cx^2)^2}\,dx$$
$$= \frac{x}{2b(b+cx^2)} + \frac{\int \frac{1}{b+cx^2}\,dx}{2b}$$
$$= \frac{x}{2b(b+cx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$\frac{x}{2b(b+cx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.80

method	result	size
default	$\frac{x}{2b(c\,x^2+b)} + \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2b\sqrt{bc}}$	36
risch	$\frac{x}{2b(c\,x^2+b)} - \frac{\ln\left(cx+\sqrt{-bc}\right)}{4\sqrt{-bc}\;b} + \frac{\ln\left(-cx+\sqrt{-bc}\right)}{4\sqrt{-bc}\;b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/b/(c*x^2+b)+1/2/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 35, normalized size = 0.78

$$\frac{x}{2(bcx^2+b^2)} + \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(b*c*x^2 + b^2) + 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b)
```

Fricas [A]
time = 0.40, size = 120, normalized size = 2.67

$$\left[\frac{2\,bcx - (cx^2+b)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{4\,(b^2c^2x^2+b^3c)}, \frac{bcx + (cx^2+b)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{2\,(b^2c^2x^2+b^3c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c*x - (c*x^2 + b)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*
x^2 + b)))/(b^2*c^2*x^2 + b^3*c), 1/2*(b*c*x + (c*x^2 + b)*sqrt(b*c)*arctan
(sqrt(b*c)*x/b))/(b^2*c^2*x^2 + b^3*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(36) = 72.
time = 0.09, size = 78, normalized size = 1.73

$$\frac{x}{2b^2+2bcx^2} - \frac{\sqrt{-\frac{1}{b^3c}}\,\log\left(-b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{b^3c}}\,\log\left(b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**2,x)
```

```
[Out] x/(2*b**2 + 2*b*c*x**2) - sqrt(-1/(b**3*c))*log(-b**2*sqrt(-1/(b**3*c)) + x
)/4 + sqrt(-1/(b**3*c))*log(b**2*sqrt(-1/(b**3*c)) + x)/4
```

Giac [A]
time = 5.92, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b} + \frac{x}{2\,(cx^2+b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) + 1/2*x/((c*x^2 + b)*b)
```

Mupad [B]
time = 0.04, size = 33, normalized size = 0.73

$$\frac{x}{2\,b\,\left(c\,x^2+b\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{3/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^2,x)
[Out] x/(2*b*(b + c*x^2)) + atan((c^(1/2)*x)/b^(1/2))/(2*b^(3/2)*c^(1/2))
```

3.199 $\int \frac{x^3}{\left(bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=38

$$\frac{1}{2b\left(b+cx^2\right)}+\frac{\log(x)}{b^2}-\frac{\log\left(b+cx^2\right)}{2b^2}$$

[Out] 1/2/b/(c*x^2+b)+ln(x)/b^2-1/2*ln(c*x^2+b)/b^2

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 46}

$$-\frac{\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)}{b^2}+\frac{1}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(b*x^2 + c*x^4)^2,x]

[Out] 1/(2*b*(b + c*x^2)) + Log[x]/b^2 - Log[b + c*x^2]/(2*b^2)

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{(bx^2+cx^4)^2}\,dx &= \int \frac{1}{x\,(b+cx^2)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^2x}-\frac{c}{b(b+cx)^2}-\frac{c}{b^2(b+cx)}\right)dx, x, x^2\right) \\
&= \frac{1}{2b\,(b+cx^2)}+\frac{\log(x)}{b^2}-\frac{\log\left(b+cx^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.87

$$
\frac{\frac{b}{b+cx^2}+2\log(x)-\log\left(b+cx^2\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b/(b + c*x^2) + 2*Log[x] - Log[b + c*x^2])/(2*b^2)
```

Maple [A]
time = 0.09, size = 42, normalized size = 1.11

method	result	size
risch	$\frac{1}{2b(c\,x^2+b)}+\frac{\ln(x)}{b^2}-\frac{\ln(c\,x^2+b)}{2b^2}$	35
norman	$-\frac{c\,x^2}{2b^2(c\,x^2+b)}+\frac{\ln(x)}{b^2}-\frac{\ln(c\,x^2+b)}{2b^2}$	39
default	$-\frac{c\left(-\frac{b}{c\left(c\,x^2+b\right)}+\frac{\ln\left(c\,x^2+b\right)}{c}\right)}{2b^2}+\frac{\ln(x)}{b^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c/b^2*(-b/c/(c*x^2+b)+ln(c*x^2+b)/c)+ln(x)/b^2
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.97

$$
\frac{1}{2\left(bcx^2+b^2\right)}-\frac{\log\left(cx^2+b\right)}{2\,b^2}+\frac{\log\left(x^2\right)}{2\,b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(b*c*x^2 + b^2) - 1/2*log(c*x^2 + b)/b^2 + 1/2*log(x^2)/b^2
```

Fricas [A]
time = 0.37, size = 47, normalized size = 1.24

$$-\frac{(cx^2+b)\log(cx^2+b)-2(cx^2+b)\log(x)-b}{2(b^2cx^2+b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((c*x^2 + b)*log(c*x^2 + b) - 2*(c*x^2 + b)*log(x) - b)/(b^2*c*x^2 + b
^3)
```

Sympy [A]
time = 0.14, size = 34, normalized size = 0.89

$$\frac{1}{2b^2+2bcx^2}+\frac{\log(x)}{b^2}-\frac{\log\left(\frac{b}{c}+x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b**2 + 2*b*c*x**2) + log(x)/b**2 - log(b/c + x**2)/(2*b**2)
```

Giac [A]
time = 6.61, size = 36, normalized size = 0.95

$$-\frac{\log(|cx^2+b|)}{2\,b^2}+\frac{\log(|x|)}{b^2}+\frac{1}{2\,(cx^2+b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*log(abs(c*x^2 + b))/b^2 + log(abs(x))/b^2 + 1/2/((c*x^2 + b)*b)
```

Mupad [B]
time = 4.18, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{b^2}+\frac{1}{2\,b\,\left(c\,x^2+b\right)}-\frac{\ln\left(c\,x^2+b\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^2,x)
```

```
[Out] log(x)/b^2 + 1/(2*b*(b + c*x^2)) - log(b + c*x^2)/(2*b^2)
```

3.200 $\int \frac{x^2}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}$$

```
[Out] -3/2/b^2/x+1/2/b/x/(c*x^2+b)-3/2*arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/b^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1598, 296, 331, 211}

$$-\frac{3\sqrt{c}\,\mathrm{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -3/(2*b^2*x) + 1/(2*b*x*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b
]])/(2*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(bx^2+cx^4)^2}\,dx &= \int \frac{1}{x^2(b+cx^2)^2}\,dx\\ &= \frac{1}{2bx(b+cx^2)}+\frac{3\int \frac{1}{x^2(b+cx^2)}\,dx}{2b}\\ &= -\frac{3}{2b^2x}+\frac{1}{2bx(b+cx^2)}-\frac{(3c)\int \frac{1}{b+cx^2}\,dx}{2b^2}\\ &= -\frac{3}{2b^2x}+\frac{1}{2bx(b+cx^2)}-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.95

$$-\frac{1}{b^2x}-\frac{cx}{2b^2(b+cx^2)}-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^2,x]
[Out] -(1/(b^2*x)) - (c*x)/(2*b^2*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(2*b^(5/2))
```

Maple [A]
time = 0.08, size = 45, normalized size = 0.79

method	result	size
default	$-\frac{c\left(\frac{x}{2c\,x^2+2b}+\frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^2}-\frac{1}{b^2x}$	45

risch	$\frac{-\frac{3 c x^2}{2 b^2}-\frac{1}{b}}{x(c x^2+b)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(b^5\_Z^2+c\right)}\_R\ln\left(\left(3\_R^2 b^5+2c\right)x+b^3\_R\right)\right)}{4}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] -c/b^2*(1/2*x/(c*x^2+b)+3/2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))-1/b^2/x
```

Maxima [A]
time = 0.52, size = 49, normalized size = 0.86

$$-\frac{3\,cx^2+2\,b}{2\,(b^2cx^3+b^3x)}-\frac{3\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] -1/2*(3*c*x^2 + 2*b)/(b^2*c*x^3 + b^3*x) - 3/2*c*arctan(c*x/sqrt(b*c))/(sqr
t(b*c)*b^2)
```

Fricas [A]
time = 0.35, size = 136, normalized size = 2.39

$$\left[-\frac{6\,cx^2-3\,(cx^3+bx)\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+4\,b}{4\,(b^2cx^3+b^3x)},-\frac{3\,cx^2+3\,(cx^3+bx)\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+2\,b}{2\,(b^2cx^3+b^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] [-1/4*(6*c*x^2 - 3*(c*x^3 + b*x)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) -
 b)/(c*x^2 + b)) + 4*b)/(b^2*c*x^3 + b^3*x), -1/2*(3*c*x^2 + 3*(c*x^3 + b*x
)*sqrt(c/b)*arctan(x*sqrt(c/b)) + 2*b)/(b^2*c*x^3 + b^3*x)]
```

Sympy [A]
time = 0.15, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{c}{b^5}}\,\log\left(-\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}-\frac{3\sqrt{-\frac{c}{b^5}}\,\log\left(\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}+\frac{-2b-3cx^2}{2b^3x+2b^2cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 3*sqrt(-c/b**5)*log(-b**3*sqrt(-c/b**5)/c + x)/4 - 3*sqrt(-c/b**5)*log(b**3
*sqrt(-c/b**5)/c + x)/4 + (-2*b - 3*c*x**2)/(2*b**3*x + 2*b**2*c*x**3)
```

Giac [A]
time = 6.70, size = 47, normalized size = 0.82

$$-\frac{3\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^2}-\frac{3\,cx^2+2\,b}{2\,(cx^3+bx)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -3/2*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/2*(3*c*x^2 + 2*b)/((c*x^3
+ b*x)*b^2)
```

Mupad [B]
time = 0.06, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{b}+\frac{3\,c\,x^2}{2\,b^2}}{c\,x^3+b\,x}-\frac{3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - (1/b + (3*c*x^2)/(2*b^2))/(b*x + c*x^3) - (3*c^(1/2)*atan((c^(1/2)*x)/b^(
1/2)))/(2*b^(5/2))
```

3.201 $\int \frac{x}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{2b^2x^2}-\frac{c}{2b^2\left(b+cx^2\right)}-\frac{2c\log(x)}{b^3}+\frac{c\log\left(b+cx^2\right)}{b^3}$$

```
[Out] -1/2/b^2/x^2-1/2*c/b^2/(c*x^2+b)-2*c*ln(x)/b^3+c*ln(c*x^2+b)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1598, 272, 46}

$$\frac{c\log\left(b+cx^2\right)}{b^3}-\frac{2c\log(x)}{b^3}-\frac{c}{2b^2\left(b+cx^2\right)}-\frac{1}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*1/(b^2*x^2) - c/(2*b^2*(b + c*x^2)) - (2*c*Log[x])/b^3 + (c*Log[b + c*
x^2])/b^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^3\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(b+cx)^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^2x^2}-\frac{2c}{b^3x}+\frac{c^2}{b^2(b+cx)^2}+\frac{2c^2}{b^3(b+cx)}\right)\,dx,x,x^2\right) \\
&= -\frac{1}{2b^2x^2}-\frac{c}{2b^2\left(b+cx^2\right)}-\frac{2c\log(x)}{b^3}+\frac{c\log\left(b+cx^2\right)}{b^3}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.84

$$
-\frac{b\left(\frac{1}{x^2}+\frac{c}{b+cx^2}\right)+4c\log(x)-2c\log\left(b+cx^2\right)}{2b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(b*(x^(-2) + c/(b + c*x^2)) + 4*c*Log[x] - 2*c*Log[b + c*x^2])/b^3
```

Maple [A]
time = 0.08, size = 55, normalized size = 1.12

method	result	size
norman	$\frac{\frac{c^2x^5}{b^3}-\frac{x}{2b}}{x^3(cx^2+b)}+\frac{c\ln(cx^2+b)}{b^3}-\frac{2c\ln(x)}{b^3}$	53
risch	$\frac{-\frac{cx^2}{b^2}-\frac{1}{2b}}{x^2(cx^2+b)}-\frac{2c\ln(x)}{b^3}+\frac{c\ln(-cx^2-b)}{b^3}$	54
default	$\frac{c^2\left(-\frac{b}{c\left(cx^2+b\right)}+\frac{2\ln\left(cx^2+b\right)}{c}\right)}{2b^3}-\frac{1}{2b^2x^2}-\frac{2c\ln(x)}{b^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c^2/b^3*(-b/c/(c*x^2+b)+2*ln(c*x^2+b)/c)-1/2/b^2/x^2-2*c*ln(x)/b^3
```

Maxima [A]
time = 0.28, size = 52, normalized size = 1.06

$$
-\frac{2\,cx^2+b}{2\left(b^2cx^4+b^3x^2\right)}+\frac{c\log\left(cx^2+b\right)}{b^3}-\frac{c\log\left(x^2\right)}{b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2) + c*log(c*x^2 + b)/b^3 - c*log(x^2
)/b^3
```

Fricas [A]
time = 0.35, size = 73, normalized size = 1.49

$$-\frac{2\,bcx^2+b^2-2\,(c^2x^4+bcx^2)\log\left(cx^2+b\right)+4\,(c^2x^4+bcx^2)\log\left(x\right)}{2\,(b^3cx^4+b^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*x^2 + b^2 - 2*(c^2*x^4 + b*c*x^2)*log(c*x^2 + b) + 4*(c^2*x^4 +
 b*c*x^2)*log(x))/(b^3*c*x^4 + b^4*x^2)
```

Sympy [A]
time = 0.19, size = 51, normalized size = 1.04

$$\frac{-b-2cx^2}{2b^3x^2+2b^2cx^4}-\frac{2c\log\left(x\right)}{b^3}+\frac{c\log\left(\frac{b}{c}+x^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-b - 2*c*x**2)/(2*b**3*x**2 + 2*b**2*c*x**4) - 2*c*log(x)/b**3 + c*log(b/c
 + x**2)/b**3
```

Giac [A]
time = 5.01, size = 50, normalized size = 1.02

$$\frac{c\log\left(|cx^2+b|\right)}{b^3}-\frac{2\,c\log\left(|x|\right)}{b^3}-\frac{2\,cx^2+b}{2\,(cx^4+bx^2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] c*log(abs(c*x^2 + b))/b^3 - 2*c*log(abs(x))/b^3 - 1/2*(2*c*x^2 + b)/((c*x^4
 + b*x^2)*b^2)
```

Mupad [B]
time = 4.21, size = 51, normalized size = 1.04

$$\frac{c\,\ln\left(c\,x^2+b\right)}{b^3}-\frac{\frac{1}{2\,b}+\frac{c\,x^2}{b^2}}{c\,x^4+b\,x^2}-\frac{2\,c\,\ln\left(x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (c*log(b + c*x^2))/b^3 - (1/(2*b) + (c*x^2)/b^2)/(b*x^2 + c*x^4) - (2*c*log
(x))/b^3
```

3.202 $\int \frac{1}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{5}{6b^2x^3}+\frac{5c}{2b^3x}+\frac{1}{2bx^3\left(b+cx^2\right)}+\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}$$

```
[Out] -5/6/b^2/x^3+5/2*c/b^3/x+1/2/b/x^3/(c*x^2+b)+5/2*c^(3/2)*arctan(x*c^(1/2)/b
^(1/2))/b^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1607, 296, 331, 211}

$$\frac{5c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{5c}{2b^3x}-\frac{5}{6b^2x^3}+\frac{1}{2bx^3\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-2),x]
```

```
[Out] -5/(6*b^2*x^3) + (5*c)/(2*b^3*x) + 1/(2*b*x^3*(b + c*x^2)) + (5*c^(3/2)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(bx^2+cx^4)^2}\,dx &= \int \frac{1}{x^4(b+cx^2)^2}\,dx\\
&= \frac{1}{2bx^3(b+cx^2)} + \frac{5\int \frac{1}{x^4(b+cx^2)}\,dx}{2b}\\
&= -\frac{5}{6b^2x^3} + \frac{1}{2bx^3(b+cx^2)} - \frac{(5c)\int \frac{1}{x^2(b+cx^2)}\,dx}{2b^2}\\
&= -\frac{5}{6b^2x^3} + \frac{5c}{2b^3x} + \frac{1}{2bx^3(b+cx^2)} + \frac{(5c^2)\int \frac{1}{b+cx^2}\,dx}{2b^3}\\
&= -\frac{5}{6b^2x^3} + \frac{5c}{2b^3x} + \frac{1}{2bx^3(b+cx^2)} + \frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.99

$$-\frac{1}{3b^2x^3} + \frac{2c}{b^3x} + \frac{c^2x}{2b^3(b+cx^2)} + \frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-2),x]
```

```
[Out] -1/3*1/(b^2*x^3) + (2*c)/(b^3*x) + (c^2*x)/(2*b^3*(b + c*x^2)) + (5*c^(3/2)
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Maple [A]
time = 0.08, size = 55, normalized size = 0.81

method	result	size

default	$\frac{c^2\left(\frac{x}{2c\,x^2+2b}+\frac{5\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^3}-\frac{1}{3b^2x^3}+\frac{2c}{b^3x}$	55
risch	$\frac{\frac{5c^2x^4}{2b^3}+\frac{5c\,x^2}{3b^2}-\frac{1}{3b}}{x^3(c\,x^2+b)}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(b^7\_Z^2+c^3\right)}\_R\ln\left(\left(3\_R^2b^7+2c^3\right)x-b^4c\_R\right)\right)}{4}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2/b^3*(1/2*x/(c*x^2+b)+5/2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))-1/3/b^2/x
^3+2*c/b^3/x
```

Maxima [A]
time = 0.53, size = 64, normalized size = 0.94

$$\frac{15\,c^2x^4+10\,bcx^2-2\,b^2}{6\,(b^3cx^5+b^4x^3)}+\frac{5\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*c^2*x^4 + 10*b*c*x^2 - 2*b^2)/(b^3*c*x^5 + b^4*x^3) + 5/2*c^2*arcta
n(c*x/sqrt(b*c))/(sqrt(b*c)*b^3)
```

Fricas [A]
time = 0.34, size = 172, normalized size = 2.53

$$\left[\frac{30\,c^2x^4+20\,bcx^2+15\,(c^2x^5+bcx^3)\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-4\,b^2}{12\,(b^3cx^5+b^4x^3)},\ \frac{15\,c^2x^4+10\,bcx^2+15\,(c^2x^5+bcx^3)\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)-2\,b^2}{6\,(b^3cx^5+b^4x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*c^2*x^4 + 20*b*c*x^2 + 15*(c^2*x^5 + b*c*x^3)*sqrt(-c/b)*log((c*x
^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) - 4*b^2)/(b^3*c*x^5 + b^4*x^3), 1/6
*(15*c^2*x^4 + 10*b*c*x^2 + 15*(c^2*x^5 + b*c*x^3)*sqrt(c/b)*arctan(x*sqrt(
c/b)) - 2*b^2)/(b^3*c*x^5 + b^4*x^3)]
```

Sympy [**A**]
time = 0.18, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4}+\frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4}+\frac{-2b^2+10bcx^2+15c^2x^4}{6b^4x^3+6b^3cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**2,x)
```

```
[Out] -5*sqrt(-c**3/b**7)*log(-b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + 5*sqrt(-c**3/b
**7)*log(b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + (-2*b**2 + 10*b*c*x**2 + 15*c*
*2*x**4)/(6*b**4*x**3 + 6*b**3*c*x**5)
```

Giac [**A**]
time = 3.56, size = 59, normalized size = 0.87

$$\frac{5\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}\,b^3}+\frac{c^2x}{2\,(cx^2+b)b^3}+\frac{6\,cx^2-b}{3\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 5/2*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/2*c^2*x/((c*x^2 + b)*b^3)
 + 1/3*(6*c*x^2 - b)/(b^3*x^3)
```

Mupad [**B**]
time = 4.17, size = 58, normalized size = 0.85

$$\frac{\frac{5\,c\,x^2}{3\,b^2}-\frac{1}{3\,b}+\frac{5\,c^2\,x^4}{2\,b^3}}{c\,x^5+b\,x^3}+\frac{5\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^2,x)
```

```
[Out] ((5*c*x^2)/(3*b^2) - 1/(3*b) + (5*c^2*x^4)/(2*b^3))/(b*x^3 + c*x^5) + (5*c^
(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*b^(7/2))
```

3.203 $\int \frac{1}{x(bx^2+cx^4)^2} dx$

Optimal. Leaf size=66

$$-\frac{1}{4b^2x^4}+\frac{c}{b^3x^2}+\frac{c^2}{2b^3\left(b+cx^2\right)}+\frac{3c^2\log(x)}{b^4}-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}$$

```
[Out] -1/4/b^2/x^4+c/b^3/x^2+1/2*c^2/b^3/(c*x^2+b)+3*c^2*ln(x)/b^4-3/2*c^2*ln(c*x
^2+b)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 46}

$$-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}+\frac{3c^2\log(x)}{b^4}+\frac{c^2}{2b^3\left(b+cx^2\right)}+\frac{c}{b^3x^2}-\frac{1}{4b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*1/(b^2*x^4) + c/(b^3*x^2) + c^2/(2*b^3*(b + c*x^2)) + (3*c^2*Log[x])/b
^4 - (3*c^2*Log[b + c*x^2])/(2*b^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx=\int \frac{1}{x^5\left(b+cx^2\right)^2}\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(b+cx)^2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^2x^3}-\frac{2c}{b^3x^2}+\frac{3c^2}{b^4x}-\frac{c^3}{b^3(b+cx)^2}-\frac{3c^3}{b^4(b+cx)}\right)\,dx,x,x^2\right)$$
$$=-\frac{1}{4b^2x^4}+\frac{c}{b^3x^2}+\frac{c^2}{2b^3\left(b+cx^2\right)}+\frac{3c^2\log(x)}{b^4}-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.86

$$\frac{b\left(-\frac{b}{x^4}+\frac{4c}{x^2}+\frac{2c^2}{b+cx^2}\right)+12c^2\log(x)-6c^2\log\left(b+cx^2\right)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^2),x]
[Out] (b*(-(b/x^4) + (4*c)/x^2 + (2*c^2)/(b + c*x^2)) + 12*c^2*Log[x] - 6*c^2*Log
[b + c*x^2])/(4*b^4)
```

Maple [A]
time = 0.09, size = 65, normalized size = 0.98

method	result	size
default	$-\frac{c^3\left(-\frac{b}{c\left(c\,x^2+b\right)}+\frac{3\ln\left(c\,x^2+b\right)}{c}\right)}{2b^4}-\frac{1}{4b^2x^4}+\frac{c}{b^3x^2}+\frac{3c^2\ln(x)}{b^4}$	65
norman	$\frac{-\frac{1}{4b}+\frac{3c\,x^2}{4b^2}-\frac{3c^3x^6}{2b^4}}{x^4\left(c\,x^2+b\right)}+\frac{3c^2\ln(x)}{b^4}-\frac{3c^2\ln\left(c\,x^2+b\right)}{2b^4}$	67
risch	$\frac{\frac{3c^2x^4}{2b^3}+\frac{3c\,x^2}{4b^2}-\frac{1}{4b}}{x^4\left(c\,x^2+b\right)}+\frac{3c^2\ln(x)}{b^4}-\frac{3c^2\ln\left(c\,x^2+b\right)}{2b^4}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*c^3/b^4*(-b/c/(c*x^2+b)+3*ln(c*x^2+b)/c)-1/4/b^2/x^4+c/b^3/x^2+3*c^2*l
n(x)/b^4
```

Maxima [A]
time = 0.30, size = 70, normalized size = 1.06

$$\frac{6\,c^2x^4+3\,bcx^2-b^2}{4\left(b^3cx^6+b^4x^4\right)}-\frac{3\,c^2\log\left(cx^2+b\right)}{2\,b^4}+\frac{3\,c^2\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*c^2*x^4 + 3*b*c*x^2 - b^2)/(b^3*c*x^6 + b^4*x^4) - 3/2*c^2*log(c*x^2
 + b)/b^4 + 3/2*c^2*log(x^2)/b^4
```

Fricas [A]
time = 0.35, size = 90, normalized size = 1.36

$$\frac{6\,bc^2x^4+3\,b^2cx^2-b^3-6\,(c^3x^6+bc^2x^4)\log\left(cx^2+b\right)+12\,(c^3x^6+bc^2x^4)\log\left(x\right)}{4\,(b^4cx^6+b^5x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b*c^2*x^4 + 3*b^2*c*x^2 - b^3 - 6*(c^3*x^6 + b*c^2*x^4)*log(c*x^2 +
b) + 12*(c^3*x^6 + b*c^2*x^4)*log(x))/(b^4*c*x^6 + b^5*x^4)
```

Sympy [A]
time = 0.22, size = 68, normalized size = 1.03

$$\frac{-b^2+3bcx^2+6c^2x^4}{4b^4x^4+4b^3cx^6}+\frac{3c^2\log\left(x\right)}{b^4}-\frac{3c^2\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-b**2 + 3*b*c*x**2 + 6*c**2*x**4)/(4*b**4*x**4 + 4*b**3*c*x**6) + 3*c**2*l
og(x)/b**4 - 3*c**2*log(b/c + x**2)/(2*b**4)
```

Giac [A]
time = 3.30, size = 86, normalized size = 1.30

$$\frac{3\,c^2\log\left(x^2\right)}{2\,b^4}-\frac{3\,c^2\log\left(|cx^2+b|\right)}{2\,b^4}+\frac{3\,c^3x^2+4\,bc^2}{2\,(cx^2+b)b^4}-\frac{9\,c^2x^4-4\,bcx^2+b^2}{4\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*c^2*log(x^2)/b^4 - 3/2*c^2*log(abs(c*x^2 + b))/b^4 + 1/2*(3*c^3*x^2 + 4
*b*c^2)/((c*x^2 + b)*b^4) - 1/4*(9*c^2*x^4 - 4*b*c*x^2 + b^2)/(b^4*x^4)
```

Mupad [B]
time = 4.17, size = 67, normalized size = 1.02

$$\frac{\frac{3\,c\,x^2}{4\,b^2}-\frac{1}{4\,b}+\frac{3\,c^2\,x^4}{2\,b^3}}{c\,x^6+b\,x^4}-\frac{3\,c^2\,\ln\left(c\,x^2+b\right)}{2\,b^4}+\frac{3\,c^2\,\ln\left(x\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((3*c*x^2)/(4*b^2) - 1/(4*b) + (3*c^2*x^4)/(2*b^3))/(b*x^4 + c*x^6) - (3*c^
2*log(b + c*x^2))/(2*b^4) + (3*c^2*log(x))/b^4
```

3.204 $\int \frac{1}{x^2(bx^2+cx^4)^2} dx$

Optimal. Leaf size=81

$$-\frac{7}{10b^2x^5}+\frac{7c}{6b^3x^3}-\frac{7c^2}{2b^4x}+\frac{1}{2bx^5(b+cx^2)}-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}$$

```
[Out] -7/10/b^2/x^5+7/6*c/b^3/x^3-7/2*c^2/b^4/x+1/2/b/x^5/(c*x^2+b)-7/2*c^(5/2)*a
rctan(x*c^(1/2)/b^(1/2))/b^(9/2)
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1598, 296, 331, 211}

$$-\frac{7c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}-\frac{7c^2}{2b^4x}+\frac{7c}{6b^3x^3}-\frac{7}{10b^2x^5}+\frac{1}{2bx^5(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -7/(10*b^2*x^5) + (7*c)/(6*b^3*x^3) - (7*c^2)/(2*b^4*x) + 1/(2*b*x^5*(b + c
*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^6\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2bx^5\left(b+cx^2\right)}+\frac{7\int \frac{1}{x^6\left(b+cx^2\right)}\,dx}{2b} \\
&= -\frac{7}{10b^2x^5}+\frac{1}{2bx^5\left(b+cx^2\right)}-\frac{(7c)\int \frac{1}{x^4\left(b+cx^2\right)}\,dx}{2b^2} \\
&= -\frac{7}{10b^2x^5}+\frac{7c}{6b^3x^3}+\frac{1}{2bx^5\left(b+cx^2\right)}+\frac{\left(7c^2\right)\int \frac{1}{x^2\left(b+cx^2\right)}\,dx}{2b^3} \\
&= -\frac{7}{10b^2x^5}+\frac{7c}{6b^3x^3}-\frac{7c^2}{2b^4x}+\frac{1}{2bx^5\left(b+cx^2\right)}-\frac{\left(7c^3\right)\int \frac{1}{b+cx^2}\,dx}{2b^4} \\
&= -\frac{7}{10b^2x^5}+\frac{7c}{6b^3x^3}-\frac{7c^2}{2b^4x}+\frac{1}{2bx^5\left(b+cx^2\right)}-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 0.99

$$-\frac{1}{5b^2x^5}+\frac{2c}{3b^3x^3}-\frac{3c^2}{b^4x}-\frac{c^3x}{2b^4\left(b+cx^2\right)}-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)^2),x]
[Out] -1/5*1/(b^2*x^5) + (2*c)/(3*b^3*x^3) - (3*c^2)/(b^4*x) - (c^3*x)/(2*b^4*(b
+ c*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Maple [A]
time = 0.11, size = 67, normalized size = 0.83

method	result	size
default	$-\frac{c^{3}\left(\frac{x}{2c\,x^{2}+2b}+\frac{7\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^{4}}-\frac{1}{5b^{2}x^{5}}-\frac{3c^{2}}{b^{4}x}+\frac{2c}{3b^{3}x^{3}}$	67
risch	$\frac{-\frac{7c^{3}x^{6}}{2b^{4}}-\frac{7c^{2}x^{4}}{3b^{3}}+\frac{7c\,x^{2}}{15b^{2}}-\frac{1}{5b}}{x^{5}\left(c\,x^{2}+b\right)}+\frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(b^{9}\_Z^{2}+c^{5}\right)}\_R\ln\left(\left(3\_R^{2}b^{9}+2c^{5}\right)x+b^{5}c^{2}\_R\right)\right)}{4}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c^3/b^4*(1/2*x/(c*x^2+b)+7/2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))-1/5/b^2/
x^5-3*c^2/b^4/x+2/3*c/b^3/x^3
```

Maxima [A]
time = 0.50, size = 75, normalized size = 0.93

$$-\frac{105\,c^{3}x^{6}+70\,bc^{2}x^{4}-14\,b^{2}cx^{2}+6\,b^{3}}{30\,\left(b^{4}cx^{7}+b^{5}x^{5}\right)}-\frac{7\,c^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(105*c^3*x^6 + 70*b*c^2*x^4 - 14*b^2*c*x^2 + 6*b^3)/(b^4*c*x^7 + b^5*
x^5) - 7/2*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

Fricas [A]
time = 0.39, size = 198, normalized size = 2.44

$$\left[-\frac{210\,c^{3}x^{6}+140\,bc^{2}x^{4}-28\,b^{2}cx^{2}+12\,b^{3}-105\,\left(c^{3}x^{7}+bc^{2}x^{5}\right)\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^{2}-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)}{60\,\left(b^{4}cx^{7}+b^{5}x^{5}\right)},\,-\frac{105\,c^{3}x^{6}+70\,bc^{2}x^{4}-14\,b^{2}cx^{2}+6\,b^{3}+105\,\left(c^{3}x^{7}+bc^{2}x^{5}\right)\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)}{30\,\left(b^{4}cx^{7}+b^{5}x^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(210*c^3*x^6 + 140*b*c^2*x^4 - 28*b^2*c*x^2 + 12*b^3 - 105*(c^3*x^7
+ b*c^2*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b
^4*c*x^7 + b^5*x^5), -1/30*(105*c^3*x^6 + 70*b*c^2*x^4 - 14*b^2*c*x^2 + 6*b
^3 + 105*(c^3*x^7 + b*c^2*x^5)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^4*c*x^7 +
b^5*x^5)]
```

Sympy [A]
time = 0.20, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{c^5}{b^9}}\log\left(-\frac{b^5\sqrt{-\frac{c^5}{b^9}}}{c^3}+x\right)}{4}-\frac{7\sqrt{-\frac{c^5}{b^9}}\log\left(\frac{b^5\sqrt{-\frac{c^5}{b^9}}}{c^3}+x\right)}{4}+\frac{-6b^3+14b^2cx^2-70bc^2x^4-105c^3x^6}{30b^5x^5+30b^4cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 7*sqrt(-c**5/b**9)*log(-b**5*sqrt(-c**5/b**9)/c**3 + x)/4 - 7*sqrt(-c**5/b*
*9)*log(b**5*sqrt(-c**5/b**9)/c**3 + x)/4 + (-6*b**3 + 14*b**2*c*x**2 - 70*
b*c**2*x**4 - 105*c**3*x**6)/(30*b**5*x**5 + 30*b**4*c*x**7)
```

Giac [A]
time = 4.55, size = 70, normalized size = 0.86

$$-\frac{7\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}\,b^4}-\frac{c^3x}{2\,(cx^2+b)b^4}-\frac{45\,c^2x^4-10\,bcx^2+3\,b^2}{15\,b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/2*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) - 1/2*c^3*x/((c*x^2 + b)*b^4
) - 1/15*(45*c^2*x^4 - 10*b*c*x^2 + 3*b^2)/(b^4*x^5)
```

Mupad [B]
time = 4.28, size = 70, normalized size = 0.86

$$-\frac{\frac{1}{5\,b}-\frac{7\,c\,x^2}{15\,b^2}+\frac{7\,c^2\,x^4}{3\,b^3}+\frac{7\,c^3\,x^6}{2\,b^4}}{c\,x^7+b\,x^5}-\frac{7\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^2),x)
```

```
[Out] - (1/(5*b) - (7*c*x^2)/(15*b^2) + (7*c^2*x^4)/(3*b^3) + (7*c^3*x^6)/(2*b^4)
)/(b*x^5 + c*x^7) - (7*c^(5/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*b^(9/2))
```

3.205 $\int \frac{x^{14}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=85

$$-\frac{35bx}{8c^4}+\frac{35x^3}{24c^3}-\frac{x^7}{4c\,(b+cx^2)^2}-\frac{7x^5}{8c^2\,(b+cx^2)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}$$

```
[Out] -35/8*b*x/c^4+35/24*x^3/c^3-1/4*x^7/c/(c*x^2+b)^2-7/8*x^5/c^2/(c*x^2+b)+35/
8*b^(3/2)*arctan(x*c^(1/2)/b^(1/2))/c^(9/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1598, 294, 308, 211}

$$\frac{35b^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{35bx}{8c^4}-\frac{7x^5}{8c^2\,(b+cx^2)}-\frac{x^7}{4c\,(b+cx^2)^2}+\frac{35x^3}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-35*b*x)/(8*c^4) + (35*x^3)/(24*c^3) - x^7/(4*c*(b + c*x^2)^2) - (7*x^5)/(
8*c^2*(b + c*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{14}}{(bx^2+cx^4)^3}\,dx &= \int \frac{x^8}{(b+cx^2)^3}\,dx \\
&= -\frac{x^7}{4c(b+cx^2)^2} + \frac{7\int \frac{x^6}{(b+cx^2)^2}\,dx}{4c} \\
&= -\frac{x^7}{4c(b+cx^2)^2} - \frac{7x^5}{8c^2(b+cx^2)} + \frac{35\int \frac{x^4}{b+cx^2}\,dx}{8c^2} \\
&= -\frac{x^7}{4c(b+cx^2)^2} - \frac{7x^5}{8c^2(b+cx^2)} + \frac{35\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2(b+cx^2)}\right)dx}{8c^2} \\
&= -\frac{35bx}{8c^4} + \frac{35x^3}{24c^3} - \frac{x^7}{4c(b+cx^2)^2} - \frac{7x^5}{8c^2(b+cx^2)} + \frac{(35b^2)\int \frac{1}{b+cx^2}\,dx}{8c^4} \\
&= -\frac{35bx}{8c^4} + \frac{35x^3}{24c^3} - \frac{x^7}{4c(b+cx^2)^2} - \frac{7x^5}{8c^2(b+cx^2)} + \frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.91

$$
-\frac{105b^3x+175b^2cx^3+56bc^2x^5-8c^3x^7}{24c^4(b+cx^2)^2} + \frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/24*(105*b^3*x + 175*b^2*c*x^3 + 56*b*c^2*x^5 - 8*c^3*x^7)/(c^4*(b + c*x^
2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Maple [A]
time = 0.10, size = 63, normalized size = 0.74

method	result	size

default	$-\frac{-\frac{1}{3}c\,x^3+3bx}{c^4}+\frac{b^2\left(\frac{-\frac{13}{8}c\,x^3-\frac{11}{8}bx}{\left(c\,x^2+b\right)^2}+\frac{35\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{c^4}$	63
risch	$\frac{x^3}{3c^3}-\frac{3bx}{c^4}+\frac{-\frac{13}{8}b^2c\,x^3-\frac{11}{8}b^3x}{c^4(c\,x^2+b)^2}+\frac{35\sqrt{-bc}\,b\ln\left(-\sqrt{-bc}\,x+b\right)}{16c^5}-\frac{35\sqrt{-bc}\,b\ln\left(\sqrt{-bc}\,x+b\right)}{16c^5}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^4*(-1/3*c*x^3+3*b*x)+b^2/c^4*((-13/8*c*x^3-11/8*b*x)/(c*x^2+b)^2+35/8/
(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Maxima [A]
time = 0.51, size = 82, normalized size = 0.96

$$-\frac{13\,b^2cx^3+11\,b^3x}{8\,(c^6x^4+2\,bc^5x^2+b^2c^4)}+\frac{35\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^4}+\frac{cx^3-9\,bx}{3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(13*b^2*c*x^3 + 11*b^3*x)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 35/8*b^2
*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/3*(c*x^3 - 9*b*x)/c^4
```

Fricas [A]
time = 0.34, size = 230, normalized size = 2.71

$$\left[\frac{16\,c^3x^7-112\,bc^2x^5-350\,b^2cx^3-210\,b^3x+105\,(bc^2x^4+2\,b^2cx^2+b^3)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{48\,(c^6x^4+2\,bc^5x^2+b^2c^4)},\ \frac{8\,c^3x^7-56\,bc^2x^5-175\,b^2cx^3-105\,b^3x+105\,(bc^2x^4+2\,b^2cx^2+b^3)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)}{24\,(c^6x^4+2\,bc^5x^2+b^2c^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^3*x^7 - 112*b*c^2*x^5 - 350*b^2*c*x^3 - 210*b^3*x + 105*(b*c^2*
x^4 + 2*b^2*c*x^2 + b^3)*sqrt(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x
^2 + b)))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4), 1/24*(8*c^3*x^7 - 56*b*c^2*x^5
 - 175*b^2*c*x^3 - 105*b^3*x + 105*(b*c^2*x^4 + 2*b^2*c*x^2 + b^3)*sqrt(b/c
)*arctan(c*x*sqrt(b/c)/b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)]
```

Sympy [A]
time = 0.22, size = 133, normalized size = 1.56

$$-\frac{3bx}{c^4} - \frac{35\sqrt{-\frac{b^3}{c^9}}\log\left(x - \frac{c^4\sqrt{-\frac{b^3}{c^9}}}{b}\right)}{16} + \frac{35\sqrt{-\frac{b^3}{c^9}}\log\left(x + \frac{c^4\sqrt{-\frac{b^3}{c^9}}}{b}\right)}{16} + \frac{-11b^3x - 13b^2cx^3}{8b^2c^4 + 16bc^5x^2 + 8c^6x^4} + \frac{x^3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*b*x/c**4 - 35*sqrt(-b**3/c**9)*log(x - c**4*sqrt(-b**3/c**9)/b)/16 + 35*
sqrt(-b**3/c**9)*log(x + c**4*sqrt(-b**3/c**9)/b)/16 + (-11*b**3*x - 13*b**
2*c*x**3)/(8*b**2*c**4 + 16*b*c**5*x**2 + 8*c**6*x**4) + x**3/(3*c**3)
```

Giac [A]
time = 5.55, size = 73, normalized size = 0.86

$$\frac{35\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^4} - \frac{13\,b^2cx^3 + 11\,b^3x}{8\,(cx^2+b)^2c^4} + \frac{c^6x^3 - 9\,bc^5x}{3\,c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 35/8*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/8*(13*b^2*c*x^3 + 11*b^3
*x)/((c*x^2 + b)^2*c^4) + 1/3*(c^6*x^3 - 9*b*c^5*x)/c^9
```

Mupad [B]
time = 4.21, size = 77, normalized size = 0.91

$$\frac{x^3}{3\,c^3} - \frac{\frac{11\,b^3\,x}{8} + \frac{13\,c\,b^2\,x^3}{8}}{b^2\,c^4 + 2\,b\,c^5\,x^2 + c^6\,x^4} + \frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,c^{9/2}} - \frac{3\,b\,x}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^3/(3*c^3) - ((11*b^3*x)/8 + (13*b^2*c*x^3)/8)/(b^2*c^4 + c^6*x^4 + 2*b*c^
5*x^2) + (35*b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*c^(9/2)) - (3*b*x)/c^4
```

3.206 $\int \frac{x^{13}}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=65

$$\frac{x^2}{2c^3} + \frac{b^3}{4c^4(b+cx^2)^2} - \frac{3b^2}{2c^4(b+cx^2)} - \frac{3b\log(b+cx^2)}{2c^4}$$

```
[Out] 1/2*x^2/c^3+1/4*b^3/c^4/(c*x^2+b)^2-3/2*b^2/c^4/(c*x^2+b)-3/2*b*ln(c*x^2+b)
/c^4
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 45}

$$\frac{b^3}{4c^4(b+cx^2)^2} - \frac{3b^2}{2c^4(b+cx^2)} - \frac{3b\log(b+cx^2)}{2c^4} + \frac{x^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^2/(2*c^3) + b^3/(4*c^4*(b + c*x^2)^2) - (3*b^2)/(2*c^4*(b + c*x^2)) - (3*
b*Log[b + c*x^2])/(2*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13}}{(bx^2+cx^4)^3}\,dx = \int \frac{x^7}{(b+cx^2)^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{c^3} - \frac{b^3}{c^3(b+cx)^3} + \frac{3b^2}{c^3(b+cx)^2} - \frac{3b}{c^3(b+cx)}\right)\,dx, x, x^2\right)$$
$$= \frac{x^2}{2c^3} + \frac{b^3}{4c^4(b+cx^2)^2} - \frac{3b^2}{2c^4(b+cx^2)} - \frac{3b\log(b+cx^2)}{2c^4}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.74

$$-\frac{-2cx^2 + \frac{b^2(5b+6cx^2)}{(b+cx^2)^2} + 6b\log(b+cx^2)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(b*x^2 + c*x^4)^3,x]
[Out] -1/4*(-2*c*x^2 + (b^2*(5*b + 6*c*x^2))/(b + c*x^2)^2 + 6*b*Log[b + c*x^2])/
c^4
```

Maple [A]
time = 0.09, size = 62, normalized size = 0.95

method	result	size
risch	$\frac{x^2}{2c^3} + \frac{-\frac{3b^2x^2}{2} - \frac{5b^3}{4c}}{c^3(cx^2+b)^2} - \frac{3b\ln(cx^2+b)}{2c^4}$	54
norman	$\frac{\frac{x^{11}}{2c} - \frac{3b^2x^7}{c^3} - \frac{9b^3x^5}{4c^4}}{x^5(cx^2+b)^2} - \frac{3b\ln(cx^2+b)}{2c^4}$	60
default	$\frac{x^2}{2c^3} - \frac{b\left(\frac{3b}{c(cx^2+b)} + \frac{3\ln(cx^2+b)}{c} - \frac{b^2}{2c(cx^2+b)^2}\right)}{2c^3}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2/c^3-1/2*b/c^3*(3*b/c/(c*x^2+b)+3*ln(c*x^2+b)/c-1/2/c*b^2/(c*x^2+b)^
2)
```

Maxima [A]
time = 0.28, size = 66, normalized size = 1.02

$$-\frac{6\,b^2cx^2 + 5\,b^3}{4\,(c^6x^4 + 2\,bc^5x^2 + b^2c^4)} + \frac{x^2}{2\,c^3} - \frac{3\,b\log(cx^2+b)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*c*x^2 + 5*b^3)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 1/2*x^2/c^3
- 3/2*b*log(c*x^2 + b)/c^4
```

Fricas [A]
time = 0.36, size = 91, normalized size = 1.40

$$\frac{2\,c^3x^6+4\,bc^2x^4-4\,b^2cx^2-5\,b^3-6\,(bc^2x^4+2\,b^2cx^2+b^3)\log{(cx^2+b)}}{4\,(c^6x^4+2\,bc^5x^2+b^2c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*x^6 + 4*b*c^2*x^4 - 4*b^2*c*x^2 - 5*b^3 - 6*(b*c^2*x^4 + 2*b^2*c
*x^2 + b^3)*log(c*x^2 + b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

Sympy [A]
time = 0.18, size = 68, normalized size = 1.05

$$-\frac{3b\log{(b+cx^2)}}{2c^4}+\frac{-5b^3-6b^2cx^2}{4b^2c^4+8bc^5x^2+4c^6x^4}+\frac{x^2}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*b*log(b + c*x**2)/(2*c**4) + (-5*b**3 - 6*b**2*c*x**2)/(4*b**2*c**4 + 8*
b*c**5*x**2 + 4*c**6*x**4) + x**2/(2*c**3)
```

Giac [A]
time = 4.84, size = 62, normalized size = 0.95

$$\frac{x^2}{2\,c^3}-\frac{3\,b\log{(|cx^2+b|)}}{2\,c^4}+\frac{9\,bc^2x^4+12\,b^2cx^2+4\,b^3}{4\,(cx^2+b)^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/c^3 - 3/2*b*log(abs(c*x^2 + b))/c^4 + 1/4*(9*b*c^2*x^4 + 12*b^2*c*x
^2 + 4*b^3)/((c*x^2 + b)^2*c^4)
```

Mupad [B]
time = 4.26, size = 68, normalized size = 1.05

$$\frac{x^2}{2\,c^3}-\frac{\frac{5\,b^3}{4\,c}+\frac{3\,b^2\,x^2}{2}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}-\frac{3\,b\,\ln{(c\,x^2+b)}}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^2/(2*c^3) - ((5*b^3)/(4*c) + (3*b^2*x^2)/2)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*
x^2) - (3*b*log(b + c*x^2))/(2*c^4)
```

3.207 $\int \frac{x^{12}}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=74

$$\frac{15x}{8c^3}-\frac{x^5}{4c\,(b+cx^2)^2}-\frac{5x^3}{8c^2\,(b+cx^2)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{7/2}}$$

```
[Out] 15/8*x/c^3-1/4*x^5/c/(c*x^2+b)^2-5/8*x^3/c^2/(c*x^2+b)-15/8*arctan(x*c^(1/2
)/b^(1/2))*b^(1/2)/c^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1598, 294, 327, 211}

$$-\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{7/2}}-\frac{5x^3}{8c^2\,(b+cx^2)}-\frac{x^5}{4c\,(b+cx^2)^2}+\frac{15x}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (15*x)/(8*c^3) - x^5/(4*c*(b + c*x^2)^2) - (5*x^3)/(8*c^2*(b + c*x^2)) - (1
5*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{12}}{(bx^2+cx^4)^3}\,dx &= \int \frac{x^6}{(b+cx^2)^3}\,dx\\ &= -\frac{x^5}{4c(b+cx^2)^2}+\frac{5\int \frac{x^4}{(b+cx^2)^2}\,dx}{4c}\\ &= -\frac{x^5}{4c(b+cx^2)^2}-\frac{5x^3}{8c^2(b+cx^2)}+\frac{15\int \frac{x^2}{b+cx^2}\,dx}{8c^2}\\ &= \frac{15x}{8c^3}-\frac{x^5}{4c(b+cx^2)^2}-\frac{5x^3}{8c^2(b+cx^2)}-\frac{(15b)\int \frac{1}{b+cx^2}\,dx}{8c^3}\\ &= \frac{15x}{8c^3}-\frac{x^5}{4c(b+cx^2)^2}-\frac{5x^3}{8c^2(b+cx^2)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.89

$$\frac{15b^2x+25bcx^3+8c^2x^5}{8c^3(b+cx^2)^2}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (15*b^2*x + 25*b*c*x^3 + 8*c^2*x^5)/(8*c^3*(b + c*x^2)^2) - (15*Sqrt[b]*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

Maple [A]
time = 0.10, size = 51, normalized size = 0.69

method	result	size

default	$\frac{x}{c^{3}}-\frac{b\left(\frac{-\frac{9}{8}c\,x^{3}-\frac{7}{8}bx}{\left(c\,x^{2}+b\right)^{2}}+\frac{15\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{c^{3}}$	51
risch	$\frac{x}{c^{3}}+\frac{\frac{9}{8}bc\,x^{3}+\frac{7}{8}b^{2}x}{c^{3}\left(c\,x^{2}+b\right)^{2}}+\frac{15\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)}{16c^{4}}-\frac{15\sqrt{-bc}\,\ln\left(\sqrt{-bc}\,x-b\right)}{16c^{4}}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^3-b/c^3*((-9/8*c*x^3-7/8*b*x)/(c*x^2+b)^2+15/8/(b*c)^(1/2)*arctan(c*x/(
b*c)^(1/2)))
```

Maxima [A]
time = 0.49, size = 68, normalized size = 0.92

$$\frac{9\,bcx^{3}+7\,b^{2}x}{8\,\left(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3}\right)}-\frac{15\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^{3}}+\frac{x}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(9*b*c*x^3 + 7*b^2*x)/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) - 15/8*b*arctan
(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + x/c^3
```

Fricas [A]
time = 0.35, size = 202, normalized size = 2.73

$$\left[\frac{16\,c^{2}x^{5}+50\,bcx^{3}+30\,b^{2}x+15\,\left(c^{2}x^{4}+2\,bcx^{2}+b^{2}\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^{2}-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^{2}+b}\right)}{16\,\left(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3}\right)},\ \frac{8\,c^{2}x^{5}+25\,bcx^{3}+15\,b^{2}x-15\,\left(c^{2}x^{4}+2\,bcx^{2}+b^{2}\right)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)}{8\,\left(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*c^2*x^5 + 50*b*c*x^3 + 30*b^2*x + 15*(c^2*x^4 + 2*b*c*x^2 + b^2)*
sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^5*x^4 + 2*b*
c^4*x^2 + b^2*c^3), 1/8*(8*c^2*x^5 + 25*b*c*x^3 + 15*b^2*x - 15*(c^2*x^4 +
2*b*c*x^2 + b^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^5*x^4 + 2*b*c^4*x^2
+ b^2*c^3)]
```

Sympy [A]
time = 0.20, size = 107, normalized size = 1.45

$$\frac{15\sqrt{-\frac{b}{c^7}}\log\left(-c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16}-\frac{15\sqrt{-\frac{b}{c^7}}\log\left(c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16}+\frac{7b^2x+9bcx^3}{8b^2c^3+16bc^4x^2+8c^5x^4}+\frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(c*x**4+b*x**2)**3,x)
```

```
[Out] 15*sqrt(-b/c**7)*log(-c**3*sqrt(-b/c**7) + x)/16 - 15*sqrt(-b/c**7)*log(c**
3*sqrt(-b/c**7) + x)/16 + (7*b**2*x + 9*b*c*x**3)/(8*b**2*c**3 + 16*b*c**4*
x**2 + 8*c**5*x**4) + x/c**3
```

Giac [A]
time = 4.61, size = 54, normalized size = 0.73

$$-\frac{15\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,c^3}+\frac{x}{c^3}+\frac{9\,bcx^3+7\,b^2x}{8\left(cx^2+b\right)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + x/c^3 + 1/8*(9*b*c*x^3 + 7*
b^2*x)/((c*x^2 + b)^2*c^3)
```

Mupad [B]
time = 4.25, size = 64, normalized size = 0.86

$$\frac{\frac{7\,b^2\,x}{8}+\frac{9\,c\,b\,x^3}{8}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{x}{c^3}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((7*b^2*x)/8 + (9*b*c*x^3)/8)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + x/c^3 - (
15*b^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*c^(7/2))
```

3.208 $\int \frac{x^{11}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=49

$$-\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

```
[Out] -1/4*b^2/c^3/(c*x^2+b)^2+b/c^3/(c*x^2+b)+1/2*ln(c*x^2+b)/c^3
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 45}

$$-\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*b^2/(c^3*(b + c*x^2)^2) + b/(c^3*(b + c*x^2)) + Log[b + c*x^2]/(2*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11}}{(bx^2+cx^4)^3}\,dx = \int \frac{x^5}{(b+cx^2)^3}\,dx$$
$$= \frac{1}{2}\mathrm{Subst}\left(\int \frac{x^2}{(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{b^2}{c^2(b+cx)^3} - \frac{2b}{c^2(b+cx)^2} + \frac{1}{c^2(b+cx)}\right)\,dx, x, x^2\right)$$
$$= -\frac{b^2}{4c^3(b+cx^2)^2} + \frac{b}{c^3(b+cx^2)} + \frac{\log(b+cx^2)}{2c^3}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.80

$$\frac{\frac{b(3b+4cx^2)}{(b+cx^2)^2} + 2\log(b+cx^2)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(b*x^2 + c*x^4)^3,x]
[Out] ((b*(3*b + 4*c*x^2))/(b + c*x^2)^2 + 2*Log[b + c*x^2])/(4*c^3)
```

Maple [A]
time = 0.10, size = 46, normalized size = 0.94

method	result	size
risch	$\frac{\frac{b x^2}{c^2}+\frac{3b^2}{4c^3}}{(c x^2+b)^2}+\frac{\ln(c x^2+b)}{2c^3}$	42
default	$-\frac{b^2}{4c^3(c x^2+b)^2}+\frac{b}{c^3(c x^2+b)}+\frac{\ln(c x^2+b)}{2c^3}$	46
norman	$\frac{\frac{b x^7}{c^2}+\frac{3b^2x^5}{4c^3}}{x^5(c x^2+b)^2}+\frac{\ln(c x^2+b)}{2c^3}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
[Out] -1/4*b^2/c^3/(c*x^2+b)^2+b/c^3/(c*x^2+b)+1/2*ln(c*x^2+b)/c^3
```

Maxima [A]
time = 0.29, size = 55, normalized size = 1.12

$$\frac{4\,bcx^2+3\,b^2}{4\,(c^5x^4+2\,bc^4x^2+b^2c^3)} + \frac{\log(cx^2+b)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*b*c*x^2 + 3*b^2)/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) + 1/2*log(c*x^2 +
 b)/c^3
```

Fricas [A]
time = 0.33, size = 69, normalized size = 1.41

$$\frac{4\,bcx^2 + 3\,b^2 + 2\,(c^2x^4 + 2\,bcx^2 + b^2)\log\left(cx^2 + b\right)}{4\,(c^5x^4 + 2\,bc^4x^2 + b^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*c*x^2 + 3*b^2 + 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2 + b))/(c^5
*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [A]
time = 0.15, size = 53, normalized size = 1.08

$$\frac{3b^2 + 4bcx^2}{4b^2c^3 + 8bc^4x^2 + 4c^5x^4} + \frac{\log\left(b + cx^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2)**3,x)
```

```
[Out] (3*b**2 + 4*b*c*x**2)/(4*b**2*c**3 + 8*b*c**4*x**2 + 4*c**5*x**4) + log(b +
 c*x**2)/(2*c**3)
```

Giac [A]
time = 4.84, size = 42, normalized size = 0.86

$$\frac{\log\left(|cx^2 + b|\right)}{2\,c^3} - \frac{3\,cx^4 + 2\,bx^2}{4\,(cx^2 + b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^2 + b))/c^3 - 1/4*(3*c*x^4 + 2*b*x^2)/((c*x^2 + b)^2*c^2)
```

Mupad [B]
time = 4.18, size = 52, normalized size = 1.06

$$\frac{\frac{3\,b^2}{4\,c^3} + \frac{b\,x^2}{c^2}}{b^2 + 2\,b\,c\,x^2 + c^2\,x^4} + \frac{\ln\left(c\,x^2 + b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*b^2)/(4*c^3) + (b*x^2)/c^2)/(b^2 + c^2*x^4 + 2*b*c*x^2) + log(b + c*x^2
)/(2*c^3)
```

3.209 $\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=64

$$-\frac{x^3}{4c(b+cx^2)^2}-\frac{3x}{8c^2(b+cx^2)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}}$$

```
[Out] -1/4*x^3/c/(c*x^2+b)^2-3/8*x/c^2/(c*x^2+b)+3/8*arctan(x*c^(1/2)/b^(1/2))/c^
(5/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 294, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}}-\frac{3x}{8c^2(b+cx^2)}-\frac{x^3}{4c(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*x^3/(c*(b + c*x^2)^2) - (3*x)/(8*c^2*(b + c*x^2)) + (3*ArcTan[(Sqrt[c]
*x)/Sqrt[b]])/(8*Sqrt[b]*c^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx &= \int \frac{x^4}{(b+cx^2)^3}\,dx \\
&= -\frac{x^3}{4c(b+cx^2)^2} + \frac{3\int \frac{x^2}{(b+cx^2)^2}\,dx}{4c} \\
&= -\frac{x^3}{4c(b+cx^2)^2} - \frac{3x}{8c^2(b+cx^2)} + \frac{3\int \frac{1}{b+cx^2}\,dx}{8c^2} \\
&= -\frac{x^3}{4c(b+cx^2)^2} - \frac{3x}{8c^2(b+cx^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 55, normalized size = 0.86

$$
-\frac{3bx+5cx^3}{8c^2(b+cx^2)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\sqrt{b}\,c^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/8*(3*b*x + 5*c*x^3)/(c^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[b]]
)/(8*Sqrt[b]*c^(5/2))
```

Maple [**A**]
time = 0.10, size = 47, normalized size = 0.73

method	result	size
default	$\frac{-\frac{5x^3}{8c}-\frac{3bx}{8c^2}}{(c\,x^2+b)^2}+\frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8c^2\sqrt{bc}}$	47
risch	$\frac{-\frac{5x^3}{8c}-\frac{3bx}{8c^2}}{(c\,x^2+b)^2}-\frac{3\ln\left(cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\,c^2}+\frac{3\ln\left(-cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\,c^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-5/8*x^3/c-3/8*b*x/c^2)/(c*x^2+b)^2+3/8/c^2/(b*c)^(1/2)*arctan(c*x/(b*c)^(
1/2))
```

Maxima [A]
time = 0.50, size = 59, normalized size = 0.92

$$-\frac{5\,cx^3+3\,bx}{8\,(c^4x^4+2\,bc^3x^2+b^2c^2)}+\frac{3\,\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(5*c*x^3 + 3*b*x)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 3/8*arctan(c*x/s
qrt(b*c))/(sqrt(b*c)*c^2)
```

Fricas [A]
time = 0.37, size = 188, normalized size = 2.94

$$\left[-\frac{10\,bc^2x^3+6\,b^2cx+3\,(c^2x^4+2\,bcx^2+b^2)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{16\,(bc^5x^4+2\,b^2c^4x^2+b^3c^3)},-\frac{5\,bc^2x^3+3\,b^2cx-3\,(c^2x^4+2\,bcx^2+b^2)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{8\,(bc^5x^4+2\,b^2c^4x^2+b^3c^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*b*c^2*x^3 + 6*b^2*c*x + 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)
*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b*c^5*x^4 + 2*b^2*c^4*x^2
+ b^3*c^3), -1/8*(5*b*c^2*x^3 + 3*b^2*c*x - 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*s
qrt(b*c)*arctan(sqrt(b*c)*x/b))/(b*c^5*x^4 + 2*b^2*c^4*x^2 + b^3*c^3)]
```

Sympy [A]
time = 0.16, size = 110, normalized size = 1.72

$$-\frac{3\sqrt{-\frac{1}{bc^5}}\,\log\left(-bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{bc^5}}\,\log\left(bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16}+\frac{-3bx-5cx^3}{8b^2c^2+16bc^3x^2+8c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b*c**5))*log(-b*c**2*sqrt(-1/(b*c**5)) + x)/16 + 3*sqrt(-1/(b*c
**5))*log(b*c**2*sqrt(-1/(b*c**5)) + x)/16 + (-3*b*x - 5*c*x**3)/(8*b**2*c*
*2 + 16*b*c**3*x**2 + 8*c**4*x**4)
```

Giac [A]
time = 3.35, size = 45, normalized size = 0.70

$$\frac{3\,\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,c^2}-\frac{5\,cx^3+3\,bx}{8\,(cx^2+b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) - 1/8*(5*c*x^3 + 3*b*x)/((c*x^2 +
 b)^2*c^2)
```

Mupad [B]
time = 4.23, size = 56, normalized size = 0.88

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,\sqrt{b}\,c^{5/2}}-\frac{\frac{5\,x^3}{8\,c}+\frac{3\,b\,x}{8\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*atan((c^(1/2)*x)/b^(1/2)))/(8*b^(1/2)*c^(5/2)) - ((5*x^3)/(8*c) + (3*b*x
)/(8*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

3.210 $\int \frac{x^9}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=19

$$\frac{x^4}{4b(b+cx^2)^2}$$

[Out] 1/4*x^4/b/(c*x^2+b)^2

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 270}

$$\frac{x^4}{4b(b+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^9/(b*x^2 + c*x^4)^3,x]

[Out] x^4/(4*b*(b + c*x^2)^2)

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(bx^2+cx^4)^3} dx &= \int \frac{x^3}{(b+cx^2)^3} dx \\ &= \frac{x^4}{4b(b+cx^2)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.26

$$-\frac{b+2cx^2}{4c^2(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(b + 2*c*x^2)/(c^2*(b + c*x^2)^2)
```

Maple [A]
time = 0.08, size = 31, normalized size = 1.63

method	result	size
gosper	$-\frac{2c\,x^2+b}{4(c\,x^2+b)^2c^2}$	23
risch	$\frac{-\frac{x^2}{2c}-\frac{b}{4c^2}}{(c\,x^2+b)^2}$	26
default	$-\frac{1}{2c^2(c\,x^2+b)}+\frac{b}{4c^2(c\,x^2+b)^2}$	31
norman	$\frac{-\frac{x^7}{2c}-\frac{b\,x^5}{4c^2}}{x^5(c\,x^2+b)^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^2/(c*x^2+b)+1/4*b/c^2/(c*x^2+b)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.
time = 0.30, size = 36, normalized size = 1.89

$$-\frac{2\,cx^2+b}{4\,(c^4x^4+2\,bc^3x^2+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.
time = 0.36, size = 36, normalized size = 1.89

$$-\frac{2\,cx^2+b}{4\,(c^4x^4+2\,bc^3x^2+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.
time = 0.13, size = 36, normalized size = 1.89

$$\frac{-b-2cx^2}{4b^2c^2+8bc^3x^2+4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-b - 2*c*x**2)/(4*b**2*c**2 + 8*b*c**3*x**2 + 4*c**4*x**4)
```

Giac [A]
time = 5.83, size = 22, normalized size = 1.16

$$-\frac{2\,cx^2+b}{4\,(cx^2+b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*c*x^2 + b)/((c*x^2 + b)^2*c^2)
```

Mupad [B]
time = 4.18, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{4\,c^2}+\frac{x^2}{2\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4)^3,x)
```

```
[Out] -(b/(4*c^2) + x^2/(2*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

3.211 $\int \frac{x^8}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=65

$$-\frac{x}{4c\,(b+cx^2)^2}+\frac{x}{8bc\,(b+cx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}$$

[Out] -1/4*x/c/(c*x^2+b)^2+1/8*x/b/c/(c*x^2+b)+1/8*arctan(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(3/2)

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1598, 294, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}+\frac{x}{8bc\,(b+cx^2)}-\frac{x}{4c\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^8/(b*x^2 + c*x^4)^3,x]

[Out] -1/4*x/(c*(b + c*x^2)^2) + x/(8*b*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(8*b^(3/2)*c^(3/2))

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{(bx^2+cx^4)^3}\,dx &= \int \frac{x^2}{(b+cx^2)^3}\,dx\\ &= -\frac{x}{4c\,(b+cx^2)^2}+\frac{\int \frac{1}{(b+cx^2)^2}\,dx}{4c}\\ &= -\frac{x}{4c\,(b+cx^2)^2}+\frac{x}{8bc\,(b+cx^2)}+\frac{\int \frac{1}{b+cx^2}\,dx}{8bc}\\ &= -\frac{x}{4c\,(b+cx^2)^2}+\frac{x}{8bc\,(b+cx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 0.89

$$\frac{\frac{\sqrt{b}\,\sqrt{c}\,x(-b+cx^2)}{(b+cx^2)^2}+\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(b*x^2 + c*x^4)^3,x]
[Out] ((Sqrt[b]*Sqrt[c]*x*(-b + c*x^2))/(b + c*x^2)^2 + ArcTan[(Sqrt[c]*x)/Sqrt[b
]])/(8*b^(3/2)*c^(3/2))
```

Maple [A]
time = 0.10, size = 49, normalized size = 0.75

method	result	size
default	$\frac{\frac{x^3}{8b}-\frac{x}{8c}}{(c\,x^2+b)^2}+\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8cb\sqrt{bc}}$	49

risch	$\frac{\frac{x^3}{8b}-\frac{x}{8c}}{(c\,x^2+b)^2}-\frac{\ln\left(cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\;cb}+\frac{\ln\left(-cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\;cb}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8/b*x^3-1/8*x/c)/(c*x^2+b)^2+1/8/c/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 62, normalized size = 0.95

$$\frac{cx^3-bx}{8\left(bc^3x^4+2\,b^2c^2x^2+b^3c\right)}+\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^3 - b*x)/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 1/8*arctan(c*x/sqrt
(b*c))/(sqrt(b*c)*b*c)
```

Fricas [A]
time = 0.33, size = 190, normalized size = 2.92

$$\left[\frac{2\,bc^2x^3-2\,b^2cx-\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{16\left(b^2c^4x^4+2\,b^3c^3x^2+b^4c^2\right)},\frac{bc^2x^3-b^2cx+\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{8\left(b^2c^4x^4+2\,b^3c^3x^2+b^4c^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^3 - 2*b^2*c*x - (c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)*log
((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b^2*c^4*x^4 + 2*b^3*c^3*x^2 +
b^4*c^2), 1/8*(b*c^2*x^3 - b^2*c*x + (c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(b*c)*
arctan(sqrt(b*c)*x/b))/(b^2*c^4*x^4 + 2*b^3*c^3*x^2 + b^4*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.
time = 0.16, size = 110, normalized size = 1.69

$$-\frac{\sqrt{-\frac{1}{b^3c^3}}\log\left(-b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{b^3c^3}}\log\left(b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{16}+\frac{-bx+cx^3}{8b^3c+16b^2c^2x^2+8bc^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**3*c**3))*log(-b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 + sqrt(-1/(b
**3*c**3))*log(b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 + (-b*x + c*x**3)/(8*b**
3*c + 16*b**2*c**2*x**2 + 8*b*c**3*x**4)
```

Giac [A]
time = 5.29, size = 50, normalized size = 0.77

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}\,bc} + \frac{cx^3 - bx}{8\,(cx^2+b)^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c) + 1/8*(c*x^3 - b*x)/((c*x^2 + b)^
2*b*c)
```

Mupad [B]
time = 4.23, size = 55, normalized size = 0.85

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{3/2}\,c^{3/2}} - \frac{\frac{x}{8\,c} - \frac{x^3}{8\,b}}{b^2 + 2\,b\,c\,x^2 + c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^2 + c*x^4)^3,x)
```

```
[Out] atan((c^(1/2)*x)/b^(1/2))/(8*b^(3/2)*c^(3/2)) - (x/(8*c) - x^3/(8*b))/(b^2
+ c^2*x^4 + 2*b*c*x^2)
```

3.212 $\int \frac{x^7}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{4c(b+cx^2)^2}$$

```
[Out] -1/4/c/(c*x^2+b)^2
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1598, 267}

$$-\frac{1}{4c(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*1/(c*(b + c*x^2)^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(bx^2+cx^4)^3}\,dx &= \int \frac{x}{(b+cx^2)^3}\,dx\\ &= -\frac{1}{4c(b+cx^2)^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{4c(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*1/(c*(b + c*x^2)^2)
```

Maple [A]
time = 0.09, size = 15, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{4c\left(c\,x^{2}+b\right)^{2}}$	15
default	$-\frac{1}{4c\left(c\,x^{2}+b\right)^{2}}$	15
norman	$-\frac{1}{4c\left(c\,x^{2}+b\right)^{2}}$	15
risch	$-\frac{1}{4c\left(c\,x^{2}+b\right)^{2}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/c/(c*x^2+b)^2
```

Maxima [A]
time = 0.27, size = 26, normalized size = 1.62

$$-\frac{1}{4\left(c^{3}x^{4}+2\,bc^{2}x^{2}+b^{2}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

Fricas [A]
time = 0.35, size = 26, normalized size = 1.62

$$-\frac{1}{4\left(c^{3}x^{4}+2\,bc^{2}x^{2}+b^{2}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

Sympy [A]
time = 0.11, size = 27, normalized size = 1.69

$$-\frac{1}{4b^{2}c+8bc^{2}x^{2}+4c^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**3,x)
```

```
[Out] -1/(4*b**2*c + 8*b*c**2*x**2 + 4*c**3*x**4)
```

Giac [A]
time = 6.10, size = 14, normalized size = 0.88

$$-\frac{1}{4\left(cx^{2}+b\right)^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4/((c*x^2 + b)^2*c)
```

Mupad [B]
time = 0.03, size = 28, normalized size = 1.75

$$-\frac{1}{4\,b^2\,c+8\,b\,c^2\,x^2+4\,c^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^3,x)
```

```
[Out] -1/(4*b^2*c + 4*c^3*x^4 + 8*b*c^2*x^2)
```

3.213 $\int \frac{x^6}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=62

$$\frac{x}{4b\left(b+cx^2\right)^2}+\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}$$

```
[Out] 1/4*x/b/(c*x^2+b)^2+3/8*x/b^2/(c*x^2+b)+3/8*arctan(x*c^(1/2)/b^(1/2))/b^(5/
2)/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 205, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}+\frac{3x}{8b^2\left(b+cx^2\right)}+\frac{x}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x/(4*b*(b + c*x^2)^2) + (3*x)/(8*b^2*(b + c*x^2)) + (3*ArcTan[(Sqrt[c]*x)/S
qrt[b]])/(8*b^(5/2)*Sqrt[c])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{(bx^2+cx^4)^3}\,dx &= \int \frac{1}{(b+cx^2)^3}\,dx\\
&= \frac{x}{4b(b+cx^2)^2} + \frac{3\int \frac{1}{(b+cx^2)^2}\,dx}{4b}\\
&= \frac{x}{4b(b+cx^2)^2} + \frac{3x}{8b^2(b+cx^2)} + \frac{3\int \frac{1}{b+cx^2}\,dx}{8b^2}\\
&= \frac{x}{4b(b+cx^2)^2} + \frac{3x}{8b^2(b+cx^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.89

$$\frac{5bx+3cx^3}{8b^2(b+cx^2)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*b*x + 3*c*x^3)/(8*b^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(
8*b^(5/2)*Sqrt[c])
```

Maple [A]
time = 0.10, size = 57, normalized size = 0.92

method	result	size
default	$\frac{x}{4b(c\,x^2+b)^2}+\frac{\frac{3x}{8b(c\,x^2+b)}+\frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8b\sqrt{bc}}}{b}$	57
risch	$\frac{\frac{3c\,x^3}{8b^2}+\frac{5x}{8b}}{(c\,x^2+b)^2}-\frac{3\ln\left(cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\;b^2}+\frac{3\ln\left(-cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\;b^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/b/(c*x^2+b)^2+3/4/b*(1/2*x/b/(c*x^2+b)+1/2/b/(b*c)^(1/2)*arctan(c*x/(
b*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 58, normalized size = 0.94

$$\frac{3\,cx^3+5\,bx}{8\,(b^2c^2x^4+2\,b^3cx^2+b^4)}+\frac{3\,\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*c*x^3 + 5*b*x)/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 3/8*arctan(c*x/sq
rt(b*c))/(sqrt(b*c)*b^2)
```

Fricas [A]
time = 0.35, size = 188, normalized size = 3.03

$$\left[\frac{6\,bc^2x^3+10\,b^2cx-3\,(c^2x^4+2\,bcx^2+b^2)\sqrt{-bc}\,\log\left(\frac{cx^2-2\,\sqrt{-bc}\,x-b}{cx^2+b}\right)}{16\,(b^3c^3x^4+2\,b^4c^2x^2+b^5c)},\frac{3\,bc^2x^3+5\,b^2cx+3\,(c^2x^4+2\,bcx^2+b^2)\sqrt{bc}\,\arctan\left(\frac{\sqrt{bc}\,x}{b}\right)}{8\,(b^3c^3x^4+2\,b^4c^2x^2+b^5c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*b*c^2*x^3 + 10*b^2*c*x - 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)*
log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b^3*c^3*x^4 + 2*b^4*c^2*x^2
 + b^5*c), 1/8*(3*b*c^2*x^3 + 5*b^2*c*x + 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqr
t(b*c)*arctan(sqrt(b*c)*x/b))/(b^3*c^3*x^4 + 2*b^4*c^2*x^2 + b^5*c)]
```

Sympy [A]
time = 0.15, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{b^5c}}\log\left(-b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{b^5c}}\log\left(b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{16}+\frac{5bx+3cx^3}{8b^4+16b^3cx^2+8b^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b**5*c))*log(-b**3*sqrt(-1/(b**5*c)) + x)/16 + 3*sqrt(-1/(b**5*
c))*log(b**3*sqrt(-1/(b**5*c)) + x)/16 + (5*b*x + 3*c*x**3)/(8*b**4 + 16*b*
*3*c*x**2 + 8*b**2*c**2*x**4)
```

Giac [A]
time = 4.80, size = 45, normalized size = 0.73

$$\frac{3\,\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^2}+\frac{3\,cx^3+5\,bx}{8\,(cx^2+b)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/8*(3*c*x^3 + 5*b*x)/((c*x^2 +
 b)^2*b^2)
```

Mupad [B]
time = 4.21, size = 55, normalized size = 0.89

$$\frac{\frac{5\,x}{8\,b}+\frac{3\,c\,x^3}{8\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{5/2}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((5*x)/(8*b) + (3*c*x^3)/(8*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) + (3*atan((c^
(1/2)*x)/b^(1/2)))/(8*b^(5/2)*c^(1/2))
```

3.214 $\int \frac{x^5}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=54

$$\frac{1}{4b\left(b+cx^2\right)^2}+\frac{1}{2b^2\left(b+cx^2\right)}+\frac{\log(x)}{b^3}-\frac{\log\left(b+cx^2\right)}{2b^3}$$

```
[Out] 1/4/b/(c*x^2+b)^2+1/2/b^2/(c*x^2+b)+ln(x)/b^3-1/2*ln(c*x^2+b)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 46}

$$-\frac{\log\left(b+cx^2\right)}{2b^3}+\frac{\log(x)}{b^3}+\frac{1}{2b^2\left(b+cx^2\right)}+\frac{1}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4)^3,x]
```

```
[Out] 1/(4*b*(b + c*x^2)^2) + 1/(2*b^2*(b + c*x^2)) + Log[x]/b^3 - Log[b + c*x^2]
/(2*b^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(bx^2+cx^4)^3}\,dx &= \int \frac{1}{x\,(b+cx^2)^3}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(b+cx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^3x}-\frac{c}{b(b+cx)^3}-\frac{c}{b^2(b+cx)^2}-\frac{c}{b^3(b+cx)}\right)\,dx,x,x^2\right)\\ &= \frac{1}{4b\,(b+cx^2)^2}+\frac{1}{2b^2\,(b+cx^2)}+\frac{\log(x)}{b^3}-\frac{\log\left(b+cx^2\right)}{2b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.80

$$\frac{\frac{b(3b+2cx^2)}{(b+cx^2)^2}+4\log(x)-2\log\left(b+cx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^3,x]
[Out] ((b*(3*b + 2*c*x^2))/(b + c*x^2)^2 + 4*Log[x] - 2*Log[b + c*x^2])/(4*b^3)
```

Maple [A]
time = 0.10, size = 59, normalized size = 1.09

method	result	size
risch	$\frac{\frac{c\,x^2}{2b^2}+\frac{3}{4b}}{(c\,x^2+b)^2}+\frac{\ln(x)}{b^3}-\frac{\ln(c\,x^2+b)}{2b^3}$	46
norman	$\frac{-\frac{c\,x^7}{b^2}-\frac{3c^2x^9}{4b^3}}{x^5(c\,x^2+b)^2}+\frac{\ln(x)}{b^3}-\frac{\ln(c\,x^2+b)}{2b^3}$	55
default	$-\frac{c\left(-\frac{b}{c\left(c\,x^2+b\right)}+\frac{\ln\left(c\,x^2+b\right)}{c}-\frac{b^2}{2c\left(c\,x^2+b\right)^2}\right)}{2b^3}+\frac{\ln(x)}{b^3}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b^3*c*(-b/c/(c*x^2+b)+ln(c*x^2+b)/c-1/2/c*b^2/(c*x^2+b)^2)+ln(x)/b^3
```

Maxima [A]
time = 0.29, size = 60, normalized size = 1.11

$$\frac{2\,cx^2+3\,b}{4\,(b^2c^2x^4+2\,b^3cx^2+b^4)}-\frac{\log\left(cx^2+b\right)}{2\,b^3}+\frac{\log\left(x^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*c*x^2 + 3*b)/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) - 1/2*log(c*x^2 + b)/
b^3 + 1/2*log(x^2)/b^3
```

Fricas [A]
time = 0.37, size = 90, normalized size = 1.67

$$\frac{2\,bcx^2+3\,b^2-2\,(c^2x^4+2\,bcx^2+b^2)\log\left(cx^2+b\right)+4\,(c^2x^4+2\,bcx^2+b^2)\log\left(x\right)}{4\,(b^3c^2x^4+2\,b^4cx^2+b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^2 + 3*b^2 - 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2 + b) + 4*(
c^2*x^4 + 2*b*c*x^2 + b^2)*log(x))/(b^3*c^2*x^4 + 2*b^4*c*x^2 + b^5)
```

Sympy [A]
time = 0.21, size = 56, normalized size = 1.04

$$\frac{3b+2cx^2}{4b^4+8b^3cx^2+4b^2c^2x^4}+\frac{\log\left(x\right)}{b^3}-\frac{\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**3,x)
```

```
[Out] (3*b + 2*c*x**2)/(4*b**4 + 8*b**3*c*x**2 + 4*b**2*c**2*x**4) + log(x)/b**3
- log(b/c + x**2)/(2*b**3)
```

Giac [A]
time = 4.55, size = 59, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,b^3}-\frac{\log\left(|cx^2+b|\right)}{2\,b^3}+\frac{3\,c^2x^4+8\,bcx^2+6\,b^2}{4\,(cx^2+b)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/b^3 - 1/2*log(abs(c*x^2 + b))/b^3 + 1/4*(3*c^2*x^4 + 8*b*c*x^2
 + 6*b^2)/((c*x^2 + b)^2*b^3)
```

Mupad [B]
time = 0.06, size = 56, normalized size = 1.04

$$\frac{\ln\left(x\right)}{b^3}+\frac{\frac{3}{4\,b}+\frac{c\,x^2}{2\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{\ln\left(c\,x^2+b\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^3,x)
```

```
[Out] log(x)/b^3 + (3/(4*b) + (c*x^2)/(2*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) - log(
b + c*x^2)/(2*b^3)
```

3.215 $\int \frac{x^4}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=76

$$-\frac{15}{8b^3x}+\frac{1}{4bx\left(b+cx^2\right)^2}+\frac{5}{8b^2x\left(b+cx^2\right)}-\frac{15\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}}$$

```
[Out] -15/8/b^3/x+1/4/b/x/(c*x^2+b)^2+5/8/b^2/x/(c*x^2+b)-15/8*arctan(x*c^(1/2)/b
^(1/2))*c^(1/2)/b^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1598, 296, 331, 211}

$$-\frac{15\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}}-\frac{15}{8b^3x}+\frac{5}{8b^2x\left(b+cx^2\right)}+\frac{1}{4bx\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -15/(8*b^3*x) + 1/(4*b*x*(b + c*x^2)^2) + 5/(8*b^2*x*(b + c*x^2)) - (15*Sqr
t[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(bx^2+cx^4)^3}\,dx &= \int \frac{1}{x^2(b+cx^2)^3}\,dx\\ &= \frac{1}{4bx(b+cx^2)^2}+\frac{5\int \frac{1}{x^2(b+cx^2)^2}\,dx}{4b}\\ &= \frac{1}{4bx(b+cx^2)^2}+\frac{5}{8b^2x(b+cx^2)}+\frac{15\int \frac{1}{x^2(b+cx^2)}\,dx}{8b^2}\\ &= -\frac{15}{8b^3x}+\frac{1}{4bx(b+cx^2)^2}+\frac{5}{8b^2x(b+cx^2)}-\frac{(15c)\int \frac{1}{b+cx^2}\,dx}{8b^3}\\ &= -\frac{15}{8b^3x}+\frac{1}{4bx(b+cx^2)^2}+\frac{5}{8b^2x(b+cx^2)}-\frac{15\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.89

$$-\frac{8b^2+25bcx^2+15c^2x^4}{8b^3x(b+cx^2)^2}-\frac{15\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/8*(8*b^2 + 25*b*c*x^2 + 15*c^2*x^4)/(b^3*x*(b + c*x^2)^2) - (15*Sqrt[c]*
ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.71

method	result	size

default	$-\frac{c\left(\frac{\frac{7}{8}c\,x^{3}+\frac{9}{8}bx}{\left(c\,x^{2}+b\right)^{2}}+\frac{15\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^{3}}-\frac{1}{b^{3}x}$	54
risch	$\frac{-\frac{15c^{2}x^{4}}{8b^{3}}-\frac{25c\,x^{2}}{8b^{2}}-\frac{1}{b}}{x(c\,x^{2}+b)^{2}}+\frac{15\sqrt{-bc}\,\ln\left(-cx+\sqrt{-bc}\right)}{16b^{4}}-\frac{15\sqrt{-bc}\,\ln\left(-cx-\sqrt{-bc}\right)}{16b^{4}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*c*((7/8*c*x^3+9/8*b*x)/(c*x^2+b)^2+15/8/(b*c)^(1/2)*arctan(c*x/(b*c)
^(1/2)))-1/b^3/x
```

Maxima [A]
time = 0.50, size = 71, normalized size = 0.93

$$-\frac{15\,c^{2}x^{4}+25\,bcx^{2}+8\,b^{2}}{8\,(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x)}-\frac{15\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(15*c^2*x^4 + 25*b*c*x^2 + 8*b^2)/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)
- 15/8*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3)
```

Fricas [A]
time = 0.33, size = 202, normalized size = 2.66

$$\left[-\frac{30\,c^{2}x^{4}+50\,bcx^{2}-15\,(c^{2}x^{5}+2\,bcx^{3}+b^{2}x)\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^{2}-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)+16\,b^{2}}{16\,(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x)},\,-\frac{15\,c^{2}x^{4}+25\,bcx^{2}+15\,(c^{2}x^{5}+2\,bcx^{3}+b^{2}x)\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)+8\,b^{2}}{8\,(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*c^2*x^4 + 50*b*c*x^2 - 15*(c^2*x^5 + 2*b*c*x^3 + b^2*x)*sqrt(-c/
b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) + 16*b^2)/(b^3*c^2*x^5 +
 2*b^4*c*x^3 + b^5*x), -1/8*(15*c^2*x^4 + 25*b*c*x^2 + 15*(c^2*x^5 + 2*b*c*
x^3 + b^2*x)*sqrt(c/b)*arctan(x*sqrt(c/b)) + 8*b^2)/(b^3*c^2*x^5 + 2*b^4*c*
x^3 + b^5*x)]
```

Sympy [A]
time = 0.22, size = 116, normalized size = 1.53

$$\frac{15\sqrt{-\frac{c}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c}{b^7}}}{c}+x\right)}{16}-\frac{15\sqrt{-\frac{c}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c}{b^7}}}{c}+x\right)}{16}+\frac{-8b^2-25bcx^2-15c^2x^4}{8b^5x+16b^4cx^3+8b^3c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**3,x)
```

```
[Out] 15*sqrt(-c/b**7)*log(-b**4*sqrt(-c/b**7)/c + x)/16 - 15*sqrt(-c/b**7)*log(b
**4*sqrt(-c/b**7)/c + x)/16 + (-8*b**2 - 25*b*c*x**2 - 15*c**2*x**4)/(8*b**
5*x + 16*b**4*c*x**3 + 8*b**3*c**2*x**5)
```

Giac [A]
time = 4.03, size = 57, normalized size = 0.75

$$-\frac{15\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^3}-\frac{7\,c^2x^3+9\,bcx}{8\,(cx^2+b)^2b^3}-\frac{1}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -15/8*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/8*(7*c^2*x^3 + 9*b*c*x)/(
(c*x^2 + b)^2*b^3) - 1/(b^3*x)
```

Mupad [B]
time = 4.26, size = 66, normalized size = 0.87

$$-\frac{\frac{1}{b}+\frac{25\,c\,x^2}{8\,b^2}+\frac{15\,c^2\,x^4}{8\,b^3}}{b^2\,x+2\,b\,c\,x^3+c^2\,x^5}-\frac{15\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (1/b + (25*c*x^2)/(8*b^2) + (15*c^2*x^4)/(8*b^3))/(b^2*x + c^2*x^5 + 2*b*
c*x^3) - (15*c^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*b^(7/2))
```

3.216 $\int \frac{x^3}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{2b^3x^2}-\frac{c}{4b^2\left(b+cx^2\right)^2}-\frac{c}{b^3\left(b+cx^2\right)}-\frac{3c\log(x)}{b^4}+\frac{3c\log\left(b+cx^2\right)}{2b^4}$$

```
[Out] -1/2/b^3/x^2-1/4*c/b^2/(c*x^2+b)^2-c/b^3/(c*x^2+b)-3*c*ln(x)/b^4+3/2*c*ln(c
*x^2+b)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 46}

$$\frac{3c\log\left(b+cx^2\right)}{2b^4}-\frac{3c\log(x)}{b^4}-\frac{c}{b^3\left(b+cx^2\right)}-\frac{1}{2b^3x^2}-\frac{c}{4b^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/2*1/(b^3*x^2) - c/(4*b^2*(b + c*x^2)^2) - c/(b^3*(b + c*x^2)) - (3*c*Log
[x])/b^4 + (3*c*Log[b + c*x^2])/(2*b^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^3}{(bx^2+cx^4)^3}\,dx = \int \frac{1}{x^3\,(b+cx^2)^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^3x^2} - \frac{3c}{b^4x} + \frac{c^2}{b^2(b+cx)^3} + \frac{2c^2}{b^3(b+cx)^2} + \frac{3c^2}{b^4(b+cx)}\right)\,dx, x, x^2\right)$$
$$= -\frac{1}{2b^3x^2} - \frac{c}{4b^2\,(b+cx^2)^2} - \frac{c}{b^3\,(b+cx^2)} - \frac{3c\log(x)}{b^4} + \frac{3c\log(b+cx^2)}{2b^4}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.88

$$-\frac{\frac{b(2b^2+9bcx^2+6c^2x^4)}{x^2(b+cx^2)^2} + 12c\log(x) - 6c\log(b+cx^2)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4)^3,x]
[Out] -1/4*((b*(2*b^2 + 9*b*c*x^2 + 6*c^2*x^4))/(x^2*(b + c*x^2)^2) + 12*c*Log[x]
 - 6*c*Log[b + c*x^2])/b^4
```

Maple [A]
time = 0.11, size = 72, normalized size = 1.07

method	result	size
risch	$\frac{-\frac{3c^2x^4}{2b^3}-\frac{9cx^2}{4b^2}-\frac{1}{2b}}{x^2(cx^2+b)^2} - \frac{3c\ln(x)}{b^4} + \frac{3c\ln(-cx^2-b)}{2b^4}$	66
norman	$\frac{\frac{3c^2x^7}{b^3}-\frac{x^3}{2b}+\frac{9c^3x^9}{4b^4}}{x^5(cx^2+b)^2} - \frac{3c\ln(x)}{b^4} + \frac{3c\ln(cx^2+b)}{2b^4}$	68
default	$\frac{c^2\left(-\frac{2b}{c(cx^2+b)}+\frac{3\ln(cx^2+b)}{c}-\frac{b^2}{2c(cx^2+b)^2}\right)}{2b^4} - \frac{1}{2b^3x^2} - \frac{3c\ln(x)}{b^4}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
[Out] 1/2/b^4*c^2*(-2*b/c/(c*x^2+b)+3*ln(c*x^2+b)/c-1/2/c*b^2/(c*x^2+b)^2)-1/2/b^
3/x^2-3*c*ln(x)/b^4
```

Maxima [A]
time = 0.29, size = 77, normalized size = 1.15

$$-\frac{6\,c^2x^4+9\,bcx^2+2\,b^2}{4\,(b^3c^2x^6+2\,b^4cx^4+b^5x^2)} + \frac{3\,c\log(cx^2+b)}{2\,b^4} - \frac{3\,c\log(x^2)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*c^2*x^4 + 9*b*c*x^2 + 2*b^2)/(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)
+ 3/2*c*log(c*x^2 + b)/b^4 - 3/2*c*log(x^2)/b^4
```

Fricas [A]
time = 0.34, size = 119, normalized size = 1.78

$$-\frac{6\,bc^2x^4+9\,b^2cx^2+2\,b^3-6\,(c^3x^6+2\,bc^2x^4+b^2cx^2)\log\left(cx^2+b\right)+12\,(c^3x^6+2\,bc^2x^4+b^2cx^2)\log\left(x\right)}{4\,(b^4c^2x^6+2\,b^5cx^4+b^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3 - 6*(c^3*x^6 + 2*b*c^2*x^4 + b^2*c*
x^2)*log(c*x^2 + b) + 12*(c^3*x^6 + 2*b*c^2*x^4 + b^2*c*x^2)*log(x))/(b^4*c
^2*x^6 + 2*b^5*c*x^4 + b^6*x^2)
```

Sympy [A]
time = 0.25, size = 80, normalized size = 1.19

$$\frac{-2b^2-9bcx^2-6c^2x^4}{4b^5x^2+8b^4cx^4+4b^3c^2x^6}-\frac{3c\log\left(x\right)}{b^4}+\frac{3c\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*b**2 - 9*b*c*x**2 - 6*c**2*x**4)/(4*b**5*x**2 + 8*b**4*c*x**4 + 4*b**3*
c**2*x**6) - 3*c*log(x)/b**4 + 3*c*log(b/c + x**2)/(2*b**4)
```

Giac [A]
time = 3.10, size = 66, normalized size = 0.99

$$\frac{3\,c\log\left(|cx^2+b|\right)}{2\,b^4}-\frac{3\,c\log\left(|x|\right)}{b^4}-\frac{6\,bc^2x^4+9\,b^2cx^2+2\,b^3}{4\,(cx^2+b)^2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/2*c*log(abs(c*x^2 + b))/b^4 - 3*c*log(abs(x))/b^4 - 1/4*(6*b*c^2*x^4 + 9*
b^2*c*x^2 + 2*b^3)/((c*x^2 + b)^2*b^4*x^2)
```

Mupad [B]
time = 0.06, size = 75, normalized size = 1.12

$$\frac{3\,c\,\ln\left(c\,x^2+b\right)}{2\,b^4}-\frac{\frac{1}{2\,b}+\frac{9\,c\,x^2}{4\,b^2}+\frac{3\,c^2\,x^4}{2\,b^3}}{b^2\,x^2+2\,b\,c\,x^4+c^2\,x^6}-\frac{3\,c\,\ln\left(x\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*c*log(b + c*x^2))/(2*b^4) - (1/(2*b) + (9*c*x^2)/(4*b^2) + (3*c^2*x^4)/(
2*b^3))/(b^2*x^2 + c^2*x^6 + 2*b*c*x^4) - (3*c*log(x))/b^4
```

3.217 $\int \frac{x^2}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=87

$$-\frac{35}{24b^3x^3}+\frac{35c}{8b^4x}+\frac{1}{4bx^3\left(b+cx^2\right)^2}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}$$

```
[Out] -35/24/b^3/x^3+35/8*c/b^4/x+1/4/b/x^3/(c*x^2+b)^2+7/8/b^2/x^3/(c*x^2+b)+35/
8*c^(3/2)*arctan(x*c^(1/2)/b^(1/2))/b^(9/2)
```

Rubi [A]
time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1598, 296, 331, 211}

$$\frac{35c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{35c}{8b^4x}-\frac{35}{24b^3x^3}+\frac{7}{8b^2x^3\left(b+cx^2\right)}+\frac{1}{4bx^3\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -35/(24*b^3*x^3) + (35*c)/(8*b^4*x) + 1/(4*b*x^3*(b + c*x^2)^2) + 7/(8*b^2*
x^3*(b + c*x^2)) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(bx^2+cx^4)^3}\,dx &= \int \frac{1}{x^4(b+cx^2)^3}\,dx\\
&= \frac{1}{4bx^3(b+cx^2)^2}+\frac{7\int \frac{1}{x^4(b+cx^2)^2}\,dx}{4b}\\
&= \frac{1}{4bx^3(b+cx^2)^2}+\frac{7}{8b^2x^3(b+cx^2)}+\frac{35\int \frac{1}{x^4(b+cx^2)}\,dx}{8b^2}\\
&= -\frac{35}{24b^3x^3}+\frac{1}{4bx^3(b+cx^2)^2}+\frac{7}{8b^2x^3(b+cx^2)}-\frac{(35c)\int \frac{1}{x^2(b+cx^2)}\,dx}{8b^3}\\
&= -\frac{35}{24b^3x^3}+\frac{35c}{8b^4x}+\frac{1}{4bx^3(b+cx^2)^2}+\frac{7}{8b^2x^3(b+cx^2)}+\frac{(35c^2)\int \frac{1}{b+cx^2}\,dx}{8b^4}\\
&= -\frac{35}{24b^3x^3}+\frac{35c}{8b^4x}+\frac{1}{4bx^3(b+cx^2)^2}+\frac{7}{8b^2x^3(b+cx^2)}+\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.91

$$\frac{-8b^3+56b^2cx^2+175bc^2x^4+105c^3x^6}{24b^4x^3(b+cx^2)^2}+\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-8*b^3 + 56*b^2*c*x^2 + 175*b*c^2*x^4 + 105*c^3*x^6)/(24*b^4*x^3*(b + c*x^
2)^2) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Maple [A]
time = 0.10, size = 64, normalized size = 0.74

method	result	size
default	$\frac{c^{2}\left(\frac{\frac{11}{8}c\,x^{3}+\frac{13}{8}bx}{\left(c\,x^{2}+b\right)^{2}}+\frac{35\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^{4}}-\frac{1}{3b^{3}x^{3}}+\frac{3c}{b^{4}x}$	64
risch	$\frac{\frac{35c^{3}x^{6}}{8b^{4}}+\frac{175c^{2}x^{4}}{24b^{3}}+\frac{7c\,x^{2}}{3b^{2}}-\frac{1}{3b}}{x^{3}\left(c\,x^{2}+b\right)^{2}}+\frac{35\left(\sum_{\_R=\mathrm{RootOf}\left(b^{9}\_Z^{2}+c^{3}\right)}\_R\ln\left(\left(3\_R^{2}b^{9}+2c^{3}\right)x-b^{5}c\_R\right)\right)}{16}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*c^2*((11/8*c*x^3+13/8*b*x)/(c*x^2+b)^2+35/8/(b*c)^(1/2)*arctan(c*x/(b
*c)^(1/2)))-1/3/b^3/x^3+3*c/b^4/x
```

Maxima [A]
time = 0.49, size = 86, normalized size = 0.99

$$\frac{105\,c^{3}x^{6}+175\,bc^{2}x^{4}+56\,b^{2}cx^{2}-8\,b^{3}}{24\,\left(b^{4}c^{2}x^{7}+2\,b^{5}cx^{5}+b^{6}x^{3}\right)}+\frac{35\,c^{2}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(105*c^3*x^6 + 175*b*c^2*x^4 + 56*b^2*c*x^2 - 8*b^3)/(b^4*c^2*x^7 + 2*
b^5*c*x^5 + b^6*x^3) + 35/8*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

Fricas [A]
time = 0.34, size = 238, normalized size = 2.74

$$\left[\frac{210\,c^{3}x^{6}+350\,bc^{2}x^{4}+112\,b^{2}cx^{2}-16\,b^{3}+105\,\left(c^{3}x^{7}+2\,bc^{2}x^{5}+b^{2}cx^{3}\right)\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^{2}+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)}{48\,\left(b^{4}c^{2}x^{7}+2\,b^{5}cx^{5}+b^{6}x^{3}\right)},\ \frac{105\,c^{3}x^{6}+175\,bc^{2}x^{4}+56\,b^{2}cx^{2}-8\,b^{3}+105\,\left(c^{3}x^{7}+2\,bc^{2}x^{5}+b^{2}cx^{3}\right)\sqrt{\frac{c}{b}}\,\arctan\left(x\sqrt{\frac{c}{b}}\right)}{24\,\left(b^{4}c^{2}x^{7}+2\,b^{5}cx^{5}+b^{6}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(210*c^3*x^6 + 350*b*c^2*x^4 + 112*b^2*c*x^2 - 16*b^3 + 105*(c^3*x^7
+ 2*b*c^2*x^5 + b^2*c*x^3)*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c
*x^2 + b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3), 1/24*(105*c^3*x^6 + 175*
b*c^2*x^4 + 56*b^2*c*x^2 - 8*b^3 + 105*(c^3*x^7 + 2*b*c^2*x^5 + b^2*c*x^3)*
sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)]
```

Sympy [A]
time = 0.24, size = 138, normalized size = 1.59

$$-\frac{35\sqrt{-\frac{c^3}{b^9}}\log\left(-\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2}+x\right)}{16}+\frac{35\sqrt{-\frac{c^3}{b^9}}\log\left(\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2}+x\right)}{16}+\frac{-8b^3+56b^2cx^2+175bc^2x^4+105c^3x^6}{24b^6x^3+48b^5cx^5+24b^4c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**3,x)
```

```
[Out] -35*sqrt(-c**3/b**9)*log(-b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + 35*sqrt(-c**
3/b**9)*log(b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + (-8*b**3 + 56*b**2*c*x**2
+ 175*b*c**2*x**4 + 105*c**3*x**6)/(24*b**6*x**3 + 48*b**5*c*x**5 + 24*b**4
*c**2*x**7)
```

Giac [A]
time = 3.20, size = 71, normalized size = 0.82

$$\frac{35\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^4}+\frac{11\,c^3x^3+13\,bc^2x}{8\,(cx^2+b)^2b^4}+\frac{9\,cx^2-b}{3\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 35/8*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) + 1/8*(11*c^3*x^3 + 13*b*c^2
*x)/((c*x^2 + b)^2*b^4) + 1/3*(9*c*x^2 - b)/(b^4*x^3)
```

Mupad [B]
time = 4.26, size = 80, normalized size = 0.92

$$\frac{\frac{7\,c\,x^2}{3\,b^2}-\frac{1}{3\,b}+\frac{175\,c^2\,x^4}{24\,b^3}+\frac{35\,c^3\,x^6}{8\,b^4}}{b^2\,x^3+2\,b\,c\,x^5+c^2\,x^7}+\frac{35\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((7*c*x^2)/(3*b^2) - 1/(3*b) + (175*c^2*x^4)/(24*b^3) + (35*c^3*x^6)/(8*b^4
))/(b^2*x^3 + c^2*x^7 + 2*b*c*x^5) + (35*c^(3/2)*atan((c^(1/2)*x)/b^(1/2)))
/(8*b^(9/2))
```

3.218 $\int \frac{x}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=86

$$-\frac{1}{4b^3x^4}+\frac{3c}{2b^4x^2}+\frac{c^2}{4b^3\left(b+cx^2\right)^2}+\frac{3c^2}{2b^4\left(b+cx^2\right)}+\frac{6c^2\log(x)}{b^5}-\frac{3c^2\log\left(b+cx^2\right)}{b^5}$$

```
[Out] -1/4/b^3/x^4+3/2*c/b^4/x^2+1/4*c^2/b^3/(c*x^2+b)^2+3/2*c^2/b^4/(c*x^2+b)+6*
c^2*ln(x)/b^5-3*c^2*ln(c*x^2+b)/b^5
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 272, 46}

$$-\frac{3c^2\log\left(b+cx^2\right)}{b^5}+\frac{6c^2\log(x)}{b^5}+\frac{3c^2}{2b^4\left(b+cx^2\right)}+\frac{3c}{2b^4x^2}+\frac{c^2}{4b^3\left(b+cx^2\right)^2}-\frac{1}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*1/(b^3*x^4) + (3*c)/(2*b^4*x^2) + c^2/(4*b^3*(b + c*x^2)^2) + (3*c^2)/
(2*b^4*(b + c*x^2)) + (6*c^2*Log[x])/b^5 - (3*c^2*Log[b + c*x^2])/b^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x}{(bx^2+cx^4)^3}\,dx = \int \frac{1}{x^5(b+cx^2)^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^3x^3} - \frac{3c}{b^4x^2} + \frac{6c^2}{b^5x} - \frac{c^3}{b^3(b+cx)^3} - \frac{3c^3}{b^4(b+cx)^2} - \frac{6c^3}{b^5(b+cx)}\right)dx, x, x^2\right)$$
$$= -\frac{1}{4b^3x^4} + \frac{3c}{2b^4x^2} + \frac{c^2}{4b^3(b+cx^2)^2} + \frac{3c^2}{2b^4(b+cx^2)} + \frac{6c^2\log(x)}{b^5} - \frac{3c^2\log(b+cx^2)}{b^5}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.86

$$\frac{\frac{b(-b^3+4b^2cx^2+18bc^2x^4+12c^3x^6)}{x^4(b+cx^2)^2} + 24c^2\log(x) - 12c^2\log(b+cx^2)}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^3,x]
[Out] ((b*(-b^3 + 4*b^2*c*x^2 + 18*b*c^2*x^4 + 12*c^3*x^6))/(x^4*(b + c*x^2)^2) +
 24*c^2*Log[x] - 12*c^2*Log[b + c*x^2])/(4*b^5)
```

Maple [A]
time = 0.13, size = 83, normalized size = 0.97

method	result	size
risch	$\frac{\frac{3c^3x^6}{b^4}+\frac{9c^2x^4}{2b^3}+\frac{cx^2}{b^2}-\frac{1}{4b}}{x^4(cx^2+b)^2}+\frac{6c^2\ln(x)}{b^5}-\frac{3c^2\ln(cx^2+b)}{b^5}$	77
norman	$\frac{\frac{cx^3}{b^2}-\frac{x}{4b}-\frac{6c^3x^7}{b^4}-\frac{9c^4x^9}{2b^5}}{x^5(cx^2+b)^2}+\frac{6c^2\ln(x)}{b^5}-\frac{3c^2\ln(cx^2+b)}{b^5}$	78
default	$-\frac{c^3\left(\frac{6\ln(cx^2+b)}{c}-\frac{b^2}{2c(cx^2+b)^2}-\frac{3b}{c(cx^2+b)}\right)}{2b^5}-\frac{1}{4b^3x^4}+\frac{3c}{2b^4x^2}+\frac{6c^2\ln(x)}{b^5}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*c^3/b^5*(6*ln(c*x^2+b)/c-1/2/c*b^2/(c*x^2+b)^2-3*b/c/(c*x^2+b))-1/4/b^
3/x^4+3/2*c/b^4/x^2+6*c^2*ln(x)/b^5
```

Maxima [A]
time = 0.29, size = 92, normalized size = 1.07

$$\frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2-b^3}{4\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)} - \frac{3\,c^2\log(cx^2+b)}{b^5} + \frac{3\,c^2\log(x^2)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*c^3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/(b^4*c^2*x^8 + 2*b^5*c*
x^6 + b^6*x^4) - 3*c^2*log(c*x^2 + b)/b^5 + 3*c^2*log(x^2)/b^5
```

Fricas [A]
time = 0.33, size = 134, normalized size = 1.56

$$\frac{12\,bc^3x^6+18\,b^2c^2x^4+4\,b^3cx^2-b^4-12\,(c^4x^8+2\,bc^3x^6+b^2c^2x^4)\log\left(cx^2+b\right)+24\,(c^4x^8+2\,bc^3x^6+b^2c^2x^4)\log\left(x\right)}{4\,(b^5c^2x^8+2\,b^6cx^6+b^7x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*b*c^3*x^6 + 18*b^2*c^2*x^4 + 4*b^3*c*x^2 - b^4 - 12*(c^4*x^8 + 2*b*
c^3*x^6 + b^2*c^2*x^4)*log(c*x^2 + b) + 24*(c^4*x^8 + 2*b*c^3*x^6 + b^2*c^2
*x^4)*log(x))/(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^7*x^4)
```

Sympy [A]
time = 0.27, size = 90, normalized size = 1.05

$$\frac{-b^3+4b^2cx^2+18bc^2x^4+12c^3x^6}{4b^6x^4+8b^5cx^6+4b^4c^2x^8}+\frac{6c^2\log\left(x\right)}{b^5}-\frac{3c^2\log\left(\frac{b}{c}+x^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-b**3 + 4*b**2*c*x**2 + 18*b*c**2*x**4 + 12*c**3*x**6)/(4*b**6*x**4 + 8*b*
*5*c*x**6 + 4*b**4*c**2*x**8) + 6*c**2*log(x)/b**5 - 3*c**2*log(b/c + x**2)
/b**5
```

Giac [A]
time = 3.67, size = 79, normalized size = 0.92

$$-\frac{3\,c^2\log\left(|cx^2+b|\right)}{b^5}+\frac{6\,c^2\log\left(|x|\right)}{b^5}+\frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2-b^3}{4\,(cx^4+bx^2)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -3*c^2*log(abs(c*x^2 + b))/b^5 + 6*c^2*log(abs(x))/b^5 + 1/4*(12*c^3*x^6 +
18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/((c*x^4 + b*x^2)^2*b^4)
```

Mupad [B]
time = 4.25, size = 88, normalized size = 1.02

$$\frac{\frac{c\,x^2}{b^2}-\frac{1}{4\,b}+\frac{9\,c^2\,x^4}{2\,b^3}+\frac{3\,c^3\,x^6}{b^4}}{b^2\,x^4+2\,b\,c\,x^6+c^2\,x^8}-\frac{3\,c^2\,\ln\left(c\,x^2+b\right)}{b^5}+\frac{6\,c^2\,\ln\left(x\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((c*x^2)/b^2 - 1/(4*b) + (9*c^2*x^4)/(2*b^3) + (3*c^3*x^6)/b^4)/(b^2*x^4 +
c^2*x^8 + 2*b*c*x^6) - (3*c^2*log(b + c*x^2))/b^5 + (6*c^2*log(x))/b^5
```

3.219 $\int \frac{1}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{63}{40b^3x^5}+\frac{21c}{8b^4x^3}-\frac{63c^2}{8b^5x}+\frac{1}{4bx^5\,(b+cx^2)^2}+\frac{9}{8b^2x^5\,(b+cx^2)}-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}$$

```
[Out] -63/40/b^3/x^5+21/8*c/b^4/x^3-63/8*c^2/b^5/x+1/4/b/x^5/(c*x^2+b)^2+9/8/b^2/
x^5/(c*x^2+b)-63/8*c^(5/2)*arctan(x*c^(1/2)/b^(1/2))/b^(11/2)
```

Rubi [A]
time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.308$, Rules used = {1607, 296, 331, 211}

$$-\frac{63c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}-\frac{63c^2}{8b^5x}+\frac{21c}{8b^4x^3}-\frac{63}{40b^3x^5}+\frac{9}{8b^2x^5\,(b+cx^2)}+\frac{1}{4bx^5\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-3),x]
```

```
[Out] -63/(40*b^3*x^5) + (21*c)/(8*b^4*x^3) - (63*c^2)/(8*b^5*x) + 1/(4*b*x^5*(b
+ c*x^2)^2) + 9/(8*b^2*x^5*(b + c*x^2)) - (63*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(8*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(bx^2+cx^4)^3}\,dx &= \int \frac{1}{x^6\,(b+cx^2)^3}\,dx \\
&= \frac{1}{4bx^5\,(b+cx^2)^2} + \frac{9\int \frac{1}{x^6(b+cx^2)^2}\,dx}{4b} \\
&= \frac{1}{4bx^5\,(b+cx^2)^2} + \frac{9}{8b^2x^5\,(b+cx^2)} + \frac{63\int \frac{1}{x^6(b+cx^2)}\,dx}{8b^2} \\
&= -\frac{63}{40b^3x^5} + \frac{1}{4bx^5\,(b+cx^2)^2} + \frac{9}{8b^2x^5\,(b+cx^2)} - \frac{(63c)\int \frac{1}{x^4(b+cx^2)}\,dx}{8b^3} \\
&= -\frac{63}{40b^3x^5} + \frac{21c}{8b^4x^3} + \frac{1}{4bx^5\,(b+cx^2)^2} + \frac{9}{8b^2x^5\,(b+cx^2)} + \frac{(63c^2)\int \frac{1}{x^2(b+cx^2)}\,dx}{8b^4} \\
&= -\frac{63}{40b^3x^5} + \frac{21c}{8b^4x^3} - \frac{63c^2}{8b^5x} + \frac{1}{4bx^5\,(b+cx^2)^2} + \frac{9}{8b^2x^5\,(b+cx^2)} - \frac{(63c^3)\int \frac{1}{b+cx^2}\,dx}{8b^5} \\
&= -\frac{63}{40b^3x^5} + \frac{21c}{8b^4x^3} - \frac{63c^2}{8b^5x} + \frac{1}{4bx^5\,(b+cx^2)^2} + \frac{9}{8b^2x^5\,(b+cx^2)} - \frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 90, normalized size = 0.90

$$
-\frac{8b^4-24b^3cx^2+168b^2c^2x^4+525bc^3x^6+315c^4x^8}{40b^5x^5\,(b+cx^2)^2} - \frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8b^{11/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-3),x]
```

```
[Out] -1/40*(8*b^4 - 24*b^3*c*x^2 + 168*b^2*c^2*x^4 + 525*b*c^3*x^6 + 315*c^4*x^8
)/(b^5*x^5*(b + c*x^2)^2) - (63*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(
11/2))
```

Maple [A]
time = 0.10, size = 75, normalized size = 0.75

method	result	size
default	$-\frac{c^{3}\left(\frac{\frac{15}{8} c\,x^{3}+\frac{17}{8} b x}{\left(c\,x^{2}+b\right)^{2}}+\frac{63 \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{8 \sqrt{b c}}\right)}{b^{5}}-\frac{1}{5 b^{3} x^{5}}+\frac{c}{b^{4} x^{3}}-\frac{6 c^{2}}{b^{5} x}$	75
risch	$\frac{-\frac{63 c^{4} x^{8}}{8 b^{5}}-\frac{105 c^{3} x^{6}}{8 b^{4}}-\frac{21 c^{2} x^{4}}{5 b^{3}}+\frac{3 c\,x^{2}}{5 b^{2}}-\frac{1}{5 b}}{x^{5} \left(c\,x^{2}+b\right)^{2}}+\frac{63 \left(\sum_{\_R =\operatorname{RootOf}\left(b^{11} \_Z^{2}+c^{5}\right)}\_R \ln\left(\left(3 \_R^{2} b^{11}+2 c^{5}\right) x +b^{6} c^{2} \_R \right)\right)}{16}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*c^3*((15/8*c*x^3+17/8*b*x)/(c*x^2+b)^2+63/8/(b*c)^(1/2)*arctan(c*x/(
b*c)^(1/2)))-1/5/b^3/x^5+c/b^4/x^3-6*c^2/b^5/x
```

Maxima [A]
time = 0.49, size = 97, normalized size = 0.97

$$-\frac{315\,c^4x^8+525\,bc^3x^6+168\,b^2c^2x^4-24\,b^3cx^2+8\,b^4}{40\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)}-\frac{63\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(315*c^4*x^8 + 525*b*c^3*x^6 + 168*b^2*c^2*x^4 - 24*b^3*c*x^2 + 8*b^4
)/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5) - 63/8*c^3*arctan(c*x/sqrt(b*c))/(s
qrt(b*c)*b^5)
```

Fricas [A]
time = 0.43, size = 264, normalized size = 2.64

$$\left[-\frac{630\,c^4x^8+1050\,bc^3x^6+336\,b^2c^2x^4-48\,b^3cx^2+16\,b^4-315\,(c^4x^9+2\,bc^3x^7+b^2c^2x^5)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{80\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)},\ -\frac{315\,c^4x^8+525\,bc^3x^6+168\,b^2c^2x^4-24\,b^3cx^2+8\,b^4+315\,(c^4x^9+2\,bc^3x^7+b^2c^2x^5)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)}{40\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/80*(630*c^4*x^8 + 1050*b*c^3*x^6 + 336*b^2*c^2*x^4 - 48*b^3*c*x^2 + 16*
b^4 - 315*(c^4*x^9 + 2*b*c^3*x^7 + b^2*c^2*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b
```

```
*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5), -1/
40*(315*c^4*x^8 + 525*b*c^3*x^6 + 168*b^2*c^2*x^4 - 24*b^3*c*x^2 + 8*b^4 +
315*(c^4*x^9 + 2*b*c^3*x^7 + b^2*c^2*x^5)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b
^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)]
```

Sympy [A]
time = 0.27, size = 150, normalized size = 1.50

$$\frac{63\sqrt{-\frac{c^5}{b^{11}}}\log\left(-\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3}+x\right)}{16}-\frac{63\sqrt{-\frac{c^5}{b^{11}}}\log\left(\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3}+x\right)}{16}+\frac{-8b^4+24b^3cx^2-168b^2c^2x^4-525bc^3x^6-315c^4x^8}{40b^7x^5+80b^6cx^7+40b^5c^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**3,x)
```

```
[Out] 63*sqrt(-c**5/b**11)*log(-b**6*sqrt(-c**5/b**11)/c**3 + x)/16 - 63*sqrt(-c*
*5/b**11)*log(b**6*sqrt(-c**5/b**11)/c**3 + x)/16 + (-8*b**4 + 24*b**3*c*x*
*2 - 168*b**2*c**2*x**4 - 525*b*c**3*x**6 - 315*c**4*x**8)/(40*b**7*x**5 +
80*b**6*c*x**7 + 40*b**5*c**2*x**9)
```

Giac [A]
time = 5.10, size = 80, normalized size = 0.80

$$-\frac{63\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}\,b^5}-\frac{15\,c^4x^3+17\,bc^3x}{8\,(cx^2+b)^2b^5}-\frac{30\,c^2x^4-5\,bcx^2+b^2}{5\,b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -63/8*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^5) - 1/8*(15*c^4*x^3 + 17*b*c^
3*x)/((c*x^2 + b)^2*b^5) - 1/5*(30*c^2*x^4 - 5*b*c*x^2 + b^2)/(b^5*x^5)
```

Mupad [B]
time = 4.24, size = 92, normalized size = 0.92

$$-\frac{\frac{1}{5\,b}-\frac{3\,c\,x^2}{5\,b^2}+\frac{21\,c^2\,x^4}{5\,b^3}+\frac{105\,c^3\,x^6}{8\,b^4}+\frac{63\,c^4\,x^8}{8\,b^5}}{b^2\,x^5+2\,b\,c\,x^7+c^2\,x^9}-\frac{63\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (1/(5*b) - (3*c*x^2)/(5*b^2) + (21*c^2*x^4)/(5*b^3) + (105*c^3*x^6)/(8*b^
4) + (63*c^4*x^8)/(8*b^5))/(b^2*x^5 + c^2*x^9 + 2*b*c*x^7) - (63*c^(5/2)*at
an((c^(1/2)*x)/b^(1/2)))/(8*b^(11/2))
```

3.220 $\int \frac{1}{x(bx^2+cx^4)^3} dx$

Optimal. Leaf size=95

$$-\frac{1}{6b^3x^6}+\frac{3c}{4b^4x^4}-\frac{3c^2}{b^5x^2}-\frac{c^3}{4b^4(b+cx^2)^2}-\frac{2c^3}{b^5(b+cx^2)}-\frac{10c^3\log(x)}{b^6}+\frac{5c^3\log(b+cx^2)}{b^6}$$

```
[Out] -1/6/b^3/x^6+3/4*c/b^4/x^4-3*c^2/b^5/x^2-1/4*c^3/b^4/(c*x^2+b)^2-2*c^3/b^5/
(c*x^2+b)-10*c^3*ln(x)/b^6+5*c^3*ln(c*x^2+b)/b^6
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1598, 272, 46}

$$\frac{5c^3\log(b+cx^2)}{b^6}-\frac{10c^3\log(x)}{b^6}-\frac{2c^3}{b^5(b+cx^2)}-\frac{3c^2}{b^5x^2}-\frac{c^3}{4b^4(b+cx^2)^2}+\frac{3c}{4b^4x^4}-\frac{1}{6b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/6*1/(b^3*x^6) + (3*c)/(4*b^4*x^4) - (3*c^2)/(b^5*x^2) - c^3/(4*b^4*(b +
c*x^2)^2) - (2*c^3)/(b^5*(b + c*x^2)) - (10*c^3*Log[x])/b^6 + (5*c^3*Log[b
+ c*x^2])/b^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^7\left(b+cx^2\right)^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(b+cx)^3}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b^3x^4} - \frac{3c}{b^4x^3} + \frac{6c^2}{b^5x^2} - \frac{10c^3}{b^6x} + \frac{c^4}{b^4(b+cx)^3} + \frac{4c^4}{b^5(b+cx)^2} + \frac{10c^4}{b^6(b+cx)}\right)\right.$$
$$= -\frac{1}{6b^3x^6} + \frac{3c}{4b^4x^4} - \frac{3c^2}{b^5x^2} - \frac{c^3}{4b^4\left(b+cx^2\right)^2} - \frac{2c^3}{b^5\left(b+cx^2\right)} - \frac{10c^3\log(x)}{b^6} + \frac{5c^3\log\left(b+\right.}{b^6}$$

Mathematica [A]
time = 0.05, size = 85, normalized size = 0.89

$$-\frac{\frac{b\left(2b^4-5b^3cx^2+20b^2c^2x^4+90bc^3x^6+60c^4x^8\right)}{x^6\left(b+cx^2\right)^2}+120c^3\log(x)-60c^3\log\left(b+cx^2\right)}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/12*((b*(2*b^4 - 5*b^3*c*x^2 + 20*b^2*c^2*x^4 + 90*b*c^3*x^6 + 60*c^4*x^8
))/(x^6*(b + c*x^2)^2) + 120*c^3*Log[x] - 60*c^3*Log[b + c*x^2])/b^6
```

Maple [A]
time = 0.09, size = 94, normalized size = 0.99

method	result	size
norman	$\frac{-\frac{1}{6b}+\frac{5c\,x^2}{12b^2}-\frac{5c^2x^4}{3b^3}+\frac{10c^4x^8}{b^5}+\frac{15c^5x^{10}}{2b^6}}{x^6\left(c\,x^2+b\right)^2}-\frac{10c^3\ln(x)}{b^6}+\frac{5c^3\ln\left(c\,x^2+b\right)}{b^6}$	89
risch	$\frac{-\frac{5c^4x^8}{b^5}-\frac{15c^3x^6}{2b^4}-\frac{5c^2x^4}{3b^3}+\frac{5c\,x^2}{12b^2}-\frac{1}{6b}}{x^6\left(c\,x^2+b\right)^2}-\frac{10c^3\ln(x)}{b^6}+\frac{5c^3\ln\left(-c\,x^2-b\right)}{b^6}$	92
default	$\frac{c^4\left(\frac{10\ln\left(c\,x^2+b\right)}{c}-\frac{b^2}{2c\left(c\,x^2+b\right)^2}-\frac{4b}{c\left(c\,x^2+b\right)}\right)}{2b^6}-\frac{1}{6b^3x^6}-\frac{10c^3\ln(x)}{b^6}-\frac{3c^2}{b^5x^2}+\frac{3c}{4b^4x^4}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c^4/b^6*(10*ln(c*x^2+b)/c-1/2/c*b^2/(c*x^2+b)^2-4*b/c/(c*x^2+b))-1/6/b^
3/x^6-10*c^3*ln(x)/b^6-3*c^2/b^5/x^2+3/4*c/b^4/x^4
```

Maxima [A]
time = 0.28, size = 103, normalized size = 1.08

$$-\frac{60\,c^4x^8+90\,bc^3x^6+20\,b^2c^2x^4-5\,b^3cx^2+2\,b^4}{12\,\left(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6\right)}+\frac{5\,c^3\log\left(cx^2+b\right)}{b^6}-\frac{5\,c^3\log\left(x^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(60*c^4*x^8 + 90*b*c^3*x^6 + 20*b^2*c^2*x^4 - 5*b^3*c*x^2 + 2*b^4)/(b
^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6) + 5*c^3*log(c*x^2 + b)/b^6 - 5*c^3*log
(x^2)/b^6
```

Fricas [A]
time = 0.36, size = 145, normalized size = 1.53

$$-\frac{60\,bc^4x^8+90\,b^2c^3x^6+20\,b^3c^2x^4-5\,b^4cx^2+2\,b^5-60\,(c^5x^{10}+2\,bc^4x^8+b^2c^3x^6)\log(cx^2+b)+120\,(c^5x^{10}+2\,bc^4x^8+b^2c^3x^6)\log(x)}{12\,(b^6c^2x^{10}+2\,b^7cx^8+b^8x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(60*b*c^4*x^8 + 90*b^2*c^3*x^6 + 20*b^3*c^2*x^4 - 5*b^4*c*x^2 + 2*b^5
 - 60*(c^5*x^10 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(c*x^2 + b) + 120*(c^5*x^10
 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(x))/(b^6*c^2*x^10 + 2*b^7*c*x^8 + b^8*x^6
)
```

Sympy [A]
time = 0.30, size = 104, normalized size = 1.09

$$\frac{-2b^4+5b^3cx^2-20b^2c^2x^4-90bc^3x^6-60c^4x^8}{12b^7x^6+24b^6cx^8+12b^5c^2x^{10}}-\frac{10c^3\log(x)}{b^6}+\frac{5c^3\log\left(\frac{b}{c}+x^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*b**4 + 5*b**3*c*x**2 - 20*b**2*c**2*x**4 - 90*b*c**3*x**6 - 60*c**4*x**
8)/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10) - 10*c**3*log(x)/b*
*6 + 5*c**3*log(b/c + x**2)/b**6
```

Giac [A]
time = 5.43, size = 110, normalized size = 1.16

$$-\frac{5\,c^3\log(x^2)}{b^6}+\frac{5\,c^3\log(|cx^2+b|)}{b^6}-\frac{30\,c^5x^4+68\,bc^4x^2+39\,b^2c^3}{4\,(cx^2+b)^2b^6}+\frac{110\,c^3x^6-36\,bc^2x^4+9\,b^2cx^2-2\,b^3}{12\,b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -5*c^3*log(x^2)/b^6 + 5*c^3*log(abs(c*x^2 + b))/b^6 - 1/4*(30*c^5*x^4 + 68*
b*c^4*x^2 + 39*b^2*c^3)/((c*x^2 + b)^2*b^6) + 1/12*(110*c^3*x^6 - 36*b*c^2*
x^4 + 9*b^2*c*x^2 - 2*b^3)/(b^6*x^6)
```

Mupad [B]
time = 0.10, size = 101, normalized size = 1.06

$$\frac{5\,c^3\,\ln\left(c\,x^2+b\right)}{b^6}-\frac{\frac{1}{6\,b}-\frac{5\,c\,x^2}{12\,b^2}+\frac{5\,c^2\,x^4}{3\,b^3}+\frac{15\,c^3\,x^6}{2\,b^4}+\frac{5\,c^4\,x^8}{b^5}}{b^2\,x^6+2\,b\,c\,x^8+c^2\,x^{10}}-\frac{10\,c^3\,\ln\left(x\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (5*c^3*log(b + c*x^2))/b^6 - (1/(6*b) - (5*c*x^2)/(12*b^2) + (5*c^2*x^4)/(3
*b^3) + (15*c^3*x^6)/(2*b^4) + (5*c^4*x^8)/b^5)/(b^2*x^6 + c^2*x^10 + 2*b*c
*x^8) - (10*c^3*log(x))/b^6
```

3.221 $\int x^5 \sqrt{bx^2+cx^4}\, dx$

Optimal. Leaf size=119

$$\frac{5b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b(bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(bx^2+cx^4)^{3/2}}{8c}-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}$$

```
[Out] -5/48*b*(c*x^4+b*x^2)^(3/2)/c^2+1/8*x^2*(c*x^4+b*x^2)^(3/2)/c-5/128*b^4*arc
tanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+5/128*b^2*(2*c*x^2+b)*(c*x^4+
b*x^2)^(1/2)/c^3
```

Rubi [A]
time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 684, 654, 626, 634, 212}

$$-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}+\frac{5b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b(bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(bx^2+cx^4)^{3/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (5*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - (5*b*(b*x^2 + c*x^4)^
(3/2))/(48*c^2) + (x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*b^4*ArcTanh[(Sqrt[
c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int x^5\sqrt{bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{x^2(bx^2+cx^4)^{3/2}}{8c}-\frac{(5b)\text{Subst}\left(\int x\sqrt{bx+cx^2}\,dx,x,x^2\right)}{16c}\\
&= -\frac{5b(bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(bx^2+cx^4)^{3/2}}{8c}+\frac{(5b^2)\,\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{32c^2}\\
&= \frac{5b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b(bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(bx^2+cx^4)^{3/2}}{8c}-\frac{(5b^4)\,\text{Subst}\left(\int \cdots\right.}{\cdots}\\
&= \frac{5b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b(bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(bx^2+cx^4)^{3/2}}{8c}-\frac{(5b^4)\,\text{Subst}\left(\int \cdots\right.}{\cdots}\\
&= \frac{5b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b(bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(bx^2+cx^4)^{3/2}}{8c}-\frac{5b^4\tanh^{-1}\left(\cdots\right.}{128c^{7\cdots}}
\end{aligned}$$

Mathematica [**A**]

time = 0.10, size = 113, normalized size = 0.95

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\,(15b^3-10b^2cx^2+8bc^2x^4+48c^3x^6)+15b^4\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{384c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(15*b^3 - 10*b^2*c*x^2 + 8*b*
c^2*x^4 + 48*c^3*x^6) + 15*b^4*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(384*c
^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 124, normalized size = 1.04

method	result
risch	$\frac{\left(48c^3x^6+8b\,c^2x^4-10b^2c\,x^2+15b^3\right)\sqrt{x^2\left(c\,x^2+b\right)}}{384c^3}-\frac{5b^4\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)\sqrt{x^2\left(c\,x^2+b\right)}}{128c^{\frac{7}{2}}x\sqrt{c\,x^2+b}}$
default	$\frac{\sqrt{c\,x^4+b\,x^2}\left(48x^5\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}-40c^{\frac{3}{2}}\left(c\,x^2+b\right)^{\frac{3}{2}}b\,x^3+30\sqrt{c}\left(c\,x^2+b\right)^{\frac{3}{2}}b^2x-15\sqrt{c}\sqrt{c\,x^2+b}\,b^3x-15\ln\left(x\sqrt{c}+\sqrt{\ldots}\right.\right.}{384x\sqrt{c\,x^2+b}\,c^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(c*x^4+b*x^2)^(1/2)*(48*x^5*(c*x^2+b)^(3/2)*c^(5/2)-40*c^(3/2)*(c*x^2
+b)^(3/2)*b*x^3+30*c^(1/2)*(c*x^2+b)^(3/2)*b^2*x-15*c^(1/2)*(c*x^2+b)^(1/2)
*b^3*x-15*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^4)/x/(c*x^2+b)^(1/2)/c^(7/2)
```

Maxima [A]
time = 0.29, size = 121, normalized size = 1.02

$$\frac{5\sqrt{cx^4+bx^2}\,b^2x^2}{64\,c^2}+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}{8\,c}-\frac{5\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{256\,c^{\frac{7}{2}}}+\frac{5\,\sqrt{cx^4+bx^2}\,b^3}{128\,c^3}-\frac{5\,\left(cx^4+bx^2\right)^{\frac{3}{2}}b}{48\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/64*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 1/8*(c*x^4 + b*x^2)^(3/2)*x^2/c - 5/
256*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 5/128*sq
rt(c*x^4 + b*x^2)*b^3/c^3 - 5/48*(c*x^4 + b*x^2)^(3/2)*b/c^2
```

Fricas [A]
time = 0.39, size = 188, normalized size = 1.58

$$\left[\frac{15\,b^4\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\left(48\,c^4x^6+8\,bc^3x^4-10\,b^2c^2x^2+15\,b^3c\right)\sqrt{cx^4+bx^2}}{768\,c^4},\ \frac{15\,b^4\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\left(48\,c^4x^6+8\,bc^3x^4-10\,b^2c^2x^2+15\,b^3c\right)\sqrt{cx^4+bx^2}}{384\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*b^4*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
2*(48*c^4*x^6 + 8*b*c^3*x^4 - 10*b^2*c^2*x^2 + 15*b^3*c)*sqrt(c*x^4 + b*x^2
))/c^4, 1/384*(15*b^4*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 +
 b)) + (48*c^4*x^6 + 8*b*c^3*x^4 - 10*b^2*c^2*x^2 + 15*b^3*c)*sqrt(c*x^4 +
b*x^2))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \sqrt{x^2 (b + cx^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 3.41, size = 101, normalized size = 0.85

$$\frac{1}{384}\left(2\left(4\left(6\,x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x^2-\frac{5\,b^2\mathrm{sgn}(x)}{c^2}\right)x^2+\frac{15\,b^3\mathrm{sgn}(x)}{c^3}\right)\sqrt{cx^2+b}\,x+\frac{5\,b^4\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{128\,c^{\frac{7}{2}}}-\frac{5\,b^4\log\left(|b|\right)\mathrm{sgn}(x)}{256\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*x^2*sgn(x) + b*sgn(x)/c)*x^2 - 5*b^2*sgn(x)/c^2)*x^2 + 15*b^
3*sgn(x)/c^3)*sqrt(c*x^2 + b)*x + 5/128*b^4*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + b)))*sgn(x)/c^(7/2) - 5/256*b^4*log(abs(b))*sgn(x)/c^(7/2)
```

Mupad [B]
time = 4.68, size = 105, normalized size = 0.88

$$\frac{x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{8\,c}-\frac{5\,b\,\left(\frac{b^3\,\ln\left(\frac{2\,c\,x^2+b}{\sqrt{c}}+2\,\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*b*((b^3*log((b + 2*c*x^2)/c^(1/2) +
2*(b*x^2 + c*x^4)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4
- 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c)
```

3.222 $\int x^3 \sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=91

$$-\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c^2}+\frac{(bx^2+cx^4)^{3/2}}{6c}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}$$

```
[Out] 1/6*(c*x^4+b*x^2)^(3/2)/c+1/16*b^3*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))
/c^(5/2)-1/16*b*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 654, 626, 634, 212}

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c^2}+\frac{(bx^2+cx^4)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/16*(b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^2 + (b*x^2 + c*x^4)^(3/2)/(6*
c) + (b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int x\sqrt{bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{(bx^2+cx^4)^{3/2}}{6c}-\frac{b\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{4c}\\
&= -\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c^2}+\frac{(bx^2+cx^4)^{3/2}}{6c}+\frac{b^3\text{Subst}\left(\int\frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{32c^2}\\
&= -\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c^2}+\frac{(bx^2+cx^4)^{3/2}}{6c}+\frac{b^3\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^2}\\
&= -\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c^2}+\frac{(bx^2+cx^4)^{3/2}}{6c}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 102, normalized size = 1.12

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\,(-3b^2+2bcx^2+8c^2x^4)-3b^3\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{48c^{5/2}\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(-3*b^2 + 2*b*c*x^2 + 8*c^2*x
^4) - 3*b^3*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(48*c^(5/2)*Sqrt[x^2*(b +
 c*x^2)])
```

Maple [A]
time = 0.09, size = 104, normalized size = 1.14

method	result	size
risch	$-\frac{\left(-8c^{2}x^{4}-2bc\,x^{2}+3b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{48c^{2}}+\frac{b^{3}\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{16c^{\frac{5}{2}}x\sqrt{c\,x^{2}+b}}$	90
default	$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\left(8x^{3}\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}-6\sqrt{c}\,\left(c\,x^{2}+b\right)^{\frac{3}{2}}bx+3\sqrt{c}\,\sqrt{c\,x^{2}+b}\,b^{2}x+3\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)b^{3}\right)}{48x\sqrt{c\,x^{2}+b}\,c^{\frac{5}{2}}}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(1/2)*(8*x^3*(c*x^2+b)^(3/2)*c^(3/2)-6*c^(1/2)*(c*x^2+b)
^(3/2)*b*x+3*c^(1/2)*(c*x^2+b)^(1/2)*b^2*x+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*
b^3)/x/(c*x^2+b)^(1/2)/c^(5/2)
```

Maxima [A]
time = 0.30, size = 97, normalized size = 1.07

$$-\frac{\sqrt{cx^{4}+bx^{2}}\,bx^{2}}{8\,c}+\frac{b^{3}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{32\,c^{\frac{5}{2}}}-\frac{\sqrt{cx^{4}+bx^{2}}\,b^{2}}{16\,c^{2}}+\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(c*x^4 + b*x^2)*b*x^2/c + 1/32*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4
+ b*x^2)*sqrt(c))/c^(5/2) - 1/16*sqrt(c*x^4 + b*x^2)*b^2/c^2 + 1/6*(c*x^4 +
 b*x^2)^(3/2)/c
```

Fricas [A]
time = 0.37, size = 167, normalized size = 1.84

$$\left[\frac{3\,b^{3}\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\,\left(8\,c^{3}x^{4}+2\,bc^{2}x^{2}-3\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}}{96\,c^{3}},-\frac{3\,b^{3}\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)-\left(8\,c^{3}x^{4}+2\,bc^{2}x^{2}-3\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}}{48\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*b^3*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
(8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^3, -1/48*(3*b^3*
sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (8*c^3*x^4 + 2*
b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{x^{2}\left(b+cx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 5.57, size = 85, normalized size = 0.93

$$\frac{1}{48}\left(2\left(4\,x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x^2-\frac{3\,b^2\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}\,x-\frac{b^3\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\,\right|\right)\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}}+\frac{b^3\log\left(|b|\right)\mathrm{sgn}(x)}{32\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2*sgn(x) + b*sgn(x)/c)*x^2 - 3*b^2*sgn(x)/c^2)*sqrt(c*x^2 + b)
*x - 1/16*b^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(5/2) + 1/32*
b^3*log(abs(b))*sgn(x)/c^(5/2)
```

Mupad [B]
time = 4.36, size = 77, normalized size = 0.85

$$\frac{b^3\,\ln\left(\frac{2\,c\,x^2+b}{\sqrt{c}}+2\,\sqrt{c\,x^4+b\,x^2}\right)}{32\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{48\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b^3*log((b + 2*c*x^2)/c^(1/2) + 2*(b*x^2 + c*x^4)^(1/2)))/(32*c^(5/2)) + (
(b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

3.223 $\int x\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=68

$$\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

```
[Out] -1/8*b^2*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/8*(2*c*x^2+b)*(
c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.235$, Rules used = {2038, 626, 634, 212}

$$\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt
[b*x^2 + c*x^4]])/(8*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \sqrt{bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{16c}\\ &= \frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c}\\ &= \frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 88, normalized size = 1.29

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\,(b+2cx^2)+b^2\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{8c^{3/2}\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^2 + c*x^4],x]
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(b + 2*c*x^2) + b^2*Log[-(Sqr
t[c]*x) + Sqrt[b + c*x^2]]))/(8*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.08, size = 84, normalized size = 1.24

method	result	size
risch	$\frac{(2cx^2+b)\sqrt{x^2\,(c\,x^2+b)}}{8c}-\frac{b^2\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)\sqrt{x^2\,(c\,x^2+b)}}{8c^{\frac{3}{2}}x\sqrt{c\,x^2+b}}$	77
default	$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(2x(c\,x^2+b)^{\frac{3}{2}}\sqrt{c}-\sqrt{c}\,\sqrt{c\,x^2+b}\,bx-\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)b^2\right)}{8x\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(2*x*(c*x^2+b)^(3/2)*c^(1/2)-c^(1/2)*(c*x^2+b)^(1/2
)*b*x-ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2)/x/(c*x^2+b)^(1/2)/c^(3/2)
```

Maxima [**A**]
time = 0.28, size = 73, normalized size = 1.07

$$\frac{1}{4}\sqrt{cx^4+bx^2}\,x^2-\frac{b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{16\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}\,b}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^4 + b*x^2)*x^2 - 1/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x
^2)*sqrt(c))/c^(3/2) + 1/8*sqrt(c*x^4 + b*x^2)*b/c
```

Fricas [**A**]
time = 0.38, size = 140, normalized size = 2.06

$$\left[\frac{b^2\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\,(2\,c^2x^2+bc)}{16\,c^2},\,\frac{b^2\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}\,(2\,c^2x^2+bc)}{8\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(b^2*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sq
rt(c*x^4 + b*x^2)*(2*c^2*x^2 + b*c))/c^2, 1/8*(b^2*sqrt(-c)*arctan(sqrt(c*x
^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + b*c))/
c^2]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\,(b+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(b + c*x**2)), x)
```

Giac [**A**]
time = 6.00, size = 69, normalized size = 1.01

$$\frac{1}{8}\sqrt{cx^2+b}\left(2\,x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x+\frac{b^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\,\right|\right)\mathrm{sgn}(x)}{8\,c^{\frac{3}{2}}}-\frac{b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^2 + b)*(2*x^2*sgn(x) + b*sgn(x)/c)*x + 1/8*b^2*log(abs(-sqrt(c
)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(3/2) - 1/16*b^2*log(abs(b))*sgn(x)/c^(3/2
)
```

Mupad [B]
time = 4.37, size = 64, normalized size = 0.94

$$\frac{\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2}}{2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/2 - (b^2*log((b/2 + c*x^2)/c^(1/2
) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2))
```

3.224 $\int \frac{\sqrt{bx^2+cx^4}}{x} \, dx$

Optimal. Leaf size=55

$$\frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] 1/2*b*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)+1/2*(c*x^4+b*x^2)^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2043, 678, 634, 212}

$$\frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/2 + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*S
qrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{1}{4}b\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{1}{2}b\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 64, normalized size = 1.16

$$\frac{1}{2}\sqrt{x^2\left(b+cx^2\right)}\left(1-\frac{b\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)}{\sqrt{c}\,x\sqrt{b+cx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(1 - (b*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]])/(Sqrt[c
]*x*Sqrt[b + c*x^2])))/2
```

Maple [**A**]
time = 0.09, size = 64, normalized size = 1.16

method	result	size
default	$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(x\sqrt{c\,x^2+b}\,\sqrt{c}+b\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)\right)}{2x\sqrt{c\,x^2+b}\,\sqrt{c}}$	64

risch	$\frac{\sqrt{x^2\,(c\,x^2+b)}}{2}+\frac{b\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)\sqrt{x^2\,(c\,x^2+b)}}{2\sqrt{c}\,x\sqrt{c\,x^2+b}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^4+b*x^2)^(1/2)*(x*(c*x^2+b)^(1/2)*c^(1/2)+b*ln(x*c^(1/2)+(c*x^2+b)
^(1/2)))/x/(c*x^2+b)^(1/2)/c^(1/2)
```

Maxima [A]
time = 0.28, size = 49, normalized size = 0.89

$$\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{4\,\sqrt{c}}+\frac{1}{2}\,\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 1/2*sqrt(c
*x^4 + b*x^2)
```

Fricas [A]
time = 0.34, size = 115, normalized size = 2.09

$$\left[\frac{b\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\,c}{4\,c},-\frac{b\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}\,c}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c, -1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(
c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*c)/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(b+cx^2)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x, x)
```

Giac [A]
time = 7.72, size = 52, normalized size = 0.95

$$\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{4\sqrt{c}}+\frac{1}{2}\left(\sqrt{cx^2+b}\,x-\frac{b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{\sqrt{c}}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="giac")
[Out] 1/4*b*log(abs(b))*sgn(x)/sqrt(c) + 1/2*(sqrt(c*x^2 + b)*x - b*log(abs(-sqrt
(c)*x + sqrt(c*x^2 + b)))/sqrt(c))*sgn(x)
```

Mupad [B]
time = 4.21, size = 50, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{2}+\frac{b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x,x)
[Out] (b*x^2 + c*x^4)^(1/2)/2 + (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1
/2)))/(4*c^(1/2))
```

3.225 $\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{bx^2+cx^4}}{x^2}+\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))*c^(1/2)-(c*x^4+b*x^2)^(1/2)/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2043, 676, 634, 212}

$$\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{\sqrt{bx^2+cx^4}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/x^2) + Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{bx^2+cx^4}}{x^2}+\frac{1}{2}c\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{bx^2+cx^4}}{x^2}+c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= -\frac{\sqrt{bx^2+cx^4}}{x^2}+\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 1.31

$$-\frac{\sqrt{b+cx^2}\left(\sqrt{b+cx^2}+\sqrt{c}\,x\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -((Sqrt[b + c*x^2]*(Sqrt[b + c*x^2] + Sqrt[c]*x*Log[-(Sqrt[c]*x) + Sqrt[b +
 c*x^2]]))/Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.11, size = 84, normalized size = 1.62

method	result	size
risch	$-\frac{\sqrt{x^2\left(c\,x^2+b\right)}}{x^2}+\frac{\sqrt{c}\,\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)\sqrt{x^2\left(c\,x^2+b\right)}}{x\sqrt{c\,x^2+b}}$	65
default	$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(c^{\frac{3}{2}}\sqrt{c\,x^2+b}\,x^2-\left(c\,x^2+b\right)^{\frac{3}{2}}\sqrt{c}+\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)bcx\right)}{x^2\sqrt{c\,x^2+b}\,b\sqrt{c}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^4+b*x^2)^(1/2)*(c^(3/2)*(c*x^2+b)^(1/2)*x^2-(c*x^2+b)^(3/2)*c^(1/2)+ln
(x*c^(1/2)+(c*x^2+b)^(1/2))*b*c*x)/x^2/(c*x^2+b)^(1/2)/b/c^(1/2)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.98

$$\frac{1}{2}\sqrt{c}\,\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-\frac{\sqrt{cx^4+bx^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - sqrt(c*x^4 +
 b*x^2)/x^2
```

Fricas [A]
time = 0.39, size = 115, normalized size = 2.21

$$\left[\frac{\sqrt{c}\,x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}}{2\,x^2},-\frac{\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}}{x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqr
t(c*x^4 + b*x^2))/x^2, -(sqrt(-c)*x^2*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(
c*x^2 + b)) + sqrt(c*x^4 + b*x^2))/x^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(b+cx^2)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**3, x)
```

Giac [A]
time = 7.38, size = 61, normalized size = 1.17

$$-\frac{1}{2}\sqrt{c}\,\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sgn}(x)+\frac{2\,b\sqrt{c}\,\mathrm{sgn}(x)}{\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2*b*sqrt(c)*sgn(
x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^3,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^3, x)
```

3.226 $\int \frac{\sqrt{bx^2+cx^4}}{x^5} dx$

Optimal. Leaf size=25

$$-\frac{(bx^2+cx^4)^{3/2}}{3bx^6}$$

```
[Out] -1/3*(c*x^4+b*x^2)^(3/2)/b/x^6
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2039}

$$-\frac{(bx^2+cx^4)^{3/2}}{3bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -1/3*(b*x^2 + c*x^4)^(3/2)/(b*x^6)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^5} dx = -\frac{(bx^2+cx^4)^{3/2}}{3bx^6}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.00

$$-\frac{(x^2(b+cx^2))^{3/2}}{3bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -1/3*(x^2*(b + c*x^2))^(3/2)/(b*x^6)
```

Maple [A]
time = 0.09, size = 29, normalized size = 1.16

method	result	size
gosper	$-\frac{(c\,x^2+b)\sqrt{c\,x^4+b\,x^2}}{3x^4b}$	29
default	$-\frac{(c\,x^2+b)\sqrt{c\,x^4+b\,x^2}}{3x^4b}$	29
trager	$-\frac{(c\,x^2+b)\sqrt{c\,x^4+b\,x^2}}{3x^4b}$	29
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,(c\,x^2+b)}{3x^4b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^4*(c*x^2+b)/b*(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 41, normalized size = 1.64

$$-\frac{\sqrt{cx^4+bx^2}\,c}{3\,bx^2}-\frac{\sqrt{cx^4+bx^2}}{3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(c*x^4 + b*x^2)*c/(b*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/x^4
```

Fricas [A]
time = 0.32, size = 28, normalized size = 1.12

$$-\frac{\sqrt{cx^4+bx^2}\,(cx^2+b)}{3\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(b*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(b+cx^2)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(21) = 42$.
time = 20.00, size = 63, normalized size = 2.52

$$\frac{2\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4 c^{\frac{3}{2}}\mathrm{sgn}(x)+b^2c^{\frac{3}{2}}\mathrm{sgn}(x)\right)}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*c^(3/2)*sgn(x) + b^2*c^(3/2)*sgn(x))
/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

Mupad [B]
time = 4.15, size = 28, normalized size = 1.12

$$-\frac{\left(c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2}}{3\,b\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^5,x)
```

```
[Out] -((b + c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b*x^4)
```

3.227 $\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx$

Optimal. Leaf size=52

$$-\frac{(bx^2+cx^4)^{3/2}}{5bx^8}+\frac{2c(bx^2+cx^4)^{3/2}}{15b^2x^6}$$

```
[Out] -1/5*(c*x^4+b*x^2)^(3/2)/b/x^8+2/15*c*(c*x^4+b*x^2)^(3/2)/b^2/x^6
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2039}

$$\frac{2c(bx^2+cx^4)^{3/2}}{15b^2x^6}-\frac{(bx^2+cx^4)^{3/2}}{5bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^7,x]
```

```
[Out] -1/5*(b*x^2 + c*x^4)^(3/2)/(b*x^8) + (2*c*(b*x^2 + c*x^4)^(3/2))/(15*b^2*x^
6)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx &= -\frac{(bx^2+cx^4)^{3/2}}{5bx^8}-\frac{(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{5b}\\ &= -\frac{(bx^2+cx^4)^{3/2}}{5bx^8}+\frac{2c(bx^2+cx^4)^{3/2}}{15b^2x^6}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 0.88

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-3b^2-bcx^2+2c^2x^4\right)}{15b^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^7,x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-3*b^2 - b*c*x^2 + 2*c^2*x^4))/(15*b^2*x^6)
```

Maple [A]
time = 0.10, size = 39, normalized size = 0.75

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+3b\right)\sqrt{c\,x^{4}+b\,x^{2}}}{15b^{2}x^{6}}$	39
default	$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+3b\right)\sqrt{c\,x^{4}+b\,x^{2}}}{15b^{2}x^{6}}$	39
trager	$-\frac{\left(-2c^{2}x^{4}+bc\,x^{2}+3b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{15b^{2}x^{6}}$	42
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-2c^{2}x^{4}+bc\,x^{2}+3b^{2}\right)}{15x^{6}b^{2}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/15*(c*x^2+b)*(-2*c*x^2+3*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^6
```

Maxima [A]
time = 0.29, size = 65, normalized size = 1.25

$$\frac{2\sqrt{cx^4+bx^2}\,c^2}{15\,b^2x^2}-\frac{\sqrt{cx^4+bx^2}\,c}{15\,bx^4}-\frac{\sqrt{cx^4+bx^2}}{5\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="maxima")
[Out] 2/15*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - 1/15*sqrt(c*x^4 + b*x^2)*c/(b*x^4)
 - 1/5*sqrt(c*x^4 + b*x^2)/x^6
```

Fricas [A]
time = 0.38, size = 42, normalized size = 0.81

$$\frac{\left(2\,c^2x^4-bcx^2-3\,b^2\right)\sqrt{cx^4+bx^2}}{15\,b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="fricas")

[Out] 1/15*(2*c^2*x^4 - b*c*x^2 - 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^2*x^6)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2)**(1/2)/x**7,x)

[Out] Integral(sqrt(x**2*(b + c*x**2))/x**7, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(44) = 88.
time = 7.44, size = 120, normalized size = 2.31

$$\frac{4\left(15\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6c^{\frac{5}{2}}\mathrm{sgn}(x)+5\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4bc^{\frac{5}{2}}\mathrm{sgn}(x)+5\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2b^2c^{\frac{5}{2}}\mathrm{sgn}(x)-b^3c^{\frac{5}{2}}\mathrm{sgn}(x)\right)}{15\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="giac")

[Out] 4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^6*c^(5/2)*sgn(x) + 5*(sqrt(c)*x - sq
rt(c*x^2 + b))^4*b*c^(5/2)*sgn(x) + 5*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^2*c
^(5/2)*sgn(x) - b^3*c^(5/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5

Mupad [B]
time = 4.26, size = 41, normalized size = 0.79

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(3\,b^2+b\,c\,x^2-2\,c^2\,x^4\right)}{15\,b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2 + c*x^4)^(1/2)/x^7,x)

[Out] -((b*x^2 + c*x^4)^(1/2)*(3*b^2 - 2*c^2*x^4 + b*c*x^2))/(15*b^2*x^6)

3.228 $\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=80

$$-\frac{(bx^2+cx^4)^{3/2}}{7bx^{10}}+\frac{4c(bx^2+cx^4)^{3/2}}{35b^2x^8}-\frac{8c^2(bx^2+cx^4)^{3/2}}{105b^3x^6}$$

```
[Out] -1/7*(c*x^4+b*x^2)^(3/2)/b/x^10+4/35*c*(c*x^4+b*x^2)^(3/2)/b^2/x^8-8/105*c^
2*(c*x^4+b*x^2)^(3/2)/b^3/x^6
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2039}

$$-\frac{8c^2(bx^2+cx^4)^{3/2}}{105b^3x^6}+\frac{4c(bx^2+cx^4)^{3/2}}{35b^2x^8}-\frac{(bx^2+cx^4)^{3/2}}{7bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -1/7*(b*x^2 + c*x^4)^(3/2)/(b*x^10) + (4*c*(b*x^2 + c*x^4)^(3/2))/(35*b^2*x
^8) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^6)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx = -\frac{(bx^2+cx^4)^{3/2}}{7bx^{10}} - \frac{(4c)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{7b}$$
$$= -\frac{(bx^2+cx^4)^{3/2}}{7bx^{10}} + \frac{4c(bx^2+cx^4)^{3/2}}{35b^2x^8} + \frac{(8c^2)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{35b^2}$$
$$= -\frac{(bx^2+cx^4)^{3/2}}{7bx^{10}} + \frac{4c(bx^2+cx^4)^{3/2}}{35b^2x^8} - \frac{8c^2(bx^2+cx^4)^{3/2}}{105b^3x^6}$$

Mathematica [A]
time = 0.06, size = 57, normalized size = 0.71

$$\frac{\sqrt{x^2(b+cx^2)}\,(-15b^3-3b^2cx^2+4bc^2x^4-8c^3x^6)}{105b^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-15*b^3 - 3*b^2*c*x^2 + 4*b*c^2*x^4 - 8*c^3*x^6))/(
105*b^3*x^8)
```

Maple [A]
time = 0.08, size = 50, normalized size = 0.62

method	result	size
gosper	$-\frac{(c\,x^2+b)(8c^2x^4-12bc\,x^2+15b^2)\sqrt{c\,x^4+b\,x^2}}{105x^8b^3}$	50
default	$-\frac{(c\,x^2+b)(8c^2x^4-12bc\,x^2+15b^2)\sqrt{c\,x^4+b\,x^2}}{105x^8b^3}$	50
trager	$-\frac{(8c^3x^6-4b\,c^2x^4+3b^2c\,x^2+15b^3)\sqrt{c\,x^4+b\,x^2}}{105x^8b^3}$	54
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(8c^3x^6-4b\,c^2x^4+3b^2c\,x^2+15b^3)}{105x^8b^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(c*x^2+b)*(8*c^2*x^4-12*b*c*x^2+15*b^2)*(c*x^4+b*x^2)^(1/2)/x^8/b^3
```

Maxima [A]
time = 0.30, size = 89, normalized size = 1.11

$$-\frac{8\sqrt{cx^4+bx^2}\,c^3}{105\,b^3x^2} + \frac{4\sqrt{cx^4+bx^2}\,c^2}{105\,b^2x^4} - \frac{\sqrt{cx^4+bx^2}\,c}{35\,bx^6} - \frac{\sqrt{cx^4+bx^2}}{7\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -8/105*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^2) + 4/105*sqrt(c*x^4 + b*x^2)*c^2/(b
^2*x^4) - 1/35*sqrt(c*x^4 + b*x^2)*c/(b*x^6) - 1/7*sqrt(c*x^4 + b*x^2)/x^8
```

Fricas [A]
time = 0.37, size = 53, normalized size = 0.66

$$-\frac{(8\,c^3x^6 - 4\,bc^2x^4 + 3\,b^2cx^2 + 15\,b^3)\sqrt{cx^4 + bx^2}}{105\,b^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/105*(8*c^3*x^6 - 4*b*c^2*x^4 + 3*b^2*c*x^2 + 15*b^3)*sqrt(c*x^4 + b*x^2)
/(b^3*x^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(b + cx^2)}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**9, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(68) = 136.
time = 7.60, size = 148, normalized size = 1.85

$$\frac{16\left(70\left(\sqrt{c}\,x - \sqrt{cx^2 + b}\right)^8 c^{\frac{7}{2}}\mathrm{sgn}(x) + 35\left(\sqrt{c}\,x - \sqrt{cx^2 + b}\right)^6 bc^{\frac{7}{2}}\mathrm{sgn}(x) + 21\left(\sqrt{c}\,x - \sqrt{cx^2 + b}\right)^4 b^2c^{\frac{7}{2}}\mathrm{sgn}(x) - 7\left(\sqrt{c}\,x - \sqrt{cx^2 + b}\right)^2 b^3c^{\frac{7}{2}}\mathrm{sgn}(x) + b^4c^{\frac{7}{2}}\mathrm{sgn}(x)\right)}{105\left(\left(\sqrt{c}\,x - \sqrt{cx^2 + b}\right)^2 - b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(7/2)*sgn(x) + 35*(sqrt(c)*x -
 sqrt(c*x^2 + b))^6*b*c^(7/2)*sgn(x) + 21*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b
^2*c^(7/2)*sgn(x) - 7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^3*c^(7/2)*sgn(x) +
b^4*c^(7/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

Mupad [B]
time = 4.34, size = 89, normalized size = 1.11

$$\frac{4\,c^2\,\sqrt{c\,x^4 + b\,x^2}}{105\,b^2\,x^4} - \frac{c\,\sqrt{c\,x^4 + b\,x^2}}{35\,b\,x^6} - \frac{\sqrt{c\,x^4 + b\,x^2}}{7\,x^8} - \frac{8\,c^3\,\sqrt{c\,x^4 + b\,x^2}}{105\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^9,x)
```

```
[Out] (4*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (c*(b*x^2 + c*x^4)^(1/2))/(35
*b*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*x^8) - (8*c^3*(b*x^2 + c*x^4)^(1/2))/(10
5*b^3*x^2)
```

3.229 $\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=108

$$-\frac{(bx^2+cx^4)^{3/2}}{9bx^{12}}+\frac{2c(bx^2+cx^4)^{3/2}}{21b^2x^{10}}-\frac{8c^2(bx^2+cx^4)^{3/2}}{105b^3x^8}+\frac{16c^3(bx^2+cx^4)^{3/2}}{315b^4x^6}$$

```
[Out] -1/9*(c*x^4+b*x^2)^(3/2)/b/x^12+2/21*c*(c*x^4+b*x^2)^(3/2)/b^2/x^10-8/105*c
^2*(c*x^4+b*x^2)^(3/2)/b^3/x^8+16/315*c^3*(c*x^4+b*x^2)^(3/2)/b^4/x^6
```

Rubi [A]
time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2041, 2039}

$$\frac{16c^3(bx^2+cx^4)^{3/2}}{315b^4x^6}-\frac{8c^2(bx^2+cx^4)^{3/2}}{105b^3x^8}+\frac{2c(bx^2+cx^4)^{3/2}}{21b^2x^{10}}-\frac{(bx^2+cx^4)^{3/2}}{9bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^11,x]
```

```
[Out] -1/9*(b*x^2 + c*x^4)^(3/2)/(b*x^12) + (2*c*(b*x^2 + c*x^4)^(3/2))/(21*b^2*x
^10) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^8) + (16*c^3*(b*x^2 + c*x^4
)^(3/2))/(315*b^4*x^6)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx = -\frac{(bx^2+cx^4)^{3/2}}{9bx^{12}} - \frac{(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx}{3b}$$
$$= -\frac{(bx^2+cx^4)^{3/2}}{9bx^{12}} + \frac{2c(bx^2+cx^4)^{3/2}}{21b^2x^{10}} + \frac{(8c^2)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{21b^2}$$
$$= -\frac{(bx^2+cx^4)^{3/2}}{9bx^{12}} + \frac{2c(bx^2+cx^4)^{3/2}}{21b^2x^{10}} - \frac{8c^2(bx^2+cx^4)^{3/2}}{105b^3x^8} - \frac{(16c^3)\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx}{105b^3}$$
$$= -\frac{(bx^2+cx^4)^{3/2}}{9bx^{12}} + \frac{2c(bx^2+cx^4)^{3/2}}{21b^2x^{10}} - \frac{8c^2(bx^2+cx^4)^{3/2}}{105b^3x^8} + \frac{16c^3(bx^2+cx^4)^{3/2}}{315b^4x^6}$$

Mathematica [A]
time = 0.06, size = 68, normalized size = 0.63

$$\frac{\sqrt{x^2(b+cx^2)}\,(-35b^4-5b^3cx^2+6b^2c^2x^4-8bc^3x^6+16c^4x^8)}{315b^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^11,x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-35*b^4 - 5*b^3*c*x^2 + 6*b^2*c^2*x^4 - 8*b*c^3*x^6
 + 16*c^4*x^8))/(315*b^4*x^10)
```

Maple [A]
time = 0.12, size = 61, normalized size = 0.56

method	result	size
gosper	$-\frac{(cx^2+b)(-16c^3x^6+24bc^2x^4-30b^2cx^2+35b^3)\sqrt{cx^4+bx^2}}{315x^{10}b^4}$	61
default	$-\frac{(cx^2+b)(-16c^3x^6+24bc^2x^4-30b^2cx^2+35b^3)\sqrt{cx^4+bx^2}}{315x^{10}b^4}$	61
trager	$-\frac{(-16c^4x^8+8bc^3x^6-6b^2c^2x^4+5b^3cx^2+35b^4)\sqrt{cx^4+bx^2}}{315x^{10}b^4}$	65
risch	$-\frac{\sqrt{x^2(cx^2+b)}\,(-16c^4x^8+8bc^3x^6-6b^2c^2x^4+5b^3cx^2+35b^4)}{315x^{10}b^4}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^11,x,method=_RETURNVERBOSE)
[Out] -1/315*(c*x^2+b)*(-16*c^3*x^6+24*b*c^2*x^4-30*b^2*c*x^2+35*b^3)*(c*x^4+b*x^
2)^(1/2)/x^10/b^4
```

Maxima [A]
time = 0.29, size = 113, normalized size = 1.05

$$\frac{16\sqrt{cx^4+bx^2}\,c^4}{315\,b^4x^2}-\frac{8\sqrt{cx^4+bx^2}\,c^3}{315\,b^3x^4}+\frac{2\sqrt{cx^4+bx^2}\,c^2}{105\,b^2x^6}-\frac{\sqrt{cx^4+bx^2}\,c}{63\,bx^8}-\frac{\sqrt{cx^4+bx^2}}{9\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] 16/315*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8/315*sqrt(c*x^4 + b*x^2)*c^3/(b
^3*x^4) + 2/105*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 1/63*sqrt(c*x^4 + b*x^2
)*c/(b*x^8) - 1/9*sqrt(c*x^4 + b*x^2)/x^10
```

Fricas [A]
time = 0.37, size = 64, normalized size = 0.59

$$\frac{\left(16\,c^4x^8-8\,bc^3x^6+6\,b^2c^2x^4-5\,b^3cx^2-35\,b^4\right)\sqrt{cx^4+bx^2}}{315\,b^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/315*(16*c^4*x^8 - 8*b*c^3*x^6 + 6*b^2*c^2*x^4 - 5*b^3*c*x^2 - 35*b^4)*sqr
t(c*x^4 + b*x^2)/(b^4*x^10)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**11, x)
```

Giac [A]
time = 5.32, size = 178, normalized size = 1.65

$$\frac{32\left(315\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}c^{\frac{9}{2}}\mathrm{sgn}(x)+189\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}bc^{\frac{9}{2}}\mathrm{sgn}(x)+84\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{6}b^2c^{\frac{9}{2}}\mathrm{sgn}(x)-36\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{4}b^3c^{\frac{9}{2}}\mathrm{sgn}(x)+9\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{2}b^4c^{\frac{9}{2}}\mathrm{sgn}(x)-b^5c^{\frac{9}{2}}\mathrm{sgn}(x)\right)}{315\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(9/2)*sgn(x) + 189*(sqrt(c)*
x - sqrt(c*x^2 + b))^8*b*c^(9/2)*sgn(x) + 84*(sqrt(c)*x - sqrt(c*x^2 + b))^
```

```
6*b^2*c^(9/2)*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^3*c^(9/2)*sgn(x
) + 9*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^4*c^(9/2)*sgn(x) - b^5*c^(9/2)*sgn(
x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

Mupad [B]
time = 4.50, size = 113, normalized size = 1.05

$$\frac{2\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^6}-\frac{c\,\sqrt{c\,x^4+b\,x^2}}{63\,b\,x^8}-\frac{\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{8\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^4}+\frac{16\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^11,x)
```

```
[Out] (2*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^6) - (c*(b*x^2 + c*x^4)^(1/2))/(63
*b*x^8) - (b*x^2 + c*x^4)^(1/2)/(9*x^10) - (8*c^3*(b*x^2 + c*x^4)^(1/2))/(3
15*b^3*x^4) + (16*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2)
```

3.230 $\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx$

Optimal. Leaf size=136

$$-\frac{(bx^2+cx^4)^{3/2}}{11bx^{14}}+\frac{8c(bx^2+cx^4)^{3/2}}{99b^2x^{12}}-\frac{16c^2(bx^2+cx^4)^{3/2}}{231b^3x^{10}}+\frac{64c^3(bx^2+cx^4)^{3/2}}{1155b^4x^8}-\frac{128c^4(bx^2+cx^4)^{3/2}}{3465b^5x^6}$$

```
[Out] -1/11*(c*x^4+b*x^2)^(3/2)/b/x^14+8/99*c*(c*x^4+b*x^2)^(3/2)/b^2/x^12-16/231
*c^2*(c*x^4+b*x^2)^(3/2)/b^3/x^10+64/1155*c^3*(c*x^4+b*x^2)^(3/2)/b^4/x^8-1
28/3465*c^4*(c*x^4+b*x^2)^(3/2)/b^5/x^6
```

Rubi [A]

time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2039}

$$-\frac{128c^4(bx^2+cx^4)^{3/2}}{3465b^5x^6}+\frac{64c^3(bx^2+cx^4)^{3/2}}{1155b^4x^8}-\frac{16c^2(bx^2+cx^4)^{3/2}}{231b^3x^{10}}+\frac{8c(bx^2+cx^4)^{3/2}}{99b^2x^{12}}-\frac{(bx^2+cx^4)^{3/2}}{11bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

```
[Out] -1/11*(b*x^2 + c*x^4)^(3/2)/(b*x^14) + (8*c*(b*x^2 + c*x^4)^(3/2))/(99*b^2*
x^12) - (16*c^2*(b*x^2 + c*x^4)^(3/2))/(231*b^3*x^10) + (64*c^3*(b*x^2 + c*
x^4)^(3/2))/(1155*b^4*x^8) - (128*c^4*(b*x^2 + c*x^4)^(3/2))/(3465*b^5*x^6)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx = -\frac{(bx^2+cx^4)^{3/2}}{11bx^{14}} - \frac{(8c)\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx}{11b}$$
$$= -\frac{(bx^2+cx^4)^{3/2}}{11bx^{14}} + \frac{8c(bx^2+cx^4)^{3/2}}{99b^2x^{12}} + \frac{(16c^2)\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx}{33b^2}$$
$$= -\frac{(bx^2+cx^4)^{3/2}}{11bx^{14}} + \frac{8c(bx^2+cx^4)^{3/2}}{99b^2x^{12}} - \frac{16c^2(bx^2+cx^4)^{3/2}}{231b^3x^{10}} - \frac{(64c^3)\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx}{231b^3}$$
$$= -\frac{(bx^2+cx^4)^{3/2}}{11bx^{14}} + \frac{8c(bx^2+cx^4)^{3/2}}{99b^2x^{12}} - \frac{16c^2(bx^2+cx^4)^{3/2}}{231b^3x^{10}} + \frac{64c^3(bx^2+cx^4)^{3/2}}{1155b^4x^8} + \frac{(128}{}$$
$$= -\frac{(bx^2+cx^4)^{3/2}}{11bx^{14}} + \frac{8c(bx^2+cx^4)^{3/2}}{99b^2x^{12}} - \frac{16c^2(bx^2+cx^4)^{3/2}}{231b^3x^{10}} + \frac{64c^3(bx^2+cx^4)^{3/2}}{1155b^4x^8} - \frac{128c}{}$$

Mathematica [A]
time = 0.07, size = 79, normalized size = 0.58

$$\frac{\sqrt{x^2(b+cx^2)}\,(-315b^5-35b^4cx^2+40b^3c^2x^4-48b^2c^3x^6+64bc^4x^8-128c^5x^{10})}{3465b^5x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-315*b^5 - 35*b^4*c*x^2 + 40*b^3*c^2*x^4 - 48*b^2*c
^3*x^6 + 64*b*c^4*x^8 - 128*c^5*x^10))/(3465*b^5*x^12)
```

Maple [A]
time = 0.11, size = 72, normalized size = 0.53

method	result	size
gosper	$-\frac{(c\,x^2+b)\,(128c^4x^8-192b\,c^3x^6+240b^2c^2x^4-280b^3c\,x^2+315b^4)\sqrt{c\,x^4+b\,x^2}}{3465x^{12}b^5}$	72
default	$-\frac{(c\,x^2+b)\,(128c^4x^8-192b\,c^3x^6+240b^2c^2x^4-280b^3c\,x^2+315b^4)\sqrt{c\,x^4+b\,x^2}}{3465x^{12}b^5}$	72
trager	$-\frac{(128c^5x^{10}-64b\,c^4x^8+48b^2c^3x^6-40b^3c^2x^4+35b^4c\,x^2+315b^5)\sqrt{c\,x^4+b\,x^2}}{3465x^{12}b^5}$	76
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,(128c^5x^{10}-64b\,c^4x^8+48b^2c^3x^6-40b^3c^2x^4+35b^4c\,x^2+315b^5)}{3465x^{12}b^5}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3465*(c*x^2+b)*(128*c^4*x^8-192*b*c^3*x^6+240*b^2*c^2*x^4-280*b^3*c*x^2+
315*b^4)*(c*x^4+b*x^2)^(1/2)/x^12/b^5
```

Maxima [A]
time = 0.30, size = 137, normalized size = 1.01

$$-\frac{128\sqrt{cx^4+bx^2}\,c^5}{3465\,b^5x^2}+\frac{64\sqrt{cx^4+bx^2}\,c^4}{3465\,b^4x^4}-\frac{16\sqrt{cx^4+bx^2}\,c^3}{1155\,b^3x^6}+\frac{8\sqrt{cx^4+bx^2}\,c^2}{693\,b^2x^8}-\frac{\sqrt{cx^4+bx^2}\,c}{99\,bx^{10}}-\frac{\sqrt{cx^4+bx^2}}{11\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] -128/3465*sqrt(c*x^4 + b*x^2)*c^5/(b^5*x^2) + 64/3465*sqrt(c*x^4 + b*x^2)*c
^4/(b^4*x^4) - 16/1155*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^6) + 8/693*sqrt(c*x^4
 + b*x^2)*c^2/(b^2*x^8) - 1/99*sqrt(c*x^4 + b*x^2)*c/(b*x^10) - 1/11*sqrt(c
*x^4 + b*x^2)/x^12
```

Fricas [A]
time = 0.38, size = 75, normalized size = 0.55

$$-\frac{\left(128\,c^5x^{10}-64\,bc^4x^8+48\,b^2c^3x^6-40\,b^3c^2x^4+35\,b^4cx^2+315\,b^5\right)\sqrt{cx^4+bx^2}}{3465\,b^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/3465*(128*c^5*x^10 - 64*b*c^4*x^8 + 48*b^2*c^3*x^6 - 40*b^3*c^2*x^4 + 35
*b^4*c*x^2 + 315*b^5)*sqrt(c*x^4 + b*x^2)/(b^5*x^12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**13, x)
```

Giac [A]
time = 5.22, size = 206, normalized size = 1.51

$$\frac{256\left(1386\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}c^{\frac{11}{2}}\operatorname{sgn}(x)+924\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{11}{2}}\operatorname{sgn}(x)+330\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{11}{2}}\operatorname{sgn}(x)-165\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{6}b^3c^{\frac{11}{2}}\operatorname{sgn}(x)+55\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{4}b^4c^{\frac{11}{2}}\operatorname{sgn}(x)-11\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{2}b^5c^{\frac{11}{2}}\operatorname{sgn}(x)+b^6c^{\frac{11}{2}}\operatorname{sgn}(x)\right)}{3465\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] 256/3465*(1386*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(11/2)*sgn(x) + 924*(sqrt
(c)*x - sqrt(c*x^2 + b))^10*b*c^(11/2)*sgn(x) + 330*(sqrt(c)*x - sqrt(c*x^2
```

```
 + b))^8*b^2*c^(11/2)*sgn(x) - 165*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^3*c^(1
1/2)*sgn(x) + 55*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(11/2)*sgn(x) - 11*(
sqrt(c)*x - sqrt(c*x^2 + b))^2*b^5*c^(11/2)*sgn(x) + b^6*c^(11/2)*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^11
```

Mupad [B]
time = 4.62, size = 137, normalized size = 1.01

$$\frac{8\,c^2\,\sqrt{c\,x^4+b\,x^2}}{693\,b^2\,x^8}-\frac{c\,\sqrt{c\,x^4+b\,x^2}}{99\,b\,x^{10}}-\frac{\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}-\frac{16\,c^3\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^6}+\frac{64\,c^4\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^4\,x^4}-\frac{128\,c^5\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^13,x)
```

```
[Out] (8*c^2*(b*x^2 + c*x^4)^(1/2))/(693*b^2*x^8) - (c*(b*x^2 + c*x^4)^(1/2))/(99
*b*x^10) - (b*x^2 + c*x^4)^(1/2)/(11*x^12) - (16*c^3*(b*x^2 + c*x^4)^(1/2))
/(1155*b^3*x^6) + (64*c^4*(b*x^2 + c*x^4)^(1/2))/(3465*b^4*x^4) - (128*c^5*
(b*x^2 + c*x^4)^(1/2))/(3465*b^5*x^2)
```

3.231 $\int x^4\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=78

$$\frac{8b^2(bx^2+cx^4)^{3/2}}{105c^3x^3}-\frac{4b(bx^2+cx^4)^{3/2}}{35c^2x}+\frac{x(bx^2+cx^4)^{3/2}}{7c}$$

```
[Out] 8/105*b^2*(c*x^4+b*x^2)^(3/2)/c^3/x^3-4/35*b*(c*x^4+b*x^2)^(3/2)/c^2/x+1/7*
x*(c*x^4+b*x^2)^(3/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2041, 2025}

$$\frac{8b^2(bx^2+cx^4)^{3/2}}{105c^3x^3}-\frac{4b(bx^2+cx^4)^{3/2}}{35c^2x}+\frac{x(bx^2+cx^4)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - (4*b*(b*x^2 + c*x^4)^(3/2))/(
35*c^2*x) + (x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x^4\sqrt{bx^2+cx^4}\,dx=\frac{x(bx^2+cx^4)^{3/2}}{7c}-\frac{(4b)\int x^2\sqrt{bx^2+cx^4}\,dx}{7c}$$
$$=-\frac{4b(bx^2+cx^4)^{3/2}}{35c^2x}+\frac{x(bx^2+cx^4)^{3/2}}{7c}+\frac{(8b^2)\int\sqrt{bx^2+cx^4}\,dx}{35c^2}$$
$$=\frac{8b^2(bx^2+cx^4)^{3/2}}{105c^3x^3}-\frac{4b(bx^2+cx^4)^{3/2}}{35c^2x}+\frac{x(bx^2+cx^4)^{3/2}}{7c}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.73

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\,(8b^3-4b^2cx^2+3bc^2x^4+15c^3x^6)}{105c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[b*x^2 + c*x^4],x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(8*b^3 - 4*b^2*c*x^2 + 3*b*c^2*x^4 + 15*c^3*x^6))/(1
05*c^3*x)
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.64

method	result	size
gosper	$\frac{(c\,x^2+b)\left(15c^2x^4-12bc\,x^2+8b^2\right)\sqrt{c\,x^4+b\,x^2}}{105c^3x}$	50
default	$\frac{(c\,x^2+b)\left(15c^2x^4-12bc\,x^2+8b^2\right)\sqrt{c\,x^4+b\,x^2}}{105c^3x}$	50
trager	$\frac{\left(15c^3x^6+3b\,c^2x^4-4b^2c\,x^2+8b^3\right)\sqrt{c\,x^4+b\,x^2}}{105c^3x}$	54
risch	$\frac{\sqrt{x^2\left(c\,x^2+b\right)}\,\left(15c^3x^6+3b\,c^2x^4-4b^2c\,x^2+8b^3\right)}{105x\,c^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/105*(c*x^2+b)*(15*c^2*x^4-12*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(1/2)/c^3/x
```

Maxima [A]
time = 0.30, size = 46, normalized size = 0.59

$$\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^2+b}}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)/c^3
```

Fricas [A]
time = 0.36, size = 53, normalized size = 0.68

$$\frac{(15 c^3x^6 + 3 bc^2x^4 - 4 b^2cx^2 + 8 b^3)\sqrt{cx^4 + bx^2}}{105 c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2)/
(c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{x^2\left(b + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 5.90, size = 60, normalized size = 0.77

$$-\frac{8 b^{\frac{7}{2}}\mathrm{sgn}(x)}{105 c^3} + \frac{15 (cx^2 + b)^{\frac{7}{2}}\mathrm{sgn}(x) - 42 (cx^2 + b)^{\frac{5}{2}}b\mathrm{sgn}(x) + 35 (cx^2 + b)^{\frac{3}{2}}b^2\mathrm{sgn}(x)}{105 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -8/105*b^(7/2)*sgn(x)/c^3 + 1/105*(15*(c*x^2 + b)^(7/2)*sgn(x) - 42*(c*x^2
+ b)^(5/2)*b*sgn(x) + 35*(c*x^2 + b)^(3/2)*b^2*sgn(x))/c^3
```

Mupad [B]
time = 4.23, size = 53, normalized size = 0.68

$$\frac{\sqrt{c\,x^4 + b\,x^2}\,\left(8\,b^3 - 4\,b^2\,c\,x^2 + 3\,b\,c^2\,x^4 + 15\,c^3\,x^6\right)}{105\,c^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(8*b^3 + 15*c^3*x^6 - 4*b^2*c*x^2 + 3*b*c^2*x^4))/(1
05*c^3*x)
```

3.232 $\int x^2\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=52

$$-\frac{2b(bx^2+cx^4)^{3/2}}{15c^2x^3}+\frac{(bx^2+cx^4)^{3/2}}{5cx}$$

```
[Out] -2/15*b*(c*x^4+b*x^2)^(3/2)/c^2/x^3+1/5*(c*x^4+b*x^2)^(3/2)/c/x
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2041, 2025}

$$\frac{(bx^2+cx^4)^{3/2}}{5cx}-\frac{2b(bx^2+cx^4)^{3/2}}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (b*x^2 + c*x^4)^(3/2)/(5*c*x)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{bx^2+cx^4}\,dx&=\frac{(bx^2+cx^4)^{3/2}}{5cx}-\frac{(2b)\int\sqrt{bx^2+cx^4}\,dx}{5c}\\&=-\frac{2b(bx^2+cx^4)^{3/2}}{15c^2x^3}+\frac{(bx^2+cx^4)^{3/2}}{5cx}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.87

$$\frac{\sqrt{x^2(b+cx^2)}\,(-2b^2+bcx^2+3c^2x^4)}{15c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^2 + c*x^4],x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-2*b^2 + b*c*x^2 + 3*c^2*x^4))/(15*c^2*x)
```

Maple [A]
time = 0.10, size = 39, normalized size = 0.75

method	result	size
gosper	$-\frac{(c\,x^2+b)(-3c\,x^2+2b)\sqrt{c\,x^4+b\,x^2}}{15c^2x}$	39
default	$-\frac{(c\,x^2+b)(-3c\,x^2+2b)\sqrt{c\,x^4+b\,x^2}}{15c^2x}$	39
trager	$-\frac{(-3c^2x^4-bc\,x^2+2b^2)\sqrt{c\,x^4+b\,x^2}}{15c^2x}$	43
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(-3c^2x^4-bc\,x^2+2b^2)}{15x\,c^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/15*(c*x^2+b)*(-3*c*x^2+2*b)*(c*x^4+b*x^2)^(1/2)/c^2/x
```

Maxima [A]
time = 0.31, size = 34, normalized size = 0.65

$$\frac{(3\,c^2x^4+bcx^2-2\,b^2)\sqrt{cx^2+b}}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 + b)/c^2
```

Fricas [A]
time = 0.35, size = 41, normalized size = 0.79

$$\frac{(3\,c^2x^4+bcx^2-2\,b^2)\sqrt{cx^4+bx^2}}{15\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^4 + b*x^2)/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 5.83, size = 44, normalized size = 0.85

$$\frac{2\,b^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,c^2}+\frac{3\,(cx^2+b)^{\frac{5}{2}}\mathrm{sgn}(x)-5\,(cx^2+b)^{\frac{3}{2}}b\mathrm{sgn}(x)}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*b^(5/2)*sgn(x)/c^2 + 1/15*(3*(c*x^2 + b)^(5/2)*sgn(x) - 5*(c*x^2 + b)^
(3/2)*b*sgn(x))/c^2
```

Mupad [B]
time = 4.14, size = 41, normalized size = 0.79

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-2\,b^2+b\,c\,x^2+3\,c^2\,x^4\right)}{15\,c^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(3*c^2*x^4 - 2*b^2 + b*c*x^2))/(15*c^2*x)
```

3.233 $\int \sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=25

$$\frac{(bx^2+cx^4)^{3/2}}{3cx^3}$$

```
[Out] 1/3*(c*x^4+b*x^2)^(3/2)/c/x^3
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2025}

$$\frac{(bx^2+cx^4)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4],x]
[Out] (b*x^2 + c*x^4)^(3/2)/(3*c*x^3)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \sqrt{bx^2+cx^4}\,dx = \frac{(bx^2+cx^4)^{3/2}}{3cx^3}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{(x^2(b+cx^2))^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4],x]
[Out] (x^2*(b + c*x^2))^(3/2)/(3*c*x^3)
```

Maple [A]
time = 0.08, size = 29, normalized size = 1.16

method	result	size
gosper	$\frac{(c\,x^2+b)\sqrt{c\,x^4+b\,x^2}}{3cx}$	29
default	$\frac{(c\,x^2+b)\sqrt{c\,x^4+b\,x^2}}{3cx}$	29
trager	$\frac{(c\,x^2+b)\sqrt{c\,x^4+b\,x^2}}{3cx}$	29
risch	$\frac{\sqrt{x^2\,(c\,x^2+b)}\,(c\,x^2+b)}{3xc}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(c*x^2+b)/c/x*(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.56

$$\frac{(cx^2+b)^{\frac{3}{2}}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(c*x^2 + b)^(3/2)/c
```

Fricas [A]
time = 0.32, size = 28, normalized size = 1.12

$$\frac{\sqrt{cx^4+bx^2}\,(cx^2+b)}{3\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + c*x**4), x)
```

Giac [A]
time = 7.83, size = 27, normalized size = 1.08

$$\frac{(cx^2+b)^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,c}-\frac{b^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(c*x^2 + b)^(3/2)*sgn(x)/c - 1/3*b^(3/2)*sgn(x)/c
```

Mupad [B]
time = 4.14, size = 29, normalized size = 1.16

$$\frac{\left(\frac{b}{3\,c}+\frac{x^2}{3}\right)\,\sqrt{c\,x^4+b\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2),x)
[Out] ((b/(3*c) + x^2/3)*(b*x^2 + c*x^4)^(1/2))/x
```

3.234 $\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b^(1/2)+(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2046, 2033, 212}

$$\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^2,x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx &= \frac{\sqrt{bx^2+cx^4}}{x} + b\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx \\ &= \frac{\sqrt{bx^2+cx^4}}{x} - b\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right) \\ &= \frac{\sqrt{bx^2+cx^4}}{x} - \sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 1.20

$$\frac{x\left(b+cx^2-\sqrt{b}\,\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^2,x]
```

```
[Out] (x*(b + c*x^2 - Sqrt[b]*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/
Sqrt[x^2*(b + c*x^2)]
```

Maple [A]
time = 0.09, size = 65, normalized size = 1.30

method	result	size
default	$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)-\sqrt{c\,x^2+b}\right)}{x\sqrt{c\,x^2+b}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)-(c*x^2+b)
^(1/2))/x/(c*x^2+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^2, x)
```

Fricas [A]
time = 0.36, size = 117, normalized size = 2.34

$$\left[\frac{\sqrt{b}\,x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}}{2\,x}, \frac{\sqrt{-b}\,x\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) +
 2*sqrt(c*x^4 + b*x^2))/x, (sqrt(-b)*x*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/
(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2))/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(b+cx^2)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**2, x)
```

Giac [A]
time = 7.39, size = 69, normalized size = 1.38

$$\frac{b\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+\sqrt{cx^2+b}\,\operatorname{sgn}(x)-\frac{\left(b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\,\sqrt{b}\right)\operatorname{sgn}(x)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + sqrt(c*x^2 + b)*sgn(x)
 - (b*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))*sgn(x)/sqrt(-b)
```

Mupad [B]
time = 4.31, size = 68, normalized size = 1.36

$$\frac{\sqrt{c\,x^4+b\,x^2}}{x}+\frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1{}\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c\,x^4+b\,x^2}\,1{}\mathrm{i}}{\sqrt{c}\,x^2\,\sqrt{\frac{b}{c\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^2,x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/x + (b^(1/2)*asin((b^(1/2)*1i)/(c^(1/2)*x))*(b*x^2 +
c*x^4)^(1/2)*1i)/(c^(1/2)*x^2*(b/(c*x^2) + 1)^(1/2))
```

3.235 $\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx$

Optimal. Leaf size=56

$$-\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

```
[Out] -1/2*c*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)-1/2*(c*x^4+b*x^2)^(1/
2)/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2045, 2033, 212}

$$-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt{bx^2+cx^4}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^4,x]
```

```
[Out] -1/2*Sqrt[b*x^2 + c*x^4]/x^3 - (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])
/(2*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{2x^3}+\frac{1}{2}c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{1}{2}c\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 1.12

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(1+\frac{cx^2\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+cx^2}}\right)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^4,x]
```

```
[Out] -1/2*(Sqrt[x^2*(b + c*x^2)]*(1 + (c*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(
Sqrt[b]*Sqrt[b + c*x^2])))/x^3
```

Maple [A]
time = 0.10, size = 85, normalized size = 1.52

method	result	size
risch	$-\frac{\sqrt{x^2\left(c\,x^2+b\right)}}{2x^3}-\frac{c\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)\sqrt{x^2\left(c\,x^2+b\right)}}{2\sqrt{b}\,x\sqrt{c\,x^2+b}}$	74
default	$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c\,x^2-\sqrt{c\,x^2+b}\,c\,x^2+\left(c\,x^2+b\right)^{\frac{3}{2}}\right)}{2x^3\sqrt{c\,x^2+b}\,b}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c*x^2
-(c*x^2+b)^(1/2)*c*x^2+(c*x^2+b)^(3/2))/x^3/(c*x^2+b)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^4, x)
```

Fricas [A]
time = 0.37, size = 134, normalized size = 2.39

$$\left[\frac{\sqrt{b}\,cx^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\,b}{4\,bx^3}, \frac{\sqrt{-b}\,cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)-\sqrt{cx^4+bx^2}\,b}{2\,bx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b*x^3), 1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c*x^
4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*b)/(b*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**4, x)
```

Giac [A]
time = 8.09, size = 50, normalized size = 0.89

$$\frac{\frac{c^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}-\frac{\sqrt{cx^2+b}\,\mathrm{csgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(c^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - sqrt(c*x^2 + b)
*c*sgn(x)/x^2)/c
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^4,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^4, x)
```

3.236 $\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}+\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}$$

```
[Out] 1/8*c^2*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/4*(c*x^4+b*x^2)^(1
/2)/x^5-1/8*c*(c*x^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2033, 212}

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -1/4*Sqrt[b*x^2 + c*x^4]/x^5 - (c*Sqrt[b*x^2 + c*x^4])/(8*b*x^3) + (c^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}+\frac{1}{4}c\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}-\frac{c^2\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}+\frac{c^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}+\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 91, normalized size = 1.08

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-\sqrt{b}\,\sqrt{b+cx^2}\left(2b+cx^2\right)+c^2x^4\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{8b^{3/2}x^5\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-(Sqrt[b]*Sqrt[b + c*x^2]*(2*b + c*x^2)) + c^2*x^4*
ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(8*b^(3/2)*x^5*Sqrt[b + c*x^2])
```

Maple [A]
time = 0.08, size = 106, normalized size = 1.26

method	result	size

risch	$-\frac{(c\,x^{2}+2b)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{8x^{5}b}+\frac{c^{2}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{8b^{\frac{3}{2}}x\sqrt{c\,x^{2}+b}}$	88
default	$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\left(\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c^{2}x^{4}-\sqrt{c\,x^{2}+b}\,c^{2}x^{4}+\left(c\,x^{2}+b\right)^{\frac{3}{2}}c\,x^{2}-2\left(c\,x^{2}+b\right)^{\frac{3}{2}}b\right)}{8x^{5}\sqrt{c\,x^{2}+b}\,b^{2}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c^2*x^
4-(c*x^2+b)^(1/2)*c^2*x^4+(c*x^2+b)^(3/2)*c*x^2-2*(c*x^2+b)^(3/2)*b)/x^5/(c
*x^2+b)^(1/2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^6, x)
```

Fricas [A]
time = 0.37, size = 159, normalized size = 1.89

$$\left[\frac{\sqrt{b}\,c^{2}x^{5}\log\left(-\frac{cx^{3}+2\,bx+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)-2\,\sqrt{cx^{4}+bx^{2}}\,(bcx^{2}+2\,b^{2})}{16\,b^{2}x^{5}},-\frac{\sqrt{-b}\,c^{2}x^{5}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)+\sqrt{cx^{4}+bx^{2}}\,(bcx^{2}+2\,b^{2})}{8\,b^{2}x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))
/x^3) - 2*sqrt(c*x^4 + b*x^2)*(b*c*x^2 + 2*b^2))/(b^2*x^5), -1/8*(sqrt(-b)*
c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
*x^2)*(b*c*x^2 + 2*b^2))/(b^2*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^{2}\left(b+cx^{2}\right)}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**6, x)
```

Giac [A]
time = 6.57, size = 78, normalized size = 0.93

$$-\frac{\frac{c^3 \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b} + \frac{(cx^2+b)^{\frac{3}{2}}c^3\mathrm{sgn}(x)+\sqrt{cx^2+b}\,bc^3\mathrm{sgn}(x)}{bc^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/8*(c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + ((c*x^2 +
b)^(3/2)*c^3*sgn(x) + sqrt(c*x^2 + b)*b*c^3*sgn(x))/(b*c^2*x^4))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^6,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^6, x)
```

3.237 $\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}$$

```
[Out] -1/16*c^3*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/6*(c*x^4+b*x^2)^
(1/2)/x^7-1/24*c*(c*x^4+b*x^2)^(1/2)/b/x^5+1/16*c^2*(c*x^4+b*x^2)^(1/2)/b^2
/x^3
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2033, 212}

$$-\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

```
[Out] -1/6*Sqrt[b*x^2 + c*x^4]/x^7 - (c*Sqrt[b*x^2 + c*x^4])/(24*b*x^5) + (c^2*Sq
rt[b*x^2 + c*x^4])/(16*b^2*x^3) - (c^3*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]])/(16*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{6x^7}+\frac{1}{6}c\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}-\frac{c^2\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{8b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}+\frac{c^3\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{16b^2}\\ &= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+c}}\right)}{16b^2}\\ &= -\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 103, normalized size = 0.92

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{b}\sqrt{b+cx^2}\left(8b^2+2bcx^2-3c^2x^4\right)+3c^3x^6\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{48b^{5/2}x^7\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

```
[Out] -1/48*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(8*b^2 + 2*b*c*x^2 -
3*c^2*x^4) + 3*c^3*x^6*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(b^(5/2)*x^7*Sqrt
[b + c*x^2])
```

Maple [A]
time = 0.12, size = 128, normalized size = 1.14

method	result
risch	$-\frac{\left(-3c^{2}x^{4}+2bc\,x^{2}+8b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{48x^{7}b^{2}}-\frac{c^{3}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{16b^{\frac{5}{2}}x\sqrt{c\,x^{2}+b}}$
default	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}\left(3\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c^{3}x^{6}-3\sqrt{c\,x^{2}+b}\,c^{3}x^{6}+3\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{2}x^{4}-6\left(c\,x^{2}+b\right)^{\frac{3}{2}}bc\,x^{2}+8\left(c\,x^{2}\right.\right.}{48x^{7}\sqrt{c\,x^{2}+b}\,b^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(c*x^4+b*x^2)^(1/2)*(3*b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c^
3*x^6-3*(c*x^2+b)^(1/2)*c^3*x^6+3*(c*x^2+b)^(3/2)*c^2*x^4-6*(c*x^2+b)^(3/2)
*b*c*x^2+8*(c*x^2+b)^(3/2)*b^2)/x^7/(c*x^2+b)^(1/2)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^8, x)
```

Fricas [A]
time = 0.35, size = 185, normalized size = 1.65

$$\left[\frac{3\sqrt{b}\,c^{3}x^{7}\log\left(-\frac{cx^{3}+2\,bx-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)+2\left(3\,bc^{2}x^{4}-2\,b^{2}cx^{2}-8\,b^{3}\right)\sqrt{cx^{4}+bx^{2}}}{96\,b^{3}x^{7}},\frac{3\sqrt{-b}\,c^{3}x^{7}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)+\left(3\,bc^{2}x^{4}-2\,b^{2}cx^{2}-8\,b^{3}\right)\sqrt{cx^{4}+bx^{2}}}{48\,b^{3}x^{7}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(b)*c^3*x^7*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*(3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^3*x
^7), 1/48*(3*sqrt(-b)*c^3*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 +
b*x)) + (3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^3*x^7)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^{2}\left(b+cx^{2}\right)}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**8,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**8, x)
```

Giac [A]
time = 4.61, size = 100, normalized size = 0.89

$$\frac{\frac{3\,c^4\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b^2}+\frac{3\,(cx^2+b)^{\frac{5}{2}}c^4\mathrm{sgn}(x)-8\,(cx^2+b)^{\frac{3}{2}}bc^4\mathrm{sgn}(x)-3\,\sqrt{cx^2+b}\,b^2c^4\mathrm{sgn}(x)}{b^2c^3x^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/48*(3*c^4*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^2) + (3*(c*
x^2 + b)^(5/2)*c^4*sgn(x) - 8*(c*x^2 + b)^(3/2)*b*c^4*sgn(x) - 3*sqrt(c*x^2
 + b)*b^2*c^4*sgn(x))/(b^2*c^3*x^6))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^8,x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^8, x)
```

3.238 $\int x^3(bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{3b^3(b+2cx^2)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)(bx^2+cx^4)^{3/2}}{32c^2}+\frac{(bx^2+cx^4)^{5/2}}{10c}-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}$$

```
[Out] -1/32*b*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^2+1/10*(c*x^4+b*x^2)^(5/2)/c-3/25
6*b^5*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+3/256*b^3*(2*c*x^2+b
)*(c*x^4+b*x^2)^(1/2)/c^3
```

Rubi [A]
time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.263$, Rules used = {2043, 654, 626, 634, 212}

$$-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{3b^3(b+2cx^2)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)(bx^2+cx^4)^{3/2}}{32c^2}+\frac{(bx^2+cx^4)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b^3*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - (b*(b + 2*c*x^2)*(b*x
^2 + c*x^4)^(3/2))/(32*c^2) + (b*x^2 + c*x^4)^(5/2)/(10*c) - (3*b^5*ArcTanh
[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int x^3\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$
$$=\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{b\text{Subst}\left(\int\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)}{4c}$$
$$=-\frac{b(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}+\frac{(3b^3)\,\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,\right.}{64c^2}$$
$$=\frac{3b^3(b+2cx^2)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{(3b^5)}{}$$
$$=\frac{3b^3(b+2cx^2)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{(3b^5)}{}$$
$$=\frac{3b^3(b+2cx^2)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{3b^5\,\text{ta}}{}$$

Mathematica [A]
time = 0.12, size = 124, normalized size = 1.00

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\left(15b^4-10b^3cx^2+8b^2c^2x^4+176bc^3x^6+128c^4x^8\right)+15b^5\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{1280c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(15*b^4 - 10*b^3*c*x^2 + 8*b^
2*c^2*x^4 + 176*b*c^3*x^6 + 128*c^4*x^8) + 15*b^5*Log[-(Sqrt[c]*x) + Sqrt[b
 + c*x^2]]))/(1280*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 142, normalized size = 1.15

method	result
risch	$\frac{\left(128c^{4}x^{8}+176b\,c^{3}x^{6}+8b^{2}c^{2}x^{4}-10b^{3}c\,x^{2}+15b^{4}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{1280c^{3}}-\frac{3b^{5}\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{256c^{\frac{7}{2}}x\sqrt{c\,x^{2}+b}}$
default	$\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(128x^{5}\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{5}{2}}-80c^{\frac{3}{2}}\left(c\,x^{2}+b\right)^{\frac{5}{2}}b\,x^{3}+40\sqrt{c}\,\left(c\,x^{2}+b\right)^{\frac{5}{2}}b^{2}x-10\sqrt{c}\,\left(c\,x^{2}+b\right)^{\frac{3}{2}}b^{3}x-15\sqrt{c}\,\sqrt{c\,x^{2}+b}\,b^{4}x-\right.}{1280x^{3}\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(128*x^5*(c*x^2+b)^(5/2)*c^(5/2)-80*c^(3/2)*(c*x
^2+b)^(5/2)*b*x^3+40*c^(1/2)*(c*x^2+b)^(5/2)*b^2*x-10*c^(1/2)*(c*x^2+b)^(3/
2)*b^3*x-15*c^(1/2)*(c*x^2+b)^(1/2)*b^4*x-15*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*
b^5)/x^3/(c*x^2+b)^(3/2)/c^(7/2)
```

Maxima [A]
time = 0.31, size = 142, normalized size = 1.15

$$\frac{3\,\sqrt{cx^{4}+bx^{2}}\,b^{3}x^{2}}{128\,c^{2}}-\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}bx^{2}}{16\,c}-\frac{3\,b^{5}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{512\,c^{\frac{7}{2}}}+\frac{3\,\sqrt{cx^{4}+bx^{2}}\,b^{4}}{256\,c^{3}}-\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}b^{2}}{32\,c^{2}}+\frac{\left(cx^{4}+bx^{2}\right)^{\frac{5}{2}}}{10\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/128*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^2 - 1/16*(c*x^4 + b*x^2)^(3/2)*b*x^2/c
- 3/512*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 3/25
6*sqrt(c*x^4 + b*x^2)*b^4/c^3 - 1/32*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 1/10*(
c*x^4 + b*x^2)^(5/2)/c
```

Fricas [A]
time = 0.35, size = 210, normalized size = 1.69

$$\left[\frac{15\,b^{5}\sqrt{c}\,\log\left(-2\,cx^{2}-b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\,\left(128\,c^{5}x^{8}+176\,bc^{4}x^{6}+8\,b^{2}c^{3}x^{4}-10\,b^{3}c^{2}x^{2}+15\,b^{4}c\right)\sqrt{cx^{4}+bx^{2}}}{2560\,c^{4}},\ \frac{15\,b^{5}\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)+\left(128\,c^{5}x^{8}+176\,bc^{4}x^{6}+8\,b^{2}c^{3}x^{4}-10\,b^{3}c^{2}x^{2}+15\,b^{4}c\right)\sqrt{cx^{4}+bx^{2}}}{1280\,c^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2560*(15*b^5*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^4 - 10*b^3*c^2*x^2 + 15*b^4*c
)*sqrt(c*x^4 + b*x^2))/c^4, 1/1280*(15*b^5*sqrt(-c)*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-c)/(c*x^2 + b)) + (128*c^5*x^8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^4 -
10*b^3*c^2*x^2 + 15*b^4*c)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 4.84, size = 115, normalized size = 0.93

$$\frac{3\,b^{5}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^{2}+b}\right|\right)\operatorname{sgn}(x)}{256\,c^{\frac{7}{2}}}-\frac{3\,b^{5}\log\left(|b|\right)\operatorname{sgn}(x)}{512\,c^{\frac{7}{2}}}+\frac{1}{1280}\left(2\left(4\left(2\left(8\,cx^{2}\operatorname{sgn}(x)+11\,b\operatorname{sgn}(x)\right)x^{2}+\frac{b^{2}\operatorname{sgn}(x)}{c}\right)x^{2}-\frac{5\,b^{3}\operatorname{sgn}(x)}{c^{2}}\right)x^{2}+\frac{15\,b^{4}\operatorname{sgn}(x)}{c^{3}}\right)\sqrt{cx^{2}+b}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/256*b^5*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(7/2) - 3/512*b^5
*log(abs(b))*sgn(x)/c^(7/2) + 1/1280*(2*(4*(2*(8*c*x^2*sgn(x) + 11*b*sgn(x)
)*x^2 + b^2*sgn(x)/c)*x^2 - 5*b^3*sgn(x)/c^2)*x^2 + 15*b^4*sgn(x)/c^3)*sqrt
(c*x^2 + b)*x
```

Mupad [B]
time = 4.35, size = 134, normalized size = 1.08

$$\frac{\left(c\,x^{4}+b\,x^{2}\right)^{5/2}}{10\,c}-\frac{b\left(\frac{x^{2}\left(c\,x^{4}+b\,x^{2}\right)^{3/2}}{4}-\frac{3\,b^{2}\left(\frac{\left(2\,c\,x^{2}+b\right)\sqrt{c\,x^{4}+b\,x^{2}}}{4\,c}-\frac{b^{2}\ln\left(\frac{c\,x^{2}+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}}\right)}{8\,c^{3/2}}\right)}{16\,c}+\frac{b\left(c\,x^{4}+b\,x^{2}\right)^{3/2}}{8\,c}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(5/2)/(10*c) - (b*((x^2*(b*x^2 + c*x^4)^(3/2))/4 - (3*b^2*(
((b + 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(4*c) - (b^2*log((b/2 + c*x^2)/c^(1/2
) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(3/2))))/(16*c) + (b*(b*x^2 + c*x^4)^(3/2)
)/(8*c)))/(4*c)
```

3.239 $\int x(bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=101

$$-\frac{3b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2}+\frac{(b+2cx^2)\,(bx^2+cx^4)^{3/2}}{16c}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}$$

```
[Out] 1/16*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c+3/128*b^4*arctanh(x^2*c^(1/2)/(c*x^4
+b*x^2)^(1/2))/c^(5/2)-3/128*b^2*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A]
time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2038, 626, 634, 212}

$$\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}-\frac{3b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2}+\frac{(b+2cx^2)\,(bx^2+cx^4)^{3/2}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^2) + ((b + 2*c*x^2)*(b*x^
2 + c*x^4)^(3/2))/(16*c) + (3*b^4*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]
])/(128*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int x\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$=\frac{(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{16c}-\frac{(3b^2)\,\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{32c}$$

$$=-\frac{3b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2}+\frac{(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{(3b^4)\,\text{Subst}\left(\int\frac{1}{\sqrt{bx+}}\right.}{256c^2}$$

$$=-\frac{3b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2}+\frac{(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{(3b^4)\,\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx\right.}{128c}$$

$$=-\frac{3b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2}+\frac{(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+c}}\right.}{128c^{5/2}}$$

Mathematica [A]
time = 0.09, size = 113, normalized size = 1.12

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\left(-3b^3+2b^2cx^2+24bc^2x^4+16c^3x^6\right)-3b^4\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{128c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(-3*b^3 + 2*b^2*c*x^2 + 24*b*
c^2*x^4 + 16*c^3*x^6) - 3*b^4*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(128*c^
(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 122, normalized size = 1.21

method	result
risch	$-\frac{\left(-16c^3x^6-24b\,c^2x^4-2b^2c\,x^2+3b^3\right)\sqrt{x^2\left(c\,x^2+b\right)}}{128c^2}+\frac{3b^4\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)\sqrt{x^2\left(c\,x^2+b\right)}}{128c^{\frac{5}{2}}x\sqrt{c\,x^2+b}}$

default	$\frac{(cx^4+bx^2)^{\frac{3}{2}}\left(16x^3(cx^2+b)^{\frac{5}{2}}c^{\frac{3}{2}}-8\sqrt{c}\,(cx^2+b)^{\frac{5}{2}}bx+2\sqrt{c}\,(cx^2+b)^{\frac{3}{2}}b^2x+3\sqrt{c}\,\sqrt{cx^2+b}\,b^3x+3\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right.\right.}{128x^3(cx^2+b)^{\frac{3}{2}}c^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(c*x^4+b*x^2)^(3/2)*(16*x^3*(c*x^2+b)^(5/2)*c^(3/2)-8*c^(1/2)*(c*x^2+
b)^(5/2)*b*x+2*c^(1/2)*(c*x^2+b)^(3/2)*b^2*x+3*c^(1/2)*(c*x^2+b)^(1/2)*b^3*
x+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^4)/x^3/(c*x^2+b)^(3/2)/c^(5/2)
```

Maxima [A]
time = 0.30, size = 118, normalized size = 1.17

$$\frac{1}{8}\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2-\frac{3\sqrt{cx^4+bx^2}\,b^2x^2}{64\,c}+\frac{3\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{256\,c^{\frac{5}{2}}}-\frac{3\sqrt{cx^4+bx^2}\,b^3}{128\,c^2}+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}b}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^4 + b*x^2)^(3/2)*x^2 - 3/64*sqrt(c*x^4 + b*x^2)*b^2*x^2/c + 3/256*
b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2) - 3/128*sqrt(c
*x^4 + b*x^2)*b^3/c^2 + 1/16*(c*x^4 + b*x^2)^(3/2)*b/c
```

Fricas [A]
time = 0.37, size = 189, normalized size = 1.87

$$\left[\frac{3\,b^4\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,(16\,c^4x^6+24\,bc^3x^4+2\,b^2c^2x^2-3\,b^3c)\sqrt{cx^4+bx^2}}{256\,c^3},-\frac{3\,b^4\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-(16\,c^4x^6+24\,bc^3x^4+2\,b^2c^2x^2-3\,b^3c)\sqrt{cx^4+bx^2}}{128\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*b^4*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*(16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2*c^2*x^2 - 3*b^3*c)*sqrt(c*x^4 + b*x^2))
/c^3, -1/128*(3*b^4*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b
)) - (16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2*c^2*x^2 - 3*b^3*c)*sqrt(c*x^4 + b*x
^2))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 3.04, size = 99, normalized size = 0.98

$$-\frac{3\,b^4\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\operatorname{sgn}(x)}{128\,c^{\frac{5}{2}}}+\frac{3\,b^4\log\left(|b|\right)\operatorname{sgn}(x)}{256\,c^{\frac{5}{2}}}+\frac{1}{128}\left(2\left(4\left(2\,cx^2\operatorname{sgn}(x)+3\,b\operatorname{sgn}(x)\right)x^2+\frac{b^2\operatorname{sgn}(x)}{c}\right)x^2-\frac{3\,b^3\operatorname{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -3/128*b^4*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(5/2) + 3/256*b^
4*log(abs(b))*sgn(x)/c^(5/2) + 1/128*(2*(4*(2*c*x^2*sgn(x) + 3*b*sgn(x))*x^
2 + b^2*sgn(x)/c)*x^2 - 3*b^3*sgn(x)/c^2)*sqrt(c*x^2 + b)*x
```

Mupad [B]
time = 4.44, size = 99, normalized size = 0.98

$$\frac{\left(c\,x^4+b\,x^2\right)^{3/2}\,\left(c\,x^2+\frac{b}{2}\right)}{8\,c}-\frac{3\,b^2\,\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{32\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(3/2)*(b/2 + c*x^2))/(8*c) - (3*b^2*((b/(4*c) + x^2/2)*(b*
x^2 + c*x^4)^(1/2) - (b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)
))/(8*c^(3/2))))/(32*c)
```

3.240 $\int \frac{(bx^2+cx^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=88

$$\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}$$

```
[Out] 1/6*(c*x^4+b*x^2)^(3/2)-1/16*b^3*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c
^(3/2)+1/16*b*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 678, 626, 634, 212}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + (b*x^2 + c*x^4)^(3/2)/6 - (b
^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^{3/2}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{6}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{4}b\text{Subst}\left(\int \sqrt{bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{32c}\\
&= \frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c}\\
&= \frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}\left(bx^2+cx^4\right)^{3/2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 102, normalized size = 1.16

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}\,x\sqrt{b+cx^2}\,(3b^2+14bcx^2+8c^2x^4)+3b^3\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{48c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(3*b^2 + 14*b*c*x^2 + 8*c^2*x
^4) + 3*b^3*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(48*c^(3/2)*Sqrt[x^2*(b +
 c*x^2)])
```

Maple [A]
time = 0.10, size = 102, normalized size = 1.16

method	result	size
risch	$\frac{(8c^2x^4+14bc\,x^2+3b^2)\sqrt{x^2\,(c\,x^2+b)}}{48c}-\frac{b^3\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)\sqrt{x^2\,(c\,x^2+b)}}{16c^{\frac{3}{2}}x\sqrt{c\,x^2+b}}$	90
default	$\frac{(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(8x(c\,x^2+b)^{\frac{5}{2}}\sqrt{c}-2\sqrt{c}\,(c\,x^2+b)^{\frac{3}{2}}bx-3\sqrt{c}\,\sqrt{c\,x^2+b}\,b^2x-3\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)b^3\right)}{48x^3(c\,x^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(8*x*(c*x^2+b)^(5/2)*c^(1/2)-2*c^(1/2)*(c*x^2+b)^(
3/2)*b*x-3*c^(1/2)*(c*x^2+b)^(1/2)*b^2*x-3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^
3)/x^3/(c*x^2+b)^(3/2)/c^(3/2)
```

Maxima [A]
time = 0.31, size = 91, normalized size = 1.03

$$\frac{1}{8}\sqrt{cx^4+bx^2}\,bx^2-\frac{b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{32\,c^{\frac{3}{2}}}+\frac{1}{6}\left(cx^4+bx^2\right)^{\frac{3}{2}}+\frac{\sqrt{cx^4+bx^2}\,b^2}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2)*b*x^2 - 1/32*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b
*x^2)*sqrt(c))/c^(3/2) + 1/6*(c*x^4 + b*x^2)^(3/2) + 1/16*sqrt(c*x^4 + b*x^
2)*b^2/c
```

Fricas [A]
time = 0.38, size = 166, normalized size = 1.89

$$\left[\frac{3\,b^3\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,(8\,c^3x^4+14\,bc^2x^2+3\,b^2c)\sqrt{cx^4+bx^2}}{96\,c^2},\frac{3\,b^3\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+(8\,c^3x^4+14\,bc^2x^2+3\,b^2c)\sqrt{cx^4+bx^2}}{48\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(3*b^3*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^2, 1/48*(3*b^3*
sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*c^3*x^4 + 14
*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x, x)
```

Giac [A]
time = 3.71, size = 84, normalized size = 0.95

$$\frac{b^{3}\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^{2}+b}\right|\right)\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}-\frac{b^{3}\log\left(|b|\right)\mathrm{sgn}(x)}{32\,c^{\frac{3}{2}}}+\frac{1}{48}\left(2\left(4\,cx^{2}\mathrm{sgn}(x)+7\,b\mathrm{sgn}(x)\right)x^{2}+\frac{3\,b^{2}\mathrm{sgn}(x)}{c}\right)\sqrt{cx^{2}+b}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/16*b^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(3/2) - 1/32*b^3*l
og(abs(b))*sgn(x)/c^(3/2) + 1/48*(2*(4*c*x^2*sgn(x) + 7*b*sgn(x))*x^2 + 3*b
^2*sgn(x)/c)*sqrt(c*x^2 + b)*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^{4}+b\,x^{2}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x, x)
```

3.241 $\int \frac{(bx^2+cx^4)^{3/2}}{x^3} dx$

Optimal. Leaf size=80

$$\frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}$$

```
[Out] 1/4*(c*x^4+b*x^2)^(3/2)/x^2+3/8*b^2*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2)
)/c^(1/2)+3/8*b*(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2043, 678, 634, 212}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}+\frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (3*b*Sqrt[b*x^2 + c*x^4])/8 + (b*x^2 + c*x^4)^(3/2)/(4*x^2) + (3*b^2*ArcTan
h[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^2}\,dx, x, x^2\right)\\ &= \frac{(bx^2+cx^4)^{3/2}}{4x^2}+\frac{1}{8}(3b)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)\\ &= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}+\frac{1}{16}\left(3b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\ &= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}+\frac{1}{8}\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 74, normalized size = 0.92

$$\frac{1}{8}\sqrt{x^2\left(b+cx^2\right)}\left(5b+2cx^2-\frac{3b^2\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)}{\sqrt{c}\,x\sqrt{b+cx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(5*b + 2*c*x^2 - (3*b^2*Log[-(Sqrt[c]*x) + Sqrt[b +
c*x^2]])/(Sqrt[c]*x*Sqrt[b + c*x^2])))/8
```

Maple [A]
time = 0.10, size = 84, normalized size = 1.05

method	result	size

risch	$\frac{(2c\,x^2+5b)\sqrt{x^2\,(c\,x^2+b)}}{8}+\frac{3b^2\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)\sqrt{x^2\,(c\,x^2+b)}}{8\sqrt{c}\,x\sqrt{c\,x^2+b}}$	76
default	$\frac{(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(2x(c\,x^2+b)^{\frac{3}{2}}\sqrt{c}+3\sqrt{c}\,\sqrt{c\,x^2+b}\,bx+3\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)b^2\right)}{8x^3(c\,x^2+b)^{\frac{3}{2}}\sqrt{c}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(3/2)*(2*x*(c*x^2+b)^(3/2)*c^(1/2)+3*c^(1/2)*(c*x^2+b)^(1
/2)*b*x+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2)/x^3/(c*x^2+b)^(3/2)/c^(1/2)
```

Maxima [A]
time = 0.29, size = 70, normalized size = 0.88

$$\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{16\,\sqrt{c}}+\frac{3}{8}\,\sqrt{cx^4+bx^2}\,b+\frac{(cx^4+bx^2)^{\frac{3}{2}}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 3/8*sqr
t(c*x^4 + b*x^2)*b + 1/4*(c*x^4 + b*x^2)^(3/2)/x^2
```

Fricas [A]
time = 0.34, size = 145, normalized size = 1.81

$$\left[\frac{3\,b^2\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\,(2\,c^2x^2+5\,bc)}{16\,c},-\frac{3\,b^2\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}\,(2\,c^2x^2+5\,bc)}{8\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^2*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 5*b*c))/c, -1/8*(3*b^2*sqrt(-c)*arctan(sqr
t(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 5
*b*c))/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**3, x)
```

Giac [A]
time = 4.34, size = 68, normalized size = 0.85

$$-\frac{3\,b^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{8\,\sqrt{c}}+\frac{3\,b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,\sqrt{c}}+\frac{1}{8}\left(2\,cx^2\mathrm{sgn}(x)+5\,b\mathrm{sgn}(x)\right)\sqrt{cx^2+b}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -3/8*b^2*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/sqrt(c) + 3/16*b^2*l
og(abs(b))*sgn(x)/sqrt(c) + 1/8*(2*c*x^2*sgn(x) + 5*b*sgn(x))*sqrt(c*x^2 +
b)*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^3,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^3, x)
```

3.242 $\int \frac{(bx^2+cx^4)^{3/2}}{x^5} dx$

Optimal. Leaf size=76

$$\frac{3}{2}c\sqrt{bx^2+cx^4}-\frac{(bx^2+cx^4)^{3/2}}{x^4}+\frac{3}{2}b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -(c*x^4+b*x^2)^(3/2)/x^4+3/2*b*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))*c^(
1/2)+3/2*c*(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 676, 678, 634, 212}

$$-\frac{(bx^2+cx^4)^{3/2}}{x^4}+\frac{3}{2}c\sqrt{bx^2+cx^4}+\frac{3}{2}b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/2 - (b*x^2 + c*x^4)^(3/2)/x^4 + (3*b*Sqrt[c]*ArcT
anh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^{3/2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(bx^2+cx^4)^{3/2}}{x^4} + \frac{1}{2}(3c)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4} - \frac{(bx^2+cx^4)^{3/2}}{x^4} + \frac{1}{4}(3bc)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4} - \frac{(bx^2+cx^4)^{3/2}}{x^4} + \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{3}{2}c\sqrt{bx^2+cx^4} - \frac{(bx^2+cx^4)^{3/2}}{x^4} + \frac{3}{2}b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 80, normalized size = 1.05

$$\frac{-2b^2-bcx^2+c^2x^4-3b\sqrt{c}\,x\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)}{2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (-2*b^2 - b*c*x^2 + c^2*x^4 - 3*b*Sqrt[c]*x*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x
) + Sqrt[b + c*x^2]])/(2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 107, normalized size = 1.41

method	result	size
risch	$-\frac{\left(-c\,x^{2}+2b\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{2x^{2}}+\frac{3b\sqrt{c}\,\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{2x\sqrt{c\,x^{2}+b}}$	77
default	$\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(2c^{\frac{3}{2}}\left(c\,x^{2}+b\right)^{\frac{3}{2}}x^{2}+3c^{\frac{3}{2}}\sqrt{c\,x^{2}+b}\,b\,x^{2}-2\left(c\,x^{2}+b\right)^{\frac{5}{2}}\sqrt{c}+3\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)b^{2}c x\right)}{2x^{4}\left(c\,x^{2}+b\right)^{\frac{3}{2}}b\sqrt{c}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^4+b*x^2)^(3/2)*(2*c^(3/2)*(c*x^2+b)^(3/2)*x^2+3*c^(3/2)*(c*x^2+b)^
(1/2)*b*x^2-2*(c*x^2+b)^(5/2)*c^(1/2)+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2*c
*x)/x^4/(c*x^2+b)^(3/2)/b/c^(1/2)
```

Maxima [A]
time = 0.30, size = 71, normalized size = 0.93

$$\frac{3}{4}\,b\sqrt{c}\,\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-\frac{3\,\sqrt{cx^{4}+bx^{2}}\,b}{2\,x^{2}}+\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{2\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 3/4*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 3/2*sqrt(c
*x^4 + b*x^2)*b/x^2 + 1/2*(c*x^4 + b*x^2)^(3/2)/x^4
```

Fricas [A]
time = 0.32, size = 139, normalized size = 1.83

$$\left[\frac{3\,b\sqrt{c}\,x^{2}\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\,\sqrt{cx^{4}+bx^{2}}\,(cx^{2}-2\,b)}{4\,x^{2}},-\frac{3\,b\sqrt{-c}\,x^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)-\sqrt{cx^{4}+bx^{2}}\,(cx^{2}-2\,b)}{2\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/4*(3*b*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b))/x^2, -1/2*(3*b*sqrt(-c)*x^2*arctan(sqrt
(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b))/
x^2]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**5, x)
```

Giac [**A**]
time = 3.52, size = 79, normalized size = 1.04

$$\frac{1}{2}\sqrt{cx^2+b}\,cx\mathrm{sgn}(x)-\frac{3}{4}b\sqrt{c}\,\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sgn}(x)+\frac{2\,b^2\sqrt{c}\,\mathrm{sgn}(x)}{\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*c*x*sgn(x) - 3/4*b*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2
+ b))^2)*sgn(x) + 2*b^2*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^5,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^5, x)
```

3.243 $\int \frac{(bx^2+cx^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=75

$$-\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{(bx^2+cx^4)^{3/2}}{3x^6}+c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -1/3*(c*x^4+b*x^2)^(3/2)/x^6+c^(3/2)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2
))-c*(c*x^4+b*x^2)^(1/2)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2043, 676, 634, 212}

$$c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{(bx^2+cx^4)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -((c*Sqrt[b*x^2 + c*x^4])/x^2) - (b*x^2 + c*x^4)^(3/2)/(3*x^6) + c^(3/2)*Ar
cTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^{3/2}}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^4}\,dx,x,x^2\right)\\ &= -\frac{(bx^2+cx^4)^{3/2}}{3x^6}+\frac{1}{2}c\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx,x,x^2\right)\\ &= -\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{(bx^2+cx^4)^{3/2}}{3x^6}+\frac{1}{2}c^2\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\ &= -\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{(bx^2+cx^4)^{3/2}}{3x^6}+c^2\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\ &= -\frac{c\sqrt{bx^2+cx^4}}{x^2}-\frac{(bx^2+cx^4)^{3/2}}{3x^6}+c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 85, normalized size = 1.13

$$-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}\,(b+4cx^2)+3c^{3/2}x^3\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{3x^4\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -1/3*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(b + 4*c*x^2) + 3*c^(3/2)*x^3*
Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(x^4*Sqrt[b + c*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(63) = 126.
time = 0.10, size = 129, normalized size = 1.72

method	result
risch	$-\frac{(4c\,x^2+b)\sqrt{x^2\,(c\,x^2+b)}}{3x^4}+\frac{c^{\frac{3}{2}}\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)\sqrt{x^2\,(c\,x^2+b)}}{x\sqrt{c\,x^2+b}}$

default	$\frac{(c\,x^{4}+b\,x^{2})^{\frac{3}{2}}\left(2c^{\frac{5}{2}}\,(c\,x^{2}+b)^{\frac{3}{2}}\,x^{4}+3c^{\frac{5}{2}}\,\sqrt{c\,x^{2}+b}\,\,b\,x^{4}-2c^{\frac{3}{2}}\,(c\,x^{2}+b)^{\frac{5}{2}}\,x^{2}+3\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)b^{2}c^{2}x^{3}-(c\,x^{2}+b)^{\frac{5}{2}}\,b\sqrt{c}\right.}{3x^{6}(c\,x^{2}+b)^{\frac{3}{2}}\,b^{2}\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x^4+b*x^2)^(3/2)*(2*c^(5/2)*(c*x^2+b)^(3/2)*x^4+3*c^(5/2)*(c*x^2+b)^
(1/2)*b*x^4-2*c^(3/2)*(c*x^2+b)^(5/2)*x^2+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b
^2*c^2*x^3-(c*x^2+b)^(5/2)*b*c^(1/2))/x^6/(c*x^2+b)^(3/2)/b^2/c^(1/2)
```

Maxima [A]
time = 0.29, size = 89, normalized size = 1.19

$$\frac{1}{2}\,c^{\frac{3}{2}}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-\frac{7\,\sqrt{cx^{4}+bx^{2}}\,c}{6\,x^{2}}-\frac{\sqrt{cx^{4}+bx^{2}}\,b}{6\,x^{4}}-\frac{(cx^{4}+bx^{2})^{\frac{3}{2}}}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7/6*sqrt(c*x
^4 + b*x^2)*c/x^2 - 1/6*sqrt(c*x^4 + b*x^2)*b/x^4 - 1/6*(c*x^4 + b*x^2)^(3/
2)/x^6
```

Fricas [A]
time = 0.37, size = 135, normalized size = 1.80

$$\left[\frac{3\,c^{\frac{3}{2}}x^{4}\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)-2\,\sqrt{cx^{4}+bx^{2}}\,(4\,cx^{2}+b)}{6\,x^{4}},-\frac{3\,\sqrt{-c}\,cx^{4}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)+\sqrt{cx^{4}+bx^{2}}\,(4\,cx^{2}+b)}{3\,x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/6*(3*c^(3/2)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*s
qrt(c*x^4 + b*x^2)*(4*c*x^2 + b))/x^4, -1/3*(3*sqrt(-c)*c*x^4*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*(4*c*x^2 + b))/x^
4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{2}(b+cx^{2})\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**7, x)
```

Giac [A]
time = 4.35, size = 122, normalized size = 1.63

$$-\frac{1}{2}\,c^{\frac{3}{2}}\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2\right)\operatorname{sgn}(x)+\frac{4\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4bc^{\frac{3}{2}}\operatorname{sgn}(x)-3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2b^2c^{\frac{3}{2}}\operatorname{sgn}(x)+2\,b^3c^{\frac{3}{2}}\operatorname{sgn}(x)\right)}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 4/3*(3*(sqrt(c)*
x - sqrt(c*x^2 + b))^4*b*c^(3/2)*sgn(x) - 3*(sqrt(c)*x - sqrt(c*x^2 + b))^2
*b^2*c^(3/2)*sgn(x) + 2*b^3*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^
2 - b)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^7,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^7, x)
```

3.244 $\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=25

$$-\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

```
[Out] -1/5*(c*x^4+b*x^2)^(5/2)/b/x^10
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2039}

$$-\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/5*(b*x^2 + c*x^4)^(5/2)/(b*x^10)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

Mathematica [A]
time = 0.06, size = 25, normalized size = 1.00

$$-\frac{(x^2(b+cx^2))^{5/2}}{5bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/5*(x^2*(b + c*x^2))^(5/2)/(b*x^10)
```

Maple [A]
time = 0.10, size = 29, normalized size = 1.16

method	result	size
gosper	$-\frac{(c x^2+b)(c x^4+b x^2)^{\frac{3}{2}}}{5x^8 b}$	29
default	$-\frac{(c x^2+b)(c x^4+b x^2)^{\frac{3}{2}}}{5x^8 b}$	29
trager	$-\frac{(c^2 x^4+2bc x^2+b^2)\sqrt{c x^4+b x^2}}{5b x^6}$	40
risch	$-\frac{\sqrt{x^2 (c x^2+b)}\,(c^2 x^4+2bc x^2+b^2)}{5x^6 b}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^8*(c*x^2+b)/b*(c*x^4+b*x^2)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(21) = 42.
time = 0.29, size = 81, normalized size = 3.24

$$-\frac{\sqrt{cx^4+bx^2}\,c^2}{5\,bx^2}+\frac{\sqrt{cx^4+bx^2}\,c}{10\,x^4}+\frac{3\,\sqrt{cx^4+bx^2}\,b}{10\,x^6}-\frac{(cx^4+bx^2)^{\frac{3}{2}}}{2\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) + 1/10*sqrt(c*x^4 + b*x^2)*c/x^4 + 3/1
0*sqrt(c*x^4 + b*x^2)*b/x^6 - 1/2*(c*x^4 + b*x^2)^(3/2)/x^8
```

Fricas [A]
time = 0.32, size = 39, normalized size = 1.56

$$-\frac{(c^2x^4+2\,bcx^2+b^2)\sqrt{cx^4+bx^2}}{5\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(b*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**9, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(21) = 42$.
time = 4.34, size = 92, normalized size = 3.68

$$\frac{2\left(5\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^8c^{\frac{5}{2}}\mathrm{sgn}(x)+10\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4b^2c^{\frac{5}{2}}\mathrm{sgn}(x)+b^4c^{\frac{5}{2}}\mathrm{sgn}(x)\right)}{5\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(5/2)*sgn(x) + 10*(sqrt(c)*x - sqr
t(c*x^2 + b))^4*b^2*c^(5/2)*sgn(x) + b^4*c^(5/2)*sgn(x))/((sqrt(c)*x - sqrt
(c*x^2 + b))^2 - b)^5
```

Mupad [B]
time = 4.38, size = 30, normalized size = 1.20

$$-\frac{{\left(c\,x^2+b\right)}^2\,\sqrt{c\,x^4+b\,x^2}}{5\,b\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^9,x)
```

```
[Out] -((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2))/(5*b*x^6)
```

3.245 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=52

$$-\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}+\frac{2c(bx^2+cx^4)^{5/2}}{35b^2x^{10}}$$

```
[Out] -1/7*(c*x^4+b*x^2)^(5/2)/b/x^12+2/35*c*(c*x^4+b*x^2)^(5/2)/b^2/x^10
```

Rubi [A]

time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2039}

$$\frac{2c(bx^2+cx^4)^{5/2}}{35b^2x^{10}}-\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] -1/7*(b*x^2 + c*x^4)^(5/2)/(b*x^12) + (2*c*(b*x^2 + c*x^4)^(5/2))/(35*b^2*x
^10)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx &= -\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}-\frac{(2c)\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx}{7b}\\ &= -\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}+\frac{2c(bx^2+cx^4)^{5/2}}{35b^2x^{10}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 35, normalized size = 0.67

$$\frac{(x^2(b+cx^2))^{5/2}(-5b+2cx^2)}{35b^2x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^11,x]
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*b + 2*c*x^2))/(35*b^2*x^12)
```

Maple [A]
time = 0.09, size = 39, normalized size = 0.75

method	result	size
gosper	$-\frac{(c\,x^2+b)\,(-2c\,x^2+5b)\,(c\,x^4+b\,x^2)^{\frac{3}{2}}}{35x^{10}b^2}$	39
default	$-\frac{(c\,x^2+b)\,(-2c\,x^2+5b)\,(c\,x^4+b\,x^2)^{\frac{3}{2}}}{35x^{10}b^2}$	39
trager	$-\frac{(-2c^3x^6+b\,c^2x^4+8b^2c\,x^2+5b^3)\sqrt{c\,x^4+b\,x^2}}{35b^2x^8}$	53
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,(-2c^3x^6+b\,c^2x^4+8b^2c\,x^2+5b^3)}{35x^8b^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^11,x,method=_RETURNVERBOSE)
[Out] -1/35*(c*x^2+b)*(-2*c*x^2+5*b)*(c*x^4+b*x^2)^(3/2)/x^10/b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(44) = 88.
time = 0.29, size = 105, normalized size = 2.02

$$\frac{2\sqrt{cx^4+bx^2}\,c^3}{35\,b^2x^2}-\frac{\sqrt{cx^4+bx^2}\,c^2}{35\,bx^4}+\frac{3\sqrt{cx^4+bx^2}\,c}{140\,x^6}+\frac{3\sqrt{cx^4+bx^2}\,b}{28\,x^8}-\frac{(cx^4+bx^2)^{\frac{3}{2}}}{4\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="maxima")
[Out] 2/35*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^2) - 1/35*sqrt(c*x^4 + b*x^2)*c^2/(b*x^
4) + 3/140*sqrt(c*x^4 + b*x^2)*c/x^6 + 3/28*sqrt(c*x^4 + b*x^2)*b/x^8 - 1/4
*(c*x^4 + b*x^2)^(3/2)/x^10
```

Fricas [A]
time = 0.33, size = 53, normalized size = 1.02

$$\frac{(2\,c^3x^6-bc^2x^4-8\,b^2cx^2-5\,b^3)\sqrt{cx^4+bx^2}}{35\,b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/35*(2*c^3*x^6 - b*c^2*x^4 - 8*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4 + b*x^2)/(b^2
*x^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**11,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**11, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(44) = 88.
time = 4.20, size = 178, normalized size = 3.42

$$\frac{4\left(35\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}c^{\frac{7}{2}}\mathrm{sgn}(x)+35\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}bc^{\frac{7}{2}}\mathrm{sgn}(x)+70\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{6}b^2c^{\frac{7}{2}}\mathrm{sgn}(x)+14\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{4}b^3c^{\frac{7}{2}}\mathrm{sgn}(x)+7\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{2}b^4c^{\frac{7}{2}}\mathrm{sgn}(x)-b^5c^{\frac{7}{2}}\mathrm{sgn}(x)\right)}{35\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] 4/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(7/2)*sgn(x) + 35*(sqrt(c)*x -
sqrt(c*x^2 + b))^8*b*c^(7/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^
2*c^(7/2)*sgn(x) + 14*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^3*c^(7/2)*sgn(x) +
7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^4*c^(7/2)*sgn(x) - b^5*c^(7/2)*sgn(x))/
((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

Mupad [B]
time = 4.58, size = 87, normalized size = 1.67

$$\frac{2\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^2}-\frac{8\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,x^6}-\frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^4}-\frac{b\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^11,x)
```

```
[Out] (2*c^3*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^2) - (8*c*(b*x^2 + c*x^4)^(1/2))/(3
5*x^6) - (c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) - (b*(b*x^2 + c*x^4)^(1/2))
/(7*x^8)
```

3.246 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=80

$$-\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}}+\frac{4c(bx^2+cx^4)^{5/2}}{63b^2x^{12}}-\frac{8c^2(bx^2+cx^4)^{5/2}}{315b^3x^{10}}$$

```
[Out] -1/9*(c*x^4+b*x^2)^(5/2)/b/x^14+4/63*c*(c*x^4+b*x^2)^(5/2)/b^2/x^12-8/315*c
^2*(c*x^4+b*x^2)^(5/2)/b^3/x^10
```

Rubi [A]
time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2039}

$$-\frac{8c^2(bx^2+cx^4)^{5/2}}{315b^3x^{10}}+\frac{4c(bx^2+cx^4)^{5/2}}{63b^2x^{12}}-\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -1/9*(b*x^2 + c*x^4)^(5/2)/(b*x^14) + (4*c*(b*x^2 + c*x^4)^(5/2))/(63*b^2*x
^12) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(315*b^3*x^10)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}} - \frac{(4c)\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx}{9b}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}} + \frac{4c(bx^2+cx^4)^{5/2}}{63b^2x^{12}} + \frac{(8c^2)\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx}{63b^2}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}} + \frac{4c(bx^2+cx^4)^{5/2}}{63b^2x^{12}} - \frac{8c^2(bx^2+cx^4)^{5/2}}{315b^3x^{10}}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 0.58

$$\frac{(x^2(b+cx^2))^{5/2}\,(-35b^2+20bcx^2-8c^2x^4)}{315b^3x^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^13,x]
[Out] ((x^2*(b + c*x^2))^(5/2)*(-35*b^2 + 20*b*c*x^2 - 8*c^2*x^4))/(315*b^3*x^14)
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.62

method	result	size
gosper	$-\frac{(c\,x^2+b)(8c^2x^4-20bc\,x^2+35b^2)(c\,x^4+b\,x^2)^{\frac{3}{2}}}{315x^{12}b^3}$	50
default	$-\frac{(c\,x^2+b)(8c^2x^4-20bc\,x^2+35b^2)(c\,x^4+b\,x^2)^{\frac{3}{2}}}{315x^{12}b^3}$	50
trager	$-\frac{(8c^4x^8-4b\,c^3x^6+3b^2c^2x^4+50b^3c\,x^2+35b^4)\sqrt{c\,x^4+b\,x^2}}{315b^3x^{10}}$	65
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,(8c^4x^8-4b\,c^3x^6+3b^2c^2x^4+50b^3c\,x^2+35b^4)}{315x^{10}b^3}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^13,x,method=_RETURNVERBOSE)
[Out] -1/315*(c*x^2+b)*(8*c^2*x^4-20*b*c*x^2+35*b^2)*(c*x^4+b*x^2)^(3/2)/x^12/b^3
```

Maxima [A]
time = 0.29, size = 129, normalized size = 1.61

$$-\frac{8\sqrt{cx^4+bx^2}\,c^4}{315\,b^3x^2} + \frac{4\sqrt{cx^4+bx^2}\,c^3}{315\,b^2x^4} - \frac{\sqrt{cx^4+bx^2}\,c^2}{105\,bx^6} + \frac{\sqrt{cx^4+bx^2}\,c}{126\,x^8} + \frac{\sqrt{cx^4+bx^2}\,b}{18\,x^{10}} - \frac{(cx^4+bx^2)^{\frac{3}{2}}}{6\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] -8/315*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2) + 4/315*sqrt(c*x^4 + b*x^2)*c^3/(b
^2*x^4) - 1/105*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6) + 1/126*sqrt(c*x^4 + b*x^2)
*c/x^8 + 1/18*sqrt(c*x^4 + b*x^2)*b/x^10 - 1/6*(c*x^4 + b*x^2)^(3/2)/x^12
```

Fricas [A]
time = 0.35, size = 64, normalized size = 0.80

$$-\frac{(8\,c^4x^8-4\,bc^3x^6+3\,b^2c^2x^4+50\,b^3cx^2+35\,b^4)\sqrt{cx^4+bx^2}}{315\,b^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/315*(8*c^4*x^8 - 4*b*c^3*x^6 + 3*b^2*c^2*x^4 + 50*b^3*c*x^2 + 35*b^4)*sq
rt(c*x^4 + b*x^2)/(b^3*x^10)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**13,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**13, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(68) = 136.
time = 3.80, size = 206, normalized size = 2.58

$$\frac{16\left(210\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}c^{\frac{9}{2}}\mathrm{sgn}(x)+315\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{9}{2}}\mathrm{sgn}(x)+441\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{9}{2}}\mathrm{sgn}(x)+126\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{6}b^3c^{\frac{9}{2}}\mathrm{sgn}(x)+36\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{4}b^4c^{\frac{9}{2}}\mathrm{sgn}(x)-9\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{2}b^5c^{\frac{9}{2}}\mathrm{sgn}(x)+b^6c^{\frac{9}{2}}\mathrm{sgn}(x)\right)}{315\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(9/2)*sgn(x) + 315*(sqrt(c)*
x - sqrt(c*x^2 + b))^10*b*c^(9/2)*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + b)
)^8*b^2*c^(9/2)*sgn(x) + 126*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^3*c^(9/2)*sg
n(x) + 36*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(9/2)*sgn(x) - 9*(sqrt(c)*x
 - sqrt(c*x^2 + b))^2*b^5*c^(9/2)*sgn(x) + b^6*c^(9/2)*sgn(x))/((sqrt(c)*x
- sqrt(c*x^2 + b))^2 - b)^9
```

Mupad [B]
time = 4.72, size = 111, normalized size = 1.39

$$\frac{4\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^2\,x^4}-\frac{10\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,x^8}-\frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b\,x^6}-\frac{b\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{8\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^13,x)
```

```
[Out] (4*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (10*c*(b*x^2 + c*x^4)^(1/2))/
(63*x^8) - (c^2*(b*x^2 + c*x^4)^(1/2))/(105*b*x^6) - (b*(b*x^2 + c*x^4)^(1/
2))/(9*x^10) - (8*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2)
```

3.247 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=108

$$-\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}}+\frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}}-\frac{8c^2(bx^2+cx^4)^{5/2}}{231b^3x^{12}}+\frac{16c^3(bx^2+cx^4)^{5/2}}{1155b^4x^{10}}$$

```
[Out] -1/11*(c*x^4+b*x^2)^(5/2)/b/x^16+2/33*c*(c*x^4+b*x^2)^(5/2)/b^2/x^14-8/231*
c^2*(c*x^4+b*x^2)^(5/2)/b^3/x^12+16/1155*c^3*(c*x^4+b*x^2)^(5/2)/b^4/x^10
```

Rubi [A]
time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2039}

$$\frac{16c^3(bx^2+cx^4)^{5/2}}{1155b^4x^{10}}-\frac{8c^2(bx^2+cx^4)^{5/2}}{231b^3x^{12}}+\frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}}-\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

```
[Out] -1/11*(b*x^2 + c*x^4)^(5/2)/(b*x^16) + (2*c*(b*x^2 + c*x^4)^(5/2))/(33*b^2*
x^14) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(231*b^3*x^12) + (16*c^3*(b*x^2 + c*x
^4)^(5/2))/(1155*b^4*x^10)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} - \frac{(6c)\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx}{11b}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}} + \frac{(8c^2)\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx}{33b^2}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}} - \frac{8c^2(bx^2+cx^4)^{5/2}}{231b^3x^{12}} - \frac{(16c^3)\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx}{231b^3}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}} - \frac{8c^2(bx^2+cx^4)^{5/2}}{231b^3x^{12}} + \frac{16c^3(bx^2+cx^4)^{5/2}}{1155b^4x^{10}}$$

Mathematica [A]
time = 0.09, size = 57, normalized size = 0.53

$$\frac{(x^2(b+cx^2))^{5/2}\left(-105b^3+70b^2cx^2-40bc^2x^4+16c^3x^6\right)}{1155b^4x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-105*b^3 + 70*b^2*c*x^2 - 40*b*c^2*x^4 + 16*c^3*x
^6))/(1155*b^4*x^16)
```

Maple [A]
time = 0.11, size = 61, normalized size = 0.56

method	result	size
gosper	$-\frac{(c\,x^2+b)\left(-16c^3x^6+40b\,c^2x^4-70b^2c\,x^2+105b^3\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{1155x^{14}b^4}$	61
default	$-\frac{(c\,x^2+b)\left(-16c^3x^6+40b\,c^2x^4-70b^2c\,x^2+105b^3\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{1155x^{14}b^4}$	61
trager	$-\frac{\left(-16c^5x^{10}+8b\,c^4x^8-6b^2c^3x^6+5b^3c^2x^4+140b^4c\,x^2+105b^5\right)\sqrt{c\,x^4+b\,x^2}}{1155b^4x^{12}}$	76
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,\left(-16c^5x^{10}+8b\,c^4x^8-6b^2c^3x^6+5b^3c^2x^4+140b^4c\,x^2+105b^5\right)}{1155x^{12}b^4}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1155*(c*x^2+b)*(-16*c^3*x^6+40*b*c^2*x^4-70*b^2*c*x^2+105*b^3)*(c*x^4+b*
x^2)^(3/2)/x^14/b^4
```

Maxima [A]
time = 0.28, size = 153, normalized size = 1.42

$$\frac{16\sqrt{cx^4+bx^2}\,c^5}{1155\,b^4x^2}-\frac{8\sqrt{cx^4+bx^2}\,c^4}{1155\,b^3x^4}+\frac{2\sqrt{cx^4+bx^2}\,c^3}{385\,b^2x^6}-\frac{\sqrt{cx^4+bx^2}\,c^2}{231\,bx^8}+\frac{\sqrt{cx^4+bx^2}\,c}{264\,x^{10}}+\frac{3\sqrt{cx^4+bx^2}\,b}{88\,x^{12}}-\frac{(cx^4+bx^2)^{\frac{3}{2}}}{8\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] 16/1155*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 8/1155*sqrt(c*x^4 + b*x^2)*c^4/
(b^3*x^4) + 2/385*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 1/231*sqrt(c*x^4 + b*
x^2)*c^2/(b*x^8) + 1/264*sqrt(c*x^4 + b*x^2)*c/x^10 + 3/88*sqrt(c*x^4 + b*x
^2)*b/x^12 - 1/8*(c*x^4 + b*x^2)^(3/2)/x^14
```

Fricas [A]
time = 0.36, size = 75, normalized size = 0.69

$$\frac{(16\,c^5x^{10}-8\,bc^4x^8+6\,b^2c^3x^6-5\,b^3c^2x^4-140\,b^4cx^2-105\,b^5)\sqrt{cx^4+bx^2}}{1155\,b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] 1/1155*(16*c^5*x^10 - 8*b*c^4*x^8 + 6*b^2*c^3*x^6 - 5*b^3*c^2*x^4 - 140*b^4
*c*x^2 - 105*b^5)*sqrt(c*x^4 + b*x^2)/(b^4*x^12)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**15,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**15, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(92) = 184.
time = 3.94, size = 236, normalized size = 2.19

$$\frac{32\left(1155\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{14}c^{\frac{11}{2}}\mathrm{sgn}(x)+2079\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}bc^{\frac{11}{2}}\mathrm{sgn}(x)+2541\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}b^2c^{\frac{11}{2}}\mathrm{sgn}(x)+825\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}b^3c^{\frac{11}{2}}\mathrm{sgn}(x)+165\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{6}b^4c^{\frac{11}{2}}\mathrm{sgn}(x)-55\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{4}b^5c^{\frac{11}{2}}\mathrm{sgn}(x)+11\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{2}b^6c^{\frac{11}{2}}\mathrm{sgn}(x)-b^7c^{\frac{11}{2}}\mathrm{sgn}(x)\right)}{1155\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] 32/1155*(1155*(sqrt(c)*x - sqrt(c*x^2 + b))^14*c^(11/2)*sgn(x) + 2079*(sqrt
(c)*x - sqrt(c*x^2 + b))^12*b*c^(11/2)*sgn(x) + 2541*(sqrt(c)*x - sqrt(c*x^
2 + b))^10*b^2*c^(11/2)*sgn(x) + 825*(sqrt(c)*x - sqrt(c*x^2 + b))^8*b^3*c^
(11/2)*sgn(x) + 165*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^4*c^(11/2)*sgn(x) - 5
5*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^5*c^(11/2)*sgn(x) + 11*(sqrt(c)*x - sqr
t(c*x^2 + b))^2*b^6*c^(11/2)*sgn(x) - b^7*c^(11/2)*sgn(x))/((sqrt(c)*x - sq
rt(c*x^2 + b))^2 - b)^11
```

Mupad [B]
time = 4.99, size = 135, normalized size = 1.25

$$\frac{2\,c^3\,\sqrt{c\,x^4+b\,x^2}}{385\,b^2\,x^6}-\frac{4\,c\,\sqrt{c\,x^4+b\,x^2}}{33\,x^{10}}-\frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{231\,b\,x^8}-\frac{b\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}-\frac{8\,c^4\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^4}+\frac{16\,c^5\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^15,x)
```

```
[Out] (2*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (4*c*(b*x^2 + c*x^4)^(1/2))/(
33*x^10) - (c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*x^8) - (b*(b*x^2 + c*x^4)^(1/
2))/(11*x^12) - (8*c^4*(b*x^2 + c*x^4)^(1/2))/(1155*b^3*x^4) + (16*c^5*(b*x
^2 + c*x^4)^(1/2))/(1155*b^4*x^2)
```

3.248 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=136

$$-\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}}+\frac{8c(bx^2+cx^4)^{5/2}}{143b^2x^{16}}-\frac{16c^2(bx^2+cx^4)^{5/2}}{429b^3x^{14}}+\frac{64c^3(bx^2+cx^4)^{5/2}}{3003b^4x^{12}}-\frac{128c^4(bx^2+cx^4)^{5/2}}{15015b^5x^{10}}$$

```
[Out] -1/13*(c*x^4+b*x^2)^(5/2)/b/x^18+8/143*c*(c*x^4+b*x^2)^(5/2)/b^2/x^16-16/42
9*c^2*(c*x^4+b*x^2)^(5/2)/b^3/x^14+64/3003*c^3*(c*x^4+b*x^2)^(5/2)/b^4/x^12
-128/15015*c^4*(c*x^4+b*x^2)^(5/2)/b^5/x^10
```

Rubi [A]
time = 0.15, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2039}

$$-\frac{128c^4(bx^2+cx^4)^{5/2}}{15015b^5x^{10}}+\frac{64c^3(bx^2+cx^4)^{5/2}}{3003b^4x^{12}}-\frac{16c^2(bx^2+cx^4)^{5/2}}{429b^3x^{14}}+\frac{8c(bx^2+cx^4)^{5/2}}{143b^2x^{16}}-\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

```
[Out] -1/13*(b*x^2 + c*x^4)^(5/2)/(b*x^18) + (8*c*(b*x^2 + c*x^4)^(5/2))/(143*b^2
*x^16) - (16*c^2*(b*x^2 + c*x^4)^(5/2))/(429*b^3*x^14) + (64*c^3*(b*x^2 + c
*x^4)^(5/2))/(3003*b^4*x^12) - (128*c^4*(b*x^2 + c*x^4)^(5/2))/(15015*b^5*x
^10)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(8c)\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx}{13b}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}} + \frac{8c(bx^2+cx^4)^{5/2}}{143b^2x^{16}} + \frac{(48c^2)\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx}{143b^2}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}} + \frac{8c(bx^2+cx^4)^{5/2}}{143b^2x^{16}} - \frac{16c^2(bx^2+cx^4)^{5/2}}{429b^3x^{14}} - \frac{(64c^3)\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx}{429b^3}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}} + \frac{8c(bx^2+cx^4)^{5/2}}{143b^2x^{16}} - \frac{16c^2(bx^2+cx^4)^{5/2}}{429b^3x^{14}} + \frac{64c^3(bx^2+cx^4)^{5/2}}{3003b^4x^{12}} + \frac{(1}{}$$
$$= -\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}} + \frac{8c(bx^2+cx^4)^{5/2}}{143b^2x^{16}} - \frac{16c^2(bx^2+cx^4)^{5/2}}{429b^3x^{14}} + \frac{64c^3(bx^2+cx^4)^{5/2}}{3003b^4x^{12}} - \frac{1}{}$$

Mathematica [A]
time = 0.10, size = 68, normalized size = 0.50

$$\frac{(x^2(b+cx^2))^{5/2}\left(-1155b^4+840b^3cx^2-560b^2c^2x^4+320bc^3x^6-128c^4x^8\right)}{15015b^5x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-1155*b^4 + 840*b^3*c*x^2 - 560*b^2*c^2*x^4 + 320
*b*c^3*x^6 - 128*c^4*x^8))/(15015*b^5*x^18)
```

Maple [A]
time = 0.11, size = 72, normalized size = 0.53

method	result	size
gosper	$-\frac{(c\,x^2+b)\left(128c^4x^8-320b\,c^3x^6+560b^2c^2x^4-840b^3c\,x^2+1155b^4\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{15015x^{16}b^5}$	72
default	$-\frac{(c\,x^2+b)\left(128c^4x^8-320b\,c^3x^6+560b^2c^2x^4-840b^3c\,x^2+1155b^4\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{15015x^{16}b^5}$	72
trager	$-\frac{\left(128c^6x^{12}-64b\,c^5x^{10}+48b^2c^4x^8-40b^3c^3x^6+35b^4c^2x^4+1470b^5c\,x^2+1155b^6\right)\sqrt{c\,x^4+b\,x^2}}{15015b^5x^{14}}$	87
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,\left(128c^6x^{12}-64b\,c^5x^{10}+48b^2c^4x^8-40b^3c^3x^6+35b^4c^2x^4+1470b^5c\,x^2+1155b^6\right)}{15015x^{14}b^5}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15015*(c*x^2+b)*(128*c^4*x^8-320*b*c^3*x^6+560*b^2*c^2*x^4-840*b^3*c*x^2
+1155*b^4)*(c*x^4+b*x^2)^(3/2)/x^16/b^5
```

Maxima [A]
time = 0.30, size = 177, normalized size = 1.30

$$-\frac{128\sqrt{cx^4+bx^2}\,c^6}{15015\,b^5x^2}+\frac{64\sqrt{cx^4+bx^2}\,c^5}{15015\,b^4x^4}-\frac{16\sqrt{cx^4+bx^2}\,c^4}{5005\,b^3x^6}+\frac{8\sqrt{cx^4+bx^2}\,c^3}{3003\,b^2x^8}-\frac{\sqrt{cx^4+bx^2}\,c^2}{429\,bx^{10}}+\frac{3\sqrt{cx^4+bx^2}\,c}{1430\,x^{12}}+\frac{3\sqrt{cx^4+bx^2}\,b}{130\,x^{14}}-\frac{(cx^4+bx^2)^{\frac{3}{2}}}{10\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] -128/15015*sqrt(c*x^4 + b*x^2)*c^6/(b^5*x^2) + 64/15015*sqrt(c*x^4 + b*x^2)
*c^5/(b^4*x^4) - 16/5005*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^6) + 8/3003*sqrt(c*
x^4 + b*x^2)*c^3/(b^2*x^8) - 1/429*sqrt(c*x^4 + b*x^2)*c^2/(b*x^10) + 3/143
0*sqrt(c*x^4 + b*x^2)*c/x^12 + 3/130*sqrt(c*x^4 + b*x^2)*b/x^14 - 1/10*(c*x
^4 + b*x^2)^(3/2)/x^16
```

Fricas [A]
time = 0.41, size = 86, normalized size = 0.63

$$-\frac{(128\,c^6x^{12}-64\,bc^5x^{10}+48\,b^2c^4x^8-40\,b^3c^3x^6+35\,b^4c^2x^4+1470\,b^5cx^2+1155\,b^6)\sqrt{cx^4+bx^2}}{15015\,b^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/15015*(128*c^6*x^12 - 64*b*c^5*x^10 + 48*b^2*c^4*x^8 - 40*b^3*c^3*x^6 +
35*b^4*c^2*x^4 + 1470*b^5*c*x^2 + 1155*b^6)*sqrt(c*x^4 + b*x^2)/(b^5*x^14)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**17,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**17, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(116) = 232.
time = 4.11, size = 264, normalized size = 1.94

$$\frac{256\left(6006\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{16}c^{\frac{13}{2}}\mathrm{sgn}(x)+12012\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{14}bc^{\frac{13}{2}}\mathrm{sgn}(x)+13728\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}b^2c^{\frac{13}{2}}\mathrm{sgn}(x)+4719\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}b^3c^{\frac{13}{2}}\mathrm{sgn}(x)+715\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}b^4c^{\frac{13}{2}}\mathrm{sgn}(x)-286\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{6}b^5c^{\frac{13}{2}}\mathrm{sgn}(x)+78\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{4}b^6c^{\frac{13}{2}}\mathrm{sgn}(x)-13\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{2}b^7c^{\frac{13}{2}}\mathrm{sgn}(x)+b^8c^{\frac{13}{2}}\mathrm{sgn}(x)\right)}{15015\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] 256/15015*(6006*(sqrt(c)*x - sqrt(c*x^2 + b))^16*c^(13/2)*sgn(x) + 12012*(s
qrt(c)*x - sqrt(c*x^2 + b))^14*b*c^(13/2)*sgn(x) + 13728*(sqrt(c)*x - sqrt(
c*x^2 + b))^12*b^2*c^(13/2)*sgn(x) + 4719*(sqrt(c)*x - sqrt(c*x^2 + b))^10*
b^3*c^(13/2)*sgn(x) + 715*(sqrt(c)*x - sqrt(c*x^2 + b))^8*b^4*c^(13/2)*sgn(
x) - 286*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^5*c^(13/2)*sgn(x) + 78*(sqrt(c)*
x - sqrt(c*x^2 + b))^4*b^6*c^(13/2)*sgn(x) - 13*(sqrt(c)*x - sqrt(c*x^2 + b
))^2*b^7*c^(13/2)*sgn(x) + b^8*c^(13/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 +
b))^2 - b)^13
```

Mupad [B]
time = 5.17, size = 159, normalized size = 1.17

$$\frac{8\,c^3\,\sqrt{c\,x^4+b\,x^2}}{3003\,b^2\,x^8}-\frac{14\,c\,\sqrt{c\,x^4+b\,x^2}}{143\,x^{12}}-\frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{429\,b\,x^{10}}-\frac{b\,\sqrt{c\,x^4+b\,x^2}}{13\,x^{14}}-\frac{16\,c^4\,\sqrt{c\,x^4+b\,x^2}}{5005\,b^3\,x^6}+\frac{64\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^4}-\frac{128\,c^6\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^17,x)
```

```
[Out] (8*c^3*(b*x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (14*c*(b*x^2 + c*x^4)^(1/2))
/(143*x^12) - (c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) - (b*(b*x^2 + c*x^4)
^(1/2))/(13*x^14) - (16*c^4*(b*x^2 + c*x^4)^(1/2))/(5005*b^3*x^6) + (64*c^5
*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (128*c^6*(b*x^2 + c*x^4)^(1/2))/(
15015*b^5*x^2)
```

3.249 $\int x^6(bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=134

$$\frac{128b^4(bx^2+cx^4)^{5/2}}{15015c^5x^5}-\frac{64b^3(bx^2+cx^4)^{5/2}}{3003c^4x^3}+\frac{16b^2(bx^2+cx^4)^{5/2}}{429c^3x}-\frac{8bx(bx^2+cx^4)^{5/2}}{143c^2}+\frac{x^3(bx^2+cx^4)^{5/2}}{13c}$$

```
[Out] 128/15015*b^4*(c*x^4+b*x^2)^(5/2)/c^5/x^5-64/3003*b^3*(c*x^4+b*x^2)^(5/2)/c
^4/x^3+16/429*b^2*(c*x^4+b*x^2)^(5/2)/c^3/x-8/143*b*x*(c*x^4+b*x^2)^(5/2)/c
^2+1/13*x^3*(c*x^4+b*x^2)^(5/2)/c
```

Rubi [A]
time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2041, 2027, 2039}

$$\frac{128b^4(bx^2+cx^4)^{5/2}}{15015c^5x^5}-\frac{64b^3(bx^2+cx^4)^{5/2}}{3003c^4x^3}+\frac{16b^2(bx^2+cx^4)^{5/2}}{429c^3x}-\frac{8bx(bx^2+cx^4)^{5/2}}{143c^2}+\frac{x^3(bx^2+cx^4)^{5/2}}{13c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (128*b^4*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (64*b^3*(b*x^2 + c*x^4)^(
5/2))/(3003*c^4*x^3) + (16*b^2*(b*x^2 + c*x^4)^(5/2))/(429*c^3*x) - (8*b*x*
(b*x^2 + c*x^4)^(5/2))/(143*c^2) + (x^3*(b*x^2 + c*x^4)^(5/2))/(13*c)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^6\left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{x^3(bx^2+cx^4)^{5/2}}{13c}-\frac{(8b)\int x^4(bx^2+cx^4)^{3/2}\,dx}{13c}\\ &= -\frac{8bx(bx^2+cx^4)^{5/2}}{143c^2}+\frac{x^3(bx^2+cx^4)^{5/2}}{13c}+\frac{(48b^2)\int x^2(bx^2+cx^4)^{3/2}\,dx}{143c^2}\\ &= \frac{16b^2(bx^2+cx^4)^{5/2}}{429c^3x}-\frac{8bx(bx^2+cx^4)^{5/2}}{143c^2}+\frac{x^3(bx^2+cx^4)^{5/2}}{13c}-\frac{(64b^3)\int (bx^2+cx^4}{429c^3}\\ &= -\frac{64b^3(bx^2+cx^4)^{5/2}}{3003c^4x^3}+\frac{16b^2(bx^2+cx^4)^{5/2}}{429c^3x}-\frac{8bx(bx^2+cx^4)^{5/2}}{143c^2}+\frac{x^3(bx^2+cx^4)^{5/}}{13c}\\ &= \frac{128b^4(bx^2+cx^4)^{5/2}}{15015c^5x^5}-\frac{64b^3(bx^2+cx^4)^{5/2}}{3003c^4x^3}+\frac{16b^2(bx^2+cx^4)^{5/2}}{429c^3x}-\frac{8bx(bx^2+cx^4)}{143c^2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 0.51

$$\frac{(x^2(b+cx^2))^{5/2}\,(128b^4-320b^3cx^2+560b^2c^2x^4-840bc^3x^6+1155c^4x^8)}{15015c^5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(b*x^2 + c*x^4)^(3/2),x]
[Out] ((x^2*(b + c*x^2))^(5/2)*(128*b^4 - 320*b^3*c*x^2 + 560*b^2*c^2*x^4 - 840*b
*c^3*x^6 + 1155*c^4*x^8))/(15015*c^5*x^5)
```

Maple [A]
time = 0.10, size = 72, normalized size = 0.54

method	result	size
gosper	$\frac{(c\,x^2+b)\,(1155c^4x^8-840b\,c^3x^6+560b^2c^2x^4-320b^3c\,x^2+128b^4)\,(c\,x^4+b\,x^2)^{\frac{3}{2}}}{15015c^5x^3}$	72
default	$\frac{(c\,x^2+b)\,(1155c^4x^8-840b\,c^3x^6+560b^2c^2x^4-320b^3c\,x^2+128b^4)\,(c\,x^4+b\,x^2)^{\frac{3}{2}}}{15015c^5x^3}$	72
trager	$\frac{(1155c^6x^{12}+1470b\,c^5x^{10}+35b^2c^4x^8-40b^3c^3x^6+48b^4c^2x^4-64b^5c\,x^2+128b^6)\sqrt{c\,x^4+b\,x^2}}{15015c^5x}$	87
risch	$\frac{\sqrt{x^2\,(c\,x^2+b)}\,(1155c^6x^{12}+1470b\,c^5x^{10}+35b^2c^4x^8-40b^3c^3x^6+48b^4c^2x^4-64b^5c\,x^2+128b^6)}{15015x\,c^5}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15015*(c*x^2+b)*(1155*c^4*x^8-840*b*c^3*x^6+560*b^2*c^2*x^4-320*b^3*c*x^2
+128*b^4)*(c*x^4+b*x^2)^(3/2)/c^5/x^3
```

Maxima [A]
time = 0.30, size = 79, normalized size = 0.59

$$\frac{\left(1155\,c^{6}x^{12}+1470\,bc^{5}x^{10}+35\,b^{2}c^{4}x^{8}-40\,b^{3}c^{3}x^{6}+48\,b^{4}c^{2}x^{4}-64\,b^{5}cx^{2}+128\,b^{6}\right)\sqrt{cx^{2}+b}}{15015\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6
+ 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^2 + b)/c^5
```

Fricas [A]
time = 0.36, size = 86, normalized size = 0.64

$$\frac{\left(1155\,c^{6}x^{12}+1470\,bc^{5}x^{10}+35\,b^{2}c^{4}x^{8}-40\,b^{3}c^{3}x^{6}+48\,b^{4}c^{2}x^{4}-64\,b^{5}cx^{2}+128\,b^{6}\right)\sqrt{cx^{4}+bx^{2}}}{15015\,c^{5}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6
+ 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^4 + b*x^2)/(c^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{6}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**6*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 4.74, size = 92, normalized size = 0.69

$$-\frac{128\,b^{\frac{13}{2}}\operatorname{sgn}(x)}{15015\,c^{5}}+\frac{1155\,(cx^{2}+b)^{\frac{13}{2}}\operatorname{sgn}(x)-5460\,(cx^{2}+b)^{\frac{11}{2}}b\operatorname{sgn}(x)+10010\,(cx^{2}+b)^{\frac{9}{2}}b^{2}\operatorname{sgn}(x)-8580\,(cx^{2}+b)^{\frac{7}{2}}b^{3}\operatorname{sgn}(x)+3003\,(cx^{2}+b)^{\frac{5}{2}}b^{4}\operatorname{sgn}(x)}{15015\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -128/15015*b^(13/2)*sgn(x)/c^5 + 1/15015*(1155*(c*x^2 + b)^(13/2)*sgn(x) -
5460*(c*x^2 + b)^(11/2)*b*sgn(x) + 10010*(c*x^2 + b)^(9/2)*b^2*sgn(x) - 858
0*(c*x^2 + b)^(7/2)*b^3*sgn(x) + 3003*(c*x^2 + b)^(5/2)*b^4*sgn(x))/c^5
```

Mupad [B]
time = 4.48, size = 73, normalized size = 0.54

$$\frac{(c\,x^2+b)^2\,\sqrt{c\,x^4+b\,x^2}\,\left(128\,b^4-320\,b^3\,c\,x^2+560\,b^2\,c^2\,x^4-840\,b\,c^3\,x^6+1155\,c^4\,x^8\right)}{15015\,c^5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(128*b^4 + 1155*c^4*x^8 - 320*b^3*c*x^
2 - 840*b*c^3*x^6 + 560*b^2*c^2*x^4))/(15015*c^5*x)
```

3.250 $\int x^4 (bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=106

$$-\frac{16b^3(bx^2+cx^4)^{5/2}}{1155c^4x^5}+\frac{8b^2(bx^2+cx^4)^{5/2}}{231c^3x^3}-\frac{2b(bx^2+cx^4)^{5/2}}{33c^2x}+\frac{x(bx^2+cx^4)^{5/2}}{11c}$$

```
[Out] -16/1155*b^3*(c*x^4+b*x^2)^(5/2)/c^4/x^5+8/231*b^2*(c*x^4+b*x^2)^(5/2)/c^3/
x^3-2/33*b*(c*x^4+b*x^2)^(5/2)/c^2/x+1/11*x*(c*x^4+b*x^2)^(5/2)/c
```

Rubi [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2041, 2027, 2039}

$$-\frac{16b^3(bx^2+cx^4)^{5/2}}{1155c^4x^5}+\frac{8b^2(bx^2+cx^4)^{5/2}}{231c^3x^3}-\frac{2b(bx^2+cx^4)^{5/2}}{33c^2x}+\frac{x(bx^2+cx^4)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-16*b^3*(b*x^2 + c*x^4)^(5/2))/(1155*c^4*x^5) + (8*b^2*(b*x^2 + c*x^4)^(5/
2))/(231*c^3*x^3) - (2*b*(b*x^2 + c*x^4)^(5/2))/(33*c^2*x) + (x*(b*x^2 + c*
x^4)^(5/2))/(11*c)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^4\left(bx^2+cx^4\right)^{3/2}\,dx &= \frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{(6b)\int x^2\left(bx^2+cx^4\right)^{3/2}\,dx}{11c}\\ &= -\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}+\frac{\left(8b^2\right)\int\left(bx^2+cx^4\right)^{3/2}\,dx}{33c^2}\\ &= \frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{\left(16b^3\right)\int\frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}\,dx}{231c^3}\\ &= -\frac{16b^3\left(bx^2+cx^4\right)^{5/2}}{1155c^4x^5}+\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.54

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(-16b^3+40b^2cx^2-70bc^2x^4+105c^3x^6\right)}{1155c^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(b*x^2 + c*x^4)^(3/2),x]
[Out] ((x^2*(b + c*x^2))^(5/2)*(-16*b^3 + 40*b^2*c*x^2 - 70*b*c^2*x^4 + 105*c^3*x
^6))/(1155*c^4*x^5)
```

Maple [A]
time = 0.08, size = 61, normalized size = 0.58

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-105c^{3}x^{6}+70b\,c^{2}x^{4}-40b^{2}c\,x^{2}+16b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{1155c^{4}x^{3}}$	61
default	$-\frac{\left(c\,x^{2}+b\right)\left(-105c^{3}x^{6}+70b\,c^{2}x^{4}-40b^{2}c\,x^{2}+16b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{1155c^{4}x^{3}}$	61
trager	$-\frac{\left(-105c^{5}x^{10}-140b\,c^{4}x^{8}-5b^{2}c^{3}x^{6}+6b^{3}c^{2}x^{4}-8b^{4}c\,x^{2}+16b^{5}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{1155c^{4}x}$	76
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-105c^{5}x^{10}-140b\,c^{4}x^{8}-5b^{2}c^{3}x^{6}+6b^{3}c^{2}x^{4}-8b^{4}c\,x^{2}+16b^{5}\right)}{1155x\,c^{4}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1155*(c*x^2+b)*(-105*c^3*x^6+70*b*c^2*x^4-40*b^2*c*x^2+16*b^3)*(c*x^4+b*
x^2)^(3/2)/c^4/x^3
```

Maxima [A]
time = 0.30, size = 68, normalized size = 0.64

$$\frac{(105\,c^5x^{10} + 140\,bc^4x^8 + 5\,b^2c^3x^6 - 6\,b^3c^2x^4 + 8\,b^4cx^2 - 16\,b^5)\sqrt{cx^2 + b}}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)/c^4
```

Fricas [A]
time = 0.33, size = 75, normalized size = 0.71

$$\frac{(105\,c^5x^{10} + 140\,bc^4x^8 + 5\,b^2c^3x^6 - 6\,b^3c^2x^4 + 8\,b^4cx^2 - 16\,b^5)\sqrt{cx^4 + bx^2}}{1155\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 3.94, size = 76, normalized size = 0.72

$$\frac{16\,b^{\frac{11}{2}}\,\mathrm{sgn}(x)}{1155\,c^4} + \frac{105\,(cx^2 + b)^{\frac{11}{2}}\,\mathrm{sgn}(x) - 385\,(cx^2 + b)^{\frac{9}{2}}b\mathrm{sgn}(x) + 495\,(cx^2 + b)^{\frac{7}{2}}b^2\mathrm{sgn}(x) - 231\,(cx^2 + b)^{\frac{5}{2}}b^3\mathrm{sgn}(x)}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 16/1155*b^(11/2)*sgn(x)/c^4 + 1/1155*(105*(c*x^2 + b)^(11/2)*sgn(x) - 385*(
c*x^2 + b)^(9/2)*b*sgn(x) + 495*(c*x^2 + b)^(7/2)*b^2*sgn(x) - 231*(c*x^2 +
 b)^(5/2)*b^3*sgn(x))/c^4
```

Mupad [B]
time = 4.30, size = 62, normalized size = 0.58

$$-\frac{(c\,x^2+b)^2\,\sqrt{c\,x^4+b\,x^2}\,\left(16\,b^3-40\,b^2\,c\,x^2+70\,b\,c^2\,x^4-105\,c^3\,x^6\right)}{1155\,c^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(16*b^3 - 105*c^3*x^6 - 40*b^2*c*x^2
+ 70*b*c^2*x^4))/(1155*c^4*x)
```

3.251 $\int x^2(bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=80

$$\frac{8b^2(bx^2+cx^4)^{5/2}}{315c^3x^5}-\frac{4b(bx^2+cx^4)^{5/2}}{63c^2x^3}+\frac{(bx^2+cx^4)^{5/2}}{9cx}$$

```
[Out] 8/315*b^2*(c*x^4+b*x^2)^(5/2)/c^3/x^5-4/63*b*(c*x^4+b*x^2)^(5/2)/c^2/x^3+1/
9*(c*x^4+b*x^2)^(5/2)/c/x
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2041, 2027, 2039}

$$\frac{8b^2(bx^2+cx^4)^{5/2}}{315c^3x^5}-\frac{4b(bx^2+cx^4)^{5/2}}{63c^2x^3}+\frac{(bx^2+cx^4)^{5/2}}{9cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - (4*b*(b*x^2 + c*x^4)^(5/2))/(
63*c^2*x^3) + (b*x^2 + c*x^4)^(5/2)/(9*c*x)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(bx^2+cx^4\right)^{3/2}dx&=\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{(4b)\int\left(bx^2+cx^4\right)^{3/2}dx}{9c}\\&=-\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}+\frac{\left(8b^2\right)\int\frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}dx}{63c^2}\\&=\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.58

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(8b^2-20bcx^2+35c^2x^4\right)}{315c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^2 + c*x^4)^(3/2),x]
[Out] ((x^2*(b + c*x^2))^(5/2)*(8*b^2 - 20*b*c*x^2 + 35*c^2*x^4))/(315*c^3*x^5)
```

Maple [A]
time = 0.08, size = 50, normalized size = 0.62

method	result	size
gosper	$\frac{\left(c\,x^{2}+b\right)\left(35c^{2}x^{4}-20bc\,x^{2}+8b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{315c^{3}x^{3}}$	50
default	$\frac{\left(c\,x^{2}+b\right)\left(35c^{2}x^{4}-20bc\,x^{2}+8b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{315c^{3}x^{3}}$	50
trager	$\frac{\left(35c^{4}x^{8}+50b\,c^{3}x^{6}+3b^{2}c^{2}x^{4}-4b^{3}c\,x^{2}+8b^{4}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{315c^{3}x}$	65
risch	$\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(35c^{4}x^{8}+50b\,c^{3}x^{6}+3b^{2}c^{2}x^{4}-4b^{3}c\,x^{2}+8b^{4}\right)}{315x\,c^{3}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/315*(c*x^2+b)*(35*c^2*x^4-20*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(3/2)/c^3/x^3
```

Maxima [A]
time = 0.31, size = 57, normalized size = 0.71

$$\frac{\left(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4\right)\sqrt{cx^2+b}}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)/c^3
```

Fricas [A]
time = 0.37, size = 64, normalized size = 0.80

$$\frac{(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4)\sqrt{cx^4+bx^2}}{315\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^4 + b*x^2)/(c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 3.62, size = 60, normalized size = 0.75

$$-\frac{8\,b^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,c^3}+\frac{35\,(cx^2+b)^{\frac{9}{2}}\mathrm{sgn}(x)-90\,(cx^2+b)^{\frac{7}{2}}b\mathrm{sgn}(x)+63\,(cx^2+b)^{\frac{5}{2}}b^2\mathrm{sgn}(x)}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/315*b^(9/2)*sgn(x)/c^3 + 1/315*(35*(c*x^2 + b)^(9/2)*sgn(x) - 90*(c*x^2
+ b)^(7/2)*b*sgn(x) + 63*(c*x^2 + b)^(5/2)*b^2*sgn(x))/c^3
```

Mupad [B]
time = 4.19, size = 51, normalized size = 0.64

$$\frac{{\left(c\,x^2+b\right)}^2\,\sqrt{c\,x^4+b\,x^2}\,\left(8\,b^2-20\,b\,c\,x^2+35\,c^2\,x^4\right)}{315\,c^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(8*b^2 + 35*c^2*x^4 - 20*b*c*x^2))/(31
5*c^3*x)
```

3.252 $\int \left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=52

$$-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}+\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}$$

```
[Out] -2/35*b*(c*x^4+b*x^2)^(5/2)/c^2/x^5+1/7*(c*x^4+b*x^2)^(5/2)/c/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {2027, 2039}

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*b*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)/(7*c*x^3)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \left(bx^2+cx^4\right)^{3/2}\,dx&=\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{(2b)\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}\,dx}{7c}\\&=-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}+\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.67

$$\frac{\left(x^2(b+cx^2)\right)^{5/2}(-2b+5cx^2)}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-2*b + 5*c*x^2))/(35*c^2*x^5)
```

Maple [A]
time = 0.12, size = 39, normalized size = 0.75

method	result	size
gosper	$-\frac{(c\,x^2+b)(-5c\,x^2+2b)(c\,x^4+b\,x^2)^{\frac{3}{2}}}{35c^2x^3}$	39
default	$-\frac{(c\,x^2+b)(-5c\,x^2+2b)(c\,x^4+b\,x^2)^{\frac{3}{2}}}{35c^2x^3}$	39
trager	$-\frac{(-5c^3x^6-8b\,c^2x^4-b^2c\,x^2+2b^3)\sqrt{c\,x^4+b\,x^2}}{35c^2x}$	54
risch	$-\frac{\sqrt{x^2\,(c\,x^2+b)}\,(-5c^3x^6-8b\,c^2x^4-b^2c\,x^2+2b^3)}{35x\,c^2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(c*x^2+b)*(-5*c*x^2+2*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.87

$$\frac{(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3)\sqrt{cx^2+b}}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)/c^2
```

Fricas [A]
time = 0.37, size = 52, normalized size = 1.00

$$\frac{(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3)\sqrt{cx^4+bx^2}}{35\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^4 + b*x^2)/(c^2
*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((b*x**2 + c*x**4)**(3/2), x)
```

Giac [A]
time = 3.37, size = 44, normalized size = 0.85

$$\frac{2\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,c^2} + \frac{5\,(cx^2 + b)^{\frac{7}{2}}\mathrm{sgn}(x) - 7\,(cx^2 + b)^{\frac{5}{2}}b\mathrm{sgn}(x)}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/35*b^(7/2)*sgn(x)/c^2 + 1/35*(5*(c*x^2 + b)^(7/2)*sgn(x) - 7*(c*x^2 + b)^
(5/2)*b*sgn(x))/c^2
```

Mupad [B]
time = 4.16, size = 40, normalized size = 0.77

$$-\frac{(c\,x^2 + b)^2\,\sqrt{c\,x^4 + b\,x^2}\,\left(2\,b - 5\,c\,x^2\right)}{35\,c^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(2*b - 5*c*x^2))/(35*c^2*x)
```

3.253 $\int \frac{(bx^2+cx^4)^{3/2}}{x^2} dx$

Optimal. Leaf size=25

$$\frac{(bx^2 + cx^4)^{5/2}}{5cx^5}$$

[Out] 1/5*(c*x^4+b*x^2)^(5/2)/c/x^5

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2039}

$$\frac{(bx^2 + cx^4)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)^(3/2)/x^2,x]

[Out] (b*x^2 + c*x^4)^(5/2)/(5*c*x^5)

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^2} dx = \frac{(bx^2 + cx^4)^{5/2}}{5cx^5}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{(x^2(b + cx^2))^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^2,x]

[Out] (x^2*(b + c*x^2))^(5/2)/(5*c*x^5)

Maple [A]
time = 0.09, size = 29, normalized size = 1.16

method	result	size
gosper	$\frac{(c\,x^2+b)\,(c\,x^4+b\,x^2)^{\frac{3}{2}}}{5c\,x^3}$	29
default	$\frac{(c\,x^2+b)\,(c\,x^4+b\,x^2)^{\frac{3}{2}}}{5c\,x^3}$	29
trager	$\frac{(c^2x^4+2bc\,x^2+b^2)\sqrt{c\,x^4+b\,x^2}}{5cx}$	40
risch	$\frac{\sqrt{x^2\,(c\,x^2+b)}\,(c^2x^4+2bc\,x^2+b^2)}{5xc}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(c*x^2+b)/c/x^3*(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 32, normalized size = 1.28

$$\frac{(c^2x^4+2\,bcx^2+b^2)\sqrt{cx^2+b}}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^2 + b)/c
```

Fricas [A]
time = 0.43, size = 39, normalized size = 1.56

$$\frac{(c^2x^4+2\,bcx^2+b^2)\sqrt{cx^4+bx^2}}{5\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**2,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**2, x)
```

Giac [A]
time = 3.99, size = 27, normalized size = 1.08

$$\frac{(cx^2+b)^{\frac{5}{2}}\operatorname{sgn}(x)}{5\,c}-\frac{b^{\frac{5}{2}}\operatorname{sgn}(x)}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="giac")
[Out] 1/5*(c*x^2 + b)^(5/2)*sgn(x)/c - 1/5*b^(5/2)*sgn(x)/c
```

Mupad [B]
time = 4.15, size = 30, normalized size = 1.20

$$\frac{(c\,x^2+b)^2\,\sqrt{c\,x^4+b\,x^2}}{5\,c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^2,x)
[Out] ((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2))/(5*c*x)
```

3.254 $\int \frac{(bx^2+cx^4)^{3/2}}{x^4} dx$

Optimal. Leaf size=73

$$\frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}-b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] 1/3*(c*x^4+b*x^2)^(3/2)/x^3-b^(3/2)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))+
b*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2046, 2033, 212}

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (b*Sqrt[b*x^2 + c*x^4])/x + (b*x^2 + c*x^4)^(3/2)/(3*x^3) - b^(3/2)*ArcTanh
[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx &= \frac{(bx^2+cx^4)^{3/2}}{3x^3}+b\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx\\ &= \frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}+b^2\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}-b^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= \frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}-b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 76, normalized size = 1.04

$$\frac{x\left(4b^2+5bcx^2+c^2x^4-3b^{3/2}\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{3\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (x*(4*b^2 + 5*b*c*x^2 + c^2*x^4 - 3*b^(3/2)*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b
+ c*x^2]/Sqrt[b]]))/(3*Sqrt[x^2*(b + c*x^2)])
```

Maple [**A**]
time = 0.10, size = 78, normalized size = 1.07

method	result	size
default	$-\frac{(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)-(c\,x^2+b)^{\frac{3}{2}}-3\sqrt{c\,x^2+b}\,b\right)}{3x^3(c\,x^2+b)^{\frac{3}{2}}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)-(c*
x^2+b)^(3/2)-3*(c*x^2+b)^(1/2)*b)/x^3/(c*x^2+b)^(3/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^4, x)
```

Fricas [A]
time = 0.40, size = 140, normalized size = 1.92

$$\left[\frac{3\,b^{\frac{3}{2}}x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\,(cx^2+4\,b)}{6\,x},\ \frac{3\,\sqrt{-b}\,bx\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,(cx^2+4\,b)}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="fricas")
[Out] [1/6*(3*b^(3/2)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)
 + 2*sqrt(c*x^4 + b*x^2)*(c*x^2 + 4*b))/x, 1/3*(3*sqrt(-b)*b*x*arctan(sqrt(
c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(c*x^2 + 4*b))
/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**4,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**4, x)
```

Giac [A]
time = 3.65, size = 89, normalized size = 1.22

$$\frac{b^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}+\frac{1}{3}\,(cx^2+b)^{\frac{3}{2}}\mathrm{sgn}(x)+\sqrt{cx^2+b}\,b\mathrm{sgn}(x)-\frac{\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\,\sqrt{-b}\,b^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{3\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="giac")
[Out] b^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 1/3*(c*x^2 + b)^(3/2
)*sgn(x) + sqrt(c*x^2 + b)*b*sgn(x) - 1/3*(3*b^2*arctan(sqrt(b)/sqrt(-b)) +
 4*sqrt(-b)*b^(3/2))*sgn(x)/sqrt(-b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^4,x)
[Out] int((b*x^2 + c*x^4)^(3/2)/x^4, x)
```

3.255 $\int \frac{(bx^2+cx^4)^{3/2}}{x^6} dx$

Optimal. Leaf size=79

$$\frac{3c\sqrt{bx^2+cx^4}}{2x} - \frac{(bx^2+cx^4)^{3/2}}{2x^5} - \frac{3}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -1/2*(c*x^4+b*x^2)^(3/2)/x^5-3/2*c*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b
^(1/2)+3/2*c*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2046, 2033, 212}

$$\frac{3c\sqrt{bx^2+cx^4}}{2x} - \frac{3}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right) - \frac{(bx^2+cx^4)^{3/2}}{2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/(2*x) - (b*x^2 + c*x^4)^(3/2)/(2*x^5) - (3*Sqrt[b
]*c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx &= -\frac{(bx^2+cx^4)^{3/2}}{2x^5}+\frac{1}{2}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx\\
&= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{(bx^2+cx^4)^{3/2}}{2x^5}+\frac{1}{2}(3bc)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{(bx^2+cx^4)^{3/2}}{2x^5}-\frac{1}{2}(3bc)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{(bx^2+cx^4)^{3/2}}{2x^5}-\frac{3}{2}\sqrt{b}\,c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 83, normalized size = 1.05

$$-\frac{\sqrt{x^2(b+cx^2)}\left((b-2cx^2)\sqrt{b+cx^2}+3\sqrt{b}\,cx^2\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{2x^3\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] -1/2*(Sqrt[x^2*(b + c*x^2)]*((b - 2*c*x^2)*Sqrt[b + c*x^2] + 3*Sqrt[b]*c*x^
2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(x^3*Sqrt[b + c*x^2])
```

Maple [A]
time = 0.09, size = 102, normalized size = 1.29

method	result	size
risch	$-\frac{b\sqrt{x^2(cx^2+b)}}{2x^3}+\frac{\left(-\frac{3\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{cx^2+b}}{x}\right)c}{2}+\sqrt{cx^2+b}\,c\right)\sqrt{x^2(cx^2+b)}}{x\sqrt{cx^2+b}}$	88

default	$-\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c\,x^{2}-\left(c\,x^{2}+b\right)^{\frac{3}{2}}c\,x^{2}+\left(c\,x^{2}+b\right)^{\frac{5}{2}}-3\sqrt{c\,x^{2}+b}\,b c\,x^{2}\right)}{2x^{5}\left(c\,x^{2}+b\right)^{\frac{3}{2}}b}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c*x
^2-(c*x^2+b)^(3/2)*c*x^2+(c*x^2+b)^(5/2)-3*(c*x^2+b)^(1/2)*b*c*x^2)/x^5/(c*
x^2+b)^(3/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^6, x)
```

Fricas [A]
time = 0.42, size = 147, normalized size = 1.86

$$\left[\frac{3\sqrt{b}\,cx^{3}\log\left(-\frac{cx^{3}+2\,bx-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)+2\,\sqrt{cx^{4}+bx^{2}}\,(2\,cx^{2}-b)}{4\,x^{3}},\ \frac{3\,\sqrt{-b}\,cx^{3}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)+\sqrt{cx^{4}+bx^{2}}\,(2\,cx^{2}-b)}{2\,x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/4*(3*sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/
x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b))/x^3, 1/2*(3*sqrt(-b)*c*x^3*arct
an(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(2*c*x
^2 - b))/x^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**6, x)
```

Giac [**A**]
time = 3.51, size = 69, normalized size = 0.87

$$\frac{\frac{3\,bc^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}+2\,\sqrt{cx^2+b}\,c^2\mathrm{sgn}(x)-\frac{\sqrt{cx^2+b}\,bc\mathrm{sgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(3*b*c^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*sqrt(c*x^
2 + b)*c^2*sgn(x) - sqrt(c*x^2 + b)*b*c*sgn(x)/x^2)/c
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^6,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^6, x)
```

3.256 $\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=81

$$-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{(bx^2+cx^4)^{3/2}}{4x^7}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}$$

```
[Out] -1/4*(c*x^4+b*x^2)^(3/2)/x^7-3/8*c^2*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))
/b^(1/2)-3/8*c*(c*x^4+b*x^2)^(1/2)/x^3
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2045, 2033, 212}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{(bx^2+cx^4)^{3/2}}{4x^7}-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(8*x^3) - (b*x^2 + c*x^4)^(3/2)/(4*x^7) - (3*c^2
*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^8}\,dx &= -\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}+\frac{1}{4}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx\\
&= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}+\frac{1}{8}\left(3c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}-\frac{1}{8}\left(3c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{\left(bx^2+cx^4\right)^{3/2}}{4x^7}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 92, normalized size = 1.14

$$
-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{b}\,\sqrt{b+cx^2}\left(2b+5cx^2\right)+3c^2x^4\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{8\sqrt{b}\,x^5\sqrt{b+cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/8*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(2*b + 5*c*x^2) + 3*c^
2*x^4*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(Sqrt[b]*x^5*Sqrt[b + c*x^2])
```

Maple [A]
time = 0.11, size = 125, normalized size = 1.54

method	result
risch	$-\frac{\left(5c\,x^2+2b\right)\sqrt{x^2\left(c\,x^2+b\right)}}{8x^5}-\frac{3c^2\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)\sqrt{x^2\left(c\,x^2+b\right)}}{8\sqrt{b}\,x\sqrt{c\,x^2+b}}$
default	$-\frac{\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^2x^4-\left(c\,x^2+b\right)^{\frac{3}{2}}c^2x^4+\left(c\,x^2+b\right)^{\frac{5}{2}}c\,x^2-3\sqrt{c\,x^2+b}\,b\,c^2x^4+2\left(c\,x^2+b\right)^{\frac{5}{2}}b\right)}{8x^7\left(c\,x^2+b\right)^{\frac{3}{2}}b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c^2
*x^4-(c*x^2+b)^(3/2)*c^2*x^4+(c*x^2+b)^(5/2)*c*x^2-3*(c*x^2+b)^(1/2)*b*c^2*
x^4+2*(c*x^2+b)^(5/2)*b)/x^7/(c*x^2+b)^(3/2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^8, x)
```

Fricas [A]
time = 0.39, size = 164, normalized size = 2.02

$$\left[\frac{3\sqrt{b}\,c^2x^5\log\left(-\frac{cx^3+2\,bx-2\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\sqrt{cx^4+bx^2}\,(5\,bcx^2+2\,b^2)}{16\,bx^5},\ \frac{3\sqrt{-b}\,c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)-\sqrt{cx^4+bx^2}\,(5\,bcx^2+2\,b^2)}{8\,bx^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(5*b*c*x^2 + 2*b^2))/(b*x^5), 1/8*(3*sqrt(-
b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4
+ b*x^2)*(5*b*c*x^2 + 2*b^2))/(b*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**8, x)
```

Giac [A]
time = 4.28, size = 76, normalized size = 0.94

$$\frac{\frac{3\,c^3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}-\frac{5\,(cx^2+b)^{\frac{3}{2}}c^3\operatorname{sgn}(x)-3\sqrt{cx^2+b}\,bc^3\operatorname{sgn}(x)}{c^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="giac")
```

[Out] 1/8*(3*c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - (5*(c*x^2 + b
)^(3/2)*c^3*sgn(x) - 3*sqrt(c*x^2 + b)*b*c^3*sgn(x))/(c^2*x^4))/c

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2 + c*x^4)^(3/2)/x^8,x)

[Out] int((b*x^2 + c*x^4)^(3/2)/x^8, x)

3.257 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}} dx$

Optimal. Leaf size=109

$$-\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{(bx^2+cx^4)^{3/2}}{6x^9}+\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}$$

```
[Out] -1/6*(c*x^4+b*x^2)^(3/2)/x^9+1/16*c^3*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2)
)/b^(3/2)-1/8*c*(c*x^4+b*x^2)^(1/2)/x^5-1/16*c^2*(c*x^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2045, 2050, 2033, 212}

$$\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{(bx^2+cx^4)^{3/2}}{6x^9}-\frac{c\sqrt{bx^2+cx^4}}{8x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

```
[Out] -1/8*(c*Sqrt[b*x^2 + c*x^4])/x^5 - (c^2*Sqrt[b*x^2 + c*x^4])/(16*b*x^3) - (
b*x^2 + c*x^4)^(3/2)/(6*x^9) + (c^3*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]
])/(16*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx &= -\frac{(bx^2+cx^4)^{3/2}}{6x^9}+\frac{1}{2}c\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{(bx^2+cx^4)^{3/2}}{6x^9}+\frac{1}{8}c^2\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{(bx^2+cx^4)^{3/2}}{6x^9}-\frac{c^3\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{16b}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{(bx^2+cx^4)^{3/2}}{6x^9}+\frac{c^3\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{16b}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{(bx^2+cx^4)^{3/2}}{6x^9}+\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 104, normalized size = 0.95

$$\frac{\sqrt{x^2(b+cx^2)}\left(-\sqrt{b}\sqrt{b+cx^2}\,(8b^2+14bcx^2+3c^2x^4)+3c^3x^6\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{48b^{3/2}x^7\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-(Sqrt[b]*Sqrt[b + c*x^2]*(8*b^2 + 14*b*c*x^2 + 3*c
^2*x^4)) + 3*c^3*x^6*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(48*b^(3/2)*x^7*Sqr
t[b + c*x^2])
```

Maple [A]
time = 0.11, size = 145, normalized size = 1.33

method	result
risch	$-\frac{(3c^2x^4+14bc\,x^2+8b^2)\sqrt{x^2\,(c\,x^2+b)}}{48x^7b}+\frac{c^3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)\sqrt{x^2\,(c\,x^2+b)}}{16b^{\frac{3}{2}}x\sqrt{c\,x^2+b}}$
default	$\frac{(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^3x^6-(c\,x^2+b)^{\frac{3}{2}}c^3x^6+(c\,x^2+b)^{\frac{5}{2}}c^2x^4-3\sqrt{c\,x^2+b}\,b\,c^3x^6+2(c\,x^2+b)^{\frac{5}{2}}bc\,x\right.}{48x^9(c\,x^2+b)^{\frac{3}{2}}b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c^3
*x^6-(c*x^2+b)^(3/2)*c^3*x^6+(c*x^2+b)^(5/2)*c^2*x^4-3*(c*x^2+b)^(1/2)*b*c^
3*x^6+2*(c*x^2+b)^(5/2)*b*c*x^2-8*(c*x^2+b)^(5/2)*b^2)/x^9/(c*x^2+b)^(3/2)/
b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^10, x)
```

Fricas [A]
time = 0.41, size = 185, normalized size = 1.70

$$\left[\frac{3\sqrt{b}\,c^3x^7\log\left(-\frac{cx^3+2\,bx+2\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,(3\,bc^2x^4+14\,b^2cx^2+8\,b^3)\sqrt{cx^4+bx^2}}{96\,b^2x^7},-\frac{3\sqrt{-b}\,c^3x^7\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+(3\,bc^2x^4+14\,b^2cx^2+8\,b^3)\sqrt{cx^4+bx^2}}{48\,b^2x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(b)*c^3*x^7*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) - 2*(3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^2*
x^7), -1/48*(3*sqrt(-b)*c^3*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3
+ b*x)) + (3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^2*x^
7)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**10,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**10, x)
```

Giac [A]
time = 3.04, size = 100, normalized size = 0.92

$$-\frac{\frac{3\,c^4\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b}+\frac{3\,(cx^2+b)^{\frac{5}{2}}c^4\mathrm{sgn}(x)+8\,(cx^2+b)^{\frac{3}{2}}bc^4\mathrm{sgn}(x)-3\,\sqrt{cx^2+b}\,b^2c^4\mathrm{sgn}(x)}{bc^3x^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="giac")
[Out] -1/48*(3*c^4*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + (3*(c*x
^2 + b)^(5/2)*c^4*sgn(x) + 8*(c*x^2 + b)^(3/2)*b*c^4*sgn(x) - 3*sqrt(c*x^2
+ b)*b^2*c^4*sgn(x))/(b*c^3*x^6))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^10,x)
[Out] int((b*x^2 + c*x^4)^(3/2)/x^10, x)
```

3.258 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}} dx$

Optimal. Leaf size=137

$$-\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}$$

```
[Out] -1/8*(c*x^4+b*x^2)^(3/2)/x^11-3/128*c^4*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/
2))/b^(5/2)-1/16*c*(c*x^4+b*x^2)^(1/2)/x^7-1/64*c^2*(c*x^4+b*x^2)^(1/2)/b/x
^5+3/128*c^3*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2033, 212}

$$-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}-\frac{c\sqrt{bx^2+cx^4}}{16x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

```
[Out] -1/16*(c*Sqrt[b*x^2 + c*x^4])/x^7 - (c^2*Sqrt[b*x^2 + c*x^4])/(64*b*x^5) +
(3*c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - (b*x^2 + c*x^4)^(3/2)/(8*x^11)
- (3*c^4*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(128*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}\,dx &= -\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}+\frac{1}{8}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}+\frac{1}{16}c^2\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}-\frac{(3c^3)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{64b}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}+\frac{(3c^4)\int \frac{}{\sqrt{bx}}}{128}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}-\frac{(3c^4)\,\mathrm{Subst}(}{}\\
&= -\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}-\frac{3c^4\tanh^{-1}(}{12}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 114, normalized size = 0.83

$$-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}\,(16b^3+24b^2cx^2+2bc^2x^4-3c^3x^6)+3c^4x^8\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{128b^{5/2}x^9\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

```
[Out] -1/128*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(16*b^3 + 24*b^2*c*x
^2 + 2*b*c^2*x^4 - 3*c^3*x^6) + 3*c^4*x^8*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])
)/(b^(5/2)*x^9*Sqrt[b + c*x^2])
```

Maple [A]
time = 0.10, size = 165, normalized size = 1.20

method	result
risch	$-\frac{\left(-3 c^{3} x^{6}+2 b\,c^{2} x^{4}+24 b^{2} c\,x^{2}+16 b^{3}\right) \sqrt{x^{2} \left(c\,x^{2}+b \right)}}{128 x^{9} b^{2}}-\frac{3 c^{4} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c\,x^{2}+b}}{x}\right) \sqrt{x^{2} \left(c\,x^{2}+b \right)}}{128 b^{\frac{5}{2}} x \sqrt{c\,x^{2}+b}}$
default	$-\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}} \left(3 b^{\frac{3}{2}} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c\,x^{2}+b}}{x}\right) c^{4} x^{8}-\left(c\,x^{2}+b \right)^{\frac{3}{2}} c^{4} x^{8}+\left(c\,x^{2}+b \right)^{\frac{5}{2}} c^{3} x^{6}-3 \sqrt{c\,x^{2}+b}\, b\,c^{4} x^{8}+2 \left(c\,x^{2}+b \right)^{\frac{5}{2}} b}{128 x^{11} \left(c\,x^{2}+b \right)^{\frac{3}{2}} b^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c
^4*x^8-(c*x^2+b)^(3/2)*c^4*x^8+(c*x^2+b)^(5/2)*c^3*x^6-3*(c*x^2+b)^(1/2)*b*
c^4*x^8+2*(c*x^2+b)^(5/2)*b*c^2*x^4-8*(c*x^2+b)^(5/2)*b^2*c*x^2+16*(c*x^2+b
)^(5/2)*b^3)/x^11/(c*x^2+b)^(3/2)/b^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^12, x)
```

Fricas [A]
time = 0.35, size = 207, normalized size = 1.51

$$\left[\frac{3 \, \sqrt{b} c^{4} x^{9} \log\left(-\frac{c x^{3} + 2 \, b x - 2 \, \sqrt{c x^{4} + b x^{2}} \sqrt{b}}{x^{3}}\right) + 2 \, {\left(3 \, b c^{3} x^{6} - 2 \, b^{2} c^{2} x^{4} - 24 \, b^{3} c x^{2} - 16 \, b^{4}\right)} \sqrt{c x^{4} + b x^{2}}}{256 \, b^{3} x^{9}}, \frac{3 \, \sqrt{-b} c^{4} x^{9} \arctan\left(\frac{\sqrt{c x^{4} + b x^{2}} \sqrt{-b}}{c x^{3} + b x}\right) + {\left(3 \, b c^{3} x^{6} - 2 \, b^{2} c^{2} x^{4} - 24 \, b^{3} c x^{2} - 16 \, b^{4}\right)} \sqrt{c x^{4} + b x^{2}}}{128 \, b^{3} x^{9}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] [1/256*(3*sqrt(b)*c^4*x^9*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(
b))/x^3) + 2*(3*b*c^3*x^6 - 2*b^2*c^2*x^4 - 24*b^3*c*x^2 - 16*b^4)*sqrt(c*x
^4 + b*x^2))/(b^3*x^9), 1/128*(3*sqrt(-b)*c^4*x^9*arctan(sqrt(c*x^4 + b*x^2
)*sqrt(-b)/(c*x^3 + b*x)) + (3*b*c^3*x^6 - 2*b^2*c^2*x^4 - 24*b^3*c*x^2 - 1
6*b^4)*sqrt(c*x^4 + b*x^2))/(b^3*x^9)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**12,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**12, x)
```

Giac [A]
time = 4.01, size = 119, normalized size = 0.87

$$\frac{\frac{3\,c^{5}\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}\,b^{2}}+\frac{3\,(cx^{2}+b)^{\frac{7}{2}}c^{5}\operatorname{sgn}(x)-11\,(cx^{2}+b)^{\frac{5}{2}}bc^{5}\operatorname{sgn}(x)-11\,(cx^{2}+b)^{\frac{3}{2}}b^{2}c^{5}\operatorname{sgn}(x)+3\,\sqrt{cx^{2}+b}\,b^{3}c^{5}\operatorname{sgn}(x)}{b^{2}c^{4}x^{8}}}{128\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/128*(3*c^5*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^2) + (3*(c
*x^2 + b)^(7/2)*c^5*sgn(x) - 11*(c*x^2 + b)^(5/2)*b*c^5*sgn(x) - 11*(c*x^2
+ b)^(3/2)*b^2*c^5*sgn(x) + 3*sqrt(c*x^2 + b)*b^3*c^5*sgn(x))/(b^2*c^4*x^8)
)/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{12}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^12,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^12, x)
```

3.259 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=165

$$-\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}+\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}$$

```
[Out] -1/10*(c*x^4+b*x^2)^(3/2)/x^13+3/256*c^5*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1
/2))/b^(7/2)-3/80*c*(c*x^4+b*x^2)^(1/2)/x^9-1/160*c^2*(c*x^4+b*x^2)^(1/2)/b
/x^7+1/128*c^3*(c*x^4+b*x^2)^(1/2)/b^2/x^5-3/256*c^4*(c*x^4+b*x^2)^(1/2)/b^
3/x^3
```

Rubi [A]
time = 0.16, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2045, 2050, 2033, 212}

$$\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}-\frac{3c\sqrt{bx^2+cx^4}}{80x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(80*x^9) - (c^2*Sqrt[b*x^2 + c*x^4])/(160*b*x^7)
 + (c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^5) - (3*c^4*Sqrt[b*x^2 + c*x^4])/(2
56*b^3*x^3) - (b*x^2 + c*x^4)^(3/2)/(10*x^13) + (3*c^5*ArcTanh[(Sqrt[b]*x)/
Sqrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
```

```
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = -\frac{(bx^2+cx^4)^{3/2}}{10x^{13}} + \frac{1}{10}(3c)\int \frac{\sqrt{bx^2+cx^4}}{x^{10}}\,dx$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{(bx^2+cx^4)^{3/2}}{10x^{13}} + \frac{1}{80}(3c^2)\int \frac{1}{x^6\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{c^2\sqrt{bx^2+cx^4}}{160bx^7} - \frac{(bx^2+cx^4)^{3/2}}{10x^{13}} - \frac{c^3\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx}{32b}$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{c^2\sqrt{bx^2+cx^4}}{160bx^7} + \frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{(bx^2+cx^4)^{3/2}}{10x^{13}} + \frac{(3c^4)\int \frac{1}{x^2\sqrt{}}}{12}$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{c^2\sqrt{bx^2+cx^4}}{160bx^7} + \frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(bx^2+cx^4}{10x^{13}}$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{c^2\sqrt{bx^2+cx^4}}{160bx^7} + \frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(bx^2+cx^4}{10x^{13}}$$

$$= -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{c^2\sqrt{bx^2+cx^4}}{160bx^7} + \frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(bx^2+cx^4}{10x^{13}}$$

Mathematica [A]
time = 0.16, size = 126, normalized size = 0.76

$$\frac{\sqrt{b+cx^2}\left(-\sqrt{b}\sqrt{b+cx^2}\,(128b^4+176b^3cx^2+8b^2c^2x^4-10bc^3x^6+15c^4x^8)+15c^5x^{10}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{1280b^{7/2}x^9\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

```
[Out] (Sqrt[b + c*x^2]*(-(Sqrt[b]*Sqrt[b + c*x^2]*(128*b^4 + 176*b^3*c*x^2 + 8*b^
2*c^2*x^4 - 10*b*c^3*x^6 + 15*c^4*x^8)) + 15*c^5*x^10*ArcTanh[Sqrt[b + c*x^
2]/Sqrt[b]]))/(1280*b^(7/2)*x^9*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 186, normalized size = 1.13

method	result
risch	$-\frac{\left(15 c^{4} x^{8}-10 b\, c^{3} x^{6}+8 b^{2} c^{2} x^{4}+176 b^{3} c\, x^{2}+128 b^{4}\right) \sqrt{x^{2} \left(c\, x^{2}+b \right)}}{1280 x^{11} b^{3}}+\frac{3 c^{5} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c\, x^{2}+b}}{x}\right) \sqrt{x^{2} \left(c\, x^{2}+b \right)}}{256 b^{\frac{7}{2}} x \sqrt{c\, x^{2}+b}}$
default	$\frac{\left(c\, x^{4}+b\, x^{2}\right)^{\frac{3}{2}} \left(15 b^{\frac{3}{2}} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c\, x^{2}+b}}{x}\right) c^{5} x^{10}-5 \left(c\, x^{2}+b \right)^{\frac{3}{2}} c^{5} x^{10}+5 \left(c\, x^{2}+b \right)^{\frac{5}{2}} c^{4} x^{8}-15 \sqrt{c\, x^{2}+b}\, b\, c^{5} x^{10}+10 \left(c\, x^{2}+ \right.}{1280 x^{13} \left(c\, x^{2}+b \right)^{\frac{3}{2}} b^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(15*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*
c^5*x^10-5*(c*x^2+b)^(3/2)*c^5*x^10+5*(c*x^2+b)^(5/2)*c^4*x^8-15*(c*x^2+b)^
(1/2)*b*c^5*x^10+10*(c*x^2+b)^(5/2)*b*c^3*x^6-40*(c*x^2+b)^(5/2)*b^2*c^2*x^
4+80*(c*x^2+b)^(5/2)*b^3*c*x^2-128*(c*x^2+b)^(5/2)*b^4)/x^13/(c*x^2+b)^(3/2
)/b^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^14, x)
```

Fricas [A]
time = 0.42, size = 229, normalized size = 1.39

$$\left[\frac{15 \sqrt{b} c^{5} x^{11} \log\left(-\frac{c x^{3} + 2 \, b x + 2 \, \sqrt{c x^{4} + b x^{2}} \sqrt{b}}{x^{3}}\right) - 2 \, {\left(15 \, b c^{4} x^{8} - 10 \, b^{2} c^{3} x^{6} + 8 \, b^{3} c^{2} x^{4} + 176 \, b^{4} c x^{2} + 128 \, b^{5}\right)} \sqrt{c x^{4} + b x^{2}}}{2560 \, b^{4} x^{11}}, -\frac{15 \, \sqrt{-b} c^{5} x^{11} \arctan\left(\frac{\sqrt{c x^{4} + b x^{2}} \sqrt{-b}}{c x^{3} + b x}\right) + {\left(15 \, b c^{4} x^{8} - 10 \, b^{2} c^{3} x^{6} + 8 \, b^{3} c^{2} x^{4} + 176 \, b^{4} c x^{2} + 128 \, b^{5}\right)} \sqrt{c x^{4} + b x^{2}}}{1280 \, b^{4} x^{11}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*sqrt(b)*c^5*x^11*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sq
rt(b))/x^3) - 2*(15*b*c^4*x^8 - 10*b^2*c^3*x^6 + 8*b^3*c^2*x^4 + 176*b^4*c*
x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))/(b^4*x^11), -1/1280*(15*sqrt(-b)*c^5*x^
```

```
11*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*b*c^4*x^8 - 10*
b^2*c^3*x^6 + 8*b^3*c^2*x^4 + 176*b^4*c*x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))
/(b^4*x^11)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**14,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**14, x)
```

Giac [A]
time = 3.65, size = 138, normalized size = 0.84

$$-\frac{\frac{15\,c^{6}\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b^{3}}+\frac{15\,(cx^{2}+b)^{\frac{9}{2}}c^{6}\mathrm{sgn}(x)-70\,(cx^{2}+b)^{\frac{7}{2}}bc^{6}\mathrm{sgn}(x)+128\,(cx^{2}+b)^{\frac{5}{2}}b^{2}c^{6}\mathrm{sgn}(x)+70\,(cx^{2}+b)^{\frac{3}{2}}b^{3}c^{6}\mathrm{sgn}(x)-15\,\sqrt{cx^{2}+b}\,b^{4}c^{6}\mathrm{sgn}(x)}{b^{3}c^{5}x^{10}}}{1280\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/1280*(15*c^6*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^3) + (1
5*(c*x^2 + b)^(9/2)*c^6*sgn(x) - 70*(c*x^2 + b)^(7/2)*b*c^6*sgn(x) + 128*(c
*x^2 + b)^(5/2)*b^2*c^6*sgn(x) + 70*(c*x^2 + b)^(3/2)*b^3*c^6*sgn(x) - 15*s
qrt(c*x^2 + b)*b^4*c^6*sgn(x))/(b^3*c^5*x^10))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^14,x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^14, x)
```

3.260 $\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=114

$$\frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}$$

```
[Out] -5/16*b^3*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+5/16*b^2*(c*x^4+
b*x^2)^(1/2)/c^3-5/24*b*x^2*(c*x^4+b*x^2)^(1/2)/c^2+1/6*x^4*(c*x^4+b*x^2)^(
1/2)/c
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 684, 654, 634, 212}

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}+\frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (5*b^2*Sqrt[b*x^2 + c*x^4])/(16*c^3) - (5*b*x^2*Sqrt[b*x^2 + c*x^4])/(24*c^
2) + (x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (5*b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*
x^2 + c*x^4]])/(16*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{(5b)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{12c}$$

$$= -\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} + \frac{(5b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^2}$$

$$= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3} - \frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,d\cdots\right.}{32c^3}$$

$$= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3} - \frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \cdots\right.}{16c^3}$$

$$= \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3} - \frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}$$

Mathematica [A]
time = 0.07, size = 102, normalized size = 0.89

$$\frac{x\left(\sqrt{c}\,x(15b^3+5b^2cx^2-2bc^2x^4+8c^3x^6)+15b^3\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{48c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(15*b^3 + 5*b^2*c*x^2 - 2*b*c^2*x^4 + 8*c^3*x^6) + 15*b^3*Sqr
t[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(48*c^(7/2)*Sqrt[x^2*(b
+ c*x^2)])
```

Maple [A]
time = 0.09, size = 105, normalized size = 0.92

method	result
risch	$\frac{x^{2}\left(8c^{2}x^{4}-10bc\,x^{2}+15b^{2}\right)\left(c\,x^{2}+b\right)}{48c^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{5b^{3}\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)x\sqrt{c\,x^{2}+b}}{16c^{\frac{7}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$
default	$\frac{x\sqrt{c\,x^{2}+b}\,\left(8x^{5}\sqrt{c\,x^{2}+b}\,c^{\frac{7}{2}}-10c^{\frac{5}{2}}\sqrt{c\,x^{2}+b}\,b\,x^{3}+15c^{\frac{3}{2}}\sqrt{c\,x^{2}+b}\,b^{2}x-15\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)b^{3}c\right)}{48\sqrt{c\,x^{4}+b\,x^{2}}\,c^{\frac{9}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*x*(c*x^2+b)^(1/2)*(8*x^5*(c*x^2+b)^(1/2)*c^(7/2)-10*c^(5/2)*(c*x^2+b)^
(1/2)*b*x^3+15*c^(3/2)*(c*x^2+b)^(1/2)*b^2*x-15*ln(x*c^(1/2)+(c*x^2+b)^(1/2
))*b^3*c)/(c*x^4+b*x^2)^(1/2)/c^(9/2)
```

Maxima [A]
time = 0.29, size = 100, normalized size = 0.88

$$\frac{\sqrt{cx^4+bx^2}\,x^4}{6\,c}-\frac{5\,\sqrt{cx^4+bx^2}\,bx^2}{24\,c^2}-\frac{5\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{32\,c^{\frac{7}{2}}}+\frac{5\,\sqrt{cx^4+bx^2}\,b^2}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(c*x^4 + b*x^2)*x^4/c - 5/24*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 5/32*b
^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 5/16*sqrt(c*x
^4 + b*x^2)*b^2/c^3
```

Fricas [A]
time = 0.35, size = 166, normalized size = 1.46

$$\left[\frac{15\,b^3\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,(8\,c^3x^4-10\,bc^2x^2+15\,b^2c)\sqrt{cx^4+bx^2}}{96\,c^4},\ \frac{15\,b^3\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+(8\,c^3x^4-10\,bc^2x^2+15\,b^2c)\sqrt{cx^4+bx^2}}{48\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*b^3*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*(8*c^3*x^4 - 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2))/c^4, 1/48*(15*b
^3*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*c^3*x^4 -
 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 4.38, size = 97, normalized size = 0.85

$$\frac{1}{48}\sqrt{cx^2+b}\left(2\,x^2\left(\frac{4\,x^2}{c\mathrm{sgn}\left(x\right)}-\frac{5\,b}{c^2\mathrm{sgn}\left(x\right)}\right)+\frac{15\,b^2}{c^3\mathrm{sgn}\left(x\right)}\right)x-\frac{5\,b^3\log\left(|b|\right)\mathrm{sgn}(x)}{32\,c^{\frac{7}{2}}}+\frac{5\,b^3\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\,\right|\right)}{16\,c^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^2 + b)*(2*x^2*(4*x^2/(c*sgn(x)) - 5*b/(c^2*sgn(x))) + 15*b^2/
(c^3*sgn(x)))*x - 5/32*b^3*log(abs(b))*sgn(x)/c^(7/2) + 5/16*b^3*log(abs(-s
qrt(c)*x + sqrt(c*x^2 + b)))/(c^(7/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^7/(b*x^2 + c*x^4)^(1/2), x)
```

3.261 $\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=86

$$-\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}+\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}$$

```
[Out] 3/8*b^2*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-3/8*b*(c*x^4+b*x^2
)^(1/2)/c^2+1/4*x^2*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 684, 654, 634, 212}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-3*b*Sqrt[b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[b*x^2 + c*x^4])/(4*c) + (3*b
^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{x^2\sqrt{bx^2+cx^4}}{4c} - \frac{(3b)\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{8c}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^2}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^2}\\
&= -\frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 91, normalized size = 1.06

$$\frac{x\left(\sqrt{c}\,x(-3b^2-bcx^2+2c^2x^4)-3b^2\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{8c^{5/2}\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(-3*b^2 - b*c*x^2 + 2*c^2*x^4) - 3*b^2*Sqrt[b + c*x^2]*Log[-(
Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(8*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 85, normalized size = 0.99

method	result	size
default	$\frac{x\sqrt{c\,x^{2}+b}\,\left(2x^{3}\sqrt{c\,x^{2}+b}\,c^{\frac{5}{2}}-3c^{\frac{3}{2}}\sqrt{c\,x^{2}+b}\,bx+3\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)b^{2}c\right)}{8\sqrt{c\,x^{4}+b\,x^{2}}\,c^{\frac{7}{2}}}$	85
risch	$-\frac{x^{2}\left(-2c\,x^{2}+3b\right)\left(c\,x^{2}+b\right)}{8c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{3b^{2}\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)x\sqrt{c\,x^{2}+b}}{8c^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(c*x^2+b)^(1/2)*(2*x^3*(c*x^2+b)^(1/2)*c^(5/2)-3*c^(3/2)*(c*x^2+b)^(1
/2)*b*x+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2*c)/(c*x^4+b*x^2)^(1/2)/c^(7/2)
```

Maxima [A]
time = 0.29, size = 76, normalized size = 0.88

$$\frac{\sqrt{cx^{4}+bx^{2}}\,x^{2}}{4\,c}+\frac{3\,b^{2}\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{16\,c^{\frac{5}{2}}}-\frac{3\,\sqrt{cx^{4}+bx^{2}}\,b}{8\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^4 + b*x^2)*x^2/c + 3/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b
*x^2)*sqrt(c))/c^(5/2) - 3/8*sqrt(c*x^4 + b*x^2)*b/c^2
```

Fricas [A]
time = 0.40, size = 145, normalized size = 1.69

$$\left[\frac{3\,b^{2}\sqrt{c}\,\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\,\sqrt{cx^{4}+bx^{2}}\,(2\,c^{2}x^{2}-3\,bc)}{16\,c^{3}},-\frac{3\,b^{2}\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)-\sqrt{cx^{4}+bx^{2}}\,(2\,c^{2}x^{2}-3\,bc)}{8\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^2*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*
sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 - 3*b*c))/c^3, -1/8*(3*b^2*sqrt(-c)*arctan(s
qrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 -
 3*b*c))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{5}}{\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 3.20, size = 79, normalized size = 0.92

$$\frac{1}{8}\sqrt{cx^2+b}\,x\left(\frac{2\,x^2}{c\mathrm{sgn}\left(x\right)}-\frac{3\,b}{c^2\mathrm{sgn}\left(x\right)}\right)+\frac{3\,b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}}-\frac{3\,b^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\,\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^2 + b)*x*(2*x^2/(c*sgn(x)) - 3*b/(c^2*sgn(x))) + 3/16*b^2*log(
abs(b))*sgn(x)/c^(5/2) - 3/8*b^2*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^
(5/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^5/(b*x^2 + c*x^4)^(1/2), x)
```

3.262 $\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] -1/2*b*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/2*(c*x^4+b*x^2)^(
1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2043, 654, 634, 212}

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/
(2*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right) \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{4c} \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c} \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 1.28

$$\frac{x\left(\sqrt{c}\,x(b+cx^2)+b\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2) + b*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c
*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 64, normalized size = 1.10

method	result	size
default	$\frac{x\sqrt{c\,x^2+b}\,\left(x\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}-b\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)c\right)}{2\sqrt{c\,x^4+b\,x^2}\,c^{\frac{5}{2}}}$	64
risch	$\frac{x^2(c\,x^2+b)}{2c\sqrt{x^2\left(c\,x^2+b\right)}}-\frac{b\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)x\sqrt{c\,x^2+b}}{2c^{\frac{3}{2}}\sqrt{x^2\left(c\,x^2+b\right)}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(c*x^2+b)^(1/2)*(x*(c*x^2+b)^(1/2)*c^(3/2)-b*ln(x*c^(1/2)+(c*x^2+b)^(
1/2))*c)/(c*x^4+b*x^2)^(1/2)/c^(5/2)
```

Maxima [A]
time = 0.29, size = 52, normalized size = 0.90

$$-\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 1/2*sqrt(
c*x^4 + b*x^2)/c
```

Fricas [A]
time = 0.42, size = 114, normalized size = 1.97

$$\left[\frac{b\sqrt{c}\,\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\,c}{4\,c^2},\ \frac{b\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}\,c}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/
(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 3.04, size = 59, normalized size = 1.02

$$-\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^2+b}\,x}{2\,c\operatorname{sgn}\left(x\right)}+\frac{b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\,\right|\right)}{2\,c^{\frac{3}{2}}\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b*log(abs(b))*sgn(x)/c^(3/2) + 1/2*sqrt(c*x^2 + b)*x/(c*sgn(x)) + 1/2*
b*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(3/2)*sgn(x))
```

Mupad [B]
time = 4.30, size = 53, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{2\,c}-\frac{b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4
)^(1/2)))/(4*c^(3/2))
```

3.263 $\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

```
[Out] arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2038, 634, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.68

$$\frac{x\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\,\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]*Sqrt[x^2*
(b + c*x^2)])
```

Maple [A]
time = 0.09, size = 44, normalized size = 1.42

method	result	size
default	$\frac{x\sqrt{c\,x^2+b}\,\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)}{\sqrt{c\,x^4+b\,x^2}\,\sqrt{c}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)*ln(x*c^(1/2)+(c*x^2+b)^(1/2))/c^(1/
2)
```

Maxima [A]
time = 0.29, size = 32, normalized size = 1.03

$$\frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c)
```

Fricas [A]
time = 0.36, size = 74, normalized size = 2.39

$$\left[\frac{\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\,\sqrt{c}},-\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c), -sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 2.66, size = 38, normalized size = 1.23

$$\frac{\log\left(|b|\right)\operatorname{sgn}(x)}{2\,\sqrt{c}}-\frac{\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{\sqrt{c}\,\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*log(abs(b))*sgn(x)/sqrt(c) - log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(sq
rt(c)*sgn(x))
```

Mupad [B]
time = 4.36, size = 33, normalized size = 1.06

$$\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^(1/2),x)
[Out] log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(1/2))
```

3.264 $\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

```
[Out] -(c*x^4+b*x^2)^(1/2)/b/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2039}

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Mathematica [A]
time = 0.03, size = 23, normalized size = 1.00

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

Maple [A]
time = 0.08, size = 26, normalized size = 1.13

method	result	size
trager	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{b\,x^{2}}$	22
gosper	$-\frac{c\,x^{2}+b}{b\sqrt{c\,x^{4}+b\,x^{2}}}$	26
default	$-\frac{c\,x^{2}+b}{b\sqrt{c\,x^{4}+b\,x^{2}}}$	26
risch	$-\frac{c\,x^{2}+b}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+b)/b/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.30, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Fricas [A]
time = 0.33, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 4.60, size = 34, normalized size = 1.48

$$\frac{2\sqrt{c}}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*sgn(x))
```

Mupad [B]
time = 4.21, size = 21, normalized size = 0.91

$$-\frac{\sqrt{c\,x^4+b\,x^2}}{b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^(1/2)),x)
[Out] -(b*x^2 + c*x^4)^(1/2)/(b*x^2)
```

3.265 $\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{bx^2+cx^4}}{3bx^4}+\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

```
[Out] -1/3*(c*x^4+b*x^2)^(1/2)/b/x^4+2/3*c*(c*x^4+b*x^2)^(1/2)/b^2/x^2
```

Rubi [A]

time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2039}

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/3*Sqrt[b*x^2 + c*x^4]/(b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{3bx^4}-\frac{(2c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{3bx^4}+\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 35, normalized size = 0.67

$$\frac{\sqrt{x^2 (b + cx^2)} (-b + 2cx^2)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

Maple [A]
time = 0.11, size = 37, normalized size = 0.71

method	result	size
trager	$-\frac{(-2c\,x^2+b)\sqrt{c\,x^4+b\,x^2}}{3b^2x^4}$	30
gosper	$-\frac{(c\,x^2+b)(-2c\,x^2+b)}{3x^2b^2\sqrt{c\,x^4+b\,x^2}}$	37
default	$-\frac{(c\,x^2+b)(-2c\,x^2+b)}{3x^2b^2\sqrt{c\,x^4+b\,x^2}}$	37
risch	$-\frac{(c\,x^2+b)(-2c\,x^2+b)}{3x^2\sqrt{x^2(c\,x^2+b)}\,b^2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(c*x^2+b)*(-2*c*x^2+b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.30, size = 44, normalized size = 0.85

$$\frac{2\sqrt{cx^4 + bx^2}\,c}{3\,b^2x^2} - \frac{\sqrt{cx^4 + bx^2}}{3\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/(b*x^4)
```

Fricas [A]
time = 0.39, size = 31, normalized size = 0.60

$$\frac{\sqrt{cx^4 + bx^2}\,(2\,cx^2 - b)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 3.77, size = 59, normalized size = 1.13

$$\frac{4\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)c^{\frac{3}{2}}}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^3\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*c^(3/2)/(((sqrt(c)*x - sqrt(c*x
^2 + b))^2 - b)^3*sgn(x))
```

Mupad [B]
time = 4.26, size = 29, normalized size = 0.56

$$-\frac{\left(b-2\,c\,x^2\right)\,\sqrt{c\,x^4+b\,x^2}}{3\,b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((b - 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b^2*x^4)
```

3.266 $\int \frac{1}{x^5 \sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{bx^2+cx^4}}{5bx^6}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}$$

```
[Out] -1/5*(c*x^4+b*x^2)^(1/2)/b/x^6+4/15*c*(c*x^4+b*x^2)^(1/2)/b^2/x^4-8/15*c^2*
(c*x^4+b*x^2)^(1/2)/b^3/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 2039}

$$-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/5*Sqrt[b*x^2 + c*x^4]/(b*x^6) + (4*c*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^4) -
 (8*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{5bx^6} - \frac{(4c)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{5b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{5bx^6} + \frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4} + \frac{(8c^2)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{5bx^6} + \frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4} - \frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 0.58

$$-\frac{\sqrt{x^2(b+cx^2)}\,(3b^2-4bcx^2+8c^2x^4)}{15b^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/15*(Sqrt[x^2*(b + c*x^2)]*(3*b^2 - 4*b*c*x^2 + 8*c^2*x^4))/(b^3*x^6)
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.62

method	result	size
trager	$-\frac{(8c^2x^4-4bc\,x^2+3b^2)\sqrt{c\,x^4+b\,x^2}}{15b^3x^6}$	43
gosper	$-\frac{(c\,x^2+b)(8c^2x^4-4bc\,x^2+3b^2)}{15x^4b^3\sqrt{c\,x^4+b\,x^2}}$	50
default	$-\frac{(c\,x^2+b)(8c^2x^4-4bc\,x^2+3b^2)}{15x^4b^3\sqrt{c\,x^4+b\,x^2}}$	50
risch	$-\frac{(c\,x^2+b)(8c^2x^4-4bc\,x^2+3b^2)}{15x^4\sqrt{x^2(c\,x^2+b)}\,b^3}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(c*x^2+b)*(8*c^2*x^4-4*b*c*x^2+3*b^2)/x^4/b^3/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 68, normalized size = 0.85

$$-\frac{8\sqrt{cx^4+bx^2}\,c^2}{15\,b^3x^2} + \frac{4\sqrt{cx^4+bx^2}\,c}{15\,b^2x^4} - \frac{\sqrt{cx^4+bx^2}}{5\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/15*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) + 4/15*sqrt(c*x^4 + b*x^2)*c/(b^2*x
^4) - 1/5*sqrt(c*x^4 + b*x^2)/(b*x^6)
```

Fricas [A]
time = 0.37, size = 42, normalized size = 0.52

$$-\frac{(8\,c^2x^4-4\,bcx^2+3\,b^2)\sqrt{cx^4+bx^2}}{15\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*c^2*x^4 - 4*b*c*x^2 + 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^3*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^5\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 3.29, size = 81, normalized size = 1.01

$$\frac{16\left(10\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4-5\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2b+b^2\right)c^{\frac{5}{2}}}{15\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^5\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 16/15*(10*(sqrt(c)*x - sqrt(c*x^2 + b))^4 - 5*(sqrt(c)*x - sqrt(c*x^2 + b))
^2*b + b^2)*c^(5/2)/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5*sgn(x))
```

Mupad [B]
time = 4.33, size = 42, normalized size = 0.52

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(3\,b^2-4\,b\,c\,x^2+8\,c^2\,x^4\right)}{15\,b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(3*b^2 + 8*c^2*x^4 - 4*b*c*x^2))/(15*b^3*x^6)
```

3.267 $\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{bx^2+cx^4}}{7bx^8}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}$$

```
[Out] -1/7*(c*x^4+b*x^2)^(1/2)/b/x^8+6/35*c*(c*x^4+b*x^2)^(1/2)/b^2/x^6-8/35*c^2*
(c*x^4+b*x^2)^(1/2)/b^3/x^4+16/35*c^3*(c*x^4+b*x^2)^(1/2)/b^4/x^2
```

Rubi [A]
time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2041, 2039}

$$\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/7*Sqrt[b*x^2 + c*x^4]/(b*x^8) + (6*c*Sqrt[b*x^2 + c*x^4])/(35*b^2*x^6) -
 (8*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^4) + (16*c^3*Sqrt[b*x^2 + c*x^4])/(3
5*b^4*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(6c)\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{7b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{(24c^2)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{35b^2}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4} - \frac{(16c^3)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{35b^3}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{7bx^8} + \frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4} + \frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}$$

Mathematica [A]
time = 0.06, size = 57, normalized size = 0.53

$$\frac{\sqrt{x^2(b+cx^2)}\,(-5b^3+6b^2cx^2-8bc^2x^4+16c^3x^6)}{35b^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-5*b^3 + 6*b^2*c*x^2 - 8*b*c^2*x^4 + 16*c^3*x^6))/(
35*b^4*x^8)
```

Maple [A]
time = 0.09, size = 61, normalized size = 0.56

method	result	size
trager	$-\frac{(-16c^3x^6+8b\,c^2x^4-6b^2c\,x^2+5b^3)\sqrt{c\,x^4+b\,x^2}}{35b^4x^8}$	54
gosper	$-\frac{(c\,x^2+b)(-16c^3x^6+8b\,c^2x^4-6b^2c\,x^2+5b^3)}{35x^6b^4\sqrt{c\,x^4+b\,x^2}}$	61
default	$-\frac{(c\,x^2+b)(-16c^3x^6+8b\,c^2x^4-6b^2c\,x^2+5b^3)}{35x^6b^4\sqrt{c\,x^4+b\,x^2}}$	61
risch	$-\frac{(c\,x^2+b)(-16c^3x^6+8b\,c^2x^4-6b^2c\,x^2+5b^3)}{35x^6\sqrt{x^2(c\,x^2+b)}\,b^4}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(c*x^2+b)*(-16*c^3*x^6+8*b*c^2*x^4-6*b^2*c*x^2+5*b^3)/x^6/b^4/(c*x^4+
b*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 92, normalized size = 0.85

$$\frac{16\sqrt{cx^4+bx^2}\,c^3}{35\,b^4x^2}-\frac{8\sqrt{cx^4+bx^2}\,c^2}{35\,b^3x^4}+\frac{6\sqrt{cx^4+bx^2}\,c}{35\,b^2x^6}-\frac{\sqrt{cx^4+bx^2}}{7\,bx^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 16/35*sqrt(c*x^4 + b*x^2)*c^3/(b^4*x^2) - 8/35*sqrt(c*x^4 + b*x^2)*c^2/(b^3
*x^4) + 6/35*sqrt(c*x^4 + b*x^2)*c/(b^2*x^6) - 1/7*sqrt(c*x^4 + b*x^2)/(b*x
^8)
```

Fricas [A]
time = 0.35, size = 53, normalized size = 0.49

$$\frac{\left(16\,c^3x^6-8\,bc^2x^4+6\,b^2cx^2-5\,b^3\right)\sqrt{cx^4+bx^2}}{35\,b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*c^3*x^6 - 8*b*c^2*x^4 + 6*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4 + b*x^2)/(
b^4*x^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^7\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 4.28, size = 107, normalized size = 0.99

$$\frac{32\left(35\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6-21\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4b+7\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2b^2-b^3\right)c^{\frac{7}{2}}}{35\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^7\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^6 - 21*(sqrt(c)*x - sqrt(c*x^2 + b)
)^4*b + 7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^2 - b^3)*c^(7/2)/(((sqrt(c)*x -
 sqrt(c*x^2 + b))^2 - b)^7*sgn(x))
```

Mupad [B]
time = 4.27, size = 92, normalized size = 0.85

$$\frac{6\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^6}-\frac{\sqrt{c\,x^4+b\,x^2}}{7\,b\,x^8}-\frac{8\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b^3\,x^4}+\frac{16\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (6*c*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*b*x^8)
- (8*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b^3*x^4) + (16*c^3*(b*x^2 + c*x^4)^(1/2
))/(35*b^4*x^2)
```

3.268 $\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=50

$$-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}+\frac{x\sqrt{bx^2+cx^4}}{3c}$$

```
[Out] -2/3*b*(c*x^4+b*x^2)^(1/2)/c^2/x+1/3*x*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]

time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2041, 1602}

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx&=\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{(2b)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c}\\&=-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}+\frac{x\sqrt{bx^2+cx^4}}{3c}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.68

$$\frac{(-2b+cx^2)\sqrt{x^2(b+cx^2)}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x^2 + c*x^4],x]
[Out] ((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

Maple [A]
time = 0.09, size = 37, normalized size = 0.74

method	result	size
trager	$-\frac{(-c\,x^2+2b)\sqrt{c\,x^4+b\,x^2}}{3c^2x}$	32
gosper	$-\frac{(c\,x^2+b)(-c\,x^2+2b)x}{3c^2\sqrt{c\,x^4+b\,x^2}}$	37
default	$-\frac{(c\,x^2+b)(-c\,x^2+2b)x}{3c^2\sqrt{c\,x^4+b\,x^2}}$	37
risch	$-\frac{x(c\,x^2+b)(-c\,x^2+2b)}{3\sqrt{x^2(c\,x^2+b)}\,c^2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(c*x^2+b)*(-c*x^2+2*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.30, size = 34, normalized size = 0.68

$$\frac{c^2x^4-bcx^2-2\,b^2}{3\,\sqrt{cx^2+b}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

Fricas [A]
time = 0.38, size = 30, normalized size = 0.60

$$\frac{\sqrt{cx^4+bx^2}\,(cx^2-2\,b)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**4+b*x**2)**(1/2),x)

[Out] Integral(x**4/sqrt(x**2*(b + c*x**2)), x)

Giac [A]
time = 4.02, size = 48, normalized size = 0.96

$$\frac{2\,b^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,c^2}+\frac{\left(cx^2+b\right)^{\frac{3}{2}}}{3\,c^2\mathrm{sgn}\left(x\right)}-\frac{\sqrt{cx^2+b}\,b}{c^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")

[Out] 2/3*b^(3/2)*sgn(x)/c^2 + 1/3*(c*x^2 + b)^(3/2)/(c^2*sgn(x)) - sqrt(c*x^2 + b)*b/(c^2*sgn(x))

Mupad [B]
time = 4.25, size = 33, normalized size = 0.66

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{2\,b}{3\,c^2}-\frac{x^2}{3\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x^2 + c*x^4)^(1/2),x)

[Out] -((b*x^2 + c*x^4)^(1/2)*((2*b)/(3*c^2) - x^2/(3*c)))/x

3.269 $\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

```
[Out] (c*x^4+b*x^2)^(1/2)/c/x
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{1602\}$

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{bx^2+cx^4}}{cx}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\sqrt{x^2\,(b+cx^2)}}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[x^2*(b + c*x^2)]/(c*x)
```

Maple [A]
time = 0.09, size = 26, normalized size = 1.18

method	result	size
trager	$\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{cx}$	21
gosper	$\frac{\left(c\,x^{2}+b\right)x}{c\sqrt{c\,x^{4}+b\,x^{2}}}$	26
default	$\frac{\left(c\,x^{2}+b\right)x}{c\sqrt{c\,x^{4}+b\,x^{2}}}$	26
risch	$\frac{x\left(c\,x^{2}+b\right)}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,c}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (c*x^2+b)/c*x/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.59

$$\frac{\sqrt{cx^2+b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(c*x^2 + b)/c
```

Fricas [A]
time = 0.37, size = 20, normalized size = 0.91

$$\frac{\sqrt{cx^4+bx^2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**2/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 4.28, size = 28, normalized size = 1.27

$$-\frac{\sqrt{b}\,\mathrm{sgn}(x)}{c}+\frac{\sqrt{cx^2+b}}{c\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] -sqrt(b)*sgn(x)/c + sqrt(c*x^2 + b)/(c*sgn(x))
```

Mupad [B]
time = 4.26, size = 20, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^(1/2),x)
[Out] (b*x^2 + c*x^4)^(1/2)/(c*x)
```

3.270 $\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] -arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2033, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^2 + c*x^4],x]
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx &= -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 1.73

$$-\frac{x\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -((x*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[x^2*(b
 + c*x^2)]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.
time = 0.09, size = 50, normalized size = 1.67

method	result	size
default	$-\frac{x\sqrt{c\,x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)}{\sqrt{c\,x^4+b\,x^2}\,\sqrt{b}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1
/2)+b)/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2), x)
```

Fricas [A]
time = 0.39, size = 80, normalized size = 2.67

$$\left[\frac{\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)/sqrt(b), sqr
t(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*x**2 + c*x**4), x)
```

Giac [A]
time = 2.86, size = 46, normalized size = 1.53

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(c*x^2 + b)/sqrt(-b)
)/(sqrt(-b)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{c\,x^4 + b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(1/(b*x^2 + c*x^4)^(1/2), x)
```

3.271 $\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}$$

```
[Out] 1/2*c*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/2*(c*x^4+b*x^2)^(1/2
)/b/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2050, 2033, 212}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^
4]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}-\frac{c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 76, normalized size = 1.29

$$\frac{-\sqrt{b}\,(b+cx^2)+cx^2\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{3/2}x\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-(Sqrt[b]*(b + c*x^2)) + c*x^2*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqr
t[b]])/(2*b^(3/2)*x*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 73, normalized size = 1.24

method	result	size
default	$-\frac{\sqrt{c\,x^2+b}\,\left(-c\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b\,x^2+\sqrt{c\,x^2+b}\,b^{\frac{3}{2}}\right)}{2x\sqrt{c\,x^4+b\,x^2}\,b^{\frac{5}{2}}}$	73
risch	$-\frac{c\,x^2+b}{2bx\sqrt{x^2\,(c\,x^2+b)}}+\frac{c\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)x\sqrt{c\,x^2+b}}{2b^{\frac{3}{2}}\sqrt{x^2\,(c\,x^2+b)}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x*(c*x^2+b)^(1/2)*(-c*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*x^2+(c*x^2
+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

Fricas [A]
time = 0.37, size = 133, normalized size = 2.25

$$\left[\frac{\sqrt{b}\,cx^{3}\log\left(-\frac{cx^{3}+2\,bx+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)-2\,\sqrt{cx^{4}+bx^{2}}\,b}{4\,b^{2}x^{3}},\,-\frac{\sqrt{-b}\,cx^{3}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)+\sqrt{cx^{4}+bx^{2}}\,b}{2\,b^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 5.47, size = 55, normalized size = 0.93

$$-\frac{\frac{c^{2}\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{\sqrt{cx^{2}+b}\,c}{bx^{2}}}{2\,c\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(c^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(c*x^2 + b)*c
/(b*x^2))/(c*sgn(x))
```

Mupad [B]
time = 4.47, size = 76, normalized size = 1.29

$$-\frac{\left(\frac{\sqrt{c}\,x^2\,\sqrt{c+\frac{b}{x^2}}}{2\,b}+\frac{c^{3/2}\,x^3\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,1\mathrm{i}}{2\,b^{3/2}}\right)\,\sqrt{\frac{b}{c\,x^2}+1}}{x\,\sqrt{c\,x^4+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((((c^(1/2)*x^2*(c + b/x^2)^(1/2))/(2*b) + (c^(3/2)*x^3*asin((b^(1/2)*1i)/(
c^(1/2)*x))*1i)/(2*b^(3/2)))*(b/(c*x^2) + 1)^(1/2))/(x*(b*x^2 + c*x^4)^(1/2
))
```

3.272 $\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}$$

```
[Out] -3/8*c^2*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/4*(c*x^4+b*x^2)^(
1/2)/b/x^5+3/8*c*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2050, 2033, 212}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/4*Sqrt[b*x^2 + c*x^4]/(b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^3) -
(3*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx &= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}-\frac{(3c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b}\\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}+\frac{(3c^2)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{(3c^2)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^2}\\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 91, normalized size = 1.05

$$
\frac{\sqrt{b}\,(-2b^2+bcx^2+3c^2x^4)-3c^2x^4\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{8b^{5/2}x^3\sqrt{x^2\,(b+cx^2)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[b]*(-2*b^2 + b*c*x^2 + 3*c^2*x^4) - 3*c^2*x^4*Sqrt[b + c*x^2]*ArcTanh
[Sqrt[b + c*x^2]/Sqrt[b]])/(8*b^(5/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.12, size = 94, normalized size = 1.08

method	result	size
default	$-\frac{\sqrt{c\,x^2+b}\,\left(3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b\,c^2x^4-3b^{\frac{3}{2}}\sqrt{c\,x^2+b}\,c\,x^2+2\sqrt{c\,x^2+b}\,b^{\frac{5}{2}}\right)}{8x^3\sqrt{c\,x^4+b\,x^2}\,b^{\frac{7}{2}}}$	94
risch	$-\frac{(c\,x^2+b)\,(-3c\,x^2+2b)}{8b^2x^3\sqrt{x^2\,(c\,x^2+b)}}-\frac{3c^2\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)x\sqrt{c\,x^2+b}}{8b^{\frac{5}{2}}\sqrt{x^2\,(c\,x^2+b)}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/x^3*(c*x^2+b)^(1/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*c^2*x^4-3
*b^(3/2)*(c*x^2+b)^(1/2)*c*x^2+2*(c*x^2+b)^(1/2)*b^(5/2))/(c*x^4+b*x^2)^(1/
2)/b^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

Fricas [A]
time = 0.38, size = 163, normalized size = 1.87

$$\left[\frac{3\sqrt{b}\,c^2x^5\log\left(-\frac{cx^3+2\,bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\sqrt{cx^4+bx^2}\,(3\,bcx^2-2\,b^2)}{16\,b^3x^5},\ \frac{3\sqrt{-b}\,c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,(3\,bcx^2-2\,b^2)}{8\,b^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5), 1/8*(3*sqrt
(-b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^
4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 3.44, size = 79, normalized size = 0.91

$$\frac{\frac{3\,c^3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^2}+\frac{3\,(cx^2+b)^{\frac{3}{2}}c^3-5\sqrt{cx^2+b}\,bc^3}{b^2c^2x^4}}{8\,\mathrm{csgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(c*x^2 + b)
^(3/2)*c^3 - 5*sqrt(c*x^2 + b)*b*c^3)/(b^2*c^2*x^4))/(c*sgn(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

3.273 $\int \frac{x^9}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{x^6}{c\sqrt{bx^2+cx^4}}-\frac{15b\sqrt{bx^2+cx^4}}{8c^3}+\frac{5x^2\sqrt{bx^2+cx^4}}{4c^2}+\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}$$

```
[Out] 15/8*b^2*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)-x^6/c/(c*x^4+b*x^
2)^(1/2)-15/8*b*(c*x^4+b*x^2)^(1/2)/c^3+5/4*x^2*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 682, 684, 654, 634, 212}

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}-\frac{15b\sqrt{bx^2+cx^4}}{8c^3}+\frac{5x^2\sqrt{bx^2+cx^4}}{4c^2}-\frac{x^6}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^6/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/(8*c^3) + (5*x^
2*Sqrt[b*x^2 + c*x^4])/(4*c^2) + (15*b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(8*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^9}{(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} + \frac{5\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{2c}$$
$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} - \frac{(15b)\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{8c^2}$$
$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{(15b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\right.}{16c^3}$$
$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{(15b^2)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x,\right.}{8c^3}$$
$$= -\frac{x^6}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{8c^3} + \frac{5x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}$$

Mathematica [A]
time = 0.09, size = 91, normalized size = 0.83

$$\frac{x\left(\sqrt{c}\,x(-15b^2-5bcx^2+2c^2x^4)-15b^2\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{8c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(Sqrt[c]*x*(-15*b^2 - 5*b*c*x^2 + 2*c^2*x^4) - 15*b^2*Sqrt[b + c*x^2]*Lo
g[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(8*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 87, normalized size = 0.80

method	result	size
default	$\frac{x^3(c\,x^2+b)\left(2x^5c^{\frac{7}{2}}-5c^{\frac{5}{2}}b\,x^3-15c^{\frac{3}{2}}b^2x+15\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)\sqrt{c\,x^2+b}\,b^2c\right)}{8(c\,x^4+b\,x^2)^{\frac{3}{2}}c^{\frac{9}{2}}}$	87

risch	$-\frac{x^2(-2cx^2+7b)(cx^2+b)}{8c^3\sqrt{x^2(cx^2+b)}}+\frac{\left(-\frac{b^2x}{c^3\sqrt{cx^2+b}}+\frac{15b^2\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)}{8c^{\frac{7}{2}}}\right)x\sqrt{cx^2+b}}{\sqrt{x^2(cx^2+b)}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^3*(c*x^2+b)*(2*x^5*c^(7/2)-5*c^(5/2)*b*x^3-15*c^(3/2)*b^2*x+15*ln(x*c
^(1/2)+(c*x^2+b)^(1/2))*(c*x^2+b)^(1/2)*b^2*c)/(c*x^4+b*x^2)^(3/2)/c^(9/2)
```

Maxima [A]
time = 0.31, size = 103, normalized size = 0.94

$$\frac{x^6}{4\sqrt{cx^4+bx^2}\,c}-\frac{5\,bx^4}{8\sqrt{cx^4+bx^2}\,c^2}-\frac{15\,b^2x^2}{8\sqrt{cx^4+bx^2}\,c^3}+\frac{15\,b^2\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{16\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^6/(sqrt(c*x^4 + b*x^2)*c) - 5/8*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) - 15/
8*b^2*x^2/(sqrt(c*x^4 + b*x^2)*c^3) + 15/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*
x^4 + b*x^2)*sqrt(c))/c^(7/2)
```

Fricas [A]
time = 0.41, size = 209, normalized size = 1.92

$$\left[\frac{15\,(b^2cx^2+b^3)\sqrt{c}\,\log\left(-2\,cx^2-b-2\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,(2\,c^3x^4-5\,bc^2x^2-15\,b^2c)\sqrt{cx^4+bx^2}}{16\,(c^5x^2+bc^4)},-\frac{15\,(b^2cx^2+b^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(2\,c^3x^4-5\,bc^2x^2-15\,b^2c)\sqrt{cx^4+bx^2}}{8\,(c^5x^2+bc^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(b^2*c*x^2 + b^3)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2
)*sqrt(c)) + 2*(2*c^3*x^4 - 5*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^4 + b*x^2))/(c
^5*x^2 + b*c^4), -1/8*(15*(b^2*c*x^2 + b^3)*sqrt(-c)*arctan(sqrt(c*x^4 + b*
x^2)*sqrt(-c)/(c*x^2 + b)) - (2*c^3*x^4 - 5*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^
4 + b*x^2))/(c^5*x^2 + b*c^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^9}{\left(x^2\,(b+cx^2)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**9/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 3.30, size = 96, normalized size = 0.88

$$\frac{\left(x^2\left(\frac{2\,x^2}{c\mathrm{sgn}(x)}-\frac{5\,b}{c^2\mathrm{sgn}(x)}\right)-\frac{15\,b^2}{c^3\mathrm{sgn}(x)}\right)x}{8\,\sqrt{cx^2+b}}+\frac{15\,b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,c^{\frac{7}{2}}}-\frac{15\,b^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\,\right|\right)}{8\,c^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(x^2*(2*x^2/(c*sgn(x)) - 5*b/(c^2*sgn(x))) - 15*b^2/(c^3*sgn(x)))*x/sqr
t(c*x^2 + b) + 15/16*b^2*log(abs(b))*sgn(x)/c^(7/2) - 15/8*b^2*log(abs(-sqr
t(c)*x + sqrt(c*x^2 + b)))/(c^(7/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^9/(b*x^2 + c*x^4)^(3/2), x)
```

3.274 $\int \frac{x^7}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{x^4}{c\sqrt{bx^2+cx^4}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}$$

```
[Out] -3/2*b*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-x^4/c/(c*x^4+b*x^2)
^(1/2)+3/2*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 682, 654, 634, 212}

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{x^4}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^4/(c*Sqrt[b*x^2 + c*x^4])) + (3*Sqrt[b*x^2 + c*x^4])/(2*c^2) - (3*b*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\left(bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}} + \frac{3\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{2c}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}} + \frac{3\sqrt{bx^2+cx^4}}{2c^2} - \frac{(3b)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{4c^2}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}} + \frac{3\sqrt{bx^2+cx^4}}{2c^2} - \frac{(3b)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^2}\\
&= -\frac{x^4}{c\sqrt{bx^2+cx^4}} + \frac{3\sqrt{bx^2+cx^4}}{2c^2} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 77, normalized size = 0.95

$$\frac{x\left(\sqrt{c}\,x(3b+cx^2)+3b\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{2c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(3*b + c*x^2) + 3*b*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b
 + c*x^2]]))/(2*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.09, size = 73, normalized size = 0.90

method	result	size
default	$\frac{x^{3} \left(c \,x^{2}+b \right) \left(x^{3} c^{\frac{5}{2}}+3 c^{\frac{3}{2}} b x -3 \ln \left(x \sqrt{c}+\sqrt{c \,x^{2}+b}\right) \sqrt{c \,x^{2}+b}\, b c \right)}{2 \left(c \,x^{4}+b \,x^{2}\right)^{\frac{3}{2}} c^{\frac{7}{2}}}$	73
risch	$\frac{x^{2} \left(c \,x^{2}+b \right)}{2 c^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}}+\frac{\left(\frac{b x}{c^{2} \sqrt{c \,x^{2}+b}}-\frac{3 b \ln \left(x \sqrt{c}+\sqrt{c \,x^{2}+b}\right)}{2 c^{\frac{5}{2}}}\right) x \sqrt{c \,x^{2}+b}}{\sqrt{x^{2} \left(c \,x^{2}+b \right)}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^3*(c*x^2+b)*(x^3*c^(5/2)+3*c^(3/2)*b*x-3*ln(x*c^(1/2)+(c*x^2+b)^(1/2)
)*(c*x^2+b)^(1/2)*b*c)/(c*x^4+b*x^2)^(3/2)/c^(7/2)
```

Maxima [A]
time = 0.29, size = 77, normalized size = 0.95

$$\frac{x^{4}}{2 \sqrt{cx^{4} + bx^{2}} c} + \frac{3 \, bx^{2}}{2 \sqrt{cx^{4} + bx^{2}} c^{2}} - \frac{3 \, b \log \left(2 \, cx^{2} + b + 2 \, \sqrt{cx^{4} + bx^{2}} \sqrt{c}\right)}{4 \, c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 3/2*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3/4
*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2)
```

Fricas [A]
time = 0.36, size = 180, normalized size = 2.22

$$\left[\frac{3 \, {\left(bcx^{2} + b^{2}\right)} \sqrt{c} \log \left(-2 \, cx^{2} - b + 2 \, \sqrt{cx^{4} + bx^{2}} \sqrt{c}\right) + 2 \, \sqrt{cx^{4} + bx^{2}} {\left(c^{2} x^{2} + 3 \, bc\right)}}{4 \, {\left(c^{4} x^{2} + bc^{3}\right)}}, \frac{3 \, {\left(bcx^{2} + b^{2}\right)} \sqrt{-c} \arctan \left(\frac{\sqrt{cx^{4} + bx^{2}} \sqrt{-c}}{cx^{2} + b}\right) + \sqrt{cx^{4} + bx^{2}} {\left(c^{2} x^{2} + 3 \, bc\right)}}{2 \, {\left(c^{4} x^{2} + bc^{3}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b*c*x^2 + b^2)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sq
rt(c)) + 2*sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c))/(c^4*x^2 + b*c^3), 1/2*(3
*(b*c*x^2 + b^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))
+ sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c))/(c^4*x^2 + b*c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**7/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 5.48, size = 74, normalized size = 0.91

$$\frac{x\left(\frac{x^2}{c\mathrm{sgn}(x)}+\frac{3\,b}{c^2\mathrm{sgn}(x)}\right)}{2\,\sqrt{cx^2+b}}-\frac{3\,b\log\left(|b|\right)\mathrm{sgn}(x)}{4\,c^{\frac{5}{2}}}+\frac{3\,b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\right|\right)}{2\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] 1/2*x*(x^2/(c*sgn(x)) + 3*b/(c^2*sgn(x)))/sqrt(c*x^2 + b) - 3/4*b*log(abs(b
))*sgn(x)/c^(5/2) + 3/2*b*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(5/2)*s
gn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^7/(b*x^2 + c*x^4)^(3/2), x)
```

3.275 $\int \frac{x^5}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{x^2}{c\sqrt{bx^2+cx^4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}$$

```
[Out] arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)-x^2/c/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2043, 666, 634, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^2/(c*Sqrt[b*x^2 + c*x^4])) + ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
/c^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[e*(d + e*x)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[
e^2*((p + 2)/(c*(p + 1))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{x^2}{c\sqrt{bx^2+cx^4}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{2c}\\
&= -\frac{x^2}{c\sqrt{bx^2+cx^4}} + \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{c}\\
&= -\frac{x^2}{c\sqrt{bx^2+cx^4}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 64, normalized size = 1.16

$$-\frac{x\left(\sqrt{c}\,x+\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{c^{3/2}\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(b*x^2 + c*x^4)^(3/2),x]
[Out] -((x*(Sqrt[c]*x + Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(c^
(3/2)*Sqrt[x^2*(b + c*x^2)]))
```

Maple [A]
time = 0.09, size = 63, normalized size = 1.15

method	result	size
default	$-\frac{x^{3}\left(c\,x^{2}+b\right)\left(x\,c^{\frac{3}{2}}-\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)c\sqrt{c\,x^{2}+b}\right)}{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^3*(c*x^2+b)*(x*c^(3/2)-ln(x*c^(1/2)+(c*x^2+b)^(1/2))*c*(c*x^2+b)^(1/2))/
(c*x^4+b*x^2)^(3/2)/c^(5/2)
```

Maxima [A]
time = 0.34, size = 54, normalized size = 0.98

$$-\frac{x^2}{\sqrt{cx^4+bx^2}\,c}+\frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/(sqrt(c*x^4 + b*x^2)*c) + 1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*
sqrt(c))/c^(3/2)
```

Fricas [A]
time = 0.38, size = 150, normalized size = 2.73

$$\left[\frac{(cx^2+b)\sqrt{c}\,\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}\,c}{2\,(c^3x^2+bc^2)},-\frac{(cx^2+b)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}\,c}{c^3x^2+bc^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*x^2 + b)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
 - 2*sqrt(c*x^4 + b*x^2)*c)/(c^3*x^2 + b*c^2), -((c*x^2 + b)*sqrt(-c)*arcta
n(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/(c^3*x
^2 + b*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\left(x^2\,(b+cx^2)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 4.20, size = 57, normalized size = 1.04

$$\frac{\log\left(|b|\right)\mathrm{sgn}(x)}{2\,c^{\frac{3}{2}}}-\frac{x}{\sqrt{cx^2+b}\,\mathrm{csgn}\,(x)}-\frac{\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\,\right|\right)}{c^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b))*sgn(x)/c^(3/2) - x/(sqrt(c*x^2 + b)*c*sgn(x)) - log(abs(-sq
rt(c)*x + sqrt(c*x^2 + b)))/(c^(3/2)*sgn(x))
```

Mupad [B]
time = 4.33, size = 55, normalized size = 1.00

$$\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,c^{3/2}}-\frac{x^2}{c\,\sqrt{c\,x^4+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(3/2)) - x^2/(c*(b*
x^2 + c*x^4)^(1/2))
```

3.276 $\int \frac{x^3}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\frac{x^2}{b\sqrt{bx^2+cx^4}}$$

```
[Out] x^2/b/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2039}

$$\frac{x^2}{b\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^2/(b*Sqrt[b*x^2 + c*x^4])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{x^2}{b\sqrt{bx^2+cx^4}}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 1.00

$$\frac{x^2}{b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^2/(b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 28, normalized size = 1.27

method	result	size
trager	$\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{b(c\,x^{2}+b)}$	27
gosper	$\frac{(c\,x^{2}+b)x^{4}}{b(c\,x^{4}+b\,x^{2})^{\frac{3}{2}}}$	28
default	$\frac{(c\,x^{2}+b)x^{4}}{b(c\,x^{4}+b\,x^{2})^{\frac{3}{2}}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] (c*x^2+b)/b*x^4/(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.32, size = 20, normalized size = 0.91

$$\frac{x^2}{\sqrt{cx^4+bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] x^2/(sqrt(c*x^4 + b*x^2)*b)
```

Fricas [A]
time = 0.33, size = 26, normalized size = 1.18

$$\frac{\sqrt{cx^4+bx^2}}{bcx^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)/(b*c*x^2 + b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**3/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 5.62, size = 18, normalized size = 0.82

$$\frac{x}{\sqrt{cx^2+b}\, b\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] x/(sqrt(c*x^2 + b)*b*sgn(x))
```

Mupad [B]
time = 4.13, size = 26, normalized size = 1.18

$$\frac{\sqrt{c\,x^4+b\,x^2}}{b\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^(3/2),x)
[Out] (b*x^2 + c*x^4)^(1/2)/(b*(b + c*x^2))
```

3.277 $\int \frac{x}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

[Out] (-2*c*x^2-b)/b^2/(c*x^4+b*x^2)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2038, 627}

$$-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x/(b*x^2 + c*x^4)^(3/2),x]

[Out] -((b + 2*c*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(bx^2+cx^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 29, normalized size = 1.04

$$\frac{-b-2cx^2}{b^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^2 + c*x^4)^(3/2),x]
[Out] (-b - 2*c*x^2)/(b^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 37, normalized size = 1.32

method	result	size
gosper	$-\frac{x^{2} \left(c \,x^{2}+b \right) \left(2 c \,x^{2}+b \right)}{b^{2} \left(c \,x^{4}+b \,x^{2}\right)^{\frac{3}{2}}}$	37
default	$-\frac{x^{2} \left(c \,x^{2}+b \right) \left(2 c \,x^{2}+b \right)}{b^{2} \left(c \,x^{4}+b \,x^{2}\right)^{\frac{3}{2}}}$	37
trager	$-\frac{\left(2 c \,x^{2}+b \right) \sqrt{c \,x^{4}+b \,x^{2}}}{\left(c \,x^{2}+b \right) b^{2} x^{2}}$	39
risch	$-\frac{c \,x^{2}+b}{b^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}}-\frac{c \,x^{2}}{b^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -x^2*(c*x^2+b)*(2*c*x^2+b)/b^2/(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 41, normalized size = 1.46

$$-\frac{2\,cx^2}{\sqrt{cx^4+bx^2}\,b^2}-\frac{1}{\sqrt{cx^4+bx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] -2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) - 1/(sqrt(c*x^4 + b*x^2)*b)
```

Fricas [A]
time = 0.42, size = 41, normalized size = 1.46

$$-\frac{\sqrt{cx^4+bx^2}\,\left(2\,cx^2+b\right)}{b^2cx^4+b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2)*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.
time = 6.11, size = 58, normalized size = 2.07

$$-\frac{cx}{\sqrt{cx^2+b}\,b^2\mathrm{sgn}\left(x\right)}+\frac{2\sqrt{c}}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)b\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -c*x/(sqrt(c*x^2 + b)*b^2*sgn(x)) + 2*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + b
))^2 - b)*b*sgn(x))
```

Mupad [B]
time = 4.13, size = 26, normalized size = 0.93

$$-\frac{2\,c\,x^2+b}{b^2\,\sqrt{c\,x^4+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -(b + 2*c*x^2)/(b^2*(b*x^2 + c*x^4)^(1/2))
```

3.278 $\int \frac{1}{x(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{1}{bx^2\sqrt{bx^2+cx^4}}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}$$

```
[Out] 1/b/x^2/(c*x^4+b*x^2)^(1/2)-4/3*(c*x^4+b*x^2)^(1/2)/b^2/x^4+8/3*c*(c*x^4+b*
x^2)^(1/2)/b^3/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2040, 2041, 2039}

$$\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{1}{bx^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^2*Sqrt[b*x^2 + c*x^4]) - (4*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^4) + (8*c*
Sqrt[b*x^2 + c*x^4])/(3*b^3*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^2\sqrt{bx^2+cx^4}}+\frac{4\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= \frac{1}{bx^2\sqrt{bx^2+cx^4}}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}-\frac{(8c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b^2}\\
&= \frac{1}{bx^2\sqrt{bx^2+cx^4}}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 0.62

$$\frac{-b^2+4bcx^2+8c^2x^4}{3b^3x^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
[Out] (-b^2 + 4*b*c*x^2 + 8*c^2*x^4)/(3*b^3*x^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 45, normalized size = 0.61

method	result	size
gosper	$-\frac{(c\,x^2+b)\left(-8c^2x^4-4bc\,x^2+b^2\right)}{3b^3\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}$	45
default	$-\frac{(c\,x^2+b)\left(-8c^2x^4-4bc\,x^2+b^2\right)}{3b^3\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}$	45
trager	$-\frac{\left(-8c^2x^4-4bc\,x^2+b^2\right)\sqrt{c\,x^4+b\,x^2}}{3(c\,x^2+b)b^3x^4}$	50
risch	$-\frac{(c\,x^2+b)\left(-5c\,x^2+b\right)}{3b^3x^2\sqrt{x^2\left(c\,x^2+b\right)}}+\frac{x^2c^2}{b^3\sqrt{x^2\left(c\,x^2+b\right)}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(c*x^2+b)*(-8*c^2*x^4-4*b*c*x^2+b^2)/b^3/(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 65, normalized size = 0.88

$$\frac{8\,c^2x^2}{3\,\sqrt{cx^4+bx^2}\,b^3}+\frac{4\,c}{3\,\sqrt{cx^4+bx^2}\,b^2}-\frac{1}{3\,\sqrt{cx^4+bx^2}\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] 8/3*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4/3*c/(sqrt(c*x^4 + b*x^2)*b^2) - 1
/3/(sqrt(c*x^4 + b*x^2)*b*x^2)
```

Fricas [A]
time = 0.34, size = 54, normalized size = 0.73

$$\frac{(8\,c^2x^4+4\,bcx^2-b^2)\sqrt{cx^4+bx^2}}{3\,(b^3cx^6+b^4x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(8*c^2*x^4 + 4*b*c*x^2 - b^2)*sqrt(c*x^4 + b*x^2)/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\,(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(1/(x*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [A]
time = 5.83, size = 114, normalized size = 1.54

$$\frac{c^2x}{\sqrt{cx^2+b}\,b^3\mathrm{sgn}\,(x)}-\frac{2\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4c^{\frac{3}{2}}-12\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2bc^{\frac{3}{2}}+5\,b^2c^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^3b^2\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] c^2*x/(sqrt(c*x^2 + b)*b^3*sgn(x)) - 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^4
*c^(3/2) - 12*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b*c^(3/2) + 5*b^2*c^(3/2))/((
(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3*b^2*sgn(x))
```

Mupad [B]
time = 4.24, size = 51, normalized size = 0.69

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-b^2+4\,b\,c\,x^2+8\,c^2\,x^4\right)}{3\,b^3\,x^4\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - b^2 + 4*b*c*x^2))/(3*b^3*x^4*(b + c*x^2
))
```

3.279 $\int \frac{1}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=102

$$\frac{1}{bx^4\sqrt{bx^2+cx^4}}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}$$

```
[Out] 1/b/x^4/(c*x^4+b*x^2)^(1/2)-6/5*(c*x^4+b*x^2)^(1/2)/b^2/x^6+8/5*c*(c*x^4+b*
x^2)^(1/2)/b^3/x^4-16/5*c^2*(c*x^4+b*x^2)^(1/2)/b^4/x^2
```

Rubi [A]
time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2040, 2041, 2039}

$$-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{1}{bx^4\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^4*Sqrt[b*x^2 + c*x^4]) - (6*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^6) + (8*c*
Sqrt[b*x^2 + c*x^4])/(5*b^3*x^4) - (16*c^2*Sqrt[b*x^2 + c*x^4])/(5*b^4*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3 (bx^2+cx^4)^{3/2}}\,dx &= \frac{1}{bx^4\sqrt{bx^2+cx^4}} + \frac{6\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{b}\\ &= \frac{1}{bx^4\sqrt{bx^2+cx^4}} - \frac{6\sqrt{bx^2+cx^4}}{5b^2x^6} - \frac{(24c)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{5b^2}\\ &= \frac{1}{bx^4\sqrt{bx^2+cx^4}} - \frac{6\sqrt{bx^2+cx^4}}{5b^2x^6} + \frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4} + \frac{(16c^2)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{5b^3}\\ &= \frac{1}{bx^4\sqrt{bx^2+cx^4}} - \frac{6\sqrt{bx^2+cx^4}}{5b^2x^6} + \frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4} - \frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 0.56

$$\frac{-b^3+2b^2cx^2-8bc^2x^4-16c^3x^6}{5b^4x^4\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-b^3 + 2*b^2*c*x^2 - 8*b*c^2*x^4 - 16*c^3*x^6)/(5*b^4*x^4*Sqrt[x^2*(b + c*
x^2)])
```

Maple [A]
time = 0.10, size = 59, normalized size = 0.58

method	result	size
gosper	$-\frac{(c x^2+b)(16c^3x^6+8b c^2x^4-2b^2c x^2+b^3)}{5x^2b^4(c x^4+b x^2)^{\frac{3}{2}}}$	59
default	$-\frac{(c x^2+b)(16c^3x^6+8b c^2x^4-2b^2c x^2+b^3)}{5x^2b^4(c x^4+b x^2)^{\frac{3}{2}}}$	59
trager	$-\frac{(16c^3x^6+8b c^2x^4-2b^2c x^2+b^3)\sqrt{c x^4+b x^2}}{5(c x^2+b)b^4x^6}$	61
risch	$-\frac{(c x^2+b)(11c^2x^4-3bc x^2+b^2)}{5b^4x^4\sqrt{x^2(c x^2+b)}}-\frac{x^2c^3}{b^4\sqrt{x^2(c x^2+b)}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(c*x^2+b)*(16*c^3*x^6+8*b*c^2*x^4-2*b^2*c*x^2+b^3)/x^2/b^4/(c*x^4+b*x^
2)^(3/2)
```

Maxima [A]
time = 0.29, size = 89, normalized size = 0.87

$$-\frac{16\,c^3x^2}{5\,\sqrt{cx^4+bx^2}\,b^4}-\frac{8\,c^2}{5\,\sqrt{cx^4+bx^2}\,b^3}+\frac{2\,c}{5\,\sqrt{cx^4+bx^2}\,b^2x^2}-\frac{1}{5\,\sqrt{cx^4+bx^2}\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -16/5*c^3*x^2/(sqrt(c*x^4 + b*x^2)*b^4) - 8/5*c^2/(sqrt(c*x^4 + b*x^2)*b^3)
 + 2/5*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2) - 1/5/(sqrt(c*x^4 + b*x^2)*b*x^4)
```

Fricas [A]
time = 0.36, size = 63, normalized size = 0.62

$$-\frac{(16\,c^3x^6+8\,bc^2x^4-2\,b^2cx^2+b^3)\sqrt{cx^4+bx^2}}{5\,(b^4cx^8+b^5x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(16*c^3*x^6 + 8*b*c^2*x^4 - 2*b^2*c*x^2 + b^3)*sqrt(c*x^4 + b*x^2)/(b^
4*c*x^8 + b^5*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\,(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [A]
time = 5.35, size = 169, normalized size = 1.66

$$-\frac{c^3x}{\sqrt{cx^2+b}\,b^4\mathrm{sgn}\,(x)}+\frac{2\left(5\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^8c^{\frac{5}{2}}-30\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^6bc^{\frac{5}{2}}+80\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^4b^2c^{\frac{5}{2}}-50\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2b^3c^{\frac{5}{2}}+11\,b^4c^{\frac{5}{2}}\right)}{5\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^5b^3\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -c^3*x/(sqrt(c*x^2 + b)*b^4*sgn(x)) + 2/5*(5*(sqrt(c)*x - sqrt(c*x^2 + b))^
8*c^(5/2) - 30*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b*c^(5/2) + 80*(sqrt(c)*x -
sqrt(c*x^2 + b))^4*b^2*c^(5/2) - 50*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^3*c^(
5/2) + 11*b^4*c^(5/2))/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5*b^3*sgn(x))
```

Mupad [B]
time = 4.31, size = 60, normalized size = 0.59

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(b^3-2\,b^2\,c\,x^2+8\,b\,c^2\,x^4+16\,c^3\,x^6\right)}{5\,b^4\,x^6\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(b^3 + 16*c^3*x^6 - 2*b^2*c*x^2 + 8*b*c^2*x^4))/(5*
b^4*x^6*(b + c*x^2))
```

3.280 $\int \frac{1}{x^5(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{1}{bx^6\sqrt{bx^2+cx^4}}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}$$

```
[Out] 1/b/x^6/(c*x^4+b*x^2)^(1/2)-8/7*(c*x^4+b*x^2)^(1/2)/b^2/x^8+48/35*c*(c*x^4+
b*x^2)^(1/2)/b^3/x^6-64/35*c^2*(c*x^4+b*x^2)^(1/2)/b^4/x^4+128/35*c^3*(c*x^
4+b*x^2)^(1/2)/b^5/x^2
```

Rubi [A]
time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2040, 2041, 2039}

$$\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{1}{bx^6\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^6*Sqrt[b*x^2 + c*x^4]) - (8*Sqrt[b*x^2 + c*x^4])/(7*b^2*x^8) + (48*c
*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^6) - (64*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^4*x
^4) + (128*c^3*Sqrt[b*x^2 + c*x^4])/(35*b^5*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{bx^6\sqrt{bx^2+cx^4}} + \frac{8\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx}{b}$$
$$= \frac{1}{bx^6\sqrt{bx^2+cx^4}} - \frac{8\sqrt{bx^2+cx^4}}{7b^2x^8} - \frac{(48c)\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx}{7b^2}$$
$$= \frac{1}{bx^6\sqrt{bx^2+cx^4}} - \frac{8\sqrt{bx^2+cx^4}}{7b^2x^8} + \frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6} + \frac{(192c^2)\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx}{35b^3}$$
$$= \frac{1}{bx^6\sqrt{bx^2+cx^4}} - \frac{8\sqrt{bx^2+cx^4}}{7b^2x^8} + \frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6} - \frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4} - \frac{(128c^3 \ldots}{\ldots}$$
$$= \frac{1}{bx^6\sqrt{bx^2+cx^4}} - \frac{8\sqrt{bx^2+cx^4}}{7b^2x^8} + \frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6} - \frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4} + \frac{128c^3 \ldots}{3 \ldots}$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 0.52

$$\frac{-5b^4+8b^3cx^2-16b^2c^2x^4+64bc^3x^6+128c^4x^8}{35b^5x^6\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
[Out] (-5*b^4 + 8*b^3*c*x^2 - 16*b^2*c^2*x^4 + 64*b*c^3*x^6 + 128*c^4*x^8)/(35*b^
5*x^6*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 72, normalized size = 0.55

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-128c^{4}x^{8}-64b\,c^{3}x^{6}+16b^{2}c^{2}x^{4}-8b^{3}c\,x^{2}+5b^{4}\right)}{35x^{4}b^{5}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	72
default	$-\frac{\left(c\,x^{2}+b\right)\left(-128c^{4}x^{8}-64b\,c^{3}x^{6}+16b^{2}c^{2}x^{4}-8b^{3}c\,x^{2}+5b^{4}\right)}{35x^{4}b^{5}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	72

trager	$-\frac{(-128c^4x^8-64b\,c^3x^6+16b^2c^2x^4-8b^3c\,x^2+5b^4)\sqrt{c\,x^4+b\,x^2}}{35(c\,x^2+b)b^5x^8}$	74
risch	$-\frac{(c\,x^2+b)(-93c^3x^6+29b\,c^2x^4-13b^2c\,x^2+5b^3)}{35b^5x^6\sqrt{x^2(c\,x^2+b)}}+\frac{x^2c^4}{b^5\sqrt{x^2(c\,x^2+b)}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(c*x^2+b)*(-128*c^4*x^8-64*b*c^3*x^6+16*b^2*c^2*x^4-8*b^3*c*x^2+5*b^4
)/x^4/b^5/(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.30, size = 113, normalized size = 0.87

$$\frac{128\,c^4x^2}{35\,\sqrt{cx^4+bx^2}\,b^5}+\frac{64\,c^3}{35\,\sqrt{cx^4+bx^2}\,b^4}-\frac{16\,c^2}{35\,\sqrt{cx^4+bx^2}\,b^3x^2}+\frac{8\,c}{35\,\sqrt{cx^4+bx^2}\,b^2x^4}-\frac{1}{7\,\sqrt{cx^4+bx^2}\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 128/35*c^4*x^2/(sqrt(c*x^4 + b*x^2)*b^5) + 64/35*c^3/(sqrt(c*x^4 + b*x^2)*b
^4) - 16/35*c^2/(sqrt(c*x^4 + b*x^2)*b^3*x^2) + 8/35*c/(sqrt(c*x^4 + b*x^2)
*b^2*x^4) - 1/7/(sqrt(c*x^4 + b*x^2)*b*x^6)
```

Fricas [A]
time = 0.38, size = 76, normalized size = 0.58

$$\frac{(128\,c^4x^8+64\,bc^3x^6-16\,b^2c^2x^4+8\,b^3cx^2-5\,b^4)\sqrt{cx^4+bx^2}}{35\,(b^5cx^{10}+b^6x^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(128*c^4*x^8 + 64*b*c^3*x^6 - 16*b^2*c^2*x^4 + 8*b^3*c*x^2 - 5*b^4)*sq
rt(c*x^4 + b*x^2)/(b^5*c*x^10 + b^6*x^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^5\,(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [A]
time = 5.86, size = 222, normalized size = 1.71

$$\frac{c^4x}{\sqrt{cx^2+b}\,b^5\mathrm{sgn}\left(x\right)}-\frac{2\left(35\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{12}c^{\frac{7}{2}}-280\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{7}{2}}+1015\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{7}{2}}-2240\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{6}b^3c^{\frac{7}{2}}+1673\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{4}b^4c^{\frac{7}{2}}-616\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{2}b^5c^{\frac{7}{2}}+93\,b^6c^{\frac{7}{2}}\right)}{35\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^{2}-b\right)^{7}b^4\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] c^4*x/(sqrt(c*x^2 + b)*b^5*sgn(x)) - 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))
^12*c^(7/2) - 280*(sqrt(c)*x - sqrt(c*x^2 + b))^10*b*c^(7/2) + 1015*(sqrt(c
)*x - sqrt(c*x^2 + b))^8*b^2*c^(7/2) - 2240*(sqrt(c)*x - sqrt(c*x^2 + b))^6
*b^3*c^(7/2) + 1673*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(7/2) - 616*(sqrt
(c)*x - sqrt(c*x^2 + b))^2*b^5*c^(7/2) + 93*b^6*c^(7/2))/(((sqrt(c)*x - sqr
t(c*x^2 + b))^2 - b)^7*b^4*sgn(x))
```

Mupad [B]
time = 4.41, size = 114, normalized size = 0.88

$$\frac{13\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b^3\,x^6}-\frac{\sqrt{c\,x^4+b\,x^2}}{7\,b^2\,x^8}-\frac{29\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^4}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{93\,c^3}{35\,b^4}+\frac{128\,c^4\,x^2}{35\,b^5}\right)}{x^2\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] (13*c*(b*x^2 + c*x^4)^(1/2))/(35*b^3*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*b^2*x^
8) - (29*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b^4*x^4) + ((b*x^2 + c*x^4)^(1/2)*(
(93*c^3)/(35*b^4) + (128*c^4*x^2)/(35*b^5)))/(x^2*(b + c*x^2))
```

3.281 $\int \frac{x^6}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{x^3}{c\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{c^2x}$$

```
[Out] -x^3/c/(c*x^4+b*x^2)^(1/2)+2*(c*x^4+b*x^2)^(1/2)/c^2/x
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2040, 1602}

$$\frac{2\sqrt{bx^2+cx^4}}{c^2x}-\frac{x^3}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(b*x^2 + c*x^4)^(3/2),x]
[Out] -(x^3/(c*Sqrt[b*x^2 + c*x^4])) + (2*Sqrt[b*x^2 + c*x^4])/(c^2*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(bx^2+cx^4)^{3/2}}\,dx &= -\frac{x^3}{c\sqrt{bx^2+cx^4}}+\frac{2\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{c}\\ &= -\frac{x^3}{c\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{c^2x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.62

$$\frac{x(2b + cx^2)}{c^2\sqrt{x^2\,(b + cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*b + c*x^2))/(c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.11, size = 37, normalized size = 0.79

method	result	size
gosper	$\frac{(c\,x^2+b)\,(c\,x^2+2b)\,x^3}{c^2\,(c\,x^4+b\,x^2)^{\frac{3}{2}}}$	37
default	$\frac{(c\,x^2+b)\,(c\,x^2+2b)\,x^3}{c^2\,(c\,x^4+b\,x^2)^{\frac{3}{2}}}$	37
trager	$\frac{(c\,x^2+2b)\,\sqrt{c\,x^4+b\,x^2}}{(c\,x^2+b)\,c^2 x}$	39
risch	$\frac{(c\,x^2+b)\,x}{c^2\,\sqrt{x^2\,(c\,x^2+b)}}+\frac{b x}{c^2\,\sqrt{x^2\,(c\,x^2+b)}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b)*(c*x^2+2*b)*x^3/c^2/(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.30, size = 22, normalized size = 0.47

$$\frac{cx^2 + 2\,b}{\sqrt{cx^2 + b}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (c*x^2 + 2*b)/(sqrt(c*x^2 + b)*c^2)
```

Fricas [A]
time = 0.38, size = 39, normalized size = 0.83

$$\frac{\sqrt{cx^4 + bx^2}\,(cx^2 + 2\,b)}{c^3x^3 + bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)*(c*x^2 + 2*b)/(c^3*x^3 + b*c^2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [**A**]
time = 4.13, size = 51, normalized size = 1.09

$$\frac{\frac{\sqrt{cx^2+b}}{c\mathrm{sgn}(x)}+\frac{b}{\sqrt{cx^2+b}\,c\mathrm{sgn}(x)}}{c}-\frac{2\sqrt{b}\,\mathrm{sgn}(x)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(c*x^2 + b)/(c*sgn(x)) + b/(sqrt(c*x^2 + b)*c*sgn(x)))/c - 2*sqrt(b)*s
gn(x)/c^2
```

Mupad [**B**]
time = 4.23, size = 38, normalized size = 0.81

$$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(c\,x^2+2\,b\right)}{c^2\,x\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(2*b + c*x^2))/(c^2*x*(b + c*x^2))
```

3.282 $\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -x/c/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1602}

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x/(c*Sqrt[b*x^2 + c*x^4]))
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x}{c\sqrt{bx^2+cx^4}}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{x}{c\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x/(c*Sqrt[x^2*(b + c*x^2)]))
```

Maple [A]
time = 0.09, size = 29, normalized size = 1.38

method	result	size
gosper	$-\frac{(c\,x^{2}+b)\,x^{3}}{c\,(c\,x^{4}+b\,x^{2})^{\frac{3}{2}}}$	29
default	$-\frac{(c\,x^{2}+b)\,x^{3}}{c\,(c\,x^{4}+b\,x^{2})^{\frac{3}{2}}}$	29
trager	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{(c\,x^{2}+b)\,c x}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+b)/c*x^3/(c*x^4+b*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.67

$$-\frac{1}{\sqrt{cx^2+b}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(c*x^2 + b)*c)
```

Fricas [A]
time = 0.34, size = 29, normalized size = 1.38

$$-\frac{\sqrt{cx^4+bx^2}}{c^2x^3+bcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2)/(c^2*x^3 + b*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**4/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 6.46, size = 28, normalized size = 1.33

$$\frac{\mathrm{sgn}(x)}{\sqrt{b}\,c} - \frac{1}{\sqrt{cx^2+b}\,c\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] sgn(x)/(sqrt(b)*c) - 1/(sqrt(c*x^2 + b)*c*sgn(x))
```

Mupad [B]
time = 4.15, size = 30, normalized size = 1.43

$$-\frac{\sqrt{c\,x^4+b\,x^2}}{c\,x\,\left(c\,x^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^(3/2),x)
[Out] -(b*x^2 + c*x^4)^(1/2)/(c*x*(b + c*x^2))
```

3.283 $\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

```
[Out] -arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)+x/b/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2048, 2033, 212}

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x/(b*Sqrt[b*x^2 + c*x^4]) - ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/b^(3/2
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx &= \frac{x}{b\sqrt{bx^2+cx^4}}+\frac{\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{b}\\ &= \frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{b}\\ &= \frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 59, normalized size = 1.16

$$\frac{x\left(\sqrt{b}-\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{b^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(Sqrt[b] - Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(b^(3/2)*S
qrt[x^2*(b + c*x^2)])
```

Maple [**A**]
time = 0.09, size = 65, normalized size = 1.27

method	result	size
default	$\frac{x^{3}\left(c\,x^{2}+b\right)\left(b^{\frac{3}{2}}-\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)b\sqrt{c\,x^{2}+b}\right)}{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] x^3*(c*x^2+b)*(b^(3/2)-ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*(c*x^2+b)^(1/2
))/(c*x^4+b*x^2)^(3/2)/b^(5/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [A]
time = 0.37, size = 162, normalized size = 3.18

$$\left[\frac{(cx^3+bx)\sqrt{b}\log\left(-\frac{cx^3+2bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\sqrt{cx^4+bx^2}\,b}{2(b^2cx^3+b^3x)}, \frac{(cx^3+bx)\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,b}{b^2cx^3+b^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*x^3 + b*x)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqr
t(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*c*x^3 + b^3*x), ((c*x^3 + b*x)*s
qrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
*x^2)*b)/(b^2*c*x^3 + b^3*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [A]
time = 4.76, size = 79, normalized size = 1.55

$$-\frac{\left(\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\right)\mathrm{sgn}(x)}{\sqrt{-b}\,b^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b\mathrm{sgn}(x)}+\frac{1}{\sqrt{cx^2+b}\,b\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(b)*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b))*sgn(x)/(sqrt(-b)*b^(3/2)) +
arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b*sgn(x)) + 1/(sqrt(c*x^2 + b)*b
*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^2/(b*x^2 + c*x^4)^(3/2), x)
```

3.284 $\int \frac{1}{(bx^2+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=81

$$\frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}$$

```
[Out] 3/2*c*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)+1/b/x/(c*x^4+b*x^2)^(1
/2)-3/2*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2031, 2050, 2033, 212}

$$\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] 1/(b*x*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2 + c*x^4])/(2*b^2*x^3) + (3*c*Ar
cTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(bx^2+cx^4)^{3/2}}\,dx &= \frac{1}{bx\sqrt{bx^2+cx^4}}+\frac{3\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}-\frac{(3c)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}\\
&= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{(3c)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b^2}\\
&= \frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 78, normalized size = 0.96

$$\frac{-\sqrt{b}\,(b+3cx^2)+3cx^2\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{5/2}x\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] (-(Sqrt[b]*(b + 3*c*x^2)) + 3*c*x^2*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]
/Sqrt[b]])/(2*b^(5/2)*x*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.13, size = 77, normalized size = 0.95

method	result	size
default	$-\frac{x(c\,x^2+b)\left(3b^{\frac{3}{2}}c\,x^2-3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)\sqrt{c\,x^2+b}\,bc\,x^2+b^{\frac{5}{2}}\right)}{2(c\,x^4+b\,x^2)^{\frac{3}{2}}b^{\frac{7}{2}}}$	77

risch	$-\frac{c\,x^{2}+b}{2 b^{2} x \sqrt{x^{2} \left(c\,x^{2}+b\right)}}+\frac{\left(-\frac{c}{b^{2} \sqrt{c\,x^{2}+b}}+\frac{3 c \ln\left(\frac{2 b+2 \sqrt{b}\, \sqrt{c\,x^{2}+b}}{x}\right)}{2 b^{\frac{5}{2}}}\right) x \sqrt{c\,x^{2}+b}}{\sqrt{x^{2} \left(c\,x^{2}+b\right)}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*(c*x^2+b)*(3*b^(3/2)*c*x^2-3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*(c*
x^2+b)^(1/2)*b*c*x^2+b^(5/2))/(c*x^4+b*x^2)^(3/2)/b^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(-3/2), x)
```

Fricas [A]
time = 0.36, size = 199, normalized size = 2.46

$$\left[\frac{3 \left(c^{2} x^{5}+b c x^{3}\right) \sqrt{b} \log\left(-\frac{c x^{3}+2 b x+2 \sqrt{c x^{4}+b x^{2}} \sqrt{b}}{x^{3}}\right)-2 \sqrt{c x^{4}+b x^{2}} \left(3 b c x^{2}+b^{2}\right)}{4 \left(b^{3} c x^{5}+b^{4} x^{3}\right)}, -\frac{3 \left(c^{2} x^{5}+b c x^{3}\right) \sqrt{-b} \arctan\left(\frac{\sqrt{c x^{4}+b x^{2}} \sqrt{-b}}{c x^{3}+b x}\right)+\sqrt{c x^{4}+b x^{2}} \left(3 b c x^{2}+b^{2}\right)}{2 \left(b^{3} c x^{5}+b^{4} x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(c^2*x^5 + b*c*x^3)*sqrt(b)*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*
x^2)*sqrt(b))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 + b^2))/(b^3*c*x^5 +
b^4*x^3), -1/2*(3*(c^2*x^5 + b*c*x^3)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*s
qrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 + b^2))/(b^3*c*x^5
+ b^4*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b x^{2}+c x^{4}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((b*x**2 + c*x**4)**(-3/2), x)
```

Giac [A]
time = 5.28, size = 80, normalized size = 0.99

$$-\frac{3\,c\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{2\,\sqrt{-b}\,b^2\mathrm{sgn}\left(x\right)}-\frac{3\,(cx^2+b)c-2\,bc}{2\left((cx^2+b)^{\frac{3}{2}}-\sqrt{cx^2+b}\,b\right)b^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -3/2*c*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2*sgn(x)) - 1/2*(3*(c*x
^2 + b)*c - 2*b*c)/((((c*x^2 + b)^(3/2) - sqrt(c*x^2 + b)*b)*b^2*sgn(x))
```

Mupad [B]
time = 4.34, size = 42, normalized size = 0.52

$$-\frac{x\,{\left(\frac{b}{c\,x^2}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{b}{c\,x^2}\right)}{5\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -(x*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -b/(c*x^2)))/(5*(b*x^2
 + c*x^4)^(3/2))
```

3.285 $\int \frac{1}{x^2(bx^2+cx^4)^{3/2}} dx$

Optimal. Leaf size=109

$$\frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}$$

```
[Out] -15/8*c^2*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(7/2)+1/b/x^3/(c*x^4+b*x
^2)^(1/2)-5/4*(c*x^4+b*x^2)^(1/2)/b^2/x^5+15/8*c*(c*x^4+b*x^2)^(1/2)/b^3/x^
3
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2048, 2050, 2033, 212}

$$-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^3*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(4*b^2*x^5) + (15*c
*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) - (15*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2
+ c*x^4]])/(8*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
```

```
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^3\sqrt{bx^2+cx^4}}+\frac{5\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}-\frac{(15c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b^2}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}+\frac{\left(15c^2\right)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^3}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{\left(15c^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x
}{8b^3}\\
&= \frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}
}{8b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 92, normalized size = 0.84

$$\frac{\sqrt{b}\left(-2b^2+5bcx^2+15c^2x^4\right)-15c^2x^4\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{8b^{7/2}x^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[b]*(-2*b^2 + 5*b*c*x^2 + 15*c^2*x^4) - 15*c^2*x^4*Sqrt[b + c*x^2]*Arc
Tanh[Sqrt[b + c*x^2]/Sqrt[b]])/(8*b^(7/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.11, size = 94, normalized size = 0.86

method	result	size
default	$-\frac{(c x^2+b)\left(-15 b^{\frac{3}{2}} c^2 x^4+15\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right)\sqrt{c x^2+b}\, b c^2 x^4-5 b^{\frac{5}{2}} c x^2+2 b^{\frac{7}{2}}\right)}{8x(c x^4+b x^2)^{\frac{3}{2}} b^{\frac{9}{2}}}$	94
risch	$-\frac{(c x^2+b)(-7c x^2+2b)}{8b^3x^3\sqrt{x^2(c x^2+b)}}+\frac{\left(\frac{c^2}{b^3\sqrt{c x^2+b}}-\frac{15c^2\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right)}{8b^{\frac{7}{2}}}\right)x\sqrt{c x^2+b}}{\sqrt{x^2(c x^2+b)}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/x*(c*x^2+b)*(-15*b^(3/2)*c^2*x^4+15*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x
)*(c*x^2+b)^(1/2)*b*c^2*x^4-5*b^(5/2)*c*x^2+2*b^(7/2))/(c*x^4+b*x^2)^(3/2)/
b^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

Fricas [A]
time = 0.37, size = 229, normalized size = 2.10

$$\left[\frac{15\,(c^3x^7+bc^2x^5)\sqrt{b}\,\log\left(-\frac{cx^3+2bx-2\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)+2\,(15\,bc^2x^4+5\,b^2cx^2-2\,b^3)\sqrt{cx^4+bx^2}}{16\,(b^4cx^7+b^5x^5)},\ \frac{15\,(c^3x^7+bc^2x^5)\sqrt{-b}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+(15\,bc^2x^4+5\,b^2cx^2-2\,b^3)\sqrt{cx^4+bx^2}}{8\,(b^4cx^7+b^5x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(c^3*x^7 + b*c^2*x^5)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4
+ b*x^2)*sqrt(b))/x^3) + 2*(15*b*c^2*x^4 + 5*b^2*c*x^2 - 2*b^3)*sqrt(c*x^4
+ b*x^2))/(b^4*c*x^7 + b^5*x^5), 1/8*(15*(c^3*x^7 + b*c^2*x^5)*sqrt(-b)*arc
tan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*b*c^2*x^4 + 5*b^2*c*x
^2 - 2*b^3)*sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [A]
time = 5.78, size = 99, normalized size = 0.91

$$\frac{15\, c^2 \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{8\, \sqrt{-b}\, b^3 \mathrm{sgn}\left(x\right)} + \frac{c^2}{\sqrt{cx^2+b}\, b^3 \mathrm{sgn}\left(x\right)} + \frac{7\left(cx^2+b\right)^{\frac{3}{2}} c^2 - 9\, \sqrt{cx^2+b}\, bc^2}{8\, b^3 c^2 x^4 \mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 15/8*c^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^3*sgn(x)) + c^2/(sqrt
(c*x^2 + b)*b^3*sgn(x)) + 1/8*(7*(c*x^2 + b)^(3/2)*c^2 - 9*sqrt(c*x^2 + b)*
b*c^2)/(b^3*c^2*x^4*sgn(x))
```

Mupad [B]
time = 4.64, size = 44, normalized size = 0.40

$$-\frac{\left(\frac{b}{c\,x^2}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{b}{c\,x^2}\right)}{7\,x\,\left(c\,x^4+b\,x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -b/(c*x^2)))/(7*x*(b*x^2
 + c*x^4)^(3/2))
```

3.286 $\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)$$

```
[Out] 3/32*arcsin(-1+8/3*x^2)-1/8*(-4*x^4+3*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2043, 654, 633, 222}

$$-\frac{3}{32}\text{ArcSin}\left(1-\frac{8x^2}{3}\right)-\frac{1}{8}\sqrt{3x^2-4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[3*x^2 - 4*x^4],x]
```

```
[Out] -1/8*Sqrt[3*x^2 - 4*x^4] - (3*ArcSin[1 - (8*x^2)/3])/32
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{3x-4x^2}}\,dx,x,x^2\right)\\ &= -\frac{1}{8}\sqrt{3x^2-4x^4}+\frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3x-4x^2}}\,dx,x,x^2\right)\\ &= -\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{1}{32}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,3-8x^2\right)\\ &= -\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 58, normalized size = 1.71

$$\frac{x\left(-6x+8x^3-3\sqrt{-3+4x^2}\,\log\left(-2x+\sqrt{-3+4x^2}\right)\right)}{16\sqrt{3x^2-4x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[3*x^2 - 4*x^4],x]
[Out] (x*(-6*x + 8*x^3 - 3*Sqrt[-3 + 4*x^2]*Log[-2*x + Sqrt[-3 + 4*x^2]]))/(16*Sq
rt[3*x^2 - 4*x^4])
```

Maple [A]
time = 0.12, size = 48, normalized size = 1.41

method	result	size
meijerg	$\frac{3i\left(\frac{2i\sqrt{\pi}\,x\sqrt{3}\,\sqrt{-\frac{4x^2}{3}+1}}{3}-i\sqrt{\pi}\,\arcsin\left(\frac{2x\sqrt{3}}{3}\right)\right)}{16\sqrt{\pi}}$	40
default	$\frac{x\sqrt{-4x^2+3}\,\left(-2x\sqrt{-4x^2+3}+3\arcsin\left(\frac{2x\sqrt{3}}{3}\right)\right)}{16\sqrt{-4x^4+3x^2}}$	48

trager	$-\frac{\sqrt{-4x^4+3x^2}}{8}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2+\sqrt{-4x^4+3x^2}}{x}\right)}{16}$	55
risch	$\frac{x^2(4x^2-3)}{8\sqrt{-x^2(4x^2-3)}}+\frac{3\arcsin\left(\frac{2x\sqrt{3}}{3}\right)x\sqrt{-4x^2+3}}{16\sqrt{-x^2(4x^2-3)}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^4+3*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x*(-4*x^2+3)^(1/2)*(-2*x*(-4*x^2+3)^(1/2)+3*arcsin(2/3*x*3^(1/2)))/(-4
*x^4+3*x^2)^(1/2)
```

Maxima [A]
time = 0.52, size = 26, normalized size = 0.76

$$-\frac{1}{8}\sqrt{-4\,x^4+3\,x^2}-\frac{3}{32}\arcsin\left(-\frac{8}{3}\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) - 3/32*arcsin(-8/3*x^2 + 1)
```

Fricas [A]
time = 0.35, size = 37, normalized size = 1.09

$$-\frac{1}{8}\sqrt{-4\,x^4+3\,x^2}-\frac{3}{16}\arctan\left(\frac{\sqrt{-4\,x^4+3\,x^2}}{2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) - 3/16*arctan(1/2*sqrt(-4*x^4 + 3*x^2)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{-x^2\cdot(4x^2-3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(4*x**2 - 3)), x)
```

Giac [A]
time = 7.48, size = 30, normalized size = 0.88

$$-\frac{\sqrt{-4\,x^2+3}\,x}{8\,\mathrm{sgn}\left(x\right)}+\frac{3\,\arcsin\left(\frac{2}{3}\,\sqrt{3}\,x\right)}{16\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-4*x^2 + 3)*x/sgn(x) + 3/16*arcsin(2/3*sqrt(3)*x)/sgn(x)
```

Mupad [B]
time = 4.33, size = 42, normalized size = 1.24

$$-\frac{\sqrt{3\,x^2-4\,x^4}}{8}-\frac{\ln\left(x^2-\frac{3}{8}-\frac{\sqrt{3-4\,x^2}\,\sqrt{x^2}\,1\mathrm{i}}{2}\right)\,3\mathrm{i}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2 - 4*x^4)^(1/2),x)
```

```
[Out] - (log(x^2 - ((3 - 4*x^2)^(1/2)*(x^2)^(1/2)*1i)/2 - 3/8)*3i)/32 - (3*x^2 -
4*x^4)^(1/2)/8
```

3.287 $\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{8}\sqrt{-3x^2-4x^4}-\frac{3}{32}\sin^{-1}\left(1+\frac{8x^2}{3}\right)$$

```
[Out] -3/32*arcsin(1+8/3*x^2)-1/8*(-4*x^4-3*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2043, 654, 633, 222}

$$-\frac{3}{32}\text{ArcSin}\left(\frac{8x^2}{3}+1\right)-\frac{1}{8}\sqrt{-4x^4-3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-3*x^2 - 4*x^4],x]
```

```
[Out] -1/8*Sqrt[-3*x^2 - 4*x^4] - (3*ArcSin[1 + (8*x^2)/3])/32
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{-3x-4x^2}}\,dx, x, x^2\right)\\ &= -\frac{1}{8}\sqrt{-3x^2-4x^4} - \frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{-3x-4x^2}}\,dx, x, x^2\right)\\ &= -\frac{1}{8}\sqrt{-3x^2-4x^4} + \frac{1}{32}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, -3-8x^2\right)\\ &= -\frac{1}{8}\sqrt{-3x^2-4x^4} - \frac{3}{32}\sin^{-1}\left(1+\frac{8x^2}{3}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 1.74

$$\frac{x\left(6x+8x^3+3\sqrt{3+4x^2}\,\log\left(-2x+\sqrt{3+4x^2}\right)\right)}{16\sqrt{-x^2(3+4x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-3*x^2 - 4*x^4],x]
[Out] (x*(6*x + 8*x^3 + 3*Sqrt[3 + 4*x^2]*Log[-2*x + Sqrt[3 + 4*x^2]]))/(16*Sqrt[
-(x^2*(3 + 4*x^2))])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(26) = 52.
time = 0.12, size = 54, normalized size = 1.59

method	result	size
meijerg	$-\frac{3i\left(\frac{2\sqrt{\pi}\,x\sqrt{3}\,\sqrt{\frac{4x^2}{3}+1}}{3}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x\sqrt{3}}{3}\right)\right)}{16\sqrt{\pi}}$	38
default	$-\frac{x\sqrt{-4x^2-3}\,\left(2x\sqrt{-4x^2-3}+3\arctan\left(\frac{2x}{\sqrt{-4x^2-3}}\right)\right)}{16\sqrt{-4x^4-3x^2}}$	54

trager	$-\frac{\sqrt{-4x^4-3x^2}}{8}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2+\sqrt{-4x^4-3x^2}}{x}\right)}{16}$	55
risch	$\frac{x^2(4x^2+3)}{8\sqrt{-x^2(4x^2+3)}}-\frac{3\arctan\left(\frac{2x}{\sqrt{-4x^2-3}}\right)x\sqrt{-4x^2-3}}{16\sqrt{-x^2(4x^2+3)}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-4*x^4-3*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*x*(-4*x^2-3)^(1/2)*(2*x*(-4*x^2-3)^(1/2)+3*arctan(2*x/(-4*x^2-3)^(1/2
)))/(-4*x^4-3*x^2)^(1/2)
```

Maxima [A]
time = 0.53, size = 26, normalized size = 0.76

$$-\frac{1}{8}\sqrt{-4x^4-3x^2}+\frac{3}{32}\arcsin\left(-\frac{8}{3}x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^4 - 3*x^2) + 3/32*arcsin(-8/3*x^2 - 1)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.35, size = 59, normalized size = 1.74

$$-\frac{1}{8}\sqrt{-4x^2-3}\,x-\frac{3}{32}i\log\left(-\frac{4\left(2x+i\sqrt{-4x^2-3}\right)}{x}\right)+\frac{3}{32}i\log\left(-\frac{4\left(2x-i\sqrt{-4x^2-3}\right)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 - 3)*x - 3/32*I*log(-4*(2*x + I*sqrt(-4*x^2 - 3))/x) + 3/3
2*I*log(-4*(2*x - I*sqrt(-4*x^2 - 3))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{-x^2\cdot(4x^2+3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-4*x**4-3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(4*x**2 + 3)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 4.62, size = 43, normalized size = 1.26

$$\frac{3}{32}i\log(3)\operatorname{sgn}(x) - \frac{i\sqrt{4x^2+3}\,x}{8\operatorname{sgn}(x)} - \frac{3i\log\left(-2x+\sqrt{4x^2+3}\right)}{16\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/32*I*log(3)*sgn(x) - 1/8*I*sqrt(4*x^2 + 3)*x/sgn(x) - 3/16*I*log(-2*x + s
qrt(4*x^2 + 3))/sgn(x)
```

Mupad [B]
time = 4.36, size = 41, normalized size = 1.21

$$-\frac{\sqrt{-4x^4-3x^2}}{8} + \frac{\ln\left(\frac{\sqrt{4x^2+3}\,\sqrt{x^2}}{2} + x^2 + \frac{3}{8}\right)3\mathrm{i}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(- 3*x^2 - 4*x^4)^(1/2),x)
```

```
[Out] (log(((4*x^2 + 3)^(1/2)*(x^2)^(1/2))/2 + x^2 + 3/8)*3i)/32 - (- 3*x^2 - 4*x
^4)^(1/2)/8
```

3.288 $\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx$

Optimal. Leaf size=45

$$\frac{1}{8}\sqrt{3x^2+4x^4}-\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{3x^2+4x^4}}\right)$$

```
[Out] -3/16*arctanh(2*x^2/(4*x^4+3*x^2)^(1/2))+1/8*(4*x^4+3*x^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.210$, Rules used = {2043, 654, 634, 212}

$$\frac{1}{8}\sqrt{4x^4+3x^2}-\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4+3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[3*x^2 + 4*x^4],x]
```

```
[Out] Sqrt[3*x^2 + 4*x^4]/8 - (3*ArcTanh[(2*x^2)/Sqrt[3*x^2 + 4*x^4]])/16
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{3x+4x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{8}\sqrt{3x^2+4x^4} - \frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3x+4x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{8}\sqrt{3x^2+4x^4} - \frac{3}{8}\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \frac{x^2}{\sqrt{3x^2+4x^4}}\right)\\
&= \frac{1}{8}\sqrt{3x^2+4x^4} - \frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{3x^2+4x^4}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 1.29

$$\frac{x\left(6x+8x^3+3\sqrt{3+4x^2}\log\left(-2x+\sqrt{3+4x^2}\right)\right)}{16\sqrt{x^2\left(3+4x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[3*x^2 + 4*x^4],x]
[Out] (x*(6*x + 8*x^3 + 3*Sqrt[3 + 4*x^2]*Log[-2*x + Sqrt[3 + 4*x^2]]))/(16*Sqrt[
x^2*(3 + 4*x^2)])
```

Maple [A]
time = 0.09, size = 48, normalized size = 1.07

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{\frac{4x^2}{3}+1}}{8}-\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x\sqrt{3}}{3}\right)}{16}}{\sqrt{\pi}}$	37
trager	$\frac{\sqrt{4x^4+3x^2}}{8}+\frac{3\ln\left(\frac{-2x^2+\sqrt{4x^4+3x^2}}{x}\right)}{16}$	43
default	$-\frac{x\sqrt{4x^2+3}\,\left(-2x\sqrt{4x^2+3}+3\operatorname{arcsinh}\left(\frac{2x\sqrt{3}}{3}\right)\right)}{16\sqrt{4x^4+3x^2}}$	48

risch	$\frac{x^{2}(4x^{2}+3)}{8\sqrt{x^{2}(4x^{2}+3)}}-\frac{3\,\mathrm{arcsinh}\left(\frac{2x\sqrt{3}}{3}\right)x\sqrt{4x^{2}+3}}{16\sqrt{x^{2}(4x^{2}+3)}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^4+3*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*x*(4*x^2+3)^(1/2)*(-2*x*(4*x^2+3)^(1/2)+3*arcsinh(2/3*x*3^(1/2)))/(4*
x^4+3*x^2)^(1/2)
```

Maxima [A]
time = 0.50, size = 41, normalized size = 0.91

$$\frac{1}{8}\,\sqrt{4\,x^{4}+3\,x^{2}}-\frac{3}{32}\,\log\left(8\,x^{2}+4\,\sqrt{4\,x^{4}+3\,x^{2}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) - 3/32*log(8*x^2 + 4*sqrt(4*x^4 + 3*x^2) + 3)
```

Fricas [A]
time = 0.38, size = 45, normalized size = 1.00

$$\frac{1}{8}\,\sqrt{4\,x^{4}+3\,x^{2}}+\frac{3}{16}\,\log\left(-\frac{2\,x^{2}-\sqrt{4\,x^{4}+3\,x^{2}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) + 3/16*log(-(2*x^2 - sqrt(4*x^4 + 3*x^2))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}}{\sqrt{x^{2}\cdot(4x^{2}+3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(4*x**2 + 3)), x)
```

Giac [A]
time = 4.91, size = 43, normalized size = 0.96

$$-\frac{3}{32}\,\log\left(3\right)\mathrm{sgn}(x)+\frac{\sqrt{4\,x^{2}+3}\,x}{8\,\mathrm{sgn}\left(x\right)}+\frac{3\,\log\left(-2\,x+\sqrt{4\,x^{2}+3}\right)}{16\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -3/32*log(3)*sgn(x) + 1/8*sqrt(4*x^2 + 3)*x/sgn(x) + 3/16*log(-2*x + sqrt(4
*x^2 + 3))/sgn(x)
```

Mupad [B]
time = 4.40, size = 40, normalized size = 0.89

$$\frac{\sqrt{4\,x^4+3\,x^2}}{8}-\frac{3\,\ln\left(\frac{\sqrt{4\,x^2+3}\,\sqrt{x^2}}{2}+x^2+\frac{3}{8}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2 + 4*x^4)^(1/2),x)
```

```
[Out] (3*x^2 + 4*x^4)^(1/2)/8 - (3*log(((4*x^2 + 3)^(1/2)*(x^2)^(1/2))/2 + x^2 +
3/8))/32
```

3.289 $\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx$

Optimal. Leaf size=45

$$\frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{-3x^2+4x^4}}\right)$$

```
[Out] 3/16*arctanh(2*x^2/(4*x^4-3*x^2)^(1/2))+1/8*(4*x^4-3*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2043, 654, 634, 212}

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[-3*x^2 + 4*x^4],x]
```

```
[Out] Sqrt[-3*x^2 + 4*x^4]/8 + (3*ArcTanh[(2*x^2)/Sqrt[-3*x^2 + 4*x^4]])/16
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{-3x+4x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{16}\text{Subst}\left(\int \frac{1}{\sqrt{-3x+4x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{1-4x^2}\,dx,x,\frac{x^2}{\sqrt{-3x^2+4x^4}}\right)\\ &= \frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{-3x^2+4x^4}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 1.29

$$\frac{x\left(-6x+8x^3-3\sqrt{-3+4x^2}\,\log\left(-2x+\sqrt{-3+4x^2}\right)\right)}{16\sqrt{x^2\left(-3+4x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[-3*x^2 + 4*x^4],x]
[Out] (x*(-6*x + 8*x^3 - 3*Sqrt[-3 + 4*x^2]*Log[-2*x + Sqrt[-3 + 4*x^2]]))/(16*Sq
rt[x^2*(-3 + 4*x^2)])
```

Maple [A]
time = 0.12, size = 60, normalized size = 1.33

method	result	size
trager	$\frac{\sqrt{4x^4-3x^2}}{8}+\frac{3\ln\left(\frac{2x^2+\sqrt{4x^4-3x^2}}{x}\right)}{16}$	43
default	$\frac{x\sqrt{4x^2-3}\,\left(3\ln\left(\sqrt{4}\,x+\sqrt{4x^2-3}\right)\sqrt{4}+4x\sqrt{4x^2-3}\right)}{32\sqrt{4x^4-3x^2}}$	60
meijerg	$\frac{3i\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{3}\right)}\left(\frac{2i\sqrt{\pi}\,x\sqrt{3}\,\sqrt{-\frac{4x^2}{3}+1}}{3}-i\sqrt{\pi}\,\arcsin\left(\frac{2x\sqrt{3}}{3}\right)\right)}{16\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{3}\right)}}$	62

risch	$\frac{x^2(4x^2-3)}{8\sqrt{x^2(4x^2-3)}}+\frac{3\ln\left(\sqrt{4}\,x+\sqrt{4x^2-3}\right)\sqrt{4}\,x\sqrt{4x^2-3}}{32\sqrt{x^2(4x^2-3)}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^4-3*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/32*x*(4*x^2-3)^(1/2)*(3*ln(4^(1/2)*x+(4*x^2-3)^(1/2))*4^(1/2)+4*x*(4*x^2-
3)^(1/2))/(4*x^4-3*x^2)^(1/2)
```

Maxima [A]
time = 0.50, size = 41, normalized size = 0.91

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{32}\log\left(8x^2+4\sqrt{4x^4-3x^2}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/8*sqrt(4*x^4 - 3*x^2) + 3/32*log(8*x^2 + 4*sqrt(4*x^4 - 3*x^2) - 3)
```

Fricas [A]
time = 0.34, size = 45, normalized size = 1.00

$$\frac{1}{8}\sqrt{4x^4-3x^2}-\frac{3}{16}\log\left(-\frac{2x^2-\sqrt{4x^4-3x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(4*x^4 - 3*x^2) - 3/16*log(-(2*x^2 - sqrt(4*x^4 - 3*x^2))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{x^2\cdot(4x^2-3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**4-3*x**2)**(1/2),x)
[Out] Integral(x**3/sqrt(x**2*(4*x**2 - 3)), x)
```

Giac [A]
time = 3.76, size = 44, normalized size = 0.98

$$\frac{3}{32}\log(3)\operatorname{sgn}(x)+\frac{\sqrt{4x^2-3}\,x}{8\operatorname{sgn}(x)}-\frac{3\log\left(\left|-2x+\sqrt{4x^2-3}\right|\right)}{16\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 3/32*log(3)*sgn(x) + 1/8*sqrt(4*x^2 - 3)*x/sgn(x) - 3/16*log(abs(-2*x + sqr
t(4*x^2 - 3)))/sgn(x)
```

Mupad [B]
time = 4.46, size = 40, normalized size = 0.89

$$\frac{3\ln\left(\frac{\sqrt{4\,x^2-3}\,\sqrt{x^2}}{2}+x^2-\frac{3}{8}\right)}{32}+\frac{\sqrt{4\,x^4-3\,x^2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^4 - 3*x^2)^(1/2),x)
```

```
[Out] (3*log(((4*x^2 - 3)^(1/2)*(x^2)^(1/2))/2 + x^2 - 3/8))/32 + (4*x^4 - 3*x^2)
^(1/2)/8
```

3.290 $\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*a*arctanh(x^2*b^(1/2)/(b*x^4+a*x^2)^(1/2))/b^(3/2)+1/2*(b*x^4+a*x^2)^(
1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.210$, Rules used = {2043, 654, 634, 212}

$$\frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^4]/(2*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^2 + b*x^4]])/
(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{ax+bx^2}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\text{Subst}\left(\int \frac{1}{\sqrt{ax+bx^2}}\,dx,x,x^2\right)}{4b}\\ &= \frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{ax^2+bx^4}}\right)}{2b}\\ &= \frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 74, normalized size = 1.28

$$\frac{x\left(\sqrt{b}\,x(a+bx^2)+a\sqrt{a+bx^2}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)\right)}{2b^{3/2}\sqrt{x^2\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^4],x]
```

```
[Out] (x*(Sqrt[b]*x*(a + b*x^2) + a*Sqrt[a + b*x^2]*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]]))/(2*b^(3/2)*Sqrt[x^2*(a + b*x^2)])
```

Maple [A]
time = 0.09, size = 64, normalized size = 1.10

method	result	size
default	$\frac{x\sqrt{b\,x^2+a}\,\left(x\sqrt{b\,x^2+a}\,b^{\frac{3}{2}}-a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)b\right)}{2\sqrt{b\,x^4+a\,x^2}\,b^{\frac{5}{2}}}$	64
risch	$\frac{x^2\,(b\,x^2+a)}{2b\sqrt{x^2\,(b\,x^2+a)}}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)x\sqrt{b\,x^2+a}}{2b^{\frac{3}{2}}\sqrt{x^2\,(b\,x^2+a)}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)*(x*(b*x^2+a)^(1/2)*b^(3/2)-a*ln(x*b^(1/2)+(b*x^2+a)^(
1/2))*b)/(b*x^4+a*x^2)^(1/2)/b^(5/2)
```

Maxima [A]
time = 0.31, size = 52, normalized size = 0.90

$$-\frac{a\log\left(2\,bx^2+a+2\,\sqrt{bx^4+ax^2}\,\sqrt{b}\right)}{4\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^4+ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*log(2*b*x^2 + a + 2*sqrt(b*x^4 + a*x^2)*sqrt(b))/b^(3/2) + 1/2*sqrt(
b*x^4 + a*x^2)/b
```

Fricas [A]
time = 0.36, size = 114, normalized size = 1.97

$$\left[\frac{a\sqrt{b}\,\log\left(-2\,bx^2-a+2\,\sqrt{bx^4+ax^2}\,\sqrt{b}\right)+2\,\sqrt{bx^4+ax^2}\,b}{4\,b^2},\frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^4+ax^2}\,\sqrt{-b}}{bx^2+a}\right)+\sqrt{bx^4+ax^2}\,b}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*log(-2*b*x^2 - a + 2*sqrt(b*x^4 + a*x^2)*sqrt(b)) + 2*sqrt(
b*x^4 + a*x^2)*b)/b^2, 1/2*(a*sqrt(-b)*arctan(sqrt(b*x^4 + a*x^2)*sqrt(-b)/
(b*x^2 + a)) + sqrt(b*x^4 + a*x^2)*b)/b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{x^2\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x**2)), x)
```

Giac [A]
time = 3.85, size = 59, normalized size = 1.02

$$-\frac{a\log\left(|a|\right)\operatorname{sgn}(x)}{4\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^2+a}\,x}{2\,b\operatorname{sgn}\left(x\right)}+\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*a*log(abs(a))*sgn(x)/b^(3/2) + 1/2*sqrt(b*x^2 + a)*x/(b*sgn(x)) + 1/2*
a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/(b^(3/2)*sgn(x))
```

Mupad [B]
time = 4.71, size = 53, normalized size = 0.91

$$\frac{\sqrt{b\,x^4+a\,x^2}}{2\,b}-\frac{a\,\ln\left(\frac{b\,x^2+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^4+a\,x^2}\right)}{4\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^4)^(1/2),x)
```

```
[Out] (a*x^2 + b*x^4)^(1/2)/(2*b) - (a*log((a/2 + b*x^2)/b^(1/2) + (a*x^2 + b*x^4
)^(1/2)))/(4*b^(3/2))
```

3.291 $\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}$$

```
[Out] 1/2*a*arctan(x^2*b^(1/2)/(-b*x^4+a*x^2)^(1/2))/b^(3/2)-1/2*(-b*x^4+a*x^2)^(
1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2043, 654, 634, 209}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{ax^2-bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 - b*x^4],x]
```

```
[Out] -1/2*Sqrt[a*x^2 - b*x^4]/b + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a*x^2 - b*x^4]])/
(2*b^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{ax-bx^2}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\text{Subst}\left(\int \frac{1}{\sqrt{ax-bx^2}}\,dx, x, x^2\right)}{4b}\\ &= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^2-bx^4}}\right)}{2b}\\ &= -\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 86, normalized size = 1.43

$$-\frac{x\left(\sqrt{-b}\,x(-a+bx^2)-a\sqrt{a-bx^2}\,\log\left(-\sqrt{-b}\,x+\sqrt{a-bx^2}\right)\right)}{2(-b)^{3/2}\sqrt{x^2\left(a-bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 - b*x^4],x]
[Out] -1/2*(x*(Sqrt[-b]*x*(-a + b*x^2) - a*Sqrt[a - b*x^2]*Log[-(Sqrt[-b]*x) + Sq
rt[a - b*x^2]]))/((-b)^(3/2)*Sqrt[x^2*(a - b*x^2)])
```

Maple [A]
time = 0.12, size = 67, normalized size = 1.12

method	result	size
default	$\frac{x\sqrt{-b\,x^2+a}\,\left(-x\sqrt{-b\,x^2+a}\,b^{\frac{3}{2}}+a\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+a}}\right)b\right)}{2\sqrt{-b\,x^4+a\,x^2}\,b^{\frac{5}{2}}}$	67

risch	$-\frac{x^2(-b\,x^2+a)}{2b\sqrt{x^2(-b\,x^2+a)}}+\frac{a\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+a}}\right)x\sqrt{-b\,x^2+a}}{2b^{\frac{3}{2}}\sqrt{x^2(-b\,x^2+a)}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-b*x^4+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(-b*x^2+a)^(1/2)*(-x*(-b*x^2+a)^(1/2)*b^(3/2)+a*arctan(b^(1/2)*x/(-b*
x^2+a)^(1/2))*b)/(-b*x^4+a*x^2)^(1/2)/b^(5/2)
```

Maxima [A]
time = 0.54, size = 42, normalized size = 0.70

$$-\frac{a\arcsin\left(-\frac{2\,bx^2-a}{a}\right)}{4\,b^{\frac{3}{2}}}-\frac{\sqrt{-bx^4+ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*arcsin(-(2*b*x^2 - a)/a)/b^(3/2) - 1/2*sqrt(-b*x^4 + a*x^2)/b
```

Fricas [A]
time = 0.34, size = 120, normalized size = 2.00

$$\left[-\frac{a\sqrt{-b}\,\log\left(2\,bx^2-a-2\,\sqrt{-bx^4+ax^2}\,\sqrt{-b}\right)+2\,\sqrt{-bx^4+ax^2}\,b}{4\,b^2},\,-\frac{a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx^4+ax^2}\,\sqrt{b}}{bx^2-a}\right)+\sqrt{-bx^4+ax^2}\,b}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*sqrt(-b)*log(2*b*x^2 - a - 2*sqrt(-b*x^4 + a*x^2)*sqrt(-b)) + 2*sq
rt(-b*x^4 + a*x^2)*b)/b^2, -1/2*(a*sqrt(b)*arctan(sqrt(-b*x^4 + a*x^2)*sqrt
(b)/(b*x^2 - a)) + sqrt(-b*x^4 + a*x^2)*b)/b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{-x^2\left(-a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-b*x**4+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(-a + b*x**2)), x)
```

Giac [A]
time = 4.47, size = 73, normalized size = 1.22

$$\frac{a\log\left(|a|\right)\operatorname{sgn}(x)}{4\sqrt{-b}\,b}-\frac{\sqrt{-bx^2+a}\,x}{2\,b\operatorname{sgn}\left(x\right)}-\frac{a\log\left(\left|-\sqrt{-b}\,x+\sqrt{-bx^2+a}\right|\right)}{2\sqrt{-b}\,b\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*a*log(abs(a))*sgn(x)/(sqrt(-b)*b) - 1/2*sqrt(-b*x^2 + a)*x/(b*sgn(x)) -
 1/2*a*log(abs(-sqrt(-b)*x + sqrt(-b*x^2 + a)))/(sqrt(-b)*b*sgn(x))
```

Mupad [B]
time = 4.62, size = 60, normalized size = 1.00

$$-\frac{\sqrt{a\,x^2-b\,x^4}}{2\,b}-\frac{a\,\ln\left(\frac{\frac{a}{2}-b\,x^2}{\sqrt{-b}}+\sqrt{a\,x^2-b\,x^4}\right)}{4\,{\left(-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 - b*x^4)^(1/2),x)
```

```
[Out] - (a*x^2 - b*x^4)^(1/2)/(2*b) - (a*log((a/2 - b*x^2)/(-b)^(1/2) + (a*x^2 -
b*x^4)^(1/2)))/(4*(-b)^(3/2))
```

3.292 $\int x^{7/2}(bx^2+cx^4)\,dx$

Optimal. Leaf size=21

$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

[Out] 2/13*b*x^(13/2)+2/17*c*x^(17/2)

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {14}

$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)*(b*x^2 + c*x^4),x]

[Out] (2*b*x^(13/2))/13 + (2*c*x^(17/2))/17

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(bx^2+cx^4\right)\,dx&=\int\left(bx^{11/2}+cx^{15/2}\right)\,dx\\&=\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{221}\left(17bx^{13/2}+13cx^{17/2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)*(b*x^2 + c*x^4),x]

[Out] (2*(17*b*x^(13/2) + 13*c*x^(17/2)))/221

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{13}{2}}}{13}+\frac{2c\,x^{\frac{17}{2}}}{17}$	14
default	$\frac{2b\,x^{\frac{13}{2}}}{13}+\frac{2c\,x^{\frac{17}{2}}}{17}$	14
gosper	$\frac{2x^{\frac{13}{2}}\left(13c\,x^{2}+17b\right)}{221}$	16
trager	$\frac{2x^{\frac{13}{2}}\left(13c\,x^{2}+17b\right)}{221}$	16
risch	$\frac{2x^{\frac{13}{2}}\left(13c\,x^{2}+17b\right)}{221}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 2/13*b*x^(13/2)+2/17*c*x^(17/2)
```

Maxima [A]
time = 0.31, size = 13, normalized size = 0.62

$$\frac{2}{17}\,cx^{\frac{17}{2}}+\frac{2}{13}\,bx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

Fricas [A]
time = 0.34, size = 18, normalized size = 0.86

$$\frac{2}{221}\left(13\,cx^{8}+17\,bx^{6}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 2/221*(13*c*x^8 + 17*b*x^6)*sqrt(x)
```

Sympy [A]
time = 0.77, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{13}{2}}}{13}+\frac{2cx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(13/2)/13 + 2*c*x**(17/2)/17
```

Giac [A]
time = 4.69, size = 13, normalized size = 0.62

$$\frac{2}{17}\,cx^{\frac{17}{2}}+\frac{2}{13}\,bx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

Mupad [B]
time = 0.04, size = 15, normalized size = 0.71

$$\frac{2\,x^{13/2}\,\left(13\,c\,x^2+17\,b\right)}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(13/2)*(17*b + 13*c*x^2))/221
```

3.293 $\int x^{5/2}(bx^2 + cx^4)\, dx$

Optimal. Leaf size=21

$$\frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2}$$

```
[Out] 2/11*b*x^(11/2)+2/15*c*x^(15/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{14\}$

$$\frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(bx^2 + cx^4\right)\, dx &= \int \left(bx^{9/2} + cx^{13/2}\right)\, dx \\ &= \frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{165}\left(15bx^{11/2} + 11cx^{15/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*(15*b*x^(11/2) + 11*c*x^(15/2)))/165
```

Maple [A]
time = 0.04, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{11}{2}}}{11}+\frac{2c\,x^{\frac{15}{2}}}{15}$	14
default	$\frac{2b\,x^{\frac{11}{2}}}{11}+\frac{2c\,x^{\frac{15}{2}}}{15}$	14
gosper	$\frac{2x^{\frac{11}{2}}\left(11c\,x^{2}+15b\right)}{165}$	16
trager	$\frac{2x^{\frac{11}{2}}\left(11c\,x^{2}+15b\right)}{165}$	16
risch	$\frac{2x^{\frac{11}{2}}\left(11c\,x^{2}+15b\right)}{165}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 2/11*b*x^(11/2)+2/15*c*x^(15/2)
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.62

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

Fricas [A]
time = 0.33, size = 18, normalized size = 0.86

$$\frac{2}{165}\left(11\,cx^{7}+15\,bx^{5}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 2/165*(11*c*x^7 + 15*b*x^5)*sqrt(x)
```

Sympy [A]
time = 0.50, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{11}{2}}}{11}+\frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

Giac [A]
time = 3.93, size = 13, normalized size = 0.62

$$\frac{2}{15}cx^{\frac{15}{2}}+\frac{2}{11}bx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{11/2}\,\left(11\,c\,x^2+15\,b\right)}{165}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(11/2)*(15*b + 11*c*x^2))/165
```

3.294 $\int x^{3/2}(bx^2+cx^4)\,dx$

Optimal. Leaf size=21

$$\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

```
[Out] 2/9*b*x^(9/2)+2/13*c*x^(13/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {14}

$$\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(bx^2+cx^4\right)\,dx &= \int \left(bx^{7/2}+cx^{11/2}\right)\,dx\\ &= \frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{117}\left(13bx^{9/2}+9cx^{13/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*(13*b*x^(9/2) + 9*c*x^(13/2)))/117
```

Maple [A]
time = 0.04, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{9}{2}}}{9}+\frac{2c\,x^{\frac{13}{2}}}{13}$	14
default	$\frac{2b\,x^{\frac{9}{2}}}{9}+\frac{2c\,x^{\frac{13}{2}}}{13}$	14
gosper	$\frac{2x^{\frac{9}{2}}\left(9c\,x^{2}+13b\right)}{117}$	16
trager	$\frac{2x^{\frac{9}{2}}\left(9c\,x^{2}+13b\right)}{117}$	16
risch	$\frac{2x^{\frac{9}{2}}\left(9c\,x^{2}+13b\right)}{117}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 2/9*b*x^(9/2)+2/13*c*x^(13/2)
```

Maxima [A]
time = 0.30, size = 13, normalized size = 0.62

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

Fricas [A]
time = 0.33, size = 18, normalized size = 0.86

$$\frac{2}{117}\left(9\,cx^{6}+13\,bx^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 2/117*(9*c*x^6 + 13*b*x^4)*sqrt(x)
```

Sympy [A]
time = 0.31, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{9}{2}}}{9}+\frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

Giac [A]
time = 5.71, size = 13, normalized size = 0.62

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{9/2}\,\left(9\,c\,x^2+13\,b\right)}{117}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(9/2)*(13*b + 9*c*x^2))/117
```

3.295 $\int \sqrt{x}\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

```
[Out] 2/7*b*x^(7/2)+2/11*c*x^(11/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {14}

$$\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4),x]
[Out] (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(bx^2+cx^4\right)\,dx &= \int \left(bx^{5/2}+cx^{9/2}\right)\,dx\\ &= \frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.90

$$\frac{2}{77}x^{7/2}\left(11b+7cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4),x]
[Out] (2*x^(7/2)*(11*b + 7*c*x^2))/77
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{7}{2}}}{7}+\frac{2c\,x^{\frac{11}{2}}}{11}$	14
default	$\frac{2b\,x^{\frac{7}{2}}}{7}+\frac{2c\,x^{\frac{11}{2}}}{11}$	14
gosper	$\frac{2x^{\frac{7}{2}}\left(7c\,x^{2}+11b\right)}{77}$	16
trager	$\frac{2x^{\frac{7}{2}}\left(7c\,x^{2}+11b\right)}{77}$	16
risch	$\frac{2x^{\frac{7}{2}}\left(7c\,x^{2}+11b\right)}{77}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 2/7*b*x^(7/2)+2/11*c*x^(11/2)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.62

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

Fricas [A]
time = 0.34, size = 18, normalized size = 0.86

$$\frac{2}{77}\left(7\,cx^{5}+11\,bx^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 2/77*(7*c*x^5 + 11*b*x^3)*sqrt(x)
```

Sympy [A]
time = 0.80, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{7}{2}}}{7}+\frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

Giac [A]
time = 4.04, size = 13, normalized size = 0.62

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{7/2}\,\left(7\,c\,x^2+11\,b\right)}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(7/2)*(11*b + 7*c*x^2))/77
```

3.296 $\int \frac{bx^2+cx^4}{\sqrt{x}} \, dx$

Optimal. Leaf size=21

$$\frac{2}{5}bx^{5/2} + \frac{2}{9}cx^{9/2}$$

```
[Out] 2/5*b*x^(5/2)+2/9*c*x^(9/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{14\}$

$$\frac{2}{5}bx^{5/2} + \frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/Sqrt[x],x]
[Out] (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{bx^2+cx^4}{\sqrt{x}} \, dx &= \int \left(bx^{3/2}+cx^{7/2}\right) \, dx \\ &= \frac{2}{5}bx^{5/2} + \frac{2}{9}cx^{9/2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{45}\left(9bx^{5/2}+5cx^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/Sqrt[x],x]
[Out] (2*(9*b*x^(5/2) + 5*c*x^(9/2)))/45
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{5}{2}}}{5}+\frac{2c\,x^{\frac{9}{2}}}{9}$	14
default	$\frac{2b\,x^{\frac{5}{2}}}{5}+\frac{2c\,x^{\frac{9}{2}}}{9}$	14
gosper	$\frac{2x^{\frac{5}{2}}\left(5c\,x^{2}+9b\right)}{45}$	16
trager	$\frac{2x^{\frac{5}{2}}\left(5c\,x^{2}+9b\right)}{45}$	16
risch	$\frac{2x^{\frac{5}{2}}\left(5c\,x^{2}+9b\right)}{45}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*b*x^(5/2)+2/9*c*x^(9/2)
```

Maxima [A]
time = 0.32, size = 13, normalized size = 0.62

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

Fricas [A]
time = 0.34, size = 18, normalized size = 0.86

$$\frac{2}{45}\left(5\,cx^{4}+9\,bx^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*c*x^4 + 9*b*x^2)*sqrt(x)
```

Sympy [A]
time = 0.15, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{5}{2}}}{5}+\frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

Giac [A]
time = 5.73, size = 13, normalized size = 0.62

$$\frac{2}{9}cx^{\frac{9}{2}} + \frac{2}{5}bx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{5/2}\,\left(5\,c\,x^2+9\,b\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(1/2),x)
```

```
[Out] (2*x^(5/2)*(9*b + 5*c*x^2))/45
```

3.297 $\int \frac{bx^2+cx^4}{x^{3/2}} \, dx$

Optimal. Leaf size=21

$$\frac{2}{3}bx^{3/2} + \frac{2}{7}cx^{7/2}$$

[Out] 2/3*b*x^(3/2)+2/7*c*x^(7/2)

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{14\}$

$$\frac{2}{3}bx^{3/2} + \frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)/x^(3/2),x]

[Out] (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{bx^2 + cx^4}{x^{3/2}} \, dx &= \int \left(b\sqrt{x} + cx^{5/2}\right) \, dx \\
&= \frac{2}{3}bx^{3/2} + \frac{2}{7}cx^{7/2}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{21}\left(7bx^{3/2} + 3cx^{7/2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^2 + c*x^4)/x^(3/2),x]

[Out] (2*(7*b*x^(3/2) + 3*c*x^(7/2)))/21

Maple [A]
time = 0.02, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$\frac{2bx^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{7}{2}}}{7}$	14
default	$\frac{2bx^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{7}{2}}}{7}$	14
gosper	$\frac{2x^{\frac{3}{2}}\left(3cx^{2}+7b\right)}{21}$	16
trager	$\frac{2x^{\frac{3}{2}}\left(3cx^{2}+7b\right)}{21}$	16
risch	$\frac{2x^{\frac{3}{2}}\left(3cx^{2}+7b\right)}{21}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*b*x^(3/2)+2/7*c*x^(7/2)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.62

$$\frac{2}{7}\,cx^{\frac{7}{2}}+\frac{2}{3}\,bx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="maxima")
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

Fricas [A]
time = 0.34, size = 16, normalized size = 0.76

$$\frac{2}{21}\left(3\,cx^{3}+7\,bx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="fricas")
[Out] 2/21*(3*c*x^3 + 7*b*x)*sqrt(x)
```

Sympy [A]
time = 0.18, size = 19, normalized size = 0.90

$$\frac{2bx^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(3/2),x)
```

```
[Out] 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

Giac [A]
time = 3.72, size = 13, normalized size = 0.62

$$\frac{2}{7}cx^{\frac{7}{2}} + \frac{2}{3}bx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.71

$$\frac{2\,x^{3/2}\,\left(3\,c\,x^2+7\,b\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(3/2),x)
```

```
[Out] (2*x^(3/2)*(7*b + 3*c*x^2))/21
```

3.298 $\int \frac{bx^2+cx^4}{x^{5/2}} \, dx$

Optimal. Leaf size=19

$$2b\sqrt{x} + \frac{2}{5}cx^{5/2}$$

```
[Out] 2/5*c*x^(5/2)+2*b*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{14\}$

$$2b\sqrt{x} + \frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^{5/2}} \, dx &= \int \left(\frac{b}{\sqrt{x}}+cx^{3/2}\right) \, dx \\ &= 2b\sqrt{x} + \frac{2}{5}cx^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.05

$$\frac{2}{5}\left(5b\sqrt{x} + cx^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] (2*(5*b*Sqrt[x] + c*x^(5/2)))/5
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.74

method	result	size
derivativedivides	$\frac{2cx^{\frac{5}{2}}}{5}+2b\sqrt{x}$	14
default	$\frac{2cx^{\frac{5}{2}}}{5}+2b\sqrt{x}$	14
gosper	$\frac{2\sqrt{x}\,(cx^2+5b)}{5}$	15
trager	$\left(\frac{2cx^2}{5}+2b\right)\sqrt{x}$	15
risch	$\frac{2\sqrt{x}\,(cx^2+5b)}{5}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/5*c*x^(5/2)+2*b*x^(1/2)
```

Maxima [A]
time = 0.33, size = 13, normalized size = 0.68

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="maxima")
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x)
```

Fricas [A]
time = 0.34, size = 14, normalized size = 0.74

$$\frac{2}{5}\left(cx^2+5\,b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="fricas")
[Out] 2/5*(c*x^2 + 5*b)*sqrt(x)
```

Sympy [A]
time = 0.23, size = 17, normalized size = 0.89

$$2b\sqrt{x}+\frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(5/2),x)
```

```
[Out] 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

Giac [A]
time = 3.71, size = 13, normalized size = 0.68

$$\frac{2}{5}\,cx^{\frac{5}{2}} + 2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x)
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.74

$$\frac{2\,\sqrt{x}\,\left(c\,x^2+5\,b\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(5/2),x)
```

```
[Out] (2*x^(1/2)*(5*b + c*x^2))/5
```

3.299 $\int \frac{bx^2+cx^4}{x^{7/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}$$

[Out] 2/3*c*x^(3/2)-2*b/x^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{3}cx^{3/2}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[(b*x^2 + c*x^4)/x^(7/2),x]

[Out] (-2*b)/Sqrt[x] + (2*c*x^(3/2))/3

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{bx^2+cx^4}{x^{7/2}}\,dx &= \int \left(\frac{b}{x^{3/2}}+c\sqrt{x}\right)dx\\ &= -\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$-\frac{2(3b-cx^2)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^2 + c*x^4)/x^(7/2),x]

```
[Out] (-2*(3*b - c*x^2))/(3*Sqrt[x])
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.74

method	result	size
derivativedivides	$\frac{2 c \,x^{\frac{3}{2}}}{3}-\frac{2 b}{\sqrt{x}}$	14
default	$\frac{2 c \,x^{\frac{3}{2}}}{3}-\frac{2 b}{\sqrt{x}}$	14
gosper	$-\frac{2\left(-c \,x^{2}+3 b \right)}{3 \sqrt{x}}$	16
trager	$-\frac{2\left(-c \,x^{2}+3 b \right)}{3 \sqrt{x}}$	16
risch	$-\frac{2\left(-c \,x^{2}+3 b \right)}{3 \sqrt{x}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/3*c*x^(3/2)-2*b/x^(1/2)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.68

$$\frac{2}{3}\, c x^{\frac{3}{2}} - \frac{2\, b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="maxima")
[Out] 2/3*c*x^(3/2) - 2*b/sqrt(x)
```

Fricas [A]
time = 0.36, size = 14, normalized size = 0.74

$$\frac{2\,(c x^2 - 3\, b)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="fricas")
[Out] 2/3*(c*x^2 - 3*b)/sqrt(x)
```

Sympy [A]
time = 0.37, size = 17, normalized size = 0.89

$$-\frac{2b}{\sqrt{x}} + \frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)/x**(7/2),x)
```

```
[Out] -2*b/sqrt(x) + 2*c*x**(3/2)/3
```

Giac [A]
time = 3.78, size = 13, normalized size = 0.68

$$\frac{2}{3}\,cx^{\frac{3}{2}} - \frac{2\,b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*c*x^(3/2) - 2*b/sqrt(x)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.79

$$-\frac{6\,b-2\,c\,x^2}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)/x^(7/2),x)
```

```
[Out] -(6*b - 2*c*x^2)/(3*x^(1/2))
```

3.300 $\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{17}b^2x^{17/2}+\frac{4}{21}bcx^{21/2}+\frac{2}{25}c^2x^{25/2}$$

```
[Out] 2/17*b^2*x^(17/2)+4/21*b*c*x^(21/2)+2/25*c^2*x^(25/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {1598, 276}

$$\frac{2}{17}b^2x^{17/2}+\frac{4}{21}bcx^{21/2}+\frac{2}{25}c^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(17/2))/17 + (4*b*c*x^(21/2))/21 + (2*c^2*x^(25/2))/25
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx&=\int x^{15/2}\left(b+cx^2\right)^2\,dx\\&=\int\left(b^2x^{15/2}+2bcx^{19/2}+c^2x^{23/2}\right)\,dx\\&=\frac{2}{17}b^2x^{17/2}+\frac{4}{21}bcx^{21/2}+\frac{2}{25}c^2x^{25/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.83

$$\frac{2x^{17/2}\left(525b^2+850bcx^2+357c^2x^4\right)}{8925}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(17/2)*(525*b^2 + 850*b*c*x^2 + 357*c^2*x^4))/8925
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{17}{2}}}{17}+\frac{4bc\,x^{\frac{21}{2}}}{21}+\frac{2c^2x^{\frac{25}{2}}}{25}$	25
default	$\frac{2b^2x^{\frac{17}{2}}}{17}+\frac{4bc\,x^{\frac{21}{2}}}{21}+\frac{2c^2x^{\frac{25}{2}}}{25}$	25
gosper	$\frac{2x^{\frac{17}{2}}\left(357c^2x^4+850bc\,x^2+525b^2\right)}{8925}$	27
trager	$\frac{2x^{\frac{17}{2}}\left(357c^2x^4+850bc\,x^2+525b^2\right)}{8925}$	27
risch	$\frac{2x^{\frac{17}{2}}\left(357c^2x^4+850bc\,x^2+525b^2\right)}{8925}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*b^2*x^(17/2)+4/21*b*c*x^(21/2)+2/25*c^2*x^(25/2)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.67

$$\frac{2}{25}\,c^2x^{\frac{25}{2}}+\frac{4}{21}\,bcx^{\frac{21}{2}}+\frac{2}{17}\,b^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)
```

Fricas [A]
time = 0.35, size = 29, normalized size = 0.81

$$\frac{2}{8925}\left(357\,c^2x^{12}+850\,bcx^{10}+525\,b^2x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/8925*(357*c^2*x^12 + 850*b*c*x^10 + 525*b^2*x^8)*sqrt(x)
```

Sympy [A]
time = 1.68, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{17}{2}}}{17} + \frac{4bcx^{\frac{21}{2}}}{21} + \frac{2c^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(17/2)/17 + 4*b*c*x**(21/2)/21 + 2*c**2*x**(25/2)/25
```

Giac [A]
time = 2.70, size = 24, normalized size = 0.67

$$\frac{2}{25}\,c^2x^{\frac{25}{2}} + \frac{4}{21}\,bcx^{\frac{21}{2}} + \frac{2}{17}\,b^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.69

$$x^{17/2}\left(\frac{2\,b^2}{17}+\frac{4\,b\,c\,x^2}{21}+\frac{2\,c^2\,x^4}{25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(17/2)*((2*b^2)/17 + (2*c^2*x^4)/25 + (4*b*c*x^2)/21)
```

3.301 $\int x^{5/2}(bx^2+cx^4)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

[Out] 2/15*b^2*x^(15/2)+4/19*b*c*x^(19/2)+2/23*c^2*x^(23/2)

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {1598, 276}

$$\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)*(b*x^2 + c*x^4)^2,x]

[Out] (2*b^2*x^(15/2))/15 + (4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx&=\int x^{13/2}\left(b+cx^2\right)^2\,dx\\&=\int\left(b^2x^{13/2}+2bcx^{17/2}+c^2x^{21/2}\right)\,dx\\&=\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.83

$$\frac{2x^{15/2}(437b^2+690bcx^2+285c^2x^4)}{6555}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(15/2)*(437*b^2 + 690*b*c*x^2 + 285*c^2*x^4))/6555
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2b^{2}x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{19}{2}}}{19}+\frac{2c^{2}x^{\frac{23}{2}}}{23}$	25
default	$\frac{2b^{2}x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{19}{2}}}{19}+\frac{2c^{2}x^{\frac{23}{2}}}{23}$	25
gosper	$\frac{2x^{\frac{15}{2}}\left(285c^{2}x^{4}+690bc\,x^{2}+437b^{2}\right)}{6555}$	27
trager	$\frac{2x^{\frac{15}{2}}\left(285c^{2}x^{4}+690bc\,x^{2}+437b^{2}\right)}{6555}$	27
risch	$\frac{2x^{\frac{15}{2}}\left(285c^{2}x^{4}+690bc\,x^{2}+437b^{2}\right)}{6555}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*b^2*x^(15/2)+4/19*b*c*x^(19/2)+2/23*c^2*x^(23/2)
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.67

$$\frac{2}{23}\,c^{2}x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^{2}x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

Fricas [A]
time = 0.35, size = 29, normalized size = 0.81

$$\frac{2}{6555}\left(285\,c^{2}x^{11}+690\,bcx^{9}+437\,b^{2}x^{7}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/6555*(285*c^2*x^11 + 690*b*c*x^9 + 437*b^2*x^7)*sqrt(x)
```

Sympy [A]
time = 1.18, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{15}{2}}}{15} + \frac{4bcx^{\frac{19}{2}}}{19} + \frac{2c^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(15/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

Giac [A]
time = 3.19, size = 24, normalized size = 0.67

$$\frac{2}{23}\,c^2x^{\frac{23}{2}} + \frac{4}{19}\,bcx^{\frac{19}{2}} + \frac{2}{15}\,b^2x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.69

$$x^{15/2}\left(\frac{2\,b^2}{15}+\frac{4\,b\,c\,x^2}{19}+\frac{2\,c^2\,x^4}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(15/2)*((2*b^2)/15 + (2*c^2*x^4)/23 + (4*b*c*x^2)/19)
```

3.302 $\int x^{3/2}(bx^2+cx^4)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

```
[Out] 2/13*b^2*x^(13/2)+4/17*b*c*x^(17/2)+2/21*c^2*x^(21/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1598, 276}

$$\frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(13/2))/13 + (4*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx&=\int x^{11/2}\left(b+cx^2\right)^2\,dx\\&=\int\left(b^2x^{11/2}+2bcx^{15/2}+c^2x^{19/2}\right)\,dx\\&=\frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.83

$$\frac{2x^{13/2}(357b^2+546bcx^2+221c^2x^4)}{4641}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(13/2)*(357*b^2 + 546*b*c*x^2 + 221*c^2*x^4))/4641
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$	25
default	$\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$	25
gosper	$\frac{2x^{\frac{13}{2}}\left(221c^2x^4+546bc\,x^2+357b^2\right)}{4641}$	27
trager	$\frac{2x^{\frac{13}{2}}\left(221c^2x^4+546bc\,x^2+357b^2\right)}{4641}$	27
risch	$\frac{2x^{\frac{13}{2}}\left(221c^2x^4+546bc\,x^2+357b^2\right)}{4641}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 2/13*b^2*x^(13/2)+4/17*b*c*x^(17/2)+2/21*c^2*x^(21/2)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.67

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

Fricas [A]
time = 0.34, size = 29, normalized size = 0.81

$$\frac{2}{4641}\left(221\,c^2x^{10}+546\,bcx^8+357\,b^2x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 2/4641*(221*c^2*x^10 + 546*b*c*x^8 + 357*b^2*x^6)*sqrt(x)
```

Sympy [A]
time = 0.81, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**2,x)
[Out] 2*b**2*x**(13/2)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

Giac [A]
time = 3.95, size = 24, normalized size = 0.67

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

Mupad [B]
time = 4.27, size = 25, normalized size = 0.69

$$x^{13/2}\left(\frac{2\,b^2}{13}+\frac{4\,b\,c\,x^2}{17}+\frac{2\,c^2\,x^4}{21}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4)^2,x)
[Out] x^(13/2)*((2*b^2)/13 + (2*c^2*x^4)/21 + (4*b*c*x^2)/17)
```

3.303 $\int \sqrt{x} \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

```
[Out] 2/11*b^2*x^(11/2)+4/15*b*c*x^(15/2)+2/19*c^2*x^(19/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1598, 276}

$$\frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(11/2))/11 + (4*b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x} \left(bx^2+cx^4\right)^2 \, dx &= \int x^{9/2}\left(b+cx^2\right)^2 \, dx \\
&= \int \left(b^2x^{9/2}+2bcx^{13/2}+c^2x^{17/2}\right) \, dx \\
&= \frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.83

$$\frac{2x^{11/2}(285b^2+418bcx^2+165c^2x^4)}{3135}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(11/2)*(285*b^2 + 418*b*c*x^2 + 165*c^2*x^4))/3135
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2b^{2}x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{15}{2}}}{15}+\frac{2c^{2}x^{\frac{19}{2}}}{19}$	25
default	$\frac{2b^{2}x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{15}{2}}}{15}+\frac{2c^{2}x^{\frac{19}{2}}}{19}$	25
gosper	$\frac{2x^{\frac{11}{2}}\left(165c^{2}x^{4}+418bc\,x^{2}+285b^{2}\right)}{3135}$	27
trager	$\frac{2x^{\frac{11}{2}}\left(165c^{2}x^{4}+418bc\,x^{2}+285b^{2}\right)}{3135}$	27
risch	$\frac{2x^{\frac{11}{2}}\left(165c^{2}x^{4}+418bc\,x^{2}+285b^{2}\right)}{3135}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 2/11*b^2*x^(11/2)+4/15*b*c*x^(15/2)+2/19*c^2*x^(19/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.67

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)
```

Fricas [A]
time = 0.35, size = 29, normalized size = 0.81

$$\frac{2}{3135}\left(165\,c^2x^9+418\,bcx^7+285\,b^2x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 2/3135*(165*c^2*x^9 + 418*b*c*x^7 + 285*b^2*x^5)*sqrt(x)
```

Sympy [A]
time = 1.31, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{11}{2}}}{11}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(11/2)/11 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/2)/19
```

Giac [A]
time = 4.32, size = 24, normalized size = 0.67

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.69

$$x^{11/2}\left(\frac{2\,b^2}{11}+\frac{4\,b\,c\,x^2}{15}+\frac{2\,c^2\,x^4}{19}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(11/2)*((2*b^2)/11 + (2*c^2*x^4)/19 + (4*b*c*x^2)/15)
```

3.304 $\int \frac{(bx^2+cx^4)^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=36

$$\frac{2}{9}b^2x^{9/2} + \frac{4}{13}bcx^{13/2} + \frac{2}{17}c^2x^{17/2}$$

```
[Out] 2/9*b^2*x^(9/2)+4/13*b*c*x^(13/2)+2/17*c^2*x^(17/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1598, 276}

$$\frac{2}{9}b^2x^{9/2} + \frac{4}{13}bcx^{13/2} + \frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] (2*b^2*x^(9/2))/9 + (4*b*c*x^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{\sqrt{x}} \, dx &= \int x^{7/2}\left(b+cx^2\right)^2 \, dx \\
&= \int \left(b^2x^{7/2} + 2bcx^{11/2} + c^2x^{15/2}\right) \, dx \\
&= \frac{2}{9}b^2x^{9/2} + \frac{4}{13}bcx^{13/2} + \frac{2}{17}c^2x^{17/2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.83

$$\frac{2x^{9/2}(221b^2 + 306bcx^2 + 117c^2x^4)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/Sqrt[x],x]
[Out] (2*x^(9/2)*(221*b^2 + 306*b*c*x^2 + 117*c^2*x^4))/1989
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{9}{2}}}{9} + \frac{4bc\,x^{\frac{13}{2}}}{13} + \frac{2c^2x^{\frac{17}{2}}}{17}$	25
default	$\frac{2b^2x^{\frac{9}{2}}}{9} + \frac{4bc\,x^{\frac{13}{2}}}{13} + \frac{2c^2x^{\frac{17}{2}}}{17}$	25
gosper	$\frac{2x^{\frac{9}{2}}\left(117c^2x^4+306bc\,x^2+221b^2\right)}{1989}$	27
trager	$\frac{2x^{\frac{9}{2}}\left(117c^2x^4+306bc\,x^2+221b^2\right)}{1989}$	27
risch	$\frac{2x^{\frac{9}{2}}\left(117c^2x^4+306bc\,x^2+221b^2\right)}{1989}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/9*b^2*x^(9/2)+4/13*b*c*x^(13/2)+2/17*c^2*x^(17/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.67

$$\frac{2}{17}\,c^2x^{\frac{17}{2}} + \frac{4}{13}\,bcx^{\frac{13}{2}} + \frac{2}{9}\,b^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

Fricas [A]
time = 0.34, size = 29, normalized size = 0.81

$$\frac{2}{1989}\left(117\,c^2x^8 + 306\,bcx^6 + 221\,b^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*c^2*x^8 + 306*b*c*x^6 + 221*b^2*x^4)*sqrt(x)
```

Sympy [A]
time = 0.49, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{9}{2}}}{9}+\frac{4bcx^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] 2*b**2*x**(9/2)/9 + 4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

Giac [A]
time = 4.56, size = 24, normalized size = 0.67

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.69

$$x^{9/2}\left(\frac{2\,b^2}{9}+\frac{4\,b\,c\,x^2}{13}+\frac{2\,c^2\,x^4}{17}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(1/2),x)
```

```
[Out] x^(9/2)*((2*b^2)/9 + (2*c^2*x^4)/17 + (4*b*c*x^2)/13)
```

3.305 $\int \frac{(bx^2+cx^4)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

```
[Out] 2/7*b^2*x^(7/2)+4/11*b*c*x^(11/2)+2/15*c^2*x^(15/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1598, 276}

$$\frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*b^2*x^(7/2))/7 + (4*b*c*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^2}{x^{3/2}}\,dx &= \int x^{5/2}(b+cx^2)^2\,dx\\
&= \int \left(b^2x^{5/2}+2bcx^{9/2}+c^2x^{13/2}\right)dx\\
&= \frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.83

$$\frac{2x^{7/2}(165b^2+210bcx^2+77c^2x^4)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(3/2),x]
[Out] (2*x^(7/2)*(165*b^2 + 210*b*c*x^2 + 77*c^2*x^4))/1155
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2b^{2}x^{\frac{7}{2}}}{7}+\frac{4bc\,x^{\frac{11}{2}}}{11}+\frac{2c^{2}x^{\frac{15}{2}}}{15}$	25
default	$\frac{2b^{2}x^{\frac{7}{2}}}{7}+\frac{4bc\,x^{\frac{11}{2}}}{11}+\frac{2c^{2}x^{\frac{15}{2}}}{15}$	25
gosper	$\frac{2x^{\frac{7}{2}}\left(77c^{2}x^{4}+210bc\,x^{2}+165b^{2}\right)}{1155}$	27
trager	$\frac{2x^{\frac{7}{2}}\left(77c^{2}x^{4}+210bc\,x^{2}+165b^{2}\right)}{1155}$	27
risch	$\frac{2x^{\frac{7}{2}}\left(77c^{2}x^{4}+210bc\,x^{2}+165b^{2}\right)}{1155}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/7*b^2*x^(7/2)+4/11*b*c*x^(11/2)+2/15*c^2*x^(15/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.67

$$\frac{2}{15}\,c^2x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\,b^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="maxima")
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)
```

Fricas [A]
time = 0.34, size = 29, normalized size = 0.81

$$\frac{2}{1155}\left(77\,c^2x^7+210\,bcx^5+165\,b^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="fricas")
[Out] 2/1155*(77*c^2*x^7 + 210*b*c*x^5 + 165*b^2*x^3)*sqrt(x)
```

Sympy [A]
time = 0.61, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(3/2),x)
[Out] 2*b**2*x**(7/2)/7 + 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

Giac [A]
time = 4.23, size = 24, normalized size = 0.67

$$\frac{2}{15}\,c^2x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\,b^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="giac")
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)
```

Mupad [B]
time = 4.44, size = 26, normalized size = 0.72

$$\frac{2\,x^{7/2}\,\left(165\,b^2+210\,b\,c\,x^2+77\,c^2\,x^4\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(3/2),x)
[Out] (2*x^(7/2)*(165*b^2 + 77*c^2*x^4 + 210*b*c*x^2))/1155
```

3.306 $\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

```
[Out] 2/5*b^2*x^(5/2)+4/9*b*c*x^(9/2)+2/13*c^2*x^(13/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1598, 276}

$$\frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^(5/2),x]
[Out] (2*b^2*x^(5/2))/5 + (4*b*c*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx &= \int x^{3/2}\left(b+cx^2\right)^2\,dx\\ &= \int \left(b^2x^{3/2}+2bcx^{7/2}+c^2x^{11/2}\right)\,dx\\ &= \frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.83

$$\frac{2}{585}x^{5/2}\left(117b^2+130bcx^2+45c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(5/2),x]
[Out] (2*x^(5/2)*(117*b^2 + 130*b*c*x^2 + 45*c^2*x^4))/585
```

Maple [A]
time = 0.11, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2b^{2}x^{\frac{5}{2}}}{5}+\frac{4bc\,x^{\frac{9}{2}}}{9}+\frac{2c^{2}x^{\frac{13}{2}}}{13}$	25
default	$\frac{2b^{2}x^{\frac{5}{2}}}{5}+\frac{4bc\,x^{\frac{9}{2}}}{9}+\frac{2c^{2}x^{\frac{13}{2}}}{13}$	25
gosper	$\frac{2x^{\frac{5}{2}}\left(45c^{2}x^{4}+130bc\,x^{2}+117b^{2}\right)}{585}$	27
trager	$\frac{2x^{\frac{5}{2}}\left(45c^{2}x^{4}+130bc\,x^{2}+117b^{2}\right)}{585}$	27
risch	$\frac{2x^{\frac{5}{2}}\left(45c^{2}x^{4}+130bc\,x^{2}+117b^{2}\right)}{585}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/5*b^2*x^(5/2)+4/9*b*c*x^(9/2)+2/13*c^2*x^(13/2)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.67

$$\frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\,b^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="maxima")
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

Fricas [A]
time = 0.35, size = 29, normalized size = 0.81

$$\frac{2}{585}\left(45\,c^2x^6+130\,bcx^4+117\,b^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="fricas")
[Out] 2/585*(45*c^2*x^6 + 130*b*c*x^4 + 117*b^2*x^2)*sqrt(x)
```

Sympy [A]
time = 0.67, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{5}{2}}}{5} + \frac{4bcx^{\frac{9}{2}}}{9} + \frac{2c^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(5/2),x)
```

```
[Out] 2*b**2*x**(5/2)/5 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

Giac [A]
time = 3.77, size = 24, normalized size = 0.67

$$\frac{2}{13}\,c^2x^{\frac{13}{2}} + \frac{4}{9}\,bcx^{\frac{9}{2}} + \frac{2}{5}\,b^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.72

$$\frac{2\,x^{5/2}\,\left(117\,b^2+130\,b\,c\,x^2+45\,c^2\,x^4\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(5/2),x)
```

```
[Out] (2*x^(5/2)*(117*b^2 + 45*c^2*x^4 + 130*b*c*x^2))/585
```

3.307 $\int \frac{(bx^2+cx^4)^2}{x^{7/2}} dx$

Optimal. Leaf size=36

$$\frac{2}{3}b^2x^{3/2} + \frac{4}{7}bcx^{7/2} + \frac{2}{11}c^2x^{11/2}$$

```
[Out] 2/3*b^2*x^(3/2)+4/7*b*c*x^(7/2)+2/11*c^2*x^(11/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1598, 276}

$$\frac{2}{3}b^2x^{3/2} + \frac{4}{7}bcx^{7/2} + \frac{2}{11}c^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*b^2*x^(3/2))/3 + (4*b*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{(bx^2+cx^4)^2}{x^{7/2}} dx &= \int \sqrt{x} \left(b+cx^2\right)^2 dx \\ &= \int \left(b^2\sqrt{x} + 2bcx^{5/2} + c^2x^{9/2}\right) dx \\ &= \frac{2}{3}b^2x^{3/2} + \frac{4}{7}bcx^{7/2} + \frac{2}{11}c^2x^{11/2} \end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.83

$$\frac{2}{231}x^{3/2}\left(77b^2 + 66bcx^2 + 21c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*x^(3/2)*(77*b^2 + 66*b*c*x^2 + 21*c^2*x^4))/231
```

Maple [A]
time = 0.11, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2b^{2}x^{\frac{3}{2}}}{3}+\frac{4bc\,x^{\frac{7}{2}}}{7}+\frac{2c^{2}x^{\frac{11}{2}}}{11}$	25
default	$\frac{2b^{2}x^{\frac{3}{2}}}{3}+\frac{4bc\,x^{\frac{7}{2}}}{7}+\frac{2c^{2}x^{\frac{11}{2}}}{11}$	25
gosper	$\frac{2x^{\frac{3}{2}}\left(21c^{2}x^{4}+66bc\,x^{2}+77b^{2}\right)}{231}$	27
trager	$\frac{2x^{\frac{3}{2}}\left(21c^{2}x^{4}+66bc\,x^{2}+77b^{2}\right)}{231}$	27
risch	$\frac{2x^{\frac{3}{2}}\left(21c^{2}x^{4}+66bc\,x^{2}+77b^{2}\right)}{231}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*b^2*x^(3/2)+4/7*b*c*x^(7/2)+2/11*c^2*x^(11/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.67

$$\frac{2}{11}\,c^{2}x^{\frac{11}{2}}+\frac{4}{7}\,bcx^{\frac{7}{2}}+\frac{2}{3}\,b^{2}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

Fricas [A]
time = 0.32, size = 27, normalized size = 0.75

$$\frac{2}{231}\left(21\,c^{2}x^{5}+66\,bcx^{3}+77\,b^{2}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*c^2*x^5 + 66*b*c*x^3 + 77*b^2*x)*sqrt(x)
```

Sympy [A]
time = 0.86, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{3}{2}}}{3} + \frac{4bcx^{\frac{7}{2}}}{7} + \frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**2/x**(7/2),x)
[Out] 2*b**2*x**(3/2)/3 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

Giac [A]
time = 5.18, size = 24, normalized size = 0.67

$$\frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{7}\,bcx^{\frac{7}{2}} + \frac{2}{3}\,b^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="giac")
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

Mupad [B]
time = 0.05, size = 26, normalized size = 0.72

$$\frac{2\,x^{3/2}\,\left(77\,b^2+66\,b\,c\,x^2+21\,c^2\,x^4\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^2/x^(7/2),x)
[Out] (2*x^(3/2)*(77*b^2 + 21*c^2*x^4 + 66*b*c*x^2))/231
```

3.308 $\int x^{7/2}(bx^2+cx^4)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

```
[Out] 2/21*b^3*x^(21/2)+6/25*b^2*c*x^(25/2)+6/29*b*c^2*x^(29/2)+2/33*c^3*x^(33/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {1598, 276}

$$\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/29 + (2*c^
3*x^(33/2))/33
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx &= \int x^{19/2}\left(b+cx^2\right)^3\,dx\\
&= \int \left(b^3x^{19/2}+3b^2cx^{23/2}+3bc^2x^{27/2}+c^3x^{31/2}\right)\,dx\\
&= \frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 1.00

$$\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/29 + (2*c^
3*x^(33/2))/33
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{21}{2}}}{21}+\frac{6b^2cx^{\frac{25}{2}}}{25}+\frac{6bc^2x^{\frac{29}{2}}}{29}+\frac{2c^3x^{\frac{33}{2}}}{33}$	36
default	$\frac{2b^3x^{\frac{21}{2}}}{21}+\frac{6b^2cx^{\frac{25}{2}}}{25}+\frac{6bc^2x^{\frac{29}{2}}}{29}+\frac{2c^3x^{\frac{33}{2}}}{33}$	36
gosper	$\frac{2x^{\frac{21}{2}}\left(5075c^3x^6+173325bc^2x^4+20097b^2cx^2+7975b^3\right)}{167475}$	38
trager	$\frac{2x^{\frac{21}{2}}\left(5075c^3x^6+173325bc^2x^4+20097b^2cx^2+7975b^3\right)}{167475}$	38
risch	$\frac{2x^{\frac{21}{2}}\left(5075c^3x^6+173325bc^2x^4+20097b^2cx^2+7975b^3\right)}{167475}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*b^3*x^(21/2)+6/25*b^2*c*x^(25/2)+6/29*b*c^2*x^(29/2)+2/33*c^3*x^(33/2)
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.69

$$\frac{2}{33}\,c^3x^{\frac{33}{2}}+\frac{6}{29}\,bc^2x^{\frac{29}{2}}+\frac{6}{25}\,b^2cx^{\frac{25}{2}}+\frac{2}{21}\,b^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2/21*b^3*x^
(21/2)
```

Fricas [A]
time = 0.34, size = 40, normalized size = 0.78

$$\frac{2}{167475}\left(5075\,c^3x^{16}+17325\,bc^2x^{14}+20097\,b^2cx^{12}+7975\,b^3x^{10}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/167475*(5075*c^3*x^16 + 17325*b*c^2*x^14 + 20097*b^2*c*x^12 + 7975*b^3*x^
10)*sqrt(x)
```

Sympy [A]
time = 3.15, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{21}{2}}}{21} + \frac{6b^2cx^{\frac{25}{2}}}{25} + \frac{6bc^2x^{\frac{29}{2}}}{29} + \frac{2c^3x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(21/2)/21 + 6*b**2*c*x**(25/2)/25 + 6*b*c**2*x**(29/2)/29 + 2*c**
3*x**(33/2)/33
```

Giac [A]
time = 4.08, size = 35, normalized size = 0.69

$$\frac{2}{33}\,c^3x^{\frac{33}{2}} + \frac{6}{29}\,bc^2x^{\frac{29}{2}} + \frac{6}{25}\,b^2cx^{\frac{25}{2}} + \frac{2}{21}\,b^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2/21*b^3*x^
(21/2)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{21/2}}{21} + \frac{2\,c^3\,x^{33/2}}{33} + \frac{6\,b^2\,c\,x^{25/2}}{25} + \frac{6\,b\,c^2\,x^{29/2}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(21/2))/21 + (2*c^3*x^(33/2))/33 + (6*b^2*c*x^(25/2))/25 + (6*b*c^
2*x^(29/2))/29
```

3.309 $\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

```
[Out] 2/19*b^3*x^(19/2)+6/23*b^2*c*x^(23/2)+2/9*b*c^2*x^(27/2)+2/31*c^3*x^(31/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {1598, 276}

$$\frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3
*x^(31/2))/31
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx&=\int x^{17/2}\left(b+cx^2\right)^3\,dx\\&=\int\left(b^3x^{17/2}+3b^2cx^{21/2}+3bc^2x^{25/2}+c^3x^{29/2}\right)dx\\&=\frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 1.00

$$\frac{2}{19}b^3x^{19/2} + \frac{6}{23}b^2cx^{23/2} + \frac{2}{9}bc^2x^{27/2} + \frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3
*x^(31/2))/31
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2b^{3}x^{\frac{19}{2}}}{19}+\frac{6b^{2}c\,x^{\frac{23}{2}}}{23}+\frac{2b\,c^{2}x^{\frac{27}{2}}}{9}+\frac{2c^{3}x^{\frac{31}{2}}}{31}$	36
default	$\frac{2b^{3}x^{\frac{19}{2}}}{19}+\frac{6b^{2}c\,x^{\frac{23}{2}}}{23}+\frac{2b\,c^{2}x^{\frac{27}{2}}}{9}+\frac{2c^{3}x^{\frac{31}{2}}}{31}$	36
gosper	$\frac{2x^{\frac{19}{2}}\left(3933c^{3}x^{6}+13547b\,c^{2}x^{4}+15903b^{2}c\,x^{2}+6417b^{3}\right)}{121923}$	38
trager	$\frac{2x^{\frac{19}{2}}\left(3933c^{3}x^{6}+13547b\,c^{2}x^{4}+15903b^{2}c\,x^{2}+6417b^{3}\right)}{121923}$	38
risch	$\frac{2x^{\frac{19}{2}}\left(3933c^{3}x^{6}+13547b\,c^{2}x^{4}+15903b^{2}c\,x^{2}+6417b^{3}\right)}{121923}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*b^3*x^(19/2)+6/23*b^2*c*x^(23/2)+2/9*b*c^2*x^(27/2)+2/31*c^3*x^(31/2)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.69

$$\frac{2}{31}\,c^3x^{\frac{31}{2}} + \frac{2}{9}\,bc^2x^{\frac{27}{2}} + \frac{6}{23}\,b^2cx^{\frac{23}{2}} + \frac{2}{19}\,b^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/19*b^3*x^(
19/2)
```

Fricas [A]
time = 0.38, size = 40, normalized size = 0.78

$$\frac{2}{121923}\left(3933\,c^3x^{15} + 13547\,bc^2x^{13} + 15903\,b^2cx^{11} + 6417\,b^3x^9\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/121923*(3933*c^3*x^15 + 13547*b*c^2*x^13 + 15903*b^2*c*x^11 + 6417*b^3*x^
9)*sqrt(x)
```

Sympy [A]
time = 2.32, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{19}{2}}}{19} + \frac{6b^2cx^{\frac{23}{2}}}{23} + \frac{2bc^2x^{\frac{27}{2}}}{9} + \frac{2c^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(19/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/9 + 2*c**3
*x**(31/2)/31
```

Giac [A]
time = 3.54, size = 35, normalized size = 0.69

$$\frac{2}{31}\,c^3x^{\frac{31}{2}} + \frac{2}{9}\,bc^2x^{\frac{27}{2}} + \frac{6}{23}\,b^2cx^{\frac{23}{2}} + \frac{2}{19}\,b^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/19*b^3*x^(
19/2)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{19/2}}{19} + \frac{2\,c^3\,x^{31/2}}{31} + \frac{6\,b^2\,c\,x^{23/2}}{23} + \frac{2\,b\,c^2\,x^{27/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(19/2))/19 + (2*c^3*x^(31/2))/31 + (6*b^2*c*x^(23/2))/23 + (2*b*c^
2*x^(27/2))/9
```

3.310 $\int x^{3/2}(bx^2+cx^4)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{17}b^3x^{17/2}+\frac{2}{7}b^2cx^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

```
[Out] 2/17*b^3*x^(17/2)+2/7*b^2*c*x^(21/2)+6/25*b*c^2*x^(25/2)+2/29*c^3*x^(29/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {1598, 276}

$$\frac{2}{17}b^3x^{17/2}+\frac{2}{7}b^2cx^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^3
*x^(29/2))/29
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(bx^2+cx^4\right)^3\,dx&=\int x^{15/2}\left(b+cx^2\right)^3\,dx\\&=\int\left(b^3x^{15/2}+3b^2cx^{19/2}+3bc^2x^{23/2}+c^3x^{27/2}\right)dx\\&=\frac{2}{17}b^3x^{17/2}+\frac{2}{7}b^2cx^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 0.92

$$\frac{2\left(5075b^3x^{17/2}+12325b^2cx^{21/2}+10353bc^2x^{25/2}+2975c^3x^{29/2}\right)}{86275}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(5075*b^3*x^(17/2) + 12325*b^2*c*x^(21/2) + 10353*b*c^2*x^(25/2) + 2975*
c^3*x^(29/2)))/86275
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{21}{2}}}{7}+\frac{6bc^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$	36
default	$\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{21}{2}}}{7}+\frac{6bc^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$	36
gosper	$\frac{2x^{\frac{17}{2}}\left(2975c^3x^6+10353bc^2x^4+12325b^2cx^2+5075b^3\right)}{86275}$	38
trager	$\frac{2x^{\frac{17}{2}}\left(2975c^3x^6+10353bc^2x^4+12325b^2cx^2+5075b^3\right)}{86275}$	38
risch	$\frac{2x^{\frac{17}{2}}\left(2975c^3x^6+10353bc^2x^4+12325b^2cx^2+5075b^3\right)}{86275}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*b^3*x^(17/2)+2/7*b^2*c*x^(21/2)+6/25*b*c^2*x^(25/2)+2/29*c^3*x^(29/2)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.69

$$\frac{2}{29}c^3x^{\frac{29}{2}}+\frac{6}{25}bc^2x^{\frac{25}{2}}+\frac{2}{7}b^2cx^{\frac{21}{2}}+\frac{2}{17}b^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/17*b^3*x^(
17/2)
```

Fricas [A]
time = 0.32, size = 40, normalized size = 0.78

$$\frac{2}{86275}\left(2975\,c^3x^{14}+10353\,bc^2x^{12}+12325\,b^2cx^{10}+5075\,b^3x^8\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/86275*(2975*c^3*x^14 + 10353*b*c^2*x^12 + 12325*b^2*c*x^10 + 5075*b^3*x^8
)*sqrt(x)
```

Sympy [A]
time = 1.70, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{21}{2}}}{7}+\frac{6bc^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(17/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/25 + 2*c**3
*x**(29/2)/29
```

Giac [A]
time = 4.31, size = 35, normalized size = 0.69

$$\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/17*b^3*x^(
17/2)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{17/2}}{17}+\frac{2\,c^3\,x^{29/2}}{29}+\frac{2\,b^2\,c\,x^{21/2}}{7}+\frac{6\,b\,c^2\,x^{25/2}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*c^3*x^(29/2))/29 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2
*x^(25/2))/25
```

3.311 $\int \sqrt{x} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

```
[Out] 2/15*b^3*x^(15/2)+6/19*b^2*c*x^(19/2)+6/23*b*c^2*x^(23/2)+2/27*c^3*x^(27/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1598, 276}

$$\frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(15/2))/15 + (6*b^2*c*x^(19/2))/19 + (6*b*c^2*x^(23/2))/23 + (2*c^
3*x^(27/2))/27
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(bx^2+cx^4\right)^3 \, dx &= \int x^{13/2}\left(b+cx^2\right)^3 \, dx\\ &= \int \left(b^3x^{13/2}+3b^2cx^{17/2}+3bc^2x^{21/2}+c^3x^{25/2}\right) dx\\ &= \frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.80

$$\frac{2x^{15/2}(3933b^3 + 9315b^2cx^2 + 7695bc^2x^4 + 2185c^3x^6)}{58995}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
[Out] (2*x^(15/2)*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^6))/58
995
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{15}{2}}}{15}+\frac{6b^2cx^{\frac{19}{2}}}{19}+\frac{6bc^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{27}{2}}}{27}$	36
default	$\frac{2b^3x^{\frac{15}{2}}}{15}+\frac{6b^2cx^{\frac{19}{2}}}{19}+\frac{6bc^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{27}{2}}}{27}$	36
gosper	$\frac{2x^{\frac{15}{2}}\left(2185c^3x^6+7695bc^2x^4+9315b^2cx^2+3933b^3\right)}{58995}$	38
trager	$\frac{2x^{\frac{15}{2}}\left(2185c^3x^6+7695bc^2x^4+9315b^2cx^2+3933b^3\right)}{58995}$	38
risch	$\frac{2x^{\frac{15}{2}}\left(2185c^3x^6+7695bc^2x^4+9315b^2cx^2+3933b^3\right)}{58995}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
[Out] 2/15*b^3*x^(15/2)+6/19*b^2*c*x^(19/2)+6/23*b*c^2*x^(23/2)+2/27*c^3*x^(27/2)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.69

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\,b^2cx^{\frac{19}{2}}+\frac{2}{15}\,b^3x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2/15*b^3*x^
(15/2)
```

Fricas [A]
time = 0.36, size = 40, normalized size = 0.78

$$\frac{2}{58995}\left(2185\,c^3x^{13}+7695\,bc^2x^{11}+9315\,b^2cx^9+3933\,b^3x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/58995*(2185*c^3*x^13 + 7695*b*c^2*x^11 + 9315*b^2*c*x^9 + 3933*b^3*x^7)*s
qrt(x)
```

Sympy [A]
time = 2.05, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{15}{2}}}{15}+\frac{6b^2cx^{\frac{19}{2}}}{19}+\frac{6bc^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(15/2)/15 + 6*b**2*c*x**(19/2)/19 + 6*b*c**2*x**(23/2)/23 + 2*c**
3*x**(27/2)/27
```

Giac [A]
time = 4.18, size = 35, normalized size = 0.69

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\,b^2cx^{\frac{19}{2}}+\frac{2}{15}\,b^3x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2/15*b^3*x^
(15/2)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{15/2}}{15}+\frac{2\,c^3\,x^{27/2}}{27}+\frac{6\,b^2\,c\,x^{19/2}}{19}+\frac{6\,b\,c^2\,x^{23/2}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (2*b^3*x^(15/2))/15 + (2*c^3*x^(27/2))/27 + (6*b^2*c*x^(19/2))/19 + (6*b*c^
2*x^(23/2))/23
```

3.312 $\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{13}b^3x^{13/2}+\frac{6}{17}b^2cx^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

```
[Out] 2/13*b^3*x^(13/2)+6/17*b^2*c*x^(17/2)+2/7*b*c^2*x^(21/2)+2/25*c^3*x^(25/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1598, 276}

$$\frac{2}{13}b^3x^{13/2}+\frac{6}{17}b^2cx^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*b^3*x^(13/2))/13 + (6*b^2*c*x^(17/2))/17 + (2*b*c^2*x^(21/2))/7 + (2*c^3
*x^(25/2))/25
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx &= \int x^{11/2}\left(b+cx^2\right)^3\,dx\\ &= \int \left(b^3x^{11/2}+3b^2cx^{15/2}+3bc^2x^{19/2}+c^3x^{23/2}\right)dx\\ &= \frac{2}{13}b^3x^{13/2}+\frac{6}{17}b^2cx^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.80

$$\frac{2x^{13/2}(2975b^3 + 6825b^2cx^2 + 5525bc^2x^4 + 1547c^3x^6)}{38675}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/Sqrt[x],x]
[Out] (2*x^(13/2)*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^6))/38
675
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{13}{2}}}{13}+\frac{6b^2cx^{\frac{17}{2}}}{17}+\frac{2bc^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{25}{2}}}{25}$	36
default	$\frac{2b^3x^{\frac{13}{2}}}{13}+\frac{6b^2cx^{\frac{17}{2}}}{17}+\frac{2bc^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{25}{2}}}{25}$	36
gosper	$\frac{2x^{\frac{13}{2}}(1547c^3x^6+5525bc^2x^4+6825b^2cx^2+2975b^3)}{38675}$	38
trager	$\frac{2x^{\frac{13}{2}}(1547c^3x^6+5525bc^2x^4+6825b^2cx^2+2975b^3)}{38675}$	38
risch	$\frac{2x^{\frac{13}{2}}(1547c^3x^6+5525bc^2x^4+6825b^2cx^2+2975b^3)}{38675}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/13*b^3*x^(13/2)+6/17*b^2*c*x^(17/2)+2/7*b*c^2*x^(21/2)+2/25*c^3*x^(25/2)
```

Maxima [A]
time = 0.30, size = 35, normalized size = 0.69

$$\frac{2}{25}c^3x^{\frac{25}{2}} + \frac{2}{7}bc^2x^{\frac{21}{2}} + \frac{6}{17}b^2cx^{\frac{17}{2}} + \frac{2}{13}b^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(
13/2)
```

Fricas [A]
time = 0.35, size = 40, normalized size = 0.78

$$\frac{2}{38675}\left(1547\,c^3x^{12} + 5525\,bc^2x^{10} + 6825\,b^2cx^8 + 2975\,b^3x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/38675*(1547*c^3*x^12 + 5525*b*c^2*x^10 + 6825*b^2*c*x^8 + 2975*b^3*x^6)*s
qrt(x)
```

Sympy [A]
time = 1.21, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{13}{2}}}{13} + \frac{6b^2cx^{\frac{17}{2}}}{17} + \frac{2bc^2x^{\frac{21}{2}}}{7} + \frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] 2*b**3*x**(13/2)/13 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/7 + 2*c**3
*x**(25/2)/25
```

Giac [A]
time = 3.69, size = 35, normalized size = 0.69

$$\frac{2}{25}\,c^3x^{\frac{25}{2}} + \frac{2}{7}\,bc^2x^{\frac{21}{2}} + \frac{6}{17}\,b^2cx^{\frac{17}{2}} + \frac{2}{13}\,b^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(
13/2)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{13/2}}{13} + \frac{2\,c^3\,x^{25/2}}{25} + \frac{6\,b^2\,c\,x^{17/2}}{17} + \frac{2\,b\,c^2\,x^{21/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^(1/2),x)
```

```
[Out] (2*b^3*x^(13/2))/13 + (2*c^3*x^(25/2))/25 + (6*b^2*c*x^(17/2))/17 + (2*b*c^
2*x^(21/2))/7
```

3.313 $\int \frac{(bx^2+cx^4)^3}{x^{3/2}} dx$

Optimal. Leaf size=51

$$\frac{2}{11}b^3x^{11/2} + \frac{2}{5}b^2cx^{15/2} + \frac{6}{19}bc^2x^{19/2} + \frac{2}{23}c^3x^{23/2}$$

```
[Out] 2/11*b^3*x^(11/2)+2/5*b^2*c*x^(15/2)+6/19*b*c^2*x^(19/2)+2/23*c^3*x^(23/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1598, 276}

$$\frac{2}{11}b^3x^{11/2} + \frac{2}{5}b^2cx^{15/2} + \frac{6}{19}bc^2x^{19/2} + \frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (2*b^3*x^(11/2))/11 + (2*b^2*c*x^(15/2))/5 + (6*b*c^2*x^(19/2))/19 + (2*c^3
*x^(23/2))/23
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^{3/2}} dx &= \int x^{9/2}(b+cx^2)^3 \, dx \\
&= \int \left(b^3x^{9/2} + 3b^2cx^{13/2} + 3bc^2x^{17/2} + c^3x^{21/2}\right) dx \\
&= \frac{2}{11}b^3x^{11/2} + \frac{2}{5}b^2cx^{15/2} + \frac{6}{19}bc^2x^{19/2} + \frac{2}{23}c^3x^{23/2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.80

$$\frac{2x^{11/2}(2185b^3+4807b^2cx^2+3795bc^2x^4+1045c^3x^6)}{24035}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(3/2),x]
[Out] (2*x^(11/2)*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^3*x^6))/24
035
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{2b^2c\,x^{\frac{15}{2}}}{5}+\frac{6b\,c^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{23}{2}}}{23}$	36
default	$\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{2b^2c\,x^{\frac{15}{2}}}{5}+\frac{6b\,c^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{23}{2}}}{23}$	36
gosper	$\frac{2x^{\frac{11}{2}}\left(1045c^3x^6+3795b\,c^2x^4+4807b^2c\,x^2+2185b^3\right)}{24035}$	38
trager	$\frac{2x^{\frac{11}{2}}\left(1045c^3x^6+3795b\,c^2x^4+4807b^2c\,x^2+2185b^3\right)}{24035}$	38
risch	$\frac{2x^{\frac{11}{2}}\left(1045c^3x^6+3795b\,c^2x^4+4807b^2c\,x^2+2185b^3\right)}{24035}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/11*b^3*x^(11/2)+2/5*b^2*c*x^(15/2)+6/19*b*c^2*x^(19/2)+2/23*c^3*x^(23/2)
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.69

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\,b^2cx^{\frac{15}{2}}+\frac{2}{11}\,b^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="maxima")
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/11*b^3*x^(
11/2)
```

Fricas [A]
time = 0.40, size = 40, normalized size = 0.78

$$\frac{2}{24035}\left(1045\,c^3x^{11}+3795\,bc^2x^9+4807\,b^2cx^7+2185\,b^3x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/24035*(1045*c^3*x^11 + 3795*b*c^2*x^9 + 4807*b^2*c*x^7 + 2185*b^3*x^5)*sq
rt(x)
```

Sympy [A]
time = 1.43, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{2b^2cx^{\frac{15}{2}}}{5}+\frac{6bc^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(3/2),x)
```

```
[Out] 2*b**3*x**(11/2)/11 + 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/19 + 2*c**3
*x**(23/2)/23
```

Giac [A]
time = 3.92, size = 35, normalized size = 0.69

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\,b^2cx^{\frac{15}{2}}+\frac{2}{11}\,b^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/11*b^3*x^(
11/2)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{11/2}}{11}+\frac{2\,c^3\,x^{23/2}}{23}+\frac{2\,b^2\,c\,x^{15/2}}{5}+\frac{6\,b\,c^2\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^(3/2),x)
```

```
[Out] (2*b^3*x^(11/2))/11 + (2*c^3*x^(23/2))/23 + (2*b^2*c*x^(15/2))/5 + (6*b*c^2
*x^(19/2))/19
```

3.314 $\int \frac{(bx^2+cx^4)^3}{x^{5/2}} \, dx$

Optimal. Leaf size=51

$$\frac{2}{9}b^3x^{9/2} + \frac{6}{13}b^2cx^{13/2} + \frac{6}{17}bc^2x^{17/2} + \frac{2}{21}c^3x^{21/2}$$

```
[Out] 2/9*b^3*x^(9/2)+6/13*b^2*c*x^(13/2)+6/17*b*c^2*x^(17/2)+2/21*c^3*x^(21/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1598, 276}

$$\frac{2}{9}b^3x^{9/2} + \frac{6}{13}b^2cx^{13/2} + \frac{6}{17}bc^2x^{17/2} + \frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*b^3*x^(9/2))/9 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2*x^(17/2))/17 + (2*c^3*
x^(21/2))/21
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{(bx^2+cx^4)^3}{x^{5/2}} \, dx &= \int x^{7/2}(b+cx^2)^3 \, dx \\ &= \int \left(b^3x^{7/2} + 3b^2cx^{11/2} + 3bc^2x^{15/2} + c^3x^{19/2}\right) dx \\ &= \frac{2}{9}b^3x^{9/2} + \frac{6}{13}b^2cx^{13/2} + \frac{6}{17}bc^2x^{17/2} + \frac{2}{21}c^3x^{21/2} \end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.80

$$\frac{2x^{9/2}(1547b^3 + 3213b^2cx^2 + 2457bc^2x^4 + 663c^3x^6)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(5/2),x]
[Out] (2*x^(9/2)*(1547*b^3 + 3213*b^2*c*x^2 + 2457*b*c^2*x^4 + 663*c^3*x^6))/1392
3
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2c\,x^{\frac{13}{2}}}{13}+\frac{6b\,c^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$	36
default	$\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2c\,x^{\frac{13}{2}}}{13}+\frac{6b\,c^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$	36
gosper	$\frac{2x^{\frac{9}{2}}\left(663c^3x^6+2457b\,c^2x^4+3213b^2c\,x^2+1547b^3\right)}{13923}$	38
trager	$\frac{2x^{\frac{9}{2}}\left(663c^3x^6+2457b\,c^2x^4+3213b^2c\,x^2+1547b^3\right)}{13923}$	38
risch	$\frac{2x^{\frac{9}{2}}\left(663c^3x^6+2457b\,c^2x^4+3213b^2c\,x^2+1547b^3\right)}{13923}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/9*b^3*x^(9/2)+6/13*b^2*c*x^(13/2)+6/17*b*c^2*x^(17/2)+2/21*c^3*x^(21/2)
```

Maxima [A]
time = 0.30, size = 35, normalized size = 0.69

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="maxima")
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2/9*b^3*x^(
9/2)
```

Fricas [A]
time = 0.35, size = 40, normalized size = 0.78

$$\frac{2}{13923}\left(663\,c^3x^{10}+2457\,bc^2x^8+3213\,b^2cx^6+1547\,b^3x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*c^3*x^10 + 2457*b*c^2*x^8 + 3213*b^2*c*x^6 + 1547*b^3*x^4)*sqr
t(x)
```

Sympy [A]
time = 1.62, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2cx^{\frac{13}{2}}}{13}+\frac{6bc^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**3/x**(5/2),x)
```

```
[Out] 2*b**3*x**(9/2)/9 + 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17 + 2*c**3*
x**(21/2)/21
```

Giac [A]
time = 4.00, size = 35, normalized size = 0.69

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2/9*b^3*x^(
9/2)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{9/2}}{9}+\frac{2\,c^3\,x^{21/2}}{21}+\frac{6\,b^2\,c\,x^{13/2}}{13}+\frac{6\,b\,c^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^3/x^(5/2),x)
```

```
[Out] (2*b^3*x^(9/2))/9 + (2*c^3*x^(21/2))/21 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2*
x^(17/2))/17
```

3.315 $\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

```
[Out] 2/7*b^3*x^(7/2)+6/11*b^2*c*x^(11/2)+2/5*b*c^2*x^(15/2)+2/19*c^3*x^(19/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1598, 276}

$$\frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (2*b^3*x^(7/2))/7 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2*x^(15/2))/5 + (2*c^3*x
^(19/2))/19
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx &= \int x^{5/2}\left(b+cx^2\right)^3\,dx\\
&= \int \left(b^3x^{5/2}+3b^2cx^{9/2}+3bc^2x^{13/2}+c^3x^{17/2}\right)dx\\
&= \frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.80

$$\frac{2x^{7/2}(1045b^3 + 1995b^2cx^2 + 1463bc^2x^4 + 385c^3x^6)}{7315}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (2*x^(7/2)*(1045*b^3 + 1995*b^2*c*x^2 + 1463*b*c^2*x^4 + 385*c^3*x^6))/7315
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$	36
default	$\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$	36
gosper	$\frac{2x^{\frac{7}{2}}\left(385c^3x^6+1463bc^2x^4+1995b^2cx^2+1045b^3\right)}{7315}$	38
trager	$\frac{2x^{\frac{7}{2}}\left(385c^3x^6+1463bc^2x^4+1995b^2cx^2+1045b^3\right)}{7315}$	38
risch	$\frac{2x^{\frac{7}{2}}\left(385c^3x^6+1463bc^2x^4+1995b^2cx^2+1045b^3\right)}{7315}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*b^3*x^(7/2)+6/11*b^2*c*x^(11/2)+2/5*b*c^2*x^(15/2)+2/19*c^3*x^(19/2)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.69

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,b^2cx^{\frac{11}{2}}+\frac{2}{7}\,b^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/7*b^3*x^(7
/2)
```

Fricas [A]
time = 0.34, size = 40, normalized size = 0.78

$$\frac{2}{7315}\left(385\,c^3x^9+1463\,bc^2x^7+1995\,b^2cx^5+1045\,b^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="fricas")

[Out] 2/7315*(385*c^3*x^9 + 1463*b*c^2*x^7 + 1995*b^2*c*x^5 + 1045*b^3*x^3)*sqrt(x)

Sympy [A]
time = 1.83, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2)**3/x**(7/2),x)

[Out] 2*b**3*x**(7/2)/7 + 6*b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5 + 2*c**3*x**(19/2)/19

Giac [A]
time = 4.07, size = 35, normalized size = 0.69

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,b^2cx^{\frac{11}{2}}+\frac{2}{7}\,b^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="giac")

[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/7*b^3*x^(7/2)

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,b^3\,x^{7/2}}{7}+\frac{2\,c^3\,x^{19/2}}{19}+\frac{6\,b^2\,c\,x^{11/2}}{11}+\frac{2\,b\,c^2\,x^{15/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2 + c*x^4)^3/x^(7/2),x)

[Out] (2*b^3*x^(7/2))/7 + (2*c^3*x^(19/2))/19 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2*x^(15/2))/5

3.316 $\int \frac{x^{13/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=217

$$-\frac{2bx^{3/2}}{3c^2}+\frac{2x^{7/2}}{7c}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}}+\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}}+\frac{b^{7/4}\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\right.}{2\sqrt{2}\,c^{1}}$$

```
[Out] -2/3*b*x^(3/2)/c^2+2/7*x^(7/2)/c-1/2*b^(7/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/
2)/b^(1/4))/c^(11/4)*2^(1/2)+1/2*b^(7/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b
^(1/4))/c^(11/4)*2^(1/2)+1/4*b^(7/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/c^(11/4)*2^(1/2)-1/4*b^(7/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)*2^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}}+\frac{b^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,c^{11/4}}+\frac{b^{7/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}}-\frac{b^{7/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}}-\frac{2bx^{3/2}}{3c^2}+\frac{2x^{7/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*b*x^(3/2))/(3*c^2) + (2*x^(7/2))/(7*c) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)*Log[Sqrt[b] - Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(7/4)*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(1
1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}}{bx^2+cx^4}\,dx = \int \frac{x^{9/2}}{b+cx^2}\,dx$$
$$= \frac{2x^{7/2}}{7c} - \frac{b\int \frac{x^{5/2}}{b+cx^2}\,dx}{c}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^2\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^2}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} - \frac{b^2\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{b^2\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x,\right.}{c^{5/2}}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^3} + \frac{b^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}}{}\right.}{2c}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} + \frac{b^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{11/4}} - \frac{b^{7/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\right.}{2\sqrt{2}\,c}$$
$$= -\frac{2bx^{3/2}}{3c^2} + \frac{2x^{7/2}}{7c} - \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}} + \frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{11/4}} + \cdots$$

Mathematica [A]
time = 0.16, size = 129, normalized size = 0.59

$$\frac{4c^{3/4}x^{3/2}(-7b+3cx^2) - 21\sqrt{2}\,b^{7/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - 21\sqrt{2}\,b^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{42c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (4*c^(3/4)*x^(3/2)*(-7*b + 3*c*x^2) - 21*Sqrt[2]*b^(7/4)*ArcTan[(Sqrt[b] -
Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 21*Sqrt[2]*b^(7/4)*ArcTanh[
(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(42*c^(11/4))
```

Maple [A]
time = 0.13, size = 128, normalized size = 0.59

method	result
derivativedivides	$-\frac{2\left(-\frac{c\,x^{\frac{7}{2}}}{7}+\frac{b\,x^{\frac{3}{2}}}{3}\right)}{c^{2}}+\frac{b^{2}\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{2\left(-\frac{c\,x^{\frac{7}{2}}}{7}+\frac{b\,x^{\frac{3}{2}}}{3}\right)}{c^{2}}+\frac{b^{2}\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$-\frac{2x^{\frac{3}{2}}\left(-3c\,x^{2}+7b\right)}{21c^{2}}+\frac{b^{2}\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\frac{b^{2}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\frac{b^{2}\sqrt{2}\,\,a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^2*(-1/7*c*x^(7/2)+1/3*b*x^(3/2))+1/4*b^2/c^3/(b/c)^(1/4)*2^(1/2)*(ln((
x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(
b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)
^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 198, normalized size = 0.91

$$\frac{b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4\,c^{2}}+\frac{2\left(3\,cx^{\frac{7}{2}}-7\,bx^{\frac{3}{2}}\right)}{21\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^
2 + 2/21*(3*c*x^(7/2) - 7*b*x^(3/2))/c^2
```

Fricas [A]
time = 0.38, size = 182, normalized size = 0.84

$$-\frac{84 c^2 \left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}} \arctan\left(-\frac{b^5 c^3 \sqrt{x} \left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}} - \sqrt{-b^7 c^5 \sqrt{-\frac{b^7}{c^{11}}} + b^{10} x}\, c^3 \left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}}{b^7}\right) - 21 c^2 \left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\log\left(c^8\left(-\frac{b^7}{c^{11}}\right)^{\frac{3}{4}} + b^5\sqrt{x}\right) + 21 c^2 \left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\log\left(-c^8\left(-\frac{b^7}{c^{11}}\right)^{\frac{3}{4}} + b^5\sqrt{x}\right) - 4\,(3\,c x^3 - 7\,b x)\sqrt{x}}{42\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/42*(84*c^2*(-b^7/c^11)^(1/4)*arctan(-(b^5*c^3*sqrt(x)*(-b^7/c^11)^(1/4)
- sqrt(-b^7*c^5*sqrt(-b^7/c^11) + b^10*x)*c^3*(-b^7/c^11)^(1/4))/b^7) - 21*
c^2*(-b^7/c^11)^(1/4)*log(c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) + 21*c^2*(-b
^7/c^11)^(1/4)*log(-c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) - 4*(3*c*x^3 - 7*b
*x)*sqrt(x))/c^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.29, size = 197, normalized size = 0.91

$$\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\, b \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^5} + \frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\, b \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^5} - \frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\, b \log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4\,c^5} + \frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\, b \log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4\,c^5} + \frac{2\left(3\,c^6 x^{\frac{7}{2}} - 7\,b c^5 x^{\frac{3}{2}}\right)}{21\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^5 + 1/2*sqrt(2)*(b*c^3)^(3/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/4*sqrt(2)*(b*c^3)^(3/4
)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/4*sqrt(2)*(b*c
^3)^(3/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 2/21*(3
*c^6*x^(7/2) - 7*b*c^5*x^(3/2))/c^7
```

Mupad [B]
time = 0.11, size = 66, normalized size = 0.30

$$\frac{2\,x^{7/2}}{7\,c}-\frac{2\,b\,x^{3/2}}{3\,c^2}+\frac{{\left(-b\right)}^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{c^{11/4}}+\frac{{\left(-b\right)}^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,1{}\mathrm{i}}{c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(7/2))/(7*c) - (2*b*x^(3/2))/(3*c^2) + ((-b)^(7/4)*atan((c^(1/4)*x^(1/
2))/(-b)^(1/4)))/c^(11/4) + ((-b)^(7/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4
))*1i)/c^(11/4)
```

3.317 $\int \frac{x^{11/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=215

$$-\frac{2b\sqrt{x}}{c^2}+\frac{2x^{5/2}}{5c}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}}+\frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}}-\frac{b^{5/4}\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\right.}{2\sqrt{2}\,c^{9}}$$

```
[Out] 2/5*x^(5/2)/c-1/2*b^(5/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(9/4)
*2^(1/2)+1/2*b^(5/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(9/4)*2^(1
/2)-1/4*b^(5/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(9/
4)*2^(1/2)+1/4*b^(5/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2)
)/c^(9/4)*2^(1/2)-2*b*x^(1/2)/c^2
```

Rubi [A]
time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}}+\frac{b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,c^{9/4}}-\frac{b^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}+\frac{b^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}}-\frac{2b\sqrt{x}}{c^2}+\frac{2x^{5/2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*b*Sqrt[x])/c^2 + (2*x^(5/2))/(5*c) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) - (b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4)) + (b^(5/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11/2}}{bx^2+cx^4}\,dx &= \int \frac{x^{7/2}}{b+cx^2}\,dx \\
&= \frac{2x^{5/2}}{5c} - \frac{b\int \frac{x^{3/2}}{b+cx^2}\,dx}{c} \\
&= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^2\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{c^2} \\
&= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2} \\
&= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^{3/2}\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2} + \frac{b^{3/2}\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx\right.}{c^2} \\
&= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} + \frac{b^{3/2}\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/2}} + \frac{b^{3/2}\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+}\right.}{} \\
&= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} - \frac{b^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{9/4}} + \frac{b^{5/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{\ }\right.}{2\sqrt{2}\,c} \\
&= -\frac{2b\sqrt{x}}{c^2} + \frac{2x^{5/2}}{5c} - \frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}} + \frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{9/4}} -
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 128, normalized size = 0.60

$$
\frac{4\sqrt[4]{c}\sqrt{x}\,(-5b+cx^2) - 5\sqrt{2}\,b^{5/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + 5\sqrt{2}\,b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{10c^{9/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (4*c^(1/4)*Sqrt[x]*(-5*b + c*x^2) - 5*Sqrt[2]*b^(5/4)*ArcTan[(Sqrt[b] - Sqr
t[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt[2]*b^(5/4)*ArcTanh[(Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(10*c^(9/4))
```

Maple [A]
time = 0.12, size = 125, normalized size = 0.58

method	result
derivativedivides	$-\frac{2\left(-\frac{c\,x^{\frac{5}{2}}}{5}+b\sqrt{x}\right)}{c^{2}}+\frac{b\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{}}{}\right.\right.}{4c^{2}}$
default	$-\frac{2\left(-\frac{c\,x^{\frac{5}{2}}}{5}+b\sqrt{x}\right)}{c^{2}}+\frac{b\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{}}{}\right.\right.}{4c^{2}}$
risch	$-\frac{2\left(-c\,x^{2}+5b\right)\sqrt{x}}{5c^{2}}+\frac{b\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4c^{2}}+\frac{b\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^2*(-1/5*c*x^(5/2)+b*x^(1/2))+1/4*b/c^2*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c
)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(
1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)
*x^(1/2)-1))
```

Maxima [A]
time = 0.52, size = 194, normalized size = 0.90

$$\frac{2\left(cx^{\frac{5}{2}}-5\,b\sqrt{x}\right)}{5\,c^{2}}+\frac{\frac{2\sqrt{2}\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,b^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,b^{\frac{5}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}\,b^{\frac{5}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}}{4\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/5*(c*x^(5/2) - 5*b*sqrt(x))/c^2 + 1/4*(2*sqrt(2)*b^(3/2)*arctan(1/2*sqrt(
2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqr
t(sqrt(b)*sqrt(c)) + 2*sqrt(2)*b^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)
*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c))
+ sqrt(2)*b^(5/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b)
)/c^(1/4) - sqrt(2)*b^(5/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*
x + sqrt(b))/c^(1/4))/c^2
```

Fricas [A]
time = 0.35, size = 170, normalized size = 0.79

$$\frac{20\,c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}\arctan\left(-\frac{bc^{7}\sqrt{x}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{3}{4}}-\sqrt{c^{4}\sqrt{-\frac{b^{5}}{c^{9}}}+b^{2}x}\,c^{7}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{3}{4}}}{b^{5}}\right)+5\,c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}\log\left(c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}+b\sqrt{x}\right)-5\,c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}\log\left(-c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}+b\sqrt{x}\right)+4\left(cx^{2}-5\,b\right)\sqrt{x}}{10\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/10*(20*c^2*(-b^5/c^9)^(1/4)*arctan(-(b*c^7*sqrt(x)*(-b^5/c^9)^(3/4) - sqr
t(c^4*sqrt(-b^5/c^9) + b^2*x)*c^7*(-b^5/c^9)^(3/4))/b^5) + 5*c^2*(-b^5/c^9)
^(1/4)*log(c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) - 5*c^2*(-b^5/c^9)^(1/4)*log(-
c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) + 4*(c*x^2 - 5*b)*sqrt(x))/c^2
```

Sympy [A]
time = 109.38, size = 136, normalized size = 0.63

$$\begin{cases} \tilde{\infty} x^{\frac{5}{2}} & \text{for}\: b = 0 \wedge c = 0 \\ \frac{2 x^{\frac{9}{2}}}{9 b} & \text{for}\: c = 0 \\ \frac{2 x^{\frac{5}{2}}}{5 c} & \text{for}\: b = 0 \\ - \frac{2 b \sqrt{x}}{c^{2}} - \frac{b \sqrt[4]{- \frac{b}{c}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{b}{c}} \right)}}{2 c^{2}} + \frac{b \sqrt[4]{- \frac{b}{c}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{b}{c}} \right)}}{2 c^{2}} + \frac{b \sqrt[4]{- \frac{b}{c}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{b}{c}}} \right)}}{c^{2}} + \frac{2 x^{\frac{5}{2}}}{5 c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(b, 0) & Eq(c, 0)), (2*x**(9/2)/(9*b), Eq(c, 0))
, (2*x**(5/2)/(5*c), Eq(b, 0)), (-2*b*sqrt(x)/c**2 - b*(-b/c)**(1/4)*log(sq
rt(x) - (-b/c)**(1/4))/(2*c**2) + b*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/
4))/(2*c**2) + b*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/c**2 + 2*x**(5/2
)/(5*c), True))
```

Giac [A]
time = 4.99, size = 196, normalized size = 0.91

$$\frac{\sqrt{2} \left(bc^{3}\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{b}{c}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2 c^{3}} + \frac{\sqrt{2} \left(bc^{3}\right)^{\frac{1}{4}} b \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{b}{c}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2 c^{3}} + \frac{\sqrt{2} \left(bc^{3}\right)^{\frac{1}{4}} b \log\left(\sqrt{2} \sqrt{x} \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4 c^{3}} - \frac{\sqrt{2} \left(bc^{3}\right)^{\frac{1}{4}} b \log\left(-\sqrt{2} \sqrt{x} \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4 c^{3}} + \frac{2 \left(c^{4} x^{\frac{5}{2}} - 5 b c^{3} \sqrt{x}\right)}{5 c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^3 + 1/4*sqrt(2)*(b*c^3)^(1/4
)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 - 1/4*sqrt(2)*(b*c
^3)^(1/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 2/5*(c^
4*x^(5/2) - 5*b*c^3*sqrt(x))/c^5
```

Mupad [B]
time = 4.49, size = 67, normalized size = 0.31

$$\frac{2\,x^{5/2}}{5\,c}-\frac{2\,b\,\sqrt{x}}{c^2}-\frac{{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{c^{9/4}}+\frac{{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,1{}\mathrm{i}}{c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(5/2))/(5*c) - (2*b*x^(1/2))/c^2 - ((-b)^(5/4)*atan((c^(1/4)*x^(1/2))/
(-b)^(1/4)))/c^(9/4) + ((-b)^(5/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*1i
)/c^(9/4)
```

3.318 $\int \frac{x^{9/2}}{bx^2+cx^4} \, dx$

Optimal. Leaf size=204

$$\frac{2x^{3/2}}{3c}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}}-\frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{2\sqrt{2}\,c^{7/4}}$$

```
[Out] 2/3*x^(3/2)/c+1/2*b^(3/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(7/4)
*2^(1/2)-1/2*b^(3/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(7/4)*2^(1
/2)-1/4*b^(3/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(7/
4)*2^(1/2)+1/4*b^(3/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2)
)/c^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}}-\frac{b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,c^{7/4}}-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}}+\frac{2x^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(Sqrt[2]*c^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(Sqrt[2]*c^(7/4)) - (b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4)) + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}}{bx^2+cx^4}\,dx = \int \frac{x^{5/2}}{b+cx^2}\,dx$$
$$= \frac{2x^{3/2}}{3c} - \frac{b\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c}$$
$$= \frac{2x^{3/2}}{3c} - \frac{(2b)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$
$$= \frac{2x^{3/2}}{3c} + \frac{b\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}} - \frac{b\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{3/2}}$$
$$= \frac{2x^{3/2}}{3c} - \frac{b\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^2} - \frac{b\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x\right.}{2c^2}$$
$$= \frac{2x^{3/2}}{3c} - \frac{b^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{7/4}} + \frac{b^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{2\sqrt{2}\,c^{7/4}}$$
$$= \frac{2x^{3/2}}{3c} + \frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}} - \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{7/4}} - \frac{b^{3/4}\log\left(\sqrt{b}\right.}{}$$

Mathematica [A]
time = 0.13, size = 119, normalized size = 0.58

$$\frac{4c^{3/4}x^{3/2}+3\sqrt{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+3\sqrt{2}\,b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{6c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (4*c^(3/4)*x^(3/2) + 3*Sqrt[2]*b^(3/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])] + 3*Sqrt[2]*b^(3/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(6*c^(7/4))
```

Maple [A]
time = 0.12, size = 116, normalized size = 0.57

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{b\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{2c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)/c-1/4*b/c^2/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1
/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1
/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]

time = 0.51, size = 186, normalized size = 0.91

$$-\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4c}+\frac{2x^{\frac{3}{2}}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c +
 2/3*x^(3/2)/c
```

Fricas [A]

time = 0.36, size = 165, normalized size = 0.81

$$\frac{12\,c\left(-\frac{b^3}{c^7}\right)^{\frac14}\arctan\left(-\frac{b^2c^2\sqrt{x}\left(-\frac{b^3}{c^7}\right)^{\frac14}-\sqrt{-b^3c^3\sqrt{-\frac{b^3}{c^7}}+b^4x}\,c^2\left(-\frac{b^3}{c^7}\right)^{\frac14}}{b^3}\right)-3\,c\left(-\frac{b^3}{c^7}\right)^{\frac14}\log\left(c^5\left(-\frac{b^3}{c^7}\right)^{\frac34}+b^2\sqrt{x}\right)+3\,c\left(-\frac{b^3}{c^7}\right)^{\frac14}\log\left(-c^5\left(-\frac{b^3}{c^7}\right)^{\frac34}+b^2\sqrt{x}\right)+4\,x^{\frac32}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(12*c*(-b^3/c^7)^(1/4)*arctan(-(b^2*c^2*sqrt(x)*(-b^3/c^7)^(1/4) - sqrt
(-b^3*c^3*sqrt(-b^3/c^7) + b^4*x)*c^2*(-b^3/c^7)^(1/4))/b^3) - 3*c*(-b^3/c^
7)^(1/4)*log(c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) + 3*c*(-b^3/c^7)^(1/4)*log
(-c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) + 4*x^(3/2))/c
```

Sympy [A]
time = 53.36, size = 124, normalized size = 0.61

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } b=0 \wedge c=0 \\ \frac{2x^{\frac{7}{2}}}{7b} & \text{for } c=0 \\ \frac{2x^{\frac{3}{2}}}{3c} & \text{for } b=0 \\ -\frac{b\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2c^2\sqrt[4]{-\frac{b}{c}}}+\frac{b\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2c^2\sqrt[4]{-\frac{b}{c}}}-\frac{b\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c^2\sqrt[4]{-\frac{b}{c}}}+\frac{2x^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(b, 0) & Eq(c, 0)), (2*x**(7/2)/(7*b), Eq(c, 0))
, (2*x**(3/2)/(3*c), Eq(b, 0)), (-b*log(sqrt(x) - (-b/c)**(1/4))/(2*c**2*(-
b/c)**(1/4)) + b*log(sqrt(x) + (-b/c)**(1/4))/(2*c**2*(-b/c)**(1/4)) - b*at
an(sqrt(x)/(-b/c)**(1/4))/(c**2*(-b/c)**(1/4)) + 2*x**(3/2)/(3*c), True))
```

Giac [A]
time = 3.71, size = 178, normalized size = 0.87

$$\frac{2\,x^{\frac32}}{3\,c}-\frac{\sqrt{2}\,(bc^3)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{2\,c^4}-\frac{\sqrt{2}\,(bc^3)^{\frac34}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{2\,c^4}+\frac{\sqrt{2}\,(bc^3)^{\frac34}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^4}-\frac{\sqrt{2}\,(bc^3)^{\frac34}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/c - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)
^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/4*sqrt(2)*
(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/4*sq
rt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4
```

Mupad [B]
time = 4.36, size = 54, normalized size = 0.26

$$\frac{2\,x^{3/2}}{3\,c}+\frac{{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{c^{7/4}}-\frac{{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(3/2))/(3*c) + ((-b)^(3/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(7/4)
 - ((-b)^(3/4)*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(7/4)
```

3.319 $\int \frac{x^{7/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$\frac{2\sqrt{x}}{c}+\frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}}+\frac{\sqrt[4]{b}\,\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}\right.}{2\sqrt{2}\,c^{5/4}}$$

```
[Out] 1/2*b^(1/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(5/4)*2^(1/2)-1/2*b
^(1/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(5/4)*2^(1/2)+1/4*b^(1/4
)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)*2^(1/2)-1/4
*b^(1/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)*2^(1
/2)+2*x^(1/2)/c
```

Rubi [A]
time = 0.11, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,c^{5/4}}+\frac{\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}}-\frac{\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}}+\frac{2\sqrt{x}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + (b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sq
rt[2]*c^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(S
qrt[2]*c^(5/4)) + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4)) - (b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx = \int \frac{x^{3/2}}{b+cx^2}\,dx$$
$$= \frac{2\sqrt{x}}{c} - \frac{b\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{c}$$
$$= \frac{2\sqrt{x}}{c} - \frac{(2b)\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$
$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}$$
$$= \frac{2\sqrt{x}}{c} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{3/2}} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+\ldots}\right.}{2c^{3/2}}$$
$$= \frac{2\sqrt{x}}{c} + \frac{\sqrt[4]{b}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,c^{5/4}} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\ldots\right.}{2\sqrt{2}\,c^{5/4}}$$
$$= \frac{2\sqrt{x}}{c} + \frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}} - \frac{\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,c^{5/4}} + \frac{\sqrt[4]{b}\,\log\left(\ldots\right.}{}$$

Mathematica [A]
time = 0.13, size = 118, normalized size = 0.58

$$\frac{4\sqrt[4]{c}\sqrt{x}+\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-\sqrt{2}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{2c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (4*c^(1/4)*Sqrt[x] + Sqrt[2]*b^(1/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x])] - Sqrt[2]*b^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(2*c^(5/4))
```

Maple [A]
time = 0.12, size = 115, normalized size = 0.57

method	result
derivativedivides	$\frac{2\sqrt{x}}{c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c}$
default	$\frac{2\sqrt{x}}{c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c}$
risch	$\frac{2\sqrt{x}}{c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\mathrm{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/c-1/4/c*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b
/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.53, size = 185, normalized size = 0.92

$$-\frac{\frac{2\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,\sqrt{b}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}}{4c}+\frac{2\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqr
t(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*sqrt
(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(
sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(1/4)*log(sqrt(2)*b^(1/
4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(1/4)*log(-sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c + 2*sqrt(x)
/c
```

Fricas [A]
time = 0.35, size = 124, normalized size = 0.61

$$-\frac{4\,c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^2\sqrt{-\frac{b}{c^5}}+x}\,c^4\left(-\frac{b}{c^5}\right)^{\frac{3}{4}}-c^4\sqrt{x}\,\left(-\frac{b}{c^5}\right)^{\frac{3}{4}}}{b}\right)+c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(-c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-4\sqrt{x}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*c*(-b/c^5)^(1/4)*arctan((sqrt(c^2*sqrt(-b/c^5) + x)*c^4*(-b/c^5)^(3
/4) - c^4*sqrt(x)*(-b/c^5)^(3/4))/b) + c*(-b/c^5)^(1/4)*log(c*(-b/c^5)^(1/4
) + sqrt(x)) - c*(-b/c^5)^(1/4)*log(-c*(-b/c^5)^(1/4) + sqrt(x)) - 4*sqrt(x
))/c
```

Sympy [A]
time = 26.40, size = 110, normalized size = 0.54

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\: b = 0 \wedge c = 0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for}\: c = 0 \\ \frac{2\sqrt{x}}{c} & \text{for}\: b = 0 \\ \frac{2\sqrt{x}}{c} + \frac{\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{b}{c}}\right)}{2c} - \frac{\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{b}{c}}\right)}{2c} - \frac{\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(b, 0) & Eq(c, 0)), (2*x**(5/2)/(5*b), Eq(c, 0)),
 (2*sqrt(x)/c, Eq(b, 0)), (2*sqrt(x)/c + (-b/c)**(1/4)*log(sqrt(x) - (-b/c)
**(1/4))/(2*c) - (-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*c) - (-b/c)*
*(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/c, True))
```

Giac [A]
time = 4.12, size = 178, normalized size = 0.88

$$-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{2}} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{2}} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^{2}} + \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^{2}} + \frac{2\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/c^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^2 - 1/4*sqrt(2)*(b*c^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^2 + 1/4*sqrt(2)*(b*c^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^2 + 2*sqrt(x)/c
```

Mupad [B]
time = 4.36, size = 55, normalized size = 0.27

$$\frac{2\sqrt{x}}{c} - \frac{(-b)^{1/4}\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{c^{5/4}} - \frac{(-b)^{1/4}\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(1/2))/c - ((-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(5/4) - (
(-b)^(1/4)*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(5/4)
```

3.320 $\int \frac{x^{5/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=192

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}+\frac{\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}-\frac{\log\left(\sqrt{b}\right.}{}$$

```
[Out] -1/2*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(3/4)*2^(1/2)+1/2*
arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(3/4)*2^(1/2)+1/4*ln(b^
(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(3/4)*2^(1/2)-1/
4*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(3/4)*2^(
1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1598, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}+\frac{\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}-\frac{\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(3/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(3/4)) +
 Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1
/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(
2*Sqrt[2]*b^(1/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}}{bx^2+cx^4}\,dx = \int \frac{\sqrt{x}}{b+cx^2}\,dx$$
$$= 2\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{c}}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c} +$$
$$= \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{3/4}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,c^{3/4}} + \frac{\text{Subst}}{}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{3/4}} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{2\sqrt{2}\sqrt[4]{b}\,c^{3/4}}$$

Mathematica [A]
time = 0.10, size = 91, normalized size = 0.47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\sqrt[4]{b}\,c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4),x]
```

```
[Out] -((ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + ArcTan
h[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(1/4
)*c^(3/4)))
```

Maple [A]
time = 0.11, size = 106, normalized size = 0.55

method	result	size

derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	106
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(
x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^
(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{4\,b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{4\,b^{\frac{1}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*
sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - 1/4*sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + 1/4*sqrt(2)*log
(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4))
```

Fricas [A]
time = 0.35, size = 126, normalized size = 0.66

$$-2\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc\sqrt{-\frac{1}{bc^3}}+x}\,c\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}-c\sqrt{x}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\right)+\frac{1}{2}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(-bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -2*(-1/(b*c^3))^(1/4)*arctan(sqrt(-b*c*sqrt(-1/(b*c^3)) + x)*c*(-1/(b*c^3))
^(1/4) - c*sqrt(x)*(-1/(b*c^3))^(1/4)) + 1/2*(-1/(b*c^3))^(1/4)*log(b*c^2*(
```

```
-1/(b*c^3))^(3/4) + sqrt(x)) - 1/2*(-1/(b*c^3))^(1/4)*log(-b*c^2*(-1/(b*c^3
))^(3/4) + sqrt(x))
```

Sympy [A]
time = 16.63, size = 104, normalized size = 0.54

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\ b = 0 \wedge c = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for}\ c = 0 \\ -\frac{2}{c\sqrt{x}} & \text{for}\ b = 0 \\ \frac{\log\left(\sqrt{x} - \sqrt[4]{-\frac{b}{c}}\right)}{2c\sqrt[4]{-\frac{b}{c}}} - \frac{\log\left(\sqrt{x} + \sqrt[4]{-\frac{b}{c}}\right)}{2c\sqrt[4]{-\frac{b}{c}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c\sqrt[4]{-\frac{b}{c}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(b, 0) & Eq(c, 0)), (2*x**(3/2)/(3*b), Eq(c, 0)),
 (-2/(c*sqrt(x)), Eq(b, 0)), (log(sqrt(x) - (-b/c)**(1/4))/(2*c*(-b/c)**(1/
4)) - log(sqrt(x) + (-b/c)**(1/4))/(2*c*(-b/c)**(1/4)) + atan(sqrt(x)/(-b/c
)**(1/4))/(c*(-b/c)**(1/4)), True))
```

Giac [A]
time = 3.81, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^3} + \frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^3} - \frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^3} + \frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c^3) + 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/4*sqrt(2)*(b*c^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) + 1/4*sqrt(2
)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3)
```

Mupad [B]
time = 0.08, size = 38, normalized size = 0.20

$$\frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{{(-b)}^{1/4}}\right)-\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{{(-b)}^{1/4}}\right)}{{(-b)}^{1/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4),x)
```

```
[Out] (atan((c^(1/4)*x^(1/2))/(-b)^(1/4)) - atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/
((-b)^(1/4)*c^(3/4))
```

3.321 $\int \frac{x^{3/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=192

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}}-\frac{\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{b}\right.}{}$$

```
[Out] -1/2*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(1/4)*2^(1/2)+1/2*
arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(1/4)*2^(1/2)-1/4*ln(b^
(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(1/4)*2^(1/2)+1/
4*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(1/4)*2^(
1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1598, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*c^(1/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*c^(1/4)) -
 Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(3
/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(
2*Sqrt[2]*b^(3/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}}{bx^2+cx^4}\,dx = \int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\sqrt{c}} -$$
$$= -\frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}} + \frac{\text{Sub}}{}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{2\sqrt{2}\,b^{3/4}\sqrt[4]{c}}$$

Mathematica [A]
time = 0.10, size = 92, normalized size = 0.48

$$\frac{-\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt{2}\,b^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + ArcTanh
[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(3/4)
*c^(1/4))
```

Maple [A]
time = 0.12, size = 106, normalized size = 0.55

method	result	size

derivativedivides	$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$	106
default	$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(
x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^
(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 172, normalized size = 0.90

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4b^{\frac{3}{4}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 1/2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*
sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 1/4*sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - 1/4*sqrt(2)*log
(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4))
```

Fricas [A]
time = 0.37, size = 126, normalized size = 0.66

$$2\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^2\sqrt{-\frac{1}{b^3c}}+x}\,b^2c\left(-\frac{1}{b^3c}\right)^{\frac{3}{4}}-b^2c\sqrt{x}\left(-\frac{1}{b^3c}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(b^3*c))^(1/4)*arctan(sqrt(b^2*sqrt(-1/(b^3*c)) + x)*b^2*c*(-1/(b^3*c
))^(3/4) - b^2*c*sqrt(x)*(-1/(b^3*c))^(3/4)) + 1/2*(-1/(b^3*c))^(1/4)*log(b
*(-1/(b^3*c))^(1/4) + sqrt(x)) - 1/2*(-1/(b^3*c))^(1/4)*log(-b*(-1/(b^3*c))
^(1/4) + sqrt(x))
```

Sympy [A]
time = 9.16, size = 104, normalized size = 0.54

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{3cx^{\frac{3}{2}}} & \text{for } b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } c = 0 \\ -\frac{\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b} + \frac{\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b} + \frac{\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(b, 0) & Eq(c, 0)), (-2/(3*c*x**(3/2)), Eq(b, 0)
), (2*sqrt(x)/b, Eq(c, 0)), (-(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2
*b) + (-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b) + (-b/c)**(1/4)*atan
(sqrt(x)/(-b/c)**(1/4))/b, True))
```

Giac [A]
time = 3.95, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc} + \frac{\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc} + \frac{\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc} - \frac{\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c) + 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c) + 1/4*sqrt(2)*(b*c^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c) - 1/4*sqrt(2)*(b*c
^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c)
```

Mupad [B]
time = 4.43, size = 37, normalized size = 0.19

$$-\frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)+\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{(-b)^{3/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4),x)
```

```
[Out] -(atan((c^(1/4)*x^(1/2))/(-b)^(1/4)) + atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))
/((-b)^(3/4)*c^(1/4))
```

3.322 $\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$-\frac{2}{b\sqrt{x}}+\frac{\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{c}\,\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}\right.}{2\sqrt{2}\,b^{5/4}}$$

```
[Out] 1/2*c^(1/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)*2^(1/2)-1/2*c
^(1/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)*2^(1/2)-1/4*c^(1/4
)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)*2^(1/2)+1/4
*c^(1/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)*2^(1
/2)-2/b/x^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{5/4}}-\frac{\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}}+\frac{\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}}-\frac{2}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4),x]
```

```
[Out] -2/(b*Sqrt[x]) + (c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(S
qrt[2]*b^(5/4)) - (c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(
Sqrt[2]*b^(5/4)) - (c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)) + (c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx = \int \frac{1}{x^{3/2}(b+cx^2)}\,dx$$
$$= -\frac{2}{b\sqrt{x}} - \frac{c\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b}$$
$$= -\frac{2}{b\sqrt{x}} - \frac{(2c)\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2}{b\sqrt{x}} + \frac{\sqrt{c}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b} - \frac{\sqrt{c}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2}{b\sqrt{x}} - \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b} - \frac{\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x,\right.}{2b}$$
$$= -\frac{2}{b\sqrt{x}} - \frac{\sqrt[4]{c}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{5/4}} + \frac{\sqrt[4]{c}\,\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{2\sqrt{2}\,b^{5/4}}$$
$$= -\frac{2}{b\sqrt{x}} + \frac{\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{5/4}} - \frac{\sqrt[4]{c}\,\log\left(\right.}{}$$

Mathematica [**A**]
time = 0.14, size = 117, normalized size = 0.58

$$\frac{-\frac{4\sqrt[4]{b}}{\sqrt{x}}+\sqrt{2}\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+\sqrt{2}\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{2b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4),x]
```

```
[Out] ((-4*b^(1/4))/Sqrt[x] + Sqrt[2]*c^(1/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x])] + Sqrt[2]*c^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(2*b^(5/4))
```

Maple [**A**]
time = 0.13, size = 115, normalized size = 0.57

method	result
derivativedivides	$-\frac{2}{b\sqrt{x}}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{2}{b\sqrt{x}}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$-\frac{2}{b\sqrt{x}}-\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4b\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{b}{c}\right.\cdots}\right.}{2b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/x^(1/2)-1/4/b/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(
b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(
b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]

time = 0.55, size = 186, normalized size = 0.92

$$-\frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4b}-\frac{2}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b -
 2/(b*sqrt(x))
```

Fricas [A]

time = 0.36, size = 142, normalized size = 0.70

$$\frac{4\,bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{bc\sqrt{x}\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}-\sqrt{-b^3c\sqrt{-\frac{c}{b^5}}+c^2x}\,b\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}}{c}\right)-bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(b^4\left(-\frac{c}{b^5}\right)^{\frac{3}{4}}+c\sqrt{x}\right)+bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(-b^4\left(-\frac{c}{b^5}\right)^{\frac{3}{4}}+c\sqrt{x}\right)-4\sqrt{x}}{2\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*x*(-c/b^5)^(1/4)*arctan(-(b*c*sqrt(x)*(-c/b^5)^(1/4) - sqrt(-b^3*c
*sqrt(-c/b^5) + c^2*x)*b*(-c/b^5)^(1/4))/c) - b*x*(-c/b^5)^(1/4)*log(b^4*(-
c/b^5)^(3/4) + c*sqrt(x)) + b*x*(-c/b^5)^(1/4)*log(-b^4*(-c/b^5)^(3/4) + c*
sqrt(x)) - 4*sqrt(x))/(b*x)
```

Sympy [A]
time = 5.89, size = 114, normalized size = 0.56

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: b = 0 \wedge c = 0 \\ - \frac{2}{5 c x^{\frac{5}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{b \sqrt{x}} & \text{for}\: c = 0 \\ - \frac{\log{\left(\sqrt{x} - \sqrt[4]{- \frac{b}{c}} \right)}}{2 b \sqrt[4]{- \frac{b}{c}}} + \frac{\log{\left(\sqrt{x} + \sqrt[4]{- \frac{b}{c}} \right)}}{2 b \sqrt[4]{- \frac{b}{c}}} - \frac{\operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{b}{c}}} \right)}}{b \sqrt[4]{- \frac{b}{c}}} - \frac{2}{b \sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(b, 0) & Eq(c, 0)), (-2/(5*c*x**(5/2)), Eq(b, 0)
), (-2/(b*sqrt(x)), Eq(c, 0)), (-log(sqrt(x) - (-b/c)**(1/4))/(2*b*(-b/c)**
(1/4)) + log(sqrt(x) + (-b/c)**(1/4))/(2*b*(-b/c)**(1/4)) - atan(sqrt(x)/(-
b/c)**(1/4))/(b*(-b/c)**(1/4)) - 2/(b*sqrt(x)), True))
```

Giac [A]
time = 4.01, size = 190, normalized size = 0.94

$$-\frac{2}{b\sqrt{x}}-\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^2}-\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^2}+\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c^2}-\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -2/(b*sqrt(x)) - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) - 1/2*sqrt(2)*(b*c^3)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) +
 1/4*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b^2*c^2) - 1/4*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
 + sqrt(b/c))/(b^2*c^2)
```

Mupad [B]
time = 4.53, size = 54, normalized size = 0.27

$$\frac{(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{5/4}}-\frac{(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{5/4}}-\frac{2}{b\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4),x)
```

```
[Out] ((-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(5/4) - ((-c)^(1/4)*atan
(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(5/4) - 2/(b*x^(1/2))
```

3.323 $\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)}\,dx$

Optimal. Leaf size=204

$$-\frac{2}{3bx^{3/2}}+\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}}-\frac{c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}}+\frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{2\sqrt{2}\,b^{7/4}}$$

```
[Out] -2/3/b/x^(3/2)+1/2*c^(3/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4
)*2^(1/2)-1/2*c^(3/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)*2^(
1/2)+1/4*c^(3/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7
/4)*2^(1/2)-1/4*c^(3/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2
))/b^(7/4)*2^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{c^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}}-\frac{c^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{7/4}}+\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}}-\frac{2}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(3*b*x^(3/2)) + (c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/
(Sqrt[2]*b^(7/4)) - (c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(Sqrt[2]*b^(7/4)) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4)) - (c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)}\,dx = \int \frac{1}{x^{5/2}(b+cx^2)}\,dx$$
$$= -\frac{2}{3bx^{3/2}} - \frac{c\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{b}$$
$$= -\frac{2}{3bx^{3/2}} - \frac{(2c)\mathrm{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2}{3bx^{3/2}} - \frac{c\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} - \frac{c\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right.}{b^{3/2}}$$
$$= -\frac{2}{3bx^{3/2}} - \frac{\sqrt{c}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}} - \frac{\sqrt{c}\,\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+\sqrt{2}\right.}{2b^{3}}$$
$$= -\frac{2}{3bx^{3/2}} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{7/4}} - \frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{}\right.}{2\sqrt{2}\,b^{7/4}}$$
$$= -\frac{2}{3bx^{3/2}} + \frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}} - \frac{c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{7/4}} + \frac{c^{3/4}}{}$$

Mathematica [A]
time = 0.14, size = 119, normalized size = 0.58

$$\frac{-\frac{4b^{3/4}}{x^{3/2}}+3\sqrt{2}\,c^{3/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-3\sqrt{2}\,c^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{6b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*b^(3/4))/x^(3/2) + 3*Sqrt[2]*c^(3/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 3*Sqrt[2]*c^(3/4)*ArcTanh[(Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(6*b^(7/4))
```

Maple [A]
time = 0.14, size = 116, normalized size = 0.57

method	result
derivativedivides	$-\frac{2}{3b\,x^{\frac{3}{2}}}-\frac{c\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.}{4b^{2}}$
default	$-\frac{2}{3b\,x^{\frac{3}{2}}}-\frac{c\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.}{4b^{2}}$
risch	$-\frac{2}{3b\,x^{\frac{3}{2}}}-\frac{c\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4b^{2}}-\frac{c\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}}-\frac{c\left(\frac{b}{c}\right)^{\frac{1}{4}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b/x^(3/2)-1/4*c/b^2*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(
1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(
1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 187, normalized size = 0.92

$$-\frac{\frac{2\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}}{4\,b}-\frac{2}{3\,bx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*
c*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sq
rt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(3/4)*log(sqrt(
2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(3/4)
*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b - 2
/3/(b*x^(3/2))
```

Fricas [A]
time = 0.40, size = 167, normalized size = 0.82

$$-\frac{12\,bx^{2}\left(-\frac{c^{3}}{b^{7}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{5}c\sqrt{x}\left(-\frac{c^{3}}{b^{7}}\right)^{\frac{3}{4}}-\sqrt{b^{4}\sqrt{-\frac{c^{3}}{b^{7}}}+c^{2}x}\,b^{5}\left(-\frac{c^{3}}{b^{7}}\right)^{\frac{3}{4}}}{c^{3}}\right)+3\,bx^{2}\left(-\frac{c^{3}}{b^{7}}\right)^{\frac{1}{4}}\log\left(b^{2}\left(-\frac{c^{3}}{b^{7}}\right)^{\frac{1}{4}}+c\sqrt{x}\right)-3\,bx^{2}\left(-\frac{c^{3}}{b^{7}}\right)^{\frac{1}{4}}\log\left(-b^{2}\left(-\frac{c^{3}}{b^{7}}\right)^{\frac{1}{4}}+c\sqrt{x}\right)+4\sqrt{x}}{6\,bx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*b*x^2*(-c^3/b^7)^(1/4)*arctan(-(b^5*c*sqrt(x)*(-c^3/b^7)^(3/4) - s
qrt(b^4*sqrt(-c^3/b^7) + c^2*x)*b^5*(-c^3/b^7)^(3/4))/c^3) + 3*b*x^2*(-c^3/
b^7)^(1/4)*log(b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) - 3*b*x^2*(-c^3/b^7)^(1/4)
*log(-b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) + 4*sqrt(x))/(b*x^2)
```

Sympy [A]
time = 7.76, size = 128, normalized size = 0.63

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: b = 0 \wedge c = 0 \\ - \frac{2}{7 c x^{\frac{7}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{3 b x^{\frac{3}{2}}} & \text{for}\: c = 0 \\ - \frac{2}{3 b x^{\frac{3}{2}}} + \frac{c \sqrt[4]{- \frac{b}{c}} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{b}{c}} \right)}}{2 b^{2}} - \frac{c \sqrt[4]{- \frac{b}{c}} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{b}{c}} \right)}}{2 b^{2}} - \frac{c \sqrt[4]{- \frac{b}{c}} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{b}{c}}} \right)}}{b^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(b, 0) & Eq(c, 0)), (-2/(7*c*x**(7/2)), Eq(b, 0)
), (-2/(3*b*x**(3/2)), Eq(c, 0)), (-2/(3*b*x**(3/2)) + c*(-b/c)**(1/4)*log(
sqrt(x) - (-b/c)**(1/4))/(2*b**2) - c*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(
1/4))/(2*b**2) - c*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b**2, True))
```

Giac [A]
time = 4.12, size = 178, normalized size = 0.87

$$-\frac{\sqrt{2} \left(b c^{3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{b}{c}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2 b^{2}} - \frac{\sqrt{2} \left(b c^{3}\right)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{b}{c}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2 b^{2}} - \frac{\sqrt{2} \left(b c^{3}\right)^{\frac{1}{4}} \log\left(\sqrt{2} \sqrt{x} \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4 b^{2}} + \frac{\sqrt{2} \left(b c^{3}\right)^{\frac{1}{4}} \log\left(-\sqrt{2} \sqrt{x} \left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4 b^{2}} - \frac{2}{3 b x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/b^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^2 - 1/4*sqrt(2)*(b*c^3)^(1/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^2 + 1/4*sqrt(2)*(b*c^3)^(
1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^2 - 2/3/(b*x^(3/2)
)
```

Mupad [B]
time = 0.10, size = 53, normalized size = 0.26

$$\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{7/4}}-\frac{2}{3\,b\,x^{3/2}}+\frac{(-c)^{3/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)),x)
[Out] ((-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(7/4) - 2/(3*b*x^(3/2)) +
 ((-c)^(3/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(7/4)
```

3.324 $\int \frac{1}{x^{3/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=215

$$-\frac{2}{5bx^{5/2}}+\frac{2c}{b^2\sqrt{x}}-\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{c^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\right.}{2\sqrt{2}\,b^{9/4}}$$

```
[Out] -2/5/b/x^(5/2)-1/2*c^(5/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4
)*2^(1/2)+1/2*c^(5/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)*2^(
1/2)+1/4*c^(5/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9
/4)*2^(1/2)-1/4*c^(5/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2
))/b^(9/4)*2^(1/2)+2*c/b^2/x^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{c^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{c^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{9/4}}+\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\frac{2c}{b^2\sqrt{x}}-\frac{2}{5bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(5*b*x^(5/2)) + (2*c)/(b^2*Sqrt[x]) - (c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4)) - (c^(5/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(bx^2+cx^4)}\,dx = \int \frac{1}{x^{7/2}(b+cx^2)}\,dx$$
$$= -\frac{2}{5bx^{5/2}} - \frac{c\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{b}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c^2\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^2}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{(2c^2)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} - \frac{c^{3/2}\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2} + \frac{c^{3/2}\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}}{b+cx^4}\right.}{b^2}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} + \frac{c\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+}\right.}{}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} + \frac{c^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{9/4}} - \frac{c^{5/4}\log\left(\sqrt{b}+\right.}{}$$
$$= -\frac{2}{5bx^{5/2}} + \frac{2c}{b^2\sqrt{x}} - \frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{9/4}} + \frac{c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,b^{9/4}}$$

Mathematica [A]
time = 0.16, size = 127, normalized size = 0.59

$$\frac{-\frac{4\sqrt[4]{b}(b-5cx^2)}{x^{5/2}} - 5\sqrt{2}\,c^{5/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - 5\sqrt{2}\,c^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{10b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*b^(1/4)*(b - 5*c*x^2))/x^(5/2) - 5*Sqrt[2]*c^(5/4)*ArcTan[(Sqrt[b] - S
qrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 5*Sqrt[2]*c^(5/4)*ArcTanh[(S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(10*b^(9/4))
```

Maple [A]
time = 0.13, size = 125, normalized size = 0.58

method	result
derivativedivides	$\frac{c\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2}{5b\,x^{\frac{5}{2}}}+$
default	$\frac{c\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2}{5b\,x^{\frac{5}{2}}}+$
risch	$-\frac{2\left(-5c\,x^{2}+b\right)}{5b^{2}x^{\frac{5}{2}}}+\frac{c\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\frac{c\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\frac{c\sqrt{2}\,\arctan}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c/b^2/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2
))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))-2/5/b/x^(5/2)+2*c/b^2
/x^(1/2)
```

Maxima [A]
time = 0.52, size = 198, normalized size = 0.92

$$\frac{c^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4\,b^{2}}+\frac{2\left(5\,cx^{2}-b\right)}{5\,b^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^
2 + 2/5*(5*c*x^2 - b)/(b^2*x^(5/2))
```

Fricas [A]
time = 0.36, size = 193, normalized size = 0.90

$$-\frac{20\,b^2x^3\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^2c^4\sqrt{x}\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}-\sqrt{-b^5c^5\sqrt{-\frac{c^5}{b^9}}+c^8x}\,b^2\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}}{c^5}\right)-5\,b^2x^3\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}\log\left(b^7\left(-\frac{c^5}{b^9}\right)^{\frac{3}{4}}+c^4\sqrt{x}\right)+5\,b^2x^3\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}\log\left(-b^7\left(-\frac{c^5}{b^9}\right)^{\frac{3}{4}}+c^4\sqrt{x}\right)-4\,(5\,cx^2-b)\sqrt{x}}{10\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*b^2*x^3*(-c^5/b^9)^(1/4)*arctan(-(b^2*c^4*sqrt(x)*(-c^5/b^9)^(1/4
) - sqrt(-b^5*c^5*sqrt(-c^5/b^9) + c^8*x)*b^2*(-c^5/b^9)^(1/4))/c^5) - 5*b^
2*x^3*(-c^5/b^9)^(1/4)*log(b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) + 5*b^2*x^3*
(-c^5/b^9)^(1/4)*log(-b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) - 4*(5*c*x^2 - b)
*sqrt(x))/(b^2*x^3)
```

Sympy [A]
time = 14.00, size = 139, normalized size = 0.65

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: b = 0 \wedge c = 0 \\ - \frac{2}{9 c x^{\frac{9}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{5 b x^{\frac{5}{2}}} & \text{for}\: c = 0 \\ - \frac{2}{5 b x^{\frac{5}{2}}} + \frac{c \log{\left(\sqrt{x} - \sqrt[4]{- \frac{b}{c}} \right)}}{2 b^{2} \sqrt[4]{- \frac{b}{c}}} - \frac{c \log{\left(\sqrt{x} + \sqrt[4]{- \frac{b}{c}} \right)}}{2 b^{2} \sqrt[4]{- \frac{b}{c}}} + \frac{c \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{b}{c}}} \right)}}{b^{2} \sqrt[4]{- \frac{b}{c}}} + \frac{2 c}{b^{2} \sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(b, 0) & Eq(c, 0)), (-2/(9*c*x**(9/2)), Eq(b, 0)
), (-2/(5*b*x**(5/2)), Eq(c, 0)), (-2/(5*b*x**(5/2)) + c*log(sqrt(x) - (-b/
c)**(1/4))/(2*b**2*(-b/c)**(1/4)) - c*log(sqrt(x) + (-b/c)**(1/4))/(2*b**2*
(-b/c)**(1/4)) + c*atan(sqrt(x)/(-b/c)**(1/4))/(b**2*(-b/c)**(1/4)) + 2*c/(
b**2*sqrt(x)), True))
```

Giac [A]
time = 4.90, size = 200, normalized size = 0.93

$$\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c}+\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c}-\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c}+\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c}+\frac{2\,(5\,cx^2-b)}{5\,b^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b^3*c) + 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) - 1/4*sqrt(2)*(b*c^3)^
(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 1/4*sqrt(2
)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) +
 2/5*(5*c*x^2 - b)/(b^2*x^(5/2))
```

Mupad [B]
time = 0.09, size = 66, normalized size = 0.31

$$\frac{(-c)^{5/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{9/4}}-\frac{(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{9/4}}-\frac{\frac{2}{5\,b}-\frac{2\,c\,x^2}{b^2}}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(9/4) - ((-c)^(5/4)*atan
(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(9/4) - (2/(5*b) - (2*c*x^2)/b^2)/x^(5/2)
```

3.325 $\int \frac{1}{x^{5/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=217

$$-\frac{2}{7bx^{7/2}}+\frac{2c}{3b^2x^{3/2}}-\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}}-\frac{c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\right.}{2\sqrt{2}\,b^{1}}$$

```
[Out] -2/7/b/x^(7/2)+2/3*c/b^2/x^(3/2)-1/2*c^(7/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/
2)/b^(1/4))/b^(11/4)*2^(1/2)+1/2*c^(7/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b
^(1/4))/b^(11/4)*2^(1/2)-1/4*c^(7/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(11/4)*2^(1/2)+1/4*c^(7/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{c^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{11/4}}-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{2c}{3b^2x^{3/2}}-\frac{2}{7bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(7*b*x^(7/2)) + (2*c)/(3*b^2*x^(3/2)) - (c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) + (c^(7/4)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) - (c^(7/4)*Log[Sqrt[b] - Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4)) + (c^(7/4)*Lo
g[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/
4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)}\,dx=\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx$$
$$=-\frac{2}{7bx^{7/2}}-\frac{c\int \frac{1}{x^{5/2}(b+cx^2)}\,dx}{b}$$
$$=-\frac{2}{7bx^{7/2}}+\frac{2c}{3b^2x^{3/2}}+\frac{c^2\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{b^2}$$
$$=-\frac{2}{7bx^{7/2}}+\frac{2c}{3b^2x^{3/2}}+\frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^2}$$
$$=-\frac{2}{7bx^{7/2}}+\frac{2c}{3b^2x^{3/2}}+\frac{c^2\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^{5/2}}+\frac{c^2\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\right.}{b^{5/2}}$$
$$=-\frac{2}{7bx^{7/2}}+\frac{2c}{3b^2x^{3/2}}+\frac{c^{3/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^{5/2}}+\frac{c^{3/2}\text{Subst}\left(\int\right.}{}$$
$$=-\frac{2}{7bx^{7/2}}+\frac{2c}{3b^2x^{3/2}}-\frac{c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}\log\left(\sqrt{b}+\right.}{}$$
$$=-\frac{2}{7bx^{7/2}}+\frac{2c}{3b^2x^{3/2}}-\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{11/4}}+\frac{c^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt{2}\,b^{11/4}}$$

Mathematica [A]
time = 0.17, size = 129, normalized size = 0.59

$$\frac{\frac{4b^{3/4}(-3b+7cx^2)}{x^{7/2}}-21\sqrt{2}\,c^{7/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+21\sqrt{2}\,c^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{42b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] ((4*b^(3/4)*(-3*b + 7*c*x^2))/x^(7/2) - 21*Sqrt[2]*c^(7/4)*ArcTan[(Sqrt[b]
- Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 21*Sqrt[2]*c^(7/4)*ArcTan
h[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(42*b^(11/4))
```

Maple [A]
time = 0.12, size = 127, normalized size = 0.59

method	result
derivativedivides	$\frac{c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^3}-\frac{}{7b}$
default	$\frac{c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^3}-\frac{}{7b}$
risch	$-\frac{2(-7c\,x^2+3b)}{21b^2x^{\frac{7}{2}}}+\frac{c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4b^3}+\frac{c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c^2/b^3*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1
/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1
/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))-2/7/b/x^(7/2)+2/3*c
/b^2/x^(3/2)
```

Maxima [A]
time = 0.50, size = 201, normalized size = 0.93

$$\frac{\frac{2\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{7}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,c^{\frac{7}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}}{4\,b^2}+\frac{2\,(7\,cx^2-3\,b)}{21\,b^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)
*c^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*log(sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(7
/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^
2 + 2/21*(7*c*x^2 - 3*b)/(b^2*x^(7/2))
```

Fricas [A]
time = 0.39, size = 189, normalized size = 0.87

$$\frac{84\,b^2x^4\left(-\frac{c^7}{b^{11}}\right)^{\frac14}\arctan\left(-\frac{b^8c^2\sqrt{x}\left(-\frac{c^7}{b^{11}}\right)^{\frac34}-\sqrt{b^6\sqrt{-\frac{c^7}{b^{11}}}+c^4x}\,b^8\left(-\frac{c^7}{b^{11}}\right)^{\frac34}}{c^7}\right)+21\,b^2x^4\left(-\frac{c^7}{b^{11}}\right)^{\frac14}\log\left(b^3\left(-\frac{c^7}{b^{11}}\right)^{\frac14}+c^2\sqrt{x}\right)-21\,b^2x^4\left(-\frac{c^7}{b^{11}}\right)^{\frac14}\log\left(-b^3\left(-\frac{c^7}{b^{11}}\right)^{\frac14}+c^2\sqrt{x}\right)+4\,(7\,cx^2-3\,b)\sqrt{x}}{42\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(84*b^2*x^4*(-c^7/b^11)^(1/4)*arctan(-(b^8*c^2*sqrt(x)*(-c^7/b^11)^(3/
4) - sqrt(b^6*sqrt(-c^7/b^11) + c^4*x)*b^8*(-c^7/b^11)^(3/4))/c^7) + 21*b^2
*x^4*(-c^7/b^11)^(1/4)*log(b^3*(-c^7/b^11)^(1/4) + c^2*sqrt(x)) - 21*b^2*x^
4*(-c^7/b^11)^(1/4)*log(-b^3*(-c^7/b^11)^(1/4) + c^2*sqrt(x)) + 4*(7*c*x^2
- 3*b)*sqrt(x))/(b^2*x^4)
```

Sympy [A]
time = 31.33, size = 146, normalized size = 0.67

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{2}}} & \text{for}\: b = 0 \wedge c = 0 \\ -\frac{2}{11cx^{\frac{11}{2}}} & \text{for}\: b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for}\: c = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} + \frac{2c}{3b^2x^{\frac{3}{2}}} - \frac{c^2\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^3} + \frac{c^2\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^3} + \frac{c^2\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(11/2), Eq(b, 0) & Eq(c, 0)), (-2/(11*c*x**(11/2)), Eq(b,
 0)), (-2/(7*b*x**(7/2)), Eq(c, 0)), (-2/(7*b*x**(7/2)) + 2*c/(3*b**2*x**(3
/2)) - c**2*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b**3) + c**2*(-b/
c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b**3) + c**2*(-b/c)**(1/4)*atan(s
qrt(x)/(-b/c)**(1/4))/b**3, True))
```

Giac [A]
time = 5.05, size = 192, normalized size = 0.88

$$\frac{\sqrt{2}\,(bc^3)^{\frac14}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{2\,b^3}+\frac{\sqrt{2}\,(bc^3)^{\frac14}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{2\,b^3}+\frac{\sqrt{2}\,(bc^3)^{\frac14}\,c\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3}-\frac{\sqrt{2}\,(bc^3)^{\frac14}\,c\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3}+\frac{2\,(7\,cx^2-3\,b)}{21\,b^2x^{\frac72}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/b^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^3 + 1/4*sqrt(2)*(b*c^3)^(1/4
)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 - 1/4*sqrt(2)*(b*c
^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 2/21*(7
*c*x^2 - 3*b)/(b^2*x^(7/2))
```

Mupad [B]
time = 4.39, size = 65, normalized size = 0.30

$$\frac{(-c)^{7/4}\operatorname{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{11/4}}-\frac{\frac{2}{7\,b}-\frac{2\,c\,x^2}{3\,b^2}}{x^{7/2}}+\frac{(-c)^{7/4}\operatorname{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(7/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(11/4) - (2/(7*b) - (2*c*
x^2)/(3*b^2))/x^(7/2) + ((-c)^(7/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^
(11/4)
```

3.326 $\int \frac{1}{x^{7/2}\left(bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=230

$$-\frac{2}{9bx^{9/2}}+\frac{2c}{5b^2x^{5/2}}-\frac{2c^2}{b^3\sqrt{x}}+\frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}}-\frac{c^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}}-\frac{c^{9/4}\log\left(\sqrt{b}\right.}{}$$

```
[Out] -2/9/b/x^(9/2)+2/5*c/b^2/x^(5/2)+1/2*c^(9/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/
2)/b^(1/4))/b^(13/4)*2^(1/2)-1/2*c^(9/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b
^(1/4))/b^(13/4)*2^(1/2)-1/4*c^(9/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/4*c^(9/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)-2*c^2/b^3/x^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{c^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}}-\frac{c^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\,b^{13/4}}-\frac{c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}+\frac{c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}}-\frac{2c^2}{b^3\sqrt{x}}+\frac{2c}{5b^2x^{5/2}}-\frac{2}{9bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(9*b*x^(9/2)) + (2*c)/(5*b^2*x^(5/2)) - (2*c^2)/(b^3*Sqrt[x]) + (c^(9/4)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) - (c^(9/
4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) - (c^(
9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]
*b^(13/4)) + (c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^{7/2}(bx^2+cx^4)}\,dx = \int \frac{1}{x^{11/2}(b+cx^2)}\,dx$$
$$= -\frac{2}{9bx^{9/2}} - \frac{c\int \frac{1}{x^{7/2}(b+cx^2)}\,dx}{b}$$
$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} + \frac{c^2\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{b^2}$$
$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^3}$$
$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{(2c^3)\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} + \frac{c^{5/2}\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3} - \frac{c^{5/2}\text{Subst}\Big(}{}$$
$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^3} - \frac{c^2\text{Sul}}{}$$
$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} - \frac{c^{9/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{2\sqrt{2}\,b^{13/4}} + \frac{c^{9/4}\log}{}$$
$$= -\frac{2}{9bx^{9/2}} + \frac{2c}{5b^2x^{5/2}} - \frac{2c^2}{b^3\sqrt{x}} + \frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\,b^{13/4}} - \frac{c^{9/4}\tan^{-1}\Big(1+\frac{\sqrt{}}{}}{\sqrt{2}\,b^{13}}$$

Mathematica [A]
time = 0.18, size = 140, normalized size = 0.61

$$\frac{-\frac{4\sqrt[4]{b}\,(5b^2-9bcx^2+45c^2x^4)}{x^{9/2}}+45\sqrt{2}\,c^{9/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+45\sqrt{2}\,c^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{90b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*b^(1/4)*(5*b^2 - 9*b*c*x^2 + 45*c^2*x^4))/x^(9/2) + 45*Sqrt[2]*c^(9/4)
*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 45*Sqrt[
2]*c^(9/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)]
)/(90*b^(13/4))
```

Maple [A]
time = 0.12, size = 138, normalized size = 0.60

method	result
derivativedivides	$-\frac{c^2\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2}{9b\,x^{\frac{9}{2}}}$
default	$-\frac{c^2\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2}{9b\,x^{\frac{9}{2}}}$
risch	$-\frac{2\left(45c^2x^4-9bc\,x^2+5b^2\right)}{45b^3x^{\frac{9}{2}}}-\frac{c^2\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{4b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{c^2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{2b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*c^2/b^3/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(
1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(
1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))-2/9/b/x^(9/2)-2*c^
2/b^3/x^(1/2)+2/5*c/b^2/x^(5/2)
```

Maxima [A]
time = 0.53, size = 209, normalized size = 0.91

$$-\frac{c^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{4\,b^3}-\frac{2\left(45\,c^2x^4-9\,bcx^2+5\,b^2\right)}{45\,b^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)
*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2
```

```
)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sq
rt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/
4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-
sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b
^3 - 2/45*(45*c^2*x^4 - 9*b*c*x^2 + 5*b^2)/(b^3*x^(9/2))
```

Fricas [A]
time = 0.39, size = 204, normalized size = 0.89

$$\frac{180\,b^{3}x^{5}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{3}c^{7}\sqrt{x}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{1}{4}}-\sqrt{-b^{7}c^{9}\sqrt{-\frac{c^{9}}{b^{13}}}+c^{14}x}\,b^{3}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{1}{4}}}{c^{9}}\right)-45\,b^{3}x^{5}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^{10}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{3}{4}}+c^{7}\sqrt{x}\right)+45\,b^{3}x^{5}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{1}{4}}\log\left(-b^{10}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{3}{4}}+c^{7}\sqrt{x}\right)-4\,(45\,c^{2}x^{4}-9\,bcx^{2}+5\,b^{2})\sqrt{x}}{90\,b^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*(180*b^3*x^5*(-c^9/b^13)^(1/4)*arctan(-(b^3*c^7*sqrt(x)*(-c^9/b^13)^(1
/4) - sqrt(-b^7*c^9*sqrt(-c^9/b^13) + c^14*x)*b^3*(-c^9/b^13)^(1/4))/c^9) -
 45*b^3*x^5*(-c^9/b^13)^(1/4)*log(b^10*(-c^9/b^13)^(3/4) + c^7*sqrt(x)) + 4
5*b^3*x^5*(-c^9/b^13)^(1/4)*log(-b^10*(-c^9/b^13)^(3/4) + c^7*sqrt(x)) - 4*
(45*c^2*x^4 - 9*b*c*x^2 + 5*b^2)*sqrt(x))/(b^3*x^5)
```

Sympy [A]
time = 76.13, size = 160, normalized size = 0.70

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{13}{2}}} & \text{for}\: b = 0 \wedge c = 0 \\ - \frac{2}{13 c x^{\frac{13}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{9 b x^{\frac{9}{2}}} & \text{for}\: c = 0 \\ - \frac{2}{9 b x^{\frac{9}{2}}} + \frac{2 c}{5 b^{2} x^{\frac{5}{2}}} - \frac{c^{2} \log{\left(\sqrt{x} - \sqrt[4]{- \frac{b}{c}} \right)}}{2 b^{3} \sqrt[4]{- \frac{b}{c}}} + \frac{c^{2} \log{\left(\sqrt{x} + \sqrt[4]{- \frac{b}{c}} \right)}}{2 b^{3} \sqrt[4]{- \frac{b}{c}}} - \frac{c^{2} \operatorname{atan}{\left(\frac{\sqrt{x}}{\sqrt[4]{- \frac{b}{c}}} \right)}}{b^{3} \sqrt[4]{- \frac{b}{c}}} - \frac{2 c^{2}}{b^{3} \sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(13/2), Eq(b, 0) & Eq(c, 0)), (-2/(13*c*x**(13/2)), Eq(b,
 0)), (-2/(9*b*x**(9/2)), Eq(c, 0)), (-2/(9*b*x**(9/2)) + 2*c/(5*b**2*x**(5
/2)) - c**2*log(sqrt(x) - (-b/c)**(1/4))/(2*b**3*(-b/c)**(1/4)) + c**2*log(
sqrt(x) + (-b/c)**(1/4))/(2*b**3*(-b/c)**(1/4)) - c**2*atan(sqrt(x)/(-b/c)*
*(1/4))/(b**3*(-b/c)**(1/4)) - 2*c**2/(b**3*sqrt(x)), True))
```

Giac [A]
time = 4.86, size = 199, normalized size = 0.87

$$-\frac{\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{4}}-\frac{\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{4}}+\frac{\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^{4}}-\frac{\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^{4}}-\frac{2\,(45\,c^{2}x^{4}-9\,bcx^{2}+5\,b^{2})}{45\,b^{3}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/b^4 - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/4*sqrt(2)*(b*c^3)^(3/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/4*sqrt(2)*(b*c^3)^(
3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 2/45*(45*c^2*x
^4 - 9*b*c*x^2 + 5*b^2)/(b^3*x^(9/2))
```

Mupad [B]
time = 4.47, size = 77, normalized size = 0.33

$$\frac{(-c)^{9/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{13/4}}-\frac{(-c)^{9/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{13/4}}-\frac{\frac{2}{9\,b}-\frac{2\,c\,x^2}{5\,b^2}+\frac{2\,c^2\,x^4}{b^3}}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(13/4) - ((-c)^(9/4)*ata
n(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(13/4) - (2/(9*b) - (2*c*x^2)/(5*b^2) +
(2*c^2*x^4)/b^3)/x^(9/2)
```

3.327 $\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=243

$$-\frac{9b\sqrt{x}}{2c^3}+\frac{9x^{5/2}}{10c^2}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{13/4}}+\frac{9b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{13/4}}-\frac{9b^{5/4}\log}{}$$

```
[Out] 9/10*x^(5/2)/c^2-1/2*x^(9/2)/c/(c*x^2+b)-9/8*b^(5/4)*arctan(1-c^(1/4)*2^(1/
2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)+9/8*b^(5/4)*arctan(1+c^(1/4)*2^(1/2)*x
^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)-9/16*b^(5/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)+9/16*b^(5/4)*ln(b^(1/2)+x*c^(1/2)
+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)-9/2*b*x^(1/2)/c^3
```

Rubi [A]
time = 0.14, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{9b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{13/4}}+\frac{9b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,c^{13/4}}-\frac{9b^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{13/4}}+\frac{9b^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{13/4}}-\frac{9b\sqrt{x}}{2c^3}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}+\frac{9x^{5/2}}{10c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-9*b*Sqrt[x])/(2*c^3) + (9*x^(5/2))/(10*c^2) - x^(9/2)/(2*c*(b + c*x^2)) -
 (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13
/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]
*c^(13/4)) - (9*b^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(8*Sqrt[2]*c^(13/4)) + (9*b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^2}\,dx = \int \frac{x^{11/2}}{(b+cx^2)^2}\,dx$$
$$= -\frac{x^{9/2}}{2c(b+cx^2)} + \frac{9\int \frac{x^{7/2}}{b+cx^2}\,dx}{4c}$$
$$= \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c(b+cx^2)} - \frac{(9b)\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c^2}$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c(b+cx^2)} + \frac{(9b^2)\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{4c^3}$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c(b+cx^2)} + \frac{(9b^2)\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^3}$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c(b+cx^2)} + \frac{(9b^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^3} + \frac{(9b^{3/2})}{}$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c(b+cx^2)} + \frac{(9b^{3/2})\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{8c^{7/2}} +$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c(b+cx^2)} - \frac{9b^{5/4}\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}\,x\right)}{8\sqrt{2}\,c^{13/4}} + \frac{9b^{5/4}}{}$$
$$= -\frac{9b\sqrt{x}}{2c^3} + \frac{9x^{5/2}}{10c^2} - \frac{x^{9/2}}{2c(b+cx^2)} - \frac{9b^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{13/4}} + \frac{9b^{5/4}\tan^{-1}\left(1 + \right.}{4\sqrt{2}}$$

Mathematica [A]

time = 0.27, size = 149, normalized size = 0.61

$$\frac{\frac{4\sqrt[4]{c}\,\sqrt{x}\,(-45b^2-36bcx^2+4c^2x^4)}{b+cx^2}-45\sqrt{2}\,b^{5/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)+45\sqrt{2}\,b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{40c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(-45*b^2 - 36*b*c*x^2 + 4*c^2*x^4))/(b + c*x^2) - 45*Sq
rt[2]*b^(5/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
)] + 45*Sqrt[2]*b^(5/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b]
+ Sqrt[c]*x)])/(40*c^(13/4))
```

Maple [A]
time = 0.14, size = 148, normalized size = 0.61

method	result
derivativedivides	$-\frac{2\left(-\frac{c\,x^{\frac{5}{2}}}{5}+2b\sqrt{x}\right)}{c^3}+\frac{2b^2\left(-\frac{\sqrt{x}}{4\left(c\,x^2+b\right)}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{32b}\right.}{c^3}$
default	$-\frac{2\left(-\frac{c\,x^{\frac{5}{2}}}{5}+2b\sqrt{x}\right)}{c^3}+\frac{2b^2\left(-\frac{\sqrt{x}}{4\left(c\,x^2+b\right)}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{32b}\right.}{c^3}$
risch	$-\frac{2\left(-c\,x^2+10b\right)\sqrt{x}}{5c^3}-\frac{b^2\sqrt{x}}{2c^3\left(c\,x^2+b\right)}+\frac{9b\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16c^3}+\frac{9b\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\text{arctan}}{8c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^3*(-1/5*c*x^(5/2)+2*b*x^(1/2))+2*b^2/c^3*(-1/4*x^(1/2)/(c*x^2+b)+9/32*
(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b
/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.50, size = 217, normalized size = 0.89

$$-\frac{b^2\sqrt{x}}{2\,(c^4x^2+bc^3)} + \frac{2\left(cx^{\frac{5}{2}} - 10\,b\sqrt{x}\right)}{5\,c^3} + \frac{9\left(\frac{2\sqrt{2}\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}\,b^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}\,b^{\frac{5}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}} - \frac{\sqrt{2}\,b^{\frac{5}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}\right)}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*sqrt(x)/(c^4*x^2 + b*c^3) + 2/5*(c*x^(5/2) - 10*b*sqrt(x))/c^3 + 9
/16*(2*sqrt(2)*b^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt
(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*b^(3/
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(5/4)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(5/4)*log(-sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c^3
```

Fricas [A]
time = 0.37, size = 227, normalized size = 0.93

$$\frac{180\,(c^4x^2+bc^3)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{bc^{10}\sqrt{x}\left(-\frac{b^5}{c^{13}}\right)^{\frac{3}{4}} - \sqrt{c^6\sqrt{-\frac{b^5}{c^{13}}}+b^2x}\,c^{10}\left(-\frac{b^5}{c^{13}}\right)^{\frac{3}{4}}}{b^5}\right) + 45\,(c^4x^2+bc^3)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\log\left(9\,c^3\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}+9\,b\sqrt{x}\right) - 45\,(c^4x^2+bc^3)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\log\left(-9\,c^3\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}+9\,b\sqrt{x}\right) + 4\,(4\,c^2x^4-36\,bcx^2-45\,b^2)\sqrt{x}}{40\,(c^4x^2+bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(180*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*arctan(-(b*c^10*sqrt(x)*(-b^5
/c^13)^(3/4) - sqrt(c^6*sqrt(-b^5/c^13) + b^2*x)*c^10*(-b^5/c^13)^(3/4))/b^
5) + 45*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*log(9*c^3*(-b^5/c^13)^(1/4) + 9
*b*sqrt(x)) - 45*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*log(-9*c^3*(-b^5/c^13)
^(1/4) + 9*b*sqrt(x)) + 4*(4*c^2*x^4 - 36*b*c*x^2 - 45*b^2)*sqrt(x))/(c^4*x
^2 + b*c^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.60, size = 216, normalized size = 0.89

$$\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{4}}+\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{4}}+\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^{4}}-\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^{4}}-\frac{b^{2}\sqrt{x}}{2\left(cx^{2}+b\right)c^{3}}+\frac{2\left(c^{8}x^{\frac{5}{2}}-10\,bc^{7}\sqrt{x}\right)}{5\,c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 9/8*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^4 + 9/8*sqrt(2)*(b*c^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 + 9/16*sqrt(2)*(b*c^3)^(1/
4)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 9/16*sqrt(2)*(b
*c^3)^(1/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/2*b
^2*sqrt(x)/((c*x^2 + b)*c^3) + 2/5*(c^8*x^(5/2) - 10*b*c^7*sqrt(x))/c^10
```

Mupad [B]
time = 0.10, size = 92, normalized size = 0.38

$$\frac{2\,x^{5/2}}{5\,c^2}-\frac{b^2\,\sqrt{x}}{2\,\left(c^4\,x^2+b\,c^3\right)}-\frac{4\,b\,\sqrt{x}}{c^3}-\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,c^{13/4}}+\frac{{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,9{}\mathrm{i}}{4\,c^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*x^(5/2))/(5*c^2) - (b^2*x^(1/2))/(2*(b*c^3 + c^4*x^2)) - (4*b*x^(1/2))/c
^3 - (9*(-b)^(5/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*c^(13/4)) + ((-b)
^(5/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*9i)/(4*c^(13/4))
```

3.328 $\int \frac{x^{17/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=230

$$\frac{7x^{3/2}}{6c^2}-\frac{x^{7/2}}{2c\,(b+cx^2)}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}}-\frac{7b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}}-\frac{7b^{3/4}\log\left(\sqrt{b}-\sqrt{\ldots}\right.}{8\sqrt{\ldots}}$$

```
[Out] 7/6*x^(3/2)/c^2-1/2*x^(7/2)/c/(c*x^2+b)+7/8*b^(3/4)*arctan(1-c^(1/4)*2^(1/2
)*x^(1/2)/b^(1/4))/c^(11/4)*2^(1/2)-7/8*b^(3/4)*arctan(1+c^(1/4)*2^(1/2)*x^
(1/2)/b^(1/4))/c^(11/4)*2^(1/2)-7/16*b^(3/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)*2^(1/2)+7/16*b^(3/4)*ln(b^(1/2)+x*c^(1/2)+
b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{7b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}}-\frac{7b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,c^{11/4}}-\frac{7b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}}+\frac{7b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}}-\frac{x^{7/2}}{2c\,(b+cx^2)}+\frac{7x^{3/2}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (7*x^(3/2))/(6*c^2) - x^(7/2)/(2*c*(b + c*x^2)) + (7*b^(3/4)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) - (7*b^(3/4)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) - (7*b^(3/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(11/
4)) + (7*b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(8*Sqrt[2]*c^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^2}\,dx = \int \frac{x^{9/2}}{(b+cx^2)^2}\,dx$$

$$= -\frac{x^{7/2}}{2c(b+cx^2)} + \frac{7\int \frac{x^{5/2}}{b+cx^2}\,dx}{4c}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c(b+cx^2)} - \frac{(7b)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c^2}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c(b+cx^2)} - \frac{(7b)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^2}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c(b+cx^2)} + \frac{(7b)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{5/2}} - \frac{(7b)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}}{b+cx^4}\right.}{4c^{5/2}}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c(b+cx^2)} - \frac{(7b)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{8c^3} - \frac{(7b)\text{Subst}\left(\int\right.}{}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c(b+cx^2)} - \frac{7b^{3/4}\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}\,x\right)}{8\sqrt{2}\,c^{11/4}} + \frac{7b^{3/4}\log\left(\sqrt{b} +\right.}{}$$

$$= \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c(b+cx^2)} + \frac{7b^{3/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{11/4}} - \frac{7b^{3/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,c^{11/4}}$$

Mathematica [A]
time = 0.26, size = 138, normalized size = 0.60

$$\frac{\frac{4c^{3/4}x^{3/2}(7b+4cx^2)}{b+cx^2} + 21\sqrt{2}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + 21\sqrt{2}\,b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{24c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(3/4)*x^(3/2)*(7*b + 4*c*x^2))/(b + c*x^2) + 21*Sqrt[2]*b^(3/4)*ArcTa
n[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 21*Sqrt[2]*b^(
3/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(24*
c^(11/4))
```

Maple [A]
time = 0.13, size = 136, normalized size = 0.59

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3c^{2}}-\frac{2b\left(-\frac{x^{\frac{3}{2}}}{4\left(c\,x^{2}+b\right)}+\frac{7\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right.\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{c^{2}}$
default	$\frac{2x^{\frac{3}{2}}}{3c^{2}}-\frac{2b\left(-\frac{x^{\frac{3}{2}}}{4\left(c\,x^{2}+b\right)}+\frac{7\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right.\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{c^{2}}$
risch	$\frac{2x^{\frac{3}{2}}}{3c^{2}}+\frac{b\,x^{\frac{3}{2}}}{2c^{2}\left(c\,x^{2}+b\right)}-\frac{7b\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{7b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{7b\sqrt{2}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)/c^2-2*b/c^2*(-1/4*x^(3/2)/(c*x^2+b)+7/32/c/(b/c)^(1/4)*2^(1/2)*
(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1
/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/
(b/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.50, size = 207, normalized size = 0.90

$$\frac{bx^{\frac{3}{2}}}{2\left(c^{3}x^{2}+bc^{2}\right)}-\frac{7\,b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,c^{2}}+\frac{2\,x^{\frac{3}{2}}}{3\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^(3/2)/(c^3*x^2 + b*c^2) - 7/16*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqr
t(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt
(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1
/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt
(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(
b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
 + sqrt(b))/(b^(1/4)*c^(3/4)))/c^2 + 2/3*x^(3/2)/c^2
```

Fricas [A]
time = 0.38, size = 229, normalized size = 1.00

$$\frac{84\,(c^3x^2+bc^2)\left(-\frac{b^3}{c^{11}}\right)^{\frac14}\arctan\left(-\frac{343\,b^2c^3\sqrt{x}\left(-\frac{b^3}{c^{11}}\right)^{\frac14}-\sqrt{-117649\,b^3c^5\sqrt{-\frac{b^3}{c^{11}}}+117649\,b^4x}\;c^3\left(-\frac{b^3}{c^{11}}\right)^{\frac14}}{343\,b^3}\right)-21\,(c^3x^2+bc^2)\left(-\frac{b^3}{c^{11}}\right)^{\frac14}\log\left(343\,c^8\left(-\frac{b^3}{c^{11}}\right)^{\frac34}+343\,b^2\sqrt{x}\right)+21\,(c^3x^2+bc^2)\left(-\frac{b^3}{c^{11}}\right)^{\frac14}\log\left(-343\,c^8\left(-\frac{b^3}{c^{11}}\right)^{\frac34}+343\,b^2\sqrt{x}\right)+4\,(4\,cx^3+7\,bx)\sqrt{x}}{24\,(c^3x^2+bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(84*(c^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*arctan(-1/343*(343*b^2*c^3*sqr
t(x)*(-b^3/c^11)^(1/4) - sqrt(-117649*b^3*c^5*sqrt(-b^3/c^11) + 117649*b^4*
x)*c^3*(-b^3/c^11)^(1/4))/b^3) - 21*(c^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*log
(343*c^8*(-b^3/c^11)^(3/4) + 343*b^2*sqrt(x)) + 21*(c^3*x^2 + b*c^2)*(-b^3/
c^11)^(1/4)*log(-343*c^8*(-b^3/c^11)^(3/4) + 343*b^2*sqrt(x)) + 4*(4*c*x^3
+ 7*b*x)*sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.40, size = 196, normalized size = 0.85

$$\frac{bx^{\frac32}}{2\,(cx^2+b)c^2}+\frac{2\,x^{\frac32}}{3\,c^2}-\frac{7\,\sqrt{2}\,(bc^3)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{8\,c^5}-\frac{7\,\sqrt{2}\,(bc^3)^{\frac34}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{8\,c^5}+\frac{7\,\sqrt{2}\,(bc^3)^{\frac34}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^5}-\frac{7\,\sqrt{2}\,(bc^3)^{\frac34}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^(3/2)/((c*x^2 + b)*c^2) + 2/3*x^(3/2)/c^2 - 7/8*sqrt(2)*(b*c^3)^(3/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 -
7/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^5 + 7/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^5 - 7/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt
(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5
```

Mupad [B]
time = 0.11, size = 80, normalized size = 0.35

$$\frac{2\,x^{3/2}}{3\,c^2}+\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,c^{11/4}}+\frac{b\,x^{3/2}}{2\,\left(c^3\,x^2+b\,c^2\right)}+\frac{{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,7{}\mathrm{i}}{4\,c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*x^(3/2))/(3*c^2) + (7*(-b)^(3/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*
c^(11/4)) + ((-b)^(3/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*7i)/(4*c^(11/
4)) + (b*x^(3/2))/(2*(b*c^2 + c^3*x^2))
```

3.329 $\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=230

$$\frac{5\sqrt{x}}{2c^2}-\frac{x^{5/2}}{2c\,(b+cx^2)}+\frac{5\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{9/4}}-\frac{5\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{9/4}}+\frac{5\sqrt[4]{b}\,\log\left(\sqrt{b}-\right.}{8}$$

```
[Out] -1/2*x^(5/2)/c/(c*x^2+b)+5/8*b^(1/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/
4))/c^(9/4)*2^(1/2)-5/8*b^(1/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c
^(9/4)*2^(1/2)+5/16*b^(1/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^
(1/2))/c^(9/4)*2^(1/2)-5/16*b^(1/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^
(1/2)*x^(1/2))/c^(9/4)*2^(1/2)+5/2*x^(1/2)/c^2
```

Rubi [A]
time = 0.12, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{5\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{9/4}}-\frac{5\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,c^{9/4}}+\frac{5\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{9/4}}-\frac{5\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{9/4}}-\frac{x^{5/2}}{2c\,(b+cx^2)}+\frac{5\sqrt{x}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (5*Sqrt[x])/(2*c^2) - x^(5/2)/(2*c*(b + c*x^2)) + (5*b^(1/4)*ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) - (5*b^(1/4)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) + (5*b^(1/4)*Log[
Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(9/4))
 - (5*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(
8*Sqrt[2]*c^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx = \int \frac{x^{7/2}}{(b+cx^2)^2}\,dx$$

$$= -\frac{x^{5/2}}{2c(b+cx^2)} + \frac{5\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c(b+cx^2)} - \frac{(5b)\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{4c^2}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c(b+cx^2)} - \frac{(5b)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^2}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c(b+cx^2)} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^2} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c(b+cx^2)} - \frac{\left(5\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^{5/2}} - \frac{\left(5\sqrt{b}\right)\text{S}}{}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c(b+cx^2)} + \frac{5\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,c^{9/4}} - \frac{5\sqrt[4]{b}\log\left(\sqrt{b}+\right.}{}$$

$$= \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c(b+cx^2)} + \frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,c^{9/4}} - \frac{5\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,c^{9/4}}$$

Mathematica [A]
time = 0.26, size = 138, normalized size = 0.60

$$\frac{\frac{4\sqrt[4]{c}\sqrt{x}\,(5b+4cx^2)}{b+cx^2} + 5\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - 5\sqrt{2}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{8c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(5*b + 4*c*x^2))/(b + c*x^2) + 5*Sqrt[2]*b^(1/4)*ArcTan
[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 5*Sqrt[2]*b^(1/
4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(8*c^(
9/4))
```

Maple [A]

time = 0.13, size = 136, normalized size = 0.59

method	result
derivativedivides	$\frac{2\sqrt{x}}{c^2}-\frac{2b\left(-\frac{\sqrt{x}}{4\left(c\,x^2+b\right)}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.\right.}{32b}\right)}{c^2}$
default	$\frac{2\sqrt{x}}{c^2}-\frac{2b\left(-\frac{\sqrt{x}}{4\left(c\,x^2+b\right)}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.\right.}{32b}\right)}{c^2}$
risch	$\frac{2\sqrt{x}}{c^2}+\frac{b\sqrt{x}}{2c^2\left(c\,x^2+b\right)}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16c^2}-\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\cdots\right.}{8c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/c^2-2*b/c^2*(-1/4*x^(1/2)/(c*x^2+b)+5/32*(b/c)^(1/4)/b*2^(1/2)*(l
n((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2
)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.52, size = 206, normalized size = 0.90

$$\frac{b\sqrt{x}}{2\left(c^3x^2+bc^2\right)}-\frac{5\left(\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}\right)}{16\,c^2}+\frac{2\sqrt{x}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*sqrt(x)/(c^3*x^2 + b*c^2) - 5/16*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2
)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt
(sqrt(b)*sqrt(c)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*
c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) +
 sqrt(2)*b^(1/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))
/c^(1/4) - sqrt(2)*b^(1/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
 + sqrt(b))/c^(1/4))/c^2 + 2*sqrt(x)/c^2
```

Fricas [A]
time = 0.38, size = 192, normalized size = 0.83

$$-\frac{20\,(c^3x^2+bc^2)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^4\sqrt{-\frac{b}{c^9}}+x}\,c^7\left(-\frac{b}{c^9}\right)^{\frac{3}{4}}-c^7\sqrt{x}\left(-\frac{b}{c^9}\right)^{\frac{3}{4}}}{b}\right)+5\,(c^3x^2+bc^2)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\log\left(5\,c^2\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}+5\sqrt{x}\right)-5\,(c^3x^2+bc^2)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\log\left(-5\,c^2\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}+5\sqrt{x}\right)-4\,(4\,cx^2+5\,b)\sqrt{x}}{8\,(c^3x^2+bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*(c^3*x^2 + b*c^2)*(-b/c^9)^(1/4)*arctan((sqrt(c^4*sqrt(-b/c^9) + x
)*c^7*(-b/c^9)^(3/4) - c^7*sqrt(x)*(-b/c^9)^(3/4))/b) + 5*(c^3*x^2 + b*c^2)
*(-b/c^9)^(1/4)*log(5*c^2*(-b/c^9)^(1/4) + 5*sqrt(x)) - 5*(c^3*x^2 + b*c^2)
*(-b/c^9)^(1/4)*log(-5*c^2*(-b/c^9)^(1/4) + 5*sqrt(x)) - 4*(4*c*x^2 + 5*b)*
sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.74, size = 196, normalized size = 0.85

$$-\frac{5\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^3}-\frac{5\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^3}-\frac{5\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^3}+\frac{5\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^3}+\frac{b\sqrt{x}}{2\,(cx^2+b)c^2}+\frac{2\sqrt{x}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/c^3 - 5/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^3 - 5/16*sqrt(2)*(b*c^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 5/16*sqrt(2)*(b*c^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 1/2*b*sqrt(x
)/((c*x^2 + b)*c^2) + 2*sqrt(x)/c^2
```

Mupad [B]
time = 4.32, size = 80, normalized size = 0.35

$$\frac{2\sqrt{x}}{c^2}-\frac{5\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,c^{9/4}}+\frac{b\,\sqrt{x}}{2\,\left(c^3\,x^2+b\,c^2\right)}+\frac{{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,5{}\mathrm{i}}{4\,c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*x^(1/2))/c^2 - (5*(-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*c^(9
/4)) + ((-b)^(1/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*5i)/(4*c^(9/4)) +
(b*x^(1/2))/(2*(b*c^2 + c^3*x^2))
```

3.330 $\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{x^{3/2}}{2c\left(b+cx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}}+\frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}$$

```
[Out] -1/2*x^(3/2)/c/(c*x^2+b)-3/8*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1
/4)/c^(7/4)*2^(1/2)+3/8*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c
^(7/4)*2^(1/2)+3/16*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b
^(1/4)/c^(7/4)*2^(1/2)-3/16*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^
(1/2))/b^(1/4)/c^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}}+\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*x^(3/2)/(c*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4)) - (3*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx = \int \frac{x^{5/2}}{(b+cx^2)^2}\,dx$$
$$= -\frac{x^{3/2}}{2c(b+cx^2)} + \frac{3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c}$$
$$= -\frac{x^{3/2}}{2c(b+cx^2)} + \frac{3\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c}$$
$$= -\frac{x^{3/2}}{2c(b+cx^2)} - \frac{3\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{3/2}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right.}{4c^{3/2}}$$
$$= -\frac{x^{3/2}}{2c(b+cx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^2} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}\right.}{8c^2}$$
$$= -\frac{x^{3/2}}{2c(b+cx^2)} + \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}} - \frac{3\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{8\sqrt{2}\sqrt[4]{b}\,c^{7/4}}$$
$$= -\frac{x^{3/2}}{2c(b+cx^2)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}\,c^{7/4}} + \frac{3\log\left(\right.}{}$$

Mathematica [A]
time = 0.25, size = 128, normalized size = 0.59

$$\frac{-\frac{4c^{3/4}x^{3/2}}{b+cx^2} - \frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{b}} - \frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt[4]{b}}}{8c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*c^(3/4)*x^(3/2))/(b + c*x^2) - (3*Sqrt[2]*ArcTan[(Sqrt[b] - Sqrt[c]*x)
/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(1/4) - (3*Sqrt[2]*ArcTanh[(Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(1/4))/(8*c^(7/4))
```

Maple [A]
time = 0.12, size = 124, normalized size = 0.57

method	result
derivativedivides	$-\frac{x^{\frac{3}{2}}}{2c\left(c\,x^{2}+b\right)}+\frac{3\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right.\right.}{16c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{x^{\frac{3}{2}}}{2c\left(c\,x^{2}+b\right)}+\frac{3\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right.\right.}{16c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(3/2)/c/(c*x^2+b)+3/16/c^2/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^
(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*a
rctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1
))
```

Maxima [A]
time = 0.52, size = 195, normalized size = 0.89

$$-\frac{x^{\frac{3}{2}}}{2\,(c^2x^2+bc)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x^(3/2)/(c^2*x^2 + b*c) + 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*s
qrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) -
 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c))
- sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/
4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sq
rt(b))/(b^(1/4)*c^(3/4)))/c
```

Fricas [A]
time = 0.37, size = 185, normalized size = 0.85

$$-\frac{12\,(c^2x^2+bc)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc^3\sqrt{-\frac{1}{bc^7}}+x}\,c^2\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}-c^2\sqrt{x}\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\right)-3\,(c^2x^2+bc)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\log\left(bc^5\left(-\frac{1}{bc^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)+3\,(c^2x^2+bc)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\log\left(-bc^5\left(-\frac{1}{bc^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)+4\,x^{\frac{3}{2}}}{8\,(c^2x^2+bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*arctan(sqrt(-b*c^3*sqrt(-1/(b*c
^7)) + x)*c^2*(-1/(b*c^7))^(1/4) - c^2*sqrt(x)*(-1/(b*c^7))^(1/4)) - 3*(c^2
*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log(b*c^5*(-1/(b*c^7))^(3/4) + sqrt(x)) + 3*
(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log(-b*c^5*(-1/(b*c^7))^(3/4) + sqrt(x))
 + 4*x^(3/2))/(c^2*x^2 + b*c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.00, size = 199, normalized size = 0.91

$$-\frac{x^{\frac{3}{2}}}{2\,(cx^2+b)c}+\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^4}+\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^4}-\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^4}+\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^(3/2)/((c*x^2 + b)*c) + 3/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) + 3/8*sqrt(2)*(b*c^
3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))
/(b*c^4) - 3/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/(b*c^4) + 3/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)
^(1/4) + x + sqrt(b/c))/(b*c^4)
```

Mupad [B]
time = 0.09, size = 64, normalized size = 0.29

$$\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{4\,{(-b)}^{1/4}\,c^{7/4}}-\frac{x^{3/2}}{2\,c\,\left(c\,x^2+b\right)}-\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{4\,{(-b)}^{1/4}\,c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(1/4)*c^(7/4)) - x^(3/2)/(2*
c*(b + c*x^2)) - (3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(1/4)*c^(7
/4))
```

3.331 $\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\right.}{8\sqrt{2}\,b^{3/4}c^{5/4}}$$

```
[Out] -1/8*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(5/4)*2^(1/2)+1/8*
arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(5/4)*2^(1/2)-1/16*ln(b
^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(5/4)*2^(1/2)+1
/16*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(5/4)*2
^(1/2)-1/2*x^(1/2)/c/(c*x^2+b)
```

Rubi [A]

time = 0.11, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*Sqrt[x]/(c*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
]/(4*Sqrt[2]*b^(3/4)*c^(5/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)]/(4*Sqrt[2]*b^(3/4)*c^(5/4)) - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^2}\,dx = \int \frac{x^{3/2}}{(b+cx^2)^2}\,dx$$
$$= -\frac{\sqrt{x}}{2c(b+cx^2)} + \frac{\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{4c}$$
$$= -\frac{\sqrt{x}}{2c(b+cx^2)} + \frac{\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c}$$
$$= -\frac{\sqrt{x}}{2c(b+cx^2)} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{b}\,c} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right.}{4\sqrt{b}\,c}$$
$$= -\frac{\sqrt{x}}{2c(b+cx^2)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{b}\,c^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}\right.}{8\sqrt{b}\,c^{3/}}$$
$$= -\frac{\sqrt{x}}{2c(b+cx^2)} - \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{3/4}c^{5/4}} + \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{8\sqrt{2}\,b^{3/4}c^{5/4}}$$
$$= -\frac{\sqrt{x}}{2c(b+cx^2)} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{3/4}c^{5/4}} - \frac{\log\left(\sqrt{b}\right.}{}$$

Mathematica [A]
time = 0.25, size = 127, normalized size = 0.58

$$\frac{-\frac{4\sqrt[4]{c}\sqrt{x}}{b+cx^2} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{b^{3/4}} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{b^{3/4}}}{8c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*c^(1/4)*Sqrt[x])/(b + c*x^2) - (Sqrt[2]*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(3/4) + (Sqrt[2]*ArcTanh[(Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(3/4))/(8*c^(5/4))
```

Maple [A]
time = 0.12, size = 127, normalized size = 0.58

method	result
derivativedivides	$-\frac{\sqrt{x}}{2c(c\,x^{2}+b)}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right)\right)}{16cb}$
default	$-\frac{\sqrt{x}}{2c(c\,x^{2}+b)}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right)\right)}{16cb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(1/2)/c/(c*x^2+b)+1/16/c*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^
(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*a
rctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1
))
```

Maxima [A]
time = 0.52, size = 195, normalized size = 0.89

$$\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,c}-\frac{\sqrt{x}}{2\,(c^{2}x^{2}+bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt
(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c - 1
/2*sqrt(x)/(c^2*x^2 + b*c)
```

Fricas [A]
time = 0.37, size = 187, normalized size = 0.86

$$\frac{4\,(c^{2}x^{2}+bc)\left(-\frac{1}{b^{3}c^{5}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^{2}c^{2}\sqrt{-\frac{1}{b^{3}c^{5}}}+x}\;b^{2}c^{4}\left(-\frac{1}{b^{3}c^{5}}\right)^{\frac{3}{4}}-b^{2}c^{4}\sqrt{x}\left(-\frac{1}{b^{3}c^{5}}\right)^{\frac{3}{4}}\right)+(c^{2}x^{2}+bc)\left(-\frac{1}{b^{3}c^{5}}\right)^{\frac{1}{4}}\log\left(bc\left(-\frac{1}{b^{3}c^{5}}\right)^{\frac{1}{4}}+\sqrt{x}\right)-(c^{2}x^{2}+bc)\left(-\frac{1}{b^{3}c^{5}}\right)^{\frac{1}{4}}\log\left(-bc\left(-\frac{1}{b^{3}c^{5}}\right)^{\frac{1}{4}}+\sqrt{x}\right)-4\sqrt{x}}{8\,(c^{2}x^{2}+bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*arctan(sqrt(b^2*c^2*sqrt(-1/(b^
3*c^5)) + x)*b^2*c^4*(-1/(b^3*c^5))^(3/4) - b^2*c^4*sqrt(x)*(-1/(b^3*c^5))^
(3/4)) + (c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log(b*c*(-1/(b^3*c^5))^(1/4)
+ sqrt(x)) - (c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log(-b*c*(-1/(b^3*c^5))^(
1/4) + sqrt(x)) - 4*sqrt(x))/(c^2*x^2 + b*c)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.12, size = 199, normalized size = 0.91

$$\frac{\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^2}+\frac{\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^2}+\frac{\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^2}-\frac{\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^2}-\frac{\sqrt{x}}{2\,(cx^2+b)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b*c^2) + 1/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) + 1/16*sqrt(2)*(b*c^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2) - 1/16*sqrt
(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2)
 - 1/2*sqrt(x)/((c*x^2 + b)*c)
```

Mupad [**B**]
time = 4.31, size = 64, normalized size = 0.29

$$-\frac{\sqrt{x}}{2\,c\,\left(c\,x^2+b\right)}-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{3/4}\,c^{5/4}}-\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{3/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - x^(1/2)/(2*c*(b + c*x^2)) - atan((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(3
/4)*c^(5/4)) - atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(3/4)*c^(5/4))
```

3.332 $\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=218

$$\frac{x^{3/2}}{2b\left(b+cx^2\right)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}}$$

```
[Out] 1/2*x^(3/2)/b/(c*x^2+b)-1/8*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/
4)/c^(3/4)*2^(1/2)+1/8*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^
(3/4)*2^(1/2)+1/16*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^
(5/4)/c^(3/4)*2^(1/2)-1/16*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(
1/2))/b^(5/4)/c^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}}-\frac{\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^(3/2)/(2*b*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(
4*Sqrt[2]*b^(5/4)*c^(3/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/
(4*Sqrt[2]*b^(5/4)*c^(3/4)) + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx = \int \frac{\sqrt{x}}{(b+cx^2)^2}\,dx$$
$$= \frac{x^{3/2}}{2b(b+cx^2)} + \frac{\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b}$$
$$= \frac{x^{3/2}}{2b(b+cx^2)} + \frac{\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b}$$
$$= \frac{x^{3/2}}{2b(b+cx^2)} - \frac{\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b\sqrt{c}}$$
$$= \frac{x^{3/2}}{2b(b+cx^2)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8bc} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\right.}{8bc}$$
$$= \frac{x^{3/2}}{2b(b+cx^2)} + \frac{\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{5/4}c^{3/4}} - \frac{\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{8\sqrt{2}\,b^{5/4}c^{3/4}}$$
$$= \frac{x^{3/2}}{2b(b+cx^2)} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{5/4}c^{3/4}} + \frac{\log\left(\sqrt{b}-\right.}{}$$

Mathematica [A]
time = 0.24, size = 128, normalized size = 0.59

$$\frac{\frac{4\sqrt[4]{b}\,x^{3/2}}{b+cx^2}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{c^{3/4}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{c^{3/4}}}{8b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*b^(1/4)*x^(3/2))/(b + c*x^2) - (Sqrt[2]*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(3/4) - (Sqrt[2]*ArcTanh[(Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(3/4))/(8*b^(5/4))
```

Maple [A]

time = 0.09, size = 127, normalized size = 0.58

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{2b(c\,x^{2}+b)}+\frac{\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{x^{\frac{3}{2}}}{2b(c\,x^{2}+b)}+\frac{\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(3/2)/b/(c*x^2+b)+1/16/b/c/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(
1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
)
```

Maxima [A]

time = 0.50, size = 194, normalized size = 0.89

$$\frac{x^{\frac{3}{2}}}{2\,(bcx^{2}+b^{2})}+\frac{\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^(3/2)/(b*c*x^2 + b^2) + 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b
^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sq
rt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) -
2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) -
 sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4
)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqr
t(b))/(b^(1/4)*c^(3/4)))/b
```

Fricas [A]

time = 0.36, size = 182, normalized size = 0.83

$$-\frac{4\,(bcx^{2}+b^{2})\left(-\frac{1}{b^{5}c^{3}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-b^{3}c\sqrt{-\frac{1}{b^{5}c^{3}}}+x}\,bc\left(-\frac{1}{b^{5}c^{3}}\right)^{\frac{1}{4}}-bc\sqrt{x}\left(-\frac{1}{b^{5}c^{3}}\right)^{\frac{1}{4}}\right)-(bcx^{2}+b^{2})\left(-\frac{1}{b^{5}c^{3}}\right)^{\frac{1}{4}}\log\left(b^{4}c^{2}\left(-\frac{1}{b^{5}c^{3}}\right)^{\frac{3}{4}}+\sqrt{x}\right)+(bcx^{2}+b^{2})\left(-\frac{1}{b^{5}c^{3}}\right)^{\frac{1}{4}}\log\left(-b^{4}c^{2}\left(-\frac{1}{b^{5}c^{3}}\right)^{\frac{3}{4}}+\sqrt{x}\right)-4\,x^{\frac{3}{2}}}{8\,(bcx^{2}+b^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*arctan(sqrt(-b^3*c*sqrt(-1/(b^
5*c^3)) + x)*b*c*(-1/(b^5*c^3))^(1/4) - b*c*sqrt(x)*(-1/(b^5*c^3))^(1/4)) -
 (b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*log(b^4*c^2*(-1/(b^5*c^3))^(3/4) + sq
rt(x)) + (b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*log(-b^4*c^2*(-1/(b^5*c^3))^(
3/4) + sqrt(x)) - 4*x^(3/2))/(b*c*x^2 + b^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.88, size = 199, normalized size = 0.91

$$\frac{x^{\frac{3}{2}}}{2\,(cx^2+b)b}+\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^3}+\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^3}-\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^3}+\frac{\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^(3/2)/((c*x^2 + b)*b) + 1/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/8*sqrt(2)*(b*c
^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4)
)/(b^2*c^3) - 1/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) +
x + sqrt(b/c))/(b^2*c^3) + 1/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3)
```

Mupad [**B**]
time = 4.34, size = 64, normalized size = 0.29

$$\frac{x^{3/2}}{2\,b\,\left(c\,x^2+b\right)}-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{5/4}\,c^{3/4}}+\frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{5/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(3/2)/(2*b*(b + c*x^2)) - atan((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(5/4
)*c^(3/4)) + atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(5/4)*c^(3/4))
```

3.333 $\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$\frac{\sqrt{x}}{2b\,(b+cx^2)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}}-\frac{3\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\right.}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}$$

```
[Out] -3/8*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(1/4)*2^(1/2)+3/8*
arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(1/4)*2^(1/2)-3/16*ln(b
^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2^(1/2)+3
/16*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2
^(1/2)+1/2*x^(1/2)/b/(c*x^2+b)
```

Rubi [A]
time = 0.11, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{7/4}\sqrt[4]{c}}-\frac{3\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{7/4}\sqrt[4]{c}}+\frac{\sqrt{x}}{2b\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] Sqrt[x]/(2*b*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) - (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*Log[Sqrt[b] + Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx = \int \frac{1}{\sqrt{x}\,(b+cx^2)^2}\,dx$$
$$= \frac{\sqrt{x}}{2b(b+cx^2)} + \frac{3\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{4b}$$
$$= \frac{\sqrt{x}}{2b(b+cx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b}$$
$$= \frac{\sqrt{x}}{2b(b+cx^2)} + \frac{3\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right.}{4b^{3/2}}$$
$$= \frac{\sqrt{x}}{2b(b+cx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{3/2}\sqrt{c}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}}\right.}{8b^{3/2}\sqrt{c}}$$
$$= \frac{\sqrt{x}}{2b(b+cx^2)} - \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{\right.}}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}$$
$$= \frac{\sqrt{x}}{2b(b+cx^2)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\right.}{}$$

Mathematica [A]
time = 0.23, size = 128, normalized size = 0.59

$$\frac{\frac{4b^{3/4}\sqrt{x}}{b+cx^2} - \frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt[4]{c}}}{8b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*b^(3/4)*Sqrt[x])/(b + c*x^2) - (3*Sqrt[2]*ArcTan[(Sqrt[b] - Sqrt[c]*x)/
(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(1/4) + (3*Sqrt[2]*ArcTanh[(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(1/4))/(8*b^(7/4))
```

Maple [A]
time = 0.12, size = 124, normalized size = 0.57

method	result
derivativedivides	$\frac{\sqrt{x}}{2b(c\,x^2+b)}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.\right.}{16b^2}$
default	$\frac{\sqrt{x}}{2b(c\,x^2+b)}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.\right.}{16b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/2)/b/(c*x^2+b)+3/16/b^2*(b/c)^(1/4)*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(
1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
)
```

Maxima [A]
time = 0.51, size = 194, normalized size = 0.89

$$\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{16\,b}+\frac{\sqrt{x}}{2\,(bcx^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt
(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b + 1
/2*sqrt(x)/(b*c*x^2 + b^2)
```

Fricas [A]
time = 0.35, size = 179, normalized size = 0.82

$$\frac{12\,(bcx^2+b^2)\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^4\sqrt{-\frac{1}{b^7c}}+x}\,b^5c\left(-\frac{1}{b^7c}\right)^{\frac{3}{4}}-b^5c\sqrt{x}\left(-\frac{1}{b^7c}\right)^{\frac{3}{4}}\right)+3\,(bcx^2+b^2)\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\,(bcx^2+b^2)\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}\log\left(-b^2\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\sqrt{x}}{8\,(bcx^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(12*(b*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*arctan(sqrt(b^4*sqrt(-1/(b^7*c))
 + x)*b^5*c*(-1/(b^7*c))^(3/4) - b^5*c*sqrt(x)*(-1/(b^7*c))^(3/4)) + 3*(b*c
*x^2 + b^2)*(-1/(b^7*c))^(1/4)*log(b^2*(-1/(b^7*c))^(1/4) + sqrt(x)) - 3*(b
*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*log(-b^2*(-1/(b^7*c))^(1/4) + sqrt(x)) + 4
*sqrt(x))/(b*c*x^2 + b^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.58, size = 199, normalized size = 0.91

$$\frac{3\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c}+\frac{3\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c}+\frac{3\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c}-\frac{3\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c}+\frac{\sqrt{x}}{2\,(cx^2+b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(
x))/(b/c)^(1/4))/(b^2*c) + 3/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 3/16*sqrt(2)*(b*c^3)
^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c) - 3/16*sqrt
(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c)
 + 1/2*sqrt(x)/((c*x^2 + b)*b)
```

Mupad [**B**]
time = 0.10, size = 64, normalized size = 0.29

$$\frac{\sqrt{x}}{2\,b\,\left(c\,x^2+b\right)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{7/4}\,c^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{7/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(1/2)/(2*b*(b + c*x^2)) + (3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^
(7/4)*c^(1/4)) + (3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(7/4)*c^(1
/4))
```

3.334 $\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{5}{2b^2\sqrt{x}}+\frac{1}{2b\sqrt{x}\,(b+cx^2)}+\frac{5\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{c}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{c}\,\log\left(\right.}{}$$

```
[Out] 5/8*c^(1/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)*2^(1/2)-5/8*c
^(1/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)*2^(1/2)-5/16*c^(1/
4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)+5/
16*c^(1/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^
(1/2)-5/2/b^2/x^(1/2)+1/2/b/(c*x^2+b)/x^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{5\sqrt[4]{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{c}\,\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{c}\,\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}}-\frac{5}{2b^2\sqrt{x}}+\frac{1}{2b\sqrt{x}\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -5/(2*b^2*Sqrt[x]) + 1/(2*b*Sqrt[x]*(b + c*x^2)) + (5*c^(1/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*c^(1/4)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) - (5*c^(1/4)*Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)
) + (5*c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(8*Sqrt[2]*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx = \int \frac{1}{x^{3/2}(b+cx^2)^2}\,dx$$

$$= \frac{1}{2b\sqrt{x}\,(b+cx^2)} + \frac{5\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{4b}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\,(b+cx^2)} - \frac{(5c)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\,(b+cx^2)} - \frac{(5c)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\,(b+cx^2)} + \frac{(5\sqrt{c})\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^2} - \frac{(5\sqrt{c})\,\text{Su}}{}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\,(b+cx^2)} - \frac{5\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{8b^2} - \frac{5\text{Subst}\Big(}{}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\,(b+cx^2)} - \frac{5\sqrt[4]{c}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{9/4}} + \frac{5\sqrt[4]{c}\,\text{log}}{}$$

$$= -\frac{5}{2b^2\sqrt{x}} + \frac{1}{2b\sqrt{x}\,(b+cx^2)} + \frac{5\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{9/4}} - \frac{5\sqrt[4]{c}\,\tan^{-1}\Big(1+\sqrt{}}{4\sqrt{2}\,b^{9/}}$$

Mathematica [A]
time = 0.28, size = 138, normalized size = 0.60

$$\frac{-\frac{4\sqrt[4]{b}\,(4b+5cx^2)}{\sqrt{x}\,(b+cx^2)} + 5\sqrt{2}\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + 5\sqrt{2}\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{8b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*b^(1/4)*(4*b + 5*c*x^2))/(Sqrt[x]*(b + c*x^2)) + 5*Sqrt[2]*c^(1/4)*Arc
Tan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt[2]*c^
(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(8*
b^(9/4))
```

Maple [A]
time = 0.14, size = 136, normalized size = 0.59

method	result
derivativedivides	$-\frac{2}{b^{2}\sqrt{x}}-\frac{2c\left(\frac{x^{\frac{3}{2}}}{4cx^{2}+4b}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{2}}$
default	$-\frac{2}{b^{2}\sqrt{x}}-\frac{2c\left(\frac{x^{\frac{3}{2}}}{4cx^{2}+4b}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{2}}$
risch	$-\frac{2}{b^{2}\sqrt{x}}-\frac{c\,x^{\frac{3}{2}}}{2b^{2}\left(cx^{2}+b\right)}-\frac{5\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{5}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^2/x^(1/2)-2*c/b^2*(1/4*x^(3/2)/(c*x^2+b)+5/32/c/(b/c)^(1/4)*2^(1/2)*(l
n((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2
)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.51, size = 208, normalized size = 0.90

$$-\frac{5\,cx^{2}+4\,b}{2\,\left(b^{2}cx^{\frac{5}{2}}+b^{3}\sqrt{x}\right)}-\frac{5\,c\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(5*c*x^2 + 4*b)/(b^2*c*x^(5/2) + b^3*sqrt(x)) - 5/16*c*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sq
rt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqr
t(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*
sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^2
```

Fricas [A]
time = 0.37, size = 208, normalized size = 0.90

$$\frac{20\,\left(b^{2}cx^{3}+b^{3}x\right)\left(-\frac{c}{b^{9}}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,b^{2}c\sqrt{x}\left(-\frac{c}{b^{9}}\right)^{\frac{1}{4}}-\sqrt{-15625\,b^{5}c\sqrt{-\frac{c}{b^{9}}}+15625\,c^{2}x}\,b^{2}\left(-\frac{c}{b^{9}}\right)^{\frac{1}{4}}}{125\,c}\right)-5\,\left(b^{2}cx^{3}+b^{3}x\right)\left(-\frac{c}{b^{9}}\right)^{\frac{1}{4}}\log\left(125\,b^{7}\left(-\frac{c}{b^{9}}\right)^{\frac{3}{4}}+125\,c\sqrt{x}\right)+5\,\left(b^{2}cx^{3}+b^{3}x\right)\left(-\frac{c}{b^{9}}\right)^{\frac{1}{4}}\log\left(-125\,b^{7}\left(-\frac{c}{b^{9}}\right)^{\frac{3}{4}}+125\,c\sqrt{x}\right)-4\,\left(5\,cx^{2}+4\,b\right)\sqrt{x}}{8\,\left(b^{2}cx^{3}+b^{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(20*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*arctan(-1/125*(125*b^2*c*sqrt(x)
*(-c/b^9)^(1/4) - sqrt(-15625*b^5*c*sqrt(-c/b^9) + 15625*c^2*x)*b^2*(-c/b^9
)^(1/4))/c) - 5*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*log(125*b^7*(-c/b^9)^(3/
4) + 125*c*sqrt(x)) + 5*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*log(-125*b^7*(-c
/b^9)^(3/4) + 125*c*sqrt(x)) - 4*(5*c*x^2 + 4*b)*sqrt(x))/(b^2*c*x^3 + b^3*
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.45, size = 210, normalized size = 0.91

$$-\frac{5\,cx^2+4\,b}{2\,\left(cx^{\frac{5}{2}}+b\sqrt{x}\right)b^2}-\frac{5\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^2}-\frac{5\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^2}+\frac{5\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c^2}-\frac{5\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*c*x^2 + 4*b)/((c*x^(5/2) + b*sqrt(x))*b^2) - 5/8*sqrt(2)*(b*c^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*
c^2) - 5/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) -
 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 5/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 5/16*sqrt(2)*(b*c^3)^(3/4
)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2)
```

Mupad [B]
time = 0.09, size = 77, normalized size = 0.33

$$\frac{5\,{(-c)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{9/4}}-\frac{5\,{(-c)}^{1/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{9/4}}-\frac{\frac{2}{b}+\frac{5\,c\,x^2}{2\,b^2}}{b\,\sqrt{x}+c\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (5*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(9/4)) - (5*(-c)^(1
/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(9/4)) - (2/b + (5*c*x^2)/(2*b
^2))/(b*x^(1/2) + c*x^(5/2))
```

3.335 $\int \frac{x^{3/2}}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=230

$$-\frac{7}{6b^2x^{3/2}}+\frac{1}{2bx^{3/2}(b+cx^2)}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}}+\frac{7c^{3/4}\log\left(\surd\right.}{}$$

```
[Out] -7/6/b^2/x^(3/2)+1/2/b/x^(3/2)/(c*x^2+b)+7/8*c^(3/4)*arctan(1-c^(1/4)*2^(1/
2)*x^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)-7/8*c^(3/4)*arctan(1+c^(1/4)*2^(1/2)*x
^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)+7/16*c^(3/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^(1/2)-7/16*c^(3/4)*ln(b^(1/2)+x*c^(1/2)
+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{7c^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7c^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{11/4}}+\frac{7c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}}-\frac{7}{6b^2x^{3/2}}+\frac{1}{2bx^{3/2}(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -7/(6*b^2*x^(3/2)) + 1/(2*b*x^(3/2)*(b + c*x^2)) + (7*c^(3/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4)) - (7*c^(3/4)*ArcTan[
1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4)) + (7*c^(3/4)*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11
/4)) - (7*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(8*Sqrt[2]*b^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^2}\,dx = \int \frac{1}{x^{5/2}(b+cx^2)^2}\,dx$$

$$= \frac{1}{2bx^{3/2}(b+cx^2)} + \frac{7\int \frac{1}{x^{5/2}(b+cx^2)}\,dx}{4b}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}(b+cx^2)} - \frac{(7c)\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{4b^2}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}(b+cx^2)} - \frac{(7c)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}(b+cx^2)} - \frac{(7c)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{5/2}} - \frac{(7c)\text{Subst}\left(\int \cdots\right.}{}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}(b+cx^2)} - \frac{(7\sqrt{c})\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{8b^{5/2}} - \frac{(7\cdots}{}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}(b+cx^2)} + \frac{7c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{11/4}} - \frac{7c^{3/4}\log\cdots}{}$$

$$= -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}(b+cx^2)} + \frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{11/4}} - \frac{7c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\cdots}{}\right)}{4\sqrt{2}\,b^{11/4}}$$

Mathematica [A]
time = 0.29, size = 138, normalized size = 0.60

$$\frac{-\frac{4b^{3/4}(4b+7cx^2)}{x^{3/2}(b+cx^2)} + 21\sqrt{2}\,c^{3/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right) - 21\sqrt{2}\,c^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{24b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*b^(3/4)*(4*b + 7*c*x^2))/(x^(3/2)*(b + c*x^2)) + 21*Sqrt[2]*c^(3/4)*Ar
cTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 21*Sqrt[2]*
c^(3/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(
24*b^(11/4))
```

Maple [A]

time = 0.14, size = 136, normalized size = 0.59

method	result
derivativedivides	$-\frac{2c\left(\frac{\sqrt{x}}{4c\,x^2+4b}+\frac{7\left(\frac{b}{c}\right)^{\frac14}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac14}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac14}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac14}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac14}}-1\right)\right)}{32b}\right)}{b^2}$
default	$-\frac{2c\left(\frac{\sqrt{x}}{4c\,x^2+4b}+\frac{7\left(\frac{b}{c}\right)^{\frac14}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac14}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac14}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac14}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac14}}-1\right)\right)}{32b}\right)}{b^2}$
risch	$-\frac{2}{3b^2x^{\frac32}}-\frac{c\sqrt{x}}{2b^2(c\,x^2+b)}-\frac{7c\left(\frac{b}{c}\right)^{\frac14}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac14}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac14}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16b^3}-\frac{7c\left(\frac{b}{c}\right)^{\frac14}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac14}}\right.}{8b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*c/b^2*(1/4*x^(1/2)/(c*x^2+b)+7/32*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/
4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))
)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1
/2)-1)))-2/3/b^2/x^(3/2)
```

Maxima [A]

time = 0.49, size = 209, normalized size = 0.91

$$-\frac{7\,cx^2+4\,b}{6\left(b^2cx^{\frac72}+b^3x^{\frac32}\right)}-\frac{7\left(\frac{2\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac14}c^{\frac14}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac14}c^{\frac14}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac34}\log\left(\sqrt{2}\,b^{\frac14}c^{\frac14}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac34}}-\frac{\sqrt{2}\,c^{\frac34}\log\left(-\sqrt{2}\,b^{\frac14}c^{\frac14}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac34}}\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(7*c*x^2 + 4*b)/(b^2*c*x^(7/2) + b^3*x^(3/2)) - 7/16*(2*sqrt(2)*c*arct
an(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*c*arctan(-1/2*sqrt(2)*
(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(
b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(3/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqr
t(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(3/4)*log(-sqrt(2)*b^(1/4)*
c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^2
```

Fricas [A]
time = 0.37, size = 228, normalized size = 0.99

$$-\frac{84\,(b^2cx^4+b^3x^2)\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^8c\sqrt{x}\left(-\frac{c^3}{b^{11}}\right)^{\frac{3}{4}}-\sqrt{b^6\sqrt{-\frac{c^3}{b^{11}}}+c^2x}\,b^8\left(-\frac{c^3}{b^{11}}\right)^{\frac{3}{4}}}{c^3}\right)+21\,(b^2cx^4+b^3x^2)\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}\log\left(7\,b^3\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}+7\,c\sqrt{x}\right)-21\,(b^2cx^4+b^3x^2)\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}\log\left(-7\,b^3\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}+7\,c\sqrt{x}\right)+4\,(7\,cx^2+4\,b)\sqrt{x}}{24\,(b^2cx^4+b^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(84*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*arctan(-(b^8*c*sqrt(x)*(-
c^3/b^11)^(3/4) - sqrt(b^6*sqrt(-c^3/b^11) + c^2*x)*b^8*(-c^3/b^11)^(3/4))/
c^3) + 21*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*log(7*b^3*(-c^3/b^11)^(1/
4) + 7*c*sqrt(x)) - 21*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*log(-7*b^3*(
-c^3/b^11)^(1/4) + 7*c*sqrt(x)) + 4*(7*c*x^2 + 4*b)*sqrt(x))/(b^2*c*x^4 + b
^3*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.61, size = 196, normalized size = 0.85

$$-\frac{7\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3}-\frac{7\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3}-\frac{7\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3}+\frac{7\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3}-\frac{c\sqrt{x}}{2\,(cx^2+b)b^2}-\frac{2}{3\,b^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/b^3 - 7/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^3 - 7/16*sqrt(2)*(b*c^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 7/16*sqrt(2)*(b*c^3)
^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 - 1/2*c*sqrt(x
)/((c*x^2 + b)*b^2) - 2/3/(b^2*x^(3/2))
```

Mupad [B]
time = 0.11, size = 77, normalized size = 0.33

$$\frac{7\,{\left(-c\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{11/4}}-\frac{\frac{2}{3\,b}+\frac{7\,c\,x^2}{6\,b^2}}{b\,x^{3/2}+c\,x^{7/2}}+\frac{7\,{\left(-c\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (7*(-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(11/4)) - (2/(3*b) +
 (7*c*x^2)/(6*b^2))/(b*x^(3/2) + c*x^(7/2)) + (7*(-c)^(3/4)*atanh(((-c)^(1/
4)*x^(1/2))/b^(1/4)))/(4*b^(11/4))
```

3.336 $\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=243

$$-\frac{9}{10b^2x^{5/2}}+\frac{9c}{2b^3\sqrt{x}}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}-\frac{9c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{9c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{13/4}}+$$

```
[Out] -9/10/b^2/x^(5/2)+1/2/b/x^(5/2)/(c*x^2+b)-9/8*c^(5/4)*arctan(1-c^(1/4)*2^(1
/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+9/8*c^(5/4)*arctan(1+c^(1/4)*2^(1/2)*
x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+9/16*c^(5/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)
*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)-9/16*c^(5/4)*ln(b^(1/2)+x*c^(1/2
)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+9/2*c/b^3/x^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{9c^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{9c^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{13/4}}+\frac{9c^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{13/4}}-\frac{9c^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{9c}{2b^3\sqrt{x}}-\frac{9}{10b^2x^{5/2}}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -9/(10*b^2*x^(5/2)) + (9*c)/(2*b^3*Sqrt[x]) + 1/(2*b*x^(5/2)*(b + c*x^2)) -
 (9*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13
/4)) + (9*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]
*b^(13/4)) + (9*c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(8*Sqrt[2]*b^(13/4)) - (9*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2}\,dx &= \int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{9\int \frac{1}{x^{7/2}(b+cx^2)}\,dx}{4b} \\
&= -\frac{9}{10b^2x^{5/2}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{(9c)\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{4b^2} \\
&= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{\left(9c^2\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^3} \\
&= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{\left(9c^2\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2b^3} \\
&= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{\left(9c^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^3} + \\
&= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{(9c)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right.}{8b^3} \\
&= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} + \frac{9c^{5/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{13/4}} \\
&= -\frac{9}{10b^2x^{5/2}} + \frac{9c}{2b^3\sqrt{x}} + \frac{1}{2bx^{5/2}\left(b+cx^2\right)} - \frac{9c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{13/4}} + \frac{9c^{5/4}\tan}{}
\end{aligned}$$

Mathematica [A]

time = 0.31, size = 149, normalized size = 0.61

$$\frac{\frac{4\sqrt[4]{b}\,(-4b^2+36bcx^2+45c^2x^4)}{x^{5/2}(b+cx^2)} - 45\sqrt{2}\,c^{5/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right) - 45\sqrt{2}\,c^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{40b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*b^(1/4)*(-4*b^2 + 36*b*c*x^2 + 45*c^2*x^4))/(x^(5/2)*(b + c*x^2)) - 45*
Sqrt[2]*c^(5/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x])] - 45*Sqrt[2]*c^(5/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b
] + Sqrt[c]*x)])/(40*b^(13/4))
```

Maple [A]
time = 0.14, size = 147, normalized size = 0.60

method	result
derivativedivides	$\frac{2c^2\left(\frac{x^{\frac{3}{2}}}{4c\,x^2+4b}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^3} -$
default	$\frac{2c^2\left(\frac{x^{\frac{3}{2}}}{4c\,x^2+4b}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^3} -$
risch	$-\frac{2(-10c\,x^2+b)}{5b^3x^{\frac{5}{2}}}+\frac{c^2x^{\frac{3}{2}}}{2b^3(c\,x^2+b)}+\frac{9c\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\frac{9c\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)}{8b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^2/b^3*(1/4*x^(3/2)/(c*x^2+b)+9/32/c/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1
/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)
))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(
1/2)-1)))-2/5/b^2/x^(5/2)+4*c/b^3/x^(1/2)
```

Maxima [A]
time = 0.51, size = 221, normalized size = 0.91

$$\frac{45\,c^2x^4+36\,bcx^2-4\,b^2}{10\left(b^3cx^{\frac{9}{2}}+b^4x^{\frac{5}{2}}\right)}+\frac{9\,c^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/10*(45*c^2*x^4 + 36*b*c*x^2 - 4*b^2)/(b^3*c*x^(9/2) + b^4*x^(5/2)) + 9/16
*c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt
(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^3
```

Fricas [A]
time = 0.37, size = 251, normalized size = 1.03

$$-\frac{180\left(b^3cx^5+b^4x^3\right)\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729\,b^3c^4\sqrt{x}\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}-\sqrt{-531441\,b^7c^5\sqrt{-\frac{c^5}{b^{13}}}+531441\,c^8x}\,b^3\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}}{729\,c^5}\right)-45\left(b^3cx^5+b^4x^3\right)\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}\log\left(729\,b^{10}\left(-\frac{c^5}{b^{13}}\right)^{\frac{3}{4}}+729\,c^4\sqrt{x}\right)+45\left(b^3cx^5+b^4x^3\right)\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}\log\left(-729\,b^{10}\left(-\frac{c^5}{b^{13}}\right)^{\frac{3}{4}}+729\,c^4\sqrt{x}\right)-4\left(45\,c^2x^4+36\,bcx^2-4\,b^2\right)\sqrt{x}}{40\left(b^3cx^5+b^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(180*(b^3*c*x^5 + b^4*x^3)*(-c^5/b^13)^(1/4)*arctan(-1/729*(729*b^3*c
^4*sqrt(x)*(-c^5/b^13)^(1/4) - sqrt(-531441*b^7*c^5*sqrt(-c^5/b^13) + 53144
1*c^8*x)*b^3*(-c^5/b^13)^(1/4))/c^5) - 45*(b^3*c*x^5 + b^4*x^3)*(-c^5/b^13)
^(1/4)*log(729*b^10*(-c^5/b^13)^(3/4) + 729*c^4*sqrt(x)) + 45*(b^3*c*x^5 +
b^4*x^3)*(-c^5/b^13)^(1/4)*log(-729*b^10*(-c^5/b^13)^(3/4) + 729*c^4*sqrt(x
)) - 4*(45*c^2*x^4 + 36*b*c*x^2 - 4*b^2)*sqrt(x))/(b^3*c*x^5 + b^4*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.18, size = 220, normalized size = 0.91

$$\frac{c^2x^{\frac{3}{2}}}{2\,(cx^2+b)b^3}+\frac{9\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c}+\frac{9\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c}-\frac{9\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c}+\frac{9\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c}+\frac{2\,(10\,cx^2-b)}{5\,b^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^(3/2)/((c*x^2 + b)*b^3) + 9/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 9/8*sqrt(2)*
(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b^4*c) - 9/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4)
+ x + sqrt(b/c))/(b^4*c) + 9/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) + 2/5*(10*c*x^2 - b)/(b^3*x^(5/2))
```

Mupad [B]
time = 4.37, size = 87, normalized size = 0.36

$$\frac{\frac{18\,c\,x^2}{5\,b^2}-\frac{2}{5\,b}+\frac{9\,c^2\,x^4}{2\,b^3}}{b\,x^{5/2}+c\,x^{9/2}}-\frac{9\,{\left(-c\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{13/4}}+\frac{9\,{\left(-c\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] ((18*c*x^2)/(5*b^2) - 2/(5*b) + (9*c^2*x^4)/(2*b^3))/(b*x^(5/2) + c*x^(9/2)
) - (9*(-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(13/4)) + (9*(-c
)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(13/4))
```

3.337 $\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=243

$$-\frac{11}{14b^2x^{7/2}}+\frac{11c}{6b^3x^{3/2}}+\frac{1}{2bx^{7/2}\,(b+cx^2)}-\frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}}+\frac{11c^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}}$$

```
[Out] -11/14/b^2/x^(7/2)+11/6*c/b^3/x^(3/2)+1/2/b/x^(7/2)/(c*x^2+b)-11/8*c^(7/4)*
arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)+11/8*c^(7/4)*arc
tan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)-11/16*c^(7/4)*ln(b^
(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)+11/16*c^(
7/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{11c^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}}+\frac{11c^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{15/4}}-\frac{11c^{7/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}+\frac{11c^{7/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{15/4}}+\frac{11c}{6b^3x^{3/2}}-\frac{11}{14b^2x^{7/2}}+\frac{1}{2bx^{7/2}\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -11/(14*b^2*x^(7/2)) + (11*c)/(6*b^3*x^(3/2)) + 1/(2*b*x^(7/2)*(b + c*x^2))
 - (11*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^
(15/4)) + (11*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqr
t[2]*b^(15/4)) - (11*c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4)) + (11*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{9/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{11\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{4b}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{(11c)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^3}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^3}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^2\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{7/2}}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} + \frac{\left(11c^{3/2}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,d\right.}{8b^{7/2}}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{11c^{7/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\right.}{8\sqrt{2}\,b^{15/4}}$$

$$= -\frac{11}{14b^2x^{7/2}} + \frac{11c}{6b^3x^{3/2}} + \frac{1}{2bx^{7/2}\left(b+cx^2\right)} - \frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{15/4}} + \frac{11c}{}$$

Mathematica [A]

time = 0.31, size = 149, normalized size = 0.61

$$\frac{\frac{4b^{3/4}(-12b^2+44bcx^2+77c^2x^4)}{x^{7/2}(b+cx^2)} - 231\sqrt{2}\,c^{7/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right) + 231\sqrt{2}\,c^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{168b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] ((4*b^(3/4)*(-12*b^2 + 44*b*c*x^2 + 77*c^2*x^4))/(x^(7/2)*(b + c*x^2)) - 23
1*Sqrt[2]*c^(7/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x])] + 231*Sqrt[2]*c^(7/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqr
t[b] + Sqrt[c]*x)])/(168*b^(15/4))
```

Maple [A]
time = 0.13, size = 147, normalized size = 0.60

method	result
derivativedivides	$\frac{2c^2\left(\frac{\sqrt{x}}{4cx^2+4b}+\frac{11\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^3}$
default	$\frac{2c^2\left(\frac{\sqrt{x}}{4cx^2+4b}+\frac{11\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^3}$
risch	$-\frac{2(-14cx^2+3b)}{21b^3x^{\frac{7}{2}}}+\frac{c^2\sqrt{x}}{2b^3(cx^2+b)}+\frac{11c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16b^4}+\frac{11c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\mathrm{arctan}}{8b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^2/b^3*(1/4*x^(1/2)/(c*x^2+b)+11/32*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(
1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2
)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^
(1/2)-1)))-2/7/b^2/x^(7/2)+4/3*c/b^3/x^(3/2)
```

Maxima [A]
time = 0.54, size = 224, normalized size = 0.92

$$\frac{77 c^2 x^4 + 44 bcx^2 - 12 b^2}{42 \left(b^3 c x^{\frac{11}{2}} + b^4 x^{\frac{7}{2}}\right)} + \frac{11\left(\frac{2\sqrt{2}\, c^2 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\, b^{\frac{1}{4}} c^{\frac{1}{4}} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}\, c^2 \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\, b^{\frac{1}{4}} c^{\frac{1}{4}} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}\, c^{\frac{7}{4}} \log\left(\sqrt{2}\, b^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x} + \sqrt{c}\, x + \sqrt{b}\right)}{b^{\frac{3}{4}}} - \frac{\sqrt{2}\, c^{\frac{7}{4}} \log\left(-\sqrt{2}\, b^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x} + \sqrt{c}\, x + \sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{16\, b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/42*(77*c^2*x^4 + 44*b*c*x^2 - 12*b^2)/(b^3*c*x^(11/2) + b^4*x^(7/2)) + 11
/16*(2*sqrt(2)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)
*c^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*log(sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(7
/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^
3
```

Fricas [A]
time = 0.40, size = 245, normalized size = 1.01

$$\frac{924 \left(b^3 c x^6 + b^4 x^4\right) \left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}} \arctan\left(-\frac{b^{11} c^2 \sqrt{x} \left(-\frac{c^7}{b^{15}}\right)^{\frac{3}{4}} - \sqrt{b^8 \sqrt{-\frac{c^7}{b^{15}}} + c^4 x}\, b^{11} \left(-\frac{c^7}{b^{15}}\right)^{\frac{3}{4}}}{c^7}\right) + 231 \left(b^3 c x^6 + b^4 x^4\right) \left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}} \log\left(11\, b^4 \left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}} + 11\, c^2 \sqrt{x}\right) - 231 \left(b^3 c x^6 + b^4 x^4\right) \left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}} \log\left(-11\, b^4 \left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}} + 11\, c^2 \sqrt{x}\right) + 4 \left(77 c^2 x^4 + 44 bcx^2 - 12 b^2\right) \sqrt{x}}{168 \left(b^3 c x^6 + b^4 x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/168*(924*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*arctan(-(b^11*c^2*sqrt(x
)*(-c^7/b^15)^(3/4) - sqrt(b^8*sqrt(-c^7/b^15) + c^4*x)*b^11*(-c^7/b^15)^(3
/4))/c^7) + 231*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*log(11*b^4*(-c^7/b^
15)^(1/4) + 11*c^2*sqrt(x)) - 231*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*l
og(-11*b^4*(-c^7/b^15)^(1/4) + 11*c^2*sqrt(x)) + 4*(77*c^2*x^4 + 44*b*c*x^2
 - 12*b^2)*sqrt(x))/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.15, size = 212, normalized size = 0.87

$$\frac{11\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4}+\frac{11\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4}+\frac{11\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4}-\frac{11\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4}+\frac{c^2\sqrt{x}}{2\,(cx^2+b)b^3}+\frac{2\,(14\,cx^2-3\,b)}{21\,b^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 11/8*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/b^4 + 11/8*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 + 11/16*sqrt(2)*(b*c^3)^
(1/4)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 11/16*sqrt(2
)*(b*c^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 1
/2*c^2*sqrt(x)/((c*x^2 + b)*b^3) + 2/21*(14*c*x^2 - 3*b)/(b^3*x^(7/2))
```

Mupad [B]
time = 0.11, size = 87, normalized size = 0.36

$$\frac{\frac{22\,c\,x^2}{21\,b^2}-\frac{2}{7\,b}+\frac{11\,c^2\,x^4}{6\,b^3}}{b\,x^{7/2}+c\,x^{11/2}}+\frac{11\,{(-c)}^{7/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{15/4}}+\frac{11\,{(-c)}^{7/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((22*c*x^2)/(21*b^2) - 2/(7*b) + (11*c^2*x^4)/(6*b^3))/(b*x^(7/2) + c*x^(11
/2)) + (11*(-c)^(7/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(15/4)) + (1
1*(-c)^(7/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(15/4))
```

3.338 $\int \frac{1}{x^{3/2}(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=258

$$-\frac{13}{18b^2x^{9/2}}+\frac{13c}{10b^3x^{5/2}}-\frac{13c^2}{2b^4\sqrt{x}}+\frac{1}{2bx^{9/2}(b+cx^2)}+\frac{13c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{17/4}}-\frac{13c^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{4\sqrt{2}\,b^{17/4}}$$

```
[Out] -13/18/b^2/x^(9/2)+13/10*c/b^3/x^(5/2)+1/2/b/x^(9/2)/(c*x^2+b)+13/8*c^(9/4)
*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-13/8*c^(9/4)*ar
ctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-13/16*c^(9/4)*ln(b
^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2)+13/16*c^
(9/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2
)-13/2*c^2/b^4/x^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{13c^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\,b^{17/4}}-\frac{13c^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\,b^{17/4}}-\frac{13c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{17/4}}+\frac{13c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{8\sqrt{2}\,b^{17/4}}-\frac{13c^2}{2b^4\sqrt{x}}+\frac{13c}{10b^3x^{5/2}}-\frac{13}{18b^2x^{9/2}}+\frac{1}{2bx^{9/2}(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -13/(18*b^2*x^(9/2)) + (13*c)/(10*b^3*x^(5/2)) - (13*c^2)/(2*b^4*Sqrt[x]) +
 1/(2*b*x^(9/2)*(b + c*x^2)) + (13*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt
[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(9/4)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4)) + (13*c^(9/4
)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^
(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{11/2}\left(b+cx^2\right)^2}\,dx$$

$$= \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{13\int \frac{1}{x^{11/2}(b+cx^2)}\,dx}{4b}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{(13c)\int \frac{1}{x^{7/2}(b+cx^2)}\,dx}{4b^2}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{(13c^2)\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{4b^3}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{(13c^3)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{(13c^3)\,\mathrm{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x,\right.}{2b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{(13c^{5/2})\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}}{b+cx^4}\right.}{4b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{(13c^2)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}}{\sqrt[4]{}}}\right.}{8b^4}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} - \frac{13c^{9/4}\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\right.}{8\sqrt{2}\,b^{17/4}}$$

$$= -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}\left(b+cx^2\right)} + \frac{13c^{9/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{4\sqrt{2}\,b^{17/4}}$$

Mathematica [A]
time = 0.33, size = 160, normalized size = 0.62

$$\frac{-\frac{4\sqrt[4]{b}\left(20b^3-52b^2cx^2+468bc^2x^4+585c^3x^6\right)}{x^{9/2}(b+cx^2)}+585\sqrt{2}\,c^{9/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)+585\sqrt{2}\,c^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{360b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*b^(1/4)*(20*b^3 - 52*b^2*c*x^2 + 468*b*c^2*x^4 + 585*c^3*x^6))/(x^(9/2
)*(b + c*x^2)) + 585*Sqrt[2]*c^(9/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x])] + 585*Sqrt[2]*c^(9/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(360*b^(17/4))
```

Maple [A]
time = 0.14, size = 158, normalized size = 0.61

method	result
derivativedivides	$-\frac{2c^3\left(\frac{x^{\frac{3}{2}}}{4c\,x^2+4b}+\frac{13\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^4}$
default	$-\frac{2c^3\left(\frac{x^{\frac{3}{2}}}{4c\,x^2+4b}+\frac{13\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^4}$
risch	$-\frac{2\left(135c^2x^4-18bc\,x^2+5b^2\right)}{45b^4x^{\frac{9}{2}}}-\frac{c^3x^{\frac{3}{2}}}{2b^4\left(c\,x^2+b\right)}-\frac{13c^2\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{16b^4\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{13c^2\sqrt{2}\,\arctan(}{8b^4(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*c^3/b^4*(1/4*x^(3/2)/(c*x^2+b)+13/32/c/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^
(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/
```

2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)-1)))-2/9/b^2/x^(9/2)-6*c^2/b^4/x^(1/2)+4/5*c/b^3/x^(5/2)

Maxima [A]
time = 0.51, size = 232, normalized size = 0.90

$$-\frac{585\,c^3x^6+468\,bc^2x^4-52\,b^2cx^2+20\,b^3}{90\left(b^4cx^{\frac{13}{2}}+b^5x^{\frac{9}{2}}\right)}-\frac{13\,c^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac14}c^{\frac14}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac14}c^{\frac14}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac14}c^{\frac14}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac14}c^{\frac34}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac14}c^{\frac14}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac14}c^{\frac34}}\right)}{16\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")

[Out] -1/90*(585*c^3*x^6 + 468*b*c^2*x^4 - 52*b^2*c*x^2 + 20*b^3)/(b^4*c*x^(13/2)
 + b^5*x^(9/2)) - 13/16*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*
c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*
sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(
c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2
)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/
4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(
b^(1/4)*c^(3/4)))/b^4

Fricas [A]
time = 0.38, size = 262, normalized size = 1.02

$$\frac{2340\left(b^4cx^7+b^5x^5\right)\left(-\frac{c^9}{b^{17}}\right)^{\frac14}\arctan\left(-\frac{2197\,b^4c^7\sqrt{x}\left(-\frac{c^9}{b^{17}}\right)^{\frac14}-\sqrt{-4826809\,b^9c^9\sqrt{-\frac{c^9}{b^{17}}}+4826809\,c^{14}x}\;b^4\left(-\frac{c^9}{b^{17}}\right)^{\frac14}}{2197\,c^9}\right)-585\left(b^4cx^7+b^5x^5\right)\left(-\frac{c^9}{b^{17}}\right)^{\frac14}\log\left(2197\,b^{13}\left(-\frac{c^9}{b^{17}}\right)^{\frac34}+2197\,c^7\sqrt{x}\right)+585\left(b^4cx^7+b^5x^5\right)\left(-\frac{c^9}{b^{17}}\right)^{\frac14}\log\left(-2197\,b^{13}\left(-\frac{c^9}{b^{17}}\right)^{\frac34}+2197\,c^7\sqrt{x}\right)-4\left(585\,c^3x^6+468\,bc^2x^4-52\,b^2cx^2+20\,b^3\right)\sqrt{x}}{360\left(b^4cx^7+b^5x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")

[Out] 1/360*(2340*(b^4*c*x^7 + b^5*x^5)*(-c^9/b^17)^(1/4)*arctan(-1/2197*(2197*b^
4*c^7*sqrt(x)*(-c^9/b^17)^(1/4) - sqrt(-4826809*b^9*c^9*sqrt(-c^9/b^17) + 4
826809*c^14*x)*b^4*(-c^9/b^17)^(1/4))/c^9) - 585*(b^4*c*x^7 + b^5*x^5)*(-c^
9/b^17)^(1/4)*log(2197*b^13*(-c^9/b^17)^(3/4) + 2197*c^7*sqrt(x)) + 585*(b^
4*c*x^7 + b^5*x^5)*(-c^9/b^17)^(1/4)*log(-2197*b^13*(-c^9/b^17)^(3/4) + 219
7*c^7*sqrt(x)) - 4*(585*c^3*x^6 + 468*b*c^2*x^4 - 52*b^2*c*x^2 + 20*b^3)*sq
rt(x))/(b^4*c*x^7 + b^5*x^5)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(3/2)/(c*x**4+b*x**2)**2,x)

[Out] Timed out

Giac [A]
time = 3.92, size = 219, normalized size = 0.85

$$-\frac{c^{3}x^{\frac{3}{2}}}{2\,(cx^{2}+b)b^{4}}-\frac{13\,\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{5}}-\frac{13\,\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{5}}+\frac{13\,\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^{5}}-\frac{13\,\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^{5}}-\frac{2\,(135\,c^{2}x^{4}-18\,bcx^{2}+5\,b^{2})}{45\,b^{4}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*c^3*x^(3/2)/((c*x^2 + b)*b^4) - 13/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*
sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^5 - 13/8*sqrt(2)*(
b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1
/4))/b^5 + 13/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
+ sqrt(b/c))/b^5 - 13/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(
1/4) + x + sqrt(b/c))/b^5 - 2/45*(135*c^2*x^4 - 18*b*c*x^2 + 5*b^2)/(b^4*x^
(9/2))
```

Mupad [B]
time = 4.37, size = 99, normalized size = 0.38

$$\frac{13\,{(-c)}^{9/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{17/4}}-\frac{13\,{(-c)}^{9/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{17/4}}-\frac{\frac{2}{9\,b}-\frac{26\,c\,x^2}{45\,b^2}+\frac{26\,c^2\,x^4}{5\,b^3}+\frac{13\,c^3\,x^6}{2\,b^4}}{b\,x^{9/2}+c\,x^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] (13*(-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(17/4)) - (13*(-c)
^(9/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(17/4)) - (2/(9*b) - (26*c*
x^2)/(45*b^2) + (26*c^2*x^4)/(5*b^3) + (13*c^3*x^6)/(2*b^4))/(b*x^(9/2) + c
*x^(13/2))
```

3.339 $\int \frac{x^{23/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=251

$$\frac{45\sqrt{x}}{16c^3}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}+\frac{45\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,c^{13/4}}-\frac{45\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,c^{13/4}}$$

```
[Out] -1/4*x^(9/2)/c/(c*x^2+b)^2-9/16*x^(5/2)/c^2/(c*x^2+b)+45/64*b^(1/4)*arctan(
1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)-45/64*b^(1/4)*arctan(1+
c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)+45/128*b^(1/4)*ln(b^(1/2)
+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)-45/128*b^(1/4)
*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)+45/
16*x^(1/2)/c^3
```

Rubi [A]

time = 0.14, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{45\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,c^{13/4}}-\frac{45\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,c^{13/4}}+\frac{45\sqrt[4]{b}\,\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,c^{13/4}}-\frac{45\sqrt[4]{b}\,\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,c^{13/4}}-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}+\frac{45\sqrt{x}}{16c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (45*Sqrt[x])/(16*c^3) - x^(9/2)/(4*c*(b + c*x^2)^2) - (9*x^(5/2))/(16*c^2*(
b + c*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*c^(13/4)) + (45*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{23/2}}{(bx^2+cx^4)^3}\,dx = \int \frac{x^{11/2}}{(b+cx^2)^3}\,dx$$

$$= -\frac{x^{9/2}}{4c(b+cx^2)^2} + \frac{9\int \frac{x^{7/2}}{(b+cx^2)^2}\,dx}{8c}$$

$$= -\frac{x^{9/2}}{4c(b+cx^2)^2} - \frac{9x^{5/2}}{16c^2(b+cx^2)} + \frac{45\int \frac{x^{3/2}}{b+cx^2}\,dx}{32c^2}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c(b+cx^2)^2} - \frac{9x^{5/2}}{16c^2(b+cx^2)} - \frac{(45b)\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{32c^3}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c(b+cx^2)^2} - \frac{9x^{5/2}}{16c^2(b+cx^2)} - \frac{(45b)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^3}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c(b+cx^2)^2} - \frac{9x^{5/2}}{16c^2(b+cx^2)} - \frac{\left(45\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32c^3}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c(b+cx^2)^2} - \frac{9x^{5/2}}{16c^2(b+cx^2)} - \frac{\left(45\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x\right.}{64c^{7/2}}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c(b+cx^2)^2} - \frac{9x^{5/2}}{16c^2(b+cx^2)} + \frac{45\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right.}{64\sqrt{2}\,c^{13/4}}$$

$$= \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c(b+cx^2)^2} - \frac{9x^{5/2}}{16c^2(b+cx^2)} + \frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,c^{13/4}} - \frac{45\sqrt[4]{b}\ldots}{}$$

Mathematica [A]

time = 0.35, size = 149, normalized size = 0.59

$$\frac{\frac{4\sqrt[4]{c}\,\sqrt{x}\,(45b^2+81bcx^2+32c^2x^4)}{(b+cx^2)^2}+45\sqrt{2}\,\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)-45\sqrt{2}\,\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{64c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(45*b^2 + 81*b*c*x^2 + 32*c^2*x^4))/(b + c*x^2)^2 + 45*
Sqrt[2]*b^(1/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x])] - 45*Sqrt[2]*b^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b
] + Sqrt[c]*x)])/(64*c^(13/4))
```

Maple [A]
time = 0.14, size = 145, normalized size = 0.58

method	result
derivativedivides	$\frac{2\sqrt{x}}{c^3}-\frac{2b\left(\frac{-\frac{17c\,x^{\frac{5}{2}}}{32}-\frac{13b\sqrt{x}}{32}}{(c\,x^2+b)^2}+\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{ar}\right.}{256b}\right.}{c^3}$
default	$\frac{2\sqrt{x}}{c^3}-\frac{2b\left(\frac{-\frac{17c\,x^{\frac{5}{2}}}{32}-\frac{13b\sqrt{x}}{32}}{(c\,x^2+b)^2}+\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{ar}\right.}{256b}\right.}{c^3}$
risch	$\frac{2\sqrt{x}}{c^3}+\frac{17b\,x^{\frac{5}{2}}}{16c^2(c\,x^2+b)^2}+\frac{13b^2\sqrt{x}}{16c^3(c\,x^2+b)^2}-\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{128c^3}-\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/c^3-2/c^3*b*((-17/32*c*x^(5/2)-13/32*b*x^(1/2))/(c*x^2+b)^2+45/25
6*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-
(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1
/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.56, size = 229, normalized size = 0.91

$$\frac{17\,bcx^{\frac{5}{2}}+13\,b^{2}\sqrt{x}}{16\,(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3})}-\frac{45\left(\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{c^{\frac{1}{4}}}\right)}{128\,c^{3}}+\frac{2\sqrt{x}}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(17*b*c*x^(5/2) + 13*b^2*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) -
45/128*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*s
qrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*sq
rt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqr
t(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(1/4)*log(sqrt(2)*b^(
1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(1/4)*log(-
sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c^3 + 2*sqr
t(x)/c^3
```

Fricas [A]
time = 0.35, size = 247, normalized size = 0.98

$$-\frac{180\,(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3})\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{c^{6}\sqrt{-\frac{b}{c^{13}}}+x}\;c^{10}\left(-\frac{b}{c^{13}}\right)^{\frac{3}{4}}-c^{10}\sqrt{x}\left(-\frac{b}{c^{13}}\right)^{\frac{3}{4}}}{b}\right)+45\,(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3})\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\log\left(45\,c^{3}\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}+45\sqrt{x}\right)-45\,(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3})\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\log\left(-45\,c^{3}\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}+45\sqrt{x}\right)-4\,(32\,c^{2}x^{4}+81\,bcx^{2}+45\,b^{2})\sqrt{x}}{64\,(c^{5}x^{4}+2\,bc^{4}x^{2}+b^{2}c^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(180*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*arctan((sqrt(c
^6*sqrt(-b/c^13) + x)*c^10*(-b/c^13)^(3/4) - c^10*sqrt(x)*(-b/c^13)^(3/4))/
b) + 45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*log(45*c^3*(-b/c^
13)^(1/4) + 45*sqrt(x)) - 45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1
/4)*log(-45*c^3*(-b/c^13)^(1/4) + 45*sqrt(x)) - 4*(32*c^2*x^4 + 81*b*c*x^2
+ 45*b^2)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.29, size = 208, normalized size = 0.83

$$-\frac{45 \sqrt{2} \left(bc^{3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{b}{c}\right)^{\frac{1}{4}}+2 \sqrt{x}\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64 c^{4}}-\frac{45 \sqrt{2} \left(bc^{3}\right)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{b}{c}\right)^{\frac{1}{4}}-2 \sqrt{x}\right)}{2 \left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64 c^{4}}-\frac{45 \sqrt{2} \left(bc^{3}\right)^{\frac{1}{4}} \log\left(\sqrt{2} \sqrt{x} \left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128 c^{4}}+\frac{45 \sqrt{2} \left(bc^{3}\right)^{\frac{1}{4}} \log\left(-\sqrt{2} \sqrt{x} \left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128 c^{4}}+\frac{2 \sqrt{x}}{c^{3}}+\frac{17 bcx^{\frac{5}{2}}+13 b^{2} \sqrt{x}}{16 \left(cx^{2}+b\right)^{2} c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -45/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/c^4 - 45/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 - 45/128*sqrt(2)*(b*c^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 45/128*sqrt(2)
*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 2*sq
rt(x)/c^3 + 1/16*(17*b*c*x^(5/2) + 13*b^2*sqrt(x))/((c*x^2 + b)^2*c^3)
```

Mupad [B]
time = 4.39, size = 101, normalized size = 0.40

$$\frac{\frac{13\,b^2\,\sqrt{x}}{16}+\frac{17\,b\,c\,x^{5/2}}{16}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{2\,\sqrt{x}}{c^3}-\frac{45\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,c^{13/4}}+\frac{{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-b\right)}^{1/4}}\right)\,45{}\mathrm{i}}{32\,c^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((13*b^2*x^(1/2))/16 + (17*b*c*x^(5/2))/16)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^
2) + (2*x^(1/2))/c^3 - (45*(-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(
32*c^(13/4)) + ((-b)^(1/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*45i)/(32*c
^(13/4))
```

3.340 $\int \frac{x^{21/2}}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{11/4}}+\frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{11/4}}+\frac{21\log\left(\sqrt{b}\right.}{}$$

```
[Out] -1/4*x^(7/2)/c/(c*x^2+b)^2-7/16*x^(3/2)/c^2/(c*x^2+b)-21/64*arctan(1-c^(1/4
)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(11/4)*2^(1/2)+21/64*arctan(1+c^(1/4)*
2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(11/4)*2^(1/2)+21/128*ln(b^(1/2)+x*c^(1/
2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)-21/128*ln(b^(1
/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1598, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{21\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{11/4}}+\frac{21\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\sqrt[4]{b}\,c^{11/4}}+\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\sqrt[4]{b}\,c^{11/4}}-\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\sqrt[4]{b}\,c^{11/4}}-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*x^(7/2)/(c*(b + c*x^2)^2) - (7*x^(3/2))/(16*c^2*(b + c*x^2)) - (21*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/4)*c^(11/4)) +
 (21*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/4)*c^(
11/4)) + (21*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(6
4*Sqrt[2]*b^(1/4)*c^(11/4)) - (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{21/2}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{x^{9/2}}{\left(b+cx^2\right)^3}\,dx\\
&= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}+\frac{7\int \frac{x^{5/2}}{\left(b+cx^2\right)^2}\,dx}{8c}\\
&= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}+\frac{21\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32c^2}\\
&= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}+\frac{21\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16c^2}\\
&= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}-\frac{21\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32c^{5/2}}+\frac{21\text{Subst}\left(\int \cdots\right.}{\cdots}\\
&= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}+\frac{21\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64c^3}+\frac{21\text{Su}\cdots}{\cdots}\\
&= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}+\frac{21\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\log\left(\cdots\right.}{\cdots}\\
&= -\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\cdots}{\cdots}\right.}{32\sqrt{2}\sqrt[4]{b}c^{1\cdots}}
\end{aligned}
$$

Mathematica [A]
time = 0.34, size = 138, normalized size = 0.58

$$
\frac{-\frac{4c^{3/4}x^{3/2}\left(7b+11cx^2\right)}{\left(b+cx^2\right)^2}-\frac{21\sqrt{2}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{b}}-\frac{21\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt[4]{b}}}{64c^{11/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*c^(3/4)*x^(3/2)*(7*b + 11*c*x^2))/(b + c*x^2)^2 - (21*Sqrt[2]*ArcTan[(
Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(1/4) - (21*Sqrt
[2]*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(1/
4))/(64*c^(11/4))
```

Maple [A]
time = 0.12, size = 136, normalized size = 0.57

method	result
derivativedivides	$\frac{-\frac{11x^{\frac{7}{2}}}{16c}-\frac{7b\,x^{\frac{3}{2}}}{16c^{2}}}{(c\,x^{2}+b)^{2}}+\frac{21\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{-\frac{11x^{\frac{7}{2}}}{16c}-\frac{7b\,x^{\frac{3}{2}}}{16c^{2}}}{(c\,x^{2}+b)^{2}}+\frac{21\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-11/32*x^(7/2)/c-7/32*b*x^(3/2)/c^2)/(c*x^2+b)^2+21/128/c^3/(b/c)^(1/4)*
2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1
/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(
2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.52, size = 218, normalized size = 0.91

$$-\frac{11\,cx^{\frac{7}{2}}+7\,bx^{\frac{3}{2}}}{16\,(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2})}+\frac{21\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(11*c*x^(7/2) + 7*b*x^(3/2))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 21/1
28*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*
sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(
1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^2
```

Fricas [A]
time = 0.36, size = 248, normalized size = 1.04

$$-\frac{84\,(c^4x^4+2\,bc^3x^2+b^2c^2)\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{-bc^5\sqrt{-\frac{1}{bc^{11}}}+x}\,c^3\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}-c^3\sqrt{x}\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\right)-21\,(c^4x^4+2\,bc^3x^2+b^2c^2)\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\log\left(bc^8\left(-\frac{1}{bc^{11}}\right)^{\frac{3}{4}}+\sqrt{x}\right)+21\,(c^4x^4+2\,bc^3x^2+b^2c^2)\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\log\left(-bc^8\left(-\frac{1}{bc^{11}}\right)^{\frac{3}{4}}+\sqrt{x}\right)+4\,(11\,cx^3+7\,bx)\sqrt{x}}{64\,(c^4x^4+2\,bc^3x^2+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(84*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11))^(1/4)*arctan(sqrt
(-b*c^5*sqrt(-1/(b*c^11)) + x)*c^3*(-1/(b*c^11))^(1/4) - c^3*sqrt(x)*(-1/(b
*c^11))^(1/4)) - 21*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11))^(1/4)*l
og(b*c^8*(-1/(b*c^11))^(3/4) + sqrt(x)) + 21*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c
^2)*(-1/(b*c^11))^(1/4)*log(-b*c^8*(-1/(b*c^11))^(3/4) + sqrt(x)) + 4*(11*c
*x^3 + 7*b*x)*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.81, size = 209, normalized size = 0.87

$$-\frac{11\,cx^{\frac{7}{2}}+7\,bx^{\frac{3}{2}}}{16\,(cx^2+b)^2c^2}+\frac{21\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^5}+\frac{21\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^5}-\frac{21\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^5}+\frac{21\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*c*x^(7/2) + 7*b*x^(3/2))/((c*x^2 + b)^2*c^2) + 21/64*sqrt(2)*(b*c
^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))
/(b*c^5) + 21/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(
1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) - 21/128*sqrt(2)*(b*c^3)^(3/4)*log(s
qrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5) + 21/128*sqrt(2)*(b*c^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
```

Mupad [B]
time = 4.28, size = 87, normalized size = 0.36

$$\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{32\,{(-b)}^{1/4}\,c^{11/4}}-\frac{\frac{11\,x^{7/2}}{16\,c}+\frac{7\,b\,x^{3/2}}{16\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{32\,{(-b)}^{1/4}\,c^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (21*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(1/4)*c^(11/4)) - ((11*x^(
7/2))/(16*c) + (7*b*x^(3/2))/(16*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (21*at
anh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(1/4)*c^(11/4))
```

3.341 $\int \frac{x^{19/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{x^{5/2}}{4c\left(b+cx^2\right)^2}-\frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{5\log\left(\sqrt{b}-\sqrt{\ldots}\right.}{64\ldots}$$

```
[Out] -1/4*x^(5/2)/c/(c*x^2+b)^2-5/64*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b
^(3/4)/c^(9/4)*2^(1/2)+5/64*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/
4)/c^(9/4)*2^(1/2)-5/128*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/b^(3/4)/c^(9/4)*2^(1/2)+5/128*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1
/2)*x^(1/2))/b^(3/4)/c^(9/4)*2^(1/2)-5/16*x^(1/2)/c^2/(c*x^2+b)
```

Rubi [A]
time = 0.13, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{5\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{5\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{3/4}c^{9/4}}-\frac{5\sqrt{x}}{16c^2\left(b+cx^2\right)}-\frac{x^{5/2}}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*x^(5/2)/(c*(b + c*x^2)^2) - (5*Sqrt[x])/(16*c^2*(b + c*x^2)) - (5*ArcT
an[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(9/4)) + (
5*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(9/4
)) - (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqr
t[2]*b^(3/4)*c^(9/4)) + (5*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^3}\,dx = \int \frac{x^{7/2}}{(b+cx^2)^3}\,dx$$
$$= -\frac{x^{5/2}}{4c(b+cx^2)^2} + \frac{5\int \frac{x^{3/2}}{(b+cx^2)^2}\,dx}{8c}$$
$$= -\frac{x^{5/2}}{4c(b+cx^2)^2} - \frac{5\sqrt{x}}{16c^2(b+cx^2)} + \frac{5\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{32c^2}$$
$$= -\frac{x^{5/2}}{4c(b+cx^2)^2} - \frac{5\sqrt{x}}{16c^2(b+cx^2)} + \frac{5\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^2}$$
$$= -\frac{x^{5/2}}{4c(b+cx^2)^2} - \frac{5\sqrt{x}}{16c^2(b+cx^2)} + \frac{5\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32\sqrt{b}c^2} + \frac{5\text{Subst}\left(\int \frac{\sqrt{b}}{}\right.}{3}$$
$$= -\frac{x^{5/2}}{4c(b+cx^2)^2} - \frac{5\sqrt{x}}{16c^2(b+cx^2)} + \frac{5\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{b}c^{5/2}} + \frac{5\text{Subs}}{}$$
$$= -\frac{x^{5/2}}{4c(b+cx^2)^2} - \frac{5\sqrt{x}}{16c^2(b+cx^2)} - \frac{5\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{9/4}} + \frac{5\log\left(\sqrt{}\right.}{}$$
$$= -\frac{x^{5/2}}{4c(b+cx^2)^2} - \frac{5\sqrt{x}}{16c^2(b+cx^2)} - \frac{5\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{9/4}} + \frac{5\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{32\sqrt{2}b^{3/4}c^{9/4}}$$

Mathematica [A]
time = 0.34, size = 138, normalized size = 0.58

$$\frac{-\frac{4\sqrt[4]{c}\sqrt{x}(5b+9cx^2)}{(b+cx^2)^2} - \frac{5\sqrt{2}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{b^{3/4}} + \frac{5\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{b^{3/4}}}{64c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*c^(1/4)*Sqrt[x]*(5*b + 9*c*x^2))/(b + c*x^2)^2 - (5*Sqrt[2]*ArcTan[(Sq
rt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(3/4) + (5*Sqrt[2]
*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(3/4))
/(64*c^(9/4))
```

Maple [A]
time = 0.11, size = 139, normalized size = 0.58

method	result
derivativedivides	$\frac{-\frac{9x^{\frac{5}{2}}}{16c}-\frac{5b\sqrt{x}}{16c^2}}{(cx^2+b)^2}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{b}{c}\right.}\right.\right.}{128c^2b}$
default	$\frac{-\frac{9x^{\frac{5}{2}}}{16c}-\frac{5b\sqrt{x}}{16c^2}}{(cx^2+b)^2}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{b}{c}\right.}\right.\right.}{128c^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-9/32*x^(5/2)/c-5/32*b*x^(1/2)/c^2)/(c*x^2+b)^2+5/128/c^2*(b/c)^(1/4)/b*
2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1
/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(
2^(1/2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.53, size = 218, normalized size = 0.91

$$-\frac{9\,cx^{\frac{5}{2}}+5\,b\sqrt{x}}{16\,(c^4x^4+2\,bc^3x^2+b^2c^2)}+\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{128\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(9*c*x^(5/2) + 5*b*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 5/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sq
rt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/
4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt(2)*
b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c^2
```

Fricas [A]
time = 0.37, size = 254, normalized size = 1.06

$$\frac{20\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)\left(-\frac{1}{b^{3}c^{9}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^{2}c^{4}\sqrt{-\frac{1}{b^{3}c^{9}}}+x}\,b^{2}c^{7}\left(-\frac{1}{b^{3}c^{9}}\right)^{\frac{3}{4}}-b^{2}c^{7}\sqrt{x}\left(-\frac{1}{b^{3}c^{9}}\right)^{\frac{3}{4}}\right)+5\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)\left(-\frac{1}{b^{3}c^{9}}\right)^{\frac{1}{4}}\log\left(bc^{2}\left(-\frac{1}{b^{3}c^{9}}\right)^{\frac{1}{4}}+\sqrt{x}\right)-5\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)\left(-\frac{1}{b^{3}c^{9}}\right)^{\frac{1}{4}}\log\left(-bc^{2}\left(-\frac{1}{b^{3}c^{9}}\right)^{\frac{1}{4}}+\sqrt{x}\right)-4\left(9\,cx^{2}+5\,b\right)\sqrt{x}}{64\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3*c^9))^(1/4)*arctan(sqrt
(b^2*c^4*sqrt(-1/(b^3*c^9)) + x)*b^2*c^7*(-1/(b^3*c^9))^(3/4) - b^2*c^7*sqr
t(x)*(-1/(b^3*c^9))^(3/4)) + 5*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3*c
^9))^(1/4)*log(b*c^2*(-1/(b^3*c^9))^(1/4) + sqrt(x)) - 5*(c^4*x^4 + 2*b*c^3
*x^2 + b^2*c^2)*(-1/(b^3*c^9))^(1/4)*log(-b*c^2*(-1/(b^3*c^9))^(1/4) + sqrt
(x)) - 4*(9*c*x^2 + 5*b)*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.58, size = 209, normalized size = 0.87

$$\frac{5\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^{3}}+\frac{5\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^{3}}+\frac{5\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^{3}}-\frac{5\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^{3}}-\frac{9\,cx^{\frac{5}{2}}+5\,b\sqrt{x}}{16\left(cx^{2}+b\right)^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b*c^3) + 5/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) + 5/128*sqrt(2)*(b*c
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) - 5/128*
sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*
c^3) - 1/16*(9*c*x^(5/2) + 5*b*sqrt(x))/((c*x^2 + b)^2*c^2)
```

Mupad [B]
time = 0.10, size = 87, normalized size = 0.36

$$-\frac{\frac{9\,x^{5/2}}{16\,c}+\frac{5\,b\,\sqrt{x}}{16\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{3/4}\,c^{9/4}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{3/4}\,c^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((9*x^(5/2))/(16*c) + (5*b*x^(1/2))/(16*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
 - (5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(3/4)*c^(9/4)) - (5*atan
h((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(3/4)*c^(9/4))
```

3.342 $\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}+\frac{3x^{3/2}}{16bc\left(b+cx^2\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{3\log\left(\sqrt{b}-\sqrt{}\right.}{64}$$

```
[Out] -1/4*x^(3/2)/c/(c*x^2+b)^2+3/16*x^(3/2)/b/c/(c*x^2+b)-3/64*arctan(1-c^(1/4)
*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+3/64*arctan(1+c^(1/4)*2^(
1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+3/128*ln(b^(1/2)+x*c^(1/2)-b^
(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(7/4)*2^(1/2)-3/128*ln(b^(1/2)+x*c
^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(7/4)*2^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}}+\frac{3x^{3/2}}{16bc\left(b+cx^2\right)}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*x^(3/2)/(c*(b + c*x^2)^2) + (3*x^(3/2))/(16*b*c*(b + c*x^2)) - (3*ArcT
an[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(7/4)) + (
3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(7/4
)) + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqr
t[2]*b^(5/4)*c^(7/4)) - (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx = \int \frac{x^{5/2}}{(b+cx^2)^3}\,dx$$

$$= -\frac{x^{3/2}}{4c(b+cx^2)^2} + \frac{3\int \frac{\sqrt{x}}{(b+cx^2)^2}\,dx}{8c}$$

$$= -\frac{x^{3/2}}{4c(b+cx^2)^2} + \frac{3x^{3/2}}{16bc(b+cx^2)} + \frac{3\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32bc}$$

$$= -\frac{x^{3/2}}{4c(b+cx^2)^2} + \frac{3x^{3/2}}{16bc(b+cx^2)} + \frac{3\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc}$$

$$= -\frac{x^{3/2}}{4c(b+cx^2)^2} + \frac{3x^{3/2}}{16bc(b+cx^2)} - \frac{3\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32bc^{3/2}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{b}}{\ldots}\right.}{\ldots}$$

$$= -\frac{x^{3/2}}{4c(b+cx^2)^2} + \frac{3x^{3/2}}{16bc(b+cx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{64bc^2} + \frac{3\text{Subs}\ldots}{\ldots}$$

$$= -\frac{x^{3/2}}{4c(b+cx^2)^2} + \frac{3x^{3/2}}{16bc(b+cx^2)} + \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{5/4}c^{7/4}} - \frac{3\log\left(\sqrt{\ldots}\right.}{\ldots}$$

$$= -\frac{x^{3/2}}{4c(b+cx^2)^2} + \frac{3x^{3/2}}{16bc(b+cx^2)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{5/4}c^{7/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\ldots}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{5/4}c^{7/4}\ldots}$$

Mathematica [A]

time = 0.33, size = 136, normalized size = 0.56

$$\frac{-\frac{4\sqrt[4]{b}\,c^{3/4}x^{3/2}(b-3cx^2)}{(b+cx^2)^2}-3\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)-3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{64b^{5/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(1/4)*c^(3/4)*x^(3/2)*(b - 3*c*x^2))/(b + c*x^2)^2 - 3*Sqrt[2]*ArcTa
n[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 3*Sqrt[2]*ArcT
anh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(5/4)*c
^(7/4))
```

Maple [A]
time = 0.11, size = 138, normalized size = 0.57

method	result
derivativedivides	$\frac{\frac{3x^{\frac{7}{2}}}{16b}-\frac{x^{\frac{3}{2}}}{16c}}{(c\,x^2+b)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128c^2b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{3x^{\frac{7}{2}}}{16b}-\frac{x^{\frac{3}{2}}}{16c}}{(c\,x^2+b)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128c^2b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/32/b*x^(7/2)-1/32*x^(3/2)/c)/(c*x^2+b)^2+3/128/c^2/b/(b/c)^(1/4)*2^(1/
2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2
^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.49, size = 222, normalized size = 0.92

$$\frac{3\,cx^{\frac{7}{2}}-bx^{\frac{3}{2}}}{16\,(bc^3x^4+2\,b^2c^2x^2+b^3c)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*c*x^(7/2) - b*x^(3/2))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqr
t(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*c)
```

Fricas [A]
time = 0.38, size = 260, normalized size = 1.07

$$-\frac{12\,(bc^3x^4+2\,b^2c^2x^2+b^3c)(-\frac{1}{b^5c^7})^{\frac{1}{4}}\arctan\left(\sqrt{-b^3c^3\sqrt{-\frac{1}{b^5c^7}}+x}\,bc^2(-\frac{1}{b^5c^7})^{\frac{1}{4}}-bc^2\sqrt{x}\,(-\frac{1}{b^5c^7})^{\frac{1}{4}}\right)-3\,(bc^3x^4+2\,b^2c^2x^2+b^3c)(-\frac{1}{b^5c^7})^{\frac{1}{4}}\log\left(b^4c^5(-\frac{1}{b^5c^7})^{\frac{3}{4}}+\sqrt{x}\right)+3\,(bc^3x^4+2\,b^2c^2x^2+b^3c)(-\frac{1}{b^5c^7})^{\frac{1}{4}}\log\left(-b^4c^5(-\frac{1}{b^5c^7})^{\frac{3}{4}}+\sqrt{x}\right)-4\,(3\,cx^3-bx)\sqrt{x}}{64\,(bc^3x^4+2\,b^2c^2x^2+b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^5*c^7))^(1/4)*arctan(s
qrt(-b^3*c^3*sqrt(-1/(b^5*c^7)) + x)*b*c^2*(-1/(b^5*c^7))^(1/4) - b*c^2*sqr
t(x)*(-1/(b^5*c^7))^(1/4)) - 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^5
*c^7))^(1/4)*log(b^4*c^5*(-1/(b^5*c^7))^(3/4) + sqrt(x)) + 3*(b*c^3*x^4 + 2
*b^2*c^2*x^2 + b^3*c)*(-1/(b^5*c^7))^(1/4)*log(-b^4*c^5*(-1/(b^5*c^7))^(3/4
) + sqrt(x)) - 4*(3*c*x^3 - b*x)*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*
c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.65, size = 212, normalized size = 0.88

$$\frac{3\,cx^{\frac{7}{2}}-bx^{\frac{3}{2}}}{16\,(cx^2+b)^2bc}+\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(\frac{b}{c})^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,(\frac{b}{c})^{\frac{1}{4}}}\right)}{64\,b^2c^4}+\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(\frac{b}{c})^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,(\frac{b}{c})^{\frac{1}{4}}}\right)}{64\,b^2c^4}-\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,(\frac{b}{c})^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^4}+\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,(\frac{b}{c})^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*c*x^(7/2) - b*x^(3/2))/((c*x^2 + b)^2*b*c) + 3/64*sqrt(2)*(b*c^3)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2
```

```
*c^4) + 3/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^4) - 3/128*sqrt(2)*(b*c^3)^(3/4)*log(sqrt
(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4) + 3/128*sqrt(2)*(b*c^3)^
(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4)
```

Mupad [B]
time = 0.09, size = 85, normalized size = 0.35

$$\frac{\frac{3\,x^{7/2}}{16\,b}-\frac{x^{3/2}}{16\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{5/4}\,c^{7/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{5/4}\,c^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*x^(7/2))/(16*b) - x^(3/2)/(16*c))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (3*atan
((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(5/4)*c^(7/4)) + (3*atanh((c^(1/4)
*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(5/4)*c^(7/4))
```

3.343 $\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{\sqrt{x}}{4c(b+cx^2)^2}+\frac{\sqrt{x}}{16bc(b+cx^2)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}}-\frac{3\log\left(\sqrt{b}-\sqrt{\ }\right.}{64}$$

```
[Out] -3/64*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(5/4)*2^(1/2)+3/6
4*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(5/4)*2^(1/2)-3/128*l
n(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(5/4)*2^(1/2
)+3/128*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(5/
4)*2^(1/2)-1/4*x^(1/2)/c/(c*x^2+b)^2+1/16*x^(1/2)/b/c/(c*x^2+b)
```

Rubi [A]
time = 0.12, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}}-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}}+\frac{\sqrt{x}}{16bc(b+cx^2)}-\frac{\sqrt{x}}{4c(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*Sqrt[x]/(c*(b + c*x^2)^2) + Sqrt[x]/(16*b*c*(b + c*x^2)) - (3*ArcTan[1
 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(5/4)) + (3*Ar
cTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(5/4)) -
 (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]
*b^(7/4)*c^(5/4)) + (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt
[c]*x])/(64*Sqrt[2]*b^(7/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx = \int \frac{x^{3/2}}{(b+cx^2)^3}\,dx$$
$$= -\frac{\sqrt{x}}{4c(b+cx^2)^2} + \frac{\int \frac{1}{\sqrt{x}\,(b+cx^2)^2}\,dx}{8c}$$
$$= -\frac{\sqrt{x}}{4c(b+cx^2)^2} + \frac{\sqrt{x}}{16bc(b+cx^2)} + \frac{3\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{32bc}$$
$$= -\frac{\sqrt{x}}{4c(b+cx^2)^2} + \frac{\sqrt{x}}{16bc(b+cx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc}$$
$$= -\frac{\sqrt{x}}{4c(b+cx^2)^2} + \frac{\sqrt{x}}{16bc(b+cx^2)} + \frac{3\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{3/2}c} + \frac{3\text{Subst}\left(\int \ldots\right.}{}$$
$$= -\frac{\sqrt{x}}{4c(b+cx^2)^2} + \frac{\sqrt{x}}{16bc(b+cx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^{3/2}c^{3/2}} + \ldots$$
$$= -\frac{\sqrt{x}}{4c(b+cx^2)^2} + \frac{\sqrt{x}}{16bc(b+cx^2)} - \frac{3\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{7/4}c^{5/4}} + \frac{3\log\left(\ldots\right.}{}$$
$$= -\frac{\sqrt{x}}{4c(b+cx^2)^2} + \frac{\sqrt{x}}{16bc(b+cx^2)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\ldots}{\ldots}\right)}{32\sqrt{2}\,b^{7/4}c^{5/4}\ldots}$$

Mathematica [A]
time = 0.32, size = 137, normalized size = 0.57

$$\frac{\frac{4b^{3/4}\sqrt[4]{c}\sqrt{x}\,(-3b+cx^2)}{(b+cx^2)^2} - 3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + 3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{64b^{7/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^(3/4)*c^(1/4)*Sqrt[x]*(-3*b + c*x^2))/(b + c*x^2)^2 - 3*Sqrt[2]*ArcTa
n[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 3*Sqrt[2]*ArcT
anh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(7/4)*c
^(5/4))
```

Maple [A]
time = 0.11, size = 138, normalized size = 0.57

method	result
derivativedivides	$\frac{\frac{x^{\frac{5}{2}}}{16b}-\frac{3\sqrt{x}}{16c}}{(cx^2+b)^2}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right.\right.}{128c\,b^2}$
default	$\frac{\frac{x^{\frac{5}{2}}}{16b}-\frac{3\sqrt{x}}{16c}}{(cx^2+b)^2}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right.\right.}{128c\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32/b*x^(5/2)-3/32*x^(1/2)/c)/(c*x^2+b)^2+3/128/c/b^2*(b/c)^(1/4)*2^(1/
2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2
^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(b/c)^(1/4)*x^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 221, normalized size = 0.91

$$\frac{cx^{\frac{5}{2}}-3b\sqrt{x}}{16\,(bc^3x^4+2\,b^2c^2x^2+b^3c)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{128\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(c*x^(5/2) - 3*b*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 3/128*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqr
t(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4
```

```
)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/(b*c)
```

Fricas [A]
time = 0.35, size = 257, normalized size = 1.06

$$\frac{12\,(bc^3x^4+2\,b^2c^2x^2+b^3c)\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^4c^2\sqrt{-\frac{1}{b^7c^5}}+x}\,b^5c^4\left(-\frac{1}{b^7c^5}\right)^{\frac{3}{4}}-b^5c^4\sqrt{x}\left(-\frac{1}{b^7c^5}\right)^{\frac{3}{4}}\right)+3\,(bc^3x^4+2\,b^2c^2x^2+b^3c)\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}\log\left(b^2c\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\,(bc^3x^4+2\,b^2c^2x^2+b^3c)\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}\log\left(-b^2c\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\,(cx^2-3\,b)\sqrt{x}}{64\,(bc^3x^4+2\,b^2c^2x^2+b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^7*c^5))^(1/4)*arctan(sq
rt(b^4*c^2*sqrt(-1/(b^7*c^5)) + x)*b^5*c^4*(-1/(b^7*c^5))^(3/4) - b^5*c^4*s
qrt(x)*(-1/(b^7*c^5))^(3/4)) + 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b
^7*c^5))^(1/4)*log(b^2*c*(-1/(b^7*c^5))^(1/4) + sqrt(x)) - 3*(b*c^3*x^4 + 2
*b^2*c^2*x^2 + b^3*c)*(-1/(b^7*c^5))^(1/4)*log(-b^2*c*(-1/(b^7*c^5))^(1/4)
+ sqrt(x)) + 4*(c*x^2 - 3*b)*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.65, size = 211, normalized size = 0.87

$$\frac{3\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^2}+\frac{3\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^2}+\frac{3\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^2}-\frac{3\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^2}+\frac{cx^{\frac{5}{2}}-3\,b\sqrt{x}}{16\,(cx^2+b)^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b^2*c^2) + 3/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 3/128*sqrt(2)*
(b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) -
3/128*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c
))/(b^2*c^2) + 1/16*(c*x^(5/2) - 3*b*sqrt(x))/((c*x^2 + b)^2*b*c)
```

Mupad [B]
time = 0.10, size = 85, normalized size = 0.35

$$\frac{\frac{x^{5/2}}{16\,b}-\frac{3\,\sqrt{x}}{16\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{32\,{(-b)}^{7/4}\,c^{5/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{32\,{(-b)}^{7/4}\,c^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(5/2)/(16*b) - (3*x^(1/2))/(16*c))/(b^2 + c^2*x^4 + 2*b*c*x^2) + (3*atan
((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(7/4)*c^(5/4)) + (3*atanh((c^(1/4)
*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(7/4)*c^(5/4))
```

3.344 $\int \frac{x^{13/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=239

$$\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}+\frac{5x^{3/2}}{16b^2\left(b+cx^2\right)}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{5\log\left(\sqrt{b}-\sqrt{2}\right.}{64\sqrt{}}$$

```
[Out] 1/4*x^(3/2)/b/(c*x^2+b)^2+5/16*x^(3/2)/b^2/(c*x^2+b)-5/64*arctan(1-c^(1/4)*
2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)/c^(3/4)*2^(1/2)+5/64*arctan(1+c^(1/4)*2^(1
/2)*x^(1/2)/b^(1/4))/b^(9/4)/c^(3/4)*2^(1/2)+5/128*ln(b^(1/2)+x*c^(1/2)-b^(
1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)-5/128*ln(b^(1/2)+x*c^
(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{5\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{5\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{5\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}}-\frac{5\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}}+\frac{5x^{3/2}}{16b^2\left(b+cx^2\right)}+\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^(3/2)/(4*b*(b + c*x^2)^2) + (5*x^(3/2))/(16*b^2*(b + c*x^2)) - (5*ArcTan[
1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(3/4)) + (5*A
rcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(3/4))
+ (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2
]*b^(9/4)*c^(3/4)) - (5*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^3}\,dx = \int \frac{\sqrt{x}}{(b+cx^2)^3}\,dx$$

$$= \frac{x^{3/2}}{4b(b+cx^2)^2} + \frac{5\int \frac{\sqrt{x}}{(b+cx^2)^2}\,dx}{8b}$$

$$= \frac{x^{3/2}}{4b(b+cx^2)^2} + \frac{5x^{3/2}}{16b^2(b+cx^2)} + \frac{5\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^2}$$

$$= \frac{x^{3/2}}{4b(b+cx^2)^2} + \frac{5x^{3/2}}{16b^2(b+cx^2)} + \frac{5\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2}$$

$$= \frac{x^{3/2}}{4b(b+cx^2)^2} + \frac{5x^{3/2}}{16b^2(b+cx^2)} - \frac{5\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^2\sqrt{c}} + \frac{5\text{Subst}\left(\int \frac{\sqrt{b}+\ldots}{b+\ldots}\right.}{32\ldots}$$

$$= \frac{x^{3/2}}{4b(b+cx^2)^2} + \frac{5x^{3/2}}{16b^2(b+cx^2)} + \frac{5\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^2c} + \frac{5\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{x^{3/2}}{4b(b+cx^2)^2} + \frac{5x^{3/2}}{16b^2(b+cx^2)} + \frac{5\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{9/4}c^{3/4}} - \frac{5\log\left(\sqrt{b}\ldots\right.}{\ldots}$$

$$= \frac{x^{3/2}}{4b(b+cx^2)^2} + \frac{5x^{3/2}}{16b^2(b+cx^2)} - \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{9/4}c^{3/4}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\ldots}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{9/4}c^{3/4}}$$

Mathematica [A]
time = 0.22, size = 138, normalized size = 0.58

$$\frac{\frac{4\sqrt[4]{b}\,x^{3/2}(9b+5cx^2)}{(b+cx^2)^2} - \frac{5\sqrt{2}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{c^{3/4}} - \frac{5\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{c^{3/4}}}{64b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```

[Out] ((4*b^(1/4)*x^(3/2)*(9*b + 5*c*x^2))/(b + c*x^2)^2 - (5*Sqrt[2]*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(3/4) - (5*Sqrt[2]*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(3/4))/(64*b^(9/4))

Maple [A]
time = 0.12, size = 150, normalized size = 0.63

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{4b(c\,x^{2}+b)^{2}}+\frac{\frac{5x^{\frac{3}{2}}}{16b(c\,x^{2}+b)}+\frac{5\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b}$
default	$\frac{x^{\frac{3}{2}}}{4b(c\,x^{2}+b)^{2}}+\frac{\frac{5x^{\frac{3}{2}}}{16b(c\,x^{2}+b)}+\frac{5\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(13/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)

[Out] 1/4*x^(3/2)/b/(c*x^2+b)^2+5/4/b*(1/4*x^(3/2)/b/(c*x^2+b)+1/32/b/c/(b/c)^(1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))

Maxima [A]
time = 0.50, size = 217, normalized size = 0.91

$$\frac{5\,cx^{\frac{7}{2}}+9\,bx^{\frac{3}{2}}}{16\,(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}+b^{4})}+\frac{5\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")

[Out] 1/16*(5*c*x^(7/2) + 9*b*x^(3/2))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 5/128*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^2

Fricas [A]
time = 0.34, size = 250, normalized size = 1.05

$$-\frac{20\left(b^{2} c^{2} x^{4}+2 b^{3} c x^{2}+b^{4}\right)\left(-\frac{1}{b^{9} c^{3}}\right)^{\frac{1}{4}} \arctan\left(\sqrt{-b^{5} c \sqrt{-\frac{1}{b^{9} c^{3}}}+x}\, b^{2} c\left(-\frac{1}{b^{9} c^{3}}\right)^{\frac{1}{4}}-b^{2} c \sqrt{x}\left(-\frac{1}{b^{9} c^{3}}\right)^{\frac{1}{4}}\right)-5\left(b^{2} c^{2} x^{4}+2 b^{3} c x^{2}+b^{4}\right)\left(-\frac{1}{b^{9} c^{3}}\right)^{\frac{1}{4}} \log\left(b^{7} c^{2}\left(-\frac{1}{b^{9} c^{3}}\right)^{\frac{3}{4}}+\sqrt{x}\right)+5\left(b^{2} c^{2} x^{4}+2 b^{3} c x^{2}+b^{4}\right)\left(-\frac{1}{b^{9} c^{3}}\right)^{\frac{1}{4}} \log\left(-b^{7} c^{2}\left(-\frac{1}{b^{9} c^{3}}\right)^{\frac{3}{4}}+\sqrt{x}\right)-4\left(5 c x^{3}+9 b x\right) \sqrt{x}}{64\left(b^{2} c^{2} x^{4}+2 b^{3} c x^{2}+b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(20*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*arctan(sqr
t(-b^5*c*sqrt(-1/(b^9*c^3)) + x)*b^2*c*(-1/(b^9*c^3))^(1/4) - b^2*c*sqrt(x)
*(-1/(b^9*c^3))^(1/4)) - 5*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))
^(1/4)*log(b^7*c^2*(-1/(b^9*c^3))^(3/4) + sqrt(x)) + 5*(b^2*c^2*x^4 + 2*b^3
*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*log(-b^7*c^2*(-1/(b^9*c^3))^(3/4) + sqrt
(x)) - 4*(5*c*x^3 + 9*b*x)*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.49, size = 209, normalized size = 0.87

$$\frac{5 c x^{\frac{7}{2}}+9 b x^{\frac{3}{2}}}{16\left(c x^{2}+b\right)^{2} b^{2}}+\frac{5 \sqrt{2}\left(b c^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2 \sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64 b^{3} c^{3}}+\frac{5 \sqrt{2}\left(b c^{3}\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2 \sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64 b^{3} c^{3}}-\frac{5 \sqrt{2}\left(b c^{3}\right)^{\frac{3}{4}} \log\left(\sqrt{2} \sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128 b^{3} c^{3}}+\frac{5 \sqrt{2}\left(b c^{3}\right)^{\frac{3}{4}} \log\left(-\sqrt{2} \sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128 b^{3} c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*c*x^(7/2) + 9*b*x^(3/2))/((c*x^2 + b)^2*b^2) + 5/64*sqrt(2)*(b*c^3)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b
^3*c^3) + 5/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^3) - 5/128*sqrt(2)*(b*c^3)^(3/4)*log(sq
rt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3) + 5/128*sqrt(2)*(b*c^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
```

Mupad [B]
time = 0.09, size = 86, normalized size = 0.36

$$\frac{\frac{9\,x^{3/2}}{16\,b}+\frac{5\,c\,x^{7/2}}{16\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{9/4}\,c^{3/4}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{9/4}\,c^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((9*x^(3/2))/(16*b) + (5*c*x^(7/2))/(16*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) +
 (5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(9/4)*c^(3/4)) - (5*atanh(
(c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(9/4)*c^(3/4))
```

3.345 $\int \frac{x^{11/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=239

$$\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}+\frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}}-\frac{21\log\left(\sqrt{b}\,-\right.}{64}$$

```
[Out] -21/64*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(1/4)*2^(1/2)+2
1/64*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(1/4)*2^(1/2)-21/
128*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)/c^(1/4)*
2^(1/2)+21/128*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/
4)/c^(1/4)*2^(1/2)+1/4*x^(1/2)/b/(c*x^2+b)^2+7/16*x^(1/2)/b^2/(c*x^2+b)
```

Rubi [A]

time = 0.12, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1598, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{21\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{21\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}}-\frac{21\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}}+\frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] Sqrt[x]/(4*b*(b + c*x^2)^2) + (7*Sqrt[x])/(16*b^2*(b + c*x^2)) - (21*ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(1/4)) + (2
1*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(1/
4)) - (21*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*S
qrt[2]*b^(11/4)*c^(1/4)) + (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{\sqrt{x}\,\left(b+cx^2\right)^3}\,dx$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\int \frac{1}{\sqrt{x}\,(b+cx^2)^2}\,dx}{8b}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{32b^2}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{5/2}} + \frac{21\text{Subst}\left(\int \frac{\sqrt{}}{}\right.}{}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} + \frac{21\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^{5/2}\sqrt{c}} + \frac{21\text{Subs}}{}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} - \frac{21\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{}\right.}{}$$

$$= \frac{\sqrt{x}}{4b\left(b+cx^2\right)^2} + \frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{32\sqrt{2}\,b^{11/4}\sqrt[4]{c}}$$

Mathematica [A]
time = 0.22, size = 138, normalized size = 0.58

$$\frac{\frac{4b^{3/4}\sqrt{x}\,(11b+7cx^2)}{(b+cx^2)^2} - \frac{21\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{c}} + \frac{21\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{\sqrt[4]{c}}}{64b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^(3/4)*Sqrt[x]*(11*b + 7*c*x^2))/(b + c*x^2)^2 - (21*Sqrt[2]*ArcTan[(S
qrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(1/4) + (21*Sqrt[
2]*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(1/4
))/(64*b^(11/4))
```

Maple [A]
time = 0.11, size = 147, normalized size = 0.62

method	result
derivativedivides	$\frac{\sqrt{x}}{4b(c\,x^2+b)^2}+\frac{\frac{7\sqrt{x}}{16b\left(c\,x^2+b\right)}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{128b^2}}{b}$
default	$\frac{\sqrt{x}}{4b(c\,x^2+b)^2}+\frac{\frac{7\sqrt{x}}{16b\left(c\,x^2+b\right)}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{128b^2}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(1/2)/b/(c*x^2+b)^2+7/4/b*(1/4*x^(1/2)/b/(c*x^2+b)+3/32/b^2*(b/c)^(1/
4)*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x
^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arct
an(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]
time = 0.55, size = 217, normalized size = 0.91

$$\frac{7\,cx^{\frac{5}{2}}+11\,b\sqrt{x}}{16\,(b^2c^2x^4+2\,b^3cx^2+b^4)}+\frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(7*c*x^(5/2) + 11*b*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 21/12
8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x
))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1
/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqrt(2)
*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^2
```

Fricas [A]
time = 0.37, size = 241, normalized size = 1.01

$$\frac{84\left(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}+b^{4}\right)\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}\arctan\left(\sqrt{b^{6}\sqrt{-\frac{1}{b^{11}c}}+x}\,b^{8}c\left(-\frac{1}{b^{11}c}\right)^{\frac{3}{4}}-b^{8}c\sqrt{x}\left(-\frac{1}{b^{11}c}\right)^{\frac{3}{4}}\right)+21\left(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}+b^{4}\right)\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}\log\left(b^{3}\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}+\sqrt{x}\right)-21\left(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}+b^{4}\right)\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}\log\left(-b^{3}\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\left(7\,cx^{2}+11\,b\right)\sqrt{x}}{64\left(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}+b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(84*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1/4)*arctan(sqrt(
b^6*sqrt(-1/(b^11*c)) + x)*b^8*c*(-1/(b^11*c))^(3/4) - b^8*c*sqrt(x)*(-1/(b
^11*c))^(3/4)) + 21*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1/4)*l
og(b^3*(-1/(b^11*c))^(1/4) + sqrt(x)) - 21*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4
)*(-1/(b^11*c))^(1/4)*log(-b^3*(-1/(b^11*c))^(1/4) + sqrt(x)) + 4*(7*c*x^2
+ 11*b)*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.54, size = 209, normalized size = 0.87

$$\frac{21\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c}+\frac{21\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c}+\frac{21\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{3}c}-\frac{21\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{3}c}+\frac{7\,cx^{\frac{5}{2}}+11\,b\sqrt{x}}{16\left(cx^{2}+b\right)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/(b^3*c) + 21/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 21/128*sqrt(2)*(
b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) - 21/
128*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b^3*c) + 1/16*(7*c*x^(5/2) + 11*b*sqrt(x))/((c*x^2 + b)^2*b^2)
```

Mupad [B]
time = 4.29, size = 86, normalized size = 0.36

$$\frac{\frac{11\,\sqrt{x}}{16\,b}+\frac{7\,c\,x^{5/2}}{16\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{11/4}\,c^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{32\,{\left(-b\right)}^{11/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((11*x^(1/2))/(16*b) + (7*c*x^(5/2))/(16*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
- (21*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(11/4)*c^(1/4)) - (21*at
anh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(11/4)*c^(1/4))
```

3.346 $\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=251

$$-\frac{45}{16b^3\sqrt{x}}+\frac{1}{4b\sqrt{x}\,(b+cx^2)^2}+\frac{9}{16b^2\sqrt{x}\,(b+cx^2)}+\frac{45\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{13/4}}-\frac{45\sqrt[4]{c}\,\tan^{-1}\left(1+\right.}{32\sqrt{2}\,b^{13/}}$$

```
[Out] 45/64*c^(1/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)-45
/64*c^(1/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)-45/1
28*c^(1/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2
^(1/2)+45/128*c^(1/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))
/b^(13/4)*2^(1/2)-45/16/b^3/x^(1/2)+1/4/b/(c*x^2+b)^2/x^(1/2)+9/16/b^2/(c*x
^2+b)/x^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{45\sqrt[4]{c}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{13/4}}-\frac{45\sqrt[4]{c}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{13/4}}-\frac{45\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}}+\frac{45\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{13/4}}-\frac{45}{16b^3\sqrt{x}}+\frac{9}{16b^2\sqrt{x}\,(b+cx^2)}+\frac{1}{4b\sqrt{x}\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -45/(16*b^3*Sqrt[x]) + 1/(4*b*Sqrt[x]*(b + c*x^2)^2) + 9/(16*b^2*Sqrt[x]*(b
 + c*x^2)) + (45*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32
*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(32*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4)) + (45*c^(1/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx = \int \frac{1}{x^{3/2}(b+cx^2)^3}\,dx$$

$$= \frac{1}{4b\sqrt{x}\,(b+cx^2)^2} + \frac{9\int \frac{1}{x^{3/2}(b+cx^2)^2}\,dx}{8b}$$

$$= \frac{1}{4b\sqrt{x}\,(b+cx^2)^2} + \frac{9}{16b^2\sqrt{x}\,(b+cx^2)} + \frac{45\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{32b^2}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\,(b+cx^2)^2} + \frac{9}{16b^2\sqrt{x}\,(b+cx^2)} - \frac{(45c)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\,(b+cx^2)^2} + \frac{9}{16b^2\sqrt{x}\,(b+cx^2)} - \frac{(45c)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\,(b+cx^2)^2} + \frac{9}{16b^2\sqrt{x}\,(b+cx^2)} + \frac{(45\sqrt{c})\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,d\right.}{32b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\,(b+cx^2)^2} + \frac{9}{16b^2\sqrt{x}\,(b+cx^2)} - \frac{45\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\right.}{64b^3}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\,(b+cx^2)^2} + \frac{9}{16b^2\sqrt{x}\,(b+cx^2)} - \frac{45\sqrt[4]{c}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{64\sqrt{2}\,b^{13/4}}$$

$$= -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\,(b+cx^2)^2} + \frac{9}{16b^2\sqrt{x}\,(b+cx^2)} + \frac{45\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{13/4}}$$

Mathematica [A]

time = 0.36, size = 149, normalized size = 0.59

$$\frac{-\frac{4\sqrt[4]{b}\,(32b^2+81bcx^2+45c^2x^4)}{\sqrt{x}\,(b+cx^2)^2}+45\sqrt{2}\,\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)+45\sqrt{2}\,\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{64b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(1/4)*(32*b^2 + 81*b*c*x^2 + 45*c^2*x^4))/(Sqrt[x]*(b + c*x^2)^2) +
45*Sqrt[2]*c^(1/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x])] + 45*Sqrt[2]*c^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqr
t[b] + Sqrt[c]*x)])/(64*b^(13/4))
```

Maple [A]
time = 0.14, size = 145, normalized size = 0.58

method	result
derivativedivides	$-\frac{2c\left(\frac{\frac{13cx^{\frac{7}{2}}}{32}+\frac{17bx^{\frac{3}{2}}}{32}}{\left(cx^2+b\right)^2}+\frac{45\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.\right.}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^3}$
default	$-\frac{2c\left(\frac{\frac{13cx^{\frac{7}{2}}}{32}+\frac{17bx^{\frac{3}{2}}}{32}}{\left(cx^2+b\right)^2}+\frac{45\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.\right.}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^3}$
risch	$-\frac{2}{b^3\sqrt{x}}-\frac{13c^2x^{\frac{7}{2}}}{16b^3(cx^2+b)^2}-\frac{17cx^{\frac{3}{2}}}{16b^2(cx^2+b)^2}-\frac{45\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{128b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{45\sqrt{2}\,\arctan\left(\right.}{64b^3\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^3*c*((13/32*c*x^(7/2)+17/32*b*x^(3/2))/(c*x^2+b)^2+45/256/c/(b/c)^(1/4
)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^
(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))-2/b^3/x^(1/2)
```

Maxima [A]
time = 0.50, size = 230, normalized size = 0.92

$$-\frac{45\,c^{2}x^{4}+81\,bcx^{2}+32\,b^{2}}{16\left(b^{3}c^{2}x^{\frac{9}{2}}+2\,b^{4}cx^{\frac{5}{2}}+b^{5}\sqrt{x}\right)}-\frac{45\,c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(45*c^2*x^4 + 81*b*c*x^2 + 32*b^2)/(b^3*c^2*x^(9/2) + 2*b^4*c*x^(5/2)
 + b^5*sqrt(x)) - 45/128*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c
^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*s
qrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c
)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)
*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4
)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(1/4)*c^(3/4)))/b^3
```

Fricas [A]
time = 0.38, size = 263, normalized size = 1.05

$$\frac{180\left(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x\right)\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\,b^{3}c\sqrt{x}\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}-\sqrt{-8303765625\,b^{7}c\sqrt{-\frac{c}{b^{13}}}+8303765625\,c^{2}x}\,b^{3}\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}}{91125\,c}\right)-45\left(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x\right)\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}\log\left(91125\,b^{10}\left(-\frac{c}{b^{13}}\right)^{\frac{3}{4}}+91125\,c\sqrt{x}\right)+45\left(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x\right)\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}\log\left(-91125\,b^{10}\left(-\frac{c}{b^{13}}\right)^{\frac{3}{4}}+91125\,c\sqrt{x}\right)-4\left(45\,c^{2}x^{4}+81\,bcx^{2}+32\,b^{2}\right)\sqrt{x}}{64\left(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(180*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-c/b^13)^(1/4)*arctan(-1/911
25*(91125*b^3*c*sqrt(x)*(-c/b^13)^(1/4) - sqrt(-8303765625*b^7*c*sqrt(-c/b^
13) + 8303765625*c^2*x)*b^3*(-c/b^13)^(1/4))/c) - 45*(b^3*c^2*x^5 + 2*b^4*c
*x^3 + b^5*x)*(-c/b^13)^(1/4)*log(91125*b^10*(-c/b^13)^(3/4) + 91125*c*sqrt
(x)) + 45*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-c/b^13)^(1/4)*log(-91125*b^
10*(-c/b^13)^(3/4) + 91125*c*sqrt(x)) - 4*(45*c^2*x^4 + 81*b*c*x^2 + 32*b^2
)*sqrt(x))/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.64, size = 220, normalized size = 0.88

$$-\frac{2}{b^{3}\sqrt{x}}-\frac{45\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{4}c^{2}}-\frac{45\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{4}c^{2}}+\frac{45\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{4}c^{2}}-\frac{45\sqrt{2}\,(bc^{3})^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{4}c^{2}}-\frac{13\,c^{2}x^{\frac{7}{2}}+17\,bcx^{\frac{3}{2}}}{16\,(cx^{2}+b)^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -2/(b^3*sqrt(x)) - 45/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) - 45/64*sqrt(2)*(b*c^3)^(3/
4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^4*
c^2) + 45/128*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + s
qrt(b/c))/(b^4*c^2) - 45/128*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 1/16*(13*c^2*x^(7/2) + 17*b*c*x^(3/2)
)/((c*x^2 + b)^2*b^3)
```

Mupad [B]
time = 4.37, size = 99, normalized size = 0.39

$$\frac{45\,{(-c)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{13/4}}-\frac{45\,{(-c)}^{1/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{13/4}}-\frac{\frac{2}{b}+\frac{81\,c\,x^2}{16\,b^2}+\frac{45\,c^2\,x^4}{16\,b^3}}{b^2\,\sqrt{x}+c^2\,x^{9/2}+2\,b\,c\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (45*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(13/4)) - (45*(-c
)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(13/4)) - (2/b + (81*c*x^
2)/(16*b^2) + (45*c^2*x^4)/(16*b^3))/(b^2*x^(1/2) + c^2*x^(9/2) + 2*b*c*x^(
5/2))
```

3.347 $\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=251

$$-\frac{77}{48b^3x^{3/2}}+\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}+\frac{11}{16b^2x^{3/2}\left(b+cx^2\right)}+\frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{15/4}}-\frac{77c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{32\sqrt{2}\,b^{15/4}}$$

```
[Out] -77/48/b^3/x^(3/2)+1/4/b/x^(3/2)/(c*x^2+b)^2+11/16/b^2/x^(3/2)/(c*x^2+b)+77
/64*c^(3/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)-77/6
4*c^(3/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)+77/128
*c^(3/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(
1/2)-77/128*c^(3/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b
^(15/4)*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{77c^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{15/4}}-\frac{77c^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{15/4}}+\frac{77c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}}-\frac{77c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{15/4}}-\frac{77}{48b^3x^{3/2}}+\frac{11}{16b^2x^{3/2}\left(b+cx^2\right)}+\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -77/(48*b^3*x^(3/2)) + 1/(4*b*x^(3/2)*(b + c*x^2)^2) + 11/(16*b^2*x^(3/2)*(
b + c*x^2)) + (77*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*b^(15/4)) + (77*c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx = \int \frac{1}{x^{5/2}(b+cx^2)^3}\,dx$$

$$= \frac{1}{4bx^{3/2}(b+cx^2)^2} + \frac{11\int \frac{1}{x^{5/2}(b+cx^2)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{3/2}(b+cx^2)^2} + \frac{11}{16b^2x^{3/2}(b+cx^2)} + \frac{77\int \frac{1}{x^{5/2}(b+cx^2)}\,dx}{32b^2}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}(b+cx^2)^2} + \frac{11}{16b^2x^{3/2}(b+cx^2)} - \frac{(77c)\int \frac{1}{\sqrt{x}\,(b+cx^2)}\,dx}{32b^3}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}(b+cx^2)^2} + \frac{11}{16b^2x^{3/2}(b+cx^2)} - \frac{(77c)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^3}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}(b+cx^2)^2} + \frac{11}{16b^2x^{3/2}(b+cx^2)} - \frac{(77c)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x,\right.}{32b^{7/2}}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}(b+cx^2)^2} + \frac{11}{16b^2x^{3/2}(b+cx^2)} - \frac{(77\sqrt{c})\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}\right.}{64b^{7/2}}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}(b+cx^2)^2} + \frac{11}{16b^2x^{3/2}(b+cx^2)} + \frac{77c^{3/4}\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,\sqrt{\right.}{64\sqrt{2}\,b^{15/4}}$$

$$= -\frac{77}{48b^3x^{3/2}} + \frac{1}{4bx^{3/2}(b+cx^2)^2} + \frac{11}{16b^2x^{3/2}(b+cx^2)} + \frac{77c^{3/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{15/4}}$$

Mathematica [A]

time = 0.37, size = 149, normalized size = 0.59

$$\frac{-\frac{4b^{3/4}(32b^2+121bcx^2+77c^2x^4)}{x^{3/2}(b+cx^2)^2}+231\sqrt{2}\,c^{3/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)-231\sqrt{2}\,c^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{192b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(3/4)*(32*b^2 + 121*b*c*x^2 + 77*c^2*x^4))/(x^(3/2)*(b + c*x^2)^2) +
 231*Sqrt[2]*c^(3/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x])] - 231*Sqrt[2]*c^(3/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(
Sqrt[b] + Sqrt[c]*x)])/(192*b^(15/4))
```

Maple [A]
time = 0.13, size = 145, normalized size = 0.58

method	result
derivativedivides	$-\frac{2c\left(\frac{\frac{15cx^{\frac{5}{2}}}{32}+\frac{19b\sqrt{x}}{32}}{(cx^2+b)^2}+\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.\right.}{256b}\cdots\right)}{b^3}$
default	$-\frac{2c\left(\frac{\frac{15cx^{\frac{5}{2}}}{32}+\frac{19b\sqrt{x}}{32}}{(cx^2+b)^2}+\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.\right.}{256b}\cdots\right)}{b^3}$
risch	$-\frac{2}{3b^3x^{\frac{3}{2}}}-\frac{15c^2x^{\frac{5}{2}}}{16b^3(cx^2+b)^2}-\frac{19c\sqrt{x}}{16b^2(cx^2+b)^2}-\frac{77c\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{128b^4}-\frac{77c\left(\frac{b}{c}\right)^{\frac{1}{4}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^3*c*((15/32*c*x^(5/2)+19/32*b*x^(1/2))/(c*x^2+b)^2+77/256*(b/c)^(1/4)/
b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^
(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))-2/3/b^3/x^(3/2)
```

Maxima [A]
time = 0.53, size = 231, normalized size = 0.92

$$-\frac{77\,c^{2}x^{4}+121\,bcx^{2}+32\,b^{2}}{48\,\left(b^{3}c^{2}x^{\frac{11}{2}}+2\,b^{4}cx^{\frac{7}{2}}+b^{5}x^{\frac{3}{2}}\right)}-\frac{77\,\left(\frac{2\,\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\,\sqrt{2}\,c\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,c^{\frac{3}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{128\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(77*c^2*x^4 + 121*b*c*x^2 + 32*b^2)/(b^3*c^2*x^(11/2) + 2*b^4*c*x^(7/
2) + b^5*x^(3/2)) - 77/128*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)
*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*
sqrt(c))) + 2*sqrt(2)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sq
rt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqr
t(2)*c^(3/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(
3/4) - sqrt(2)*c^(3/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/b^(3/4))/b^3
```

Fricas [A]
time = 0.38, size = 283, normalized size = 1.13

$$-\frac{924\,\left(b^{3}c^{2}x^{6}+2\,b^{4}cx^{4}+b^{5}x^{2}\right)\left(-\frac{c^{3}}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{11}c\sqrt{x}\left(-\frac{c^{3}}{b^{15}}\right)^{\frac{3}{4}}-\sqrt{b^{8}\sqrt{-\frac{c^{3}}{b^{15}}}+c^{2}x}\,b^{11}\left(-\frac{c^{3}}{b^{15}}\right)^{\frac{3}{4}}}{c^{3}}\right)+231\,\left(b^{3}c^{2}x^{6}+2\,b^{4}cx^{4}+b^{5}x^{2}\right)\left(-\frac{c^{3}}{b^{15}}\right)^{\frac{1}{4}}\log\left(77\,b^{4}\left(-\frac{c^{3}}{b^{15}}\right)^{\frac{1}{4}}+77\,c\sqrt{x}\right)-231\,\left(b^{3}c^{2}x^{6}+2\,b^{4}cx^{4}+b^{5}x^{2}\right)\left(-\frac{c^{3}}{b^{15}}\right)^{\frac{1}{4}}\log\left(-77\,b^{4}\left(-\frac{c^{3}}{b^{15}}\right)^{\frac{1}{4}}+77\,c\sqrt{x}\right)+4\,\left(77\,c^{2}x^{4}+121\,bcx^{2}+32\,b^{2}\right)\sqrt{x}}{192\,\left(b^{3}c^{2}x^{6}+2\,b^{4}cx^{4}+b^{5}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(924*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/b^15)^(1/4)*arctan(
-(b^11*c*sqrt(x)*(-c^3/b^15)^(3/4) - sqrt(b^8*sqrt(-c^3/b^15) + c^2*x)*b^11
*(-c^3/b^15)^(3/4))/c^3) + 231*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/
b^15)^(1/4)*log(77*b^4*(-c^3/b^15)^(1/4) + 77*c*sqrt(x)) - 231*(b^3*c^2*x^6
 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/b^15)^(1/4)*log(-77*b^4*(-c^3/b^15)^(1/4) +
 77*c*sqrt(x)) + 4*(77*c^2*x^4 + 121*b*c*x^2 + 32*b^2)*sqrt(x))/(b^3*c^2*x^
6 + 2*b^4*c*x^4 + b^5*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.06, size = 208, normalized size = 0.83

$$-\frac{77\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{4}}-\frac{77\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{4}}-\frac{77\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{4}}+\frac{77\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{4}}-\frac{15\,c^{2}x^{\frac{5}{2}}+19\,bc\sqrt{x}}{16\left(cx^{2}+b\right)^{2}b^{3}}-\frac{2}{3\,b^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/b^4 - 77/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 - 77/128*sqrt(2)*(b*c^3)^
(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 77/128*sqrt(2)
*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/16
*(15*c^2*x^(5/2) + 19*b*c*sqrt(x))/((c*x^2 + b)^2*b^3) - 2/3/(b^3*x^(3/2))
```

Mupad [B]
time = 0.13, size = 99, normalized size = 0.39

$$\frac{77\,{\left(-c\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{15/4}}-\frac{\frac{2}{3\,b}+\frac{121\,c\,x^2}{48\,b^2}+\frac{77\,c^2\,x^4}{48\,b^3}}{b^2\,x^{3/2}+c^2\,x^{11/2}+2\,b\,c\,x^{7/2}}+\frac{77\,{\left(-c\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (77*(-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(15/4)) - (2/(3*b)
 + (121*c*x^2)/(48*b^2) + (77*c^2*x^4)/(48*b^3))/(b^2*x^(3/2) + c^2*x^(11/2
) + 2*b*c*x^(7/2)) + (77*(-c)^(3/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(3
2*b^(15/4))
```

3.348 $\int \frac{x^{5/2}}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=264

$$-\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}(b+cx^2)^2}+\frac{13}{16b^2x^{5/2}(b+cx^2)}-\frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}}+\frac{117c^{5/4}\text{ta}}{}$$

```
[Out] -117/80/b^3/x^(5/2)+1/4/b/x^(5/2)/(c*x^2+b)^2+13/16/b^2/x^(5/2)/(c*x^2+b)-1
17/64*c^(5/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+11
7/64*c^(5/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+117
/128*c^(5/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)
*2^(1/2)-117/128*c^(5/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/b^(17/4)*2^(1/2)+117/16*c/b^4/x^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{117c^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}}+\frac{117c^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{17/4}}+\frac{117c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}-\frac{117c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}+\frac{117c}{16b^4\sqrt{x}}-\frac{117}{80b^3x^{5/2}}+\frac{13}{16b^2x^{5/2}(b+cx^2)}+\frac{1}{4bx^{5/2}(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -117/(80*b^3*x^(5/2)) + (117*c)/(16*b^4*Sqrt[x]) + 1/(4*b*x^(5/2)*(b + c*x^
2)^2) + 13/(16*b^2*x^(5/2)*(b + c*x^2)) - (117*c^(5/4)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17
/4)) - (117*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(64*Sqrt[2]*b^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^3}\,dx = \int \frac{1}{x^{7/2}(b+cx^2)^3}\,dx$$
$$= \frac{1}{4bx^{5/2}(b+cx^2)^2} + \frac{13\int \frac{1}{x^{7/2}(b+cx^2)^2}\,dx}{8b}$$
$$= \frac{1}{4bx^{5/2}(b+cx^2)^2} + \frac{13}{16b^2x^{5/2}(b+cx^2)} + \frac{117\int \frac{1}{x^{7/2}(b+cx^2)}\,dx}{32b^2}$$
$$= -\frac{117}{80b^3x^{5/2}} + \frac{1}{4bx^{5/2}(b+cx^2)^2} + \frac{13}{16b^2x^{5/2}(b+cx^2)} - \frac{(117c)\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{32b^3}$$
$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}(b+cx^2)^2} + \frac{13}{16b^2x^{5/2}(b+cx^2)} + \frac{(117c^2)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^4}$$
$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}(b+cx^2)^2} + \frac{13}{16b^2x^{5/2}(b+cx^2)} + \frac{(117c^2)\,\text{Subst}\Big(\int \frac{}{b+}$$
$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}(b+cx^2)^2} + \frac{13}{16b^2x^{5/2}(b+cx^2)} - \frac{(117c^{3/2})\,\text{Subst}\Big(\int$$
$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}(b+cx^2)^2} + \frac{13}{16b^2x^{5/2}(b+cx^2)} + \frac{(117c)\text{Subst}\Bigg(\int \frac{}{\sqrt{}}$$
$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}(b+cx^2)^2} + \frac{13}{16b^2x^{5/2}(b+cx^2)} + \frac{117c^{5/4}\log\left(\sqrt{b}\right. -}{6}$$
$$= -\frac{117}{80b^3x^{5/2}} + \frac{117c}{16b^4\sqrt{x}} + \frac{1}{4bx^{5/2}(b+cx^2)^2} + \frac{13}{16b^2x^{5/2}(b+cx^2)} - \frac{117c^{5/4}\tan^{-1}\left(1 -\right.}{32\sqrt{2}\,b}$$

Mathematica [A]
time = 0.38, size = 160, normalized size = 0.61

$$\frac{\frac{4\sqrt[4]{b}\,(-32b^3+416b^2cx^2+1053bc^2x^4+585c^3x^6)}{x^{5/2}(b+cx^2)^2} - 585\sqrt{2}\,c^{5/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right) - 585\sqrt{2}\,c^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{320b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^(1/4)*(-32*b^3 + 416*b^2*c*x^2 + 1053*b*c^2*x^4 + 585*c^3*x^6))/(x^(5
/2)*(b + c*x^2)^2) - 585*Sqrt[2]*c^(5/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 585*Sqrt[2]*c^(5/4)*ArcTanh[(Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(320*b^(17/4))
```

Maple [A]

time = 0.15, size = 156, normalized size = 0.59

method	result
derivativedivides	$\frac{2c^{2}\left(\frac{\frac{21 c x^{\frac{7}{2}}}{32}+\frac{25 b x^{\frac{3}{2}}}{32}}{\left(c x^{2}+b\right)^{2}}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{4}}$
default	$\frac{2c^{2}\left(\frac{\frac{21 c x^{\frac{7}{2}}}{32}+\frac{25 b x^{\frac{3}{2}}}{32}}{\left(c x^{2}+b\right)^{2}}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{4}}$
risch	$-\frac{2\left(-15 c x^{2}+b\right)}{5 b^{4} x^{\frac{5}{2}}}+\frac{21 c^{3} x^{\frac{7}{2}}}{16 b^{4}\left(c x^{2}+b\right)^{2}}+\frac{25 c^{2} x^{\frac{3}{2}}}{16 b^{3}\left(c x^{2}+b\right)^{2}}+\frac{117 c\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{128 b^{4}\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\frac{117 c\sqrt{2}\ \cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*c^2*((21/32*c*x^(7/2)+25/32*b*x^(3/2))/(c*x^2+b)^2+117/256/c/(b/c)^(1
/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*
x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arc
tan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))-2/5/b^3/x^(5/2)+6*c/b^4/x^(1/2)
```

Maxima [A]

time = 0.50, size = 243, normalized size = 0.92

$$\frac{585\,c^3x^6+1053\,bc^2x^4+416\,b^2cx^2-32\,b^3}{80\left(b^4c^2x^{\frac{13}{2}}+2\,b^5cx^{\frac{9}{2}}+b^6x^{\frac{5}{2}}\right)}+\frac{117\,c^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/80*(585*c^3*x^6 + 1053*b*c^2*x^4 + 416*b^2*c*x^2 - 32*b^3)/(b^4*c^2*x^(13
/2) + 2*b^5*c*x^(9/2) + b^6*x^(5/2)) + 117/128*c^2*(2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/
(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^
(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqr
t(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x +
sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
 sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^4
```

Fricas [A]
time = 0.38, size = 306, normalized size = 1.16

$$-\frac{2340\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)\left(-\frac{c^5}{b^{17}}\right)^{\frac14}\arctan\left(-\frac{1601613\,b^4c^4\sqrt{x}\left(-\frac{c^5}{b^{17}}\right)^{\frac14}-\sqrt{-2565164201769\,b^9c^5\sqrt{-\frac{c^5}{b^{17}}}+2565164201769\,c^8x}\,b^4\left(-\frac{c^5}{b^{17}}\right)^{\frac14}}{1601613\,c^5}\right)-585\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)\left(-\frac{c^5}{b^{17}}\right)^{\frac14}\log\left(1601613\,b^{13}\left(-\frac{c^5}{b^{17}}\right)^{\frac34}+1601613\,c^4\sqrt{x}\right)+585\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)\left(-\frac{c^5}{b^{17}}\right)^{\frac14}\log\left(-1601613\,b^{13}\left(-\frac{c^5}{b^{17}}\right)^{\frac34}+1601613\,c^4\sqrt{x}\right)-4\,(585\,c^3x^6+1053\,bc^2x^4+416\,b^2cx^2-32\,b^3)\sqrt{x}}{320\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(2340*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*arctan
(-1/1601613*(1601613*b^4*c^4*sqrt(x)*(-c^5/b^17)^(1/4) - sqrt(-256516420176
9*b^9*c^5*sqrt(-c^5/b^17) + 2565164201769*c^8*x)*b^4*(-c^5/b^17)^(1/4))/c^5
) - 585*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*log(1601613
*b^13*(-c^5/b^17)^(3/4) + 1601613*c^4*sqrt(x)) + 585*(b^4*c^2*x^7 + 2*b^5*c
*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*log(-1601613*b^13*(-c^5/b^17)^(3/4) + 160
1613*c^4*sqrt(x)) - 4*(585*c^3*x^6 + 1053*b*c^2*x^4 + 416*b^2*c*x^2 - 32*b^
3)*sqrt(x))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.95, size = 232, normalized size = 0.88

$$\frac{117\,\sqrt{2}\,(bc^3)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{64\,b^5c}+\frac{117\,\sqrt{2}\,(bc^3)^{\frac34}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{64\,b^5c}-\frac{117\,\sqrt{2}\,(bc^3)^{\frac34}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5c}+\frac{117\,\sqrt{2}\,(bc^3)^{\frac34}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5c}+\frac{21\,c^3x^{\frac72}+25\,bc^2x^{\frac32}}{16\,(cx^2+b)^2b^4}+\frac{2\,(15\,cx^2-b)}{5\,b^4x^{\frac52}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/(b^5*c) + 117/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) - 117/128*sqrt(2
)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c) +
117/128*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b
/c))/(b^5*c) + 1/16*(21*c^3*x^(7/2) + 25*b*c^2*x^(3/2))/((c*x^2 + b)^2*b^4)
 + 2/5*(15*c*x^2 - b)/(b^4*x^(5/2))
```

Mupad [B]
time = 0.12, size = 109, normalized size = 0.41

$$\frac{\frac{26\,c\,x^2}{5\,b^2}-\frac{2}{5\,b}+\frac{1053\,c^2\,x^4}{80\,b^3}+\frac{117\,c^3\,x^6}{16\,b^4}}{b^2\,x^{5/2}+c^2\,x^{13/2}+2\,b\,c\,x^{9/2}}-\frac{117\,{\left(-c\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{17/4}}+\frac{117\,{\left(-c\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((26*c*x^2)/(5*b^2) - 2/(5*b) + (1053*c^2*x^4)/(80*b^3) + (117*c^3*x^6)/(16
*b^4))/(b^2*x^(5/2) + c^2*x^(13/2) + 2*b*c*x^(9/2)) - (117*(-c)^(5/4)*atan(
((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(17/4)) + (117*(-c)^(5/4)*atanh(((-c)^
(1/4)*x^(1/2))/b^(1/4)))/(32*b^(17/4))
```

3.349 $\int \frac{x^{3/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=264

$$-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{19/4}}+\frac{165c^{7/4}}{}$$

```
[Out] -165/112/b^3/x^(7/2)+55/16*c/b^4/x^(3/2)+1/4/b/x^(7/2)/(c*x^2+b)^2+15/16/b^
2/x^(7/2)/(c*x^2+b)-165/64*c^(7/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4)
)/b^(19/4)*2^(1/2)+165/64*c^(7/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))
/b^(19/4)*2^(1/2)-165/128*c^(7/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1
/2)*x^(1/2))/b^(19/4)*2^(1/2)+165/128*c^(7/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(19/4)*2^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{165c^{7/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{19/4}}+\frac{165c^{7/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{19/4}}-\frac{165c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}+\frac{165c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{19/4}}+\frac{55c}{16b^4x^{3/2}}-\frac{165}{112b^3x^{7/2}}+\frac{15}{16b^2x^{7/2}\left(b+cx^2\right)}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -165/(112*b^3*x^(7/2)) + (55*c)/(16*b^4*x^(3/2)) + 1/(4*b*x^(7/2)*(b + c*x^
2)^2) + 15/(16*b^2*x^(7/2)*(b + c*x^2)) - (165*c^(7/4)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) + (165*c^(7/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) - (165*c^(7/4)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19
/4)) + (165*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(64*Sqrt[2]*b^(19/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^3}\,dx &= \int \frac{1}{x^{9/2}\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{4bx^{7/2}\left(b+cx^2\right)^2} + \frac{15\int \frac{1}{x^{9/2}(b+cx^2)^2}\,dx}{8b} \\
&= \frac{1}{4bx^{7/2}\left(b+cx^2\right)^2} + \frac{15}{16b^2x^{7/2}\left(b+cx^2\right)} + \frac{165\int \frac{1}{x^{9/2}(b+cx^2)}\,dx}{32b^2} \\
&= -\frac{165}{112b^3x^{7/2}} + \frac{1}{4bx^{7/2}\left(b+cx^2\right)^2} + \frac{15}{16b^2x^{7/2}\left(b+cx^2\right)} - \frac{(165c)\int \frac{1}{x^{5/2}(b+cx^2)}\,dx}{32b^3} \\
&= -\frac{165}{112b^3x^{7/2}} + \frac{55c}{16b^4x^{3/2}} + \frac{1}{4bx^{7/2}\left(b+cx^2\right)^2} + \frac{15}{16b^2x^{7/2}\left(b+cx^2\right)} + \frac{(165c^2)\int \frac{1}{\sqrt{x}\,(b+cx^2 \cdots}}{32b^4 \cdots} \\
&= -\frac{165}{112b^3x^{7/2}} + \frac{55c}{16b^4x^{3/2}} + \frac{1}{4bx^{7/2}\left(b+cx^2\right)^2} + \frac{15}{16b^2x^{7/2}\left(b+cx^2\right)} + \frac{(165c^2)\,\mathrm{Subst}\big(\int \frac{1}{b+c \cdots}}{16b^4 \cdots} \\
&= -\frac{165}{112b^3x^{7/2}} + \frac{55c}{16b^4x^{3/2}} + \frac{1}{4bx^{7/2}\left(b+cx^2\right)^2} + \frac{15}{16b^2x^{7/2}\left(b+cx^2\right)} + \frac{(165c^2)\,\mathrm{Subst}\Big(\int \frac{\sqrt{\cdots}}{\cdots}}{32 \cdots} \\
&= -\frac{165}{112b^3x^{7/2}} + \frac{55c}{16b^4x^{3/2}} + \frac{1}{4bx^{7/2}\left(b+cx^2\right)^2} + \frac{15}{16b^2x^{7/2}\left(b+cx^2\right)} + \frac{(165c^{3/2})\,\mathrm{Subst}\Bigg(\int \cdots}{\cdots} \\
&= -\frac{165}{112b^3x^{7/2}} + \frac{55c}{16b^4x^{3/2}} + \frac{1}{4bx^{7/2}\left(b+cx^2\right)^2} + \frac{15}{16b^2x^{7/2}\left(b+cx^2\right)} - \frac{165c^{7/4}\log\left(\sqrt{b} - \cdots\right.}{64 \cdots} \\
&= -\frac{165}{112b^3x^{7/2}} + \frac{55c}{16b^4x^{3/2}} + \frac{1}{4bx^{7/2}\left(b+cx^2\right)^2} + \frac{15}{16b^2x^{7/2}\left(b+cx^2\right)} - \frac{165c^{7/4}\tan^{-1}\left(1 - \cdots\right.}{32\sqrt{2}\,b \cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.37, size = 160, normalized size = 0.61

$$
\frac{\frac{4b^{3/4}\left(-32b^3+160b^2cx^2+605bc^2x^4+385c^3x^6\right)}{x^{7/2}\left(b+cx^2\right)^2} - 1155\sqrt{2}\,c^{7/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right) + 1155\sqrt{2}\,c^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{448b^{19/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^(3/4)*(-32*b^3 + 160*b^2*c*x^2 + 605*b*c^2*x^4 + 385*c^3*x^6))/(x^(7/
2)*(b + c*x^2)^2) - 1155*Sqrt[2]*c^(7/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 1155*Sqrt[2]*c^(7/4)*ArcTanh[(Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(448*b^(19/4))
```

Maple [A]

time = 0.14, size = 156, normalized size = 0.59

method	result
derivativedivides	$-\frac{2}{7b^3x^{\frac{7}{2}}}+\frac{2c}{b^4x^{\frac{3}{2}}}+\frac{2c^2\left(\frac{\frac{23c\,x^{\frac{5}{2}}}{32}+\frac{27b\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{165\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right)\cdots\right)}{256b}\cdots\right)}{b^4}\cdots$
default	$-\frac{2}{7b^3x^{\frac{7}{2}}}+\frac{2c}{b^4x^{\frac{3}{2}}}+\frac{2c^2\left(\frac{\frac{23c\,x^{\frac{5}{2}}}{32}+\frac{27b\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{165\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right)\cdots\right)}{256b}\cdots\right)}{b^4}\cdots$
risch	$-\frac{2\left(-7c\,x^2+b\right)}{7b^4x^{\frac{7}{2}}}+\frac{23c^3x^{\frac{5}{2}}}{16b^4\left(c\,x^2+b\right)^2}+\frac{27c^2\sqrt{x}}{16b^3\left(c\,x^2+b\right)^2}+\frac{165c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{128b^5}+\frac{165\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/b^3/x^(7/2)+2*c/b^4/x^(3/2)+2/b^4*c^2*((23/32*c*x^(5/2)+27/32*b*x^(1/2
))/(c*x^2+b)^2+165/256*(b/c)^(1/4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(
1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(
1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))
```

Maxima [A]

time = 0.53, size = 246, normalized size = 0.93

$$\frac{385\,c^3x^6+605\,bc^2x^4+160\,b^2cx^2-32\,b^3}{112\left(b^4c^2x^{\frac{15}{2}}+2\,b^5cx^{\frac{11}{2}}+b^6x^{\frac{7}{2}}\right)}+\frac{165\left(\frac{2\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{7}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,c^{\frac{7}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/112*(385*c^3*x^6 + 605*b*c^2*x^4 + 160*b^2*c*x^2 - 32*b^3)/(b^4*c^2*x^(15
/2) + 2*b^5*c*x^(11/2) + b^6*x^(7/2)) + 165/128*(2*sqrt(2)*c^2*arctan(1/2*s
qrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))
/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*c^2*arctan(-1/2*sqrt(2)*(sqrt(
2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqr
t(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
 sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(7/4)*log(-sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^4
```

Fricas [A]
time = 0.38, size = 300, normalized size = 1.14

$$\frac{4620\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{14}c^2\sqrt{x}\left(-\frac{c^7}{b^{19}}\right)^{\frac{3}{4}}-\sqrt{b^{10}\sqrt{-\frac{c^7}{b^{19}}}+c^4x}\,b^{14}\left(-\frac{c^7}{b^{19}}\right)^{\frac{3}{4}}}{c^7}\right)+1155\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}\log\left(165\,b^5\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}+165\,c^2\sqrt{x}\right)-1155\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}\log\left(-165\,b^5\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}+165\,c^2\sqrt{x}\right)+4\,(385\,c^3x^6+605\,bc^2x^4+160\,b^2cx^2-32\,b^3)\sqrt{x}}{448\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/448*(4620*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-c^7/b^19)^(1/4)*arctan(
-(b^14*c^2*sqrt(x)*(-c^7/b^19)^(3/4) - sqrt(b^10*sqrt(-c^7/b^19) + c^4*x)*b
^14*(-c^7/b^19)^(3/4))/c^7) + 1155*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-
c^7/b^19)^(1/4)*log(165*b^5*(-c^7/b^19)^(1/4) + 165*c^2*sqrt(x)) - 1155*(b^
4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-c^7/b^19)^(1/4)*log(-165*b^5*(-c^7/b^1
9)^(1/4) + 165*c^2*sqrt(x)) + 4*(385*c^3*x^6 + 605*b*c^2*x^4 + 160*b^2*c*x^
2 - 32*b^3)*sqrt(x))/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.13, size = 224, normalized size = 0.85

$$\frac{165\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5}+\frac{165\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5}+\frac{165\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5}-\frac{165\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5}+\frac{23\,c^3x^{\frac{5}{2}}+27\,bc^2\sqrt{x}}{16\,(cx^2+b)^2b^4}+\frac{2\,(7\,cx^2-b)}{7\,b^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 165/64*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*
sqrt(x))/(b/c)^(1/4))/b^5 + 165/64*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^5 + 165/128*sqrt(2)*(b
*c^3)^(1/4)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 165/12
8*sqrt(2)*(b*c^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/b^5 + 1/16*(23*c^3*x^(5/2) + 27*b*c^2*sqrt(x))/((c*x^2 + b)^2*b^4) + 2/7*(
7*c*x^2 - b)/(b^4*x^(7/2))
```

Mupad [B]
time = 4.35, size = 109, normalized size = 0.41

$$\frac{\frac{10\,c\,x^2}{7\,b^2}-\frac{2}{7\,b}+\frac{605\,c^2\,x^4}{112\,b^3}+\frac{55\,c^3\,x^6}{16\,b^4}}{b^2\,x^{7/2}+c^2\,x^{15/2}+2\,b\,c\,x^{11/2}}+\frac{165\,{\left(-c\right)}^{7/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{19/4}}+\frac{165\,{\left(-c\right)}^{7/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{19/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((10*c*x^2)/(7*b^2) - 2/(7*b) + (605*c^2*x^4)/(112*b^3) + (55*c^3*x^6)/(16*
b^4))/(b^2*x^(7/2) + c^2*x^(15/2) + 2*b*c*x^(11/2)) + (165*(-c)^(7/4)*atan(
((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(19/4)) + (165*(-c)^(7/4)*atanh(((-c)^
(1/4)*x^(1/2))/b^(1/4)))/(32*b^(19/4))
```

3.350 $\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=279

$$-\frac{221}{144b^3x^{9/2}}+\frac{221c}{80b^4x^{5/2}}-\frac{221c^2}{16b^5\sqrt{x}}+\frac{1}{4bx^{9/2}(b+cx^2)^2}+\frac{17}{16b^2x^{9/2}(b+cx^2)}+\frac{221c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{21/4}}$$

```
[Out] -221/144/b^3/x^(9/2)+221/80*c/b^4/x^(5/2)+1/4/b/x^(9/2)/(c*x^2+b)^2+17/16/b
^2/x^(9/2)/(c*x^2+b)+221/64*c^(9/4)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4
))/b^(21/4)*2^(1/2)-221/64*c^(9/4)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4)
)/b^(21/4)*2^(1/2)-221/128*c^(9/4)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(
1/2)*x^(1/2))/b^(21/4)*2^(1/2)+221/128*c^(9/4)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)
*c^(1/4)*2^(1/2)*x^(1/2))/b^(21/4)*2^(1/2)-221/16*c^2/b^5/x^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{221c^{9/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{21/4}}-\frac{221c^{9/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{21/4}}-\frac{221c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{21/4}}+\frac{221c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{21/4}}-\frac{221c^2}{16b^5\sqrt{x}}+\frac{221c}{80b^4x^{5/2}}-\frac{221}{144b^3x^{9/2}}+\frac{17}{16b^2x^{9/2}(b+cx^2)}+\frac{1}{4bx^{9/2}(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -221/(144*b^3*x^(9/2)) + (221*c)/(80*b^4*x^(5/2)) - (221*c^2)/(16*b^5*Sqrt[
x]) + 1/(4*b*x^(9/2)*(b + c*x^2)^2) + 17/(16*b^2*x^(9/2)*(b + c*x^2)) + (22
1*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/
4)) - (221*c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[
2]*b^(21/4)) - (221*c^(9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4)) + (221*c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^3}\,dx = \int \frac{1}{x^{11/2}\left(b+cx^2\right)^3}\,dx$$

$$= \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17\int \frac{1}{x^{11/2}\left(b+cx^2\right)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{221\int \frac{1}{x^{11/2}\left(b+cx^2\right)}\,dx}{32b^2}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{(221c)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{\left(221c^2\right)\int \frac{1}{x^{3/2}(b+cx}}{32b^4}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{(221c}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{(221c}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{(221c}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{(221c}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} - \frac{221c^{9/}}{}$$

$$= -\frac{221}{144b^3x^{9/2}} + \frac{221c}{80b^4x^{5/2}} - \frac{221c^2}{16b^5\sqrt{x}} + \frac{1}{4bx^{9/2}\left(b+cx^2\right)^2} + \frac{17}{16b^2x^{9/2}\left(b+cx^2\right)} + \frac{221c^{9/}}{}$$

<u>Mathematica</u> [A]

time = 0.39, size = 171, normalized size = 0.61

$$\frac{-\frac{4\sqrt[4]{b}\left(160b^4-544b^3cx^2+7072b^2c^2x^4+17901bc^3x^6+9945c^4x^8\right)}{x^{9/2}\left(b+cx^2\right)^2} + 9945\sqrt{2}\,c^{9/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right) + 9945\sqrt{2}\,c^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{2880b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(1/4)*(160*b^4 - 544*b^3*c*x^2 + 7072*b^2*c^2*x^4 + 17901*b*c^3*x^6
+ 9945*c^4*x^8))/(x^(9/2)*(b + c*x^2)^2) + 9945*Sqrt[2]*c^(9/4)*ArcTan[(Sqr
t[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 9945*Sqrt[2]*c^(9/4)
*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(2880*b^
(21/4))
```

Maple [A]
time = 0.13, size = 167, normalized size = 0.60

method	result
derivativedivides	$-\frac{2c^3\left(\frac{\frac{29c\,x^{\frac{7}{2}}}{32}+\frac{33b\,x^{\frac{3}{2}}}{32}}{\left(c\,x^2+b\right)^2}+\frac{221\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.\right.}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^5}$
default	$-\frac{2c^3\left(\frac{\frac{29c\,x^{\frac{7}{2}}}{32}+\frac{33b\,x^{\frac{3}{2}}}{32}}{\left(c\,x^2+b\right)^2}+\frac{221\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.\right.}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^5}$
risch	$-\frac{2(270c^2x^4-27bc\,x^2+5b^2)}{45b^5x^{\frac{9}{2}}}-\frac{29c^4x^{\frac{7}{2}}}{16b^5(c\,x^2+b)^2}-\frac{33c^3x^{\frac{3}{2}}}{16b^4(c\,x^2+b)^2}-\frac{221c^2\sqrt{2}\,\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)}{128b^5\left(\frac{b}{c}\right)^{\frac{1}{4}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^5*c^3*((29/32*c*x^(7/2)+33/32*b*x^(3/2))/(c*x^2+b)^2+221/256/c/(b/c)^(
1/4)*2^(1/2)*(ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)
*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))-2/9/b^3/x^(9/2)-12*c^2/b^5/x^(1/2)+6/
5*c/b^4/x^(5/2)
```

Maxima [A]
time = 0.54, size = 254, normalized size = 0.91

$$-\frac{9945\,c^4x^8+17901\,bc^3x^6+7072\,b^2c^2x^4-544\,b^3cx^2+160\,b^4}{720\left(b^5c^2x^{\frac{17}{2}}+2\,b^6cx^{\frac{13}{2}}+b^7x^{\frac{9}{2}}\right)}-\frac{221\,c^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{128\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/720*(9945*c^4*x^8 + 17901*b*c^3*x^6 + 7072*b^2*c^2*x^4 - 544*b^3*c*x^2 +
 160*b^4)/(b^5*c^2*x^(17/2) + 2*b^6*c*x^(13/2) + b^7*x^(9/2)) - 221/128*c^3
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sq
rt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/
4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*
b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^5
```

Fricas [**A**]
time = 0.37, size = 317, normalized size = 1.14

$$\frac{39780\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}\arctan\left(-\frac{10793861\,b^5c^7\sqrt{x}\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}-\sqrt{-116507435287321\,b^{11}c^9\sqrt{-\frac{c^9}{b^{21}}}+116507435287321\,c^{14}x}\,b^5\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}}{10793861\,c^9}\right)-9945\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}\log\left(10793861\,b^{16}\left(-\frac{c^9}{b^{21}}\right)^{\frac{3}{4}}+10793861\,c^7\sqrt{x}\right)+9945\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}\log\left(-10793861\,b^{16}\left(-\frac{c^9}{b^{21}}\right)^{\frac{3}{4}}+10793861\,c^7\sqrt{x}\right)-4\,(9945\,c^4x^8+17901\,bc^3x^6+7072\,b^2c^2x^4-544\,b^3cx^2+160\,b^4)\sqrt{x}}{2880\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2880*(39780*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*arcta
n(-1/10793861*(10793861*b^5*c^7*sqrt(x)*(-c^9/b^21)^(1/4) - sqrt(-116507435
287321*b^11*c^9*sqrt(-c^9/b^21) + 116507435287321*c^14*x)*b^5*(-c^9/b^21)^(
1/4))/c^9) - 9945*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*l
og(10793861*b^16*(-c^9/b^21)^(3/4) + 10793861*c^7*sqrt(x)) + 9945*(b^5*c^2*
x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*log(-10793861*b^16*(-c^9/b^2
1)^(3/4) + 10793861*c^7*sqrt(x)) - 4*(9945*c^4*x^8 + 17901*b*c^3*x^6 + 7072
*b^2*c^2*x^4 - 544*b^3*c*x^2 + 160*b^4)*sqrt(x))/(b^5*c^2*x^9 + 2*b^6*c*x^7
 + b^7*x^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.45, size = 231, normalized size = 0.83

$$-\frac{221\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}-\frac{221\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}+\frac{221\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6}-\frac{221\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6}-\frac{29\,c^4x^{\frac{7}{2}}+33\,bc^3x^{\frac{3}{2}}}{16\,(cx^2+b)^2b^5}-\frac{2\,(270\,c^2x^4-27\,bcx^2+5\,b^2)}{45\,b^5x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -221/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/b^6 - 221/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 + 221/128*sqrt(2)*(b*c^
3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 - 221/128*sqr
t(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 -
1/16*(29*c^4*x^(7/2) + 33*b*c^3*x^(3/2))/((c*x^2 + b)^2*b^5) - 2/45*(270*c^
2*x^4 - 27*b*c*x^2 + 5*b^2)/(b^5*x^(9/2))
```

Mupad [B]

time = 0.14, size = 121, normalized size = 0.43

$$\frac{221\,{\left(-c\right)}^{9/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{21/4}}-\frac{221\,{\left(-c\right)}^{9/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{21/4}}-\frac{\frac{2}{9\,b}-\frac{34\,c\,x^2}{45\,b^2}+\frac{442\,c^2\,x^4}{45\,b^3}+\frac{1989\,c^3\,x^6}{80\,b^4}+\frac{221\,c^4\,x^8}{16\,b^5}}{b^2\,x^{9/2}+c^2\,x^{17/2}+2\,b\,c\,x^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (221*(-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(21/4)) - (221*(
-c)^(9/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(21/4)) - (2/(9*b) - (3
4*c*x^2)/(45*b^2) + (442*c^2*x^4)/(45*b^3) + (1989*c^3*x^6)/(80*b^4) + (221
*c^4*x^8)/(16*b^5))/(b^2*x^(9/2) + c^2*x^(17/2) + 2*b*c*x^(13/2))
```

3.351 $\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=279

$$-\frac{285}{176b^3x^{11/2}}+\frac{285c}{112b^4x^{7/2}}-\frac{95c^2}{16b^5x^{3/2}}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}+\frac{285c^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{32\sqrt{2}\,b^{23/4}}$$

```
[Out] -285/176/b^3/x^(11/2)+285/112*c/b^4/x^(7/2)-95/16*c^2/b^5/x^(3/2)+1/4/b/x^(
11/2)/(c*x^2+b)^2+19/16/b^2/x^(11/2)/(c*x^2+b)+285/64*c^(11/4)*arctan(1-c^(
1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^(1/2)-285/64*c^(11/4)*arctan(1+c^(
1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^(1/2)+285/128*c^(11/4)*ln(b^(1/2)+
x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(23/4)*2^(1/2)-285/128*c^(11/4
)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(23/4)*2^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1598, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{285c^{11/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\,b^{23/4}}-\frac{285c^{11/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\,b^{23/4}}+\frac{285c^{11/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{23/4}}-\frac{285c^{11/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}\,x\right)}{64\sqrt{2}\,b^{23/4}}-\frac{95c^2}{16b^5x^{3/2}}+\frac{285c}{112b^4x^{7/2}}-\frac{285}{176b^3x^{11/2}}+\frac{19}{16b^2x^{11/2}\left(b+cx^2\right)}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -285/(176*b^3*x^(11/2)) + (285*c)/(112*b^4*x^(7/2)) - (95*c^2)/(16*b^5*x^(3
/2)) + 1/(4*b*x^(11/2)*(b + c*x^2)^2) + 19/(16*b^2*x^(11/2)*(b + c*x^2)) +
(285*c^(11/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^
(23/4)) - (285*c^(11/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*
Sqrt[2]*b^(23/4)) + (285*c^(11/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4)) - (285*c^(11/4)*Log[Sqrt[b] + Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = \int \frac{1}{x^{13/2}\,(b+cx^2)^3}\,dx$$

$$= \frac{1}{4bx^{11/2}\,(b+cx^2)^2} + \frac{19\int \frac{1}{x^{13/2}(b+cx^2)^2}\,dx}{8b}$$

$$= \frac{1}{4bx^{11/2}\,(b+cx^2)^2} + \frac{19}{16b^2x^{11/2}\,(b+cx^2)} + \frac{285\int \frac{1}{x^{13/2}(b+cx^2)}\,dx}{32b^2}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{1}{4bx^{11/2}\,(b+cx^2)^2} + \frac{19}{16b^2x^{11/2}\,(b+cx^2)} - \frac{(285c)\int \frac{1}{x^{9/2}(b+cx^2)}\,dx}{32b^3}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} + \frac{1}{4bx^{11/2}\,(b+cx^2)^2} + \frac{19}{16b^2x^{11/2}\,(b+cx^2)} + \frac{(285c^2)\int \frac{}{x}}{32}$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\,(b+cx^2)^2} + \frac{19}{16b^2x^{11/2}\,(b+cx^2)} -$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\,(b+cx^2)^2} + \frac{19}{16b^2x^{11/2}\,(b+cx^2)} -$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\,(b+cx^2)^2} + \frac{19}{16b^2x^{11/2}\,(b+cx^2)} -$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\,(b+cx^2)^2} + \frac{19}{16b^2x^{11/2}\,(b+cx^2)} -$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\,(b+cx^2)^2} + \frac{19}{16b^2x^{11/2}\,(b+cx^2)} +$$

$$= -\frac{285}{176b^3x^{11/2}} + \frac{285c}{112b^4x^{7/2}} - \frac{95c^2}{16b^5x^{3/2}} + \frac{1}{4bx^{11/2}\,(b+cx^2)^2} + \frac{19}{16b^2x^{11/2}\,(b+cx^2)} +$$

Mathematica [A]
time = 0.39, size = 171, normalized size = 0.61

$$\frac{-\frac{4b^{3/4}(224b^4-608b^3cx^2+3040b^2c^2x^4+11495bc^3x^6+7315c^4x^8)}{x^{11/2}(b+cx^2)^2}+21945\sqrt{2}\,c^{11/4}\tan^{-1}\left(\frac{\sqrt{b}-\sqrt{c}\,x}{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}\right)-21945\sqrt{2}\,c^{11/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt{b}+\sqrt{c}\,x}\right)}{4928b^{23/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-4*b^(3/4)*(224*b^4 - 608*b^3*c*x^2 + 3040*b^2*c^2*x^4 + 11495*b*c^3*x^6
+ 7315*c^4*x^8))/(x^(11/2)*(b + c*x^2)^2) + 21945*Sqrt[2]*c^(11/4)*ArcTan[(
Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 21945*Sqrt[2]*c^(
11/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(49
28*b^(23/4))
```

Maple [A]

time = 0.12, size = 167, normalized size = 0.60

method	result
derivativedivides	$-\frac{2c^3\left(\frac{\frac{31c\,x^{\frac{5}{2}}}{32}+\frac{35b\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{285\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\sqrt{}\right.\right.}{256b}\right.}{b^5}$
default	$-\frac{2c^3\left(\frac{\frac{31c\,x^{\frac{5}{2}}}{32}+\frac{35b\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{285\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\sqrt{}\right.\right.}{256b}\right.}{b^5}$
risch	$-\frac{2\left(154c^2x^4-33bc\,x^2+7b^2\right)}{77b^5x^{\frac{11}{2}}}-\frac{31c^4x^{\frac{5}{2}}}{16b^5\left(c\,x^2+b\right)^2}-\frac{35c^3\sqrt{x}}{16b^4\left(c\,x^2+b\right)^2}-\frac{285c^3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}\right.}{128b^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^5*c^3*((31/32*c*x^(5/2)+35/32*b*x^(1/2))/(c*x^2+b)^2+285/256*(b/c)^(1/
4)/b*2^(1/2)*(ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)
*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+2*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+2*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)))-2/11/b^3/x^(11/2)-4*c^2/b^5/x^(3/2)+6
/7*c/b^4/x^(7/2)
```

Maxima [A]

time = 0.53, size = 257, normalized size = 0.92

$$-\frac{7315\,c^4x^8+11495\,bc^3x^6+3040\,b^2c^2x^4-608\,b^3cx^2+224\,b^4}{1232\left(b^5c^2x^{\frac{19}{2}}+2\,b^6cx^{\frac{15}{2}}+b^7x^{\frac{11}{2}}\right)}-\frac{285\left(\frac{2\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{2\sqrt{2}\,c^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\,\sqrt{x}\right)}{2\sqrt{\sqrt{b}\,\sqrt{c}}}\right)}{\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{c}}}+\frac{\sqrt{2}\,c^{\frac{11}{4}}\log\left(\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}\,c^{\frac{11}{4}}\log\left(-\sqrt{2}\,b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\,x+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right)}{128\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/1232*(7315*c^4*x^8 + 11495*b*c^3*x^6 + 3040*b^2*c^2*x^4 - 608*b^3*c*x^2
+ 224*b^4)/(b^5*c^2*x^(19/2) + 2*b^6*c*x^(15/2) + b^7*x^(11/2)) - 285/128*(
2*sqrt(2)*c^3*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*c^3*
arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(11/4)*log(sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(11/4)
*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^5
```

Fricas [A]
time = 0.38, size = 311, normalized size = 1.11

$$-\frac{87780\,(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6)\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^{17}c^3\sqrt{x}\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{3}{4}}-\sqrt{b^{12}\sqrt{-\frac{c^{11}}{b^{23}}}+c^6x}\,b^{17}\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{3}{4}}}{c^{11}}\right)+21945\,(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6)\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}\log\left(285\,b^6\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}+285\,c^3\sqrt{x}\right)-21945\,(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6)\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}\log\left(-285\,b^6\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}+285\,c^3\sqrt{x}\right)+4\,(7315\,c^4x^8+11495\,bc^3x^6+3040\,b^2c^2x^4-608\,b^3cx^2+224\,b^4)\sqrt{x}}{4928\,(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4928*(87780*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-c^11/b^23)^(1/4)*ar
ctan(-(b^17*c^3*sqrt(x)*(-c^11/b^23)^(3/4) - sqrt(b^12*sqrt(-c^11/b^23) + c
^6*x)*b^17*(-c^11/b^23)^(3/4))/c^11) + 21945*(b^5*c^2*x^10 + 2*b^6*c*x^8 +
b^7*x^6)*(-c^11/b^23)^(1/4)*log(285*b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x
)) - 21945*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-c^11/b^23)^(1/4)*log(-2
85*b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x)) + 4*(7315*c^4*x^8 + 11495*b*c^
3*x^6 + 3040*b^2*c^2*x^4 - 608*b^3*c*x^2 + 224*b^4)*sqrt(x))/(b^5*c^2*x^10
+ 2*b^6*c*x^8 + b^7*x^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.32, size = 243, normalized size = 0.87

$$-\frac{285\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}-\frac{285\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}-\frac{285\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c^2\log\left(\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6}+\frac{285\,\sqrt{2}\,(bc^3)^{\frac{1}{4}}\,c^2\log\left(-\sqrt{2}\,\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6}-\frac{31\,c^4x^{\frac{5}{2}}+35\,bc^3\sqrt{x}}{16\,(cx^2+b)^2b^5}-\frac{2\,(154\,c^2x^4-33\,bcx^2+7\,b^2)}{77\,b^5x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] -285/64*sqrt(2)*(b*c^3)^(1/4)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) +
 2*sqrt(x))/(b/c)^(1/4))/b^6 - 285/64*sqrt(2)*(b*c^3)^(1/4)*c^2*arctan(-1/2
*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 - 285/128*sqrt(
2)*(b*c^3)^(1/4)*c^2*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 +
 285/128*sqrt(2)*(b*c^3)^(1/4)*c^2*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + s
qrt(b/c))/b^6 - 1/16*(31*c^4*x^(5/2) + 35*b*c^3*sqrt(x))/((c*x^2 + b)^2*b^5
) - 2/77*(154*c^2*x^4 - 33*b*c*x^2 + 7*b^2)/(b^5*x^(11/2))
```

Mupad [B]
time = 4.39, size = 121, normalized size = 0.43

$$\frac{285\,{\left(-c\right)}^{11/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{23/4}}-\frac{\frac{2}{11\,b}-\frac{38\,c\,x^2}{77\,b^2}+\frac{190\,c^2\,x^4}{77\,b^3}+\frac{1045\,c^3\,x^6}{112\,b^4}+\frac{95\,c^4\,x^8}{16\,b^5}}{b^2\,x^{11/2}+c^2\,x^{19/2}+2\,b\,c\,x^{15/2}}+\frac{285\,{\left(-c\right)}^{11/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{23/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (285*(-c)^(11/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(23/4)) - (2/(11
*b) - (38*c*x^2)/(77*b^2) + (190*c^2*x^4)/(77*b^3) + (1045*c^3*x^6)/(112*b^
4) + (95*c^4*x^8)/(16*b^5))/(b^2*x^(11/2) + c^2*x^(19/2) + 2*b*c*x^(15/2))
+ (285*(-c)^(11/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(23/4))
```

3.352 $\int x^{7/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=323

$$\frac{28b^3x^{3/2}(b+cx^2)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{28b^2\sqrt{x}\,\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}-\frac{28b^{13}}{}$$

```
[Out] 28/195*b^3*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2
)+4/117*b*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c+2/13*x^(9/2)*(c*x^4+b*x^2)^(1/2)-28
/585*b^2*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2-28/195*b^(13/4)*x*(cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ell
ipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/
2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+14
/195*b^(13/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)
/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2046, 2049, 2057, 335, 311, 226, 1210}

$$\frac{14b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}-\frac{28b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}+\frac{28b^3x^{3/2}(b+cx^2)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{28b^2\sqrt{x}\,\sqrt{bx^2+cx^4}}{585c^2}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (28*b^3*x^(3/2)*(b + c*x^2))/(195*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
+ c*x^4]) - (28*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(585*c^2) + (4*b*x^(5/2)*S
qrt[b*x^2 + c*x^4])/(117*c) + (2*x^(9/2)*Sqrt[b*x^2 + c*x^4])/13 - (28*b^(1
3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[b*x^2 +
 c*x^4]) + (14*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^
(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx = \frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{1}{13}(2b)\int\frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$=\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}-\frac{(14b^2)\int\frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{117c}$$

$$=-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{(14b^3)\int \ldots}{195\ldots}$$

$$=-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{\left(14b^3x\sqrt{b+\ldots}\right.}{195\ldots}$$

$$=-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{\left(28b^3x\sqrt{b+\ldots}\right.}{\ldots}$$

$$=-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{\left(28b^{7/2}x\sqrt{b}\ldots\right.}{\ldots}$$

$$=\frac{28b^3x^{3/2}(b+cx^2)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 102, normalized size = 0.32

$$\frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(\sqrt{1+\frac{cx^2}{b}}\,(-7b^2+2bcx^2+9c^2x^4)+7b^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{117c^2\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(Sqrt[1 + (c*x^2)/b]*(-7*b^2 + 2*b*c*x^2 +
 9*c^2*x^4) + 7*b^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(117*
c^2*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.13, size = 237, normalized size = 0.73

method	result
default	$\frac{2\sqrt{c\,x^{4}+b\,x^{2}}\,\left(45c^{4}x^{8}+55b\,c^{3}x^{6}+42b^{4}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{}}{\sqrt{-}}}\right.\right.}{5}$
	$14b^{3}\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{}$
risch	$-\frac{2\sqrt{x}\,\left(-45c^{2}x^{4}-10bc\,x^{2}+14b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{585c^{2}}+\frac{}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/585*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^3*(45*c^4*x^8+55*b*c^3*x^6+42
*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-21*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(
1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*b^2*c
^2*x^4-14*b^3*c*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 73, normalized size = 0.23

$$-\frac{2\left(42\,b^{3}\sqrt{c}\ \text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(45\,c^{3}x^{4}+10\,bc^{2}x^{2}-14\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}\,\sqrt{x}\right)}{585\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/585*(42*b^3*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/
c, 0, x)) - (45*c^3*x^4 + 10*b*c^2*x^2 - 14*b^2*c)*sqrt(c*x^4 + b*x^2)*sqrt
(x))/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{7}{2}}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)*sqrt(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{7/2}\,\sqrt{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(7/2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.353 $\int x^{5/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=176

$$-\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{10b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}}{231c^{9/4}\sqrt{bx^2+cx}}$$

```
[Out] 4/77*b*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c+2/11*x^(7/2)*(c*x^4+b*x^2)^(1/2)-20/23
1*b^2*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+10/231*b^(11/4)*x*(cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipt
icF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))
*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2046, 2049, 2057, 335, 226}

$$\frac{10b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-20*b^2*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (4*b*x^(3/2)*Sqrt[b*x^2 +
 c*x^4])/(77*c) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/11 + (10*b^(11/4)*x*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}\sqrt{bx^2+cx^4}\,dx &= \frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{1}{11}(2b)\int\frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}-\frac{(10b^2)\int\frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c}\\
&= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{(10b^3)\int\frac{\sqrt{x}}{\sqrt{bx^2+}}}{231c^2}\\
&= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{\left(10b^3x\sqrt{b+cx^2}\right.}{231c^2}\\
&= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{\left(20b^3x\sqrt{b+cx^2}\right.}{}\\
&= -\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{10b^{11/4}x\left(\sqrt{b}+\right.}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 102, normalized size = 0.58

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{1+\frac{cx^2}{b}}\left(-5b^2+2bcx^2+7c^2x^4\right)+5b^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{77c^2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(Sqrt[1 + (c*x^2)/b]*(-5*b^2 + 2*b*c*x^2 + 7*c^2*x
^4) + 5*b^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(77*c^2*Sqrt[
x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.14, size = 157, normalized size = 0.89

method	result
default	$\frac{2\sqrt{c\,x^4+b\,x^2}\left(21c^4x^7+5b^3\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-}}{\sqrt{-b}}}\right.\right.}{231x^{\frac{3}{2}}(c\,x^2+b)c^3}$
risch	$-\frac{2\left(-21c^2x^4-6bc\,x^2+10b^2\right)\sqrt{x^2\left(c\,x^2+b\right)}}{231\sqrt{x}\,c^2}+\frac{10b^3\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{x}{\sqrt{-}}}}{231c^3\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(21*c^4*x^7+5*b^3*(-b*c)^(1/2)*
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2),1/2*2^(1/2))+27*b*c^3*x^5-4*b^2*c^2*x^3-10*b^3*c*x)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 68, normalized size = 0.39

$$\frac{2\left(10\,b^3\sqrt{c}\,x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(21\,c^3x^4+6\,bc^2x^2-10\,b^2c\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{231\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/231*(10*b^3*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + (21*c^3*x^4 + 6
*b*c^2*x^2 - 10*b^2*c)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**(5/2)*sqrt(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(5/2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.354 $\int x^{3/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=293

$$-\frac{4b^2x^{3/2}(b+cx^2)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}+\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\cdots}}{15\cdots}$$

```
[Out] -4/15*b^2*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)
+2/9*x^(5/2)*(c*x^4+b*x^2)^(1/2)+4/45*b*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+4/15*
b^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2
*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/
4)/(c*x^4+b*x^2)^(1/2)-2/15*b^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/
2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2046, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}+\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}-\frac{4b^2x^{3/2}(b+cx^2)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-4*b^2*x^(3/2)*(b + c*x^2))/(15*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c) + (2*x^(5/2)*Sqrt[b*x^2
 + c*x^4])/9 + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c
^(7/4)*Sqrt[b*x^2 + c*x^4]) - (2*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx = \frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4} + \frac{1}{9}(2b)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c} + \frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4} - \frac{(2b^2)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c}$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c} + \frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4} - \frac{\left(2b^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15c\sqrt{bx^2+cx^4}}$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c} + \frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4} - \frac{\left(4b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\right.}{15c\sqrt{bx^2+cx^4}}$$

$$= \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c} + \frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4} - \frac{\left(4b^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx}}\right.}{15c^{3/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{4b^2x^{3/2}(b+cx^2)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c} + \frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 86, normalized size = 0.29

$$\frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}} - b\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{9c\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b] - b*Hyper
geometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(9*c*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.10, size = 226, normalized size = 0.77

method	result

default	$-\frac{2\sqrt{c\,x^4+b\,x^2}\left(-5c^3x^6+6b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\right.\right.}{45x^{\frac{3}{2}}(}$
risch	$\frac{2\sqrt{x}\,(5c\,x^2+2b)\sqrt{x^2\,(c\,x^2+b)}}{45c}-\frac{2b^2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(2\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^2*(-5*c^3*x^6+6*b^3*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))-3*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((
-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-7*b*c^2*x^4-2*b^2*c*x^
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 61, normalized size = 0.21

$$\frac{2\left(6\,b^2\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,(5\,c^2x^2+2\,bc)\sqrt{x}\right)}{45\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(6*b^2*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c,
0, x)) + sqrt(c*x^4 + b*x^2)*(5*c^2*x^2 + 2*b*c)*sqrt(x))/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\sqrt{x^2\,(b+cx^2)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**(3/2)*sqrt(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\,\sqrt{c\,x^4+b\,x^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(3/2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.355 $\int \sqrt{x}\,\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=146

$$\frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}-\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/7*x^(3/2)*(c*x^4+b*x^2)^(1/2)+4/21*b*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-2/21*b
^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*
2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(5/4
)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2046, 2049, 2057, 335, 226}

$$-\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(21*c*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/
7 - (2*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*c^(5/4)*Sqrt
[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt{x}\,\sqrt{bx^2+cx^4}\,dx = \frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}+\frac{1}{7}(2b)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}-\frac{(2b^2)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21c}$$

$$= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}-\frac{\left(2b^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{21c\sqrt{bx^2+cx^4}}$$

$$= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}-\frac{\left(4b^2x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,$$

$$= \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}-\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}}{21c^{5/4}\sqrt{bx^2+cx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 9.54, size = 86, normalized size = 0.59

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\left(b+cx^2\right)\sqrt{1+\frac{cx^2}{b}}-b\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{7c\sqrt{x}\,\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b] - b*Hypergeometri
c2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(7*c*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.13, size = 145, normalized size = 0.99

method	result
default	$-\frac{2\sqrt{c\,x^4+b\,x^2}\left(b^2\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\sqrt{\right.\right.}{21x^{\frac{3}{2}}\left(c\,x^2+b\right)c^2}$
risch	$\frac{2\left(3c\,x^2+2b\right)\sqrt{x^2\left(c\,x^2+b\right)}}{21\sqrt{x}\,c}-\frac{2b^2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\right.}{21c^2\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}\left(c\,x\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(b^2*(-b*c)^(1/2)*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2))-3*c^3*x^5-5*b*c^2*x^3-2*b^2*c*x)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 58, normalized size = 0.40

$$-\frac{2\left(2\,b^{2}\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^{4}+bx^{2}}\,(3\,c^{2}x^{2}+2\,bc)\sqrt{x}\right)}{21\,c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] -2/21*(2*b^2*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x
^2)*(3*c^2*x^2 + 2*b*c)*sqrt(x))/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x}\,\sqrt{x^{2}\,(b+cx^{2})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(sqrt(x)*sqrt(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{x}\,\sqrt{c\,x^{4}+b\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(1/2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.356 $\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$

Optimal. Leaf size=263

$$\frac{4bx^{3/2}(b+cx^2)}{5\sqrt{c}\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4}-\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\right.}{5c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 4/5*b*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+2/5
*x^(1/2)*(c*x^4+b*x^2)^(1/2)-4/5*b^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)+2/5*b^(5/4)*x*(cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1
/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2
)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2046, 2057, 335, 311, 226, 1210}

$$\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}-\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4}+\frac{4bx^{3/2}(b+cx^2)}{5\sqrt{c}\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/Sqrt[x],x]
```

```
[Out] (4*b*x^(3/2)*(b + c*x^2))/(5*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x
^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (4*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2*b^(5/4)*x*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{1}{5}(2b)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{\left(2bx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}}$$
$$= \frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{\left(4bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{bx^2+cx^4}}$$
$$= \frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{\left(4b^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{c}\,\sqrt{bx^2+cx^4}} - \frac{\left(4b^{3/2}x\sqrt{\right.}{}$$
$$= \frac{4bx^{3/2}(b+cx^2)}{5\sqrt{c}\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{2}{5}\sqrt{x}\,\sqrt{bx^2+cx^4} - \frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{}{\left(\sqrt{\right.}}}}{5c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.82, size = 57, normalized size = 0.22

$$\frac{2\sqrt{x}\,\sqrt{x^2(b+cx^2)}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{3\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2
)/b)])/(3*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.10, size = 213, normalized size = 0.81

method	result
default	$\frac{2\sqrt{c\,x^4+b\,x^2}\left(2b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-b^2\right.}{5x^{\frac{3}{2}}(c\,x^2+b)c}$

risch	$\frac{2\sqrt{x}\,\sqrt{x^2\,(c\,x^2+b)}}{5}+\frac{2b\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\ \mathrm{Ell}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c*(2*b^2*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*
c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))-b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+c^2*x^4+b*c*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 48, normalized size = 0.18

$$-\frac{2\left(2\,b\sqrt{c}\ \mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}\,c\sqrt{x}\right)}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(2*b*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0,
 x)) - sqrt(c*x^4 + b*x^2)*c*sqrt(x))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(b+cx^2)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/sqrt(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(1/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(1/2), x)
```

3.357 $\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}$$

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(1/2)+2/3*b^(3/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b
)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2046, 2057, 335, 226}

$$\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) + (2*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx &= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{1}{3}(2b)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{\left(2bx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{3\sqrt{bx^2+cx^4}}\\
&= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{\left(4bx\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{bx^2+cx^4}}\\
&= \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}+\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.85, size = 55, normalized size = 0.47

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)}{\sqrt{x}\,\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)])/(
Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.09, size = 130, normalized size = 1.10

method	result
default	$\frac{2\sqrt{c\,x^{4}+b\,x^{2}}\left(b\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right.\right.}{3x^{\frac{3}{2}}\left(c\,x^{2}+b\right)c}$
risch	$\frac{2\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{3\sqrt{x}}+\frac{2b\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)}{\sqrt{-bc}}}\right.}{3c\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(b*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x
*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))+c^2*x^3+b*c*x)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 43, normalized size = 0.36

$$\frac{2\left(2\,b\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^{4}+bx^{2}}\,c\sqrt{x}\right)}{3\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^2)*
c*sqrt(x))/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(b+cx^2)}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(3/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(3/2),x)
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(3/2), x)
```

3.358 $\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$

Optimal. Leaf size=254

$$\frac{4\sqrt{c}\,x^{3/2}(b+cx^2)}{\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}-\frac{4\sqrt[4]{b}\,\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{\sqrt{bx^2+cx^4}}$$

```
[Out] 4*x^(3/2)*(c*x^2+b)*c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2*(c*x^
4+b*x^2)^(1/2)/x^(3/2)-4*b^(1/4)*c^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)+2*b^(1/4)*c^(1/4)*x*(cos(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1
/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2
)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2045, 2057, 335, 311, 226, 1210}

$$\frac{2\sqrt[4]{b}\,\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}}-\frac{4\sqrt[4]{b}\,\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}}+\frac{4\sqrt{c}\,x^{3/2}(b+cx^2)}{\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(5/2),x]
```

```
[Out] (4*Sqrt[c]*x^(3/2)*(b + c*x^2))/((Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4])
 - (2*Sqrt[b*x^2 + c*x^4])/x^(3/2) - (4*b^(1/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*
Sqrt[x])/b^(1/4)], 1/2])/Sqrt[b*x^2 + c*x^4] + (2*b^(1/4)*c^(1/4)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/Sqrt[b*x^2 + c*x^4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + (2c)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + \frac{\left(2cx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + \frac{\left(4cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} + \frac{\left(4\sqrt{b}\sqrt{c}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx^2+cx^4}} - \frac{\left(4\sqrt{b}\right.}{}$$

$$= \frac{4\sqrt{c}\,x^{3/2}(b+cx^2)}{\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} - \frac{4\sqrt[4]{b}\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+}{\left(\sqrt{b}+\right.}}}{\sqrt{bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 55, normalized size = 0.22

$$-\frac{2\sqrt{x^2(b+cx^2)}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)}{x^{3/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(5/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((c*x^2)/b)])
/(x^(3/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.12, size = 202, normalized size = 0.80

method	result
default	$\frac{2\sqrt{c\,x^4+b\,x^2}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b-\sqrt{\right.}{x^{\frac{3}{2}}(c\,x^2+b)}$

risch	$-\frac{2\sqrt{x^2(cx^2+b)}}{x^{\frac{3}{2}}}+\frac{2\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\ \mathrm{EllipticE}\left(\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2),1/2*2^(1/2))*b-c*x^2-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 48, normalized size = 0.19

$$-\frac{2\left(2\sqrt{c}\,x^2\mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2*(2*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0,
 x)) + sqrt(c*x^4 + b*x^2)*sqrt(x))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(5/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(5/2), x)
```

3.359 $\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{bx^2+cx^4}}$$

```
[Out] -2/3*(c*x^4+b*x^2)^(1/2)/x^(5/2)+2/3*c^(3/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+
b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2045, 2057, 335, 226}

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(7/2),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*x^(5/2)) + (2*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{1}{3}(2c)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{\left(2cx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3\sqrt{bx^2+cx^4}}\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{\left(4cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{bx^2+cx^4}}\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\right.}{3\sqrt[4]{b}\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.48

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)}{3x^{5/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(7/2),x]
```

[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((c*x^2)/b)])
/(3*x^(5/2)*Sqrt[1 + (c*x^2)/b])

Maple [A]
time = 0.11, size = 125, normalized size = 1.06

method	result
default	$\frac{2\sqrt{c\,x^{4}+b\,x^{2}}\,\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}}{3x^{\frac{5}{2}}\left(c\,x^{2}+b\right)}$
risch	$-\frac{2\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{3x^{\frac{5}{2}}}+\frac{2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\sqrt{-}\right.}{\sqrt{-}}}\right.}{3\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(5/2)/(c*x^2+b)*(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*
c)^(1/2)*x-c*x^2-b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(7/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 41, normalized size = 0.35

$$\frac{2\left(2\sqrt{c}\,x^{3}\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\sqrt{cx^{4}+bx^{2}}\,\sqrt{x}\right)}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="fricas")

[Out] 2/3*(2*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x^2)*
sqrt(x))/x^3

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 (b + cx^2)}}{x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(7/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4 + b\,x^2}}{x^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(7/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(7/2), x)
```

3.360 $\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$

Optimal. Leaf size=293

$$\frac{4c^{3/2}x^{3/2}(b+cx^2)}{5b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}-\frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}}-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 4/5*c^(3/2)*x^(3/2)*(c*x^2+b)/b/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/5
*(c*x^4+b*x^2)^(1/2)/x^(7/2)-4/5*c*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-4/5*c^(5/4
)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/
2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*
x^4+b*x^2)^(1/2)+2/5*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^
(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2045, 2050, 2057, 335, 311, 226, 1210}

$$\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}+\frac{4c^{3/2}x^{3/2}(b+cx^2)}{5b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}}-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(9/2),x]
```

```
[Out] (4*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x
^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*x^(7/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(5*b
*x^(3/2)) - (4*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/
4)*Sqrt[b*x^2 + c*x^4]) + (2*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^
2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
1/2])/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} + \frac{1}{5}(2c)\int \frac{1}{\sqrt{x}\,\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{(2c^2)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{\left(2c^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{\left(4c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} + \frac{\left(4c^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right.}{5\sqrt{b}\,\sqrt{bx^2+cx^4}}$$

$$= \frac{4c^{3/2}x^{3/2}(b+cx^2)}{5b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 57, normalized size = 0.19

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\;{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5x^{7/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(9/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((c*x^2)/b)]
)/(5*x^(7/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.11, size = 224, normalized size = 0.76

method	result
default	$\frac{2\sqrt{c\,x^4+b\,x^2}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^2-\right.}{5x^{\frac{7}{2}}\,(c\,x}$
risch	$-\frac{2(2c\,x^2+b)\sqrt{x^2\,(c\,x^2+b)}}{5x^{\frac{7}{2}}b}+\frac{2c\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-b}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(7/2)/(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*
c*x^2-((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-2*c^2*x^4-3*b*c*x^2-b^2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 59, normalized size = 0.20

$$-\frac{2\left(2\,c^{\frac{3}{2}}x^4\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,(2\,cx^2+b)\sqrt{x}\right)}{5\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/5*(2*c^(3/2)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c,
0, x)) + sqrt(c*x^4 + b*x^2)*(2*c*x^2 + b)*sqrt(x))/(b*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(9/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(9/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(9/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(9/2), x)
```

3.361 $\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/7*(c*x^4+b*x^2)^(1/2)/x^(9/2)-4/21*c*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)-2/21*
c^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2
*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/
4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2045, 2050, 2057, 335, 226}

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(11/2),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*x^(9/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(21*b*x^(5
/2)) - (2*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*b^(5/4)*S
qrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx &= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}+\frac{1}{7}(2c)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{(2c^2)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21b}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{\left(2c^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{21b\sqrt{bx^2+cx^4}}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{\left(4c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{21b\sqrt{bx^2+cx^4}}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan\right.}{21b^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 57, normalized size = 0.39

$$-\frac{2\sqrt{x^2(b+cx^2)}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7x^{9/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(11/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-7/4, -1/2, -3/4, -((c*x^2)/b)]
)/(7*x^(9/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.11, size = 142, normalized size = 0.97

method	result
default	$-\frac{2\sqrt{c\,x^4+b\,x^2}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-}\right.}{21x^{\frac{9}{2}}(c\,x^2+b)b}$
risch	$-\frac{2(2c\,x^2+3b)\sqrt{x^2\,(c\,x^2+b)}}{21x^{\frac{9}{2}}b}-\frac{2c\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}}{21b\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(c*x^4+b*x^2)^(1/2)/x^(9/2)/(c*x^2+b)*(((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-
b*c)^(1/2)*c*x^3+2*c^2*x^4+5*b*c*x^2+3*b^2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 53, normalized size = 0.36

$$-\frac{2\left(2\,c^{\frac{3}{2}}x^{5}\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^{4}+bx^{2}}\,(2\,cx^{2}+3\,b)\sqrt{x}\right)}{21\,bx^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] -2/21*(2*c^(3/2)*x^5*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^2
)*(2*c*x^2 + 3*b)*sqrt(x))/(b*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^{2}\,(b+cx^{2})}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(11/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(11/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(11/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(11/2), x)
```

3.362 $\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

Optimal. Leaf size=323

$$-\frac{4c^{5/2}x^{3/2}(b+cx^2)}{15b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}-\frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}+\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right.}{}$$

```
[Out] -4/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)
-2/9*(c*x^4+b*x^2)^(1/2)/x^(11/2)-4/45*c*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)+4/15
*c^2*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)+4/15*c^(9/4)*x*(cos(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(
sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c
*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)-2/15*c^(9/
4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1
/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c
*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2045, 2050, 2057, 335, 311, 226, 1210}

$$-\frac{2c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}+\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/2}x^{3/2}(b+cx^2)}{15b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}-\frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}}-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(13/2),x]
```

```
[Out] (-4*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*x^(11/2)) - (4*c*Sqrt[b*x^2 + c*x^4])
/(45*b*x^(7/2)) + (4*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^(3/2)) + (4*c^(9/4)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*x^2 + c*x^4
]) - (2*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqr
t[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}+\frac{1}{9}(2c)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}-\frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}}-\frac{(2c^2)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{15b}$$
$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}-\frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}-\frac{(2c^3)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$
$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}-\frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}-\frac{\left(2c^3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}}{15b^2\sqrt{bx^2+cx^4}}$$
$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}-\frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}-\frac{\left(4c^3x\sqrt{b+cx^2}\right)\text{Subst}\Big(\int}{15b^2\sqrt{bx^2+}}$$
$$= -\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}-\frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}-\frac{\left(4c^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\Big(\int}{15b^{3/2}\sqrt{bx^2}}$$
$$= -\frac{4c^{5/2}x^{3/2}(b+cx^2)}{15b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}-\frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}}+\frac{4c^2\sqrt{bx^2+}}{15b^2x^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 57, normalized size = 0.18

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\ {}_2F_1\left(-\frac{9}{4},-\frac{1}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9x^{11/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(13/2),x]
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-9/4, -1/2, -5/4, -((c*x^2)/b)]
)/(9*x^(11/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.12, size = 239, normalized size = 0.74

method	result
default	$-\frac{2\sqrt{c\,x^4+b\,x^2}\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^2x^4 \right.}{45}$
	$2c^2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}$
risch	$-\frac{2(-6c^2x^4+2bc\,x^2+5b^2)\sqrt{x^2\left(c\,x^2+b\right)}}{45x^{\frac{11}{2}}b^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(c*x^4+b*x^2)^(1/2)/x^(11/2)/(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*b*c^2*x^4-3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-6*c^3*x^6-4*b*c^2*x^4+7*
b^2*c*x^2+5*b^3)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 72, normalized size = 0.22

$$\frac{2\left(6\,c^{\frac{5}{2}}x^6\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(6\,c^2x^4-2\,bcx^2-5\,b^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{45\,b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] 2/45*(6*c^(5/2)*x^6*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c,
0, x)) + (6*c^2*x^4 - 2*b*c*x^2 - 5*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^2*
x^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(13/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(13/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(13/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(13/2), x)
```

3.363 $\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}+\frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\right.}{231b^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/11*(c*x^4+b*x^2)^(1/2)/x^(13/2)-4/77*c*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)+20/
231*c^2*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)+10/231*c^(11/4)*x*(cos(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Elli
pticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2
))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2045, 2050, 2057, 335, 226}

$$\frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + c*x^4]/x^(15/2),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*x^(13/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/(77*b*x^
(9/2)) + (20*c^2*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) + (10*c^(11/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}+\frac{1}{11}(2c)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{(10c^2)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b}$$
$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}+\frac{(10c^3)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231b^2}$$
$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}+\frac{\left(10c^3x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b}}}{231b^2\sqrt{bx^2+cx^4}}$$
$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}+\frac{\left(20c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\right.}{231b^2\sqrt{bx^2+}}$$
$$= -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}+\frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{}{(\sqrt{}}}}{231}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 57, normalized size = 0.32

$$-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{2};-\frac{7}{4};-\frac{cx^2}{b}\right)}{11x^{13/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + c*x^4]/x^(15/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((c*x^2)/b)
])/(11*x^(13/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.12, size = 156, normalized size = 0.89

method	result
default	$\frac{2\sqrt{c\,x^4+b\,x^2}\left(5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\right.}{231x^{\frac{13}{2}}\left(c\,x^2+b\right)b^2}$

risch	$-\frac{2(-10c^2x^4+6bc\,x^2+21b^2)\sqrt{x^2\,(c\,x^2+b)}}{231x^{\frac{13}{2}}b^2}+\frac{10c^2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{x\cdots}{\sqrt{-\cdots}}}}{231b^2\sqrt{c\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(1/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231*(c*x^4+b*x^2)^(1/2)/x^(13/2)/(c*x^2+b)*(5*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*(-b*c)^(1/2)*c^2*x^5+10*c^3*x^6+4*b*c^2*x^4-27*b^2*c*x^2-21*b^3)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 64, normalized size = 0.36

$$\frac{2\left(10\,c^{\frac{5}{2}}x^7\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(10\,c^2x^4-6\,bcx^2-21\,b^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{231\,b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] 2/231*(10*c^(5/2)*x^7*weierstrassPInverse(-4*b/c, 0, x) + (10*c^2*x^4 - 6*b
*c*x^2 - 21*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^2*x^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(1/2)/x**(15/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(1/2)/x^(15/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(1/2)/x^(15/2), x)
```

3.364 $\int x^{3/2}(bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=350

$$\frac{56b^4x^{3/2}(b+cx^2)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{56b^3\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+$$

```
[Out] 2/17*x^(5/2)*(c*x^4+b*x^2)^(3/2)+56/1105*b^4*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(
1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+8/663*b^2*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c
+12/221*b*x^(9/2)*(c*x^4+b*x^2)^(1/2)-56/3315*b^3*x^(1/2)*(c*x^4+b*x^2)^(1/
2)/c^2-56/1105*b^(17/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2)
)^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+28/1105*b^(17/4)*x*(cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ell
ipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/
2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2046, 2049, 2057, 335, 311, 226, 1210}

$$\frac{28b^{17/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}-\frac{56b^{17/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}+\frac{56b^4x^{3/2}(b+cx^2)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{56b^3\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (56*b^4*x^(3/2)*(b + c*x^2))/(1105*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
 + c*x^4]) - (56*b^3*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(3315*c^2) + (8*b^2*x^(5/
2)*Sqrt[b*x^2 + c*x^4])/(663*c) + (12*b*x^(9/2)*Sqrt[b*x^2 + c*x^4])/221 +
(2*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/17 - (56*b^(17/4)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (28*b^(17/4)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{17}(6b)\int x^{7/2}\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{221}\left(12b^2\right)\int\frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx$$
$$=\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(28b^3\right)\int}{}$$
$$=-\frac{56b^3\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/}$$
$$=-\frac{56b^3\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/}$$
$$=-\frac{56b^3\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/}$$
$$=-\frac{56b^3\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/}$$
$$=\frac{56b^4x^{3/2}(b+cx^2)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{56b^3\sqrt{x}\,\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+}}{663c}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 101, normalized size = 0.29

$$\frac{2\sqrt{x}\,\sqrt{x^2\left(b+cx^2\right)}\left(-\left(\left(7b-13cx^2\right)\left(b+cx^2\right)^2\sqrt{1+\frac{cx^2}{b}}\right)+7b^3\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{221c^2\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^(3/2),x]

[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((7*b - 13*c*x^2)*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]) + 7*b^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(221*c^2*Sqrt[1 + (c*x^2)/b])

Maple [A]
time = 0.10, size = 248, normalized size = 0.71

method	result
default	$\frac{2(cx^4+bx^2)^{\frac{3}{2}}\left(195c^5x^{10}+480bc^4x^8+305b^2c^3x^6+84b^5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\ldots}\right.\right.}{\ldots}$
	$28b^4\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-b\ldots}}{c}\right.}{\sqrt{-bc\ldots}}}$
risch	$-\frac{2\sqrt{x}\left(-195c^3x^6-285bc^2x^4-20b^2cx^2+28b^3\right)\sqrt{x^2\left(cx^2+b\right)}}{3315c^2}+\frac{\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/3315*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^3*(195*c^5*x^10+480*b*c^4*x^8+305*b^2*c^3*x^6+84*b^5*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-42*b^5*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-8*b^3*c^2*x^4-28*b^4*c*x^2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 84, normalized size = 0.24

$$-\frac{2\left(84\,b^4\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(195\,c^4x^6+285\,bc^3x^4+20\,b^2c^2x^2-28\,b^3c\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{3315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3315*(84*b^4*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b
/c, 0, x)) - (195*c^4*x^6 + 285*b*c^3*x^4 + 20*b^2*c^2*x^2 - 28*b^3*c)*sqrt
(c*x^4 + b*x^2)*sqrt(x))/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(3/2)*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\,{\left(c\,x^4+b\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(3/2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.365 $\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=203

$$-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\cdots}{\cdots}$$

```
[Out] 2/15*x^(3/2)*(c*x^4+b*x^2)^(3/2)+8/385*b^2*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c+4/
55*b*x^(7/2)*(c*x^4+b*x^2)^(1/2)-8/231*b^3*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+
4/231*b^(15/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2046, 2049, 2057, 335, 226}

$$\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-8*b^3*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (8*b^2*x^(3/2)*Sqrt[b*x^2
+ c*x^4])/(385*c) + (4*b*x^(7/2)*Sqrt[b*x^2 + c*x^4])/55 + (2*x^(3/2)*(b*x^
2 + c*x^4)^(3/2))/15 + (4*b^(15/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{5}(2b)\int x^{5/2}\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{1}{55}\left(4b^2\right)\int\frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx$$
$$=\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}-\frac{\left(4b^3\right)\int\frac{}{\sqrt{}}}{}$$
$$=-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+\right.$$
$$=-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+\right.$$
$$=-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+\right.$$
$$=-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 101, normalized size = 0.50

$$\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(-\left(\left(5b-11cx^2\right)\left(b+cx^2\right)^2\sqrt{1+\frac{cx^2}{b}}\right)+5b^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{165c^2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((5*b - 11*c*x^2)*(b + c*x^2)^2*Sqrt[1 + (c*x^2)
/b]) + 5*b^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(165*c^2*Sqr
t[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.12, size = 168, normalized size = 0.83

method	result
default	$\frac{2(cx^4+bx^2)^{\frac{3}{2}}\left(77c^5x^9+196b\,c^4x^7+10b^4\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticF}\left(\sqrt{\frac{c}{}}\right.\right.}{1155x^{\frac{7}{2}}(c\,x^2+b)^2c^3}$
risch	$-\frac{2(-77c^3x^6-119b\,c^2x^4-12b^2c\,x^2+20b^3)\sqrt{x^2(c\,x^2+b)}}{1155\sqrt{x}\,c^2}+\frac{4b^4\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(77*c^5*x^9+196*b*c^4*x^7+10
*b^4*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+131*b^2*c^3*x^5-8*b^3*c^2*x^3
-20*b^4*c*x)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 79, normalized size = 0.39

$$\frac{2\left(20\,b^4\sqrt{c}\,x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(77\,c^4x^6+119\,bc^3x^4+12\,b^2c^2x^2-20\,b^3c)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{1155\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(20*b^4*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + (77*c^4*x^6 +
119*b*c^3*x^4 + 12*b^2*c^2*x^2 - 20*b^3*c)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^
3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x} \, \left(c\,x^4 + b\,x^2\right)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(1/2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.366 $\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=320

$$-\frac{8b^3x^{3/2}(b+cx^2)}{65c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{x}\,\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\,\left(bx^2+cx^4\right)^{3/2}+$$

```
[Out] 2/13*(c*x^4+b*x^2)^(3/2)*x^(1/2)-8/65*b^3*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2
)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+4/39*b*x^(5/2)*(c*x^4+b*x^2)^(1/2)+8/195*b
^2*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+8/65*b^(13/4)*x*(cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-4/65*b^(13/4)
*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2
))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x
^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2046, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65c^{7/4}\sqrt{bx^2+cx^4}}+\frac{8b^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65c^{7/4}\sqrt{bx^2+cx^4}}-\frac{8b^3x^{3/2}(b+cx^2)}{65c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{x}\,\sqrt{bx^2+cx^4}}{195c}+\frac{2}{13}\sqrt{x}\,\left(bx^2+cx^4\right)^{3/2}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/Sqrt[x],x]
```

```
[Out] (-8*b^3*x^(3/2)*(b + c*x^2))/(65*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) + (8*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(195*c) + (4*b*x^(5/2)*Sqrt[
b*x^2 + c*x^4])/39 + (2*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/13 + (8*b^(13/4)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*c^(7/4)*Sqrt[b*x^2 + c*x^4]) -
 (4*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)
^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*c^(7/4)*Sqrt[b
*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \frac{2}{13}\sqrt{x}\,(bx^2+cx^4)^{3/2}+\frac{1}{13}(6b)\int x^{3/2}\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\,(bx^2+cx^4)^{3/2}+\frac{1}{39}(4b^2)\int\frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\,(bx^2+cx^4)^{3/2}-\frac{(4b^3)\int\frac{}{\sqrt{bx^2}}}{65}$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\,(bx^2+cx^4)^{3/2}-\frac{\left(4b^3x\sqrt{b+c}\right.}{65c}$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\,(bx^2+cx^4)^{3/2}-\frac{\left(8b^3x\sqrt{b+c}\right.}{}$$

$$= \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}+\frac{2}{13}\sqrt{x}\,(bx^2+cx^4)^{3/2}-\frac{\left(8b^{7/2}x\sqrt{b+}\right.}{}$$

$$= -\frac{8b^3x^{3/2}(b+cx^2)}{65c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 90, normalized size = 0.28

$$\frac{2\sqrt{x}\,\sqrt{x^2\,(b+cx^2)}\left((b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}-b^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{13c\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/Sqrt[x],x]
```

[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(13*c*Sqrt[1 + (c*x^2)/b])

Maple [A]
time = 0.12, size = 237, normalized size = 0.74

method	result
default	$-\frac{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(-15c^{4}x^{8}-40b\,c^{3}x^{6}+12b^{4}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-}}{\sqrt{-bc}}}\right.\right.}{195}$
	$4b^{3}\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-b}}}$
risch	$\frac{2\sqrt{x}\,\left(15c^{2}x^{4}+25bc\,x^{2}+4b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{195c}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/195*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^2*(-15*c^4*x^8-40*b*c^3*x^6+12*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-6*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-29*b^2*c^2*x^4-4*b^3*c*x^2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 72, normalized size = 0.22

$$\frac{2\left(12\,b^{3}\sqrt{c}\,\operatorname{weierstrassZeta}\left(-\frac{4\,b}{c},0,\operatorname{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(15\,c^{3}x^{4}+25\,bc^{2}x^{2}+4\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}\,\sqrt{x}\right)}{195\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/195*(12*b^3*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c
, 0, x)) + (15*c^3*x^4 + 25*b*c^2*x^2 + 4*b^2*c)*sqrt(c*x^4 + b*x^2)*sqrt(x
))/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/sqrt(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(1/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(1/2), x)
```

3.367 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}}+\frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4}+\frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}}-\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan\right.}{77c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/11*(c*x^4+b*x^2)^(3/2)/x^(1/2)+12/77*b*x^(3/2)*(c*x^4+b*x^2)^(1/2)+8/77*b
^2*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-4/77*b^(11/4)*x*(cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2046, 2049, 2057, 335, 226}

$$-\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}}+\frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}}+\frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(3/2),x]
```

```
[Out] (8*b^2*Sqrt[b*x^2 + c*x^4])/(77*c*Sqrt[x]) + (12*b*x^(3/2)*Sqrt[b*x^2 + c*x
^4])/77 + (2*(b*x^2 + c*x^4)^(3/2))/(11*Sqrt[x]) - (4*b^(11/4)*x*(Sqrt[b] +
 Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c
^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}} + \frac{1}{11}(6b)\int \sqrt{x}\,\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}} + \frac{1}{77}(12b^2)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}} - \frac{(4b^3)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77c}$$

$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}} - \frac{\left(4b^3x\sqrt{b+cx^2}\right)\int \frac{}{\sqrt{x}}}{77c\sqrt{bx^2+c}}$$

$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}} - \frac{\left(8b^3x\sqrt{b+cx^2}\right)\text{Subst}}{77c\sqrt{b}}$$

$$= \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}} - \frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 90, normalized size = 0.52

$$\frac{2\sqrt{x^2(b+cx^2)}\left((b+cx^2)^2\sqrt{1+\frac{cx^2}{b}} - b^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{11c\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b^2*Hypergeom
etric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(11*c*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.10, size = 157, normalized size = 0.91

method	result

default	$-\frac{2(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(-7c^4x^7+2b^3\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-b}}{\sqrt{-bc}}}\right.\right.}{77x^{\frac{7}{2}}(c\,x^2+b)^2c^2}$
risch	$\frac{2(7c^2x^4+13bc\,x^2+4b^2)\sqrt{x^2\,(c\,x^2+b)}}{77\sqrt{x}\,c}-\frac{4b^3\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}}{77c^2\sqrt{c\,x^3+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(-7*c^4*x^7+2*b^3*(-b*c)^(1/2
)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))-20*b*c^3*x^5-17*b^2*c^2*x^3-4*b^3*c*x)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 69, normalized size = 0.40

$$-\frac{2\left(4\,b^3\sqrt{c}\,x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-(7\,c^3x^4+13\,bc^2x^2+4\,b^2c)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{77\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*b^3*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - (7*c^3*x^4 + 13*
b*c^2*x^2 + 4*b^2*c)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(3/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(3/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(3/2), x)
```

3.368 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}} \, dx$

Optimal. Leaf size=290

$$\frac{8b^2x^{3/2}(b+cx^2)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{4}{15}b\sqrt{x}\,\sqrt{bx^2+cx^4}+\frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}-\frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b\cdots}{\left(\sqrt{b\cdots}\right.}}}{15c^{3\cdots}}$$

```
[Out] 2/9*(c*x^4+b*x^2)^(3/2)/x^(3/2)+8/15*b^2*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+4/15*b*x^(1/2)*(c*x^4+b*x^2)^(1/2)-8/15*b^(
9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^
(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/
(c*x^4+b*x^2)^(1/2)+4/15*b^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+
x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2046, 2057, 335, 311, 226, 1210}

$$\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}-\frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}+\frac{8b^2x^{3/2}(b+cx^2)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{4}{15}b\sqrt{x}\,\sqrt{bx^2+cx^4}+\frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(5/2),x]
```

```
[Out] (8*b^2*x^(3/2)*(b + c*x^2))/(15*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/15 + (2*(b*x^2 + c*x^4)^(3/2))/
(9*x^(3/2)) - (8*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^
(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} + \frac{1}{3}(2b)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$

$$= \frac{4}{15}b\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} + \frac{1}{15}(4b^2)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4}{15}b\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} + \frac{\left(4b^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15\sqrt{bx^2+cx^4}}$$

$$= \frac{4}{15}b\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} + \frac{\left(8b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,\right.}{15\sqrt{bx^2+cx^4}}$$

$$= \frac{4}{15}b\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} + \frac{\left(8b^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,d\right.}{15\sqrt{c}\,\sqrt{bx^2+cx^4}}$$

$$8b^{9}$$

$$= \frac{8b^2x^{3/2}(b+cx^2)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{4}{15}b\sqrt{x}\,\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 58, normalized size = 0.20

$$\frac{2b\sqrt{x}\,\sqrt{x^2(b+cx^2)}\;{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{3\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x
^2)/b)])/(3*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.10, size = 226, normalized size = 0.78

method	result

default	$\frac{2(cx^4+bx^2)^{\frac{3}{2}}\left(5c^3x^6+12b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-\cdots\right.}{45x^{\frac{7}{2}}(cx^2\cdots}$
risch	$\frac{2\sqrt{x}\,(5cx^2+11b)\sqrt{x^2(cx^2+b)}}{45}+\frac{4b^2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\left(\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c*(5*c^3*x^6+12*b^3*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))-6*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF((
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+16*b*c^2*x^4+11*b^2*c*x
^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 62, normalized size = 0.21

$$-\frac{2\left(12\,b^2\sqrt{c}\,\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}\,(5\,c^2x^2+11\,bc)\sqrt{x}\right)}{45\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/45*(12*b^2*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c
, 0, x)) - sqrt(c*x^4 + b*x^2)*(5*c^2*x^2 + 11*b*c)*sqrt(x))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(5/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(5/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(5/2), x)
```

3.369 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx$

Optimal. Leaf size=143

$$\frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}+\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}$$

```
[Out] 2/7*(c*x^4+b*x^2)^(3/2)/x^(5/2)+4/7*b*(c*x^4+b*x^2)^(1/2)/x^(1/2)+4/7*b^(7/
4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1
/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(1/4)/(c
*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2046, 2057, 335, 226}

$$\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(7/2),x]
```

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(7*Sqrt[x]) + (2*(b*x^2 + c*x^4)^(3/2))/(7*x^(5/2
)) + (4*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(7*c^(1/4)*Sqrt
[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}} + \frac{1}{7}(6b)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$$

$$= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}} + \frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}} + \frac{1}{7}(4b^2)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}} + \frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}} + \frac{\left(4b^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{7\sqrt{bx^2+cx^4}}$$

$$= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}} + \frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}} + \frac{\left(8b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{\ldots}\right.}{7\sqrt{bx^2+cx^4}}$$

$$= \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}} + \frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}} + \frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\ldots\right.}{7\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.02, size = 56, normalized size = 0.39

$$\frac{2b\sqrt{x^2(b+cx^2)}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^2}{b}\right)}{\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(7/2),x]
```

```
[Out] (2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)])
/(Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.10, size = 145, normalized size = 1.01

method	result
default	$\frac{2(cx^4+bx^2)^{\frac{3}{2}}\Bigg(2b^2\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\Bigg(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\Bigg)+}{7x^{\frac{7}{2}}(cx^2+b)^2c}$
risch	$\frac{2(cx^2+3b)\sqrt{x^2(cx^2+b)}}{7\sqrt{x}}+\frac{4b^2\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\Bigg(}{7c\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(cx^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(2*b^2*(-b*c)^(1/2)*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))+c^3*x^5+4*b*c^2*x^3+3*b^2*c*x)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 56, normalized size = 0.39

$$\frac{2\left(4\,b^2\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}\,(c^2x^2+3\,bc)\sqrt{x}\right)}{7\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7*(4*b^2*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^2
)*(c^2*x^2 + 3*b*c)*sqrt(x))/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(7/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(7/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(7/2), x)
```

3.370 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx$

Optimal. Leaf size=286

$$\frac{24b\sqrt{c}\,x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4}-\frac{2(bx^2+cx^4)^{3/2}}{x^{7/2}}-\frac{24b^{5/4}\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b-}{\left(\sqrt{b}-\right.}}}{5\sqrt{bx^2}}$$

```
[Out] -2*(c*x^4+b*x^2)^(3/2)/x^(7/2)+24/5*b*x^(3/2)*(c*x^2+b)*c^(1/2)/(b^(1/2)+x*
c^(1/2))/(c*x^4+b*x^2)^(1/2)+12/5*c*x^(1/2)*(c*x^4+b*x^2)^(1/2)-24/5*b^(5/4
)*c^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1
/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(c*
x^4+b*x^2)^(1/2)+12/5*b^(5/4)*c^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(
1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2045, 2046, 2057, 335, 311, 226, 1210}

$$\frac{12b^{5/4}\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{24b^{5/4}\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}+\frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4}+\frac{24b\sqrt{c}\,x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2(bx^2+cx^4)^{3/2}}{x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(9/2),x]
```

```
[Out] (24*b*Sqrt[c]*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) + (12*c*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (2*(b*x^2 + c*x^4)^(3/2))/x^
(7/2) - (24*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*
Sqrt[b*x^2 + c*x^4]) + (12*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
+ c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1
/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx = -\frac{2(bx^2+cx^4)^{3/2}}{x^{7/2}} + (6c)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$

$$= \frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4} - \frac{2(bx^2+cx^4)^{3/2}}{x^{7/2}} + \frac{1}{5}(12bc)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4} - \frac{2(bx^2+cx^4)^{3/2}}{x^{7/2}} + \frac{\left(12bcx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4} - \frac{2(bx^2+cx^4)^{3/2}}{x^{7/2}} + \frac{\left(24bcx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,\right.}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4} - \frac{2(bx^2+cx^4)^{3/2}}{x^{7/2}} + \frac{\left(24b^{3/2}\sqrt{c}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx}}\right.}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{24b\sqrt{c}\,x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{12}{5}c\sqrt{x}\,\sqrt{bx^2+cx^4} - \frac{2(bx^2+cx^4)^{3/2}}{x^{7/2}} - \frac{24b^{5/4}\sqrt[4]{c}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 56, normalized size = 0.20

$$-\frac{2b\sqrt{x^2(b+cx^2)}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{cx^2}{b}\right)}{x^{3/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(9/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((c*x^2)/b)
])/(x^(3/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.12, size = 216, normalized size = 0.76

method	result

default	$\frac{2(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(12b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-6b^2\sqrt{\right.}{5x^{\frac{7}{2}}(c\,x^2+b}$
risch	$-\frac{2(-c\,x^2+5b)\sqrt{x^2\,(c\,x^2+b)}}{5x^{\frac{3}{2}}}+\frac{12b\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(2\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(12*b^2*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))-6*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+c^2*x^4-4*b*c*x^2-5*b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 59, normalized size = 0.21

$$-\frac{2\left(12\,b\sqrt{c}\,x^2\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}\,(cx^2-5\,b)\sqrt{x}\right)}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/5*(12*b*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/
c, 0, x)) - sqrt(c*x^4 + b*x^2)*(c*x^2 - 5*b)*sqrt(x))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(9/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(9/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(9/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(9/2), x)
```

3.371 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx$

Optimal. Leaf size=143

$$\frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}}-\frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}}+\frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{bx^2+cx^4}}$$

```
[Out] -2/3*(c*x^4+b*x^2)^(3/2)/x^(9/2)+4/3*c*(c*x^4+b*x^2)^(1/2)/x^(1/2)+4/3*b^(3
/4)*c^(3/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
,1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(
c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2045, 2046, 2057, 335, 226}

$$\frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{bx^2+cx^4}}+\frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}}-\frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(11/2),x]
```

```
[Out] (4*c*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) - (2*(b*x^2 + c*x^4)^(3/2))/(3*x^(9/2
)) + (4*b^(3/4)*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*Sqrt
[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx &= -\frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}} + (2c)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx \\
&= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}} + \frac{1}{3}(4bc)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx \\
&= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}} + \frac{\left(4bcx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3\sqrt{bx^2+cx^4}} \\
&= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}} + \frac{\left(8bcx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right.}{3\sqrt{bx^2+cx^4}} \\
&= \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}} + \frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(\right.}{3\sqrt{bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.01, size = 58, normalized size = 0.41

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\ {}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{cx^2}{b}\right)}{3x^{5/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(11/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((c*x^2)/b)
])/(3*x^(5/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.12, size = 130, normalized size = 0.91

method	result
default	$\frac{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,b\cdots\right.}{3x^{\frac{9}{2}}\left(c\,x^{2}+b\right)^{2}}$
risch	$-\frac{2\left(-c\,x^{2}+b\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{3x^{\frac{5}{2}}}+\frac{4b\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\cdots}{3\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(3/2)/x^(9/2)/(c*x^2+b)^2*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
(-b*c)^(1/2)*b*x+c^2*x^4-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 50, normalized size = 0.35

$$\frac{2\left(4\,b\sqrt{c}\,x^3\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}\,(cx^2-b)\sqrt{x}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/3*(4*b*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^2
)*(c*x^2 - b)*sqrt(x))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(11/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(11/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(c\,x^4+b\,x^2)^{3/2}}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(11/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(11/2), x)
```

3.372 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx$

Optimal. Leaf size=287

$$\frac{24c^{3/2}x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}}-\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}}-\frac{24\sqrt[4]{b}\,c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c}{\left(\sqrt{b}+\right.}}}{5\sqrt{bx^2+}}$$

```
[Out] -2/5*(c*x^4+b*x^2)^(3/2)/x^(11/2)+24/5*c^(3/2)*x^(3/2)*(c*x^2+b)/(b^(1/2)+x
*c^(1/2))/(c*x^4+b*x^2)^(1/2)-12/5*c*(c*x^4+b*x^2)^(1/2)/x^(3/2)-24/5*b^(1/
4)*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),
1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(c
*x^4+b*x^2)^(1/2)+12/5*b^(1/4)*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arcta
n(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^
(1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2045, 2057, 335, 311, 226, 1210}

$$\frac{12\sqrt[4]{b}\,c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}-\frac{24\sqrt[4]{b}\,c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}+\frac{24c^{3/2}x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}}-\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(13/2),x]
```

```
[Out] (24*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^
4]) - (12*c*Sqrt[b*x^2 + c*x^4])/(5*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(5
*x^(11/2)) - (24*b^(1/4)*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(
Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(5*Sqrt[b*x^2 + c*x^4]) + (12*b^(1/4)*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = -\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} + \frac{1}{5}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} + \frac{1}{5}\left(12c^2\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} + \frac{\left(12c^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}}$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} + \frac{\left(24c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx,x \ldots}{5\sqrt{bx^2+cx^4}}$$

$$= -\frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} + \frac{\left(24\sqrt{b}\,c^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}} \ldots\right.}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{24c^{3/2}x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} - \frac{24\sqrt[4]{b}\,c^{5/4}x\left(\ldots\right.}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 58, normalized size = 0.20

$$-\frac{2b\sqrt{x^2(b+cx^2)}\;{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5x^{7/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(13/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((c*x^2)/b
)])/(5*x^(7/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.12, size = 221, normalized size = 0.77

method	result

risch	$-\frac{2(7cx^2+b)\sqrt{x^2(cx^2+b)}}{5x^{\frac{7}{2}}}+\frac{12c\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-b}}{}\right.}{}$
default	$\frac{2(cx^4+bx^2)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bcx^2-6\sqrt{}\right.}{5x^{\frac{11}{2}}(cx^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(3/2)/x^(11/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)
^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)*b*c*x^2-6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-7*c^2*x^4-8*b*c*x^2-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 56, normalized size = 0.20

$$-\frac{2\left(12\,c^{\frac{3}{2}}x^4\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,(7\,cx^2+b)\sqrt{x}\right)}{5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] -2/5*(12*c^(3/2)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c,
 0, x)) + sqrt(c*x^4 + b*x^2)*(7*c*x^2 + b)*sqrt(x))/x^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(13/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(13/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(13/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(13/2), x)
```

3.373 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2(bx^2+cx^4)^{3/2}}{7x^{13/2}}+\frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/7*(c*x^4+b*x^2)^(3/2)/x^(13/2)-4/7*c*(c*x^4+b*x^2)^(1/2)/x^(5/2)+4/7*c^(
7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^
(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/
(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2045, 2057, 335, 226}

$$\frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2(bx^2+cx^4)^{3/2}}{7x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(15/2),x]
```

```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(7*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(7*x^(13
/2)) + (4*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(7*b^(1/4)*Sq
rt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = -\frac{2(bx^2+cx^4)^{3/2}}{7x^{13/2}} + \frac{1}{7}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx$$
$$= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{7x^{13/2}} + \frac{1}{7}(4c^2)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{7x^{13/2}} + \frac{\left(4c^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{7\sqrt{bx^2+cx^4}}$$
$$= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{7x^{13/2}} + \frac{\left(8c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \cdot\right.}{7\sqrt{bx^2+cx^4}}$$
$$= -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{7x^{13/2}} + \frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(\right.}{7\sqrt[4]{b}\sqrt{bx^2+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 58, normalized size = 0.41

$$-\frac{2b\sqrt{x^2(b+cx^2)}\,{}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7x^{9/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(15/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((c*x^2)/b
)])/(7*x^(9/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.11, size = 140, normalized size = 0.98

method	result
default	$\frac{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,c\,x^{3}\right.}{7x^{\frac{13}{2}}\left(c\,x^{2}+b\right)^{2}}$
risch	$-\frac{2\left(3c\,x^{2}+b\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{7x^{\frac{9}{2}}}+\frac{4c\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\right.}{7\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(c*x^4+b*x^2)^(3/2)/x^(13/2)/(c*x^2+b)^2*(2*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*(-b*c)^(1/2)*c*x^3-3*c^2*x^4-4*b*c*x^2-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 49, normalized size = 0.34

$$\frac{2\left(4\,c^{\frac{3}{2}}x^{5}\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^{4}+bx^{2}}\left(3\,cx^{2}+b\right)\sqrt{x}\right)}{7\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] 2/7*(4*c^(3/2)*x^5*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x^2)*
(3*c*x^2 + b)*sqrt(x))/x^5
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(15/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(15/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(15/2), x)
```

3.374 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$

Optimal. Leaf size=320

$$\frac{8c^{5/2}x^{3/2}(b+cx^2)}{15b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}}-\frac{8c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)}{}$$

```
[Out] -2/9*(c*x^4+b*x^2)^(3/2)/x^(15/2)+8/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/15*c*(c*x^4+b*x^2)^(1/2)/x^(7/2)-8/15*c^2
*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-8/15*c^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b
)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+4/15*c^(9/4)*x*(
cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(
b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b
*x^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2045, 2050, 2057, 335, 311, 226, 1210}

$$\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}-\frac{8c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}+\frac{8c^{5/2}x^{3/2}(b+cx^2)}{15b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(17/2),x]
```

```
[Out] (8*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(15*x^(7/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4]
)/(15*b*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(9*x^(15/2)) - (8*c^(9/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x^2 + c*x^4]) +
(4*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x
^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = -\frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}}+\frac{1}{3}(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}}+\frac{1}{15}(4c^2)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}}+\frac{(4c^3)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}}+\frac{\left(4c^3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+c}}}{15b\sqrt{bx^2+cx^4}}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}}+\frac{\left(8c^3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\right.}{15b\sqrt{bx^2-}}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}}+\frac{\left(8c^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\right.}{15\sqrt{b}\sqrt{bx}}$$

$$= \frac{8c^{5/2}x^{3/2}(b+cx^2)}{15b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}}-\frac{2(bx^2+cx^4)}{9x^{15/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 58, normalized size = 0.18

$$-\frac{2b\sqrt{x^2(b+cx^2)}\;{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9x^{11/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(17/2),x]
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((c*x^2)/b
)])/(9*x^(11/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.13, size = 239, normalized size = 0.75

method	result
default	$\frac{2(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\;\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^2x^4-6}{45}$
	$4c^2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}$
risch	$-\frac{2(12c^2x^4+11bc\,x^2+5b^2)\sqrt{x^2\,(c\,x^2+b)}}{45x^{\frac{11}{2}}\,b}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(17/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(15/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*b*c^2*x^4-6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-12*c^3*x^6-23*b*c^2*x^
4-16*b^2*c*x^2-5*b^3)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 72, normalized size = 0.22

$$-\frac{2\left(12\,c^{\frac{5}{2}}x^6\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(12\,c^2x^4+11\,bcx^2+5\,b^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{45\,bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")
```

```
[Out] -2/45*(12*c^(5/2)*x^6*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c
, 0, x)) + (12*c^2*x^4 + 11*b*c*x^2 + 5*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(
b*x^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(17/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{17/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(17/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(17/2), x)
```

3.375 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{19/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}}-\frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan\right.}{77b^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/11*(c*x^4+b*x^2)^(3/2)/x^(17/2)-12/77*c*(c*x^4+b*x^2)^(1/2)/x^(9/2)-8/77
*c^2*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)-4/77*c^(11/4)*x*(cos(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(s
in(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*
x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2045, 2050, 2057, 335, 226}

$$-\frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{bx^2+cx^4}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(19/2),x]
```

```
[Out] (-12*c*Sqrt[b*x^2 + c*x^4])/(77*x^(9/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(77*
b*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(11*x^(17/2)) - (4*c^(11/4)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{19/2}}\,dx = -\frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} + \frac{1}{11}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$$
$$= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}} - \frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} + \frac{1}{77}(12c^2)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} - \frac{(4c^3)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77b}$$
$$= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} - \frac{\left(4c^3x\sqrt{b+cx^2}\right)\int \frac{}{\sqrt{x}}}{77b\sqrt{bx^2+cx}}$$
$$= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} - \frac{\left(8c^3x\sqrt{b+cx^2}\right)\mathrm{Subst}\Big(}{77b\sqrt{bx}}$$
$$= -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} - \frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 58, normalized size = 0.34

$$-\frac{2b\sqrt{x^2\left(b+cx^2\right)}\,{}_2F_1\left(-\frac{11}{4},-\frac{3}{2};-\frac{7}{4};-\frac{cx^2}{b}\right)}{11x^{13/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(19/2),x]
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((c*x^2)/
b)])/(11*x^(13/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.12, size = 156, normalized size = 0.90

method	result
default	$-\frac{2\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\right.}{77x^{\frac{17}{2}}\left(c\,x^2+b\right)^2 b}$

risch	$-\frac{2(4c^2x^4+13bc\,x^2+7b^2)\sqrt{x^2\,(c\,x^2+b)}}{77x^{\frac{13}{2}}b}-\frac{4c^2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}}{77b\sqrt{c\,x^3+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(19/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*(c*x^4+b*x^2)^(3/2)/x^(17/2)/(c*x^2+b)^2*(2*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*(-b*c)^(1/2)*c^2*x^5+4*c^3*x^6+17*b*c^2*x^4+20*b^2*c*x^2+7*b^3)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 64, normalized size = 0.37

$$-\frac{2\left(4\,c^{\frac{5}{2}}x^7\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(4\,c^2x^4+13\,bcx^2+7\,b^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{77\,bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*c^(5/2)*x^7*weierstrassPInverse(-4*b/c, 0, x) + (4*c^2*x^4 + 13*b*
c*x^2 + 7*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b*x^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(19/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{19/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(19/2),x)
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(19/2), x)
```

3.376 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}}\,dx$

Optimal. Leaf size=350

$$-\frac{8c^{7/2}x^{3/2}(b+cx^2)}{65b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}}+ \cdots 8$$

```
[Out] -2/13*(c*x^4+b*x^2)^(3/2)/x^(19/2)-8/65*c^(7/2)*x^(3/2)*(c*x^2+b)/b^2/(b^(1
/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/39*c*(c*x^4+b*x^2)^(1/2)/x^(11/2)-8/19
5*c^2*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)+8/65*c^3*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2
)+8/65*c^(13/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/
2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)-4/65*c^(13/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+
b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2045, 2050, 2057, 335, 311, 226, 1210}

$$-\frac{4c^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}}+\frac{8c^{13/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}}-\frac{8c^{7/2}x^{3/2}(b+cx^2)}{65b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}-\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(21/2),x]
```

```
[Out] (-8*c^(7/2)*x^(3/2)*(b + c*x^2))/(65*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(39*x^(11/2)) - (8*c^2*Sqrt[b*x^2 + c*
x^4])/(195*b*x^(7/2)) + (8*c^3*Sqrt[b*x^2 + c*x^4])/(65*b^2*x^(3/2)) - (2*(
b*x^2 + c*x^4)^(3/2))/(13*x^(19/2)) + (8*c^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (4*c^(13/4)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}}\,dx = -\frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}} + \frac{1}{13}(6c)\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}} - \frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}} + \frac{1}{39}(4c^2)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}} - \frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}} - \frac{(4c^3)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{65b}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}} - \frac{(4c^4)\int \cdots}{\cdots}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}} - \frac{\left(4c^4x\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}} - \frac{\left(8c^4x\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}} - \frac{\left(8c^{7/2}x\cdots\right.}{\cdots}$$

$$= -\frac{8c^{7/2}x^{3/2}(b+cx^2)}{65b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}} + \frac{8c^3\sqrt{bx^2-\cdots}}{65b^2x^{3\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 58, normalized size = 0.17

$$-\frac{2b\sqrt{x^2(b+cx^2)}\ {}_2F_1\left(-\frac{13}{4},-\frac{3}{2};-\frac{9}{4};-\frac{cx^2}{b}\right)}{13x^{15/2}\sqrt{1+\frac{cx^2}{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(21/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-13/4, -3/2, -9/4, -((c*x^2)/
b)])/(13*x^(15/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.12, size = 250, normalized size = 0.71

method	result
default	$-\frac{2(cx^4+bx^2)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\ \mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc^3x^6-\right.}{}$ $4c^3\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}$
risch	$-\frac{2(-12c^3x^6+4bc^2x^4+25b^2cx^2+15b^3)\sqrt{x^2(cx^2+b)}}{195x^{\frac{15}{2}}b^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(21/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/195*(c*x^4+b*x^2)^(3/2)/x^(19/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*b*c^3*x^6-6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^3*x^6-12*c^4*x^8-8*b*c^3*x
^6+29*b^2*c^2*x^4+40*b^3*c*x^2+15*b^4)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 83, normalized size = 0.24

$$\frac{2\left(12\,c^{\frac{7}{2}}x^8\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(12\,c^3x^6-4\,bc^2x^4-25\,b^2cx^2-15\,b^3\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{195\,b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="fricas")
```

```
[Out] 2/195*(12*c^(7/2)*x^8*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c
, 0, x)) + (12*c^3*x^6 - 4*b*c^2*x^4 - 25*b^2*c*x^2 - 15*b^3)*sqrt(c*x^4 +
b*x^2)*sqrt(x))/(b^2*x^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(21/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{21/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(21/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(21/2), x)
```

3.377 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{23/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}+\frac{4c^{15/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+c}{\left(\sqrt{b}+\right.}}}{231b^{9/4}\sqrt{}}$$

```
[Out] -2/15*(c*x^4+b*x^2)^(3/2)/x^(21/2)-4/55*c*(c*x^4+b*x^2)^(1/2)/x^(13/2)-8/38
5*c^2*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)+8/231*c^3*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/
2)+4/231*c^(15/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(
1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(
1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2045, 2050, 2057, 335, 226}

$$\frac{4c^{15/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}+\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^(23/2),x]
```

```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(55*x^(13/2)) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(385
*b*x^(9/2)) + (8*c^3*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) - (2*(b*x^2 + c
*x^4)^(3/2))/(15*x^(21/2)) + (4*c^(15/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx &= -\frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{1}{5}(2c)\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx \\
&= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{1}{55}\left(4c^2\right)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx \\
&= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} - \frac{\left(4c^3\right)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b} \\
&= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{\left(4c^4\right)\int}{} \\
&= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{\left(4c^4x\sqrt{}\right.}{} \\
&= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{\left(8c^4x\sqrt{}\right.}{} \\
&= -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{4c^{15/4}x}{}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 58, normalized size = 0.29

$$
-\frac{2b\sqrt{x^2(b+cx^2)}\;{}_2F_1\left(-\frac{15}{4},-\frac{3}{2};-\frac{11}{4};-\frac{cx^2}{b}\right)}{15x^{17/2}\sqrt{1+\frac{cx^2}{b}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)^(3/2)/x^(23/2),x]
```

```
[Out] (-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-15/4, -3/2, -11/4, -((c*x^2)
/b)])/(15*x^(17/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A]
time = 0.11, size = 167, normalized size = 0.82

method	result

default	$\frac{2(c x^4+b x^2)^{\frac{3}{2}} \left(10 \sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\, \sqrt{2}\, \sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\, \sqrt{-\frac{xc}{\sqrt{-bc}}}\, \text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right) \sqrt{-bc}\, c^3\right.}{1155 x^{\frac{21}{2}} (c x^2+b)^2 b^2}$
risch	$-\frac{2(-20 c^3 x^6+12 b c^2 x^4+119 b^2 c x^2+77 b^3) \sqrt{x^2 (c x^2+b)}}{1155 x^{\frac{17}{2}} b^2}+\frac{4 c^3 \sqrt{-bc} \sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right) c}{\sqrt{-bc}}} \sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right) c}{\sqrt{-bc}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^(3/2)/x^(23/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155*(c*x^4+b*x^2)^(3/2)/x^(21/2)/(c*x^2+b)^2*(10*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*(-b*c)^(1/2)*c^3*x^7+20*c^4*x^8+8*b*c^3*x^6-131*b^2*c^2*x^4-196*b^3*c*
x^2-77*b^4)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 75, normalized size = 0.37

$$\frac{2\left(20\, c^{\frac{7}{2}} x^9 \text{weierstrassPInverse}\left(-\frac{4b}{c}, 0, x\right) + (20\, c^3 x^6 - 12\, b c^2 x^4 - 119\, b^2 c x^2 - 77\, b^3)\sqrt{c x^4 + b x^2}\, \sqrt{x}\right)}{1155\, b^2 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(20*c^(7/2)*x^9*weierstrassPInverse(-4*b/c, 0, x) + (20*c^3*x^6 - 12
*b*c^2*x^4 - 119*b^2*c*x^2 - 77*b^3)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^2*x^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2)**(3/2)/x**(23/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{23/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + c*x^4)^(3/2)/x^(23/2),x)
```

```
[Out] int((b*x^2 + c*x^4)^(3/2)/x^(23/2), x)
```

3.378 $\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=179

$$\frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}}-\frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2}+\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\Big(}{77c^{13/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -18/77*b*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2+2/11*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c
+30/77*b^2*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-15/77*b^(11/4)*x*(cos(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1
/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2049, 2057, 335, 226}

$$-\frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{13/4}\sqrt{bx^2+cx^4}}+\frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}}-\frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2}+\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (30*b^2*Sqrt[b*x^2 + c*x^4])/(77*c^3*Sqrt[x]) - (18*b*x^(3/2)*Sqrt[b*x^2 +
c*x^4])/(77*c^2) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(11*c) - (15*b^(11/4)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{(9b)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{11c}$$

$$= -\frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} + \frac{(45b^2)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c^2}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{(15b^3)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}}{77c^3}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(15b^3x\sqrt{b+cx^2}\right)\int}{77c^3\sqrt{bx^2}}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(30b^3x\sqrt{b+cx^2}\right)\mathrm{S}}{77c}$$

$$= \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}\right.}{}$$

Mathematica **[C]** Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 97, normalized size = 0.54

$$\frac{2x^{3/2}\left(15b^3+6b^2cx^2-2bc^2x^4+7c^3x^6-15b^3\sqrt{1+\frac{cx^2}{b}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{77c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(15*b^3 + 6*b^2*c*x^2 - 2*b*c^2*x^4 + 7*c^3*x^6 - 15*b^3*Sqrt[1
+ (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(77*c^3*Sqrt[
x^2*(b + c*x^2)])
```

Maple [A]
time = 0.11, size = 148, normalized size = 0.83

method	result
default	$-\frac{\sqrt{x}\left(-14c^4x^7+15b^3\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\cdots\right.\right.}{77\sqrt{c\,x^4+b\,x^2}\,c^4}$
risch	$\frac{2\left(7c^2x^4-9bc\,x^2+15b^2\right)x^{\frac{3}{2}}\left(c\,x^2+b\right)}{77c^3\sqrt{x^2\left(c\,x^2+b\right)}}-\frac{15b^3\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\ \mathrm{EllipticF}\cdots}{77c^4\sqrt{c\,x^3+bx}\sqrt{x^2\left(c\,x^2+\cdots\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/77/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-14*c^4*x^7+15*b^3*(-b*c)^(1/2)*((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))+4*b*c^3*x^5-12*b^2*c^2*x^3-30*b^3*c*x)/c^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 69, normalized size = 0.39

$$-\frac{2\left(15\,b^3\sqrt{c}\,x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\left(7\,c^3x^4-9\,bc^2x^2+15\,b^2c\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{77\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/77*(15*b^3*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - (7*c^3*x^4 - 9*
b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(13/2)/(b*x^2 + c*x^4)^(1/2), x)
```

3.379 $\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=296

$$\frac{14b^2x^{3/2}(b+cx^2)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{14b\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2}+\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{}{\left(\sqrt{\right.}}}}{15c}$$

```
[Out] 14/15*b^2*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)
+2/9*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c-14/45*b*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2-
14/15*b^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)
/c^(11/4)/(c*x^4+b*x^2)^(1/2)+7/15*b^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/
b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*a
rctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)
/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2049, 2057, 335, 311, 226, 1210}

$$\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}-\frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}+\frac{14b^2x^{3/2}(b+cx^2)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{14b\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2}+\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (14*b^2*x^(3/2)*(b + c*x^2))/(15*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (14*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c^2) + (2*x^(5/2)*Sqrt[b*
x^2 + c*x^4])/(9*c) - (14*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (7*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{(7b)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{9c}$$

$$= -\frac{14b\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{(7b^2)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c^2}$$

$$= -\frac{14b\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(7b^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{14b\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(14b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\right.}{15c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{14b\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(14b^{5/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\right.}{15c^{5/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{14b^2x^{3/2}(b+cx^2)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{14b\sqrt{x}\,\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{14b^{9}\cdots}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 86, normalized size = 0.29

$$\frac{2x^{5/2}\left(-7b^2-2bcx^2+5c^2x^4+7b^2\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{45c^2\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(-7*b^2 - 2*b*c*x^2 + 5*c^2*x^4 + 7*b^2*Sqrt[1 + (c*x^2)/b]*Hype
rgeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(45*c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.11, size = 217, normalized size = 0.73

method	result

default	$\frac{\sqrt{x}\left(10c^3x^6+42b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-21b^3\right.}{45\sqrt{c\,x^4+b}}$
risch	$-\frac{2x^{\frac{5}{2}}\left(-5c\,x^2+7b\right)\left(c\,x^2+b\right)}{45c^2\sqrt{x^2\left(c\,x^2+b\right)}}+\frac{7b^2\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\mathrm{E}}{}\right.}{15c^3\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^3*(10*c^3*x^6+42*b^3*((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c
/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2))-21*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*b*c^2*x^4-14*b^2*c*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 62, normalized size = 0.21

$$-\frac{2\left(21\,b^2\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}\left(5\,c^2x^2-7\,bc\right)\sqrt{x}\right)}{45\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(21*b^2*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c
, 0, x)) - sqrt(c*x^4 + b*x^2)*(5*c^2*x^2 - 7*b*c)*sqrt(x))/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{x^2 (b + cx^2)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(11/2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11/2}}{\sqrt{c\,x^4 + b\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(11/2)/(b*x^2 + c*x^4)^(1/2), x)
```

3.380 $\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=149

$$-\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}+\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/7*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c-10/21*b*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+5
/21*b^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
,1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c
^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2049, 2057, 335, 226}

$$\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-10*b*Sqrt[b*x^2 + c*x^4])/(21*c^2*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*x^2 + c*x^
4])/(7*c) + (5*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*c^(9
/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} - \frac{(5b)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{7c}$$

$$= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{(5b^2)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21c^2}$$

$$= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(5b^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{21c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(10b^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x}{21c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\big(}{21c^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 86, normalized size = 0.58

$$\frac{2x^{3/2}\left(-5b^2-2bcx^2+3c^2x^4+5b^2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{21c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(9/2)/Sqrt[b*x^2 + c*x^4],x]

[Out] (2*x^(3/2)*(-5*b^2 - 2*b*c*x^2 + 3*c^2*x^4 + 5*b^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(21*c^2*Sqrt[x^2*(b + c*x^2)])

Maple [A]
time = 0.12, size = 137, normalized size = 0.92

method	result
default	$\frac{\sqrt{x}\left(5b^2\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+6c^3x^5\right.}{21\sqrt{cx^4+bx^2}\,c^3}$
risch	$-\frac{2(-3cx^2+5b)x^{\frac{3}{2}}(cx^2+b)}{21c^2\sqrt{x^2(cx^2+b)}}+\frac{5b^2\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\right.}\right.}{21c^3\sqrt{cx^3+bx}\sqrt{x^2(cx^2+b)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(9/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/21/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(5*b^2*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+6*c^3*x^5-4*b*c^2*x^3-10*b^2*c*x)/c^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(9/2)/sqrt(c*x^4 + b*x^2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 57, normalized size = 0.38

$$\frac{2\left(5\,b^2\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}\,(3\,c^2x^2-5\,bc)\sqrt{x}\right)}{21\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*b^2*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^
2)*(3*c^2*x^2 - 5*b*c)*sqrt(x))/(c^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(9/2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{9/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^(9/2)/(b*x^2 + c*x^4)^(1/2), x)
```

3.381 $\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=266

$$-\frac{6bx^{3/2}(b+cx^2)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c}+\frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\Big(2\,\mathrm{ta}}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -6/5*b*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+2/
5*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+6/5*b^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b
)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-3/5*b^(5/4)*x*(c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b
^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*
x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2049, 2057, 335, 311, 226, 1210}

$$-\frac{3b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}+\frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}-\frac{6bx^{3/2}(b+cx^2)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-6*b*x^(3/2)*(b + c*x^2))/(5*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*
x^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) + (6*b^(5/4)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/
4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4]) - (3*b^(5/4)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{(3b)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{\left(3bx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{\left(6bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5c\sqrt{bx^2+cx^4}}$$

$$= \frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} - \frac{\left(6b^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5c^{3/2}\sqrt{bx^2+cx^4}} + \frac{\left(6b^{3/2}x\right.}{}$$

$$= -\frac{6bx^{3/2}(b+cx^2)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{2\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c} + \frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\cdots}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.26

$$\frac{2x^{5/2}\left(b+cx^2-b\sqrt{1+\frac{cx^2}{b}}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{5c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(b + c*x^2 - b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7
/4, -((c*x^2)/b)]))/(5*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.12, size = 206, normalized size = 0.77

method	result

default	$-\frac{\sqrt{x}\left(6b^{2}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3b^{2}\sqrt{\frac{cx+\sqrt{-}}{\sqrt{-}}}\right.}{5\sqrt{c\,x^{4}+b\,x^{2}}\,c^{2}}$
risch	$\frac{2x^{\frac{5}{2}}\left(c\,x^{2}+b\right)}{5c\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{3b\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\mathrm{EllipticE}\left(\right.}{}\right.}{5c^{2}\sqrt{c\,x^{3}+b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^2*(6*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2
))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-3*b
^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2),1/2*2^(1/2))-2*c^2*x^4-2*b*c*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 47, normalized size = 0.18

$$\frac{2\left(3\,b\sqrt{c}\ \mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^{4}+bx^{2}}\,c\sqrt{x}\right)}{5\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

[Out] 2/5*(3*b*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*c*sqrt(x))/c^2

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{x^2 \left(b + cx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**(7/2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7/2}}{\sqrt{c\,x^4 + b\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(7/2)/(b*x^2 + c*x^4)^(1/2), x)
```

3.382 $\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-1/3*b^(3/4)*x*(cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2
+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2049, 2057, 335, 226}

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx &= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3c}\\ &= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{\left(bx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{3c\sqrt{bx^2+cx^4}}\\ &= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{\left(2bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3c\sqrt{bx^2+cx^4}}\\ &= \frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 70, normalized size = 0.58

$$\frac{2x^{3/2}\left(b+cx^2-b\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{3c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[b*x^2 + c*x^4],x]
```

[Out] (2*x^(3/2)*(b + c*x^2 - b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5
/4, -((c*x^2)/b)]))/(3*c*Sqrt[x^2*(b + c*x^2)])

Maple [A]
time = 0.12, size = 123, normalized size = 1.02

method	result
default	$-\frac{\sqrt{x}\,\left(b\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-2c^{2}x^{3}-\right.}{3\sqrt{c\,x^{4}+b\,x^{2}}\,c^{2}}$
risch	$\frac{2x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}{3c\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-}}{c}\right.}{\sqrt{-b}}}\right.}{3c^{2}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(b*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)
^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)-2*c^2*x^3-2*b*c*x)/c^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(5/2)/sqrt(c*x^4 + b*x^2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 43, normalized size = 0.36

$$-\frac{2\left(b\sqrt{c}\,x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^{4}+bx^{2}}\,c\sqrt{x}\right)}{3\,c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")

[Out] -2/3*(b*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x^2)*c
*sqrt(x))/(c^2*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**(5/2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(5/2)/(b*x^2 + c*x^4)^(1/2), x)
```

3.383 $\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=231

$$\frac{2x^{3/2}(b+cx^2)}{\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}+\cdots$$

```
[Out] 2*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2*b^(1/
4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1
/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c
*x^4+b*x^2)^(1/2)+b^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/
2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2057, 335, 311, 226, 1210}

$$\frac{\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}+\frac{2x^{3/2}(b+cx^2)}{\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(b + c*x^2))/(Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4])
 - (2*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(3/4)*Sqrt[b*x
^2 + c*x^4]) + (b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(3/4
)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{\left(x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{\sqrt{bx^2+cx^4}}$$

$$= \frac{\left(2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx^2+cx^4}}$$

$$= \frac{\left(2\sqrt{b}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{c}\,\sqrt{bx^2+cx^4}} - \frac{\left(2\sqrt{b}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \cdots\right.}{\sqrt{c}\,\sqrt{bx^2+\cdots}}$$

$$= \frac{2x^{3/2}(b+cx^2)}{\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan\cdots\right.}{c^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.25

$$\frac{2x^{5/2}\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^2}{b}\right)}{3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b
)])/(3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 131, normalized size = 0.57

method	result
default	$\frac{\sqrt{x}\,b\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(2\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\sqrt{\frac{cx+\cdots}{\cdots}}\right.\right.}{\sqrt{c\,x^4+b\,x^2}\,c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*b/c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*(
2*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-EllipticF(
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 22, normalized size = 0.10

$$-\frac{2\,\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x))/sqrt(c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(3/2)/(b*x^2 + c*x^4)^(1/2), x)
```

3.384 $\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=90

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}$$

```
[Out] x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2)
)*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(1/
4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2057, 335, 226}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(1/4)*c^(1/4)*Sqrt[b*x^2 +
c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx &= \frac{\left(x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{\sqrt{bx^2+cx^4}}\\ &= \frac{\left(2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{bx^2+cx^4}}\\ &= \frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 55, normalized size = 0.61

$$\frac{2x^{3/2}\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b
)])/Sqrt[x^2*(b + c*x^2)]
```

Maple [A]
time = 0.08, size = 106, normalized size = 1.18

method	result	size

default	$\frac{\sqrt{x}\,\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{c\,x^4+b\,x^2}\,c}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 14, normalized size = 0.16

$$\frac{2\,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*weierstrassPInverse(-4*b/c, 0, x)/sqrt(c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x**2*(b + c*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(1/2)/(b*x^2 + c*x^4)^(1/2), x)
```

3.385 $\int \frac{1}{\sqrt{x}\,\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=259

$$\frac{2\sqrt{c}\,x^{3/2}(b+cx^2)}{b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}-\frac{2\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{b^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2*x^(3/2)*(c*x^2+b)*c^(1/2)/b/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2*(c*
x^4+b*x^2)^(1/2)/b/x^(3/2)-2*c^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+c^(1/4)*x*(cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^
(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2050, 2057, 335, 311, 226, 1210}

$$\frac{\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{c}\,x^{3/2}(b+cx^2)}{b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[c]*x^(3/2)*(b + c*x^2))/(b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4
]) - (2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) - (2*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (c^(1/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{c\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(cx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2\sqrt{c}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{bx^2+cx^4}} - \frac{\left(2\sqrt{c}\right.}{}$$

$$= \frac{2\sqrt{c}\,x^{3/2}(b+cx^2)}{b\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} - \frac{2\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b}{\left(\sqrt{b}\right.}}}{b^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 55, normalized size = 0.21

$$-\frac{2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{cx^2}{b}\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((c*x^2)
/b)])/Sqrt[x^2*(b + c*x^2)]
```

Maple [A]
time = 0.12, size = 195, normalized size = 0.75

method	result
default	$\frac{\sqrt{x}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b-\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{\sqrt{c\,x^4+b\,x^2}\,b}$

risch	$-\frac{2(c\,x^2+b)\sqrt{x}}{b\sqrt{x^2\,(c\,x^2+b)}}+\frac{\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\text{EllipticE}\left(\ldots\right.}{\ldots}\right.}{b\sqrt{c\,x^3+b\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^
(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*El
lipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*b-2*c*x^2-2*b)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 50, normalized size = 0.19

$$-\frac{2\left(\sqrt{c}\,x^2\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x
)) + sqrt(c*x^4 + b*x^2)*sqrt(x))/(b*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x}\,\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.386 $\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=121

$$-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/3*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)-1/3*c^(3/4)*x*(cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2050, 2057, 335, 226}

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx &= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}-\frac{c\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3b}\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}-\frac{\left(cx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3b\sqrt{bx^2+cx^4}}\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}-\frac{\left(2cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3b\sqrt{bx^2+cx^4}}\\ &= -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{3b^{5/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.47

$$-\frac{2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{cx^2}{b}\right)}{3\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((c*x^2)/b)])/(3
*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.09, size = 119, normalized size = 0.98

method	result
default	$-\frac{\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,x+2cx^2+2b}{3\sqrt{cx^4+bx^2}\,\sqrt{x}\,b}$
risch	$-\frac{2(cx^2+b)}{3b\sqrt{x}\,\sqrt{x^2(cx^2+b)}}-\frac{\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{(x}{}}\right.}{3b\sqrt{cx^3+bx}\,\sqrt{x^2(cx^2+b)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(c*x^4+b*x^2)^(1/2)/x^(1/2)*(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*E
llipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*
x+2*c*x^2+2*b)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 42, normalized size = 0.35

$$-\frac{2\left(\sqrt{c}\,x^3\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{3\,bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^2)*s
qrt(x))/(b*x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.387 $\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=296

$$-\frac{6c^{3/2}x^{3/2}(b+cx^2)}{5b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}+\frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}}+\frac{6c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}}{5b^{7/4}\sqrt{bx^2+c}}$$

```
[Out] -6/5*c^(3/2)*x^(3/2)*(c*x^2+b)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-
2/5*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)+6/5*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)+6/5
*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/
2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7
/4)/(c*x^4+b*x^2)^(1/2)-3/5*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/
2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2050, 2057, 335, 311, 226, 1210}

$$-\frac{3c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}+\frac{6c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}-\frac{6c^{3/2}x^{3/2}(b+cx^2)}{5b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-6*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) + (6*c*Sqrt[b*x^2 + c*x^4])
/(5*b^2*x^(3/2)) + (6*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(
5*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (3*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{(3c)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{5b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{(3c^2)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{\left(3c^2x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{\left(6c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{\right.}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{\left(6c^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x,\right.}{5b^{3/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{6c^{3/2}x^{3/2}(b+cx^2)}{5b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{6c^{5/4}x\left(\sqrt{b\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.19

$$-\frac{2\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5x^{3/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((c*x^2)/b)])/(
5*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.11, size = 215, normalized size = 0.73

method	result

default	$-\frac{6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^2-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\ldots}{5\sqrt{c\,x^4+b\,x^2}\,x\ldots}$
risch	$-\frac{2(c\,x^2+b)(-3c\,x^2+b)}{5b^2x^{\frac{3}{2}}\sqrt{x^2\,(c\,x^2+b)}}-\frac{3c\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\ldots}{\ldots}\right.}{5b^2\sqrt{c\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/(c*x^4+b*x^2)^(1/2)/x^(3/2)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)
*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-3*(
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-6*c^2*x^4-4*b*c*x^2+2*b^2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 61, normalized size = 0.21

$$\frac{2\left(3\,c^{\frac{3}{2}}x^4\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,(3\,cx^2-b)\sqrt{x}\right)}{5\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*c^(3/2)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0
, x)) + sqrt(c*x^4 + b*x^2)*(3*c*x^2 - b)*sqrt(x))/(b^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{5/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.388 $\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=149

$$-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}+\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/7*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)+10/21*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)+
5/21*c^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/
b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2050, 2057, 335, 226}

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) + (10*c*Sqrt[b*x^2 + c*x^4])/(21*b^2
*x^(5/2)) + (5*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(21*b^(9
/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx &= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}-\frac{(5c)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{7b}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}+\frac{(5c^2)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21b^2}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}+\frac{\left(5c^2x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21b^2\sqrt{bx^2+cx^4}}\\
&= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}+\frac{\left(10c^2x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x
\end{aligned}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}+\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\big(}{21b^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.38

$$-\frac{2\sqrt{1+\frac{cx^2}{b}}\ {}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7x^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((c*x^2)/b)])/(
7*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.10, size = 134, normalized size = 0.90

method	result
default	$\frac{5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,c\,x^{3}+10c^{2}x^{4}+4bc}{21\sqrt{c\,x^{4}+b\,x^{2}}\,x^{\frac{5}{2}}b^{2}}$
risch	$-\frac{2\left(c\,x^{2}+b\right)\left(-5c\,x^{2}+3b\right)}{21b^{2}x^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{5c\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\right.}{21b^{2}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/(c*x^4+b*x^2)^(1/2)/x^(5/2)*(5*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)
*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2
)*c*x^3+10*c^2*x^4+4*b*c*x^2-6*b^2)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 53, normalized size = 0.36

$$\frac{2\left(5\,c^{\frac{3}{2}}x^{5}\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^{4}+bx^{2}}\,(5\,cx^{2}-3\,b)\sqrt{x}\right)}{21\,b^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*c^(3/2)*x^5*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^2)
*(5*c*x^2 - 3*b)*sqrt(x))/(b^2*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{7/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.389 $\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=326

$$\frac{14c^{5/2}x^{3/2}(b+cx^2)}{15b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}+\frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}}-\frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}}-\frac{14c^{9/4}x\left(\sqrt{b}+\sqrt{c}\right.}{}$$

```
[Out] 14/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b^3/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)
-2/9*(c*x^4+b*x^2)^(1/2)/b/x^(11/2)+14/45*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(7/2)
-14/15*c^2*(c*x^4+b*x^2)^(1/2)/b^3/x^(3/2)-14/15*c^(9/4)*x*(cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ell
ipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/
2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)+7/
15*c^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),
1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^
(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2050, 2057, 335, 311, 226, 1210}

$$\frac{7c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}-\frac{14c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}+\frac{14c^{5/2}x^{3/2}(b+cx^2)}{15b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}}+\frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (14*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^3*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 +
 c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) + (14*c*Sqrt[b*x^2 + c*x^
4])/(45*b^2*x^(7/2)) - (14*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^(3/2)) - (14*
c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[b*x^2
 + c*x^4]) + (7*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(
11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{(7c)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{9b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{(7c^2)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{(7c^3)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b^3}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{\left(7c^3x\sqrt{b+cx^2}\right)\int \frac{}{\sqrt{}}}{15b^3\sqrt{bx^2+cx}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{\left(14c^3x\sqrt{b+cx^2}\right)\text{Sub}}{15b^3\sqrt{}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{\left(14c^{5/2}x\sqrt{b+cx^2}\right)\text{Su}}{15b^{5/}}$$

$$= \frac{14c^{5/2}x^{3/2}(b+cx^2)}{15b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx}}{15b^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 57, normalized size = 0.17

$$-\frac{2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(-\frac{9}{4},\frac{1}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9x^{7/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-9/4, 1/2, -5/4, -((c*x^2)/b)])/(
9*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.12, size = 230, normalized size = 0.71

method	result
default	$\frac{42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^2x^4-21\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}}{45\sqrt{c\,x^4+b\,x}}$
risch	$-\frac{2(c\,x^2+b)\,(21c^2x^4-7bc\,x^2+5b^2)}{45b^3x^{\frac{7}{2}}\sqrt{x^2\,(c\,x^2+b)}}+\frac{7c^2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-b}}{}\right.}{15b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45/(c*x^4+b*x^2)^(1/2)/x^(7/2)*(42*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2
)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-
21*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-42*c^3*x^6-28*b*c^2*x^4+4*b^2*c*x^
2-10*b^3)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 72, normalized size = 0.22

$$-\frac{2\left(21\,c^{\frac{5}{2}}x^6\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(21\,c^2x^4-7\,bcx^2+5\,b^2\right)\sqrt{cx^4+bx^2}\,\sqrt{x}\right)}{45\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(21*c^(5/2)*x^6*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c
, 0, x)) + (21*c^2*x^4 - 7*b*c*x^2 + 5*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b
^3*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{9}{2}}\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(9/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{9/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.390 $\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=179

$$-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}+\frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}-\frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}}-\frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{\cdot}\right.}{77b^{13/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/11*(c*x^4+b*x^2)^(1/2)/b/x^(13/2)+18/77*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(9/2
)-30/77*c^2*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)-15/77*c^(11/4)*x*(cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*E
llipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(
1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2050, 2057, 335, 226}

$$-\frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{13/4}\sqrt{bx^2+cx^4}}-\frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}}+\frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) + (18*c*Sqrt[b*x^2 + c*x^4])/(77*b
^2*x^(9/2)) - (30*c^2*Sqrt[b*x^2 + c*x^4])/(77*b^3*x^(5/2)) - (15*c^(11/4)*
x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF
[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77*b^(13/4)*Sqrt[b*x^2 + c*x^4
])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{(9c)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx}{11b}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{(45c^2)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b^2}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{(15c^3)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}}{77b^3}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{\left(15c^3x\sqrt{b+cx^2}\right)\int}{77b^3\sqrt{bx^2}}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{\left(30c^3x\sqrt{b+cx^2}\right)\text{Su}}{77b^3}$$

$$= -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}} + \frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}\,x\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 57, normalized size = 0.32

$$-\frac{2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(-\frac{11}{4},\frac{1}{2};-\frac{7}{4};-\frac{cx^2}{b}\right)}{11x^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-11/4, 1/2, -7/4, -((c*x^2)/b)])/
(11*x^(9/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.13, size = 147, normalized size = 0.82

method	result
default	$-\frac{15\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,c^2x^5+30c^3x^6+1}{77\sqrt{cx^4+bx^2}\,x^{\frac{9}{2}}b^3}$
risch	$-\frac{2\left(cx^2+b\right)\left(15c^2x^4-9bcx^2+7b^2\right)}{77b^3x^{\frac{9}{2}}\sqrt{x^2\left(cx^2+b\right)}}-\frac{15c^2\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\right.}{77b^3\sqrt{cx^3+bx}\sqrt{x^2\left(cx^2+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/77/(c*x^4+b*x^2)^(1/2)/x^(9/2)*(15*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1
/2)*c^2*x^5+30*c^3*x^6+12*b*c^2*x^4-4*b^2*c*x^2+14*b^3)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 64, normalized size = 0.36

$$-\frac{2\left(15\,c^{\frac{5}{2}}x^{7}\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(15\,c^{2}x^{4}-9\,bcx^{2}+7\,b^{2}\right)\sqrt{cx^{4}+bx^{2}}\,\sqrt{x}\right)}{77\,b^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/77*(15*c^(5/2)*x^7*weierstrassPInverse(-4*b/c, 0, x) + (15*c^2*x^4 - 9*b
*c*x^2 + 7*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^3*x^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{11}{2}}\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(11/2)*sqrt(x**2*(b + c*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^{11/2}\,\sqrt{c\,x^{4}+b\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.391 $\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}-\frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}}+\frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2}+\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-}\right.}{14c^{13/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -x^(11/2)/c/(c*x^4+b*x^2)^(1/2)+9/7*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2-15/7*b*
(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)+15/14*b^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2
+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2047, 2049, 2057, 335, 226}

$$\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14c^{13/4}\sqrt{bx^2+cx^4}}-\frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}}+\frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(11/2)/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/(7*c^3*Sqr
t[x]) + (9*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c^2) + (15*b^(7/4)*x*(Sqrt[b] +
Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^
(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} + \frac{9\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} - \frac{(45b)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{14c^2}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{(15b^2)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{14c^3}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{\left(15b^2x\sqrt{b+cx^2}\right)\int \frac{}{\sqrt{x}}}{14c^3\sqrt{bx^2+cx}}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{\left(15b^2x\sqrt{b+cx^2}\right)\text{Subst}\Big(}{7c^3\sqrt{bx^2}}$$

$$= -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 86, normalized size = 0.49

$$\frac{x^{3/2}\left(-15b^2-6bcx^2+2c^2x^4+15b^2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{7c^3\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-15*b^2 - 6*b*c*x^2 + 2*c^2*x^4 + 15*b^2*Sqrt[1 + (c*x^2)/b]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(7*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.14, size = 144, normalized size = 0.83

method	result

default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(15b^{2}\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right.}{14\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}c^{4}}$
risch	$-\frac{2\left(-c\,x^{2}+4b\right)\left(c\,x^{2}+b\right)x^{\frac{3}{2}}}{7c^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{b^{2}\left(\frac{11\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\right.\right.}{c\sqrt{c\,x^{3}+bx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(15*b^2*(-b*c)^(1/2)*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))+4*c^3*x^5-12*b*c^2*x^3-30*b^2*c*x)/c^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 88, normalized size = 0.51

$$\frac{15\left(b^{2}cx^{3}+b^{3}x\right)\sqrt{c}\ \mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(2\,c^{3}x^{4}-6\,bc^{2}x^{2}-15\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}\,\sqrt{x}}{7\left(c^{5}x^{3}+bc^{4}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/7*(15*(b^2*c*x^3 + b^3*x)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) + (2*
c^3*x^4 - 6*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^5*x^3 + b
*c^4*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(c*x**4+b*x**2)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7771 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{17/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^(17/2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.392 $\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}-\frac{21bx^{3/2}(b+cx^2)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{7\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c^2}+\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+\cdots}{\left(\sqrt{b}+\cdots\right.}}\cdots}{5c^{11/4}\sqrt{\cdots}}$$

```
[Out] -x^(9/2)/c/(c*x^4+b*x^2)^(1/2)-21/5*b*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+x*
c^(1/2))/(c*x^4+b*x^2)^(1/2)+7/5*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2+21/5*b^(5/
4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1
/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(
c*x^4+b*x^2)^(1/2)-21/10*b^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+
x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2047, 2049, 2057, 335, 311, 226, 1210}

$$-\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}+\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{11/4}\sqrt{bx^2+cx^4}}-\frac{21bx^{3/2}(b+cx^2)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{7\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c^2}-\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(9/2)/(c*Sqrt[b*x^2 + c*x^4])) - (21*b*x^(3/2)*(b + c*x^2))/(5*c^(5/2)*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (7*Sqrt[x]*Sqrt[b*x^2 + c*x^4]
)/(5*c^2) + (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(1
1/4)*Sqrt[b*x^2 + c*x^4]) - (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(10*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} + \frac{7\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} + \frac{7\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c^2} - \frac{(21b)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{10c^2}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} + \frac{7\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c^2} - \frac{\left(21bx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{10c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} + \frac{7\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c^2} - \frac{\left(21bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} + \frac{7\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c^2} - \frac{\left(21b^{3/2}x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx$$

$$21b^{5/4}x$$

$$= -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}} - \frac{21bx^{3/2}(b+cx^2)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{7\sqrt{x}\,\sqrt{bx^2+cx^4}}{5c^2} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 72, normalized size = 0.25

$$\frac{2x^{5/2}\left(-7b+cx^2+7b\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{5c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*(-7*b + c*x^2 + 7*b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3
/2, 7/4, -((c*x^2)/b)]))/(5*c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.14, size = 213, normalized size = 0.73

method	result
default	$-\frac{x^{\frac{5}{2}}(c\,x^2+b)\left(42b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-21b^2\sqrt{\cdots}\right.}{10(c\,x^4+b\,x^2)^{\frac{3}{2}}}$
risch	$\frac{2x^{\frac{5}{2}}(c\,x^2+b)}{5c^2\sqrt{x^2\,(c\,x^2+b)}}-\frac{b\left(\frac{8\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\text{EllipticE}\cdots}{\cdots}\right.}{c\sqrt{c\,x^3+b\cdots}}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(42*b^2*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))-21*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*c^2*x^4-14*b*c*x^2)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 80, normalized size = 0.27

$$\frac{21\left(bcx^{2}+b^{2}\right)\sqrt{c}\,\text{weierstrassZeta}\left(-\frac{4b}{c},0,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\sqrt{cx^{4}+bx^{2}}\left(2c^{2}x^{2}+7bc\right)\sqrt{x}}{5\left(c^{4}x^{2}+bc^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(21*(b*c*x^2 + b^2)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInve
rse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 7*b*c)*sqrt(x))/(c^4*
x^2 + b*c^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5457 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{15/2}}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(15/2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.393 $\int \frac{x^{13/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}+\frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}}-\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6c^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -x^(7/2)/c/(c*x^4+b*x^2)^(1/2)+5/3*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)-5/6*b^(3
/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(
1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(
c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2047, 2049, 2057, 335, 226}

$$-\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6c^{9/4}\sqrt{bx^2+cx^4}}+\frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(7/2)/(c*Sqrt[b*x^2 + c*x^4])) + (5*Sqrt[b*x^2 + c*x^4])/(3*c^2*Sqrt[x]
) - (5*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(6*c^(9/4)*Sqrt[
b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{(5b)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6c^2}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{\left(5bx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{6c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{\left(5bx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}} + \frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\Big(2\,\text{ta}}{6c^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.50

$$\frac{x^{3/2}\left(5b+2cx^2-5b\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{3c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(5*b + 2*c*x^2 - 5*b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/
2, 5/4, -((c*x^2)/b)]))/(3*c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.13, size = 131, normalized size = 0.90

method	result
default	$-\frac{x^{\frac{5}{2}}\left(c\,x^2+b\right)\left(5b\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-4\right.}{6\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}c^3}$

risch	$\frac{2x^{\frac{3}{2}}(c\,x^2+b)}{3c^2\sqrt{x^2\,(c\,x^2+b)}} - \frac{b\left(\frac{4\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\cdots\right.}{\sqrt{\cdots}}}\right.}{c\sqrt{c\,x^3+bx}}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(5*b*(-b*c)^(1/2)*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
,1/2*2^(1/2))-4*c^2*x^3-10*b*c*x)/c^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 76, normalized size = 0.52

$$-\frac{5\,(bcx^3+b^2x)\sqrt{c}\;\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}\,(2\,c^2x^2+5\,bc)\sqrt{x}}{3\,(c^4x^3+bc^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(5*(b*c*x^3 + b^2*x)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) - sqrt(
c*x^4 + b*x^2)*(2*c^2*x^2 + 5*b*c)*sqrt(x))/(c^4*x^3 + b*c^3*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(13/2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.394 $\int \frac{x^{11/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=259

$$-\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}+\frac{3x^{3/2}(b+cx^2)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -x^(5/2)/c/(c*x^4+b*x^2)^(1/2)+3*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+x*c^(1/
2))/(c*x^4+b*x^2)^(1/2)-3*b^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)
+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)+3/2*b^(1/4)*x*(cos(2*arcta
n(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c
^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2047, 2057, 335, 311, 226, 1210}

$$\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2c^{7/4}\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{bx^2+cx^4}}+\frac{3x^{3/2}(b+cx^2)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(5/2)/(c*Sqrt[b*x^2 + c*x^4])) + (3*x^(3/2)*(b + c*x^2))/(c^(3/2)*(Sqrt
[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(c^(7/4)*Sqrt[b*x^2 + c*x^4]) + (3*b^(1/4)*x*(Sqrt[b] +
 Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c
^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}} + \frac{3\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2c}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}} + \frac{\left(3x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2c\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}} + \frac{\left(3x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{c\sqrt{bx^2+cx^4}}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}} + \frac{\left(3\sqrt{b}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{c^{3/2}\sqrt{bx^2+cx^4}} - \frac{\left(3\sqrt{b}\,x\cdots\right.}{\cdots}$$

$$= -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}} + \frac{3x^{3/2}(b+cx^2)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{3\sqrt[4]{b}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{\cdots}{\left(\sqrt{b}\cdots\right.}}}{c^{7/\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 61, normalized size = 0.24

$$-\frac{2x^{5/2}\left(-1+\sqrt{1+\frac{cx^2}{b}}\ {}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)\right)}{c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x^(5/2)*(-1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c
*x^2)/b)]))/(c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]

time = 0.11, size = 200, normalized size = 0.77

method	result

default	$\frac{x^{\frac{5}{2}}(c\,x^{2}+b)\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b-3\sqrt{\frac{cx+\sqrt{-}}{\sqrt{-}}}\right.}{2(c\,x^{4}+b\,x^{2})^{\frac{3}{2}}c^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-
3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))*b-2*c*x^2)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 65, normalized size = 0.25

$$-\frac{3\,(cx^{2}+b)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^{4}+bx^{2}}\,c\sqrt{x}}{c^{3}x^{2}+bc^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(3*(c*x^2 + b)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b
/c, 0, x)) + sqrt(c*x^4 + b*x^2)*c*sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{11}{2}}}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(11/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11/2}}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^(11/2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.395 $\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -x^(3/2)/c/(c*x^4+b*x^2)^(1/2)+1/2*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
)^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2
)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2047, 2057, 335, 226}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(3/2)/(c*Sqrt[b*x^2 + c*x^4])) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(2*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx &= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{2c}\\ &= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{2c\sqrt{bx^2+cx^4}}\\ &= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{c\sqrt{bx^2+cx^4}}\\ &= -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}\,c^{5/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 60, normalized size = 0.50

$$\frac{x^{3/2}\left(-1+\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{c\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^
2)/b)]))/(c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.13, size = 120, normalized size = 1.01

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(\sqrt{-b c}\,\sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}\,\sqrt{2}\,\sqrt{\frac{-c x+\sqrt{-b c}}{\sqrt{-b c}}}\,\sqrt{-\frac{x c}{\sqrt{-b c}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}, \frac{\sqrt{2}}{2}\right)-2 c x\right)}{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}} c^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c
/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2))-2*c*x)/c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 59, normalized size = 0.50

$$\frac{(cx^3+bx)\sqrt{c}\,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\sqrt{cx^4+bx^2}\,c\sqrt{x}}{c^3x^3+bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((c*x^3 + b*x)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x
^2)*c*sqrt(x))/(c^3*x^3 + b*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{9}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**(9/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{9/2}}{{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^(9/2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.396 $\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=260

$$\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x^{3/2}(b+cx^2)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] x^(5/2)/b/(c*x^4+b*x^2)^(1/2)-x^(3/2)*(c*x^2+b)/b/c^(1/2)/(b^(1/2)+x*c^(1/2
))/(c*x^4+b*x^2)^(1/2)+x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))
^2)^(1/2)/b^(3/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)-1/2*x*(cos(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(
sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c
*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2048, 2057, 335, 311, 226, 1210}

$$-\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x^{3/2}(b+cx^2)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^(5/2)/(b*Sqrt[b*x^2 + c*x^4]) - (x^(3/2)*(b + c*x^2))/(b*Sqrt[c]*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4]) - (x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(2*b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\left(x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2b\sqrt{bx^2+cx^4}}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\left(x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{\left(x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\,\sqrt{c}\,\sqrt{bx^2+cx^4}} + \frac{\left(x\sqrt{b+cx^2}\right)\text{Su}}{\sqrt{b}}$$

$$= \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{x^{3/2}(b+cx^2)}{b\sqrt{c}\,\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} + \frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\right.}}}{b^{3/4}c^{3/4}\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 60, normalized size = 0.23

$$\frac{2x^{5/2}\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^2}{b}\right)}{3b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^2)/b
)])/(3*b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.09, size = 203, normalized size = 0.78

method	result

default	$-\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b-\sqrt{\frac{cx+}{\sqrt{}}}\right.}{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}cb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1
/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b
-((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2),1/2*2^(1/2))*b-2*c*x^2)/c/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 64, normalized size = 0.25

$$\frac{(cx^2+b)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,c\sqrt{x}}{bc^2x^2+b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((c*x^2 + b)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c,
 0, x)) + sqrt(c*x^4 + b*x^2)*c*sqrt(x))/(b*c^2*x^2 + b^2*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{7}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(7/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7/2}}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^(7/2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.397 $\int \frac{x^{5/2}}{\left(bx^2+cx^4\right)^{3/2}} \, dx$

Optimal. Leaf size=118

$$\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}$$

```
[Out] x^(3/2)/b/(c*x^4+b*x^2)^(1/2)+1/2*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)
+x*c^(1/2))^2)^(1/2)/b^(5/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2048, 2057, 335, 226}

$$\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^(3/2)/(b*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(2*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx &= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{2b}\\
&= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{b+cx^2}}\,dx}{2b\sqrt{bx^2+cx^4}}\\
&= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{\left(x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}\\
&= \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\,\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.88, size = 60, normalized size = 0.51

$$\frac{x^{3/2}\left(1+\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^2}{b}\right)\right)}{b\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2
)/b)]))/(b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.11, size = 123, normalized size = 1.04

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+2cx\right.}{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}cb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c
/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2))+2*c*x)/c/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 59, normalized size = 0.50

$$\frac{(cx^3+bx)\sqrt{c}\ \mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+\sqrt{cx^4+bx^2}\,c\sqrt{x}}{bc^2x^3+b^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((c*x^3 + b*x)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x
^2)*c*sqrt(x))/(b*c^2*x^3 + b^2*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{5}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(5/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(5/2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.398 $\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}+\frac{3\sqrt{c}\,x^{3/2}(b+cx^2)}{b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}}-\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}}{b^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 3*x^(3/2)*(c*x^2+b)*c^(1/2)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+x^(
1/2)/b/(c*x^4+b*x^2)^(1/2)-3*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)-3*c^(1/4)*x*(c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b
^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*
x^2)^(1/2)+3/2*c^(1/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))
^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2048, 2050, 2057, 335, 311, 226, 1210}

$$\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{bx^2+cx^4}}-\frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{bx^2+cx^4}}+\frac{3\sqrt{c}\,x^{3/2}(b+cx^2)}{b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] Sqrt[x]/(b*Sqrt[b*x^2 + c*x^4]) + (3*Sqrt[c]*x^(3/2)*(b + c*x^2))/(b^2*(Sqr
t[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2 + c*x^4])/(b^2*x^(3/
2)) - (3*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c
]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(7/4)*Sqrt[
b*x^2 + c*x^4]) + (3*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt
[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2
*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{3\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} + \frac{(3c)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} + \frac{\left(3cx\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} + \frac{\left(3cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} + \frac{\left(3\sqrt{c}\,x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right.}{b^{3/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{3\sqrt{c}\,x^{3/2}(b+cx^2)}{b^2\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} - \frac{3\sqrt[4]{c}\,x\left(\sqrt{b}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 58, normalized size = 0.20

$$-\frac{2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{cx^2}{b}\right)}{b\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((c*x^2)
/b)])/(b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.12, size = 203, normalized size = 0.71

method	result
default	$\frac{x^{\frac{5}{2}}(c\,x^2+b)\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b-3\sqrt{\frac{cx+\sqrt{-}}{\sqrt{-}}}\right.}{2(c\,x^4+b\,x^2)^{\frac{3}{2}}b^2}$
risch	$-\frac{2(c\,x^2+b)\sqrt{x}}{b^2\sqrt{x^2\,(c\,x^2+b)}}+\frac{c^2\left(\frac{\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\mathrm{Elliptic}}{\quad}\right.}{c^2\sqrt{c\,x^3+}}\right.}{\quad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
```

```
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-
3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))*b-6*c*x^2-4*b)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 81, normalized size = 0.28

$$-\frac{3\,(cx^4+bx^2)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\,(3\,cx^2+2\,b)\sqrt{x}}{b^2cx^4+b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(3*(c*x^4 + b*x^2)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(
-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*(3*c*x^2 + 2*b)*sqrt(x))/(b^2*c*x^4 +
b^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(3/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(3/2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.399 $\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{1}{b\sqrt{x}\,\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}}-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/b/x^(1/2)/(c*x^4+b*x^2)^(1/2)-5/3*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)-5/6*c^(
3/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^
(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(9/4)/
(c*x^4+b*x^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2048, 2050, 2057, 335, 226}

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}}+\frac{1}{b\sqrt{x}\,\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] 1/(b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^(5/2))
 - (5*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(6*b^(9/4)*Sqrt[b
*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{5\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{(5c)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6b^2}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{\left(5cx\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{6b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{\left(5cx\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{\ }\right.}{3b^2\sqrt{bx^2+cx^4}}$$

$$= \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(\right.}{6b^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 60, normalized size = 0.41

$$-\frac{2\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{cx^2}{b}\right)}{3b\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((c*x^2)/b)])/(3
*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.12, size = 127, normalized size = 0.88

method	result
default	$-\frac{x^{\frac{3}{2}}\left(c\,x^2+b\right)\left(5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,x\right.}{6\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}b^2}$

risch	$-\frac{2(c\,x^2+b)}{3b^2\sqrt{x}\,\sqrt{x^2\,(c\,x^2+b)}}-\frac{c\left(\frac{\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\cdots}\right.}{c\sqrt{c\,x^3+bx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(3/2)*(c*x^2+b)*(5*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1
/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(
-b*c)^(1/2)*x+10*c*x^2+4*b)/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 73, normalized size = 0.50

$$-\frac{5\,(cx^5+bx^3)\sqrt{c}\ \mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}\,(5\,cx^2+2\,b)\sqrt{x}}{3\,(b^2cx^5+b^3x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(5*(c*x^5 + b*x^3)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*
x^4 + b*x^2)*(5*c*x^2 + 2*b)*sqrt(x))/(b^2*c*x^5 + b^3*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(x**2*(b + c*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^(1/2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.400 $\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=320

$$\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}-\frac{21c^{3/2}x^{3/2}(b+cx^2)}{5b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}+\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right.}{}$$

```
[Out] 1/b/x^(3/2)/(c*x^4+b*x^2)^(1/2)-21/5*c^(3/2)*x^(3/2)*(c*x^2+b)/b^3/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-7/5*(c*x^4+b*x^2)^(1/2)/b^2/x^(7/2)+21/5*c*
(c*x^4+b*x^2)^(1/2)/b^3/x^(3/2)+21/5*c^(5/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+
b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)-21/10*c^(5/4)*
x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2)
)*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x
^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2048, 2050, 2057, 335, 311, 226, 1210}

$$-\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}+\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{bx^2+cx^4}}-\frac{21c^{3/2}x^{3/2}(b+cx^2)}{5b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) - (21*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b^3
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (7*Sqrt[b*x^2 + c*x^4])/(5*b^
2*x^(7/2)) + (21*c*Sqrt[b*x^2 + c*x^4])/(5*b^3*x^(3/2)) + (21*c^(5/4)*x*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[b*x^2 + c*x^4]) - (
21*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(10*b^(11/4)*Sqrt[b*
x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{7\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}} - \frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}} - \frac{(21c)\int \frac{1}{\sqrt{x}\,\sqrt{bx^2+cx^4}}\,dx}{10b^2}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}} - \frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}} + \frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} - \frac{(21c^2)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,d}{10b^3}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}} - \frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}} + \frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} - \frac{\left(21c^2x\sqrt{b+cx^2}\right)\int -}{10b^3\sqrt{bx^2+c}}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}} - \frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}} + \frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} - \frac{\left(21c^2x\sqrt{b+cx^2}\right)\text{Sub}}{5b^3\sqrt{}}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}} - \frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}} + \frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} - \frac{\left(21c^{3/2}x\sqrt{b+cx^2}\right)\text{Su}}{5b^{5/2}}$$

$$= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}} - \frac{21c^{3/2}x^{3/2}(b+cx^2)}{5b^3\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}} + \frac{21c\sqrt{bx}}{5b^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 60, normalized size = 0.19

$$-\frac{2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{cx^2}{b}\right)}{5bx^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((c*x^2)/b)])/(
5*b*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.14, size = 222, normalized size = 0.69

method	result
default	$-\frac{\sqrt{x}\,(c x^2+b)\left(42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^2-\right.}{10(c\,x}$
risch	$-\frac{2(c x^2+b)(-8c x^2+b)}{5b^3x^{\frac{3}{2}}\sqrt{x^2(c x^2+b)}}-\frac{c^2\left(\frac{8\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}}{}\right.}{c\sqrt{c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(1/2)*(c*x^2+b)*(42*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*b*c*x^2-21*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-42*c^2*x^4-28*b*c*x^2+4*b^2
)/b^3
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 95, normalized size = 0.30

$$\frac{21\,(c^2x^6+bcx^4)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+(21\,c^2x^4+14\,bcx^2-2\,b^2)\sqrt{cx^4+bx^2}\,\sqrt{x}}{5\,(b^3cx^6+b^4x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(21*(c^2*x^6 + b*c*x^4)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassP
Inverse(-4*b/c, 0, x)) + (21*c^2*x^4 + 14*b*c*x^2 - 2*b^2)*sqrt(c*x^4 + b*x
^2)*sqrt(x))/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\left(x^2\,(b+cx^2)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x}\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.401 $\int \frac{1}{x^{3/2}(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}-\frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}}+\frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}}+\frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\tan^{-}\right.}{14b^{13/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/b/x^(5/2)/(c*x^4+b*x^2)^(1/2)-9/7*(c*x^4+b*x^2)^(1/2)/b^2/x^(9/2)+15/7*c*
(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)+15/14*c^(7/4)*x*(cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2
+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2048, 2050, 2057, 335, 226}

$$\frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14b^{13/4}\sqrt{bx^2+cx^4}}+\frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}}-\frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) - (9*Sqrt[b*x^2 + c*x^4])/(7*b^2*x^(9/2))
 + (15*c*Sqrt[b*x^2 + c*x^4])/(7*b^3*x^(5/2)) + (15*c^(7/4)*x*(Sqrt[b] + Sq
rt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1
/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{9\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} - \frac{(45c)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{14b^2}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{(15c^2)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}}{14b^3}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{\left(15c^2x\sqrt{b+cx^2}\right)\int}{14b^3\sqrt{bx^2}}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{\left(15c^2x\sqrt{b+cx^2}\right)\text{Su}}{7b^3}$$

$$= \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}} - \frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}} + \frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 60, normalized size = 0.35

$$-\frac{2\sqrt{1+\frac{cx^2}{b}}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{2};-\frac{3}{4};-\frac{cx^2}{b}\right)}{7bx^{5/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-7/4, 3/2, -3/4, -((c*x^2)/b)])/(
7*b*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.13, size = 141, normalized size = 0.82

method	result
default	$\frac{(cx^2+b)\left(15\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,cx^3+\right.}{14(cx^4+bx^2)^{\frac{3}{2}}\sqrt{x}\,b^3}$

risch	$-\frac{2(c x^2+b)(-4c x^2+b)}{7b^3x^{\frac{5}{2}}\sqrt{x^2(c x^2+b)}}+\frac{c^2\left(\frac{4\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\ldots}\right.}{c\sqrt{c x^3+bx}}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14/(c*x^4+b*x^2)^(3/2)/x^(1/2)*(c*x^2+b)*(15*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
(-b*c)^(1/2)*c*x^3+30*c^2*x^4+12*b*c*x^2-4*b^2)/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 87, normalized size = 0.50

$$\frac{15\,(c^2x^7+bcx^5)\sqrt{c}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(15\,c^2x^4+6\,bcx^2-2\,b^2)\sqrt{cx^4+bx^2}\,\sqrt{x}}{7\,(b^3cx^7+b^4x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/7*(15*(c^2*x^7 + b*c*x^5)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) + (15
*c^2*x^4 + 6*b*c*x^2 - 2*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^3*c*x^7 + b^4
*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{3}{2}}\,(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\,{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.402 $\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=350

$$\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}+\frac{77c^{5/2}x^{3/2}(b+cx^2)}{15b^4\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}}+\frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}}-\frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}}-$$

```
[Out] 1/b/x^(7/2)/(c*x^4+b*x^2)^(1/2)+77/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b^4/(b^(1/2
)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-11/9*(c*x^4+b*x^2)^(1/2)/b^2/x^(11/2)+77/4
5*c*(c*x^4+b*x^2)^(1/2)/b^3/x^(7/2)-77/15*c^2*(c*x^4+b*x^2)^(1/2)/b^4/x^(3/
2)-77/15*c^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1
/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1
/2)/b^(15/4)/(c*x^4+b*x^2)^(1/2)+77/30*c^(9/4)*x*(cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(15/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2048, 2050, 2057, 335, 311, 226, 1210}

$$\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}-\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}\,x\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{15/4}\sqrt{bx^2+cx^4}}+\frac{77c^{5/2}x^{3/2}(b+cx^2)}{15b^4\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}}-\frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}}+\frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}}-\frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}}+\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (77*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^
4*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (11*Sqrt[b*x^2 + c*x^4])/(9*
b^2*x^(11/2)) + (77*c*Sqrt[b*x^2 + c*x^4])/(45*b^3*x^(7/2)) - (77*c^2*Sqrt[
b*x^2 + c*x^4])/(15*b^4*x^(3/2)) - (77*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(15*b^(15/4)*Sqrt[b*x^2 + c*x^4]) + (77*c^(9/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(30*b^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{11\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx}{2b}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} - \frac{(77c)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{18b^2}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{(77c^2)\int \frac{1}{\sqrt{x}\sqrt{bx^2+c}}}{30b^3}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{(7}{}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{(7}{}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{(7}{}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{(7}{}$$

$$= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{77c^{5/2}x^{3/2}(b+cx^2)}{15b^4\left(\sqrt{b}+\sqrt{c}\,x\right)\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{77c\sqrt{}}{45}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 60, normalized size = 0.17

$$-\frac{2\sqrt{1+\frac{cx^2}{b}}\;{}_2F_1\left(-\frac{9}{4},\frac{3}{2};-\frac{5}{4};-\frac{cx^2}{b}\right)}{9bx^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-9/4, 3/2, -5/4, -((c*x^2)/b)])/(
9*b*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.13, size = 237, normalized size = 0.68

method	result
default	$\frac{\left(c\,x^{2}+b\right)\left(462\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^{2}x^{4}-231\sqrt{}\right.}{90(}$
risch	$-\frac{2\left(c\,x^{2}+b\right)\left(93c^{2}x^{4}-16bc\,x^{2}+5b^{2}\right)}{45b^{4}x^{\frac{7}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{c^{3}\left(\frac{31\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(2\sqrt{}\ -\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/90/(c*x^4+b*x^2)^(3/2)/x^(3/2)*(c*x^2+b)*(462*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*b*c^2*x^4-231*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-462*c^3*x^6-308*b*c^2*
x^4+44*b^2*c*x^2-20*b^3)/b^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 108, normalized size = 0.31

$$-\frac{231\left(c^{3}x^{8}+bc^{2}x^{6}\right)\sqrt{c}\ \mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\left(231\,c^{3}x^{6}+154\,bc^{2}x^{4}-22\,b^{2}cx^{2}+10\,b^{3}\right)\sqrt{cx^{4}+bx^{2}}\ \sqrt{x}}{45\left(b^{4}cx^{8}+b^{5}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/45*(231*(c^3*x^8 + b*c^2*x^6)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierst
rassPInverse(-4*b/c, 0, x)) + (231*c^3*x^6 + 154*b*c^2*x^4 - 22*b^2*c*x^2 +
 10*b^3)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^4*c*x^8 + b^5*x^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**(5/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{5/2}\,{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.403 $\int (cx)^m (bx^2 + cx^4)^3 \, dx$

Optimal. Leaf size=73

$$\frac{b^3x^7(cx)^m}{7+m} + \frac{3b^2cx^9(cx)^m}{9+m} + \frac{3bc^2x^{11}(cx)^m}{11+m} + \frac{c^3x^{13}(cx)^m}{13+m}$$

```
[Out] b^3*x^7*(c*x)^m/(7+m)+3*b^2*c*x^9*(c*x)^m/(9+m)+3*b*c^2*x^11*(c*x)^m/(11+m)
+c^3*x^13*(c*x)^m/(13+m)
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1156, 1598, 276}

$$\frac{b^3x^7(cx)^m}{m+7} + \frac{3b^2cx^9(cx)^m}{m+9} + \frac{3bc^2x^{11}(cx)^m}{m+11} + \frac{c^3x^{13}(cx)^m}{m+13}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (b^3*x^7*(c*x)^m)/(7 + m) + (3*b^2*c*x^9*(c*x)^m)/(9 + m) + (3*b*c^2*x^11*(
c*x)^m)/(11 + m) + (c^3*x^13*(c*x)^m)/(13 + m)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(bx^2+cx^4\right)^3\,dx &= \left(x^{-m}(cx)^m\right)\text{Subst}\left(\int x^m\left(bx^2+cx^4\right)^3\,dx,x,x\right)\\ &= \left(x^{-m}(cx)^m\right)\text{Subst}\left(\int x^{6+m}\left(b+cx^2\right)^3\,dx,x,x\right)\\ &= \left(x^{-m}(cx)^m\right)\text{Subst}\left(\int \left(b^3x^{6+m}+3b^2cx^{8+m}+3bc^2x^{10+m}+c^3x^{12+m}\right)dx,x,x\right)\\ &= \frac{b^3x^7(cx)^m}{7+m}+\frac{3b^2cx^9(cx)^m}{9+m}+\frac{3bc^2x^{11}(cx)^m}{11+m}+\frac{c^3x^{13}(cx)^m}{13+m}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 59, normalized size = 0.81

$$x^7(cx)^m\left(\frac{b^3}{7+m}+\frac{3b^2cx^2}{9+m}+\frac{3bc^2x^4}{11+m}+\frac{c^3x^6}{13+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^7*(c*x)^m*(b^3/(7 + m) + (3*b^2*c*x^2)/(9 + m) + (3*b*c^2*x^4)/(11 + m) +
 (c^3*x^6)/(13 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(73) = 146.
time = 0.10, size = 181, normalized size = 2.48

method	result
gosper	$\frac{(cx)^m\left(c^3m^3x^6+27c^3m^2x^6+3b\,c^2m^3x^4+239m\,x^6c^3+87b\,c^2m^2x^4+693c^3x^6+3b^2c\,m^3x^2+813b\,c^2m\,x^4+93b^2c\,m^2x^2+2457b\,c^2x^4+b^3m\right.}{(13+m)(11+m)(9+m)(7+m)}$
risch	$\frac{(cx)^m\left(c^3m^3x^6+27c^3m^2x^6+3b\,c^2m^3x^4+239m\,x^6c^3+87b\,c^2m^2x^4+693c^3x^6+3b^2c\,m^3x^2+813b\,c^2m\,x^4+93b^2c\,m^2x^2+2457b\,c^2x^4+b^3m\right.}{(13+m)(11+m)(9+m)(7+m)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x)^m*(c^3*m^3*x^6+27*c^3*m^2*x^6+3*b*c^2*m^3*x^4+239*c^3*m*x^6+87*b*c^2*
m^2*x^4+693*c^3*x^6+3*b^2*c*m^3*x^2+813*b*c^2*m*x^4+93*b^2*c*m^2*x^2+2457*b
*c^2*x^4+b^3*m^3+933*b^2*c*m*x^2+33*b^3*m^2+3003*b^2*c*x^2+359*b^3*m+1287*b
^3)*x^7/(13+m)/(11+m)/(9+m)/(7+m)
```

Maxima [A]
time = 0.28, size = 76, normalized size = 1.04

$$\frac{c^{m+3}x^{13}x^m}{m+13}+\frac{3\,bc^{m+2}x^{11}x^m}{m+11}+\frac{3\,b^2c^{m+1}x^9x^m}{m+9}+\frac{b^3c^mx^7x^m}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] c^(m + 3)*x^13*x^m/(m + 13) + 3*b*c^(m + 2)*x^11*x^m/(m + 11) + 3*b^2*c^(m
+ 1)*x^9*x^m/(m + 9) + b^3*c^m*x^7*x^m/(m + 7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(73) = 146$.
time = 0.35, size = 161, normalized size = 2.21

$$\frac{\left(\left(c^{3} m^{3} + 27 \, c^{3} m^{2} + 239 \, c^{3} m + 693 \, c^{3}\right) x^{13} + 3 \, \left(b c^{2} m^{3} + 29 \, b c^{2} m^{2} + 271 \, b c^{2} m + 819 \, b c^{2}\right) x^{11} + 3 \, \left(b^{2} c m^{3} + 31 \, b^{2} c m^{2} + 311 \, b^{2} c m + 1001 \, b^{2} c\right) x^{9} + \left(b^{3} m^{3} + 33 \, b^{3} m^{2} + 359 \, b^{3} m + 1287 \, b^{3}\right) x^{7}\right) \left(c x\right)^{m}}{m^{4} + 40 \, m^{3} + 590 \, m^{2} + 3800 \, m + 9009}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^3 + 27*c^3*m^2 + 239*c^3*m + 693*c^3)*x^13 + 3*(b*c^2*m^3 + 29*b*c^
2*m^2 + 271*b*c^2*m + 819*b*c^2)*x^11 + 3*(b^2*c*m^3 + 31*b^2*c*m^2 + 311*b
^2*c*m + 1001*b^2*c)*x^9 + (b^3*m^3 + 33*b^3*m^2 + 359*b^3*m + 1287*b^3)*x^
7)*(c*x)^m/(m^4 + 40*m^3 + 590*m^2 + 3800*m + 9009)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(66) = 132$.
time = 0.74, size = 731, normalized size = 10.01

$$\begin{cases} \frac{- \frac{b^{3}}{6 x^{6}} - \frac{3 b^{2} c}{4 x^{4}} - \frac{3 b c^{2}}{2 x^{2}} + c^{3} \log{\left(x \right)}}{c^{13}} & \text{for}\: m = -13 \\ \frac{- \frac{b^{3}}{4 x^{4}} - \frac{3 b^{2} c}{2 x^{2}} + 3 b c^{2} \log{\left(x \right)} + \frac{c^{3} x^{2}}{2}}{c^{11}} & \text{for}\: m = -11 \\ \frac{- \frac{b^{3}}{2 x^{2}} + 3 b^{2} c \log{\left(x \right)} + \frac{3 b c^{2} x^{2}}{2} + \frac{c^{3} x^{4}}{4}}{c^{9}} & \text{for}\: m = -9 \\ \frac{b^{3} \log{\left(x \right)} + \frac{3 b^{2} c x^{2}}{2} + \frac{3 b c^{2} x^{4}}{4} + \frac{c^{3} x^{6}}{6}}{c^{7}} & \text{for}\: m = -7 \\ \frac{b^{3} m^{3} x^{7} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{33 b^{3} m^{2} x^{7} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{359 b^{3} m x^{7} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{1287 b^{3} x^{7} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{3 b^{2} c m^{3} x^{9} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{93 b^{2} c m^{2} x^{9} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{933 b^{2} c m x^{9} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{3003 b^{2} c x^{9} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{3 b c^{2} m^{3} x^{11} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{87 b c^{2} m^{2} x^{11} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{813 b c^{2} m x^{11} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{2457 b c^{2} x^{11} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{c^{3} m^{3} x^{13} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{27 c^{3} m^{2} x^{13} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{239 c^{3} m x^{13} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} + \frac{693 c^{3} x^{13} \left(c x\right)^{m}}{m^{4} + 40 m^{3} + 590 m^{2} + 3800 m + 9009} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2)**3,x)
```

```
[Out] Piecewise(((-b**3/(6*x**6) - 3*b**2*c/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*l
og(x))/c**13, Eq(m, -13)), ((-b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b*c**2*
log(x) + c**3*x**2/2)/c**11, Eq(m, -11)), ((-b**3/(2*x**2) + 3*b**2*c*log(x
) + 3*b*c**2*x**2/2 + c**3*x**4/4)/c**9, Eq(m, -9)), ((b**3*log(x) + 3*b**2
*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6)/c**7, Eq(m, -7)), (b**3*m**3*x**
7*(c*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 33*b**3*m**2*x**7*
(c*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 359*b**3*m*x**7*(c*x
)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 1287*b**3*x**7*(c*x)**m/
(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3*b**2*c*m**3*x**9*(c*x)**m/(
m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 93*b**2*c*m**2*x**9*(c*x)**m/(
m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 933*b**2*c*m*x**9*(c*x)**m/(m*
*4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3003*b**2*c*x**9*(c*x)**m/(m**4
+ 40*m**3 + 590*m**2 + 3800*m + 9009) + 3*b*c**2*m**3*x**11*(c*x)**m/(m**4
+ 40*m**3 + 590*m**2 + 3800*m + 9009) + 87*b*c**2*m**2*x**11*(c*x)**m/(m**4
 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 813*b*c**2*m*x**11*(c*x)**m/(m**4
+ 40*m**3 + 590*m**2 + 3800*m + 9009) + 2457*b*c**2*x**11*(c*x)**m/(m**4 +
```

```
40*m**3 + 590*m**2 + 3800*m + 9009) + c**3*m**3*x**13*(c*x)**m/(m**4 + 40*m
**3 + 590*m**2 + 3800*m + 9009) + 27*c**3*m**2*x**13*(c*x)**m/(m**4 + 40*m*
*3 + 590*m**2 + 3800*m + 9009) + 239*c**3*m*x**13*(c*x)**m/(m**4 + 40*m**3
+ 590*m**2 + 3800*m + 9009) + 693*c**3*x**13*(c*x)**m/(m**4 + 40*m**3 + 590
*m**2 + 3800*m + 9009), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(73) = 146.
time = 5.69, size = 264, normalized size = 3.62

$$\frac{(cx)^m c^3m^3x^{13} + 27 (cx)^m c^3m^2x^{13} + 3 (cx)^m bc^2m^3x^{11} + 239 (cx)^m c^3mx^{13} + 87 (cx)^m bc^2m^2x^{11} + 693 (cx)^m c^3x^{13} + 3 (cx)^m b^2cm^3x^9 + 813 (cx)^m bc^2mx^{11} + 93 (cx)^m b^2cm^2x^9 + 2457 (cx)^m bc^2x^{11} + (cx)^m b^3m^3x^7 + 933 (cx)^m b^2cmx^9 + 33 (cx)^m b^3m^2x^7 + 3003 (cx)^m b^2cx^9 + 359 (cx)^m b^3mx^7 + 1287 (cx)^m b^3x^7}{m^4 + 40 m^3 + 590 m^2 + 3800 m + 9009}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] ((c*x)^m*c^3*m^3*x^13 + 27*(c*x)^m*c^3*m^2*x^13 + 3*(c*x)^m*b*c^2*m^3*x^11
+ 239*(c*x)^m*c^3*m*x^13 + 87*(c*x)^m*b*c^2*m^2*x^11 + 693*(c*x)^m*c^3*x^13
 + 3*(c*x)^m*b^2*c*m^3*x^9 + 813*(c*x)^m*b*c^2*m*x^11 + 93*(c*x)^m*b^2*c*m^
2*x^9 + 2457*(c*x)^m*b*c^2*x^11 + (c*x)^m*b^3*m^3*x^7 + 933*(c*x)^m*b^2*c*m
*x^9 + 33*(c*x)^m*b^3*m^2*x^7 + 3003*(c*x)^m*b^2*c*x^9 + 359*(c*x)^m*b^3*m*
x^7 + 1287*(c*x)^m*b^3*x^7)/(m^4 + 40*m^3 + 590*m^2 + 3800*m + 9009)
```

Mupad [B]
time = 4.29, size = 171, normalized size = 2.34

$$(c\,x)^m\,\left(\frac{b^3\,x^7\,\left(m^3+33\,m^2+359\,m+1287\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009}+\frac{c^3\,x^{13}\,\left(m^3+27\,m^2+239\,m+693\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009}+\frac{3\,b\,c^2\,x^{11}\,\left(m^3+29\,m^2+271\,m+819\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009}+\frac{3\,b^2\,c\,x^9\,\left(m^3+31\,m^2+311\,m+1001\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (c*x)^m*((b^3*x^7*(359*m + 33*m^2 + m^3 + 1287))/(3800*m + 590*m^2 + 40*m^3
 + m^4 + 9009) + (c^3*x^13*(239*m + 27*m^2 + m^3 + 693))/(3800*m + 590*m^2
+ 40*m^3 + m^4 + 9009) + (3*b*c^2*x^11*(271*m + 29*m^2 + m^3 + 819))/(3800*
m + 590*m^2 + 40*m^3 + m^4 + 9009) + (3*b^2*c*x^9*(311*m + 31*m^2 + m^3 + 1
001))/(3800*m + 590*m^2 + 40*m^3 + m^4 + 9009))
```

3.404 $\int (cx)^m (bx^2 + cx^4)^2 \, dx$

Optimal. Leaf size=52

$$\frac{b^2x^5(cx)^m}{5+m} + \frac{2bcx^7(cx)^m}{7+m} + \frac{c^2x^9(cx)^m}{9+m}$$

```
[Out] b^2*x^5*(c*x)^m/(5+m)+2*b*c*x^7*(c*x)^m/(7+m)+c^2*x^9*(c*x)^m/(9+m)
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1156, 1598, 276}

$$\frac{b^2x^5(cx)^m}{m+5} + \frac{2bcx^7(cx)^m}{m+7} + \frac{c^2x^9(cx)^m}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5*(c*x)^m)/(5 + m) + (2*b*c*x^7*(c*x)^m)/(7 + m) + (c^2*x^9*(c*x)^m)
/(9 + m)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int (cx)^m \left(bx^2+cx^4\right)^2 \, dx &= \left(x^{-m}(cx)^m\right) \text{Subst}\left(\int x^m \left(bx^2+cx^4\right)^2 \, dx, x, x\right) \\
&= \left(x^{-m}(cx)^m\right) \text{Subst}\left(\int x^{4+m} \left(b+cx^2\right)^2 \, dx, x, x\right) \\
&= \left(x^{-m}(cx)^m\right) \text{Subst}\left(\int \left(b^2x^{4+m}+2bcx^{6+m}+c^2x^{8+m}\right) \, dx, x, x\right) \\
&= \frac{b^2x^5(cx)^m}{5+m}+\frac{2bcx^7(cx)^m}{7+m}+\frac{c^2x^9(cx)^m}{9+m}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.83

$$
x^5(cx)^m\left(\frac{b^2}{5+m}+\frac{2bcx^2}{7+m}+\frac{c^2x^4}{9+m}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4)^2,x]
```

```
[Out] x^5*(c*x)^m*(b^2/(5 + m) + (2*b*c*x^2)/(7 + m) + (c^2*x^4)/(9 + m))
```

Maple [A]
time = 0.10, size = 96, normalized size = 1.85

method	result	size
gosper	$\frac{(cx)^m\left(c^2m^2x^4+12m\,x^4c^2+2bc\,m^2x^2+35c^2x^4+28bcm\,x^2+b^2m^2+90bc\,x^2+16b^2m+63b^2\right)x^5}{(9+m)(7+m)(5+m)}$	96
risch	$\frac{(cx)^m\left(c^2m^2x^4+12m\,x^4c^2+2bc\,m^2x^2+35c^2x^4+28bcm\,x^2+b^2m^2+90bc\,x^2+16b^2m+63b^2\right)x^5}{(9+m)(7+m)(5+m)}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x)^m*(c^2*m^2*x^4+12*c^2*m*x^4+2*b*c*m^2*x^2+35*c^2*x^4+28*b*c*m*x^2+b^2
*m^2+90*b*c*x^2+16*b^2*m+63*b^2)*x^5/(9+m)/(7+m)/(5+m)
```

Maxima [A]
time = 0.29, size = 55, normalized size = 1.06

$$
\frac{c^{m+2}x^9x^m}{m+9}+\frac{2\,bc^{m+1}x^7x^m}{m+7}+\frac{b^2c^mx^5x^m}{m+5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] c^(m + 2)*x^9*x^m/(m + 9) + 2*b*c^(m + 1)*x^7*x^m/(m + 7) + b^2*c^m*x^5*x^m
/(m + 5)
```

Fricas [A]
time = 0.37, size = 89, normalized size = 1.71

$$\frac{((c^2m^2 + 12\,c^2m + 35\,c^2)x^9 + 2\,(bcm^2 + 14\,bcm + 45\,bc)x^7 + (b^2m^2 + 16\,b^2m + 63\,b^2)x^5)(cx)^m}{m^3 + 21\,m^2 + 143\,m + 315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^2 + 12*c^2*m + 35*c^2)*x^9 + 2*(b*c*m^2 + 14*b*c*m + 45*b*c)*x^7 +
(b^2*m^2 + 16*b^2*m + 63*b^2)*x^5)*(c*x)^m/(m^3 + 21*m^2 + 143*m + 315)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(46) = 92.
time = 0.43, size = 337, normalized size = 6.48

$$\begin{cases} \frac{- \frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)}{c^9} & \text{for}\ m = -9 \\ \frac{- \frac{b^2}{2x^2} + 2bc \log(x) + \frac{c^2x^2}{2}}{c^7} & \text{for}\ m = -7 \\ \frac{b^2 \log(x) + bcx^2 + \frac{c^2x^4}{4}}{c^5} & \text{for}\ m = -5 \\ \frac{b^2m^2x^5(cx)^m}{m^3+21m^2+143m+315} + \frac{16b^2mx^5(cx)^m}{m^3+21m^2+143m+315} + \frac{63b^2x^5(cx)^m}{m^3+21m^2+143m+315} + \frac{2bcm^2x^7(cx)^m}{m^3+21m^2+143m+315} + \frac{28bcmx^7(cx)^m}{m^3+21m^2+143m+315} + \frac{90bcx^7(cx)^m}{m^3+21m^2+143m+315} + \frac{c^2m^2x^9(cx)^m}{m^3+21m^2+143m+315} + \frac{12c^2mx^9(cx)^m}{m^3+21m^2+143m+315} + \frac{35c^2x^9(cx)^m}{m^3+21m^2+143m+315} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2)**2,x)
```

```
[Out] Piecewise(((-b**2/(4*x**4) - b*c/x**2 + c**2*log(x))/c**9, Eq(m, -9)), ((-b
**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2)/c**7, Eq(m, -7)), ((b**2*log(x)
+ b*c*x**2 + c**2*x**4/4)/c**5, Eq(m, -5)), (b**2*m**2*x**5*(c*x)**m/(m**3
+ 21*m**2 + 143*m + 315) + 16*b**2*m*x**5*(c*x)**m/(m**3 + 21*m**2 + 143*m
+ 315) + 63*b**2*x**5*(c*x)**m/(m**3 + 21*m**2 + 143*m + 315) + 2*b*c*m**2*
x**7*(c*x)**m/(m**3 + 21*m**2 + 143*m + 315) + 28*b*c*m*x**7*(c*x)**m/(m**3
 + 21*m**2 + 143*m + 315) + 90*b*c*x**7*(c*x)**m/(m**3 + 21*m**2 + 143*m +
315) + c**2*m**2*x**9*(c*x)**m/(m**3 + 21*m**2 + 143*m + 315) + 12*c**2*m*x
**9*(c*x)**m/(m**3 + 21*m**2 + 143*m + 315) + 35*c**2*x**9*(c*x)**m/(m**3 +
 21*m**2 + 143*m + 315), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(52) = 104.
time = 5.02, size = 141, normalized size = 2.71

$$\frac{(cx)^m\,c^2m^2x^9 + 12\,(cx)^m\,c^2mx^9 + 2\,(cx)^m\,bcm^2x^7 + 35\,(cx)^m\,c^2x^9 + 28\,(cx)^m\,bcmx^7 + (cx)^m\,b^2m^2x^5 + 90\,(cx)^m\,bcx^7 + 16\,(cx)^m\,b^2mx^5 + 63\,(cx)^m\,b^2x^5}{m^3 + 21\,m^2 + 143\,m + 315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] ((c*x)^m*c^2*m^2*x^9 + 12*(c*x)^m*c^2*m*x^9 + 2*(c*x)^m*b*c*m^2*x^7 + 35*(c
*x)^m*c^2*x^9 + 28*(c*x)^m*b*c*m*x^7 + (c*x)^m*b^2*m^2*x^5 + 90*(c*x)^m*b*c
*x^7 + 16*(c*x)^m*b^2*m*x^5 + 63*(c*x)^m*b^2*x^5)/(m^3 + 21*m^2 + 143*m + 3
15)
```

Mupad [B]
time = 4.19, size = 97, normalized size = 1.87

$$(c\,x)^m\,\left(\frac{b^2\,x^5\,\left(m^2+16\,m+63\right)}{m^3+21\,m^2+143\,m+315}+\frac{c^2\,x^9\,\left(m^2+12\,m+35\right)}{m^3+21\,m^2+143\,m+315}+\frac{2\,b\,c\,x^7\,\left(m^2+14\,m+45\right)}{m^3+21\,m^2+143\,m+315}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2 + c*x^4)^2,x)
```

```
[Out] (c*x)^m*((b^2*x^5*(16*m + m^2 + 63))/(143*m + 21*m^2 + m^3 + 315) + (c^2*x^
9*(12*m + m^2 + 35))/(143*m + 21*m^2 + m^3 + 315) + (2*b*c*x^7*(14*m + m^2
+ 45))/(143*m + 21*m^2 + m^3 + 315))
```

3.405 $\int (cx)^m \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=34

$$\frac{b(cx)^{3+m}}{c^3(3+m)}+\frac{(cx)^{5+m}}{c^4(5+m)}$$

```
[Out] b*(c*x)^(3+m)/c^3/(3+m)+(c*x)^(5+m)/c^4/(5+m)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = $\{14\}$

$$\frac{b(cx)^{m+3}}{c^3(m+3)}+\frac{(cx)^{m+5}}{c^4(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(b*x^2 + c*x^4),x]
```

```
[Out] (b*(c*x)^(3 + m))/(c^3*(3 + m)) + (c*x)^(5 + m)/(c^4*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (cx)^m \left(bx^2+cx^4\right) dx &= \int \left(\frac{b(cx)^{2+m}}{c^2}+\frac{(cx)^{4+m}}{c^3}\right) dx\\ &= \frac{b(cx)^{3+m}}{c^3(3+m)}+\frac{(cx)^{5+m}}{c^4(5+m)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 27, normalized size = 0.79

$$x^3(cx)^m\left(\frac{b}{3+m}+\frac{cx^2}{5+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(b*x^2 + c*x^4),x]
```

```
[Out] x^3*(c*x)^m*(b/(3 + m) + (c*x^2)/(5 + m))
```

Maple [A]
time = 0.01, size = 36, normalized size = 1.06

method	result	size
norman	$\frac{b\,x^{3} {\mathrm e}^{m \ln \left(c x \right)}}{3+m}+\frac{c\,x^{5} {\mathrm e}^{m \ln \left(c x \right)}}{5+m}$	36
gosper	$\frac{\left(c x \right)^{m} \left(c m\,x^{2}+3 c\,x^{2}+b m +5 b \right) x^{3}}{\left(5+m \right) \left(3+m \right)}$	39
risch	$\frac{\left(c x \right)^{m} \left(c m\,x^{2}+3 c\,x^{2}+b m +5 b \right) x^{3}}{\left(5+m \right) \left(3+m \right)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] b/(3+m)*x^3*exp(m*ln(c*x))+c/(5+m)*x^5*exp(m*ln(c*x))
```

Maxima [A]
time = 0.28, size = 34, normalized size = 1.00

$$\frac{c^{m+1}x^5x^m}{m+5}+\frac{bc^mx^3x^m}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] c^(m + 1)*x^5*x^m/(m + 5) + b*c^m*x^3*x^m/(m + 3)
```

Fricas [A]
time = 0.36, size = 39, normalized size = 1.15

$$\frac{((cm+3\,c)x^5+(bm+5\,b)x^3)(cx)^m}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] ((c*m + 3*c)*x^5 + (b*m + 5*b)*x^3)*(c*x)^m/(m^2 + 8*m + 15)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(27) = 54$.
time = 0.19, size = 112, normalized size = 3.29

$$\begin{cases} \frac{-\frac{b}{2x^2}+c\log(x)}{c^5} & \text{for}\ m=-5 \\ \frac{b\log(x)+\frac{cx^2}{2}}{c^3} & \text{for}\ m=-3 \\ \frac{bmx^3(cx)^m}{m^2+8m+15}+\frac{5bx^3(cx)^m}{m^2+8m+15}+\frac{cmx^5(cx)^m}{m^2+8m+15}+\frac{3cx^5(cx)^m}{m^2+8m+15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(c*x**4+b*x**2),x)
```

```
[Out] Piecewise(((-b/(2*x**2) + c*log(x))/c**5, Eq(m, -5)), ((b*log(x) + c*x**2/2
)/c**3, Eq(m, -3)), (b*m*x**3*(c*x)**m/(m**2 + 8*m + 15) + 5*b*x**3*(c*x)**
m/(m**2 + 8*m + 15) + c*m*x**5*(c*x)**m/(m**2 + 8*m + 15) + 3*c*x**5*(c*x)*
*m/(m**2 + 8*m + 15), True))
```

Giac [A]
time = 7.17, size = 56, normalized size = 1.65

$$\frac{(cx)^m\,cmx^5+3\,(cx)^m\,cx^5+(cx)^m\,bmx^3+5\,(cx)^m\,bx^3}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] ((c*x)^m*c*m*x^5 + 3*(c*x)^m*c*x^5 + (c*x)^m*b*m*x^3 + 5*(c*x)^m*b*x^3)/(m^
2 + 8*m + 15)
```

Mupad [B]
time = 4.15, size = 38, normalized size = 1.12

$$\frac{x^3\,{\left(c\,x\right)}^m\,\left(5\,b+b\,m+3\,c\,x^2+c\,m\,x^2\right)}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x^2 + c*x^4),x)
```

```
[Out] (x^3*(c*x)^m*(5*b + b*m + 3*c*x^2 + c*m*x^2))/(8*m + m^2 + 15)
```

3.406 $\int \frac{(cx)^m}{bx^2+cx^4} \, dx$

Optimal. Leaf size=45

$$-\frac{(cx)^m \, {}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{cx^2}{b}\right)}{b(1-m)x}$$

```
[Out] -(c*x)^m*hypergeom([1, -1/2+1/2*m],[1/2+1/2*m],-c*x^2/b)/b/(1-m)/x
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1156, 1598, 371}

$$-\frac{(cx)^m \, {}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{b(1-m)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^2 + c*x^4),x]
```

```
[Out] -(((c*x)^m*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*(1
 - m)*x))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(cx)^m}{bx^2+cx^4}\,dx = \left(x^{-m}(cx)^m\right)\mathrm{Subst}\left(\int \frac{x^m}{bx^2+cx^4}\,dx,x,x\right)$$
$$= \left(x^{-m}(cx)^m\right)\mathrm{Subst}\left(\int \frac{x^{-2+m}}{b+cx^2}\,dx,x,x\right)$$
$$= -\frac{(cx)^m\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{cx^2}{b}\right)}{b(1-m)x}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.93

$$\frac{(cx)^m\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{cx^2}{b}\right)}{b(-1+m)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^2 + c*x^4),x]
[Out] ((c*x)^m*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*(-1
+ m)*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(c*x^4+b*x^2),x)
[Out] int((c*x)^m/(c*x^4+b*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] integrate((c*x)^m/(c*x^4 + b*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] integral((c*x)^m/(c*x^4 + b*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{x^2\,(b+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(c*x**4+b*x**2),x)
[Out] Integral((c*x)**m/(x**2*(b + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2),x, algorithm="giac")
[Out] integrate((c*x)^m/(c*x^4 + b*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^m}{c\,x^4+b\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2 + c*x^4),x)
[Out] int((c*x)^m/(b*x^2 + c*x^4), x)
```

3.407 $\int \frac{(cx)^m}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=45

$$-\frac{(cx)^m \,_2F_1\left(2,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\frac{cx^2}{b}\right)}{b^2(3-m)x^3}$$

```
[Out] -(c*x)^m*hypergeom([2, -3/2+1/2*m],[-1/2+1/2*m],-c*x^2/b)/b^2/(3-m)/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1156, 1598, 371}

$$-\frac{(cx)^m \,_2F_1\left(2,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{b^2(3-m)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(((c*x)^m*Hypergeometric2F1[2, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)])/(b^2
*(3 - m)*x^3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(cx)^m}{(bx^2+cx^4)^2}\,dx = (x^{-m}(cx)^m)\,\text{Subst}\left(\int \frac{x^m}{(bx^2+cx^4)^2}\,dx, x, x\right)$$
$$= (x^{-m}(cx)^m)\,\text{Subst}\left(\int \frac{x^{-4+m}}{(b+cx^2)^2}\,dx, x, x\right)$$
$$= -\frac{(cx)^m\,{}_2F_1\left(2, \frac{1}{2}(-3+m); \frac{1}{2}(-1+m); -\frac{cx^2}{b}\right)}{b^2(3-m)x^3}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.98

$$\frac{(cx)^m\,{}_2F_1\left(2, \frac{1}{2}(-3+m); 1+\frac{1}{2}(-3+m); -\frac{cx^2}{b}\right)}{b^2(-3+m)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^2 + c*x^4)^2,x]
[Out] ((c*x)^m*Hypergeometric2F1[2, (-3 + m)/2, 1 + (-3 + m)/2, -((c*x^2)/b)])/(b
^2*(-3 + m)*x^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(c\,x^4+b\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(c*x^4+b*x^2)^2,x)
[Out] int((c*x)^m/(c*x^4+b*x^2)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] integral((c*x)^m/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{x^4 (b + cx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(c*x**4+b*x**2)**2,x)
[Out] Integral((c*x)**m/(x**4*(b + c*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^m}{(c\,x^4 + b\,x^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2 + c*x^4)^2,x)
[Out] int((c*x)^m/(b*x^2 + c*x^4)^2, x)
```

3.408 $\int \frac{(cx)^m}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=45

$$-\frac{(cx)^m\,{}_2F_1\left(3,\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);-\frac{cx^2}{b}\right)}{b^3(5-m)x^5}$$

```
[Out] -(c*x)^m*hypergeom([3, -5/2+1/2*m],[-3/2+1/2*m],-c*x^2/b)/b^3/(5-m)/x^5
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1156, 1598, 371}

$$-\frac{(cx)^m\,{}_2F_1\left(3,\frac{m-5}{2};\frac{m-3}{2};-\frac{cx^2}{b}\right)}{b^3(5-m)x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(((c*x)^m*Hypergeometric2F1[3, (-5 + m)/2, (-3 + m)/2, -((c*x^2)/b)])/(b^3
*(5 - m)*x^5))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{(cx)^m}{(bx^2+cx^4)^3}\,dx = (x^{-m}(cx)^m)\,\mathrm{Subst}\left(\int \frac{x^m}{(bx^2+cx^4)^3}\,dx, x, x\right)$$
$$= (x^{-m}(cx)^m)\,\mathrm{Subst}\left(\int \frac{x^{-6+m}}{(b+cx^2)^3}\,dx, x, x\right)$$
$$= -\frac{(cx)^m\,{}_2F_1\left(3, \frac{1}{2}(-5+m); \frac{1}{2}(-3+m); -\frac{cx^2}{b}\right)}{b^3(5-m)x^5}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.98

$$\frac{(cx)^m\,{}_2F_1\left(3, \frac{1}{2}(-5+m); 1+\frac{1}{2}(-5+m); -\frac{cx^2}{b}\right)}{b^3(-5+m)x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((c*x)^m*Hypergeometric2F1[3, (-5 + m)/2, 1 + (-5 + m)/2, -((c*x^2)/b)])/(b
^3*(-5 + m)*x^5)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(c\,x^4+b\,x^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(c*x^4+b*x^2)^3,x)
```

```
[Out] int((c*x)^m/(c*x^4+b*x^2)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^3,x, algorithm="fricas")
[Out] integral((c*x)^m/(c^3*x^12 + 3*b*c^2*x^10 + 3*b^2*c*x^8 + b^3*x^6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{x^6\,(b+cx^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(c*x**4+b*x**2)**3,x)
[Out] Integral((c*x)**m/(x**6*(b + c*x**2)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(c*x^4+b*x^2)^3,x, algorithm="giac")
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^m}{(c\,x^4+b\,x^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(b*x^2 + c*x^4)^3,x)
[Out] int((c*x)^m/(b*x^2 + c*x^4)^3, x)
```

3.409 $\int x^3(a^2+2abx^2+b^2x^4)\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = $\{14\}$

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^2+b^2x^4\right)\,dx&=\int\left(a^2x^3+2abx^5+b^2x^7\right)\,dx\\&=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Maple [A]
time = 0.06, size = 25, normalized size = 0.83

method	result	size
default	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
gosper	$\frac{x^4(3b^2x^4+8ab\,x^2+6a^2)}{24}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Fricas [A]
time = 0.31, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.80

$$\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

Giac [A]
time = 5.22, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

3.410 $\int x^2(a^2 + 2abx^2 + b^2x^4)\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{14\}$

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2 + 2abx^2 + b^2x^4\right)\,dx &= \int \left(a^2x^2 + 2abx^4 + b^2x^6\right)\,dx\\ &= \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.83

method	result	size
default	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
norman	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
risch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
gosper	$\frac{x^3(15b^2x^4+42ab\,x^2+35a^2)}{105}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Maxima [A]
time = 0.30, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Fricas [A]
time = 0.34, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

Giac [A]
time = 3.58, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

3.411 $\int x(a^2+2abx^2+b^2x^4)\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}$$

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*b^2*x^6
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {14}

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + (b^2*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)\,dx&=\int \left(a^2x+2abx^3+b^2x^5\right)\,dx\\&=\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 0.53

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (a + b*x^2)^3/(6*b)
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.83

method	result	size
default	$\frac{1}{2}a^2x^2+\frac{1}{2}ab\,x^4+\frac{1}{6}b^2x^6$	25
norman	$\frac{1}{2}a^2x^2+\frac{1}{2}ab\,x^4+\frac{1}{6}b^2x^6$	25
risch	$\frac{1}{2}a^2x^2+\frac{1}{2}ab\,x^4+\frac{1}{6}b^2x^6$	25
gosper	$\frac{x^2(b^2x^4+3ab\,x^2+3a^2)}{6}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*b^2*x^6
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Fricas [A]
time = 0.34, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.80

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

Giac [A]
time = 4.90, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^2}{2} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

3.412 $\int (a^2 + 2abx^2 + b^2x^4) \, dx$

Optimal. Leaf size=25

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi steps

$$\int (a^2 + 2abx^2 + b^2x^4) \, dx = a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Maple [A]
time = 0.01, size = 22, normalized size = 0.88

method	result	size
default	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
norman	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22

risch	$a^2x + \frac{2}{3}ab\,x^3 + \frac{1}{5}b^2x^5$	22
gosper	$\frac{x(3b^2x^4+10ab\,x^2+15a^2)}{15}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^2*x^4+2*a*b*x^2+a^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Fricas [A]
time = 0.31, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A]
time = 0.01, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**2*x**4+2*a*b*x**2+a**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Giac [A]
time = 15.23, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x+\frac{2\,a\,b\,x^3}{3}+\frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^2 + b^2*x^4 + 2*a*b*x^2,x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

3.413 $\int \frac{a^2+2abx^2+b^2x^4}{x}\,dx$

Optimal. Leaf size=23

$$abx^2+\frac{b^2x^4}{4}+a^2\log(x)$$

```
[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = $\{14\}$

$$a^2\log(x)+abx^2+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x}\,dx &= \int \left(\frac{a^2}{x}+2abx+b^2x^3\right)dx\\ &= abx^2+\frac{b^2x^4}{4}+a^2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$abx^2+\frac{b^2x^4}{4}+a^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Maple [A]
time = 0.02, size = 22, normalized size = 0.96

method	result	size
default	$ab\,x^2+\frac{b^2x^4}{4}+a^2\ln\left(x\right)$	22
norman	$ab\,x^2+\frac{b^2x^4}{4}+a^2\ln\left(x\right)$	22
risch	$\frac{b^2x^4}{4}+ab\,x^2+a^2+a^2\ln\left(x\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x,x,method=_RETURNVERBOSE)
[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 1.04

$$\frac{1}{4}\,b^2x^4+abx^2+\frac{1}{2}\,a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="maxima")
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Fricas [A]
time = 0.37, size = 21, normalized size = 0.91

$$\frac{1}{4}\,b^2x^4+abx^2+a^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="fricas")
[Out] 1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

Sympy [A]
time = 0.02, size = 20, normalized size = 0.87

$$a^2\log\left(x\right)+abx^2+\frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x,x)
[Out] a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

Giac [A]
time = 12.32, size = 24, normalized size = 1.04

$$\frac{1}{4}b^2x^4 + abx^2 + \frac{1}{2}a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Mupad [B]
time = 4.10, size = 21, normalized size = 0.91

$$a^2\ln(x) + \frac{b^2x^4}{4} + a\,b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x,x)
```

```
[Out] a^2*log(x) + (b^2*x^4)/4 + a*b*x^2
```

3.414 $\int \frac{a^2+2abx^2+b^2x^4}{x^2} \, dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{14\}$

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]

[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^2}\,dx &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]

[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3

Maple [A]
time = 0.01, size = 23, normalized size = 0.96

method	result	size
default	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
risch	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
norman	$\frac{\frac{1}{3}b^2x^4+2ab\,x^2-a^2}{x}$	26
gosper	$-\frac{-b^2x^4-6ab\,x^2+3a^2}{3x}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^2,x,method=_RETURNVERBOSE)
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

Maxima [A]
time = 0.30, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Fricas [A]
time = 0.33, size = 25, normalized size = 1.04

$$\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="fricas")
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A]
time = 0.02, size = 19, normalized size = 0.79

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

Giac [**A**]
time = 14.26, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Mupad [**B**]
time = 0.03, size = 22, normalized size = 0.92

$$\frac{b^2\,x^3}{3} - \frac{a^2}{x} + 2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^2,x)
```

```
[Out] (b^2*x^3)/3 - a^2/x + 2*a*b*x
```

3.415 $\int \frac{a^2+2abx^2+b^2x^4}{x^3}\,dx$

Optimal. Leaf size=27

$$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)$$

```
[Out] -1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = $\{14\}$

$$-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]
[Out] -1/2*a^2/x^2 + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^3}\,dx &= \int \left(\frac{a^2}{x^3}+\frac{2ab}{x}+b^2x\right)dx\\ &= -\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]
[Out] -1/2*a^2/x^2 + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Maple [A]
time = 0.01, size = 24, normalized size = 0.89

method	result	size
default	$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\ln(x)$	24
risch	$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\ln(x)$	24
norman	$\frac{-\frac{a^2}{2}+\frac{b^2x^4}{2}}{x^2}+2ab\ln(x)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.89

$$\frac{1}{2}\,b^2x^2+ab\log\left(x^2\right)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="maxima")
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*a^2/x^2
```

Fricas [A]
time = 0.33, size = 27, normalized size = 1.00

$$\frac{b^2x^4+4\,abx^2\log\left(x\right)-a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="fricas")
[Out] 1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

Sympy [A]
time = 0.04, size = 24, normalized size = 0.89

$$-\frac{a^2}{2x^2}+2ab\log\left(x\right)+\frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**3,x)
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

Giac [A]
time = 7.87, size = 32, normalized size = 1.19

$$\frac{1}{2}\,b^{2}x^{2} + ab\log\left(x^{2}\right) - \frac{2\,abx^{2} + a^{2}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

Mupad [B]
time = 0.03, size = 23, normalized size = 0.85

$$\frac{b^2\,x^2}{2} - \frac{a^2}{2\,x^2} + 2\,a\,b\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^3,x)
```

```
[Out] (b^2*x^2)/2 - a^2/(2*x^2) + 2*a*b*log(x)
```

3.416 $\int \frac{a^2+2abx^2+b^2x^4}{x^4}\,dx$

Optimal. Leaf size=23

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

```
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {14}

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^4}\,dx &= \int \left(b^2+\frac{a^2}{x^4}+\frac{2ab}{x^2}\right)dx\\ &= -\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

Maple [A]
time = 0.01, size = 22, normalized size = 0.96

method	result	size
default	$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$	22
risch	$b^2x+\frac{-2ab\,x^2-\frac{1}{3}a^2}{x^3}$	24
gosper	$-\frac{-3b^2x^4+6ab\,x^2+a^2}{3x^3}$	25
norman	$\frac{b^2x^4-2ab\,x^2-\frac{1}{3}a^2}{x^3}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

Maxima [A]
time = 0.29, size = 22, normalized size = 0.96

$$b^2x-\frac{6\,abx^2+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="maxima")
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Fricas [A]
time = 0.34, size = 26, normalized size = 1.13

$$\frac{3\,b^2x^4-6\,abx^2-a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

Sympy [A]
time = 0.04, size = 22, normalized size = 0.96

$$b^2x+\frac{-a^2-6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**4,x)
```

```
[Out] b**2*x + (-a**2 - 6*a*b*x**2)/(3*x**3)
```

Giac [A]
time = 4.97, size = 22, normalized size = 0.96

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="giac")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Mupad [B]
time = 4.11, size = 24, normalized size = 1.04

$$b^2\,x - \frac{\frac{a^2}{3} + 2\,b\,a\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^4,x)
```

```
[Out] b^2*x - (a^2/3 + 2*a*b*x^2)/x^3
```

3.417 $\int \frac{a^2+2abx^2+b^2x^4}{x^5} dx$

Optimal. Leaf size=24

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

```
[Out] -1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{14\}$

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]
[Out] -1/4*a^2/x^4 - (a*b)/x^2 + b^2*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^5}\,dx &= \int \left(\frac{a^2}{x^5}+\frac{2ab}{x^3}+\frac{b^2}{x}\right)dx\\ &= -\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]
[Out] -1/4*a^2/x^4 - (a*b)/x^2 + b^2*Log[x]
```

Maple [A]
time = 0.01, size = 23, normalized size = 0.96

method	result	size
default	$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\ln(x)$	23
norman	$\frac{-\frac{1}{4}a^2-ab\,x^2}{x^4}+b^2\ln(x)$	25
risch	$\frac{-\frac{1}{4}a^2-ab\,x^2}{x^4}+b^2\ln(x)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 1.08

$$\frac{1}{2}\,b^2\log\left(x^2\right)-\frac{4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="maxima")
[Out] 1/2*b^2*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/x^4
```

Fricas [A]
time = 0.34, size = 28, normalized size = 1.17

$$\frac{4\,b^2x^4\log(x)-4\,abx^2-a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="fricas")
[Out] 1/4*(4*b^2*x^4*log(x) - 4*a*b*x^2 - a^2)/x^4
```

Sympy [A]
time = 0.06, size = 24, normalized size = 1.00

$$b^2\log(x)+\frac{-a^2-4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**5,x)
[Out] b**2*log(x) + (-a**2 - 4*a*b*x**2)/(4*x**4)
```

Giac [A]
time = 3.49, size = 34, normalized size = 1.42

$$\frac{1}{2}\,b^2\log\left(x^2\right)-\frac{3\,b^2x^4+4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="giac")
[Out] 1/2*b^2*log(x^2) - 1/4*(3*b^2*x^4 + 4*a*b*x^2 + a^2)/x^4
```

Mupad [B]
time = 0.04, size = 24, normalized size = 1.00

$$b^2\,\ln\left(x\right)-\frac{\frac{a^2}{4}+b\,a\,x^2}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^5,x)
[Out] b^2*log(x) - (a^2/4 + a*b*x^2)/x^4
```

3.418 $\int \frac{a^2+2abx^2+b^2x^4}{x^6}\,dx$

Optimal. Leaf size=28

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = $\{14\}$

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^6}\,dx &= \int \left(\frac{a^2}{x^6}+\frac{2ab}{x^4}+\frac{b^2}{x^2}\right)dx\\ &= -\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}\end{aligned}$$

Mathematica [A]

time = 0.00, size = 28, normalized size = 1.00

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.89

method	result	size
default	$-\frac{a^{2}}{5x^{5}}-\frac{2ab}{3x^{3}}-\frac{b^{2}}{x}$	25
norman	$\frac{-b^{2}x^{4}-\frac{2}{3}ab\,x^{2}-\frac{1}{5}a^{2}}{x^{5}}$	26
risch	$\frac{-b^{2}x^{4}-\frac{2}{3}ab\,x^{2}-\frac{1}{5}a^{2}}{x^{5}}$	26
gosper	$-\frac{15b^{2}x^{4}+10ab\,x^{2}+3a^{2}}{15x^{5}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.93

$$-\frac{15\,b^{2}x^{4}+10\,abx^{2}+3\,a^{2}}{15\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Fricas [A]
time = 0.33, size = 26, normalized size = 0.93

$$-\frac{15\,b^{2}x^{4}+10\,abx^{2}+3\,a^{2}}{15\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Sympy [A]
time = 0.06, size = 27, normalized size = 0.96

$$\frac{-3a^{2}-10abx^{2}-15b^{2}x^{4}}{15x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**6,x)
```

```
[Out] (-3*a**2 - 10*a*b*x**2 - 15*b**2*x**4)/(15*x**5)
```

Giac [A]
time = 4.30, size = 26, normalized size = 0.93

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.89

$$-\frac{\frac{a^2}{5}+\frac{2\,a\,b\,x^2}{3}+b^2\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^6,x)
```

```
[Out] -(a^2/5 + b^2*x^4 + (2*a*b*x^2)/3)/x^5
```

3.419 $\int \frac{a^2+2abx^2+b^2x^4}{x^7}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

```
[Out] -1/6*a^2/x^6-1/2*a*b/x^4-1/2*b^2/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = $\{14\}$

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]
[Out] -1/6*a^2/x^6 - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{x^7}\,dx &= \int \left(\frac{a^2}{x^7}+\frac{2ab}{x^5}+\frac{b^2}{x^3}\right)dx\\ &= -\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]
[Out] -1/6*a^2/x^6 - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.83

method	result	size
gosper	$-\frac{3b^2x^4+3ab\,x^2+a^2}{6x^6}$	25
default	$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$	25
norman	$\frac{-\frac{1}{2}b^2x^4-\frac{1}{2}ab\,x^2-\frac{1}{6}a^2}{x^6}$	26
risch	$\frac{-\frac{1}{2}b^2x^4-\frac{1}{2}ab\,x^2-\frac{1}{6}a^2}{x^6}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^2/x^6-1/2*a*b/x^4-1/2*b^2/x^2
```

Maxima [A]
time = 0.30, size = 24, normalized size = 0.80

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Fricas [A]
time = 0.34, size = 24, normalized size = 0.80

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Sympy [A]
time = 0.07, size = 26, normalized size = 0.87

$$\frac{-a^2-3abx^2-3b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**7,x)
```

```
[Out] (-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*x**6)
```

Giac [A]
time = 3.65, size = 24, normalized size = 0.80

$$-\frac{3 b^2 x^4 + 3 a b x^2 + a^2}{6 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Mupad [B]
time = 0.03, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{6} + \frac{a b x^2}{2} + \frac{b^2 x^4}{2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^7,x)
```

```
[Out] -(a^2/6 + (b^2*x^4)/2 + (a*b*x^2)/2)/x^6
```

3.420 $\int \frac{a^2+2abx^2+b^2x^4}{x^8} \, dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

```
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{14\}$

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a^2+2abx^2+b^2x^4}{x^8} \, dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2}{x^4}\right) \, dx \\ &= -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Maple [A]
time = 0.02, size = 25, normalized size = 0.83

method	result	size
default	$-\frac{a^{2}}{7x^{7}}-\frac{2ab}{5x^{5}}-\frac{b^{2}}{3x^{3}}$	25
norman	$\frac{-\frac{1}{3}b^{2}x^{4}-\frac{2}{5}ab\,x^{2}-\frac{1}{7}a^{2}}{x^{7}}$	26
risch	$\frac{-\frac{1}{3}b^{2}x^{4}-\frac{2}{5}ab\,x^{2}-\frac{1}{7}a^{2}}{x^{7}}$	26
gosper	$-\frac{35b^{2}x^{4}+42ab\,x^{2}+15a^{2}}{105x^{7}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Maxima [A]
time = 0.31, size = 26, normalized size = 0.87

$$-\frac{35\,b^{2}x^{4}+42\,abx^{2}+15\,a^{2}}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="maxima")
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Fricas [A]
time = 0.33, size = 26, normalized size = 0.87

$$-\frac{35\,b^{2}x^{4}+42\,abx^{2}+15\,a^{2}}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="fricas")
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Sympy [A]
time = 0.07, size = 27, normalized size = 0.90

$$\frac{-15a^{2}-42abx^{2}-35b^{2}x^{4}}{105x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/x**8,x)
```

[Out] (-15*a**2 - 42*a*b*x**2 - 35*b**2*x**4)/(105*x**7)

Giac [A]
time = 3.95, size = 26, normalized size = 0.87

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Mupad [B]
time = 0.03, size = 26, normalized size = 0.87

$$-\frac{\frac{a^2}{7}+\frac{2\,a\,b\,x^2}{5}+\frac{b^2\,x^4}{3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^8,x)
```

```
[Out] -(a^2/7 + (b^2*x^4)/3 + (2*a*b*x^2)/5)/x^7
```

3.421 $\int x^6(a^2+2abx^2+b^2x^4)^2\,dx$

Optimal. Leaf size=56

$$\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

```
[Out] 1/7*a^4*x^7+4/9*a^3*b*x^9+6/11*a^2*b^2*x^11+4/13*a*b^3*x^13+1/15*b^4*x^15
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 276}

$$\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^13)/13 + (
b^4*x^15)/15
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6\left(a^2+2abx^2+b^2x^4\right)^2\,dx&=\frac{\int x^6\left(ab+b^2x^2\right)^4\,dx}{b^4}\\&=\frac{\int\left(a^4b^4x^6+4a^3b^5x^8+6a^2b^6x^{10}+4ab^7x^{12}+b^8x^{14}\right)dx}{b^4}\\&=\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.00

$$\frac{a^4x^7}{7} + \frac{4}{9}a^3bx^9 + \frac{6}{11}a^2b^2x^{11} + \frac{4}{13}ab^3x^{13} + \frac{b^4x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^13)/13 + (
b^4*x^15)/15
```

Maple [A]
time = 0.05, size = 47, normalized size = 0.84

method	result	size
default	$\frac{1}{7}a^4x^7 + \frac{4}{9}a^3b\,x^9 + \frac{6}{11}a^2b^2x^{11} + \frac{4}{13}a\,b^3x^{13} + \frac{1}{15}b^4x^{15}$	47
norman	$\frac{1}{7}a^4x^7 + \frac{4}{9}a^3b\,x^9 + \frac{6}{11}a^2b^2x^{11} + \frac{4}{13}a\,b^3x^{13} + \frac{1}{15}b^4x^{15}$	47
risch	$\frac{1}{7}a^4x^7 + \frac{4}{9}a^3b\,x^9 + \frac{6}{11}a^2b^2x^{11} + \frac{4}{13}a\,b^3x^{13} + \frac{1}{15}b^4x^{15}$	47
gosper	$\frac{x^7\left(3003b^4x^8+13860a\,b^3x^6+24570a^2b^2x^4+20020a^3b\,x^2+6435a^4\right)}{45045}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^4*x^7+4/9*a^3*b*x^9+6/11*a^2*b^2*x^11+4/13*a*b^3*x^13+1/15*b^4*x^15
```

Maxima [A]
time = 0.30, size = 46, normalized size = 0.82

$$\frac{1}{15}\,b^4x^{15} + \frac{4}{13}\,ab^3x^{13} + \frac{6}{11}\,a^2b^2x^{11} + \frac{4}{9}\,a^3bx^9 + \frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*a
^4*x^7
```

Fricas [A]
time = 0.32, size = 46, normalized size = 0.82

$$\frac{1}{15}\,b^4x^{15} + \frac{4}{13}\,ab^3x^{13} + \frac{6}{11}\,a^2b^2x^{11} + \frac{4}{9}\,a^3bx^9 + \frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*a
^4*x^7
```

Sympy [A]
time = 0.01, size = 53, normalized size = 0.95

$$\frac{a^4x^7}{7}+\frac{4a^3bx^9}{9}+\frac{6a^2b^2x^{11}}{11}+\frac{4ab^3x^{13}}{13}+\frac{b^4x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**7/7 + 4*a**3*b*x**9/9 + 6*a**2*b**2*x**11/11 + 4*a*b**3*x**13/13 +
b**4*x**15/15
```

Giac [A]
time = 2.98, size = 46, normalized size = 0.82

$$\frac{1}{15}\,b^4x^{15}+\frac{4}{13}\,ab^3x^{13}+\frac{6}{11}\,a^2b^2x^{11}+\frac{4}{9}\,a^3bx^9+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*a
^4*x^7
```

Mupad [B]
time = 0.02, size = 46, normalized size = 0.82

$$\frac{a^4\,x^7}{7}+\frac{4\,a^3\,b\,x^9}{9}+\frac{6\,a^2\,b^2\,x^{11}}{11}+\frac{4\,a\,b^3\,x^{13}}{13}+\frac{b^4\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^7)/7 + (b^4*x^15)/15 + (4*a^3*b*x^9)/9 + (4*a*b^3*x^13)/13 + (6*a^2*
b^2*x^11)/11
```

3.422 $\int x^5 (a^2+2abx^2+b^2x^4)^2 \, dx$

Optimal. Leaf size=53

$$\frac{a^2(a+bx^2)^5}{10b^3}-\frac{a(a+bx^2)^6}{6b^3}+\frac{(a+bx^2)^7}{14b^3}$$

```
[Out] 1/10*a^2*(b*x^2+a)^5/b^3-1/6*a*(b*x^2+a)^6/b^3+1/14*(b*x^2+a)^7/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$\frac{a^2(a+bx^2)^5}{10b^3}+\frac{(a+bx^2)^7}{14b^3}-\frac{a(a+bx^2)^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^2*(a + b*x^2)^5)/(10*b^3) - (a*(a + b*x^2)^6)/(6*b^3) + (a + b*x^2)^7/(1
4*b^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{\int x^5(ab+b^2x^2)^4\,dx}{b^4}$$
$$=\frac{\text{Subst}\left(\int x^2(ab+b^2x)^4\,dx,x,x^2\right)}{2b^4}$$
$$=\frac{\text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^4}{b^2}-\frac{2a(ab+b^2x)^5}{b^3}+\frac{(ab+b^2x)^6}{b^4}\right)dx,x,x^2\right)}{2b^4}$$
$$=\frac{a^2(a+bx^2)^5}{10b^3}-\frac{a(a+bx^2)^6}{6b^3}+\frac{(a+bx^2)^7}{14b^3}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.06

$$\frac{a^4x^6}{6}+\frac{1}{2}a^3bx^8+\frac{3}{5}a^2b^2x^{10}+\frac{1}{3}ab^3x^{12}+\frac{b^4x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^6)/6 + (a^3*b*x^8)/2 + (3*a^2*b^2*x^10)/5 + (a*b^3*x^12)/3 + (b^4*x^
14)/14
```

Maple [A]
time = 0.05, size = 47, normalized size = 0.89

method	result	size
default	$\frac{1}{6}a^4x^6+\frac{1}{2}a^3b\,x^8+\frac{3}{5}a^2b^2x^{10}+\frac{1}{3}a\,b^3x^{12}+\frac{1}{14}b^4x^{14}$	47
norman	$\frac{1}{6}a^4x^6+\frac{1}{2}a^3b\,x^8+\frac{3}{5}a^2b^2x^{10}+\frac{1}{3}a\,b^3x^{12}+\frac{1}{14}b^4x^{14}$	47
risch	$\frac{1}{6}a^4x^6+\frac{1}{2}a^3b\,x^8+\frac{3}{5}a^2b^2x^{10}+\frac{1}{3}a\,b^3x^{12}+\frac{1}{14}b^4x^{14}$	47
gosper	$\frac{x^6\left(15b^4x^8+70a\,b^3x^6+126a^2b^2x^4+105a^3b\,x^2+35a^4\right)}{210}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^4*x^6+1/2*a^3*b*x^8+3/5*a^2*b^2*x^10+1/3*a*b^3*x^12+1/14*b^4*x^14
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.87

$$\frac{1}{14}\,b^4x^{14}+\frac{1}{3}\,ab^3x^{12}+\frac{3}{5}\,a^2b^2x^{10}+\frac{1}{2}\,a^3bx^8+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^4
*x^6
```

Fricas [A]
time = 0.32, size = 46, normalized size = 0.87

$$\frac{1}{14}\,b^4x^{14}+\frac{1}{3}\,ab^3x^{12}+\frac{3}{5}\,a^2b^2x^{10}+\frac{1}{2}\,a^3bx^8+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^4
*x^6
```

Sympy [A]
time = 0.01, size = 49, normalized size = 0.92

$$\frac{a^4x^6}{6}+\frac{a^3bx^8}{2}+\frac{3a^2b^2x^{10}}{5}+\frac{ab^3x^{12}}{3}+\frac{b^4x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**6/6 + a**3*b*x**8/2 + 3*a**2*b**2*x**10/5 + a*b**3*x**12/3 + b**4*x
**14/14
```

Giac [A]
time = 3.64, size = 46, normalized size = 0.87

$$\frac{1}{14}\,b^4x^{14}+\frac{1}{3}\,ab^3x^{12}+\frac{3}{5}\,a^2b^2x^{10}+\frac{1}{2}\,a^3bx^8+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^4
*x^6
```

Mupad [B]
time = 0.02, size = 46, normalized size = 0.87

$$\frac{a^4\,x^6}{6}+\frac{a^3\,b\,x^8}{2}+\frac{3\,a^2\,b^2\,x^{10}}{5}+\frac{a\,b^3\,x^{12}}{3}+\frac{b^4\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^6)/6 + (b^4*x^14)/14 + (a^3*b*x^8)/2 + (a*b^3*x^12)/3 + (3*a^2*b^2*x
^10)/5
```

3.423 $\int x^4(a^2+2abx^2+b^2x^4)^2\,dx$

Optimal. Leaf size=56

$$\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

```
[Out] 1/5*a^4*x^5+4/7*a^3*b*x^7+2/3*a^2*b^2*x^9+4/11*a*b^3*x^11+1/13*b^4*x^13
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 276}

$$\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11)/11 + (b^
4*x^13)/13
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a^2+2abx^2+b^2x^4\right)^2\,dx &= \frac{\int x^4\left(ab+b^2x^2\right)^4\,dx}{b^4}\\ &= \frac{\int \left(a^4b^4x^4+4a^3b^5x^6+6a^2b^6x^8+4ab^7x^{10}+b^8x^{12}\right)dx}{b^4}\\ &= \frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.00

$$\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11)/11 + (b^
4*x^13)/13
```

Maple [A]
time = 0.05, size = 47, normalized size = 0.84

method	result	size
default	$\frac{1}{5}a^4x^5+\frac{4}{7}a^3b\,x^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}a\,b^3x^{11}+\frac{1}{13}b^4x^{13}$	47
norman	$\frac{1}{5}a^4x^5+\frac{4}{7}a^3b\,x^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}a\,b^3x^{11}+\frac{1}{13}b^4x^{13}$	47
risch	$\frac{1}{5}a^4x^5+\frac{4}{7}a^3b\,x^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}a\,b^3x^{11}+\frac{1}{13}b^4x^{13}$	47
gosper	$\frac{x^5\left(1155b^4x^8+5460a\,b^3x^6+10010a^2b^2x^4+8580a^3b\,x^2+3003a^4\right)}{15015}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^4*x^5+4/7*a^3*b*x^7+2/3*a^2*b^2*x^9+4/11*a*b^3*x^11+1/13*b^4*x^13
```

Maxima [A]
time = 0.30, size = 46, normalized size = 0.82

$$\frac{1}{13}\,b^4x^{13}+\frac{4}{11}\,ab^3x^{11}+\frac{2}{3}\,a^2b^2x^9+\frac{4}{7}\,a^3bx^7+\frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^4
*x^5
```

Fricas [A]
time = 0.34, size = 46, normalized size = 0.82

$$\frac{1}{13}\,b^4x^{13}+\frac{4}{11}\,ab^3x^{11}+\frac{2}{3}\,a^2b^2x^9+\frac{4}{7}\,a^3bx^7+\frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^4
*x^5
```

Sympy [A]
time = 0.01, size = 53, normalized size = 0.95

$$\frac{a^4x^5}{5} + \frac{4a^3bx^7}{7} + \frac{2a^2b^2x^9}{3} + \frac{4ab^3x^{11}}{11} + \frac{b^4x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**5/5 + 4*a**3*b*x**7/7 + 2*a**2*b**2*x**9/3 + 4*a*b**3*x**11/11 + b*
*4*x**13/13
```

Giac [A]
time = 3.39, size = 46, normalized size = 0.82

$$\frac{1}{13}\,b^4x^{13} + \frac{4}{11}\,ab^3x^{11} + \frac{2}{3}\,a^2b^2x^9 + \frac{4}{7}\,a^3bx^7 + \frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^4
*x^5
```

Mupad [B]
time = 0.02, size = 46, normalized size = 0.82

$$\frac{a^4\,x^5}{5} + \frac{4\,a^3\,b\,x^7}{7} + \frac{2\,a^2\,b^2\,x^9}{3} + \frac{4\,a\,b^3\,x^{11}}{11} + \frac{b^4\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^5)/5 + (b^4*x^13)/13 + (4*a^3*b*x^7)/7 + (4*a*b^3*x^11)/11 + (2*a^2*
b^2*x^9)/3
```

3.424 $\int x^3(a^2 + 2abx^2 + b^2x^4)^2\, dx$

Optimal. Leaf size=34

$$-\frac{a(a + bx^2)^5}{10b^2} + \frac{(a + bx^2)^6}{12b^2}$$

```
[Out] -1/10*a*(b*x^2+a)^5/b^2+1/12*(b*x^2+a)^6/b^2
```

Rubi [A]

time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$\frac{(a + bx^2)^6}{12b^2} - \frac{a(a + bx^2)^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/10*(a*(a + b*x^2)^5)/b^2 + (a + b*x^2)^6/(12*b^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{\int x^3(ab+b^2x^2)^4\,dx}{b^4}$$
$$=\frac{\text{Subst}\left(\int x(ab+b^2x)^4\,dx,x,x^2\right)}{2b^4}$$
$$=\frac{\text{Subst}\left(\int\left(-\frac{a(ab+b^2x)^4}{b}+\frac{(ab+b^2x)^5}{b^2}\right)\,dx,x,x^2\right)}{2b^4}$$
$$=-\frac{a(a+bx^2)^5}{10b^2}+\frac{(a+bx^2)^6}{12b^2}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.65

$$\frac{a^4x^4}{4}+\frac{2}{3}a^3bx^6+\frac{3}{4}a^2b^2x^8+\frac{2}{5}ab^3x^{10}+\frac{b^4x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^4)/4 + (2*a^3*b*x^6)/3 + (3*a^2*b^2*x^8)/4 + (2*a*b^3*x^10)/5 + (b^4
*x^12)/12
```

Maple [A]
time = 0.05, size = 47, normalized size = 1.38

method	result	size
default	$\frac{1}{12}b^4x^{12}+\frac{2}{5}a\,b^3x^{10}+\frac{3}{4}a^2b^2x^8+\frac{2}{3}a^3b\,x^6+\frac{1}{4}a^4x^4$	47
norman	$\frac{1}{12}b^4x^{12}+\frac{2}{5}a\,b^3x^{10}+\frac{3}{4}a^2b^2x^8+\frac{2}{3}a^3b\,x^6+\frac{1}{4}a^4x^4$	47
risch	$\frac{1}{12}b^4x^{12}+\frac{2}{5}a\,b^3x^{10}+\frac{3}{4}a^2b^2x^8+\frac{2}{3}a^3b\,x^6+\frac{1}{4}a^4x^4$	47
gosper	$\frac{x^4\left(5b^4x^8+24a\,b^3x^6+45a^2b^2x^4+40a^3b\,x^2+15a^4\right)}{60}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^4*x^12+2/5*a*b^3*x^10+3/4*a^2*b^2*x^8+2/3*a^3*b*x^6+1/4*a^4*x^4
```

Maxima [A]
time = 0.29, size = 46, normalized size = 1.35

$$\frac{1}{12}\,b^4x^{12}+\frac{2}{5}\,ab^3x^{10}+\frac{3}{4}\,a^2b^2x^8+\frac{2}{3}\,a^3bx^6+\frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4*
x^4
```

Fricas [A]
time = 0.35, size = 46, normalized size = 1.35

$$\frac{1}{12}\,b^4x^{12}+\frac{2}{5}\,ab^3x^{10}+\frac{3}{4}\,a^2b^2x^8+\frac{2}{3}\,a^3bx^6+\frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4*
x^4
```

Sympy [A]
time = 0.01, size = 53, normalized size = 1.56

$$\frac{a^4x^4}{4}+\frac{2a^3bx^6}{3}+\frac{3a^2b^2x^8}{4}+\frac{2ab^3x^{10}}{5}+\frac{b^4x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**4/4 + 2*a**3*b*x**6/3 + 3*a**2*b**2*x**8/4 + 2*a*b**3*x**10/5 + b**
4*x**12/12
```

Giac [A]
time = 4.20, size = 46, normalized size = 1.35

$$\frac{1}{12}\,b^4x^{12}+\frac{2}{5}\,ab^3x^{10}+\frac{3}{4}\,a^2b^2x^8+\frac{2}{3}\,a^3bx^6+\frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4*
x^4
```

Mupad [B]
time = 0.02, size = 46, normalized size = 1.35

$$\frac{a^4\,x^4}{4}+\frac{2\,a^3\,b\,x^6}{3}+\frac{3\,a^2\,b^2\,x^8}{4}+\frac{2\,a\,b^3\,x^{10}}{5}+\frac{b^4\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^4)/4 + (b^4*x^12)/12 + (2*a^3*b*x^6)/3 + (2*a*b^3*x^10)/5 + (3*a^2*b
^2*x^8)/4
```

3.425 $\int x^2(a^2+2abx^2+b^2x^4)^2\,dx$

Optimal. Leaf size=56

$$\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

```
[Out] 1/3*a^4*x^3+4/5*a^3*b*x^5+6/7*a^2*b^2*x^7+4/9*a*b^3*x^9+1/11*b^4*x^11
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {28, 276}

$$\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)/9 + (b^4*
x^11)/11
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^2+b^2x^4\right)^2\,dx&=\frac{\int x^2\left(ab+b^2x^2\right)^4\,dx}{b^4}\\&=\frac{\int\left(a^4b^4x^2+4a^3b^5x^4+6a^2b^6x^6+4ab^7x^8+b^8x^{10}\right)dx}{b^4}\\&=\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.00

$$\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)/9 + (b^4*
x^11)/11
```

Maple [A]
time = 0.05, size = 47, normalized size = 0.84

method	result	size
default	$\frac{1}{3}a^4x^3+\frac{4}{5}a^3b\,x^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}a\,b^3x^9+\frac{1}{11}b^4x^{11}$	47
norman	$\frac{1}{3}a^4x^3+\frac{4}{5}a^3b\,x^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}a\,b^3x^9+\frac{1}{11}b^4x^{11}$	47
risch	$\frac{1}{3}a^4x^3+\frac{4}{5}a^3b\,x^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}a\,b^3x^9+\frac{1}{11}b^4x^{11}$	47
gosper	$\frac{x^3\left(315b^4x^8+1540a\,b^3x^6+2970a^2b^2x^4+2772a^3b\,x^2+1155a^4\right)}{3465}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^4*x^3+4/5*a^3*b*x^5+6/7*a^2*b^2*x^7+4/9*a*b^3*x^9+1/11*b^4*x^11
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.82

$$\frac{1}{11}\,b^4x^{11}+\frac{4}{9}\,ab^3x^9+\frac{6}{7}\,a^2b^2x^7+\frac{4}{5}\,a^3bx^5+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*x
^3
```

Fricas [A]
time = 0.34, size = 46, normalized size = 0.82

$$\frac{1}{11}\,b^4x^{11}+\frac{4}{9}\,ab^3x^9+\frac{6}{7}\,a^2b^2x^7+\frac{4}{5}\,a^3bx^5+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*x
^3
```

Sympy [A]
time = 0.01, size = 53, normalized size = 0.95

$$\frac{a^4x^3}{3}+\frac{4a^3bx^5}{5}+\frac{6a^2b^2x^7}{7}+\frac{4ab^3x^9}{9}+\frac{b^4x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**3/3 + 4*a**3*b*x**5/5 + 6*a**2*b**2*x**7/7 + 4*a*b**3*x**9/9 + b**4
*x**11/11
```

Giac [A]
time = 4.28, size = 46, normalized size = 0.82

$$\frac{1}{11}\,b^4x^{11}+\frac{4}{9}\,ab^3x^9+\frac{6}{7}\,a^2b^2x^7+\frac{4}{5}\,a^3bx^5+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*x
^3
```

Mupad [B]
time = 0.02, size = 46, normalized size = 0.82

$$\frac{a^4\,x^3}{3}+\frac{4\,a^3\,b\,x^5}{5}+\frac{6\,a^2\,b^2\,x^7}{7}+\frac{4\,a\,b^3\,x^9}{9}+\frac{b^4\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^3)/3 + (b^4*x^11)/11 + (4*a^3*b*x^5)/5 + (4*a*b^3*x^9)/9 + (6*a^2*b^
2*x^7)/7
```

3.426 $\int x(a^2+2abx^2+b^2x^4)^2\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^2)^5}{10b}$$

```
[Out] 1/10*(b*x^2+a)^5/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {28, 267}

$$\frac{(a+bx^2)^5}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a + b*x^2)^5/(10*b)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx &= \frac{\int x\left(ab+b^2x^2\right)^4\,dx}{b^4}\\ &= \frac{\left(a+bx^2\right)^5}{10b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{(a+bx^2)^5}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a + b*x^2)^5/(10*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(14) = 28$.
time = 0.04, size = 45, normalized size = 2.81

method	result	size
default	$\frac{1}{10}b^4x^{10}+\frac{1}{2}a\,b^3x^8+a^2b^2x^6+a^3b\,x^4+\frac{1}{2}a^4x^2$	45
norman	$\frac{1}{10}b^4x^{10}+\frac{1}{2}a\,b^3x^8+a^2b^2x^6+a^3b\,x^4+\frac{1}{2}a^4x^2$	45
risch	$\frac{1}{10}b^4x^{10}+\frac{1}{2}a\,b^3x^8+a^2b^2x^6+a^3b\,x^4+\frac{1}{2}a^4x^2$	45
gosper	$\frac{x^2\left(b^4x^8+5a\,b^3x^6+10a^2b^2x^4+10a^3b\,x^2+5a^4\right)}{10}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^4*x^10+1/2*a*b^3*x^8+a^2*b^2*x^6+a^3*b*x^4+1/2*a^4*x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(14) = 28$.
time = 0.28, size = 44, normalized size = 2.75

$$\frac{1}{10}\,b^4x^{10}+\frac{1}{2}\,ab^3x^8+a^2b^2x^6+a^3bx^4+\frac{1}{2}\,a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(14) = 28$.
time = 0.34, size = 44, normalized size = 2.75

$$\frac{1}{10}\,b^4x^{10}+\frac{1}{2}\,ab^3x^8+a^2b^2x^6+a^3bx^4+\frac{1}{2}\,a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(10) = 20$.
time = 0.01, size = 44, normalized size = 2.75

$$\frac{a^4x^2}{2} + a^3bx^4 + a^2b^2x^6 + \frac{ab^3x^8}{2} + \frac{b^4x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**2/2 + a**3*b*x**4 + a**2*b**2*x**6 + a*b**3*x**8/2 + b**4*x**10/10
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(14) = 28$.
time = 4.20, size = 44, normalized size = 2.75

$$\frac{1}{10}b^4x^{10} + \frac{1}{2}ab^3x^8 + a^2b^2x^6 + a^3bx^4 + \frac{1}{2}a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

Mupad [B]
time = 0.02, size = 44, normalized size = 2.75

$$\frac{a^4\,x^2}{2} + a^3\,b\,x^4 + a^2\,b^2\,x^6 + \frac{a\,b^3\,x^8}{2} + \frac{b^4\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (a^4*x^2)/2 + (b^4*x^10)/10 + a^3*b*x^4 + (a*b^3*x^8)/2 + a^2*b^2*x^6
```

3.427 $\int (a^2+2abx^2+b^2x^4)^2\,dx$

Optimal. Leaf size=51

$$a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

```
[Out] a^4*x+4/3*a^3*b*x^3+6/5*a^2*b^2*x^5+4/7*a*b^3*x^7+1/9*b^4*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {28, 200}

$$a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/9
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^2+b^2x^4\right)^2\,dx &= \frac{\int \left(ab+b^2x^2\right)^4\,dx}{b^4}\\ &= \frac{\int \left(a^4b^4+4a^3b^5x^2+6a^2b^6x^4+4ab^7x^6+b^8x^8\right)\,dx}{b^4}\\ &= a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 51, normalized size = 1.00

$$a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/9
```

Maple [A]
time = 0.01, size = 44, normalized size = 0.86

method	result	size
default	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
norman	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
risch	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
gosper	$\frac{x\left(35b^4x^8+180a\,b^3x^6+378a^2b^2x^4+420a^3b\,x^2+315a^4\right)}{315}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
[Out] a^4*x+4/3*a^3*b*x^3+6/5*a^2*b^2*x^5+4/7*a*b^3*x^7+1/9*b^4*x^9
```

Maxima [A]
time = 0.28, size = 55, normalized size = 1.08

$$\frac{1}{9}\,b^4x^9+\frac{4}{7}\,ab^3x^7+\frac{4}{5}\,a^2b^2x^5+a^4x+\frac{2}{15}\,\left(3\,b^2x^5+10\,abx^3\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
[Out] 1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 4/5*a^2*b^2*x^5 + a^4*x + 2/15*(3*b^2*x^5 + 1
0*a*b*x^3)*a^2
```

Fricas [A]
time = 0.33, size = 43, normalized size = 0.84

$$\frac{1}{9}\,b^4x^9+\frac{4}{7}\,ab^3x^7+\frac{6}{5}\,a^2b^2x^5+\frac{4}{3}\,a^3bx^3+a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
[Out] 1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 + 4/3*a^3*b*x^3 + a^4*x
```

Sympy [A]
time = 0.01, size = 49, normalized size = 0.96

$$a^4x+\frac{4a^3bx^3}{3}+\frac{6a^2b^2x^5}{5}+\frac{4ab^3x^7}{7}+\frac{b^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x + 4*a**3*b*x**3/3 + 6*a**2*b**2*x**5/5 + 4*a*b**3*x**7/7 + b**4*x**9
/9
```

Giac [A]
time = 5.41, size = 43, normalized size = 0.84

$$\frac{1}{9}b^4x^9 + \frac{4}{7}ab^3x^7 + \frac{6}{5}a^2b^2x^5 + \frac{4}{3}a^3bx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 + 4/3*a^3*b*x^3 + a^4*x
```

Mupad [B]
time = 0.02, size = 43, normalized size = 0.84

$$a^4\,x+\frac{4\,a^3\,b\,x^3}{3}+\frac{6\,a^2\,b^2\,x^5}{5}+\frac{4\,a\,b^3\,x^7}{7}+\frac{b^4\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] a^4*x + (b^4*x^9)/9 + (4*a^3*b*x^3)/3 + (4*a*b^3*x^7)/7 + (6*a^2*b^2*x^5)/5
```

3.428 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x}\,dx$

Optimal. Leaf size=50

$$2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}+a^4\log(x)$$

```
[Out] 2*a^3*b*x^2+3/2*a^2*b^2*x^4+2/3*a*b^3*x^6+1/8*b^4*x^8+a^4*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$a^4\log(x)+2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 + a^4*Log[x
]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x}\,dx}{b^4}$$

$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x}\,dx, x, x^2\right)}{2b^4}$$

$$= \frac{\text{Subst}\left(\int \left(4a^3b^5+\frac{a^4b^4}{x}+6a^2b^6x+4ab^7x^2+b^8x^3\right)\,dx, x, x^2\right)}{2b^4}$$

$$= 2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}+a^4\log(x)$$

Mathematica [A]
time = 0.00, size = 50, normalized size = 1.00

$$2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}+a^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 + a^4*Log[x
]
```

Maple [A]
time = 0.04, size = 45, normalized size = 0.90

method	result	size
default	$2a^3b\,x^2+\frac{3a^2b^2x^4}{2}+\frac{2a\,b^3x^6}{3}+\frac{b^4x^8}{8}+a^4\ln(x)$	45
norman	$2a^3b\,x^2+\frac{3a^2b^2x^4}{2}+\frac{2a\,b^3x^6}{3}+\frac{b^4x^8}{8}+a^4\ln(x)$	45
risch	$\frac{3a^2b^2x^4}{2}+2a^3b\,x^2+\frac{4a^4}{3}+\frac{2a\,b^3x^6}{3}+\frac{b^4x^8}{8}+a^4\ln(x)$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^3*b*x^2+3/2*a^2*b^2*x^4+2/3*a*b^3*x^6+1/8*b^4*x^8+a^4*ln(x)
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.94

$$\frac{1}{8}\,b^4x^8+\frac{2}{3}\,ab^3x^6+\frac{3}{2}\,a^2b^2x^4+2\,a^3bx^2+\frac{1}{2}\,a^4\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2*a^4*log(x
^2)
```

Fricas [A]
time = 0.37, size = 44, normalized size = 0.88

$$\frac{1}{8}\,b^4x^8+\frac{2}{3}\,ab^3x^6+\frac{3}{2}\,a^2b^2x^4+2\,a^3bx^2+a^4\log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4*log(x)
```

Sympy [A]
time = 0.03, size = 49, normalized size = 0.98

$$a^4\log{(x)}+2a^3bx^2+\frac{3a^2b^2x^4}{2}+\frac{2ab^3x^6}{3}+\frac{b^4x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x,x)
```

```
[Out] a**4*log(x) + 2*a**3*b*x**2 + 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/3 + b**4*x
**8/8
```

Giac [A]
time = 6.36, size = 47, normalized size = 0.94

$$\frac{1}{8}\,b^4x^8+\frac{2}{3}\,ab^3x^6+\frac{3}{2}\,a^2b^2x^4+2\,a^3bx^2+\frac{1}{2}\,a^4\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2*a^4*log(x
^2)
```

Mupad [B]
time = 0.03, size = 44, normalized size = 0.88

$$a^4\,\ln{(x)}+\frac{b^4\,x^8}{8}+2\,a^3\,b\,x^2+\frac{2\,a\,b^3\,x^6}{3}+\frac{3\,a^2\,b^2\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x,x)
```

```
[Out] a^4*log(x) + (b^4*x^8)/8 + 2*a^3*b*x^2 + (2*a*b^3*x^6)/3 + (3*a^2*b^2*x^4)/
2
```

3.429 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

```
[Out] -a^4/x+4*a^3*b*x+2*a^2*b^2*x^3+4/5*a*b^3*x^5+1/7*b^4*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 276}

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
```

```
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7)/7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^2}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^2}\,dx}{b^4}\\
&= \frac{\int \left(4a^3b^5+\frac{a^4b^4}{x^2}+6a^2b^6x^2+4ab^7x^4+b^8x^6\right)\,dx}{b^4}\\
&= -\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.00

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
```

```
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7)/7
```

Maple [A]
time = 0.04, size = 45, normalized size = 0.94

method	result	size
default	$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4a\,b^3x^5}{5}+\frac{b^4x^7}{7}$	45
risch	$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4a\,b^3x^5}{5}+\frac{b^4x^7}{7}$	45
norman	$\frac{\frac{1}{7}b^4x^8+\frac{4}{5}a\,b^3x^6+2a^2b^2x^4+4a^3b\,x^2-a^4}{x}$	48
gosper	$-\frac{-5b^4x^8-28a\,b^3x^6-70a^2b^2x^4-140a^3b\,x^2+35a^4}{35x}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^4/x+4*a^3*b*x+2*a^2*b^2*x^3+4/5*a*b^3*x^5+1/7*b^4*x^7
```

Maxima [A]
time = 0.29, size = 44, normalized size = 0.92

$$\frac{1}{7}\,b^4x^7+\frac{4}{5}\,ab^3x^5+2\,a^2b^2x^3+4\,a^3bx-\frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

Fricas [A]
time = 0.35, size = 48, normalized size = 1.00

$$\frac{5\,b^4x^8+28\,ab^3x^6+70\,a^2b^2x^4+140\,a^3bx^2-35\,a^4}{35\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^4*x^8 + 28*a*b^3*x^6 + 70*a^2*b^2*x^4 + 140*a^3*b*x^2 - 35*a^4)/x
```

Sympy [A]
time = 0.03, size = 44, normalized size = 0.92

$$-\frac{a^4}{x} + 4a^3bx + 2a^2b^2x^3 + \frac{4ab^3x^5}{5} + \frac{b^4x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**2,x)
```

```
[Out] -a**4/x + 4*a**3*b*x + 2*a**2*b**2*x**3 + 4*a*b**3*x**5/5 + b**4*x**7/7
```

Giac [A]
time = 7.39, size = 44, normalized size = 0.92

$$\frac{1}{7}b^4x^7 + \frac{4}{5}ab^3x^5 + 2a^2b^2x^3 + 4a^3bx - \frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

Mupad [B]
time = 0.02, size = 44, normalized size = 0.92

$$\frac{b^4x^7}{7} - \frac{a^4}{x} + \frac{4ab^3x^5}{5} + 2a^2b^2x^3 + 4a^3bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^2,x)
```

```
[Out] (b^4*x^7)/7 - a^4/x + (4*a*b^3*x^5)/5 + 2*a^2*b^2*x^3 + 4*a^3*b*x
```

3.430 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3} dx$

Optimal. Leaf size=48

$$-\frac{a^4}{2x^2} + 3a^2b^2x^2 + ab^3x^4 + \frac{b^4x^6}{6} + 4a^3b\log(x)$$

```
[Out] -1/2*a^4/x^2+3*a^2*b^2*x^2+a*b^3*x^4+1/6*b^4*x^6+4*a^3*b*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$-\frac{a^4}{2x^2} + 4a^3b\log(x) + 3a^2b^2x^2 + ab^3x^4 + \frac{b^4x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
```

```
[Out] -1/2*a^4/x^2 + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^3}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x^2}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(6a^2b^6+\frac{a^4b^4}{x^2}+\frac{4a^3b^5}{x}+4ab^7x+b^8x^2\right)\,dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{2x^2}+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}+4a^3b\log(x)$$

Mathematica [A]
time = 0.00, size = 48, normalized size = 1.00

$$-\frac{a^4}{2x^2}+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}+4a^3b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
```

```
[Out] -1/2*a^4/x^2 + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*Log[x]
```

Maple [A]
time = 0.02, size = 45, normalized size = 0.94

method	result	size
default	$-\frac{a^4}{2x^2}+3a^2b^2x^2+a\,b^3x^4+\frac{b^4x^6}{6}+4a^3b\ln(x)$	45
risch	$-\frac{a^4}{2x^2}+3a^2b^2x^2+a\,b^3x^4+\frac{b^4x^6}{6}+4a^3b\ln(x)$	45
norman	$\frac{a\,b^3x^6-\frac{1}{2}a^4+\frac{1}{6}b^4x^8+3a^2b^2x^4}{x^2}+4a^3b\ln(x)$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^4/x^2+3*a^2*b^2*x^2+a*b^3*x^4+1/6*b^4*x^6+4*a^3*b*ln(x)
```

Maxima [A]
time = 0.30, size = 46, normalized size = 0.96

$$\frac{1}{6}\,b^4x^6+ab^3x^4+3\,a^2b^2x^2+2\,a^3b\log\left(x^2\right)-\frac{a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*log(x^2) - 1/2*a^4/x^2
```

Fricas [A]
time = 0.35, size = 49, normalized size = 1.02

$$\frac{b^4x^8 + 6\,ab^3x^6 + 18\,a^2b^2x^4 + 24\,a^3bx^2\log\left(x\right) - 3\,a^4}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 + 6*a*b^3*x^6 + 18*a^2*b^2*x^4 + 24*a^3*b*x^2*log(x) - 3*a^4)/
x^2
```

Sympy [A]
time = 0.05, size = 46, normalized size = 0.96

$$-\frac{a^4}{2x^2} + 4a^3b\log\left(x\right) + 3a^2b^2x^2 + ab^3x^4 + \frac{b^4x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**3,x)
```

```
[Out] -a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4 + b**4*x*
*6/6
```

Giac [A]
time = 3.82, size = 56, normalized size = 1.17

$$\frac{1}{6}\,b^4x^6 + ab^3x^4 + 3\,a^2b^2x^2 + 2\,a^3b\log\left(x^2\right) - \frac{4\,a^3bx^2 + a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*log(x^2) - 1/2*(4*a^3*b*x
^2 + a^4)/x^2
```

Mupad [B]
time = 0.03, size = 44, normalized size = 0.92

$$\frac{b^4\,x^6}{6} - \frac{a^4}{2\,x^2} + a\,b^3\,x^4 + 4\,a^3\,b\,\ln\left(x\right) + 3\,a^2\,b^2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^3,x)
```

```
[Out] (b^4*x^6)/6 - a^4/(2*x^2) + a*b^3*x^4 + 4*a^3*b*log(x) + 3*a^2*b^2*x^2
```

3.431 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^4} dx$

Optimal. Leaf size=50

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

```
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 276}

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
```

```
[Out] -1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^4}\,dx &= \frac{\int \frac{(ab+b^2x^2)^4}{x^4}\,dx}{b^4}\\
&= \frac{\int \left(6a^2b^6+\frac{a^4b^4}{x^4}+\frac{4a^3b^5}{x^2}+4ab^7x^2+b^8x^4\right)dx}{b^4}\\
&= -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 50, normalized size = 1.00

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
[Out] -1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Maple [A]
time = 0.02, size = 45, normalized size = 0.90

method	result	size
default	$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4a\,b^3x^3}{3}+\frac{b^4x^5}{5}$	45
risch	$\frac{b^4x^5}{5}+\frac{4a\,b^3x^3}{3}+6a^2b^2x+\frac{-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	47
norman	$\frac{\frac{1}{5}b^4x^8+\frac{4}{3}a\,b^3x^6+6a^2b^2x^4-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	48
gosper	$-\frac{-3b^4x^8-20a\,b^3x^6-90a^2b^2x^4+60a^3b\,x^2+5a^4}{15x^3}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Maxima [A]
time = 0.29, size = 45, normalized size = 0.90

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{12\,a^3bx^2+a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="maxima")
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

Fricas [A]
time = 0.37, size = 48, normalized size = 0.96

$$\frac{3\,b^4x^8+20\,ab^3x^6+90\,a^2b^2x^4-60\,a^3bx^2-5\,a^4}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^3
```

Sympy [A]
time = 0.06, size = 49, normalized size = 0.98

$$6a^2b^2x + \frac{4ab^3x^3}{3} + \frac{b^4x^5}{5} + \frac{-a^4 - 12a^3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**4,x)
```

```
[Out] 6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 + (-a**4 - 12*a**3*b*x**2)/(3
*x**3)
```

Giac [A]
time = 5.96, size = 45, normalized size = 0.90

$$\frac{1}{5}b^4x^5 + \frac{4}{3}ab^3x^3 + 6a^2b^2x - \frac{12a^3bx^2 + a^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

Mupad [B]
time = 0.04, size = 47, normalized size = 0.94

$$\frac{b^4x^5}{5} - \frac{\frac{a^4}{3} + 4ba^3x^2}{x^3} + 6a^2b^2x + \frac{4ab^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^4,x)
```

```
[Out] (b^4*x^5)/5 - (a^4/3 + 4*a^3*b*x^2)/x^3 + 6*a^2*b^2*x + (4*a*b^3*x^3)/3
```

3.432 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5} dx$

Optimal. Leaf size=49

$$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+2ab^3x^2+\frac{b^4x^4}{4}+6a^2b^2\log(x)$$

```
[Out] -1/4*a^4/x^4-2*a^3*b/x^2+2*a*b^3*x^2+1/4*b^4*x^4+6*a^2*b^2*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+6a^2b^2\log(x)+2ab^3x^2+\frac{b^4x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
```

```
[Out] -1/4*a^4/x^4 - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*b^2*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^5}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x^3}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(4ab^7+\frac{a^4b^4}{x^3}+\frac{4a^3b^5}{x^2}+\frac{6a^2b^6}{x}+b^8x\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+2ab^3x^2+\frac{b^4x^4}{4}+6a^2b^2\log(x)$$

Mathematica [A]
time = 0.00, size = 49, normalized size = 1.00

$$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+2ab^3x^2+\frac{b^4x^4}{4}+6a^2b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
[Out] -1/4*a^4/x^4 - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*b^2*Log[x]
```

Maple [A]
time = 0.03, size = 46, normalized size = 0.94

method	result	size
default	$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+2a\,b^3x^2+\frac{b^4x^4}{4}+6a^2b^2\ln\left(x\right)$	46
norman	$\frac{-\frac{1}{4}a^4+\frac{1}{4}b^4x^8+2a\,b^3x^6-2a^3b\,x^2}{x^4}+6a^2b^2\ln\left(x\right)$	48
risch	$\frac{b^4x^4}{4}+2a\,b^3x^2+4a^2b^2+\frac{-2a^3b\,x^2-\frac{1}{4}a^4}{x^4}+6a^2b^2\ln\left(x\right)$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a^4/x^4-2*a^3*b/x^2+2*a*b^3*x^2+1/4*b^4*x^4+6*a^2*b^2*ln(x)
```

Maxima [A]
time = 0.30, size = 48, normalized size = 0.98

$$\frac{1}{4}\,b^4x^4+2\,ab^3x^2+3\,a^2b^2\log\left(x^2\right)-\frac{8\,a^3bx^2+a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*log(x^2) - 1/4*(8*a^3*b*x^2 + a^4)/x^
4
```

Fricas [A]
time = 0.39, size = 49, normalized size = 1.00

$$\frac{b^4x^8 + 8\,ab^3x^6 + 24\,a^2b^2x^4\log\left(x\right) - 8\,a^3bx^2 - a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^8 + 8*a*b^3*x^6 + 24*a^2*b^2*x^4*log(x) - 8*a^3*b*x^2 - a^4)/x^4
```

Sympy [A]
time = 0.08, size = 49, normalized size = 1.00

$$6a^2b^2\log\left(x\right) + 2ab^3x^2 + \frac{b^4x^4}{4} + \frac{-a^4 - 8a^3bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**5,x)
```

```
[Out] 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4 + (-a**4 - 8*a**3*b*x**2)/
(4*x**4)
```

Giac [A]
time = 3.19, size = 59, normalized size = 1.20

$$\frac{1}{4}\,b^4x^4 + 2\,ab^3x^2 + 3\,a^2b^2\log\left(x^2\right) - \frac{18\,a^2b^2x^4 + 8\,a^3bx^2 + a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*log(x^2) - 1/4*(18*a^2*b^2*x^4 + 8*a^
3*b*x^2 + a^4)/x^4
```

Mupad [B]
time = 0.04, size = 48, normalized size = 0.98

$$\frac{b^4\,x^4}{4} - \frac{\frac{a^4}{4} + 2\,b\,a^3\,x^2}{x^4} + 2\,a\,b^3\,x^2 + 6\,a^2\,b^2\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^5,x)
```

```
[Out] (b^4*x^4)/4 - (a^4/4 + 2*a^3*b*x^2)/x^4 + 2*a*b^3*x^2 + 6*a^2*b^2*log(x)
```

3.433 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^6} dx$

Optimal. Leaf size=50

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

```
[Out] -1/5*a^4/x^5-4/3*a^3*b/x^3-6*a^2*b^2/x+4*a*b^3*x+1/3*b^4*x^3
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
```

```
[Out] -1/5*a^4/x^5 - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b^4*x^3)/3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^6} dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^6} dx}{b^4}\\
&= \frac{\int \left(4ab^7+\frac{a^4b^4}{x^6}+\frac{4a^3b^5}{x^4}+\frac{6a^2b^6}{x^2}+b^8x^2\right) dx}{b^4}\\
&= -\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
[Out] -1/5*a^4/x^5 - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b^4*x^3)/3
```

Maple [A]
time = 0.02, size = 45, normalized size = 0.90

method	result	size
default	$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4a\,b^3x+\frac{b^4x^3}{3}$	45
risch	$\frac{b^4x^3}{3}+4a\,b^3x+\frac{-6a^2b^2x^4-\frac{4}{3}a^3b\,x^2-\frac{1}{5}a^4}{x^5}$	47
norman	$\frac{\frac{1}{3}b^4x^8+4a\,b^3x^6-6a^2b^2x^4-\frac{4}{3}a^3b\,x^2-\frac{1}{5}a^4}{x^5}$	48
gosper	$-\frac{-5b^4x^8-60a\,b^3x^6+90a^2b^2x^4+20a^3b\,x^2+3a^4}{15x^5}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x,method=_RETURNVERBOSE)
[Out] -1/5*a^4/x^5-4/3*a^3*b/x^3-6*a^2*b^2/x+4*a*b^3*x+1/3*b^4*x^3
```

Maxima [A]
time = 0.31, size = 47, normalized size = 0.94

$$\frac{1}{3}\,b^4x^3+4\,ab^3x-\frac{90\,a^2b^2x^4+20\,a^3bx^2+3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="maxima")
[Out] 1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3*a^4)/x^5
```

Fricas [A]
time = 0.34, size = 48, normalized size = 0.96

$$\frac{5\,b^4x^8+60\,ab^3x^6-90\,a^2b^2x^4-20\,a^3bx^2-3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(5*b^4*x^8 + 60*a*b^3*x^6 - 90*a^2*b^2*x^4 - 20*a^3*b*x^2 - 3*a^4)/x^5
```

Sympy [A]
time = 0.08, size = 49, normalized size = 0.98

$$4ab^3x + \frac{b^4x^3}{3} + \frac{-3a^4 - 20a^3bx^2 - 90a^2b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**6,x)
```

```
[Out] 4*a*b**3*x + b**4*x**3/3 + (-3*a**4 - 20*a**3*b*x**2 - 90*a**2*b**2*x**4)/(
15*x**5)
```

Giac [A]
time = 3.33, size = 47, normalized size = 0.94

$$\frac{1}{3}\,b^4x^3 + 4\,ab^3x - \frac{90\,a^2b^2x^4 + 20\,a^3bx^2 + 3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3*a^4)/x^5
```

Mupad [B]
time = 0.04, size = 47, normalized size = 0.94

$$\frac{b^4\,x^3}{3} - \frac{\frac{a^4}{5} + \frac{4\,a^3\,b\,x^2}{3} + 6\,a^2\,b^2\,x^4}{x^5} + 4\,a\,b^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^6,x)
```

```
[Out] (b^4*x^3)/3 - (a^4/5 + (4*a^3*b*x^2)/3 + 6*a^2*b^2*x^4)/x^5 + 4*a*b^3*x
```

3.434 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^7}\,dx$

Optimal. Leaf size=49

$$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+\frac{b^4x^2}{2}+4ab^3\log(x)$$

```
[Out] -1/6*a^4/x^6-a^3*b/x^4-3*a^2*b^2/x^2+1/2*b^4*x^2+4*a*b^3*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
```

```
[Out] -1/6*a^4/x^6 - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*b^3*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^7}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^7}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x^4}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(b^8+\frac{a^4b^4}{x^4}+\frac{4a^3b^5}{x^3}+\frac{6a^2b^6}{x^2}+\frac{4ab^7}{x}\right)\,dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+\frac{b^4x^2}{2}+4ab^3\log(x)$$

Mathematica [A]
time = 0.00, size = 49, normalized size = 1.00

$$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+\frac{b^4x^2}{2}+4ab^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
```

```
[Out] -1/6*a^4/x^6 - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*b^3*Log[x]
```

Maple [A]
time = 0.02, size = 46, normalized size = 0.94

method	result	size
default	$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+\frac{b^4x^2}{2}+4a\,b^3\ln(x)$	46
norman	$\frac{-\frac{1}{6}a^4+\frac{1}{2}b^4x^8-3a^2b^2x^4-a^3b\,x^2}{x^6}+4a\,b^3\ln(x)$	48
risch	$\frac{b^4x^2}{2}+\frac{-3a^2b^2x^4-a^3b\,x^2-\frac{1}{6}a^4}{x^6}+4a\,b^3\ln(x)$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^4/x^6-a^3*b/x^4-3*a^2*b^2/x^2+1/2*b^4*x^2+4*a*b^3*ln(x)
```

Maxima [A]
time = 0.30, size = 48, normalized size = 0.98

$$\frac{1}{2}\,b^4x^2+2\,ab^3\log\left(x^2\right)-\frac{18\,a^2b^2x^4+6\,a^3bx^2+a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*x^2 + 2*a*b^3*log(x^2) - 1/6*(18*a^2*b^2*x^4 + 6*a^3*b*x^2 + a^4)/x
^6
```

Fricas [A]
time = 0.33, size = 50, normalized size = 1.02

$$\frac{3\,b^4x^8+24\,ab^3x^6\log\left(x\right)-18\,a^2b^2x^4-6\,a^3bx^2-a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*x^8 + 24*a*b^3*x^6*log(x) - 18*a^2*b^2*x^4 - 6*a^3*b*x^2 - a^4)/
x^6
```

Sympy [A]
time = 0.11, size = 49, normalized size = 1.00

$$4ab^3\log\left(x\right)+\frac{b^4x^2}{2}+\frac{-a^4-6a^3bx^2-18a^2b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**7,x)
```

```
[Out] 4*a*b**3*log(x) + b**4*x**2/2 + (-a**4 - 6*a**3*b*x**2 - 18*a**2*b**2*x**4)
/(6*x**6)
```

Giac [A]
time = 3.44, size = 57, normalized size = 1.16

$$\frac{1}{2}\,b^4x^2+2\,ab^3\log\left(x^2\right)-\frac{22\,ab^3x^6+18\,a^2b^2x^4+6\,a^3bx^2+a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^4*x^2 + 2*a*b^3*log(x^2) - 1/6*(22*a*b^3*x^6 + 18*a^2*b^2*x^4 + 6*a^3
*b*x^2 + a^4)/x^6
```

Mupad [B]
time = 0.04, size = 47, normalized size = 0.96

$$\frac{b^4\,x^2}{2}-\frac{\frac{a^4}{6}+a^3\,b\,x^2+3\,a^2\,b^2\,x^4}{x^6}+4\,a\,b^3\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^7,x)
```

```
[Out] (b^4*x^2)/2 - (a^4/6 + a^3*b*x^2 + 3*a^2*b^2*x^4)/x^6 + 4*a*b^3*log(x)
```

3.435 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^8}\,dx$

Optimal. Leaf size=47

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

```
[Out] -1/7*a^4/x^7-4/5*a^3*b/x^5-2*a^2*b^2/x^3-4*a*b^3/x+b^4*x
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 276}

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
```

```
[Out] -1/7*a^4/x^7 - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x + b^4*x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^8}\,dx &= \frac{\int \frac{(ab+b^2x^2)^4}{x^8}\,dx}{b^4}\\
&= \frac{\int \left(b^8+\frac{a^4b^4}{x^8}+\frac{4a^3b^5}{x^6}+\frac{6a^2b^6}{x^4}+\frac{4ab^7}{x^2}\right)dx}{b^4}\\
&= -\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 47, normalized size = 1.00

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
[Out] -1/7*a^4/x^7 - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x + b^4*x
```

Maple [A]
time = 0.02, size = 44, normalized size = 0.94

method	result	size
default	$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4a\,b^3}{x}+b^4x$	44
risch	$b^4x+\frac{-4a\,b^3x^6-2a^2b^2x^4-\frac{4}{5}a^3b\,x^2-\frac{1}{7}a^4}{x^7}$	46
norman	$\frac{b^4x^8-4a\,b^3x^6-2a^2b^2x^4-\frac{4}{5}a^3b\,x^2-\frac{1}{7}a^4}{x^7}$	47
gosper	$-\frac{-35b^4x^8+140a\,b^3x^6+70a^2b^2x^4+28a^3b\,x^2+5a^4}{35x^7}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x,method=_RETURNVERBOSE)
[Out] -1/7*a^4/x^7-4/5*a^3*b/x^5-2*a^2*b^2/x^3-4*a*b^3/x+b^4*x
```

Maxima [A]
time = 0.32, size = 46, normalized size = 0.98

$$b^4x-\frac{140\,ab^3x^6+70\,a^2b^2x^4+28\,a^3bx^2+5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="maxima")
[Out] b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a^4)/x^7
```

Fricas [A]
time = 0.32, size = 48, normalized size = 1.02

$$\frac{35\,b^4x^8-140\,ab^3x^6-70\,a^2b^2x^4-28\,a^3bx^2-5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 - 140*a*b^3*x^6 - 70*a^2*b^2*x^4 - 28*a^3*b*x^2 - 5*a^4)/x
^7
```

Sympy [A]
time = 0.12, size = 48, normalized size = 1.02

$$b^4x + \frac{-5a^4 - 28a^3bx^2 - 70a^2b^2x^4 - 140ab^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**8,x)
```

```
[Out] b**4*x + (-5*a**4 - 28*a**3*b*x**2 - 70*a**2*b**2*x**4 - 140*a*b**3*x**6)/(
35*x**7)
```

Giac [A]
time = 3.74, size = 46, normalized size = 0.98

$$b^4x - \frac{140\,ab^3x^6 + 70\,a^2b^2x^4 + 28\,a^3bx^2 + 5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="giac")
```

```
[Out] b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a^4)/x^7
```

Mupad [B]
time = 4.19, size = 46, normalized size = 0.98

$$b^4\,x - \frac{\frac{a^4}{7} + \frac{4\,a^3\,b\,x^2}{5} + 2\,a^2\,b^2\,x^4 + 4\,a\,b^3\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^8,x)
```

```
[Out] b^4*x - (a^4/7 + (4*a^3*b*x^2)/5 + 4*a*b^3*x^6 + 2*a^2*b^2*x^4)/x^7
```

3.436 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9}\,dx$

Optimal. Leaf size=50

$$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

```
[Out] -1/8*a^4/x^8-2/3*a^3*b/x^6-3/2*a^2*b^2/x^4-2*a*b^3/x^2+b^4*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 45}

$$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

```
[Out] -1/8*a^4/x^8 - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3)/x^2 + b^
4*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^9}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x^5}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4b^4}{x^5}+\frac{4a^3b^5}{x^4}+\frac{6a^2b^6}{x^3}+\frac{4ab^7}{x^2}+\frac{b^8}{x}\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Mathematica [A]
time = 0.00, size = 50, normalized size = 1.00

$$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

```
[Out] -1/8*a^4/x^8 - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3)/x^2 + b^
4*Log[x]
```

Maple [A]
time = 0.02, size = 45, normalized size = 0.90

method	result	size
default	$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2a\,b^3}{x^2}+b^4\ln(x)$	45
norman	$\frac{-\frac{1}{8}a^4-2a\,b^3x^6-\frac{3}{2}a^2b^2x^4-\frac{2}{3}a^3b\,x^2}{x^8}+b^4\ln(x)$	47
risch	$\frac{-\frac{1}{8}a^4-2a\,b^3x^6-\frac{3}{2}a^2b^2x^4-\frac{2}{3}a^3b\,x^2}{x^8}+b^4\ln(x)$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a^4/x^8-2/3*a^3*b/x^6-3/2*a^2*b^2/x^4-2*a*b^3/x^2+b^4*ln(x)
```

Maxima [A]
time = 0.32, size = 50, normalized size = 1.00

$$\frac{1}{2}\,b^4\log\left(x^2\right)-\frac{48\,ab^3x^6+36\,a^2b^2x^4+16\,a^3bx^2+3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(x^2) - 1/24*(48*a*b^3*x^6 + 36*a^2*b^2*x^4 + 16*a^3*b*x^2 + 3*a
^4)/x^8
```

Fricas [A]
time = 0.37, size = 50, normalized size = 1.00

$$\frac{24\,b^4x^8\log\left(x\right)-48\,ab^3x^6-36\,a^2b^2x^4-16\,a^3bx^2-3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^4*x^8*log(x) - 48*a*b^3*x^6 - 36*a^2*b^2*x^4 - 16*a^3*b*x^2 - 3*
a^4)/x^8
```

Sympy [A]
time = 0.15, size = 49, normalized size = 0.98

$$b^4\log\left(x\right)+\frac{-3a^4-16a^3bx^2-36a^2b^2x^4-48ab^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**9,x)
```

```
[Out] b**4*log(x) + (-3*a**4 - 16*a**3*b*x**2 - 36*a**2*b**2*x**4 - 48*a*b**3*x**
6)/(24*x**8)
```

Giac [A]
time = 3.89, size = 58, normalized size = 1.16

$$\frac{1}{2}\,b^4\log\left(x^2\right)-\frac{25\,b^4x^8+48\,ab^3x^6+36\,a^2b^2x^4+16\,a^3bx^2+3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^4*log(x^2) - 1/24*(25*b^4*x^8 + 48*a*b^3*x^6 + 36*a^2*b^2*x^4 + 16*a^
3*b*x^2 + 3*a^4)/x^8
```

Mupad [B]
time = 0.05, size = 47, normalized size = 0.94

$$b^4\,\ln\left(x\right)-\frac{\frac{a^4}{8}+\frac{2\,a^3\,b\,x^2}{3}+\frac{3\,a^2\,b^2\,x^4}{2}+2\,a\,b^3\,x^6}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^9,x)
```

```
[Out] b^4*log(x) - (a^4/8 + (2*a^3*b*x^2)/3 + 2*a*b^3*x^6 + (3*a^2*b^2*x^4)/2)/x^
8
```

3.437 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{10}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

```
[Out] -1/9*a^4/x^9-4/7*a^3*b/x^7-6/5*a^2*b^2/x^5-4/3*a*b^3/x^3-b^4/x
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 276}

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

```
[Out] -1/9*a^4/x^9 - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3)/(3*x^3)
- b^4/x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{10}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^4}{x^{10}}\,dx}{b^4}\\ &= \frac{\int \left(\frac{a^4b^4}{x^{10}}+\frac{4a^3b^5}{x^8}+\frac{6a^2b^6}{x^6}+\frac{4ab^7}{x^4}+\frac{b^8}{x^2}\right)dx}{b^4}\\ &= -\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

```
[Out] -1/9*a^4/x^9 - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3)/(3*x^3)
- b^4/x
```

Maple [A]
time = 0.01, size = 47, normalized size = 0.87

method	result	size
default	$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4a\,b^3}{3x^3}-\frac{b^4}{x}$	47
norman	$\frac{-b^4x^8-\frac{4}{3}a\,b^3x^6-\frac{6}{5}a^2b^2x^4-\frac{4}{7}a^3b\,x^2-\frac{1}{9}a^4}{x^9}$	48
risch	$\frac{-b^4x^8-\frac{4}{3}a\,b^3x^6-\frac{6}{5}a^2b^2x^4-\frac{4}{7}a^3b\,x^2-\frac{1}{9}a^4}{x^9}$	48
gosper	$-\frac{315b^4x^8+420a\,b^3x^6+378a^2b^2x^4+180a^3b\,x^2+35a^4}{315x^9}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^4/x^9-4/7*a^3*b/x^7-6/5*a^2*b^2/x^5-4/3*a*b^3/x^3-b^4/x
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.89

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

Fricas [A]
time = 0.33, size = 48, normalized size = 0.89

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

Sympy [A]
time = 0.14, size = 51, normalized size = 0.94

$$\frac{-35a^4 - 180a^3bx^2 - 378a^2b^2x^4 - 420ab^3x^6 - 315b^4x^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**10,x)
```

```
[Out] (-35*a**4 - 180*a**3*b*x**2 - 378*a**2*b**2*x**4 - 420*a*b**3*x**6 - 315*b*
*4*x**8)/(315*x**9)
```

Giac [A]
time = 3.38, size = 48, normalized size = 0.89

$$-\frac{315\,b^4x^8 + 420\,ab^3x^6 + 378\,a^2b^2x^4 + 180\,a^3bx^2 + 35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35*
a^4)/x^9
```

Mupad [B]
time = 0.03, size = 47, normalized size = 0.87

$$-\frac{\frac{a^4}{9} + \frac{4\,a^3\,b\,x^2}{7} + \frac{6\,a^2\,b^2\,x^4}{5} + \frac{4\,a\,b^3\,x^6}{3} + b^4\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^10,x)
```

```
[Out] -(a^4/9 + b^4*x^8 + (4*a^3*b*x^2)/7 + (4*a*b^3*x^6)/3 + (6*a^2*b^2*x^4)/5)/
x^9
```

3.438 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^5}{10ax^{10}}$$

```
[Out] -1/10*(b*x^2+a)^5/a/x^10
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 270}

$$-\frac{(a+bx^2)^5}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
```

```
[Out] -1/10*(a + b*x^2)^5/(a*x^10)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^4}{x^{11}}\,dx}{b^4}\\ &= -\frac{(a+bx^2)^5}{10ax^{10}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(19) = 38.
time = 0.00, size = 52, normalized size = 2.74

$$-\frac{a^4}{10x^{10}}-\frac{a^3b}{2x^8}-\frac{a^2b^2}{x^6}-\frac{ab^3}{x^4}-\frac{b^4}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
```

```
[Out] -1/10*a^4/x^10 - (a^3*b)/(2*x^8) - (a^2*b^2)/x^6 - (a*b^3)/x^4 - b^4/(2*x^2
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.
time = 0.01, size = 47, normalized size = 2.47

method	result	size
gosper	$-\frac{5 b^{4} x^{8}+10 a \,b^{3} x^{6}+10 a^{2} b^{2} x^{4}+5 a^{3} b \,x^{2}+a^{4}}{10 x^{10}}$	47
default	$-\frac{a \,b^{3}}{x^{4}}-\frac{b^{4}}{2 x^{2}}-\frac{a^{3} b}{2 x^{8}}-\frac{a^{4}}{10 x^{10}}-\frac{a^{2} b^{2}}{x^{6}}$	47
norman	$\frac{-\frac{1}{2} b^{4} x^{8}-a \,b^{3} x^{6}-a^{2} b^{2} x^{4}-\frac{1}{2} a^{3} b \,x^{2}-\frac{1}{10} a^{4}}{x^{10}}$	48
risch	$\frac{-\frac{1}{2} b^{4} x^{8}-a \,b^{3} x^{6}-a^{2} b^{2} x^{4}-\frac{1}{2} a^{3} b \,x^{2}-\frac{1}{10} a^{4}}{x^{10}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -a*b^3/x^4-1/2*b^4/x^2-1/2*a^3*b/x^8-1/10*a^4/x^10-a^2*b^2/x^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.
time = 0.38, size = 46, normalized size = 2.42

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.
time = 0.37, size = 46, normalized size = 2.42

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(15) = 30$.
time = 0.16, size = 49, normalized size = 2.58

$$\frac{-a^4 - 5a^3bx^2 - 10a^2b^2x^4 - 10ab^3x^6 - 5b^4x^8}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**11,x)
```

```
[Out] (-a**4 - 5*a**3*b*x**2 - 10*a**2*b**2*x**4 - 10*a*b**3*x**6 - 5*b**4*x**8)/
(10*x**10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.
time = 3.62, size = 46, normalized size = 2.42

$$-\frac{5\,b^4x^8 + 10\,ab^3x^6 + 10\,a^2b^2x^4 + 5\,a^3bx^2 + a^4}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

Mupad [B]
time = 0.03, size = 46, normalized size = 2.42

$$-\frac{\frac{a^4}{10} + \frac{a^3\,b\,x^2}{2} + a^2\,b^2\,x^4 + a\,b^3\,x^6 + \frac{b^4\,x^8}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^11,x)
```

```
[Out] -(a^4/10 + (b^4*x^8)/2 + (a^3*b*x^2)/2 + a*b^3*x^6 + a^2*b^2*x^4)/x^10
```

3.439 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}} dx$

Optimal. Leaf size=56

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

```
[Out] -1/11*a^4/x^11-4/9*a^3*b/x^9-6/7*a^2*b^2/x^7-4/5*a*b^3/x^5-1/3*b^4/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

```
[Out] -1/11*a^4/x^11 - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b^3)/(5*x^5
) - b^4/(3*x^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}} dx &= \frac{\int \frac{(ab+b^2x^2)^4}{x^{12}} dx}{b^4}\\
&= \frac{\int \left(\frac{a^4b^4}{x^{12}}+\frac{4a^3b^5}{x^{10}}+\frac{6a^2b^6}{x^8}+\frac{4ab^7}{x^6}+\frac{b^8}{x^4}\right) dx}{b^4}\\
&= -\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

```
[Out] -1/11*a^4/x^11 - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b^3)/(5*x^5
) - b^4/(3*x^3)
```

Maple [A]
time = 0.02, size = 47, normalized size = 0.84

method	result	size
default	$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4a\,b^3}{5x^5}-\frac{b^4}{3x^3}$	47
norman	$\frac{-\frac{1}{3}b^4x^8-\frac{4}{5}a\,b^3x^6-\frac{6}{7}a^2b^2x^4-\frac{4}{9}a^3b\,x^2-\frac{1}{11}a^4}{x^{11}}$	48
risch	$\frac{-\frac{1}{3}b^4x^8-\frac{4}{5}a\,b^3x^6-\frac{6}{7}a^2b^2x^4-\frac{4}{9}a^3b\,x^2-\frac{1}{11}a^4}{x^{11}}$	48
gosper	$-\frac{1155b^4x^8+2772a\,b^3x^6+2970a^2b^2x^4+1540a^3b\,x^2+315a^4}{3465x^{11}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*a^4/x^11-4/9*a^3*b/x^9-6/7*a^2*b^2/x^7-4/5*a*b^3/x^5-1/3*b^4/x^3
```

Maxima [A]
time = 0.31, size = 48, normalized size = 0.86

$$-\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

Fricas [A]
time = 0.34, size = 48, normalized size = 0.86

$$-\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

Sympy [A]
time = 0.16, size = 51, normalized size = 0.91

$$\frac{-315a^4 - 1540a^3bx^2 - 2970a^2b^2x^4 - 2772ab^3x^6 - 1155b^4x^8}{3465x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**12,x)
```

```
[Out] (-315*a**4 - 1540*a**3*b*x**2 - 2970*a**2*b**2*x**4 - 2772*a*b**3*x**6 - 11
55*b**4*x**8)/(3465*x**11)
```

Giac [A]
time = 4.03, size = 48, normalized size = 0.86

$$-\frac{1155\,b^4x^8 + 2772\,ab^3x^6 + 2970\,a^2b^2x^4 + 1540\,a^3bx^2 + 315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
+ 315*a^4)/x^11
```

Mupad [B]
time = 4.84, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{11} + \frac{4\,a^3\,b\,x^2}{9} + \frac{6\,a^2\,b^2\,x^4}{7} + \frac{4\,a\,b^3\,x^6}{5} + \frac{b^4\,x^8}{3}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^12,x)
```

```
[Out] -(a^4/11 + (b^4*x^8)/3 + (4*a^3*b*x^2)/9 + (4*a*b^3*x^6)/5 + (6*a^2*b^2*x^4
)/7)/x^11
```

3.440 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}} dx$

Optimal. Leaf size=40

$$-\frac{(a+bx^2)^5}{12ax^{12}}+\frac{b(a+bx^2)^5}{60a^2x^{10}}$$

```
[Out] -1/12*(b*x^2+a)^5/a/x^12+1/60*b*(b*x^2+a)^5/a^2/x^10
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 272, 47, 37}

$$\frac{b(a+bx^2)^5}{60a^2x^{10}}-\frac{(a+bx^2)^5}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
```

```
[Out] -1/12*(a + b*x^2)^5/(a*x^12) + (b*(a + b*x^2)^5)/(60*a^2*x^10)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^4}{x^{13}}\,dx}{b^4}\\ &= \frac{\mathrm{Subst}\left(\int \frac{(ab+b^2x)^4}{x^7}\,dx,x,x^2\right)}{2b^4}\\ &= -\frac{(a+bx^2)^5}{12ax^{12}}-\frac{\mathrm{Subst}\left(\int \frac{(ab+b^2x)^4}{x^6}\,dx,x,x^2\right)}{12ab^3}\\ &= -\frac{(a+bx^2)^5}{12ax^{12}}+\frac{b(a+bx^2)^5}{60a^2x^{10}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.40

$$-\frac{a^4}{12x^{12}}-\frac{2a^3b}{5x^{10}}-\frac{3a^2b^2}{4x^8}-\frac{2ab^3}{3x^6}-\frac{b^4}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
```

```
[Out] -1/12*a^4/x^12 - (2*a^3*b)/(5*x^10) - (3*a^2*b^2)/(4*x^8) - (2*a*b^3)/(3*x^
6) - b^4/(4*x^4)
```

Maple [A]
time = 0.02, size = 47, normalized size = 1.18

method	result	size
default	$-\frac{b^4}{4x^4}-\frac{a^4}{12x^{12}}-\frac{3a^2b^2}{4x^8}-\frac{2a^3b}{5x^{10}}-\frac{2a\,b^3}{3x^6}$	47
norman	$\frac{-\frac{1}{4}b^4x^8-\frac{2}{3}a\,b^3x^6-\frac{3}{4}a^2b^2x^4-\frac{2}{5}a^3b\,x^2-\frac{1}{12}a^4}{x^{12}}$	48
risch	$\frac{-\frac{1}{4}b^4x^8-\frac{2}{3}a\,b^3x^6-\frac{3}{4}a^2b^2x^4-\frac{2}{5}a^3b\,x^2-\frac{1}{12}a^4}{x^{12}}$	48
gosper	$-\frac{15b^4x^8+40a\,b^3x^6+45a^2b^2x^4+24a^3b\,x^2+5a^4}{60x^{12}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x,method=_RETURNVERBOSE)
```

[Out] -1/4*b^4/x^4-1/12*a^4/x^12-3/4*a^2*b^2/x^8-2/5*a^3*b/x^10-2/3*a*b^3/x^6

Maxima [A]
time = 0.34, size = 48, normalized size = 1.20

$$-\frac{15\,b^4x^8+40\,ab^3x^6+45\,a^2b^2x^4+24\,a^3bx^2+5\,a^4}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="maxima")

[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12

Fricas [A]
time = 0.31, size = 48, normalized size = 1.20

$$-\frac{15\,b^4x^8+40\,ab^3x^6+45\,a^2b^2x^4+24\,a^3bx^2+5\,a^4}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="fricas")

[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12

Sympy [A]
time = 0.17, size = 51, normalized size = 1.28

$$\frac{-5a^4-24a^3bx^2-45a^2b^2x^4-40ab^3x^6-15b^4x^8}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**13,x)

[Out] (-5*a**4 - 24*a**3*b*x**2 - 45*a**2*b**2*x**4 - 40*a*b**3*x**6 - 15*b**4*x**
*8)/(60*x**12)

Giac [A]
time = 4.03, size = 48, normalized size = 1.20

$$-\frac{15\,b^4x^8+40\,ab^3x^6+45\,a^2b^2x^4+24\,a^3bx^2+5\,a^4}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="giac")

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/x
^12
```

Mupad [B]
time = 4.22, size = 48, normalized size = 1.20

$$-\frac{\frac{a^4}{12}+\frac{2\,a^3\,b\,x^2}{5}+\frac{3\,a^2\,b^2\,x^4}{4}+\frac{2\,a\,b^3\,x^6}{3}+\frac{b^4\,x^8}{4}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^13,x)
```

```
[Out] -(a^4/12 + (b^4*x^8)/4 + (2*a^3*b*x^2)/5 + (2*a*b^3*x^6)/3 + (3*a^2*b^2*x^4
)/4)/x^12
```

3.441 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

```
[Out] -1/13*a^4/x^13-4/11*a^3*b/x^11-2/3*a^2*b^2/x^9-4/7*a*b^3/x^7-1/5*b^4/x^5
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
```

```
[Out] -1/13*a^4/x^13 - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a*b^3)/(7*x
^7) - b^4/(5*x^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^4}{x^{14}}\,dx}{b^4}\\
&= \frac{\int \left(\frac{a^4b^4}{x^{14}}+\frac{4a^3b^5}{x^{12}}+\frac{6a^2b^6}{x^{10}}+\frac{4ab^7}{x^8}+\frac{b^8}{x^6}\right)dx}{b^4}\\
&= -\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
[Out] -1/13*a^4/x^13 - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a*b^3)/(7*x
^7) - b^4/(5*x^5)
```

Maple [A]
time = 0.02, size = 47, normalized size = 0.84

method	result	size
default	$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4a\,b^3}{7x^7}-\frac{b^4}{5x^5}$	47
norman	$\frac{-\frac{1}{5}b^4x^8-\frac{4}{7}a\,b^3x^6-\frac{2}{3}a^2b^2x^4-\frac{4}{11}a^3b\,x^2-\frac{1}{13}a^4}{x^{13}}$	48
risch	$\frac{-\frac{1}{5}b^4x^8-\frac{4}{7}a\,b^3x^6-\frac{2}{3}a^2b^2x^4-\frac{4}{11}a^3b\,x^2-\frac{1}{13}a^4}{x^{13}}$	48
gosper	$-\frac{3003b^4x^8+8580a\,b^3x^6+10010a^2b^2x^4+5460a^3b\,x^2+1155a^4}{15015x^{13}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x,method=_RETURNVERBOSE)
[Out] -1/13*a^4/x^13-4/11*a^3*b/x^11-2/3*a^2*b^2/x^9-4/7*a*b^3/x^7-1/5*b^4/x^5
```

Maxima [A]
time = 0.30, size = 48, normalized size = 0.86

$$-\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="maxima")
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^
2 + 1155*a^4)/x^13
```

Fricas [A]
time = 0.32, size = 48, normalized size = 0.86

$$-\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="fricas")

[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^2 + 1155*a^4)/x^13

Sympy [A]
time = 0.17, size = 51, normalized size = 0.91

$$\frac{-1155a^4 - 5460a^3bx^2 - 10010a^2b^2x^4 - 8580ab^3x^6 - 3003b^4x^8}{15015x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**14,x)

[Out] (-1155*a**4 - 5460*a**3*b*x**2 - 10010*a**2*b**2*x**4 - 8580*a*b**3*x**6 - 3003*b**4*x**8)/(15015*x**13)

Giac [A]
time = 4.02, size = 48, normalized size = 0.86

$$-\frac{3003\,b^4x^8 + 8580\,ab^3x^6 + 10010\,a^2b^2x^4 + 5460\,a^3bx^2 + 1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="giac")

[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x^2 + 1155*a^4)/x^13

Mupad [B]
time = 0.04, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{13} + \frac{4\,a^3\,b\,x^2}{11} + \frac{2\,a^2\,b^2\,x^4}{3} + \frac{4\,a\,b^3\,x^6}{7} + \frac{b^4\,x^8}{5}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^14,x)

[Out] -(a^4/13 + (b^4*x^8)/5 + (4*a^3*b*x^2)/11 + (4*a*b^3*x^6)/7 + (2*a^2*b^2*x^4)/3)/x^13

3.442 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

```
[Out] -1/14*a^4/x^14-1/3*a^3*b/x^12-3/5*a^2*b^2/x^10-1/2*a*b^3/x^8-1/6*b^4/x^6
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

```
[Out] -1/14*a^4/x^14 - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3)/(2*x^8)
- b^4/(6*x^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{x^{15}}\,dx}{b^4}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^4}{x^8}\,dx, x, x^2\right)}{2b^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4b^4}{x^8}+\frac{4a^3b^5}{x^7}+\frac{6a^2b^6}{x^6}+\frac{4ab^7}{x^5}+\frac{b^8}{x^4}\right)dx, x, x^2\right)}{2b^4}$$
$$= -\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.00

$$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

```
[Out] -1/14*a^4/x^14 - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3)/(2*x^8)
- b^4/(6*x^6)
```

Maple [A]
time = 0.02, size = 47, normalized size = 0.84

method	result	size
default	$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{a\,b^3}{2x^8}-\frac{b^4}{6x^6}$	47
norman	$\frac{-\frac{1}{14}a^4-\frac{1}{3}a^3b\,x^2-\frac{3}{5}a^2b^2x^4-\frac{1}{2}a\,b^3x^6-\frac{1}{6}b^4x^8}{x^{14}}$	48
risch	$\frac{-\frac{1}{14}a^4-\frac{1}{3}a^3b\,x^2-\frac{3}{5}a^2b^2x^4-\frac{1}{2}a\,b^3x^6-\frac{1}{6}b^4x^8}{x^{14}}$	48
gosper	$-\frac{35b^4x^8+105a\,b^3x^6+126a^2b^2x^4+70a^3b\,x^2+15a^4}{210x^{14}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14*a^4/x^14-1/3*a^3*b/x^12-3/5*a^2*b^2/x^10-1/2*a*b^3/x^8-1/6*b^4/x^6
```

Maxima [A]
time = 0.30, size = 48, normalized size = 0.86

$$-\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="maxima")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

Fricas [A]
time = 0.32, size = 48, normalized size = 0.86

$$-\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="fricas")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

Sympy [A]
time = 0.19, size = 51, normalized size = 0.91

$$\frac{-15a^4-70a^3bx^2-126a^2b^2x^4-105ab^3x^6-35b^4x^8}{210x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**15,x)
```

```
[Out] (-15*a**4 - 70*a**3*b*x**2 - 126*a**2*b**2*x**4 - 105*a*b**3*x**6 - 35*b**4
*x**8)/(210*x**14)
```

Giac [A]
time = 4.18, size = 48, normalized size = 0.86

$$-\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="giac")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a^
4)/x^14
```

Mupad [B]
time = 4.33, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{14}+\frac{a^3\,b\,x^2}{3}+\frac{3\,a^2\,b^2\,x^4}{5}+\frac{a\,b^3\,x^6}{2}+\frac{b^4\,x^8}{6}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^15,x)
```

```
[Out] -(a^4/14 + (b^4*x^8)/6 + (a^3*b*x^2)/3 + (a*b^3*x^6)/2 + (3*a^2*b^2*x^4)/5)
/x^14
```

3.443 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{16}} dx$

Optimal. Leaf size=56

$$-\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

```
[Out] -1/15*a^4/x^15-4/13*a^3*b/x^13-6/11*a^2*b^2/x^11-4/9*a*b^3/x^9-1/7*b^4/x^7
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16,x]
```

```
[Out] -1/15*a^4/x^15 - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4*a*b^3)/(9
*x^9) - b^4/(7*x^7)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{16}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^4}{x^{16}}\,dx}{b^4}\\
&= \frac{\int \left(\frac{a^4b^4}{x^{16}}+\frac{4a^3b^5}{x^{14}}+\frac{6a^2b^6}{x^{12}}+\frac{4ab^7}{x^{10}}+\frac{b^8}{x^8}\right)dx}{b^4}\\
&= -\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16,x]
```

```
[Out] -1/15*a^4/x^15 - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4*a*b^3)/(9
*x^9) - b^4/(7*x^7)
```

Maple [A]
time = 0.02, size = 47, normalized size = 0.84

method	result	size
default	$-\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4a\,b^3}{9x^9}-\frac{b^4}{7x^7}$	47
norman	$\frac{-\frac{1}{15}a^4-\frac{4}{13}a^3b\,x^2-\frac{6}{11}a^2b^2x^4-\frac{4}{9}a\,b^3x^6-\frac{1}{7}b^4x^8}{x^{15}}$	48
risch	$\frac{-\frac{1}{15}a^4-\frac{4}{13}a^3b\,x^2-\frac{6}{11}a^2b^2x^4-\frac{4}{9}a\,b^3x^6-\frac{1}{7}b^4x^8}{x^{15}}$	48
gosper	$-\frac{6435b^4x^8+20020a\,b^3x^6+24570a^2b^2x^4+13860a^3b\,x^2+3003a^4}{45045x^{15}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*a^4/x^15-4/13*a^3*b/x^13-6/11*a^2*b^2/x^11-4/9*a*b^3/x^9-1/7*b^4/x^7
```

Maxima [A]
time = 0.43, size = 48, normalized size = 0.86

$$-\frac{6435\,b^4x^8+20020\,ab^3x^6+24570\,a^2b^2x^4+13860\,a^3bx^2+3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

Fricas [A]
time = 0.33, size = 48, normalized size = 0.86

$$-\frac{6435\,b^4x^8+20020\,ab^3x^6+24570\,a^2b^2x^4+13860\,a^3bx^2+3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

Sympy [A]
time = 0.19, size = 51, normalized size = 0.91

$$\frac{-3003a^4 - 13860a^3bx^2 - 24570a^2b^2x^4 - 20020ab^3x^6 - 6435b^4x^8}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**16,x)
```

```
[Out] (-3003*a**4 - 13860*a**3*b*x**2 - 24570*a**2*b**2*x**4 - 20020*a*b**3*x**6
- 6435*b**4*x**8)/(45045*x**15)
```

Giac [A]
time = 3.90, size = 48, normalized size = 0.86

$$-\frac{6435\,b^4x^8 + 20020\,ab^3x^6 + 24570\,a^2b^2x^4 + 13860\,a^3bx^2 + 3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13860*a^3*b*
x^2 + 3003*a^4)/x^15
```

Mupad [B]
time = 4.35, size = 48, normalized size = 0.86

$$-\frac{\frac{a^4}{15} + \frac{4\,a^3\,b\,x^2}{13} + \frac{6\,a^2\,b^2\,x^4}{11} + \frac{4\,a\,b^3\,x^6}{9} + \frac{b^4\,x^8}{7}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^16,x)
```

```
[Out] -(a^4/15 + (b^4*x^8)/7 + (4*a^3*b*x^2)/13 + (4*a*b^3*x^6)/9 + (6*a^2*b^2*x^
4)/11)/x^15
```

3.444 $\int x^8(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal. Leaf size=82

$$\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

```
[Out] 1/9*a^6*x^9+6/11*a^5*b*x^11+15/13*a^4*b^2*x^13+4/3*a^3*b^3*x^15+15/17*a^2*b
^4*x^17+6/19*a*b^5*x^19+1/21*b^6*x^21
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {28, 276}

$$\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3
 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21)/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int x^8\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\int \left(a^6b^6x^8+6a^5b^7x^{10}+15a^4b^8x^{12}+20a^3b^9x^{14}+15a^2b^{10}x^{16}+6ab^{11}x^{18}+b\right.}{b^6}\\ &= \frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{2}}{21}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 82, normalized size = 1.00

$$\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3
 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21)/21
```

Maple [A]
time = 0.05, size = 69, normalized size = 0.84

method	result	size
default	$\frac{1}{9}a^6x^9+\frac{6}{11}a^5b\,x^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}a\,b^5x^{19}+\frac{1}{21}b^6x^{21}$	69
norman	$\frac{1}{9}a^6x^9+\frac{6}{11}a^5b\,x^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}a\,b^5x^{19}+\frac{1}{21}b^6x^{21}$	69
risch	$\frac{1}{9}a^6x^9+\frac{6}{11}a^5b\,x^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}a\,b^5x^{19}+\frac{1}{21}b^6x^{21}$	69
gosper	$\frac{x^9\left(138567b^6x^{12}+918918a\,b^5x^{10}+2567565a^2b^4x^8+3879876a^3b^3x^6+3357585a^4b^2x^4+1587222a^5b\,x^2+323323a^6\right)}{2909907}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a^6*x^9+6/11*a^5*b*x^11+15/13*a^4*b^2*x^13+4/3*a^3*b^3*x^15+15/17*a^2*b
^4*x^17+6/19*a*b^5*x^19+1/21*b^6*x^21
```

Maxima [A]
time = 0.54, size = 68, normalized size = 0.83

$$\frac{1}{21}\,b^6x^{21}+\frac{6}{19}\,ab^5x^{19}+\frac{15}{17}\,a^2b^4x^{17}+\frac{4}{3}\,a^3b^3x^{15}+\frac{15}{13}\,a^4b^2x^{13}+\frac{6}{11}\,a^5bx^{11}+\frac{1}{9}\,a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 + 1
5/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

Fricas [A]
time = 0.36, size = 68, normalized size = 0.83

$$\frac{1}{21}\,b^6x^{21}+\frac{6}{19}\,ab^5x^{19}+\frac{15}{17}\,a^2b^4x^{17}+\frac{4}{3}\,a^3b^3x^{15}+\frac{15}{13}\,a^4b^2x^{13}+\frac{6}{11}\,a^5bx^{11}+\frac{1}{9}\,a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 + 1
5/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

Sympy [A]
time = 0.01, size = 80, normalized size = 0.98

$$\frac{a^6x^9}{9}+\frac{6a^5bx^{11}}{11}+\frac{15a^4b^2x^{13}}{13}+\frac{4a^3b^3x^{15}}{3}+\frac{15a^2b^4x^{17}}{17}+\frac{6ab^5x^{19}}{19}+\frac{b^6x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**9/9 + 6*a**5*b*x**11/11 + 15*a**4*b**2*x**13/13 + 4*a**3*b**3*x**15
/3 + 15*a**2*b**4*x**17/17 + 6*a*b**5*x**19/19 + b**6*x**21/21
```

Giac [A]
time = 3.32, size = 68, normalized size = 0.83

$$\frac{1}{21}\,b^6x^{21}+\frac{6}{19}\,ab^5x^{19}+\frac{15}{17}\,a^2b^4x^{17}+\frac{4}{3}\,a^3b^3x^{15}+\frac{15}{13}\,a^4b^2x^{13}+\frac{6}{11}\,a^5bx^{11}+\frac{1}{9}\,a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 + 1
5/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

Mupad [B]
time = 0.03, size = 68, normalized size = 0.83

$$\frac{a^6\,x^9}{9}+\frac{6\,a^5\,b\,x^{11}}{11}+\frac{15\,a^4\,b^2\,x^{13}}{13}+\frac{4\,a^3\,b^3\,x^{15}}{3}+\frac{15\,a^2\,b^4\,x^{17}}{17}+\frac{6\,a\,b^5\,x^{19}}{19}+\frac{b^6\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^9)/9 + (b^6*x^21)/21 + (6*a^5*b*x^11)/11 + (6*a*b^5*x^19)/19 + (15*a
^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3 + (15*a^2*b^4*x^17)/17
```

3.445 $\int x^7(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal. Leaf size=72

$$-\frac{a^3(a+bx^2)^7}{14b^4}+\frac{3a^2(a+bx^2)^8}{16b^4}-\frac{a(a+bx^2)^9}{6b^4}+\frac{(a+bx^2)^{10}}{20b^4}$$

```
[Out] -1/14*a^3*(b*x^2+a)^7/b^4+3/16*a^2*(b*x^2+a)^8/b^4-1/6*a*(b*x^2+a)^9/b^4+1/
20*(b*x^2+a)^10/b^4
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {28, 272, 45}

$$-\frac{a^3(a+bx^2)^7}{14b^4}+\frac{3a^2(a+bx^2)^8}{16b^4}+\frac{(a+bx^2)^{10}}{20b^4}-\frac{a(a+bx^2)^9}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/14*(a^3*(a + b*x^2)^7)/b^4 + (3*a^2*(a + b*x^2)^8)/(16*b^4) - (a*(a + b*
x^2)^9)/(6*b^4) + (a + b*x^2)^10/(20*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{\int x^7\left(ab+b^2x^2\right)^6\,dx}{b^6}$$
$$=\frac{\text{Subst}\left(\int x^3\left(ab+b^2x\right)^6\,dx,x,x^2\right)}{2b^6}$$
$$=\frac{\text{Subst}\left(\int\left(-\frac{a^3(ab+b^2x)^6}{b^3}+\frac{3a^2(ab+b^2x)^7}{b^4}-\frac{3a(ab+b^2x)^8}{b^5}+\frac{(ab+b^2x)^9}{b^6}\right)dx,x,x^2\right)}{2b^6}$$
$$=-\frac{a^3(a+bx^2)^7}{14b^4}+\frac{3a^2(a+bx^2)^8}{16b^4}-\frac{a(a+bx^2)^9}{6b^4}+\frac{(a+bx^2)^{10}}{20b^4}$$

Mathematica [A]
time = 0.00, size = 82, normalized size = 1.14

$$\frac{a^6x^8}{8}+\frac{3}{5}a^5bx^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}ab^5x^{18}+\frac{b^6x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^8)/8 + (3*a^5*b*x^10)/5 + (5*a^4*b^2*x^12)/4 + (10*a^3*b^3*x^14)/7 +
 (15*a^2*b^4*x^16)/16 + (a*b^5*x^18)/3 + (b^6*x^20)/20
```

Maple [A]
time = 0.05, size = 69, normalized size = 0.96

method	result	size
default	$\frac{1}{8}a^6x^8+\frac{3}{5}a^5b\,x^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}a\,b^5x^{18}+\frac{1}{20}b^6x^{20}$	69
norman	$\frac{1}{8}a^6x^8+\frac{3}{5}a^5b\,x^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}a\,b^5x^{18}+\frac{1}{20}b^6x^{20}$	69
risch	$\frac{1}{8}a^6x^8+\frac{3}{5}a^5b\,x^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}a\,b^5x^{18}+\frac{1}{20}b^6x^{20}$	69
gosper	$\frac{x^8\left(84b^6x^{12}+560a\,b^5x^{10}+1575a^2b^4x^8+2400a^3b^3x^6+2100a^4b^2x^4+1008a^5b\,x^2+210a^6\right)}{1680}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a^6*x^8+3/5*a^5*b*x^10+5/4*a^4*b^2*x^12+10/7*a^3*b^3*x^14+15/16*a^2*b^4
*x^16+1/3*a*b^5*x^18+1/20*b^6*x^20
```

Maxima [A]
time = 0.31, size = 68, normalized size = 0.94

$$\frac{1}{20}\,b^6x^{20}+\frac{1}{3}\,ab^5x^{18}+\frac{15}{16}\,a^2b^4x^{16}+\frac{10}{7}\,a^3b^3x^{14}+\frac{5}{4}\,a^4b^2x^{12}+\frac{3}{5}\,a^5bx^{10}+\frac{1}{8}\,a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 + 5
/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

Fricas [A]
time = 0.32, size = 68, normalized size = 0.94

$$\frac{1}{20}\,b^6x^{20}+\frac{1}{3}\,ab^5x^{18}+\frac{15}{16}\,a^2b^4x^{16}+\frac{10}{7}\,a^3b^3x^{14}+\frac{5}{4}\,a^4b^2x^{12}+\frac{3}{5}\,a^5bx^{10}+\frac{1}{8}\,a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 + 5
/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

Sympy [A]
time = 0.02, size = 78, normalized size = 1.08

$$\frac{a^6x^8}{8}+\frac{3a^5bx^{10}}{5}+\frac{5a^4b^2x^{12}}{4}+\frac{10a^3b^3x^{14}}{7}+\frac{15a^2b^4x^{16}}{16}+\frac{ab^5x^{18}}{3}+\frac{b^6x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**8/8 + 3*a**5*b*x**10/5 + 5*a**4*b**2*x**12/4 + 10*a**3*b**3*x**14/7
 + 15*a**2*b**4*x**16/16 + a*b**5*x**18/3 + b**6*x**20/20
```

Giac [A]
time = 3.35, size = 68, normalized size = 0.94

$$\frac{1}{20}\,b^6x^{20}+\frac{1}{3}\,ab^5x^{18}+\frac{15}{16}\,a^2b^4x^{16}+\frac{10}{7}\,a^3b^3x^{14}+\frac{5}{4}\,a^4b^2x^{12}+\frac{3}{5}\,a^5bx^{10}+\frac{1}{8}\,a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 + 5
/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

Mupad [B]
time = 0.03, size = 68, normalized size = 0.94

$$\frac{a^6\,x^8}{8}+\frac{3\,a^5\,b\,x^{10}}{5}+\frac{5\,a^4\,b^2\,x^{12}}{4}+\frac{10\,a^3\,b^3\,x^{14}}{7}+\frac{15\,a^2\,b^4\,x^{16}}{16}+\frac{a\,b^5\,x^{18}}{3}+\frac{b^6\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^8)/8 + (b^6*x^20)/20 + (3*a^5*b*x^10)/5 + (a*b^5*x^18)/3 + (5*a^4*b^
2*x^12)/4 + (10*a^3*b^3*x^14)/7 + (15*a^2*b^4*x^16)/16
```

3.446 $\int x^6(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal. Leaf size=79

$$\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

```
[Out] 1/7*a^6*x^7+2/3*a^5*b*x^9+15/11*a^4*b^2*x^11+20/13*a^3*b^3*x^13+a^2*b^4*x^1
5+6/17*a*b^5*x^17+1/19*b^6*x^19
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 276}

$$\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13
 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^6\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int x^6\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\int \left(a^6b^6x^6+6a^5b^7x^8+15a^4b^8x^{10}+20a^3b^9x^{12}+15a^2b^{10}x^{14}+6ab^{11}x^{16}+b^{1}\right.}{b^6}\\ &= \frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 79, normalized size = 1.00

$$\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13
 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Maple [A]
time = 0.05, size = 68, normalized size = 0.86

method	result	size
default	$\frac{1}{7}a^6x^7+\frac{2}{3}a^5b\,x^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}a\,b^5x^{17}+\frac{1}{19}b^6x^{19}$	68
norman	$\frac{1}{7}a^6x^7+\frac{2}{3}a^5b\,x^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}a\,b^5x^{17}+\frac{1}{19}b^6x^{19}$	68
risch	$\frac{1}{7}a^6x^7+\frac{2}{3}a^5b\,x^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}a\,b^5x^{17}+\frac{1}{19}b^6x^{19}$	68
gosper	$\frac{x^7\left(51051b^6x^{12}+342342a\,b^5x^{10}+969969a^2b^4x^8+1492260a^3b^3x^6+1322685a^4b^2x^4+646646a^5b\,x^2+138567a^6\right)}{969969}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^6*x^7+2/3*a^5*b*x^9+15/11*a^4*b^2*x^11+20/13*a^3*b^3*x^13+a^2*b^4*x^1
5+6/17*a*b^5*x^17+1/19*b^6*x^19
```

Maxima [A]
time = 0.35, size = 67, normalized size = 0.85

$$\frac{1}{19}\,b^6x^{19}+\frac{6}{17}\,ab^5x^{17}+a^2b^4x^{15}+\frac{20}{13}\,a^3b^3x^{13}+\frac{15}{11}\,a^4b^2x^{11}+\frac{2}{3}\,a^5bx^9+\frac{1}{7}\,a^6x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/11
*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

Fricas [A]
time = 0.34, size = 67, normalized size = 0.85

$$\frac{1}{19}\,b^6x^{19}+\frac{6}{17}\,ab^5x^{17}+a^2b^4x^{15}+\frac{20}{13}\,a^3b^3x^{13}+\frac{15}{11}\,a^4b^2x^{11}+\frac{2}{3}\,a^5bx^9+\frac{1}{7}\,a^6x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/11
*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

Sympy [A]
time = 0.01, size = 76, normalized size = 0.96

$$\frac{a^6x^7}{7}+\frac{2a^5bx^9}{3}+\frac{15a^4b^2x^{11}}{11}+\frac{20a^3b^3x^{13}}{13}+a^2b^4x^{15}+\frac{6ab^5x^{17}}{17}+\frac{b^6x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**7/7 + 2*a**5*b*x**9/3 + 15*a**4*b**2*x**11/11 + 20*a**3*b**3*x**13/
13 + a**2*b**4*x**15 + 6*a*b**5*x**17/17 + b**6*x**19/19
```

Giac [A]
time = 4.27, size = 67, normalized size = 0.85

$$\frac{1}{19}b^6x^{19}+\frac{6}{17}ab^5x^{17}+a^2b^4x^{15}+\frac{20}{13}a^3b^3x^{13}+\frac{15}{11}a^4b^2x^{11}+\frac{2}{3}a^5bx^9+\frac{1}{7}a^6x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/11
*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

Mupad [B]
time = 0.03, size = 67, normalized size = 0.85

$$\frac{a^6\,x^7}{7}+\frac{2\,a^5\,b\,x^9}{3}+\frac{15\,a^4\,b^2\,x^{11}}{11}+\frac{20\,a^3\,b^3\,x^{13}}{13}+a^2\,b^4\,x^{15}+\frac{6\,a\,b^5\,x^{17}}{17}+\frac{b^6\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^7)/7 + (b^6*x^19)/19 + (2*a^5*b*x^9)/3 + (6*a*b^5*x^17)/17 + (15*a^4
*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13 + a^2*b^4*x^15
```

3.447 $\int x^5(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal. Leaf size=53

$$\frac{a^2(a+bx^2)^7}{14b^3}-\frac{a(a+bx^2)^8}{8b^3}+\frac{(a+bx^2)^9}{18b^3}$$

```
[Out] 1/14*a^2*(b*x^2+a)^7/b^3-1/8*a*(b*x^2+a)^8/b^3+1/18*(b*x^2+a)^9/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {28, 272, 45}

$$\frac{a^2(a+bx^2)^7}{14b^3}+\frac{(a+bx^2)^9}{18b^3}-\frac{a(a+bx^2)^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^2*(a + b*x^2)^7)/(14*b^3) - (a*(a + b*x^2)^8)/(8*b^3) + (a + b*x^2)^9/(1
8*b^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^3\,dx = \frac{\int x^5\left(ab+b^2x^2\right)^6\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int x^2\left(ab+b^2x\right)^6\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2(ab+b^2x)^6}{b^2}-\frac{2a(ab+b^2x)^7}{b^3}+\frac{(ab+b^2x)^8}{b^4}\right)\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{a^2(a+bx^2)^7}{14b^3}-\frac{a(a+bx^2)^8}{8b^3}+\frac{(a+bx^2)^9}{18b^3}$$

Mathematica [A]
time = 0.00, size = 82, normalized size = 1.55

$$\frac{a^6x^6}{6}+\frac{3}{4}a^5bx^8+\frac{3}{2}a^4b^2x^{10}+\frac{5}{3}a^3b^3x^{12}+\frac{15}{14}a^2b^4x^{14}+\frac{3}{8}ab^5x^{16}+\frac{b^6x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^6)/6 + (3*a^5*b*x^8)/4 + (3*a^4*b^2*x^10)/2 + (5*a^3*b^3*x^12)/3 + (
15*a^2*b^4*x^14)/14 + (3*a*b^5*x^16)/8 + (b^6*x^18)/18
```

Maple [A]
time = 0.05, size = 69, normalized size = 1.30

method	result	size
default	$\frac{1}{6}a^6x^6+\frac{3}{4}a^5b\,x^8+\frac{3}{2}a^4b^2x^{10}+\frac{5}{3}a^3b^3x^{12}+\frac{15}{14}a^2b^4x^{14}+\frac{3}{8}a\,b^5x^{16}+\frac{1}{18}b^6x^{18}$	69
norman	$\frac{1}{6}a^6x^6+\frac{3}{4}a^5b\,x^8+\frac{3}{2}a^4b^2x^{10}+\frac{5}{3}a^3b^3x^{12}+\frac{15}{14}a^2b^4x^{14}+\frac{3}{8}a\,b^5x^{16}+\frac{1}{18}b^6x^{18}$	69
risch	$\frac{1}{6}a^6x^6+\frac{3}{4}a^5b\,x^8+\frac{3}{2}a^4b^2x^{10}+\frac{5}{3}a^3b^3x^{12}+\frac{15}{14}a^2b^4x^{14}+\frac{3}{8}a\,b^5x^{16}+\frac{1}{18}b^6x^{18}$	69
gosper	$\frac{x^6\left(28b^6x^{12}+189a\,b^5x^{10}+540a^2b^4x^8+840a^3b^3x^6+756a^4b^2x^4+378a^5b\,x^2+84a^6\right)}{504}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^6*x^6+3/4*a^5*b*x^8+3/2*a^4*b^2*x^10+5/3*a^3*b^3*x^12+15/14*a^2*b^4*x
^14+3/8*a*b^5*x^16+1/18*b^6*x^18
```

Maxima [A]
time = 0.31, size = 68, normalized size = 1.28

$$\frac{1}{18}\,b^6x^{18}+\frac{3}{8}\,ab^5x^{16}+\frac{15}{14}\,a^2b^4x^{14}+\frac{5}{3}\,a^3b^3x^{12}+\frac{3}{2}\,a^4b^2x^{10}+\frac{3}{4}\,a^5bx^8+\frac{1}{6}\,a^6x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3/
2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

Fricas [A]
time = 0.35, size = 68, normalized size = 1.28

$$\frac{1}{18}\,b^{6}x^{18}+\frac{3}{8}\,ab^{5}x^{16}+\frac{15}{14}\,a^{2}b^{4}x^{14}+\frac{5}{3}\,a^{3}b^{3}x^{12}+\frac{3}{2}\,a^{4}b^{2}x^{10}+\frac{3}{4}\,a^{5}bx^{8}+\frac{1}{6}\,a^{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3/
2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

Sympy [A]
time = 0.02, size = 80, normalized size = 1.51

$$\frac{a^{6}x^{6}}{6}+\frac{3a^{5}bx^{8}}{4}+\frac{3a^{4}b^{2}x^{10}}{2}+\frac{5a^{3}b^{3}x^{12}}{3}+\frac{15a^{2}b^{4}x^{14}}{14}+\frac{3ab^{5}x^{16}}{8}+\frac{b^{6}x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**6/6 + 3*a**5*b*x**8/4 + 3*a**4*b**2*x**10/2 + 5*a**3*b**3*x**12/3 +
 15*a**2*b**4*x**14/14 + 3*a*b**5*x**16/8 + b**6*x**18/18
```

Giac [A]
time = 4.58, size = 68, normalized size = 1.28

$$\frac{1}{18}\,b^{6}x^{18}+\frac{3}{8}\,ab^{5}x^{16}+\frac{15}{14}\,a^{2}b^{4}x^{14}+\frac{5}{3}\,a^{3}b^{3}x^{12}+\frac{3}{2}\,a^{4}b^{2}x^{10}+\frac{3}{4}\,a^{5}bx^{8}+\frac{1}{6}\,a^{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3/
2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

Mupad [B]
time = 0.03, size = 68, normalized size = 1.28

$$\frac{a^6\,x^6}{6}+\frac{3\,a^5\,b\,x^8}{4}+\frac{3\,a^4\,b^2\,x^{10}}{2}+\frac{5\,a^3\,b^3\,x^{12}}{3}+\frac{15\,a^2\,b^4\,x^{14}}{14}+\frac{3\,a\,b^5\,x^{16}}{8}+\frac{b^6\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^6)/6 + (b^6*x^18)/18 + (3*a^5*b*x^8)/4 + (3*a*b^5*x^16)/8 + (3*a^4*b
^2*x^10)/2 + (5*a^3*b^3*x^12)/3 + (15*a^2*b^4*x^14)/14
```

3.448 $\int x^4(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal. Leaf size=82

$$\frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

```
[Out] 1/5*a^6*x^5+6/7*a^5*b*x^7+5/3*a^4*b^2*x^9+20/11*a^3*b^3*x^11+15/13*a^2*b^4*
x^13+2/5*a*b^5*x^15+1/17*b^6*x^17
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {28, 276}

$$\frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 +
(15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int x^4\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\int \left(a^6b^6x^4+6a^5b^7x^6+15a^4b^8x^8+20a^3b^9x^{10}+15a^2b^{10}x^{12}+6ab^{11}x^{14}+b^{12}\right.}{b^6}\\ &= \frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 82, normalized size = 1.00

$$\frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 +
(15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Maple [A]
time = 0.05, size = 69, normalized size = 0.84

method	result	size
default	$\frac{1}{5}a^6x^5+\frac{6}{7}a^5b\,x^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}a\,b^5x^{15}+\frac{1}{17}b^6x^{17}$	69
norman	$\frac{1}{5}a^6x^5+\frac{6}{7}a^5b\,x^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}a\,b^5x^{15}+\frac{1}{17}b^6x^{17}$	69
risch	$\frac{1}{5}a^6x^5+\frac{6}{7}a^5b\,x^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}a\,b^5x^{15}+\frac{1}{17}b^6x^{17}$	69
gosper	$\frac{x^5(15015b^6x^{12}+102102a\,b^5x^{10}+294525a^2b^4x^8+464100a^3b^3x^6+425425a^4b^2x^4+218790a^5b\,x^2+51051a^6)}{255255}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^6*x^5+6/7*a^5*b*x^7+5/3*a^4*b^2*x^9+20/11*a^3*b^3*x^11+15/13*a^2*b^4*
x^13+2/5*a*b^5*x^15+1/17*b^6*x^17
```

Maxima [A]
time = 0.30, size = 68, normalized size = 0.83

$$\frac{1}{17}\,b^6x^{17}+\frac{2}{5}\,ab^5x^{15}+\frac{15}{13}\,a^2b^4x^{13}+\frac{20}{11}\,a^3b^3x^{11}+\frac{5}{3}\,a^4b^2x^9+\frac{6}{7}\,a^5bx^7+\frac{1}{5}\,a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

Fricas [A]
time = 0.33, size = 68, normalized size = 0.83

$$\frac{1}{17}\,b^6x^{17}+\frac{2}{5}\,ab^5x^{15}+\frac{15}{13}\,a^2b^4x^{13}+\frac{20}{11}\,a^3b^3x^{11}+\frac{5}{3}\,a^4b^2x^9+\frac{6}{7}\,a^5bx^7+\frac{1}{5}\,a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

Sympy [A]
time = 0.01, size = 80, normalized size = 0.98

$$\frac{a^6x^5}{5} + \frac{6a^5bx^7}{7} + \frac{5a^4b^2x^9}{3} + \frac{20a^3b^3x^{11}}{11} + \frac{15a^2b^4x^{13}}{13} + \frac{2ab^5x^{15}}{5} + \frac{b^6x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**5/5 + 6*a**5*b*x**7/7 + 5*a**4*b**2*x**9/3 + 20*a**3*b**3*x**11/11
+ 15*a**2*b**4*x**13/13 + 2*a*b**5*x**15/5 + b**6*x**17/17
```

Giac [A]
time = 4.13, size = 68, normalized size = 0.83

$$\frac{1}{17}\,b^6x^{17} + \frac{2}{5}\,ab^5x^{15} + \frac{15}{13}\,a^2b^4x^{13} + \frac{20}{11}\,a^3b^3x^{11} + \frac{5}{3}\,a^4b^2x^9 + \frac{6}{7}\,a^5bx^7 + \frac{1}{5}\,a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

Mupad [B]
time = 0.03, size = 68, normalized size = 0.83

$$\frac{a^6\,x^5}{5} + \frac{6\,a^5\,b\,x^7}{7} + \frac{5\,a^4\,b^2\,x^9}{3} + \frac{20\,a^3\,b^3\,x^{11}}{11} + \frac{15\,a^2\,b^4\,x^{13}}{13} + \frac{2\,a\,b^5\,x^{15}}{5} + \frac{b^6\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^5)/5 + (b^6*x^17)/17 + (6*a^5*b*x^7)/7 + (2*a*b^5*x^15)/5 + (5*a^4*b
^2*x^9)/3 + (20*a^3*b^3*x^11)/11 + (15*a^2*b^4*x^13)/13
```

3.449 $\int x^3(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal. Leaf size=34

$$-\frac{a(a+bx^2)^7}{14b^2}+\frac{(a+bx^2)^8}{16b^2}$$

```
[Out] -1/14*a*(b*x^2+a)^7/b^2+1/16*(b*x^2+a)^8/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$\frac{(a+bx^2)^8}{16b^2}-\frac{a(a+bx^2)^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/14*(a*(a + b*x^2)^7)/b^2 + (a + b*x^2)^8/(16*b^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3 (a^2 + 2abx^2 + b^2x^4)^3 \, dx = \frac{\int x^3 (ab + b^2x^2)^6 \, dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int x(ab + b^2x)^6 \, dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a(ab+b^2x)^6}{b} + \frac{(ab+b^2x)^7}{b^2}\right) dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a(a + bx^2)^7}{14b^2} + \frac{(a + bx^2)^8}{16b^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.
time = 0.00, size = 77, normalized size = 2.26

$$\frac{a^6x^4}{4} + a^5bx^6 + \frac{15}{8}a^4b^2x^8 + 2a^3b^3x^{10} + \frac{5}{4}a^2b^4x^{12} + \frac{3}{7}ab^5x^{14} + \frac{b^6x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^4)/4 + a^5*b*x^6 + (15*a^4*b^2*x^8)/8 + 2*a^3*b^3*x^10 + (5*a^2*b^4*
x^12)/4 + (3*a*b^5*x^14)/7 + (b^6*x^16)/16
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.
time = 0.05, size = 68, normalized size = 2.00

method	result	size
default	$\frac{1}{4}a^6x^4 + a^5b\,x^6 + \frac{15}{8}a^4b^2x^8 + 2a^3b^3x^{10} + \frac{5}{4}a^2b^4x^{12} + \frac{3}{7}a\,b^5x^{14} + \frac{1}{16}b^6x^{16}$	68
norman	$\frac{1}{4}a^6x^4 + a^5b\,x^6 + \frac{15}{8}a^4b^2x^8 + 2a^3b^3x^{10} + \frac{5}{4}a^2b^4x^{12} + \frac{3}{7}a\,b^5x^{14} + \frac{1}{16}b^6x^{16}$	68
risch	$\frac{1}{4}a^6x^4 + a^5b\,x^6 + \frac{15}{8}a^4b^2x^8 + 2a^3b^3x^{10} + \frac{5}{4}a^2b^4x^{12} + \frac{3}{7}a\,b^5x^{14} + \frac{1}{16}b^6x^{16}$	68
gosper	$\frac{x^4(7b^6x^{12}+48a\,b^5x^{10}+140a^2b^4x^8+224a^3b^3x^6+210a^4b^2x^4+112a^5b\,x^2+28a^6)}{112}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^6*x^4+a^5*b*x^6+15/8*a^4*b^2*x^8+2*a^3*b^3*x^10+5/4*a^2*b^4*x^12+3/7*
a*b^5*x^14+1/16*b^6*x^16
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.
time = 0.29, size = 67, normalized size = 1.97

$$\frac{1}{16}\,b^6x^{16} + \frac{3}{7}\,ab^5x^{14} + \frac{5}{4}\,a^2b^4x^{12} + 2\,a^3b^3x^{10} + \frac{15}{8}\,a^4b^2x^8 + a^5bx^6 + \frac{1}{4}\,a^6x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*a
^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.
time = 0.34, size = 67, normalized size = 1.97

$$\frac{1}{16}\,b^6x^{16} + \frac{3}{7}\,ab^5x^{14} + \frac{5}{4}\,a^2b^4x^{12} + 2\,a^3b^3x^{10} + \frac{15}{8}\,a^4b^2x^8 + a^5bx^6 + \frac{1}{4}\,a^6x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*a
^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(27) = 54$.
time = 0.02, size = 75, normalized size = 2.21

$$\frac{a^6x^4}{4} + a^5bx^6 + \frac{15a^4b^2x^8}{8} + 2a^3b^3x^{10} + \frac{5a^2b^4x^{12}}{4} + \frac{3ab^5x^{14}}{7} + \frac{b^6x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**4/4 + a**5*b*x**6 + 15*a**4*b**2*x**8/8 + 2*a**3*b**3*x**10 + 5*a**
2*b**4*x**12/4 + 3*a*b**5*x**14/7 + b**6*x**16/16
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.
time = 3.95, size = 67, normalized size = 1.97

$$\frac{1}{16}\,b^6x^{16} + \frac{3}{7}\,ab^5x^{14} + \frac{5}{4}\,a^2b^4x^{12} + 2\,a^3b^3x^{10} + \frac{15}{8}\,a^4b^2x^8 + a^5bx^6 + \frac{1}{4}\,a^6x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*a
^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

Mupad [B]
time = 0.03, size = 67, normalized size = 1.97

$$\frac{a^6\,x^4}{4}+a^5\,b\,x^6+\frac{15\,a^4\,b^2\,x^8}{8}+2\,a^3\,b^3\,x^{10}+\frac{5\,a^2\,b^4\,x^{12}}{4}+\frac{3\,a\,b^5\,x^{14}}{7}+\frac{b^6\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^4)/4 + (b^6*x^16)/16 + a^5*b*x^6 + (3*a*b^5*x^14)/7 + (15*a^4*b^2*x^
8)/8 + 2*a^3*b^3*x^10 + (5*a^2*b^4*x^12)/4
```

3.450 $\int x^2(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal. Leaf size=82

$$\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

```
[Out] 1/3*a^6*x^3+6/5*a^5*b*x^5+15/7*a^4*b^2*x^7+20/9*a^3*b^3*x^9+15/11*a^2*b^4*x
^11+6/13*a*b^5*x^13+1/15*b^6*x^15
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 276}

$$\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (
15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int x^2\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\int \left(a^6b^6x^2+6a^5b^7x^4+15a^4b^8x^6+20a^3b^9x^8+15a^2b^{10}x^{10}+6ab^{11}x^{12}+b^{12}x^{1}\right.}{b^6}\\ &= \frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 82, normalized size = 1.00

$$\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (
15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Maple [A]
time = 0.05, size = 69, normalized size = 0.84

method	result	size
default	$\frac{1}{3}a^6x^3+\frac{6}{5}a^5b\,x^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}a\,b^5x^{13}+\frac{1}{15}b^6x^{15}$	69
norman	$\frac{1}{3}a^6x^3+\frac{6}{5}a^5b\,x^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}a\,b^5x^{13}+\frac{1}{15}b^6x^{15}$	69
risch	$\frac{1}{3}a^6x^3+\frac{6}{5}a^5b\,x^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}a\,b^5x^{13}+\frac{1}{15}b^6x^{15}$	69
gosper	$\frac{x^3\left(3003b^6x^{12}+20790a\,b^5x^{10}+61425a^2b^4x^8+100100a^3b^3x^6+96525a^4b^2x^4+54054a^5b\,x^2+15015a^6\right)}{45045}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^6*x^3+6/5*a^5*b*x^5+15/7*a^4*b^2*x^7+20/9*a^3*b^3*x^9+15/11*a^2*b^4*x
^11+6/13*a*b^5*x^13+1/15*b^6*x^15
```

Maxima [A]
time = 0.29, size = 68, normalized size = 0.83

$$\frac{1}{15}\,b^6x^{15}+\frac{6}{13}\,ab^5x^{13}+\frac{15}{11}\,a^2b^4x^{11}+\frac{20}{9}\,a^3b^3x^9+\frac{15}{7}\,a^4b^2x^7+\frac{6}{5}\,a^5bx^5+\frac{1}{3}\,a^6x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 + 1
5/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

Fricas [A]
time = 0.36, size = 68, normalized size = 0.83

$$\frac{1}{15}\,b^6x^{15}+\frac{6}{13}\,ab^5x^{13}+\frac{15}{11}\,a^2b^4x^{11}+\frac{20}{9}\,a^3b^3x^9+\frac{15}{7}\,a^4b^2x^7+\frac{6}{5}\,a^5bx^5+\frac{1}{3}\,a^6x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 + 1
5/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

Sympy [A]
time = 0.01, size = 80, normalized size = 0.98

$$\frac{a^6x^3}{3}+\frac{6a^5bx^5}{5}+\frac{15a^4b^2x^7}{7}+\frac{20a^3b^3x^9}{9}+\frac{15a^2b^4x^{11}}{11}+\frac{6ab^5x^{13}}{13}+\frac{b^6x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**3/3 + 6*a**5*b*x**5/5 + 15*a**4*b**2*x**7/7 + 20*a**3*b**3*x**9/9 +
 15*a**2*b**4*x**11/11 + 6*a*b**5*x**13/13 + b**6*x**15/15
```

Giac [A]
time = 5.43, size = 68, normalized size = 0.83

$$\frac{1}{15}\,b^6x^{15}+\frac{6}{13}\,ab^5x^{13}+\frac{15}{11}\,a^2b^4x^{11}+\frac{20}{9}\,a^3b^3x^9+\frac{15}{7}\,a^4b^2x^7+\frac{6}{5}\,a^5bx^5+\frac{1}{3}\,a^6x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 + 1
5/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

Mupad [B]
time = 0.03, size = 68, normalized size = 0.83

$$\frac{a^6\,x^3}{3}+\frac{6\,a^5\,b\,x^5}{5}+\frac{15\,a^4\,b^2\,x^7}{7}+\frac{20\,a^3\,b^3\,x^9}{9}+\frac{15\,a^2\,b^4\,x^{11}}{11}+\frac{6\,a\,b^5\,x^{13}}{13}+\frac{b^6\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^3)/3 + (b^6*x^15)/15 + (6*a^5*b*x^5)/5 + (6*a*b^5*x^13)/13 + (15*a^4
*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (15*a^2*b^4*x^11)/11
```

3.451 $\int x(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal. Leaf size=16

$$\frac{(a+bx^2)^7}{14b}$$

```
[Out] 1/14*(b*x^2+a)^7/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {28, 267}

$$\frac{(a+bx^2)^7}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a + b*x^2)^7/(14*b)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int x\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\left(a+bx^2\right)^7}{14b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{(a+bx^2)^7}{14b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a + b*x^2)^7/(14*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(14) = 28$.
time = 0.05, size = 69, normalized size = 4.31

method	result	size
default	$\frac{1}{2}a^6x^2+\frac{3}{2}a^5b\,x^4+\frac{5}{2}a^4b^2x^6+\frac{5}{2}a^3b^3x^8+\frac{3}{2}a^2b^4x^{10}+\frac{1}{2}a\,b^5x^{12}+\frac{1}{14}b^6x^{14}$	69
norman	$\frac{1}{2}a^6x^2+\frac{3}{2}a^5b\,x^4+\frac{5}{2}a^4b^2x^6+\frac{5}{2}a^3b^3x^8+\frac{3}{2}a^2b^4x^{10}+\frac{1}{2}a\,b^5x^{12}+\frac{1}{14}b^6x^{14}$	69
risch	$\frac{1}{2}a^6x^2+\frac{3}{2}a^5b\,x^4+\frac{5}{2}a^4b^2x^6+\frac{5}{2}a^3b^3x^8+\frac{3}{2}a^2b^4x^{10}+\frac{1}{2}a\,b^5x^{12}+\frac{1}{14}b^6x^{14}$	69
gosper	$\frac{x^2\left(b^6x^{12}+7a\,b^5x^{10}+21a^2b^4x^8+35a^3b^3x^6+35a^4b^2x^4+21a^5b\,x^2+7a^6\right)}{14}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^6*x^2+3/2*a^5*b*x^4+5/2*a^4*b^2*x^6+5/2*a^3*b^3*x^8+3/2*a^2*b^4*x^10+
1/2*a*b^5*x^12+1/14*b^6*x^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(14) = 28$.
time = 0.30, size = 68, normalized size = 4.25

$$\frac{1}{14}\,b^6x^{14}+\frac{1}{2}\,ab^5x^{12}+\frac{3}{2}\,a^2b^4x^{10}+\frac{5}{2}\,a^3b^3x^8+\frac{5}{2}\,a^4b^2x^6+\frac{3}{2}\,a^5bx^4+\frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*a
^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(14) = 28$.
time = 0.33, size = 68, normalized size = 4.25

$$\frac{1}{14}\,b^6x^{14}+\frac{1}{2}\,ab^5x^{12}+\frac{3}{2}\,a^2b^4x^{10}+\frac{5}{2}\,a^3b^3x^8+\frac{5}{2}\,a^4b^2x^6+\frac{3}{2}\,a^5bx^4+\frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*a
^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(10) = 20$.
time = 0.02, size = 78, normalized size = 4.88

$$\frac{a^6x^2}{2} + \frac{3a^5bx^4}{2} + \frac{5a^4b^2x^6}{2} + \frac{5a^3b^3x^8}{2} + \frac{3a^2b^4x^{10}}{2} + \frac{ab^5x^{12}}{2} + \frac{b^6x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**2/2 + 3*a**5*b*x**4/2 + 5*a**4*b**2*x**6/2 + 5*a**3*b**3*x**8/2 + 3
*a**2*b**4*x**10/2 + a*b**5*x**12/2 + b**6*x**14/14
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(14) = 28$.
time = 4.93, size = 68, normalized size = 4.25

$$\frac{1}{14}\,b^6x^{14} + \frac{1}{2}\,ab^5x^{12} + \frac{3}{2}\,a^2b^4x^{10} + \frac{5}{2}\,a^3b^3x^8 + \frac{5}{2}\,a^4b^2x^6 + \frac{3}{2}\,a^5bx^4 + \frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*a
^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

Mupad [B]
time = 0.03, size = 68, normalized size = 4.25

$$\frac{a^6\,x^2}{2} + \frac{3\,a^5\,b\,x^4}{2} + \frac{5\,a^4\,b^2\,x^6}{2} + \frac{5\,a^3\,b^3\,x^8}{2} + \frac{3\,a^2\,b^4\,x^{10}}{2} + \frac{a\,b^5\,x^{12}}{2} + \frac{b^6\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x^2)/2 + (b^6*x^14)/14 + (3*a^5*b*x^4)/2 + (a*b^5*x^12)/2 + (5*a^4*b^2
*x^6)/2 + (5*a^3*b^3*x^8)/2 + (3*a^2*b^4*x^10)/2
```

3.452 $\int \left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=73

$$a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

```
[Out] a^6*x+2*a^5*b*x^3+3*a^4*b^2*x^5+20/7*a^3*b^3*x^7+5/3*a^2*b^4*x^9+6/11*a*b^5
*x^11+1/13*b^6*x^13
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {28, 200}

$$a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/
3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int \left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\int \left(a^6b^6+6a^5b^7x^2+15a^4b^8x^4+20a^3b^9x^6+15a^2b^{10}x^8+6ab^{11}x^{10}+b^{12}x^{12}\right)dx}{b^6}\\ &= a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 73, normalized size = 1.00

$$a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/
3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

Maple [A]
time = 0.01, size = 66, normalized size = 0.90

method	result	size
default	$a^6x+2a^5b\,x^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}a\,b^5x^{11}+\frac{1}{13}b^6x^{13}$	66
norman	$a^6x+2a^5b\,x^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}a\,b^5x^{11}+\frac{1}{13}b^6x^{13}$	66
risch	$a^6x+2a^5b\,x^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}a\,b^5x^{11}+\frac{1}{13}b^6x^{13}$	66
gosper	$\frac{x\left(231b^6x^{12}+1638a\,b^5x^{10}+5005a^2b^4x^8+8580a^3b^3x^6+9009a^4b^2x^4+6006a^5b\,x^2+3003a^6\right)}{3003}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^6*x+2*a^5*b*x^3+3*a^4*b^2*x^5+20/7*a^3*b^3*x^7+5/3*a^2*b^4*x^9+6/11*a*b^5
*x^11+1/13*b^6*x^13
```

Maxima [A]
time = 0.35, size = 100, normalized size = 1.37

$$\frac{1}{13}\,b^6x^{13}+\frac{6}{11}\,ab^5x^{11}+\frac{4}{3}\,a^2b^4x^9+\frac{8}{7}\,a^3b^3x^7+a^6x+\frac{1}{5}\,\left(3\,b^2x^5+10\,abx^3\right)a^4+\frac{1}{105}\,\left(35\,b^4x^9+180\,ab^3x^7+252\,a^2b^2x^5\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 4/3*a^2*b^4*x^9 + 8/7*a^3*b^3*x^7 + a^6*x
 + 1/5*(3*b^2*x^5 + 10*a*b*x^3)*a^4 + 1/105*(35*b^4*x^9 + 180*a*b^3*x^7 + 2
52*a^2*b^2*x^5)*a^2
```

Fricas [A]
time = 0.36, size = 65, normalized size = 0.89

$$\frac{1}{13}\,b^6x^{13}+\frac{6}{11}\,ab^5x^{11}+\frac{5}{3}\,a^2b^4x^9+\frac{20}{7}\,a^3b^3x^7+3\,a^4b^2x^5+2\,a^5bx^3+a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 5/3*a^2*b^4*x^9 + 20/7*a^3*b^3*x^7 + 3*a^
4*b^2*x^5 + 2*a^5*b*x^3 + a^6*x
```

Sympy [A]
time = 0.01, size = 73, normalized size = 1.00

$$a^{6}x + 2a^{5}bx^{3} + 3a^{4}b^{2}x^{5} + \frac{20a^{3}b^{3}x^{7}}{7} + \frac{5a^{2}b^{4}x^{9}}{3} + \frac{6ab^{5}x^{11}}{11} + \frac{b^{6}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x + 2*a**5*b*x**3 + 3*a**4*b**2*x**5 + 20*a**3*b**3*x**7/7 + 5*a**2*b*
*4*x**9/3 + 6*a*b**5*x**11/11 + b**6*x**13/13
```

Giac [A]
time = 6.16, size = 65, normalized size = 0.89

$$\frac{1}{13}\,b^{6}x^{13} + \frac{6}{11}\,ab^{5}x^{11} + \frac{5}{3}\,a^{2}b^{4}x^{9} + \frac{20}{7}\,a^{3}b^{3}x^{7} + 3\,a^{4}b^{2}x^{5} + 2\,a^{5}bx^{3} + a^{6}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 5/3*a^2*b^4*x^9 + 20/7*a^3*b^3*x^7 + 3*a^
4*b^2*x^5 + 2*a^5*b*x^3 + a^6*x
```

Mupad [B]
time = 0.03, size = 65, normalized size = 0.89

$$a^{6}\,x + 2\,a^{5}\,b\,x^{3} + 3\,a^{4}\,b^{2}\,x^{5} + \frac{20\,a^{3}\,b^{3}\,x^{7}}{7} + \frac{5\,a^{2}\,b^{4}\,x^{9}}{3} + \frac{6\,a\,b^{5}\,x^{11}}{11} + \frac{b^{6}\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] a^6*x + (b^6*x^13)/13 + 2*a^5*b*x^3 + (6*a*b^5*x^11)/11 + 3*a^4*b^2*x^5 + (
20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/3
```

3.453 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x}\,dx$

Optimal. Leaf size=76

$$3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}+a^6\log(x)$$

```
[Out] 3*a^5*b*x^2+15/4*a^4*b^2*x^4+10/3*a^3*b^3*x^6+15/8*a^2*b^4*x^8+3/5*a*b^5*x^
10+1/12*b^6*x^12+a^6*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 45}

$$a^6\log(x)+3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
+ (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x}\,dx = \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^6}{x}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(6a^5b^7+\frac{a^6b^6}{x}+15a^4b^8x+20a^3b^9x^2+15a^2b^{10}x^3+6ab^{11}x^4+b^{12}x^5\right)\right.}{2b^6}$$
$$= 3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}+a^6\log(x)$$

Mathematica [A]
time = 0.00, size = 76, normalized size = 1.00

$$3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}+a^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
+ (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

Maple [A]
time = 0.06, size = 67, normalized size = 0.88

method	result	size
default	$3a^5b\,x^2+\frac{15a^4b^2x^4}{4}+\frac{10a^3b^3x^6}{3}+\frac{15a^2b^4x^8}{8}+\frac{3a\,b^5x^{10}}{5}+\frac{b^6x^{12}}{12}+a^6\ln\left(x\right)$	67
norman	$3a^5b\,x^2+\frac{15a^4b^2x^4}{4}+\frac{10a^3b^3x^6}{3}+\frac{15a^2b^4x^8}{8}+\frac{3a\,b^5x^{10}}{5}+\frac{b^6x^{12}}{12}+a^6\ln\left(x\right)$	67
risch	$a^6\ln\left(x\right)+\frac{b^6x^{12}}{12}+\frac{23a^6}{15}+3a^5b\,x^2+\frac{15a^4b^2x^4}{4}+\frac{10a^3b^3x^6}{3}+\frac{15a^2b^4x^8}{8}+\frac{3a\,b^5x^{10}}{5}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a^5*b*x^2+15/4*a^4*b^2*x^4+10/3*a^3*b^3*x^6+15/8*a^2*b^4*x^8+3/5*a*b^5*x^
10+1/12*b^6*x^12+a^6*ln(x)
```

Maxima [A]
time = 0.36, size = 69, normalized size = 0.91

$$\frac{1}{12}\,b^6x^{12}+\frac{3}{5}\,ab^5x^{10}+\frac{15}{8}\,a^2b^4x^8+\frac{10}{3}\,a^3b^3x^6+\frac{15}{4}\,a^4b^2x^4+3\,a^5bx^2+\frac{1}{2}\,a^6\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="maxima")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*log(x^2)
```

Fricas [A]
time = 0.33, size = 66, normalized size = 0.87

$$\frac{1}{12}\,b^6x^{12}+\frac{3}{5}\,ab^5x^{10}+\frac{15}{8}\,a^2b^4x^8+\frac{10}{3}\,a^3b^3x^6+\frac{15}{4}\,a^4b^2x^4+3\,a^5bx^2+a^6\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6*log(x)
```

Sympy [A]
time = 0.04, size = 76, normalized size = 1.00

$$a^6\log\left(x\right)+3a^5bx^2+\frac{15a^4b^2x^4}{4}+\frac{10a^3b^3x^6}{3}+\frac{15a^2b^4x^8}{8}+\frac{3ab^5x^{10}}{5}+\frac{b^6x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x,x)
```

```
[Out] a**6*log(x) + 3*a**5*b*x**2 + 15*a**4*b**2*x**4/4 + 10*a**3*b**3*x**6/3 + 1
5*a**2*b**4*x**8/8 + 3*a*b**5*x**10/5 + b**6*x**12/12
```

Giac [A]
time = 4.01, size = 69, normalized size = 0.91

$$\frac{1}{12}\,b^6x^{12}+\frac{3}{5}\,ab^5x^{10}+\frac{15}{8}\,a^2b^4x^8+\frac{10}{3}\,a^3b^3x^6+\frac{15}{4}\,a^4b^2x^4+3\,a^5bx^2+\frac{1}{2}\,a^6\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="giac")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/4
*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*log(x^2)
```

Mupad [B]
time = 0.04, size = 66, normalized size = 0.87

$$a^6\,\ln\left(x\right)+\frac{b^6\,x^{12}}{12}+3\,a^5\,b\,x^2+\frac{3\,a\,b^5\,x^{10}}{5}+\frac{15\,a^4\,b^2\,x^4}{4}+\frac{10\,a^3\,b^3\,x^6}{3}+\frac{15\,a^2\,b^4\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x,x)
```

```
[Out] a^6*log(x) + (b^6*x^12)/12 + 3*a^5*b*x^2 + (3*a*b^5*x^10)/5 + (15*a^4*b^2*x
^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
```

3.454 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

```
[Out] -a^6/x+6*a^5*b*x+5*a^4*b^2*x^3+4*a^3*b^3*x^5+15/7*a^2*b^4*x^7+2/3*a*b^5*x^9
+1/11*b^6*x^11
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^4*x^7)/7 +
 (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^2}\,dx}{b^6}\\ &= \frac{\int \left(6a^5b^7+\frac{a^6b^6}{x^2}+15a^4b^8x^2+20a^3b^9x^4+15a^2b^{10}x^6+6ab^{11}x^8+b^{12}x^{10}\right)dx}{b^6}\\ &= -\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 72, normalized size = 1.00

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^4*x^7)/7 +
 (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Maple [A]
time = 0.09, size = 67, normalized size = 0.93

method	result	size
default	$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15a^2b^4x^7}{7}+\frac{2a\,b^5x^9}{3}+\frac{b^6x^{11}}{11}$	67
risch	$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15a^2b^4x^7}{7}+\frac{2a\,b^5x^9}{3}+\frac{b^6x^{11}}{11}$	67
norman	$\frac{\frac{1}{11}b^6x^{12}+\frac{2}{3}a\,b^5x^{10}+\frac{15}{7}a^2b^4x^8+4a^3b^3x^6+5a^4b^2x^4+6a^5b\,x^2-a^6}{x}$	70
gosper	$-\frac{-21b^6x^{12}-154a\,b^5x^{10}-495a^2b^4x^8-924a^3b^3x^6-1155a^4b^2x^4-1386a^5b\,x^2+231a^6}{231x}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^6/x+6*a^5*b*x+5*a^4*b^2*x^3+4*a^3*b^3*x^5+15/7*a^2*b^4*x^7+2/3*a*b^5*x^9
+1/11*b^6*x^11
```

Maxima [A]
time = 0.29, size = 66, normalized size = 0.92

$$\frac{1}{11}\,b^6x^{11}+\frac{2}{3}\,ab^5x^9+\frac{15}{7}\,a^2b^4x^7+4\,a^3b^3x^5+5\,a^4b^2x^3+6\,a^5bx-\frac{a^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5 + 5*a^4*b^
2*x^3 + 6*a^5*b*x - a^6/x
```

Fricas [A]
time = 0.31, size = 70, normalized size = 0.97

$$\frac{21\,b^6x^{12}+154\,ab^5x^{10}+495\,a^2b^4x^8+924\,a^3b^3x^6+1155\,a^4b^2x^4+1386\,a^5bx^2-231\,a^6}{231\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/231*(21*b^6*x^12 + 154*a*b^5*x^10 + 495*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 1
155*a^4*b^2*x^4 + 1386*a^5*b*x^2 - 231*a^6)/x
```

Sympy [A]
time = 0.04, size = 70, normalized size = 0.97

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15a^2b^4x^7}{7}+\frac{2ab^5x^9}{3}+\frac{b^6x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**2,x)
```

```
[Out] -a**6/x + 6*a**5*b*x + 5*a**4*b**2*x**3 + 4*a**3*b**3*x**5 + 15*a**2*b**4*x
**7/7 + 2*a*b**5*x**9/3 + b**6*x**11/11
```

Giac [A]
time = 7.55, size = 66, normalized size = 0.92

$$\frac{1}{11}\,b^6x^{11}+\frac{2}{3}\,ab^5x^9+\frac{15}{7}\,a^2b^4x^7+4\,a^3b^3x^5+5\,a^4b^2x^3+6\,a^5bx-\frac{a^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5 + 5*a^4*b^
2*x^3 + 6*a^5*b*x - a^6/x
```

Mupad [B]
time = 0.03, size = 66, normalized size = 0.92

$$\frac{b^6\,x^{11}}{11}-\frac{a^6}{x}+\frac{2\,a\,b^5\,x^9}{3}+5\,a^4\,b^2\,x^3+4\,a^3\,b^3\,x^5+\frac{15\,a^2\,b^4\,x^7}{7}+6\,a^5\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^2,x)
```

```
[Out] (b^6*x^11)/11 - a^6/x + (2*a*b^5*x^9)/3 + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (
15*a^2*b^4*x^7)/7 + 6*a^5*b*x
```

3.455 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx$

Optimal. Leaf size=77

$$-\frac{a^6}{2x^2}+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}+6a^5b\log(x)$$

```
[Out] -1/2*a^6/x^2+15/2*a^4*b^2*x^2+5*a^3*b^3*x^4+5/2*a^2*b^4*x^6+3/4*a*b^5*x^8+1
/10*b^6*x^10+6*a^5*b*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 45}

$$-\frac{a^6}{2x^2}+6a^5b\log(x)+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

```
[Out] -1/2*a^6/x^2 + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2 + (3*
a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^3}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^2}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(15a^4b^8+\frac{a^6b^6}{x^2}+\frac{6a^5b^7}{x}+20a^3b^9x+15a^2b^{10}x^2+6ab^{11}x^3+b^{12}x^4\right)\,dx,\right.}{2b^6}$$
$$= -\frac{a^6}{2x^2}+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}+6a^5b\log(x)$$

Mathematica [A]
time = 0.01, size = 77, normalized size = 1.00

$$-\frac{a^6}{2x^2}+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}+6a^5b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

```
[Out] -1/2*a^6/x^2 + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2 + (3*
a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

Maple [A]
time = 0.02, size = 68, normalized size = 0.88

method	result	size
default	$-\frac{a^6}{2x^2}+\frac{15a^4b^2x^2}{2}+5a^3b^3x^4+\frac{5a^2b^4x^6}{2}+\frac{3a\,b^5x^8}{4}+\frac{b^6x^{10}}{10}+6a^5b\ln(x)$	68
risch	$-\frac{a^6}{2x^2}+\frac{15a^4b^2x^2}{2}+5a^3b^3x^4+\frac{5a^2b^4x^6}{2}+\frac{3a\,b^5x^8}{4}+\frac{b^6x^{10}}{10}+6a^5b\ln(x)$	68
norman	$\frac{-\frac{1}{2}a^6+\frac{1}{10}b^6x^{12}+\frac{3}{4}a\,b^5x^{10}+\frac{5}{2}a^2b^4x^8+5a^3b^3x^6+\frac{15}{2}a^4b^2x^4}{x^2}+6a^5b\ln(x)$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^6/x^2+15/2*a^4*b^2*x^2+5*a^3*b^3*x^4+5/2*a^2*b^4*x^6+3/4*a*b^5*x^8+1
/10*b^6*x^10+6*a^5*b*ln(x)
```

Maxima [A]
time = 0.30, size = 69, normalized size = 0.90

$$\frac{1}{10}\,b^6x^{10}+\frac{3}{4}\,ab^5x^8+\frac{5}{2}\,a^2b^4x^6+5\,a^3b^3x^4+\frac{15}{2}\,a^4b^2x^2+3\,a^5b\log\left(x^2\right)-\frac{a^6}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 + 15/2*a^4*
b^2*x^2 + 3*a^5*b*log(x^2) - 1/2*a^6/x^2
```

Fricas [A]
time = 0.34, size = 72, normalized size = 0.94

$$\frac{2\,b^6x^{12}+15\,ab^5x^{10}+50\,a^2b^4x^8+100\,a^3b^3x^6+150\,a^4b^2x^4+120\,a^5bx^2\log\left(x\right)-10\,a^6}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^6*x^12 + 15*a*b^5*x^10 + 50*a^2*b^4*x^8 + 100*a^3*b^3*x^6 + 150*a
^4*b^2*x^4 + 120*a^5*b*x^2*log(x) - 10*a^6)/x^2
```

Sympy [A]
time = 0.06, size = 76, normalized size = 0.99

$$-\frac{a^6}{2x^2}+6a^5b\log\left(x\right)+\frac{15a^4b^2x^2}{2}+5a^3b^3x^4+\frac{5a^2b^4x^6}{2}+\frac{3ab^5x^8}{4}+\frac{b^6x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**3,x)
```

```
[Out] -a**6/(2*x**2) + 6*a**5*b*log(x) + 15*a**4*b**2*x**2/2 + 5*a**3*b**3*x**4 +
 5*a**2*b**4*x**6/2 + 3*a*b**5*x**8/4 + b**6*x**10/10
```

Giac [A]
time = 11.15, size = 79, normalized size = 1.03

$$\frac{1}{10}\,b^6x^{10}+\frac{3}{4}\,ab^5x^8+\frac{5}{2}\,a^2b^4x^6+5\,a^3b^3x^4+\frac{15}{2}\,a^4b^2x^2+3\,a^5b\log\left(x^2\right)-\frac{6\,a^5bx^2+a^6}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 + 15/2*a^4*
b^2*x^2 + 3*a^5*b*log(x^2) - 1/2*(6*a^5*b*x^2 + a^6)/x^2
```

Mupad [B]
time = 0.04, size = 67, normalized size = 0.87

$$\frac{b^6\,x^{10}}{10}-\frac{a^6}{2\,x^2}+\frac{3\,a\,b^5\,x^8}{4}+6\,a^5\,b\,\ln\left(x\right)+\frac{15\,a^4\,b^2\,x^2}{2}+5\,a^3\,b^3\,x^4+\frac{5\,a^2\,b^4\,x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^3,x)
```

```
[Out] (b^6*x^10)/10 - a^6/(2*x^2) + (3*a*b^5*x^8)/4 + 6*a^5*b*log(x) + (15*a^4*b^
2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2
```

3.456 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4}\,dx$

Optimal. Leaf size=74

$$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

```
[Out] -1/3*a^6/x^3-6*a^5*b/x+15*a^4*b^2*x+20/3*a^3*b^3*x^3+3*a^2*b^4*x^5+6/7*a*b^
5*x^7+1/9*b^6*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

```
[Out] -1/3*a^6/x^3 - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 + 3*a^2*b^4*
x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^4}\,dx}{b^6}\\
&= \frac{\int \left(15a^4b^8+\frac{a^6b^6}{x^4}+\frac{6a^5b^7}{x^2}+20a^3b^9x^2+15a^2b^{10}x^4+6ab^{11}x^6+b^{12}x^8\right)dx}{b^6}\\
&= -\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 74, normalized size = 1.00

$$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

```
[Out] -1/3*a^6/x^3 - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 + 3*a^2*b^4*
x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Maple [A]
time = 0.02, size = 67, normalized size = 0.91

method	result	size
default	$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20a^3b^3x^3}{3}+3a^2b^4x^5+\frac{6a\,b^5x^7}{7}+\frac{b^6x^9}{9}$	67
risch	$\frac{b^6x^9}{9}+\frac{6a\,b^5x^7}{7}+3a^2b^4x^5+\frac{20a^3b^3x^3}{3}+15a^4b^2x+\frac{-6a^5b\,x^2-\frac{1}{3}a^6}{x^3}$	69
norman	$\frac{\frac{1}{9}b^6x^{12}+\frac{6}{7}a\,b^5x^{10}+3a^2b^4x^8+\frac{20}{3}a^3b^3x^6+15a^4b^2x^4-6a^5b\,x^2-\frac{1}{3}a^6}{x^3}$	70
gosper	$-\frac{-7b^6x^{12}-54a\,b^5x^{10}-189a^2b^4x^8-420a^3b^3x^6-945a^4b^2x^4+378a^5b\,x^2+21a^6}{63x^3}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^6/x^3-6*a^5*b/x+15*a^4*b^2*x+20/3*a^3*b^3*x^3+3*a^2*b^4*x^5+6/7*a*b^
5*x^7+1/9*b^6*x^9
```

Maxima [A]
time = 0.36, size = 67, normalized size = 0.91

$$\frac{1}{9}\,b^6x^9+\frac{6}{7}\,ab^5x^7+3\,a^2b^4x^5+\frac{20}{3}\,a^3b^3x^3+15\,a^4b^2x-\frac{18\,a^5bx^2+a^6}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 + 15*a^4*b^2
*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

Fricas [A]
time = 0.33, size = 70, normalized size = 0.95

$$\frac{7\,b^6x^{12}+54\,ab^5x^{10}+189\,a^2b^4x^8+420\,a^3b^3x^6+945\,a^4b^2x^4-378\,a^5bx^2-21\,a^6}{63\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^6*x^12 + 54*a*b^5*x^10 + 189*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 945*
a^4*b^2*x^4 - 378*a^5*b*x^2 - 21*a^6)/x^3
```

Sympy [A]
time = 0.06, size = 75, normalized size = 1.01

$$15a^4b^2x + \frac{20a^3b^3x^3}{3} + 3a^2b^4x^5 + \frac{6ab^5x^7}{7} + \frac{b^6x^9}{9} + \frac{-a^6 - 18a^5bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**4,x)
```

```
[Out] 15*a**4*b**2*x + 20*a**3*b**3*x**3/3 + 3*a**2*b**4*x**5 + 6*a*b**5*x**7/7 +
 b**6*x**9/9 + (-a**6 - 18*a**5*b*x**2)/(3*x**3)
```

Giac [A]
time = 4.09, size = 67, normalized size = 0.91

$$\frac{1}{9}\,b^6x^9 + \frac{6}{7}\,ab^5x^7 + 3\,a^2b^4x^5 + \frac{20}{3}\,a^3b^3x^3 + 15\,a^4b^2x - \frac{18\,a^5bx^2 + a^6}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 + 15*a^4*b^2
*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.93

$$\frac{b^6\,x^9}{9} - \frac{\frac{a^6}{3} + 6\,b\,a^5\,x^2}{x^3} + 15\,a^4\,b^2\,x + \frac{6\,a\,b^5\,x^7}{7} + \frac{20\,a^3\,b^3\,x^3}{3} + 3\,a^2\,b^4\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^4,x)
```

```
[Out] (b^6*x^9)/9 - (a^6/3 + 6*a^5*b*x^2)/x^3 + 15*a^4*b^2*x + (6*a*b^5*x^7)/7 +
(20*a^3*b^3*x^3)/3 + 3*a^2*b^4*x^5
```

3.457 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}+15a^4b^2\log(x)$$

```
[Out] -1/4*a^6/x^4-3*a^5*b/x^2+10*a^3*b^3*x^2+15/4*a^2*b^4*x^4+a*b^5*x^6+1/8*b^6*
x^8+15*a^4*b^2*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+15a^4b^2\log(x)+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

```
[Out] -1/4*a^6/x^4 - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/4 + a*b^5*
x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^5}\,dx}{b^6}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(ab+b^2x)^6}{x^3}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\operatorname{Subst}\left(\int \left(20a^3b^9+\frac{a^6b^6}{x^3}+\frac{6a^5b^7}{x^2}+\frac{15a^4b^8}{x}+15a^2b^{10}x+6ab^{11}x^2+b^{12}x^3\right)\,dx, x,}{2b^6}$$
$$= -\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}+15a^4b^2\log(x)$$

Mathematica [A]
time = 0.00, size = 72, normalized size = 1.00

$$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}+15a^4b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

```
[Out] -1/4*a^6/x^4 - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/4 + a*b^5*
x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

Maple [A]
time = 0.04, size = 67, normalized size = 0.93

method	result	size
default	$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+10a^3b^3x^2+\frac{15a^2b^4x^4}{4}+a\,b^5x^6+\frac{b^6x^8}{8}+15a^4b^2\ln(x)$	67
norman	$\frac{a\,b^5x^{10}-\frac{1}{4}a^6+\frac{1}{8}b^6x^{12}+\frac{15}{4}a^2b^4x^8+10a^3b^3x^6-3a^5b\,x^2}{x^4}+15a^4b^2\ln(x)$	69
risch	$\frac{b^6x^8}{8}+a\,b^5x^6+\frac{15a^2b^4x^4}{4}+10a^3b^3x^2+\frac{-3a^5b\,x^2-\frac{1}{4}a^6}{x^4}+15a^4b^2\ln(x)$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^6/x^4-3*a^5*b/x^2+10*a^3*b^3*x^2+15/4*a^2*b^4*x^4+a*b^5*x^6+1/8*b^6*
x^8+15*a^4*b^2*ln(x)
```

Maxima [A]
time = 0.29, size = 69, normalized size = 0.96

$$\frac{1}{8}\,b^6x^8+ab^5x^6+\frac{15}{4}\,a^2b^4x^4+10\,a^3b^3x^2+\frac{15}{2}\,a^4b^2\log\left(x^2\right)-\frac{12\,a^5bx^2+a^6}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/2*a^4*b^2*
log(x^2) - 1/4*(12*a^5*b*x^2 + a^6)/x^4
```

Fricas [A]
time = 0.35, size = 71, normalized size = 0.99

$$\frac{b^6x^{12} + 8\,ab^5x^{10} + 30\,a^2b^4x^8 + 80\,a^3b^3x^6 + 120\,a^4b^2x^4\log\left(x\right) - 24\,a^5bx^2 - 2\,a^6}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(b^6*x^12 + 8*a*b^5*x^10 + 30*a^2*b^4*x^8 + 80*a^3*b^3*x^6 + 120*a^4*b^
2*x^4*log(x) - 24*a^5*b*x^2 - 2*a^6)/x^4
```

Sympy [A]
time = 0.09, size = 73, normalized size = 1.01

$$15a^4b^2\log\left(x\right) + 10a^3b^3x^2 + \frac{15a^2b^4x^4}{4} + ab^5x^6 + \frac{b^6x^8}{8} + \frac{-a^6 - 12a^5bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**5,x)
```

```
[Out] 15*a**4*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a*b**5*x**6
 + b**6*x**8/8 + (-a**6 - 12*a**5*b*x**2)/(4*x**4)
```

Giac [A]
time = 3.29, size = 80, normalized size = 1.11

$$\frac{1}{8}\,b^6x^8 + ab^5x^6 + \frac{15}{4}\,a^2b^4x^4 + 10\,a^3b^3x^2 + \frac{15}{2}\,a^4b^2\log\left(x^2\right) - \frac{45\,a^4b^2x^4 + 12\,a^5bx^2 + a^6}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/2*a^4*b^2*
log(x^2) - 1/4*(45*a^4*b^2*x^4 + 12*a^5*b*x^2 + a^6)/x^4
```

Mupad [B]
time = 0.04, size = 69, normalized size = 0.96

$$\frac{b^6\,x^8}{8} - \frac{\frac{a^6}{4} + 3\,b\,a^5\,x^2}{x^4} + a\,b^5\,x^6 + 10\,a^3\,b^3\,x^2 + \frac{15\,a^2\,b^4\,x^4}{4} + 15\,a^4\,b^2\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^5,x)
```

```
[Out] (b^6*x^8)/8 - (a^6/4 + 3*a^5*b*x^2)/x^4 + a*b^5*x^6 + 10*a^3*b^3*x^2 + (15*
a^2*b^4*x^4)/4 + 15*a^4*b^2*log(x)
```

3.458 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^6}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

```
[Out] -1/5*a^6/x^5-2*a^5*b/x^3-15*a^4*b^2/x+20*a^3*b^3*x+5*a^2*b^4*x^3+6/5*a*b^5*
x^5+1/7*b^6*x^7
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {28, 276}

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

```
[Out] -1/5*a^6/x^5 - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*a^2*b^4*x^
3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^6}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^6}\,dx}{b^6}\\ &= \frac{\int \left(20a^3b^9+\frac{a^6b^6}{x^6}+\frac{6a^5b^7}{x^4}+\frac{15a^4b^8}{x^2}+15a^2b^{10}x^2+6ab^{11}x^4+b^{12}x^6\right)dx}{b^6}\\ &= -\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 72, normalized size = 1.00

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

```
[Out] -1/5*a^6/x^5 - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*a^2*b^4*x^
3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

Maple [A]
time = 0.03, size = 67, normalized size = 0.93

method	result	size
default	$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6a\,b^5x^5}{5}+\frac{b^6x^7}{7}$	67
risch	$\frac{b^6x^7}{7}+\frac{6a\,b^5x^5}{5}+5a^2b^4x^3+20a^3b^3x+\frac{-15a^4b^2x^4-2a^5b\,x^2-\frac{1}{5}a^6}{x^5}$	69
norman	$\frac{\frac{1}{7}b^6x^{12}+\frac{6}{5}a\,b^5x^{10}+5a^2b^4x^8+20a^3b^3x^6-15a^4b^2x^4-2a^5b\,x^2-\frac{1}{5}a^6}{x^5}$	70
gosper	$-\frac{-5b^6x^{12}-42a\,b^5x^{10}-175a^2b^4x^8-700a^3b^3x^6+525a^4b^2x^4+70a^5b\,x^2+7a^6}{35x^5}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^6/x^5-2*a^5*b/x^3-15*a^4*b^2/x+20*a^3*b^3*x+5*a^2*b^4*x^3+6/5*a*b^5*
x^5+1/7*b^6*x^7
```

Maxima [A]
time = 0.29, size = 67, normalized size = 0.93

$$\frac{1}{7}\,b^6x^7+\frac{6}{5}\,ab^5x^5+5\,a^2b^4x^3+20\,a^3b^3x-\frac{75\,a^4b^2x^4+10\,a^5bx^2+a^6}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*(75*a^4*b^
2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

Fricas [A]
time = 0.33, size = 70, normalized size = 0.97

$$\frac{5\,b^6x^{12}+42\,ab^5x^{10}+175\,a^2b^4x^8+700\,a^3b^3x^6-525\,a^4b^2x^4-70\,a^5bx^2-7\,a^6}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^6*x^12 + 42*a*b^5*x^10 + 175*a^2*b^4*x^8 + 700*a^3*b^3*x^6 - 525*
a^4*b^2*x^4 - 70*a^5*b*x^2 - 7*a^6)/x^5
```

Sympy [A]
time = 0.09, size = 73, normalized size = 1.01

$$20a^3b^3x + 5a^2b^4x^3 + \frac{6ab^5x^5}{5} + \frac{b^6x^7}{7} + \frac{-a^6 - 10a^5bx^2 - 75a^4b^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**6,x)
```

```
[Out] 20*a**3*b**3*x + 5*a**2*b**4*x**3 + 6*a*b**5*x**5/5 + b**6*x**7/7 + (-a**6
- 10*a**5*b*x**2 - 75*a**4*b**2*x**4)/(5*x**5)
```

Giac [A]
time = 3.57, size = 67, normalized size = 0.93

$$\frac{1}{7}\,b^6x^7 + \frac{6}{5}\,ab^5x^5 + 5\,a^2b^4x^3 + 20\,a^3b^3x - \frac{75\,a^4b^2x^4 + 10\,a^5bx^2 + a^6}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*(75*a^4*b^
2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

Mupad [B]
time = 0.03, size = 69, normalized size = 0.96

$$\frac{b^6\,x^7}{7} - \frac{\frac{a^6}{5} + 2\,a^5\,b\,x^2 + 15\,a^4\,b^2\,x^4}{x^5} + 20\,a^3\,b^3\,x + \frac{6\,a\,b^5\,x^5}{5} + 5\,a^2\,b^4\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^6,x)
```

```
[Out] (b^6*x^7)/7 - (a^6/5 + 2*a^5*b*x^2 + 15*a^4*b^2*x^4)/x^5 + 20*a^3*b^3*x + (
6*a*b^5*x^5)/5 + 5*a^2*b^4*x^3
```

3.459 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7}\,dx$

Optimal. Leaf size=79

$$-\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}+20a^3b^3\log(x)$$

```
[Out] -1/6*a^6/x^6-3/2*a^5*b/x^4-15/2*a^4*b^2/x^2+15/2*a^2*b^4*x^2+3/2*a*b^5*x^4+
1/6*b^6*x^6+20*a^3*b^3*ln(x)
```

Rubi [A]

time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 45}

$$-\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+20a^3b^3\log(x)+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

```
[Out] -1/6*a^6/x^6 - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2*b^4*x^2)/
2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^7}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^4}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(15a^2b^{10} + \frac{a^6b^6}{x^4} + \frac{6a^5b^7}{x^3} + \frac{15a^4b^8}{x^2} + \frac{20a^3b^9}{x} + 6ab^{11}x + b^{12}x^2\right)\,dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{6x^6} - \frac{3a^5b}{2x^4} - \frac{15a^4b^2}{2x^2} + \frac{15}{2}a^2b^4x^2 + \frac{3}{2}ab^5x^4 + \frac{b^6x^6}{6} + 20a^3b^3\log(x)$$

Mathematica [A]
time = 0.00, size = 79, normalized size = 1.00

$$-\frac{a^6}{6x^6} - \frac{3a^5b}{2x^4} - \frac{15a^4b^2}{2x^2} + \frac{15}{2}a^2b^4x^2 + \frac{3}{2}ab^5x^4 + \frac{b^6x^6}{6} + 20a^3b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

```
[Out] -1/6*a^6/x^6 - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2*b^4*x^2)/
2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

Maple [A]
time = 0.02, size = 68, normalized size = 0.86

method	result	size
default	$-\frac{a^6}{6x^6} - \frac{3a^5b}{2x^4} - \frac{15a^4b^2}{2x^2} + \frac{15a^2b^4x^2}{2} + \frac{3a\,b^5x^4}{2} + \frac{b^6x^6}{6} + 20a^3b^3\ln(x)$	68
norman	$\frac{-\frac{1}{6}a^6+\frac{1}{6}b^6x^{12}+\frac{3}{2}a\,b^5x^{10}+\frac{15}{2}a^2b^4x^8-\frac{15}{2}a^4b^2x^4-\frac{3}{2}a^5b\,x^2}{x^6} + 20a^3b^3\ln(x)$	70
risch	$\frac{b^6x^6}{6} + \frac{3a\,b^5x^4}{2} + \frac{15a^2b^4x^2}{2} + \frac{-\frac{15}{2}a^4b^2x^4-\frac{3}{2}a^5b\,x^2-\frac{1}{6}a^6}{x^6} + 20a^3b^3\ln(x)$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^6/x^6-3/2*a^5*b/x^4-15/2*a^4*b^2/x^2+15/2*a^2*b^4*x^2+3/2*a*b^5*x^4+
1/6*b^6*x^6+20*a^3*b^3*ln(x)
```

Maxima [A]
time = 0.33, size = 70, normalized size = 0.89

$$\frac{1}{6}\,b^6x^6 + \frac{3}{2}\,ab^5x^4 + \frac{15}{2}\,a^2b^4x^2 + 10\,a^3b^3\log\left(x^2\right) - \frac{45\,a^4b^2x^4 + 9\,a^5bx^2 + a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*log(x^2) - 1/6*
(45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x^6
```

Fricas [A]
time = 0.33, size = 71, normalized size = 0.90

$$\frac{b^6x^{12} + 9\,ab^5x^{10} + 45\,a^2b^4x^8 + 120\,a^3b^3x^6\log\left(x\right) - 45\,a^4b^2x^4 - 9\,a^5bx^2 - a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(b^6*x^12 + 9*a*b^5*x^10 + 45*a^2*b^4*x^8 + 120*a^3*b^3*x^6*log(x) - 45
*a^4*b^2*x^4 - 9*a^5*b*x^2 - a^6)/x^6
```

Sympy [A]
time = 0.12, size = 76, normalized size = 0.96

$$20a^3b^3\log\left(x\right) + \frac{15a^2b^4x^2}{2} + \frac{3ab^5x^4}{2} + \frac{b^6x^6}{6} + \frac{-a^6 - 9a^5bx^2 - 45a^4b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**7,x)
```

```
[Out] 20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5*x**4/2 + b**6*x**6/6 +
 (-a**6 - 9*a**5*b*x**2 - 45*a**4*b**2*x**4)/(6*x**6)
```

Giac [A]
time = 4.32, size = 81, normalized size = 1.03

$$\frac{1}{6}\,b^6x^6 + \frac{3}{2}\,ab^5x^4 + \frac{15}{2}\,a^2b^4x^2 + 10\,a^3b^3\log\left(x^2\right) - \frac{110\,a^3b^3x^6 + 45\,a^4b^2x^4 + 9\,a^5bx^2 + a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*log(x^2) - 1/6*
(110*a^3*b^3*x^6 + 45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x^6
```

Mupad [B]
time = 4.34, size = 70, normalized size = 0.89

$$\frac{b^6\,x^6}{6} - \frac{\frac{a^6}{6} + \frac{3\,a^5\,b\,x^2}{2} + \frac{15\,a^4\,b^2\,x^4}{2}}{x^6} + \frac{3\,a\,b^5\,x^4}{2} + \frac{15\,a^2\,b^4\,x^2}{2} + 20\,a^3\,b^3\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^7,x)
```

```
[Out] (b^6*x^6)/6 - (a^6/6 + (3*a^5*b*x^2)/2 + (15*a^4*b^2*x^4)/2)/x^6 + (3*a*b^5
*x^4)/2 + (15*a^2*b^4*x^2)/2 + 20*a^3*b^3*log(x)
```

3.460 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

```
[Out] -1/7*a^6/x^7-6/5*a^5*b/x^5-5*a^4*b^2/x^3-20*a^3*b^3/x+15*a^2*b^4*x+2*a*b^5*
x^3+1/5*b^6*x^5
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

```
[Out] -1/7*a^6/x^7 - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)/x + 15*a^
2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^8}\,dx}{b^6}\\
&= \frac{\int \left(15a^2b^{10}+\frac{a^6b^6}{x^8}+\frac{6a^5b^7}{x^6}+\frac{15a^4b^8}{x^4}+\frac{20a^3b^9}{x^2}+6ab^{11}x^2+b^{12}x^4\right)dx}{b^6}\\
&= -\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 72, normalized size = 1.00

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

```
[Out] -1/7*a^6/x^7 - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)/x + 15*a^
2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Maple [A]
time = 0.02, size = 67, normalized size = 0.93

method	result	size
default	$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2a\,b^5x^3+\frac{b^6x^5}{5}$	67
risch	$\frac{b^6x^5}{5}+2a\,b^5x^3+15a^2b^4x+\frac{-20a^3b^3x^6-5a^4b^2x^4-\frac{6}{5}a^5b\,x^2-\frac{1}{7}a^6}{x^7}$	69
norman	$\frac{\frac{1}{5}b^6x^{12}+2a\,b^5x^{10}+15a^2b^4x^8-20a^3b^3x^6-5a^4b^2x^4-\frac{6}{5}a^5b\,x^2-\frac{1}{7}a^6}{x^7}$	70
gosper	$-\frac{-7b^6x^{12}-70a\,b^5x^{10}-525a^2b^4x^8+700a^3b^3x^6+175a^4b^2x^4+42a^5b\,x^2+5a^6}{35x^7}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^6/x^7-6/5*a^5*b/x^5-5*a^4*b^2/x^3-20*a^3*b^3/x+15*a^2*b^4*x+2*a*b^5*
x^3+1/5*b^6*x^5
```

Maxima [A]
time = 0.29, size = 69, normalized size = 0.96

$$\frac{1}{5}\,b^6x^5+2\,ab^5x^3+15\,a^2b^4x-\frac{700\,a^3b^3x^6+175\,a^4b^2x^4+42\,a^5bx^2+5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6 + 175*a^4*
b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

Fricas [A]
time = 0.36, size = 70, normalized size = 0.97

$$\frac{7\,b^6x^{12}+70\,ab^5x^{10}+525\,a^2b^4x^8-700\,a^3b^3x^6-175\,a^4b^2x^4-42\,a^5bx^2-5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(7*b^6*x^12 + 70*a*b^5*x^10 + 525*a^2*b^4*x^8 - 700*a^3*b^3*x^6 - 175*
a^4*b^2*x^4 - 42*a^5*b*x^2 - 5*a^6)/x^7
```

Sympy [A]
time = 0.13, size = 73, normalized size = 1.01

$$15a^2b^4x + 2ab^5x^3 + \frac{b^6x^5}{5} + \frac{-5a^6 - 42a^5bx^2 - 175a^4b^2x^4 - 700a^3b^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**8,x)
```

```
[Out] 15*a**2*b**4*x + 2*a*b**5*x**3 + b**6*x**5/5 + (-5*a**6 - 42*a**5*b*x**2 -
175*a**4*b**2*x**4 - 700*a**3*b**3*x**6)/(35*x**7)
```

Giac [A]
time = 3.46, size = 69, normalized size = 0.96

$$\frac{1}{5}\,b^6x^5 + 2\,ab^5x^3 + 15\,a^2b^4x - \frac{700\,a^3b^3x^6 + 175\,a^4b^2x^4 + 42\,a^5bx^2 + 5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="giac")
```

```
[Out] 1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6 + 175*a^4*
b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

Mupad [B]
time = 0.05, size = 69, normalized size = 0.96

$$\frac{b^6\,x^5}{5} - \frac{\frac{a^6}{7} + \frac{6\,a^5\,b\,x^2}{5} + 5\,a^4\,b^2\,x^4 + 20\,a^3\,b^3\,x^6}{x^7} + 15\,a^2\,b^4\,x + 2\,a\,b^5\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^8,x)
```

```
[Out] (b^6*x^5)/5 - (a^6/7 + (6*a^5*b*x^2)/5 + 5*a^4*b^2*x^4 + 20*a^3*b^3*x^6)/x^
7 + 15*a^2*b^4*x + 2*a*b^5*x^3
```

3.461 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9}\,dx$

Optimal. Leaf size=73

$$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+3ab^5x^2+\frac{b^6x^4}{4}+15a^2b^4\log(x)$$

```
[Out] -1/8*a^6/x^8-a^5*b/x^6-15/4*a^4*b^2/x^4-10*a^3*b^3/x^2+3*a*b^5*x^2+1/4*b^6*
x^4+15*a^2*b^4*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 45}

$$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)+3ab^5x^2+\frac{b^6x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

```
[Out] -1/8*a^6/x^8 - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/x^2 + 3*a*
b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^9}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^5}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(6ab^{11}+\frac{a^6b^6}{x^5}+\frac{6a^5b^7}{x^4}+\frac{15a^4b^8}{x^3}+\frac{20a^3b^9}{x^2}+\frac{15a^2b^{10}}{x}+b^{12}x\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+3ab^5x^2+\frac{b^6x^4}{4}+15a^2b^4\log(x)$$

Mathematica [A]
time = 0.01, size = 73, normalized size = 1.00

$$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+3ab^5x^2+\frac{b^6x^4}{4}+15a^2b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

```
[Out] -1/8*a^6/x^8 - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/x^2 + 3*a*
b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

Maple [A]
time = 0.03, size = 68, normalized size = 0.93

method	result	size
default	$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+3a\,b^5x^2+\frac{b^6x^4}{4}+15a^2b^4\ln(x)$	68
norman	$\frac{-\frac{1}{8}a^6+\frac{1}{4}b^6x^{12}+3a\,b^5x^{10}-10a^3b^3x^6-\frac{15}{4}a^4b^2x^4-a^5b\,x^2}{x^8}+15a^2b^4\ln(x)$	70
risch	$\frac{b^6x^4}{4}+3a\,b^5x^2+9a^2b^4+\frac{-10a^3b^3x^6-\frac{15}{4}a^4b^2x^4-a^5b\,x^2-\frac{1}{8}a^6}{x^8}+15a^2b^4\ln(x)$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a^6/x^8-a^5*b/x^6-15/4*a^4*b^2/x^4-10*a^3*b^3/x^2+3*a*b^5*x^2+1/4*b^6*
x^4+15*a^2*b^4*ln(x)
```

Maxima [A]
time = 0.30, size = 70, normalized size = 0.96

$$\frac{1}{4}\,b^6x^4+3\,ab^5x^2+\frac{15}{2}\,a^2b^4\log\left(x^2\right)-\frac{80\,a^3b^3x^6+30\,a^4b^2x^4+8\,a^5bx^2+a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="maxima")
[Out] 1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*log(x^2) - 1/8*(80*a^3*b^3*x^6 + 3
0*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8
```

Fricas [A]
time = 0.41, size = 72, normalized size = 0.99

$$\frac{2\,b^6x^{12}+24\,ab^5x^{10}+120\,a^2b^4x^8\log\left(x\right)-80\,a^3b^3x^6-30\,a^4b^2x^4-8\,a^5bx^2-a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="fricas")
[Out] 1/8*(2*b^6*x^12 + 24*a*b^5*x^10 + 120*a^2*b^4*x^8*log(x) - 80*a^3*b^3*x^6 -
 30*a^4*b^2*x^4 - 8*a^5*b*x^2 - a^6)/x^8
```

Sympy [A]
time = 0.16, size = 73, normalized size = 1.00

$$15a^2b^4\log\left(x\right)+3ab^5x^2+\frac{b^6x^4}{4}+\frac{-a^6-8a^5bx^2-30a^4b^2x^4-80a^3b^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**9,x)
[Out] 15*a**2*b**4*log(x) + 3*a*b**5*x**2 + b**6*x**4/4 + (-a**6 - 8*a**5*b*x**2
- 30*a**4*b**2*x**4 - 80*a**3*b**3*x**6)/(8*x**8)
```

Giac [A]
time = 3.56, size = 81, normalized size = 1.11

$$\frac{1}{4}\,b^6x^4+3\,ab^5x^2+\frac{15}{2}\,a^2b^4\log\left(x^2\right)-\frac{125\,a^2b^4x^8+80\,a^3b^3x^6+30\,a^4b^2x^4+8\,a^5bx^2+a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="giac")
[Out] 1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*log(x^2) - 1/8*(125*a^2*b^4*x^8 +
80*a^3*b^3*x^6 + 30*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8
```

Mupad [B]
time = 0.05, size = 69, normalized size = 0.95

$$\frac{b^6\,x^4}{4}-\frac{\frac{a^6}{8}+a^5\,b\,x^2+\frac{15\,a^4\,b^2\,x^4}{4}+10\,a^3\,b^3\,x^6}{x^8}+3\,a\,b^5\,x^2+15\,a^2\,b^4\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^9,x)
[Out] (b^6*x^4)/4 - (a^6/8 + a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + 10*a^3*b^3*x^6)/x^8
 + 3*a*b^5*x^2 + 15*a^2*b^4*log(x)
```

3.462 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{10}}\,dx$

Optimal. Leaf size=74

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

```
[Out] -1/9*a^6/x^9-6/7*a^5*b/x^7-3*a^4*b^2/x^5-20/3*a^3*b^3/x^3-15*a^2*b^4/x+6*a*
b^5*x+1/3*b^6*x^3
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

```
[Out] -1/9*a^6/x^9 - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)/(3*x^3) -
 (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{10}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^{10}}\,dx}{b^6}\\
&= \frac{\int \left(6ab^{11}+\frac{a^6b^6}{x^{10}}+\frac{6a^5b^7}{x^8}+\frac{15a^4b^8}{x^6}+\frac{20a^3b^9}{x^4}+\frac{15a^2b^{10}}{x^2}+b^{12}x^2\right)dx}{b^6}\\
&= -\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 74, normalized size = 1.00

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

```
[Out] -1/9*a^6/x^9 - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)/(3*x^3) -
 (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Maple [A]
time = 0.02, size = 67, normalized size = 0.91

method	result	size
default	$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6a\,b^5x+\frac{b^6x^3}{3}$	67
risch	$\frac{b^6x^3}{3}+6a\,b^5x+\frac{-15a^2b^4x^8-\frac{20}{3}a^3b^3x^6-3a^4b^2x^4-\frac{6}{7}a^5b\,x^2-\frac{1}{9}a^6}{x^9}$	69
norman	$\frac{\frac{1}{3}b^6x^{12}+6a\,b^5x^{10}-15a^2b^4x^8-\frac{20}{3}a^3b^3x^6-3a^4b^2x^4-\frac{6}{7}a^5b\,x^2-\frac{1}{9}a^6}{x^9}$	70
gosper	$-\frac{-21b^6x^{12}-378a\,b^5x^{10}+945a^2b^4x^8+420a^3b^3x^6+189a^4b^2x^4+54a^5b\,x^2+7a^6}{63x^9}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^6/x^9-6/7*a^5*b/x^7-3*a^4*b^2/x^5-20/3*a^3*b^3/x^3-15*a^2*b^4/x+6*a*
b^5*x+1/3*b^6*x^3
```

Maxima [A]
time = 0.30, size = 69, normalized size = 0.93

$$\frac{1}{3}\,b^6x^3+6\,ab^5x-\frac{945\,a^2b^4x^8+420\,a^3b^3x^6+189\,a^4b^2x^4+54\,a^5bx^2+7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 189*a^4
*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

Fricas [A]
time = 0.34, size = 70, normalized size = 0.95

$$\frac{21\,b^6x^{12}+378\,ab^5x^{10}-945\,a^2b^4x^8-420\,a^3b^3x^6-189\,a^4b^2x^4-54\,a^5bx^2-7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(21*b^6*x^12 + 378*a*b^5*x^10 - 945*a^2*b^4*x^8 - 420*a^3*b^3*x^6 - 18
9*a^4*b^2*x^4 - 54*a^5*b*x^2 - 7*a^6)/x^9
```

Sympy [A]
time = 0.16, size = 73, normalized size = 0.99

$$6ab^5x + \frac{b^6x^3}{3} + \frac{-7a^6 - 54a^5bx^2 - 189a^4b^2x^4 - 420a^3b^3x^6 - 945a^2b^4x^8}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**10,x)
```

```
[Out] 6*a*b**5*x + b**6*x**3/3 + (-7*a**6 - 54*a**5*b*x**2 - 189*a**4*b**2*x**4 -
 420*a**3*b**3*x**6 - 945*a**2*b**4*x**8)/(63*x**9)
```

Giac [A]
time = 4.20, size = 69, normalized size = 0.93

$$\frac{1}{3}\,b^6x^3 + 6\,ab^5x - \frac{945\,a^2b^4x^8 + 420\,a^3b^3x^6 + 189\,a^4b^2x^4 + 54\,a^5bx^2 + 7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 189*a^4
*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

Mupad [B]
time = 0.05, size = 70, normalized size = 0.95

$$-\frac{\frac{a^6}{9} + \frac{6\,a^5\,b\,x^2}{7} + 3\,a^4\,b^2\,x^4 + \frac{20\,a^3\,b^3\,x^6}{3} + 15\,a^2\,b^4\,x^8 - 6\,a\,b^5\,x^{10} - \frac{b^6\,x^{12}}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^10,x)
```

```
[Out] -(a^6/9 - (b^6*x^12)/3 + (6*a^5*b*x^2)/7 - 6*a*b^5*x^10 + 3*a^4*b^2*x^4 + (
20*a^3*b^3*x^6)/3 + 15*a^2*b^4*x^8)/x^9
```

3.463 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{11}}\,dx$

Optimal. Leaf size=77

$$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+\frac{b^6x^2}{2}+6ab^5\log(x)$$

```
[Out] -1/10*a^6/x^10-3/4*a^5*b/x^8-5/2*a^4*b^2/x^6-5*a^3*b^3/x^4-15/2*a^2*b^4/x^2
+1/2*b^6*x^2+6*a*b^5*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

```
[Out] -1/10*a^6/x^10 - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3*b^3)/x^4
- (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{11}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^{11}}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^6}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\text{Subst}\left(\int \left(b^{12}+\frac{a^6b^6}{x^6}+\frac{6a^5b^7}{x^5}+\frac{15a^4b^8}{x^4}+\frac{20a^3b^9}{x^3}+\frac{15a^2b^{10}}{x^2}+\frac{6ab^{11}}{x}\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+\frac{b^6x^2}{2}+6ab^5\log(x)$$

Mathematica [A]
time = 0.00, size = 77, normalized size = 1.00

$$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+\frac{b^6x^2}{2}+6ab^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

```
[Out] -1/10*a^6/x^10 - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3*b^3)/x^4
- (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

Maple [A]
time = 0.02, size = 68, normalized size = 0.88

method	result	size
default	$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+\frac{b^6x^2}{2}+6a\,b^5\ln(x)$	68
norman	$\frac{-\frac{1}{10}a^6+\frac{1}{2}b^6x^{12}-\frac{15}{2}a^2b^4x^8-5a^3b^3x^6-\frac{5}{2}a^4b^2x^4-\frac{3}{4}a^5b\,x^2}{x^{10}}+6a\,b^5\ln(x)$	70
risch	$\frac{b^6x^2}{2}+\frac{-\frac{15}{2}a^2b^4x^8-5a^3b^3x^6-\frac{5}{2}a^4b^2x^4-\frac{3}{4}a^5b\,x^2-\frac{1}{10}a^6}{x^{10}}+6a\,b^5\ln(x)$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^6/x^10-3/4*a^5*b/x^8-5/2*a^4*b^2/x^6-5*a^3*b^3/x^4-15/2*a^2*b^4/x^2
+1/2*b^6*x^2+6*a*b^5*ln(x)
```

Maxima [A]
time = 0.30, size = 72, normalized size = 0.94

$$\frac{1}{2}\,b^6x^2+3\,ab^5\log\left(x^2\right)-\frac{150\,a^2b^4x^8+100\,a^3b^3x^6+50\,a^4b^2x^4+15\,a^5bx^2+2\,a^6}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*b^6*x^2 + 3*a*b^5*log(x^2) - 1/20*(150*a^2*b^4*x^8 + 100*a^3*b^3*x^6 +
50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/x^10
```

Fricas [A]
time = 0.33, size = 72, normalized size = 0.94

$$\frac{10\,b^6x^{12}+120\,ab^5x^{10}\log\left(x\right)-150\,a^2b^4x^8-100\,a^3b^3x^6-50\,a^4b^2x^4-15\,a^5bx^2-2\,a^6}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^6*x^12 + 120*a*b^5*x^10*log(x) - 150*a^2*b^4*x^8 - 100*a^3*b^3*x
^6 - 50*a^4*b^2*x^4 - 15*a^5*b*x^2 - 2*a^6)/x^10
```

Sympy [A]
time = 0.21, size = 75, normalized size = 0.97

$$6ab^5\log\left(x\right)+\frac{b^6x^2}{2}+\frac{-2a^6-15a^5bx^2-50a^4b^2x^4-100a^3b^3x^6-150a^2b^4x^8}{20x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**11,x)
```

```
[Out] 6*a*b**5*log(x) + b**6*x**2/2 + (-2*a**6 - 15*a**5*b*x**2 - 50*a**4*b**2*x*
*4 - 100*a**3*b**3*x**6 - 150*a**2*b**4*x**8)/(20*x**10)
```

Giac [A]
time = 3.90, size = 81, normalized size = 1.05

$$\frac{1}{2}\,b^6x^2+3\,ab^5\log\left(x^2\right)-\frac{137\,ab^5x^{10}+150\,a^2b^4x^8+100\,a^3b^3x^6+50\,a^4b^2x^4+15\,a^5bx^2+2\,a^6}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^6*x^2 + 3*a*b^5*log(x^2) - 1/20*(137*a*b^5*x^10 + 150*a^2*b^4*x^8 + 1
00*a^3*b^3*x^6 + 50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/x^10
```

Mupad [B]
time = 4.40, size = 70, normalized size = 0.91

$$\frac{b^6\,x^2}{2}-\frac{\frac{a^6}{10}+\frac{3\,a^5\,b\,x^2}{4}+\frac{5\,a^4\,b^2\,x^4}{2}+5\,a^3\,b^3\,x^6+\frac{15\,a^2\,b^4\,x^8}{2}}{x^{10}}+6\,a\,b^5\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^11,x)
```

```
[Out] (b^6*x^2)/2 - (a^6/10 + (3*a^5*b*x^2)/4 + (5*a^4*b^2*x^4)/2 + 5*a^3*b^3*x^6
 + (15*a^2*b^4*x^8)/2)/x^10 + 6*a*b^5*log(x)
```

3.464 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{12}}\,dx$

Optimal. Leaf size=71

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

```
[Out] -1/11*a^6/x^11-2/3*a^5*b/x^9-15/7*a^4*b^2/x^7-4*a^3*b^3/x^5-5*a^2*b^4/x^3-6
*a*b^5/x+b^6*x
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {28, 276}

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

```
[Out] -1/11*a^6/x^11 - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^3*b^3)/x^5
 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{12}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{12}}\,dx}{b^6}\\
&= \frac{\int \left(b^{12}+\frac{a^6b^6}{x^{12}}+\frac{6a^5b^7}{x^{10}}+\frac{15a^4b^8}{x^8}+\frac{20a^3b^9}{x^6}+\frac{15a^2b^{10}}{x^4}+\frac{6ab^{11}}{x^2}\right)dx}{b^6}\\
&= -\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 71, normalized size = 1.00

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

```
[Out] -1/11*a^6/x^11 - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^3*b^3)/x^5
 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Maple [A]
time = 0.02, size = 66, normalized size = 0.93

method	result	size
default	$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6a\,b^5}{x}+b^6x$	66
risch	$b^6x+\frac{-6a\,b^5x^{10}-5a^2b^4x^8-4a^3b^3x^6-\frac{15}{7}a^4b^2x^4-\frac{2}{3}a^5b\,x^2-\frac{1}{11}a^6}{x^{11}}$	68
norman	$\frac{b^6x^{12}-6a\,b^5x^{10}-5a^2b^4x^8-4a^3b^3x^6-\frac{15}{7}a^4b^2x^4-\frac{2}{3}a^5b\,x^2-\frac{1}{11}a^6}{x^{11}}$	69
gosper	$-\frac{-231b^6x^{12}+1386a\,b^5x^{10}+1155a^2b^4x^8+924a^3b^3x^6+495a^4b^2x^4+154a^5b\,x^2+21a^6}{231x^{11}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*a^6/x^11-2/3*a^5*b/x^9-15/7*a^4*b^2/x^7-4*a^3*b^3/x^5-5*a^2*b^4/x^3-6
*a*b^5/x+b^6*x
```

Maxima [A]
time = 0.28, size = 68, normalized size = 0.96

$$b^6x-\frac{1386\,ab^5x^{10}+1155\,a^2b^4x^8+924\,a^3b^3x^6+495\,a^4b^2x^4+154\,a^5bx^2+21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 495*a
^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

Fricas [A]
time = 0.35, size = 70, normalized size = 0.99

$$\frac{231\,b^6x^{12}-1386\,ab^5x^{10}-1155\,a^2b^4x^8-924\,a^3b^3x^6-495\,a^4b^2x^4-154\,a^5bx^2-21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/231*(231*b^6*x^12 - 1386*a*b^5*x^10 - 1155*a^2*b^4*x^8 - 924*a^3*b^3*x^6
- 495*a^4*b^2*x^4 - 154*a^5*b*x^2 - 21*a^6)/x^11
```

Sympy [A]
time = 0.21, size = 71, normalized size = 1.00

$$b^6x + \frac{-21a^6 - 154a^5bx^2 - 495a^4b^2x^4 - 924a^3b^3x^6 - 1155a^2b^4x^8 - 1386ab^5x^{10}}{231x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**12,x)
```

```
[Out] b**6*x + (-21*a**6 - 154*a**5*b*x**2 - 495*a**4*b**2*x**4 - 924*a**3*b**3*x
**6 - 1155*a**2*b**4*x**8 - 1386*a*b**5*x**10)/(231*x**11)
```

Giac [A]
time = 4.03, size = 68, normalized size = 0.96

$$b^6x - \frac{1386\,ab^5x^{10} + 1155\,a^2b^4x^8 + 924\,a^3b^3x^6 + 495\,a^4b^2x^4 + 154\,a^5bx^2 + 21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="giac")
```

```
[Out] b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 495*a
^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

Mupad [B]
time = 4.30, size = 68, normalized size = 0.96

$$b^6\,x - \frac{\frac{a^6}{11} + \frac{2\,a^5\,b\,x^2}{3} + \frac{15\,a^4\,b^2\,x^4}{7} + 4\,a^3\,b^3\,x^6 + 5\,a^2\,b^4\,x^8 + 6\,a\,b^5\,x^{10}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^12,x)
```

```
[Out] b^6*x - (a^6/11 + (2*a^5*b*x^2)/3 + 6*a*b^5*x^10 + (15*a^4*b^2*x^4)/7 + 4*a
^3*b^3*x^6 + 5*a^2*b^4*x^8)/x^11
```

3.465 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{13}}\,dx$

Optimal. Leaf size=76

$$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

```
[Out] -1/12*a^6/x^12-3/5*a^5*b/x^10-15/8*a^4*b^2/x^8-10/3*a^3*b^3/x^6-15/4*a^2*b^
4/x^4-3*a*b^5/x^2+b^6*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

```
[Out] -1/12*a^6/x^12 - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*a^3*b^3)/(
3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{13}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^{13}}\,dx}{b^6}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(ab+b^2x)^6}{x^7}\,dx, x, x^2\right)}{2b^6}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^6b^6}{x^7}+\frac{6a^5b^7}{x^6}+\frac{15a^4b^8}{x^5}+\frac{20a^3b^9}{x^4}+\frac{15a^2b^{10}}{x^3}+\frac{6ab^{11}}{x^2}+\frac{b^{12}}{x}\right)dx, x, x^2\right)}{2b^6}$$
$$= -\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Mathematica [A]
time = 0.00, size = 76, normalized size = 1.00

$$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
[Out] -1/12*a^6/x^12 - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*a^3*b^3)/(
3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log[x]
```

Maple [A]
time = 0.02, size = 67, normalized size = 0.88

method	result	size
default	$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3a\,b^5}{x^2}+b^6\ln(x)$	67
norman	$\frac{-\frac{1}{12}a^6-3a\,b^5x^{10}-\frac{15}{4}a^2b^4x^8-\frac{10}{3}a^3b^3x^6-\frac{15}{8}a^4b^2x^4-\frac{3}{5}a^5b\,x^2}{x^{12}}+b^6\ln(x)$	69
risch	$\frac{-\frac{1}{12}a^6-3a\,b^5x^{10}-\frac{15}{4}a^2b^4x^8-\frac{10}{3}a^3b^3x^6-\frac{15}{8}a^4b^2x^4-\frac{3}{5}a^5b\,x^2}{x^{12}}+b^6\ln(x)$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x,method=_RETURNVERBOSE)
[Out] -1/12*a^6/x^12-3/5*a^5*b/x^10-15/8*a^4*b^2/x^8-10/3*a^3*b^3/x^6-15/4*a^2*b^
4/x^4-3*a*b^5/x^2+b^6*ln(x)
```

Maxima [A]
time = 0.32, size = 72, normalized size = 0.95

$$\frac{1}{2}b^6\log\left(x^2\right)-\frac{360\,ab^5x^{10}+450\,a^2b^4x^8+400\,a^3b^3x^6+225\,a^4b^2x^4+72\,a^5bx^2+10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*b^6*log(x^2) - 1/120*(360*a*b^5*x^10 + 450*a^2*b^4*x^8 + 400*a^3*b^3*x^
6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a^6)/x^12
```

Fricas [A]
time = 0.35, size = 72, normalized size = 0.95

$$\frac{120\,b^6x^{12}\log\left(x\right)-360\,ab^5x^{10}-450\,a^2b^4x^8-400\,a^3b^3x^6-225\,a^4b^2x^4-72\,a^5bx^2-10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^6*x^12*log(x) - 360*a*b^5*x^10 - 450*a^2*b^4*x^8 - 400*a^3*b^3
*x^6 - 225*a^4*b^2*x^4 - 72*a^5*b*x^2 - 10*a^6)/x^12
```

Sympy [A]
time = 0.26, size = 73, normalized size = 0.96

$$b^6\log\left(x\right)+\frac{-10a^6-72a^5bx^2-225a^4b^2x^4-400a^3b^3x^6-450a^2b^4x^8-360ab^5x^{10}}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**13,x)
```

```
[Out] b**6*log(x) + (-10*a**6 - 72*a**5*b*x**2 - 225*a**4*b**2*x**4 - 400*a**3*b*
*3*x**6 - 450*a**2*b**4*x**8 - 360*a*b**5*x**10)/(120*x**12)
```

Giac [A]
time = 5.02, size = 80, normalized size = 1.05

$$\frac{1}{2}\,b^6\log\left(x^2\right)-\frac{147\,b^6x^{12}+360\,ab^5x^{10}+450\,a^2b^4x^8+400\,a^3b^3x^6+225\,a^4b^2x^4+72\,a^5bx^2+10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*b^6*log(x^2) - 1/120*(147*b^6*x^12 + 360*a*b^5*x^10 + 450*a^2*b^4*x^8 +
 400*a^3*b^3*x^6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a^6)/x^12
```

Mupad [B]
time = 0.07, size = 69, normalized size = 0.91

$$b^6\,\ln\left(x\right)-\frac{\frac{a^6}{12}+\frac{3\,a^5\,b\,x^2}{5}+\frac{15\,a^4\,b^2\,x^4}{8}+\frac{10\,a^3\,b^3\,x^6}{3}+\frac{15\,a^2\,b^4\,x^8}{4}+3\,a\,b^5\,x^{10}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^13,x)
```

```
[Out] b^6*log(x) - (a^6/12 + (3*a^5*b*x^2)/5 + 3*a*b^5*x^10 + (15*a^4*b^2*x^4)/8
+ (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/4)/x^12
```

3.466 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{14}}\,dx$

Optimal. Leaf size=76

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

```
[Out] -1/13*a^6/x^13-6/11*a^5*b/x^11-5/3*a^4*b^2/x^9-20/7*a^3*b^3/x^7-3*a^2*b^4/x
^5-2*a*b^5/x^3-b^6/x
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

```
[Out] -1/13*a^6/x^13 - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*a^3*b^3)/(
7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{14}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^{14}}\,dx}{b^6} \\
&= \frac{\int \left(\frac{a^6b^6}{x^{14}}+\frac{6a^5b^7}{x^{12}}+\frac{15a^4b^8}{x^{10}}+\frac{20a^3b^9}{x^8}+\frac{15a^2b^{10}}{x^6}+\frac{6ab^{11}}{x^4}+\frac{b^{12}}{x^2}\right)dx}{b^6} \\
&= -\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 76, normalized size = 1.00

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

```
[Out] -1/13*a^6/x^13 - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*a^3*b^3)/(
7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Maple [A]
time = 0.02, size = 69, normalized size = 0.91

method	result	size
default	$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2a\,b^5}{x^3}-\frac{b^6}{x}$	69
norman	$\frac{-b^6x^{12}-2a\,b^5x^{10}-3a^2b^4x^8-\frac{20}{7}a^3b^3x^6-\frac{5}{3}a^4b^2x^4-\frac{6}{11}a^5b\,x^2-\frac{1}{13}a^6}{x^{13}}$	70
risch	$\frac{-b^6x^{12}-2a\,b^5x^{10}-3a^2b^4x^8-\frac{20}{7}a^3b^3x^6-\frac{5}{3}a^4b^2x^4-\frac{6}{11}a^5b\,x^2-\frac{1}{13}a^6}{x^{13}}$	70
gosper	$-\frac{3003b^6x^{12}+6006a\,b^5x^{10}+9009a^2b^4x^8+8580a^3b^3x^6+5005a^4b^2x^4+1638a^5b\,x^2+231a^6}{3003x^{13}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*a^6/x^13-6/11*a^5*b/x^11-5/3*a^4*b^2/x^9-20/7*a^3*b^3/x^7-3*a^2*b^4/x
^5-2*a*b^5/x^3-b^6/x
```

Maxima [A]
time = 0.38, size = 70, normalized size = 0.92

$$-\frac{3003\,b^6x^{12}+6006\,ab^5x^{10}+9009\,a^2b^4x^8+8580\,a^3b^3x^6+5005\,a^4b^2x^4+1638\,a^5bx^2+231\,a^6}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="maxima")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

Fricas [A]
time = 0.32, size = 70, normalized size = 0.92

$$-\frac{3003\,b^6x^{12}+6006\,ab^5x^{10}+9009\,a^2b^4x^8+8580\,a^3b^3x^6+5005\,a^4b^2x^4+1638\,a^5bx^2+231\,a^6}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="fricas")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

Sympy [A]
time = 0.24, size = 75, normalized size = 0.99

$$\frac{-231a^6 - 1638a^5bx^2 - 5005a^4b^2x^4 - 8580a^3b^3x^6 - 9009a^2b^4x^8 - 6006ab^5x^{10} - 3003b^6x^{12}}{3003x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**14,x)
```

```
[Out] (-231*a**6 - 1638*a**5*b*x**2 - 5005*a**4*b**2*x**4 - 8580*a**3*b**3*x**6 -
 9009*a**2*b**4*x**8 - 6006*a*b**5*x**10 - 3003*b**6*x**12)/(3003*x**13)
```

Giac [A]
time = 4.09, size = 70, normalized size = 0.92

$$-\frac{3003\,b^6x^{12} + 6006\,ab^5x^{10} + 9009\,a^2b^4x^8 + 8580\,a^3b^3x^6 + 5005\,a^4b^2x^4 + 1638\,a^5bx^2 + 231\,a^6}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="giac")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3*
x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

Mupad [B]
time = 0.05, size = 69, normalized size = 0.91

$$-\frac{\frac{a^6}{13} + \frac{6\,a^5\,b\,x^2}{11} + \frac{5\,a^4\,b^2\,x^4}{3} + \frac{20\,a^3\,b^3\,x^6}{7} + 3\,a^2\,b^4\,x^8 + 2\,a\,b^5\,x^{10} + b^6\,x^{12}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^14,x)
```

```
[Out] -(a^6/13 + b^6*x^12 + (6*a^5*b*x^2)/11 + 2*a*b^5*x^10 + (5*a^4*b^2*x^4)/3 +
 (20*a^3*b^3*x^6)/7 + 3*a^2*b^4*x^8)/x^13
```

3.467 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{15}}\,dx$

Optimal. Leaf size=19

$$-\frac{\left(a+bx^2\right)^7}{14ax^{14}}$$

```
[Out] -1/14*(b*x^2+a)^7/a/x^14
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 270}

$$-\frac{\left(a+bx^2\right)^7}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

```
[Out] -1/14*(a + b*x^2)^7/(a*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{15}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{15}}\,dx}{b^6}\\ &= -\frac{\left(a+bx^2\right)^7}{14ax^{14}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(19) = 38.
time = 0.01, size = 82, normalized size = 4.32

$$-\frac{a^6}{14x^{14}}-\frac{a^5b}{2x^{12}}-\frac{3a^4b^2}{2x^{10}}-\frac{5a^3b^3}{2x^8}-\frac{5a^2b^4}{2x^6}-\frac{3ab^5}{2x^4}-\frac{b^6}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

```
[Out] -1/14*a^6/x^14 - (a^5*b)/(2*x^12) - (3*a^4*b^2)/(2*x^10) - (5*a^3*b^3)/(2*x
^8) - (5*a^2*b^4)/(2*x^6) - (3*a*b^5)/(2*x^4) - b^6/(2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.
time = 0.02, size = 69, normalized size = 3.63

method	result	size
gosper	$-\frac{7b^6x^{12}+21a\,b^5x^{10}+35a^2b^4x^8+35a^3b^3x^6+21a^4b^2x^4+7a^5b\,x^2+a^6}{14x^{14}}$	69
default	$-\frac{3a\,b^5}{2x^4}-\frac{a^5b}{2x^{12}}-\frac{b^6}{2x^2}-\frac{5a^3b^3}{2x^8}-\frac{3a^4b^2}{2x^{10}}-\frac{5a^2b^4}{2x^6}-\frac{a^6}{14x^{14}}$	69
norman	$\frac{-\frac{1}{14}a^6-\frac{1}{2}a^5b\,x^2-\frac{3}{2}a^4b^2x^4-\frac{5}{2}a^3b^3x^6-\frac{5}{2}a^2b^4x^8-\frac{3}{2}a\,b^5x^{10}-\frac{1}{2}b^6x^{12}}{x^{14}}$	70
risch	$\frac{-\frac{1}{14}a^6-\frac{1}{2}a^5b\,x^2-\frac{3}{2}a^4b^2x^4-\frac{5}{2}a^3b^3x^6-\frac{5}{2}a^2b^4x^8-\frac{3}{2}a\,b^5x^{10}-\frac{1}{2}b^6x^{12}}{x^{14}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*a*b^5/x^4-1/2*a^5*b/x^12-1/2*b^6/x^2-5/2*a^3*b^3/x^8-3/2*a^4*b^2/x^10-
5/2*a^2*b^4/x^6-1/14*a^6/x^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.
time = 0.37, size = 68, normalized size = 3.58

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="maxima")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.
time = 0.36, size = 68, normalized size = 3.58

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="fricas")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(15) = 30$.
time = 0.27, size = 73, normalized size = 3.84

$$\frac{-a^6 - 7a^5bx^2 - 21a^4b^2x^4 - 35a^3b^3x^6 - 35a^2b^4x^8 - 21ab^5x^{10} - 7b^6x^{12}}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**15,x)
```

```
[Out] (-a**6 - 7*a**5*b*x**2 - 21*a**4*b**2*x**4 - 35*a**3*b**3*x**6 - 35*a**2*b*
*4*x**8 - 21*a*b**5*x**10 - 7*b**6*x**12)/(14*x**14)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.
time = 4.04, size = 68, normalized size = 3.58

$$-\frac{7\,b^6x^{12} + 21\,ab^5x^{10} + 35\,a^2b^4x^8 + 35\,a^3b^3x^6 + 21\,a^4b^2x^4 + 7\,a^5bx^2 + a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="giac")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a^
4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

Mupad [B]
time = 4.36, size = 70, normalized size = 3.68

$$-\frac{\frac{a^6}{14} + \frac{a^5\,b\,x^2}{2} + \frac{3\,a^4\,b^2\,x^4}{2} + \frac{5\,a^3\,b^3\,x^6}{2} + \frac{5\,a^2\,b^4\,x^8}{2} + \frac{3\,a\,b^5\,x^{10}}{2} + \frac{b^6\,x^{12}}{2}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^15,x)
```

```
[Out] -(a^6/14 + (b^6*x^12)/2 + (a^5*b*x^2)/2 + (3*a*b^5*x^10)/2 + (3*a^4*b^2*x^4
)/2 + (5*a^3*b^3*x^6)/2 + (5*a^2*b^4*x^8)/2)/x^14
```

3.468 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

```
[Out] -1/15*a^6/x^15-6/13*a^5*b/x^13-15/11*a^4*b^2/x^11-20/9*a^3*b^3/x^9-15/7*a^2
*b^4/x^7-6/5*a*b^5/x^5-1/3*b^6/x^3
```

Rubi [A]

time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

```
[Out] -1/15*a^6/x^15 - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (20*a^3*b^3
)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) - b^6/(3*x^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^{16}}\,dx}{b^6}\\
&= \frac{\int \left(\frac{a^6b^6}{x^{16}}+\frac{6a^5b^7}{x^{14}}+\frac{15a^4b^8}{x^{12}}+\frac{20a^3b^9}{x^{10}}+\frac{15a^2b^{10}}{x^8}+\frac{6ab^{11}}{x^6}+\frac{b^{12}}{x^4}\right)dx}{b^6}\\
&= -\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.00

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

```
[Out] -1/15*a^6/x^15 - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (20*a^3*b^3
)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) - b^6/(3*x^3)
```

Maple [A]
time = 0.04, size = 69, normalized size = 0.84

method	result	size
default	$-\frac{a^{6}}{15x^{15}}-\frac{6a^{5}b}{13x^{13}}-\frac{15a^{4}b^{2}}{11x^{11}}-\frac{20a^{3}b^{3}}{9x^{9}}-\frac{15a^{2}b^{4}}{7x^{7}}-\frac{6a\,b^{5}}{5x^{5}}-\frac{b^{6}}{3x^{3}}$	69
norman	$\frac{-\frac{1}{15}a^{6}-\frac{6}{13}a^{5}b\,x^{2}-\frac{15}{11}a^{4}b^{2}x^{4}-\frac{20}{9}a^{3}b^{3}x^{6}-\frac{15}{7}a^{2}b^{4}x^{8}-\frac{6}{5}a\,b^{5}x^{10}-\frac{1}{3}b^{6}x^{12}}{x^{15}}$	70
risch	$\frac{-\frac{1}{15}a^{6}-\frac{6}{13}a^{5}b\,x^{2}-\frac{15}{11}a^{4}b^{2}x^{4}-\frac{20}{9}a^{3}b^{3}x^{6}-\frac{15}{7}a^{2}b^{4}x^{8}-\frac{6}{5}a\,b^{5}x^{10}-\frac{1}{3}b^{6}x^{12}}{x^{15}}$	70
gosper	$-\frac{15015b^{6}x^{12}+54054a\,b^{5}x^{10}+96525a^{2}b^{4}x^{8}+100100a^{3}b^{3}x^{6}+61425a^{4}b^{2}x^{4}+20790a^{5}b\,x^{2}+3003a^{6}}{45045x^{15}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*a^6/x^15-6/13*a^5*b/x^13-15/11*a^4*b^2/x^11-20/9*a^3*b^3/x^9-15/7*a^2
*b^4/x^7-6/5*a*b^5/x^5-1/3*b^6/x^3
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.85

$$-\frac{15015\,b^6x^{12}+54054\,ab^5x^{10}+96525\,a^2b^4x^8+100100\,a^3b^3x^6+61425\,a^4b^2x^4+20790\,a^5bx^2+3003\,a^6}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

Fricas [A]
time = 0.35, size = 70, normalized size = 0.85

$$-\frac{15015\,b^6x^{12}+54054\,ab^5x^{10}+96525\,a^2b^4x^8+100100\,a^3b^3x^6+61425\,a^4b^2x^4+20790\,a^5bx^2+3003\,a^6}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

Sympy [A]
time = 0.26, size = 75, normalized size = 0.91

$$\frac{-3003a^6 - 20790a^5bx^2 - 61425a^4b^2x^4 - 100100a^3b^3x^6 - 96525a^2b^4x^8 - 54054ab^5x^{10} - 15015b^6x^{12}}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**16,x)
```

```
[Out] (-3003*a**6 - 20790*a**5*b*x**2 - 61425*a**4*b**2*x**4 - 100100*a**3*b**3*x
**6 - 96525*a**2*b**4*x**8 - 54054*a*b**5*x**10 - 15015*b**6*x**12)/(45045*
x**15)
```

Giac [A]
time = 3.73, size = 70, normalized size = 0.85

$$-\frac{15015\, b^6x^{12} + 54054\, ab^5x^{10} + 96525\, a^2b^4x^8 + 100100\, a^3b^3x^6 + 61425\, a^4b^2x^4 + 20790\, a^5bx^2 + 3003\, a^6}{45045\, x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a^
3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

Mupad [B]
time = 0.05, size = 70, normalized size = 0.85

$$-\frac{\frac{a^6}{15} + \frac{6\,a^5\,b\,x^2}{13} + \frac{15\,a^4\,b^2\,x^4}{11} + \frac{20\,a^3\,b^3\,x^6}{9} + \frac{15\,a^2\,b^4\,x^8}{7} + \frac{6\,a\,b^5\,x^{10}}{5} + \frac{b^6\,x^{12}}{3}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^16,x)
```

```
[Out] -(a^6/15 + (b^6*x^12)/3 + (6*a^5*b*x^2)/13 + (6*a*b^5*x^10)/5 + (15*a^4*b^2
*x^4)/11 + (20*a^3*b^3*x^6)/9 + (15*a^2*b^4*x^8)/7)/x^15
```

3.469 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{17}}\,dx$

Optimal. Leaf size=40

$$-\frac{(a+bx^2)^7}{16ax^{16}}+\frac{b(a+bx^2)^7}{112a^2x^{14}}$$

```
[Out] -1/16*(b*x^2+a)^7/a/x^16+1/112*b*(b*x^2+a)^7/a^2/x^14
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 272, 47, 37}

$$\frac{b(a+bx^2)^7}{112a^2x^{14}}-\frac{(a+bx^2)^7}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

```
[Out] -1/16*(a + b*x^2)^7/(a*x^16) + (b*(a + b*x^2)^7)/(112*a^2*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{17}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^{17}}\,dx}{b^6}\\ &= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^9}\,dx, x, x^2\right)}{2b^6}\\ &= -\frac{(a+bx^2)^7}{16ax^{16}} - \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^8}\,dx, x, x^2\right)}{16ab^5}\\ &= -\frac{(a+bx^2)^7}{16ax^{16}} + \frac{b(a+bx^2)^7}{112a^2x^{14}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 78, normalized size = 1.95

$$-\frac{a^6}{16x^{16}} - \frac{3a^5b}{7x^{14}} - \frac{5a^4b^2}{4x^{12}} - \frac{2a^3b^3}{x^{10}} - \frac{15a^2b^4}{8x^8} - \frac{ab^5}{x^6} - \frac{b^6}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

```
[Out] -1/16*a^6/x^16 - (3*a^5*b)/(7*x^14) - (5*a^4*b^2)/(4*x^12) - (2*a^3*b^3)/x^
10 - (15*a^2*b^4)/(8*x^8) - (a*b^5)/x^6 - b^6/(4*x^4)
```

Maple [A]
time = 0.02, size = 69, normalized size = 1.72

method	result	size
default	$-\frac{b^6}{4x^4}-\frac{5a^4b^2}{4x^{12}}-\frac{15a^2b^4}{8x^8}-\frac{a^6}{16x^{16}}-\frac{2a^3b^3}{x^{10}}-\frac{a\,b^5}{x^6}-\frac{3a^5b}{7x^{14}}$	69
norman	$\frac{-\frac{1}{16}a^6-\frac{3}{7}a^5b\,x^2-\frac{5}{4}a^4b^2x^4-2a^3b^3x^6-\frac{15}{8}a^2b^4x^8-a\,b^5x^{10}-\frac{1}{4}b^6x^{12}}{x^{16}}$	70
risch	$\frac{-\frac{1}{16}a^6-\frac{3}{7}a^5b\,x^2-\frac{5}{4}a^4b^2x^4-2a^3b^3x^6-\frac{15}{8}a^2b^4x^8-a\,b^5x^{10}-\frac{1}{4}b^6x^{12}}{x^{16}}$	70
gosper	$-\frac{28b^6x^{12}+112a\,b^5x^{10}+210a^2b^4x^8+224a^3b^3x^6+140a^4b^2x^4+48a^5b\,x^2+7a^6}{112x^{16}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b^6/x^4-5/4*a^4*b^2/x^12-15/8*a^2*b^4/x^8-1/16*a^6/x^16-2*a^3*b^3/x^10
-a*b^5/x^6-3/7*a^5*b/x^14
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.75

$$-\frac{28\,b^6x^{12}+112\,ab^5x^{10}+210\,a^2b^4x^8+224\,a^3b^3x^6+140\,a^4b^2x^4+48\,a^5bx^2+7\,a^6}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="maxima")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

Fricas [A]
time = 0.34, size = 70, normalized size = 1.75

$$-\frac{28\,b^6x^{12}+112\,ab^5x^{10}+210\,a^2b^4x^8+224\,a^3b^3x^6+140\,a^4b^2x^4+48\,a^5bx^2+7\,a^6}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(32) = 64$.
time = 0.30, size = 75, normalized size = 1.88

$$\frac{-7a^6-48a^5bx^2-140a^4b^2x^4-224a^3b^3x^6-210a^2b^4x^8-112ab^5x^{10}-28b^6x^{12}}{112x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**17,x)
```

```
[Out] (-7*a**6 - 48*a**5*b*x**2 - 140*a**4*b**2*x**4 - 224*a**3*b**3*x**6 - 210*a
**2*b**4*x**8 - 112*a*b**5*x**10 - 28*b**6*x**12)/(112*x**16)
```

Giac [A]
time = 4.02, size = 70, normalized size = 1.75

$$-\frac{28\,b^6x^{12}+112\,ab^5x^{10}+210\,a^2b^4x^8+224\,a^3b^3x^6+140\,a^4b^2x^4+48\,a^5bx^2+7\,a^6}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="giac")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

Mupad [B]
time = 4.37, size = 69, normalized size = 1.72

$$-\frac{\frac{a^6}{16}+\frac{3\,a^5\,b\,x^2}{7}+\frac{5\,a^4\,b^2\,x^4}{4}+2\,a^3\,b^3\,x^6+\frac{15\,a^2\,b^4\,x^8}{8}+a\,b^5\,x^{10}+\frac{b^6\,x^{12}}{4}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^17,x)
```

```
[Out] -(a^6/16 + (b^6*x^12)/4 + (3*a^5*b*x^2)/7 + a*b^5*x^10 + (5*a^4*b^2*x^4)/4
+ 2*a^3*b^3*x^6 + (15*a^2*b^4*x^8)/8)/x^16
```

3.470 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{18}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

```
[Out] -1/17*a^6/x^17-2/5*a^5*b/x^15-15/13*a^4*b^2/x^13-20/11*a^3*b^3/x^11-5/3*a^2
*b^4/x^9-6/7*a*b^5/x^7-1/5*b^6/x^5
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18,x]
```

```
[Out] -1/17*a^6/x^17 - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (20*a^3*b^3)
/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) - b^6/(5*x^5)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{18}}\,dx &= \frac{\int \frac{\left(ab+b^2x^2\right)^6}{x^{18}}\,dx}{b^6}\\
&= \frac{\int \left(\frac{a^6b^6}{x^{18}}+\frac{6a^5b^7}{x^{16}}+\frac{15a^4b^8}{x^{14}}+\frac{20a^3b^9}{x^{12}}+\frac{15a^2b^{10}}{x^{10}}+\frac{6ab^{11}}{x^8}+\frac{b^{12}}{x^6}\right)dx}{b^6}\\
&= -\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.00

$$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18,x]
```

```
[Out] -1/17*a^6/x^17 - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (20*a^3*b^3)
/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) - b^6/(5*x^5)
```

Maple [A]
time = 0.02, size = 69, normalized size = 0.84

method	result	size
default	$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$	69
norman	$\frac{-\frac{1}{17}a^6-\frac{2}{5}a^5bx^2-\frac{15}{13}a^4b^2x^4-\frac{20}{11}a^3b^3x^6-\frac{5}{3}a^2b^4x^8-\frac{6}{7}ab^5x^{10}-\frac{1}{5}b^6x^{12}}{x^{17}}$	70
risch	$\frac{-\frac{1}{17}a^6-\frac{2}{5}a^5bx^2-\frac{15}{13}a^4b^2x^4-\frac{20}{11}a^3b^3x^6-\frac{5}{3}a^2b^4x^8-\frac{6}{7}ab^5x^{10}-\frac{1}{5}b^6x^{12}}{x^{17}}$	70
gosper	$-\frac{51051b^6x^{12}+218790ab^5x^{10}+425425a^2b^4x^8+464100a^3b^3x^6+294525a^4b^2x^4+102102a^5bx^2+15015a^6}{255255x^{17}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x,method=_RETURNVERBOSE)
```

```
[Out] -1/17*a^6/x^17-2/5*a^5*b/x^15-15/13*a^4*b^2/x^13-20/11*a^3*b^3/x^11-5/3*a^2
*b^4/x^9-6/7*a*b^5/x^7-1/5*b^6/x^5
```

Maxima [A]
time = 0.29, size = 70, normalized size = 0.85

$$-\frac{51051\,b^6x^{12}+218790\,ab^5x^{10}+425425\,a^2b^4x^8+464100\,a^3b^3x^6+294525\,a^4b^2x^4+102102\,a^5bx^2+15015\,a^6}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="maxima")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

Fricas [A]
time = 0.35, size = 70, normalized size = 0.85

$$-\frac{51051\,b^6x^{12}+218790\,ab^5x^{10}+425425\,a^2b^4x^8+464100\,a^3b^3x^6+294525\,a^4b^2x^4+102102\,a^5bx^2+15015\,a^6}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="fricas")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

Sympy [A]
time = 0.28, size = 75, normalized size = 0.91

$$\frac{-15015a^6-102102a^5bx^2-294525a^4b^2x^4-464100a^3b^3x^6-425425a^2b^4x^8-218790ab^5x^{10}-51051b^6x^{12}}{255255x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**18,x)
```

```
[Out] (-15015*a**6 - 102102*a**5*b*x**2 - 294525*a**4*b**2*x**4 - 464100*a**3*b**
3*x**6 - 425425*a**2*b**4*x**8 - 218790*a*b**5*x**10 - 51051*b**6*x**12)/(2
55255*x**17)
```

Giac [A]
time = 4.49, size = 70, normalized size = 0.85

$$-\frac{51051\,b^6x^{12}+218790\,ab^5x^{10}+425425\,a^2b^4x^8+464100\,a^3b^3x^6+294525\,a^4b^2x^4+102102\,a^5bx^2+15015\,a^6}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x, algorithm="giac")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^8 + 464100
*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 + 15015*a^6)/x^17
```

Mupad [B]
time = 0.05, size = 70, normalized size = 0.85

$$-\frac{\frac{a^6}{17}+\frac{2\,a^5\,b\,x^2}{5}+\frac{15\,a^4\,b^2\,x^4}{13}+\frac{20\,a^3\,b^3\,x^6}{11}+\frac{5\,a^2\,b^4\,x^8}{3}+\frac{6\,a\,b^5\,x^{10}}{7}+\frac{b^6\,x^{12}}{5}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^18,x)
```

```
[Out] -(a^6/17 + (b^6*x^12)/5 + (2*a^5*b*x^2)/5 + (6*a*b^5*x^10)/7 + (15*a^4*b^2*
x^4)/13 + (20*a^3*b^3*x^6)/11 + (5*a^2*b^4*x^8)/3)/x^17
```

3.471 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx$

Optimal. Leaf size=62

$$-\frac{(a+bx^2)^7}{18ax^{18}}+\frac{b(a+bx^2)^7}{72a^2x^{16}}-\frac{b^2(a+bx^2)^7}{504a^3x^{14}}$$

```
[Out] -1/18*(b*x^2+a)^7/a/x^18+1/72*b*(b*x^2+a)^7/a^2/x^16-1/504*b^2*(b*x^2+a)^7/
a^3/x^14
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 272, 47, 37}

$$-\frac{b^2(a+bx^2)^7}{504a^3x^{14}}+\frac{b(a+bx^2)^7}{72a^2x^{16}}-\frac{(a+bx^2)^7}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

```
[Out] -1/18*(a + b*x^2)^7/(a*x^18) + (b*(a + b*x^2)^7)/(72*a^2*x^16) - (b^2*(a +
b*x^2)^7)/(504*a^3*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{x^{19}}\,dx}{b^6}$$
$$= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^{10}}\,dx, x, x^2\right)}{2b^6}$$
$$= -\frac{(a+bx^2)^7}{18ax^{18}} - \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^9}\,dx, x, x^2\right)}{9ab^5}$$
$$= -\frac{(a+bx^2)^7}{18ax^{18}} + \frac{b(a+bx^2)^7}{72a^2x^{16}} + \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^8}\,dx, x, x^2\right)}{72a^2b^4}$$
$$= -\frac{(a+bx^2)^7}{18ax^{18}} + \frac{b(a+bx^2)^7}{72a^2x^{16}} - \frac{b^2(a+bx^2)^7}{504a^3x^{14}}$$

Mathematica [A]
time = 0.00, size = 82, normalized size = 1.32

$$-\frac{a^6}{18x^{18}} - \frac{3a^5b}{8x^{16}} - \frac{15a^4b^2}{14x^{14}} - \frac{5a^3b^3}{3x^{12}} - \frac{3a^2b^4}{2x^{10}} - \frac{3ab^5}{4x^8} - \frac{b^6}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

```
[Out] -1/18*a^6/x^18 - (3*a^5*b)/(8*x^16) - (15*a^4*b^2)/(14*x^14) - (5*a^3*b^3)/
(3*x^12) - (3*a^2*b^4)/(2*x^10) - (3*a*b^5)/(4*x^8) - b^6/(6*x^6)
```

Maple [A]
time = 0.02, size = 69, normalized size = 1.11

method	result	size
default	$-\frac{5a^3b^3}{3x^{12}} - \frac{3a\,b^5}{4x^8} - \frac{3a^5b}{8x^{16}} - \frac{3a^2b^4}{2x^{10}} - \frac{b^6}{6x^6} - \frac{15a^4b^2}{14x^{14}} - \frac{a^6}{18x^{18}}$	69
norman	$\frac{-\frac{1}{18}a^6 - \frac{3}{8}a^5b\,x^2 - \frac{15}{14}a^4b^2x^4 - \frac{5}{3}a^3b^3x^6 - \frac{3}{2}a^2b^4x^8 - \frac{3}{4}a\,b^5x^{10} - \frac{1}{6}b^6x^{12}}{x^{18}}$	70
risch	$\frac{-\frac{1}{18}a^6 - \frac{3}{8}a^5b\,x^2 - \frac{15}{14}a^4b^2x^4 - \frac{5}{3}a^3b^3x^6 - \frac{3}{2}a^2b^4x^8 - \frac{3}{4}a\,b^5x^{10} - \frac{1}{6}b^6x^{12}}{x^{18}}$	70

gosper	$-\frac{84b^6x^{12}+378a\,b^5x^{10}+756a^2b^4x^8+840a^3b^3x^6+540a^4b^2x^4+189a^5b\,x^2+28a^6}{504x^{18}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x,method=_RETURNVERBOSE)
```

```
[Out] -5/3*a^3*b^3/x^12-3/4*a*b^5/x^8-3/8*a^5*b/x^16-3/2*a^2*b^4/x^10-1/6*b^6/x^6
-15/14*a^4*b^2/x^14-1/18*a^6/x^18
```

Maxima [A]
time = 0.29, size = 70, normalized size = 1.13

$$-\frac{84\,b^6x^{12}+378\,ab^5x^{10}+756\,a^2b^4x^8+840\,a^3b^3x^6+540\,a^4b^2x^4+189\,a^5bx^2+28\,a^6}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

Fricas [A]
time = 0.32, size = 70, normalized size = 1.13

$$-\frac{84\,b^6x^{12}+378\,ab^5x^{10}+756\,a^2b^4x^8+840\,a^3b^3x^6+540\,a^4b^2x^4+189\,a^5bx^2+28\,a^6}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

Sympy [A]
time = 0.31, size = 75, normalized size = 1.21

$$\frac{-28a^6-189a^5bx^2-540a^4b^2x^4-840a^3b^3x^6-756a^2b^4x^8-378ab^5x^{10}-84b^6x^{12}}{504x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**19,x)
```

```
[Out] (-28*a**6 - 189*a**5*b*x**2 - 540*a**4*b**2*x**4 - 840*a**3*b**3*x**6 - 756
*a**2*b**4*x**8 - 378*a*b**5*x**10 - 84*b**6*x**12)/(504*x**18)
```

Giac [A]
time = 4.67, size = 70, normalized size = 1.13

$$-\frac{84\,b^6x^{12}+378\,ab^5x^{10}+756\,a^2b^4x^8+840\,a^3b^3x^6+540\,a^4b^2x^4+189\,a^5bx^2+28\,a^6}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

Mupad [B]
time = 4.31, size = 70, normalized size = 1.13

$$-\frac{\frac{a^6}{18}+\frac{3\,a^5\,b\,x^2}{8}+\frac{15\,a^4\,b^2\,x^4}{14}+\frac{5\,a^3\,b^3\,x^6}{3}+\frac{3\,a^2\,b^4\,x^8}{2}+\frac{3\,a\,b^5\,x^{10}}{4}+\frac{b^6\,x^{12}}{6}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^19,x)
```

```
[Out] -(a^6/18 + (b^6*x^12)/6 + (3*a^5*b*x^2)/8 + (3*a*b^5*x^10)/4 + (15*a^4*b^2*
x^4)/14 + (5*a^3*b^3*x^6)/3 + (3*a^2*b^4*x^8)/2)/x^18
```

3.472 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{20}}\,dx$

Optimal. Leaf size=80

$$-\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

```
[Out] -1/19*a^6/x^19-6/17*a^5*b/x^17-a^4*b^2/x^15-20/13*a^3*b^3/x^13-15/11*a^2*b^
4/x^11-2/3*a*b^5/x^9-1/7*b^6/x^7
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 276}

$$-\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20,x]
```

```
[Out] -1/19*a^6/x^19 - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b^3)/(13*x^
13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/(7*x^7)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{20}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^{20}}\,dx}{b^6}\\
&= \frac{\int \left(\frac{a^6b^6}{x^{20}}+\frac{6a^5b^7}{x^{18}}+\frac{15a^4b^8}{x^{16}}+\frac{20a^3b^9}{x^{14}}+\frac{15a^2b^{10}}{x^{12}}+\frac{6ab^{11}}{x^{10}}+\frac{b^{12}}{x^8}\right)dx}{b^6}\\
&= -\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 80, normalized size = 1.00

$$-\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20,x]
```

```
[Out] -1/19*a^6/x^19 - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b^3)/(13*x^
13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/(7*x^7)
```

Maple [A]
time = 0.02, size = 69, normalized size = 0.86

method	result	size
default	$-\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2a\,b^5}{3x^9}-\frac{b^6}{7x^7}$	69
norman	$\frac{-\frac{1}{19}a^6-\frac{6}{17}a^5b\,x^2-a^4b^2x^4-\frac{20}{13}a^3b^3x^6-\frac{15}{11}a^2b^4x^8-\frac{2}{3}a\,b^5x^{10}-\frac{1}{7}b^6x^{12}}{x^{19}}$	70
risch	$\frac{-\frac{1}{19}a^6-\frac{6}{17}a^5b\,x^2-a^4b^2x^4-\frac{20}{13}a^3b^3x^6-\frac{15}{11}a^2b^4x^8-\frac{2}{3}a\,b^5x^{10}-\frac{1}{7}b^6x^{12}}{x^{19}}$	70
gosper	$-\frac{138567b^6x^{12}+646646a\,b^5x^{10}+1322685a^2b^4x^8+1492260a^3b^3x^6+969969a^4b^2x^4+342342a^5b\,x^2+51051a^6}{969969x^{19}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x,method=_RETURNVERBOSE)
```

```
[Out] -1/19*a^6/x^19-6/17*a^5*b/x^17-a^4*b^2/x^15-20/13*a^3*b^3/x^13-15/11*a^2*b^
4/x^11-2/3*a*b^5/x^9-1/7*b^6/x^7
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.88

$$-\frac{138567\,b^6x^{12}+646646\,ab^5x^{10}+1322685\,a^2b^4x^8+1492260\,a^3b^3x^6+969969\,a^4b^2x^4+342342\,a^5bx^2+51051\,a^6}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="maxima")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

Fricas [A]
time = 0.32, size = 70, normalized size = 0.88

$$-\frac{138567\,b^6x^{12}+646646\,ab^5x^{10}+1322685\,a^2b^4x^8+1492260\,a^3b^3x^6+969969\,a^4b^2x^4+342342\,a^5bx^2+51051\,a^6}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="fricas")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

Sympy [A]
time = 0.30, size = 75, normalized size = 0.94

$$\frac{-51051a^6 - 342342a^5bx^2 - 969969a^4b^2x^4 - 1492260a^3b^3x^6 - 1322685a^2b^4x^8 - 646646ab^5x^{10} - 138567b^6x^{12}}{969969x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**20,x)
```

```
[Out] (-51051*a**6 - 342342*a**5*b*x**2 - 969969*a**4*b**2*x**4 - 1492260*a**3*b*
*3*x**6 - 1322685*a**2*b**4*x**8 - 646646*a*b**5*x**10 - 138567*b**6*x**12)
/(969969*x**19)
```

Giac [A]
time = 4.25, size = 70, normalized size = 0.88

$$-\frac{138567\,b^6x^{12} + 646646\,ab^5x^{10} + 1322685\,a^2b^4x^8 + 1492260\,a^3b^3x^6 + 969969\,a^4b^2x^4 + 342342\,a^5bx^2 + 51051\,a^6}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x, algorithm="giac")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*x^8 + 1492
260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2 + 51051*a^6)/x^19
```

Mupad [B]
time = 0.05, size = 69, normalized size = 0.86

$$-\frac{\frac{a^6}{19} + \frac{6\,a^5\,b\,x^2}{17} + a^4\,b^2\,x^4 + \frac{20\,a^3\,b^3\,x^6}{13} + \frac{15\,a^2\,b^4\,x^8}{11} + \frac{2\,a\,b^5\,x^{10}}{3} + \frac{b^6\,x^{12}}{7}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^20,x)
```

```
[Out] -(a^6/19 + (b^6*x^12)/7 + (6*a^5*b*x^2)/17 + (2*a*b^5*x^10)/3 + a^4*b^2*x^4
 + (20*a^3*b^3*x^6)/13 + (15*a^2*b^4*x^8)/11)/x^19
```

3.473 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{21}} dx$

Optimal. Leaf size=84

$$-\frac{(a+bx^2)^7}{20ax^{20}}+\frac{b(a+bx^2)^7}{60a^2x^{18}}-\frac{b^2(a+bx^2)^7}{240a^3x^{16}}+\frac{b^3(a+bx^2)^7}{1680a^4x^{14}}$$

```
[Out] -1/20*(b*x^2+a)^7/a/x^20+1/60*b*(b*x^2+a)^7/a^2/x^18-1/240*b^2*(b*x^2+a)^7/
a^3/x^16+1/1680*b^3*(b*x^2+a)^7/a^4/x^14
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 272, 47, 37}

$$\frac{b^3(a+bx^2)^7}{1680a^4x^{14}}-\frac{b^2(a+bx^2)^7}{240a^3x^{16}}+\frac{b(a+bx^2)^7}{60a^2x^{18}}-\frac{(a+bx^2)^7}{20ax^{20}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

```
[Out] -1/20*(a + b*x^2)^7/(a*x^20) + (b*(a + b*x^2)^7)/(60*a^2*x^18) - (b^2*(a +
b*x^2)^7)/(240*a^3*x^16) + (b^3*(a + b*x^2)^7)/(1680*a^4*x^14)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{21}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^{21}}\,dx}{b^6} \\
&= \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^{11}}\,dx, x, x^2\right)}{2b^6} \\
&= -\frac{(a+bx^2)^7}{20ax^{20}} - \frac{3\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^{10}}\,dx, x, x^2\right)}{20ab^5} \\
&= -\frac{(a+bx^2)^7}{20ax^{20}} + \frac{b(a+bx^2)^7}{60a^2x^{18}} + \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^9}\,dx, x, x^2\right)}{30a^2b^4} \\
&= -\frac{(a+bx^2)^7}{20ax^{20}} + \frac{b(a+bx^2)^7}{60a^2x^{18}} - \frac{b^2(a+bx^2)^7}{240a^3x^{16}} - \frac{\text{Subst}\left(\int \frac{(ab+b^2x)^6}{x^8}\,dx, x, x^2\right)}{240a^3b^3} \\
&= -\frac{(a+bx^2)^7}{20ax^{20}} + \frac{b(a+bx^2)^7}{60a^2x^{18}} - \frac{b^2(a+bx^2)^7}{240a^3x^{16}} + \frac{b^3(a+bx^2)^7}{1680a^4x^{14}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 0.98

$$-\frac{a^6}{20x^{20}} - \frac{a^5b}{3x^{18}} - \frac{15a^4b^2}{16x^{16}} - \frac{10a^3b^3}{7x^{14}} - \frac{5a^2b^4}{4x^{12}} - \frac{3ab^5}{5x^{10}} - \frac{b^6}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

```
[Out] -1/20*a^6/x^20 - (a^5*b)/(3*x^18) - (15*a^4*b^2)/(16*x^16) - (10*a^3*b^3)/(
7*x^14) - (5*a^2*b^4)/(4*x^12) - (3*a*b^5)/(5*x^10) - b^6/(8*x^8)
```

Maple [A]
time = 0.03, size = 69, normalized size = 0.82

method	result	size
default	$-\frac{5a^2b^4}{4x^{12}} - \frac{a^6}{20x^{20}} - \frac{b^6}{8x^8} - \frac{15a^4b^2}{16x^{16}} - \frac{3a\,b^5}{5x^{10}} - \frac{10a^3b^3}{7x^{14}} - \frac{a^5b}{3x^{18}}$	69

norman	$\frac{-\frac{1}{20}a^6-\frac{1}{3}a^5b\,x^2-\frac{15}{16}a^4b^2x^4-\frac{10}{7}a^3b^3x^6-\frac{5}{4}a^2b^4x^8-\frac{3}{5}a\,b^5x^{10}-\frac{1}{8}b^6x^{12}}{x^{20}}$	70
risch	$\frac{-\frac{1}{20}a^6-\frac{1}{3}a^5b\,x^2-\frac{15}{16}a^4b^2x^4-\frac{10}{7}a^3b^3x^6-\frac{5}{4}a^2b^4x^8-\frac{3}{5}a\,b^5x^{10}-\frac{1}{8}b^6x^{12}}{x^{20}}$	70
gosper	$-\frac{210b^6x^{12}+1008a\,b^5x^{10}+2100a^2b^4x^8+2400a^3b^3x^6+1575a^4b^2x^4+560a^5b\,x^2+84a^6}{1680x^{20}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x,method=_RETURNVERBOSE)
```

```
[Out] -5/4*a^2*b^4/x^12-1/20*a^6/x^20-1/8*b^6/x^8-15/16*a^4*b^2/x^16-3/5*a*b^5/x^
10-10/7*a^3*b^3/x^14-1/3*a^5*b/x^18
```

Maxima [A]
time = 0.29, size = 70, normalized size = 0.83

$$-\frac{210\,b^6x^{12}+1008\,ab^5x^{10}+2100\,a^2b^4x^8+2400\,a^3b^3x^6+1575\,a^4b^2x^4+560\,a^5bx^2+84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="maxima")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

Fricas [A]
time = 0.35, size = 70, normalized size = 0.83

$$-\frac{210\,b^6x^{12}+1008\,ab^5x^{10}+2100\,a^2b^4x^8+2400\,a^3b^3x^6+1575\,a^4b^2x^4+560\,a^5bx^2+84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="fricas")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

Sympy [A]
time = 0.33, size = 75, normalized size = 0.89

$$\frac{-84a^6-560a^5bx^2-1575a^4b^2x^4-2400a^3b^3x^6-2100a^2b^4x^8-1008ab^5x^{10}-210b^6x^{12}}{1680x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**21,x)
```

```
[Out] (-84*a**6 - 560*a**5*b*x**2 - 1575*a**4*b**2*x**4 - 2400*a**3*b**3*x**6 - 2
100*a**2*b**4*x**8 - 1008*a*b**5*x**10 - 210*b**6*x**12)/(1680*x**20)
```

Giac [A]
time = 3.87, size = 70, normalized size = 0.83

$$-\frac{210\,b^6x^{12}+1008\,ab^5x^{10}+2100\,a^2b^4x^8+2400\,a^3b^3x^6+1575\,a^4b^2x^4+560\,a^5bx^2+84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="giac")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*x
^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

Mupad [B]
time = 0.05, size = 70, normalized size = 0.83

$$-\frac{\frac{a^6}{20}+\frac{a^5\,b\,x^2}{3}+\frac{15\,a^4\,b^2\,x^4}{16}+\frac{10\,a^3\,b^3\,x^6}{7}+\frac{5\,a^2\,b^4\,x^8}{4}+\frac{3\,a\,b^5\,x^{10}}{5}+\frac{b^6\,x^{12}}{8}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^21,x)
```

```
[Out] -(a^6/20 + (b^6*x^12)/8 + (a^5*b*x^2)/3 + (3*a*b^5*x^10)/5 + (15*a^4*b^2*x^
4)/16 + (10*a^3*b^3*x^6)/7 + (5*a^2*b^4*x^8)/4)/x^20
```

3.474 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{22}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

```
[Out] -1/21*a^6/x^21-6/19*a^5*b/x^19-15/17*a^4*b^2/x^17-4/3*a^3*b^3/x^15-15/13*a^
2*b^4/x^13-6/11*a*b^5/x^11-1/9*b^6/x^9
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {28, 276}

$$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22,x]
```

```
[Out] -1/21*a^6/x^21 - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (4*a^3*b^3)
/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11) - b^6/(9*x^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{22}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^6}{x^{22}}\,dx}{b^6}\\
&= \frac{\int \left(\frac{a^6b^6}{x^{22}}+\frac{6a^5b^7}{x^{20}}+\frac{15a^4b^8}{x^{18}}+\frac{20a^3b^9}{x^{16}}+\frac{15a^2b^{10}}{x^{14}}+\frac{6ab^{11}}{x^{12}}+\frac{b^{12}}{x^{10}}\right)dx}{b^6}\\
&= -\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.00

$$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22,x]
```

```
[Out] -1/21*a^6/x^21 - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (4*a^3*b^3)
/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11) - b^6/(9*x^9)
```

Maple [A]
time = 0.02, size = 69, normalized size = 0.84

method	result	size
default	$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6a\,b^5}{11x^{11}}-\frac{b^6}{9x^9}$	69
norman	$\frac{-\frac{1}{21}a^6-\frac{6}{19}a^5b\,x^2-\frac{15}{17}a^4b^2x^4-\frac{4}{3}a^3b^3x^6-\frac{15}{13}a^2b^4x^8-\frac{6}{11}a\,b^5x^{10}-\frac{1}{9}b^6x^{12}}{x^{21}}$	70
risch	$\frac{-\frac{1}{21}a^6-\frac{6}{19}a^5b\,x^2-\frac{15}{17}a^4b^2x^4-\frac{4}{3}a^3b^3x^6-\frac{15}{13}a^2b^4x^8-\frac{6}{11}a\,b^5x^{10}-\frac{1}{9}b^6x^{12}}{x^{21}}$	70
gosper	$-\frac{323323b^6x^{12}+1587222a\,b^5x^{10}+3357585a^2b^4x^8+3879876a^3b^3x^6+2567565a^4b^2x^4+918918a^5b\,x^2+138567a^6}{2909907x^{21}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21*a^6/x^21-6/19*a^5*b/x^19-15/17*a^4*b^2/x^17-4/3*a^3*b^3/x^15-15/13*a^
2*b^4/x^13-6/11*a*b^5/x^11-1/9*b^6/x^9
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.85

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="maxima")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

Fricas [A]
time = 0.34, size = 70, normalized size = 0.85

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="fricas")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

Sympy [A]
time = 0.31, size = 75, normalized size = 0.91

$$\frac{-138567a^6 - 918918a^5bx^2 - 2567565a^4b^2x^4 - 3879876a^3b^3x^6 - 3357585a^2b^4x^8 - 1587222ab^5x^{10} - 323323b^6x^{12}}{2909907x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**22,x)
```

```
[Out] (-138567*a**6 - 918918*a**5*b*x**2 - 2567565*a**4*b**2*x**4 - 3879876*a**3*
b**3*x**6 - 3357585*a**2*b**4*x**8 - 1587222*a*b**5*x**10 - 323323*b**6*x**
12)/(2909907*x**21)
```

Giac [A]
time = 3.88, size = 70, normalized size = 0.85

$$-\frac{323323\,b^6x^{12} + 1587222\,ab^5x^{10} + 3357585\,a^2b^4x^8 + 3879876\,a^3b^3x^6 + 2567565\,a^4b^2x^4 + 918918\,a^5bx^2 + 138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x, algorithm="giac")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^4*x^8 + 38
79876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*x^2 + 138567*a^6)/x^
21
```

Mupad [B]
time = 0.05, size = 70, normalized size = 0.85

$$-\frac{\frac{a^6}{21}+\frac{6\,a^5\,b\,x^2}{19}+\frac{15\,a^4\,b^2\,x^4}{17}+\frac{4\,a^3\,b^3\,x^6}{3}+\frac{15\,a^2\,b^4\,x^8}{13}+\frac{6\,a\,b^5\,x^{10}}{11}+\frac{b^6\,x^{12}}{9}}{x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^22,x)
```

```
[Out] -(a^6/21 + (b^6*x^12)/9 + (6*a^5*b*x^2)/19 + (6*a*b^5*x^10)/11 + (15*a^4*b^
2*x^4)/17 + (4*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/13)/x^21
```

3.475 $\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=83

$$-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}$$

```
[Out] -2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^2
+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 45}

$$\frac{a^5}{2b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^
5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^{11}}{(ab+b^2x^2)^2}\,dx$$
$$= \frac{1}{2}b^2\text{Subst}\left(\int \frac{x^5}{(ab+b^2x)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\text{Subst}\left(\int \left(-\frac{4a^3}{b^7}+\frac{3a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{b^4}-\frac{a^5}{b^7(a+bx)^2}+\frac{5a^4}{b^7(a+bx)}\right)dx, x,$$
$$= -\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6(a+bx^2)}+\frac{5a^4\log(a+bx^2)}{2b^6}$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 0.87

$$\frac{-48a^3bx^2+18a^2b^2x^4-8ab^3x^6+3b^4x^8+\frac{12a^5}{a+bx^2}+60a^4\log(a+bx^2)}{24b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a^5)/(a + b
*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)
```

Maple [A]
time = 0.03, size = 75, normalized size = 0.90

method	result	size
risch	$-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{a\,x^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6(b\,x^2+a)}+\frac{5a^4\ln(b\,x^2+a)}{2b^6}$	74
default	$-\frac{-\frac{1}{4}b^3x^8+\frac{2}{3}a\,b^2x^6-\frac{3}{2}a^2b\,x^4+4a^3x^2}{2b^5}+\frac{a^5}{2b^6(b\,x^2+a)}+\frac{5a^4\ln(b\,x^2+a)}{2b^6}$	75
norman	$\frac{\frac{x^{10}}{8b}-\frac{5a\,x^8}{24b^2}+\frac{5a^5}{2b^6}+\frac{5a^2x^6}{12b^3}-\frac{5a^3x^4}{4b^4}}{b\,x^2+a}+\frac{5a^4\ln(b\,x^2+a)}{2b^6}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^5*(-1/4*b^3*x^8+2/3*a*b^2*x^6-3/2*a^2*b*x^4+4*a^3*x^2)+1/2*a^5/b^6/(
b*x^2+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Maxima [A]
time = 0.28, size = 77, normalized size = 0.93

$$\frac{a^5}{2(b^7x^2+ab^6)}+\frac{5a^4\log(bx^2+a)}{2b^6}+\frac{3b^3x^8-8ab^2x^6+18a^2bx^4-48a^3x^2}{24b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*log(b*x^2 + a)/b^6 + 1/24*(3*b^3*x^8 -
8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5
```

Fricas [A]
time = 0.34, size = 93, normalized size = 1.12

$$\frac{3\,b^5x^{10}-5\,ab^4x^8+10\,a^2b^3x^6-30\,a^3b^2x^4-48\,a^4bx^2+12\,a^5+60\,(a^4bx^2+a^5)\log{(bx^2+a)}}{24\,(b^7x^2+ab^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4 - 48*a^4*b
*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

Sympy [A]
time = 0.13, size = 80, normalized size = 0.96

$$\frac{a^5}{2ab^6+2b^7x^2}+\frac{5a^4\log{(a+bx^2)}}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*a**3*x*
*2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)
```

Giac [A]
time = 5.36, size = 92, normalized size = 1.11

$$\frac{5\,a^4\log{(|bx^2+a|)}}{2\,b^6}-\frac{5\,a^4bx^2+4\,a^5}{2\,(bx^2+a)b^6}+\frac{3\,b^6x^8-8\,ab^5x^6+18\,a^2b^4x^4-48\,a^3b^3x^2}{24\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 5/2*a^4*log(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^2 + a)*b^
6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48*a^3*b^3*x^2)/b^8
```

Mupad [B]
time = 4.36, size = 79, normalized size = 0.95

$$\frac{x^8}{8\,b^2}+\frac{a^5}{2\,b\,\left(b^6\,x^2+a\,b^5\right)}-\frac{a\,x^6}{3\,b^3}+\frac{5\,a^4\,\ln{(b\,x^2+a)}}{2\,b^6}+\frac{3\,a^2\,x^4}{4\,b^4}-\frac{2\,a^3\,x^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^8/(8*b^2) + a^5/(2*b*(a*b^5 + b^6*x^2)) - (a*x^6)/(3*b^3) + (5*a^4*log(a
+ b*x^2))/(2*b^6) + (3*a^2*x^4)/(4*b^4) - (2*a^3*x^2)/b^5
```

3.476 $\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=70

$$\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}$$

```
[Out] 3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b*
x^2+a)/b^5
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 45}

$$-\frac{a^4}{2b^5\left(a+bx^2\right)}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2
)) - (2*a^3*Log[a + b*x^2])/b^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^9}{(ab+b^2x^2)^2}\,dx$$
$$= \frac{1}{2}b^2\text{Subst}\left(\int \frac{x^4}{(ab+b^2x)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\text{Subst}\left(\int \left(\frac{3a^2}{b^6}-\frac{2ax}{b^5}+\frac{x^2}{b^4}+\frac{a^4}{b^6(a+bx)^2}-\frac{4a^3}{b^6(a+bx)}\right)dx, x, x^2\right)$$
$$= \frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}-\frac{a^4}{2b^5(a+bx^2)}-\frac{2a^3\log(a+bx^2)}{b^5}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 0.86

$$\frac{9a^2bx^2-3ab^2x^4+b^3x^6-\frac{3a^4}{a+bx^2}-12a^3\log(a+bx^2)}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a^3*Log[a +
 b*x^2])/(6*b^5)
```

Maple [A]
time = 0.02, size = 64, normalized size = 0.91

method	result	size
risch	$\frac{3a^2x^2}{2b^4}-\frac{a\,x^4}{2b^3}+\frac{x^6}{6b^2}-\frac{a^4}{2b^5(b\,x^2+a)}-\frac{2a^3\ln(b\,x^2+a)}{b^5}$	63
default	$\frac{\frac{1}{3}b^2x^6-ab\,x^4+3a^2x^2}{2b^4}-\frac{a^4}{2b^5(b\,x^2+a)}-\frac{2a^3\ln(b\,x^2+a)}{b^5}$	64
norman	$\frac{\frac{a^2x^4}{b^3}+\frac{x^8}{6b}-\frac{a\,x^6}{3b^2}-\frac{2a^4}{b^5}}{b\,x^2+a}-\frac{2a^3\ln(b\,x^2+a)}{b^5}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^4*(1/3*b^2*x^6-a*b*x^4+3*a^2*x^2)-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b*x^
2+a)/b^5
```

Maxima [A]
time = 0.27, size = 65, normalized size = 0.93

$$-\frac{a^4}{2(b^6x^2+ab^5)}-\frac{2\,a^3\log(bx^2+a)}{b^5}+\frac{b^2x^6-3\,abx^4+9\,a^2x^2}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^4/(b^6*x^2 + a*b^5) - 2*a^3*log(b*x^2 + a)/b^5 + 1/6*(b^2*x^6 - 3*a*
b*x^4 + 9*a^2*x^2)/b^4
```

Fricas [A]
time = 0.34, size = 81, normalized size = 1.16

$$\frac{b^4x^8 - 2\,ab^3x^6 + 6\,a^2b^2x^4 + 9\,a^3bx^2 - 3\,a^4 - 12\,(a^3bx^2 + a^4)\log\left(bx^2 + a\right)}{6\,(b^6x^2 + ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 - 2*a*b^3*x^6 + 6*a^2*b^2*x^4 + 9*a^3*b*x^2 - 3*a^4 - 12*(a^3*
b*x^2 + a^4)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

Sympy [A]
time = 0.12, size = 66, normalized size = 0.94

$$-\frac{a^4}{2ab^5 + 2b^6x^2} - \frac{2a^3\log\left(a + bx^2\right)}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**2*x**2/
(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)
```

Giac [A]
time = 4.42, size = 80, normalized size = 1.14

$$-\frac{2\,a^3\log\left(|bx^2 + a|\right)}{b^5} + \frac{b^4x^6 - 3\,ab^3x^4 + 9\,a^2b^2x^2}{6\,b^6} + \frac{4\,a^3bx^2 + 3\,a^4}{2\,(bx^2 + a)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(b*x^2 + a))/b^5 + 1/6*(b^4*x^6 - 3*a*b^3*x^4 + 9*a^2*b^2*x^2
)/b^6 + 1/2*(4*a^3*b*x^2 + 3*a^4)/((b*x^2 + a)*b^5)
```

Mupad [B]
time = 0.04, size = 68, normalized size = 0.97

$$\frac{x^6}{6\,b^2} - \frac{a^4}{2\,b\,\left(b^5\,x^2 + a\,b^4\right)} - \frac{a\,x^4}{2\,b^3} - \frac{2\,a^3\,\ln\left(b\,x^2 + a\right)}{b^5} + \frac{3\,a^2\,x^2}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^6/(6*b^2) - a^4/(2*b*(a*b^4 + b^5*x^2)) - (a*x^4)/(2*b^3) - (2*a^3*log(a
+ b*x^2))/b^5 + (3*a^2*x^2)/(2*b^4)
```

3.477 $\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=57

$$-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}+\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}$$

```
[Out] -a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 45}

$$\frac{a^3}{2b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/(2*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^7}{(ab+b^2x^2)^2}\,dx\\
&= \frac{1}{2}b^2\text{Subst}\left(\int \frac{x^3}{(ab+b^2x)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}b^2\text{Subst}\left(\int \left(-\frac{2a}{b^5}+\frac{x}{b^4}-\frac{a^3}{b^5(a+bx)^2}+\frac{3a^2}{b^5(a+bx)}\right)\,dx,x,x^2\right)\\
&= -\frac{ax^2}{b^3}+\frac{x^4}{4b^2}+\frac{a^3}{2b^4(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{2b^4}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.86

$$\frac{-4abx^2+b^2x^4+\frac{2a^3}{a+bx^2}+6a^2\log(a+bx^2)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (-4*a*b*x^2 + b^2*x^4 + (2*a^3)/(a + b*x^2) + 6*a^2*Log[a + b*x^2])/(4*b^4)
```

Maple [A]
time = 0.03, size = 53, normalized size = 0.93

method	result	size
default	$-\frac{-\frac{1}{2}b\,x^4+2a\,x^2}{2b^3}+\frac{a^3}{2b^4(b\,x^2+a)}+\frac{3a^2\ln(b\,x^2+a)}{2b^4}$	53
norman	$\frac{\frac{x^6}{4b}-\frac{3a\,x^4}{4b^2}+\frac{3a^3}{2b^4}}{b\,x^2+a}+\frac{3a^2\ln(b\,x^2+a)}{2b^4}$	54
risch	$\frac{x^4}{4b^2}-\frac{a\,x^2}{b^3}+\frac{a^2}{b^4}+\frac{a^3}{2b^4(b\,x^2+a)}+\frac{3a^2\ln(b\,x^2+a)}{2b^4}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
[Out] -1/2/b^3*(-1/2*b*x^4+2*a*x^2)+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

Maxima [A]
time = 0.28, size = 54, normalized size = 0.95

$$\frac{a^3}{2(b^5x^2+ab^4)}+\frac{3\,a^2\log(bx^2+a)}{2\,b^4}+\frac{bx^4-4\,ax^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^3/(b^5*x^2 + a*b^4) + 3/2*a^2*log(b*x^2 + a)/b^4 + 1/4*(b*x^4 - 4*a*x
^2)/b^3
```

Fricas [A]
time = 0.36, size = 70, normalized size = 1.23

$$\frac{b^3x^6 - 3\,ab^2x^4 - 4\,a^2bx^2 + 2\,a^3 + 6\,(a^2bx^2 + a^3)\log\left(bx^2 + a\right)}{4\,(b^5x^2 + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 - 3*a*b^2*x^4 - 4*a^2*b*x^2 + 2*a^3 + 6*(a^2*b*x^2 + a^3)*log(
b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A]
time = 0.11, size = 53, normalized size = 0.93

$$\frac{a^3}{2ab^4 + 2b^5x^2} + \frac{3a^2\log\left(a + bx^2\right)}{2b^4} - \frac{ax^2}{b^3} + \frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**3/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*log(a + b*x**2)/(2*b**4) - a*x**2/b*
*3 + x**4/(4*b**2)
```

Giac [A]
time = 5.23, size = 67, normalized size = 1.18

$$\frac{3\,a^2\log\left(|bx^2 + a|\right)}{2\,b^4} + \frac{b^2x^4 - 4\,abx^2}{4\,b^4} - \frac{3\,a^2bx^2 + 2\,a^3}{2\,(bx^2 + a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 3/2*a^2*log(abs(b*x^2 + a))/b^4 + 1/4*(b^2*x^4 - 4*a*b*x^2)/b^4 - 1/2*(3*a^
2*b*x^2 + 2*a^3)/((b*x^2 + a)*b^4)
```

Mupad [B]
time = 0.05, size = 57, normalized size = 1.00

$$\frac{x^4}{4\,b^2} + \frac{a^3}{2\,b\,\left(b^4\,x^2 + a\,b^3\right)} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,\ln\left(b\,x^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^4/(4*b^2) + a^3/(2*b*(a*b^3 + b^4*x^2)) - (a*x^2)/b^3 + (3*a^2*log(a + b*
x^2))/(2*b^4)
```

3.478 $\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=44

$$\frac{x^2}{2b^2}-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}$$

```
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^5}{(ab+b^2x^2)^2}\,dx\\ &= \frac{1}{2}b^2\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}b^2\text{Subst}\left(\int \left(\frac{1}{b^4}+\frac{a^2}{b^4(a+bx)^2}-\frac{2a}{b^4(a+bx)}\right)\,dx,x,x^2\right)\\ &= \frac{x^2}{2b^2}-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.86

$$\frac{bx^2-\frac{a^2}{a+bx^2}-2a\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)
```

Maple [A]
time = 0.02, size = 41, normalized size = 0.93

method	result	size
default	$\frac{x^2}{2b^2}-\frac{a^2}{2b^3(b\,x^2+a)}-\frac{a\ln(b\,x^2+a)}{b^3}$	41
risch	$\frac{x^2}{2b^2}-\frac{a^2}{2b^3(b\,x^2+a)}-\frac{a\ln(b\,x^2+a)}{b^3}$	41
norman	$\frac{\frac{x^4}{2b}-\frac{a^2}{b^3}}{b\,x^2+a}-\frac{a\ln(b\,x^2+a)}{b^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.98

$$-\frac{a^2}{2\left(b^4x^2+ab^3\right)}+\frac{x^2}{2\,b^2}-\frac{a\log\left(bx^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*log(b*x^2 + a)/b^3
```

Fricas [A]
time = 0.35, size = 56, normalized size = 1.27

$$\frac{b^2x^4 + abx^2 - a^2 - 2\,(abx^2 + a^2)\log\left(bx^2 + a\right)}{2\,(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*log(b*x^2 + a))/(b^4*x^2 +
 a*b^3)
```

Sympy [A]
time = 0.09, size = 39, normalized size = 0.89

$$-\frac{a^2}{2ab^3 + 2b^4x^2} - \frac{a\log\left(a + bx^2\right)}{b^3} + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**2/(2*a*b**3 + 2*b**4*x**2) - a*log(a + b*x**2)/b**3 + x**2/(2*b**2)
```

Giac [A]
time = 6.37, size = 49, normalized size = 1.11

$$\frac{x^2}{2\,b^2} - \frac{a\log\left(|bx^2 + a|\right)}{b^3} + \frac{2\,abx^2 + a^2}{2\,(bx^2 + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^2 - a*log(abs(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((b*x^2 + a
)*b^3)
```

Mupad [B]
time = 0.05, size = 45, normalized size = 1.02

$$\frac{x^2}{2\,b^2} - \frac{a^2}{2\,\left(b^4\,x^2 + a\,b^3\right)} - \frac{a\,\ln\left(b\,x^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^2/(2*b^2) - a^2/(2*(a*b^3 + b^4*x^2)) - (a*log(a + b*x^2))/b^3
```

3.479 $\int \frac{x^3}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=33

$$\frac{a}{2b^2(a+bx^2)}+\frac{\log(a+bx^2)}{2b^2}$$

[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$\frac{a}{2b^2(a+bx^2)}+\frac{\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]

[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^3}{(ab+b^2x^2)^2}\,dx$$
$$= \frac{1}{2}b^2\text{Subst}\left(\int \frac{x}{(ab+b^2x)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\text{Subst}\left(\int \left(-\frac{a}{b^3(a+bx)^2}+\frac{1}{b^3(a+bx)}\right)\,dx, x, x^2\right)$$
$$= \frac{a}{2b^2(a+bx^2)}+\frac{\log(a+bx^2)}{2b^2}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^2}+\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a/(a + b*x^2) + Log[a + b*x^2])/(2*b^2)
```

Maple [A]
time = 0.04, size = 30, normalized size = 0.91

method	result	size
default	$\frac{a}{2b^2(bx^2+a)}+\frac{\ln(bx^2+a)}{2b^2}$	30
norman	$\frac{a}{2b^2(bx^2+a)}+\frac{\ln(bx^2+a)}{2b^2}$	30
risch	$\frac{a}{2b^2(bx^2+a)}+\frac{\ln(bx^2+a)}{2b^2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Maxima [A]
time = 0.29, size = 32, normalized size = 0.97

$$\frac{a}{2(b^3x^2+ab^2)}+\frac{\log(bx^2+a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] 1/2*a/(b^3*x^2 + a*b^2) + 1/2*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 0.34, size = 35, normalized size = 1.06

$$\frac{(bx^2 + a)\log(bx^2 + a) + a}{2(b^3x^2 + ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] 1/2*((b*x^2 + a)*log(b*x^2 + a) + a)/(b^3*x^2 + a*b^2)
```

Sympy [A]
time = 0.07, size = 29, normalized size = 0.88

$$\frac{a}{2ab^2 + 2b^3x^2} + \frac{\log(a + bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
[Out] a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2)
```

Giac [A]
time = 2.60, size = 30, normalized size = 0.91

$$\frac{\log(|bx^2 + a|)}{2b^2} + \frac{a}{2(bx^2 + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))/b^2 + 1/2*a/((b*x^2 + a)*b^2)
```

Mupad [B]
time = 0.05, size = 29, normalized size = 0.88

$$\frac{\ln(b\,x^2 + a)}{2\,b^2} + \frac{a}{2\,b^2\,(b\,x^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] log(a + b*x^2)/(2*b^2) + a/(2*b^2*(a + b*x^2))
```

3.480 $\int \frac{x}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\left(a+bx^2\right)}$$

[Out] -1/2/b/(b*x^2+a)

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {28, 267}

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]

[Out] -1/2*1/(b*(a + b*x^2))

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{a^2+2abx^2+b^2x^4} \, dx &= b^2 \int \frac{x}{\left(ab+b^2x^2\right)^2} \, dx \\ &= -\frac{1}{2b\left(a+bx^2\right)} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*1/(b*(a + b*x^2))
```

Maple [A]
time = 0.01, size = 15, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{2b(b\,x^{2}+a)}$	15
default	$-\frac{1}{2b(b\,x^{2}+a)}$	15
norman	$-\frac{1}{2b(b\,x^{2}+a)}$	15
risch	$-\frac{1}{2b(b\,x^{2}+a)}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/(b*x^2+a)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.94

$$-\frac{1}{2\,(b^{2}x^{2}+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

Fricas [A]
time = 0.34, size = 15, normalized size = 0.94

$$-\frac{1}{2\,(b^{2}x^{2}+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.94

$$-\frac{1}{2ab+2b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -1/(2*a*b + 2*b**2*x**2)
```

Giac [A]
time = 2.99, size = 14, normalized size = 0.88

$$-\frac{1}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

Mupad [B]
time = 4.32, size = 14, normalized size = 0.88

$$-\frac{1}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] -1/(2*b*(a + b*x^2))
```

3.481 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=38

$$\frac{1}{2a(a+bx^2)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx^2)}{2a^2}$$

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 46}

$$-\frac{\log(a+bx^2)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x\left(ab+b^2x^2\right)^2}\,dx \\
&= \frac{1}{2}b^2\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}b^2\text{Subst}\left(\int \left(\frac{1}{a^2b^2x}-\frac{1}{ab(a+bx)^2}-\frac{1}{a^2b(a+bx)}\right)dx,x,x^2\right) \\
&= \frac{1}{2a\left(a+bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2\right)}{2a^2}
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2}+2\log(x)-\log\left(a+bx^2\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

Maple [**A**]
time = 0.03, size = 42, normalized size = 1.11

method	result	size
risch	$\frac{1}{2a(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	35
norman	$-\frac{b\,x^2}{2a^2(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	39
default	$-\frac{b\left(-\frac{a}{b\left(b\,x^2+a\right)}+\frac{\ln\left(b\,x^2+a\right)}{b}\right)}{2a^2}+\frac{\ln(x)}{a^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
[Out] -1/2*b/a^2*(-a/b/(b*x^2+a)+1/b*ln(b*x^2+a))+ln(x)/a^2
```

Maxima [**A**]
time = 0.27, size = 37, normalized size = 0.97

$$\frac{1}{2\left(abx^2+a^2\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

Fricas [A]
time = 0.35, size = 47, normalized size = 1.24

$$-\frac{(bx^2+a)\log(bx^2+a)-2(bx^2+a)\log(x)-a}{2(a^2bx^2+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a
^3)
```

Sympy [A]
time = 0.13, size = 34, normalized size = 0.89

$$\frac{1}{2a^2+2abx^2}+\frac{\log(x)}{a^2}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

Giac [A]
time = 3.56, size = 47, normalized size = 1.24

$$\frac{\log(x^2)}{2a^2}-\frac{\log(|bx^2+a|)}{2a^2}+\frac{bx^2+2a}{2(bx^2+a)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
+ a)*a^2)
```

Mupad [B]
time = 4.39, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2}+\frac{1}{2a\,(b\,x^2+a)}-\frac{\ln(b\,x^2+a)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

3.482 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}$$

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 46}

$$\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/2*1/(a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*
x^2])/a^3
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)} dx = b^2 \int \frac{1}{x^3 (ab + b^2x^2)^2} dx$$
$$= \frac{1}{2} b^2 \text{Subst}\left(\int \frac{1}{x^2 (ab + b^2x)^2} dx, x, x^2 \right)$$
$$= \frac{1}{2} b^2 \text{Subst}\left(\int \left(\frac{1}{a^2b^2x^2} - \frac{2}{a^3bx} + \frac{1}{a^2(a+bx)^2} + \frac{2}{a^3(a+bx)} \right) dx, x, x^2 \right)$$
$$= -\frac{1}{2a^2x^2} - \frac{b}{2a^2 (a + bx^2)} - \frac{2b \log(x)}{a^3} + \frac{b \log (a + bx^2)}{a^3}$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.84

$$-\frac{a\left(\frac{1}{x^2} + \frac{b}{a+bx^2}\right) + 4b \log(x) - 2b \log (a + bx^2)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
[Out] -1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

Maple [A]
time = 0.03, size = 55, normalized size = 1.12

method	result	size
norman	$\frac{-\frac{b\,x^2}{a^2}-\frac{1}{2a}}{x^2(b\,x^2+a)}+\frac{b\ln(b\,x^2+a)}{a^3}-\frac{2b\ln(x)}{a^3}$	51
risch	$\frac{-\frac{b\,x^2}{a^2}-\frac{1}{2a}}{x^2(b\,x^2+a)}-\frac{2b\ln(x)}{a^3}+\frac{b\ln(-b\,x^2-a)}{a^3}$	54
default	$\frac{b^2\left(-\frac{a}{b(b\,x^2+a)}+\frac{2\ln(b\,x^2+a)}{b}\right)}{2a^3}-\frac{1}{2a^2x^2}-\frac{2b\ln(x)}{a^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
[Out] 1/2*b^2/a^3*(-a/b/(b*x^2+a)+2/b*ln(b*x^2+a))-1/2/a^2/x^2-2*b*ln(x)/a^3
```

Maxima [A]
time = 0.28, size = 52, normalized size = 1.06

$$-\frac{2\,bx^2 + a}{2\,(a^2bx^4 + a^3x^2)} + \frac{b \log (bx^2 + a)}{a^3} - \frac{b \log (x^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - b*log(x^2
)/a^3
```

Fricas [A]
time = 0.33, size = 73, normalized size = 1.49

$$-\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log\left(x\right)}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A]
time = 0.17, size = 51, normalized size = 1.04

$$\frac{-a-2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log\left(x\right)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

Giac [A]
time = 3.39, size = 51, normalized size = 1.04

$$-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bx^2+a}{2\left(bx^4+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

Mupad [B]
time = 0.08, size = 51, normalized size = 1.04

$$\frac{b\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{\frac{1}{2\,a}+\frac{b\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{2\,b\,\ln\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*log
(x))/a^3
```

3.483 $\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}$$

```
[Out] -1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*x
^2+a)/a^4
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 46}

$$-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/4*1/(a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*Log[x])/a
^4 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{x^5\left(ab+b^2x^2\right)^2}\,dx$$
$$= \frac{1}{2}b^2\text{Subst}\left(\int \frac{1}{x^3\left(ab+b^2x\right)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^2\text{Subst}\left(\int \left(\frac{1}{a^2b^2x^3}-\frac{2}{a^3bx^2}+\frac{3}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{3b}{a^4(a+bx)}\right) dx, x, x$$
$$= -\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}$$

Mathematica [**A**]
time = 0.04, size = 57, normalized size = 0.86

$$\frac{a\left(-\frac{a}{x^4}+\frac{4b}{x^2}+\frac{2b^2}{a+bx^2}\right)+12b^2\log(x)-6b^2\log\left(a+bx^2\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*Log[x] - 6*b^2*Log
[a + b*x^2])/(4*a^4)
```

Maple [**A**]
time = 0.03, size = 65, normalized size = 0.98

method	result	size
default	$-\frac{b^3\left(-\frac{a}{b\left(bx^2+a\right)}+\frac{3\ln\left(bx^2+a\right)}{b}\right)}{2a^4}-\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{3b^2\ln(x)}{a^4}$	65
norman	$\frac{-\frac{1}{4a}+\frac{3bx^2}{4a^2}-\frac{3b^3x^6}{2a^4}}{x^4\left(bx^2+a\right)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln\left(bx^2+a\right)}{2a^4}$	67
risch	$\frac{\frac{3b^2x^4}{2a^3}+\frac{3bx^2}{4a^2}-\frac{1}{4a}}{x^4\left(bx^2+a\right)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln\left(bx^2+a\right)}{2a^4}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^3/a^4*(-a/b/(b*x^2+a)+3/b*ln(b*x^2+a))-1/4/a^2/x^4+b/a^3/x^2+3*b^2*l
n(x)/a^4
```

Maxima [**A**]
time = 0.29, size = 70, normalized size = 1.06

$$\frac{6\,b^2x^4+3\,abx^2-a^2}{4\left(a^3bx^6+a^4x^4\right)}-\frac{3\,b^2\log\left(bx^2+a\right)}{2\,a^4}+\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2*log(b*x^2
 + a)/a^4 + 3/2*b^2*log(x^2)/a^4
```

Fricas [A]
time = 0.33, size = 90, normalized size = 1.36

$$\frac{6\,ab^2x^4+3\,a^2bx^2-a^3-6\,(b^3x^6+ab^2x^4)\log\left(bx^2+a\right)+12\,(b^3x^6+ab^2x^4)\log\left(x\right)}{4\,(a^4bx^6+a^5x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*log(b*x^2 +
a) + 12*(b^3*x^6 + a*b^2*x^4)*log(x))/(a^4*b*x^6 + a^5*x^4)
```

Sympy [A]
time = 0.20, size = 68, normalized size = 1.03

$$\frac{-a^2+3abx^2+6b^2x^4}{4a^4x^4+4a^3bx^6}+\frac{3b^2\log\left(x\right)}{a^4}-\frac{3b^2\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6) + 3*b**2*l
og(x)/a**4 - 3*b**2*log(a/b + x**2)/(2*a**4)
```

Giac [A]
time = 4.05, size = 86, normalized size = 1.30

$$\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}-\frac{3\,b^2\log\left(|bx^2+a|\right)}{2\,a^4}+\frac{3\,b^3x^2+4\,ab^2}{2\,(bx^2+a)a^4}-\frac{9\,b^2x^4-4\,abx^2+a^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 3/2*b^2*log(x^2)/a^4 - 3/2*b^2*log(abs(b*x^2 + a))/a^4 + 1/2*(3*b^3*x^2 + 4
*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

Mupad [B]
time = 0.07, size = 67, normalized size = 1.02

$$\frac{\frac{3\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{3\,b^2\,x^4}{2\,a^3}}{b\,x^6+a\,x^4}-\frac{3\,b^2\,\ln\left(b\,x^2+a\right)}{2\,a^4}+\frac{3\,b^2\,\ln\left(x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] ((3*b*x^2)/(4*a^2) - 1/(4*a) + (3*b^2*x^4)/(2*a^3))/(a*x^4 + b*x^6) - (3*b^
2*log(a + b*x^2))/(2*a^4) + (3*b^2*log(x))/a^4
```

3.484 $\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=92

$$-\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b(a+bx^2)}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}$$

```
[Out] -9/2*a^3*x/b^5+3/2*a^2*x^3/b^4-9/10*a*x^5/b^3+9/14*x^7/b^2-1/2*x^9/b/(b*x^2
+a)+9/2*a^(7/2)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 308, 211}

$$\frac{9a^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b(a+bx^2)}+\frac{9x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14
*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
b^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^{10}}{\left(ab+b^2x^2\right)^2}\,dx$$
$$= -\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9}{2}\int \frac{x^8}{ab+b^2x^2}\,dx$$
$$= -\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9}{2}\int \left(-\frac{a^3}{b^5}+\frac{a^2x^2}{b^4}-\frac{ax^4}{b^3}+\frac{x^6}{b^2}+\frac{a^4}{b^4\left(ab+b^2x^2\right)}\right)dx$$
$$= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{\left(9a^4\right)\int \frac{1}{ab+b^2x^2}\,dx}{2b^4}$$
$$= -\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}+\frac{9x^7}{14b^2}-\frac{x^9}{2b\left(a+bx^2\right)}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Mathematica [A]
time = 0.04, size = 82, normalized size = 0.89

$$\frac{x\left(-280a^3+70a^2bx^2-28ab^2x^4+10b^3x^6-\frac{35a^4}{a+bx^2}\right)}{70b^5}+\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4)/(a + b*x
^2)))/(70*b^5) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Maple [A]
time = 0.04, size = 76, normalized size = 0.83

method	result	si
default	$-\frac{-\frac{1}{7}b^3x^7+\frac{2}{5}ab^2x^5-a^2bx^3+4a^3x}{b^5}+\frac{a^4\left(-\frac{x}{2\left(bx^2+a\right)}+\frac{9\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^5}$	7

risch	$\frac{x^7}{7b^2}-\frac{2a\,x^5}{5b^3}+\frac{a^2x^3}{b^4}-\frac{4a^3x}{b^5}-\frac{a^4x}{2b^5(b\,x^2+a)}+\frac{9\sqrt{-ab}\,a^3\ln\left(-\sqrt{-ab}\,x+a\right)}{4b^6}-\frac{9\sqrt{-ab}\,a^3\ln\left(\sqrt{-ab}\,x+a\right)}{4b^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
[Out] -1/b^5*(-1/7*b^3*x^7+2/5*a*b^2*x^5-a^2*b*x^3+4*a^3*x)+a^4/b^5*(-1/2*x/(b*x^
2+a)+9/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 82, normalized size = 0.89

$$-\frac{a^4x}{2\left(b^6x^2+ab^5\right)}+\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}+\frac{5\,b^3x^7-14\,ab^2x^5+35\,a^2bx^3-140\,a^3x}{35\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] -1/2*a^4*x/(b^6*x^2 + a*b^5) + 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5
) + 1/35*(5*b^3*x^7 - 14*a*b^2*x^5 + 35*a^2*b*x^3 - 140*a^3*x)/b^5
```

Fricas [A]
time = 0.37, size = 212, normalized size = 2.30

$$\left[\frac{20\,b^4x^9-36\,ab^3x^7+84\,a^2b^2x^5-420\,a^3bx^3-630\,a^4x+315\,\left(a^3bx^2+a^4\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{140\,\left(b^6x^2+ab^5\right)},\ \frac{10\,b^4x^9-18\,ab^3x^7+42\,a^2b^2x^5-210\,a^3bx^3-315\,a^4x+315\,\left(a^3bx^2+a^4\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{70\,\left(b^6x^2+ab^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] [1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^3 - 630*a^
4*x + 315*(a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*x^9 - 18*a*b^3*x^7 + 42*a^2*b^
2*x^5 - 210*a^3*b*x^3 - 315*a^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a))/(b^6*x^2 + a*b^5)]
```

Sympy [A]
time = 0.14, size = 134, normalized size = 1.46

$$-\frac{a^4x}{2ab^5+2b^6x^2}-\frac{4a^3x}{b^5}+\frac{a^2x^3}{b^4}-\frac{2ax^5}{5b^3}-\frac{9\sqrt{-\frac{a^7}{b^{11}}}\,\log\left(x-\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{9\sqrt{-\frac{a^7}{b^{11}}}\,\log\left(x+\frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4}+\frac{x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4 - 2*a*x**
5/(5*b**3) - 9*sqrt(-a**7/b**11)*log(x - b**5*sqrt(-a**7/b**11)/a**3)/4 + 9
*sqrt(-a**7/b**11)*log(x + b**5*sqrt(-a**7/b**11)/a**3)/4 + x**7/(7*b**2)
```

Giac [A]
time = 4.68, size = 84, normalized size = 0.91

$$\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}-\frac{a^4x}{2\,(bx^2+a)b^5}+\frac{5\,b^{12}x^7-14\,ab^{11}x^5+35\,a^2b^{10}x^3-140\,a^3b^9x}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*a^4*x/((b*x^2 + a)*b^5)
 + 1/35*(5*b^12*x^7 - 14*a*b^11*x^5 + 35*a^2*b^10*x^3 - 140*a^3*b^9*x)/b^14
```

Mupad [B]
time = 0.04, size = 77, normalized size = 0.84

$$\frac{x^7}{7\,b^2}-\frac{2\,a\,x^5}{5\,b^3}-\frac{4\,a^3\,x}{b^5}+\frac{9\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{11/2}}+\frac{a^2\,x^3}{b^4}-\frac{a^4\,x}{2\,\left(b^6\,x^2+a\,b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^7/(7*b^2) - (2*a*x^5)/(5*b^3) - (4*a^3*x)/b^5 + (9*a^(7/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(11/2)) + (a^2*x^3)/b^4 - (a^4*x)/(2*(a*b^5 + b^6*x^2))
```

3.485 $\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b\,(a+bx^2)}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

```
[Out] 7/2*a^2*x/b^4-7/6*a*x^3/b^3+7/10*x^5/b^2-1/2*x^7/b/(b*x^2+a)-7/2*a^(5/2)*ar
ctan(x*b^(1/2)/a^(1/2))/b^(9/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 294, 308, 211}

$$-\frac{7a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\,(a+bx^2)}+\frac{7x^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2*b*(a + b*
x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^8}{(ab+b^2x^2)^2}\,dx\\
&= -\frac{x^7}{2b(a+bx^2)}+\frac{7}{2}\int \frac{x^6}{ab+b^2x^2}\,dx\\
&= -\frac{x^7}{2b(a+bx^2)}+\frac{7}{2}\int \left(\frac{a^2}{b^4}-\frac{ax^2}{b^3}+\frac{x^4}{b^2}-\frac{a^3}{b^3(ab+b^2x^2)}\right)dx\\
&= \frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b(a+bx^2)}-\frac{(7a^3)\int \frac{1}{ab+b^2x^2}\,dx}{2b^3}\\
&= \frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}+\frac{7x^5}{10b^2}-\frac{x^7}{2b(a+bx^2)}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 0.90

$$\frac{x\left(90a^2-20abx^2+6b^2x^4+\frac{15a^3}{a+bx^2}\right)}{30b^4}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(90*a^2 - 20*a*b*x^2 + 6*b^2*x^4 + (15*a^3)/(a + b*x^2)))/(30*b^4) - (7*
a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Maple [A]
time = 0.06, size = 65, normalized size = 0.82

method	result	size
default	$\frac{\frac{1}{5}b^2x^5-\frac{2}{3}ab\,x^3+3a^2x}{b^4}-\frac{a^3\left(-\frac{x}{2\left(b\,x^2+a\right)}+\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}$	65

risch	$\frac{x^5}{5b^2} - \frac{2a\,x^3}{3b^3} + \frac{3a^2x}{b^4} + \frac{a^3x}{2b^4(b\,x^2+a)} + \frac{7\sqrt{-ab}\;a^2\ln\left(-\sqrt{-ab}\;x-a\right)}{4b^5} - \frac{7\sqrt{-ab}\;a^2\ln\left(\sqrt{-ab}\;x-a\right)}{4b^5}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/5*b^2*x^5-2/3*a*b*x^3+3*a^2*x)-a^3/b^4*(-1/2*x/(b*x^2+a)+7/2/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 71, normalized size = 0.90

$$\frac{a^3x}{2\,(b^5x^2+ab^4)} - \frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4} + \frac{3\,b^2x^5 - 10\,abx^3 + 45\,a^2x}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^3*x/(b^5*x^2 + a*b^4) - 7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
 + 1/15*(3*b^2*x^5 - 10*a*b*x^3 + 45*a^2*x)/b^4
```

Fricas [A]
time = 0.33, size = 190, normalized size = 2.41

$$\left[\frac{12\,b^3x^7 - 28\,ab^2x^5 + 140\,a^2bx^3 + 210\,a^3x + 105\,(a^2bx^2 + a^3)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right)}{60\,(b^5x^2 + ab^4)},\ \frac{6\,b^3x^7 - 14\,ab^2x^5 + 70\,a^2bx^3 + 105\,a^3x - 105\,(a^2bx^2 + a^3)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{30\,(b^5x^2 + ab^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*x^7 - 28*a*b^2*x^5 + 140*a^2*b*x^3 + 210*a^3*x + 105*(a^2*b*x
^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*
x^2 + a*b^4), 1/30*(6*b^3*x^7 - 14*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*x - 1
05*(a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^2 + a*b^4)]
```

Sympy [A]
time = 0.13, size = 124, normalized size = 1.57

$$\frac{a^3x}{2ab^4 + 2b^5x^2} + \frac{3a^2x}{b^4} - \frac{2ax^3}{3b^3} + \frac{7\sqrt{-\frac{a^5}{b^9}}\,\log\left(x - \frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4} - \frac{7\sqrt{-\frac{a^5}{b^9}}\,\log\left(x + \frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4} + \frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**3*x/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*x/b**4 - 2*a*x**3/(3*b**3) + 7*sqr
t(-a**5/b**9)*log(x - b**4*sqrt(-a**5/b**9)/a**2)/4 - 7*sqrt(-a**5/b**9)*lo
g(x + b**4*sqrt(-a**5/b**9)/a**2)/4 + x**5/(5*b**2)
```

Giac [A]
time = 4.65, size = 73, normalized size = 0.92

$$-\frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}+\frac{a^3x}{2\,(bx^2+a)b^4}+\frac{3\,b^8x^5-10\,ab^7x^3+45\,a^2b^6x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

Mupad [B]
time = 4.27, size = 66, normalized size = 0.84

$$\frac{x^5}{5\,b^2}-\frac{2\,a\,x^3}{3\,b^3}+\frac{3\,a^2\,x}{b^4}-\frac{7\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{9/2}}+\frac{a^3\,x}{2\,\left(b^5\,x^2+a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^5/(5*b^2) - (2*a*x^3)/(3*b^3) + (3*a^2*x)/b^4 - (7*a^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/(2*b^(9/2)) + (a^3*x)/(2*(a*b^4 + b^5*x^2))
```

3.486 $\int \frac{x^6}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=66

$$-\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}$$

```
[Out] -5/2*a*x/b^3+5/6*x^3/b^2-1/2*x^5/b/(b*x^2+a)+5/2*a^(3/2)*arctan(x*b^(1/2)/a
^(1/2))/b^(7/2)
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 308, 211}

$$\frac{5a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^6}{(ab+b^2x^2)^2}\,dx\\ &= -\frac{x^5}{2b(a+bx^2)}+\frac{5}{2}\int \frac{x^4}{ab+b^2x^2}\,dx\\ &= -\frac{x^5}{2b(a+bx^2)}+\frac{5}{2}\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a^2}{b^2(ab+b^2x^2)}\right)dx\\ &= -\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b(a+bx^2)}+\frac{(5a^2)\int \frac{1}{ab+b^2x^2}\,dx}{2b^2}\\ &= -\frac{5ax}{2b^3}+\frac{5x^3}{6b^2}-\frac{x^5}{2b(a+bx^2)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.91

$$\frac{x\left(-12a+2bx^2-\frac{3a^2}{a+bx^2}\right)}{6b^3}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^(3/2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Maple [A]
time = 0.03, size = 54, normalized size = 0.82

method	result	size
default	$-\frac{-\frac{1}{3}bx^3+2ax}{b^3}+\frac{a^2\left(-\frac{x}{2(bx^2+a)}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$	54

risch	$\frac{x^3}{3b^2} - \frac{2ax}{b^3} - \frac{a^2x}{2b^3(b\,x^2+a)} + \frac{5\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x+a\right)}{4b^4} - \frac{5\sqrt{-ab}\,a\ln\left(\sqrt{-ab}\,x+a\right)}{4b^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/3*b*x^3+2*a*x)+a^2/b^3*(-1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan(
b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 59, normalized size = 0.89

$$-\frac{a^2x}{2\,(b^4x^2+ab^3)} + \frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3} + \frac{bx^3-6\,ax}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*x/(b^4*x^2 + a*b^3) + 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3
) + 1/3*(b*x^3 - 6*a*x)/b^3
```

Fricas [A]
time = 0.36, size = 164, normalized size = 2.48

$$\left[\frac{4\,b^2x^5-20\,abx^3-30\,a^2x+15\,(abx^2+a^2)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{12\,(b^4x^2+ab^3)}, \frac{2\,b^2x^5-10\,abx^3-15\,a^2x+15\,(abx^2+a^2)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{6\,(b^4x^2+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*x^5 - 20*a*b*x^3 - 30*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(-a/b)*lo
g((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^4*x^2 + a*b^3), 1/6*(2*b^
2*x^5 - 10*a*b*x^3 - 15*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqr
t(a/b)/a))/(b^4*x^2 + a*b^3)]
```

Sympy [A]
time = 0.12, size = 107, normalized size = 1.62

$$-\frac{a^2x}{2ab^3+2b^4x^2} - \frac{2ax}{b^3} - \frac{5\sqrt{-\frac{a^3}{b^7}}\,\log\left(x-\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{5\sqrt{-\frac{a^3}{b^7}}\,\log\left(x+\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**2*x/(2*a*b**3 + 2*b**4*x**2) - 2*a*x/b**3 - 5*sqrt(-a**3/b**7)*log(x -
b**3*sqrt(-a**3/b**7)/a)/4 + 5*sqrt(-a**3/b**7)*log(x + b**3*sqrt(-a**3/b**
7)/a)/4 + x**3/(3*b**2)
```

Giac [A]
time = 4.48, size = 61, normalized size = 0.92

$$\frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}-\frac{a^2x}{2\,(bx^2+a)b^3}+\frac{b^4x^3-6\,ab^3x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2 + a)*b^3)
 + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6
```

Mupad [B]
time = 0.06, size = 56, normalized size = 0.85

$$\frac{x^3}{3\,b^2}+\frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{7/2}}-\frac{a^2\,x}{2\,\left(b^4\,x^2+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x^3/(3*b^2) + (5*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(7/2)) - (a^2*x)/(
2*(a*b^3 + b^4*x^2)) - (2*a*x)/b^3
```

3.487 $\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=55

$$\frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}$$

```
[Out] 3/2*x/b^2-1/2*x^3/b/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 327, 211}

$$-\frac{3\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{x^4}{\left(ab+b^2x^2\right)^2}\,dx\\ &= -\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3}{2}\int \frac{x^2}{ab+b^2x^2}\,dx\\ &= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{(3a)\int \frac{1}{ab+b^2x^2}\,dx}{2b}\\ &= \frac{3x}{2b^2}-\frac{x^3}{2b\left(a+bx^2\right)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 0.93

$$\frac{x}{b^2}+\frac{ax}{2b^2\left(a+bx^2\right)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] x/b^2 + (a*x)/(2*b^2*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*b^(5/2))
```

Maple [A]
time = 0.03, size = 42, normalized size = 0.76

method	result	size
default	$\frac{x}{b^2}-\frac{a\left(-\frac{x}{2\left(b\,x^2+a\right)}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	42

risch	$\frac{x}{b^2}+\frac{ax}{2b^2(b\,x^2+a)}+\frac{3\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{4b^3}-\frac{3\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{4b^3}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] x/b^2-a/b^2*(-1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 45, normalized size = 0.82

$$\frac{ax}{2\,(b^3x^2+ab^2)}-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*x/(b^3*x^2 + a*b^2) - 3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + x
/b^2
```

Fricas [A]
time = 0.37, size = 136, normalized size = 2.47

$$\left[\frac{4\,bx^3+3\,(bx^2+a)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,ax}{4\,(b^3x^2+ab^2)},\ \frac{2\,bx^3-3\,(bx^2+a)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,ax}{2\,(b^3x^2+ab^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*x^3 + 3*(b*x^2 + a)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)) + 6*a*x)/(b^3*x^2 + a*b^2), 1/2*(2*b*x^3 - 3*(b*x^2 + a)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*a*x)/(b^3*x^2 + a*b^2)]
```

Sympy [A]
time = 0.11, size = 83, normalized size = 1.51

$$\frac{ax}{2ab^2+2b^3x^2}+\frac{3\sqrt{-\frac{a}{b^5}}\,\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}-\frac{3\sqrt{-\frac{a}{b^5}}\,\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a*x/(2*a*b**2 + 2*b**3*x**2) + 3*sqrt(-a/b**5)*log(-b**2*sqrt(-a/b**5) + x)
/4 - 3*sqrt(-a/b**5)*log(b**2*sqrt(-a/b**5) + x)/4 + x/b**2
```

Giac [A]
time = 4.46, size = 42, normalized size = 0.76

$$-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{ax}{2\,(bx^2+a)b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 + a)*b^2) +
x/b^2
```

Mupad [B]
time = 4.29, size = 43, normalized size = 0.78

$$\frac{x}{b^2}+\frac{a\,x}{2\,\left(b^3\,x^2+a\,b^2\right)}-\frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] x/b^2 + (a*x)/(2*(a*b^2 + b^3*x^2)) - (3*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))
/(2*b^(5/2))
```

3.488 $\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=45

$$-\frac{x}{2b\,(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

```
[Out] -1/2*x/b/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 294, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{x^2}{(ab+b^2x^2)^2}\,dx$$
$$= -\frac{x}{2b\,(a+bx^2)} + \frac{1}{2}\int \frac{1}{ab+b^2x^2}\,dx$$
$$= -\frac{x}{2b\,(a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$-\frac{x}{2b\,(a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Maple [A]
time = 0.03, size = 36, normalized size = 0.80

method	result	size
default	$-\frac{x}{2b(b\,x^2+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}$	36
risch	$-\frac{x}{2b(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\;b}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\;b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 36, normalized size = 0.80

$$-\frac{x}{2\,(b^2x^2+ab)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*x/(b^2*x^2 + a*b) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Fricas [A]
time = 0.34, size = 120, normalized size = 2.67

$$\left[-\frac{2\,abx+(bx^2+a)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\,(ab^3x^2+a^2b^2)}, -\frac{abx-(bx^2+a)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,(ab^3x^2+a^2b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*x + (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*x - (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.
time = 0.08, size = 78, normalized size = 1.73

$$-\frac{x}{2ab+2b^2x^2}-\frac{\sqrt{-\frac{1}{ab^3}}\,\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^3}}\,\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3)) + x
)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4
```

Giac [A]
time = 7.09, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b}-\frac{x}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*x/((b*x^2 + a)*b)
```

Mupad [B]
time = 0.05, size = 33, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}-\frac{x}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] atan((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2)) - x/(2*b*(a + b*x^2))
```

3.489 $\int \frac{1}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=45

$$\frac{x}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

```
[Out] 1/2*x/a/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {28, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{1}{(ab+b^2x^2)^2}\,dx$$
$$= \frac{x}{2a(a+bx^2)} + \frac{b\int \frac{1}{ab+b^2x^2}\,dx}{2a}$$
$$= \frac{x}{2a(a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$\frac{x}{2a(a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.80

method	result	size
default	$\frac{x}{2a(b\,x^2+a)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	36
risch	$\frac{x}{2a(b\,x^2+a)} - \frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\;a} + \frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\;a}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 35, normalized size = 0.78

$$\frac{x}{2(abx^2+a^2)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

Fricas [A]
time = 0.36, size = 120, normalized size = 2.67

$$\left[\frac{2\,abx - (bx^2 + a)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right)}{4\,(a^2b^2x^2 + a^3b)}, \frac{abx + (bx^2 + a)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,(a^2b^2x^2 + a^3b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.
time = 0.09, size = 78, normalized size = 1.73

$$\frac{x}{2a^2 + 2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(-a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\,\log\left(a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

Giac [A]
time = 4.12, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a} + \frac{x}{2\,(bx^2 + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

Mupad [B]
time = 0.04, size = 33, normalized size = 0.73

$$\frac{x}{2\,a\,\left(b\,x^2+a\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

3.490 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=57

$$-\frac{3}{2a^2x}+\frac{1}{2ax\,(a+bx^2)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}$$

```
[Out] -3/2/a^2/x+1/2/a/x/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 296, 331, 211}

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^2\left(ab+b^2x^2\right)^2}\,dx\\
&= \frac{1}{2ax\left(a+bx^2\right)}+\frac{(3b)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a}\\
&= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{\left(3b^2\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^2}\\
&= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.95

$$
-\frac{1}{a^2x}-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

Maple [A]
time = 0.03, size = 45, normalized size = 0.79

method	result	size
default	$-\frac{b\left(\frac{x}{2bx^2+2a}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{1}{a^2x}$	45

risch	$\frac{-\frac{3b\,x^2}{2a^2}-\frac{1}{a}}{x(b\,x^2+a)}+\frac{3\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{4a^3}-\frac{3\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{4a^3}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^2*(1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/a^2/x
```

Maxima [A]
time = 0.54, size = 49, normalized size = 0.86

$$-\frac{3\,bx^2+2\,a}{2\,(a^2bx^3+a^3x)}-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2)
```

Fricas [A]
time = 0.35, size = 136, normalized size = 2.39

$$\left[-\frac{6\,bx^2-3\,(bx^3+ax)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4\,a}{4\,(a^2bx^3+a^3x)},-\frac{3\,bx^2+3\,(bx^3+ax)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,a}{2\,(a^2bx^3+a^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A]
time = 0.13, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{b}{a^5}}\,\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\,\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{-2a-3bx^2}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

Giac [A]
time = 3.95, size = 47, normalized size = 0.82

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{3\,bx^2+2\,a}{2\,(bx^3+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

Mupad [B]
time = 4.49, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{a}+\frac{3\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] - (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^(
1/2)))/(2*a^(5/2))
```

3.491 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=68

$$-\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

```
[Out] -5/6/a^2/x^3+5/2*b/a^3/x+1/2/a/x^3/(b*x^2+a)+5/2*b^(3/2)*arctan(x*b^(1/2)/a
^(1/2))/a^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 296, 331, 211}

$$\frac{5b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^4\left(ab+b^2x^2\right)^2}\,dx\\
&= \frac{1}{2ax^3\left(a+bx^2\right)}+\frac{(5b)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{2a}\\
&= -\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}-\frac{\left(5b^2\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a^2}\\
&= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{\left(5b^3\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^3}\\
&= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.99

$$-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}+\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Maple [A]
time = 0.04, size = 55, normalized size = 0.81

method	result	size

default	$\frac{b^2\left(\frac{x}{2b\,x^2+2a}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}$	55
risch	$\frac{\frac{5b^2x^4}{2a^3}+\frac{5b\,x^2}{3a^2}-\frac{1}{3a}}{x^3(b\,x^2+a)}+\frac{5\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{4a^4}-\frac{5\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{4a^4}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^3*(1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/3/a^2/x
^3+2*b/a^3/x
```

Maxima [A]
time = 0.52, size = 64, normalized size = 0.94

$$\frac{15\,b^2x^4+10\,abx^2-2\,a^2}{6\,(a^3bx^5+a^4x^3)}+\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.38, size = 172, normalized size = 2.53

$$\left[\frac{30\,b^2x^4+20\,abx^2+15\,(b^2x^5+abx^3)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-4\,a^2}{12\,(a^3bx^5+a^4x^3)},\ \frac{15\,b^2x^4+10\,abx^2+15\,(b^2x^5+abx^3)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)-2\,a^2}{6\,(a^3bx^5+a^4x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [A]
time = 0.16, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Giac [A]
time = 3.97, size = 59, normalized size = 0.87

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}+\frac{b^2x}{2\,(bx^2+a)a^3}+\frac{6\,bx^2-a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

Mupad [B]
time = 4.43, size = 58, normalized size = 0.85

$$\frac{\frac{5\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{5\,b^2\,x^4}{2\,a^3}}{b\,x^5+a\,x^3}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] ((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b^
(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

3.492 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=81

$$-\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}$$

```
[Out] -7/10/a^2/x^5+7/6*b/a^3/x^3-7/2*b^2/a^4/x+1/2/a/x^5/(b*x^2+a)-7/2*b^(5/2)*a
rctan(x*b^(1/2)/a^(1/2))/a^(9/2)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 296, 331, 211}

$$-\frac{7b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{7b^2}{2a^4x}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b
*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{x^6\left(ab+b^2x^2\right)^2}\,dx\\
&= \frac{1}{2ax^5\left(a+bx^2\right)}+\frac{(7b)\int \frac{1}{x^6\left(ab+b^2x^2\right)}\,dx}{2a}\\
&= -\frac{7}{10a^2x^5}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{\left(7b^2\right)\int \frac{1}{x^4\left(ab+b^2x^2\right)}\,dx}{2a^2}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}+\frac{1}{2ax^5\left(a+bx^2\right)}+\frac{\left(7b^3\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)}\,dx}{2a^3}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{\left(7b^4\right)\int \frac{1}{ab+b^2x^2}\,dx}{2a^4}\\
&= -\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}+\frac{1}{2ax^5\left(a+bx^2\right)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 0.99

$$-\frac{1}{5a^2x^5}+\frac{2b}{3a^3x^3}-\frac{3b^2}{a^4x}-\frac{b^3x}{2a^4\left(a+bx^2\right)}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/5*1/(a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2*a^4*(a
+ b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Maple [A]
time = 0.04, size = 67, normalized size = 0.83

method	result	size

default	$-\frac{b^{3}\left(\frac{x}{2 b \,x^{2}+2 a}+\frac{7 \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{a^{4}}-\frac{1}{5 a^{2} x^{5}}-\frac{3 b^{2}}{a^{4} x}+\frac{2 b}{3 a^{3} x^{3}}$	67
risch	$\frac{-\frac{7 b^{3} x^{6}}{2 a^{4}}-\frac{7 b^{2} x^{4}}{3 a^{3}}+\frac{7 b \,x^{2}}{15 a^{2}}-\frac{1}{5 a}}{x^{5} \left(b \,x^{2}+a \right)}+\frac{7 \sqrt{-a b}\, b^{2} \ln \left(-b x +\sqrt{-a b}\right)}{4 a^{5}}-\frac{7 \sqrt{-a b}\, b^{2} \ln \left(-b x -\sqrt{-a b}\right)}{4 a^{5}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
[Out] -b^3/a^4*(1/2*x/(b*x^2+a)+7/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/5/a^2/
x^5-3*b^2/a^4/x+2/3*b/a^3/x^3
```

Maxima [A]
time = 0.51, size = 75, normalized size = 0.93

$$-\frac{105 \, b^{3} x^{6} + 70 \, a b^{2} x^{4} - 14 \, a^{2} b x^{2} + 6 \, a^{3}}{30 \, {\left(a^{4} b x^{7} + a^{5} x^{5}\right)}} - \frac{7 \, b^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, \sqrt{a b} a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a^3)/(a^4*b*x^7 + a^5*
x^5) - 7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 0.36, size = 198, normalized size = 2.44

$$\left[-\frac{210 \, b^{3} x^{6} + 140 \, a b^{2} x^{4} - 28 \, a^{2} b x^{2} + 12 \, a^{3} - 105 \, {\left(b^{3} x^{7} + a b^{2} x^{5}\right)} \sqrt{-\frac{b}{a}} \log\left(\frac{b x^{2} - 2 \, a x \sqrt{-\frac{b}{a}} - a}{b x^{2} + a}\right)}{60 \, {\left(a^{4} b x^{7} + a^{5} x^{5}\right)}}, -\frac{105 \, b^{3} x^{6} + 70 \, a b^{2} x^{4} - 14 \, a^{2} b x^{2} + 6 \, a^{3} + 105 \, {\left(b^{3} x^{7} + a b^{2} x^{5}\right)} \sqrt{\frac{b}{a}} \arctan\left(x \sqrt{\frac{b}{a}}\right)}{30 \, {\left(a^{4} b x^{7} + a^{5} x^{5}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] [-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105*(b^3*x^7
+ a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a
^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a
^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b*x^7 +
a^5*x^5)]
```

Sympy [A]
time = 0.18, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}-\frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4}+\frac{-6a^3+14a^2bx^2-70ab^2x^4-105b^3x^6}{30a^5x^5+30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqrt(-b**5/a*
*9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 + (-6*a**3 + 14*a**2*b*x**2 - 70*
a*b**2*x**4 - 105*b**3*x**6)/(30*a**5*x**5 + 30*a**4*b*x**7)
```

Giac [A]
time = 3.42, size = 70, normalized size = 0.86

$$-\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}-\frac{b^3x}{2\,(bx^2+a)a^4}-\frac{45\,b^2x^4-10\,abx^2+3\,a^2}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^2 + a)*a^4
) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

Mupad [B]
time = 4.71, size = 70, normalized size = 0.86

$$-\frac{\frac{1}{5\,a}-\frac{7\,b\,x^2}{15\,a^2}+\frac{7\,b^2\,x^4}{3\,a^3}+\frac{7\,b^3\,x^6}{2\,a^4}}{b\,x^7+a\,x^5}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] - (1/(5*a) - (7*b*x^2)/(15*a^2) + (7*b^2*x^4)/(3*a^3) + (7*b^3*x^6)/(2*a^4)
)/(a*x^5 + b*x^7) - (7*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(9/2))
```

3.493 $\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}+\frac{a^5}{6b^6(a+bx^2)^3}-\frac{5a^4}{4b^6(a+bx^2)^2}+\frac{5a^3}{b^6(a+bx^2)}+\frac{5a^2\log(a+bx^2)}{b^6}$$

```
[Out] -2*a*x^2/b^5+1/4*x^4/b^4+1/6*a^5/b^6/(b*x^2+a)^3-5/4*a^4/b^6/(b*x^2+a)^2+5*
a^3/b^6/(b*x^2+a)+5*a^2*ln(b*x^2+a)/b^6
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$\frac{a^5}{6b^6(a+bx^2)^3}-\frac{5a^4}{4b^6(a+bx^2)^2}+\frac{5a^3}{b^6(a+bx^2)}+\frac{5a^2\log(a+bx^2)}{b^6}-\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-2*a*x^2)/b^5 + x^4/(4*b^4) + a^5/(6*b^6*(a + b*x^2)^3) - (5*a^4)/(4*b^6*(
a + b*x^2)^2) + (5*a^3)/(b^6*(a + b*x^2)) + (5*a^2*Log[a + b*x^2])/b^6
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{x^{11}}{(ab+b^2x^2)^4}\,dx$$
$$= \frac{1}{2}b^4\text{Subst}\left(\int \frac{x^5}{(ab+b^2x)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\text{Subst}\left(\int \left(-\frac{4a}{b^9}+\frac{x}{b^8}-\frac{a^5}{b^9(a+bx)^4}+\frac{5a^4}{b^9(a+bx)^3}-\frac{10a^3}{b^9(a+bx)^2}+\frac{10a}{b^9(a+\right.\right.$$
$$= -\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}+\frac{a^5}{6b^6(a+bx^2)^3}-\frac{5a^4}{4b^6(a+bx^2)^2}+\frac{5a^3}{b^6(a+bx^2)}+\frac{5a^2\log(a+}{b^6}$$

Mathematica [A]
time = 0.02, size = 78, normalized size = 0.86

$$\frac{-24abx^2+3b^2x^4+\frac{2a^5}{(a+bx^2)^3}-\frac{15a^4}{(a+bx^2)^2}+\frac{60a^3}{a+bx^2}+60a^2\log(a+bx^2)}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-24*a*b*x^2 + 3*b^2*x^4 + (2*a^5)/(a + b*x^2)^3 - (15*a^4)/(a + b*x^2)^2 +
 (60*a^3)/(a + b*x^2) + 60*a^2*Log[a + b*x^2])/(12*b^6)
```

Maple [A]
time = 0.06, size = 90, normalized size = 0.99

method	result	size
norman	$\frac{\frac{x^{10}}{4b}-\frac{5a\,x^8}{4b^2}+\frac{55a^5}{6b^6}+\frac{15a^3x^4}{b^4}+\frac{45a^4x^2}{2b^5}}{(b\,x^2+a)^3}+\frac{5a^2\ln(b\,x^2+a)}{b^6}$	76
default	$\frac{(-b\,x^2+4a)^2}{4b^6}+\frac{a^2\left(\frac{a^3}{3b(b\,x^2+a)^3}+\frac{10a}{b(b\,x^2+a)}-\frac{5a^2}{2b(b\,x^2+a)^2}+\frac{10\ln(b\,x^2+a)}{b}\right)}{2b^5}$	90
risch	$\frac{x^4}{4b^4}-\frac{2a\,x^2}{b^5}+\frac{4a^2}{b^6}+\frac{5a^3b\,x^4+\frac{35a^4x^2}{4}+\frac{47a^5}{12b}}{b^5(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}+\frac{5a^2\ln(b\,x^2+a)}{b^6}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-b*x^2+4*a)^2/b^6+1/2*a^2/b^5*(1/3*a^3/b/(b*x^2+a)^3+10*a/b/(b*x^2+a)-
5/2*a^2/b/(b*x^2+a)^2+10/b*ln(b*x^2+a))
```

Maxima [A]
time = 0.29, size = 99, normalized size = 1.09

$$\frac{60\,a^3b^2x^4+105\,a^4bx^2+47\,a^5}{12\,(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)}+\frac{5\,a^2\log(bx^2+a)}{b^6}+\frac{bx^4-8\,ax^2}{4\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*a^3*b^2*x^4 + 105*a^4*b*x^2 + 47*a^5)/(b^9*x^6 + 3*a*b^8*x^4 + 3*a
^2*b^7*x^2 + a^3*b^6) + 5*a^2*log(b*x^2 + a)/b^6 + 1/4*(b*x^4 - 8*a*x^2)/b^
5
```

Fricas [A]
time = 0.32, size = 137, normalized size = 1.51

$$\frac{3\,b^{5}x^{10}-15\,ab^{4}x^{8}-63\,a^{2}b^{3}x^{6}-9\,a^{3}b^{2}x^{4}+81\,a^{4}bx^{2}+47\,a^{5}+60\,\left(a^{2}b^{3}x^{6}+3\,a^{3}b^{2}x^{4}+3\,a^{4}bx^{2}+a^{5}\right)\log\left(bx^{2}+a\right)}{12\,\left(b^{9}x^{6}+3\,ab^{8}x^{4}+3\,a^{2}b^{7}x^{2}+a^{3}b^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 - 15*a*b^4*x^8 - 63*a^2*b^3*x^6 - 9*a^3*b^2*x^4 + 81*a^4*b
*x^2 + 47*a^5 + 60*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*log(b*
x^2 + a))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)
```

Sympy [A]
time = 0.28, size = 100, normalized size = 1.10

$$\frac{5a^{2}\log\left(a+bx^{2}\right)}{b^{6}}-\frac{2ax^{2}}{b^{5}}+\frac{47a^{5}+105a^{4}bx^{2}+60a^{3}b^{2}x^{4}}{12a^{3}b^{6}+36a^{2}b^{7}x^{2}+36ab^{8}x^{4}+12b^{9}x^{6}}+\frac{x^{4}}{4b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 5*a**2*log(a + b*x**2)/b**6 - 2*a*x**2/b**5 + (47*a**5 + 105*a**4*b*x**2 +
60*a**3*b**2*x**4)/(12*a**3*b**6 + 36*a**2*b**7*x**2 + 36*a*b**8*x**4 + 12*
b**9*x**6) + x**4/(4*b**4)
```

Giac [A]
time = 4.54, size = 91, normalized size = 1.00

$$\frac{5\,a^{2}\log\left(|bx^{2}+a|\right)}{b^{6}}+\frac{b^{4}x^{4}-8\,ab^{3}x^{2}}{4\,b^{8}}-\frac{110\,a^{2}b^{3}x^{6}+270\,a^{3}b^{2}x^{4}+225\,a^{4}bx^{2}+63\,a^{5}}{12\,\left(bx^{2}+a\right)^{3}b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*log(abs(b*x^2 + a))/b^6 + 1/4*(b^4*x^4 - 8*a*b^3*x^2)/b^8 - 1/12*(110
*a^2*b^3*x^6 + 270*a^3*b^2*x^4 + 225*a^4*b*x^2 + 63*a^5)/((b*x^2 + a)^3*b^6
)
```

Mupad [B]
time = 4.48, size = 98, normalized size = 1.08

$$\frac{\frac{47\,a^5}{12\,b}+\frac{35\,a^4\,x^2}{4}+5\,a^3\,b\,x^4}{a^3\,b^5+3\,a^2\,b^6\,x^2+3\,a\,b^7\,x^4+b^8\,x^6}+\frac{x^4}{4\,b^4}-\frac{2\,a\,x^2}{b^5}+\frac{5\,a^2\,\ln\left(b\,x^2+a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((47*a^5)/(12*b) + (35*a^4*x^2)/4 + 5*a^3*b*x^4)/(a^3*b^5 + b^8*x^6 + 3*a*b
^7*x^4 + 3*a^2*b^6*x^2) + x^4/(4*b^4) - (2*a*x^2)/b^5 + (5*a^2*log(a + b*x^
2))/b^6
```

3.494 $\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=77

$$\frac{x^2}{2b^4}-\frac{a^4}{6b^5\left(a+bx^2\right)^3}+\frac{a^3}{b^5\left(a+bx^2\right)^2}-\frac{3a^2}{b^5\left(a+bx^2\right)}-\frac{2a\log\left(a+bx^2\right)}{b^5}$$

```
[Out] 1/2*x^2/b^4-1/6*a^4/b^5/(b*x^2+a)^3+a^3/b^5/(b*x^2+a)^2-3*a^2/b^5/(b*x^2+a)
-2*a*ln(b*x^2+a)/b^5
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 45}

$$-\frac{a^4}{6b^5\left(a+bx^2\right)^3}+\frac{a^3}{b^5\left(a+bx^2\right)^2}-\frac{3a^2}{b^5\left(a+bx^2\right)}-\frac{2a\log\left(a+bx^2\right)}{b^5}+\frac{x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x^2/(2*b^4) - a^4/(6*b^5*(a + b*x^2)^3) + a^3/(b^5*(a + b*x^2)^2) - (3*a^2)
/(b^5*(a + b*x^2)) - (2*a*Log[a + b*x^2])/b^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{x^9}{(ab+b^2x^2)^4}\,dx$$
$$= \frac{1}{2}b^4\text{Subst}\left(\int \frac{x^4}{(ab+b^2x)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\text{Subst}\left(\int \left(\frac{1}{b^8}+\frac{a^4}{b^8(a+bx)^4}-\frac{4a^3}{b^8(a+bx)^3}+\frac{6a^2}{b^8(a+bx)^2}-\frac{4a}{b^8(a+bx)}\right)\right.$$
$$= \frac{x^2}{2b^4}-\frac{a^4}{6b^5(a+bx^2)^3}+\frac{a^3}{b^5(a+bx^2)^2}-\frac{3a^2}{b^5(a+bx^2)}-\frac{2a\log(a+bx^2)}{b^5}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.77

$$-\frac{-3bx^2+\frac{a^2(13a^2+30abx^2+18b^2x^4)}{(a+bx^2)^3}+12a\log(a+bx^2)}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] -1/6*(-3*b*x^2 + (a^2*(13*a^2 + 30*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)^3 + 1
2*a*Log[a + b*x^2])/b^5
```

Maple [A]
time = 0.03, size = 79, normalized size = 1.03

method	result	size
norman	$\frac{\frac{x^8}{2b}-\frac{11a^4}{3b^5}-\frac{6a^2x^4}{b^3}-\frac{9a^3x^2}{b^4}}{(b\,x^2+a)^3}-\frac{2a\ln(b\,x^2+a)}{b^5}$	65
default	$\frac{x^2}{2b^4}-\frac{a\left(-\frac{2a^2}{b(b\,x^2+a)^2}+\frac{a^3}{3b(b\,x^2+a)^3}+\frac{6a}{b(b\,x^2+a)}+\frac{4\ln(b\,x^2+a)}{b}\right)}{2b^4}$	79
risch	$\frac{x^2}{2b^4}+\frac{-3a^2b\,x^4-5a^3x^2-\frac{13a^4}{6b}}{b^4(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}-\frac{2a\ln(b\,x^2+a)}{b^5}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2/b^4-1/2*a/b^4*(-2*a^2/b/(b*x^2+a)^2+1/3*a^3/b/(b*x^2+a)^3+6*a/b/(b*
x^2+a)+4/b*ln(b*x^2+a))
```

Maxima [A]
time = 0.29, size = 88, normalized size = 1.14

$$-\frac{18\,a^2b^2x^4+30\,a^3bx^2+13\,a^4}{6\,(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)}+\frac{x^2}{2\,b^4}-\frac{2\,a\log(bx^2+a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(18*a^2*b^2*x^4 + 30*a^3*b*x^2 + 13*a^4)/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^
2*b^6*x^2 + a^3*b^5) + 1/2*x^2/b^4 - 2*a*log(b*x^2 + a)/b^5
```

Fricas [A]
time = 0.33, size = 124, normalized size = 1.61

$$\frac{3\,b^4x^8 + 9\,ab^3x^6 - 9\,a^2b^2x^4 - 27\,a^3bx^2 - 13\,a^4 - 12\,(ab^3x^6 + 3\,a^2b^2x^4 + 3\,a^3bx^2 + a^4)\log\left(bx^2 + a\right)}{6\,(b^8x^6 + 3\,ab^7x^4 + 3\,a^2b^6x^2 + a^3b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*x^8 + 9*a*b^3*x^6 - 9*a^2*b^2*x^4 - 27*a^3*b*x^2 - 13*a^4 - 12*(
a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^8*x^6 + 3
*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)
```

Sympy [A]
time = 0.25, size = 90, normalized size = 1.17

$$-\frac{2a\log\left(a + bx^2\right)}{b^5} + \frac{-13a^4 - 30a^3bx^2 - 18a^2b^2x^4}{6a^3b^5 + 18a^2b^6x^2 + 18ab^7x^4 + 6b^8x^6} + \frac{x^2}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -2*a*log(a + b*x**2)/b**5 + (-13*a**4 - 30*a**3*b*x**2 - 18*a**2*b**2*x**4)
/(6*a**3*b**5 + 18*a**2*b**6*x**2 + 18*a*b**7*x**4 + 6*b**8*x**6) + x**2/(2
*b**4)
```

Giac [A]
time = 3.59, size = 73, normalized size = 0.95

$$\frac{x^2}{2\,b^4} - \frac{2\,a\log\left(|bx^2 + a|\right)}{b^5} + \frac{22\,ab^3x^6 + 48\,a^2b^2x^4 + 36\,a^3bx^2 + 9\,a^4}{6\,(bx^2 + a)^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^4 - 2*a*log(abs(b*x^2 + a))/b^5 + 1/6*(22*a*b^3*x^6 + 48*a^2*b^2*
x^4 + 36*a^3*b*x^2 + 9*a^4)/((b*x^2 + a)^3*b^5)
```

Mupad [B]
time = 4.51, size = 88, normalized size = 1.14

$$\frac{x^2}{2\,b^4} - \frac{\frac{13\,a^4}{6\,b} + 5\,a^3\,x^2 + 3\,a^2\,b\,x^4}{a^3\,b^4 + 3\,a^2\,b^5\,x^2 + 3\,a\,b^6\,x^4 + b^7\,x^6} - \frac{2\,a\,\ln\left(b\,x^2 + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] x^2/(2*b^4) - ((13*a^4)/(6*b) + 5*a^3*x^2 + 3*a^2*b*x^4)/(a^3*b^4 + b^7*x^6
 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2) - (2*a*log(a + b*x^2))/b^5
```

3.495 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{a^3}{6b^4\left(a+bx^2\right)^3}-\frac{3a^2}{4b^4\left(a+bx^2\right)^2}+\frac{3a}{2b^4\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

```
[Out] 1/6*a^3/b^4/(b*x^2+a)^3-3/4*a^2/b^4/(b*x^2+a)^2+3/2*a/b^4/(b*x^2+a)+1/2*ln(
b*x^2+a)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 45}

$$\frac{a^3}{6b^4\left(a+bx^2\right)^3}-\frac{3a^2}{4b^4\left(a+bx^2\right)^2}+\frac{3a}{2b^4\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^3/(6*b^4*(a + b*x^2)^3) - (3*a^2)/(4*b^4*(a + b*x^2)^2) + (3*a)/(2*b^4*(a
 + b*x^2)) + Log[a + b*x^2]/(2*b^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{x^7}{(ab+b^2x^2)^4}\,dx$$
$$= \frac{1}{2}b^4\text{Subst}\left(\int \frac{x^3}{(ab+b^2x)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\text{Subst}\left(\int \left(-\frac{a^3}{b^7(a+bx)^4}+\frac{3a^2}{b^7(a+bx)^3}-\frac{3a}{b^7(a+bx)^2}+\frac{1}{b^7(a+bx)}\right)\,dx,$$
$$= \frac{a^3}{6b^4(a+bx^2)^3}-\frac{3a^2}{4b^4(a+bx^2)^2}+\frac{3a}{2b^4(a+bx^2)}+\frac{\log(a+bx^2)}{2b^4}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.70

$$\frac{\frac{a(11a^2+27abx^2+18b^2x^4)}{(a+bx^2)^3}+6\log(a+bx^2)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
[Out] ((a*(11*a^2 + 27*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)^3 + 6*Log[a + b*x^2])/(
12*b^4)
```

Maple [A]
time = 0.05, size = 64, normalized size = 0.90

method	result	size
norman	$\frac{\frac{11a^3}{12b^4}+\frac{3a\,x^4}{2b^2}+\frac{9a^2x^2}{4b^3}}{(b\,x^2+a)^3}+\frac{\ln(b\,x^2+a)}{2b^4}$	54
default	$\frac{a^3}{6b^4(b\,x^2+a)^3}-\frac{3a^2}{4b^4(b\,x^2+a)^2}+\frac{3a}{2b^4(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^4}$	64
risch	$\frac{\frac{11a^3}{12b^4}+\frac{3a\,x^4}{2b^2}+\frac{9a^2x^2}{4b^3}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}+\frac{\ln(b\,x^2+a)}{2b^4}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/6*a^3/b^4/(b*x^2+a)^3-3/4*a^2/b^4/(b*x^2+a)^2+3/2*a/b^4/(b*x^2+a)+1/2*ln(
b*x^2+a)/b^4
```

Maxima [A]
time = 0.29, size = 77, normalized size = 1.08

$$\frac{18\,ab^2x^4+27\,a^2bx^2+11\,a^3}{12\,(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)}+\frac{\log(bx^2+a)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3)/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*
b^5*x^2 + a^3*b^4) + 1/2*log(b*x^2 + a)/b^4
```

Fricas [A]
time = 0.36, size = 102, normalized size = 1.44

$$\frac{18\,ab^2x^4+27\,a^2bx^2+11\,a^3+6\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\log\left(bx^2+a\right)}{12\,(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3 + 6*(b^3*x^6 + 3*a*b^2*x^4 + 3*a
^2*b*x^2 + a^3)*log(b*x^2 + a))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^
3*b^4)
```

Sympy [A]
time = 0.21, size = 76, normalized size = 1.07

$$\frac{11a^3+27a^2bx^2+18ab^2x^4}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (11*a**3 + 27*a**2*b*x**2 + 18*a*b**2*x**4)/(12*a**3*b**4 + 36*a**2*b**5*x*
*2 + 36*a*b**6*x**4 + 12*b**7*x**6) + log(a + b*x**2)/(2*b**4)
```

Giac [A]
time = 3.94, size = 53, normalized size = 0.75

$$\frac{\log\left(|bx^2+a|\right)}{2\,b^4}-\frac{11\,b^2x^6+15\,abx^4+6\,a^2x^2}{12\,(bx^2+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^4 - 1/12*(11*b^2*x^6 + 15*a*b*x^4 + 6*a^2*x^2)/((
b*x^2 + a)^3*b^3)
```

Mupad [B]
time = 4.33, size = 75, normalized size = 1.06

$$\frac{\frac{11\,a^3}{12\,b^4}+\frac{3\,a\,x^4}{2\,b^2}+\frac{9\,a^2\,x^2}{4\,b^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}+\frac{\ln\left(b\,x^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((11*a^3)/(12*b^4) + (3*a*x^4)/(2*b^2) + (9*a^2*x^2)/(4*b^3))/(a^3 + b^3*x^
6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) + log(a + b*x^2)/(2*b^4)
```

3.496 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^2} \, dx$

Optimal. Leaf size=19

$$\frac{x^6}{6a\left(a+bx^2\right)^3}$$

```
[Out] 1/6*x^6/a/(b*x^2+a)^3
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 270}

$$\frac{x^6}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x^6/(6*a*(a + b*x^2)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^2} \, dx &= b^4 \int \frac{x^5}{\left(ab+b^2x^2\right)^4} \, dx\\ &= \frac{x^6}{6a\left(a+bx^2\right)^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.84

$$-\frac{a^2+3abx^2+3b^2x^4}{6b^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*(a^2 + 3*a*b*x^2 + 3*b^2*x^4)/(b^3*(a + b*x^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.
time = 0.03, size = 48, normalized size = 2.53

method	result	size
norman	$\frac{-\frac{x^4}{2b}-\frac{a x^2}{2b^2}-\frac{a^2}{6b^3}}{\left(b x^2+a\right)^3}$	37
default	$\frac{a}{2b^3\left(b x^2+a\right)^2}-\frac{1}{2b^3\left(b x^2+a\right)}-\frac{a^2}{6b^3\left(b x^2+a\right)^3}$	48
gosper	$-\frac{3b^2x^4+3ab x^2+a^2}{6(b x^2+a)(b^2x^4+2ab x^2+a^2)b^3}$	54
risch	$\frac{-\frac{x^4}{2b}-\frac{a x^2}{2b^2}-\frac{a^2}{6b^3}}{(b x^2+a)(b^2x^4+2ab x^2+a^2)}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/b^3/(b*x^2+a)^2-1/2/b^3/(b*x^2+a)-1/6*a^2/b^3/(b*x^2+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(17) = 34$.
time = 0.33, size = 58, normalized size = 3.05

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 +
 a^3*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(17) = 34$.
time = 0.34, size = 58, normalized size = 3.05

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 +
 a^3*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(14) = 28$.
time = 0.17, size = 60, normalized size = 3.16

$$\frac{-a^2 - 3abx^2 - 3b^2x^4}{6a^3b^3 + 18a^2b^4x^2 + 18ab^5x^4 + 6b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*a**3*b**3 + 18*a**2*b**4*x**2 + 18*a*
b**5*x**4 + 6*b**6*x**6)
```

Giac [A]
time = 4.02, size = 33, normalized size = 1.74

$$-\frac{3\,b^2x^4 + 3\,abx^2 + a^2}{6\,(bx^2 + a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/((b*x^2 + a)^3*b^3)
```

Mupad [B]
time = 4.29, size = 60, normalized size = 3.16

$$-\frac{a^2 + 3\,a\,b\,x^2 + 3\,b^2\,x^4}{6\,a^3\,b^3 + 18\,a^2\,b^4\,x^2 + 18\,a\,b^5\,x^4 + 6\,b^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] -(a^2 + 3*b^2*x^4 + 3*a*b*x^2)/(6*a^3*b^3 + 6*b^6*x^6 + 18*a*b^5*x^4 + 18*a
^2*b^4*x^2)
```

3.497 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=34

$$\frac{a}{6b^2\left(a+bx^2\right)^3}-\frac{1}{4b^2\left(a+bx^2\right)^2}$$

```
[Out] 1/6*a/b^2/(b*x^2+a)^3-1/4/b^2/(b*x^2+a)^2
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$\frac{a}{6b^2\left(a+bx^2\right)^3}-\frac{1}{4b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a/(6*b^2*(a + b*x^2)^3) - 1/(4*b^2*(a + b*x^2)^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{x^3}{(ab+b^2x^2)^4}\,dx$$
$$= \frac{1}{2}b^4\text{Subst}\left(\int \frac{x}{(ab+b^2x)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\text{Subst}\left(\int \left(-\frac{a}{b^5(a+bx)^4}+\frac{1}{b^5(a+bx)^3}\right)\,dx, x, x^2\right)$$
$$= \frac{a}{6b^2(a+bx^2)^3}-\frac{1}{4b^2(a+bx^2)^2}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.71

$$-\frac{a+3bx^2}{12b^2(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/12*(a + 3*b*x^2)/(b^2*(a + b*x^2)^3)
```

Maple [A]
time = 0.03, size = 31, normalized size = 0.91

method	result	size
norman	$\frac{-\frac{x^2}{4b}-\frac{a}{12b^2}}{(b\,x^2+a)^3}$	26
default	$\frac{a}{6b^2(b\,x^2+a)^3}-\frac{1}{4b^2(b\,x^2+a)^2}$	31
gosper	$-\frac{3b\,x^2+a}{12(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)b^2}$	43
risch	$\frac{-\frac{x^2}{4b}-\frac{a}{12b^2}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a/b^2/(b*x^2+a)^3-1/4/b^2/(b*x^2+a)^2
```

Maxima [A]
time = 0.30, size = 47, normalized size = 1.38

$$-\frac{3\,bx^2+a}{12\,(b^5x^6+3\,ab^4x^4+3\,a^2b^3x^2+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*b^2)
```

Fricas [A]
time = 0.35, size = 47, normalized size = 1.38

$$-\frac{3\,bx^2+a}{12\,(b^5x^6+3\,ab^4x^4+3\,a^2b^3x^2+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [A]
time = 0.16, size = 48, normalized size = 1.41

$$\frac{-a-3bx^2}{12a^3b^2+36a^2b^3x^2+36ab^4x^4+12b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-a - 3*b*x**2)/(12*a**3*b**2 + 36*a**2*b**3*x**2 + 36*a*b**4*x**4 + 12*b**
5*x**6)
```

Giac [A]
time = 3.75, size = 22, normalized size = 0.65

$$-\frac{3\,bx^2+a}{12\,(bx^2+a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*(3*b*x^2 + a)/((b*x^2 + a)^3*b^2)
```

Mupad [B]
time = 4.23, size = 48, normalized size = 1.41

$$-\frac{\frac{a}{12\,b^2}+\frac{x^2}{4\,b}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] -(a/(12*b^2) + x^2/(4*b))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4)
```

3.498 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{6b\left(a+bx^2\right)^3}$$

```
[Out] -1/6/b/(b*x^2+a)^3
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {28, 267}

$$-\frac{1}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*1/(b*(a + b*x^2)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{x}{\left(ab+b^2x^2\right)^4}\,dx\\ &= -\frac{1}{6b\left(a+bx^2\right)^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*1/(b*(a + b*x^2)^3)
```

Maple [A]
time = 0.02, size = 15, normalized size = 0.94

method	result	size
default	$-\frac{1}{6b\left(b\,x^{2}+a\right)^{3}}$	15
norman	$-\frac{1}{6b\left(b\,x^{2}+a\right)^{3}}$	15
gosper	$-\frac{1}{6\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)b}$	35
risch	$-\frac{1}{6\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)b}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b/(b*x^2+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.
time = 0.32, size = 37, normalized size = 2.31

$$-\frac{1}{6\left(b^{4}x^{6}+3\,ab^{3}x^{4}+3\,a^{2}b^{2}x^{2}+a^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.
time = 0.32, size = 37, normalized size = 2.31

$$-\frac{1}{6\left(b^{4}x^{6}+3\,ab^{3}x^{4}+3\,a^{2}b^{2}x^{2}+a^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

time = 0.14, size = 39, normalized size = 2.44

$$-\frac{1}{6a^3b + 18a^2b^2x^2 + 18ab^3x^4 + 6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -1/(6*a**3*b + 18*a**2*b**2*x**2 + 18*a*b**3*x**4 + 6*b**4*x**6)
```

Giac [A]

time = 2.81, size = 14, normalized size = 0.88

$$-\frac{1}{6(bx^2 + a)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/6/((b*x^2 + a)^3*b)
```

Mupad [B]

time = 4.28, size = 39, normalized size = 2.44

$$-\frac{1}{6\,a^3\,b + 18\,a^2\,b^2\,x^2 + 18\,a\,b^3\,x^4 + 6\,b^4\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] -1/(6*a^3*b + 6*b^4*x^6 + 18*a*b^3*x^4 + 18*a^2*b^2*x^2)
```

3.499 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=70

$$\frac{1}{6a\left(a+bx^2\right)^3}+\frac{1}{4a^2\left(a+bx^2\right)^2}+\frac{1}{2a^3\left(a+bx^2\right)}+\frac{\log(x)}{a^4}-\frac{\log\left(a+bx^2\right)}{2a^4}$$

```
[Out] 1/6/a/(b*x^2+a)^3+1/4/a^2/(b*x^2+a)^2+1/2/a^3/(b*x^2+a)+ln(x)/a^4-1/2*ln(b*
x^2+a)/a^4
```

Rubi [A]

time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 46}

$$-\frac{\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)}{a^4}+\frac{1}{2a^3\left(a+bx^2\right)}+\frac{1}{4a^2\left(a+bx^2\right)^2}+\frac{1}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] 1/(6*a*(a + b*x^2)^3) + 1/(4*a^2*(a + b*x^2)^2) + 1/(2*a^3*(a + b*x^2)) + L
og[x]/a^4 - Log[a + b*x^2]/(2*a^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{1}{x(ab+b^2x^2)^4}\,dx$$
$$= \frac{1}{2}b^4\text{Subst}\left(\int \frac{1}{x(ab+b^2x)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^4\text{Subst}\left(\int \left(\frac{1}{a^4b^4x} - \frac{1}{ab^3(a+bx)^4} - \frac{1}{a^2b^3(a+bx)^3} - \frac{1}{a^3b^3(a+bx)^2} - \frac{}{a}\right.\right.$$
$$= \frac{1}{6a(a+bx^2)^3} + \frac{1}{4a^2(a+bx^2)^2} + \frac{1}{2a^3(a+bx^2)} + \frac{\log(x)}{a^4} - \frac{\log(a+bx^2)}{2a^4}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.77

$$\frac{\frac{a(11a^2+15abx^2+6b^2x^4)}{(a+bx^2)^3} + 12\log(x) - 6\log(a+bx^2)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((a*(11*a^2 + 15*a*b*x^2 + 6*b^2*x^4))/(a + b*x^2)^3 + 12*Log[x] - 6*Log[a
+ b*x^2])/(12*a^4)
```

Maple [A]
time = 0.04, size = 76, normalized size = 1.09

method	result	size
norman	$\frac{-\frac{3bx^2}{2a^2}-\frac{9b^2x^4}{4a^3}-\frac{11b^3x^6}{12a^4}}{(bx^2+a)^3}+\frac{\ln(x)}{a^4}-\frac{\ln(bx^2+a)}{2a^4}$	63
default	$-\frac{b\left(-\frac{a^2}{2b(bx^2+a)^2}-\frac{a}{b(bx^2+a)}-\frac{a^3}{3b(bx^2+a)^3}+\frac{\ln(bx^2+a)}{b}\right)}{2a^4}+\frac{\ln(x)}{a^4}$	76
risch	$\frac{\frac{b^2x^4}{2a^3}+\frac{5bx^2}{4a^2}+\frac{11}{12a}}{(bx^2+a)(b^2x^4+2abx^2+a^2)}+\frac{\ln(x)}{a^4}-\frac{\ln(bx^2+a)}{2a^4}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/a^4*(-1/2*a^2/b/(b*x^2+a)^2-a/b/(b*x^2+a)-1/3*a^3/b/(b*x^2+a)^3+1/b*
ln(b*x^2+a))+ln(x)/a^4
```

Maxima [A]
time = 0.30, size = 82, normalized size = 1.17

$$\frac{6\,b^2x^4+15\,abx^2+11\,a^2}{12\,(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6)} - \frac{\log(bx^2+a)}{2\,a^4} + \frac{\log(x^2)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^2*x^4 + 15*a*b*x^2 + 11*a^2)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5
*b*x^2 + a^6) - 1/2*log(b*x^2 + a)/a^4 + 1/2*log(x^2)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(62) = 124$.
time = 0.35, size = 134, normalized size = 1.91

$$\frac{6\,ab^2x^4+15\,a^2bx^2+11\,a^3-6\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\log\left(bx^2+a\right)+12\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\log\left(x\right)}{12\,(a^4b^3x^6+3\,a^5b^2x^4+3\,a^6bx^2+a^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*b^2*x^4 + 15*a^2*b*x^2 + 11*a^3 - 6*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^
2*b*x^2 + a^3)*log(b*x^2 + a) + 12*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3)*log(x))/(a^4*b^3*x^6 + 3*a^5*b^2*x^4 + 3*a^6*b*x^2 + a^7)
```

Sympy [A]
time = 0.25, size = 80, normalized size = 1.14

$$\frac{11a^2+15abx^2+6b^2x^4}{12a^6+36a^5bx^2+36a^4b^2x^4+12a^3b^3x^6}+\frac{\log\left(x\right)}{a^4}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (11*a**2 + 15*a*b*x**2 + 6*b**2*x**4)/(12*a**6 + 36*a**5*b*x**2 + 36*a**4*b
**2*x**4 + 12*a**3*b**3*x**6) + log(x)/a**4 - log(a/b + x**2)/(2*a**4)
```

Giac [A]
time = 2.79, size = 70, normalized size = 1.00

$$\frac{\log\left(x^2\right)}{2\,a^4}-\frac{\log\left(|bx^2+a|\right)}{2\,a^4}+\frac{11\,b^3x^6+39\,ab^2x^4+48\,a^2bx^2+22\,a^3}{12\,(bx^2+a)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^4 - 1/2*log(abs(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6 + 39*a*b^
2*x^4 + 48*a^2*b*x^2 + 22*a^3)/((b*x^2 + a)^3*a^4)
```

Mupad [B]
time = 4.47, size = 78, normalized size = 1.11

$$\frac{\ln\left(x\right)}{a^4}+\frac{\frac{11}{12\,a}+\frac{5\,b\,x^2}{4\,a^2}+\frac{b^2\,x^4}{2\,a^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] log(x)/a^4 + (11/(12*a) + (5*b*x^2)/(4*a^2) + (b^2*x^4)/(2*a^3))/(a^3 + b^3
*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) - log(a + b*x^2)/(2*a^4)
```

3.500 $\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{1}{2a^4x^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{4b\log(x)}{a^5}+\frac{2b\log\left(a+bx^2\right)}{a^5}$$

```
[Out] -1/2/a^4/x^2-1/6*b/a^2/(b*x^2+a)^3-1/2*b/a^3/(b*x^2+a)^2-3/2*b/a^4/(b*x^2+a
)-4*b*ln(x)/a^5+2*b*ln(b*x^2+a)/a^5
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 46}

$$\frac{2b\log\left(a+bx^2\right)}{a^5}-\frac{4b\log(x)}{a^5}-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{1}{2a^4x^2}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/2*1/(a^4*x^2) - b/(6*a^2*(a + b*x^2)^3) - b/(2*a^3*(a + b*x^2)^2) - (3*b
)/(2*a^4*(a + b*x^2)) - (4*b*Log[x])/a^5 + (2*b*Log[a + b*x^2])/a^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^2} dx = b^4 \int \frac{1}{x^3 (ab + b^2x^2)^4} dx$$
$$= \frac{1}{2} b^4 \text{Subst}\left(\int \frac{1}{x^2 (ab + b^2x)^4} dx, x, x^2\right)$$
$$= \frac{1}{2} b^4 \text{Subst}\left(\int \left(\frac{1}{a^4b^4x^2} - \frac{4}{a^5b^3x} + \frac{1}{a^2b^2(a+bx)^4} + \frac{2}{a^3b^2(a+bx)^3} + \frac{}{a^4b^2(a}\right.\right.$$
$$= -\frac{1}{2a^4x^2} - \frac{b}{6a^2 (a + bx^2)^3} - \frac{b}{2a^3 (a + bx^2)^2} - \frac{3b}{2a^4 (a + bx^2)} - \frac{4b \log(x)}{a^5} +$$

Mathematica [A]
time = 0.05, size = 70, normalized size = 0.83

$$-\frac{\frac{a(3a^3+22a^2bx^2+30ab^2x^4+12b^3x^6)}{x^2(a+bx^2)^3} + 24b\log(x) - 12b\log(a + bx^2)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/6*((a*(3*a^3 + 22*a^2*b*x^2 + 30*a*b^2*x^4 + 12*b^3*x^6))/(x^2*(a + b*x^
2)^3) + 24*b*Log[x] - 12*b*Log[a + b*x^2])/a^5
```

Maple [A]
time = 0.04, size = 89, normalized size = 1.06

method	result	size
norman	$\frac{-\frac{1}{2a}+\frac{6b^2x^4}{a^3}+\frac{9b^3x^6}{a^4}+\frac{11b^4x^8}{3a^5}}{x^2(b x^2+a)^3} - \frac{4b\ln(x)}{a^5} + \frac{2b\ln(b x^2+a)}{a^5}$	76
default	$\frac{b^2\left(-\frac{3a}{b(b x^2+a)} - \frac{a^2}{b(b x^2+a)^2} - \frac{a^3}{3b(b x^2+a)^3} + \frac{4\ln(b x^2+a)}{b}\right)}{2a^5} - \frac{1}{2a^4x^2} - \frac{4b\ln(x)}{a^5}$	89
risch	$\frac{-\frac{2b^3x^6}{a^4} - \frac{5b^2x^4}{a^3} - \frac{11b x^2}{3a^2} - \frac{1}{2a}}{x^2(b^2x^4+2ab x^2+a^2)(b x^2+a)} - \frac{4b\ln(x)}{a^5} + \frac{2b\ln(-b x^2-a)}{a^5}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/a^5*(-3*a/b/(b*x^2+a)-a^2/b/(b*x^2+a)^2-1/3*a^3/b/(b*x^2+a)^3+4/b*l
n(b*x^2+a))-1/2/a^4/x^2-4*b*ln(x)/a^5
```

Maxima [A]
time = 0.30, size = 99, normalized size = 1.18

$$-\frac{12 b^3x^6 + 30 ab^2x^4 + 22 a^2bx^2 + 3 a^3}{6 (a^4b^3x^8 + 3 a^5b^2x^6 + 3 a^6bx^4 + a^7x^2)} + \frac{2 b \log(bx^2 + a)}{a^5} - \frac{2 b \log(x^2)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(12*b^3*x^6 + 30*a*b^2*x^4 + 22*a^2*b*x^2 + 3*a^3)/(a^4*b^3*x^8 + 3*a^
5*b^2*x^6 + 3*a^6*b*x^4 + a^7*x^2) + 2*b*log(b*x^2 + a)/a^5 - 2*b*log(x^2)/
a^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(76) = 152$.
time = 0.35, size = 163, normalized size = 1.94

$$-\frac{12\,ab^3x^6 + 30\,a^2b^2x^4 + 22\,a^3bx^2 + 3\,a^4 - 12\,(b^4x^8 + 3\,ab^3x^6 + 3\,a^2b^2x^4 + a^3bx^2)\log\left(bx^2 + a\right) + 24\,(b^4x^8 + 3\,ab^3x^6 + 3\,a^2b^2x^4 + a^3bx^2)\log\left(x\right)}{6\,(a^5b^3x^8 + 3\,a^6b^2x^6 + 3\,a^7bx^4 + a^8x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*b^3*x^6 + 30*a^2*b^2*x^4 + 22*a^3*b*x^2 + 3*a^4 - 12*(b^4*x^8 +
3*a*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(b*x^2 + a) + 24*(b^4*x^8 + 3*a
*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(x))/(a^5*b^3*x^8 + 3*a^6*b^2*x^6
+ 3*a^7*b*x^4 + a^8*x^2)
```

Sympy [A]
time = 0.30, size = 102, normalized size = 1.21

$$\frac{-3a^3 - 22a^2bx^2 - 30ab^2x^4 - 12b^3x^6}{6a^7x^2 + 18a^6bx^4 + 18a^5b^2x^6 + 6a^4b^3x^8} - \frac{4b\log\left(x\right)}{a^5} + \frac{2b\log\left(\frac{a}{b} + x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-3*a**3 - 22*a**2*b*x**2 - 30*a*b**2*x**4 - 12*b**3*x**6)/(6*a**7*x**2 + 1
8*a**6*b*x**4 + 18*a**5*b**2*x**6 + 6*a**4*b**3*x**8) - 4*b*log(x)/a**5 + 2
*b*log(a/b + x**2)/a**5
```

Giac [A]
time = 3.94, size = 93, normalized size = 1.11

$$-\frac{2\,b\log\left(x^2\right)}{a^5} + \frac{2\,b\log\left(|bx^2 + a|\right)}{a^5} + \frac{4\,bx^2 - a}{2\,a^5x^2} - \frac{22\,b^4x^6 + 75\,ab^3x^4 + 87\,a^2b^2x^2 + 35\,a^3b}{6\,(bx^2 + a)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -2*b*log(x^2)/a^5 + 2*b*log(abs(b*x^2 + a))/a^5 + 1/2*(4*b*x^2 - a)/(a^5*x^
2) - 1/6*(22*b^4*x^6 + 75*a*b^3*x^4 + 87*a^2*b^2*x^2 + 35*a^3*b)/((b*x^2 +
a)^3*a^5)
```

Mupad [B]
time = 0.15, size = 97, normalized size = 1.15

$$\frac{2\,b\,\ln\left(b\,x^2+a\right)}{a^5}-\frac{\frac{1}{2\,a}+\frac{11\,b\,x^2}{3\,a^2}+\frac{5\,b^2\,x^4}{a^3}+\frac{2\,b^3\,x^6}{a^4}}{a^3\,x^2+3\,a^2\,b\,x^4+3\,a\,b^2\,x^6+b^3\,x^8}-\frac{4\,b\,\ln\left(x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] (2*b*log(a + b*x^2))/a^5 - (1/(2*a) + (11*b*x^2)/(3*a^2) + (5*b^2*x^4)/a^3
+ (2*b^3*x^6)/a^4)/(a^3*x^2 + b^3*x^8 + 3*a^2*b*x^4 + 3*a*b^2*x^6) - (4*b*l
og(x))/a^5
```

3.501 $\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{4a^4x^4}+\frac{2b}{a^5x^2}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}+\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{10b^2\log(x)}{a^6}-\frac{5b^2\log\left(a+bx^2\right)}{a^6}$$

```
[Out] -1/4/a^4/x^4+2*b/a^5/x^2+1/6*b^2/a^3/(b*x^2+a)^3+3/4*b^2/a^4/(b*x^2+a)^2+3*
b^2/a^5/(b*x^2+a)+10*b^2*ln(x)/a^6-5*b^2*ln(b*x^2+a)/a^6
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 272, 46}

$$-\frac{5b^2\log\left(a+bx^2\right)}{a^6}+\frac{10b^2\log(x)}{a^6}+\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{2b}{a^5x^2}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}-\frac{1}{4a^4x^4}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/4*1/(a^4*x^4) + (2*b)/(a^5*x^2) + b^2/(6*a^3*(a + b*x^2)^3) + (3*b^2)/(4
*a^4*(a + b*x^2)^2) + (3*b^2)/(a^5*(a + b*x^2)) + (10*b^2*Log[x])/a^6 - (5*
b^2*Log[a + b*x^2])/a^6
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5 (a^2+2abx^2+b^2x^4)^2}\,dx = b^4 \int \frac{1}{x^5 (ab+b^2x^2)^4}\,dx$$
$$= \frac{1}{2} b^4 \text{Subst}\left(\int \frac{1}{x^3 (ab+b^2x)^4}\,dx, x, x^2\right)$$
$$= \frac{1}{2} b^4 \text{Subst}\left(\int \left(\frac{1}{a^4b^4x^3} - \frac{4}{a^5b^3x^2} + \frac{10}{a^6b^2x} - \frac{1}{a^3b(a+bx)^4} - \frac{3}{a^4b(a+bx)^3} - \right.\right.$$
$$= -\frac{1}{4a^4x^4} + \frac{2b}{a^5x^2} + \frac{b^2}{6a^3 (a+bx^2)^3} + \frac{3b^2}{4a^4 (a+bx^2)^2} + \frac{3b^2}{a^5 (a+bx^2)} + \frac{10b^2 \mathrm{l}}{a}$$

Mathematica [A]
time = 0.04, size = 85, normalized size = 0.84

$$\frac{\frac{a(-3a^4+15a^3bx^2+110a^2b^2x^4+150ab^3x^6+60b^4x^8)}{x^4(a+bx^2)^3} + 120b^2 \log(x) - 60b^2 \log(a+bx^2)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((a*(-3*a^4 + 15*a^3*b*x^2 + 110*a^2*b^2*x^4 + 150*a*b^3*x^6 + 60*b^4*x^8))
/(x^4*(a + b*x^2)^3) + 120*b^2*Log[x] - 60*b^2*Log[a + b*x^2])/(12*a^6)
```

Maple [A]
time = 0.05, size = 100, normalized size = 0.99

method	result	size
norman	$\frac{-\frac{1}{4a}+\frac{5b\,x^2}{4a^2}-\frac{15b^3x^6}{a^4}-\frac{45b^4x^8}{2a^5}-\frac{55b^5x^{10}}{6a^6}}{x^4(b\,x^2+a)^3}+\frac{10b^2\ln(x)}{a^6}-\frac{5b^2\ln(b\,x^2+a)}{a^6}$	89
default	$-\frac{b^3\left(-\frac{6a}{b(b\,x^2+a)}-\frac{a^3}{3b(b\,x^2+a)^3}-\frac{3a^2}{2b(b\,x^2+a)^2}+\frac{10\ln(b\,x^2+a)}{b}\right)}{2a^6}-\frac{1}{4a^4x^4}+\frac{2b}{a^5x^2}+\frac{10b^2\ln(x)}{a^6}$	100
risch	$\frac{\frac{5b^4x^8}{a^5}+\frac{25b^3x^6}{2a^4}+\frac{55b^2x^4}{6a^3}+\frac{5b\,x^2}{4a^2}-\frac{1}{4a}}{x^4(b^2x^4+2ab\,x^2+a^2)(b\,x^2+a)}+\frac{10b^2\ln(x)}{a^6}-\frac{5b^2\ln(b\,x^2+a)}{a^6}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^3/a^6*(-6*a/b/(b*x^2+a)-1/3*a^3/b/(b*x^2+a)^3-3/2*a^2/b/(b*x^2+a)^2+
10/b*ln(b*x^2+a))-1/4/a^4/x^4+2*b/a^5/x^2+10*b^2*ln(x)/a^6
```

Maxima [A]
time = 0.30, size = 114, normalized size = 1.13

$$\frac{60\,b^4x^8+150\,ab^3x^6+110\,a^2b^2x^4+15\,a^3bx^2-3\,a^4}{12\,(a^5b^3x^{10}+3\,a^6b^2x^8+3\,a^7bx^6+a^8x^4)} - \frac{5\,b^2\log(bx^2+a)}{a^6} + \frac{5\,b^2\log(x^2)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^8 + 150*a*b^3*x^6 + 110*a^2*b^2*x^4 + 15*a^3*b*x^2 - 3*a^4)/
(a^5*b^3*x^10 + 3*a^6*b^2*x^8 + 3*a^7*b*x^6 + a^8*x^4) - 5*b^2*log(b*x^2 +
a)/a^6 + 5*b^2*log(x^2)/a^6
```

Fricas [A]
time = 0.34, size = 178, normalized size = 1.76

$$\frac{60\,ab^4x^8+150\,a^2b^3x^6+110\,a^3b^2x^4+15\,a^4bx^2-3\,a^5-60\,(b^5x^{10}+3\,ab^4x^8+3\,a^2b^3x^6+a^3b^2x^4)\log\left(bx^2+a\right)+120\,(b^5x^{10}+3\,ab^4x^8+3\,a^2b^3x^6+a^3b^2x^4)\log\left(x\right)}{12\,(a^6b^3x^{10}+3\,a^7b^2x^8+3\,a^8bx^6+a^9x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^8 + 150*a^2*b^3*x^6 + 110*a^3*b^2*x^4 + 15*a^4*b*x^2 - 3*a
^5 - 60*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*x^6 + a^3*b^2*x^4)*log(b*x^2 +
a) + 120*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*x^6 + a^3*b^2*x^4)*log(x))/(a^
6*b^3*x^10 + 3*a^7*b^2*x^8 + 3*a^8*b*x^6 + a^9*x^4)
```

Sympy [A]
time = 0.33, size = 116, normalized size = 1.15

$$\frac{-3a^4+15a^3bx^2+110a^2b^2x^4+150ab^3x^6+60b^4x^8}{12a^8x^4+36a^7bx^6+36a^6b^2x^8+12a^5b^3x^{10}}+\frac{10b^2\log\left(x\right)}{a^6}-\frac{5b^2\log\left(\frac{a}{b}+x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-3*a**4 + 15*a**3*b*x**2 + 110*a**2*b**2*x**4 + 150*a*b**3*x**6 + 60*b**4*
x**8)/(12*a**8*x**4 + 36*a**7*b*x**6 + 36*a**6*b**2*x**8 + 12*a**5*b**3*x**
10) + 10*b**2*log(x)/a**6 - 5*b**2*log(a/b + x**2)/a**6
```

Giac [A]
time = 5.89, size = 108, normalized size = 1.07

$$\frac{5\,b^2\log\left(x^2\right)}{a^6}-\frac{5\,b^2\log\left(|bx^2+a|\right)}{a^6}+\frac{110\,b^5x^6+366\,ab^4x^4+411\,a^2b^3x^2+157\,a^3b^2}{12\,(bx^2+a)^3a^6}-\frac{30\,b^2x^4-8\,abx^2+a^2}{4\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5*b^2*log(x^2)/a^6 - 5*b^2*log(abs(b*x^2 + a))/a^6 + 1/12*(110*b^5*x^6 + 36
6*a*b^4*x^4 + 411*a^2*b^3*x^2 + 157*a^3*b^2)/((b*x^2 + a)^3*a^6) - 1/4*(30*
b^2*x^4 - 8*a*b*x^2 + a^2)/(a^6*x^4)
```

Mupad [B]
time = 4.66, size = 111, normalized size = 1.10

$$\frac{\frac{5\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{55\,b^2\,x^4}{6\,a^3}+\frac{25\,b^3\,x^6}{2\,a^4}+\frac{5\,b^4\,x^8}{a^5}}{a^3\,x^4+3\,a^2\,b\,x^6+3\,a\,b^2\,x^8+b^3\,x^{10}}-\frac{5\,b^2\,\ln\left(b\,x^2+a\right)}{a^6}+\frac{10\,b^2\,\ln\left(x\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((5*b*x^2)/(4*a^2) - 1/(4*a) + (55*b^2*x^4)/(6*a^3) + (25*b^3*x^6)/(2*a^4)
+ (5*b^4*x^8)/a^5)/(a^3*x^4 + b^3*x^10 + 3*a^2*b*x^6 + 3*a*b^2*x^8) - (5*b^
2*log(a + b*x^2))/a^6 + (10*b^2*log(x))/a^6
```

3.502 $\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=117

$$\frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}+\frac{231x^5}{80b^4}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{13/2}}$$

```
[Out] 231/16*a^2*x/b^6-77/16*a*x^3/b^5+231/80*x^5/b^4-1/6*x^11/b/(b*x^2+a)^3-11/2
4*x^9/b^2/(b*x^2+a)^2-33/16*x^7/b^3/(b*x^2+a)-231/16*a^(5/2)*arctan(x*b^(1/
2)/a^(1/2))/b^(13/2)
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 308, 211}

$$-\frac{231a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{13/2}}+\frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}-\frac{33x^7}{16b^3\left(a+bx^2\right)}-\frac{11x^9}{24b^2\left(a+bx^2\right)^2}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}+\frac{231x^5}{80b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (231*a^2*x)/(16*b^6) - (77*a*x^3)/(16*b^5) + (231*x^5)/(80*b^4) - x^11/(6*b
*(a + b*x^2)^3) - (11*x^9)/(24*b^2*(a + b*x^2)^2) - (33*x^7)/(16*b^3*(a + b
*x^2)) - (231*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{x^{12}}{(ab+b^2x^2)^4}\,dx$$
$$= -\frac{x^{11}}{6b(a+bx^2)^3}+\frac{1}{6}(11b^2)\int \frac{x^{10}}{(ab+b^2x^2)^3}\,dx$$
$$= -\frac{x^{11}}{6b(a+bx^2)^3}-\frac{11x^9}{24b^2(a+bx^2)^2}+\frac{33}{8}\int \frac{x^8}{(ab+b^2x^2)^2}\,dx$$
$$= -\frac{x^{11}}{6b(a+bx^2)^3}-\frac{11x^9}{24b^2(a+bx^2)^2}-\frac{33x^7}{16b^3(a+bx^2)}+\frac{231\int \frac{x^6}{ab+b^2x^2}\,dx}{16b^2}$$
$$= -\frac{x^{11}}{6b(a+bx^2)^3}-\frac{11x^9}{24b^2(a+bx^2)^2}-\frac{33x^7}{16b^3(a+bx^2)}+\frac{231\int \left(\frac{a^2}{b^4}-\frac{ax^2}{b^3}+\frac{x^4}{b^2}-\right.}{16b^2}$$
$$= \frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}+\frac{231x^5}{80b^4}-\frac{x^{11}}{6b(a+bx^2)^3}-\frac{11x^9}{24b^2(a+bx^2)^2}-\frac{33x^7}{16b^3(a+bx^2)}$$
$$= \frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}+\frac{231x^5}{80b^4}-\frac{x^{11}}{6b(a+bx^2)^3}-\frac{11x^9}{24b^2(a+bx^2)^2}-\frac{33x^7}{16b^3(a+bx^2)}$$

Mathematica [A]
time = 0.04, size = 99, normalized size = 0.85

$$\frac{3465a^5x+9240a^4bx^3+7623a^3b^2x^5+1584a^2b^3x^7-176ab^4x^9+48b^5x^{11}}{240b^6(a+bx^2)^3}-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (3465*a^5*x + 9240*a^4*b*x^3 + 7623*a^3*b^2*x^5 + 1584*a^2*b^3*x^7 - 176*a*
b^4*x^9 + 48*b^5*x^11)/(240*b^6*(a + b*x^2)^3) - (231*a^(5/2)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(16*b^(13/2))
```

Maple [A]
time = 0.05, size = 85, normalized size = 0.73

method	result
default	$\frac{\frac{1}{5}b^2x^5-\frac{4}{3}ab\,x^3+10a^2x}{b^6}-\frac{a^3\left(\frac{-\frac{89}{16}b^2x^5-\frac{59}{6}ab\,x^3-\frac{71}{16}a^2x}{\left(b\,x^2+a\right)^3}+\frac{231\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{b^6}$
risch	$\frac{x^5}{5b^4}-\frac{4a\,x^3}{3b^5}+\frac{10a^2x}{b^6}+\frac{\frac{89}{16}a^3b^2x^5+\frac{59}{6}a^4b\,x^3+\frac{71}{16}a^5x}{b^6(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}+\frac{231\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\,x-a\right)}{32b^7}-\frac{231\sqrt{-ab}\,a^2\ln\left(\sqrt{-a\right.}}{32b^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/5*b^2*x^5-4/3*a*b*x^3+10*a^2*x)-1/b^6*a^3*((-89/16*b^2*x^5-59/6*a*
b*x^3-71/16*a^2*x)/(b*x^2+a)^3+231/16/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 116, normalized size = 0.99

$$\frac{267\,a^3b^2x^5+472\,a^4bx^3+213\,a^5x}{48\,(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)}-\frac{231\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^6}+\frac{3\,b^2x^5-20\,abx^3+150\,a^2x}{15\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(267*a^3*b^2*x^5 + 472*a^4*b*x^3 + 213*a^5*x)/(b^9*x^6 + 3*a*b^8*x^4 +
 3*a^2*b^7*x^2 + a^3*b^6) - 231/16*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6
) + 1/15*(3*b^2*x^5 - 20*a*b*x^3 + 150*a^2*x)/b^6
```

Fricas [A]
time = 0.35, size = 322, normalized size = 2.75

$$\left[\frac{96\,b^5x^{11}-352\,ab^4x^9+3168\,a^2b^3x^7+15246\,a^3b^2x^5+18480\,a^4bx^3+6930\,a^5x+3465\,(a^2b^3x^6+3\,a^3b^2x^4+3\,a^4bx^2+a^5)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{480\,(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)},\ \frac{48\,b^5x^{11}-176\,ab^4x^9+1584\,a^2b^3x^7+7623\,a^3b^2x^5+9240\,a^4bx^3+3465\,a^5x-3465\,(a^2b^3x^6+3\,a^3b^2x^4+3\,a^4bx^2+a^5)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{240\,(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/480*(96*b^5*x^11 - 352*a*b^4*x^9 + 3168*a^2*b^3*x^7 + 15246*a^3*b^2*x^5
+ 18480*a^4*b*x^3 + 6930*a^5*x + 3465*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*
b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b
^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6), 1/240*(48*b^5*x^11 - 176*a
```

```
*b^4*x^9 + 1584*a^2*b^3*x^7 + 7623*a^3*b^2*x^5 + 9240*a^4*b*x^3 + 3465*a^5*
x - 3465*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*sqrt(a/b)*arctan
(b*x*sqrt(a/b)/a))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)]
```

Sympy [A]
time = 0.30, size = 172, normalized size = 1.47

$$\frac{10a^2x}{b^6} - \frac{4ax^3}{3b^5} + \frac{231\sqrt{-\frac{a^5}{b^{13}}}\log\left(x - \frac{b^6\sqrt{-\frac{a^5}{b^{13}}}}{a^2}\right)}{32} - \frac{231\sqrt{-\frac{a^5}{b^{13}}}\log\left(x + \frac{b^6\sqrt{-\frac{a^5}{b^{13}}}}{a^2}\right)}{32} + \frac{213a^5x + 472a^4bx^3 + 267a^3b^2x^5}{48a^3b^6 + 144a^2b^7x^2 + 144ab^8x^4 + 48b^9x^6} + \frac{x^5}{5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 10*a**2*x/b**6 - 4*a*x**3/(3*b**5) + 231*sqrt(-a**5/b**13)*log(x - b**6*sqr
t(-a**5/b**13)/a**2)/32 - 231*sqrt(-a**5/b**13)*log(x + b**6*sqrt(-a**5/b**
13)/a**2)/32 + (213*a**5*x + 472*a**4*b*x**3 + 267*a**3*b**2*x**5)/(48*a**3
*b**6 + 144*a**2*b**7*x**2 + 144*a*b**8*x**4 + 48*b**9*x**6) + x**5/(5*b**4
)
```

Giac [A]
time = 4.52, size = 96, normalized size = 0.82

$$-\frac{231\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^6} + \frac{267\,a^3b^2x^5 + 472\,a^4bx^3 + 213\,a^5x}{48\,(bx^2+a)^3b^6} + \frac{3\,b^{16}x^5 - 20\,ab^{15}x^3 + 150\,a^2b^{14}x}{15\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -231/16*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/48*(267*a^3*b^2*x^5 +
 472*a^4*b*x^3 + 213*a^5*x)/((b*x^2 + a)^3*b^6) + 1/15*(3*b^16*x^5 - 20*a*b
^15*x^3 + 150*a^2*b^14*x)/b^20
```

Mupad [B]
time = 0.06, size = 109, normalized size = 0.93

$$\frac{\frac{71\,a^5\,x}{16} + \frac{59\,a^4\,b\,x^3}{6} + \frac{89\,a^3\,b^2\,x^5}{16}}{a^3\,b^6 + 3\,a^2\,b^7\,x^2 + 3\,a\,b^8\,x^4 + b^9\,x^6} + \frac{x^5}{5\,b^4} - \frac{4\,a\,x^3}{3\,b^5} + \frac{10\,a^2\,x}{b^6} - \frac{231\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((71*a^5*x)/16 + (59*a^4*b*x^3)/6 + (89*a^3*b^2*x^5)/16)/(a^3*b^6 + b^9*x^6
 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2) + x^5/(5*b^4) - (4*a*x^3)/(3*b^5) + (10*a^2
*x)/b^6 - (231*a^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(16*b^(13/2))
```

3.503 $\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=104

$$-\frac{105ax}{16b^5}+\frac{35x^3}{16b^4}-\frac{x^9}{6b\left(a+bx^2\right)^3}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{21x^5}{16b^3\left(a+bx^2\right)}+\frac{105a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{11/2}}$$

```
[Out] -105/16*a*x/b^5+35/16*x^3/b^4-1/6*x^9/b/(b*x^2+a)^3-3/8*x^7/b^2/(b*x^2+a)^2
-21/16*x^5/b^3/(b*x^2+a)+105/16*a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {28, 294, 308, 211}

$$\frac{105a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{11/2}}-\frac{105ax}{16b^5}-\frac{21x^5}{16b^3\left(a+bx^2\right)}-\frac{3x^7}{8b^2\left(a+bx^2\right)^2}-\frac{x^9}{6b\left(a+bx^2\right)^3}+\frac{35x^3}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-105*a*x)/(16*b^5) + (35*x^3)/(16*b^4) - x^9/(6*b*(a + b*x^2)^3) - (3*x^7)
/(8*b^2*(a + b*x^2)^2) - (21*x^5)/(16*b^3*(a + b*x^2)) + (105*a^(3/2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2}\,dx &= b^4\int \frac{x^{10}}{(ab+b^2x^2)^4}\,dx\\
&= -\frac{x^9}{6b\,(a+bx^2)^3}+\frac{1}{2}(3b^2)\int \frac{x^8}{(ab+b^2x^2)^3}\,dx\\
&= -\frac{x^9}{6b\,(a+bx^2)^3}-\frac{3x^7}{8b^2\,(a+bx^2)^2}+\frac{21}{8}\int \frac{x^6}{(ab+b^2x^2)^2}\,dx\\
&= -\frac{x^9}{6b\,(a+bx^2)^3}-\frac{3x^7}{8b^2\,(a+bx^2)^2}-\frac{21x^5}{16b^3\,(a+bx^2)}+\frac{105\int \frac{x^4}{ab+b^2x^2}\,dx}{16b^2}\\
&= -\frac{x^9}{6b\,(a+bx^2)^3}-\frac{3x^7}{8b^2\,(a+bx^2)^2}-\frac{21x^5}{16b^3\,(a+bx^2)}+\frac{105\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a}{b^2(ab+\cdots\right.}{16b^2}\\
&= -\frac{105ax}{16b^5}+\frac{35x^3}{16b^4}-\frac{x^9}{6b\,(a+bx^2)^3}-\frac{3x^7}{8b^2\,(a+bx^2)^2}-\frac{21x^5}{16b^3\,(a+bx^2)}+\frac{(105a^2\cdots}{}\\
&= -\frac{105ax}{16b^5}+\frac{35x^3}{16b^4}-\frac{x^9}{6b\,(a+bx^2)^3}-\frac{3x^7}{8b^2\,(a+bx^2)^2}-\frac{21x^5}{16b^3\,(a+bx^2)}+\frac{105a^{3/\cdots}}{}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 89, normalized size = 0.86

$$\frac{\frac{\sqrt{b}\,x(-315a^4-840a^3bx^2-693a^2b^2x^4-144ab^3x^6+16b^4x^8)}{(a+bx^2)^3}+315a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{48b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((Sqrt[b]*x*(-315*a^4 - 840*a^3*b*x^2 - 693*a^2*b^2*x^4 - 144*a*b^3*x^6 + 1
6*b^4*x^8))/(a + b*x^2)^3 + 315*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(48*b^
(11/2))
```

Maple [A]
time = 0.04, size = 74, normalized size = 0.71

method	result	si
default	$-\frac{-\frac{1}{3} b\,x^{3}+4 a x}{b^{5}}+\frac{a^{2} \left(\frac{-\frac{55}{16} b^{2} x^{5}-\frac{35}{6} a b\,x^{3}-\frac{41}{16} a^{2} x}{\left(b\,x^{2}+a \right)^{3}}+\frac{105 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{16 \sqrt{a b}}\right)}{b^{5}}$	7
risch	$\frac{x^{3}}{3 b^{4}}-\frac{4 a x}{b^{5}}+\frac{-\frac{55}{16} a^{2} b^{2} x^{5}-\frac{35}{6} a^{3} b\,x^{3}-\frac{41}{16} a^{4} x}{b^{5} \left(b\,x^{2}+a \right) \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)}+\frac{105 \sqrt{-a b}\, a \ln \left(-\sqrt{-a b}\, x+a \right)}{32 b^{6}}-\frac{105 \sqrt{-a b}\, a \ln \left(\sqrt{-a b}\, x+a \right)}{32 b^{6}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/3*b*x^3+4*a*x)+1/b^5*a^2*((-55/16*b^2*x^5-35/6*a*b*x^3-41/16*a^2
*x)/(b*x^2+a)^3+105/16/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 104, normalized size = 1.00

$$-\frac{165\,a^2b^2x^5+280\,a^3bx^3+123\,a^4x}{48\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)}+\frac{105\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^5}+\frac{bx^3-12\,ax}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(165*a^2*b^2*x^5 + 280*a^3*b*x^3 + 123*a^4*x)/(b^8*x^6 + 3*a*b^7*x^4
+ 3*a^2*b^6*x^2 + a^3*b^5) + 105/16*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^
5) + 1/3*(b*x^3 - 12*a*x)/b^5
```

Fricas [A]
time = 0.37, size = 296, normalized size = 2.85

$$\left[\frac{32\,b^4x^9-288\,ab^3x^7-1386\,a^2b^2x^5-1680\,a^3bx^3-630\,a^4x+315\left(ab^3x^6+3\,a^2b^2x^4+3\,a^3bx^2+a^4\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{96\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)},\ \frac{16\,b^4x^9-144\,ab^3x^7-693\,a^2b^2x^5-840\,a^3bx^3-315\,a^4x+315\left(ab^3x^6+3\,a^2b^2x^4+3\,a^3bx^2+a^4\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{48\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(32*b^4*x^9 - 288*a*b^3*x^7 - 1386*a^2*b^2*x^5 - 1680*a^3*b*x^3 - 630
*a^4*x + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*sqrt(-a/b)*log
((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^
2*b^6*x^2 + a^3*b^5), 1/48*(16*b^4*x^9 - 144*a*b^3*x^7 - 693*a^2*b^2*x^5 -
840*a^3*b*x^3 - 315*a^4*x + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 +
```

```
a^4)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*
x^2 + a^3*b^5)]
```

Sympy [A]
time = 0.29, size = 156, normalized size = 1.50

$$-\frac{4ax}{b^5}-\frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32}+\frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32}+\frac{-123a^4x-280a^3bx^3-165a^2b^2x^5}{48a^3b^5+144a^2b^6x^2+144ab^7x^4+48b^8x^6}+\frac{x^3}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -4*a*x/b**5 - 105*sqrt(-a**3/b**11)*log(x - b**5*sqrt(-a**3/b**11)/a)/32 +
105*sqrt(-a**3/b**11)*log(x + b**5*sqrt(-a**3/b**11)/a)/32 + (-123*a**4*x -
 280*a**3*b*x**3 - 165*a**2*b**2*x**5)/(48*a**3*b**5 + 144*a**2*b**6*x**2 +
 144*a*b**7*x**4 + 48*b**8*x**6) + x**3/(3*b**4)
```

Giac [A]
time = 4.02, size = 84, normalized size = 0.81

$$\frac{105\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^5}-\frac{165\,a^2b^2x^5+280\,a^3bx^3+123\,a^4x}{48\,(bx^2+a)^3b^5}+\frac{b^8x^3-12\,ab^7x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 105/16*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/48*(165*a^2*b^2*x^5 +
280*a^3*b*x^3 + 123*a^4*x)/((b*x^2 + a)^3*b^5) + 1/3*(b^8*x^3 - 12*a*b^7*x)
/b^12
```

Mupad [B]
time = 4.36, size = 99, normalized size = 0.95

$$\frac{x^3}{3\,b^4}-\frac{\frac{41\,a^4\,x}{16}+\frac{35\,a^3\,b\,x^3}{6}+\frac{55\,a^2\,b^2\,x^5}{16}}{a^3\,b^5+3\,a^2\,b^6\,x^2+3\,a\,b^7\,x^4+b^8\,x^6}+\frac{105\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{11/2}}-\frac{4\,a\,x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] x^3/(3*b^4) - ((41*a^4*x)/16 + (35*a^3*b*x^3)/6 + (55*a^2*b^2*x^5)/16)/(a^3
*b^5 + b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2) + (105*a^(3/2)*atan((b^(1/2)*
x)/a^(1/2)))/(16*b^(11/2)) - (4*a*x)/b^5
```

3.504 $\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=93

$$\frac{35x}{16b^4}-\frac{x^7}{6b\,(a+bx^2)^3}-\frac{7x^5}{24b^2\,(a+bx^2)^2}-\frac{35x^3}{48b^3\,(a+bx^2)}-\frac{35\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{9/2}}$$

```
[Out] 35/16*x/b^4-1/6*x^7/b/(b*x^2+a)^3-7/24*x^5/b^2/(b*x^2+a)^2-35/48*x^3/b^3/(b
*x^2+a)-35/16*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(9/2)
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 327, 211}

$$-\frac{35\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{9/2}}-\frac{35x^3}{48b^3\,(a+bx^2)}-\frac{7x^5}{24b^2\,(a+bx^2)^2}-\frac{x^7}{6b\,(a+bx^2)^3}+\frac{35x}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (35*x)/(16*b^4) - x^7/(6*b*(a + b*x^2)^3) - (7*x^5)/(24*b^2*(a + b*x^2)^2)
- (35*x^3)/(48*b^3*(a + b*x^2)) - (35*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(16*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx &= b^4\int \frac{x^8}{(ab+b^2x^2)^4}\,dx\\
&= -\frac{x^7}{6b\,(a+bx^2)^3}+\frac{1}{6}(7b^2)\int \frac{x^6}{(ab+b^2x^2)^3}\,dx\\
&= -\frac{x^7}{6b\,(a+bx^2)^3}-\frac{7x^5}{24b^2\,(a+bx^2)^2}+\frac{35}{24}\int \frac{x^4}{(ab+b^2x^2)^2}\,dx\\
&= -\frac{x^7}{6b\,(a+bx^2)^3}-\frac{7x^5}{24b^2\,(a+bx^2)^2}-\frac{35x^3}{48b^3\,(a+bx^2)}+\frac{35\int \frac{x^2}{ab+b^2x^2}\,dx}{16b^2}\\
&= \frac{35x}{16b^4}-\frac{x^7}{6b\,(a+bx^2)^3}-\frac{7x^5}{24b^2\,(a+bx^2)^2}-\frac{35x^3}{48b^3\,(a+bx^2)}-\frac{(35a)\int \frac{1}{ab+b^2x^2}\,dx}{16b^3}\\
&= \frac{35x}{16b^4}-\frac{x^7}{6b\,(a+bx^2)^3}-\frac{7x^5}{24b^2\,(a+bx^2)^2}-\frac{35x^3}{48b^3\,(a+bx^2)}-\frac{35\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{16b^{9/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.83

$$
\frac{105a^3x+280a^2bx^3+231ab^2x^5+48b^3x^7}{48b^4\,(a+bx^2)^3}-\frac{35\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (105*a^3*x + 280*a^2*b*x^3 + 231*a*b^2*x^5 + 48*b^3*x^7)/(48*b^4*(a + b*x^2
)^3) - (35*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(9/2))
```

Maple [A]
time = 0.04, size = 62, normalized size = 0.67

method	result	size
default	$\frac{x}{b^4}-\frac{a\left(\frac{-\frac{29}{16}b^2x^5-\frac{17}{6}ab\,x^3-\frac{19}{16}a^2x}{\left(b\,x^2+a\right)^3}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{b^4}$	62
risch	$\frac{x}{b^4}+\frac{\frac{29}{16}a\,b^2x^5+\frac{17}{6}a^2b\,x^3+\frac{19}{16}a^3x}{b^4(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}+\frac{35\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{32b^5}-\frac{35\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{32b^5}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^4-1/b^4*a*((-29/16*b^2*x^5-17/6*a*b*x^3-19/16*a^2*x)/(b*x^2+a)^3+35/16/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.54, size = 90, normalized size = 0.97

$$\frac{87\,ab^2x^5+136\,a^2bx^3+57\,a^3x}{48\,(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)}-\frac{35\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^4}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(87*a*b^2*x^5 + 136*a^2*b*x^3 + 57*a^3*x)/(b^7*x^6 + 3*a*b^6*x^4 + 3*a
^2*b^5*x^2 + a^3*b^4) - 35/16*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + x/b
^4
```

Fricas [A]
time = 0.33, size = 268, normalized size = 2.88

$$\left[\frac{96\,b^3x^7+462\,ab^2x^5+560\,a^2bx^3+210\,a^3x+105\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{96\,(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)},\ \frac{48\,b^3x^7+231\,ab^2x^5+280\,a^2bx^3+105\,a^3x-105\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{48\,(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(96*b^3*x^7 + 462*a*b^2*x^5 + 560*a^2*b*x^3 + 210*a^3*x + 105*(b^3*x^
6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/
b) - a)/(b*x^2 + a)))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4), 1/
48*(48*b^3*x^7 + 231*a*b^2*x^5 + 280*a^2*b*x^3 + 105*a^3*x - 105*(b^3*x^6 +
```

```
 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^7*x
^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [A]
time = 0.26, size = 131, normalized size = 1.41

$$\frac{35\sqrt{-\frac{a}{b^9}}\log\left(-b^4\sqrt{-\frac{a}{b^9}}+x\right)}{32}-\frac{35\sqrt{-\frac{a}{b^9}}\log\left(b^4\sqrt{-\frac{a}{b^9}}+x\right)}{32}+\frac{57a^3x+136a^2bx^3+87ab^2x^5}{48a^3b^4+144a^2b^5x^2+144ab^6x^4+48b^7x^6}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 35*sqrt(-a/b**9)*log(-b**4*sqrt(-a/b**9) + x)/32 - 35*sqrt(-a/b**9)*log(b**
4*sqrt(-a/b**9) + x)/32 + (57*a**3*x + 136*a**2*b*x**3 + 87*a*b**2*x**5)/(4
8*a**3*b**4 + 144*a**2*b**5*x**2 + 144*a*b**6*x**4 + 48*b**7*x**6) + x/b**4
```

Giac [A]
time = 4.37, size = 65, normalized size = 0.70

$$-\frac{35\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^4}+\frac{x}{b^4}+\frac{87\,ab^2x^5+136\,a^2bx^3+57\,a^3x}{48\,(bx^2+a)^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -35/16*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + x/b^4 + 1/48*(87*a*b^2*x^5
 + 136*a^2*b*x^3 + 57*a^3*x)/((b*x^2 + a)^3*b^4)
```

Mupad [B]
time = 0.10, size = 86, normalized size = 0.92

$$\frac{x}{b^4}+\frac{\frac{19\,a^3\,x}{16}+\frac{17\,a^2\,b\,x^3}{6}+\frac{29\,a\,b^2\,x^5}{16}}{a^3\,b^4+3\,a^2\,b^5\,x^2+3\,a\,b^6\,x^4+b^7\,x^6}-\frac{35\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] x/b^4 + ((19*a^3*x)/16 + (17*a^2*b*x^3)/6 + (29*a*b^2*x^5)/16)/(a^3*b^4 + b
^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2) - (35*a^(1/2)*atan((b^(1/2)*x)/a^(1/2
)))/(16*b^(9/2))
```

3.505 $\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{x^5}{6b\,(a+bx^2)^3}-\frac{5x^3}{24b^2\,(a+bx^2)^2}-\frac{5x}{16b^3\,(a+bx^2)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}$$

```
[Out] -1/6*x^5/b/(b*x^2+a)^3-5/24*x^3/b^2/(b*x^2+a)^2-5/16*x/b^3/(b*x^2+a)+5/16*a
rctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {28, 294, 211}

$$\frac{5\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}-\frac{5x}{16b^3\,(a+bx^2)}-\frac{5x^3}{24b^2\,(a+bx^2)^2}-\frac{x^5}{6b\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*x^5/(b*(a + b*x^2)^3) - (5*x^3)/(24*b^2*(a + b*x^2)^2) - (5*x)/(16*b^3
*(a + b*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^2}\,dx &= b^4\int \frac{x^6}{(ab+b^2x^2)^4}\,dx\\
&= -\frac{x^5}{6b\,(a+bx^2)^3}+\frac{1}{6}(5b^2)\int \frac{x^4}{(ab+b^2x^2)^3}\,dx\\
&= -\frac{x^5}{6b\,(a+bx^2)^3}-\frac{5x^3}{24b^2\,(a+bx^2)^2}+\frac{5}{8}\int \frac{x^2}{(ab+b^2x^2)^2}\,dx\\
&= -\frac{x^5}{6b\,(a+bx^2)^3}-\frac{5x^3}{24b^2\,(a+bx^2)^2}-\frac{5x}{16b^3\,(a+bx^2)}+\frac{5\int \frac{1}{ab+b^2x^2}\,dx}{16b^2}\\
&= -\frac{x^5}{6b\,(a+bx^2)^3}-\frac{5x^3}{24b^2\,(a+bx^2)^2}-\frac{5x}{16b^3\,(a+bx^2)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 66, normalized size = 0.80

$$
-\frac{x(15a^2+40abx^2+33b^2x^4)}{48b^3\,(a+bx^2)^3}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{a}\,b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/48*(x*(15*a^2 + 40*a*b*x^2 + 33*b^2*x^4))/(b^3*(a + b*x^2)^3) + (5*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(7/2))
```

Maple [**A**]
time = 0.04, size = 58, normalized size = 0.70

method	result	size
default	$\frac{-\frac{11x^5}{16b}-\frac{5a\,x^3}{6b^2}-\frac{5a^2x}{16b^3}}{(b\,x^2+a)^3}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16b^3\sqrt{ab}}$	58
risch	$\frac{-\frac{11x^5}{16b}-\frac{5a\,x^3}{6b^2}-\frac{5a^2x}{16b^3}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}-\frac{5\ln\left(bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\;b^3}+\frac{5\ln\left(-bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\;b^3}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-11/16/b*x^5-5/6*a/b^2*x^3-5/16*a^2/b^3*x)/(b*x^2+a)^3+5/16/b^3/(a*b)^(1/2
)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 81, normalized size = 0.98

$$-\frac{33\,b^2x^5+40\,abx^3+15\,a^2x}{48\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)}+\frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(33*b^2*x^5 + 40*a*b*x^3 + 15*a^2*x)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b
^4*x^2 + a^3*b^3) + 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
```

Fricas [A]
time = 0.35, size = 254, normalized size = 3.06

$$\left[-\frac{66\,ab^3x^5+80\,a^2b^2x^3+30\,a^3bx+15\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{96\,(ab^7x^6+3\,a^2b^6x^4+3\,a^3b^5x^2+a^4b^4)},\,-\frac{33\,ab^3x^5+40\,a^2b^2x^3+15\,a^3bx-15\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{48\,(ab^7x^6+3\,a^2b^6x^4+3\,a^3b^5x^2+a^4b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(66*a*b^3*x^5 + 80*a^2*b^2*x^3 + 30*a^3*b*x + 15*(b^3*x^6 + 3*a*b^2*
x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)))/(a*b^7*x^6 + 3*a^2*b^6*x^4 + 3*a^3*b^5*x^2 + a^4*b^4), -1/48*(33*a*
b^3*x^5 + 40*a^2*b^2*x^3 + 15*a^3*b*x - 15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b
*x^2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^7*x^6 + 3*a^2*b^6*x^4 + 3
*a^3*b^5*x^2 + a^4*b^4)]
```

Sympy [A]
time = 0.22, size = 134, normalized size = 1.61

$$-\frac{5\sqrt{-\frac{1}{ab^7}}\,\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{32}+\frac{5\sqrt{-\frac{1}{ab^7}}\,\log\left(ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{32}+\frac{-15a^2x-40abx^3-33b^2x^5}{48a^3b^3+144a^2b^4x^2+144ab^5x^4+48b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -5*sqrt(-1/(a*b**7))*log(-a*b**3*sqrt(-1/(a*b**7)) + x)/32 + 5*sqrt(-1/(a*b
**7))*log(a*b**3*sqrt(-1/(a*b**7)) + x)/32 + (-15*a**2*x - 40*a*b*x**3 - 33
*b**2*x**5)/(48*a**3*b**3 + 144*a**2*b**4*x**2 + 144*a*b**5*x**4 + 48*b**6*
x**6)
```

Giac [A]
time = 3.98, size = 56, normalized size = 0.67

$$\frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \sqrt{ab}\, b^3} - \frac{33\, b^2 x^5 + 40\, abx^3 + 15\, a^2 x}{48 \left(bx^2 + a\right)^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/48*(33*b^2*x^5 + 40*a*b*x^3
+ 15*a^2*x)/((b*x^2 + a)^3*b^3)
```

Mupad [B]
time = 4.39, size = 78, normalized size = 0.94

$$\frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,\sqrt{a}\,b^{7/2}} - \frac{\frac{11\,x^5}{16\,b} + \frac{5\,a\,x^3}{6\,b^2} + \frac{5\,a^2\,x}{16\,b^3}}{a^3 + 3\,a^2\,b\,x^2 + 3\,a\,b^2\,x^4 + b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (5*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(1/2)*b^(7/2)) - ((11*x^5)/(16*b) + (5*
a*x^3)/(6*b^2) + (5*a^2*x)/(16*b^3))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2
*x^4)
```

3.506 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{x^3}{6b\left(a+bx^2\right)^3}-\frac{x}{8b^2\left(a+bx^2\right)^2}+\frac{x}{16ab^2\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}$$

```
[Out] -1/6*x^3/b/(b*x^2+a)^3-1/8*x/b^2/(b*x^2+a)^2+1/16*x/a/b^2/(b*x^2+a)+1/16*ar
ctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{x}{16ab^2\left(a+bx^2\right)}-\frac{x}{8b^2\left(a+bx^2\right)^2}-\frac{x^3}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*x^3/(b*(a + b*x^2)^3) - x/(8*b^2*(a + b*x^2)^2) + x/(16*a*b^2*(a + b*x
^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx &= b^4\int \frac{x^4}{(ab+b^2x^2)^4}\,dx\\
&= -\frac{x^3}{6b\,(a+bx^2)^3}+\frac{1}{2}b^2\int \frac{x^2}{(ab+b^2x^2)^3}\,dx\\
&= -\frac{x^3}{6b\,(a+bx^2)^3}-\frac{x}{8b^2\,(a+bx^2)^2}+\frac{1}{8}\int \frac{1}{(ab+b^2x^2)^2}\,dx\\
&= -\frac{x^3}{6b\,(a+bx^2)^3}-\frac{x}{8b^2\,(a+bx^2)^2}+\frac{x}{16ab^2\,(a+bx^2)}+\frac{\int \frac{1}{ab+b^2x^2}\,dx}{16ab}\\
&= -\frac{x^3}{6b\,(a+bx^2)^3}-\frac{x}{8b^2\,(a+bx^2)^2}+\frac{x}{16ab^2\,(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.82

$$\frac{-3a^2x-8abx^3+3b^2x^5}{48ab^2\,(a+bx^2)^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-3*a^2*x - 8*a*b*x^3 + 3*b^2*x^5)/(48*a*b^2*(a + b*x^2)^3) + ArcTan[(Sqrt[
b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.69

method	result	size

default	$\frac{\frac{x^5}{16a}-\frac{x^3}{6b}-\frac{ax}{16b^2}}{(b\,x^2+a)^3}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16a\,b^2\sqrt{ab}}$	58
risch	$\frac{\frac{x^5}{16a}-\frac{x^3}{6b}-\frac{ax}{16b^2}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\;b^2a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\;b^2a}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16/a*x^5-1/6/b*x^3-1/16*a/b^2*x)/(b*x^2+a)^3+1/16/a/b^2/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.54, size = 87, normalized size = 1.04

$$\frac{3\,b^2x^5-8\,abx^3-3\,a^2x}{48\,(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^2*x^5 - 8*a*b*x^3 - 3*a^2*x)/(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b
^3*x^2 + a^4*b^2) + 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

Fricas [A]
time = 0.37, size = 258, normalized size = 3.07

$$\left[\frac{6\,ab^3x^5-16\,a^2b^2x^3-6\,a^3bx-3\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{96\,(a^2b^6x^6+3\,a^3b^5x^4+3\,a^4b^4x^2+a^5b^3)},\frac{3\,ab^3x^5-8\,a^2b^2x^3-3\,a^3bx+3\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{48\,(a^2b^6x^6+3\,a^3b^5x^4+3\,a^4b^4x^2+a^5b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(6*a*b^3*x^5 - 16*a^2*b^2*x^3 - 6*a^3*b*x - 3*(b^3*x^6 + 3*a*b^2*x^4
+ 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)))/(a^2*b^6*x^6 + 3*a^3*b^5*x^4 + 3*a^4*b^4*x^2 + a^5*b^3), 1/48*(3*a*b^3*
x^5 - 8*a^2*b^2*x^3 - 3*a^3*b*x + 3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 +
a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^6*x^6 + 3*a^3*b^5*x^4 + 3*a^4*
b^4*x^2 + a^5*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(70) = 140$.

time = 0.19, size = 143, normalized size = 1.70

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32}+\frac{-3a^2x-8abx^3+3b^2x^5}{48a^4b^2+144a^3b^3x^2+144a^2b^4x^4+48ab^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/32 + sqrt(-1
/(a**3*b**5))*log(a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/32 + (-3*a**2*x - 8*a
*b*x**3 + 3*b**2*x**5)/(48*a**4*b**2 + 144*a**3*b**3*x**2 + 144*a**2*b**4*x
**4 + 48*a*b**5*x**6)
```

Giac [A]
time = 4.06, size = 62, normalized size = 0.74

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\,ab^2}+\frac{3\,b^2x^5-8\,abx^3-3\,a^2x}{48\,(bx^2+a)^3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/48*(3*b^2*x^5 - 8*a*b*x^3
- 3*a^2*x)/((b*x^2 + a)^3*a*b^2)
```

Mupad [B]
time = 4.35, size = 75, normalized size = 0.89

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{3/2}\,b^{5/2}}-\frac{\frac{x^3}{6\,b}-\frac{x^5}{16\,a}+\frac{a\,x}{16\,b^2}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(16*a^(3/2)*b^(5/2)) - (x^3/(6*b) - x^5/(16*a) +
(a*x)/(16*b^2))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4)
```

3.507 $\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{x}{6b\,(a+bx^2)^3}+\frac{x}{24ab\,(a+bx^2)^2}+\frac{x}{16a^2b\,(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}$$

```
[Out] -1/6*x/b/(b*x^2+a)^3+1/24*x/a/b/(b*x^2+a)^2+1/16*x/a^2/b/(b*x^2+a)+1/16*arc
tan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x}{16a^2b\,(a+bx^2)}+\frac{x}{24ab\,(a+bx^2)^2}-\frac{x}{6b\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*x/(b*(a + b*x^2)^3) + x/(24*a*b*(a + b*x^2)^2) + x/(16*a^2*b*(a + b*x^
2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2}\,dx &= b^4\int \frac{x^2}{(ab+b^2x^2)^4}\,dx\\ &= -\frac{x}{6b\,(a+bx^2)^3}+\frac{1}{6}b^2\int \frac{1}{(ab+b^2x^2)^3}\,dx\\ &= -\frac{x}{6b\,(a+bx^2)^3}+\frac{x}{24ab\,(a+bx^2)^2}+\frac{b\int \frac{1}{(ab+b^2x^2)^2}\,dx}{8a}\\ &= -\frac{x}{6b\,(a+bx^2)^3}+\frac{x}{24ab\,(a+bx^2)^2}+\frac{x}{16a^2b\,(a+bx^2)}+\frac{\int \frac{1}{ab+b^2x^2}\,dx}{16a^2}\\ &= -\frac{x}{6b\,(a+bx^2)^3}+\frac{x}{24ab\,(a+bx^2)^2}+\frac{x}{16a^2b\,(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 69, normalized size = 0.81

$$\frac{-3a^2x+8abx^3+3b^2x^5}{48a^2b\,(a+bx^2)^3}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-3*a^2*x + 8*a*b*x^3 + 3*b^2*x^5)/(48*a^2*b*(a + b*x^2)^3) + ArcTan[(Sqrt[
b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

Maple [**A**]
time = 0.04, size = 58, normalized size = 0.68

method	result	size

default	$\frac{\frac{b\,x^5}{16a^2}+\frac{x^3}{6a}-\frac{x}{16b}}{(b\,x^2+a)^3}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16a^2b\sqrt{ab}}$	58
risch	$\frac{\frac{b\,x^5}{16a^2}+\frac{x^3}{6a}-\frac{x}{16b}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\,b\,a^2}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\,b\,a^2}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*b/a^2*x^5+1/6/a*x^3-1/16/b*x)/(b*x^2+a)^3+1/16/a^2/b/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 87, normalized size = 1.02

$$\frac{3\,b^2x^5+8\,abx^3-3\,a^2x}{48\,(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^2*x^5 + 8*a*b*x^3 - 3*a^2*x)/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4
*b^2*x^2 + a^5*b) + 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Fricas [A]
time = 0.34, size = 258, normalized size = 3.04

$$\left[\frac{6\,ab^3x^5+16\,a^2b^2x^3-6\,a^3bx-3\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{96\,(a^3b^5x^6+3\,a^4b^4x^4+3\,a^5b^3x^2+a^6b^2)},\frac{3\,ab^3x^5+8\,a^2b^2x^3-3\,a^3bx+3\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{48\,(a^3b^5x^6+3\,a^4b^4x^4+3\,a^5b^3x^2+a^6b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(6*a*b^3*x^5 + 16*a^2*b^2*x^3 - 6*a^3*b*x - 3*(b^3*x^6 + 3*a*b^2*x^4
+ 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)))/(a^3*b^5*x^6 + 3*a^4*b^4*x^4 + 3*a^5*b^3*x^2 + a^6*b^2), 1/48*(3*a*b^3*
x^5 + 8*a^2*b^2*x^3 - 3*a^3*b*x + 3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 +
a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^5*x^6 + 3*a^4*b^4*x^4 + 3*a^5*
b^3*x^2 + a^6*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(68) = 136$.

time = 0.19, size = 139, normalized size = 1.64

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^5b^3}}\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32}+\frac{-3a^2x+8abx^3+3b^2x^5}{48a^5b+144a^4b^2x^2+144a^3b^3x^4+48a^2b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 + sqrt(-1/(a
**5*b**3))*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 + (-3*a**2*x + 8*a*b*x**
3 + 3*b**2*x**5)/(48*a**5*b + 144*a**4*b**2*x**2 + 144*a**3*b**3*x**4 + 48*
a**2*b**4*x**6)
```

Giac [A]
time = 3.86, size = 62, normalized size = 0.73

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\,a^2b}+\frac{3\,b^2x^5+8\,abx^3-3\,a^2x}{48\,(bx^2+a)^3a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/48*(3*b^2*x^5 + 8*a*b*x^3
- 3*a^2*x)/((b*x^2 + a)^3*a^2*b)
```

Mupad [B]
time = 4.31, size = 74, normalized size = 0.87

$$\frac{\frac{x^3}{6\,a}-\frac{x}{16\,b}+\frac{b\,x^5}{16\,a^2}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (x^3/(6*a) - x/(16*b) + (b*x^5)/(16*a^2))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*
a*b^2*x^4) + atan((b^(1/2)*x)/a^(1/2))/(16*a^(5/2)*b^(3/2))
```

3.508 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=79

$$\frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}$$

```
[Out] 1/6*x/a/(b*x^2+a)^3+5/24*x/a^2/(b*x^2+a)^2+5/16*x/a^3/(b*x^2+a)+5/16*arctan
(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {28, 205, 211}

$$\frac{5\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{x}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
```

```
[Out] x/(6*a*(a + b*x^2)^3) + (5*x)/(24*a^2*(a + b*x^2)^2) + (5*x)/(16*a^3*(a + b
*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= b^4\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx\\
&= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{\left(5b^3\right)\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{6a}\\
&= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{\left(5b^2\right)\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{8a^2}\\
&= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{(5b)\int \frac{1}{ab+b^2x^2}\,dx}{16a^3}\\
&= \frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.84

$$\frac{33a^2x+40abx^3+15b^2x^5}{48a^3\left(a+bx^2\right)^3}+\frac{5\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
```

```
[Out] (33*a^2*x + 40*a*b*x^3 + 15*b^2*x^5)/(48*a^3*(a + b*x^2)^3) + (5*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

Maple [A]
time = 0.04, size = 78, normalized size = 0.99

method	result	size
default	$\frac{x}{6a(b\,x^2+a)^3}+\frac{\frac{5x}{24a\left(b\,x^2+a\right)^2}+\frac{5\left(\frac{3x}{8a\left(b\,x^2+a\right)}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)}{6a}}{a}$	78
risch	$\frac{\frac{5b^2x^5}{16a^3}+\frac{5b\,x^3}{6a^2}+\frac{11x}{16a}}{(b^2x^4+2ab\,x^2+a^2)(b\,x^2+a)}-\frac{5\ln\left(bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\;a^3}+\frac{5\ln\left(-bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\;a^3}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x/a/(b*x^2+a)^3+5/6/a*(1/4*x/a/(b*x^2+a)^2+3/4/a*(1/2*x/a/(b*x^2+a)+1/2
/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Maxima [A]
time = 0.49, size = 80, normalized size = 1.01

$$\frac{15\,b^2x^5+40\,abx^3+33\,a^2x}{48\,(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6)}+\frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(15*b^2*x^5 + 40*a*b*x^3 + 33*a^2*x)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*
a^5*b*x^2 + a^6) + 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.34, size = 254, normalized size = 3.22

$$\left[\frac{30\,ab^3x^5+80\,a^2b^2x^3+66\,a^3bx-15\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{96\,(a^4b^4x^6+3\,a^5b^3x^4+3\,a^6b^2x^2+a^7b)},\ \frac{15\,ab^3x^5+40\,a^2b^2x^3+33\,a^3bx+15\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{48\,(a^4b^4x^6+3\,a^5b^3x^4+3\,a^6b^2x^2+a^7b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(30*a*b^3*x^5 + 80*a^2*b^2*x^3 + 66*a^3*b*x - 15*(b^3*x^6 + 3*a*b^2*x
^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
+ a)))/(a^4*b^4*x^6 + 3*a^5*b^3*x^4 + 3*a^6*b^2*x^2 + a^7*b), 1/48*(15*a*b^
3*x^5 + 40*a^2*b^2*x^3 + 33*a^3*b*x + 15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x
^2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^4*x^6 + 3*a^5*b^3*x^4 + 3
*a^6*b^2*x^2 + a^7*b)]
```

Sympy [A]
time = 0.19, size = 129, normalized size = 1.63

$$-\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32}+\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32}+\frac{33a^2x+40abx^3+15b^2x^5}{48a^6+144a^5bx^2+144a^4b^2x^4+48a^3b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -5*sqrt(-1/(a**7*b))*log(-a**4*sqrt(-1/(a**7*b)) + x)/32 + 5*sqrt(-1/(a**7*
b))*log(a**4*sqrt(-1/(a**7*b)) + x)/32 + (33*a**2*x + 40*a*b*x**3 + 15*b**2
*x**5)/(48*a**6 + 144*a**5*b*x**2 + 144*a**4*b**2*x**4 + 48*a**3*b**3*x**6)
```

Giac [A]
time = 4.10, size = 56, normalized size = 0.71

$$\frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}\,a^3} + \frac{15\,b^2x^5 + 40\,abx^3 + 33\,a^2x}{48\,(bx^2+a)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/48*(15*b^2*x^5 + 40*a*b*x^3
+ 33*a^2*x)/((b*x^2 + a)^3*a^3)
```

Mupad [B]
time = 4.36, size = 77, normalized size = 0.97

$$\frac{\frac{11\,x}{16\,a} + \frac{5\,b\,x^3}{6\,a^2} + \frac{5\,b^2\,x^5}{16\,a^3}}{a^3 + 3\,a^2\,b\,x^2 + 3\,a\,b^2\,x^4 + b^3\,x^6} + \frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{7/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((11*x)/(16*a) + (5*b*x^3)/(6*a^2) + (5*b^2*x^5)/(16*a^3))/(a^3 + b^3*x^6 +
 3*a^2*b*x^2 + 3*a*b^2*x^4) + (5*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(7/2)*b^(
1/2))
```

3.509 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{35}{16a^4x}+\frac{1}{6ax\left(a+bx^2\right)^3}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{35}{48a^3x\left(a+bx^2\right)}-\frac{35\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{9/2}}$$

```
[Out] -35/16/a^4/x+1/6/a/x/(b*x^2+a)^3+7/24/a^2/x/(b*x^2+a)^2+35/48/a^3/x/(b*x^2+
a)-35/16*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(9/2)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 296, 331, 211}

$$-\frac{35\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{35}{16a^4x}+\frac{35}{48a^3x\left(a+bx^2\right)}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{1}{6ax\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -35/(16*a^4*x) + 1/(6*a*x*(a + b*x^2)^3) + 7/(24*a^2*x*(a + b*x^2)^2) + 35/
(48*a^3*x*(a + b*x^2)) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(9/
2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^2\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ax\left(a+bx^2\right)^3}+\frac{\left(7b^3\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^3}\,dx}{6a}$$

$$= \frac{1}{6ax\left(a+bx^2\right)^3}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{\left(35b^2\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^2}\,dx}{24a^2}$$

$$= \frac{1}{6ax\left(a+bx^2\right)^3}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{35}{48a^3x\left(a+bx^2\right)}+\frac{(35b)\int \frac{1}{x^2\left(ab+b^2x^2\right)}}{16a^3}$$

$$= -\frac{35}{16a^4x}+\frac{1}{6ax\left(a+bx^2\right)^3}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{35}{48a^3x\left(a+bx^2\right)}-\frac{\left(35b^2\right)}{}$$

$$= -\frac{35}{16a^4x}+\frac{1}{6ax\left(a+bx^2\right)^3}+\frac{7}{24a^2x\left(a+bx^2\right)^2}+\frac{35}{48a^3x\left(a+bx^2\right)}-\frac{35\sqrt{b}}{}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.83

$$-\frac{48a^3+231a^2bx^2+280ab^2x^4+105b^3x^6}{48a^4x\left(a+bx^2\right)^3}-\frac{35\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/48*(48*a^3 + 231*a^2*b*x^2 + 280*a*b^2*x^4 + 105*b^3*x^6)/(a^4*x*(a + b*
x^2)^3) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(9/2))
```

Maple [A]
time = 0.04, size = 65, normalized size = 0.68

method	result	size
default	$-\frac{b\left(\frac{\frac{19}{16}b^{2}x^{5}+\frac{17}{6}ab\,x^{3}+\frac{29}{16}a^{2}x}{\left(b\,x^{2}+a\right)^{3}}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{a^{4}}-\frac{1}{a^{4}x}$	65
risch	$\frac{-\frac{35b^{3}x^{6}}{16a^{4}}-\frac{35b^{2}x^{4}}{6a^{3}}-\frac{77b\,x^{2}}{16a^{2}}-\frac{1}{a}}{x(b^{2}x^{4}+2ab\,x^{2}+a^{2})(b\,x^{2}+a)}+\frac{35\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{32a^{5}}-\frac{35\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{32a^{5}}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^4*((19/16*b^2*x^5+17/6*a*b*x^3+29/16*a^2*x)/(b*x^2+a)^3+35/16/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2)))-1/a^4/x
```

Maxima [A]
time = 0.54, size = 93, normalized size = 0.98

$$-\frac{105\,b^{3}x^{6}+280\,ab^{2}x^{4}+231\,a^{2}bx^{2}+48\,a^{3}}{48\,(a^{4}b^{3}x^{7}+3\,a^{5}b^{2}x^{5}+3\,a^{6}bx^{3}+a^{7}x)}-\frac{35\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(105*b^3*x^6 + 280*a*b^2*x^4 + 231*a^2*b*x^2 + 48*a^3)/(a^4*b^3*x^7 +
 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x) - 35/16*b*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^4)
```

Fricas [A]
time = 0.36, size = 268, normalized size = 2.82

$$\left[-\frac{210\,b^{3}x^{6}+560\,ab^{2}x^{4}+462\,a^{2}bx^{2}+96\,a^{3}-105\,(b^{3}x^{7}+3\,ab^{2}x^{5}+3\,a^{2}bx^{3}+a^{3}x)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{96\,(a^{4}b^{3}x^{7}+3\,a^{5}b^{2}x^{5}+3\,a^{6}bx^{3}+a^{7}x)},\ -\frac{105\,b^{3}x^{6}+280\,ab^{2}x^{4}+231\,a^{2}bx^{2}+48\,a^{3}+105\,(b^{3}x^{7}+3\,ab^{2}x^{5}+3\,a^{2}bx^{3}+a^{3}x)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{48\,(a^{4}b^{3}x^{7}+3\,a^{5}b^{2}x^{5}+3\,a^{6}bx^{3}+a^{7}x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(210*b^3*x^6 + 560*a*b^2*x^4 + 462*a^2*b*x^2 + 96*a^3 - 105*(b^3*x^7
 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b
/a) - a)/(b*x^2 + a)))/(a^4*b^3*x^7 + 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x),
 -1/48*(105*b^3*x^6 + 280*a*b^2*x^4 + 231*a^2*b*x^2 + 48*a^3 + 105*(b^3*x^7
```

```
 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b
^3*x^7 + 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x)]
```

Sympy [A]
time = 0.26, size = 139, normalized size = 1.46

$$\frac{35\sqrt{-\frac{b}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}}{b}+x\right)}{32}-\frac{35\sqrt{-\frac{b}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}}{b}+x\right)}{32}+\frac{-48a^3-231a^2bx^2-280ab^2x^4-105b^3x^6}{48a^7x+144a^6bx^3+144a^5b^2x^5+48a^4b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 35*sqrt(-b/a**9)*log(-a**5*sqrt(-b/a**9)/b + x)/32 - 35*sqrt(-b/a**9)*log(a
**5*sqrt(-b/a**9)/b + x)/32 + (-48*a**3 - 231*a**2*b*x**2 - 280*a*b**2*x**4
 - 105*b**3*x**6)/(48*a**7*x + 144*a**6*b*x**3 + 144*a**5*b**2*x**5 + 48*a*
*4*b**3*x**7)
```

Giac [A]
time = 3.72, size = 68, normalized size = 0.72

$$-\frac{35\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^4}-\frac{1}{a^4x}-\frac{57\,b^3x^5+136\,ab^2x^3+87\,a^2bx}{48\,(bx^2+a)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -35/16*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/(a^4*x) - 1/48*(57*b^3*x
^5 + 136*a*b^2*x^3 + 87*a^2*b*x)/((b*x^2 + a)^3*a^4)
```

Mupad [B]
time = 4.44, size = 88, normalized size = 0.93

$$-\frac{\frac{1}{a}+\frac{77\,b\,x^2}{16\,a^2}+\frac{35\,b^2\,x^4}{6\,a^3}+\frac{35\,b^3\,x^6}{16\,a^4}}{a^3\,x+3\,a^2\,b\,x^3+3\,a\,b^2\,x^5+b^3\,x^7}-\frac{35\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] - (1/a + (77*b*x^2)/(16*a^2) + (35*b^2*x^4)/(6*a^3) + (35*b^3*x^6)/(16*a^4)
)/(a^3*x + b^3*x^7 + 3*a^2*b*x^3 + 3*a*b^2*x^5) - (35*b^(1/2)*atan((b^(1/2)
*x)/a^(1/2)))/(16*a^(9/2))
```

3.510 $\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=106

$$-\frac{35}{16a^4x^3}+\frac{105b}{16a^5x}+\frac{1}{6ax^3\left(a+bx^2\right)^3}+\frac{3}{8a^2x^3\left(a+bx^2\right)^2}+\frac{21}{16a^3x^3\left(a+bx^2\right)}+\frac{105b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{11/2}}$$

```
[Out] -35/16/a^4/x^3+105/16*b/a^5/x+1/6/a/x^3/(b*x^2+a)^3+3/8/a^2/x^3/(b*x^2+a)^2
+21/16/a^3/x^3/(b*x^2+a)+105/16*b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 296, 331, 211}

$$\frac{105b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{11/2}}+\frac{105b}{16a^5x}-\frac{35}{16a^4x^3}+\frac{21}{16a^3x^3\left(a+bx^2\right)}+\frac{3}{8a^2x^3\left(a+bx^2\right)^2}+\frac{1}{6ax^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -35/(16*a^4*x^3) + (105*b)/(16*a^5*x) + 1/(6*a*x^3*(a + b*x^2)^3) + 3/(8*a^
2*x^3*(a + b*x^2)^2) + 21/(16*a^3*x^3*(a + b*x^2)) + (105*b^(3/2)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(16*a^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{1}{x^4(ab+b^2x^2)^4}\,dx$$
$$= \frac{1}{6ax^3(a+bx^2)^3} + \frac{(3b^3)\int \frac{1}{x^4(ab+b^2x^2)^3}\,dx}{2a}$$
$$= \frac{1}{6ax^3(a+bx^2)^3} + \frac{3}{8a^2x^3(a+bx^2)^2} + \frac{(21b^2)\int \frac{1}{x^4(ab+b^2x^2)^2}\,dx}{8a^2}$$
$$= \frac{1}{6ax^3(a+bx^2)^3} + \frac{3}{8a^2x^3(a+bx^2)^2} + \frac{21}{16a^3x^3(a+bx^2)} + \frac{(105b)\int \frac{1}{x^4(ab+b^2}}{16a^3}$$
$$= -\frac{35}{16a^4x^3} + \frac{1}{6ax^3(a+bx^2)^3} + \frac{3}{8a^2x^3(a+bx^2)^2} + \frac{21}{16a^3x^3(a+bx^2)} - (105$$
$$= -\frac{35}{16a^4x^3} + \frac{105b}{16a^5x} + \frac{1}{6ax^3(a+bx^2)^3} + \frac{3}{8a^2x^3(a+bx^2)^2} + \frac{21}{16a^3x^3(a+bx}$$
$$= -\frac{35}{16a^4x^3} + \frac{105b}{16a^5x} + \frac{1}{6ax^3(a+bx^2)^3} + \frac{3}{8a^2x^3(a+bx^2)^2} + \frac{21}{16a^3x^3(a+bx}$$

Mathematica [A]
time = 0.03, size = 91, normalized size = 0.86

$$\frac{\frac{\sqrt{a}\,(-16a^4+144a^3bx^2+693a^2b^2x^4+840ab^3x^6+315b^4x^8)}{x^3(a+bx^2)^3} + 315b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{48a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((Sqrt[a]*(-16*a^4 + 144*a^3*b*x^2 + 693*a^2*b^2*x^4 + 840*a*b^3*x^6 + 315*
b^4*x^8))/(x^3*(a + b*x^2)^3) + 315*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(4
8*a^(11/2))
```

Maple [A]

time = 0.05, size = 75, normalized size = 0.71

method	result	size
default	$\frac{b^{2}\left(\frac{\frac{41}{16}b^{2}x^{5}+\frac{35}{6}ab\,x^{3}+\frac{55}{16}a^{2}x}{\left(b\,x^{2}+a\right)^{3}}+\frac{105\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{a^{5}}-\frac{1}{3a^{4}x^{3}}+\frac{4b}{a^{5}x}$	75
risch	$\frac{\frac{105b^{4}x^{8}}{16a^{5}}+\frac{35b^{3}x^{6}}{2a^{4}}+\frac{231b^{2}x^{4}}{16a^{3}}+\frac{3b\,x^{2}}{a^{2}}-\frac{1}{3a}}{x^{3}(b^{2}x^{4}+2ab\,x^{2}+a^{2})(b\,x^{2}+a)}+\frac{105\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^{2}+b^{3}\right)}\_R\ln\left(\left(3\_R^{2}a^{11}+2b^{3}\right)x-a^{6}b\_R\right)\right)}{32}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^5*((41/16*b^2*x^5+35/6*a*b*x^3+55/16*a^2*x)/(b*x^2+a)^3+105/16/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2)))-1/3/a^4/x^3+4*b/a^5/x
```

Maxima [A]

time = 0.53, size = 108, normalized size = 1.02

$$\frac{315\,b^{4}x^{8}+840\,ab^{3}x^{6}+693\,a^{2}b^{2}x^{4}+144\,a^{3}bx^{2}-16\,a^{4}}{48\,(a^{5}b^{3}x^{9}+3\,a^{6}b^{2}x^{7}+3\,a^{7}bx^{5}+a^{8}x^{3})}+\frac{105\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(315*b^4*x^8 + 840*a*b^3*x^6 + 693*a^2*b^2*x^4 + 144*a^3*b*x^2 - 16*a^
4)/(a^5*b^3*x^9 + 3*a^6*b^2*x^7 + 3*a^7*b*x^5 + a^8*x^3) + 105/16*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Fricas [A]

time = 0.36, size = 304, normalized size = 2.87

$$\left[\frac{630\,b^{4}x^{8}+1680\,ab^{3}x^{6}+1386\,a^{2}b^{2}x^{4}+288\,a^{3}bx^{2}-32\,a^{4}+315\,(b^{4}x^{9}+3\,ab^{3}x^{7}+3\,a^{2}b^{2}x^{5}+a^{3}bx^{3})\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{96\,(a^{5}b^{3}x^{9}+3\,a^{6}b^{2}x^{7}+3\,a^{7}bx^{5}+a^{8}x^{3})},\ \frac{315\,b^{4}x^{8}+840\,ab^{3}x^{6}+693\,a^{2}b^{2}x^{4}+144\,a^{3}bx^{2}-16\,a^{4}+315\,(b^{4}x^{9}+3\,ab^{3}x^{7}+3\,a^{2}b^{2}x^{5}+a^{3}bx^{3})\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{48\,(a^{5}b^{3}x^{9}+3\,a^{6}b^{2}x^{7}+3\,a^{7}bx^{5}+a^{8}x^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(630*b^4*x^8 + 1680*a*b^3*x^6 + 1386*a^2*b^2*x^4 + 288*a^3*b*x^2 - 32
*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2*x^5 + a^3*b*x^3)*sqrt(-b/a)*l
```

```
og((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^3*x^9 + 3*a^6*b^2*x^
7 + 3*a^7*b*x^5 + a^8*x^3), 1/48*(315*b^4*x^8 + 840*a*b^3*x^6 + 693*a^2*b^2
*x^4 + 144*a^3*b*x^2 - 16*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2*x^5
+ a^3*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^5*b^3*x^9 + 3*a^6*b^2*x^7 +
3*a^7*b*x^5 + a^8*x^3)]
```

Sympy [A]
time = 0.33, size = 162, normalized size = 1.53

$$-\frac{105\sqrt{-\frac{b^3}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2}+x\right)}{32}+\frac{105\sqrt{-\frac{b^3}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2}+x\right)}{32}+\frac{-16a^4+144a^3bx^2+693a^2b^2x^4+840ab^3x^6+315b^4x^8}{48a^8x^3+144a^7bx^5+144a^6b^2x^7+48a^5b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -105*sqrt(-b**3/a**11)*log(-a**6*sqrt(-b**3/a**11)/b**2 + x)/32 + 105*sqrt(
-b**3/a**11)*log(a**6*sqrt(-b**3/a**11)/b**2 + x)/32 + (-16*a**4 + 144*a**3
*b*x**2 + 693*a**2*b**2*x**4 + 840*a*b**3*x**6 + 315*b**4*x**8)/(48*a**8*x*
*3 + 144*a**7*b*x**5 + 144*a**6*b**2*x**7 + 48*a**5*b**3*x**9)
```

Giac [A]
time = 3.66, size = 82, normalized size = 0.77

$$\frac{105\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^5}+\frac{315\,b^4x^8+840\,ab^3x^6+693\,a^2b^2x^4+144\,a^3bx^2-16\,a^4}{48\,(bx^3+ax)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 105/16*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/48*(315*b^4*x^8 + 840*
a*b^3*x^6 + 693*a^2*b^2*x^4 + 144*a^3*b*x^2 - 16*a^4)/((b*x^3 + a*x)^3*a^5)
```

Mupad [B]
time = 4.45, size = 102, normalized size = 0.96

$$\frac{\frac{3\,b\,x^2}{a^2}-\frac{1}{3\,a}+\frac{231\,b^2\,x^4}{16\,a^3}+\frac{35\,b^3\,x^6}{2\,a^4}+\frac{105\,b^4\,x^8}{16\,a^5}}{a^3\,x^3+3\,a^2\,b\,x^5+3\,a\,b^2\,x^7+b^3\,x^9}+\frac{105\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((3*b*x^2)/a^2 - 1/(3*a) + (231*b^2*x^4)/(16*a^3) + (35*b^3*x^6)/(2*a^4) +
(105*b^4*x^8)/(16*a^5))/(a^3*x^3 + b^3*x^9 + 3*a^2*b*x^5 + 3*a*b^2*x^7) + (
105*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(11/2))
```

3.511 $\int \frac{1}{x^6 (a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=119

$$-\frac{231}{80a^4x^5}+\frac{77b}{16a^5x^3}-\frac{231b^2}{16a^6x}+\frac{1}{6ax^5(a+bx^2)^3}+\frac{11}{24a^2x^5(a+bx^2)^2}+\frac{33}{16a^3x^5(a+bx^2)}-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{13/2}}$$

```
[Out] -231/80/a^4/x^5+77/16*b/a^5/x^3-231/16*b^2/a^6/x+1/6/a/x^5/(b*x^2+a)^3+11/2
4/a^2/x^5/(b*x^2+a)^2+33/16/a^3/x^5/(b*x^2+a)-231/16*b^(5/2)*arctan(x*b^(1/
2)/a^(1/2))/a^(13/2)
```

Rubi [A]
time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 296, 331, 211}

$$-\frac{231b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{13/2}}-\frac{231b^2}{16a^6x}+\frac{77b}{16a^5x^3}-\frac{231}{80a^4x^5}+\frac{33}{16a^3x^5(a+bx^2)}+\frac{11}{24a^2x^5(a+bx^2)^2}+\frac{1}{6ax^5(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -231/(80*a^4*x^5) + (77*b)/(16*a^5*x^3) - (231*b^2)/(16*a^6*x) + 1/(6*a*x^5
*(a + b*x^2)^3) + 11/(24*a^2*x^5*(a + b*x^2)^2) + 33/(16*a^3*x^5*(a + b*x^2
)) - (231*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{x^6\left(ab+b^2x^2\right)^4}\,dx$$
$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{\left(11b^3\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^3}\,dx}{6a}$$
$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{\left(33b^2\right)\int \frac{1}{x^6\left(ab+b^2x^2\right)^2}\,dx}{8a^2}$$
$$= \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)} + \frac{(231b)\int \frac{1}{x^6(ab+b}}{16a^3}$$
$$= -\frac{231}{80a^4x^5} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5\left(a+bx^2\right)} - (23$$
$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{33}{16a^3x^5(a+}$$
$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} - \frac{231b^2}{16a^6x} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{}{16a}$$
$$= -\frac{231}{80a^4x^5} + \frac{77b}{16a^5x^3} - \frac{231b^2}{16a^6x} + \frac{1}{6ax^5\left(a+bx^2\right)^3} + \frac{11}{24a^2x^5\left(a+bx^2\right)^2} + \frac{}{16a}$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 0.85

$$-\frac{48a^5-176a^4bx^2+1584a^3b^2x^4+7623a^2b^3x^6+9240ab^4x^8+3465b^5x^{10}}{240a^6x^5\left(a+bx^2\right)^3} - \frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/240*(48*a^5 - 176*a^4*b*x^2 + 1584*a^3*b^2*x^4 + 7623*a^2*b^3*x^6 + 9240
*a*b^4*x^8 + 3465*b^5*x^10)/(a^6*x^5*(a + b*x^2)^3) - (231*b^(5/2)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(16*a^(13/2))
```

Maple [A]
time = 0.05, size = 87, normalized size = 0.73

method	result
default	$-\frac{b^{3} \left(\frac{\frac{71}{16} b^{2} x^{5}+\frac{59}{6} a b \,x^{3}+\frac{89}{16} a^{2} x}{\left(b \,x^{2}+a \right)^{3}}+\frac{231 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{16 \sqrt{a b}}\right)}{a^{6}}-\frac{1}{5 a^{4} x^{5}}-\frac{10 b^{2}}{a^{6} x}+\frac{4 b}{3 a^{5} x^{3}}$
risch	$\frac{-\frac{231 b^{5} x^{10}}{16 a^{6}}-\frac{77 b^{4} x^{8}}{2 a^{5}}-\frac{2541 b^{3} x^{6}}{80 a^{4}}-\frac{33 b^{2} x^{4}}{5 a^{3}}+\frac{11 b \,x^{2}}{15 a^{2}}-\frac{1}{5 a}}{x^{5} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right) \left(b \,x^{2}+a \right)}+\frac{231 \left(\sum_{\_R =\operatorname{RootOf}\left(a^{13} \_Z^{2}+b^{5}\right)}\_R \ln \left(\left(3 \_R^{2} a^{13}+2 b^{5}\right) x +a^{7} b^{2} \_R \right)\right)}{32}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^6*b^3*((71/16*b^2*x^5+59/6*a*b*x^3+89/16*a^2*x)/(b*x^2+a)^3+231/16/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/5/a^4/x^5-10*b^2/a^6/x+4/3*b/a^5/x^3
```

Maxima [A]
time = 0.51, size = 119, normalized size = 1.00

$$-\frac{3465 \, b^{5} x^{10} + 9240 \, a b^{4} x^{8} + 7623 \, a^{2} b^{3} x^{6} + 1584 \, a^{3} b^{2} x^{4} - 176 \, a^{4} b x^{2} + 48 \, a^{5}}{240 \, {\left(a^{6} b^{3} x^{11} + 3 \, a^{7} b^{2} x^{9} + 3 \, a^{8} b x^{7} + a^{9} x^{5}\right)}} - \frac{231 \, b^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{16 \, \sqrt{a b} a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/240*(3465*b^5*x^10 + 9240*a*b^4*x^8 + 7623*a^2*b^3*x^6 + 1584*a^3*b^2*x^
4 - 176*a^4*b*x^2 + 48*a^5)/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a
^9*x^5) - 231/16*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

Fricas [A]
time = 0.36, size = 330, normalized size = 2.77

$$\left[-\frac{6930 \, b^{5} x^{10} + 18480 \, a b^{4} x^{8} + 15246 \, a^{2} b^{3} x^{6} + 3168 \, a^{3} b^{2} x^{4} - 352 \, a^{4} b x^{2} + 96 \, a^{5} - 3465 \, {\left(b^{5} x^{11} + 3 \, a b^{4} x^{9} + 3 \, a^{2} b^{3} x^{7} + a^{3} b^{2} x^{5}\right)} \sqrt{-\frac{b}{a}} \log\left(\frac{b x^{2} - 2 \, a x \sqrt{-\frac{b}{a}} - a}{b x^{2} + a}\right)}{480 \, {\left(a^{6} b^{3} x^{11} + 3 \, a^{7} b^{2} x^{9} + 3 \, a^{8} b x^{7} + a^{9} x^{5}\right)}}, -\frac{3465 \, b^{5} x^{10} + 9240 \, a b^{4} x^{8} + 7623 \, a^{2} b^{3} x^{6} + 1584 \, a^{3} b^{2} x^{4} - 176 \, a^{4} b x^{2} + 48 \, a^{5} + 3465 \, {\left(b^{5} x^{11} + 3 \, a b^{4} x^{9} + 3 \, a^{2} b^{3} x^{7} + a^{3} b^{2} x^{5}\right)} \sqrt{\frac{b}{a}} \arctan\left(x \sqrt{\frac{b}{a}}\right)}{240 \, {\left(a^{6} b^{3} x^{11} + 3 \, a^{7} b^{2} x^{9} + 3 \, a^{8} b x^{7} + a^{9} x^{5}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/480*(6930*b^5*x^10 + 18480*a*b^4*x^8 + 15246*a^2*b^3*x^6 + 3168*a^3*b^2
*x^4 - 352*a^4*b*x^2 + 96*a^5 - 3465*(b^5*x^11 + 3*a*b^4*x^9 + 3*a^2*b^3*x^
7 + a^3*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a))
)/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a^9*x^5), -1/240*(3465*b^5*
x^10 + 9240*a*b^4*x^8 + 7623*a^2*b^3*x^6 + 1584*a^3*b^2*x^4 - 176*a^4*b*x^2
 + 48*a^5 + 3465*(b^5*x^11 + 3*a*b^4*x^9 + 3*a^2*b^3*x^7 + a^3*b^2*x^5)*sqr
t(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a
^9*x^5)]
```

Sympy [A]
time = 0.33, size = 173, normalized size = 1.45

$$\frac{231\sqrt{-\frac{b^5}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3}+x\right)}{32}-\frac{231\sqrt{-\frac{b^5}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3}+x\right)}{32}+\frac{-48a^5+176a^4bx^2-1584a^3b^2x^4-7623a^2b^3x^6-9240ab^4x^8-3465b^5x^{10}}{240a^9x^5+720a^8bx^7+720a^7b^2x^9+240a^6b^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 231*sqrt(-b**5/a**13)*log(-a**7*sqrt(-b**5/a**13)/b**3 + x)/32 - 231*sqrt(-
b**5/a**13)*log(a**7*sqrt(-b**5/a**13)/b**3 + x)/32 + (-48*a**5 + 176*a**4*
b*x**2 - 1584*a**3*b**2*x**4 - 7623*a**2*b**3*x**6 - 9240*a*b**4*x**8 - 346
5*b**5*x**10)/(240*a**9*x**5 + 720*a**8*b*x**7 + 720*a**7*b**2*x**9 + 240*a
**6*b**3*x**11)
```

Giac [A]
time = 3.97, size = 93, normalized size = 0.78

$$-\frac{231\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}\,a^6}-\frac{213\,b^5x^5+472\,ab^4x^3+267\,a^2b^3x}{48\,(bx^2+a)^3a^6}-\frac{150\,b^2x^4-20\,abx^2+3\,a^2}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -231/16*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/48*(213*b^5*x^5 + 472
*a*b^4*x^3 + 267*a^2*b^3*x)/((b*x^2 + a)^3*a^6) - 1/15*(150*b^2*x^4 - 20*a*
b*x^2 + 3*a^2)/(a^6*x^5)
```

Mupad [B]
time = 4.46, size = 114, normalized size = 0.96

$$-\frac{\frac{1}{5\,a}-\frac{11\,b\,x^2}{15\,a^2}+\frac{33\,b^2\,x^4}{5\,a^3}+\frac{2541\,b^3\,x^6}{80\,a^4}+\frac{77\,b^4\,x^8}{2\,a^5}+\frac{231\,b^5\,x^{10}}{16\,a^6}}{a^3\,x^5+3\,a^2\,b\,x^7+3\,a\,b^2\,x^9+b^3\,x^{11}}-\frac{231\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] - (1/(5*a) - (11*b*x^2)/(15*a^2) + (33*b^2*x^4)/(5*a^3) + (2541*b^3*x^6)/(8
0*a^4) + (77*b^4*x^8)/(2*a^5) + (231*b^5*x^10)/(16*a^6))/(a^3*x^5 + b^3*x^1
1 + 3*a^2*b*x^7 + 3*a*b^2*x^9) - (231*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(1
6*a^(13/2))
```

3.512 $\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=133

$$-\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}+\frac{a^7}{10b^8(a+bx^2)^5}-\frac{7a^6}{8b^8(a+bx^2)^4}+\frac{7a^5}{2b^8(a+bx^2)^3}-\frac{35a^4}{4b^8(a+bx^2)^2}+\frac{35a^3}{2b^8(a+bx^2)}+\frac{21a^2\log(a+$$

[Out] -3*a*x^2/b^7+1/4*x^4/b^6+1/10*a^7/b^8/(b*x^2+a)^5-7/8*a^6/b^8/(b*x^2+a)^4+7/2*a^5/b^8/(b*x^2+a)^3-35/4*a^4/b^8/(b*x^2+a)^2+35/2*a^3/b^8/(b*x^2+a)+21/2*a^2*ln(b*x^2+a)/b^8

Rubi [A]

time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$\frac{a^7}{10b^8(a+bx^2)^5}-\frac{7a^6}{8b^8(a+bx^2)^4}+\frac{7a^5}{2b^8(a+bx^2)^3}-\frac{35a^4}{4b^8(a+bx^2)^2}+\frac{35a^3}{2b^8(a+bx^2)}+\frac{21a^2\log(a+bx^2)}{2b^8}-\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}$$

Antiderivative was successfully verified.

[In] Int[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] (-3*a*x^2)/b^7 + x^4/(4*b^6) + a^7/(10*b^8*(a + b*x^2)^5) - (7*a^6)/(8*b^8*(a + b*x^2)^4) + (7*a^5)/(2*b^8*(a + b*x^2)^3) - (35*a^4)/(4*b^8*(a + b*x^2)^2) + (35*a^3)/(2*b^8*(a + b*x^2)) + (21*a^2*Log[a + b*x^2])/(2*b^8)

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{x^{15}}{(ab+b^2x^2)^6}\,dx$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \frac{x^7}{(ab+b^2x)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \left(-\frac{6a}{b^{13}}+\frac{x}{b^{12}}-\frac{a^7}{b^{13}(a+bx)^6}+\frac{7a^6}{b^{13}(a+bx)^5}-\frac{21a^5}{b^{13}(a+bx)^4}+\frac{}{b^{1}}\right.\right.$$
$$= -\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}+\frac{a^7}{10b^8(a+bx^2)^5}-\frac{7a^6}{8b^8(a+bx^2)^4}+\frac{7a^5}{2b^8(a+bx^2)^3}-\frac{35a^4}{4b^8(a+b}$$

Mathematica [A]
time = 0.02, size = 114, normalized size = 0.86

$$\frac{459a^7+1875a^6bx^2+2700a^5b^2x^4+1300a^4b^3x^6-400a^3b^4x^8-500a^2b^5x^{10}-70ab^6x^{12}+10b^7x^{14}+420a^2(a+bx^2)^5\log(a+bx^2)}{40b^8(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (459*a^7 + 1875*a^6*b*x^2 + 2700*a^5*b^2*x^4 + 1300*a^4*b^3*x^6 - 400*a^3*b
^4*x^8 - 500*a^2*b^5*x^10 - 70*a*b^6*x^12 + 10*b^7*x^14 + 420*a^2*(a + b*x^
2)^5*Log[a + b*x^2])/(40*b^8*(a + b*x^2)^5)
```

Maple [A]
time = 0.08, size = 124, normalized size = 0.93

method	result	size
norman	$\frac{\frac{x^{14}}{4b}-\frac{7a\,x^{12}}{4b^2}+\frac{959a^7}{40b^8}+\frac{105a^3x^8}{2b^4}+\frac{315a^4x^6}{2b^5}+\frac{385a^5x^4}{2b^6}+\frac{875a^6x^2}{8b^7}}{(b\,x^2+a)^5}+\frac{21a^2\ln(b\,x^2+a)}{2b^8}$	98
default	$\frac{(-b\,x^2+6a)^2}{4b^8}+\frac{a^2\left(-\frac{7a^4}{4b(b\,x^2+a)^4}+\frac{35a}{b(b\,x^2+a)}+\frac{a^5}{5b(b\,x^2+a)^5}-\frac{35a^2}{2b(b\,x^2+a)^2}+\frac{7a^3}{b(b\,x^2+a)^3}+\frac{21\ln(b\,x^2+a)}{b}\right)}{2b^7}$	124
risch	$\frac{x^4}{4b^6}-\frac{3a\,x^2}{b^7}+\frac{9a^2}{b^8}+\frac{\frac{35a^3b^3x^8}{2}+\frac{245a^4b^2x^6}{4}+\frac{329a^5b\,x^4}{4}+\frac{399a^6x^2}{8}+\frac{459a^7}{40b}}{b^7(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}+\frac{21a^2\ln(b\,x^2+a)}{2b^8}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-b*x^2+6*a)^2/b^8+1/2*a^2/b^7*(-7/4*a^4/b/(b*x^2+a)^4+35*a/b/(b*x^2+a)
+1/5*a^5/b/(b*x^2+a)^5-35/2*a^2/b/(b*x^2+a)^2+7*a^3/b/(b*x^2+a)^3+21/b*ln(b
*x^2+a))
```

Maxima [A]
time = 0.28, size = 143, normalized size = 1.08

$$\frac{700\,a^{3}b^{4}x^{8}+2450\,a^{4}b^{3}x^{6}+3290\,a^{5}b^{2}x^{4}+1995\,a^{6}bx^{2}+459\,a^{7}}{40\,(b^{13}x^{10}+5\,ab^{12}x^{8}+10\,a^{2}b^{11}x^{6}+10\,a^{3}b^{10}x^{4}+5\,a^{4}b^{9}x^{2}+a^{5}b^{8})}+\frac{21\,a^{2}\log\left(bx^{2}+a\right)}{2\,b^{8}}+\frac{bx^{4}-12\,ax^{2}}{4\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/40*(700*a^3*b^4*x^8 + 2450*a^4*b^3*x^6 + 3290*a^5*b^2*x^4 + 1995*a^6*b*x^
2 + 459*a^7)/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4
+ 5*a^4*b^9*x^2 + a^5*b^8) + 21/2*a^2*log(b*x^2 + a)/b^8 + 1/4*(b*x^4 - 12*
a*x^2)/b^7
```

Fricas [A]
time = 0.35, size = 203, normalized size = 1.53

$$\frac{10\,b^{7}x^{14}-70\,ab^{6}x^{12}-500\,a^{2}b^{5}x^{10}-400\,a^{3}b^{4}x^{8}+1300\,a^{4}b^{3}x^{6}+2700\,a^{5}b^{2}x^{4}+1875\,a^{6}bx^{2}+459\,a^{7}+420\,(a^{2}b^{5}x^{10}+5\,a^{3}b^{4}x^{8}+10\,a^{4}b^{3}x^{6}+10\,a^{5}b^{2}x^{4}+5\,a^{6}bx^{2}+a^{7})\log\left(bx^{2}+a\right)}{40\,(b^{13}x^{10}+5\,ab^{12}x^{8}+10\,a^{2}b^{11}x^{6}+10\,a^{3}b^{10}x^{4}+5\,a^{4}b^{9}x^{2}+a^{5}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(10*b^7*x^14 - 70*a*b^6*x^12 - 500*a^2*b^5*x^10 - 400*a^3*b^4*x^8 + 13
00*a^4*b^3*x^6 + 2700*a^5*b^2*x^4 + 1875*a^6*b*x^2 + 459*a^7 + 420*(a^2*b^5
*x^10 + 5*a^3*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 + a^7
)*log(b*x^2 + a))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10
*x^4 + 5*a^4*b^9*x^2 + a^5*b^8)
```

Sympy [A]
time = 0.48, size = 150, normalized size = 1.13

$$\frac{21a^{2}\log\left(a+bx^{2}\right)}{2b^{8}}-\frac{3ax^{2}}{b^{7}}+\frac{459a^{7}+1995a^{6}bx^{2}+3290a^{5}b^{2}x^{4}+2450a^{4}b^{3}x^{6}+700a^{3}b^{4}x^{8}}{40a^{5}b^{8}+200a^{4}b^{9}x^{2}+400a^{3}b^{10}x^{4}+400a^{2}b^{11}x^{6}+200ab^{12}x^{8}+40b^{13}x^{10}}+\frac{x^{4}}{4b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 21*a**2*log(a + b*x**2)/(2*b**8) - 3*a*x**2/b**7 + (459*a**7 + 1995*a**6*b*
x**2 + 3290*a**5*b**2*x**4 + 2450*a**4*b**3*x**6 + 700*a**3*b**4*x**8)/(40*
a**5*b**8 + 200*a**4*b**9*x**2 + 400*a**3*b**10*x**4 + 400*a**2*b**11*x**6
+ 200*a*b**12*x**8 + 40*b**13*x**10) + x**4/(4*b**6)
```

Giac [A]
time = 4.13, size = 113, normalized size = 0.85

$$\frac{21\,a^{2}\log\left(|bx^{2}+a|\right)}{2\,b^{8}}+\frac{b^{6}x^{4}-12\,ab^{5}x^{2}}{4\,b^{12}}-\frac{959\,a^{2}b^{5}x^{10}+4095\,a^{3}b^{4}x^{8}+7140\,a^{4}b^{3}x^{6}+6300\,a^{5}b^{2}x^{4}+2800\,a^{6}bx^{2}+500\,a^{7}}{40\,(bx^{2}+a)^{5}b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 21/2*a^2*log(abs(b*x^2 + a))/b^8 + 1/4*(b^6*x^4 - 12*a*b^5*x^2)/b^12 - 1/40
*(959*a^2*b^5*x^10 + 4095*a^3*b^4*x^8 + 7140*a^4*b^3*x^6 + 6300*a^5*b^2*x^4
 + 2800*a^6*b*x^2 + 500*a^7)/((b*x^2 + a)^5*b^8)
```

Mupad [B]
time = 0.13, size = 142, normalized size = 1.07

$$\frac{\frac{459\,a^7}{40\,b}+\frac{399\,a^6\,x^2}{8}+\frac{329\,a^5\,b\,x^4}{4}+\frac{245\,a^4\,b^2\,x^6}{4}+\frac{35\,a^3\,b^3\,x^8}{2}}{a^5\,b^7+5\,a^4\,b^8\,x^2+10\,a^3\,b^9\,x^4+10\,a^2\,b^{10}\,x^6+5\,a\,b^{11}\,x^8+b^{12}\,x^{10}}+\frac{x^4}{4\,b^6}-\frac{3\,a\,x^2}{b^7}+\frac{21\,a^2\,\ln\left(b\,x^2+a\right)}{2\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((459*a^7)/(40*b) + (399*a^6*x^2)/8 + (329*a^5*b*x^4)/4 + (245*a^4*b^2*x^6)
/4 + (35*a^3*b^3*x^8)/2)/(a^5*b^7 + b^12*x^10 + 5*a*b^11*x^8 + 5*a^4*b^8*x^
2 + 10*a^3*b^9*x^4 + 10*a^2*b^10*x^6) + x^4/(4*b^6) - (3*a*x^2)/b^7 + (21*a
^2*log(a + b*x^2))/(2*b^8)
```

3.513 $\int \frac{x^{13}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=118

$$\frac{x^2}{2b^6}-\frac{a^6}{10b^7(a+bx^2)^5}+\frac{3a^5}{4b^7(a+bx^2)^4}-\frac{5a^4}{2b^7(a+bx^2)^3}+\frac{5a^3}{b^7(a+bx^2)^2}-\frac{15a^2}{2b^7(a+bx^2)}-\frac{3a\log(a+bx^2)}{b^7}$$

```
[Out] 1/2*x^2/b^6-1/10*a^6/b^7/(b*x^2+a)^5+3/4*a^5/b^7/(b*x^2+a)^4-5/2*a^4/b^7/(b
*x^2+a)^3+5*a^3/b^7/(b*x^2+a)^2-15/2*a^2/b^7/(b*x^2+a)-3*a*ln(b*x^2+a)/b^7
```

Rubi [A]
time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$-\frac{a^6}{10b^7(a+bx^2)^5}+\frac{3a^5}{4b^7(a+bx^2)^4}-\frac{5a^4}{2b^7(a+bx^2)^3}+\frac{5a^3}{b^7(a+bx^2)^2}-\frac{15a^2}{2b^7(a+bx^2)}-\frac{3a\log(a+bx^2)}{b^7}+\frac{x^2}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^2/(2*b^6) - a^6/(10*b^7*(a + b*x^2)^5) + (3*a^5)/(4*b^7*(a + b*x^2)^4) -
(5*a^4)/(2*b^7*(a + b*x^2)^3) + (5*a^3)/(b^7*(a + b*x^2)^2) - (15*a^2)/(2*b
^7*(a + b*x^2)) - (3*a*Log[a + b*x^2])/b^7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{13}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{x^{13}}{(ab+b^2x^2)^6}\,dx$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \frac{x^6}{(ab+b^2x)^6}\,dx,x,x^2\right)$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \left(\frac{1}{b^{12}}+\frac{a^6}{b^{12}(a+bx)^6}-\frac{6a^5}{b^{12}(a+bx)^5}+\frac{15a^4}{b^{12}(a+bx)^4}-\frac{20a^3}{b^{12}(a+bx}\right.\right.$$
$$= \frac{x^2}{2b^6}-\frac{a^6}{10b^7(a+bx^2)^5}+\frac{3a^5}{4b^7(a+bx^2)^4}-\frac{5a^4}{2b^7(a+bx^2)^3}+\frac{5a^3}{b^7(a+bx^2)^2}-\frac{}{2b^7}$$

Mathematica [A]
time = 0.02, size = 101, normalized size = 0.86

$$-\frac{87a^6+375a^5bx^2+600a^4b^2x^4+400a^3b^3x^6+50a^2b^4x^8-50ab^5x^{10}-10b^6x^{12}+60a(a+bx^2)^5\log(a+bx^2)}{20b^7(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/20*(87*a^6 + 375*a^5*b*x^2 + 600*a^4*b^2*x^4 + 400*a^3*b^3*x^6 + 50*a^2*
b^4*x^8 - 50*a*b^5*x^10 - 10*b^6*x^12 + 60*a*(a + b*x^2)^5*Log[a + b*x^2])/
(b^7*(a + b*x^2)^5)
```

Maple [A]
time = 0.05, size = 113, normalized size = 0.96

method	result	size
norman	$\frac{\frac{x^{12}}{2b}-\frac{137a^6}{20b^7}-\frac{15a^2x^8}{b^3}-\frac{45a^3x^6}{b^4}-\frac{55a^4x^4}{b^5}-\frac{125a^5x^2}{4b^6}}{(bx^2+a)^5}-\frac{3a\ln(bx^2+a)}{b^7}$	87
risch	$\frac{x^2}{2b^6}+\frac{-\frac{15a^2b^3x^8}{2}-25b^2a^3x^6-\frac{65ba^4x^4}{2}-\frac{77a^5x^2}{4}-\frac{87a^6}{20b}}{b^6(bx^2+a)(b^2x^4+2abx^2+a^2)^2}-\frac{3a\ln(bx^2+a)}{b^7}$	105
default	$\frac{x^2}{2b^6}-\frac{a\left(\frac{15a}{b(bx^2+a)}+\frac{a^5}{5b(bx^2+a)^5}-\frac{10a^2}{b(bx^2+a)^2}+\frac{5a^3}{b(bx^2+a)^3}-\frac{3a^4}{2b(bx^2+a)^4}+\frac{6\ln(bx^2+a)}{b}\right)}{2b^6}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/b^6-1/2*a/b^6*(15*a/b/(b*x^2+a)+1/5*a^5/b/(b*x^2+a)^5-10*a^2/b/(b*x
^2+a)^2+5*a^3/b/(b*x^2+a)^3-3/2*a^4/b/(b*x^2+a)^4+6/b*ln(b*x^2+a))
```

Maxima [A]
time = 0.29, size = 132, normalized size = 1.12

$$-\frac{150\,a^2b^4x^8+500\,a^3b^3x^6+650\,a^4b^2x^4+385\,a^5bx^2+87\,a^6}{20\,(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7)}+\frac{x^2}{2\,b^6}-\frac{3\,a\log\left(bx^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(150*a^2*b^4*x^8 + 500*a^3*b^3*x^6 + 650*a^4*b^2*x^4 + 385*a^5*b*x^2
+ 87*a^6)/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*
a^4*b^8*x^2 + a^5*b^7) + 1/2*x^2/b^6 - 3*a*log(b*x^2 + a)/b^7
```

Fricas [A]
time = 0.38, size = 190, normalized size = 1.61

$$\frac{10\,b^6x^{12}+50\,ab^5x^{10}-50\,a^2b^4x^8-400\,a^3b^3x^6-600\,a^4b^2x^4-375\,a^5bx^2-87\,a^6-60\,(ab^5x^{10}+5\,a^2b^4x^8+10\,a^3b^3x^6+10\,a^4b^2x^4+5\,a^5bx^2+a^6)\log\left(bx^2+a\right)}{20\,(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^6*x^12 + 50*a*b^5*x^10 - 50*a^2*b^4*x^8 - 400*a^3*b^3*x^6 - 600*
a^4*b^2*x^4 - 375*a^5*b*x^2 - 87*a^6 - 60*(a*b^5*x^10 + 5*a^2*b^4*x^8 + 10*
a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 + a^6)*log(b*x^2 + a))/(b^12*x^1
0 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b
^7)
```

Sympy [A]
time = 0.45, size = 138, normalized size = 1.17

$$-\frac{3a\log\left(a+bx^2\right)}{b^7}+\frac{-87a^6-385a^5bx^2-650a^4b^2x^4-500a^3b^3x^6-150a^2b^4x^8}{20a^5b^7+100a^4b^8x^2+200a^3b^9x^4+200a^2b^{10}x^6+100ab^{11}x^8+20b^{12}x^{10}}+\frac{x^2}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*a*log(a + b*x**2)/b**7 + (-87*a**6 - 385*a**5*b*x**2 - 650*a**4*b**2*x**
4 - 500*a**3*b**3*x**6 - 150*a**2*b**4*x**8)/(20*a**5*b**7 + 100*a**4*b**8*
x**2 + 200*a**3*b**9*x**4 + 200*a**2*b**10*x**6 + 100*a*b**11*x**8 + 20*b**
12*x**10) + x**2/(2*b**6)
```

Giac [A]
time = 3.24, size = 95, normalized size = 0.81

$$\frac{x^2}{2\,b^6}-\frac{3\,a\log\left(|bx^2+a|\right)}{b^7}+\frac{137\,ab^5x^{10}+535\,a^2b^4x^8+870\,a^3b^3x^6+720\,a^4b^2x^4+300\,a^5bx^2+50\,a^6}{20\,(bx^2+a)^5b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^6 - 3*a*log(abs(b*x^2 + a))/b^7 + 1/20*(137*a*b^5*x^10 + 535*a^2*
b^4*x^8 + 870*a^3*b^3*x^6 + 720*a^4*b^2*x^4 + 300*a^5*b*x^2 + 50*a^6)/((b*x
^2 + a)^5*b^7)
```

Mupad [B]
time = 4.60, size = 132, normalized size = 1.12

$$\frac{x^2}{2\,b^6}-\frac{\frac{87\,a^6}{20\,b}+\frac{77\,a^5\,x^2}{4}+\frac{65\,a^4\,b\,x^4}{2}+25\,a^3\,b^2\,x^6+\frac{15\,a^2\,b^3\,x^8}{2}}{a^5\,b^6+5\,a^4\,b^7\,x^2+10\,a^3\,b^8\,x^4+10\,a^2\,b^9\,x^6+5\,a\,b^{10}\,x^8+b^{11}\,x^{10}}-\frac{3\,a\,\ln\left(b\,x^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] x^2/(2*b^6) - ((87*a^6)/(20*b) + (77*a^5*x^2)/4 + (65*a^4*b*x^4)/2 + 25*a^3
*b^2*x^6 + (15*a^2*b^3*x^8)/2)/(a^5*b^6 + b^11*x^10 + 5*a*b^10*x^8 + 5*a^4*
b^7*x^2 + 10*a^3*b^8*x^4 + 10*a^2*b^9*x^6) - (3*a*log(a + b*x^2))/b^7
```

3.514 $\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=109

$$\frac{a^5}{10b^6(a+bx^2)^5}-\frac{5a^4}{8b^6(a+bx^2)^4}+\frac{5a^3}{3b^6(a+bx^2)^3}-\frac{5a^2}{2b^6(a+bx^2)^2}+\frac{5a}{2b^6(a+bx^2)}+\frac{\log(a+bx^2)}{2b^6}$$

```
[Out] 1/10*a^5/b^6/(b*x^2+a)^5-5/8*a^4/b^6/(b*x^2+a)^4+5/3*a^3/b^6/(b*x^2+a)^3-5/
2*a^2/b^6/(b*x^2+a)^2+5/2*a/b^6/(b*x^2+a)+1/2*ln(b*x^2+a)/b^6
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$\frac{a^5}{10b^6(a+bx^2)^5}-\frac{5a^4}{8b^6(a+bx^2)^4}+\frac{5a^3}{3b^6(a+bx^2)^3}-\frac{5a^2}{2b^6(a+bx^2)^2}+\frac{5a}{2b^6(a+bx^2)}+\frac{\log(a+bx^2)}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^5/(10*b^6*(a + b*x^2)^5) - (5*a^4)/(8*b^6*(a + b*x^2)^4) + (5*a^3)/(3*b^6
*(a + b*x^2)^3) - (5*a^2)/(2*b^6*(a + b*x^2)^2) + (5*a)/(2*b^6*(a + b*x^2))
 + Log[a + b*x^2]/(2*b^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{x^{11}}{(ab+b^2x^2)^6}\,dx$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \frac{x^5}{(ab+b^2x)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \left(-\frac{a^5}{b^{11}(a+bx)^6}+\frac{5a^4}{b^{11}(a+bx)^5}-\frac{10a^3}{b^{11}(a+bx)^4}+\frac{10a^2}{b^{11}(a+bx)^3}-\right.\right.$$
$$= \frac{a^5}{10b^6(a+bx^2)^5}-\frac{5a^4}{8b^6(a+bx^2)^4}+\frac{5a^3}{3b^6(a+bx^2)^3}-\frac{5a^2}{2b^6(a+bx^2)^2}+\frac{5a}{2b^6(a+}$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 0.66

$$\frac{\frac{a(137a^4+625a^3bx^2+1100a^2b^2x^4+900ab^3x^6+300b^4x^8)}{(a+bx^2)^5}+60\log(a+bx^2)}{120b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((a*(137*a^4 + 625*a^3*b*x^2 + 1100*a^2*b^2*x^4 + 900*a*b^3*x^6 + 300*b^4*x
^8))/(a + b*x^2)^5 + 60*Log[a + b*x^2])/(120*b^6)
```

Maple [A]
time = 0.06, size = 98, normalized size = 0.90

method	result	size
norman	$\frac{\frac{137a^5}{120b^6}+\frac{5a\,x^8}{2b^2}+\frac{15a^2x^6}{2b^3}+\frac{55a^3x^4}{6b^4}+\frac{125a^4x^2}{24b^5}}{(b\,x^2+a)^5}+\frac{\ln(b\,x^2+a)}{2b^6}$	76
risch	$\frac{\frac{137a^5}{120b^6}+\frac{5a\,x^8}{2b^2}+\frac{15a^2x^6}{2b^3}+\frac{55a^3x^4}{6b^4}+\frac{125a^4x^2}{24b^5}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}+\frac{\ln(b\,x^2+a)}{2b^6}$	96
default	$\frac{a^5}{10b^6(b\,x^2+a)^5}-\frac{5a^4}{8b^6(b\,x^2+a)^4}+\frac{5a^3}{3b^6(b\,x^2+a)^3}-\frac{5a^2}{2b^6(b\,x^2+a)^2}+\frac{5a}{2b^6(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^6}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*a^5/b^6/(b*x^2+a)^5-5/8*a^4/b^6/(b*x^2+a)^4+5/3*a^3/b^6/(b*x^2+a)^3-5/
2*a^2/b^6/(b*x^2+a)^2+5/2*a/b^6/(b*x^2+a)+1/2*ln(b*x^2+a)/b^6
```

Maxima [A]
time = 0.28, size = 121, normalized size = 1.11

$$\frac{300\,ab^4x^8+900\,a^2b^3x^6+1100\,a^3b^2x^4+625\,a^4bx^2+137\,a^5}{120\,(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)}+\frac{\log(bx^2+a)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a^4*b*x^2 +
 137*a^5)/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a
^4*b^7*x^2 + a^5*b^6) + 1/2*log(b*x^2 + a)/b^6
```

Fricas [A]
time = 0.33, size = 168, normalized size = 1.54

$$\frac{300\,ab^4x^8+900\,a^2b^3x^6+1100\,a^3b^2x^4+625\,a^4bx^2+137\,a^5+60\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\log\left(bx^2+a\right)}{120\,(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a^4*b*x^2 +
 137*a^5 + 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5
*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^
6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
```

Sympy [A]
time = 0.37, size = 124, normalized size = 1.14

$$\frac{137a^5+625a^4bx^2+1100a^3b^2x^4+900a^2b^3x^6+300ab^4x^8}{120a^5b^6+600a^4b^7x^2+1200a^3b^8x^4+1200a^2b^9x^6+600ab^{10}x^8+120b^{11}x^{10}}+\frac{\log\left(a+bx^2\right)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (137*a**5 + 625*a**4*b*x**2 + 1100*a**3*b**2*x**4 + 900*a**2*b**3*x**6 + 30
0*a*b**4*x**8)/(120*a**5*b**6 + 600*a**4*b**7*x**2 + 1200*a**3*b**8*x**4 +
1200*a**2*b**9*x**6 + 600*a*b**10*x**8 + 120*b**11*x**10) + log(a + b*x**2)
/(2*b**6)
```

Giac [A]
time = 3.05, size = 75, normalized size = 0.69

$$\frac{\log\left(|bx^2+a|\right)}{2\,b^6}-\frac{137\,b^4x^{10}+385\,ab^3x^8+470\,a^2b^2x^6+270\,a^3bx^4+60\,a^4x^2}{120\,(bx^2+a)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b^6 - 1/120*(137*b^4*x^10 + 385*a*b^3*x^8 + 470*a^2
*b^2*x^6 + 270*a^3*b*x^4 + 60*a^4*x^2)/((b*x^2 + a)^5*b^5)
```

Mupad [B]

time = 4.37, size = 119, normalized size = 1.09

$$\frac{\frac{137\,a^5}{120\,b^6}+\frac{5\,a\,x^8}{2\,b^2}+\frac{15\,a^2\,x^6}{2\,b^3}+\frac{55\,a^3\,x^4}{6\,b^4}+\frac{125\,a^4\,x^2}{24\,b^5}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}+\frac{\ln\left(b\,x^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((137*a^5)/(120*b^6) + (5*a*x^8)/(2*b^2) + (15*a^2*x^6)/(2*b^3) + (55*a^3*x
^4)/(6*b^4) + (125*a^4*x^2)/(24*b^5))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b
^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + log(a + b*x^2)/(2*b^6)
```

3.515 $\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal. Leaf size=19

$$\frac{x^{10}}{10a(a+bx^2)^5}$$

[Out] 1/10*x^10/a/(b*x^2+a)^5

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {28, 270}

$$\frac{x^{10}}{10a(a+bx^2)^5}$$

Antiderivative was successfully verified.

[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] x^10/(10*a*(a + b*x^2)^5)

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^3}\,dx &= b^6\int \frac{x^9}{(ab+b^2x^2)^6}\,dx\\ &= \frac{x^{10}}{10a(a+bx^2)^5}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(19) = 38.

time = 0.01, size = 57, normalized size = 3.00

$$-\frac{a^4+5a^3bx^2+10a^2b^2x^4+10ab^3x^6+5b^4x^8}{10b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(a^4 + 5*a^3*b*x^2 + 10*a^2*b^2*x^4 + 10*a*b^3*x^6 + 5*b^4*x^8)/(b^5*
(a + b*x^2)^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(17) = 34$.
time = 0.04, size = 81, normalized size = 4.26

method	result	size
norman	$\frac{-\frac{x^8}{2b}-\frac{a\,x^6}{b^2}-\frac{a^2x^4}{b^3}-\frac{a^3x^2}{2b^4}-\frac{a^4}{10b^5}}{\left(b\,x^2+a\right)^5}$	59
gosper	$-\frac{5b^4x^8+10a\,b^3x^6+10a^2b^2x^4+5a^3b\,x^2+a^4}{10\left(b\,x^2+a\right)\left(b^2x^4+2ab\,x^2+a^2\right)^2b^5}$	76
risch	$\frac{-\frac{x^8}{2b}-\frac{a\,x^6}{b^2}-\frac{a^2x^4}{b^3}-\frac{a^3x^2}{2b^4}-\frac{a^4}{10b^5}}{\left(b\,x^2+a\right)\left(b^2x^4+2ab\,x^2+a^2\right)^2}$	79
default	$\frac{a^3}{2b^5\left(b\,x^2+a\right)^4}-\frac{1}{2b^5\left(b\,x^2+a\right)}-\frac{a^2}{b^5\left(b\,x^2+a\right)^3}-\frac{a^4}{10b^5\left(b\,x^2+a\right)^5}+\frac{a}{b^5\left(b\,x^2+a\right)^2}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^3/b^5/(b*x^2+a)^4-1/2/b^5/(b*x^2+a)-a^2/b^5/(b*x^2+a)^3-1/10*a^4/b^5/
(b*x^2+a)^5+a/b^5/(b*x^2+a)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(17) = 34$.
time = 0.28, size = 102, normalized size = 5.37

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,\left(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/(b^10
*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5
*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(17) = 34.
time = 0.35, size = 102, normalized size = 5.37

$$-\frac{5 b^{4} x^{8} + 10 a b^{3} x^{6} + 10 a^{2} b^{2} x^{4} + 5 a^{3} b x^{2} + a^{4}}{10 \left(b^{10} x^{10} + 5 a b^{9} x^{8} + 10 a^{2} b^{8} x^{6} + 10 a^{3} b^{7} x^{4} + 5 a^{4} b^{6} x^{2} + a^{5} b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/(b^10
*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5
*b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(14) = 28.
time = 0.32, size = 107, normalized size = 5.63

$$\frac{-a^{4} - 5a^{3}bx^{2} - 10a^{2}b^{2}x^{4} - 10ab^{3}x^{6} - 5b^{4}x^{8}}{10a^{5}b^{5} + 50a^{4}b^{6}x^{2} + 100a^{3}b^{7}x^{4} + 100a^{2}b^{8}x^{6} + 50ab^{9}x^{8} + 10b^{10}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
[Out] (-a**4 - 5*a**3*b*x**2 - 10*a**2*b**2*x**4 - 10*a*b**3*x**6 - 5*b**4*x**8)/
(10*a**5*b**5 + 50*a**4*b**6*x**2 + 100*a**3*b**7*x**4 + 100*a**2*b**8*x**6
 + 50*a*b**9*x**8 + 10*b**10*x**10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(17) = 34.
time = 3.87, size = 55, normalized size = 2.89

$$-\frac{5 b^{4} x^{8} + 10 a b^{3} x^{6} + 10 a^{2} b^{2} x^{4} + 5 a^{3} b x^{2} + a^{4}}{10 \left(b x^{2} + a\right)^{5} b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/((b*x
^2 + a)^5*b^5)
```

Mupad [B]
time = 4.45, size = 104, normalized size = 5.47

$$-\frac{a^4 + 5 a^3 b x^2 + 10 a^2 b^2 x^4 + 10 a b^3 x^6 + 5 b^4 x^8}{10 a^5 b^5 + 50 a^4 b^6 x^2 + 100 a^3 b^7 x^4 + 100 a^2 b^8 x^6 + 50 a b^9 x^8 + 10 b^{10} x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a^4 + 5*b^4*x^8 + 5*a^3*b*x^2 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4)/(10*a^5*b^
5 + 10*b^10*x^10 + 50*a*b^9*x^8 + 50*a^4*b^6*x^2 + 100*a^3*b^7*x^4 + 100*a^
2*b^8*x^6)
```

3.516 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^3} dx$

Optimal. Leaf size=39

$$\frac{x^8}{10a\left(a+bx^2\right)^5}+\frac{x^8}{40a^2\left(a+bx^2\right)^4}$$

```
[Out] 1/10*x^8/a/(b*x^2+a)^5+1/40*x^8/a^2/(b*x^2+a)^4
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {28, 272, 47, 37}

$$\frac{x^8}{40a^2\left(a+bx^2\right)^4}+\frac{x^8}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^8/(10*a*(a + b*x^2)^5) + x^8/(40*a^2*(a + b*x^2)^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^3}\,dx &= b^6\int \frac{x^7}{(ab+b^2x^2)^6}\,dx\\ &= \frac{1}{2}b^6\text{Subst}\left(\int \frac{x^3}{(ab+b^2x)^6}\,dx,x,x^2\right)\\ &= \frac{x^8}{10a\,(a+bx^2)^5}+\frac{b^5\text{Subst}\left(\int \frac{x^3}{(ab+b^2x)^5}\,dx,x,x^2\right)}{10a}\\ &= \frac{x^8}{10a\,(a+bx^2)^5}+\frac{x^8}{40a^2\,(a+bx^2)^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.18

$$-\frac{a^3+5a^2bx^2+10ab^2x^4+10b^3x^6}{40b^4\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
[Out] -1/40*(a^3 + 5*a^2*b*x^2 + 10*a*b^2*x^4 + 10*b^3*x^6)/(b^4*(a + b*x^2)^5)
```

Maple [A]
time = 0.04, size = 65, normalized size = 1.67

method	result	size
norman	$\frac{-\frac{x^6}{4b}-\frac{a\,x^4}{4b^2}-\frac{a^2x^2}{8b^3}-\frac{a^3}{40b^4}}{(b\,x^2+a)^5}$	48
gosper	$-\frac{10b^3x^6+10a\,b^2x^4+5a^2b\,x^2+a^3}{40(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2b^4}$	65
default	$-\frac{1}{4b^4(b\,x^2+a)^2}+\frac{a^3}{10b^4(b\,x^2+a)^5}+\frac{a}{2b^4(b\,x^2+a)^3}-\frac{3a^2}{8b^4(b\,x^2+a)^4}$	65
risch	$\frac{-\frac{x^6}{4b}-\frac{a\,x^4}{4b^2}-\frac{a^2x^2}{8b^3}-\frac{a^3}{40b^4}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^4/(b*x^2+a)^2+1/10*a^3/b^4/(b*x^2+a)^5+1/2*a/b^4/(b*x^2+a)^3-3/8*a^2
/b^4/(b*x^2+a)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.
time = 0.32, size = 91, normalized size = 2.33

$$-\frac{10\,b^3x^6 + 10\,ab^2x^4 + 5\,a^2bx^2 + a^3}{40\,(b^9x^{10} + 5\,ab^8x^8 + 10\,a^2b^7x^6 + 10\,a^3b^6x^4 + 5\,a^4b^5x^2 + a^5b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 + 5*a*b^8*x
^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 + a^5*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.
time = 0.34, size = 91, normalized size = 2.33

$$-\frac{10\,b^3x^6 + 10\,ab^2x^4 + 5\,a^2bx^2 + a^3}{40\,(b^9x^{10} + 5\,ab^8x^8 + 10\,a^2b^7x^6 + 10\,a^3b^6x^4 + 5\,a^4b^5x^2 + a^5b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 + 5*a*b^8*x
^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 + a^5*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(31) = 62$.
time = 0.29, size = 95, normalized size = 2.44

$$\frac{-a^3 - 5a^2bx^2 - 10ab^2x^4 - 10b^3x^6}{40a^5b^4 + 200a^4b^5x^2 + 400a^3b^6x^4 + 400a^2b^7x^6 + 200ab^8x^8 + 40b^9x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-a**3 - 5*a**2*b*x**2 - 10*a*b**2*x**4 - 10*b**3*x**6)/(40*a**5*b**4 + 200
*a**4*b**5*x**2 + 400*a**3*b**6*x**4 + 400*a**2*b**7*x**6 + 200*a*b**8*x**8
 + 40*b**9*x**10)
```

Giac [A]
time = 3.83, size = 44, normalized size = 1.13

$$-\frac{10\,b^3x^6 + 10\,ab^2x^4 + 5\,a^2bx^2 + a^3}{40\,(bx^2 + a)^5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/((b*x^2 + a)^5*b^4)
```

Mupad [B]
time = 0.05, size = 93, normalized size = 2.38

$$-\frac{a^3+5\,a^2\,b\,x^2+10\,a\,b^2\,x^4+10\,b^3\,x^6}{40\,a^5\,b^4+200\,a^4\,b^5\,x^2+400\,a^3\,b^6\,x^4+400\,a^2\,b^7\,x^6+200\,a\,b^8\,x^8+40\,b^9\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a^3 + 10*b^3*x^6 + 5*a^2*b*x^2 + 10*a*b^2*x^4)/(40*a^5*b^4 + 40*b^9*x^10
+ 200*a*b^8*x^8 + 200*a^4*b^5*x^2 + 400*a^3*b^6*x^4 + 400*a^2*b^7*x^6)
```

3.517 $\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2}{10b^3\left(a+bx^2\right)^5}+\frac{a}{4b^3\left(a+bx^2\right)^4}-\frac{1}{6b^3\left(a+bx^2\right)^3}$$

```
[Out] -1/10*a^2/b^3/(b*x^2+a)^5+1/4*a/b^3/(b*x^2+a)^4-1/6/b^3/(b*x^2+a)^3
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$-\frac{a^2}{10b^3\left(a+bx^2\right)^5}+\frac{a}{4b^3\left(a+bx^2\right)^4}-\frac{1}{6b^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*a^2/(b^3*(a + b*x^2)^5) + a/(4*b^3*(a + b*x^2)^4) - 1/(6*b^3*(a + b*x
^2)^3)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6 \int \frac{x^5}{(ab+b^2x^2)^6}\,dx$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \left(\frac{a^2}{b^8(a+bx)^6} - \frac{2a}{b^8(a+bx)^5} + \frac{1}{b^8(a+bx)^4}\right)\,dx, x, x^2\right)$$
$$= -\frac{a^2}{10b^3\,(a+bx^2)^5} + \frac{a}{4b^3\,(a+bx^2)^4} - \frac{1}{6b^3\,(a+bx^2)^3}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.66

$$-\frac{a^2+5abx^2+10b^2x^4}{60b^3\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/60*(a^2 + 5*a*b*x^2 + 10*b^2*x^4)/(b^3*(a + b*x^2)^5)
```

Maple [A]
time = 0.04, size = 48, normalized size = 0.91

method	result	size
norman	$\frac{-\frac{x^4}{6b}-\frac{a\,x^2}{12b^2}-\frac{a^2}{60b^3}}{(b\,x^2+a)^5}$	37
default	$-\frac{a^2}{10b^3(b\,x^2+a)^5}+\frac{a}{4b^3(b\,x^2+a)^4}-\frac{1}{6b^3(b\,x^2+a)^3}$	48
gosper	$-\frac{10b^2x^4+5ab\,x^2+a^2}{60(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2b^3}$	54
risch	$\frac{-\frac{x^4}{6b}-\frac{a\,x^2}{12b^2}-\frac{a^2}{60b^3}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^2/b^3/(b*x^2+a)^5+1/4*a/b^3/(b*x^2+a)^4-1/6/b^3/(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 80, normalized size = 1.51

$$-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\,(b^8x^{10}+5\,ab^7x^8+10\,a^2b^6x^6+10\,a^3b^5x^4+5\,a^4b^4x^2+a^5b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10*a^2*b^6*x
^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

Fricas [A]
time = 0.32, size = 80, normalized size = 1.51

$$-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\,(b^8x^{10}+5\,ab^7x^8+10\,a^2b^6x^6+10\,a^3b^5x^4+5\,a^4b^4x^2+a^5b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10*a^2*b^6*x
^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

Sympy [A]
time = 0.26, size = 83, normalized size = 1.57

$$\frac{-a^2-5abx^2-10b^2x^4}{60a^5b^3+300a^4b^4x^2+600a^3b^5x^4+600a^2b^6x^6+300ab^7x^8+60b^8x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-a**2 - 5*a*b*x**2 - 10*b**2*x**4)/(60*a**5*b**3 + 300*a**4*b**4*x**2 + 60
0*a**3*b**5*x**4 + 600*a**2*b**6*x**6 + 300*a*b**7*x**8 + 60*b**8*x**10)
```

Giac [A]
time = 4.11, size = 33, normalized size = 0.62

$$-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\,(bx^2+a)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/((b*x^2 + a)^5*b^3)
```

Mupad [B]
time = 4.62, size = 81, normalized size = 1.53

$$-\frac{\frac{a^2}{60\,b^3}+\frac{x^4}{6\,b}+\frac{a\,x^2}{12\,b^2}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a^2/(60*b^3) + x^4/(6*b) + (a*x^2)/(12*b^2))/(a^5 + b^5*x^10 + 5*a^4*b*x^
2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

3.518 $\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^3} \, dx$

Optimal. Leaf size=34

$$\frac{a}{10b^2\left(a+bx^2\right)^5}-\frac{1}{8b^2\left(a+bx^2\right)^4}$$

```
[Out] 1/10*a/b^2/(b*x^2+a)^5-1/8/b^2/(b*x^2+a)^4
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 45}

$$\frac{a}{10b^2\left(a+bx^2\right)^5}-\frac{1}{8b^2\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a/(10*b^2*(a + b*x^2)^5) - 1/(8*b^2*(a + b*x^2)^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^3}\,dx &= b^6\int \frac{x^3}{(ab+b^2x^2)^6}\,dx\\ &= \frac{1}{2}b^6\text{Subst}\left(\int \frac{x}{(ab+b^2x)^6}\,dx, x, x^2\right)\\ &= \frac{1}{2}b^6\text{Subst}\left(\int \left(-\frac{a}{b^7(a+bx)^6}+\frac{1}{b^7(a+bx)^5}\right)dx, x, x^2\right)\\ &= \frac{a}{10b^2(a+bx^2)^5}-\frac{1}{8b^2(a+bx^2)^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.71

$$-\frac{a+5bx^2}{40b^2(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/40*(a + 5*b*x^2)/(b^2*(a + b*x^2)^5)
```

Maple [A]
time = 0.04, size = 31, normalized size = 0.91

method	result	size
norman	$\frac{-\frac{x^2}{8b}-\frac{a}{40b^2}}{(b\,x^2+a)^5}$	26
default	$\frac{a}{10b^2(b\,x^2+a)^5}-\frac{1}{8b^2(b\,x^2+a)^4}$	31
gosper	$-\frac{5b\,x^2+a}{40(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2b^2}$	43
risch	$\frac{-\frac{x^2}{8b}-\frac{a}{40b^2}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*a/b^2/(b*x^2+a)^5-1/8/b^2/(b*x^2+a)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(30) = 60.
time = 0.28, size = 69, normalized size = 2.03

$$-\frac{5\,bx^2+a}{40\,(b^7x^{10}+5\,ab^6x^8+10\,a^2b^5x^6+10\,a^3b^4x^4+5\,a^4b^3x^2+a^5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10*a^3*b^4*x
^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(30) = 60$.
time = 0.32, size = 69, normalized size = 2.03

$$-\frac{5\,bx^2+a}{40\,(b^7x^{10}+5\,ab^6x^8+10\,a^2b^5x^6+10\,a^3b^4x^4+5\,a^4b^3x^2+a^5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10*a^3*b^4*x
^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(29) = 58$.
time = 0.25, size = 71, normalized size = 2.09

$$\frac{-a-5bx^2}{40a^5b^2+200a^4b^3x^2+400a^3b^4x^4+400a^2b^5x^6+200ab^6x^8+40b^7x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-a - 5*b*x**2)/(40*a**5*b**2 + 200*a**4*b**3*x**2 + 400*a**3*b**4*x**4 + 4
00*a**2*b**5*x**6 + 200*a*b**6*x**8 + 40*b**7*x**10)
```

Giac [A]
time = 3.79, size = 22, normalized size = 0.65

$$-\frac{5\,bx^2+a}{40\,(bx^2+a)^5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/40*(5*b*x^2 + a)/((b*x^2 + a)^5*b^2)
```

Mupad [B]
time = 4.48, size = 70, normalized size = 2.06

$$-\frac{\frac{a}{40\,b^2}+\frac{x^2}{8\,b}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -(a/(40*b^2) + x^2/(8*b))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*
a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

3.519 $\int \frac{x}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal. Leaf size=16

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

[Out] -1/10/b/(b*x^2+a)^5

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {28, 267}

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] -1/10*1/(b*(a + b*x^2)^5)

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6\int \frac{x}{\left(ab+b^2x^2\right)^6}\,dx\\ &= -\frac{1}{10b\left(a+bx^2\right)^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*1/(b*(a + b*x^2)^5)
```

Maple [A]
time = 0.04, size = 15, normalized size = 0.94

method	result	size
default	$-\frac{1}{10b\left(b\,x^{2}+a\right)^{5}}$	15
norman	$-\frac{1}{10b\left(b\,x^{2}+a\right)^{5}}$	15
gosper	$-\frac{1}{10\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}b}$	35
risch	$-\frac{1}{10\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}b}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/b/(b*x^2+a)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(14) = 28.
time = 0.29, size = 59, normalized size = 3.69

$$-\frac{1}{10\left(b^{6}x^{10}+5\,ab^{5}x^{8}+10\,a^{2}b^{4}x^{6}+10\,a^{3}b^{3}x^{4}+5\,a^{4}b^{2}x^{2}+a^{5}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 + 5*a^4*b^2
*x^2 + a^5*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(14) = 28.
time = 0.36, size = 59, normalized size = 3.69

$$-\frac{1}{10\left(b^{6}x^{10}+5\,ab^{5}x^{8}+10\,a^{2}b^{4}x^{6}+10\,a^{3}b^{3}x^{4}+5\,a^{4}b^{2}x^{2}+a^{5}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 + 5*a^4*b^2
*x^2 + a^5*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(14) = 28$.
time = 0.23, size = 63, normalized size = 3.94

$$-\frac{1}{10a^5b + 50a^4b^2x^2 + 100a^3b^3x^4 + 100a^2b^4x^6 + 50ab^5x^8 + 10b^6x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -1/(10*a**5*b + 50*a**4*b**2*x**2 + 100*a**3*b**3*x**4 + 100*a**2*b**4*x**6
 + 50*a*b**5*x**8 + 10*b**6*x**10)
```

Giac [A]
time = 5.52, size = 14, normalized size = 0.88

$$-\frac{1}{10\left(bx^2+a\right)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/10/((b*x^2 + a)^5*b)
```

Mupad [B]
time = 0.06, size = 61, normalized size = 3.81

$$-\frac{1}{10\,a^5\,b+50\,a^4\,b^2\,x^2+100\,a^3\,b^3\,x^4+100\,a^2\,b^4\,x^6+50\,a\,b^5\,x^8+10\,b^6\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] -1/(10*a^5*b + 10*b^6*x^10 + 50*a*b^5*x^8 + 50*a^4*b^2*x^2 + 100*a^3*b^3*x^
4 + 100*a^2*b^4*x^6)
```

3.520 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=102

$$\frac{1}{10a\left(a+bx^2\right)^5}+\frac{1}{8a^2\left(a+bx^2\right)^4}+\frac{1}{6a^3\left(a+bx^2\right)^3}+\frac{1}{4a^4\left(a+bx^2\right)^2}+\frac{1}{2a^5\left(a+bx^2\right)}+\frac{\log(x)}{a^6}-\frac{\log\left(a+bx^2\right)}{2a^6}$$

```
[Out] 1/10/a/(b*x^2+a)^5+1/8/a^2/(b*x^2+a)^4+1/6/a^3/(b*x^2+a)^3+1/4/a^4/(b*x^2+a
)^2+1/2/a^5/(b*x^2+a)+ln(x)/a^6-1/2*ln(b*x^2+a)/a^6
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 46}

$$-\frac{\log\left(a+bx^2\right)}{2a^6}+\frac{\log(x)}{a^6}+\frac{1}{2a^5\left(a+bx^2\right)}+\frac{1}{4a^4\left(a+bx^2\right)^2}+\frac{1}{6a^3\left(a+bx^2\right)^3}+\frac{1}{8a^2\left(a+bx^2\right)^4}+\frac{1}{10a\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] 1/(10*a*(a + b*x^2)^5) + 1/(8*a^2*(a + b*x^2)^4) + 1/(6*a^3*(a + b*x^2)^3)
+ 1/(4*a^4*(a + b*x^2)^2) + 1/(2*a^5*(a + b*x^2)) + Log[x]/a^6 - Log[a + b*
x^2]/(2*a^6)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{1}{x(ab+b^2x^2)^6}\,dx$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \frac{1}{x(ab+b^2x)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \left(\frac{1}{a^6b^6x} - \frac{1}{ab^5(a+bx)^6} - \frac{1}{a^2b^5(a+bx)^5} - \frac{1}{a^3b^5(a+bx)^4} - \frac{1}{a^4}\right.\right.$$
$$= \frac{1}{10a(a+bx^2)^5} + \frac{1}{8a^2(a+bx^2)^4} + \frac{1}{6a^3(a+bx^2)^3} + \frac{1}{4a^4(a+bx^2)^2} + \frac{1}{2a^5(a}$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 0.75

$$\frac{\frac{a(137a^4+385a^3bx^2+470a^2b^2x^4+270ab^3x^6+60b^4x^8)}{(a+bx^2)^5}+120\log(x)-60\log(a+bx^2)}{120a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((a*(137*a^4 + 385*a^3*b*x^2 + 470*a^2*b^2*x^4 + 270*a*b^3*x^6 + 60*b^4*x^8
))/(a + b*x^2)^5 + 120*Log[x] - 60*Log[a + b*x^2])/(120*a^6)
```

Maple [A]
time = 0.06, size = 110, normalized size = 1.08

method	result	size
norman	$\frac{-\frac{5bx^2}{2a^2}-\frac{15b^2x^4}{2a^3}-\frac{55b^3x^6}{6a^4}-\frac{125b^4x^8}{24a^5}-\frac{137b^5x^{10}}{120a^6}}{(bx^2+a)^5}+\frac{\ln(x)}{a^6}-\frac{\ln(bx^2+a)}{2a^6}$	85
risch	$\frac{\frac{b^4x^8}{2a^5}+\frac{9b^3x^6}{4a^4}+\frac{47b^2x^4}{12a^3}+\frac{77bx^2}{24a^2}+\frac{137}{120a}}{(bx^2+a)(b^2x^4+2abx^2+a^2)^2}+\frac{\ln(x)}{a^6}-\frac{\ln(bx^2+a)}{2a^6}$	99
default	$-\frac{b\left(-\frac{a^2}{2b(bx^2+a)^2}-\frac{a^4}{4b(bx^2+a)^4}-\frac{a}{b(bx^2+a)}-\frac{a^5}{5b(bx^2+a)^5}-\frac{a^3}{3b(bx^2+a)^3}+\frac{\ln(bx^2+a)}{b}\right)}{2a^6}+\frac{\ln(x)}{a^6}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/a^6*(-1/2*a^2/b/(b*x^2+a)^2-1/4*a^4/b/(b*x^2+a)^4-a/b/(b*x^2+a)-1/5*
a^5/b/(b*x^2+a)^5-1/3*a^3/b/(b*x^2+a)^3+1/b*ln(b*x^2+a))+ln(x)/a^6
```

Maxima [A]
time = 0.28, size = 126, normalized size = 1.24

$$\frac{60\,b^4x^8+270\,ab^3x^6+470\,a^2b^2x^4+385\,a^3bx^2+137\,a^4}{120\,(a^5b^5x^{10}+5\,a^6b^4x^8+10\,a^7b^3x^6+10\,a^8b^2x^4+5\,a^9bx^2+a^{10})} - \frac{\log(bx^2+a)}{2\,a^6} + \frac{\log(x^2)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(60*b^4*x^8 + 270*a*b^3*x^6 + 470*a^2*b^2*x^4 + 385*a^3*b*x^2 + 137*a
^4)/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9
*b*x^2 + a^10) - 1/2*log(b*x^2 + a)/a^6 + 1/2*log(x^2)/a^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(90) = 180$.
time = 0.34, size = 222, normalized size = 2.18

$$\frac{60\,ab^4x^8 + 270\,a^2b^3x^6 + 470\,a^3b^2x^4 + 385\,a^4bx^2 + 137\,a^5 - 60\,(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a^4bx^2 + a^5)\log\left(bx^2 + a\right) + 120\,(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a^4bx^2 + a^5)\log\left(x\right)}{120\,(a^6b^5x^{10} + 5\,a^7b^4x^8 + 10\,a^8b^3x^6 + 10\,a^9b^2x^4 + 5\,a^{10}bx^2 + a^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(60*a*b^4*x^8 + 270*a^2*b^3*x^6 + 470*a^3*b^2*x^4 + 385*a^4*b*x^2 + 1
37*a^5 - 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a
^4*b*x^2 + a^5)*log(b*x^2 + a) + 120*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x
^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*log(x))/(a^6*b^5*x^10 + 5*a^7*b^4*
x^8 + 10*a^8*b^3*x^6 + 10*a^9*b^2*x^4 + 5*a^10*b*x^2 + a^11)
```

Sympy [A]
time = 0.36, size = 128, normalized size = 1.25

$$\frac{137a^4 + 385a^3bx^2 + 470a^2b^2x^4 + 270ab^3x^6 + 60b^4x^8}{120a^{10} + 600a^9bx^2 + 1200a^8b^2x^4 + 1200a^7b^3x^6 + 600a^6b^4x^8 + 120a^5b^5x^{10}} + \frac{\log\left(x\right)}{a^6} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (137*a**4 + 385*a**3*b*x**2 + 470*a**2*b**2*x**4 + 270*a*b**3*x**6 + 60*b**
4*x**8)/(120*a**10 + 600*a**9*b*x**2 + 1200*a**8*b**2*x**4 + 1200*a**7*b**3
*x**6 + 600*a**6*b**4*x**8 + 120*a**5*b**5*x**10) + log(x)/a**6 - log(a/b +
 x**2)/(2*a**6)
```

Giac [A]
time = 3.58, size = 92, normalized size = 0.90

$$\frac{\log\left(x^2\right)}{2\,a^6} - \frac{\log\left(|bx^2 + a|\right)}{2\,a^6} + \frac{137\,b^5x^{10} + 745\,ab^4x^8 + 1640\,a^2b^3x^6 + 1840\,a^3b^2x^4 + 1070\,a^4bx^2 + 274\,a^5}{120\,(bx^2 + a)^5a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^6 - 1/2*log(abs(b*x^2 + a))/a^6 + 1/120*(137*b^5*x^10 + 745*
a*b^4*x^8 + 1640*a^2*b^3*x^6 + 1840*a^3*b^2*x^4 + 1070*a^4*b*x^2 + 274*a^5)
/((b*x^2 + a)^5*a^6)
```

Mupad [B]
time = 0.24, size = 122, normalized size = 1.20

$$\frac{\ln(x)}{a^6} - \frac{\ln(b\,x^2+a)}{2\,a^6} + \frac{\frac{137}{120\,a} + \frac{77\,b\,x^2}{24\,a^2} + \frac{47\,b^2\,x^4}{12\,a^3} + \frac{9\,b^3\,x^6}{4\,a^4} + \frac{b^4\,x^8}{2\,a^5}}{a^5 + 5\,a^4\,b\,x^2 + 10\,a^3\,b^2\,x^4 + 10\,a^2\,b^3\,x^6 + 5\,a\,b^4\,x^8 + b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] log(x)/a^6 - log(a + b*x^2)/(2*a^6) + (137/(120*a) + (77*b*x^2)/(24*a^2) +
(47*b^2*x^4)/(12*a^3) + (9*b^3*x^6)/(4*a^4) + (b^4*x^8)/(2*a^5))/(a^5 + b^5
*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

3.521 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=116

$$-\frac{1}{2a^6x^2}-\frac{b}{10a^2\left(a+bx^2\right)^5}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{a^5\left(a+bx^2\right)^2}-\frac{5b}{2a^6\left(a+bx^2\right)}-\frac{6b\log(x)}{a^7}+\frac{3b\log}{}$$

```
[Out] -1/2/a^6/x^2-1/10*b/a^2/(b*x^2+a)^5-1/4*b/a^3/(b*x^2+a)^4-1/2*b/a^4/(b*x^2+
a)^3-b/a^5/(b*x^2+a)^2-5/2*b/a^6/(b*x^2+a)-6*b*ln(x)/a^7+3*b*ln(b*x^2+a)/a^
7
```

Rubi [A]
time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 46}

$$\frac{3b\log\left(a+bx^2\right)}{a^7}-\frac{6b\log(x)}{a^7}-\frac{5b}{2a^6\left(a+bx^2\right)}-\frac{1}{2a^6x^2}-\frac{b}{a^5\left(a+bx^2\right)^2}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{10a^2\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/2*1/(a^6*x^2) - b/(10*a^2*(a + b*x^2)^5) - b/(4*a^3*(a + b*x^2)^4) - b/(
2*a^4*(a + b*x^2)^3) - b/(a^5*(a + b*x^2)^2) - (5*b)/(2*a^6*(a + b*x^2)) -
(6*b*Log[x])/a^7 + (3*b*Log[a + b*x^2])/a^7
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{1}{x^3(ab+b^2x^2)^6}\,dx$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \frac{1}{x^2(ab+b^2x)^6}\,dx, x, x^2\right)$$
$$= \frac{1}{2}b^6\text{Subst}\left(\int \left(\frac{1}{a^6b^6x^2} - \frac{6}{a^7b^5x} + \frac{1}{a^2b^4(a+bx)^6} + \frac{2}{a^3b^4(a+bx)^5} + \frac{3}{a^4b^4(a-\right.\right.$$
$$= -\frac{1}{2a^6x^2} - \frac{b}{10a^2(a+bx^2)^5} - \frac{b}{4a^3(a+bx^2)^4} - \frac{b}{2a^4(a+bx^2)^3} - \frac{b}{a^5(a+bx^2)}$$

Mathematica [A]
time = 0.06, size = 92, normalized size = 0.79

$$-\frac{\frac{a(10a^5+137a^4bx^2+385a^3b^2x^4+470a^2b^3x^6+270ab^4x^8+60b^5x^{10})}{x^2(a+bx^2)^5}+120b\log(x)-60b\log(a+bx^2)}{20a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/20*((a*(10*a^5 + 137*a^4*b*x^2 + 385*a^3*b^2*x^4 + 470*a^2*b^3*x^6 + 270
*a*b^4*x^8 + 60*b^5*x^10))/(x^2*(a + b*x^2)^5) + 120*b*Log[x] - 60*b*Log[a
+ b*x^2])/a^7
```

Maple [A]
time = 0.07, size = 123, normalized size = 1.06

method	result	size
norman	$\frac{-\frac{1}{2a}+\frac{15b^2x^4}{a^3}+\frac{45b^3x^6}{a^4}+\frac{55b^4x^8}{a^5}+\frac{125b^5x^{10}}{4a^6}+\frac{137b^6x^{12}}{20a^7}}{x^2(bx^2+a)^5}-\frac{6b\ln(x)}{a^7}+\frac{3b\ln(bx^2+a)}{a^7}$	98
risch	$\frac{-\frac{3b^5x^{10}}{a^6}-\frac{27b^4x^8}{2a^5}-\frac{47b^3x^6}{2a^4}-\frac{77b^2x^4}{4a^3}-\frac{137bx^2}{20a^2}-\frac{1}{2a}}{x^2(b^2x^4+2abx^2+a^2)^2(bx^2+a)}-\frac{6b\ln(x)}{a^7}+\frac{3b\ln(-bx^2-a)}{a^7}$	119
default	$\frac{b^2\left(-\frac{a^4}{2b(bx^2+a)^4}-\frac{5a}{b(bx^2+a)}-\frac{a^5}{5b(bx^2+a)^5}-\frac{2a^2}{b(bx^2+a)^2}-\frac{a^3}{b(bx^2+a)^3}+\frac{6\ln(bx^2+a)}{b}\right)}{2a^7}-\frac{1}{2a^6x^2}-\frac{6b\ln(x)}{a^7}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/a^7*(-1/2*a^4/b/(b*x^2+a)^4-5*a/b/(b*x^2+a)-1/5*a^5/b/(b*x^2+a)^5-2
*a^2/b/(b*x^2+a)^2-a^3/b/(b*x^2+a)^3+6/b*ln(b*x^2+a))-1/2/a^6/x^2-6*b*ln(x)
/a^7
```

Maxima [A]
time = 0.29, size = 143, normalized size = 1.23

$$-\frac{60\,b^5x^{10}+270\,ab^4x^8+470\,a^2b^3x^6+385\,a^3b^2x^4+137\,a^4bx^2+10\,a^5}{20\,(a^6b^5x^{12}+5\,a^7b^4x^{10}+10\,a^8b^3x^8+10\,a^9b^2x^6+5\,a^{10}bx^4+a^{11}x^2)}+\frac{3\,b\log\left(bx^2+a\right)}{a^7}-\frac{3\,b\log\left(x^2\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(60*b^5*x^10 + 270*a*b^4*x^8 + 470*a^2*b^3*x^6 + 385*a^3*b^2*x^4 + 13
7*a^4*b*x^2 + 10*a^5)/(a^6*b^5*x^12 + 5*a^7*b^4*x^10 + 10*a^8*b^3*x^8 + 10*
a^9*b^2*x^6 + 5*a^10*b*x^4 + a^11*x^2) + 3*b*log(b*x^2 + a)/a^7 - 3*b*log(x
^2)/a^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(106) = 212.
time = 0.38, size = 251, normalized size = 2.16

$$-\frac{60\,ab^5x^{10}+270\,a^2b^4x^8+470\,a^3b^3x^6+385\,a^4b^2x^4+137\,a^5bx^2+10\,a^6-60\,(b^6x^{12}+5\,ab^5x^{10}+10\,a^2b^4x^8+10\,a^3b^3x^6+5\,a^4b^2x^4+a^5bx^2)\log\left(bx^2+a\right)+120\,(b^6x^{12}+5\,ab^5x^{10}+10\,a^2b^4x^8+10\,a^3b^3x^6+5\,a^4b^2x^4+a^5bx^2)\log\left(x\right)}{20\,(a^7b^5x^{12}+5\,a^8b^4x^{10}+10\,a^9b^3x^8+10\,a^{10}b^2x^6+5\,a^{11}bx^4+a^{12}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/20*(60*a*b^5*x^10 + 270*a^2*b^4*x^8 + 470*a^3*b^3*x^6 + 385*a^4*b^2*x^4
+ 137*a^5*b*x^2 + 10*a^6 - 60*(b^6*x^12 + 5*a*b^5*x^10 + 10*a^2*b^4*x^8 + 1
0*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b*x^2)*log(b*x^2 + a) + 120*(b^6*x^12 +
 5*a*b^5*x^10 + 10*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b*x^2
)*log(x))/(a^7*b^5*x^12 + 5*a^8*b^4*x^10 + 10*a^9*b^3*x^8 + 10*a^10*b^2*x^6
 + 5*a^11*b*x^4 + a^12*x^2)
```

Sympy [A]
time = 0.43, size = 150, normalized size = 1.29

$$\frac{-10a^5-137a^4bx^2-385a^3b^2x^4-470a^2b^3x^6-270ab^4x^8-60b^5x^{10}}{20a^{11}x^2+100a^{10}bx^4+200a^9b^2x^6+200a^8b^3x^8+100a^7b^4x^{10}+20a^6b^5x^{12}}-\frac{6b\log\left(x\right)}{a^7}+\frac{3b\log\left(\frac{a}{b}+x^2\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-10*a**5 - 137*a**4*b*x**2 - 385*a**3*b**2*x**4 - 470*a**2*b**3*x**6 - 270
*a*b**4*x**8 - 60*b**5*x**10)/(20*a**11*x**2 + 100*a**10*b*x**4 + 200*a**9*
b**2*x**6 + 200*a**8*b**3*x**8 + 100*a**7*b**4*x**10 + 20*a**6*b**5*x**12)
- 6*b*log(x)/a**7 + 3*b*log(a/b + x**2)/a**7
```

Giac [A]
time = 3.57, size = 115, normalized size = 0.99

$$-\frac{3\,b\log\left(x^2\right)}{a^7}+\frac{3\,b\log\left(|bx^2+a|\right)}{a^7}+\frac{6\,bx^2-a}{2\,a^7x^2}-\frac{137\,b^6x^{10}+735\,ab^5x^8+1590\,a^2b^4x^6+1740\,a^3b^3x^4+970\,a^4b^2x^2+224\,a^5b}{20\,(bx^2+a)^5a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -3*b*log(x^2)/a^7 + 3*b*log(abs(b*x^2 + a))/a^7 + 1/2*(6*b*x^2 - a)/(a^7*x^
2) - 1/20*(137*b^6*x^10 + 735*a*b^5*x^8 + 1590*a^2*b^4*x^6 + 1740*a^3*b^3*x
^4 + 970*a^4*b^2*x^2 + 224*a^5*b)/((b*x^2 + a)^5*a^7)
```

Mupad [B]
time = 4.68, size = 141, normalized size = 1.22

$$\frac{3\,b\,\ln\left(b\,x^2+a\right)}{a^7}-\frac{\frac{1}{2\,a}+\frac{137\,b\,x^2}{20\,a^2}+\frac{77\,b^2\,x^4}{4\,a^3}+\frac{47\,b^3\,x^6}{2\,a^4}+\frac{27\,b^4\,x^8}{2\,a^5}+\frac{3\,b^5\,x^{10}}{a^6}}{a^5\,x^2+5\,a^4\,b\,x^4+10\,a^3\,b^2\,x^6+10\,a^2\,b^3\,x^8+5\,a\,b^4\,x^{10}+b^5\,x^{12}}-\frac{6\,b\,\ln\left(x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] (3*b*log(a + b*x^2))/a^7 - (1/(2*a) + (137*b*x^2)/(20*a^2) + (77*b^2*x^4)/(
4*a^3) + (47*b^3*x^6)/(2*a^4) + (27*b^4*x^8)/(2*a^5) + (3*b^5*x^10)/a^6)/(a
^5*x^2 + b^5*x^12 + 5*a^4*b*x^4 + 5*a*b^4*x^10 + 10*a^3*b^2*x^6 + 10*a^2*b^
3*x^8) - (6*b*log(x))/a^7
```

3.522 $\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{1}{4a^6x^4}+\frac{3b}{a^7x^2}+\frac{b^2}{10a^3\,(a+bx^2)^5}+\frac{3b^2}{8a^4\,(a+bx^2)^4}+\frac{b^2}{a^5\,(a+bx^2)^3}+\frac{5b^2}{2a^6\,(a+bx^2)^2}+\frac{15b^2}{2a^7\,(a+bx^2)}+\frac{21b^2\log(x}{a^8}$$

```
[Out] -1/4/a^6/x^4+3*b/a^7/x^2+1/10*b^2/a^3/(b*x^2+a)^5+3/8*b^2/a^4/(b*x^2+a)^4+b
^2/a^5/(b*x^2+a)^3+5/2*b^2/a^6/(b*x^2+a)^2+15/2*b^2/a^7/(b*x^2+a)+21*b^2*ln
(x)/a^8-21/2*b^2*ln(b*x^2+a)/a^8
```

Rubi [A]
time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 272, 46}

$$-\frac{21b^2\log\left(a+bx^2\right)}{2a^8}+\frac{21b^2\log(x)}{a^8}+\frac{15b^2}{2a^7\,(a+bx^2)}+\frac{3b}{a^7x^2}+\frac{5b^2}{2a^6\,(a+bx^2)^2}-\frac{1}{4a^6x^4}+\frac{b^2}{a^5\,(a+bx^2)^3}+\frac{3b^2}{8a^4\,(a+bx^2)^4}+\frac{b^2}{10a^3\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/4*1/(a^6*x^4) + (3*b)/(a^7*x^2) + b^2/(10*a^3*(a + b*x^2)^5) + (3*b^2)/(
8*a^4*(a + b*x^2)^4) + b^2/(a^5*(a + b*x^2)^3) + (5*b^2)/(2*a^6*(a + b*x^2)
^2) + (15*b^2)/(2*a^7*(a + b*x^2)) + (21*b^2*Log[x])/a^8 - (21*b^2*Log[a +
b*x^2])/(2*a^8)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5 (a^2+2abx^2+b^2x^4)^3} dx = b^6 \int \frac{1}{x^5 (ab+b^2x^2)^6} dx$$
$$= \frac{1}{2} b^6 \text{Subst}\left(\int \frac{1}{x^3 (ab+b^2x)^6} dx, x, x^2\right)$$
$$= \frac{1}{2} b^6 \text{Subst}\left(\int \left(\frac{1}{a^6b^6x^3} - \frac{6}{a^7b^5x^2} + \frac{21}{a^8b^4x} - \frac{1}{a^3b^3(a+bx)^6} - \frac{3}{a^4b^3(a+bx)^5}\right.\right.$$
$$= -\frac{1}{4a^6x^4} + \frac{3b}{a^7x^2} + \frac{b^2}{10a^3 (a+bx^2)^5} + \frac{3b^2}{8a^4 (a+bx^2)^4} + \frac{b^2}{a^5 (a+bx^2)^3} + \frac{}{2a^6 (}$$

Mathematica [A]
time = 0.04, size = 107, normalized size = 0.76

$$\frac{\frac{a(-10a^6+70a^5bx^2+959a^4b^2x^4+2695a^3b^3x^6+3290a^2b^4x^8+1890ab^5x^{10}+420b^6x^{12})}{x^4(a+bx^2)^5} + 840b^2\log(x) - 420b^2\log(a+bx^2)}{40a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((a*(-10*a^6 + 70*a^5*b*x^2 + 959*a^4*b^2*x^4 + 2695*a^3*b^3*x^6 + 3290*a^2
*b^4*x^8 + 1890*a*b^5*x^10 + 420*b^6*x^12))/(x^4*(a + b*x^2)^5) + 840*b^2*L
og[x] - 420*b^2*Log[a + b*x^2])/(40*a^8)
```

Maple [A]
time = 0.07, size = 134, normalized size = 0.96

method	result	size
norman	$\frac{-\frac{1}{4a}+\frac{7b x^2}{4a^2}-\frac{105b^3x^6}{2a^4}-\frac{315b^4x^8}{2a^5}-\frac{385b^5x^{10}}{2a^6}-\frac{875b^6x^{12}}{8a^7}-\frac{959b^7x^{14}}{40a^8}}{x^4(b x^2+a)^5}+\frac{21b^2\ln(x)}{a^8}-\frac{21b^2\ln(b x^2+a)}{2a^8}$	111
risch	$\frac{\frac{21b^6x^{12}}{2a^7}+\frac{189b^5x^{10}}{4a^6}+\frac{329b^4x^8}{4a^5}+\frac{539b^3x^6}{8a^4}+\frac{959b^2x^4}{40a^3}+\frac{7b x^2}{4a^2}-\frac{1}{4a}}{x^4(b^2x^4+2ab x^2+a^2)^2(b x^2+a)}+\frac{21b^2\ln(x)}{a^8}-\frac{21b^2\ln(b x^2+a)}{2a^8}$	131
default	$-\frac{b^3\left(-\frac{5a^2}{b(b x^2+a)^2}-\frac{3a^4}{4b(b x^2+a)^4}-\frac{15a}{b(b x^2+a)}-\frac{2a^3}{b(b x^2+a)^3}-\frac{a^5}{5b(b x^2+a)^5}+\frac{21\ln(b x^2+a)}{b}\right)}{2a^8}-\frac{1}{4a^6x^4}+\frac{3b}{a^7x^2}+\frac{21b^2\ln(x)}{a^8}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^3/a^8*(-5*a^2/b/(b*x^2+a)^2-3/4*a^4/b/(b*x^2+a)^4-15*a/b/(b*x^2+a)-2
*a^3/b/(b*x^2+a)^3-1/5*a^5/b/(b*x^2+a)^5+21/b*ln(b*x^2+a))-1/4/a^6/x^4+3*b/
a^7/x^2+21*b^2*ln(x)/a^8
```

Maxima [**A**]
time = 0.29, size = 158, normalized size = 1.13

$$\frac{420\,b^6x^{12}+1890\,ab^5x^{10}+3290\,a^2b^4x^8+2695\,a^3b^3x^6+959\,a^4b^2x^4+70\,a^5bx^2-10\,a^6}{40\,(a^7b^5x^{14}+5\,a^8b^4x^{12}+10\,a^9b^3x^{10}+10\,a^{10}b^2x^8+5\,a^{11}bx^6+a^{12}x^4)}-\frac{21\,b^2\log\left(bx^2+a\right)}{2\,a^8}+\frac{21\,b^2\log\left(x^2\right)}{2\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/40*(420*b^6*x^12 + 1890*a*b^5*x^10 + 3290*a^2*b^4*x^8 + 2695*a^3*b^3*x^6
+ 959*a^4*b^2*x^4 + 70*a^5*b*x^2 - 10*a^6)/(a^7*b^5*x^14 + 5*a^8*b^4*x^12 +
 10*a^9*b^3*x^10 + 10*a^10*b^2*x^8 + 5*a^11*b*x^6 + a^12*x^4) - 21/2*b^2*lo
g(b*x^2 + a)/a^8 + 21/2*b^2*log(x^2)/a^8
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(128) = 256$.
time = 0.38, size = 266, normalized size = 1.90

$$\frac{420\,ab^6x^{12}+1890\,a^2b^5x^{10}+3290\,a^3b^4x^8+2695\,a^4b^3x^6+959\,a^5b^2x^4+70\,a^6bx^2-10\,a^7-420\,(b^7x^{14}+5\,ab^6x^{12}+10\,a^2b^5x^{10}+10\,a^3b^4x^8+5\,a^4b^3x^6+a^5b^2x^4)\log\left(bx^2+a\right)+840\,(b^7x^{14}+5\,ab^6x^{12}+10\,a^2b^5x^{10}+10\,a^3b^4x^8+5\,a^4b^3x^6+a^5b^2x^4)\log\left(x\right)}{40\,(a^8b^5x^{14}+5\,a^9b^4x^{12}+10\,a^{10}b^3x^{10}+10\,a^{11}b^2x^8+5\,a^{12}bx^6+a^{13}x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(420*a*b^6*x^12 + 1890*a^2*b^5*x^10 + 3290*a^3*b^4*x^8 + 2695*a^4*b^3*
x^6 + 959*a^5*b^2*x^4 + 70*a^6*b*x^2 - 10*a^7 - 420*(b^7*x^14 + 5*a*b^6*x^1
2 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x^8 + 5*a^4*b^3*x^6 + a^5*b^2*x^4)*log(b*x
^2 + a) + 840*(b^7*x^14 + 5*a*b^6*x^12 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x^8 +
 5*a^4*b^3*x^6 + a^5*b^2*x^4)*log(x))/(a^8*b^5*x^14 + 5*a^9*b^4*x^12 + 10*a
^10*b^3*x^10 + 10*a^11*b^2*x^8 + 5*a^12*b*x^6 + a^13*x^4)
```

Sympy [**A**]
time = 0.47, size = 165, normalized size = 1.18

$$\frac{-10a^6+70a^5bx^2+959a^4b^2x^4+2695a^3b^3x^6+3290a^2b^4x^8+1890ab^5x^{10}+420b^6x^{12}}{40a^{12}x^4+200a^{11}bx^6+400a^{10}b^2x^8+400a^9b^3x^{10}+200a^8b^4x^{12}+40a^7b^5x^{14}}+\frac{21b^2\log\left(x\right)}{a^8}-\frac{21b^2\log\left(\frac{a}{b}+x^2\right)}{2a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-10*a**6 + 70*a**5*b*x**2 + 959*a**4*b**2*x**4 + 2695*a**3*b**3*x**6 + 329
0*a**2*b**4*x**8 + 1890*a*b**5*x**10 + 420*b**6*x**12)/(40*a**12*x**4 + 200
*a**11*b*x**6 + 400*a**10*b**2*x**8 + 400*a**9*b**3*x**10 + 200*a**8*b**4*x
**12 + 40*a**7*b**5*x**14) + 21*b**2*log(x)/a**8 - 21*b**2*log(a/b + x**2)/
(2*a**8)
```

Giac [**A**]
time = 3.31, size = 130, normalized size = 0.93

$$\frac{21\,b^2\log\left(x^2\right)}{2\,a^8}-\frac{21\,b^2\log\left(|bx^2+a|\right)}{2\,a^8}-\frac{63\,b^2x^4-12\,abx^2+a^2}{4\,a^8x^4}+\frac{959\,b^7x^{10}+5095\,ab^6x^8+10890\,a^2b^5x^6+11730\,a^3b^4x^4+6390\,a^4b^3x^2+1418\,a^5b^2}{40\,(bx^2+a)^5a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 21/2*b^2*log(x^2)/a^8 - 21/2*b^2*log(abs(b*x^2 + a))/a^8 - 1/4*(63*b^2*x^4
- 12*a*b*x^2 + a^2)/(a^8*x^4) + 1/40*(959*b^7*x^10 + 5095*a*b^6*x^8 + 10890
*a^2*b^5*x^6 + 11730*a^3*b^4*x^4 + 6390*a^4*b^3*x^2 + 1418*a^5*b^2)/((b*x^2
 + a)^5*a^8)
```

Mupad [B]
time = 4.91, size = 155, normalized size = 1.11

$$\frac{\frac{7\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{959\,b^2\,x^4}{40\,a^3}+\frac{539\,b^3\,x^6}{8\,a^4}+\frac{329\,b^4\,x^8}{4\,a^5}+\frac{189\,b^5\,x^{10}}{4\,a^6}+\frac{21\,b^6\,x^{12}}{2\,a^7}}{a^5\,x^4+5\,a^4\,b\,x^6+10\,a^3\,b^2\,x^8+10\,a^2\,b^3\,x^{10}+5\,a\,b^4\,x^{12}+b^5\,x^{14}}-\frac{21\,b^2\,\ln\left(b\,x^2+a\right)}{2\,a^8}+\frac{21\,b^2\,\ln\left(x\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((7*b*x^2)/(4*a^2) - 1/(4*a) + (959*b^2*x^4)/(40*a^3) + (539*b^3*x^6)/(8*a^
4) + (329*b^4*x^8)/(4*a^5) + (189*b^5*x^10)/(4*a^6) + (21*b^6*x^12)/(2*a^7)
)/(a^5*x^4 + b^5*x^14 + 5*a^4*b*x^6 + 5*a*b^4*x^12 + 10*a^3*b^2*x^8 + 10*a^
2*b^3*x^10) - (21*b^2*log(a + b*x^2))/(2*a^8) + (21*b^2*log(x))/a^8
```

3.523 $\int \frac{x^{16}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=155

$$\frac{9009a^2x}{256b^8}-\frac{3003ax^3}{256b^7}+\frac{9009x^5}{1280b^6}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{128}{256b^5\,(}$$

```
[Out] 9009/256*a^2*x/b^8-3003/256*a*x^3/b^7+9009/1280*x^5/b^6-1/10*x^15/b/(b*x^2+
a)^5-3/16*x^13/b^2/(b*x^2+a)^4-13/32*x^11/b^3/(b*x^2+a)^3-143/128*x^9/b^4/(
b*x^2+a)^2-1287/256*x^7/b^5/(b*x^2+a)-9009/256*a^(5/2)*arctan(x*b^(1/2)/a^(
1/2))/b^(17/2)
```

Rubi [A]

time = 0.07, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 308, 211}

$$-\frac{9009a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{17/2}}+\frac{9009a^2x}{256b^8}-\frac{3003ax^3}{256b^7}-\frac{1287x^7}{256b^5\left(a+bx^2\right)}-\frac{143x^9}{128b^4\left(a+bx^2\right)^2}-\frac{13x^{11}}{32b^3\left(a+bx^2\right)^3}-\frac{3x^{13}}{16b^2\left(a+bx^2\right)^4}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}+\frac{9009x^5}{1280b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (9009*a^2*x)/(256*b^8) - (3003*a*x^3)/(256*b^7) + (9009*x^5)/(1280*b^6) - x
^15/(10*b*(a + b*x^2)^5) - (3*x^13)/(16*b^2*(a + b*x^2)^4) - (13*x^11)/(32*
b^3*(a + b*x^2)^3) - (143*x^9)/(128*b^4*(a + b*x^2)^2) - (1287*x^7)/(256*b^
5*(a + b*x^2)) - (9009*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(17/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{x^{16}}{(ab+b^2x^2)^6}\,dx$$
$$= -\frac{x^{15}}{10b(a+bx^2)^5} + \frac{1}{2}(3b^4)\int \frac{x^{14}}{(ab+b^2x^2)^5}\,dx$$
$$= -\frac{x^{15}}{10b(a+bx^2)^5} - \frac{3x^{13}}{16b^2(a+bx^2)^4} + \frac{1}{16}(39b^2)\int \frac{x^{12}}{(ab+b^2x^2)^4}\,dx$$
$$= -\frac{x^{15}}{10b(a+bx^2)^5} - \frac{3x^{13}}{16b^2(a+bx^2)^4} - \frac{13x^{11}}{32b^3(a+bx^2)^3} + \frac{143}{32}\int \frac{x^{10}}{(ab+b^2x^2)^3}\,dx$$
$$= -\frac{x^{15}}{10b(a+bx^2)^5} - \frac{3x^{13}}{16b^2(a+bx^2)^4} - \frac{13x^{11}}{32b^3(a+bx^2)^3} - \frac{143x^9}{128b^4(a+bx^2)^2} + \frac{128}{}$$
$$= -\frac{x^{15}}{10b(a+bx^2)^5} - \frac{3x^{13}}{16b^2(a+bx^2)^4} - \frac{13x^{11}}{32b^3(a+bx^2)^3} - \frac{143x^9}{128b^4(a+bx^2)^2} - \frac{}{256}$$
$$= -\frac{x^{15}}{10b(a+bx^2)^5} - \frac{3x^{13}}{16b^2(a+bx^2)^4} - \frac{13x^{11}}{32b^3(a+bx^2)^3} - \frac{143x^9}{128b^4(a+bx^2)^2} - \frac{}{256}$$
$$= \frac{9009a^2x}{256b^8} - \frac{3003ax^3}{256b^7} + \frac{9009x^5}{1280b^6} - \frac{x^{15}}{10b(a+bx^2)^5} - \frac{3x^{13}}{16b^2(a+bx^2)^4} - \frac{13x^1}{32b^3(a+}$$
$$= \frac{9009a^2x}{256b^8} - \frac{3003ax^3}{256b^7} + \frac{9009x^5}{1280b^6} - \frac{x^{15}}{10b(a+bx^2)^5} - \frac{3x^{13}}{16b^2(a+bx^2)^4} - \frac{13x^1}{32b^3(a+}$$

Mathematica [A]

time = 0.04, size = 122, normalized size = 0.79

$$\frac{\frac{\sqrt{b}\,x(45045a^7+210210a^6bx^2+384384a^5b^2x^4+338910a^4b^3x^6+137995a^3b^4x^8+16640a^2b^5x^{10}-1280ab^6x^{12}+256b^7x^{14})}{(a+bx^2)^5} - 45045a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{1280b^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(45045*a^7 + 210210*a^6*b*x^2 + 384384*a^5*b^2*x^4 + 338910*a^4
*b^3*x^6 + 137995*a^3*b^4*x^8 + 16640*a^2*b^5*x^10 - 1280*a*b^6*x^12 + 256*
b^7*x^14))/(a + b*x^2)^5 - 45045*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(1280
*b^(17/2))
```

Maple [A]
time = 0.06, size = 107, normalized size = 0.69

method	result
default	$\frac{\frac{1}{5}b^2x^5-2ab\,x^3+21a^2x}{b^8}-\frac{a^3\left(\frac{-\frac{5327}{256}b^4x^9-\frac{9443}{128}a\,b^3x^7-\frac{1001}{10}a^2b^2x^5-\frac{7837}{128}a^3b\,x^3-\frac{3633}{256}a^4x}{\left(b\,x^2+a\right)^5}+\frac{9009\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}}\right)}{b^8}$
risch	$\frac{x^5}{5b^6}-\frac{2a\,x^3}{b^7}+\frac{21a^2x}{b^8}+\frac{\frac{5327}{256}a^3b^4x^9+\frac{9443}{128}a^4b^3x^7+\frac{1001}{10}a^5b^2x^5+\frac{7837}{128}a^6b\,x^3+\frac{3633}{256}a^7x}{b^8(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}+\frac{9009\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\,x-\right.}{512b^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^8*(1/5*b^2*x^5-2*a*b*x^3+21*a^2*x)-1/b^8*a^3*((-5327/256*b^4*x^9-9443/1
28*a*b^3*x^7-1001/10*a^2*b^2*x^5-7837/128*a^3*b*x^3-3633/256*a^4*x)/(b*x^2+
a)^5+9009/256/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 159, normalized size = 1.03

$$\frac{26635\,a^3b^4x^9+94430\,a^4b^3x^7+128128\,a^5b^2x^5+78370\,a^6bx^3+18165\,a^7x}{1280\,(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8)}-\frac{9009\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^8}+\frac{b^2x^5-10\,abx^3+105\,a^2x}{5\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(26635*a^3*b^4*x^9 + 94430*a^4*b^3*x^7 + 128128*a^5*b^2*x^5 + 78370*
a^6*b*x^3 + 18165*a^7*x)/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a
^3*b^10*x^4 + 5*a^4*b^9*x^2 + a^5*b^8) - 9009/256*a^3*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^8) + 1/5*(b^2*x^5 - 10*a*b*x^3 + 105*a^2*x)/b^8
```

Fricas [A]
time = 0.35, size = 454, normalized size = 2.93

$$\left[\frac{512\,b^7x^{15}-2560\,ab^6x^{13}+33280\,a^2b^5x^{11}+275990\,a^3b^4x^9+677820\,a^4b^3x^7+768768\,a^5b^2x^5+420420\,a^6bx^3+90090\,a^7x+45045\,(a^2b^5x^{10}+5\,a^3b^4x^8+10\,a^4b^3x^6+10\,a^5b^2x^4+5\,a^6bx^2+a^7)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{2560\,(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8)},\ \frac{256\,b^7x^{15}-1280\,ab^6x^{13}+16640\,a^2b^5x^{11}+137995\,a^3b^4x^9+338910\,a^4b^3x^7+384384\,a^5b^2x^5+210210\,a^6bx^3+45045\,a^7x-45045\,(a^2b^5x^{10}+5\,a^3b^4x^8+10\,a^4b^3x^6+10\,a^5b^2x^4+5\,a^6bx^2+a^7)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{1280\,(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(512*b^7*x^15 - 2560*a*b^6*x^13 + 33280*a^2*b^5*x^11 + 275990*a^3*b
^4*x^9 + 677820*a^4*b^3*x^7 + 768768*a^5*b^2*x^5 + 420420*a^6*b*x^3 + 90090
*a^7*x + 45045*(a^2*b^5*x^10 + 5*a^3*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a^5*b^2*
x^4 + 5*a^6*b*x^2 + a^7)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4 + 5
*a^4*b^9*x^2 + a^5*b^8), 1/1280*(256*b^7*x^15 - 1280*a*b^6*x^13 + 16640*a^2
*b^5*x^11 + 137995*a^3*b^4*x^9 + 338910*a^4*b^3*x^7 + 384384*a^5*b^2*x^5 +
210210*a^6*b*x^3 + 45045*a^7*x - 45045*(a^2*b^5*x^10 + 5*a^3*b^4*x^8 + 10*a
^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 + a^7)*sqrt(a/b)*arctan(b*x*sqrt(
a/b)/a))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4 + 5*
a^4*b^9*x^2 + a^5*b^8)]
```

Sympy [A]
time = 0.49, size = 218, normalized size = 1.41

$$\frac{21a^2x}{b^8} - \frac{2ax^3}{b^7} + \frac{9009\sqrt{-\frac{a^5}{b^{17}}}\log\left(x - \frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512} - \frac{9009\sqrt{-\frac{a^5}{b^{17}}}\log\left(x + \frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512} + \frac{18165a^7x + 78370a^6bx^3 + 128128a^5b^2x^5 + 94430a^4b^3x^7 + 26635a^3b^4x^9}{1280a^5b^8 + 6400a^4b^9x^2 + 12800a^3b^{10}x^4 + 12800a^2b^{11}x^6 + 6400ab^{12}x^8 + 1280b^{13}x^{10}} + \frac{x^5}{5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**16/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 21*a**2*x/b**8 - 2*a*x**3/b**7 + 9009*sqrt(-a**5/b**17)*log(x - b**8*sqrt(-
a**5/b**17)/a**2)/512 - 9009*sqrt(-a**5/b**17)*log(x + b**8*sqrt(-a**5/b**1
7)/a**2)/512 + (18165*a**7*x + 78370*a**6*b*x**3 + 128128*a**5*b**2*x**5 +
94430*a**4*b**3*x**7 + 26635*a**3*b**4*x**9)/(1280*a**5*b**8 + 6400*a**4*b*
*9*x**2 + 12800*a**3*b**10*x**4 + 12800*a**2*b**11*x**6 + 6400*a*b**12*x**8
 + 1280*b**13*x**10) + x**5/(5*b**6)
```

Giac [A]
time = 2.86, size = 117, normalized size = 0.75

$$-\frac{9009\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^8} + \frac{26635\,a^3b^4x^9 + 94430\,a^4b^3x^7 + 128128\,a^5b^2x^5 + 78370\,a^6bx^3 + 18165\,a^7x}{1280\,(bx^2+a)^5b^8} + \frac{b^{24}x^5 - 10\,ab^{23}x^3 + 105\,a^2b^{22}x}{5\,b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -9009/256*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^8) + 1/1280*(26635*a^3*b^4
*x^9 + 94430*a^4*b^3*x^7 + 128128*a^5*b^2*x^5 + 78370*a^6*b*x^3 + 18165*a^7
*x)/((b*x^2 + a)^5*b^8) + 1/5*(b^24*x^5 - 10*a*b^23*x^3 + 105*a^2*b^22*x)/b
^30
```

Mupad [B]
time = 0.11, size = 153, normalized size = 0.99

$$\frac{\frac{3633\,a^7\,x}{256} + \frac{7837\,a^6\,b\,x^3}{128} + \frac{1001\,a^5\,b^2\,x^5}{10} + \frac{9443\,a^4\,b^3\,x^7}{128} + \frac{5327\,a^3\,b^4\,x^9}{256}}{a^5\,b^8 + 5\,a^4\,b^9\,x^2 + 10\,a^3\,b^{10}\,x^4 + 10\,a^2\,b^{11}\,x^6 + 5\,a\,b^{12}\,x^8 + b^{13}\,x^{10}} + \frac{x^5}{5\,b^6} - \frac{2\,a\,x^3}{b^7} + \frac{21\,a^2\,x}{b^8} - \frac{9009\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{17/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^16/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)

[Out] ((3633*a^7*x)/256 + (7837*a^6*b*x^3)/128 + (1001*a^5*b^2*x^5)/10 + (9443*a^4*b^3*x^7)/128 + (5327*a^3*b^4*x^9)/256)/(a^5*b^8 + b^13*x^10 + 5*a*b^12*x^8 + 5*a^4*b^9*x^2 + 10*a^3*b^10*x^4 + 10*a^2*b^11*x^6) + x^5/(5*b^6) - (2*a*x^3)/b^7 + (21*a^2*x)/b^8 - (9009*a^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(256*b^(17/2))

3.524 $\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=142

$$-\frac{3003ax}{256b^7}+\frac{1001x^3}{256b^6}-\frac{x^{13}}{10b\,(a+bx^2)^5}-\frac{13x^{11}}{80b^2\,(a+bx^2)^4}-\frac{143x^9}{480b^3\,(a+bx^2)^3}-\frac{429x^7}{640b^4\,(a+bx^2)^2}-\frac{3003x^5}{1280b^5\,(a+bx^2)}-$$

```
[Out] -3003/256*a*x/b^7+1001/256*x^3/b^6-1/10*x^13/b/(b*x^2+a)^5-13/80*x^11/b^2/(
b*x^2+a)^4-143/480*x^9/b^3/(b*x^2+a)^3-429/640*x^7/b^4/(b*x^2+a)^2-3003/128
0*x^5/b^5/(b*x^2+a)+3003/256*a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(15/2)
```

Rubi [A]
time = 0.06, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 308, 211}

$$\frac{3003a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{15/2}}-\frac{3003ax}{256b^7}-\frac{3003x^5}{1280b^5\,(a+bx^2)}-\frac{429x^7}{640b^4\,(a+bx^2)^2}-\frac{143x^9}{480b^3\,(a+bx^2)^3}-\frac{13x^{11}}{80b^2\,(a+bx^2)^4}-\frac{x^{13}}{10b\,(a+bx^2)^5}+\frac{1001x^3}{256b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-3003*a*x)/(256*b^7) + (1001*x^3)/(256*b^6) - x^13/(10*b*(a + b*x^2)^5) -
(13*x^11)/(80*b^2*(a + b*x^2)^4) - (143*x^9)/(480*b^3*(a + b*x^2)^3) - (429
*x^7)/(640*b^4*(a + b*x^2)^2) - (3003*x^5)/(1280*b^5*(a + b*x^2)) + (3003*a
^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(15/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{x^{14}}{(ab+b^2x^2)^6}\,dx$$
$$= -\frac{x^{13}}{10b(a+bx^2)^5}+\frac{1}{10}(13b^4)\int \frac{x^{12}}{(ab+b^2x^2)^5}\,dx$$
$$= -\frac{x^{13}}{10b(a+bx^2)^5}-\frac{13x^{11}}{80b^2(a+bx^2)^4}+\frac{1}{80}(143b^2)\int \frac{x^{10}}{(ab+b^2x^2)^4}\,dx$$
$$= -\frac{x^{13}}{10b(a+bx^2)^5}-\frac{13x^{11}}{80b^2(a+bx^2)^4}-\frac{143x^9}{480b^3(a+bx^2)^3}+\frac{429}{160}\int \frac{x^8}{(ab+b^2x^2)^3}$$
$$= -\frac{x^{13}}{10b(a+bx^2)^5}-\frac{13x^{11}}{80b^2(a+bx^2)^4}-\frac{143x^9}{480b^3(a+bx^2)^3}-\frac{429x^7}{640b^4(a+bx^2)^2}+$$
$$= -\frac{x^{13}}{10b(a+bx^2)^5}-\frac{13x^{11}}{80b^2(a+bx^2)^4}-\frac{143x^9}{480b^3(a+bx^2)^3}-\frac{429x^7}{640b^4(a+bx^2)^2}-$$
$$= -\frac{x^{13}}{10b(a+bx^2)^5}-\frac{13x^{11}}{80b^2(a+bx^2)^4}-\frac{143x^9}{480b^3(a+bx^2)^3}-\frac{429x^7}{640b^4(a+bx^2)^2}-$$
$$= -\frac{3003ax}{256b^7}+\frac{1001x^3}{256b^6}-\frac{x^{13}}{10b(a+bx^2)^5}-\frac{13x^{11}}{80b^2(a+bx^2)^4}-\frac{143x^9}{480b^3(a+bx^2)^3}-$$
$$= -\frac{3003ax}{256b^7}+\frac{1001x^3}{256b^6}-\frac{x^{13}}{10b(a+bx^2)^5}-\frac{13x^{11}}{80b^2(a+bx^2)^4}-\frac{143x^9}{480b^3(a+bx^2)^3}-$$

Mathematica [A]
time = 0.04, size = 111, normalized size = 0.78

$$\frac{\frac{\sqrt{b}\,x(-45045a^6-210210a^5bx^2-384384a^4b^2x^4-338910a^3b^3x^6-137995a^2b^4x^8-16640ab^5x^{10}+1280b^6x^{12})}{(a+bx^2)^5}+45045a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3840b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(-45045*a^6 - 210210*a^5*b*x^2 - 384384*a^4*b^2*x^4 - 338910*a^
3*b^3*x^6 - 137995*a^2*b^4*x^8 - 16640*a*b^5*x^10 + 1280*b^6*x^12))/(a + b*
x^2)^5 + 45045*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3840*b^(15/2))
```

Maple [A]
time = 0.06, size = 96, normalized size = 0.68

method	result
default	$-\frac{-\frac{1}{3} b\,x^{3}+6 a x}{b^{7}}+\frac{a^{2} \left(\frac{-\frac{2373}{256} b^{4} x^{9}-\frac{12131}{384} a\,b^{3} x^{7}-\frac{1253}{30} a^{2} b^{2} x^{5}-\frac{9629}{384} a^{3} b\,x^{3}-\frac{1467}{256} a^{4} x}{\left(b\,x^{2}+a\right)^{5}}+\frac{3003 \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{256 \sqrt{a b}}\right)}{b^{7}}$
risch	$\frac{x^{3}}{3 b^{6}}-\frac{6 a x}{b^{7}}+\frac{-\frac{2373}{256} a^{2} b^{4} x^{9}-\frac{12131}{384} a^{3} b^{3} x^{7}-\frac{1253}{30} a^{4} b^{2} x^{5}-\frac{9629}{384} a^{5} b\,x^{3}-\frac{1467}{256} a^{6} x}{b^{7} \left(b\,x^{2}+a\right) \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{2}}+\frac{3003 \sqrt{-a b}\, a \ln\left(-\sqrt{-a b}\, x+a\right)}{512 b^{8}}-\frac{3003}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^7*(-1/3*b*x^3+6*a*x)+1/b^7*a^2*((-2373/256*b^4*x^9-12131/384*a*b^3*x^7
-1253/30*a^2*b^2*x^5-9629/384*a^3*b*x^3-1467/256*a^4*x)/(b*x^2+a)^5+3003/25
6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 148, normalized size = 1.04

$$-\frac{35595 \, a^{2} b^{4} x^{9} + 121310 \, a^{3} b^{3} x^{7} + 160384 \, a^{4} b^{2} x^{5} + 96290 \, a^{5} b x^{3} + 22005 \, a^{6} x}{3840 \, {\left(b^{12} x^{10} + 5 \, a b^{11} x^{8} + 10 \, a^{2} b^{10} x^{6} + 10 \, a^{3} b^{9} x^{4} + 5 \, a^{4} b^{8} x^{2} + a^{5} b^{7}\right)}} + \frac{3003 \, a^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{256 \, \sqrt{a b} b^{7}} + \frac{b x^{3} - 18 \, a x}{3 \, b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/3840*(35595*a^2*b^4*x^9 + 121310*a^3*b^3*x^7 + 160384*a^4*b^2*x^5 + 9629
0*a^5*b*x^3 + 22005*a^6*x)/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10
*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7) + 3003/256*a^2*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^7) + 1/3*(b*x^3 - 18*a*x)/b^7
```

Fricas [A]
time = 0.34, size = 428, normalized size = 3.01

$$\left[\frac{2560 \, b^{6} x^{13} - 33280 \, a b^{5} x^{11} - 275990 \, a^{2} b^{4} x^{9} - 677820 \, a^{3} b^{3} x^{7} - 768768 \, a^{4} b^{2} x^{5} - 420420 \, a^{5} b x^{3} - 90090 \, a^{6} x + 45045 \, {\left(a b^{5} x^{10} + 5 \, a^{2} b^{4} x^{8} + 10 \, a^{3} b^{3} x^{6} + 10 \, a^{4} b^{2} x^{4} + 5 \, a^{5} b x^{2} + a^{6}\right)} \sqrt{-\frac{a}{b}} \log\left(\frac{b x^{2} + 2 \, b x \sqrt{-\frac{a}{b}} - a}{b x^{2} + a}\right)}{7680 \, {\left(b^{12} x^{10} + 5 \, a b^{11} x^{8} + 10 \, a^{2} b^{10} x^{6} + 10 \, a^{3} b^{9} x^{4} + 5 \, a^{4} b^{8} x^{2} + a^{5} b^{7}\right)}}, \frac{1280 \, b^{6} x^{13} - 16640 \, a b^{5} x^{11} - 137995 \, a^{2} b^{4} x^{9} - 338910 \, a^{3} b^{3} x^{7} - 384384 \, a^{4} b^{2} x^{5} - 210210 \, a^{5} b x^{3} - 45045 \, a^{6} x + 45045 \, {\left(a b^{5} x^{10} + 5 \, a^{2} b^{4} x^{8} + 10 \, a^{3} b^{3} x^{6} + 10 \, a^{4} b^{2} x^{4} + 5 \, a^{5} b x^{2} + a^{6}\right)} \sqrt{\frac{a}{b}} \arctan\left(\frac{b x \sqrt{\frac{a}{b}}}{a}\right)}{3840 \, {\left(b^{12} x^{10} + 5 \, a b^{11} x^{8} + 10 \, a^{2} b^{10} x^{6} + 10 \, a^{3} b^{9} x^{4} + 5 \, a^{4} b^{8} x^{2} + a^{5} b^{7}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(2560*b^6*x^13 - 33280*a*b^5*x^11 - 275990*a^2*b^4*x^9 - 677820*a^3
*b^3*x^7 - 768768*a^4*b^2*x^5 - 420420*a^5*b*x^3 - 90090*a^6*x + 45045*(a*b
^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 + a
^6)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^12*x^10
+ 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7
), 1/3840*(1280*b^6*x^13 - 16640*a*b^5*x^11 - 137995*a^2*b^4*x^9 - 338910*a
^3*b^3*x^7 - 384384*a^4*b^2*x^5 - 210210*a^5*b*x^3 - 45045*a^6*x + 45045*(a
*b^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 +
 a^6)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2
*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)]
```

Sympy [A]
time = 0.48, size = 204, normalized size = 1.44

$$-\frac{6ax}{b^7} - \frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x - \frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512} + \frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x + \frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512} + \frac{-22005a^6x - 96290a^5bx^3 - 160384a^4b^2x^5 - 121310a^3b^3x^7 - 35595a^2b^4x^9}{3840a^5b^7 + 19200a^4b^8x^2 + 38400a^3b^9x^4 + 38400a^2b^{10}x^6 + 19200ab^{11}x^8 + 3840b^{12}x^{10}} + \frac{x^3}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -6*a*x/b**7 - 3003*sqrt(-a**3/b**15)*log(x - b**7*sqrt(-a**3/b**15)/a)/512
+ 3003*sqrt(-a**3/b**15)*log(x + b**7*sqrt(-a**3/b**15)/a)/512 + (-22005*a*
*6*x - 96290*a**5*b*x**3 - 160384*a**4*b**2*x**5 - 121310*a**3*b**3*x**7 -
35595*a**2*b**4*x**9)/(3840*a**5*b**7 + 19200*a**4*b**8*x**2 + 38400*a**3*b
**9*x**4 + 38400*a**2*b**10*x**6 + 19200*a*b**11*x**8 + 3840*b**12*x**10) +
 x**3/(3*b**6)
```

Giac [A]
time = 3.00, size = 106, normalized size = 0.75

$$\frac{3003\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^7} - \frac{35595\,a^2b^4x^9 + 121310\,a^3b^3x^7 + 160384\,a^4b^2x^5 + 96290\,a^5bx^3 + 22005\,a^6x}{3840\,(bx^2+a)^5b^7} + \frac{b^{12}x^3 - 18\,ab^{11}x}{3\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3003/256*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) - 1/3840*(35595*a^2*b^4*
x^9 + 121310*a^3*b^3*x^7 + 160384*a^4*b^2*x^5 + 96290*a^5*b*x^3 + 22005*a^6
*x)/((b*x^2 + a)^5*b^7) + 1/3*(b^12*x^3 - 18*a*b^11*x)/b^18
```

Mupad [B]
time = 4.52, size = 143, normalized size = 1.01

$$\frac{x^3}{3\,b^6} - \frac{\frac{1467\,a^6\,x}{256} + \frac{9629\,a^5\,b\,x^3}{384} + \frac{1253\,a^4\,b^2\,x^5}{30} + \frac{12131\,a^3\,b^3\,x^7}{384} + \frac{2373\,a^2\,b^4\,x^9}{256}}{a^5\,b^7 + 5\,a^4\,b^8\,x^2 + 10\,a^3\,b^9\,x^4 + 10\,a^2\,b^{10}\,x^6 + 5\,a\,b^{11}\,x^8 + b^{12}\,x^{10}} + \frac{3003\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{15/2}} - \frac{6\,a\,x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] x^3/(3*b^6) - ((1467*a^6*x)/256 + (9629*a^5*b*x^3)/384 + (1253*a^4*b^2*x^5)
/30 + (12131*a^3*b^3*x^7)/384 + (2373*a^2*b^4*x^9)/256)/(a^5*b^7 + b^12*x^1
0 + 5*a*b^11*x^8 + 5*a^4*b^8*x^2 + 10*a^3*b^9*x^4 + 10*a^2*b^10*x^6) + (300
3*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(256*b^(15/2)) - (6*a*x)/b^7
```

3.525 $\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=131

$$\frac{693x}{256b^6}-\frac{x^{11}}{10b(a+bx^2)^5}-\frac{11x^9}{80b^2(a+bx^2)^4}-\frac{33x^7}{160b^3(a+bx^2)^3}-\frac{231x^5}{640b^4(a+bx^2)^2}-\frac{231x^3}{256b^5(a+bx^2)}-\frac{693\sqrt{a}\ \mathrm{tan}}{256}$$

[Out] 693/256*x/b^6-1/10*x^11/b/(b*x^2+a)^5-11/80*x^9/b^2/(b*x^2+a)^4-33/160*x^7/b^3/(b*x^2+a)^3-231/640*x^5/b^4/(b*x^2+a)^2-231/256*x^3/b^5/(b*x^2+a)-693/256*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(13/2)

Rubi [A]
time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 327, 211}

$$-\frac{693\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{13/2}}-\frac{231x^3}{256b^5(a+bx^2)}-\frac{231x^5}{640b^4(a+bx^2)^2}-\frac{33x^7}{160b^3(a+bx^2)^3}-\frac{11x^9}{80b^2(a+bx^2)^4}-\frac{x^{11}}{10b(a+bx^2)^5}+\frac{693x}{256b^6}$$

Antiderivative was successfully verified.

[In] Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] (693*x)/(256*b^6) - x^11/(10*b*(a + b*x^2)^5) - (11*x^9)/(80*b^2*(a + b*x^2)^4) - (33*x^7)/(160*b^3*(a + b*x^2)^3) - (231*x^5)/(640*b^4*(a + b*x^2)^2) - (231*x^3)/(256*b^5*(a + b*x^2)) - (693*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(13/2))

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{x^{12}}{(ab+b^2x^2)^6}\,dx$$
$$= -\frac{x^{11}}{10b\,(a+bx^2)^5}+\frac{1}{10}(11b^4)\int \frac{x^{10}}{(ab+b^2x^2)^5}\,dx$$
$$= -\frac{x^{11}}{10b\,(a+bx^2)^5}-\frac{11x^9}{80b^2\,(a+bx^2)^4}+\frac{1}{80}(99b^2)\int \frac{x^8}{(ab+b^2x^2)^4}\,dx$$
$$= -\frac{x^{11}}{10b\,(a+bx^2)^5}-\frac{11x^9}{80b^2\,(a+bx^2)^4}-\frac{33x^7}{160b^3\,(a+bx^2)^3}+\frac{231}{160}\int \frac{x^6}{(ab+b^2x^2)^3}\,d$$
$$= -\frac{x^{11}}{10b\,(a+bx^2)^5}-\frac{11x^9}{80b^2\,(a+bx^2)^4}-\frac{33x^7}{160b^3\,(a+bx^2)^3}-\frac{231x^5}{640b^4\,(a+bx^2)^2}+$$
$$= -\frac{x^{11}}{10b\,(a+bx^2)^5}-\frac{11x^9}{80b^2\,(a+bx^2)^4}-\frac{33x^7}{160b^3\,(a+bx^2)^3}-\frac{231x^5}{640b^4\,(a+bx^2)^2}-$$
$$= \frac{693x}{256b^6}-\frac{x^{11}}{10b\,(a+bx^2)^5}-\frac{11x^9}{80b^2\,(a+bx^2)^4}-\frac{33x^7}{160b^3\,(a+bx^2)^3}-\frac{231x^5}{640b^4\,(a+bx^2}$$
$$= \frac{693x}{256b^6}-\frac{x^{11}}{10b\,(a+bx^2)^5}-\frac{11x^9}{80b^2\,(a+bx^2)^4}-\frac{33x^7}{160b^3\,(a+bx^2)^3}-\frac{231x^5}{640b^4\,(a+bx^2}$$

Mathematica [A]
time = 0.04, size = 100, normalized size = 0.76

$$\frac{\frac{\sqrt{b}\,x(3465a^5+16170a^4bx^2+29568a^3b^2x^4+26070a^2b^3x^6+10615ab^4x^8+1280b^5x^{10})}{(a+bx^2)^5}-3465\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{1280b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(3465*a^5 + 16170*a^4*b*x^2 + 29568*a^3*b^2*x^4 + 26070*a^2*b^3
*x^6 + 10615*a*b^4*x^8 + 1280*b^5*x^10))/(a + b*x^2)^5 - 3465*Sqrt[a]*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(1280*b^(13/2))
```

Maple [A]
time = 0.06, size = 84, normalized size = 0.64

method	result
default	$\frac{x}{b^6}-\frac{a\left(\frac{-\frac{843}{256}b^4x^9-\frac{1327}{128}a\,b^3x^7-\frac{131}{10}a^2b^2x^5-\frac{977}{128}a^3b\,x^3-\frac{437}{256}a^4x}{\left(b\,x^2+a\right)^5}+\frac{693\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}}\right)}{b^6}$
risch	$\frac{x}{b^6}+\frac{\frac{843}{256}a\,b^4x^9+\frac{1327}{128}a^2b^3x^7+\frac{131}{10}a^3b^2x^5+\frac{977}{128}a^4b\,x^3+\frac{437}{256}a^5x}{b^6(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}+\frac{693\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{512b^7}-\frac{693\sqrt{-ab}\,\ln\left(\sqrt{-\right.}}{512b^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^6-1/b^6*a*((-843/256*b^4*x^9-1327/128*a*b^3*x^7-131/10*a^2*b^2*x^5-977/
128*a^3*b*x^3-437/256*a^4*x)/(b*x^2+a)^5+693/256/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 134, normalized size = 1.02

$$\frac{4215\,ab^4x^9+13270\,a^2b^3x^7+16768\,a^3b^2x^5+9770\,a^4bx^3+2185\,a^5x}{1280\,(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)}-\frac{693\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^6}+\frac{x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(4215*a*b^4*x^9 + 13270*a^2*b^3*x^7 + 16768*a^3*b^2*x^5 + 9770*a^4*b
*x^3 + 2185*a^5*x)/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*
x^4 + 5*a^4*b^7*x^2 + a^5*b^6) - 693/256*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + x/b^6
```

Fricas [A]
time = 0.36, size = 400, normalized size = 3.05

$$\left[\frac{2560\,b^5x^{11}+21230\,ab^4x^9+52140\,a^2b^3x^7+59136\,a^3b^2x^5+32340\,a^4bx^3+6930\,a^5x+3465\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{2560\,(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)},\ \frac{1280\,b^5x^{11}+10615\,ab^4x^9+26070\,a^2b^3x^7+29568\,a^3b^2x^5+16170\,a^4bx^3+3465\,a^5x-3465\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{1280\,(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(2560*b^5*x^11 + 21230*a*b^4*x^9 + 52140*a^2*b^3*x^7 + 59136*a^3*b^
2*x^5 + 32340*a^4*b*x^3 + 6930*a^5*x + 3465*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^
2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2 - 2*b
*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6
 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6), 1/1280*(1280*b^5*x^11 + 10615
*a*b^4*x^9 + 26070*a^2*b^3*x^7 + 29568*a^3*b^2*x^5 + 16170*a^4*b*x^3 + 3465
*a^5*x - 3465*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5
*a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^11*x^10 + 5*a*b^10*
x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)]
```

Sympy [A]
time = 0.43, size = 178, normalized size = 1.36

$$\frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(-b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{512}-\frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{512}+\frac{2185a^5x+9770a^4bx^3+16768a^3b^2x^5+13270a^2b^3x^7+4215ab^4x^9}{1280a^5b^6+6400a^4b^7x^2+12800a^3b^8x^4+12800a^2b^9x^6+6400ab^{10}x^8+1280b^{11}x^{10}}+\frac{x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 693*sqrt(-a/b**13)*log(-b**6*sqrt(-a/b**13) + x)/512 - 693*sqrt(-a/b**13)*l
og(b**6*sqrt(-a/b**13) + x)/512 + (2185*a**5*x + 9770*a**4*b*x**3 + 16768*a
**3*b**2*x**5 + 13270*a**2*b**3*x**7 + 4215*a*b**4*x**9)/(1280*a**5*b**6 +
6400*a**4*b**7*x**2 + 12800*a**3*b**8*x**4 + 12800*a**2*b**9*x**6 + 6400*a*
b**10*x**8 + 1280*b**11*x**10) + x/b**6
```

Giac [A]
time = 3.59, size = 87, normalized size = 0.66

$$-\frac{693\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^6}+\frac{x}{b^6}+\frac{4215\,ab^4x^9+13270\,a^2b^3x^7+16768\,a^3b^2x^5+9770\,a^4bx^3+2185\,a^5x}{1280\,(bx^2+a)^5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -693/256*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + x/b^6 + 1/1280*(4215*a*b
^4*x^9 + 13270*a^2*b^3*x^7 + 16768*a^3*b^2*x^5 + 9770*a^4*b*x^3 + 2185*a^5*
x)/((b*x^2 + a)^5*b^6)
```

Mupad [B]
time = 0.16, size = 130, normalized size = 0.99

$$\frac{\frac{437\,a^5\,x}{256}+\frac{977\,a^4\,b\,x^3}{128}+\frac{131\,a^3\,b^2\,x^5}{10}+\frac{1327\,a^2\,b^3\,x^7}{128}+\frac{843\,a\,b^4\,x^9}{256}}{a^5\,b^6+5\,a^4\,b^7\,x^2+10\,a^3\,b^8\,x^4+10\,a^2\,b^9\,x^6+5\,a\,b^{10}\,x^8+b^{11}\,x^{10}}+\frac{x}{b^6}-\frac{693\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((437*a^5*x)/256 + (977*a^4*b*x^3)/128 + (843*a*b^4*x^9)/256 + (131*a^3*b^2
*x^5)/10 + (1327*a^2*b^3*x^7)/128)/(a^5*b^6 + b^11*x^10 + 5*a*b^10*x^8 + 5*
a^4*b^7*x^2 + 10*a^3*b^8*x^4 + 10*a^2*b^9*x^6) + x/b^6 - (693*a^(1/2)*atan(
(b^(1/2)*x)/a^(1/2)))/(256*b^(13/2))
```

3.526 $\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=121

$$-\frac{x^9}{10b\,(a+bx^2)^5}-\frac{9x^7}{80b^2\,(a+bx^2)^4}-\frac{21x^5}{160b^3\,(a+bx^2)^3}-\frac{21x^3}{128b^4\,(a+bx^2)^2}-\frac{63x}{256b^5\,(a+bx^2)}+\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{a}\,b^{11/2}}$$

```
[Out] -1/10*x^9/b/(b*x^2+a)^5-9/80*x^7/b^2/(b*x^2+a)^4-21/160*x^5/b^3/(b*x^2+a)^3
-21/128*x^3/b^4/(b*x^2+a)^2-63/256*x/b^5/(b*x^2+a)+63/256*arctan(x*b^(1/2)/
a^(1/2))/b^(11/2)/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 294, 211}

$$\frac{63\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{a}\,b^{11/2}}-\frac{63x}{256b^5\,(a+bx^2)}-\frac{21x^3}{128b^4\,(a+bx^2)^2}-\frac{21x^5}{160b^3\,(a+bx^2)^3}-\frac{9x^7}{80b^2\,(a+bx^2)^4}-\frac{x^9}{10b\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*x^9/(b*(a + b*x^2)^5) - (9*x^7)/(80*b^2*(a + b*x^2)^4) - (21*x^5)/(16
0*b^3*(a + b*x^2)^3) - (21*x^3)/(128*b^4*(a + b*x^2)^2) - (63*x)/(256*b^5*(
a + b*x^2)) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*Sqrt[a]*b^(11/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= b^6 \int \frac{x^{10}}{\left(ab+b^2x^2\right)^6}\,dx \\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}+\frac{1}{10}\left(9b^4\right)\int \frac{x^8}{\left(ab+b^2x^2\right)^5}\,dx \\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}+\frac{1}{80}\left(63b^2\right)\int \frac{x^6}{\left(ab+b^2x^2\right)^4}\,dx \\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}+\frac{21}{32}\int \frac{x^4}{\left(ab+b^2x^2\right)^3}\,d \\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}+ \\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}- \\
&= -\frac{x^9}{10b\left(a+bx^2\right)^5}-\frac{9x^7}{80b^2\left(a+bx^2\right)^4}-\frac{21x^5}{160b^3\left(a+bx^2\right)^3}-\frac{21x^3}{128b^4\left(a+bx^2\right)^2}-
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.73

$$
-\frac{x\left(315a^4+1470a^3bx^2+2688a^2b^2x^4+2370ab^3x^6+965b^4x^8\right)}{1280b^5\left(a+bx^2\right)^5}+\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{a}\,b^{11/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/1280*(x*(315*a^4 + 1470*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 2370*a*b^3*x^6 +
965*b^4*x^8))/(b^5*(a + b*x^2)^5) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*S
qrt[a]*b^(11/2))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.66

method	result	size
default	$\frac{-\frac{193x^9}{256b}-\frac{237a\,x^7}{128b^2}-\frac{21a^2x^5}{10b^3}-\frac{147a^3x^3}{128b^4}-\frac{63a^4x}{256b^5}}{\left(b\,x^2+a\right)^5}+\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256b^5\sqrt{ab}}$	80

risch	$\frac{-\frac{193x^9}{256b}-\frac{237a\,x^7}{128b^2}-\frac{21a^2x^5}{10b^3}-\frac{147a^3x^3}{128b^4}-\frac{63a^4x}{256b^5}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}-\frac{63\ln\left(bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^5}+\frac{63\ln\left(-bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^5}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-193/256/b*x^9-237/128*a/b^2*x^7-21/10*a^2/b^3*x^5-147/128*a^3/b^4*x^3-63/
256*a^4/b^5*x)/(b*x^2+a)^5+63/256/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 125, normalized size = 1.03

$$-\frac{965\,b^4x^9+2370\,ab^3x^7+2688\,a^2b^2x^5+1470\,a^3bx^3+315\,a^4x}{1280\,(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5)}+\frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(965*b^4*x^9 + 2370*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 1470*a^3*b*x^3 +
 315*a^4*x)/(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*
a^4*b^6*x^2 + a^5*b^5) + 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5)
```

Fricas [A]
time = 0.43, size = 386, normalized size = 3.19

$$\left[-\frac{1930\,ab^5x^9+4740\,a^2b^4x^7+5376\,a^3b^3x^5+2940\,a^4b^2x^3+630\,a^5bx+315\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2560\,(ab^{11}x^{10}+5\,a^2b^{10}x^8+10\,a^3b^9x^6+10\,a^4b^8x^4+5\,a^5b^7x^2+a^6b^6)},-\frac{965\,ab^5x^9+2370\,a^2b^4x^7+2688\,a^3b^3x^5+1470\,a^4b^2x^3+315\,a^5bx-315\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{1280\,(ab^{11}x^{10}+5\,a^2b^{10}x^8+10\,a^3b^9x^6+10\,a^4b^8x^4+5\,a^5b^7x^2+a^6b^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(1930*a*b^5*x^9 + 4740*a^2*b^4*x^7 + 5376*a^3*b^3*x^5 + 2940*a^4*b
^2*x^3 + 630*a^5*b*x + 315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^
3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)))/(a*b^11*x^10 + 5*a^2*b^10*x^8 + 10*a^3*b^9*x^6 + 10*a^4*b^8*x
^4 + 5*a^5*b^7*x^2 + a^6*b^6), -1/1280*(965*a*b^5*x^9 + 2370*a^2*b^4*x^7 +
2688*a^3*b^3*x^5 + 1470*a^4*b^2*x^3 + 315*a^5*b*x - 315*(b^5*x^10 + 5*a*b^4
*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arcta
n(sqrt(a*b)*x/a))/(a*b^11*x^10 + 5*a^2*b^10*x^8 + 10*a^3*b^9*x^6 + 10*a^4*b
^8*x^4 + 5*a^5*b^7*x^2 + a^6*b^6)]
```

Sympy [A]
time = 0.36, size = 182, normalized size = 1.50

$$-\frac{63\sqrt{-\frac{1}{ab^{11}}}\,\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}+\frac{63\sqrt{-\frac{1}{ab^{11}}}\,\log\left(ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}+\frac{-315a^4x-1470a^3bx^3-2688a^2b^2x^5-2370ab^3x^7-965b^4x^9}{1280a^5b^5+6400a^4b^6x^2+12800a^3b^7x^4+12800a^2b^8x^6+6400ab^9x^8+1280b^{10}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -63*sqrt(-1/(a*b**11))*log(-a*b**5*sqrt(-1/(a*b**11)) + x)/512 + 63*sqrt(-1
/(a*b**11))*log(a*b**5*sqrt(-1/(a*b**11)) + x)/512 + (-315*a**4*x - 1470*a*
*3*b*x**3 - 2688*a**2*b**2*x**5 - 2370*a*b**3*x**7 - 965*b**4*x**9)/(1280*a
**5*b**5 + 6400*a**4*b**6*x**2 + 12800*a**3*b**7*x**4 + 12800*a**2*b**8*x**
6 + 6400*a*b**9*x**8 + 1280*b**10*x**10)
```

Giac [A]
time = 4.70, size = 78, normalized size = 0.64

$$\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,b^5}-\frac{965\,b^4x^9+2370\,ab^3x^7+2688\,a^2b^2x^5+1470\,a^3bx^3+315\,a^4x}{1280\,(bx^2+a)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/1280*(965*b^4*x^9 + 2370*a
*b^3*x^7 + 2688*a^2*b^2*x^5 + 1470*a^3*b*x^3 + 315*a^4*x)/((b*x^2 + a)^5*b^
5)
```

Mupad [B]
time = 4.52, size = 122, normalized size = 1.01

$$\frac{63\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,\sqrt{a}\,b^{11/2}}-\frac{\frac{193\,x^9}{256\,b}+\frac{237\,a\,x^7}{128\,b^2}+\frac{63\,a^4\,x}{256\,b^5}+\frac{21\,a^2\,x^5}{10\,b^3}+\frac{147\,a^3\,x^3}{128\,b^4}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (63*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(1/2)*b^(11/2)) - ((193*x^9)/(256*b)
+ (237*a*x^7)/(128*b^2) + (63*a^4*x)/(256*b^5) + (21*a^2*x^5)/(10*b^3) + (1
47*a^3*x^3)/(128*b^4))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3
*b^2*x^4 + 10*a^2*b^3*x^6)
```

3.527 $\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=122

$$-\frac{x^7}{10b(a+bx^2)^5}-\frac{7x^5}{80b^2(a+bx^2)^4}-\frac{7x^3}{96b^3(a+bx^2)^3}-\frac{7x}{128b^4(a+bx^2)^2}+\frac{7x}{256ab^4(a+bx^2)}+\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}$$

```
[Out] -1/10*x^7/b/(b*x^2+a)^5-7/80*x^5/b^2/(b*x^2+a)^4-7/96*x^3/b^3/(b*x^2+a)^3-7
/128*x/b^4/(b*x^2+a)^2+7/256*x/a/b^4/(b*x^2+a)+7/256*arctan(x*b^(1/2)/a^(1/
2))/a^(3/2)/b^(9/2)
```

Rubi [A]
time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 205, 211}

$$\frac{7\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}+\frac{7x}{256ab^4(a+bx^2)}-\frac{7x}{128b^4(a+bx^2)^2}-\frac{7x^3}{96b^3(a+bx^2)^3}-\frac{7x^5}{80b^2(a+bx^2)^4}-\frac{x^7}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*x^7/(b*(a + b*x^2)^5) - (7*x^5)/(80*b^2*(a + b*x^2)^4) - (7*x^3)/(96*
b^3*(a + b*x^2)^3) - (7*x)/(128*b^4*(a + b*x^2)^2) + (7*x)/(256*a*b^4*(a +
b*x^2)) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(3/2)*b^(9/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{x^8}{(ab+b^2x^2)^6}\,dx$$

$$= -\frac{x^7}{10b\,(a+bx^2)^5} + \frac{1}{10}(7b^4)\int \frac{x^6}{(ab+b^2x^2)^5}\,dx$$

$$= -\frac{x^7}{10b\,(a+bx^2)^5} - \frac{7x^5}{80b^2\,(a+bx^2)^4} + \frac{1}{16}(7b^2)\int \frac{x^4}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{x^7}{10b\,(a+bx^2)^5} - \frac{7x^5}{80b^2\,(a+bx^2)^4} - \frac{7x^3}{96b^3\,(a+bx^2)^3} + \frac{7}{32}\int \frac{x^2}{(ab+b^2x^2)^3}\,dx$$

$$= -\frac{x^7}{10b\,(a+bx^2)^5} - \frac{7x^5}{80b^2\,(a+bx^2)^4} - \frac{7x^3}{96b^3\,(a+bx^2)^3} - \frac{7x}{128b^4\,(a+bx^2)^2} + \frac{7}{}$$

$$= -\frac{x^7}{10b\,(a+bx^2)^5} - \frac{7x^5}{80b^2\,(a+bx^2)^4} - \frac{7x^3}{96b^3\,(a+bx^2)^3} - \frac{7x}{128b^4\,(a+bx^2)^2} + \frac{}{2}$$

$$= -\frac{x^7}{10b\,(a+bx^2)^5} - \frac{7x^5}{80b^2\,(a+bx^2)^4} - \frac{7x^3}{96b^3\,(a+bx^2)^3} - \frac{7x}{128b^4\,(a+bx^2)^2} + \frac{}{2}$$

Mathematica [A]
time = 0.04, size = 91, normalized size = 0.75

$$-\frac{x(105a^4+490a^3bx^2+896a^2b^2x^4+790ab^3x^6-105b^4x^8)}{3840ab^4\,(a+bx^2)^5} + \frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/3840*(x*(105*a^4 + 490*a^3*b*x^2 + 896*a^2*b^2*x^4 + 790*a*b^3*x^6 - 105
*b^4*x^8))/(a*b^4*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(
3/2)*b^(9/2))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.66

method	result	size
default	$\frac{\frac{7x^9}{256a}-\frac{79x^7}{384b}-\frac{7a\,x^5}{30b^2}-\frac{49a^2x^3}{384b^3}-\frac{7a^3x}{256b^4}}{(b\,x^2+a)^5}+\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256b^4a\sqrt{ab}}$	80
risch	$\frac{\frac{7x^9}{256a}-\frac{79x^7}{384b}-\frac{7a\,x^5}{30b^2}-\frac{49a^2x^3}{384b^3}-\frac{7a^3x}{256b^4}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}-\frac{7\ln\left(bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^4a}+\frac{7\ln\left(-bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^4a}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (7/256/a*x^9-79/384/b*x^7-7/30*a/b^2*x^5-49/384*a^2/b^3*x^3-7/256*a^3/b^4*x
)/(b*x^2+a)^5+7/256/b^4/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 131, normalized size = 1.07

$$\frac{105\,b^4x^9-790\,ab^3x^7-896\,a^2b^2x^5-490\,a^3bx^3-105\,a^4x}{3840\,(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4)}+\frac{7\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(105*b^4*x^9 - 790*a*b^3*x^7 - 896*a^2*b^2*x^5 - 490*a^3*b*x^3 - 105
*a^4*x)/(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a
^5*b^5*x^2 + a^6*b^4) + 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

Fricas [A]
time = 0.35, size = 390, normalized size = 3.20

$$\left[\frac{210\,ab^5x^9-1580\,a^2b^4x^7-1792\,a^3b^3x^5-980\,a^4b^2x^3-210\,a^5bx-105\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{7680\,(a^2b^{10}x^{10}+5\,a^3b^9x^8+10\,a^4b^8x^6+10\,a^5b^7x^4+5\,a^6b^6x^2+a^7b^5)},\ \frac{105\,ab^5x^9-790\,a^2b^4x^7-896\,a^3b^3x^5-490\,a^4b^2x^3-105\,a^5bx+105\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{3840\,(a^2b^{10}x^{10}+5\,a^3b^9x^8+10\,a^4b^8x^6+10\,a^5b^7x^4+5\,a^6b^6x^2+a^7b^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(210*a*b^5*x^9 - 1580*a^2*b^4*x^7 - 1792*a^3*b^3*x^5 - 980*a^4*b^2*
x^3 - 210*a^5*b*x - 105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b
^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^10*x^10 + 5*a^3*b^9*x^8 + 10*a^4*b^8*x^6 + 10*a^5*b^7*x^4
 + 5*a^6*b^6*x^2 + a^7*b^5), 1/3840*(105*a*b^5*x^9 - 790*a^2*b^4*x^7 - 896*
a^3*b^3*x^5 - 490*a^4*b^2*x^3 - 105*a^5*b*x + 105*(b^5*x^10 + 5*a*b^4*x^8 +
```

```
 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt
(a*b)*x/a))/(a^2*b^10*x^10 + 5*a^3*b^9*x^8 + 10*a^4*b^8*x^6 + 10*a^5*b^7*x^
4 + 5*a^6*b^6*x^2 + a^7*b^5)]
```

Sympy [A]
time = 0.34, size = 194, normalized size = 1.59

$$-\frac{7\sqrt{-\frac{1}{a^3b^9}}\log\left(-a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{512}+\frac{7\sqrt{-\frac{1}{a^3b^9}}\log\left(a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{512}+\frac{-105a^4x-490a^3bx^3-896a^2b^2x^5-790ab^3x^7+105b^4x^9}{3840a^6b^4+19200a^5b^5x^2+38400a^4b^6x^4+38400a^3b^7x^6+19200a^2b^8x^8+3840ab^9x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -7*sqrt(-1/(a**3*b**9))*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/512 + 7*sq
rt(-1/(a**3*b**9))*log(a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/512 + (-105*a**4
*x - 490*a**3*b*x**3 - 896*a**2*b**2*x**5 - 790*a*b**3*x**7 + 105*b**4*x**9
)/(3840*a**6*b**4 + 19200*a**5*b**5*x**2 + 38400*a**4*b**6*x**4 + 38400*a**
3*b**7*x**6 + 19200*a**2*b**8*x**8 + 3840*a*b**9*x**10)
```

Giac [A]
time = 5.48, size = 84, normalized size = 0.69

$$\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,ab^4}+\frac{105\,b^4x^9-790\,ab^3x^7-896\,a^2b^2x^5-490\,a^3bx^3-105\,a^4x}{3840\,(bx^2+a)^5ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/3840*(105*b^4*x^9 - 790*a
*b^3*x^7 - 896*a^2*b^2*x^5 - 490*a^3*b*x^3 - 105*a^4*x)/((b*x^2 + a)^5*a*b^
4)
```

Mupad [B]
time = 4.42, size = 119, normalized size = 0.98

$$\frac{7\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{3/2}\,b^{9/2}}-\frac{\frac{79\,x^7}{384\,b}-\frac{7\,x^9}{256\,a}+\frac{7\,a\,x^5}{30\,b^2}+\frac{7\,a^3\,x}{256\,b^4}+\frac{49\,a^2\,x^3}{384\,b^3}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (7*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(3/2)*b^(9/2)) - ((79*x^7)/(384*b) - (
7*x^9)/(256*a) + (7*a*x^5)/(30*b^2) + (7*a^3*x)/(256*b^4) + (49*a^2*x^3)/(3
84*b^3))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*
a^2*b^3*x^6)
```

3.528 $\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x}{32b^3\left(a+bx^2\right)^3}+\frac{x}{128ab^3\left(a+bx^2\right)^2}+\frac{3x}{256a^2b^3\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}$$

```
[Out] -1/10*x^5/b/(b*x^2+a)^5-1/16*x^3/b^2/(b*x^2+a)^4-1/32*x/b^3/(b*x^2+a)^3+1/1
28*x/a/b^3/(b*x^2+a)^2+3/256*x/a^2/b^3/(b*x^2+a)+3/256*arctan(x*b^(1/2)/a^(
1/2))/a^(5/2)/b^(7/2)
```

Rubi [A]
time = 0.05, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 205, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}+\frac{3x}{256a^2b^3\left(a+bx^2\right)}+\frac{x}{128ab^3\left(a+bx^2\right)^2}-\frac{x}{32b^3\left(a+bx^2\right)^3}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x^5}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*x^5/(b*(a + b*x^2)^5) - x^3/(16*b^2*(a + b*x^2)^4) - x/(32*b^3*(a + b
*x^2)^3) + x/(128*a*b^3*(a + b*x^2)^2) + (3*x)/(256*a^2*b^3*(a + b*x^2)) +
(3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)*b^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^3}\,dx &= b^6 \int \frac{x^6}{(ab+b^2x^2)^6}\,dx \\
&= -\frac{x^5}{10b\,(a+bx^2)^5} + \frac{1}{2}b^4 \int \frac{x^4}{(ab+b^2x^2)^5}\,dx \\
&= -\frac{x^5}{10b\,(a+bx^2)^5} - \frac{x^3}{16b^2\,(a+bx^2)^4} + \frac{1}{16}(3b^2) \int \frac{x^2}{(ab+b^2x^2)^4}\,dx \\
&= -\frac{x^5}{10b\,(a+bx^2)^5} - \frac{x^3}{16b^2\,(a+bx^2)^4} - \frac{x}{32b^3\,(a+bx^2)^3} + \frac{1}{32} \int \frac{1}{(ab+b^2x^2)^3}\,dx \\
&= -\frac{x^5}{10b\,(a+bx^2)^5} - \frac{x^3}{16b^2\,(a+bx^2)^4} - \frac{x}{32b^3\,(a+bx^2)^3} + \frac{x}{128ab^3\,(a+bx^2)^2} + \\
&= -\frac{x^5}{10b\,(a+bx^2)^5} - \frac{x^3}{16b^2\,(a+bx^2)^4} - \frac{x}{32b^3\,(a+bx^2)^3} + \frac{x}{128ab^3\,(a+bx^2)^2} + \\
&= -\frac{x^5}{10b\,(a+bx^2)^5} - \frac{x^3}{16b^2\,(a+bx^2)^4} - \frac{x}{32b^3\,(a+bx^2)^3} + \frac{x}{128ab^3\,(a+bx^2)^2} +
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 91, normalized size = 0.74

$$\frac{-15a^4x-70a^3bx^3-128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^2b^3\,(a+bx^2)^5} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-15*a^4*x - 70*a^3*b*x^3 - 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b^4*x^9)/(1
280*a^2*b^3*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)*b
^(7/2))
```

Maple [A]
time = 0.06, size = 78, normalized size = 0.63

method	result	size
default	$\frac{\frac{3b\,x^{9}}{256a^{2}}+\frac{7x^{7}}{128a}-\frac{x^{5}}{10b}-\frac{7a\,x^{3}}{128b^{2}}-\frac{3a^{2}x}{256b^{3}}}{\left(b\,x^{2}+a\right)^{5}}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256a^{2}b^{3}\sqrt{ab}}$	78
risch	$\frac{\frac{3b\,x^{9}}{256a^{2}}+\frac{7x^{7}}{128a}-\frac{x^{5}}{10b}-\frac{7a\,x^{3}}{128b^{2}}-\frac{3a^{2}x}{256b^{3}}}{\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}-\frac{3\ln\left(bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^{3}a^{2}}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^{3}a^{2}}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (3/256*b/a^2*x^9+7/128/a*x^7-1/10/b*x^5-7/128*a/b^2*x^3-3/256*a^2/b^3*x)/(b
*x^2+a)^5+3/256/a^2/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 133, normalized size = 1.08

$$\frac{15\,b^{4}x^{9}+70\,ab^{3}x^{7}-128\,a^{2}b^{2}x^{5}-70\,a^{3}bx^{3}-15\,a^{4}x}{1280\,\left(a^{2}b^{8}x^{10}+5\,a^{3}b^{7}x^{8}+10\,a^{4}b^{6}x^{6}+10\,a^{5}b^{5}x^{4}+5\,a^{6}b^{4}x^{2}+a^{7}b^{3}\right)}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(15*b^4*x^9 + 70*a*b^3*x^7 - 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4
*x)/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6
*b^4*x^2 + a^7*b^3) + 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3)
```

Fricas [A]
time = 0.34, size = 390, normalized size = 3.17

$$\left[\frac{30\,ab^{5}x^{9}+140\,a^{2}b^{4}x^{7}-256\,a^{3}b^{3}x^{5}-140\,a^{4}b^{2}x^{3}-30\,a^{5}bx-15\,\left(b^{5}x^{10}+5\,ab^{4}x^{8}+10\,a^{2}b^{3}x^{6}+10\,a^{3}b^{2}x^{4}+5\,a^{4}bx^{2}+a^{5}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{2560\,\left(a^{3}b^{9}x^{10}+5\,a^{4}b^{8}x^{8}+10\,a^{5}b^{7}x^{6}+10\,a^{6}b^{6}x^{4}+5\,a^{7}b^{5}x^{2}+a^{8}b^{4}\right)},\ \frac{15\,ab^{5}x^{9}+70\,a^{2}b^{4}x^{7}-128\,a^{3}b^{3}x^{5}-70\,a^{4}b^{2}x^{3}-15\,a^{5}bx+15\,\left(b^{5}x^{10}+5\,ab^{4}x^{8}+10\,a^{2}b^{3}x^{6}+10\,a^{3}b^{2}x^{4}+5\,a^{4}bx^{2}+a^{5}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{1280\,\left(a^{3}b^{9}x^{10}+5\,a^{4}b^{8}x^{8}+10\,a^{5}b^{7}x^{6}+10\,a^{6}b^{6}x^{4}+5\,a^{7}b^{5}x^{2}+a^{8}b^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(30*a*b^5*x^9 + 140*a^2*b^4*x^7 - 256*a^3*b^3*x^5 - 140*a^4*b^2*x^3
 - 30*a^5*b*x - 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^
4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)))/(a^3*b^9*x^10 + 5*a^4*b^8*x^8 + 10*a^5*b^7*x^6 + 10*a^6*b^6*x^4 + 5*a
^7*b^5*x^2 + a^8*b^4), 1/1280*(15*a*b^5*x^9 + 70*a^2*b^4*x^7 - 128*a^3*b^3*
x^5 - 70*a^4*b^2*x^3 - 15*a^5*b*x + 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3
```

```
*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))
/(a^3*b^9*x^10 + 5*a^4*b^8*x^8 + 10*a^5*b^7*x^6 + 10*a^6*b^6*x^4 + 5*a^7*b^
5*x^2 + a^8*b^4)]
```

Sympy [A]
time = 0.32, size = 196, normalized size = 1.59

$$-\frac{3\sqrt{-\frac{1}{a^5b^7}}\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{512}+\frac{3\sqrt{-\frac{1}{a^5b^7}}\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{512}+\frac{-15a^4x-70a^3bx^3-128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^7b^3+6400a^6b^4x^2+12800a^5b^5x^4+12800a^4b^6x^6+6400a^3b^7x^8+1280a^2b^8x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*b**7))*log(-a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/512 + 3*sq
rt(-1/(a**5*b**7))*log(a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/512 + (-15*a**4*
x - 70*a**3*b*x**3 - 128*a**2*b**2*x**5 + 70*a*b**3*x**7 + 15*b**4*x**9)/(1
280*a**7*b**3 + 6400*a**6*b**4*x**2 + 12800*a**5*b**5*x**4 + 12800*a**4*b**
6*x**6 + 6400*a**3*b**7*x**8 + 1280*a**2*b**8*x**10)
```

Giac [A]
time = 3.46, size = 84, normalized size = 0.68

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^2b^3}+\frac{15\,b^4x^9+70\,ab^3x^7-128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\,(bx^2+a)^5a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/1280*(15*b^4*x^9 + 70*a
*b^3*x^7 - 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/((b*x^2 + a)^5*a^2*b^
3)
```

Mupad [B]
time = 4.50, size = 117, normalized size = 0.95

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{5/2}\,b^{7/2}}-\frac{\frac{x^5}{10\,b}-\frac{7\,x^7}{128\,a}+\frac{7\,a\,x^3}{128\,b^2}+\frac{3\,a^2\,x}{256\,b^3}-\frac{3\,b\,x^9}{256\,a^2}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (3*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(5/2)*b^(7/2)) - (x^5/(10*b) - (7*x^7)
/(128*a) + (7*a*x^3)/(128*b^2) + (3*a^2*x)/(256*b^3) - (3*b*x^9)/(256*a^2))
/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*
x^6)
```

3.529 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=124

$$-\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\frac{3x}{256a^3b^2\left(a+bx^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}$$

```
[Out] -1/10*x^3/b/(b*x^2+a)^5-3/80*x/b^2/(b*x^2+a)^4+1/160*x/a/b^2/(b*x^2+a)^3+1/
128*x/a^2/b^2/(b*x^2+a)^2+3/256*x/a^3/b^2/(b*x^2+a)+3/256*arctan(x*b^(1/2)/
a^(1/2))/a^(7/2)/b^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 205, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{3x}{256a^3b^2\left(a+bx^2\right)}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\frac{x}{160ab^2\left(a+bx^2\right)^3}-\frac{3x}{80b^2\left(a+bx^2\right)^4}-\frac{x^3}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*x^3/(b*(a + b*x^2)^5) - (3*x)/(80*b^2*(a + b*x^2)^4) + x/(160*a*b^2*(
a + b*x^2)^3) + x/(128*a^2*b^2*(a + b*x^2)^2) + (3*x)/(256*a^3*b^2*(a + b*x
^2)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(7/2)*b^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^3}\,dx &= b^6\int \frac{x^4}{(ab+b^2x^2)^6}\,dx\\
&= -\frac{x^3}{10b\,(a+bx^2)^5}+\frac{1}{10}(3b^4)\int \frac{x^2}{(ab+b^2x^2)^5}\,dx\\
&= -\frac{x^3}{10b\,(a+bx^2)^5}-\frac{3x}{80b^2\,(a+bx^2)^4}+\frac{1}{80}(3b^2)\int \frac{1}{(ab+b^2x^2)^4}\,dx\\
&= -\frac{x^3}{10b\,(a+bx^2)^5}-\frac{3x}{80b^2\,(a+bx^2)^4}+\frac{x}{160ab^2\,(a+bx^2)^3}+\frac{b\int \frac{1}{(ab+b^2x^2)^3}\,dx}{32a}\\
&= -\frac{x^3}{10b\,(a+bx^2)^5}-\frac{3x}{80b^2\,(a+bx^2)^4}+\frac{x}{160ab^2\,(a+bx^2)^3}+\frac{x}{128a^2b^2\,(a+bx^2)^2}\\
&= -\frac{x^3}{10b\,(a+bx^2)^5}-\frac{3x}{80b^2\,(a+bx^2)^4}+\frac{x}{160ab^2\,(a+bx^2)^3}+\frac{x}{128a^2b^2\,(a+bx^2)^2}\\
&= -\frac{x^3}{10b\,(a+bx^2)^5}-\frac{3x}{80b^2\,(a+bx^2)^4}+\frac{x}{160ab^2\,(a+bx^2)^3}+\frac{x}{128a^2b^2\,(a+bx^2)^2}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 91, normalized size = 0.73

$$
\frac{-15a^4x-70a^3bx^3+128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^3b^2\,(a+bx^2)^5}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-15*a^4*x - 70*a^3*b*x^3 + 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b^4*x^9)/(1
280*a^3*b^2*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(7/2)*b
^(5/2))
```

Maple [A]
time = 0.05, size = 78, normalized size = 0.63

method	result	size
default	$\frac{\frac{3b^2x^9}{256a^3}+\frac{7b\,x^7}{128a^2}+\frac{x^5}{10a}-\frac{7x^3}{128b}-\frac{3ax}{256b^2}}{(b\,x^2+a)^5}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256b^2a^3\sqrt{ab}}$	78
risch	$\frac{\frac{3b^2x^9}{256a^3}+\frac{7b\,x^7}{128a^2}+\frac{x^5}{10a}-\frac{7x^3}{128b}-\frac{3ax}{256b^2}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}-\frac{3\ln\left(bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^2a^3}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^2a^3}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (3/256*b^2/a^3*x^9+7/128*b/a^2*x^7+1/10/a*x^5-7/128/b*x^3-3/256*a/b^2*x)/(b
*x^2+a)^5+3/256/b^2/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 133, normalized size = 1.07

$$\frac{15\,b^4x^9+70\,ab^3x^7+128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\,(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2)}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(15*b^4*x^9 + 70*a*b^3*x^7 + 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4
*x)/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7
*b^3*x^2 + a^8*b^2) + 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

Fricas [A]
time = 0.33, size = 390, normalized size = 3.15

$$\left[\frac{30\,ab^5x^9+140\,a^2b^4x^7+256\,a^3b^3x^5-140\,a^4b^2x^3-30\,a^5bx-15\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2560\,(a^4b^8x^{10}+5\,a^5b^7x^8+10\,a^6b^6x^6+10\,a^7b^5x^4+5\,a^8b^4x^2+a^9b^3)},\ \frac{15\,ab^5x^9+70\,a^2b^4x^7+128\,a^3b^3x^5-70\,a^4b^2x^3-15\,a^5bx+15\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{1280\,(a^4b^8x^{10}+5\,a^5b^7x^8+10\,a^6b^6x^6+10\,a^7b^5x^4+5\,a^8b^4x^2+a^9b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(30*a*b^5*x^9 + 140*a^2*b^4*x^7 + 256*a^3*b^3*x^5 - 140*a^4*b^2*x^3
 - 30*a^5*b*x - 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^
4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)))/(a^4*b^8*x^10 + 5*a^5*b^7*x^8 + 10*a^6*b^6*x^6 + 10*a^7*b^5*x^4 + 5*a
^8*b^4*x^2 + a^9*b^3), 1/1280*(15*a*b^5*x^9 + 70*a^2*b^4*x^7 + 128*a^3*b^3*
x^5 - 70*a^4*b^2*x^3 - 15*a^5*b*x + 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3
```

```
*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))
/(a^4*b^8*x^10 + 5*a^5*b^7*x^8 + 10*a^6*b^6*x^6 + 10*a^7*b^5*x^4 + 5*a^8*b^
4*x^2 + a^9*b^3)]
```

Sympy [A]
time = 0.30, size = 196, normalized size = 1.58

$$-\frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{512}+\frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{512}+\frac{-15a^4x-70a^3bx^3+128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^8b^2+6400a^7b^3x^2+12800a^6b^4x^4+12800a^5b^5x^6+6400a^4b^6x^8+1280a^3b^7x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*sqrt(-1/(a**7*b**5))*log(-a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/512 + 3*sq
rt(-1/(a**7*b**5))*log(a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/512 + (-15*a**4*
x - 70*a**3*b*x**3 + 128*a**2*b**2*x**5 + 70*a*b**3*x**7 + 15*b**4*x**9)/(1
280*a**8*b**2 + 6400*a**7*b**3*x**2 + 12800*a**6*b**4*x**4 + 12800*a**5*b**
5*x**6 + 6400*a**4*b**6*x**8 + 1280*a**3*b**7*x**10)
```

Giac [A]
time = 3.47, size = 84, normalized size = 0.68

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^3b^2}+\frac{15\,b^4x^9+70\,ab^3x^7+128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\,(bx^2+a)^5a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2) + 1/1280*(15*b^4*x^9 + 70*a
*b^3*x^7 + 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/((b*x^2 + a)^5*a^3*b^
2)
```

Mupad [B]
time = 4.47, size = 116, normalized size = 0.94

$$\frac{\frac{x^5}{10\,a}-\frac{7\,x^3}{128\,b}+\frac{7\,b\,x^7}{128\,a^2}+\frac{3\,b^2\,x^9}{256\,a^3}-\frac{3\,a\,x}{256\,b^2}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{7/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (x^5/(10*a) - (7*x^3)/(128*b) + (7*b*x^7)/(128*a^2) + (3*b^2*x^9)/(256*a^3)
 - (3*a*x)/(256*b^2))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*
b^2*x^4 + 10*a^2*b^3*x^6) + (3*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(7/2)*b^(5
/2))
```

3.530 $\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=125

$$-\frac{x}{10b(a+bx^2)^5}+\frac{x}{80ab(a+bx^2)^4}+\frac{7x}{480a^2b(a+bx^2)^3}+\frac{7x}{384a^3b(a+bx^2)^2}+\frac{7x}{256a^4b(a+bx^2)}+\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}$$

```
[Out] -1/10*x/b/(b*x^2+a)^5+1/80*x/a/b/(b*x^2+a)^4+7/480*x/a^2/b/(b*x^2+a)^3+7/38
4*x/a^3/b/(b*x^2+a)^2+7/256*x/a^4/b/(b*x^2+a)+7/256*arctan(x*b^(1/2)/a^(1/2
))/a^(9/2)/b^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 294, 205, 211}

$$\frac{7\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{7x}{256a^4b(a+bx^2)}+\frac{7x}{384a^3b(a+bx^2)^2}+\frac{7x}{480a^2b(a+bx^2)^3}+\frac{x}{80ab(a+bx^2)^4}-\frac{x}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*x/(b*(a + b*x^2)^5) + x/(80*a*b*(a + b*x^2)^4) + (7*x)/(480*a^2*b*(a
+ b*x^2)^3) + (7*x)/(384*a^3*b*(a + b*x^2)^2) + (7*x)/(256*a^4*b*(a + b*x^2
)) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/2)*b^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{x^2}{(ab+b^2x^2)^6}\,dx$$
$$= -\frac{x}{10b\,(a+bx^2)^5}+\frac{1}{10}b^4\int \frac{1}{(ab+b^2x^2)^5}\,dx$$
$$= -\frac{x}{10b\,(a+bx^2)^5}+\frac{x}{80ab\,(a+bx^2)^4}+\frac{(7b^3)\int \frac{1}{(ab+b^2x^2)^4}\,dx}{80a}$$
$$= -\frac{x}{10b\,(a+bx^2)^5}+\frac{x}{80ab\,(a+bx^2)^4}+\frac{7x}{480a^2b\,(a+bx^2)^3}+\frac{(7b^2)\int \frac{1}{(ab+b^2x^2)^3}\,dx}{96a^2}$$
$$= -\frac{x}{10b\,(a+bx^2)^5}+\frac{x}{80ab\,(a+bx^2)^4}+\frac{7x}{480a^2b\,(a+bx^2)^3}+\frac{7x}{384a^3b\,(a+bx^2)^2}$$
$$= -\frac{x}{10b\,(a+bx^2)^5}+\frac{x}{80ab\,(a+bx^2)^4}+\frac{7x}{480a^2b\,(a+bx^2)^3}+\frac{7x}{384a^3b\,(a+bx^2)^2}$$
$$= -\frac{x}{10b\,(a+bx^2)^5}+\frac{x}{80ab\,(a+bx^2)^4}+\frac{7x}{480a^2b\,(a+bx^2)^3}+\frac{7x}{384a^3b\,(a+bx^2)^2}$$

Mathematica [A]
time = 0.04, size = 91, normalized size = 0.73

$$\frac{-105a^4x+790a^3bx^3+896a^2b^2x^5+490ab^3x^7+105b^4x^9}{3840a^4b\,(a+bx^2)^5}+\frac{7\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-105*a^4*x + 790*a^3*b*x^3 + 896*a^2*b^2*x^5 + 490*a*b^3*x^7 + 105*b^4*x^9
)/(3840*a^4*b*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/2)
*b^(3/2))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.64

method	result	size
default	$\frac{\frac{7b^3x^9}{256a^4}+\frac{49b^2x^7}{384a^3}+\frac{7b\,x^5}{30a^2}+\frac{79x^3}{384a}-\frac{7x}{256b}}{(b\,x^2+a)^5}+\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256b\,a^4\sqrt{ab}}$	80
risch	$\frac{\frac{7b^3x^9}{256a^4}+\frac{49b^2x^7}{384a^3}+\frac{7b\,x^5}{30a^2}+\frac{79x^3}{384a}-\frac{7x}{256b}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}-\frac{7\ln\left(bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b\,a^4}+\frac{7\ln\left(-bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b\,a^4}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (7/256*b^3/a^4*x^9+49/384*b^2/a^3*x^7+7/30*b/a^2*x^5+79/384/a*x^3-7/256/b*x
)/(b*x^2+a)^5+7/256/b/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 131, normalized size = 1.05

$$\frac{105\,b^4x^9+490\,ab^3x^7+896\,a^2b^2x^5+790\,a^3bx^3-105\,a^4x}{3840\,(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b)}+\frac{7\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(105*b^4*x^9 + 490*a*b^3*x^7 + 896*a^2*b^2*x^5 + 790*a^3*b*x^3 - 105
*a^4*x)/(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5
*a^8*b^2*x^2 + a^9*b) + 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b)
```

Fricas [A]
time = 0.36, size = 390, normalized size = 3.12

$$\left[\frac{210\,ab^5x^9+980\,a^2b^4x^7+1792\,a^3b^3x^5+1580\,a^4b^2x^3-210\,a^5bx-105\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{7680\,(a^5b^7x^{10}+5\,a^6b^6x^8+10\,a^7b^5x^6+10\,a^8b^4x^4+5\,a^9b^3x^2+a^{10}b^2)},\ \frac{105\,ab^5x^9+490\,a^2b^4x^7+896\,a^3b^3x^5+790\,a^4b^2x^3-105\,a^5bx+105\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{3840\,(a^5b^7x^{10}+5\,a^6b^6x^8+10\,a^7b^5x^6+10\,a^8b^4x^4+5\,a^9b^3x^2+a^{10}b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(210*a*b^5*x^9 + 980*a^2*b^4*x^7 + 1792*a^3*b^3*x^5 + 1580*a^4*b^2*
x^3 - 210*a^5*b*x - 105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b
^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^5*b^7*x^10 + 5*a^6*b^6*x^8 + 10*a^7*b^5*x^6 + 10*a^8*b^4*x^4
+ 5*a^9*b^3*x^2 + a^10*b^2), 1/3840*(105*a*b^5*x^9 + 490*a^2*b^4*x^7 + 896*
a^3*b^3*x^5 + 790*a^4*b^2*x^3 - 105*a^5*b*x + 105*(b^5*x^10 + 5*a*b^4*x^8 +
```

```
 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt
(a*b)*x/a))/(a^5*b^7*x^10 + 5*a^6*b^6*x^8 + 10*a^7*b^5*x^6 + 10*a^8*b^4*x^4
 + 5*a^9*b^3*x^2 + a^10*b^2)]
```

Sympy [A]
time = 0.29, size = 190, normalized size = 1.52

$$-\frac{7\sqrt{-\frac{1}{a^9b^3}}\log\left(-a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{512}+\frac{7\sqrt{-\frac{1}{a^9b^3}}\log\left(a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{512}+\frac{-105a^4x+790a^3bx^3+896a^2b^2x^5+490ab^3x^7+105b^4x^9}{3840a^9b+19200a^8b^2x^2+38400a^7b^3x^4+38400a^6b^4x^6+19200a^5b^5x^8+3840a^4b^6x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -7*sqrt(-1/(a**9*b**3))*log(-a**5*b*sqrt(-1/(a**9*b**3)) + x)/512 + 7*sqrt(
-1/(a**9*b**3))*log(a**5*b*sqrt(-1/(a**9*b**3)) + x)/512 + (-105*a**4*x + 7
90*a**3*b*x**3 + 896*a**2*b**2*x**5 + 490*a*b**3*x**7 + 105*b**4*x**9)/(384
0*a**9*b + 19200*a**8*b**2*x**2 + 38400*a**7*b**3*x**4 + 38400*a**6*b**4*x*
*6 + 19200*a**5*b**5*x**8 + 3840*a**4*b**6*x**10)
```

Giac [A]
time = 3.99, size = 84, normalized size = 0.67

$$\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^4b}+\frac{105\,b^4x^9+490\,ab^3x^7+896\,a^2b^2x^5+790\,a^3bx^3-105\,a^4x}{3840\,(bx^2+a)^5a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b) + 1/3840*(105*b^4*x^9 + 490*a
*b^3*x^7 + 896*a^2*b^2*x^5 + 790*a^3*b*x^3 - 105*a^4*x)/((b*x^2 + a)^5*a^4*
b)
```

Mupad [B]
time = 4.48, size = 118, normalized size = 0.94

$$\frac{\frac{79\,x^3}{384\,a}-\frac{7\,x}{256\,b}+\frac{7\,b\,x^5}{30\,a^2}+\frac{49\,b^2\,x^7}{384\,a^3}+\frac{7\,b^3\,x^9}{256\,a^4}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}+\frac{7\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{9/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((79*x^3)/(384*a) - (7*x)/(256*b) + (7*b*x^5)/(30*a^2) + (49*b^2*x^7)/(384*
a^3) + (7*b^3*x^9)/(256*a^4))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 +
 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + (7*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(9
/2)*b^(3/2))
```

3.531 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=113

$$\frac{x}{10a(a+bx^2)^5}+\frac{9x}{80a^2(a+bx^2)^4}+\frac{21x}{160a^3(a+bx^2)^3}+\frac{21x}{128a^4(a+bx^2)^2}+\frac{63x}{256a^5(a+bx^2)}+\frac{63\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{11/2}\sqrt{b}}$$

```
[Out] 1/10*x/a/(b*x^2+a)^5+9/80*x/a^2/(b*x^2+a)^4+21/160*x/a^3/(b*x^2+a)^3+21/128
*x/a^4/(b*x^2+a)^2+63/256*x/a^5/(b*x^2+a)+63/256*arctan(x*b^(1/2)/a^(1/2))/
a^(11/2)/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {28, 205, 211}

$$\frac{63\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{11/2}\sqrt{b}}+\frac{63x}{256a^5(a+bx^2)}+\frac{21x}{128a^4(a+bx^2)^2}+\frac{21x}{160a^3(a+bx^2)^3}+\frac{9x}{80a^2(a+bx^2)^4}+\frac{x}{10a(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

```
[Out] x/(10*a*(a + b*x^2)^5) + (9*x)/(80*a^2*(a + b*x^2)^4) + (21*x)/(160*a^3*(a
 + b*x^2)^3) + (21*x)/(128*a^4*(a + b*x^2)^2) + (63*x)/(256*a^5*(a + b*x^2))
 + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(11/2)*Sqrt[b])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{\left(9b^5\right)\int \frac{1}{\left(ab+b^2x^2\right)^5}\,dx}{10a}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{\left(63b^4\right)\int \frac{1}{\left(ab+b^2x^2\right)^4}\,dx}{80a^2}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{\left(21b^3\right)\int \frac{1}{\left(ab+b^2x^2\right)^3}\,dx}{32a^3}$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{21x}{128a^4\left(a+bx^2\right)^2} +$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{21x}{128a^4\left(a+bx^2\right)^2} +$$

$$= \frac{x}{10a\left(a+bx^2\right)^5} + \frac{9x}{80a^2\left(a+bx^2\right)^4} + \frac{21x}{160a^3\left(a+bx^2\right)^3} + \frac{21x}{128a^4\left(a+bx^2\right)^2} +$$

Mathematica [A]
time = 0.03, size = 89, normalized size = 0.79

$$\frac{\frac{\sqrt{a}\,x\left(965a^4+2370a^3bx^2+2688a^2b^2x^4+1470ab^3x^6+315b^4x^8\right)}{\left(a+bx^2\right)^5}+\frac{315\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}}{1280a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

```
[Out] ((Sqrt[a]*x*(965*a^4 + 2370*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 1470*a*b^3*x^6 +
 315*b^4*x^8))/(a + b*x^2)^5 + (315*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(
1280*a^(11/2))
```

Maple [A]
time = 0.07, size = 120, normalized size = 1.06

method	result	size

default	$\frac{x}{10a(b\,x^{2}+a)^{5}}+\frac{\frac{9x}{80a\left(b\,x^{2}+a\right)^{4}}+\frac{9\left(\frac{7x}{48a\left(b\,x^{2}+a\right)^{3}}+\frac{7\left(\frac{5x}{24a\left(b\,x^{2}+a\right)^{2}}+\frac{5\left(\frac{3x}{8a\left(b\,x^{2}+a\right)}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)}{6a}\right)}{8a}\right)}{10a}}{a}$	120
risch	$\frac{\frac{63b^{4}x^{9}}{256a^{5}}+\frac{147b^{3}x^{7}}{128a^{4}}+\frac{21b^{2}x^{5}}{10a^{3}}+\frac{237b\,x^{3}}{128a^{2}}+\frac{193x}{256a}}{(b^{2}x^{4}+2ab\,x^{2}+a^{2})^{2}(b\,x^{2}+a)}-\frac{63\ln\left(bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,a^{5}}+\frac{63\ln\left(-bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,a^{5}}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x/a/(b*x^2+a)^5+9/10/a*(1/8*x/a/(b*x^2+a)^4+7/8/a*(1/6*x/a/(b*x^2+a)^3
+5/6/a*(1/4*x/a/(b*x^2+a)^2+3/4/a*(1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))))))
```

Maxima [A]
time = 0.51, size = 124, normalized size = 1.10

$$\frac{315\,b^{4}x^{9}+1470\,ab^{3}x^{7}+2688\,a^{2}b^{2}x^{5}+2370\,a^{3}bx^{3}+965\,a^{4}x}{1280\,(a^{5}b^{5}x^{10}+5\,a^{6}b^{4}x^{8}+10\,a^{7}b^{3}x^{6}+10\,a^{8}b^{2}x^{4}+5\,a^{9}bx^{2}+a^{10})}+\frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/1280*(315*b^4*x^9 + 1470*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 2370*a^3*b*x^3 +
965*a^4*x)/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4
+ 5*a^9*b*x^2 + a^10) + 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Fricas [A]
time = 0.35, size = 386, normalized size = 3.42

$$\left[\frac{630\,ab^{5}x^{9}+2940\,a^{2}b^{4}x^{7}+5376\,a^{3}b^{3}x^{5}+4740\,a^{4}b^{2}x^{3}+1930\,a^{5}bx-315\,(b^{5}x^{10}+5\,ab^{4}x^{8}+10\,a^{2}b^{3}x^{6}+10\,a^{3}b^{2}x^{4}+5\,a^{4}bx^{2}+a^{5})\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{2560\,(a^{6}b^{6}x^{10}+5\,a^{7}b^{5}x^{8}+10\,a^{8}b^{4}x^{6}+10\,a^{9}b^{3}x^{4}+5\,a^{10}b^{2}x^{2}+a^{11}b)},\ \frac{315\,ab^{5}x^{9}+1470\,a^{2}b^{4}x^{7}+2688\,a^{3}b^{3}x^{5}+2370\,a^{4}b^{2}x^{3}+965\,a^{5}bx+315\,(b^{5}x^{10}+5\,ab^{4}x^{8}+10\,a^{2}b^{3}x^{6}+10\,a^{3}b^{2}x^{4}+5\,a^{4}bx^{2}+a^{5})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{1280\,(a^{6}b^{6}x^{10}+5\,a^{7}b^{5}x^{8}+10\,a^{8}b^{4}x^{6}+10\,a^{9}b^{3}x^{4}+5\,a^{10}b^{2}x^{2}+a^{11}b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(630*a*b^5*x^9 + 2940*a^2*b^4*x^7 + 5376*a^3*b^3*x^5 + 4740*a^4*b^2
*x^3 + 1930*a^5*b*x - 315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3
*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a^6*b^6*x^10 + 5*a^7*b^5*x^8 + 10*a^8*b^4*x^6 + 10*a^9*b^3*x^
4 + 5*a^10*b^2*x^2 + a^11*b), 1/1280*(315*a*b^5*x^9 + 1470*a^2*b^4*x^7 + 26
88*a^3*b^3*x^5 + 2370*a^4*b^2*x^3 + 965*a^5*b*x + 315*(b^5*x^10 + 5*a*b^4*x
^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a))/(a^6*b^6*x^10 + 5*a^7*b^5*x^8 + 10*a^8*b^4*x^6 + 10*a^9*b^3
*x^4 + 5*a^10*b^2*x^2 + a^11*b)]
```

Sympy [A]
time = 0.30, size = 177, normalized size = 1.57

$$-\frac{63\sqrt{-\frac{1}{a^{11}b}}\log\left(-a^{6}\sqrt{-\frac{1}{a^{11}b}}+x\right)}{512}+\frac{63\sqrt{-\frac{1}{a^{11}b}}\log\left(a^{6}\sqrt{-\frac{1}{a^{11}b}}+x\right)}{512}+\frac{965a^{4}x+2370a^{3}bx^{3}+2688a^{2}b^{2}x^{5}+1470ab^{3}x^{7}+315b^{4}x^{9}}{1280a^{10}+6400a^{9}bx^{2}+12800a^{8}b^{2}x^{4}+12800a^{7}b^{3}x^{6}+6400a^{6}b^{4}x^{8}+1280a^{5}b^{5}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -63*sqrt(-1/(a**11*b))*log(-a**6*sqrt(-1/(a**11*b)) + x)/512 + 63*sqrt(-1/(
a**11*b))*log(a**6*sqrt(-1/(a**11*b)) + x)/512 + (965*a**4*x + 2370*a**3*b*
x**3 + 2688*a**2*b**2*x**5 + 1470*a*b**3*x**7 + 315*b**4*x**9)/(1280*a**10
+ 6400*a**9*b*x**2 + 12800*a**8*b**2*x**4 + 12800*a**7*b**3*x**6 + 6400*a**
6*b**4*x**8 + 1280*a**5*b**5*x**10)
```

Giac [A]
time = 3.83, size = 78, normalized size = 0.69

$$\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}\,a^{5}}+\frac{315\,b^{4}x^{9}+1470\,ab^{3}x^{7}+2688\,a^{2}b^{2}x^{5}+2370\,a^{3}bx^{3}+965\,a^{4}x}{1280\,(bx^{2}+a)^{5}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/1280*(315*b^4*x^9 + 1470*a
*b^3*x^7 + 2688*a^2*b^2*x^5 + 2370*a^3*b*x^3 + 965*a^4*x)/((b*x^2 + a)^5*a^
5)
```

Mupad [B]
time = 4.71, size = 121, normalized size = 1.07

$$\frac{\frac{193\,x}{256\,a}+\frac{237\,b\,x^{3}}{128\,a^{2}}+\frac{21\,b^{2}\,x^{5}}{10\,a^{3}}+\frac{147\,b^{3}\,x^{7}}{128\,a^{4}}+\frac{63\,b^{4}\,x^{9}}{256\,a^{5}}}{a^{5}+5\,a^{4}\,b\,x^{2}+10\,a^{3}\,b^{2}\,x^{4}+10\,a^{2}\,b^{3}\,x^{6}+5\,a\,b^{4}\,x^{8}+b^{5}\,x^{10}}+\frac{63\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{11/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((193*x)/(256*a) + (237*b*x^3)/(128*a^2) + (21*b^2*x^5)/(10*a^3) + (147*b^3
*x^7)/(128*a^4) + (63*b^4*x^9)/(256*a^5))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5
*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + (63*atan((b^(1/2)*x)/a^(1/2
)))/(256*a^(11/2)*b^(1/2))
```

3.532 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=133

$$-\frac{693}{256a^6x}+\frac{1}{10ax\left(a+bx^2\right)^5}+\frac{11}{80a^2x\left(a+bx^2\right)^4}+\frac{33}{160a^3x\left(a+bx^2\right)^3}+\frac{231}{640a^4x\left(a+bx^2\right)^2}+\frac{231}{256a^5x\left(a+bx^2\right)}-$$

```
[Out] -693/256/a^6/x+1/10/a/x/(b*x^2+a)^5+11/80/a^2/x/(b*x^2+a)^4+33/160/a^3/x/(b
*x^2+a)^3+231/640/a^4/x/(b*x^2+a)^2+231/256/a^5/x/(b*x^2+a)-693/256*arctan(
x*b^(1/2)/a^(1/2))*b^(1/2)/a^(13/2)
```

Rubi [A]
time = 0.06, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 296, 331, 211}

$$-\frac{693\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{13/2}}-\frac{693}{256a^6x}+\frac{231}{256a^5x\left(a+bx^2\right)}+\frac{231}{640a^4x\left(a+bx^2\right)^2}+\frac{33}{160a^3x\left(a+bx^2\right)^3}+\frac{11}{80a^2x\left(a+bx^2\right)^4}+\frac{1}{10ax\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -693/(256*a^6*x) + 1/(10*a*x*(a + b*x^2)^5) + 11/(80*a^2*x*(a + b*x^2)^4) +
 33/(160*a^3*x*(a + b*x^2)^3) + 231/(640*a^4*x*(a + b*x^2)^2) + 231/(256*a^
5*x*(a + b*x^2)) - (693*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(13/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{x^2\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{\left(11b^5\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^5}\,dx}{10a}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{\left(99b^4\right)\int \frac{1}{x^2\left(ab+b^2x^2\right)^4}\,dx}{80a^2}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{\left(231b^3\right)\int \frac{1}{x^2(ab+b}}{160a^3}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{231}{640a^4x\left(a+bx^2\right.}$$

$$= \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{231}{640a^4x\left(a+bx^2\right.}$$

$$= -\frac{693}{256a^6x} + \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{}{640}$$

$$= -\frac{693}{256a^6x} + \frac{1}{10ax\left(a+bx^2\right)^5} + \frac{11}{80a^2x\left(a+bx^2\right)^4} + \frac{33}{160a^3x\left(a+bx^2\right)^3} + \frac{}{640}$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 0.76

$$-\frac{1280a^5+10615a^4bx^2+26070a^3b^2x^4+29568a^2b^3x^6+16170ab^4x^8+3465b^5x^{10}}{1280a^6x\left(a+bx^2\right)^5}-\frac{693\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/1280*(1280*a^5 + 10615*a^4*b*x^2 + 26070*a^3*b^2*x^4 + 29568*a^2*b^3*x^6
 + 16170*a*b^4*x^8 + 3465*b^5*x^10)/(a^6*x*(a + b*x^2)^5) - (693*Sqrt[b]*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(13/2))
```

Maple [A]
time = 0.06, size = 87, normalized size = 0.65

method	result
default	$-\frac{b\left(\frac{\frac{437}{256}b^{4}x^{9}+\frac{977}{128}a\,b^{3}x^{7}+\frac{131}{10}a^{2}b^{2}x^{5}+\frac{1327}{128}a^{3}b\,x^{3}+\frac{843}{256}a^{4}x}{\left(b\,x^{2}+a\right)^{5}}+\frac{693\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}}\right)}{a^{6}}-\frac{1}{a^{6}x}$
risch	$\frac{-\frac{693b^{5}x^{10}}{256a^{6}}-\frac{1617b^{4}x^{8}}{128a^{5}}-\frac{231b^{3}x^{6}}{10a^{4}}-\frac{2607b^{2}x^{4}}{128a^{3}}-\frac{2123b\,x^{2}}{256a^{2}}-\frac{1}{a}}{x(b^{2}x^{4}+2ab\,x^{2}+a^{2})^{2}(b\,x^{2}+a)}+\frac{693\left(\sum_{\_R=\mathrm{RootOf}\left(a^{13}\_Z^{2}+b\right)}\_R\ln\left(\left(3\_R^{2}a^{13}+2b\right)x+a^{7}\_R\right)\right)}{512}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^6*((437/256*b^4*x^9+977/128*a*b^3*x^7+131/10*a^2*b^2*x^5+1327/128*a^3*
b*x^3+843/256*a^4*x)/(b*x^2+a)^5+693/256/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
))-1/a^6/x
```

Maxima [A]
time = 0.50, size = 137, normalized size = 1.03

$$-\frac{3465\,b^{5}x^{10}+16170\,ab^{4}x^{8}+29568\,a^{2}b^{3}x^{6}+26070\,a^{3}b^{2}x^{4}+10615\,a^{4}bx^{2}+1280\,a^{5}}{1280\,(a^{6}b^{5}x^{11}+5\,a^{7}b^{4}x^{9}+10\,a^{8}b^{3}x^{7}+10\,a^{9}b^{2}x^{5}+5\,a^{10}bx^{3}+a^{11}x)}-\frac{693\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(3465*b^5*x^10 + 16170*a*b^4*x^8 + 29568*a^2*b^3*x^6 + 26070*a^3*b^
2*x^4 + 10615*a^4*b*x^2 + 1280*a^5)/(a^6*b^5*x^11 + 5*a^7*b^4*x^9 + 10*a^8*
b^3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x) - 693/256*b*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^6)
```

Fricas [A]
time = 0.35, size = 400, normalized size = 3.01

$$\left[-\frac{6930\,b^{5}x^{10}+32340\,ab^{4}x^{8}+59136\,a^{2}b^{3}x^{6}+52140\,a^{3}b^{2}x^{4}+21230\,a^{4}bx^{2}+2560\,a^{5}-3465\,(b^{5}x^{11}+5\,ab^{4}x^{9}+10\,a^{2}b^{3}x^{7}+10\,a^{3}b^{2}x^{5}+5\,a^{4}bx^{3}+a^{5}x)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{2560\,(a^{6}b^{5}x^{11}+5\,a^{7}b^{4}x^{9}+10\,a^{8}b^{3}x^{7}+10\,a^{9}b^{2}x^{5}+5\,a^{10}bx^{3}+a^{11}x)},\ -\frac{3465\,b^{5}x^{10}+16170\,ab^{4}x^{8}+29568\,a^{2}b^{3}x^{6}+26070\,a^{3}b^{2}x^{4}+10615\,a^{4}bx^{2}+1280\,a^{5}+3465\,(b^{5}x^{11}+5\,ab^{4}x^{9}+10\,a^{2}b^{3}x^{7}+10\,a^{3}b^{2}x^{5}+5\,a^{4}bx^{3}+a^{5}x)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{1280\,(a^{6}b^{5}x^{11}+5\,a^{7}b^{4}x^{9}+10\,a^{8}b^{3}x^{7}+10\,a^{9}b^{2}x^{5}+5\,a^{10}bx^{3}+a^{11}x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(6930*b^5*x^10 + 32340*a*b^4*x^8 + 59136*a^2*b^3*x^6 + 52140*a^3*b
^2*x^4 + 21230*a^4*b*x^2 + 2560*a^5 - 3465*(b^5*x^11 + 5*a*b^4*x^9 + 10*a^2
*b^3*x^7 + 10*a^3*b^2*x^5 + 5*a^4*b*x^3 + a^5*x)*sqrt(-b/a)*log((b*x^2 - 2*
a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^5*x^11 + 5*a^7*b^4*x^9 + 10*a^8*b^
3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x), -1/1280*(3465*b^5*x^10 + 1
6170*a*b^4*x^8 + 29568*a^2*b^3*x^6 + 26070*a^3*b^2*x^4 + 10615*a^4*b*x^2 +
1280*a^5 + 3465*(b^5*x^11 + 5*a*b^4*x^9 + 10*a^2*b^3*x^7 + 10*a^3*b^2*x^5 +
 5*a^4*b*x^3 + a^5*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^5*x^11 + 5*a^7*
b^4*x^9 + 10*a^8*b^3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x)]
```

Sympy [A]
time = 0.39, size = 187, normalized size = 1.41

$$\frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(-\frac{a^{7}\sqrt{-\frac{b}{a^{13}}}}{b}+x\right)}{512}-\frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(\frac{a^{7}\sqrt{-\frac{b}{a^{13}}}}{b}+x\right)}{512}+\frac{-1280a^{5}-10615a^{4}bx^{2}-26070a^{3}b^{2}x^{4}-29568a^{2}b^{3}x^{6}-16170ab^{4}x^{8}-3465b^{5}x^{10}}{1280a^{11}x+6400a^{10}bx^{3}+12800a^{9}b^{2}x^{5}+12800a^{8}b^{3}x^{7}+6400a^{7}b^{4}x^{9}+1280a^{6}b^{5}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 693*sqrt(-b/a**13)*log(-a**7*sqrt(-b/a**13)/b + x)/512 - 693*sqrt(-b/a**13)
*log(a**7*sqrt(-b/a**13)/b + x)/512 + (-1280*a**5 - 10615*a**4*b*x**2 - 260
70*a**3*b**2*x**4 - 29568*a**2*b**3*x**6 - 16170*a*b**4*x**8 - 3465*b**5*x*
*10)/(1280*a**11*x + 6400*a**10*b*x**3 + 12800*a**9*b**2*x**5 + 12800*a**8*
b**3*x**7 + 6400*a**7*b**4*x**9 + 1280*a**6*b**5*x**11)
```

Giac [A]
time = 3.77, size = 90, normalized size = 0.68

$$-\frac{693\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^{6}}-\frac{1}{a^{6}x}-\frac{2185\,b^{5}x^{9}+9770\,ab^{4}x^{7}+16768\,a^{2}b^{3}x^{5}+13270\,a^{3}b^{2}x^{3}+4215\,a^{4}bx}{1280\,\left(bx^{2}+a\right)^{5}a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -693/256*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/(a^6*x) - 1/1280*(2185
*b^5*x^9 + 9770*a*b^4*x^7 + 16768*a^2*b^3*x^5 + 13270*a^3*b^2*x^3 + 4215*a^
4*b*x)/((b*x^2 + a)^5*a^6)
```

Mupad [B]
time = 4.58, size = 132, normalized size = 0.99

$$-\frac{\frac{1}{a}+\frac{2123\,b\,x^{2}}{256\,a^{2}}+\frac{2607\,b^{2}\,x^{4}}{128\,a^{3}}+\frac{231\,b^{3}\,x^{6}}{10\,a^{4}}+\frac{1617\,b^{4}\,x^{8}}{128\,a^{5}}+\frac{693\,b^{5}\,x^{10}}{256\,a^{6}}}{a^{5}\,x+5\,a^{4}\,b\,x^{3}+10\,a^{3}\,b^{2}\,x^{5}+10\,a^{2}\,b^{3}\,x^{7}+5\,a\,b^{4}\,x^{9}+b^{5}\,x^{11}}-\frac{693\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] - (1/a + (2123*b*x^2)/(256*a^2) + (2607*b^2*x^4)/(128*a^3) + (231*b^3*x^6)/
(10*a^4) + (1617*b^4*x^8)/(128*a^5) + (693*b^5*x^10)/(256*a^6))/(a^5*x + b^
5*x^11 + 5*a^4*b*x^3 + 5*a*b^4*x^9 + 10*a^3*b^2*x^5 + 10*a^2*b^3*x^7) - (69
3*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(13/2))
```

3.533 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=144

$$-\frac{1001}{256a^6x^3}+\frac{3003b}{256a^7x}+\frac{1}{10ax^3\left(a+bx^2\right)^5}+\frac{13}{80a^2x^3\left(a+bx^2\right)^4}+\frac{143}{480a^3x^3\left(a+bx^2\right)^3}+\frac{429}{640a^4x^3\left(a+bx^2\right)^2}+\frac{1280a^5}{}$$

```
[Out] -1001/256/a^6/x^3+3003/256*b/a^7/x+1/10/a/x^3/(b*x^2+a)^5+13/80/a^2/x^3/(b*
x^2+a)^4+143/480/a^3/x^3/(b*x^2+a)^3+429/640/a^4/x^3/(b*x^2+a)^2+3003/1280/
a^5/x^3/(b*x^2+a)+3003/256*b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(15/2)
```

Rubi [A]
time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 296, 331, 211}

$$\frac{3003b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{15/2}}+\frac{3003b}{256a^7x}-\frac{1001}{256a^6x^3}+\frac{3003}{1280a^5x^3\left(a+bx^2\right)}+\frac{429}{640a^4x^3\left(a+bx^2\right)^2}+\frac{143}{480a^3x^3\left(a+bx^2\right)^3}+\frac{13}{80a^2x^3\left(a+bx^2\right)^4}+\frac{1}{10ax^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1001/(256*a^6*x^3) + (3003*b)/(256*a^7*x) + 1/(10*a*x^3*(a + b*x^2)^5) + 1
3/(80*a^2*x^3*(a + b*x^2)^4) + 143/(480*a^3*x^3*(a + b*x^2)^3) + 429/(640*a
^4*x^3*(a + b*x^2)^2) + 3003/(1280*a^5*x^3*(a + b*x^2)) + (3003*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(15/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4 (a^2+2abx^2+b^2x^4)^3}\,dx = b^6 \int \frac{1}{x^4 (ab+b^2x^2)^6}\,dx$$

$$= \frac{1}{10ax^3 (a+bx^2)^5} + \frac{(13b^5)\int \frac{1}{x^4(ab+b^2x^2)^5}\,dx}{10a}$$

$$= \frac{1}{10ax^3 (a+bx^2)^5} + \frac{13}{80a^2x^3 (a+bx^2)^4} + \frac{(143b^4)\int \frac{1}{x^4(ab+b^2x^2)^4}\,dx}{80a^2}$$

$$= \frac{1}{10ax^3 (a+bx^2)^5} + \frac{13}{80a^2x^3 (a+bx^2)^4} + \frac{143}{480a^3x^3 (a+bx^2)^3} + \frac{(429b^3)\int \frac{}{x^4}}{16}$$

$$= \frac{1}{10ax^3 (a+bx^2)^5} + \frac{13}{80a^2x^3 (a+bx^2)^4} + \frac{143}{480a^3x^3 (a+bx^2)^3} + \frac{429}{640a^4x^3 (a}$$

$$= \frac{1}{10ax^3 (a+bx^2)^5} + \frac{13}{80a^2x^3 (a+bx^2)^4} + \frac{143}{480a^3x^3 (a+bx^2)^3} + \frac{429}{640a^4x^3 (a}$$

$$= -\frac{1001}{256a^6x^3} + \frac{1}{10ax^3 (a+bx^2)^5} + \frac{13}{80a^2x^3 (a+bx^2)^4} + \frac{143}{480a^3x^3 (a+bx^2)^3}$$

$$= -\frac{1001}{256a^6x^3} + \frac{3003b}{256a^7x} + \frac{1}{10ax^3 (a+bx^2)^5} + \frac{13}{80a^2x^3 (a+bx^2)^4} + \frac{14}{480a^3x^3 (}$$

$$= -\frac{1001}{256a^6x^3} + \frac{3003b}{256a^7x} + \frac{1}{10ax^3 (a+bx^2)^5} + \frac{13}{80a^2x^3 (a+bx^2)^4} + \frac{14}{480a^3x^3 (}$$

Mathematica [A]
time = 0.04, size = 113, normalized size = 0.78

$$\frac{\frac{\sqrt{a}\,(-1280a^6+16640a^5bx^2+137995a^4b^2x^4+338910a^3b^3x^6+384384a^2b^4x^8+210210ab^5x^{10}+45045b^6x^{12})}{x^3(a+bx^2)^5} + 45045b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3840a^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((Sqrt[a]*(-1280*a^6 + 16640*a^5*b*x^2 + 137995*a^4*b^2*x^4 + 338910*a^3*b^
3*x^6 + 384384*a^2*b^4*x^8 + 210210*a*b^5*x^10 + 45045*b^6*x^12))/(x^3*(a +
 b*x^2)^5) + 45045*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3840*a^(15/2))
```

Maple [A]
time = 0.06, size = 97, normalized size = 0.67

method	result
default	$\frac{b^{2}\left(\frac{\frac{1467}{256}b^{4}x^{9}+\frac{9629}{384}a\,b^{3}x^{7}+\frac{1253}{30}a^{2}b^{2}x^{5}+\frac{12131}{384}a^{3}b\,x^{3}+\frac{2373}{256}a^{4}x}{\left(b\,x^{2}+a\right)^{5}}+\frac{3003\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}}\right)}{a^{7}}-\frac{1}{3a^{6}x^{3}}+\frac{6b}{a^{7}x}$
risch	$\frac{\frac{3003b^{6}x^{12}}{256a^{7}}+\frac{7007b^{5}x^{10}}{128a^{6}}+\frac{1001b^{4}x^{8}}{10a^{5}}+\frac{11297b^{3}x^{6}}{128a^{4}}+\frac{27599b^{2}x^{4}}{768a^{3}}+\frac{13b\,x^{2}}{3a^{2}}-\frac{1}{3a}}{x^{3}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}\left(b\,x^{2}+a\right)}+\frac{3003\left(\sum_{\_R=\mathrm{RootOf}\left(a^{15}\_Z^{2}+b^{3}\right)}\_R\ln\left(\left(3\_R^{2}a^{15}+2b^{3}\right)\right.\right.}{512}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^7*((1467/256*b^4*x^9+9629/384*a*b^3*x^7+1253/30*a^2*b^2*x^5+12131/384
*a^3*b*x^3+2373/256*a^4*x)/(b*x^2+a)^5+3003/256/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2)))-1/3/a^6/x^3+6*b/a^7/x
```

Maxima [A]
time = 0.51, size = 152, normalized size = 1.06

$$\frac{45045\,b^{6}x^{12}+210210\,ab^{5}x^{10}+384384\,a^{2}b^{4}x^{8}+338910\,a^{3}b^{3}x^{6}+137995\,a^{4}b^{2}x^{4}+16640\,a^{5}bx^{2}-1280\,a^{6}}{3840\,\left(a^{7}b^{5}x^{13}+5\,a^{8}b^{4}x^{11}+10\,a^{9}b^{3}x^{9}+10\,a^{10}b^{2}x^{7}+5\,a^{11}bx^{5}+a^{12}x^{3}\right)}+\frac{3003\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/3840*(45045*b^6*x^12 + 210210*a*b^5*x^10 + 384384*a^2*b^4*x^8 + 338910*a^
3*b^3*x^6 + 137995*a^4*b^2*x^4 + 16640*a^5*b*x^2 - 1280*a^6)/(a^7*b^5*x^13
+ 5*a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 + a^12*x
^3) + 3003/256*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7)
```

Fricas [A]
time = 0.36, size = 436, normalized size = 3.03

$$\left[\frac{90090\,b^{6}x^{12}+420420\,ab^{5}x^{10}+768768\,a^{2}b^{4}x^{8}+677820\,a^{3}b^{3}x^{6}+275990\,a^{4}b^{2}x^{4}+33280\,a^{5}bx^{2}-2560\,a^{6}+45045\,\left(b^{6}x^{13}+5\,ab^{5}x^{11}+10\,a^{2}b^{4}x^{9}+10\,a^{3}b^{3}x^{7}+5\,a^{4}b^{2}x^{5}+a^{5}bx^{3}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{7680\,\left(a^{7}b^{5}x^{13}+5\,a^{8}b^{4}x^{11}+10\,a^{9}b^{3}x^{9}+10\,a^{10}b^{2}x^{7}+5\,a^{11}bx^{5}+a^{12}x^{3}\right)},\ \frac{45045\,b^{6}x^{12}+210210\,ab^{5}x^{10}+384384\,a^{2}b^{4}x^{8}+338910\,a^{3}b^{3}x^{6}+137995\,a^{4}b^{2}x^{4}+16640\,a^{5}bx^{2}-1280\,a^{6}+45045\,\left(b^{6}x^{13}+5\,ab^{5}x^{11}+10\,a^{2}b^{4}x^{9}+10\,a^{3}b^{3}x^{7}+5\,a^{4}b^{2}x^{5}+a^{5}bx^{3}\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{3840\,\left(a^{7}b^{5}x^{13}+5\,a^{8}b^{4}x^{11}+10\,a^{9}b^{3}x^{9}+10\,a^{10}b^{2}x^{7}+5\,a^{11}bx^{5}+a^{12}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(90090*b^6*x^12 + 420420*a*b^5*x^10 + 768768*a^2*b^4*x^8 + 677820*a
^3*b^3*x^6 + 275990*a^4*b^2*x^4 + 33280*a^5*b*x^2 - 2560*a^6 + 45045*(b^6*x
^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*b^3*x^7 + 5*a^4*b^2*x^5 + a^5*
b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^5
*x^13 + 5*a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 +
a^12*x^3), 1/3840*(45045*b^6*x^12 + 210210*a*b^5*x^10 + 384384*a^2*b^4*x^8
+ 338910*a^3*b^3*x^6 + 137995*a^4*b^2*x^4 + 16640*a^5*b*x^2 - 1280*a^6 + 45
045*(b^6*x^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*b^3*x^7 + 5*a^4*b^2*
x^5 + a^5*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^7*b^5*x^13 + 5*a^8*b^4*x
^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 + a^12*x^3)]
```

Sympy [**A**]
time = 0.41, size = 209, normalized size = 1.45

$$-\frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(-\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512}+\frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512}+\frac{-1280a^6+16640a^5bx^2+137995a^4b^2x^4+338910a^3b^3x^6+384384a^2b^4x^8+210210ab^5x^{10}+45045b^6x^{12}}{3840a^{12}x^3+19200a^{11}bx^5+38400a^{10}b^2x^7+38400a^9b^3x^9+19200a^8b^4x^{11}+3840a^7b^5x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3003*sqrt(-b**3/a**15)*log(-a**8*sqrt(-b**3/a**15)/b**2 + x)/512 + 3003*sq
rt(-b**3/a**15)*log(a**8*sqrt(-b**3/a**15)/b**2 + x)/512 + (-1280*a**6 + 16
640*a**5*b*x**2 + 137995*a**4*b**2*x**4 + 338910*a**3*b**3*x**6 + 384384*a*
*2*b**4*x**8 + 210210*a*b**5*x**10 + 45045*b**6*x**12)/(3840*a**12*x**3 + 1
9200*a**11*b*x**5 + 38400*a**10*b**2*x**7 + 38400*a**9*b**3*x**9 + 19200*a*
*8*b**4*x**11 + 3840*a**7*b**5*x**13)
```

Giac [**A**]
time = 4.10, size = 104, normalized size = 0.72

$$\frac{3003\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^7}+\frac{18\,bx^2-a}{3\,a^7x^3}+\frac{22005\,b^6x^9+96290\,ab^5x^7+160384\,a^2b^4x^5+121310\,a^3b^3x^3+35595\,a^4b^2x}{3840\,(bx^2+a)^5a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 3003/256*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7) + 1/3*(18*b*x^2 - a)/(a^
7*x^3) + 1/3840*(22005*b^6*x^9 + 96290*a*b^5*x^7 + 160384*a^2*b^4*x^5 + 121
310*a^3*b^3*x^3 + 35595*a^4*b^2*x)/((b*x^2 + a)^5*a^7)
```

Mupad [B]
time = 4.62, size = 146, normalized size = 1.01

$$\frac{\frac{13\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{27599\,b^2\,x^4}{768\,a^3}+\frac{11297\,b^3\,x^6}{128\,a^4}+\frac{1001\,b^4\,x^8}{10\,a^5}+\frac{7007\,b^5\,x^{10}}{128\,a^6}+\frac{3003\,b^6\,x^{12}}{256\,a^7}}{a^5\,x^3+5\,a^4\,b\,x^5+10\,a^3\,b^2\,x^7+10\,a^2\,b^3\,x^9+5\,a\,b^4\,x^{11}+b^5\,x^{13}}+\frac{3003\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{15/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((13*b*x^2)/(3*a^2) - 1/(3*a) + (27599*b^2*x^4)/(768*a^3) + (11297*b^3*x^6)
/(128*a^4) + (1001*b^4*x^8)/(10*a^5) + (7007*b^5*x^10)/(128*a^6) + (3003*b^
6*x^12)/(256*a^7))/(a^5*x^3 + b^5*x^13 + 5*a^4*b*x^5 + 5*a*b^4*x^11 + 10*a^
3*b^2*x^7 + 10*a^2*b^3*x^9) + (3003*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(256
*a^(15/2))
```

3.534 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{9009}{1280a^6x^5}+\frac{3003b}{256a^7x^3}-\frac{9009b^2}{256a^8x}+\frac{1}{10ax^5\,(a+bx^2)^5}+\frac{3}{16a^2x^5\,(a+bx^2)^4}+\frac{13}{32a^3x^5\,(a+bx^2)^3}+\frac{143}{128a^4x^5\,(a+bx$$

```
[Out] -9009/1280/a^6/x^5+3003/256*b/a^7/x^3-9009/256*b^2/a^8/x+1/10/a/x^5/(b*x^2+
a)^5+3/16/a^2/x^5/(b*x^2+a)^4+13/32/a^3/x^5/(b*x^2+a)^3+143/128/a^4/x^5/(b*
x^2+a)^2+1287/256/a^5/x^5/(b*x^2+a)-9009/256*b^(5/2)*arctan(x*b^(1/2)/a^(1/
2))/a^(17/2)
```

Rubi [A]
time = 0.08, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {28, 296, 331, 211}

$$-\frac{9009b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{17/2}}-\frac{9009b^2}{256a^8x}+\frac{3003b}{256a^7x^3}-\frac{9009}{1280a^6x^5}+\frac{1287}{256a^5x^5\,(a+bx^2)}+\frac{143}{128a^4x^5\,(a+bx^2)^2}+\frac{13}{32a^3x^5\,(a+bx^2)^3}+\frac{3}{16a^2x^5\,(a+bx^2)^4}+\frac{1}{10ax^5\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -9009/(1280*a^6*x^5) + (3003*b)/(256*a^7*x^3) - (9009*b^2)/(256*a^8*x) + 1/
(10*a*x^5*(a + b*x^2)^5) + 3/(16*a^2*x^5*(a + b*x^2)^4) + 13/(32*a^3*x^5*(a
 + b*x^2)^3) + 143/(128*a^4*x^5*(a + b*x^2)^2) + 1287/(256*a^5*x^5*(a + b*x
^2)) - (9009*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(17/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^6 (a^2+2abx^2+b^2x^4)^3}\,dx = b^6 \int \frac{1}{x^6 (ab+b^2x^2)^6}\,dx$$

$$= \frac{1}{10ax^5 (a+bx^2)^5} + \frac{(3b^5)\int \frac{1}{x^6(ab+b^2x^2)^5}\,dx}{2a}$$

$$= \frac{1}{10ax^5 (a+bx^2)^5} + \frac{3}{16a^2x^5 (a+bx^2)^4} + \frac{(39b^4)\int \frac{1}{x^6(ab+b^2x^2)^4}\,dx}{16a^2}$$

$$= \frac{1}{10ax^5 (a+bx^2)^5} + \frac{3}{16a^2x^5 (a+bx^2)^4} + \frac{13}{32a^3x^5 (a+bx^2)^3} + \frac{(143b^3)\int \frac{1}{x^6(ab}{32a^3}$$

$$= \frac{1}{10ax^5 (a+bx^2)^5} + \frac{3}{16a^2x^5 (a+bx^2)^4} + \frac{13}{32a^3x^5 (a+bx^2)^3} + \frac{143}{128a^4x^5 (a+b}$$

$$= \frac{1}{10ax^5 (a+bx^2)^5} + \frac{3}{16a^2x^5 (a+bx^2)^4} + \frac{13}{32a^3x^5 (a+bx^2)^3} + \frac{143}{128a^4x^5 (a+b}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{1}{10ax^5 (a+bx^2)^5} + \frac{3}{16a^2x^5 (a+bx^2)^4} + \frac{13}{32a^3x^5 (a+bx^2)^3} +$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} + \frac{1}{10ax^5 (a+bx^2)^5} + \frac{3}{16a^2x^5 (a+bx^2)^4} + \frac{13}{32a^3x^5 (a}$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} - \frac{9009b^2}{256a^8x} + \frac{1}{10ax^5 (a+bx^2)^5} + \frac{3}{16a^2x^5 (a+bx^2)^4} +$$

$$= -\frac{9009}{1280a^6x^5} + \frac{3003b}{256a^7x^3} - \frac{9009b^2}{256a^8x} + \frac{1}{10ax^5 (a+bx^2)^5} + \frac{3}{16a^2x^5 (a+bx^2)^4} +$$

Mathematica [A]

time = 0.04, size = 123, normalized size = 0.78

$$-\frac{256a^7-1280a^6bx^2+16640a^5b^2x^4+137995a^4b^3x^6+338910a^3b^4x^8+384384a^2b^5x^{10}+210210ab^6x^{12}+45045b^7x^{14}}{1280a^8x^5\left(a+bx^2\right)^5}-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256a^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/1280*(256*a^7 - 1280*a^6*b*x^2 + 16640*a^5*b^2*x^4 + 137995*a^4*b^3*x^6
+ 338910*a^3*b^4*x^8 + 384384*a^2*b^5*x^10 + 210210*a*b^6*x^12 + 45045*b^7*
x^14)/(a^8*x^5*(a + b*x^2)^5) - (9009*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(256*a^(17/2))
```

Maple [A]
time = 0.07, size = 109, normalized size = 0.69

method	result
default	$-\frac{b^3\left(\frac{\frac{3633}{256}b^4x^9+\frac{7837}{128}a\,b^3x^7+\frac{1001}{10}a^2b^2x^5+\frac{9443}{128}a^3b\,x^3+\frac{5327}{256}a^4x}{\left(b\,x^2+a\right)^5}+\frac{9009\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}}\right)}{a^8}-\frac{1}{5a^6x^5}-\frac{21b^2}{a^8x}+\frac{2b}{a^7x^3}$
risch	$\frac{-\frac{1}{5a}+\frac{b\,x^2}{a^2}-\frac{13b^2x^4}{a^3}-\frac{27599b^3x^6}{256a^4}-\frac{33891b^4x^8}{128a^5}-\frac{3003b^5x^{10}}{10a^6}-\frac{21021b^6x^{12}}{128a^7}-\frac{9009b^7x^{14}}{256a^8}}{x^5(b^2x^4+2ab\,x^2+a^2)^2(b\,x^2+a)}+\frac{9009\left(\sum_{\_R=\mathrm{RootOf}\left(a^{17}\_Z^2+b^5\right)}\_R\ln\left(\left(3\_\right.\right.\right.}{512}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^8*b^3*((3633/256*b^4*x^9+7837/128*a*b^3*x^7+1001/10*a^2*b^2*x^5+9443/1
28*a^3*b*x^3+5327/256*a^4*x)/(b*x^2+a)^5+9009/256/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2)))-1/5/a^6/x^5-21*b^2/a^8/x+2*b/a^7/x^3
```

Maxima [A]
time = 0.52, size = 163, normalized size = 1.04

$$-\frac{45045\,b^7x^{14}+210210\,ab^6x^{12}+384384\,a^2b^5x^{10}+338910\,a^3b^4x^8+137995\,a^4b^3x^6+16640\,a^5b^2x^4-1280\,a^6bx^2+256\,a^7}{1280\left(a^8b^5x^{15}+5\,a^9b^4x^{13}+10\,a^{10}b^3x^{11}+10\,a^{11}b^2x^9+5\,a^{12}bx^7+a^{13}x^5\right)}-\frac{9009\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(45045*b^7*x^14 + 210210*a*b^6*x^12 + 384384*a^2*b^5*x^10 + 338910*
a^3*b^4*x^8 + 137995*a^4*b^3*x^6 + 16640*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256
*a^7)/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10*a^11*b^2*x^9 +
```

```
5*a^12*b*x^7 + a^13*x^5) - 9009/256*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a
^8)
```

Fricas [A]
time = 0.44, size = 462, normalized size = 2.94

$$\left[-\frac{90090\,b^7x^{14}+420420\,ab^6x^{12}+768768\,a^2b^5x^{10}+677820\,a^3b^4x^8+275990\,a^4b^3x^6+33280\,a^5b^2x^4-2560\,a^6bx^2+512\,a^7-45045\,(b^7x^{15}+5\,ab^6x^{13}+10\,a^2b^5x^{11}+10\,a^3b^4x^9+5\,a^4b^3x^7+a^5b^2x^5)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2560\,(a^8b^5x^{15}+5\,a^9b^4x^{13}+10\,a^{10}b^3x^{11}+10\,a^{11}b^2x^9+5\,a^{12}bx^7+a^{13}x^5)},\ -\frac{45045\,b^7x^{14}+210210\,ab^6x^{12}+384384\,a^2b^5x^{10}+338910\,a^3b^4x^8+137995\,a^4b^3x^6+16640\,a^5b^2x^4-1280\,a^6bx^2+256\,a^7+45045\,(b^7x^{15}+5\,ab^6x^{13}+10\,a^2b^5x^{11}+10\,a^3b^4x^9+5\,a^4b^3x^7+a^5b^2x^5)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{1280\,(a^8b^5x^{15}+5\,a^9b^4x^{13}+10\,a^{10}b^3x^{11}+10\,a^{11}b^2x^9+5\,a^{12}bx^7+a^{13}x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(90090*b^7*x^14 + 420420*a*b^6*x^12 + 768768*a^2*b^5*x^10 + 677820
*a^3*b^4*x^8 + 275990*a^4*b^3*x^6 + 33280*a^5*b^2*x^4 - 2560*a^6*b*x^2 + 51
2*a^7 - 45045*(b^7*x^15 + 5*a*b^6*x^13 + 10*a^2*b^5*x^11 + 10*a^3*b^4*x^9 +
 5*a^4*b^3*x^7 + a^5*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)
/(b*x^2 + a)))/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10*a^11*
b^2*x^9 + 5*a^12*b*x^7 + a^13*x^5), -1/1280*(45045*b^7*x^14 + 210210*a*b^6*
x^12 + 384384*a^2*b^5*x^10 + 338910*a^3*b^4*x^8 + 137995*a^4*b^3*x^6 + 1664
0*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256*a^7 + 45045*(b^7*x^15 + 5*a*b^6*x^13 +
 10*a^2*b^5*x^11 + 10*a^3*b^4*x^9 + 5*a^4*b^3*x^7 + a^5*b^2*x^5)*sqrt(b/a)*
arctan(x*sqrt(b/a)))/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10
*a^11*b^2*x^9 + 5*a^12*b*x^7 + a^13*x^5)]
```

Sympy [A]
time = 0.45, size = 221, normalized size = 1.41

$$\frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(-\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3}+x\right)}{512}-\frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3}+x\right)}{512}+\frac{-256a^7+1280a^6bx^2-16640a^5b^2x^4-137995a^4b^3x^6-338910a^3b^4x^8-384384a^2b^5x^{10}-210210ab^6x^{12}-45045b^7x^{14}}{1280a^{13}x^5+6400a^{12}bx^7+12800a^{11}b^2x^9+12800a^{10}b^3x^{11}+6400a^9b^4x^{13}+1280a^8b^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 9009*sqrt(-b**5/a**17)*log(-a**9*sqrt(-b**5/a**17)/b**3 + x)/512 - 9009*sqr
t(-b**5/a**17)*log(a**9*sqrt(-b**5/a**17)/b**3 + x)/512 + (-256*a**7 + 1280
*a**6*b*x**2 - 16640*a**5*b**2*x**4 - 137995*a**4*b**3*x**6 - 338910*a**3*b
**4*x**8 - 384384*a**2*b**5*x**10 - 210210*a*b**6*x**12 - 45045*b**7*x**14)
/(1280*a**13*x**5 + 6400*a**12*b*x**7 + 12800*a**11*b**2*x**9 + 12800*a**10
*b**3*x**11 + 6400*a**9*b**4*x**13 + 1280*a**8*b**5*x**15)
```

Giac [A]
time = 4.22, size = 115, normalized size = 0.73

$$-\frac{9009\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}\,a^8}-\frac{45045\,b^7x^{14}+210210\,ab^6x^{12}+384384\,a^2b^5x^{10}+338910\,a^3b^4x^8+137995\,a^4b^3x^6+16640\,a^5b^2x^4-1280\,a^6bx^2+256\,a^7}{1280\,(bx^3+ax)^5a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -9009/256*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8) - 1/1280*(45045*b^7*x^1
4 + 210210*a*b^6*x^12 + 384384*a^2*b^5*x^10 + 338910*a^3*b^4*x^8 + 137995*a
^4*b^3*x^6 + 16640*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256*a^7)/((b*x^3 + a*x)^5
*a^8)
```

Mupad [B]
time = 4.65, size = 158, normalized size = 1.01

$$-\frac{\frac{1}{5\,a}-\frac{b\,x^2}{a^2}+\frac{13\,b^2\,x^4}{a^3}+\frac{27599\,b^3\,x^6}{256\,a^4}+\frac{33891\,b^4\,x^8}{128\,a^5}+\frac{3003\,b^5\,x^{10}}{10\,a^6}+\frac{21021\,b^6\,x^{12}}{128\,a^7}+\frac{9009\,b^7\,x^{14}}{256\,a^8}}{a^5\,x^5+5\,a^4\,b\,x^7+10\,a^3\,b^2\,x^9+10\,a^2\,b^3\,x^{11}+5\,a\,b^4\,x^{13}+b^5\,x^{15}}-\frac{9009\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{17/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] - (1/(5*a) - (b*x^2)/a^2 + (13*b^2*x^4)/a^3 + (27599*b^3*x^6)/(256*a^4) + (
33891*b^4*x^8)/(128*a^5) + (3003*b^5*x^10)/(10*a^6) + (21021*b^6*x^12)/(128
*a^7) + (9009*b^7*x^14)/(256*a^8))/(a^5*x^5 + b^5*x^15 + 5*a^4*b*x^7 + 5*a*
b^4*x^13 + 10*a^3*b^2*x^9 + 10*a^2*b^3*x^11) - (9009*b^(5/2)*atan((b^(1/2)*
x)/a^(1/2)))/(256*a^(17/2))
```

3.535 $\int \frac{1}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=19

$$\frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 205, 209}

$$\frac{\text{ArcTan}(x)}{2}+\frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1+2x^2+x^4}\,dx = \int \frac{1}{(1+x^2)^2}\,dx$$
$$= \frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{x}{2(1+x^2)} + \frac{1}{2}\tan^{-1}(x)$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 0.84

$$\frac{1}{2}\left(\frac{x}{1+x^2}+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + x^4)^(-1),x]
[Out] (x/(1 + x^2) + ArcTan[x])/2
```

Maple [A]
time = 0.01, size = 16, normalized size = 0.84

method	result	size
default	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16
risch	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A]
time = 0.49, size = 15, normalized size = 0.79

$$\frac{x}{2(x^2+1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A]
time = 0.32, size = 19, normalized size = 1.00

$$\frac{(x^2+1)\arctan(x)+x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.63

$$\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*x**2+1),x)
[Out] x/(2*x**2 + 2) + atan(x)/2
```

Giac [A]
time = 4.10, size = 15, normalized size = 0.79

$$\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="giac")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.84

$$\frac{\operatorname{atan}(x)}{2}+\frac{x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + x^4 + 1),x)
[Out] atan(x)/2 + x/(2*(x^2 + 1))
```

3.536 $\int \frac{x}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2(1+x^2)}$$

[Out] -1/2/(x^2+1)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 267}

$$-\frac{1}{2(x^2+1)}$$

Antiderivative was successfully verified.

[In] Int[x/(1 + 2*x^2 + x^4),x]

[Out] -1/2*1/(1 + x^2)

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+2x^2+x^4}\,dx &= \int \frac{x}{(1+x^2)^2}\,dx\\ &= -\frac{1}{2(1+x^2)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-\frac{1}{2(1+x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/2*1/(1 + x^2)
```

Maple [A]
time = 0.02, size = 10, normalized size = 0.91

method	result	size
gosper	$-\frac{1}{2(x^2+1)}$	10
default	$-\frac{1}{2(x^2+1)}$	10
norman	$-\frac{1}{2(x^2+1)}$	10
risch	$-\frac{1}{2(x^2+1)}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(x^2+1)
```

Maxima [A]
time = 0.30, size = 9, normalized size = 0.82

$$-\frac{1}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2/(x^2 + 1)
```

Fricas [A]
time = 0.36, size = 9, normalized size = 0.82

$$-\frac{1}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2/(x^2 + 1)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.73

$$-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**2+1),x)
```

```
[Out] -1/(2*x**2 + 2)
```

Giac [A]
time = 3.88, size = 9, normalized size = 0.82

$$-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2/(x^2 + 1)
```

Mupad [B]
time = 0.02, size = 11, normalized size = 1.00

$$-\frac{1}{2\ \left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2 + x^4 + 1),x)
```

```
[Out] -1/(2*(x^2 + 1))
```

3.537 $\int \frac{x^2}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=19

$$-\frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Rubi [A]

time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 294, 209}

$$\frac{\text{ArcTan}(x)}{2}-\frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{1+2x^2+x^4}\,dx = \int \frac{x^2}{(1+x^2)^2}\,dx$$
$$= -\frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= -\frac{x}{2(1+x^2)} + \frac{1}{2}\tan^{-1}(x)$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$-\frac{x}{2(1+x^2)} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 2*x^2 + x^4),x]
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Maple [A]
time = 0.02, size = 16, normalized size = 0.84

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A]
time = 0.52, size = 15, normalized size = 0.79

$$-\frac{x}{2(x^2+1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A]
time = 0.38, size = 21, normalized size = 1.11

$$\frac{(x^2+1)\arctan(x)-x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+2*x**2+1),x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

Giac [A]
time = 3.69, size = 15, normalized size = 0.79

$$-\frac{x}{2\,(x^2+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}(x)}{2}-\frac{x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^2 + x^4 + 1),x)
```

```
[Out] atan(x)/2 - x/(2*(x^2 + 1))
```

3.538 $\int \frac{x^3}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2(1+x^2)}+\frac{1}{2}\log\left(1+x^2\right)$$

[Out] 1/2/(x^2+1)+1/2*ln(x^2+1)

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 272, 45}

$$\frac{1}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 + 2*x^2 + x^4),x]

[Out] 1/(2*(1 + x^2)) + Log[1 + x^2]/2

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1+2x^2+x^4}\,dx &= \int \frac{x^3}{(1+x^2)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)\,dx, x, x^2\right)\\ &= \frac{1}{2(1+x^2)}+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 0.82

$$\frac{1}{2}\left(\frac{1}{1+x^2}+\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 2*x^2 + x^4),x]
[Out] ((1 + x^2)^(-1) + Log[1 + x^2])/2
```

Maple [A]
time = 0.01, size = 19, normalized size = 0.86

method	result	size
default	$\frac{1}{2x^2+2}+\frac{\ln(x^2+1)}{2}$	19
norman	$\frac{1}{2x^2+2}+\frac{\ln(x^2+1)}{2}$	19
risch	$\frac{1}{2x^2+2}+\frac{\ln(x^2+1)}{2}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2/(x^2+1)+1/2*ln(x^2+1)
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.82

$$\frac{1}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] 1/2/(x^2 + 1) + 1/2*log(x^2 + 1)
```

Fricas [A]
time = 0.36, size = 23, normalized size = 1.05

$$\frac{(x^2+1)\log(x^2+1)+1}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*log(x^2 + 1) + 1)/(x^2 + 1)
```

Sympy [A]
time = 0.02, size = 15, normalized size = 0.68

$$\frac{\log(x^2+1)}{2}+\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4+2*x**2+1),x)
[Out] log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

Giac [A]
time = 3.86, size = 18, normalized size = 0.82

$$\frac{1}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4+2*x^2+1),x, algorithm="giac")
[Out] 1/2/(x^2 + 1) + 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.04, size = 18, normalized size = 0.82

$$\frac{\ln(x^2+1)}{2}+\frac{1}{2\ (x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*x^2 + x^4 + 1),x)
[Out] log(x^2 + 1)/2 + 1/(2*(x^2 + 1))
```

3.539 $\int \frac{x}{81-18x^2+x^4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{2(9-x^2)}$$

```
[Out] 1/2/(-x^2+9)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 267}

$$\frac{1}{2(9-x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(81 - 18*x^2 + x^4),x]
```

```
[Out] 1/(2*(9 - x^2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{81-18x^2+x^4}\,dx &= \int \frac{x}{(-9+x^2)^2}\,dx\\ &= \frac{1}{2(9-x^2)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 0.85

$$-\frac{1}{2(-9+x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(81 - 18*x^2 + x^4),x]
```

```
[Out] -1/2*1/(-9 + x^2)
```

Maple [A]
time = 0.03, size = 10, normalized size = 0.77

method	result	size
gosper	$-\frac{1}{2(x^2-9)}$	10
default	$-\frac{1}{2(x^2-9)}$	10
norman	$-\frac{1}{2(x^2-9)}$	10
risch	$-\frac{1}{2(x^2-9)}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4-18*x^2+81),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(x^2-9)
```

Maxima [A]
time = 0.28, size = 9, normalized size = 0.69

$$-\frac{1}{2(x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="maxima")
```

```
[Out] -1/2/(x^2 - 9)
```

Fricas [A]
time = 0.33, size = 9, normalized size = 0.69

$$-\frac{1}{2(x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="fricas")
```

```
[Out] -1/2/(x^2 - 9)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.62

$$-\frac{1}{2x^2-18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4-18*x**2+81),x)
[Out] -1/(2*x**2 - 18)
```

Giac [A]
time = 4.10, size = 9, normalized size = 0.69

$$-\frac{1}{2\left(x^{2}-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-18*x^2+81),x, algorithm="giac")
[Out] -1/2/(x^2 - 9)
```

Mupad [B]
time = 0.05, size = 11, normalized size = 0.85

$$-\frac{1}{2\,\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 - 18*x^2 + 81),x)
[Out] -1/(2*(x^2 - 9))
```

3.540 $\int \frac{x^3}{16-8x^2+x^4} \, dx$

Optimal. Leaf size=24

$$\frac{2}{4-x^2}+\frac{1}{2}\log\left(4-x^2\right)$$

```
[Out] 2/(-x^2+4)+1/2*ln(-x^2+4)
```

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 272, 45}

$$\frac{2}{4-x^2}+\frac{1}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(16 - 8*x^2 + x^4),x]
```

```
[Out] 2/(4 - x^2) + Log[4 - x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{16-8x^2+x^4}\,dx &= \int \frac{x^3}{(-4+x^2)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(-4+x)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{4}{(-4+x)^2}+\frac{1}{-4+x}\right)\,dx, x, x^2\right) \\
&= \frac{2}{4-x^2}+\frac{1}{2}\log\left(4-x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.83

$$-\frac{2}{-4+x^2}+\frac{1}{2}\log\left(-4+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(16 - 8*x^2 + x^4),x]
```

```
[Out] -2/(-4 + x^2) + Log[-4 + x^2]/2
```

Maple [A]
time = 0.01, size = 19, normalized size = 0.79

method	result	size
default	$-\frac{2}{x^2-4}+\frac{\ln(x^2-4)}{2}$	19
risch	$-\frac{2}{x^2-4}+\frac{\ln(x^2-4)}{2}$	19
norman	$-\frac{2}{x^2-4}+\frac{\ln(x-2)}{2}+\frac{\ln(x+2)}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4-8*x^2+16),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(x^2-4)+1/2*ln(x^2-4)
```

Maxima [A]
time = 0.29, size = 18, normalized size = 0.75

$$-\frac{2}{x^2-4}+\frac{1}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="maxima")
[Out] -2/(x^2 - 4) + 1/2*log(x^2 - 4)
```

Fricas [A]
time = 0.32, size = 23, normalized size = 0.96

$$\frac{(x^2-4)\log(x^2-4)-4}{2(x^2-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="fricas")
[Out] 1/2*((x^2 - 4)*log(x^2 - 4) - 4)/(x^2 - 4)
```

Sympy [A]
time = 0.02, size = 14, normalized size = 0.58

$$\frac{\log(x^2-4)}{2}-\frac{2}{x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4-8*x**2+16),x)
[Out] log(x**2 - 4)/2 - 2/(x**2 - 4)
```

Giac [A]
time = 3.88, size = 19, normalized size = 0.79

$$-\frac{2}{x^2-4}+\frac{1}{2}\log(|x^2-4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-8*x^2+16),x, algorithm="giac")
[Out] -2/(x^2 - 4) + 1/2*log(abs(x^2 - 4))
```

Mupad [B]
time = 4.23, size = 18, normalized size = 0.75

$$\frac{\ln(x^2-4)}{2}-\frac{2}{x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 - 8*x^2 + 16),x)
[Out] log(x^2 - 4)/2 - 2/(x^2 - 4)
```

3.541 $\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}$$

```
[Out] 1/6*a*x^6*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/8*b*x^8*((b*x^2+a)^2)^(1/2)/(b*x^
2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {1125, 660, 45}

$$\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}+\frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (b*x^8*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$
\begin{aligned}
\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^2(ab+b^2x)\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(abx^2+b^2x^3\right)\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(4ax^6+3bx^8\right)}{24\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(4*a*x^6 + 3*b*x^8))/(24*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.46

method	result	size
gosper	$\frac{x^6\left(3b\,x^2+4a\right)\sqrt{\left(b\,x^2+a\right)^2}}{24b\,x^2+24a}$	36
default	$\frac{x^6\left(3b\,x^2+4a\right)\sqrt{\left(b\,x^2+a\right)^2}}{24b\,x^2+24a}$	36
risch	$\frac{a\,x^6\sqrt{\left(b\,x^2+a\right)^2}}{6b\,x^2+6a}+\frac{b\,x^8\sqrt{\left(b\,x^2+a\right)^2}}{8b\,x^2+8a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/24*x^6*(3*b*x^2+4*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^8+\frac{1}{6}\,ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/8*b*x^8 + 1/6*a*x^6
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^8+\frac{1}{6}\,ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/8*b*x^8 + 1/6*a*x^6
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.15

$$\frac{ax^6}{6}+\frac{bx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*((b*x**2+a)**2)**(1/2),x)
[Out] a*x**6/6 + b*x**8/8
```

Giac [A]
time = 3.60, size = 29, normalized size = 0.37

$$\frac{1}{8}\,bx^8\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{6}\,ax^6\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/8*b*x^8*sgn(b*x^2 + a) + 1/6*a*x^6*sgn(b*x^2 + a)
```

Mupad [B]
time = 4.45, size = 71, normalized size = 0.90

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(a^3-4\,a^2\,b\,x^2-5\,a\,b^2\,x^4+3\,b\,x^2\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)\right)}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a^3 - 4*a^2*b*x^2 - 5*a*b^2*x^4 + 3*b*x
^2*(a^2 + b^2*x^4 + 2*a*b*x^2)))/(24*b^3)
```

3.542 $\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=67

$$-\frac{a(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}+\frac{(a^2+2abx^2+b^2x^4)^{3/2}}{6b^2}$$

```
[Out] 1/6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/b^2-1/4*a*(b*x^2+a)*((b*x^2+a)^2)^(1/2)/b
^2
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 654, 623}

$$\frac{(a^2+2abx^2+b^2x^4)^{3/2}}{6b^2}-\frac{a(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/4*(a*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^2 + (a^2 + 2*a*b*x^2
 + b^2*x^4)^(3/2)/(6*b^2)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)$$
$$= \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{6b^2} - \frac{a\text{Subst}\left(\int\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)}{2b}$$
$$= -\frac{a(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2} + \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{6b^2}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.58

$$\frac{\sqrt{(a+bx^2)^2}\,(3ax^4+2bx^6)}{12\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x^4 + 2*b*x^6))/(12*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.54

method	result	size
gosper	$\frac{x^4(2b\,x^2+3a)\sqrt{(b\,x^2+a)^2}}{12b\,x^2+12a}$	36
default	$\frac{x^4(2b\,x^2+3a)\sqrt{(b\,x^2+a)^2}}{12b\,x^2+12a}$	36
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,b\,x^6}{6b\,x^2+6a}+\frac{\sqrt{(b\,x^2+a)^2}\,a\,x^4}{4b\,x^2+4a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^4*(2*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A]
time = 0.30, size = 13, normalized size = 0.19

$$\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Fricas [A]
time = 0.35, size = 13, normalized size = 0.19

$$\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.18

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

Giac [A]
time = 3.12, size = 23, normalized size = 0.34

$$\frac{1}{12}\left(2\,bx^6+3\,ax^4\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*b*x^6 + 3*a*x^4)*sgn(b*x^2 + a)
```

Mupad [B]
time = 4.31, size = 59, normalized size = 0.88

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(8\,b^2\,\left(a^2+b^2\,x^4\right)-12\,a^2\,b^2+4\,a\,b^3\,x^2\right)}{48\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(8*b^2*(a^2 + b^2*x^4) - 12*a^2*b^2 + 4*
a*b^3*x^2))/(48*b^4)
```

3.543 $\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

```
[Out] 1/4*(b*x^2+a)*((b*x^2+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1121, 623}

$$\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)\\ &= \frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.06

$$\frac{\sqrt{(a+bx^2)^2}\,(2ax^2+bx^4)}{4\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(2*a*x^2 + b*x^4))/(4*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 24, normalized size = 0.67

method	result	size
default	$\frac{\left(b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{4b}$	24
risch	$\frac{\left(b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{4b}$	24
gosper	$\frac{x^{2}\left(b\,x^{2}+2a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{4b\,x^{2}+4a}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(b*x^2+a)*((b*x^2+a)^2)^(1/2)/b
```

Maxima [A]
time = 0.30, size = 14, normalized size = 0.39

$$\frac{\left(bx^{2}+a\right)^{2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/4*(b*x^2 + a)^2/b
```

Fricas [A]
time = 0.36, size = 13, normalized size = 0.36

$$\frac{1}{4}\,bx^{4}+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.33

$$\frac{ax^{2}}{2}+\frac{bx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

Giac [A]
time = 3.13, size = 22, normalized size = 0.61

$$\frac{1}{4}\left(bx^4 + 2\,ax^2\right)\operatorname{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(b*x^4 + 2*a*x^2)*sgn(b*x^2 + a)
```

Mupad [B]
time = 4.35, size = 33, normalized size = 0.92

$$\left(\frac{a}{4\,b}+\frac{x^2}{4}\right)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] (a/(4*b) + x^2/4)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
```

3.544 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

Optimal. Leaf size=75

$$\frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,\log(x)}{a+bx^2}$$

```
[Out] 1/2*b*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+
a)
```

Rubi [A]

time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{1126, 14\}$

$$\frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{a\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
```

```
[Out] (b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (a*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x}\,dx}{ab+b^2x^2}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x}+b^2x\right)\,dx}{ab+b^2x^2}\\ &= \frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,\log(x)}{a+bx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.49

$$\frac{\sqrt{(a+bx^2)^2}\,(bx^2+2a\log(x))}{2\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2 + 2*a*Log[x]))/(2*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.45

method	result	size
default	$\frac{\sqrt{(b\,x^2+a)^2}\,(b\,x^2+2a\ln(x))}{2b\,x^2+2a}$	34
risch	$\frac{b\,x^2\sqrt{(b\,x^2+a)^2}}{2b\,x^2+2a}+\frac{a\ln(x)\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(b*x^2+2*a*ln(x))/(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.19

$$\frac{1}{2}\,bx^2+\frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

Fricas [A]
time = 0.33, size = 11, normalized size = 0.15

$$\frac{1}{2}\,bx^2+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.13

$$a\log(x) + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x,x)
[Out] a*log(x) + b*x**2/2
```

Giac [A]
time = 3.31, size = 30, normalized size = 0.40

$$\frac{1}{2}\,bx^2\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{2}\,a\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
[Out] 1/2*b*x^2*sgn(b*x^2 + a) + 1/2*a*log(x^2)*sgn(b*x^2 + a)
```

Mupad [B]
time = 4.39, size = 109, normalized size = 1.45

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2}-\frac{\ln\left(a\,b+\frac{a^2}{x^2}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^2}\right)\,\sqrt{a^2}}{2}+\frac{a\,b\,\ln\left(a\,b+\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{b^2}+b^2\,x^2\right)}{2\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x,x)
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/2 - (log(a*b + a^2/x^2 + ((a^2)^(1/2)*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/x^2)*(a^2)^(1/2))/2 + (a*b*log(a*b + ((a +
b*x^2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(1/2))
```

3.545 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal. Leaf size=75

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,\log(x)}{a+bx^2}$$

```
[Out] -1/2*a*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+b*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2
+a)
```

Rubi [A]
time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 14}

$$\frac{b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (b*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^3}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^3}+\frac{b^2}{x}\right)\,dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\,(a+bx^2)} + \frac{b\sqrt{a^2+2abx^2+b^2x^4}\,\log(x)}{a+bx^2}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.52

$$-\frac{\sqrt{(a+bx^2)^2}\,(a-2bx^2\log(x))}{2x^2\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
[Out] -1/2*(Sqrt[(a + b*x^2)^2]*(a - 2*b*x^2*Log[x]))/(x^2*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 38, normalized size = 0.51

method	result	size
default	$\frac{\sqrt{(b\,x^2+a)^2}\,(2b\ln(x)x^2-a)}{2x^2(b\,x^2+a)}$	38
risch	$-\frac{a\sqrt{(b\,x^2+a)^2}}{2x^2(b\,x^2+a)}+\frac{b\ln(x)\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(2*b*ln(x)*x^2-a)/x^2/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.19

$$\frac{1}{2}\,b\log\left(x^2\right) - \frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*log(x^2) - 1/2*a/x^2
```

Fricas [A]
time = 0.35, size = 17, normalized size = 0.23

$$\frac{2\,bx^2\log\left(x\right)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*log(x) - a)/x^2
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.13

$$-\frac{a}{2x^2}+b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x)
```

Giac [A]
time = 3.60, size = 45, normalized size = 0.60

$$\frac{1}{2}\,b\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{bx^2\mathrm{sgn}(bx^2+a)+a\mathrm{sgn}(bx^2+a)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*log(x^2)*sgn(b*x^2 + a) - 1/2*(b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a
))/x^2
```

Mupad [B]
time = 4.45, size = 112, normalized size = 1.49

$$\frac{\ln\left(a\,b+\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{b^2}+b^2\,x^2\right)\,\sqrt{b^2}}{2}-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^2}-\frac{a\,b\,\ln\left(a\,b+\frac{a^2}{x^2}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^2}\right)}{2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^3,x)
```

```
[Out] (log(a*b + ((a + b*x^2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2)*(b^2)^(1/2))/2 - (a
^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*x^2) - (a*b*log(a*b + a^2/x^2 + ((a^2)^(
1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/x^2))/(2*(a^2)^(1/2))
```

3.546 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5} dx$

Optimal. Leaf size=39

$$-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

```
[Out] -1/4*(b*x^2+a)*((b*x^2+a)^2)^(1/2)/a/x^4
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 37}

$$-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
```

```
[Out] -1/4*((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^3}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{ab+b^2x}{x^3}\,dx, x, x^2\right)}{2(ab+b^2x^2)}$$
$$= -\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.95

$$-\frac{\sqrt{(a+bx^2)^2}\,(a+2bx^2)}{4x^4(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
[Out] -1/4*(Sqrt[(a + b*x^2)^2]*(a + 2*b*x^2))/(x^4*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.87

method	result	size
gosper	$-\frac{(2bx^2+a)\sqrt{(bx^2+a)^2}}{4x^4(bx^2+a)}$	34
default	$-\frac{(2bx^2+a)\sqrt{(bx^2+a)^2}}{4x^4(bx^2+a)}$	34
risch	$\frac{\left(-\frac{bx^2}{2}-\frac{a}{4}\right)\sqrt{(bx^2+a)^2}}{x^4(bx^2+a)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*(2*b*x^2+a)*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.33

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="maxima")
[Out] -1/4*(2*b*x^2 + a)/x^4
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.33

$$-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="fricas")
[Out] -1/4*(2*b*x^2 + a)/x^4
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.36

$$\frac{-a-2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**5,x)
[Out] (-a - 2*b*x**2)/(4*x**4)
```

Giac [A]
time = 4.26, size = 30, normalized size = 0.77

$$-\frac{2\,bx^2\mathrm{sgn}(bx^2+a)+a\mathrm{sgn}(bx^2+a)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="giac")
[Out] -1/4*(2*b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a))/x^4
```

Mupad [B]
time = 4.21, size = 33, normalized size = 0.85

$$-\frac{\left(2\,b\,x^2+a\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{4\,x^4\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^5,x)
[Out] -((a + 2*b*x^2)*((a + b*x^2)^2)^(1/2))/(4*x^4*(a + b*x^2))
```

3.547 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx$

Optimal. Leaf size=72

$$-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}+\frac{(a^2+2abx^2+b^2x^4)^{3/2}}{12a^2x^6}$$

```
[Out] 1/12*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/a^2/x^6-1/4*(b*x^2+a)*((b*x^2+a)^2)^(1/2
)/a/x^6
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1124}

$$\frac{(a^2+2abx^2+b^2x^4)^{3/2}}{12a^2x^6}-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
```

```
[Out] -1/4*((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a*x^6) + (a^2 + 2*a*b*x
^2 + b^2*x^4)^(3/2)/(12*a^2*x^6)
```

Rule 1124

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(4*a*d*(p + 1)*(2*p + 1
))), x] - Simp[(d*x)^(m + 1)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^p/(4*a*d*(2
*p + 1))), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx=-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}+\frac{(a^2+2abx^2+b^2x^4)^{3/2}}{12a^2x^6}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.54

$$-\frac{\sqrt{(a+bx^2)^2}\,(2a+3bx^2)}{12x^6\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
[Out] -1/12*(Sqrt[(a + b*x^2)^2]*(2*a + 3*b*x^2))/(x^6*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.50

method	result	size
risch	$\frac{\left(-\frac{b\,x^{2}}{4}-\frac{a}{6}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x^{6}\left(b\,x^{2}+a\right)}$	35
gosper	$-\frac{\left(3b\,x^{2}+2a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{12x^{6}\left(b\,x^{2}+a\right)}$	36
default	$-\frac{\left(3b\,x^{2}+2a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{12x^{6}\left(b\,x^{2}+a\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/12*(3*b*x^2+2*a)*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.21

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="maxima")
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

Fricas [A]
time = 0.32, size = 15, normalized size = 0.21

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="fricas")
[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

Sympy [**A**]
time = 0.05, size = 15, normalized size = 0.21

$$\frac{-2a - 3bx^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**7,x)
```

```
[Out] (-2*a - 3*b*x**2)/(12*x**6)
```

Giac [**A**]
time = 4.32, size = 31, normalized size = 0.43

$$-\frac{3\,bx^2\mathrm{sgn}(bx^2+a)+2\,a\mathrm{sgn}(bx^2+a)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(3*b*x^2*sgn(b*x^2 + a) + 2*a*sgn(b*x^2 + a))/x^6
```

Mupad [**B**]
time = 4.24, size = 35, normalized size = 0.49

$$-\frac{(3\,b\,x^2+2\,a)\,\sqrt{{(b\,x^2+a)}^2}}{12\,x^6\,(b\,x^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^7,x)
```

```
[Out] -((2*a + 3*b*x^2)*((a + b*x^2)^2)^(1/2))/(12*x^6*(a + b*x^2))
```

3.548 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\,(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\,(a+bx^2)}$$

```
[Out] -1/8*a*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)-1/6*b*((b*x^2+a)^2)^(1/2)/x^6/(b*x
^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1125, 660, 45}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\,(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

```
[Out] -1/8*(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^8*(a + b*x^2)) - (b*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^5}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{ab+b^2x}{x^5}\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{ab}{x^5}+\frac{b^2}{x^4}\right)\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 39, normalized size = 0.49

$$
-\frac{\sqrt{\left(a+bx^2\right)^2}\,\left(3a+4bx^2\right)}{24x^8\left(a+bx^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

```
[Out] -1/24*(Sqrt[(a + b*x^2)^2]*(3*a + 4*b*x^2))/(x^8*(a + b*x^2))
```

Maple [**A**]
time = 0.02, size = 36, normalized size = 0.46

method	result	size
risch	$\frac{\left(-\frac{b\,x^2}{6}-\frac{a}{8}\right)\sqrt{\left(b\,x^2+a\right)^2}}{x^8\left(b\,x^2+a\right)}$	35
gosper	$-\frac{\left(4b\,x^2+3a\right)\sqrt{\left(b\,x^2+a\right)^2}}{24x^8\left(b\,x^2+a\right)}$	36
default	$-\frac{\left(4b\,x^2+3a\right)\sqrt{\left(b\,x^2+a\right)^2}}{24x^8\left(b\,x^2+a\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(4*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)
```

Maxima [A]
time = 0.31, size = 15, normalized size = 0.19

$$-\frac{4\,bx^2+3\,a}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/24*(4*b*x^2 + 3*a)/x^8
```

Fricas [A]
time = 0.32, size = 15, normalized size = 0.19

$$-\frac{4\,bx^2+3\,a}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(4*b*x^2 + 3*a)/x^8
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.19

$$\frac{-3a-4bx^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**9,x)
```

```
[Out] (-3*a - 4*b*x**2)/(24*x**8)
```

Giac [A]
time = 4.31, size = 31, normalized size = 0.39

$$-\frac{4\,bx^2\mathrm{sgn}(bx^2+a)+3\,a\mathrm{sgn}(bx^2+a)}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/24*(4*b*x^2*sgn(b*x^2 + a) + 3*a*sgn(b*x^2 + a))/x^8
```

Mupad [B]
time = 4.24, size = 35, normalized size = 0.44

$$-\frac{\left(4\,b\,x^2+3\,a\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{24\,x^8\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^9,x)
[Out] -((3*a + 4*b*x^2)*((a + b*x^2)^2)^(1/2))/(24*x^8*(a + b*x^2))
```

3.549 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

```
[Out] -1/10*a*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)-1/8*b*((b*x^2+a)^2)^(1/2)/x^8/(b
*x^2+a)
```

Rubi [A]

time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

```
[Out] -1/10*(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^10*(a + b*x^2)) - (b*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^6}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{ab+b^2x}{x^6}\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{ab}{x^6}+\frac{b^2}{x^5}\right)\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 39, normalized size = 0.49

$$
-\frac{\sqrt{\left(a+bx^2\right)^2}\,\left(4a+5bx^2\right)}{40x^{10}\left(a+bx^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

```
[Out] -1/40*(Sqrt[(a + b*x^2)^2]*(4*a + 5*b*x^2))/(x^10*(a + b*x^2))
```

Maple [**A**]
time = 0.02, size = 36, normalized size = 0.46

method	result	size
risch	$\frac{\left(-\frac{b\,x^2}{8}-\frac{a}{10}\right)\sqrt{\left(b\,x^2+a\right)^2}}{x^{10}\left(b\,x^2+a\right)}$	35
gosper	$-\frac{\left(5b\,x^2+4a\right)\sqrt{\left(b\,x^2+a\right)^2}}{40x^{10}\left(b\,x^2+a\right)}$	36
default	$-\frac{\left(5b\,x^2+4a\right)\sqrt{\left(b\,x^2+a\right)^2}}{40x^{10}\left(b\,x^2+a\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(5*b*x^2+4*a)*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.19

$$-\frac{5\,bx^2+4\,a}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/40*(5*b*x^2 + 4*a)/x^10
```

Fricas [A]
time = 0.36, size = 15, normalized size = 0.19

$$-\frac{5\,bx^2+4\,a}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/40*(5*b*x^2 + 4*a)/x^10
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.19

$$\frac{-4a-5bx^2}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**11,x)
```

```
[Out] (-4*a - 5*b*x**2)/(40*x**10)
```

Giac [A]
time = 4.52, size = 31, normalized size = 0.39

$$-\frac{5\,bx^2\mathrm{sgn}(bx^2+a)+4\,a\mathrm{sgn}(bx^2+a)}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/40*(5*b*x^2*sgn(b*x^2 + a) + 4*a*sgn(b*x^2 + a))/x^10
```

Mupad [B]
time = 4.21, size = 35, normalized size = 0.44

$$-\frac{\left(5\,b\,x^2+4\,a\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{40\,x^{10}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^11,x)
[Out] -((4*a + 5*b*x^2)*((a + b*x^2)^2)^(1/2))/(40*x^10*(a + b*x^2))
```

3.550 $\int x^4 \sqrt{a^2 + 2abx^2 + b^2x^4} \, dx$

Optimal. Leaf size=79

$$\frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}+\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}$$

```
[Out] 1/5*a*x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/7*b*x^7*((b*x^2+a)^2)^(1/2)/(b*x^
2+a)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 14}

$$\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (b*x^7*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^4(ab+b^2x^2)\,dx}{ab+b^2x^2}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (abx^4+b^2x^6)\,dx}{ab+b^2x^2}\\
&= \frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}+\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a+bx^2)^2}\,(7ax^5+5bx^7)}{35\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(7*a*x^5 + 5*b*x^7))/(35*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.46

method	result	size
gosper	$\frac{x^{5}\left(5b\,x^{2}+7a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{35b\,x^{2}+35a}$	36
default	$\frac{x^{5}\left(5b\,x^{2}+7a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{35b\,x^{2}+35a}$	36
risch	$\frac{a\,x^{5}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{5b\,x^{2}+5a}+\frac{b\,x^{7}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{7b\,x^{2}+7a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/35*x^5*(5*b*x^2+7*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.16

$$\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

Fricas [A]
time = 0.35, size = 13, normalized size = 0.16

$$\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.15

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**5/5 + b*x**7/7
```

Giac [A]
time = 5.98, size = 29, normalized size = 0.37

$$\frac{1}{7}\,bx^7 \mathrm{sgn}\left(bx^2 + a\right) + \frac{1}{5}\,ax^5 \mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7*b*x^7*sgn(b*x^2 + a) + 1/5*a*x^5*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^4*((a + b*x^2)^2)^(1/2), x)
```

3.551 $\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

```
[Out] 1/3*a*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/5*b*x^5*((b*x^2+a)^2)^(1/2)/(b*x^
2+a)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1126, 14}

$$\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b*x^5*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx&=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2\left(ab+b^2x^2\right)\,dx}{ab+b^2x^2}\\&=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(abx^2+b^2x^4\right)\,dx}{ab+b^2x^2}\\&=\frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a+bx^2)^2}\,(5ax^3+3bx^5)}{15\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(5*a*x^3 + 3*b*x^5))/(15*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.46

method	result	size
gosper	$\frac{x^3(3b\,x^2+5a)\sqrt{\left(b\,x^2+a\right)^2}}{15b\,x^2+15a}$	36
default	$\frac{x^3(3b\,x^2+5a)\sqrt{\left(b\,x^2+a\right)^2}}{15b\,x^2+15a}$	36
risch	$\frac{a\,x^3\sqrt{\left(b\,x^2+a\right)^2}}{3b\,x^2+3a}+\frac{b\,x^5\sqrt{\left(b\,x^2+a\right)^2}}{5b\,x^2+5a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*x^3*(3*b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.16

$$\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A]
time = 0.35, size = 13, normalized size = 0.16

$$\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.15

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

Giac [A]
time = 3.66, size = 29, normalized size = 0.37

$$\frac{1}{5}\,bx^5\mathrm{sgn}\left(bx^2 + a\right) + \frac{1}{3}\,ax^3\mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5*sgn(b*x^2 + a) + 1/3*a*x^3*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^2*((a + b*x^2)^2)^(1/2), x)
```

3.552 $\int \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=74

$$\frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}$$

```
[Out] a*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*b*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1102}

$$\frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*x^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rule 1102

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (2ab+2b^2x^2)\,dx}{2ab+2b^2x^2}\\ &= \frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 36, normalized size = 0.49

$$\frac{\sqrt{(a+bx^2)^2}\,(3ax+bx^3)}{3\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x + b*x^3))/(3*(a + b*x^2))
```

Maple [A]
time = 0.01, size = 33, normalized size = 0.45

method	result	size
gosper	$\frac{x(b\,x^2+3a)\sqrt{(b\,x^2+a)^2}}{3b\,x^2+3a}$	33
default	$\frac{x(b\,x^2+3a)\sqrt{(b\,x^2+a)^2}}{3b\,x^2+3a}$	33
risch	$\frac{ax\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}+\frac{b\,x^3\sqrt{(b\,x^2+a)^2}}{3b\,x^2+3a}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.14

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*b*x^3 + a*x
```

Fricas [A]
time = 0.39, size = 10, normalized size = 0.14

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*b*x^3 + a*x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.11

$$ax+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x + b*x**3/3
```

Giac [A]
time = 6.43, size = 20, normalized size = 0.27

$$\frac{1}{3}\left(bx^3 + 3\,ax\right)\mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x^3 + 3*a*x)*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(((a + b*x^2)^2)^(1/2), x)
```

3.553 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal. Leaf size=72

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] -a*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+b*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1126, 14}

$$\frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (b*x*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^2}\,dx}{ab+b^2x^2}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^2+\frac{ab}{x^2}\right)\,dx}{ab+b^2x^2}\\ &= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.49

$$\frac{(-a+bx^2)\sqrt{(a+bx^2)^2}}{x\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
```

```
[Out] ((-a + b*x^2)*Sqrt[(a + b*x^2)^2])/(x*(a + b*x^2))
```

Maple [A]
time = 0.03, size = 34, normalized size = 0.47

method	result	size
gosper	$-\frac{(-b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{x(b\,x^2+a)}$	34
default	$-\frac{(-b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{x(b\,x^2+a)}$	34
risch	$-\frac{a\sqrt{(b\,x^2+a)^2}}{x(b\,x^2+a)}+\frac{bx\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-b*x^2+a)*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.14

$$bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] b*x - a/x
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.18

$$\frac{bx^2-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (b*x^2 - a)/x
```

Sympy [**A**]
time = 0.02, size = 5, normalized size = 0.07

$$-\frac{a}{x} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] -a/x + b*x
```

Giac [**A**]
time = 4.02, size = 26, normalized size = 0.36

$$bx\mathrm{sgn}\left(bx^2 + a\right) - \frac{a\mathrm{sgn}(bx^2 + a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] b*x*sgn(b*x^2 + a) - a*sgn(b*x^2 + a)/x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{{\left(b\,x^2+a\right)}^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^2,x)
```

```
[Out] int(((a + b*x^2)^2)^(1/2)/x^2, x)
```

3.554 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$

Optimal. Leaf size=77

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\,(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\,(a+bx^2)}$$

```
[Out] -1/3*a*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-b*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)
```

Rubi [A]
time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$,
Rules used = {1126, 14}

$$-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\,(a+bx^2)}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
```

```
[Out] -1/3*(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (b*Sqrt[a^2 + 
2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^4}\,dx}{ab+b^2x^2}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^4}+\frac{b^2}{x^2}\right)\,dx}{ab+b^2x^2}\\
&= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\,(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\,(a+bx^2)}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.48

$$-\frac{\sqrt{(a+bx^2)^2}\,(a+3bx^2)}{3x^3\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
```

```
[Out] -1/3*(Sqrt[(a + b*x^2)^2]*(a + 3*b*x^2))/(x^3*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.44

method	result	size
gosper	$-\frac{(3b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{3(b\,x^2+a)x^3}$	34
default	$-\frac{(3b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{3(b\,x^2+a)x^3}$	34
risch	$\frac{(-b\,x^2-\frac{a}{3})\sqrt{(b\,x^2+a)^2}}{x^3(b\,x^2+a)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(3*b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/x^3
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.17

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.17

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*x^2 + a)/x^3
```

Sympy [**A**]
time = 0.04, size = 14, normalized size = 0.18

$$\frac{-a-3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**4,x)
```

```
[Out] (-a - 3*b*x**2)/(3*x**3)
```

Giac [**A**]
time = 3.96, size = 30, normalized size = 0.39

$$-\frac{3\,bx^2\mathrm{sgn}(bx^2+a)+a\mathrm{sgn}(bx^2+a)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a))/x^3
```

Mupad [**B**]
time = 4.24, size = 33, normalized size = 0.43

$$-\frac{\left(3\,b\,x^2+a\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{3\,x^3\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^4,x)
```

```
[Out] -((a + 3*b*x^2)*((a + b*x^2)^2)^(1/2))/(3*x^3*(a + b*x^2))
```

3.555 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

```
[Out] -1/5*a*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-1/3*b*((b*x^2+a)^2)^(1/2)/x^3/(b*x
^2+a)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{1126, 14\}$

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
```

```
[Out] -1/5*(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (b*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^6}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^6}+\frac{b^2}{x^4}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\,(a+bx^2)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\,(a+bx^2)}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a+bx^2)^2}\,(3a+5bx^2)}{15x^5\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
[Out] -1/15*(Sqrt[(a + b*x^2)^2]*(3*a + 5*b*x^2))/(x^5*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.46

method	result	size
risch	$\frac{\left(-\frac{b\,x^2}{3}-\frac{a}{5}\right)\sqrt{\left(b\,x^2+a\right)^2}}{x^5\left(b\,x^2+a\right)}$	35
gosper	$-\frac{\left(5b\,x^2+3a\right)\sqrt{\left(b\,x^2+a\right)^2}}{15\left(b\,x^2+a\right)x^5}$	36
default	$-\frac{\left(5b\,x^2+3a\right)\sqrt{\left(b\,x^2+a\right)^2}}{15\left(b\,x^2+a\right)x^5}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
[Out] -1/15*(5*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/x^5
```

Maxima [A]
time = 0.29, size = 15, normalized size = 0.19

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

Fricas [A]
time = 0.41, size = 15, normalized size = 0.19

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.19

$$\frac{-3a-5bx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**6,x)
```

```
[Out] (-3*a - 5*b*x**2)/(15*x**5)
```

Giac [A]
time = 4.22, size = 31, normalized size = 0.39

$$-\frac{5\,bx^2\mathrm{sgn}(bx^2+a)+3\,a\mathrm{sgn}(bx^2+a)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(5*b*x^2*sgn(b*x^2 + a) + 3*a*sgn(b*x^2 + a))/x^5
```

Mupad [B]
time = 4.21, size = 35, normalized size = 0.44

$$-\frac{\left(5\,b\,x^2+3\,a\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{15\,x^5\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^6,x)
```

```
[Out] -((3*a + 5*b*x^2)*((a + b*x^2)^2)^(1/2))/(15*x^5*(a + b*x^2))
```

3.556 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

```
[Out] -1/7*a*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-1/5*b*((b*x^2+a)^2)^(1/2)/x^5/(b*x
^2+a)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1126, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
```

```
[Out] -1/7*(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^7*(a + b*x^2)) - (b*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^8}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^8}+\frac{b^2}{x^6}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a+bx^2)^2}\,(5a+7bx^2)}{35x^7(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
[Out] -1/35*(Sqrt[(a + b*x^2)^2]*(5*a + 7*b*x^2))/(x^7*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.46

method	result	size
risch	$\frac{\left(-\frac{b\,x^2}{5}-\frac{a}{7}\right)\sqrt{\left(b\,x^2+a\right)^2}}{x^7\left(b\,x^2+a\right)}$	35
gosper	$-\frac{\left(7b\,x^2+5a\right)\sqrt{\left(b\,x^2+a\right)^2}}{35x^7\left(b\,x^2+a\right)}$	36
default	$-\frac{\left(7b\,x^2+5a\right)\sqrt{\left(b\,x^2+a\right)^2}}{35x^7\left(b\,x^2+a\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/35*(7*b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.19

$$-\frac{7\,bx^2+5\,a}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/35*(7*b*x^2 + 5*a)/x^7
```

Fricas [A]
time = 0.34, size = 15, normalized size = 0.19

$$-\frac{7\,bx^2+5\,a}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*(7*b*x^2 + 5*a)/x^7
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.19

$$\frac{-5a-7bx^2}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**8,x)
```

```
[Out] (-5*a - 7*b*x**2)/(35*x**7)
```

Giac [A]
time = 4.21, size = 31, normalized size = 0.39

$$-\frac{7\,bx^2\mathrm{sgn}(bx^2+a)+5\,a\mathrm{sgn}(bx^2+a)}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] -1/35*(7*b*x^2*sgn(b*x^2 + a) + 5*a*sgn(b*x^2 + a))/x^7
```

Mupad [B]
time = 4.18, size = 35, normalized size = 0.44

$$-\frac{(7\,b\,x^2+5\,a)\,\sqrt{{(b\,x^2+a)}^2}}{35\,x^7\,(b\,x^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^8,x)
```

```
[Out] -((5*a + 7*b*x^2)*((a + b*x^2)^2)^(1/2))/(35*x^7*(a + b*x^2))
```

3.557 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

```
[Out] -1/9*a*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-1/7*b*((b*x^2+a)^2)^(1/2)/x^7/(b*x
^2+a)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1126, 14}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
```

```
[Out] -1/9*(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^9*(a + b*x^2)) - (b*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{x^{10}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{x^{10}}+\frac{b^2}{x^8}\right)dx}{ab+b^2x^2}$$
$$= -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a+bx^2)^2}\,(7a+9bx^2)}{63x^9(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
[Out] -1/63*(Sqrt[(a + b*x^2)^2]*(7*a + 9*b*x^2))/(x^9*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.46

method	result	size
risch	$\frac{\left(-\frac{b\,x^2}{7}-\frac{a}{9}\right)\sqrt{\left(b\,x^2+a\right)^2}}{x^9\left(b\,x^2+a\right)}$	35
gosper	$-\frac{\left(9b\,x^2+7a\right)\sqrt{\left(b\,x^2+a\right)^2}}{63x^9\left(b\,x^2+a\right)}$	36
default	$-\frac{\left(9b\,x^2+7a\right)\sqrt{\left(b\,x^2+a\right)^2}}{63x^9\left(b\,x^2+a\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/x^10,x,method=_RETURNVERBOSE)
[Out] -1/63*(9*b*x^2+7*a)*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.19

$$-\frac{9\,bx^2+7\,a}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/63*(9*b*x^2 + 7*a)/x^9
```

Fricas [A]
time = 0.37, size = 15, normalized size = 0.19

$$-\frac{9\,bx^2+7\,a}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/63*(9*b*x^2 + 7*a)/x^9
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.19

$$\frac{-7a-9bx^2}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/x**10,x)
```

```
[Out] (-7*a - 9*b*x**2)/(63*x**9)
```

Giac [A]
time = 4.03, size = 31, normalized size = 0.39

$$-\frac{9\,bx^2\mathrm{sgn}(bx^2+a)+7\,a\mathrm{sgn}(bx^2+a)}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/63*(9*b*x^2*sgn(b*x^2 + a) + 7*a*sgn(b*x^2 + a))/x^9
```

Mupad [B]
time = 4.20, size = 35, normalized size = 0.44

$$-\frac{(9\,b\,x^2+7\,a)\,\sqrt{(b\,x^2+a)^2}}{63\,x^9\,(b\,x^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/x^10,x)
```

```
[Out] -((7*a + 9*b*x^2)*((a + b*x^2)^2)^(1/2))/(63*x^9*(a + b*x^2))
```

3.558 $\int x^9(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\,(a+bx^2)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\,(a+bx^2)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\,(a+bx^2)}+\frac{b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\,(a+bx^2)}$$

```
[Out] 1/10*a^3*x^10*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/4*a^2*b*x^12*((b*x^2+a)^2)^(1
/2)/(b*x^2+a)+3/14*a*b^2*x^14*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/16*b^3*x^16*(
(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.08, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\,(a+bx^2)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\,(a+bx^2)}+\frac{b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\,(a+bx^2)}+\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a^2*b*x^12*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a*b^2*x^14*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(16*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^9\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{1}{2}\mathrm{Subst}\left(\int x^4\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int x^4\left(ab+b^2x\right)^3\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int\left(a^3b^3x^4+3a^2b^4x^5+3ab^5x^6+b^6x^7\right)\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$=\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3ab^2x^{14}\sqrt{a^2}}{14\left(a\right.}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{x^{10}\sqrt{\left(a+bx^2\right)^2}\left(56a^3+140a^2bx^2+120ab^2x^4+35b^3x^6\right)}{560\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^2)^2]*(56*a^3 + 140*a^2*b*x^2 + 120*a*b^2*x^4 + 35*b^3*
x^6))/(560*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^{10}\left(35b^3x^6+120a\,b^2x^4+140a^2b\,x^2+56a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{560\left(b\,x^2+a\right)^3}$	58
default	$\frac{x^{10}\left(35b^3x^6+120a\,b^2x^4+140a^2b\,x^2+56a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{560\left(b\,x^2+a\right)^3}$	58
risch	$\frac{a^3x^{10}\sqrt{\left(b\,x^2+a\right)^2}}{10b\,x^2+10a}+\frac{a^2b\,x^{12}\sqrt{\left(b\,x^2+a\right)^2}}{4b\,x^2+4a}+\frac{3a\,b^2x^{14}\sqrt{\left(b\,x^2+a\right)^2}}{14\left(b\,x^2+a\right)}+\frac{b^3x^{16}\sqrt{\left(b\,x^2+a\right)^2}}{16b\,x^2+16a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/560*x^10*(35*b^3*x^6+120*a*b^2*x^4+140*a^2*b*x^2+56*a^3)*((b*x^2+a)^2)^(3
/2)/(b*x^2+a)^3
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.21

$$\frac{1}{16}b^3x^{16}+\frac{3}{14}ab^2x^{14}+\frac{1}{4}a^2bx^{12}+\frac{1}{10}a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Fricas [A]
time = 0.34, size = 35, normalized size = 0.21

$$\frac{1}{16}b^3x^{16}+\frac{3}{14}ab^2x^{14}+\frac{1}{4}a^2bx^{12}+\frac{1}{10}a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^9\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**9*((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 3.90, size = 67, normalized size = 0.40

$$\frac{1}{16}b^3x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{14}ab^2x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}a^2bx^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{10}a^3x^{10}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*b^3*x^16*sgn(b*x^2 + a) + 3/14*a*b^2*x^14*sgn(b*x^2 + a) + 1/4*a^2*b*x
^12*sgn(b*x^2 + a) + 1/10*a^3*x^10*sgn(b*x^2 + a)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^9 \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.559 $\int x^7(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10(a+bx^2)}+\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{b^3x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14(a+bx^2)}$$

```
[Out] 1/8*a^3*x^8*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/10*a^2*b*x^10*((b*x^2+a)^2)^(1/
2)/(b*x^2+a)+1/4*a*b^2*x^12*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/14*b^3*x^14*((b
*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.08, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10(a+bx^2)}+\frac{b^3x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14(a+bx^2)}+\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (3*a^2*b*x^10*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a*b^2*x^12*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(14*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^3\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^3\left(ab+b^2x\right)^3\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(a^3b^3x^3+3a^2b^4x^4+3ab^5x^5+b^6x^6\right)\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$=\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{ab^2x^{12}\sqrt{a^2+\cdots}}{4\left(a\cdots\right.}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{x^8\sqrt{\left(a+bx^2\right)^2}\left(35a^3+84a^2bx^2+70ab^2x^4+20b^3x^6\right)}{280\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(35*a^3 + 84*a^2*b*x^2 + 70*a*b^2*x^4 + 20*b^3*x^6
))/(280*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^8\left(20b^3x^6+70a\,b^2x^4+84a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{280\left(b\,x^2+a\right)^3}$	58
default	$\frac{x^8\left(20b^3x^6+70a\,b^2x^4+84a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{280\left(b\,x^2+a\right)^3}$	58
risch	$\frac{a^3x^8\sqrt{\left(b\,x^2+a\right)^2}}{8b\,x^2+8a}+\frac{3a^2b\,x^{10}\sqrt{\left(b\,x^2+a\right)^2}}{10\left(b\,x^2+a\right)}+\frac{a\,b^2x^{12}\sqrt{\left(b\,x^2+a\right)^2}}{4b\,x^2+4a}+\frac{b^3x^{14}\sqrt{\left(b\,x^2+a\right)^2}}{14b\,x^2+14a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/280*x^8*(20*b^3*x^6+70*a*b^2*x^4+84*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.21

$$\frac{1}{14}\,b^3x^{14}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Fricas [A]
time = 0.38, size = 35, normalized size = 0.21

$$\frac{1}{14}\,b^3x^{14}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**7*((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 3.76, size = 67, normalized size = 0.40

$$\frac{1}{14}\,b^3x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}\,ab^2x^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{10}\,a^2bx^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{8}\,a^3x^8\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*sgn(b*x^2 + a) + 1/4*a*b^2*x^12*sgn(b*x^2 + a) + 3/10*a^2*b*x
^10*sgn(b*x^2 + a) + 1/8*a^3*x^8*sgn(b*x^2 + a)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^7 \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.560 $\int x^5(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal. Leaf size=106

$$\frac{a^2(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^3}-\frac{a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{5b^3}+\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^3}$$

```
[Out] 1/8*a^2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/b^3-1/5*a*(b^2*x^4+2*a*b*x^
2+a^2)^(5/2)/b^3+1/12*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1125, 659}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{12b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^4}{5b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^3}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^2*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^3) - (a*(a + b*x^2
)^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*b^3) + ((a + b*x^2)^5*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(12*b^3)
```

Rule 659

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x^2 \left(a^2+2abx+b^2x^2\right)^{3/2} dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^2(ab+b^2x)^3}{b^2} - \frac{2a(ab+b^2x)^4}{b^3} + \frac{(ab+b^2x)^5}{b^4}\right) dx, x\right.}{2b^2\left(ab+b^2x^2\right)}$$

$$= \frac{a^2(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}{8b^3} - \frac{a(a+bx^2)^4\sqrt{a^2+2abx^2+b^2x^4}}{5b^3} +$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.58

$$\frac{x^6\sqrt{(a+bx^2)^2}\,(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6)}{120\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^2)^2]*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3*x^6
))/(120*(a + b*x^2))
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.55

method	result	size
gosper	$\frac{x^6\left(10b^3x^6+36a\,b^2x^4+45a^2b\,x^2+20a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{120\left(b\,x^2+a\right)^3}$	58
default	$\frac{x^6\left(10b^3x^6+36a\,b^2x^4+45a^2b\,x^2+20a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{120\left(b\,x^2+a\right)^3}$	58
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,b^3x^{12}}{12b\,x^2+12a}+\frac{3\sqrt{\left(b\,x^2+a\right)^2}\,a\,b^2x^{10}}{10\left(b\,x^2+a\right)}+\frac{3\sqrt{\left(b\,x^2+a\right)^2}\,a^2b\,x^8}{8\left(b\,x^2+a\right)}+\frac{\sqrt{\left(b\,x^2+a\right)^2}\,a^3x^6}{6b\,x^2+6a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.33

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Fricas [A]
time = 0.35, size = 35, normalized size = 0.33

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**5*((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 3.89, size = 67, normalized size = 0.63

$$\frac{1}{12}\,b^3x^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{10}\,ab^2x^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{8}\,a^2bx^8\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{6}\,a^3x^6\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*b^3*x^12*sgn(b*x^2 + a) + 3/10*a*b^2*x^10*sgn(b*x^2 + a) + 3/8*a^2*b*x
^8*sgn(b*x^2 + a) + 1/6*a^3*x^6*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.561 $\int x^3(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal. Leaf size=67

$$-\frac{a(a+bx^2)\,(a^2+2abx^2+b^2x^4)^{3/2}}{8b^2}+\frac{(a^2+2abx^2+b^2x^4)^{5/2}}{10b^2}$$

```
[Out] -1/8*a*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/b^2+1/10*(b^2*x^4+2*a*b*x^2+
a^2)^(5/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {1125, 654, 623}

$$\frac{(a^2+2abx^2+b^2x^4)^{5/2}}{10b^2}-\frac{a(a+bx^2)\,(a^2+2abx^2+b^2x^4)^{3/2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/8*(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/b^2 + (a^2 + 2*a*b*x
^2 + b^2*x^4)^(5/2)/(10*b^2)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^3(a^2+2abx^2+b^2x^4)^{3/2}\,dx = \frac{1}{2}\text{Subst}\left(\int x(a^2+2abx+b^2x^2)^{3/2}\,dx, x, x^2\right)$$
$$= \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{10b^2} - \frac{a\text{Subst}\left(\int (a^2+2abx+b^2x^2)^{3/2}\,dx, x, x^2\right)}{2b}$$
$$= -\frac{a(a+bx^2)(a^2+2abx^2+b^2x^4)^{3/2}}{8b^2} + \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{10b^2}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.91

$$\frac{x^4\sqrt{(a+bx^2)^2}\,(10a^3+20a^2bx^2+15ab^2x^4+4b^3x^6)}{40\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*x^6)
)/(40*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.87

method	result	size
gosper	$\frac{x^4(4b^3x^6+15a\,b^2x^4+20a^2b\,x^2+10a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{40(b\,x^2+a)^3}$	58
default	$\frac{x^4(4b^3x^6+15a\,b^2x^4+20a^2b\,x^2+10a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{40(b\,x^2+a)^3}$	58
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,b^3x^{10}}{10b\,x^2+10a}+\frac{3\sqrt{(b\,x^2+a)^2}\,a\,b^2x^8}{8(b\,x^2+a)}+\frac{\sqrt{(b\,x^2+a)^2}\,a^2b\,x^6}{2b\,x^2+2a}+\frac{\sqrt{(b\,x^2+a)^2}\,a^3x^4}{4b\,x^2+4a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*((b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3
```

Maxima [A]
time = 0.32, size = 35, normalized size = 0.52

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{2}\,a^2bx^6+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Fricas [**A**]
time = 0.35, size = 35, normalized size = 0.52

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{2}\,a^2bx^6+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral(x**3*((a + b*x**2)**2)**(3/2), x)
```

Giac [**A**]
time = 4.21, size = 45, normalized size = 0.67

$$\frac{1}{40}\left(4\,b^3x^{10}+15\,ab^2x^8+20\,a^2bx^6+10\,a^3x^4\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] 1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*sgn(b*x^2 + a)
```

Mupad [**B**]
time = 4.29, size = 46, normalized size = 0.69

$$\frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\left(-a^2+3\,a\,b\,x^2+4\,b^2\,x^4\right)}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)*(4*b^2*x^4 - a^2 + 3*a*b*x^2))/(40*b^2)
```

3.562 $\int x(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}$$

```
[Out] 1/8*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1121, 623}

$$\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx, x, x^2\right)\\ &= \frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.75

$$\frac{(a+bx^2)\left((a+bx^2)^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] ((a + b*x^2)*((a + b*x^2)^2)^(3/2))/(8*b)
```

Maple [A]
time = 0.04, size = 24, normalized size = 0.67

method	result	size
default	$\frac{(b\,x^{2}+a)\left((b\,x^{2}+a)^{2}\right)^{\frac{3}{2}}}{8b}$	24
risch	$\frac{\sqrt{(b\,x^{2}+a)^{2}}\;(b\,x^{2}+a)^{3}}{8b}$	26
gosper	$\frac{x^{2}(b^{3}x^{6}+4a\,b^{2}x^{4}+6a^{2}b\,x^{2}+4a^{3})\left((b\,x^{2}+a)^{2}\right)^{\frac{3}{2}}}{8(b\,x^{2}+a)^{3}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/8*(b*x^2+a)*((b*x^2+a)^2)^(3/2)/b
```

Maxima [A]
time = 0.31, size = 35, normalized size = 0.97

$$\frac{1}{8}\,b^{3}x^{8}+\frac{1}{2}\,ab^{2}x^{6}+\frac{3}{4}\,a^{2}bx^{4}+\frac{1}{2}\,a^{3}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

Fricas [A]
time = 0.32, size = 35, normalized size = 0.97

$$\frac{1}{8}\,b^{3}x^{8}+\frac{1}{2}\,ab^{2}x^{6}+\frac{3}{4}\,a^{2}bx^{4}+\frac{1}{2}\,a^{3}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 3.65, size = 44, normalized size = 1.22

$$\frac{1}{8}\left(2\left(bx^4+2\,ax^2\right)a^2+\left(bx^4+2\,ax^2\right)^2b\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^4 + 2*a*x^2)*a^2 + (b*x^4 + 2*a*x^2)^2*b)*sgn(b*x^2 + a)
```

Mupad [B]
time = 4.25, size = 36, normalized size = 1.00

$$\frac{\left(b^2\,x^2+a\,b\right)\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] ((a*b + b^2*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2))/(8*b^2)
```

3.563 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=163

$$\frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{a^3\sqrt{a^2+2abx^2+b^2 \ldots}}{a+bx^2}$$

```
[Out] 3/2*a^2*b*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/4*a*b^2*x^4*((b*x^2+a)^2)^(1/
2)/(b*x^2+a)+1/6*b^3*x^6*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^3*ln(x)*((b*x^2+a)
^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 45}

$$\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{a^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

```
[Out] (3*a^2*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*x^
4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x^6*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*
Log[x])/(a + b*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{b^3x^6\sqrt{a^2+2ab}}{6\left(a+b\right.}$$

Mathematica [**A**]
time = 0.01, size = 60, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\,\left(bx^2\left(18a^2+9abx^2+2b^2x^4\right)+12a^3\log(x)\right)}{12\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2*(18*a^2 + 9*a*b*x^2 + 2*b^2*x^4) + 12*a^3*Log[x
]))/(12*(a + b*x^2))
```

Maple [**A**]
time = 0.02, size = 57, normalized size = 0.35

method	result	size
default	$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(2b^3x^6+9a\,b^2x^4+18a^2b\,x^2+12a^3\ln(x)\right)}{12\left(b\,x^2+a\right)^3}$	57
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,b\left(\frac{1}{6}b^2x^6+\frac{3}{4}ab\,x^4+\frac{3}{2}a^2x^2\right)}{b\,x^2+a}+\frac{a^3\ln(x)\sqrt{\left(b\,x^2+a\right)^2}}{b\,x^2+a}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*((b*x^2+a)^2)^(3/2)*(2*b^3*x^6+9*a*b^2*x^4+18*a^2*b*x^2+12*a^3*ln(x))/
(b*x^2+a)^3
```

Maxima [A]
time = 0.29, size = 33, normalized size = 0.20

$$\frac{1}{6}\,b^3x^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{2}\,a^2bx^2 + a^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

Fricas [A]
time = 0.33, size = 33, normalized size = 0.20

$$\frac{1}{6}\,b^3x^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{2}\,a^2bx^2 + a^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x, x)
```

Giac [A]
time = 3.44, size = 68, normalized size = 0.42

$$\frac{1}{6}\,b^3x^6\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{4}\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{2}\,a^3\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sgn(b*x^2 + a) + 3/4*a*b^2*x^4*sgn(b*x^2 + a) + 3/2*a^2*b*x^2*s
gn(b*x^2 + a) + 1/2*a^3*log(x^2)*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x, x)
```

3.564 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=164

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{3a^2b\sqrt{a^2+2abx^2+b^2}}{a+bx^2}$$

```
[Out] -1/2*a^3*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+3/2*a*b^2*x^2*((b*x^2+a)^2)^(1/2
)/(b*x^2+a)+1/4*b^3*x^4*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3*a^2*b*ln(x)*((b*x^2
+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]

time = 0.03, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 45}

$$\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```

```
[Out] -1/2*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (3*a*b^2*x^2
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (b^3*x^4*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]*Log[x])/(a + b*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{x^3}\,dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^2}\,dx,x,x^2\right)}{2b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(3ab^5+\frac{a^3b^3}{x^2}+\frac{3a^2b^4}{x}+b^6x\right)dx,x,x^2\right)}{2b^2(ab+b^2x^2)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{b^3x^4\sqrt{a^2+2abx^2}}{4(a+bx^2}$$

Mathematica [**A**]
time = 0.02, size = 62, normalized size = 0.38

$$\frac{\sqrt{(a+bx^2)^2}\,(-2a^3+6ab^2x^4+b^3x^6+12a^2bx^2\log(x))}{4x^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
[Out] (Sqrt[(a + b*x^2)^2]*(-2*a^3 + 6*a*b^2*x^4 + b^3*x^6 + 12*a^2*b*x^2*Log[x])
)/(4*x^2*(a + b*x^2))
```

Maple [**A**]
time = 0.04, size = 59, normalized size = 0.36

method	result	size
default	$\frac{\left((b\,x^2+a)^2\right)^{\frac{3}{2}}(b^3x^6+6a\,b^2x^4+12a^2b\ln(x)x^2-2a^3)}{4(b\,x^2+a)^3x^2}$	59
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,b(b\,x^2+3a)^2}{4b\,x^2+4a}-\frac{a^3\sqrt{(b\,x^2+a)^2}}{2x^2(b\,x^2+a)}+\frac{3a^2b\ln(x)\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/4*((b*x^2+a)^2)^(3/2)*(b^3*x^6+6*a*b^2*x^4+12*a^2*b*ln(x)*x^2-2*a^3)/(b*x
^2+a)^3/x^2
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.21

$$\frac{1}{4}\,b^3x^4+\frac{3}{2}\,ab^2x^2+3\,a^2b\log\left(x\right)-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*log(x) - 1/2*a^3/x^2
```

Fricas [A]
time = 0.38, size = 38, normalized size = 0.23

$$\frac{b^3x^6+6\,ab^2x^4+12\,a^2bx^2\log\left(x\right)-2\,a^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**3, x)
```

Giac [A]
time = 4.04, size = 87, normalized size = 0.53

$$\frac{1}{4}\,b^3x^4\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{2}\,ab^2x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{2}\,a^2b\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{3\,a^2bx^2\mathrm{sgn}(bx^2+a)+a^3\mathrm{sgn}(bx^2+a)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*sgn(b*x^2 + a) + 3/2*a*b^2*x^2*sgn(b*x^2 + a) + 3/2*a^2*b*log(x
^2)*sgn(b*x^2 + a) - 1/2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/
x^2
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^3,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^3, x)
```

3.565 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^5} \, dx$

Optimal. Leaf size=164

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] -1/4*a^3*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-3/2*a^2*b*((b*x^2+a)^2)^(1/2)/x^
2/(b*x^2+a)+1/2*b^3*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3*a*b^2*ln(x)*((b*x^2
+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 45}

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

```
[Out] -1/4*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^4*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*x^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]*Log[x])/(a + b*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{x^5}\,dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)^3}{x^3}\,dx, x, x^2\right)}{2b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(b^6+\frac{a^3b^3}{x^3}+\frac{3a^2b^4}{x^2}+\frac{3ab^5}{x}\right)dx, x, x^2\right)}{2b^2(ab+b^2x^2)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)} + \frac{b^3x^2\sqrt{a^2+2abx^2+}}{2(a+bx^2)}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(a^3+6a^2bx^2-2b^3x^6-12ab^2x^4\log(x))}{4x^4(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

```
[Out] -1/4*(Sqrt[(a + b*x^2)^2]*(a^3 + 6*a^2*b*x^2 - 2*b^3*x^6 - 12*a*b^2*x^4*Log
[x]))/(x^4*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 60, normalized size = 0.37

method	result	size
default	$\frac{\left((b\,x^2+a)^2\right)^{\frac{3}{2}}(2b^3x^6+12a\,b^2\ln(x)x^4-6a^2b\,x^2-a^3)}{4(b\,x^2+a)^3x^4}$	60
risch	$\frac{b^3x^2\sqrt{(b\,x^2+a)^2}}{2b\,x^2+2a}+\frac{\sqrt{(b\,x^2+a)^2}\,(-\frac{3}{2}a^2b\,x^2-\frac{1}{4}a^3)}{(b\,x^2+a)x^4}+\frac{3a\,b^2\ln(x)\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((b*x^2+a)^2)^(3/2)*(2*b^3*x^6+12*a*b^2*ln(x)*x^4-6*a^2*b*x^2-a^3)/(b*x
^2+a)^3/x^4
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.21

$$\frac{1}{2}\,b^3x^2+3\,ab^2\log\left(x\right)-\frac{3\,a^2b}{2\,x^2}-\frac{a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="maxima")
[Out] 1/2*b^3*x^2 + 3*a*b^2*log(x) - 3/2*a^2*b/x^2 - 1/4*a^3/x^4
```

Fricas [A]
time = 0.36, size = 39, normalized size = 0.24

$$\frac{2\,b^3x^6+12\,ab^2x^4\log\left(x\right)-6\,a^2bx^2-a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="fricas")
[Out] 1/4*(2*b^3*x^6 + 12*a*b^2*x^4*log(x) - 6*a^2*b*x^2 - a^3)/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**5,x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**5, x)
```

Giac [A]
time = 3.60, size = 87, normalized size = 0.53

$$\frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{2}\,ab^2\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{9\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+6\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+a^3\mathrm{sgn}\left(bx^2+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="giac")
[Out] 1/2*b^3*x^2*sgn(b*x^2 + a) + 3/2*a*b^2*log(x^2)*sgn(b*x^2 + a) - 1/4*(9*a*b
^2*x^4*sgn(b*x^2 + a) + 6*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^
4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^5,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^5, x)
```

3.566 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=163

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{lo}}{a+bx^2}$$

```
[Out] -1/6*a^3*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)-3/4*a^2*b*((b*x^2+a)^2)^(1/2)/x^
4/(b*x^2+a)-3/2*a*b^2*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+b^3*ln(x)*((b*x^2+a
)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 45}

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

```
[Out] -1/6*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^6*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]*Log[x])/(a + b*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^7}\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{\left(ab+b^2x\right)^3}{x^4}\,dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^3}+\frac{3ab^5}{x^2}+\frac{b^6}{x}\right)dx,x,x^2\right)}{2b^2\left(ab+b^2x^2\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+}}{2x^2\left(a+bx^2\right)}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 0.39

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a\left(2a^2+9abx^2+18b^2x^4\right)-12b^3x^6\log(x)\right)}{12x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

```
[Out] -1/12*(Sqrt[(a + b*x^2)^2]*(a*(2*a^2 + 9*a*b*x^2 + 18*b^2*x^4) - 12*b^3*x^6
*Log[x]))/(x^6*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 60, normalized size = 0.37

method	result	size
default	$\frac{\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(12b^3\ln(x)x^6-18a\,b^2x^4-9a^2b\,x^2-2a^3\right)}{12\left(b\,x^2+a\right)^3x^6}$	60
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(-\frac{3}{2}a\,b^2x^4-\frac{3}{4}a^2b\,x^2-\frac{1}{6}a^3\right)}{\left(b\,x^2+a\right)x^6}+\frac{b^3\ln(x)\sqrt{\left(b\,x^2+a\right)^2}}{b\,x^2+a}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*((b*x^2+a)^2)^(3/2)*(12*b^3*ln(x)*x^6-18*a*b^2*x^4-9*a^2*b*x^2-2*a^3)/
(b*x^2+a)^3/x^6
```

Maxima [A]
time = 0.30, size = 33, normalized size = 0.20

$$b^3 \log(x) - \frac{3\,ab^2}{2\,x^2} - \frac{3\,a^2b}{4\,x^4} - \frac{a^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] b^3*log(x) - 3/2*a*b^2/x^2 - 3/4*a^2*b/x^4 - 1/6*a^3/x^6
```

Fricas [A]
time = 0.35, size = 39, normalized size = 0.24

$$\frac{12\,b^3x^6 \log(x) - 18\,ab^2x^4 - 9\,a^2bx^2 - 2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^3*x^6*log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**7, x)
```

Giac [A]
time = 4.34, size = 87, normalized size = 0.53

$$\frac{1}{2}\,b^3 \log(x^2)\,\mathrm{sgn}(bx^2+a) - \frac{11\,b^3x^6\mathrm{sgn}(bx^2+a) + 18\,ab^2x^4\mathrm{sgn}(bx^2+a) + 9\,a^2bx^2\mathrm{sgn}(bx^2+a) + 2\,a^3\mathrm{sgn}(bx^2+a)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(x^2)*sgn(b*x^2 + a) - 1/12*(11*b^3*x^6*sgn(b*x^2 + a) + 18*a*b^
2*x^4*sgn(b*x^2 + a) + 9*a^2*b*x^2*sgn(b*x^2 + a) + 2*a^3*sgn(b*x^2 + a))/x
^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^7,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^7, x)
```

3.567 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=41

$$-\frac{(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

```
[Out] -1/8*(b*x^2+a)^3*((b*x^2+a)^2)^(1/2)/a/x^8
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 37}

$$-\frac{(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/8*((a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^5}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^5}\,dx, x, x^2\right)}{2b^2(ab+b^2x^2)}$$
$$= -\frac{(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.44

$$-\frac{\sqrt{(a+bx^2)^2}\,(a^3+4a^2bx^2+6ab^2x^4+4b^3x^6)}{8x^8(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/8*(Sqrt[(a + b*x^2)^2]*(a^3 + 4*a^2*b*x^2 + 6*a*b^2*x^4 + 4*b^3*x^6))/(x
^8*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 56, normalized size = 1.37

method	result	size
gosper	$-\frac{(4b^3x^6+6a\,b^2x^4+4a^2b\,x^2+a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{8x^8(b\,x^2+a)^3}$	56
default	$-\frac{(4b^3x^6+6a\,b^2x^4+4a^2b\,x^2+a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{8x^8(b\,x^2+a)^3}$	56
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{2}b^3x^6-\frac{3}{4}a\,b^2x^4-\frac{1}{2}a^2b\,x^2-\frac{1}{8}a^3\right)}{(b\,x^2+a)x^8}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(4*b^3*x^6+6*a*b^2*x^4+4*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^8/(b*x^2
+a)^3
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.85

$$-\frac{b^3}{2\,x^2}-\frac{3\,ab^2}{4\,x^4}-\frac{a^2b}{2\,x^6}-\frac{a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="maxima")
[Out] -1/2*b^3/x^2 - 3/4*a*b^2/x^4 - 1/2*a^2*b/x^6 - 1/8*a^3/x^8
```

Fricas [A]
time = 0.37, size = 35, normalized size = 0.85

$$-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="fricas")
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**9,x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**9, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.
time = 4.01, size = 68, normalized size = 1.66

$$-\frac{4\,b^3x^6\mathrm{sgn}(bx^2+a)+6\,ab^2x^4\mathrm{sgn}(bx^2+a)+4\,a^2bx^2\mathrm{sgn}(bx^2+a)+a^3\mathrm{sgn}(bx^2+a)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="giac")
[Out] -1/8*(4*b^3*x^6*sgn(b*x^2 + a) + 6*a*b^2*x^4*sgn(b*x^2 + a) + 4*a^2*b*x^2*s
gn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^8
```

Mupad [B]
time = 4.24, size = 151, normalized size = 3.68

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^2\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\,\left(b\,x^2+a\right)}-\frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^6\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^9,x)
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (b^3*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*x^2*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(2*x^6*(a + b*x^2))
```

3.568 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=72

$$-\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}$$

```
[Out] -1/8*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/a/x^10+1/40*(b^2*x^4+2*a*b*x^2
+a^2)^(5/2)/a^2/x^10
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1124}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}-\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

```
[Out] -1/8*((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(a*x^10) + (a^2 + 2*a*
b*x^2 + b^2*x^4)^(5/2)/(40*a^2*x^10)
```

Rule 1124

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(4*a*d*(p + 1)*(2*p + 1
))), x] - Simp[(d*x)^(m + 1)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^p/(4*a*d*(2
*p + 1))), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{11}}\,dx=-\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.85

$$-\frac{\sqrt{(a+bx^2)^2}\left(4a^3+15a^2bx^2+20ab^2x^4+10b^3x^6\right)}{40x^{10}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

```
[Out] -1/40*(Sqrt[(a + b*x^2)^2]*(4*a^3 + 15*a^2*b*x^2 + 20*a*b^2*x^4 + 10*b^3*x^
6))/(x^10*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.81

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{4}b^{3}x^{6}-\frac{1}{2}a\,b^{2}x^{4}-\frac{3}{8}a^{2}b\,x^{2}-\frac{1}{10}a^{3}\right)}{\left(b\,x^{2}+a\right)x^{10}}$	57
gosper	$-\frac{\left(10b^{3}x^{6}+20a\,b^{2}x^{4}+15a^{2}b\,x^{2}+4a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{40x^{10}\left(b\,x^{2}+a\right)^{3}}$	58
default	$-\frac{\left(10b^{3}x^{6}+20a\,b^{2}x^{4}+15a^{2}b\,x^{2}+4a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{40x^{10}\left(b\,x^{2}+a\right)^{3}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(10*b^3*x^6+20*a*b^2*x^4+15*a^2*b*x^2+4*a^3)*((b*x^2+a)^2)^(3/2)/x^10
/(b*x^2+a)^3
```

Maxima [A]
time = 0.30, size = 35, normalized size = 0.49

$$-\frac{b^{3}}{4\,x^{4}}-\frac{ab^{2}}{2\,x^{6}}-\frac{3\,a^{2}b}{8\,x^{8}}-\frac{a^{3}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/4*b^3/x^4 - 1/2*a*b^2/x^6 - 3/8*a^2*b/x^8 - 1/10*a^3/x^10
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.51

$$-\frac{10\,b^{3}x^{6}+20\,ab^{2}x^{4}+15\,a^{2}bx^{2}+4\,a^{3}}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**11, x)
```

Giac [A]
time = 4.25, size = 69, normalized size = 0.96

$$-\frac{10\,b^3x^6\mathrm{sgn}(bx^2+a)+20\,ab^2x^4\mathrm{sgn}(bx^2+a)+15\,a^2bx^2\mathrm{sgn}(bx^2+a)+4\,a^3\mathrm{sgn}(bx^2+a)}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/40*(10*b^3*x^6*sgn(b*x^2 + a) + 20*a*b^2*x^4*sgn(b*x^2 + a) + 15*a^2*b*x
^2*sgn(b*x^2 + a) + 4*a^3*sgn(b*x^2 + a))/x^10
```

Mupad [B]
time = 4.20, size = 151, normalized size = 2.10

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\,\left(b\,x^2+a\right)}-\frac{a\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^6\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^11,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(2*x^6*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(8*x^8*(a + b*x^2))
```

3.569 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}$$

```
[Out] -1/12*a^3*((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)-3/10*a^2*b*((b*x^2+a)^2)^(1/2)
/x^10/(b*x^2+a)-3/8*a*b^2*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)-1/6*b^3*((b*x^2
+a)^2)^(1/2)/x^6/(b*x^2+a)
```

Rubi [A]
time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```

```
[Out] -1/12*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^12*(a + b*x^2)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(6*x^6*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{13}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^7}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^7}\,dx,x,x^2\right)}{2b^2(ab+b^2x^2)}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^3b^3}{x^7}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^5}+\frac{b^6}{x^4}\right)dx,x,x^2\right)}{2b^2(ab+b^2x^2)}\\
&= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+}}{8x^8(a+bx^2)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(10a^3+36a^2bx^2+45ab^2x^4+20b^3x^6)}{120x^{12}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
[Out] -1/120*(Sqrt[(a + b*x^2)^2]*(10*a^3 + 36*a^2*b*x^2 + 45*a*b^2*x^4 + 20*b^3*
x^6))/(x^12*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{6}b^3x^6-\frac{3}{8}a\,b^2x^4-\frac{3}{10}a^2b\,x^2-\frac{1}{12}a^3\right)}{(b\,x^2+a)x^{12}}$	57
gosper	$-\frac{(20b^3x^6+45a\,b^2x^4+36a^2b\,x^2+10a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{120x^{12}(b\,x^2+a)^3}$	58
default	$-\frac{(20b^3x^6+45a\,b^2x^4+36a^2b\,x^2+10a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{120x^{12}(b\,x^2+a)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*(20*b^3*x^6+45*a*b^2*x^4+36*a^2*b*x^2+10*a^3)*((b*x^2+a)^2)^(3/2)/x^
12/(b*x^2+a)^3
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.21

$$-\frac{b^3}{6\,x^6}-\frac{3\,ab^2}{8\,x^8}-\frac{3\,a^2b}{10\,x^{10}}-\frac{a^3}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] -1/6*b^3/x^6 - 3/8*a*b^2/x^8 - 3/10*a^2*b/x^10 - 1/12*a^3/x^12
```

Fricas [A]
time = 0.36, size = 37, normalized size = 0.22

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**13, x)
```

Giac [A]
time = 3.75, size = 69, normalized size = 0.41

$$-\frac{20\,b^3x^6\mathrm{sgn}(bx^2+a)+45\,ab^2x^4\mathrm{sgn}(bx^2+a)+36\,a^2bx^2\mathrm{sgn}(bx^2+a)+10\,a^3\mathrm{sgn}(bx^2+a)}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/120*(20*b^3*x^6*sgn(b*x^2 + a) + 45*a*b^2*x^4*sgn(b*x^2 + a) + 36*a^2*b*
x^2*sgn(b*x^2 + a) + 10*a^3*sgn(b*x^2 + a))/x^12
```

Mupad [B]
time = 4.21, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^6\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^13,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(10*x^10*(a + b*x^2))
```

3.570 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}$$

```
[Out] -1/14*a^3*((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)-1/4*a^2*b*((b*x^2+a)^2)^(1/2)/
x^12/(b*x^2+a)-3/10*a*b^2*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)-1/8*b^3*((b*x^
2+a)^2)^(1/2)/x^8/(b*x^2+a)
```

Rubi [A]

time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

```
[Out] -1/14*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^14*(a + b*x^2)) - (a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(8*x^8*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{15}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^8}\,dx,x,x^2\right)\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^8}\,dx,x,x^2\right)}{2b^2(ab+b^2x^2)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^7}+\frac{3ab^5}{x^6}+\frac{b^6}{x^5}\right)dx,x,x^2\right)}{2b^2(ab+b^2x^2)}\\ &= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b}}{10x^{10}(a+bx^2)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(20a^3+70a^2bx^2+84ab^2x^4+35b^3x^6)}{280x^{14}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
[Out] -1/280*(Sqrt[(a + b*x^2)^2]*(20*a^3 + 70*a^2*b*x^2 + 84*a*b^2*x^4 + 35*b^3*
x^6))/(x^14*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{14}a^3-\frac{1}{4}a^2b\,x^2-\frac{3}{10}a\,b^2x^4-\frac{1}{8}b^3x^6\right)}{(b\,x^2+a)x^{14}}$	57
gosper	$-\frac{(35b^3x^6+84a\,b^2x^4+70a^2b\,x^2+20a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{280x^{14}(b\,x^2+a)^3}$	58
default	$-\frac{(35b^3x^6+84a\,b^2x^4+70a^2b\,x^2+20a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{280x^{14}(b\,x^2+a)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/280*(35*b^3*x^6+84*a*b^2*x^4+70*a^2*b*x^2+20*a^3)*((b*x^2+a)^2)^(3/2)/x^
14/(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.21

$$-\frac{b^3}{8\,x^8}-\frac{3\,ab^2}{10\,x^{10}}-\frac{a^2b}{4\,x^{12}}-\frac{a^3}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/8*b^3/x^8 - 3/10*a*b^2/x^10 - 1/4*a^2*b/x^12 - 1/14*a^3/x^14
```

Fricas [A]
time = 0.42, size = 37, normalized size = 0.22

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**15, x)
```

Giac [A]
time = 4.19, size = 69, normalized size = 0.41

$$-\frac{35\,b^3x^6\mathrm{sgn}(bx^2+a)+84\,ab^2x^4\mathrm{sgn}(bx^2+a)+70\,a^2bx^2\mathrm{sgn}(bx^2+a)+20\,a^3\mathrm{sgn}(bx^2+a)}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/280*(35*b^3*x^6*sgn(b*x^2 + a) + 84*a*b^2*x^4*sgn(b*x^2 + a) + 70*a^2*b*
x^2*sgn(b*x^2 + a) + 20*a^3*sgn(b*x^2 + a))/x^14
```

Mupad [B]
time = 4.21, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}-\frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^{12}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^15,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(4*x^12*(a + b*x^2))
```

3.571 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}$$

```
[Out] -1/16*a^3*((b*x^2+a)^2)^(1/2)/x^16/(b*x^2+a)-3/14*a^2*b*((b*x^2+a)^2)^(1/2)
/x^14/(b*x^2+a)-1/4*a*b^2*((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)-1/10*b^3*((b*x
^2+a)^2)^(1/2)/x^10/(b*x^2+a)
```

Rubi [A]
time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

```
[Out] -1/16*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^16*(a + b*x^2)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(10*x^10*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{17}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a^2+2abx+b^2x^2)^{3/2}}{x^9}\,dx, x, x^2\right)\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^9}\,dx, x, x^2\right)}{2b^2\,(ab+b^2x^2)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^3b^3}{x^9}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^7}+\frac{b^6}{x^6}\right)dx, x, x^2\right)}{2b^2\,(ab+b^2x^2)}\\ &= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\,(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\,(a+bx^2)} - \frac{ab^2\sqrt{a^2+2abx^2+b}}{4x^{12}\,(a+bx^2)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(35a^3+120a^2bx^2+140ab^2x^4+56b^3x^6)}{560x^{16}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
[Out] -1/560*(Sqrt[(a + b*x^2)^2]*(35*a^3 + 120*a^2*b*x^2 + 140*a*b^2*x^4 + 56*b^
3*x^6))/(x^16*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{(bx^2+a)^2}\,\left(-\frac{1}{16}a^3-\frac{3}{14}a^2bx^2-\frac{1}{4}ab^2x^4-\frac{1}{10}b^3x^6\right)}{(bx^2+a)x^{16}}$	57
gosper	$-\frac{(56b^3x^6+140ab^2x^4+120a^2bx^2+35a^3)\left((bx^2+a)^2\right)^{\frac{3}{2}}}{560x^{16}(bx^2+a)^3}$	58
default	$-\frac{(56b^3x^6+140ab^2x^4+120a^2bx^2+35a^3)\left((bx^2+a)^2\right)^{\frac{3}{2}}}{560x^{16}(bx^2+a)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/560*(56*b^3*x^6+140*a*b^2*x^4+120*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)/
x^16/(b*x^2+a)^3
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.21

$$-\frac{b^3}{10\,x^{10}}-\frac{ab^2}{4\,x^{12}}-\frac{3\,a^2b}{14\,x^{14}}-\frac{a^3}{16\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/10*b^3/x^10 - 1/4*a*b^2/x^12 - 3/14*a^2*b/x^14 - 1/16*a^3/x^16
```

Fricas [A]
time = 0.36, size = 37, normalized size = 0.22

$$-\frac{56\,b^3x^6+140\,ab^2x^4+120\,a^2bx^2+35\,a^3}{560\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/560*(56*b^3*x^6 + 140*a*b^2*x^4 + 120*a^2*b*x^2 + 35*a^3)/x^16
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**17, x)
```

Giac [A]
time = 3.94, size = 69, normalized size = 0.41

$$-\frac{56\,b^3x^6\mathrm{sgn}(bx^2+a)+140\,ab^2x^4\mathrm{sgn}(bx^2+a)+120\,a^2bx^2\mathrm{sgn}(bx^2+a)+35\,a^3\mathrm{sgn}(bx^2+a)}{560\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/560*(56*b^3*x^6*sgn(b*x^2 + a) + 140*a*b^2*x^4*sgn(b*x^2 + a) + 120*a^2*
b*x^2*sgn(b*x^2 + a) + 35*a^3*sgn(b*x^2 + a))/x^16
```

Mupad [B]
time = 4.23, size = 151, normalized size = 0.90

$$-\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}-\frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^{12}\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^17,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(4*x^12*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(14*x^14*(a + b*x^2))
```

3.572 $\int x^8(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11(a+bx^2)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)}+\frac{b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15(a+bx^2)}$$

```
[Out] 1/9*a^3*x^9*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/11*a^2*b*x^11*((b*x^2+a)^2)^(1/
2)/(b*x^2+a)+3/13*a*b^2*x^13*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/15*b^3*x^15*((
b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$\frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11(a+bx^2)}+\frac{b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15(a+bx^2)}+\frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (3*a^2*b*x^11*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (3*a*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (b^3*x^15*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(15*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^8 \left(ab+b^2x^2\right)^3 dx}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3x^8+3a^2b^4x^{10}+3ab^5x^{12}+b^6x^{14}\right) dx}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)} + \frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)} + \frac{3ab^2x^{13}\sqrt{a^2+}}{13\,(a}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{x^9\sqrt{\left(a+bx^2\right)^2}\left(715a^3+1755a^2bx^2+1485ab^2x^4+429b^3x^6\right)}{6435\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^2)^2]*(715*a^3 + 1755*a^2*b*x^2 + 1485*a*b^2*x^4 + 429*b
^3*x^6))/(6435*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^9\left(429b^3x^6+1485a\,b^2x^4+1755a^2b\,x^2+715a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{6435\left(b\,x^2+a\right)^3}$	58
default	$\frac{x^9\left(429b^3x^6+1485a\,b^2x^4+1755a^2b\,x^2+715a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{6435\left(b\,x^2+a\right)^3}$	58
risch	$\frac{a^3x^9\sqrt{\left(b\,x^2+a\right)^2}}{9b\,x^2+9a}+\frac{3a^2b\,x^{11}\sqrt{\left(b\,x^2+a\right)^2}}{11\left(b\,x^2+a\right)}+\frac{3a\,b^2x^{13}\sqrt{\left(b\,x^2+a\right)^2}}{13\left(b\,x^2+a\right)}+\frac{b^3x^{15}\sqrt{\left(b\,x^2+a\right)^2}}{15b\,x^2+15a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6435*x^9*(429*b^3*x^6+1485*a*b^2*x^4+1755*a^2*b*x^2+715*a^3)*((b*x^2+a)^2
)^(3/2)/(b*x^2+a)^3
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.21

$$\frac{1}{15}\,b^3x^{15}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

Fricas [A]
time = 0.42, size = 35, normalized size = 0.21

$$\frac{1}{15}\,b^3x^{15}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**8*((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 4.04, size = 67, normalized size = 0.40

$$\frac{1}{15}\,b^3x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{13}\,ab^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{11}\,a^2bx^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{9}\,a^3x^9\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*b^3*x^15*sgn(b*x^2 + a) + 3/13*a*b^2*x^13*sgn(b*x^2 + a) + 3/11*a^2*b*
x^11*sgn(b*x^2 + a) + 1/9*a^3*x^9*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.573 $\int x^6(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11(a+bx^2)}+\frac{b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)}$$

```
[Out] 1/7*a^3*x^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*a^2*b*x^9*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)+3/11*a*b^2*x^11*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/13*b^3*x^13*((b*
x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1126, 276}

$$\frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11(a+bx^2)}+\frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)}+\frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a^2*b*x^9*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a*b^2*x^11*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(13*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^6 (ab+b^2x^2)^3 dx}{b^2 (ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (a^3b^3x^6+3a^2b^4x^8+3ab^5x^{10}+b^6x^{12}) dx}{b^2 (ab+b^2x^2)}$$
$$= \frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7 (a+bx^2)} + \frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3 (a+bx^2)} + \frac{3ab^2x^{11}\sqrt{a^2+}}{11 (a}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{x^7\sqrt{(a+bx^2)^2} (429a^3+1001a^2bx^2+819ab^2x^4+231b^3x^6)}{3003 (a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^2)^2]*(429*a^3 + 1001*a^2*b*x^2 + 819*a*b^2*x^4 + 231*b^
3*x^6))/(3003*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^{7} \left(231 b^{3} x^{6}+819 a \,b^{2} x^{4}+1001 a^{2} b \,x^{2}+429 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{3003 \left(b \,x^{2}+a \right)^{3}}$	58
default	$\frac{x^{7} \left(231 b^{3} x^{6}+819 a \,b^{2} x^{4}+1001 a^{2} b \,x^{2}+429 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{3003 \left(b \,x^{2}+a \right)^{3}}$	58
risch	$\frac{a^{3} x^{7} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{7 b \,x^{2}+7 a}+\frac{a^{2} b \,x^{9} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{3 b \,x^{2}+3 a}+\frac{3 a \,b^{2} x^{11} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{11 \left(b \,x^{2}+a \right)}+\frac{b^{3} x^{13} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{13 b \,x^{2}+13 a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3003*x^7*(231*b^3*x^6+819*a*b^2*x^4+1001*a^2*b*x^2+429*a^3)*((b*x^2+a)^2)
^(3/2)/(b*x^2+a)^3
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.21

$$\frac{1}{13} b^3x^{13} + \frac{3}{11} ab^2x^{11} + \frac{1}{3} a^2bx^9 + \frac{1}{7} a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Fricas [A]
time = 0.36, size = 35, normalized size = 0.21

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**6*((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 4.37, size = 67, normalized size = 0.40

$$\frac{1}{13}\,b^3x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{11}\,ab^2x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,a^2bx^9\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{7}\,a^3x^7\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*sgn(b*x^2 + a) + 3/11*a*b^2*x^11*sgn(b*x^2 + a) + 1/3*a^2*b*x
^9*sgn(b*x^2 + a) + 1/7*a^3*x^7*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.574 $\int x^4(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\,(a+bx^2)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}$$

```
[Out] 1/5*a^3*x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/7*a^2*b*x^7*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)+1/3*a*b^2*x^9*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/11*b^3*x^11*((b*x^
2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1126, 276}

$$\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}+\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a^2*b*x^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a*b^2*x^9*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(11*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int x^4(ab+b^2x^2)^3\,dx}{b^2\,(ab+b^2x^2)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int\left(a^3b^3x^4+3a^2b^4x^6+3ab^5x^8+b^6x^{10}\right)dx}{b^2\,(ab+b^2x^2)}$$
$$=\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\,(a+bx^2)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{ab^2x^9\sqrt{a^2+2a}}{3\,(a+}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{x^5\sqrt{(a+bx^2)^2}\,(231a^3+495a^2bx^2+385ab^2x^4+105b^3x^6)}{1155\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^2)^2]*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3
*x^6))/(1155*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^5(105b^3x^6+385a\,b^2x^4+495a^2b\,x^2+231a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{1155(b\,x^2+a)^3}$	58
default	$\frac{x^5(105b^3x^6+385a\,b^2x^4+495a^2b\,x^2+231a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{1155(b\,x^2+a)^3}$	58
risch	$\frac{a^3x^5\sqrt{(b\,x^2+a)^2}}{5b\,x^2+5a}+\frac{3a^2b\,x^7\sqrt{(b\,x^2+a)^2}}{7(b\,x^2+a)}+\frac{a\,b^2x^9\sqrt{(b\,x^2+a)^2}}{3b\,x^2+3a}+\frac{b^3x^{11}\sqrt{(b\,x^2+a)^2}}{11b\,x^2+11a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*((b*x^2+a)^2)^
(3/2)/(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.21

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Fricas [A]
time = 0.38, size = 35, normalized size = 0.21

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 3.14, size = 67, normalized size = 0.40

$$\frac{1}{11}\,b^3x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,ab^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{7}\,a^2bx^7\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{5}\,a^3x^5\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*sgn(b*x^2 + a) + 1/3*a*b^2*x^9*sgn(b*x^2 + a) + 3/7*a^2*b*x^7
*sgn(b*x^2 + a) + 1/5*a^3*x^5*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.575 $\int x^2(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}+\frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{b^3x^9\sqrt{a^2+2abx^2+b}}{9(a+bx^2)}$$

```
[Out] 1/3*a^3*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/5*a^2*b*x^5*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)+3/7*a*b^2*x^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/9*b^3*x^9*((b*x^2+
a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1126, 276}

$$\frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}+\frac{b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}+\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a^2*b*x^5*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a*b^2*x^7*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(9*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2(ab+b^2x^2)^3\,dx}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^3b^3x^2+3a^2b^4x^4+3ab^5x^6+b^6x^8\right)dx}{b^2\left(ab+b^2x^2\right)}$$
$$=\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{3ab^2x^7\sqrt{a^2+}}{7\left(a\right.}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(105a^3x^3+189a^2bx^5+135ab^2x^7+35b^3x^9\right)}{315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7 + 35*b^3*
x^9))/(315*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^3\left(35b^3x^6+135a\,b^2x^4+189a^2b\,x^2+105a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{315\left(b\,x^2+a\right)^3}$	58
default	$\frac{x^3\left(35b^3x^6+135a\,b^2x^4+189a^2b\,x^2+105a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{315\left(b\,x^2+a\right)^3}$	58
risch	$\frac{a^3x^3\sqrt{\left(b\,x^2+a\right)^2}}{3b\,x^2+3a}+\frac{3a^2b\,x^5\sqrt{\left(b\,x^2+a\right)^2}}{5\left(b\,x^2+a\right)}+\frac{3a\,b^2x^7\sqrt{\left(b\,x^2+a\right)^2}}{7\left(b\,x^2+a\right)}+\frac{b^3x^9\sqrt{\left(b\,x^2+a\right)^2}}{9b\,x^2+9a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*((b*x^2+a)^2)^(3
/2)/(b*x^2+a)^3
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.21

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Fricas [A]
time = 0.34, size = 35, normalized size = 0.21

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 2.88, size = 67, normalized size = 0.40

$$\frac{1}{9}\,b^3x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{7}\,ab^2x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{5}\,a^2bx^5\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,a^3x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*sgn(b*x^2 + a) + 3/7*a*b^2*x^7*sgn(b*x^2 + a) + 3/5*a^2*b*x^5*s
gn(b*x^2 + a) + 1/3*a^3*x^3*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.576 $\int (a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal. Leaf size=159

$$\frac{a^3x(a^2+2abx^2+b^2x^4)^{3/2}}{(a+bx^2)^3}+\frac{a^2bx^3(a^2+2abx^2+b^2x^4)^{3/2}}{(a+bx^2)^3}+\frac{3ab^2x^5(a^2+2abx^2+b^2x^4)^{3/2}}{5\,(a+bx^2)^3}+\frac{b^3x^7(a^2+2abx^2+b^2x^4)^{3/2}}{7\,(a+bx^2)^3}$$

```
[Out] a^3*x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(b*x^2+a)^3+a^2*b*x^3*(b^2*x^4+2*a*b*x^
2+a^2)^(3/2)/(b*x^2+a)^3+3/5*a*b^2*x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(b*x^2
+a)^3+1/7*b^3*x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(b*x^2+a)^3
```

Rubi [A]
time = 0.02, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1102, 200}

$$\frac{3ab^2x^5(a^2+2abx^2+b^2x^4)^{3/2}}{5\,(a+bx^2)^3}+\frac{a^2bx^3(a^2+2abx^2+b^2x^4)^{3/2}}{(a+bx^2)^3}+\frac{b^3x^7(a^2+2abx^2+b^2x^4)^{3/2}}{7\,(a+bx^2)^3}+\frac{a^3x(a^2+2abx^2+b^2x^4)^{3/2}}{(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (a^2*b*x^3*(a^2 +
 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (3*a*b^2*x^5*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(3/2))/(5*(a + b*x^2)^3) + (b^3*x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^
(3/2))/(7*(a + b*x^2)^3)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1102

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx = \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}\int \left(2ab+2b^2x^2\right)^3\,dx}{\left(2ab+2b^2x^2\right)^3}$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}\int \left(8a^3b^3+24a^2b^4x^2+24ab^5x^4+8b^6x^6\right)\,dx}{\left(2ab+2b^2x^2\right)^3}$$
$$= \frac{a^3x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{3ab^2x^5\left(a^2+2ab\right.}{5\left(a+b\right.}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 0.37

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(35a^3x+35a^2bx^3+21ab^2x^5+5b^3x^7\right)}{35\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*b^3*x^7))/
(35*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 56, normalized size = 0.35

method	result	size
gosper	$\frac{x\left(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{35\left(b\,x^2+a\right)^3}$	56
default	$\frac{x\left(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{35\left(b\,x^2+a\right)^3}$	56
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\;b^3x^7}{7b\,x^2+7a}+\frac{3\sqrt{\left(b\,x^2+a\right)^2}\;a\,b^2x^5}{5\left(b\,x^2+a\right)}+\frac{\sqrt{\left(b\,x^2+a\right)^2}\;a^2b\,x^3}{b\,x^2+a}+\frac{\sqrt{\left(b\,x^2+a\right)^2}\;a^3x}{b\,x^2+a}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)/(b*
x^2+a)^3
```

Maxima [A]
time = 0.30, size = 31, normalized size = 0.19

$$\frac{1}{7}\,b^3x^7+\frac{3}{5}\,ab^2x^5+a^2bx^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Fricas [A]
time = 0.37, size = 31, normalized size = 0.19

$$\frac{1}{7}b^3x^7 + \frac{3}{5}ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2), x)
```

Giac [A]
time = 4.07, size = 63, normalized size = 0.40

$$\frac{1}{7}b^3x^7\text{sgn}\left(bx^2 + a\right) + \frac{3}{5}ab^2x^5\text{sgn}\left(bx^2 + a\right) + a^2bx^3\text{sgn}\left(bx^2 + a\right) + a^3x\text{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7*sgn(b*x^2 + a) + 3/5*a*b^2*x^5*sgn(b*x^2 + a) + a^2*b*x^3*sgn(b
*x^2 + a) + a^3*x*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.577 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^2} dx$

Optimal. Leaf size=158

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}+\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^5\sqrt{a^2+2abx^2+b^2x}}{5(a+bx^2)}$$

```
[Out] -a^3*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+3*a^2*b*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a
)+a*b^2*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/5*b^3*x^5*((b*x^2+a)^2)^(1/2)/(
b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (3*a^2*b*x*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (a*b^2*x^3*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(a + b*x^2) + (b^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a +
b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{x^2}\,dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(3a^2b^4+\frac{a^3b^3}{x^2}+3ab^5x^2+b^6x^4\right)dx}{b^2(ab+b^2x^2)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)} + \frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{ab^2x^3\sqrt{a^2+2abx}}{a+bx^2}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 0.38

$$\frac{\sqrt{(a+bx^2)^2}\,(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6)}{5x(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(5*x*
(a + b*x^2))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.37

method	result	size
gosper	$-\frac{(-b^3x^6-5a\,b^2x^4-15a^2b\,x^2+5a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{5x(b\,x^2+a)^3}$	58
default	$-\frac{(-b^3x^6-5a\,b^2x^4-15a^2b\,x^2+5a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{5x(b\,x^2+a)^3}$	58
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,b(\frac{1}{5}b^2x^5+ab\,x^3+3a^2x)}{b\,x^2+a} - \frac{a^3\sqrt{(b\,x^2+a)^2}}{x(b\,x^2+a)}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*((b*x^2+a)^2)^(3/2)/x/(b*x^2
+a)^3
```

Maxima [A]
time = 0.28, size = 32, normalized size = 0.20

$$\frac{1}{5}\,b^3x^5+ab^2x^3+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

Fricas [A]
time = 0.41, size = 36, normalized size = 0.23

$$\frac{b^3x^6 + 5\,ab^2x^4 + 15\,a^2bx^2 - 5\,a^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^2)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**2, x)
```

Giac [A]
time = 4.97, size = 64, normalized size = 0.41

$$\frac{1}{5}\,b^3x^5\mathrm{sgn}\left(bx^2 + a\right) + ab^2x^3\mathrm{sgn}\left(bx^2 + a\right) + 3\,a^2bx\mathrm{sgn}\left(bx^2 + a\right) - \frac{a^3\mathrm{sgn}(bx^2 + a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5*sgn(b*x^2 + a) + a*b^2*x^3*sgn(b*x^2 + a) + 3*a^2*b*x*sgn(b*x^2
 + a) - a^3*sgn(b*x^2 + a)/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^2, x)
```

3.578 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=161

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}+\frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}$$

```
[Out] -1/3*a^3*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-3*a^2*b*((b*x^2+a)^2)^(1/2)/x/(b
*x^2+a)+3*a*b^2*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*b^3*x^3*((b*x^2+a)^2)^(
1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}+\frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

```
[Out] -1/3*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (3*a*b^2*x*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(a + b*x^2) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3
*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{x^4}\,dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(3ab^5+\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^2}+b^6x^2\right)dx}{b^2(ab+b^2x^2)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)} + \frac{3ab^2x\sqrt{a^2+2abx^2}}{a+bx^2}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(a^3+9a^2bx^2-9ab^2x^4-b^3x^6)}{3x^3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

```
[Out] -1/3*(Sqrt[(a + b*x^2)^2]*(a^3 + 9*a^2*b*x^2 - 9*a*b^2*x^4 - b^3*x^6))/(x^3
*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 56, normalized size = 0.35

method	result	size
gosper	$-\frac{(-b^3x^6-9a\,b^2x^4+9a^2b\,x^2+a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{3x^3(b\,x^2+a)^3}$	56
default	$-\frac{(-b^3x^6-9a\,b^2x^4+9a^2b\,x^2+a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{3x^3(b\,x^2+a)^3}$	56
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,b^2(\frac{1}{3}b\,x^3+3ax)}{b\,x^2+a}+\frac{\sqrt{(b\,x^2+a)^2}\,(-3a^2b\,x^2-\frac{1}{3}a^3)}{(b\,x^2+a)x^3}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-b^3*x^6-9*a*b^2*x^4+9*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^3/(b*x^2+
a)^3
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.20

$$\frac{1}{3}\,b^3x^3+3\,ab^2x-\frac{3\,a^2b}{x}-\frac{a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3*a*b^2*x - 3*a^2*b/x - 1/3*a^3/x^3
```

Fricas [A]
time = 0.36, size = 36, normalized size = 0.22

$$\frac{b^3x^6 + 9\,ab^2x^4 - 9\,a^2bx^2 - a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**4, x)
```

Giac [A]
time = 3.10, size = 67, normalized size = 0.42

$$\frac{1}{3}\,b^3x^3\mathrm{sgn}\left(bx^2 + a\right) + 3\,ab^2x\mathrm{sgn}\left(bx^2 + a\right) - \frac{9\,a^2bx^2\mathrm{sgn}(bx^2 + a) + a^3\mathrm{sgn}(bx^2 + a)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sgn(b*x^2 + a) + 3*a*b^2*x*sgn(b*x^2 + a) - 1/3*(9*a^2*b*x^2*sg
n(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^4,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^4, x)
```

3.579 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=158

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] -1/5*a^3*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-a^2*b*((b*x^2+a)^2)^(1/2)/x^3/(b
*x^2+a)-3*a*b^2*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+b^3*x*((b*x^2+a)^2)^(1/2)/(
b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}+\frac{b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

```
[Out] -1/5*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(x*(a + b*x^2)) + (b^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a
 + b*x^2)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{x^6}\,dx}{b^2(ab+b^2x^2)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^6+\frac{a^3b^3}{x^6}+\frac{3a^2b^4}{x^4}+\frac{3ab^5}{x^2}\right)dx}{b^2(ab+b^2x^2)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)} - \frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3(a+bx^2)} - \frac{3ab^2\sqrt{a^2+2abx^2+}}{x(a+bx^2)}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(a^3+5a^2bx^2+15ab^2x^4-5b^3x^6)}{5x^5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
[Out] -1/5*(Sqrt[(a + b*x^2)^2]*(a^3 + 5*a^2*b*x^2 + 15*a*b^2*x^4 - 5*b^3*x^6))/(
x^5*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 56, normalized size = 0.35

method	result	size
gosper	$-\frac{(-5b^3x^6+15a\,b^2x^4+5a^2b\,x^2+a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{5(b\,x^2+a)^3x^5}$	56
default	$-\frac{(-5b^3x^6+15a\,b^2x^4+5a^2b\,x^2+a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{5(b\,x^2+a)^3x^5}$	56
risch	$\frac{b^3x\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}+\frac{\sqrt{(b\,x^2+a)^2}\,(-3a\,b^2x^4-a^2b\,x^2-\frac{1}{5}a^3)}{(b\,x^2+a)x^5}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
[Out] -1/5*(-5*b^3*x^6+15*a*b^2*x^4+5*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/(b*x^2+a
)^3/x^5
```

Maxima [A]
time = 0.29, size = 32, normalized size = 0.20

$$b^3x-\frac{3\,ab^2}{x}-\frac{a^2b}{x^3}-\frac{a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] b^3*x - 3*a*b^2/x - a^2*b/x^3 - 1/5*a^3/x^5
```

Fricas [A]
time = 0.37, size = 37, normalized size = 0.23

$$\frac{5\,b^3x^6-15\,ab^2x^4-5\,a^2bx^2-a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**6, x)
```

Giac [A]
time = 3.12, size = 66, normalized size = 0.42

$$b^3x\mathrm{sgn}\left(bx^2+a\right)-\frac{15\,ab^2x^4\mathrm{sgn}(bx^2+a)+5\,a^2bx^2\mathrm{sgn}(bx^2+a)+a^3\mathrm{sgn}(bx^2+a)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] b^3*x*sgn(b*x^2 + a) - 1/5*(15*a*b^2*x^4*sgn(b*x^2 + a) + 5*a^2*b*x^2*sgn(b
*x^2 + a) + a^3*sgn(b*x^2 + a))/x^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^6, x)
```

3.580 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=163

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

```
[Out] -1/7*a^3*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-3/5*a^2*b*((b*x^2+a)^2)^(1/2)/x^
5/(b*x^2+a)-a*b^2*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-b^3*((b*x^2+a)^2)^(1/2)
/x/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/7*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^7*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/
(x*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{x^8}\,dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^4}+\frac{b^6}{x^2}\right)dx}{b^2(ab+b^2x^2)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)} - \frac{ab^2\sqrt{a^2+2abx^2+b}}{x^3(a+bx^2)}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(5a^3+21a^2bx^2+35ab^2x^4+35b^3x^6)}{35x^7(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/35*(Sqrt[(a + b*x^2)^2]*(5*a^3 + 21*a^2*b*x^2 + 35*a*b^2*x^4 + 35*b^3*x^
6))/(x^7*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.36

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-b^3x^6-a\,b^2x^4-\frac{3}{5}a^2b\,x^2-\frac{1}{7}a^3\right)}{(b\,x^2+a)x^7}$	57
gosper	$-\frac{(35b^3x^6+35a\,b^2x^4+21a^2b\,x^2+5a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{35x^7(b\,x^2+a)^3}$	58
default	$-\frac{(35b^3x^6+35a\,b^2x^4+21a^2b\,x^2+5a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{35x^7(b\,x^2+a)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(35*b^3*x^6+35*a*b^2*x^4+21*a^2*b*x^2+5*a^3)*((b*x^2+a)^2)^(3/2)/x^7/
(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.21

$$-\frac{b^3}{x}-\frac{ab^2}{x^3}-\frac{3\,a^2b}{5\,x^5}-\frac{a^3}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="maxima")
[Out] -b^3/x - a*b^2/x^3 - 3/5*a^2*b/x^5 - 1/7*a^3/x^7
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.23

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="fricas")
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**8,x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**8, x)
```

Giac [A]
time = 4.84, size = 69, normalized size = 0.42

$$-\frac{35\,b^3x^6\mathrm{sgn}(bx^2+a)+35\,ab^2x^4\mathrm{sgn}(bx^2+a)+21\,a^2bx^2\mathrm{sgn}(bx^2+a)+5\,a^3\mathrm{sgn}(bx^2+a)}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="giac")
[Out] -1/35*(35*b^3*x^6*sgn(b*x^2 + a) + 35*a*b^2*x^4*sgn(b*x^2 + a) + 21*a^2*b*x
^2*sgn(b*x^2 + a) + 5*a^3*sgn(b*x^2 + a))/x^7
```

Mupad [B]
time = 4.25, size = 151, normalized size = 0.93

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x\,\left(b\,x^2+a\right)}-\frac{a\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^3\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{5\,x^5\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^8,x)
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (b^3*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(x*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(x^3*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))/(5*x^5*(a + b*x^2))
```

3.581 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{10}} dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}$$

```
[Out] -1/9*a^3*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-3/7*a^2*b*((b*x^2+a)^2)^(1/2)/x^
7/(b*x^2+a)-3/5*a*b^2*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-1/3*b^3*((b*x^2+a)^
2)^(1/2)/x^3/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

```
[Out] -1/9*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^9*(a + b*x^2)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(3*x^3*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{x^{10}}\,dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{10}}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^6}+\frac{b^6}{x^4}\right)dx}{b^2(ab+b^2x^2)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)} - \frac{3ab^2\sqrt{a^2+2abx^2}}{5x^5(a+bx^2}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(35a^3+135a^2bx^2+189ab^2x^4+105b^3x^6)}{315x^9(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

```
[Out] -1/315*(Sqrt[(a + b*x^2)^2]*(35*a^3 + 135*a^2*b*x^2 + 189*a*b^2*x^4 + 105*b
^3*x^6))/(x^9*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{3}b^3x^6-\frac{3}{5}a\,b^2x^4-\frac{3}{7}a^2b\,x^2-\frac{1}{9}a^3\right)}{(b\,x^2+a)x^9}$	57
gosper	$-\frac{(105b^3x^6+189a\,b^2x^4+135a^2b\,x^2+35a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{315(b\,x^2+a)^3x^9}$	58
default	$-\frac{(105b^3x^6+189a\,b^2x^4+135a^2b\,x^2+35a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{315(b\,x^2+a)^3x^9}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(105*b^3*x^6+189*a*b^2*x^4+135*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3/x^9
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.21

$$-\frac{b^3}{3\,x^3}-\frac{3\,ab^2}{5\,x^5}-\frac{3\,a^2b}{7\,x^7}-\frac{a^3}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="maxima")
[Out] -1/3*b^3/x^3 - 3/5*a*b^2/x^5 - 3/7*a^2*b/x^7 - 1/9*a^3/x^9
```

Fricas [A]
time = 0.32, size = 37, normalized size = 0.22

$$-\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="fricas")
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**10,x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**10, x)
```

Giac [A]
time = 6.16, size = 69, normalized size = 0.41

$$-\frac{105\,b^3x^6\mathrm{sgn}(bx^2+a)+189\,ab^2x^4\mathrm{sgn}(bx^2+a)+135\,a^2bx^2\mathrm{sgn}(bx^2+a)+35\,a^3\mathrm{sgn}(bx^2+a)}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="giac")
[Out] -1/315*(105*b^3*x^6*sgn(b*x^2 + a) + 189*a*b^2*x^4*sgn(b*x^2 + a) + 135*a^2
*b*x^2*sgn(b*x^2 + a) + 35*a^3*sgn(b*x^2 + a))/x^9
```

Mupad [B]
time = 4.26, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^3\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{5\,x^5\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^10,x)
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (b^3*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(7*x^7*(a + b*x^2))
```

3.582 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}$$

```
[Out] -1/11*a^3*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-1/3*a^2*b*((b*x^2+a)^2)^(1/2)/
x^9/(b*x^2+a)-3/7*a*b^2*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-1/5*b^3*((b*x^2+a
)^2)^(1/2)/x^5/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

```
[Out] -1/11*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^11*(a + b*x^2)) - (a^2*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(5*x^5*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{x^{12}}\,dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{12}}+\frac{3a^2b^4}{x^{10}}+\frac{3ab^5}{x^8}+\frac{b^6}{x^6}\right)dx}{b^2(ab+b^2x^2)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)} - \frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)} - \frac{3ab^2\sqrt{a^2+2abx^2+b}}{7x^7(a+bx^2)}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(105a^3+385a^2bx^2+495ab^2x^4+231b^3x^6)}{1155x^{11}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

```
[Out] -1/1155*(Sqrt[(a + b*x^2)^2]*(105*a^3 + 385*a^2*b*x^2 + 495*a*b^2*x^4 + 231
*b^3*x^6))/(x^11*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{5}b^3x^6-\frac{3}{7}a\,b^2x^4-\frac{1}{3}a^2b\,x^2-\frac{1}{11}a^3\right)}{(b\,x^2+a)x^{11}}$	57
gosper	$-\frac{(231b^3x^6+495a\,b^2x^4+385a^2b\,x^2+105a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{1155x^{11}(b\,x^2+a)^3}$	58
default	$-\frac{(231b^3x^6+495a\,b^2x^4+385a^2b\,x^2+105a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{1155x^{11}(b\,x^2+a)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1155*(231*b^3*x^6+495*a*b^2*x^4+385*a^2*b*x^2+105*a^3)*((b*x^2+a)^2)^(3/
2)/x^11/(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.21

$$-\frac{b^3}{5\,x^5}-\frac{3\,ab^2}{7\,x^7}-\frac{a^2b}{3\,x^9}-\frac{a^3}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="maxima")
[Out] -1/5*b^3/x^5 - 3/7*a*b^2/x^7 - 1/3*a^2*b/x^9 - 1/11*a^3/x^11
```

Fricas [A]
time = 0.34, size = 37, normalized size = 0.22

$$-\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="fricas")
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**12,x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**12, x)
```

Giac [A]
time = 4.10, size = 69, normalized size = 0.41

$$-\frac{231\,b^3x^6\mathrm{sgn}(bx^2+a)+495\,ab^2x^4\mathrm{sgn}(bx^2+a)+385\,a^2bx^2\mathrm{sgn}(bx^2+a)+105\,a^3\mathrm{sgn}(bx^2+a)}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="giac")
[Out] -1/1155*(231*b^3*x^6*sgn(b*x^2 + a) + 495*a*b^2*x^4*sgn(b*x^2 + a) + 385*a^
2*b*x^2*sgn(b*x^2 + a) + 105*a^3*sgn(b*x^2 + a))/x^11
```

Mupad [B]
time = 4.55, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{5\,x^5\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}-\frac{a^2\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^9\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^12,x)
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(3*x^9*(a + b*x^2))
```

3.583 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

```
[Out] -1/13*a^3*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-3/11*a^2*b*((b*x^2+a)^2)^(1/2)
/x^11/(b*x^2+a)-1/3*a*b^2*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-1/7*b^3*((b*x^2
+a)^2)^(1/2)/x^7/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

```
[Out] -1/13*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^13*(a + b*x^2)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (a*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(7*x^7*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{14}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{x^{14}}dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{14}}+\frac{3a^2b^4}{x^{12}}+\frac{3ab^5}{x^{10}}+\frac{b^6}{x^8}\right)dx}{b^2(ab+b^2x^2)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)} - \frac{ab^2\sqrt{a^2+2abx^2+}}{3x^9(a+bx^2}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(231a^3+819a^2bx^2+1001ab^2x^4+429b^3x^6)}{3003x^{13}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

```
[Out] -1/3003*(Sqrt[(a + b*x^2)^2]*(231*a^3 + 819*a^2*b*x^2 + 1001*a*b^2*x^4 + 42
9*b^3*x^6))/(x^13*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{7}b^3x^6-\frac{1}{3}a\,b^2x^4-\frac{3}{11}a^2b\,x^2-\frac{1}{13}a^3\right)}{(b\,x^2+a)x^{13}}$	57
gosper	$-\frac{(429b^3x^6+1001a\,b^2x^4+819a^2b\,x^2+231a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{3003x^{13}(b\,x^2+a)^3}$	58
default	$-\frac{(429b^3x^6+1001a\,b^2x^4+819a^2b\,x^2+231a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{3003x^{13}(b\,x^2+a)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3003*(429*b^3*x^6+1001*a*b^2*x^4+819*a^2*b*x^2+231*a^3)*((b*x^2+a)^2)^(3
/2)/x^13/(b*x^2+a)^3
```

Maxima [A]
time = 0.32, size = 35, normalized size = 0.21

$$-\frac{b^3}{7\,x^7}-\frac{ab^2}{3\,x^9}-\frac{3\,a^2b}{11\,x^{11}}-\frac{a^3}{13\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="maxima")
[Out] -1/7*b^3/x^7 - 1/3*a*b^2/x^9 - 3/11*a^2*b/x^11 - 1/13*a^3/x^13
```

Fricas [A]
time = 0.33, size = 37, normalized size = 0.22

$$-\frac{429\,b^3x^6+1001\,ab^2x^4+819\,a^2bx^2+231\,a^3}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="fricas")
[Out] -1/3003*(429*b^3*x^6 + 1001*a*b^2*x^4 + 819*a^2*b*x^2 + 231*a^3)/x^13
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**14,x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**14, x)
```

Giac [A]
time = 5.69, size = 69, normalized size = 0.41

$$-\frac{429\,b^3x^6\mathrm{sgn}(bx^2+a)+1001\,ab^2x^4\mathrm{sgn}(bx^2+a)+819\,a^2bx^2\mathrm{sgn}(bx^2+a)+231\,a^3\mathrm{sgn}(bx^2+a)}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="giac")
[Out] -1/3003*(429*b^3*x^6*sgn(b*x^2 + a) + 1001*a*b^2*x^4*sgn(b*x^2 + a) + 819*a
^2*b*x^2*sgn(b*x^2 + a) + 231*a^3*sgn(b*x^2 + a))/x^13
```

Mupad [B]
time = 4.63, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}-\frac{a\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^9\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^14,x)
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(3*x^9*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(11*x^11*(a + b*x^2))
```

3.584 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{16}} dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}$$

```
[Out] -1/15*a^3*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-3/13*a^2*b*((b*x^2+a)^2)^(1/2)
/x^13/(b*x^2+a)-3/11*a*b^2*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-1/9*b^3*((b*x
^2+a)^2)^(1/2)/x^9/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```

```
[Out] -1/15*(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^15*(a + b*x^2)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (3*a*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (b^3*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(9*x^9*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^3}{x^{16}}\,dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{x^{16}}+\frac{3a^2b^4}{x^{14}}+\frac{3ab^5}{x^{12}}+\frac{b^6}{x^{10}}\right)dx}{b^2\left(ab+b^2x^2\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{3ab^2\sqrt{a^2+2abx^2+}}{11x^{11}\left(a+bx^2\right.}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(429a^3+1485a^2bx^2+1755ab^2x^4+715b^3x^6\right)}{6435x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```

```
[Out] -1/6435*(Sqrt[(a + b*x^2)^2]*(429*a^3 + 1485*a^2*b*x^2 + 1755*a*b^2*x^4 + 7
15*b^3*x^6))/(x^15*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(-\frac{1}{15}a^3-\frac{3}{13}a^2b\,x^2-\frac{3}{11}a\,b^2x^4-\frac{1}{9}b^3x^6\right)}{\left(b\,x^2+a\right)x^{15}}$	57
gosper	$-\frac{\left(715b^3x^6+1755a\,b^2x^4+1485a^2b\,x^2+429a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{6435x^{15}\left(b\,x^2+a\right)^3}$	58
default	$-\frac{\left(715b^3x^6+1755a\,b^2x^4+1485a^2b\,x^2+429a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{6435x^{15}\left(b\,x^2+a\right)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6435*(715*b^3*x^6+1755*a*b^2*x^4+1485*a^2*b*x^2+429*a^3)*((b*x^2+a)^2)^(
3/2)/x^15/(b*x^2+a)^3
```

Maxima [A]
time = 0.30, size = 35, normalized size = 0.21

$$-\frac{b^3}{9\,x^9}-\frac{3\,ab^2}{11\,x^{11}}-\frac{3\,a^2b}{13\,x^{13}}-\frac{a^3}{15\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="maxima")
[Out] -1/9*b^3/x^9 - 3/11*a*b^2/x^11 - 3/13*a^2*b/x^13 - 1/15*a^3/x^15
```

Fricas [A]
time = 0.33, size = 37, normalized size = 0.22

$$-\frac{715\,b^3x^6+1755\,ab^2x^4+1485\,a^2bx^2+429\,a^3}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="fricas")
[Out] -1/6435*(715*b^3*x^6 + 1755*a*b^2*x^4 + 1485*a^2*b*x^2 + 429*a^3)/x^15
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**16,x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**16, x)
```

Giac [A]
time = 4.96, size = 69, normalized size = 0.41

$$-\frac{715\,b^3x^6\mathrm{sgn}(bx^2+a)+1755\,ab^2x^4\mathrm{sgn}(bx^2+a)+1485\,a^2bx^2\mathrm{sgn}(bx^2+a)+429\,a^3\mathrm{sgn}(bx^2+a)}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="giac")
[Out] -1/6435*(715*b^3*x^6*sgn(b*x^2 + a) + 1755*a*b^2*x^4*sgn(b*x^2 + a) + 1485*
a^2*b*x^2*sgn(b*x^2 + a) + 429*a^3*sgn(b*x^2 + a))/x^15
```

Mupad [B]
time = 4.30, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{15\,x^{15}\,\left(b\,x^2+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^16,x)
[Out] - (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(15*x^15*(a + b*x^2)) - (b^3*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2))/(13*x^13*(a + b*x^2))
```

3.585 $\int x^{13}(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{5a^3b^2x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}$$

```
[Out] 1/14*a^5*x^14*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/16*a^4*b*x^16*((b*x^2+a)^2)^(
1/2)/(b*x^2+a)+5/9*a^3*b^2*x^18*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/2*a^2*b^3*x
^20*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/22*a*b^4*x^22*((b*x^2+a)^2)^(1/2)/(b*x^
2+a)+1/24*b^5*x^24*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.11, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$\frac{b^5x^{24}\sqrt{a^2+2abx^2+b^2x^4}}{24\left(a+bx^2\right)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\left(a+bx^2\right)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{5a^3b^2x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*a^4*b*x^16
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*(a + b*x^2)) + (5*a^3*b^2*x^18*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(22*(a + b*x^2)) + (b^5*x^24*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*(a
 + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^6\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^6(ab+b^2x)^5\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(a^5b^5x^6+5a^4b^6x^7+10a^3b^7x^8+10a^2b^8x^9\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{5a^3b^2x^{18}\sqrt{}}{9}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{14}\sqrt{\left(a+bx^2\right)^2}\left(792a^5+3465a^4bx^2+6160a^3b^2x^4+5544a^2b^3x^6+2520ab^4x^8+462b^5x^{10}\right)}{11088\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
[Out] (x^14*Sqrt[(a + b*x^2)^2]*(792*a^5 + 3465*a^4*b*x^2 + 6160*a^3*b^2*x^4 + 55
44*a^2*b^3*x^6 + 2520*a*b^4*x^8 + 462*b^5*x^10))/(11088*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^{14}\left(462b^5x^{10}+2520b^4a\,x^8+5544a^2b^3x^6+6160b^2a^3x^4+3465b\,a^4x^2+792a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{11088\left(b\,x^2+a\right)^5}$
default	$\frac{x^{14}\left(462b^5x^{10}+2520b^4a\,x^8+5544a^2b^3x^6+6160b^2a^3x^4+3465b\,a^4x^2+792a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{11088\left(b\,x^2+a\right)^5}$
risch	$\frac{a^5x^{14}\sqrt{\left(b\,x^2+a\right)^2}}{14b\,x^2+14a}+\frac{5a^4b\,x^{16}\sqrt{\left(b\,x^2+a\right)^2}}{16\left(b\,x^2+a\right)}+\frac{5a^3b^2x^{18}\sqrt{\left(b\,x^2+a\right)^2}}{9\left(b\,x^2+a\right)}+\frac{a^2b^3x^{20}\sqrt{\left(b\,x^2+a\right)^2}}{2b\,x^2+2a}+\frac{5a\,b^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11088*x^14*(462*b^5*x^10+2520*a*b^4*x^8+5544*a^2*b^3*x^6+6160*a^3*b^2*x^4
+3465*a^4*b*x^2+792*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.22

$$\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

Fricas [A]
time = 0.33, size = 57, normalized size = 0.22

$$\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/1
6*a^4*b*x^16 + 1/14*a^5*x^14
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{13}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**13*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.41, size = 105, normalized size = 0.41

$$\frac{1}{24}\,b^5x^{24}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{22}\,ab^4x^{22}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^2b^3x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^3b^2x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{16}\,a^4bx^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{14}\,a^5x^{14}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24*sgn(b*x^2 + a) + 5/22*a*b^4*x^22*sgn(b*x^2 + a) + 1/2*a^2*b^3
*x^20*sgn(b*x^2 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^2 + a) + 5/16*a^4*b*x^16*sg
n(b*x^2 + a) + 1/14*a^5*x^14*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{13}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^13*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.586 $\int x^{11}(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12\,(a+bx^2)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\,(a+bx^2)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{8\,(a+bx^2)}+\frac{5a^2b^3x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}$$

```
[Out] 1/12*a^5*x^12*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/14*a^4*b*x^14*((b*x^2+a)^2)^(
1/2)/(b*x^2+a)+5/8*a^3*b^2*x^16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/9*a^2*b^3*x
^18*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/4*a*b^4*x^20*((b*x^2+a)^2)^(1/2)/(b*x^2
+a)+1/22*b^5*x^22*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.11, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$\frac{b^5x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\,(a+bx^2)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{4\,(a+bx^2)}+\frac{5a^2b^3x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}+\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12\,(a+bx^2)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\,(a+bx^2)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{8\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*(a + b*x^2)) + (5*a^4*b*x^14
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*a^3*b^2*x^16*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (5*a^2*b^3*x^18*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(4*(a + b*x^2)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*(a
+ b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^5\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^5\left(ab+b^2x\right)^5\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(a^5b^5x^5+5a^4b^6x^6+10a^3b^7x^7+10a^2b^8x^8\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$
$$=\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12\left(a+bx^2\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{5a^3b^2x^{16}\sqrt{}}{8}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^{12}\sqrt{\left(a+bx^2\right)^2}\left(462a^5+1980a^4bx^2+3465a^3b^2x^4+3080a^2b^3x^6+1386ab^4x^8+252b^5x^{10}\right)}{5544\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
[Out] (x^12*Sqrt[(a + b*x^2)^2]*(462*a^5 + 1980*a^4*b*x^2 + 3465*a^3*b^2*x^4 + 30
80*a^2*b^3*x^6 + 1386*a*b^4*x^8 + 252*b^5*x^10))/(5544*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^{12}\left(252b^5x^{10}+1386b^4a\,x^8+3080a^2b^3x^6+3465b^2a^3x^4+1980b\,a^4x^2+462a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{5544\left(b\,x^2+a\right)^5}$
default	$\frac{x^{12}\left(252b^5x^{10}+1386b^4a\,x^8+3080a^2b^3x^6+3465b^2a^3x^4+1980b\,a^4x^2+462a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{5544\left(b\,x^2+a\right)^5}$
risch	$\frac{a^5x^{12}\sqrt{\left(b\,x^2+a\right)^2}}{12b\,x^2+12a}+\frac{5a^4b\,x^{14}\sqrt{\left(b\,x^2+a\right)^2}}{14\left(b\,x^2+a\right)}+\frac{5a^3b^2x^{16}\sqrt{\left(b\,x^2+a\right)^2}}{8\left(b\,x^2+a\right)}+\frac{5a^2b^3x^{18}\sqrt{\left(b\,x^2+a\right)^2}}{9\left(b\,x^2+a\right)}+\frac{a\,b^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5544*x^12*(252*b^5*x^10+1386*a*b^4*x^8+3080*a^2*b^3*x^6+3465*a^3*b^2*x^4+
1980*a^4*b*x^2+462*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.31, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

Fricas [A]
time = 0.32, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/14
*a^4*b*x^14 + 1/12*a^5*x^12
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**11*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.05, size = 105, normalized size = 0.41

$$\frac{1}{22}\,b^5x^{22}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}\,ab^4x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^2b^3x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^3b^2x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{14}\,a^4bx^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{12}\,a^5x^{12}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22*sgn(b*x^2 + a) + 1/4*a*b^4*x^20*sgn(b*x^2 + a) + 5/9*a^2*b^3*
x^18*sgn(b*x^2 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^2 + a) + 5/14*a^4*b*x^14*sgn
(b*x^2 + a) + 1/12*a^5*x^12*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{11}\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^11*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.587 $\int x^9(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal. Leaf size=201

$$\frac{a^4(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^5}-\frac{2a^3(a+bx^2)^6\sqrt{a^2+2abx^2+b^2x^4}}{7b^5}+\frac{3a^2(a+bx^2)^7\sqrt{a^2+2abx^2+b^2x^4}}{8b^5}$$

```
[Out] 1/12*a^4*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/b^5-2/7*a^3*(b*x^2+a)^6*((b*x^2+a)
^2)^(1/2)/b^5+3/8*a^2*(b*x^2+a)^7*((b*x^2+a)^2)^(1/2)/b^5-2/9*a*(b*x^2+a)^8
*((b*x^2+a)^2)^(1/2)/b^5+1/20*(b*x^2+a)^9*((b*x^2+a)^2)^(1/2)/b^5
```

Rubi [A]
time = 0.09, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1125, 659}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^9}{20b^5}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^8}{9b^5}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^7}{8b^5}+\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{12b^5}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^6}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^4*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^5) - (2*a^3*(a +
b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^5) + (3*a^2*(a + b*x^2)^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^5) - (2*a*(a + b*x^2)^8*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(9*b^5) + ((a + b*x^2)^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
/(20*b^5)
```

Rule 659

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^9 \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \frac{1}{2}\text{Subst}\left(\int x^4 \left(a^2 + 2abx + b^2x^2\right)^{5/2} dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^4(ab+b^2x)^5}{b^4} - \frac{4a^3(ab+b^2x)^6}{b^5} + \frac{6a^2(ab+b^2x)^7}{b^6} - \right.\right.}{2b^4\left(ab + b^2x^2\right)}$$

$$= \frac{a^4(a+bx^2)^5\sqrt{a^2 + 2abx^2 + b^2x^4}}{12b^5} - \frac{2a^3(a+bx^2)^6\sqrt{a^2 + 2abx^2 + b^2x^4}}{7b^5}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.41

$$\frac{x^{10}\sqrt{(a+bx^2)^2}\left(252a^5 + 1050a^4bx^2 + 1800a^3b^2x^4 + 1575a^2b^3x^6 + 700ab^4x^8 + 126b^5x^{10}\right)}{2520\left(a + bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^2)^2]*(252*a^5 + 1050*a^4*b*x^2 + 1800*a^3*b^2*x^4 + 15
75*a^2*b^3*x^6 + 700*a*b^4*x^8 + 126*b^5*x^10))/(2520*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.40

method	result
gosper	$\frac{x^{10}\left(126b^5x^{10}+700b^4a\,x^8+1575a^2b^3x^6+1800b^2a^3x^4+1050b\,a^4x^2+252a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{2520\left(b\,x^2+a\right)^5}$
default	$\frac{x^{10}\left(126b^5x^{10}+700b^4a\,x^8+1575a^2b^3x^6+1800b^2a^3x^4+1050b\,a^4x^2+252a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{2520\left(b\,x^2+a\right)^5}$
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,a^5x^{10}}{10b\,x^2+10a} + \frac{5\sqrt{\left(b\,x^2+a\right)^2}\,b\,a^4x^{12}}{12\left(b\,x^2+a\right)} + \frac{5\sqrt{\left(b\,x^2+a\right)^2}\,b^2a^3x^{14}}{7\left(b\,x^2+a\right)} + \frac{5\sqrt{\left(b\,x^2+a\right)^2}\,a^2b^3x^{16}}{8\left(b\,x^2+a\right)} + -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2520*x^10*(126*b^5*x^10+700*a*b^4*x^8+1575*a^2*b^3*x^6+1800*a^3*b^2*x^4+1
050*a^4*b*x^2+252*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.33, size = 57, normalized size = 0.28

$$\frac{1}{20}\,b^5x^{20} + \frac{5}{18}\,ab^4x^{18} + \frac{5}{8}\,a^2b^3x^{16} + \frac{5}{7}\,a^3b^2x^{14} + \frac{5}{12}\,a^4bx^{12} + \frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

Fricas [A]
time = 0.33, size = 57, normalized size = 0.28

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{18}\,ab^4x^{18}+\frac{5}{8}\,a^2b^3x^{16}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/1
2*a^4*b*x^12 + 1/10*a^5*x^10
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^9\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**9*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.60, size = 105, normalized size = 0.52

$$\frac{1}{20}\,b^5x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{18}\,ab^4x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^2b^3x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}\,a^3b^2x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{12}\,a^4bx^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{10}\,a^5x^{10}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20*sgn(b*x^2 + a) + 5/18*a*b^4*x^18*sgn(b*x^2 + a) + 5/8*a^2*b^3
*x^16*sgn(b*x^2 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^2 + a) + 5/12*a^4*b*x^12*sg
n(b*x^2 + a) + 1/10*a^5*x^10*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^9\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.588 $\int x^7(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal. Leaf size=160

$$-\frac{a^3(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^4}+\frac{3a^2(a+bx^2)^6\sqrt{a^2+2abx^2+b^2x^4}}{14b^4}-\frac{3a(a+bx^2)^7\sqrt{a^2+2abx^2+b^2x^4}}{16b^4}$$

```
[Out] -1/12*a^3*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/b^4+3/14*a^2*(b*x^2+a)^6*((b*x^2+
a)^2)^(1/2)/b^4-3/16*a*(b*x^2+a)^7*((b*x^2+a)^2)^(1/2)/b^4+1/18*(b*x^2+a)^8
*((b*x^2+a)^2)^(1/2)/b^4
```

Rubi [A]
time = 0.08, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^8}{18b^4}-\frac{3a\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^7}{16b^4}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^6}{14b^4}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/12*(a^3*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^4 + (3*a^2*(a +
 b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*b^4) - (3*a*(a + b*x^2)^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*b^4) + ((a + b*x^2)^8*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(18*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^3\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^3(ab+b^2x)^5\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(-\frac{a^3(ab+b^2x)^5}{b^3}+\frac{3a^2(ab+b^2x)^6}{b^4}-\frac{3a(ab+b^2x)^7}{b^5}+\right.\right.}{2b^4\left(ab+b^2x^2\right)}$$
$$=-\frac{a^3(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^4}+\frac{3a^2(a+bx^2)^6\sqrt{a^2+2abx^2+b^2x^4}}{14b^4}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.52

$$\frac{x^8\sqrt{(a+bx^2)^2}\left(126a^5+504a^4bx^2+840a^3b^2x^4+720a^2b^3x^6+315ab^4x^8+56b^5x^{10}\right)}{1008\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(126*a^5 + 504*a^4*b*x^2 + 840*a^3*b^2*x^4 + 720*a
^2*b^3*x^6 + 315*a*b^4*x^8 + 56*b^5*x^10))/(1008*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.50

method	result
gosper	$\frac{x^8\left(56b^5x^{10}+315b^4a\,x^8+720a^2b^3x^6+840b^2a^3x^4+504b\,a^4x^2+126a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{1008\left(b\,x^2+a\right)^5}$
default	$\frac{x^8\left(56b^5x^{10}+315b^4a\,x^8+720a^2b^3x^6+840b^2a^3x^4+504b\,a^4x^2+126a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{1008\left(b\,x^2+a\right)^5}$
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,a^5x^8}{8b\,x^2+8a}+\frac{\sqrt{\left(b\,x^2+a\right)^2}\,b\,a^4x^{10}}{2b\,x^2+2a}+\frac{5\sqrt{\left(b\,x^2+a\right)^2}\,b^2a^3x^{12}}{6\left(b\,x^2+a\right)}+\frac{5\sqrt{\left(b\,x^2+a\right)^2}\,a^2b^3x^{14}}{7\left(b\,x^2+a\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1008*x^8*(56*b^5*x^10+315*a*b^4*x^8+720*a^2*b^3*x^6+840*a^3*b^2*x^4+504*a
^4*b*x^2+126*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.30, size = 57, normalized size = 0.36

$$\frac{1}{18}\,b^5x^{18}+\frac{5}{16}\,ab^4x^{16}+\frac{5}{7}\,a^2b^3x^{14}+\frac{5}{6}\,a^3b^2x^{12}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

Fricas [A]
time = 0.35, size = 57, normalized size = 0.36

$$\frac{1}{18}\,b^5x^{18}+\frac{5}{16}\,ab^4x^{16}+\frac{5}{7}\,a^2b^3x^{14}+\frac{5}{6}\,a^3b^2x^{12}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/2
*a^4*b*x^10 + 1/8*a^5*x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**7*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.10, size = 105, normalized size = 0.66

$$\frac{1}{18}\,b^5x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{16}\,ab^4x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}\,a^2b^3x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{6}\,a^3b^2x^{12}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^4bx^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{8}\,a^5x^8\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/18*b^5*x^18*sgn(b*x^2 + a) + 5/16*a*b^4*x^16*sgn(b*x^2 + a) + 5/7*a^2*b^3
*x^14*sgn(b*x^2 + a) + 5/6*a^3*b^2*x^12*sgn(b*x^2 + a) + 1/2*a^4*b*x^10*sgn
(b*x^2 + a) + 1/8*a^5*x^8*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^7 \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.589 $\int x^5(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal. Leaf size=119

$$\frac{a^2(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^3}-\frac{a(a+bx^2)^6\sqrt{a^2+2abx^2+b^2x^4}}{7b^3}+\frac{(a+bx^2)^7\sqrt{a^2+2abx^2+b^2x^4}}{16b^3}$$

```
[Out] 1/12*a^2*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/b^3-1/7*a*(b*x^2+a)^6*((b*x^2+a)^2
)^(1/2)/b^3+1/16*(b*x^2+a)^7*((b*x^2+a)^2)^(1/2)/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^7}{16b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^6}{7b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^3) - (a*(a + b*x^
2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^3) + ((a + b*x^2)^7*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(16*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int x^2\left(ab+b^2x\right)^5\,dx,x,x^2\right)}{2b^4\left(ab+b^2x^2\right)}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^5}{b^2}-\frac{2a\left(ab+b^2x\right)^6}{b^3}+\frac{\left(ab+b^2x\right)^7}{b^4}\right)\,dx,x,\right.}{2b^4\left(ab+b^2x^2\right)}$$

$$=\frac{a^2\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^3}-\frac{a\left(a+bx^2\right)^6\sqrt{a^2+2abx^2+b^2x^4}}{7b^3}+\frac{(}{}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.70

$$\frac{x^6\sqrt{\left(a+bx^2\right)^2}\,\left(56a^5+210a^4bx^2+336a^3b^2x^4+280a^2b^3x^6+120ab^4x^8+21b^5x^{10}\right)}{336\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^2)^2]*(56*a^5 + 210*a^4*b*x^2 + 336*a^3*b^2*x^4 + 280*a^
2*b^3*x^6 + 120*a*b^4*x^8 + 21*b^5*x^10))/(336*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.67

method	result
gosper	$\frac{x^6\left(21b^5x^{10}+120b^4a\,x^8+280a^2b^3x^6+336b^2a^3x^4+210b\,a^4x^2+56a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{336\left(b\,x^2+a\right)^5}$
default	$\frac{x^6\left(21b^5x^{10}+120b^4a\,x^8+280a^2b^3x^6+336b^2a^3x^4+210b\,a^4x^2+56a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{336\left(b\,x^2+a\right)^5}$
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,a^5x^6}{6b\,x^2+6a}+\frac{5\sqrt{\left(b\,x^2+a\right)^2}\,b\,a^4x^8}{8\left(b\,x^2+a\right)}+\frac{\sqrt{\left(b\,x^2+a\right)^2}\,b^2a^3x^{10}}{b\,x^2+a}+\frac{5\sqrt{\left(b\,x^2+a\right)^2}\,a^2b^3x^{12}}{6\left(b\,x^2+a\right)}+\frac{5\sqrt{(b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/336*x^6*(21*b^5*x^10+120*a*b^4*x^8+280*a^2*b^3*x^6+336*a^3*b^2*x^4+210*a^
4*b*x^2+56*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.29, size = 56, normalized size = 0.47

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

Fricas [A]
time = 0.34, size = 56, normalized size = 0.47

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^4
*b*x^8 + 1/6*a^5*x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**5*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.21, size = 104, normalized size = 0.87

$$\frac{1}{16}\,b^5x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{14}\,ab^4x^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{6}\,a^2b^3x^{12}\mathrm{sgn}\left(bx^2+a\right)+a^3b^2x^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^4bx^8\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{6}\,a^5x^6\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16*sgn(b*x^2 + a) + 5/14*a*b^4*x^14*sgn(b*x^2 + a) + 5/6*a^2*b^3
*x^12*sgn(b*x^2 + a) + a^3*b^2*x^10*sgn(b*x^2 + a) + 5/8*a^4*b*x^8*sgn(b*x^
2 + a) + 1/6*a^5*x^6*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.590 $\int x^3(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal. Leaf size=67

$$-\frac{a(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}$$

```
[Out] -1/12*a*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/b^2+1/14*(b^2*x^4+2*a*b*x^2
+a^2)^(7/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {1125, 654, 623}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}-\frac{a(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/12*(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/b^2 + (a^2 + 2*a*b*
x^2 + b^2*x^4)^(7/2)/(14*b^2)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{2}\text{Subst}\left(\int x\left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^2\right)$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2} - \frac{a\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, x^2\right)}{2b}$$
$$= -\frac{a(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2} + \frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 1.24

$$\frac{x^4\sqrt{(a+bx^2)^2}\left(21a^5+70a^4bx^2+105a^3b^2x^4+84a^2b^3x^6+35ab^4x^8+6b^5x^{10}\right)}{84(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(21*a^5 + 70*a^4*b*x^2 + 105*a^3*b^2*x^4 + 84*a^2*
b^3*x^6 + 35*a*b^4*x^8 + 6*b^5*x^10))/(84*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 80, normalized size = 1.19

method	result
gosper	$\frac{x^4(6b^5x^{10}+35b^4a\,x^8+84a^2b^3x^6+105b^2a^3x^4+70b\,a^4x^2+21a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{84(b\,x^2+a)^5}$
default	$\frac{x^4(6b^5x^{10}+35b^4a\,x^8+84a^2b^3x^6+105b^2a^3x^4+70b\,a^4x^2+21a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{84(b\,x^2+a)^5}$
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,a^5x^4}{4b\,x^2+4a}+\frac{5\sqrt{(b\,x^2+a)^2}\,b\,a^4x^6}{6(b\,x^2+a)}+\frac{5\sqrt{(b\,x^2+a)^2}\,b^2a^3x^8}{4(b\,x^2+a)}+\frac{\sqrt{(b\,x^2+a)^2}\,a^2b^3x^{10}}{b\,x^2+a}+\frac{5\sqrt{(b\,$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/84*x^4*(6*b^5*x^10+35*a*b^4*x^8+84*a^2*b^3*x^6+105*a^3*b^2*x^4+70*a^4*b*x
^2+21*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.84

$$\frac{1}{14}b^5x^{14}+\frac{5}{12}ab^4x^{12}+a^2b^3x^{10}+\frac{5}{4}a^3b^2x^8+\frac{5}{6}a^4bx^6+\frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

Fricas [A]
time = 0.34, size = 56, normalized size = 0.84

$$\frac{1}{14}\,b^5x^{14}+\frac{5}{12}\,ab^4x^{12}+a^2b^3x^{10}+\frac{5}{4}\,a^3b^2x^8+\frac{5}{6}\,a^4bx^6+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4*
b*x^6 + 1/4*a^5*x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**3*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.32, size = 67, normalized size = 1.00

$$\frac{1}{84}\left(6\,b^5x^{14}+35\,ab^4x^{12}+84\,a^2b^3x^{10}+105\,a^3b^2x^8+70\,a^4bx^6+21\,a^5x^4\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/84*(6*b^5*x^14 + 35*a*b^4*x^12 + 84*a^2*b^3*x^10 + 105*a^3*b^2*x^8 + 70*a
^4*b*x^6 + 21*a^5*x^4)*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.591 $\int x(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^2)\,(a^2+2abx^2+b^2x^4)^{5/2}}{12b}$$

```
[Out] 1/12*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1121, 623}

$$\frac{(a+bx^2)\,(a^2+2abx^2+b^2x^4)^{5/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^2\right)\\ &= \frac{(a+bx^2)\,(a^2+2abx^2+b^2x^4)^{5/2}}{12b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.75

$$\frac{(a+bx^2)\left((a+bx^2)^2\right)^{5/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^(5/2))/(12*b)
```

Maple [A]
time = 0.04, size = 24, normalized size = 0.67

method	result	size
default	$\frac{(b\,x^2+a)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{12b}$	24
risch	$\frac{\sqrt{(b\,x^2+a)^2}\;(b\,x^2+a)^5}{12b}$	26
gosper	$\frac{x^2\left(b^5x^{10}+6b^4a\,x^8+15a^2b^3x^6+20b^2a^3x^4+15b\,a^4x^2+6a^5\right)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{12(b\,x^2+a)^5}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(b*x^2+a)*((b*x^2+a)^2)^(5/2)/b
```

Maxima [A]
time = 0.29, size = 57, normalized size = 1.58

$$\frac{1}{12}\,b^5x^{12}+\frac{1}{2}\,ab^4x^{10}+\frac{5}{4}\,a^2b^3x^8+\frac{5}{3}\,a^3b^2x^6+\frac{5}{4}\,a^4bx^4+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^6 + 5/4*a^
4*b*x^4 + 1/2*a^5*x^2
```

Fricas [A]
time = 0.35, size = 57, normalized size = 1.58

$$\frac{1}{12}\,b^5x^{12}+\frac{1}{2}\,ab^4x^{10}+\frac{5}{4}\,a^2b^3x^8+\frac{5}{3}\,a^3b^2x^6+\frac{5}{4}\,a^4bx^4+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^6 + 5/4*a^
4*b*x^4 + 1/2*a^5*x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x**2)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(32) = 64.
time = 3.46, size = 66, normalized size = 1.83

$$\frac{1}{12}\left(3\left(bx^4+2\,ax^2\right)a^4+3\left(bx^4+2\,ax^2\right)^2a^2b+\left(bx^4+2\,ax^2\right)^3b^2\right)\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*(b*x^4 + 2*a*x^2)*a^4 + 3*(b*x^4 + 2*a*x^2)^2*a^2*b + (b*x^4 + 2*a*
x^2)^3*b^2)*sgn(b*x^2 + a)
```

Mupad [B]
time = 4.40, size = 36, normalized size = 1.00

$$\frac{\left(b^2\,x^2+a\,b\right)\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] ((a*b + b^2*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2))/(12*b^2)
```

3.592 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x}\,dx$

Optimal. Leaf size=251

$$\frac{5a^4bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\,(a+bx^2)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\,(a+bx^2)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{5ab^4x^8\sqrt{a^2+2a}}{8\,(a+b}$$

```
[Out] 5/2*a^4*b*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/2*a^3*b^2*x^4*((b*x^2+a)^2)^(
1/2)/(b*x^2+a)+5/3*a^2*b^3*x^6*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/8*a*b^4*x^8*
((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/10*b^5*x^10*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a
^5*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 45}

$$\frac{b^5x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\,(a+bx^2)}+\frac{5ab^4x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\,(a+bx^2)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{a^5\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5a^4bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\,(a+bx^2)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

```
[Out] (5*a^4*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a^3*b^2*
x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a^2*b^3*x^6*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (b^5*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(10*(a + b*x^2)) + (a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x
^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x}\,dx}{b^4(ab+b^2x^2)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x}\,dx, x, x^2\right)}{2b^4(ab+b^2x^2)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(5a^4b^6+\frac{a^5b^5}{x}+10a^3b^7x+10a^2b^8x^2+5ab^9x^3 \right.\right.}{2b^4(ab+b^2x^2)}$$

$$= \frac{5a^4bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{5a^2b^3x^6\sqrt{a^2+}}{3(a}$$

Mathematica [A]
time = 0.02, size = 82, normalized size = 0.33

$$\frac{\sqrt{(a+bx^2)^2}\,(bx^2(300a^4+300a^3bx^2+200a^2b^2x^4+75ab^3x^6+12b^4x^8)+120a^5\log(x))}{120(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2*(300*a^4 + 300*a^3*b*x^2 + 200*a^2*b^2*x^4 + 75
*a*b^3*x^6 + 12*b^4*x^8) + 120*a^5*Log[x]))/(120*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 79, normalized size = 0.31

method	result	size
default	$\frac{\left((b\,x^2+a)^2\right)^{\frac{5}{2}}(12b^5x^{10}+75b^4a\,x^8+200a^2b^3x^6+300b^2a^3x^4+300b\,a^4x^2+120a^5\ln(x))}{120(b\,x^2+a)^5}$	79
risch	$\frac{\sqrt{(b\,x^2+a)^2}\;b(\frac{1}{10}b^4x^{10}+\frac{5}{8}a\,b^3x^8+\frac{5}{3}a^2b^2x^6+\frac{5}{2}a^3b\,x^4+\frac{5}{2}a^4x^2)}{b\,x^2+a}+\frac{a^5\ln(x)\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*((b*x^2+a)^2)^(5/2)*(12*b^5*x^10+75*b^4*a*x^8+200*a^2*b^3*x^6+300*b^2
*a^3*x^4+300*b*a^4*x^2+120*a^5*ln(x))/(b*x^2+a)^5
```

Maxima [A]
time = 0.29, size = 55, normalized size = 0.22

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+a^5\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + a^5*log(x)
```

Fricas [A]
time = 0.34, size = 55, normalized size = 0.22

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+a^5\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4
*b*x^2 + a^5*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x, x)
```

Giac [A]
time = 3.71, size = 106, normalized size = 0.42

$$\frac{1}{10}\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{3}\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^5\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sgn(b*x^2 + a) + 5/8*a*b^4*x^8*sgn(b*x^2 + a) + 5/3*a^2*b^3*x
^6*sgn(b*x^2 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^2 + a) + 5/2*a^4*b*x^2*sgn(b*x^
2 + a) + 1/2*a^5*log(x^2)*sgn(b*x^2 + a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x, x)
```

3.593 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=250

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}$$

```
[Out] -1/2*a^5*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+5*a^3*b^2*x^2*((b*x^2+a)^2)^(1/2
)/(b*x^2+a)+5/2*a^2*b^3*x^4*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/6*a*b^4*x^6*((b
*x^2+a)^2)^(1/2)/(b*x^2+a)+1/8*b^5*x^8*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5*a^4*
b*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1126, 272, 45}

$$\frac{b^5x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{5a^4b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

```
[Out] -1/2*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (5*a^3*b^2*x
^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a^2*b^3*x^4*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*x^6*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(6*(a + b*x^2)) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*
(a + b*x^2)) + (5*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^3}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^2}\,dx,x,x^2\right)}{2b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(10a^3b^7+\frac{a^5b^5}{x^2}+\frac{5a^4b^6}{x}+10a^2b^8x+5ab^9x^2+b\right.\right.}{2b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5a^2b^3x^4\sqrt{a^2+2a}}{2(a+b}$$

Mathematica [A]
time = 0.02, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^2)^2}\,(-12a^5+120a^3b^2x^4+60a^2b^3x^6+20ab^4x^8+3b^5x^{10}+120a^4bx^2\log(x))}{24x^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-12*a^5 + 120*a^3*b^2*x^4 + 60*a^2*b^3*x^6 + 20*a*b^4
*x^8 + 3*b^5*x^10 + 120*a^4*b*x^2*Log[x]))/(24*x^2*(a + b*x^2))
```

Maple [A]
time = 0.03, size = 82, normalized size = 0.33

method	result	size
default	$\frac{\left((b\,x^2+a)^2\right)^{\frac{5}{2}}(3b^5x^{10}+20b^4a\,x^8+60a^2b^3x^6+120b^2a^3x^4+120b\,a^4\ln(x)x^2-12a^5)}{24(b\,x^2+a)^5x^2}$	82
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,b^2(\frac{1}{8}b^3x^8+\frac{5}{6}a\,b^2x^6+\frac{5}{2}a^2b\,x^4+5a^3x^2)}{b\,x^2+a}-\frac{a^5\sqrt{(b\,x^2+a)^2}}{2x^2(b\,x^2+a)}+\frac{5a^4b\ln(x)\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*((b*x^2+a)^2)^(5/2)*(3*b^5*x^10+20*b^4*a*x^8+60*a^2*b^3*x^6+120*b^2*a^
3*x^4+120*b*a^4*ln(x)*x^2-12*a^5)/(b*x^2+a)^5/x^2
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.22

$$\frac{1}{8}\,b^{5}x^{8}+\frac{5}{6}\,ab^{4}x^{6}+\frac{5}{2}\,a^{2}b^{3}x^{4}+5\,a^{3}b^{2}x^{2}+5\,a^{4}b\log\left(x\right)-\frac{a^{5}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5*a^4*b*log
(x) - 1/2*a^5/x^2
```

Fricas [A]
time = 0.35, size = 61, normalized size = 0.24

$$\frac{3\,b^{5}x^{10}+20\,ab^{4}x^{8}+60\,a^{2}b^{3}x^{6}+120\,a^{3}b^{2}x^{4}+120\,a^{4}bx^{2}\log\left(x\right)-12\,a^{5}}{24\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4 + 120*a^
4*b*x^2*log(x) - 12*a^5)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**3, x)
```

Giac [A]
time = 3.92, size = 125, normalized size = 0.50

$$\frac{1}{8}\,b^{5}x^{8}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{5}{6}\,ab^{4}x^{6}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{5}{2}\,a^{2}b^{3}x^{4}\mathrm{sgn}\left(bx^{2}+a\right)+5\,a^{3}b^{2}x^{2}\mathrm{sgn}\left(bx^{2}+a\right)+\frac{5}{2}\,a^{4}b\log\left(x^{2}\right)\mathrm{sgn}\left(bx^{2}+a\right)-\frac{5\,a^{4}bx^{2}\mathrm{sgn}(bx^{2}+a)+a^{5}\mathrm{sgn}(bx^{2}+a)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*sgn(b*x^2 + a) + 5/6*a*b^4*x^6*sgn(b*x^2 + a) + 5/2*a^2*b^3*x^4
*sgn(b*x^2 + a) + 5*a^3*b^2*x^2*sgn(b*x^2 + a) + 5/2*a^4*b*log(x^2)*sgn(b*x
^2 + a) - 1/2*(5*a^4*b*x^2*sgn(b*x^2 + a) + a^5*sgn(b*x^2 + a))/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^3, x)
```

3.594 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=250

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5ab^4x^4\sqrt{a^2+2abx^2+}}{4\left(a+bx^2\right)}$$

```
[Out] -1/4*a^5*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-5/2*a^4*b*((b*x^2+a)^2)^(1/2)/x^
2/(b*x^2+a)+5*a^2*b^3*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/4*a*b^4*x^4*((b*x
^2+a)^2)^(1/2)/(b*x^2+a)+1/6*b^5*x^6*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10*a^3*b
^2*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 45}

$$\frac{b^5x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5ab^4x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{10a^3b^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```

```
[Out] -1/4*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^4*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a^2*b^3*x^2*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(4*(a + b*x^2)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a
 + b*x^2)) + (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^5}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^3}\,dx,x,x^2\right)}{2b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(10a^2b^8+\frac{a^5b^5}{x^3}+\frac{5a^4b^6}{x^2}+\frac{10a^3b^7}{x}+5ab^9x+b^{10}x^{\cdots}\right.\right.}{2b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)} + \frac{5a^2b^3x^2\sqrt{a^2+2abx\cdots}}{a+bx^2}$$

Mathematica [A]
time = 0.02, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^2)^2}\,(-3a^5-30a^4bx^2+60a^2b^3x^6+15ab^4x^8+2b^5x^{10}+120a^3b^2x^4\log(x))}{12x^4(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 30*a^4*b*x^2 + 60*a^2*b^3*x^6 + 15*a*b^4*x^8
 + 2*b^5*x^10 + 120*a^3*b^2*x^4*Log[x]))/(12*x^4*(a + b*x^2))
```

Maple [A]
time = 0.03, size = 82, normalized size = 0.33

method	result	size
default	$\frac{\left((b\,x^2+a)^2\right)^{\frac{5}{2}}(2b^5x^{10}+15b^4a\,x^8+60a^2b^3x^6+120b^2a^3\ln(x)x^4-30b\,a^4x^2-3a^5)}{12(b\,x^2+a)^5x^4}$	82
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,b^3(\frac{1}{6}b^2x^6+\frac{5}{4}ab\,x^4+5a^2x^2)}{b\,x^2+a}+\frac{\sqrt{(b\,x^2+a)^2}\,(-\frac{5}{2}b\,a^4x^2-\frac{1}{4}a^5)}{(b\,x^2+a)x^4}+\frac{10a^3b^2\ln(x)\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

[Out] 1/12*((b*x^2+a)^2)^(5/2)*(2*b^5*x^10+15*b^4*a*x^8+60*a^2*b^3*x^6+120*b^2*a^3*ln(x)*x^4-30*b*a^4*x^2-3*a^5)/(b*x^2+a)^5/x^4

Maxima [A]
time = 0.30, size = 56, normalized size = 0.22

$$\frac{1}{6}\,b^5x^6+\frac{5}{4}\,ab^4x^4+5\,a^2b^3x^2+10\,a^3b^2\log\left(x\right)-\frac{5\,a^4b}{2\,x^2}-\frac{a^5}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 10*a^3*b^2*log(x) - 5/2*a^4*b/x^2 - 1/4*a^5/x^4
```

Fricas [A]
time = 0.33, size = 61, normalized size = 0.24

$$\frac{2\,b^5x^{10}+15\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4\log\left(x\right)-30\,a^4bx^2-3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4*log(x) - 30*a^4*b*x^2 - 3*a^5)/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**5, x)
```

Giac [A]
time = 4.32, size = 127, normalized size = 0.51

$$\frac{1}{6}\,b^5x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{4}\,ab^4x^4\mathrm{sgn}\left(bx^2+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^2+a\right)+5\,a^3b^2\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{30\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+10\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+a^5\mathrm{sgn}\left(bx^2+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sgn(b*x^2 + a) + 5/4*a*b^4*x^4*sgn(b*x^2 + a) + 5*a^2*b^3*x^2*s
gn(b*x^2 + a) + 5*a^3*b^2*log(x^2)*sgn(b*x^2 + a) - 1/4*(30*a^3*b^2*x^4*sgn
(b*x^2 + a) + 10*a^4*b*x^2*sgn(b*x^2 + a) + a^5*sgn(b*x^2 + a))/x^4
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^5,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^5, x)
```

3.595 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^7} dx$

Optimal. Leaf size=250

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2(a+bx^2)}+\frac{5ab^4x^2\sqrt{a^2+2abx^2+b}}{2(a+bx^2)}$$

```
[Out] -1/6*a^5*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)-5/4*a^4*b*((b*x^2+a)^2)^(1/2)/x^
4/(b*x^2+a)-5*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+5/2*a*b^4*x^2*((b*x
^2+a)^2)^(1/2)/(b*x^2+a)+1/4*b^5*x^4*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10*a^2*b
^3*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 45}

$$\frac{b^5x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{5ab^4x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

```
[Out] -1/6*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^6*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(2*(a + b*x^2)) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*
(a + b*x^2)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x
^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^7}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)^5}{x^4}\,dx, x, x^2\right)}{2b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(5ab^9+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^3}+\frac{10a^3b^7}{x^2}+\frac{10a^2b^8}{x}+b^{10}x\right) d\right.}{2b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2}}{x^2(a+bx^2)}$$

Mathematica [A]
time = 0.02, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^2)^2}\,(-2a^5-15a^4bx^2-60a^3b^2x^4+30ab^4x^8+3b^5x^{10}+120a^2b^3x^6\log(x))}{12x^6(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-2*a^5 - 15*a^4*b*x^2 - 60*a^3*b^2*x^4 + 30*a*b^4*x^8
 + 3*b^5*x^10 + 120*a^2*b^3*x^6*Log[x]))/(12*x^6*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 82, normalized size = 0.33

method	result	size
default	$\frac{\left((b\,x^2+a)^2\right)^{\frac{5}{2}}(3b^5x^{10}+30b^4a\,x^8+120a^2b^3\ln(x)x^6-60b^2a^3x^4-15b\,a^4x^2-2a^5)}{12(b\,x^2+a)^5x^6}$	82
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,b^3(b\,x^2+5a)^2}{4b\,x^2+4a}+\frac{\sqrt{(b\,x^2+a)^2}\,(-5b^2a^3x^4-\frac{5}{4}b\,a^4x^2-\frac{1}{6}a^5)}{(b\,x^2+a)x^6}+\frac{10a^2b^3\ln(x)\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*((b*x^2+a)^2)^(5/2)*(3*b^5*x^10+30*b^4*a*x^8+120*a^2*b^3*ln(x)*x^6-60*
b^2*a^3*x^4-15*b*a^4*x^2-2*a^5)/(b*x^2+a)^5/x^6
```

Maxima [A]
time = 0.29, size = 56, normalized size = 0.22

$$\frac{1}{4}\,b^5x^4+\frac{5}{2}\,ab^4x^2+10\,a^2b^3\log\left(x\right)-\frac{5\,a^3b^2}{x^2}-\frac{5\,a^4b}{4\,x^4}-\frac{a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 10*a^2*b^3*log(x) - 5*a^3*b^2/x^2 - 5/4*a^4*b
/x^4 - 1/6*a^5/x^6
```

Fricas [A]
time = 0.33, size = 61, normalized size = 0.24

$$\frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3x^6\log\left(x\right)-60\,a^3b^2x^4-15\,a^4bx^2-2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*log(x) - 60*a^3*b^2*x^4 -
 15*a^4*b*x^2 - 2*a^5)/x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**7, x)
```

Giac [A]
time = 2.91, size = 128, normalized size = 0.51

$$\frac{1}{4}\,b^5x^4\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,ab^4x^2\mathrm{sgn}\left(bx^2+a\right)+5\,a^2b^3\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{110\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+60\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+15\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+2\,a^5\mathrm{sgn}\left(bx^2+a\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4*sgn(b*x^2 + a) + 5/2*a*b^4*x^2*sgn(b*x^2 + a) + 5*a^2*b^3*log(x
^2)*sgn(b*x^2 + a) - 1/12*(110*a^2*b^3*x^6*sgn(b*x^2 + a) + 60*a^3*b^2*x^4*
sgn(b*x^2 + a) + 15*a^4*b*x^2*sgn(b*x^2 + a) + 2*a^5*sgn(b*x^2 + a))/x^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^7,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^7, x)
```

3.596 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=250

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2}}{x^2\left(a+bx^2\right)}$$

```
[Out] -1/8*a^5*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)-5/6*a^4*b*((b*x^2+a)^2)^(1/2)/x^
6/(b*x^2+a)-5/2*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-5*a^2*b^3*((b*x^2
+a)^2)^(1/2)/x^2/(b*x^2+a)+1/2*b^5*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5*a*b^
4*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 45}

$$\frac{b^5x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

```
[Out] -1/8*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^8*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(x^2*(a + b*x^2)) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
2*(a + b*x^2)) + (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^
2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^9}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^9}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)^5}{x^5}\,dx, x, x^2\right)}{2b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \left(b^{10}+\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^3}+\frac{10a^2b^8}{x^2}+\frac{5ab^9}{x}\right)\,dx\right.}{2b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2}}{2x^4(a+bx^2)}$$

Mathematica [A]
time = 0.01, size = 85, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^2)^2}\,(3a^5+20a^4bx^2+60a^3b^2x^4+120a^2b^3x^6-12b^5x^{10}-120ab^4x^8\log(x))}{24x^8(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

```
[Out] -1/24*(Sqrt[(a + b*x^2)^2]*(3*a^5 + 20*a^4*b*x^2 + 60*a^3*b^2*x^4 + 120*a^2
*b^3*x^6 - 12*b^5*x^10 - 120*a*b^4*x^8*Log[x]))/(x^8*(a + b*x^2))
```

Maple [A]
time = 0.03, size = 82, normalized size = 0.33

method	result	size
default	$\frac{\left((b\,x^2+a)^2\right)^{\frac{5}{2}}(12b^5x^{10}+120b^4a\ln(x)x^8-120a^2b^3x^6-60b^2a^3x^4-20b\,a^4x^2-3a^5)}{24(b\,x^2+a)^5x^8}$	82
risch	$\frac{b^5x^2\sqrt{(b\,x^2+a)^2}}{2b\,x^2+2a}+\frac{\sqrt{(b\,x^2+a)^2}\,(-5a^2b^3x^6-\frac{5}{2}b^2a^3x^4-\frac{5}{6}b\,a^4x^2-\frac{1}{8}a^5)}{(b\,x^2+a)x^8}+\frac{5a\,b^4\ln(x)\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*((b*x^2+a)^2)^(5/2)*(12*b^5*x^10+120*b^4*a*ln(x)*x^8-120*a^2*b^3*x^6-6
0*b^2*a^3*x^4-20*b*a^4*x^2-3*a^5)/(b*x^2+a)^5/x^8
```

Maxima [A]
time = 0.30, size = 56, normalized size = 0.22

$$\frac{1}{2}\,b^5x^2+5\,ab^4\log\left(x\right)-\frac{5\,a^2b^3}{x^2}-\frac{5\,a^3b^2}{2\,x^4}-\frac{5\,a^4b}{6\,x^6}-\frac{a^5}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*log(x) - 5*a^2*b^3/x^2 - 5/2*a^3*b^2/x^4 - 5/6*a^4*b/
x^6 - 1/8*a^5/x^8
```

Fricas [A]
time = 0.33, size = 61, normalized size = 0.24

$$\frac{12\,b^5x^{10}+120\,ab^4x^8\log\left(x\right)-120\,a^2b^3x^6-60\,a^3b^2x^4-20\,a^4bx^2-3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 120*a*b^4*x^8*log(x) - 120*a^2*b^3*x^6 - 60*a^3*b^2*x^4
 - 20*a^4*b*x^2 - 3*a^5)/x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**9, x)
```

Giac [A]
time = 3.98, size = 126, normalized size = 0.50

$$\frac{1}{2}\,b^5x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,ab^4\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{125\,ab^4x^8\mathrm{sgn}(bx^2+a)+120\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+60\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+20\,a^4bx^2\mathrm{sgn}(bx^2+a)+3\,a^5\mathrm{sgn}(bx^2+a)}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sgn(b*x^2 + a) + 5/2*a*b^4*log(x^2)*sgn(b*x^2 + a) - 1/24*(125*
a*b^4*x^8*sgn(b*x^2 + a) + 120*a^2*b^3*x^6*sgn(b*x^2 + a) + 60*a^3*b^2*x^4*
sgn(b*x^2 + a) + 20*a^4*b*x^2*sgn(b*x^2 + a) + 3*a^5*sgn(b*x^2 + a))/x^8
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^9,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^9, x)
```

3.597 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=251

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^6\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2}}{2x^4\left(a+bx^2\right)}$$

```
[Out] -1/10*a^5*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)-5/8*a^4*b*((b*x^2+a)^2)^(1/2)/
x^8/(b*x^2+a)-5/3*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)-5/2*a^2*b^3*((b
*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-5/2*a*b^4*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+
b^5*ln(x)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 45}

$$\frac{b^5\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^4\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

```
[Out] -1/10*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^10*(a + b*x^2)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(3*x^6*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(2*x^2*(a + b*x^2)) + (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a +
b*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{11}}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^{11}}\,dx}{b^4(ab+b^2x^2)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^6}\,dx,x,x^2\right)}{2b^4(ab+b^2x^2)}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^3}+\frac{5ab^9}{x^2}+\frac{b^{10}}{x}\right)\,dx\right.}{2b^4(ab+b^2x^2)}\\ &= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2\ }}{3x^6(a+bx^2)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 85, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^2)^2}\,(a(12a^4+75a^3bx^2+200a^2b^2x^4+300ab^3x^6+300b^4x^8)-120b^5x^{10}\log(x))}{120x^{10}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

```
[Out] -1/120*(Sqrt[(a + b*x^2)^2]*(a*(12*a^4 + 75*a^3*b*x^2 + 200*a^2*b^2*x^4 + 3
00*a*b^3*x^6 + 300*b^4*x^8) - 120*b^5*x^10*Log[x]))/(x^10*(a + b*x^2))
```

Maple [A]
time = 0.03, size = 82, normalized size = 0.33

method	result	size
default	$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}\left(120 b^{5} \ln \left(x \right) x^{10}-300 b^{4} a\,x^{8}-300 a^{2} b^{3} x^{6}-200 b^{2} a^{3} x^{4}-75 b\,a^{4} x^{2}-12 a^{5}\right)}{120 \left(b\,x^{2}+a\right)^{5} x^{10}}$	82
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{5}{2} b^{4} a\,x^{8}-\frac{5}{2} a^{2} b^{3} x^{6}-\frac{5}{3} b^{2} a^{3} x^{4}-\frac{5}{8} b\,a^{4} x^{2}-\frac{1}{10} a^{5}\right)}{\left(b\,x^{2}+a\right) x^{10}}+\frac{b^{5} \ln \left(x \right) \sqrt{\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*((b*x^2+a)^2)^(5/2)*(120*b^5*ln(x)*x^10-300*b^4*a*x^8-300*a^2*b^3*x^6
-200*b^2*a^3*x^4-75*b*a^4*x^2-12*a^5)/(b*x^2+a)^5/x^10
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.22

$$b^5 \log(x) - \frac{5\,ab^4}{2\,x^2} - \frac{5\,a^2b^3}{2\,x^4} - \frac{5\,a^3b^2}{3\,x^6} - \frac{5\,a^4b}{8\,x^8} - \frac{a^5}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] b^5*log(x) - 5/2*a*b^4/x^2 - 5/2*a^2*b^3/x^4 - 5/3*a^3*b^2/x^6 - 5/8*a^4*b/
x^8 - 1/10*a^5/x^10
```

Fricas [A]
time = 0.34, size = 61, normalized size = 0.24

$$\frac{120\,b^5x^{10}\log(x) - 300\,ab^4x^8 - 300\,a^2b^3x^6 - 200\,a^3b^2x^4 - 75\,a^4bx^2 - 12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^5*x^10*log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 200*a^3*b^2*
x^4 - 75*a^4*b*x^2 - 12*a^5)/x^10
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**11, x)
```

Giac [A]
time = 4.19, size = 125, normalized size = 0.50

$$\frac{1}{2}\,b^5\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right) - \frac{137\,b^5x^{10}\mathrm{sgn}(bx^2+a) + 300\,ab^4x^8\mathrm{sgn}(bx^2+a) + 300\,a^2b^3x^6\mathrm{sgn}(bx^2+a) + 200\,a^3b^2x^4\mathrm{sgn}(bx^2+a) + 75\,a^4bx^2\mathrm{sgn}(bx^2+a) + 12\,a^5\mathrm{sgn}(bx^2+a)}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(x^2)*sgn(b*x^2 + a) - 1/120*(137*b^5*x^10*sgn(b*x^2 + a) + 300*
a*b^4*x^8*sgn(b*x^2 + a) + 300*a^2*b^3*x^6*sgn(b*x^2 + a) + 200*a^3*b^2*x^4
*sgn(b*x^2 + a) + 75*a^4*b*x^2*sgn(b*x^2 + a) + 12*a^5*sgn(b*x^2 + a))/x^10
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^11,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^11, x)
```

3.598 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{13}}\,dx$

Optimal. Leaf size=41

$$-\frac{(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

```
[Out] -1/12*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/a/x^12
```

Rubi [A]

time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 37}

$$-\frac{(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

```
[Out] -1/12*((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{13}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^7}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^7}\,dx, x, x^2\right)}{2b^4(ab+b^2x^2)}$$
$$= -\frac{(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 1.98

$$-\frac{\sqrt{(a+bx^2)^2}\,(a^5+6a^4bx^2+15a^3b^2x^4+20a^2b^3x^6+15ab^4x^8+6b^5x^{10})}{12x^{12}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

```
[Out] -1/12*(Sqrt[(a + b*x^2)^2]*(a^5 + 6*a^4*b*x^2 + 15*a^3*b^2*x^4 + 20*a^2*b^3
*x^6 + 15*a*b^4*x^8 + 6*b^5*x^10))/(x^12*(a + b*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(28) = 56$.
time = 0.04, size = 78, normalized size = 1.90

method	result	size
gosper	$-\frac{(6b^5x^{10}+15b^4a\,x^8+20a^2b^3x^6+15b^2a^3x^4+6b\,a^4x^2+a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{12x^{12}(b\,x^2+a)^5}$	78
default	$-\frac{(6b^5x^{10}+15b^4a\,x^8+20a^2b^3x^6+15b^2a^3x^4+6b\,a^4x^2+a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{12x^{12}(b\,x^2+a)^5}$	78
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{2}b^5x^{10}-\frac{5}{4}b^4a\,x^8-\frac{5}{3}a^2b^3x^6-\frac{5}{4}b^2a^3x^4-\frac{1}{2}b\,a^4x^2-\frac{1}{12}a^5\right)}{(b\,x^2+a)x^{12}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(6*b^5*x^10+15*a*b^4*x^8+20*a^2*b^3*x^6+15*a^3*b^2*x^4+6*a^4*b*x^2+a^
5)*((b*x^2+a)^2)^(5/2)/x^12/(b*x^2+a)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(28) = 56$.

time = 0.27, size = 57, normalized size = 1.39

$$-\frac{b^5}{2\,x^2}-\frac{5\,ab^4}{4\,x^4}-\frac{5\,a^2b^3}{3\,x^6}-\frac{5\,a^3b^2}{4\,x^8}-\frac{a^4b}{2\,x^{10}}-\frac{a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

```
[Out] -1/2*b^5/x^2 - 5/4*a*b^4/x^4 - 5/3*a^2*b^3/x^6 - 5/4*a^3*b^2/x^8 - 1/2*a^4*
b/x^10 - 1/12*a^5/x^12
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(28) = 56.
time = 0.36, size = 57, normalized size = 1.39

$$-\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*
b*x^2 + a^5)/x^12
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**13, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(28) = 56.
time = 4.48, size = 106, normalized size = 2.59

$$-\frac{6\,b^5x^{10}\mathrm{sgn}(bx^2+a)+15\,ab^4x^8\mathrm{sgn}(bx^2+a)+20\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+15\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+6\,a^4bx^2\mathrm{sgn}(bx^2+a)+a^5\mathrm{sgn}(bx^2+a)}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*x^10*sgn(b*x^2 + a) + 15*a*b^4*x^8*sgn(b*x^2 + a) + 20*a^2*b^3
*x^6*sgn(b*x^2 + a) + 15*a^3*b^2*x^4*sgn(b*x^2 + a) + 6*a^4*b*x^2*sgn(b*x^2
 + a) + a^5*sgn(b*x^2 + a))/x^12
```

Mupad [B]
time = 4.18, size = 231, normalized size = 5.63

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^2\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\,\left(b\,x^2+a\right)}-\frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^{10}\,\left(b\,x^2+a\right)}-\frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^6\,\left(b\,x^2+a\right)}-\frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^8\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^13,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*x^2*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(2*x^10*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1
/2))/(3*x^6*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4
*x^8*(a + b*x^2))
```

3.599 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{15}}\,dx$

Optimal. Leaf size=72

$$-\frac{(a+bx^2)(a^2+2abx^2+b^2x^4)^{5/2}}{12ax^{14}}+\frac{(a^2+2abx^2+b^2x^4)^{7/2}}{84a^2x^{14}}$$

```
[Out] -1/12*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/a/x^14+1/84*(b^2*x^4+2*a*b*x^
2+a^2)^(7/2)/a^2/x^14
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1124}

$$\frac{(a^2+2abx^2+b^2x^4)^{7/2}}{84a^2x^{14}}-\frac{(a+bx^2)(a^2+2abx^2+b^2x^4)^{5/2}}{12ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

```
[Out] -1/12*((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(a*x^14) + (a^2 + 2*a
*b*x^2 + b^2*x^4)^(7/2)/(84*a^2*x^14)
```

Rule 1124

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(4*a*d*(p + 1)*(2*p + 1
))), x] - Simp[(d*x)^(m + 1)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^p/(4*a*d*(2
*p + 1))), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && NeQ[p, -2^(-1)]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{15}}\,dx=-\frac{(a+bx^2)(a^2+2abx^2+b^2x^4)^{5/2}}{12ax^{14}}+\frac{(a^2+2abx^2+b^2x^4)^{7/2}}{84a^2x^{14}}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 1.15

$$-\frac{\sqrt{(a+bx^2)^2}\,(6a^5+35a^4bx^2+84a^3b^2x^4+105a^2b^3x^6+70ab^4x^8+21b^5x^{10})}{84x^{14}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

```
[Out] -1/84*(Sqrt[(a + b*x^2)^2]*(6*a^5 + 35*a^4*b*x^2 + 84*a^3*b^2*x^4 + 105*a^2
*b^3*x^6 + 70*a*b^4*x^8 + 21*b^5*x^10))/(x^14*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 80, normalized size = 1.11

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{14}a^{5}-\frac{5}{12}b\,a^{4}x^{2}-b^{2}a^{3}x^{4}-\frac{5}{4}a^{2}b^{3}x^{6}-\frac{5}{6}b^{4}a\,x^{8}-\frac{1}{4}b^{5}x^{10}\right)}{\left(b\,x^{2}+a\right)x^{14}}$	79
gosper	$-\frac{\left(21b^{5}x^{10}+70b^{4}a\,x^{8}+105a^{2}b^{3}x^{6}+84b^{2}a^{3}x^{4}+35b\,a^{4}x^{2}+6a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{84x^{14}\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(21b^{5}x^{10}+70b^{4}a\,x^{8}+105a^{2}b^{3}x^{6}+84b^{2}a^{3}x^{4}+35b\,a^{4}x^{2}+6a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{84x^{14}\left(b\,x^{2}+a\right)^{5}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/84*(21*b^5*x^10+70*a*b^4*x^8+105*a^2*b^3*x^6+84*a^3*b^2*x^4+35*a^4*b*x^2
+6*a^5)*((b*x^2+a)^2)^(5/2)/x^14/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.79

$$-\frac{b^{5}}{4\,x^{4}}-\frac{5\,ab^{4}}{6\,x^{6}}-\frac{5\,a^{2}b^{3}}{4\,x^{8}}-\frac{a^{3}b^{2}}{x^{10}}-\frac{5\,a^{4}b}{12\,x^{12}}-\frac{a^{5}}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/4*b^5/x^4 - 5/6*a*b^4/x^6 - 5/4*a^2*b^3/x^8 - a^3*b^2/x^10 - 5/12*a^4*b/
x^12 - 1/14*a^5/x^14
```

Fricas [A]
time = 0.32, size = 59, normalized size = 0.82

$$-\frac{21\,b^{5}x^{10}+70\,ab^{4}x^{8}+105\,a^{2}b^{3}x^{6}+84\,a^{3}b^{2}x^{4}+35\,a^{4}bx^{2}+6\,a^{5}}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a
^4*b*x^2 + 6*a^5)/x^14
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**15, x)
```

Giac [A]
time = 4.48, size = 107, normalized size = 1.49

$$-\frac{21\,b^5x^{10}\mathrm{sgn}(bx^2+a)+70\,ab^4x^8\mathrm{sgn}(bx^2+a)+105\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+84\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+35\,a^4bx^2\mathrm{sgn}(bx^2+a)+6\,a^5\mathrm{sgn}(bx^2+a)}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(21*b^5*x^10*sgn(b*x^2 + a) + 70*a*b^4*x^8*sgn(b*x^2 + a) + 105*a^2*b
^3*x^6*sgn(b*x^2 + a) + 84*a^3*b^2*x^4*sgn(b*x^2 + a) + 35*a^4*b*x^2*sgn(b*
x^2 + a) + 6*a^5*sgn(b*x^2 + a))/x^14
```

Mupad [B]
time = 4.22, size = 231, normalized size = 3.21

$$-\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,\left(b\,x^2+a\right)}-\frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^6\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,\left(b\,x^2+a\right)}-\frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^8\,\left(b\,x^2+a\right)}-\frac{a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^{10}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^15,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)
^(1/2))/(4*x^8*(a + b*x^2)) - (a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(
x^10*(a + b*x^2))
```

3.600 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{17}}\,dx$

Optimal. Leaf size=128

$$-\frac{(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}}+\frac{b(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{56a^2x^{14}}-\frac{b^2(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{336a^3x^{12}}$$

```
[Out] -1/16*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/a/x^16+1/56*b*(b*x^2+a)^5*((b*x^2+a)^
2)^(1/2)/a^2/x^14-1/336*b^2*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/a^3/x^12
```

Rubi [A]
time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1125, 660, 47, 37}

$$-\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{16ax^{16}}+\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{56a^2x^{14}}-\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{336a^3x^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

```
[Out] -1/16*((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a*x^16) + (b*(a + b*
x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(56*a^2*x^14) - (b^2*(a + b*x^2)^5*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(336*a^3*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
```

```
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{17}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^9}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^9}\,dx, x, x^2\right)}{2b^4\,(ab+b^2x^2)}$$

$$= -\frac{(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} - \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^8} \cdots\right.}{8ab^3\,(ab+b^2x^2)}$$

$$= -\frac{(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} + \frac{b(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{56a^2x^{14}} + \cdots$$

$$= -\frac{(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} + \frac{b(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{56a^2x^{14}} - \cdots$$

Mathematica [**A**]
time = 0.01, size = 83, normalized size = 0.65

$$-\frac{\sqrt{(a+bx^2)^2}\,(21a^5+120a^4bx^2+280a^3b^2x^4+336a^2b^3x^6+210ab^4x^8+56b^5x^{10})}{336x^{16}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

```
[Out] -1/336*(Sqrt[(a + b*x^2)^2]*(21*a^5 + 120*a^4*b*x^2 + 280*a^3*b^2*x^4 + 336
*a^2*b^3*x^6 + 210*a*b^4*x^8 + 56*b^5*x^10))/(x^16*(a + b*x^2))
```

Maple [**A**]
time = 0.02, size = 80, normalized size = 0.62

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{16}a^{5}-\frac{5}{14}b\,a^{4}x^{2}-\frac{5}{6}b^{2}a^{3}x^{4}-a^{2}b^{3}x^{6}-\frac{5}{8}b^{4}a\,x^{8}-\frac{1}{6}b^{5}x^{10}\right)}{\left(b\,x^{2}+a\right)x^{16}}$	79
gosper	$-\frac{\left(56b^{5}x^{10}+210b^{4}a\,x^{8}+336a^{2}b^{3}x^{6}+280b^{2}a^{3}x^{4}+120b\,a^{4}x^{2}+21a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{336x^{16}\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(56b^{5}x^{10}+210b^{4}a\,x^{8}+336a^{2}b^{3}x^{6}+280b^{2}a^{3}x^{4}+120b\,a^{4}x^{2}+21a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{336x^{16}\left(b\,x^{2}+a\right)^{5}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/336*(56*b^5*x^10+210*a*b^4*x^8+336*a^2*b^3*x^6+280*a^3*b^2*x^4+120*a^4*b
*x^2+21*a^5)*((b*x^2+a)^2)^(5/2)/x^16/(b*x^2+a)^5
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.45

$$-\frac{b^{5}}{6\,x^{6}}-\frac{5\,ab^{4}}{8\,x^{8}}-\frac{a^{2}b^{3}}{x^{10}}-\frac{5\,a^{3}b^{2}}{6\,x^{12}}-\frac{5\,a^{4}b}{14\,x^{14}}-\frac{a^{5}}{16\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/6*b^5/x^6 - 5/8*a*b^4/x^8 - a^2*b^3/x^10 - 5/6*a^3*b^2/x^12 - 5/14*a^4*b
/x^14 - 1/16*a^5/x^16
```

Fricas [A]
time = 0.37, size = 59, normalized size = 0.46

$$-\frac{56\,b^{5}x^{10}+210\,ab^{4}x^{8}+336\,a^{2}b^{3}x^{6}+280\,a^{3}b^{2}x^{4}+120\,a^{4}bx^{2}+21\,a^{5}}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 1
20*a^4*b*x^2 + 21*a^5)/x^16
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**17, x)
```

Giac [A]
time = 4.05, size = 107, normalized size = 0.84

$$-\frac{56\,b^5x^{10}\mathrm{sgn}(bx^2+a)+210\,ab^4x^8\mathrm{sgn}(bx^2+a)+336\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+280\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+120\,a^4bx^2\mathrm{sgn}(bx^2+a)+21\,a^5\mathrm{sgn}(bx^2+a)}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(56*b^5*x^10*sgn(b*x^2 + a) + 210*a*b^4*x^8*sgn(b*x^2 + a) + 336*a^2
*b^3*x^6*sgn(b*x^2 + a) + 280*a^3*b^2*x^4*sgn(b*x^2 + a) + 120*a^4*b*x^2*sg
n(b*x^2 + a) + 21*a^5*sgn(b*x^2 + a))/x^16
```

Mupad [B]
time = 4.24, size = 231, normalized size = 1.80

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^6\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,\left(b\,x^2+a\right)}-\frac{a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^{10}\,\left(b\,x^2+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^{12}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^17,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(x^10*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(6
*x^12*(a + b*x^2))
```

3.601 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{19}} dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^{12}(a+bx^2)}$$

```
[Out] -1/18*a^5*((b*x^2+a)^2)^(1/2)/x^18/(b*x^2+a)-5/16*a^4*b*((b*x^2+a)^2)^(1/2)
/x^16/(b*x^2+a)-5/7*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)-5/6*a^2*b^3*
((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)-1/2*a*b^4*((b*x^2+a)^2)^(1/2)/x^10/(b*x^
2+a)-1/8*b^5*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)
```

Rubi [A]
time = 0.10, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^{10}(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^{12}(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

```
[Out] -1/18*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^18*(a + b*x^2)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(6*x^12*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(2*x^10*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a
 + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{19}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{10}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^{10}}\,dx, x, x^2\right)}{2b^4\,(ab+b^2x^2)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^{10}}+\frac{5a^4b^6}{x^9}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^7}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^5}\right) d\right.}{2b^4\,(ab+b^2x^2)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\,(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\,(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2}}{7x^{14}\,(a+bx}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(56a^5+315a^4bx^2+720a^3b^2x^4+840a^2b^3x^6+504ab^4x^8+126b^5x^{10})}{1008x^{18}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
[Out] -1/1008*(Sqrt[(a + b*x^2)^2]*(56*a^5 + 315*a^4*b*x^2 + 720*a^3*b^2*x^4 + 84
0*a^2*b^3*x^6 + 504*a*b^4*x^8 + 126*b^5*x^10))/(x^18*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,(-\frac{1}{18}a^5-\frac{5}{16}b\,a^4x^2-\frac{5}{7}b^2a^3x^4-\frac{5}{6}a^2b^3x^6-\frac{1}{2}b^4a\,x^8-\frac{1}{8}b^5x^{10})}{(b\,x^2+a)x^{18}}$	79
gosper	$-\frac{(126b^5x^{10}+504b^4a\,x^8+840a^2b^3x^6+720b^2a^3x^4+315b\,a^4x^2+56a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{1008x^{18}(b\,x^2+a)^5}$	80

default	$-\frac{\left(126 b^{5} x^{10}+504 b^{4} a\,x^{8}+840 a^{2} b^{3} x^{6}+720 b^{2} a^{3} x^{4}+315 b\,a^{4} x^{2}+56 a^{5}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{1008 x^{18} \left(b\,x^{2}+a\right)^{5}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1008*(126*b^5*x^10+504*a*b^4*x^8+840*a^2*b^3*x^6+720*a^3*b^2*x^4+315*a^4
*b*x^2+56*a^5)*((b*x^2+a)^2)^(5/2)/x^18/(b*x^2+a)^5
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.22

$$-\frac{b^{5}}{8\,x^{8}}-\frac{ab^{4}}{2\,x^{10}}-\frac{5\,a^{2}b^{3}}{6\,x^{12}}-\frac{5\,a^{3}b^{2}}{7\,x^{14}}-\frac{5\,a^{4}b}{16\,x^{16}}-\frac{a^{5}}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="maxima")
```

```
[Out] -1/8*b^5/x^8 - 1/2*a*b^4/x^10 - 5/6*a^2*b^3/x^12 - 5/7*a^3*b^2/x^14 - 5/16*
a^4*b/x^16 - 1/18*a^5/x^18
```

Fricas [A]
time = 0.33, size = 59, normalized size = 0.23

$$-\frac{126\,b^{5}x^{10}+504\,ab^{4}x^{8}+840\,a^{2}b^{3}x^{6}+720\,a^{3}b^{2}x^{4}+315\,a^{4}bx^{2}+56\,a^{5}}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 +
 315*a^4*b*x^2 + 56*a^5)/x^18
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**19,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**19, x)
```

Giac [A]
time = 4.97, size = 107, normalized size = 0.42

$$-\frac{126\,b^5x^{10}\mathrm{sgn}(bx^2+a)+504\,ab^4x^8\mathrm{sgn}(bx^2+a)+840\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+720\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+315\,a^4bx^2\mathrm{sgn}(bx^2+a)+56\,a^5\mathrm{sgn}(bx^2+a)}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10*sgn(b*x^2 + a) + 504*a*b^4*x^8*sgn(b*x^2 + a) + 840*a
^2*b^3*x^6*sgn(b*x^2 + a) + 720*a^3*b^2*x^4*sgn(b*x^2 + a) + 315*a^4*b*x^2*
sgn(b*x^2 + a) + 56*a^5*sgn(b*x^2 + a))/x^18
```

Mupad [B]
time = 4.27, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{18\,x^{18}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^{10}\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,\left(b\,x^2+a\right)}-\frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^{12}\,\left(b\,x^2+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^{14}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^19,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(18*x^18*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (a*b^4*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(2*x^10*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^
(1/2))/(6*x^12*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))
/(7*x^14*(a + b*x^2))
```

3.602 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{21}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}$$

```
[Out] -1/20*a^5*((b*x^2+a)^2)^(1/2)/x^20/(b*x^2+a)-5/18*a^4*b*((b*x^2+a)^2)^(1/2)
/x^18/(b*x^2+a)-5/8*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^16/(b*x^2+a)-5/7*a^2*b^3*
((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)-5/12*a*b^4*((b*x^2+a)^2)^(1/2)/x^12/(b*x
^2+a)-1/10*b^5*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)
```

Rubi [A]
time = 0.11, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1125, 660, 45}

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```

```
[Out] -1/20*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^20*(a + b*x^2)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/(8*x^16*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(12*x^12*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^
10*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{21}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{11}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^{11}}\,dx, x, x^2\right)}{2b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^{11}}+\frac{5a^4b^6}{x^{10}}+\frac{10a^3b^7}{x^9}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^7}+\frac{b^{10}}{x^6}\right) d\right.}{2b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2}}{8x^{16}(a+bx}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(126a^5+700a^4bx^2+1575a^3b^2x^4+1800a^2b^3x^6+1050ab^4x^8+252b^5x^{10})}{2520x^{20}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
[Out] -1/2520*(Sqrt[(a + b*x^2)^2]*(126*a^5 + 700*a^4*b*x^2 + 1575*a^3*b^2*x^4 +
1800*a^2*b^3*x^6 + 1050*a*b^4*x^8 + 252*b^5*x^10))/(x^20*(a + b*x^2))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{20}a^5-\frac{5}{18}b\,a^4x^2-\frac{5}{8}b^2a^3x^4-\frac{5}{7}a^2b^3x^6-\frac{5}{12}b^4a\,x^8-\frac{1}{10}b^5x^{10}\right)}{(b\,x^2+a)x^{20}}$	79
gosper	$-\frac{(252b^5x^{10}+1050b^4a\,x^8+1800a^2b^3x^6+1575b^2a^3x^4+700b\,a^4x^2+126a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{2520x^{20}(b\,x^2+a)^5}$	80

default	$-\frac{\left(252 b^{5} x^{10}+1050 b^{4} a \,x^{8}+1800 a^{2} b^{3} x^{6}+1575 b^{2} a^{3} x^{4}+700 b \,a^{4} x^{2}+126 a^{5}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}{2520 x^{20} \left(b \,x^{2}+a \right)^{5}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2520*(252*b^5*x^10+1050*a*b^4*x^8+1800*a^2*b^3*x^6+1575*a^3*b^2*x^4+700*
a^4*b*x^2+126*a^5)*((b*x^2+a)^2)^(5/2)/x^20/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.22

$$-\frac{b^{5}}{10\,x^{10}}-\frac{5\,ab^{4}}{12\,x^{12}}-\frac{5\,a^{2}b^{3}}{7\,x^{14}}-\frac{5\,a^{3}b^{2}}{8\,x^{16}}-\frac{5\,a^{4}b}{18\,x^{18}}-\frac{a^{5}}{20\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="maxima")
```

```
[Out] -1/10*b^5/x^10 - 5/12*a*b^4/x^12 - 5/7*a^2*b^3/x^14 - 5/8*a^3*b^2/x^16 - 5/
18*a^4*b/x^18 - 1/20*a^5/x^20
```

Fricas [A]
time = 0.34, size = 59, normalized size = 0.23

$$-\frac{252\,b^{5}x^{10}+1050\,ab^{4}x^{8}+1800\,a^{2}b^{3}x^{6}+1575\,a^{3}b^{2}x^{4}+700\,a^{4}bx^{2}+126\,a^{5}}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^
4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{21}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**21,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**21, x)
```

Giac [A]
time = 4.81, size = 107, normalized size = 0.42

$$-\frac{252\,b^5x^{10}\mathrm{sgn}(bx^2+a)+1050\,ab^4x^8\mathrm{sgn}(bx^2+a)+1800\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+1575\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+700\,a^4bx^2\mathrm{sgn}(bx^2+a)+126\,a^5\mathrm{sgn}(bx^2+a)}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="giac")
```

```
[Out] -1/2520*(252*b^5*x^10*sgn(b*x^2 + a) + 1050*a*b^4*x^8*sgn(b*x^2 + a) + 1800
*a^2*b^3*x^6*sgn(b*x^2 + a) + 1575*a^3*b^2*x^4*sgn(b*x^2 + a) + 700*a^4*b*x
^2*sgn(b*x^2 + a) + 126*a^5*sgn(b*x^2 + a))/x^20
```

Mupad [B]
time = 4.22, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{20\,x^{20}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{18\,x^{18}\,\left(b\,x^2+a\right)}-\frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^{14}\,\left(b\,x^2+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^{16}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^21,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(20*x^20*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(1/2))/(18*x^18*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(7*x^14*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(8*x^16*(a + b*x^2))
```

3.603 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{23}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}(a+bx^2)}$$

```
[Out] -1/22*a^5*((b*x^2+a)^2)^(1/2)/x^22/(b*x^2+a)-1/4*a^4*b*((b*x^2+a)^2)^(1/2)/
x^20/(b*x^2+a)-5/9*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^18/(b*x^2+a)-5/8*a^2*b^3*(
(b*x^2+a)^2)^(1/2)/x^16/(b*x^2+a)-5/14*a*b^4*((b*x^2+a)^2)^(1/2)/x^14/(b*x^
2+a)-1/12*b^5*((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)
```

Rubi [A]
time = 0.11, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```

```
[Out] -1/22*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^22*(a + b*x^2)) - (a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^20*(a + b*x^2)) - (5*a^3*b^2*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])/(8*x^16*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(14*x^14*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*
(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{23}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{12}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^{12}}\,dx, x, x^2\right)}{2b^4(ab+b^2x^2)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^{11}}+\frac{10a^3b^7}{x^{10}}+\frac{10a^2b^8}{x^9}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^7}\right) d\right.}{2b^4(ab+b^2x^2)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}(a+bx^2)} - \frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2}}{9x^{18}(a+bx^2}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(252a^5+1386a^4bx^2+3080a^3b^2x^4+3465a^2b^3x^6+1980ab^4x^8+462b^5x^{10})}{5544x^{22}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
[Out] -1/5544*(Sqrt[(a + b*x^2)^2]*(252*a^5 + 1386*a^4*b*x^2 + 3080*a^3*b^2*x^4 +
 3465*a^2*b^3*x^6 + 1980*a*b^4*x^8 + 462*b^5*x^10))/(x^22*(a + b*x^2))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,(-\frac{1}{22}a^5-\frac{1}{4}b\,a^4x^2-\frac{5}{9}b^2a^3x^4-\frac{5}{8}a^2b^3x^6-\frac{5}{14}b^4a\,x^8-\frac{1}{12}b^5x^{10})}{(b\,x^2+a)x^{22}}$	79
gosper	$-\frac{(462b^5x^{10}+1980b^4a\,x^8+3465a^2b^3x^6+3080b^2a^3x^4+1386b\,a^4x^2+252a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{5544x^{22}(b\,x^2+a)^5}$	80

default	$-\frac{\left(462 b^{5} x^{10}+1980 b^{4} a\,x^{8}+3465 a^{2} b^{3} x^{6}+3080 b^{2} a^{3} x^{4}+1386 b\,a^{4} x^{2}+252 a^{5}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{5544 x^{22} \left(b\,x^{2}+a\right)^{5}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5544*(462*b^5*x^10+1980*a*b^4*x^8+3465*a^2*b^3*x^6+3080*a^3*b^2*x^4+1386
*a^4*b*x^2+252*a^5)*((b*x^2+a)^2)^(5/2)/x^22/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.22

$$-\frac{b^{5}}{12\,x^{12}}-\frac{5\,ab^{4}}{14\,x^{14}}-\frac{5\,a^{2}b^{3}}{8\,x^{16}}-\frac{5\,a^{3}b^{2}}{9\,x^{18}}-\frac{a^{4}b}{4\,x^{20}}-\frac{a^{5}}{22\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="maxima")
```

```
[Out] -1/12*b^5/x^12 - 5/14*a*b^4/x^14 - 5/8*a^2*b^3/x^16 - 5/9*a^3*b^2/x^18 - 1/
4*a^4*b/x^20 - 1/22*a^5/x^22
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.23

$$-\frac{462\,b^{5}x^{10}+1980\,ab^{4}x^{8}+3465\,a^{2}b^{3}x^{6}+3080\,a^{3}b^{2}x^{4}+1386\,a^{4}bx^{2}+252\,a^{5}}{5544\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="fricas")
```

```
[Out] -1/5544*(462*b^5*x^10 + 1980*a*b^4*x^8 + 3465*a^2*b^3*x^6 + 3080*a^3*b^2*x^
4 + 1386*a^4*b*x^2 + 252*a^5)/x^22
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{23}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**23,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**23, x)
```

Giac [A]
time = 3.44, size = 107, normalized size = 0.42

$$-\frac{462\,b^{5}x^{10}\mathrm{sgn}(bx^{2}+a)+1980\,ab^{4}x^{8}\mathrm{sgn}(bx^{2}+a)+3465\,a^{2}b^{3}x^{6}\mathrm{sgn}(bx^{2}+a)+3080\,a^{3}b^{2}x^{4}\mathrm{sgn}(bx^{2}+a)+1386\,a^{4}bx^{2}\mathrm{sgn}(bx^{2}+a)+252\,a^{5}\mathrm{sgn}(bx^{2}+a)}{5544\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="giac")
```

```
[Out] -1/5544*(462*b^5*x^10*sgn(b*x^2 + a) + 1980*a*b^4*x^8*sgn(b*x^2 + a) + 3465
*a^2*b^3*x^6*sgn(b*x^2 + a) + 3080*a^3*b^2*x^4*sgn(b*x^2 + a) + 1386*a^4*b*
x^2*sgn(b*x^2 + a) + 252*a^5*sgn(b*x^2 + a))/x^22
```

Mupad [B]
time = 4.23, size = 231, normalized size = 0.91

$$-\frac{a^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{22\,x^{22}\,\left(b\,x^{2}+a\right)}-\frac{b^{5}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{12\,x^{12}\,\left(b\,x^{2}+a\right)}-\frac{5\,a\,b^{4}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{14\,x^{14}\,\left(b\,x^{2}+a\right)}-\frac{a^{4}\,b\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{4\,x^{20}\,\left(b\,x^{2}+a\right)}-\frac{5\,a^{2}\,b^{3}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{8\,x^{16}\,\left(b\,x^{2}+a\right)}-\frac{5\,a^{3}\,b^{2}\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{9\,x^{18}\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^23,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(22*x^22*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2))/(4*x^20*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(8*x^16*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))/(9*x^18*(a + b*x^2))
```

3.604 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{25}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}(a+bx^2)}-\frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}(a+bx^2)}$$

```
[Out] -1/24*a^5*((b*x^2+a)^2)^(1/2)/x^24/(b*x^2+a)-5/22*a^4*b*((b*x^2+a)^2)^(1/2)
/x^22/(b*x^2+a)-1/2*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^20/(b*x^2+a)-5/9*a^2*b^3*
((b*x^2+a)^2)^(1/2)/x^18/(b*x^2+a)-5/16*a*b^4*((b*x^2+a)^2)^(1/2)/x^16/(b*x
^2+a)-1/14*b^5*((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)
```

Rubi [A]
time = 0.11, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}(a+bx^2)}-\frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

```
[Out] -1/24*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^24*(a + b*x^2)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(2*x^20*(a + b*x^2)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(16*x^16*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14
*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{25}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a^2+2abx+b^2x^2)^{5/2}}{x^{13}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^{13}}\,dx, x, x^2\right)}{2b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(\frac{a^5b^5}{x^{13}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^{11}}+\frac{10a^2b^8}{x^{10}}+\frac{5ab^9}{x^9}+\frac{b^{10}}{x^8}\right) d\right.}{2b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}(a+bx^2)} - \frac{a^3b^2\sqrt{a^2+2abx^2}}{2x^{20}(a+bx^2}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(462a^5+2520a^4bx^2+5544a^3b^2x^4+6160a^2b^3x^6+3465ab^4x^8+792b^5x^{10})}{11088x^{24}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

```
[Out] -1/11088*(Sqrt[(a + b*x^2)^2]*(462*a^5 + 2520*a^4*b*x^2 + 5544*a^3*b^2*x^4
+ 6160*a^2*b^3*x^6 + 3465*a*b^4*x^8 + 792*b^5*x^10))/(x^24*(a + b*x^2))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{14}b^5x^{10}-\frac{5}{16}b^4a\,x^8-\frac{1}{2}b^2a^3x^4-\frac{5}{9}a^2b^3x^6-\frac{5}{22}b\,a^4x^2-\frac{1}{24}a^5\right)}{(b\,x^2+a)x^{24}}$	79
gosper	$-\frac{(792b^5x^{10}+3465b^4a\,x^8+6160a^2b^3x^6+5544b^2a^3x^4+2520b\,a^4x^2+462a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{11088x^{24}(b\,x^2+a)^5}$	80

default	$-\frac{\left(792 b^{5} x^{10}+3465 b^{4} a \,x^{8}+6160 a^{2} b^{3} x^{6}+5544 b^{2} a^{3} x^{4}+2520 b \,a^{4} x^{2}+462 a^{5}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}{11088 x^{24} \left(b \,x^{2}+a \right)^{5}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x,method=_RETURNVERBOSE)
[Out] -1/11088*(792*b^5*x^10+3465*a*b^4*x^8+6160*a^2*b^3*x^6+5544*a^3*b^2*x^4+252
0*a^4*b*x^2+462*a^5)*((b*x^2+a)^2)^(5/2)/x^24/(b*x^2+a)^5
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.22

$$-\frac{b^{5}}{14\,x^{14}}-\frac{5\,ab^{4}}{16\,x^{16}}-\frac{5\,a^{2}b^{3}}{9\,x^{18}}-\frac{a^{3}b^{2}}{2\,x^{20}}-\frac{5\,a^{4}b}{22\,x^{22}}-\frac{a^{5}}{24\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="maxima")
[Out] -1/14*b^5/x^14 - 5/16*a*b^4/x^16 - 5/9*a^2*b^3/x^18 - 1/2*a^3*b^2/x^20 - 5/
22*a^4*b/x^22 - 1/24*a^5/x^24
```

Fricas [A]
time = 0.36, size = 59, normalized size = 0.23

$$-\frac{792\,b^{5}x^{10}+3465\,ab^{4}x^{8}+6160\,a^{2}b^{3}x^{6}+5544\,a^{3}b^{2}x^{4}+2520\,a^{4}bx^{2}+462\,a^{5}}{11088\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="fricas")
[Out] -1/11088*(792*b^5*x^10 + 3465*a*b^4*x^8 + 6160*a^2*b^3*x^6 + 5544*a^3*b^2*x
^4 + 2520*a^4*b*x^2 + 462*a^5)/x^24
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{25}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**25,x)
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**25, x)
```

Giac [A]
time = 3.47, size = 107, normalized size = 0.42

$$-\frac{792\,b^5x^{10}\mathrm{sgn}(bx^2+a)+3465\,ab^4x^8\mathrm{sgn}(bx^2+a)+6160\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+5544\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+2520\,a^4bx^2\mathrm{sgn}(bx^2+a)+462\,a^5\mathrm{sgn}(bx^2+a)}{11088\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="giac")
```

```
[Out] -1/11088*(792*b^5*x^10*sgn(b*x^2 + a) + 3465*a*b^4*x^8*sgn(b*x^2 + a) + 616
0*a^2*b^3*x^6*sgn(b*x^2 + a) + 5544*a^3*b^2*x^4*sgn(b*x^2 + a) + 2520*a^4*b
*x^2*sgn(b*x^2 + a) + 462*a^5*sgn(b*x^2 + a))/x^24
```

Mupad [B]
time = 4.22, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{24\,x^{24}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{22\,x^{22}\,\left(b\,x^2+a\right)}-\frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^{18}\,\left(b\,x^2+a\right)}-\frac{a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^{20}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^25,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(24*x^24*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(1/2))/(22*x^22*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(9*x^18*(a + b*x^2)) - (a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/
2))/(2*x^20*(a + b*x^2))
```

3.605 $\int x^{12}(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{a^4bx^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\,(a+bx^2)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2}}{19\,(a+}$$

```
[Out] 1/13*a^5*x^13*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*a^4*b*x^15*((b*x^2+a)^2)^(1
/2)/(b*x^2+a)+10/17*a^3*b^2*x^17*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/19*a^2*b^
3*x^19*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/21*a*b^4*x^21*((b*x^2+a)^2)^(1/2)/(b
*x^2+a)+1/23*b^5*x^23*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]

time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$\frac{b^5x^{23}\sqrt{a^2+2abx^2+b^2x^4}}{23\,(a+bx^2)}+\frac{5ab^4x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\,(a+bx^2)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\,(a+bx^2)}+\frac{a^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{a^4bx^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a^4*b*x^15*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a^3*b^2*x^17*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (10*a^2*b^3*x^19*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(19*(a + b*x^2)) + (5*a*b^4*x^21*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(21*(a + b*x^2)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23
*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int x^{12}(ab+b^2x^2)^5\,dx}{b^4\,(ab+b^2x^2)}$$

$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int (a^5b^5x^{12}+5a^4b^6x^{14}+10a^3b^7x^{16}+10a^2b^8x^{18}+5}{b^4\,(ab+b^2x^2)}$$

$$=\frac{a^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{a^4bx^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{10a^3b^2x^{17}\sqrt{}}{17}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{13}\sqrt{(a+bx^2)^2}\,(156009a^5+676039a^4bx^2+1193010a^3b^2x^4+1067430a^2b^3x^6+482885ab^4x^8+88179b^5x^{10})}{2028117\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^13*Sqrt[(a + b*x^2)^2]*(156009*a^5 + 676039*a^4*b*x^2 + 1193010*a^3*b^2*
x^4 + 1067430*a^2*b^3*x^6 + 482885*a*b^4*x^8 + 88179*b^5*x^10))/(2028117*(a
 + b*x^2))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^{13}\left(88179b^5x^{10}+482885b^4a\,x^8+1067430a^2b^3x^6+1193010b^2a^3x^4+676039b\,a^4x^2+156009a^5\right)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{2028117(b\,x^2+a)^5}$
default	$\frac{x^{13}\left(88179b^5x^{10}+482885b^4a\,x^8+1067430a^2b^3x^6+1193010b^2a^3x^4+676039b\,a^4x^2+156009a^5\right)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{2028117(b\,x^2+a)^5}$
risch	$\frac{a^5x^{13}\sqrt{(b\,x^2+a)^2}}{13b\,x^2+13a}+\frac{a^4b\,x^{15}\sqrt{(b\,x^2+a)^2}}{3b\,x^2+3a}+\frac{10a^3b^2x^{17}\sqrt{(b\,x^2+a)^2}}{17(b\,x^2+a)}+\frac{10a^2b^3x^{19}\sqrt{(b\,x^2+a)^2}}{19(b\,x^2+a)}+\frac{5a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2028117*x^13*(88179*b^5*x^10+482885*a*b^4*x^8+1067430*a^2*b^3*x^6+1193010
*a^3*b^2*x^4+676039*a^4*b*x^2+156009*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23}+\frac{5}{21}\,ab^4x^{21}+\frac{10}{19}\,a^2b^3x^{19}+\frac{10}{17}\,a^3b^2x^{17}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*b^2*x^17 +
 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

Fricas [A]
time = 0.33, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23}+\frac{5}{21}\,ab^4x^{21}+\frac{10}{19}\,a^2b^3x^{19}+\frac{10}{17}\,a^3b^2x^{17}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*b^2*x^17 +
 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{12}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**12*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.01, size = 105, normalized size = 0.41

$$\frac{1}{23}\,b^5x^{23}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{21}\,ab^4x^{21}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{19}\,a^2b^3x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{17}\,a^3b^2x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,a^4bx^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{13}\,a^5x^{13}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/23*b^5*x^23*sgn(b*x^2 + a) + 5/21*a*b^4*x^21*sgn(b*x^2 + a) + 10/19*a^2*b
^3*x^19*sgn(b*x^2 + a) + 10/17*a^3*b^2*x^17*sgn(b*x^2 + a) + 1/3*a^4*b*x^15
*sgn(b*x^2 + a) + 1/13*a^5*x^13*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{12}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^12*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.606 $\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{2a^3b^2x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+}}{17\left(a+\right.}$$

```
[Out] 1/11*a^5*x^11*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/13*a^4*b*x^13*((b*x^2+a)^2)^(
1/2)/(b*x^2+a)+2/3*a^3*b^2*x^15*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/17*a^2*b^3
*x^17*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/19*a*b^4*x^19*((b*x^2+a)^2)^(1/2)/(b*
x^2+a)+1/21*b^5*x^21*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1126, 276}

$$\frac{b^5x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{a^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{2a^3b^2x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*a^4*b*x^13
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2*a^3*b^2*x^15*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(19*(a + b*x^2)) + (b^5*x^21*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2
1*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int x^{10}(ab+b^2x^2)^5\,dx}{b^4\,(ab+b^2x^2)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int(a^5b^5x^{10}+5a^4b^6x^{12}+10a^3b^7x^{14}+10a^2b^8x^{16}+5a}{b^4\,(ab+b^2x^2)}$$
$$=\frac{a^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{2a^3b^2x^{15}\sqrt{a}}{3\,(}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^{11}\sqrt{(a+bx^2)^2}\,(88179a^5+373065a^4bx^2+646646a^3b^2x^4+570570a^2b^3x^6+255255ab^4x^8+46189b^5x^{10})}{969969\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^11*Sqrt[(a + b*x^2)^2]*(88179*a^5 + 373065*a^4*b*x^2 + 646646*a^3*b^2*x^
4 + 570570*a^2*b^3*x^6 + 255255*a*b^4*x^8 + 46189*b^5*x^10))/(969969*(a + b
*x^2))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^{11}\left(46189b^5x^{10}+255255b^4a\,x^8+570570a^2b^3x^6+646646b^2a^3x^4+373065b\,a^4x^2+88179a^5\right)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{969969(b\,x^2+a)^5}$
default	$\frac{x^{11}\left(46189b^5x^{10}+255255b^4a\,x^8+570570a^2b^3x^6+646646b^2a^3x^4+373065b\,a^4x^2+88179a^5\right)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{969969(b\,x^2+a)^5}$
risch	$\frac{a^5x^{11}\sqrt{(b\,x^2+a)^2}}{11b\,x^2+11a}+\frac{5a^4b\,x^{13}\sqrt{(b\,x^2+a)^2}}{13(b\,x^2+a)}+\frac{2a^3b^2x^{15}\sqrt{(b\,x^2+a)^2}}{3(b\,x^2+a)}+\frac{10a^2b^3x^{17}\sqrt{(b\,x^2+a)^2}}{17(b\,x^2+a)}+\frac{5a\,b^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/969969*x^11*(46189*b^5*x^10+255255*a*b^4*x^8+570570*a^2*b^3*x^6+646646*a^
3*b^2*x^4+373065*a^4*b*x^2+88179*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.22

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{19}\,ab^4x^{19}+\frac{10}{17}\,a^2b^3x^{17}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^2*x^15 + 5
/13*a^4*b*x^13 + 1/11*a^5*x^11
```

Fricas [A]
time = 0.35, size = 57, normalized size = 0.22

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{19}\,ab^4x^{19}+\frac{10}{17}\,a^2b^3x^{17}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^2*x^15 + 5
/13*a^4*b*x^13 + 1/11*a^5*x^11
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{10}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**10*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.03, size = 105, normalized size = 0.41

$$\frac{1}{21}\,b^5x^{21}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{19}\,ab^4x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{17}\,a^2b^3x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\,a^3b^2x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{13}\,a^4bx^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{11}\,a^5x^{11}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/21*b^5*x^21*sgn(b*x^2 + a) + 5/19*a*b^4*x^19*sgn(b*x^2 + a) + 10/17*a^2*b
^3*x^17*sgn(b*x^2 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^2 + a) + 5/13*a^4*b*x^13*
sgn(b*x^2 + a) + 1/11*a^5*x^11*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{10}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^10*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.607 $\int x^8(a^2 + 2abx^2 + b^2x^4)^{5/2}\, dx$

Optimal. Leaf size=255

$$\frac{a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{2a^2b^3x^{15}\sqrt{a^2+2a}}{3\left(a+b\right.}$$

```
[Out] 1/9*a^5*x^9*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/11*a^4*b*x^11*((b*x^2+a)^2)^(1/
2)/(b*x^2+a)+10/13*a^3*b^2*x^13*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+2/3*a^2*b^3*x
^15*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/17*a*b^4*x^17*((b*x^2+a)^2)^(1/2)/(b*x^
2+a)+1/19*b^5*x^19*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$\frac{b^5x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{2a^2b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (5*a^4*b*x^11*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*a^3*b^2*x^13*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2*a^2*b^3*x^15*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(17*(a + b*x^2)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*
(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^8\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int x^8\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int \left(a^5b^5x^8+5a^4b^6x^{10}+10a^3b^7x^{12}+10a^2b^8x^{14}+5a\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)} + \frac{5a^4bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)} + \frac{10a^3b^2x^{13}\sqrt{}}{13}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^9\sqrt{\left(a+bx^2\right)^2}\left(46189a^5+188955a^4bx^2+319770a^3b^2x^4+277134a^2b^3x^6+122265ab^4x^8+21879b^5x^{10}\right)}{415701\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
[Out] (x^9*Sqrt[(a + b*x^2)^2]*(46189*a^5 + 188955*a^4*b*x^2 + 319770*a^3*b^2*x^4
 + 277134*a^2*b^3*x^6 + 122265*a*b^4*x^8 + 21879*b^5*x^10))/(415701*(a + b*
x^2))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^9\left(21879b^5x^{10}+122265b^4a\,x^8+277134a^2b^3x^6+319770b^2a^3x^4+188955b\,a^4x^2+46189a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{415701\left(b\,x^2+a\right)^5}$
default	$\frac{x^9\left(21879b^5x^{10}+122265b^4a\,x^8+277134a^2b^3x^6+319770b^2a^3x^4+188955b\,a^4x^2+46189a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{415701\left(b\,x^2+a\right)^5}$
risch	$\frac{a^5x^9\sqrt{\left(b\,x^2+a\right)^2}}{9b\,x^2+9a}+\frac{5a^4b\,x^{11}\sqrt{\left(b\,x^2+a\right)^2}}{11\left(b\,x^2+a\right)}+\frac{10a^3b^2x^{13}\sqrt{\left(b\,x^2+a\right)^2}}{13\left(b\,x^2+a\right)}+\frac{2a^2b^3x^{15}\sqrt{\left(b\,x^2+a\right)^2}}{3\left(b\,x^2+a\right)}+\frac{5a\,b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/415701*x^9*(21879*b^5*x^10+122265*a*b^4*x^8+277134*a^2*b^3*x^6+319770*a^3
*b^2*x^4+188955*a^4*b*x^2+46189*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.22

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Fricas [A]
time = 0.32, size = 57, normalized size = 0.22

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 + 5
/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**8*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.31, size = 105, normalized size = 0.41

$$\frac{1}{19}\,b^5x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{17}\,ab^4x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\,a^2b^3x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{13}\,a^3b^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{11}\,a^4bx^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{9}\,a^5x^9\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19*sgn(b*x^2 + a) + 5/17*a*b^4*x^17*sgn(b*x^2 + a) + 2/3*a^2*b^3
*x^15*sgn(b*x^2 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^2 + a) + 5/11*a^4*b*x^11*
sgn(b*x^2 + a) + 1/9*a^5*x^9*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.608 $\int x^6(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{5a^4bx^9\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11(a+bx^2)}+\frac{10a^2b^3x^{13}\sqrt{a^2+}}{13(a+}$$

```
[Out] 1/7*a^5*x^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/9*a^4*b*x^9*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)+10/11*a^3*b^2*x^11*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/13*a^2*b^3*x
^13*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*a*b^4*x^15*((b*x^2+a)^2)^(1/2)/(b*x^2
+a)+1/17*b^5*x^17*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$\frac{b^5x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17(a+bx^2)}+\frac{ab^4x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)}+\frac{a^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{5a^4bx^9\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (5*a^4*b*x^9*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^3*b^2*x^11*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a*b^4*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(3*(a + b*x^2)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a
+ b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^6 (ab+b^2x^2)^5 dx}{b^4 (ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (a^5b^5x^6+5a^4b^6x^8+10a^3b^7x^{10}+10a^2b^8x^{12}+5ab^9x}{b^4 (ab+b^2x^2)}$$
$$= \frac{a^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)} + \frac{5a^4bx^9\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)} + \frac{10a^3b^2x^{11}\sqrt{a^2}}{11(a}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^7\sqrt{(a+bx^2)^2}\,(21879a^5+85085a^4bx^2+139230a^3b^2x^4+117810a^2b^3x^6+51051ab^4x^8+9009b^5x^{10})}{153153\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^2)^2]*(21879*a^5 + 85085*a^4*b*x^2 + 139230*a^3*b^2*x^4
+ 117810*a^2*b^3*x^6 + 51051*a*b^4*x^8 + 9009*b^5*x^10))/(153153*(a + b*x^2
))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^7(9009b^5x^{10}+51051b^4a\,x^8+117810a^2b^3x^6+139230b^2a^3x^4+85085b\,a^4x^2+21879a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{153153(b\,x^2+a)^5}$
default	$\frac{x^7(9009b^5x^{10}+51051b^4a\,x^8+117810a^2b^3x^6+139230b^2a^3x^4+85085b\,a^4x^2+21879a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{153153(b\,x^2+a)^5}$
risch	$\frac{a^5x^7\sqrt{(b\,x^2+a)^2}}{7b\,x^2+7a}+\frac{5a^4b\,x^9\sqrt{(b\,x^2+a)^2}}{9(b\,x^2+a)}+\frac{10a^3b^2x^{11}\sqrt{(b\,x^2+a)^2}}{11(b\,x^2+a)}+\frac{10a^2b^3x^{13}\sqrt{(b\,x^2+a)^2}}{13(b\,x^2+a)}+\frac{a\,b^4x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/153153*x^7*(9009*b^5*x^10+51051*a*b^4*x^8+117810*a^2*b^3*x^6+139230*a^3*b
^2*x^4+85085*a^4*b*x^2+21879*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.22

$$\frac{1}{17}\,b^5x^{17}+\frac{1}{3}\,ab^4x^{15}+\frac{10}{13}\,a^2b^3x^{13}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{9}\,a^4bx^9+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Fricas [A]
time = 0.34, size = 57, normalized size = 0.22

$$\frac{1}{17}\,b^5x^{17}+\frac{1}{3}\,ab^4x^{15}+\frac{10}{13}\,a^2b^3x^{13}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{9}\,a^4bx^9+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**6*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.00, size = 105, normalized size = 0.41

$$\frac{1}{17}\,b^5x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,ab^4x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{13}\,a^2b^3x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{11}\,a^3b^2x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^4bx^9\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{7}\,a^5x^7\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17*sgn(b*x^2 + a) + 1/3*a*b^4*x^15*sgn(b*x^2 + a) + 10/13*a^2*b^
3*x^13*sgn(b*x^2 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^2 + a) + 5/9*a^4*b*x^9*s
gn(b*x^2 + a) + 1/7*a^5*x^7*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.609 $\int x^4(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\,(a+bx^2)}+\frac{5a^4bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{10a^3b^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2a}}{11\,(a+bx}$$

```
[Out] 1/5*a^5*x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/7*a^4*b*x^7*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)+10/9*a^3*b^2*x^9*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/11*a^2*b^3*x^1
1*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5/13*a*b^4*x^13*((b*x^2+a)^2)^(1/2)/(b*x^2+
a)+1/15*b^5*x^15*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1126, 276}

$$\frac{b^5x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\,(a+bx^2)}+\frac{5ab^4x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}+\frac{a^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\,(a+bx^2)}+\frac{5a^4bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{10a^3b^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (5*a^4*b*x^7*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^3*b^2*x^9*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*a*b^4*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(13*(a + b*x^2)) + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*(a
 + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int x^4(ab+b^2x^2)^5\,dx}{b^4\,(ab+b^2x^2)}$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int \left(a^5b^5x^4+5a^4b^6x^6+10a^3b^7x^8+10a^2b^8x^{10}+5ab^9\right.}{b^4\,(ab+b^2x^2)}$$
$$=\frac{a^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\,(a+bx^2)}+\frac{5a^4bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{10a^3b^2x^9\sqrt{a^2}}{9\,(a}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^5\sqrt{(a+bx^2)^2}\,(9009a^5+32175a^4bx^2+50050a^3b^2x^4+40950a^2b^3x^6+17325ab^4x^8+3003b^5x^{10})}{45045\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^2)^2]*(9009*a^5 + 32175*a^4*b*x^2 + 50050*a^3*b^2*x^4 +
40950*a^2*b^3*x^6 + 17325*a*b^4*x^8 + 3003*b^5*x^10))/(45045*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^5\left(3003b^5x^{10}+17325b^4a\,x^8+40950a^2b^3x^6+50050b^2a^3x^4+32175b\,a^4x^2+9009a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{45045\left(b\,x^2+a\right)^5}$
default	$\frac{x^5\left(3003b^5x^{10}+17325b^4a\,x^8+40950a^2b^3x^6+50050b^2a^3x^4+32175b\,a^4x^2+9009a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{45045\left(b\,x^2+a\right)^5}$
risch	$\frac{a^5x^5\sqrt{\left(b\,x^2+a\right)^2}}{5b\,x^2+5a}+\frac{5a^4b\,x^7\sqrt{\left(b\,x^2+a\right)^2}}{7\left(b\,x^2+a\right)}+\frac{10a^3b^2x^9\sqrt{\left(b\,x^2+a\right)^2}}{9\left(b\,x^2+a\right)}+\frac{10a^2b^3x^{11}\sqrt{\left(b\,x^2+a\right)^2}}{11\left(b\,x^2+a\right)}+\frac{5a\,b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45045*x^5*(3003*b^5*x^10+17325*a*b^4*x^8+40950*a^2*b^3*x^6+50050*a^3*b^2*
x^4+32175*a^4*b*x^2+9009*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.22

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Fricas [A]
time = 0.33, size = 57, normalized size = 0.22

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 + 5
/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.75, size = 105, normalized size = 0.41

$$\frac{1}{15}\,b^5x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{13}\,ab^4x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{11}\,a^2b^3x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{9}\,a^3b^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}\,a^4bx^7\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{5}\,a^5x^5\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15*sgn(b*x^2 + a) + 5/13*a*b^4*x^13*sgn(b*x^2 + a) + 10/11*a^2*b
^3*x^11*sgn(b*x^2 + a) + 10/9*a^3*b^2*x^9*sgn(b*x^2 + a) + 5/7*a^4*b*x^7*sg
n(b*x^2 + a) + 1/5*a^5*x^5*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.610 $\int x^2(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal. Leaf size=252

$$\frac{a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{a^4bx^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{10a^2b^3x^9\sqrt{a^2+2a}}{9\,(a+bx}$$

```
[Out] 1/3*a^5*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^4*b*x^5*((b*x^2+a)^2)^(1/2)/(b*
x^2+a)+10/7*a^3*b^2*x^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/9*a^2*b^3*x^9*((b*
x^2+a)^2)^(1/2)/(b*x^2+a)+5/11*a*b^4*x^11*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/1
3*b^5*x^13*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$\frac{b^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}+\frac{a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{a^4bx^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (a^4*b*x^5*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x^7*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^2*b^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(9*(a + b*x^2)) + (5*a*b^4*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
11*(a + b*x^2)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2
))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int x^2 \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^2 (ab+b^2x^2)^5 dx}{b^4 (ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (a^5b^5x^2 + 5a^4b^6x^4 + 10a^3b^7x^6 + 10a^2b^8x^8 + 5ab^9x^1}{b^4 (ab+b^2x^2)}$$
$$= \frac{a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)} + \frac{a^4bx^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{10a^3b^2x^7\sqrt{a^2+}}{7(a+}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^3\sqrt{(a+bx^2)^2} (3003a^5 + 9009a^4bx^2 + 12870a^3b^2x^4 + 10010a^2b^3x^6 + 4095ab^4x^8 + 693b^5x^{10})}{9009(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^2)^2]*(3003*a^5 + 9009*a^4*b*x^2 + 12870*a^3*b^2*x^4 + 1
0010*a^2*b^3*x^6 + 4095*a*b^4*x^8 + 693*b^5*x^10))/(9009*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 80, normalized size = 0.32

method	result
gosper	$\frac{x^3(693b^5x^{10}+4095b^4a\,x^8+10010a^2b^3x^6+12870b^2a^3x^4+9009b\,a^4x^2+3003a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{9009(b\,x^2+a)^5}$
default	$\frac{x^3(693b^5x^{10}+4095b^4a\,x^8+10010a^2b^3x^6+12870b^2a^3x^4+9009b\,a^4x^2+3003a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{9009(b\,x^2+a)^5}$
risch	$\frac{a^5x^3\sqrt{(b\,x^2+a)^2}}{3b\,x^2+3a} + \frac{a^4b\,x^5\sqrt{(b\,x^2+a)^2}}{b\,x^2+a} + \frac{10a^3b^2x^7\sqrt{(b\,x^2+a)^2}}{7(b\,x^2+a)} + \frac{10a^2b^3x^9\sqrt{(b\,x^2+a)^2}}{9(b\,x^2+a)} + \frac{5a\,b^4x^{11}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9009*x^3*(693*b^5*x^10+4095*a*b^4*x^8+10010*a^2*b^3*x^6+12870*a^3*b^2*x^4
+9009*a^4*b*x^2+3003*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.22

$$\frac{1}{13}\,b^5x^{13} + \frac{5}{11}\,ab^4x^{11} + \frac{10}{9}\,a^2b^3x^9 + \frac{10}{7}\,a^3b^2x^7 + a^4bx^5 + \frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4 *b*x^5 + 1/3*a^5*x^3

Fricas [A]
time = 0.34, size = 56, normalized size = 0.22

$$\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^4 *b*x^5 + 1/3*a^5*x^3

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)

[Out] Integral(x**2*((a + b*x**2)**2)**(5/2), x)

Giac [A]
time = 3.91, size = 104, normalized size = 0.41

$$\frac{1}{13}\,b^5x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{11}\,ab^4x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{9}\,a^2b^3x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{7}\,a^3b^2x^7\mathrm{sgn}\left(bx^2+a\right)+a^4bx^5\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,a^5x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")

[Out] 1/13*b^5*x^13*sgn(b*x^2 + a) + 5/11*a*b^4*x^11*sgn(b*x^2 + a) + 10/9*a^2*b^ 3*x^9*sgn(b*x^2 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^2 + a) + a^4*b*x^5*sgn(b*x^ 2 + a) + 1/3*a^5*x^3*sgn(b*x^2 + a)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)

[Out] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)

3.611 $\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=248

$$\frac{a^5x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5}+\frac{5a^4bx^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{3\left(a+bx^2\right)^5}+\frac{2a^3b^2x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5}+\frac{10a^2b^3x^7\left(a^2+2a\right.}{7\left(a+\right.}$$

```
[Out] a^5*x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(b*x^2+a)^5+5/3*a^4*b*x^3*(b^2*x^4+2*a*
b*x^2+a^2)^(5/2)/(b*x^2+a)^5+2*a^3*b^2*x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(b
*x^2+a)^5+10/7*a^2*b^3*x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(b*x^2+a)^5+5/9*a*
b^4*x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(b*x^2+a)^5+1/11*b^5*x^11*(b^2*x^4+2*
a*b*x^2+a^2)^(5/2)/(b*x^2+a)^5
```

Rubi [A]

time = 0.03, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1102, 200}

$$\frac{b^5x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{11\left(a+bx^2\right)^5}+\frac{5ab^4x^9\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{9\left(a+bx^2\right)^5}+\frac{10a^2b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{7\left(a+bx^2\right)^5}+\frac{a^5x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5}+\frac{5a^4bx^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{3\left(a+bx^2\right)^5}+\frac{2a^3b^2x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (5*a^4*b*x^3*(a^2
 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(3*(a + b*x^2)^5) + (2*a^3*b^2*x^5*(a^2 + 2*
a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (10*a^2*b^3*x^7*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(5/2))/(7*(a + b*x^2)^5) + (5*a*b^4*x^9*(a^2 + 2*a*b*x^2 + b^2*x
^4)^(5/2))/(9*(a + b*x^2)^5) + (b^5*x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
/(11*(a + b*x^2)^5)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1102

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2} \int \left(2ab+2b^2x^2\right)^5 dx}{\left(2ab+2b^2x^2\right)^5}$$
$$= \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2} \int \left(32a^5b^5+160a^4b^6x^2+320a^3b^7x^4+320a^2b^8x^6+1\right.}{\left(2ab+2b^2x^2\right)^5}$$
$$= \frac{a^5x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5} + \frac{5a^4bx^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{3\left(a+bx^2\right)^5} + \frac{2a^3b^2x^5\left(a^2+\right.}{\left(a\right.}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 0.33

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(693a^5x+1155a^4bx^3+1386a^3b^2x^5+990a^2b^3x^7+385ab^4x^9+63b^5x^{11}\right)}{693\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(693*a^5*x + 1155*a^4*b*x^3 + 1386*a^3*b^2*x^5 + 990*a
^2*b^3*x^7 + 385*a*b^4*x^9 + 63*b^5*x^11))/(693*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 78, normalized size = 0.31

method	result
gosper	$\frac{x\left(63b^5x^{10}+385b^4a\,x^8+990a^2b^3x^6+1386b^2a^3x^4+1155b\,a^4x^2+693a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{693\left(b\,x^2+a\right)^5}$
default	$\frac{x\left(63b^5x^{10}+385b^4a\,x^8+990a^2b^3x^6+1386b^2a^3x^4+1155b\,a^4x^2+693a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{693\left(b\,x^2+a\right)^5}$
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,b^5x^{11}}{11b\,x^2+11a}+\frac{5\sqrt{\left(b\,x^2+a\right)^2}\,a\,b^4x^9}{9\left(b\,x^2+a\right)}+\frac{10\sqrt{\left(b\,x^2+a\right)^2}\,a^2b^3x^7}{7\left(b\,x^2+a\right)}+\frac{2\sqrt{\left(b\,x^2+a\right)^2}\,a^3b^2x^5}{b\,x^2+a}+\frac{5\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/693*x*(63*b^5*x^10+385*a*b^4*x^8+990*a^2*b^3*x^6+1386*a^3*b^2*x^4+1155*a^
4*b*x^2+693*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 54, normalized size = 0.22

$$\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

Fricas [A]
time = 0.32, size = 54, normalized size = 0.22

$$\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4*
b*x^3 + a^5*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2), x)
```

Giac [A]
time = 3.97, size = 102, normalized size = 0.41

$$\frac{1}{11}\,b^5x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,ab^4x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{7}\,a^2b^3x^7\mathrm{sgn}\left(bx^2+a\right)+2\,a^3b^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{3}\,a^4bx^3\mathrm{sgn}\left(bx^2+a\right)+a^5x\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*sgn(b*x^2 + a) + 5/9*a*b^4*x^9*sgn(b*x^2 + a) + 10/7*a^2*b^3*
x^7*sgn(b*x^2 + a) + 2*a^3*b^2*x^5*sgn(b*x^2 + a) + 5/3*a^4*b*x^3*sgn(b*x^2
 + a) + a^5*x*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.612 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=247

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{5a^4bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2ab}}{a+bx^2}$$

```
[Out] -a^5*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+5*a^4*b*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a
)+10/3*a^3*b^2*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+2*a^2*b^3*x^5*((b*x^2+a)^2
)^(1/2)/(b*x^2+a)+5/7*a*b^4*x^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/9*b^5*x^9*(
(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$\frac{b^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5ab^4x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{5a^4bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (5*a^4*b*x*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x^3*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])/(3*(a + b*x^2)) + (2*a^2*b^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(a + b*x^2) + (5*a*b^4*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a +
b*x^2)) + (b^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^2}\,dx}{b^4\,(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(5a^4b^6+\frac{a^5b^5}{x^2}+10a^3b^7x^2+10a^2b^8x^4+5ab^9x^6+b^{10}x\right.}{b^4\,(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x\,(a+bx^2)}+\frac{5a^4bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2a}}{3\,(a+bx}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.34

$$\frac{\sqrt{(a+bx^2)^2}\,(-63a^5+315a^4bx^2+210a^3b^2x^4+126a^2b^3x^6+45ab^4x^8+7b^5x^{10})}{63x\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-63*a^5 + 315*a^4*b*x^2 + 210*a^3*b^2*x^4 + 126*a^2*b
^3*x^6 + 45*a*b^4*x^8 + 7*b^5*x^10))/(63*x*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-7b^5x^{10}-45b^4a\,x^8-126a^2b^3x^6-210b^2a^3x^4-315b\,a^4x^2+63a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{63x(b\,x^2+a)^5}$	80
default	$-\frac{(-7b^5x^{10}-45b^4a\,x^8-126a^2b^3x^6-210b^2a^3x^4-315b\,a^4x^2+63a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{63x(b\,x^2+a)^5}$	80
risch	$\frac{\sqrt{(b\,x^2+a)^2}\;b(\frac{1}{9}b^4x^9+\frac{5}{7}a\,b^3x^7+2a^2b^2x^5+\frac{10}{3}a^3b\,x^3+5a^4x)}{b\,x^2+a}-\frac{a^5\sqrt{(b\,x^2+a)^2}}{x(b\,x^2+a)}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/63*(-7*b^5*x^10-45*a*b^4*x^8-126*a^2*b^3*x^6-210*a^3*b^2*x^4-315*a^4*b*x
^2+63*a^5)*((b*x^2+a)^2)^(5/2)/x/(b*x^2+a)^5
```

Maxima [A]
time = 0.31, size = 55, normalized size = 0.22

$$\frac{1}{9}\,b^5x^9+\frac{5}{7}\,ab^4x^7+2\,a^2b^3x^5+\frac{10}{3}\,a^3b^2x^3+5\,a^4bx-\frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
- a^5/x
```

Fricas [A]
time = 0.31, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{10}+45\,ab^4x^8+126\,a^2b^3x^6+210\,a^3b^2x^4+315\,a^4bx^2-63\,a^5}{63\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*a
^4*b*x^2 - 63*a^5)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**2, x)
```

Giac [A]
time = 3.49, size = 103, normalized size = 0.42

$$\frac{1}{9}\,b^5x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{7}\,ab^4x^7\mathrm{sgn}\left(bx^2+a\right)+2\,a^2b^3x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{3}\,a^3b^2x^3\mathrm{sgn}\left(bx^2+a\right)+5\,a^4bx\mathrm{sgn}\left(bx^2+a\right)-\frac{a^5\mathrm{sgn}\left(bx^2+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*sgn(b*x^2 + a) + 5/7*a*b^4*x^7*sgn(b*x^2 + a) + 2*a^2*b^3*x^5*s
gn(b*x^2 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^2 + a) + 5*a^4*b*x*sgn(b*x^2 + a)
- a^5*sgn(b*x^2 + a)/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^2, x)
```

3.613 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=246

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^2}}{3\left(a+bx^2\right)}$$

```
[Out] -1/3*a^5*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-5*a^4*b*((b*x^2+a)^2)^(1/2)/x/(b
*x^2+a)+10*a^3*b^2*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10/3*a^2*b^3*x^3*((b*x^2
+a)^2)^(1/2)/(b*x^2+a)+a*b^4*x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/7*b^5*x^7*
((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$\frac{b^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

```
[Out] -1/3*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^3*b^2*x*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^2*b^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(3*(a + b*x^2)) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a +
b*x^2) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^4}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(10a^3b^7+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^2}+10a^2b^8x^2+5ab^9x^4+b^{10}x^6\right.}{b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)} + \frac{10a^3b^2x\sqrt{a^2+2ab}}{a+bx^2}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 0.34

$$\frac{\sqrt{(a+bx^2)^2}\,(-7a^5-105a^4bx^2+210a^3b^2x^4+70a^2b^3x^6+21ab^4x^8+3b^5x^{10})}{21x^3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-7*a^5 - 105*a^4*b*x^2 + 210*a^3*b^2*x^4 + 70*a^2*b^3
*x^6 + 21*a*b^4*x^8 + 3*b^5*x^10))/(21*x^3*(a + b*x^2))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.33

method	result	size
gosper	$-\frac{(-3b^5x^{10}-21b^4a\,x^8-70a^2b^3x^6-210b^2a^3x^4+105b\,a^4x^2+7a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{21x^3(b\,x^2+a)^5}$	80
default	$-\frac{(-3b^5x^{10}-21b^4a\,x^8-70a^2b^3x^6-210b^2a^3x^4+105b\,a^4x^2+7a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{21x^3(b\,x^2+a)^5}$	80
risch	$\frac{\sqrt{(b\,x^2+a)^2}\;b^2(\frac{1}{7}b^3x^7+a\,b^2x^5+\frac{10}{3}a^2b\,x^3+10a^3x)}{b\,x^2+a}+\frac{\sqrt{(b\,x^2+a)^2}\;(-5b\,a^4x^2-\frac{1}{3}a^5)}{(b\,x^2+a)x^3}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21*(-3*b^5*x^10-21*a*b^4*x^8-70*a^2*b^3*x^6-210*a^3*b^2*x^4+105*a^4*b*x^
2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^3/(b*x^2+a)^5
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.22

$$\frac{1}{7}\,b^5x^7+ab^4x^5+\frac{10}{3}\,a^2b^3x^3+10\,a^3b^2x-\frac{5\,a^4b}{x}-\frac{a^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="maxima")
[Out] 1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 5*a^4*b/x - 1/3
*a^5/x^3
```

Fricas [A]
time = 0.33, size = 59, normalized size = 0.24

$$\frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="fricas")
[Out] 1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a^
4*b*x^2 - 7*a^5)/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**4,x)
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**4, x)
```

Giac [A]
time = 5.90, size = 104, normalized size = 0.42

$$\frac{1}{7}\,b^5x^7\mathrm{sgn}\left(bx^2+a\right)+ab^4x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{3}\,a^2b^3x^3\mathrm{sgn}\left(bx^2+a\right)+10\,a^3b^2x\mathrm{sgn}\left(bx^2+a\right)-\frac{15\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+a^5\mathrm{sgn}\left(bx^2+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="giac")
[Out] 1/7*b^5*x^7*sgn(b*x^2 + a) + a*b^4*x^5*sgn(b*x^2 + a) + 10/3*a^2*b^3*x^3*sg
n(b*x^2 + a) + 10*a^3*b^2*x*sgn(b*x^2 + a) - 1/3*(15*a^4*b*x^2*sgn(b*x^2 +
a) + a^5*sgn(b*x^2 + a))/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^4,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^4, x)
```

3.614 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=249

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}+\frac{10a^2b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] -1/5*a^5*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-5/3*a^4*b*((b*x^2+a)^2)^(1/2)/x^
3/(b*x^2+a)-10*a^3*b^2*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+10*a^2*b^3*x*((b*x^2
+a)^2)^(1/2)/(b*x^2+a)+5/3*a*b^4*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/5*b^5*
x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$,
Rules used = {1126, 276}

$$\frac{b^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}+\frac{5ab^4x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{10a^2b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

```
[Out] -1/5*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*
(a + b*x^2)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^6}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(10a^2b^8+\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^2}+5ab^9x^2+b^{10}x^4\right)dx}{b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx^2}}{x(a+bx^2)}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^2)^2}\,(-3a^5-25a^4bx^2-150a^3b^2x^4+150a^2b^3x^6+25ab^4x^8+3b^5x^{10})}{15x^5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 25*a^4*b*x^2 - 150*a^3*b^2*x^4 + 150*a^2*b^3
*x^6 + 25*a*b^4*x^8 + 3*b^5*x^10))/(15*x^5*(a + b*x^2))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-3b^5x^{10}-25b^4a\,x^8-150a^2b^3x^6+150b^2a^3x^4+25b\,a^4x^2+3a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{15x^5(b\,x^2+a)^5}$	80
default	$-\frac{(-3b^5x^{10}-25b^4a\,x^8-150a^2b^3x^6+150b^2a^3x^4+25b\,a^4x^2+3a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{15x^5(b\,x^2+a)^5}$	80
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,b^3(\frac{1}{5}b^2x^5+\frac{5}{3}ab\,x^3+10a^2x)}{b\,x^2+a}+\frac{\sqrt{(b\,x^2+a)^2}\,(-10b^2a^3x^4-\frac{5}{3}b\,a^4x^2-\frac{1}{5}a^5)}{(b\,x^2+a)x^5}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-3*b^5*x^10-25*a*b^4*x^8-150*a^2*b^3*x^6+150*a^3*b^2*x^4+25*a^4*b*x^
2+3*a^5)*((b*x^2+a)^2)^(5/2)/x^5/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.22

$$\frac{1}{5}\,b^5x^5+\frac{5}{3}\,ab^4x^3+10\,a^2b^3x-\frac{10\,a^3b^2}{x}-\frac{5\,a^4b}{3\,x^3}-\frac{a^5}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 10*a^3*b^2/x - 5/3*a^4*b/x^3 -
 1/5*a^5/x^5
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.24

$$\frac{3\,b^5x^{10}+25\,ab^4x^8+150\,a^2b^3x^6-150\,a^3b^2x^4-25\,a^4bx^2-3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a^
4*b*x^2 - 3*a^5)/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**6, x)
```

Giac [A]
time = 4.61, size = 106, normalized size = 0.43

$$\frac{1}{5}\,b^5x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{3}\,ab^4x^3\mathrm{sgn}\left(bx^2+a\right)+10\,a^2b^3x\mathrm{sgn}\left(bx^2+a\right)-\frac{150\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+25\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+3\,a^5\mathrm{sgn}\left(bx^2+a\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5*sgn(b*x^2 + a) + 5/3*a*b^4*x^3*sgn(b*x^2 + a) + 10*a^2*b^3*x*sg
n(b*x^2 + a) - 1/15*(150*a^3*b^2*x^4*sgn(b*x^2 + a) + 25*a^4*b*x^2*sgn(b*x^
2 + a) + 3*a^5*sgn(b*x^2 + a))/x^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^6, x)
```

3.615 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^8}\,dx$

Optimal. Leaf size=247

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x}}{x(a+bx^2)}$$

```
[Out] -1/7*a^5*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-a^4*b*((b*x^2+a)^2)^(1/2)/x^5/(b
*x^2+a)-10/3*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-10*a^2*b^3*((b*x^2+a
)^2)^(1/2)/x/(b*x^2+a)+5*a*b^4*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/3*b^5*x^3*
((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$\frac{b^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{5ab^4x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

```
[Out] -1/7*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^7*(a + b*x^2)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(x*(a + b*x^2)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a
+ b*x^2) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^8}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(5ab^9+\frac{a^5b^5}{x^8}+\frac{5a^4b^6}{x^6}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^2}+b^{10}x^2\right)dx}{b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)} - \frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx^2}}{3x^3(a+bx^2}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^2)^2}\,(3a^5+21a^4bx^2+70a^3b^2x^4+210a^2b^3x^6-105ab^4x^8-7b^5x^{10})}{21x^7(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

```
[Out] -1/21*(Sqrt[(a + b*x^2)^2]*(3*a^5 + 21*a^4*b*x^2 + 70*a^3*b^2*x^4 + 210*a^2
*b^3*x^6 - 105*a*b^4*x^8 - 7*b^5*x^10))/(x^7*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-7b^5x^{10}-105b^4a\,x^8+210a^2b^3x^6+70b^2a^3x^4+21b\,a^4x^2+3a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{21(b\,x^2+a)^5x^7}$	80
default	$-\frac{(-7b^5x^{10}-105b^4a\,x^8+210a^2b^3x^6+70b^2a^3x^4+21b\,a^4x^2+3a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{21(b\,x^2+a)^5x^7}$	80
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,b^4(\frac{1}{3}b\,x^3+5ax)}{b\,x^2+a}+\frac{\sqrt{(b\,x^2+a)^2}\,(-10a^2b^3x^6-\frac{10}{3}b^2a^3x^4-b\,a^4x^2-\frac{1}{7}a^5)}{(b\,x^2+a)x^7}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21*(-7*b^5*x^10-105*a*b^4*x^8+210*a^2*b^3*x^6+70*a^3*b^2*x^4+21*a^4*b*x^
2+3*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/x^7
```

Maxima [A]
time = 0.29, size = 55, normalized size = 0.22

$$\frac{1}{3}\,b^5x^3+5\,ab^4x-\frac{10\,a^2b^3}{x}-\frac{10\,a^3b^2}{3\,x^3}-\frac{a^4b}{x^5}-\frac{a^5}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5*a*b^4*x - 10*a^2*b^3/x - 10/3*a^3*b^2/x^3 - a^4*b/x^5 - 1/7
*a^5/x^7
```

Fricas [A]
time = 0.34, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{10}+105\,ab^4x^8-210\,a^2b^3x^6-70\,a^3b^2x^4-21\,a^4bx^2-3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a^
4*b*x^2 - 3*a^5)/x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**8, x)
```

Giac [A]
time = 5.29, size = 106, normalized size = 0.43

$$\frac{1}{3}\,b^5x^3\mathrm{sgn}\left(bx^2+a\right)+5\,ab^4x\mathrm{sgn}\left(bx^2+a\right)-\frac{210\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+70\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+21\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+3\,a^5\mathrm{sgn}\left(bx^2+a\right)}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sgn(b*x^2 + a) + 5*a*b^4*x*sgn(b*x^2 + a) - 1/21*(210*a^2*b^3*x
^6*sgn(b*x^2 + a) + 70*a^3*b^2*x^4*sgn(b*x^2 + a) + 21*a^4*b*x^2*sgn(b*x^2
+ a) + 3*a^5*sgn(b*x^2 + a))/x^7
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^8,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^8, x)
```

3.616 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=246

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\,(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\,(a+bx^2)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5\,(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b}}{3x^3\,(a+bx^2)}$$

```
[Out] -1/9*a^5*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-5/7*a^4*b*((b*x^2+a)^2)^(1/2)/x^
7/(b*x^2+a)-2*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-10/3*a^2*b^3*((b*x^
2+a)^2)^(1/2)/x^3/(b*x^2+a)-5*a*b^4*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+b^5*x*(
(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$\frac{b^5x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x\,(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\,(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\,(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\,(a+bx^2)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

```
[Out] -1/9*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^9*(a + b*x^2)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^3*b^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/
(x*(a + b*x^2)) + (b^5*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^{10}}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^{10}+\frac{a^5b^5}{x^{10}}+\frac{5a^4b^6}{x^8}+\frac{10a^3b^7}{x^6}+\frac{10a^2b^8}{x^4}+\frac{5ab^9}{x^2}\right)dx}{b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)} - \frac{2a^3b^2\sqrt{a^2+2abx^2 -}}{x^5(a+bx^2)}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^2)^2}\,(7a^5+45a^4bx^2+126a^3b^2x^4+210a^2b^3x^6+315ab^4x^8-63b^5x^{10})}{63x^9(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

```
[Out] -1/63*(Sqrt[(a + b*x^2)^2]*(7*a^5 + 45*a^4*b*x^2 + 126*a^3*b^2*x^4 + 210*a^
2*b^3*x^6 + 315*a*b^4*x^8 - 63*b^5*x^10))/(x^9*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.33

method	result	size
gosper	$-\frac{(-63b^5x^{10}+315b^4a\,x^8+210a^2b^3x^6+126b^2a^3x^4+45b\,a^4x^2+7a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{63x^9(b\,x^2+a)^5}$	80
default	$-\frac{(-63b^5x^{10}+315b^4a\,x^8+210a^2b^3x^6+126b^2a^3x^4+45b\,a^4x^2+7a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{63x^9(b\,x^2+a)^5}$	80
risch	$\frac{b^5x\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}+\frac{\sqrt{(b\,x^2+a)^2}\,(-5b^4a\,x^8-\frac{10}{3}a^2b^3x^6-2b^2a^3x^4-\frac{5}{7}b\,a^4x^2-\frac{1}{9}a^5)}{(b\,x^2+a)x^9}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/63*(-63*b^5*x^10+315*a*b^4*x^8+210*a^2*b^3*x^6+126*a^3*b^2*x^4+45*a^4*b*
x^2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^9/(b*x^2+a)^5
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.22

$$b^5x - \frac{5\,ab^4}{x} - \frac{10\,a^2b^3}{3\,x^3} - \frac{2\,a^3b^2}{x^5} - \frac{5\,a^4b}{7\,x^7} - \frac{a^5}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] b^5*x - 5*a*b^4/x - 10/3*a^2*b^3/x^3 - 2*a^3*b^2/x^5 - 5/7*a^4*b/x^7 - 1/9*
a^5/x^9
```

Fricas [A]
time = 0.36, size = 59, normalized size = 0.24

$$\frac{63\,b^5x^{10}-315\,ab^4x^8-210\,a^2b^3x^6-126\,a^3b^2x^4-45\,a^4bx^2-7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45*
a^4*b*x^2 - 7*a^5)/x^9
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**10, x)
```

Giac [A]
time = 6.08, size = 105, normalized size = 0.43

$$b^5x\mathrm{sgn}\left(bx^2+a\right)-\frac{315\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+210\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+126\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+45\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+7\,a^5\mathrm{sgn}\left(bx^2+a\right)}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] b^5*x*sgn(b*x^2 + a) - 1/63*(315*a*b^4*x^8*sgn(b*x^2 + a) + 210*a^2*b^3*x^6
*sgn(b*x^2 + a) + 126*a^3*b^2*x^4*sgn(b*x^2 + a) + 45*a^4*b*x^2*sgn(b*x^2 +
 a) + 7*a^5*sgn(b*x^2 + a))/x^9
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^10,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^10, x)
```

3.617 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=251

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}$$

```
[Out] -1/11*a^5*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-5/9*a^4*b*((b*x^2+a)^2)^(1/2)/
x^9/(b*x^2+a)-10/7*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-2*a^2*b^3*((b*
x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-5/3*a*b^4*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-b
^5*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

```
[Out] -1/11*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^11*(a + b*x^2)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(3*x^3*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2
))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^{12}}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^{10}}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^6}+\frac{5ab^9}{x^4}+\frac{b^{10}}{x^2}\right)dx}{b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx^2}}{7x^7(a+bx}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(63a^5+385a^4bx^2+990a^3b^2x^4+1386a^2b^3x^6+1155ab^4x^8+693b^5x^{10})}{693x^{11}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

```
[Out] -1/693*(Sqrt[(a + b*x^2)^2]*(63*a^5 + 385*a^4*b*x^2 + 990*a^3*b^2*x^4 + 138
6*a^2*b^3*x^6 + 1155*a*b^4*x^8 + 693*b^5*x^10))/(x^11*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.32

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-b^5x^{10}-\frac{5}{3}b^4a\,x^8-2a^2b^3x^6-\frac{10}{7}b^2a^3x^4-\frac{5}{9}b\,a^4x^2-\frac{1}{11}a^5\right)}{(b\,x^2+a)x^{11}}$	79
gosper	$-\frac{(693b^5x^{10}+1155b^4a\,x^8+1386a^2b^3x^6+990b^2a^3x^4+385b\,a^4x^2+63a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{693(b\,x^2+a)^5x^{11}}$	80
default	$-\frac{(693b^5x^{10}+1155b^4a\,x^8+1386a^2b^3x^6+990b^2a^3x^4+385b\,a^4x^2+63a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{693(b\,x^2+a)^5x^{11}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/693*(693*b^5*x^10+1155*a*b^4*x^8+1386*a^2*b^3*x^6+990*a^3*b^2*x^4+385*a^
4*b*x^2+63*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/x^11
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.23

$$-\frac{b^5}{x}-\frac{5\,ab^4}{3\,x^3}-\frac{2\,a^2b^3}{x^5}-\frac{10\,a^3b^2}{7\,x^7}-\frac{5\,a^4b}{9\,x^9}-\frac{a^5}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] -b^5/x - 5/3*a*b^4/x^3 - 2*a^2*b^3/x^5 - 10/7*a^3*b^2/x^7 - 5/9*a^4*b/x^9 -
 1/11*a^5/x^11
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.24

$$-\frac{693\, b^{5} x^{10} + 1155\, a b^{4} x^{8} + 1386\, a^{2} b^{3} x^{6} + 990\, a^{3} b^{2} x^{4} + 385\, a^{4} b x^{2} + 63\, a^{5}}{693\, x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
+ 385*a^4*b*x^2 + 63*a^5)/x^11
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**12, x)
```

Giac [A]
time = 5.07, size = 107, normalized size = 0.43

$$-\frac{693\, b^{5} x^{10} \mathrm{sgn}\left(b x^{2} + a\right) + 1155\, a b^{4} x^{8} \mathrm{sgn}\left(b x^{2} + a\right) + 1386\, a^{2} b^{3} x^{6} \mathrm{sgn}\left(b x^{2} + a\right) + 990\, a^{3} b^{2} x^{4} \mathrm{sgn}\left(b x^{2} + a\right) + 385\, a^{4} b x^{2} \mathrm{sgn}\left(b x^{2} + a\right) + 63\, a^{5} \mathrm{sgn}\left(b x^{2} + a\right)}{693\, x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/693*(693*b^5*x^10*sgn(b*x^2 + a) + 1155*a*b^4*x^8*sgn(b*x^2 + a) + 1386*
a^2*b^3*x^6*sgn(b*x^2 + a) + 990*a^3*b^2*x^4*sgn(b*x^2 + a) + 385*a^4*b*x^2
*sgn(b*x^2 + a) + 63*a^5*sgn(b*x^2 + a))/x^11
```

Mupad [B]
time = 4.22, size = 231, normalized size = 0.92

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^3\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}-\frac{2\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^5\,\left(b\,x^2+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^12,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(x*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^4 +
2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(9*x^9*(a + b*x^2)) - (2*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
)/(x^5*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7
*(a + b*x^2))
```

3.618 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{14}}\,dx$

Optimal. Leaf size=253

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b}}{7x^7(a+bx^2)}$$

```
[Out] -1/13*a^5*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-5/11*a^4*b*((b*x^2+a)^2)^(1/2)
/x^11/(b*x^2+a)-10/9*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-10/7*a^2*b^3
*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-a*b^4*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-
1/3*b^5*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)
```

Rubi [A]

time = 0.04, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```

```
[Out] -1/13*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^13*(a + b*x^2)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(x^5*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a +
b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{14}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^{14}}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{14}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^{10}}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^4}\right)dx}{b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2aba}}{9x^9(a+ba}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(693a^5+4095a^4bx^2+10010a^3b^2x^4+12870a^2b^3x^6+9009ab^4x^8+3003b^5x^{10})}{9009x^{13}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
[Out] -1/9009*(Sqrt[(a + b*x^2)^2]*(693*a^5 + 4095*a^4*b*x^2 + 10010*a^3*b^2*x^4
+ 12870*a^2*b^3*x^6 + 9009*a*b^4*x^8 + 3003*b^5*x^10))/(x^13*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.32

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,(-\frac{1}{3}b^5x^{10}-b^4a\,x^8-\frac{10}{7}a^2b^3x^6-\frac{10}{9}b^2a^3x^4-\frac{5}{11}b\,a^4x^2-\frac{1}{13}a^5)}{(b\,x^2+a)x^{13}}$	79
gosper	$-\frac{(3003b^5x^{10}+9009b^4a\,x^8+12870a^2b^3x^6+10010b^2a^3x^4+4095b\,a^4x^2+693a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{9009x^{13}(b\,x^2+a)^5}$	80
default	$-\frac{(3003b^5x^{10}+9009b^4a\,x^8+12870a^2b^3x^6+10010b^2a^3x^4+4095b\,a^4x^2+693a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{9009x^{13}(b\,x^2+a)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x,method=_RETURNVERBOSE)
[Out] -1/9009*(3003*b^5*x^10+9009*a*b^4*x^8+12870*a^2*b^3*x^6+10010*a^3*b^2*x^4+4
095*a^4*b*x^2+693*a^5)*((b*x^2+a)^2)^(5/2)/x^13/(b*x^2+a)^5
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.23

$$-\frac{b^5}{3\,x^3}-\frac{ab^4}{x^5}-\frac{10\,a^2b^3}{7\,x^7}-\frac{10\,a^3b^2}{9\,x^9}-\frac{5\,a^4b}{11\,x^{11}}-\frac{a^5}{13\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/3*b^5/x^3 - a*b^4/x^5 - 10/7*a^2*b^3/x^7 - 10/9*a^3*b^2/x^9 - 5/11*a^4*b
/x^11 - 1/13*a^5/x^13
```

Fricas [A]
time = 0.34, size = 59, normalized size = 0.23

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2
*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**14, x)
```

Giac [A]
time = 3.93, size = 107, normalized size = 0.42

$$-\frac{3003\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+9009\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+12870\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+10010\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+4095\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+693\,a^5\mathrm{sgn}\left(bx^2+a\right)}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(3003*b^5*x^10*sgn(b*x^2 + a) + 9009*a*b^4*x^8*sgn(b*x^2 + a) + 128
70*a^2*b^3*x^6*sgn(b*x^2 + a) + 10010*a^3*b^2*x^4*sgn(b*x^2 + a) + 4095*a^4
*b*x^2*sgn(b*x^2 + a) + 693*a^5*sgn(b*x^2 + a))/x^13
```

Mupad [B]
time = 4.35, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^3\,\left(b\,x^2+a\right)}-\frac{a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^5\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^14,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (a*b^4*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(x^5*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(11*x^11*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1
/2))/(7*x^7*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(
9*x^9*(a + b*x^2))
```

3.619 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}$$

```
[Out] -1/15*a^5*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-5/13*a^4*b*((b*x^2+a)^2)^(1/2)
/x^13/(b*x^2+a)-10/11*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-10/9*a^2*b
^3*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-5/7*a*b^4*((b*x^2+a)^2)^(1/2)/x^7/(b*x
^2+a)-1/5*b^5*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)
```

Rubi [A]

time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

```
[Out] -1/15*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^15*(a + b*x^2)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(7*x^7*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5
*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^{16}}\,dx}{b^4\,(ab+b^2x^2)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{16}}+\frac{5a^4b^6}{x^{14}}+\frac{10a^3b^7}{x^{12}}+\frac{10a^2b^8}{x^{10}}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^6}\right)dx}{b^4\,(ab+b^2x^2)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\,(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\,(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx^2}}{11x^{11}\,(a+b}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(3003a^5+17325a^4bx^2+40950a^3b^2x^4+50050a^2b^3x^6+32175ab^4x^8+9009b^5x^{10})}{45045x^{15}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

```
[Out] -1/45045*(Sqrt[(a + b*x^2)^2]*(3003*a^5 + 17325*a^4*b*x^2 + 40950*a^3*b^2*x
^4 + 50050*a^2*b^3*x^6 + 32175*a*b^4*x^8 + 9009*b^5*x^10))/(x^15*(a + b*x^2
))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{15}a^5-\frac{5}{13}b\,a^4x^2-\frac{10}{11}b^2a^3x^4-\frac{10}{9}a^2b^3x^6-\frac{5}{7}b^4a\,x^8-\frac{1}{5}b^5x^{10}\right)}{(b\,x^2+a)x^{15}}$	79
gosper	$-\frac{(9009b^5x^{10}+32175b^4a\,x^8+50050a^2b^3x^6+40950b^2a^3x^4+17325b\,a^4x^2+3003a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{45045x^{15}(b\,x^2+a)^5}$	80
default	$-\frac{(9009b^5x^{10}+32175b^4a\,x^8+50050a^2b^3x^6+40950b^2a^3x^4+17325b\,a^4x^2+3003a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{45045x^{15}(b\,x^2+a)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/45045*(9009*b^5*x^10+32175*a*b^4*x^8+50050*a^2*b^3*x^6+40950*a^3*b^2*x^4
+17325*a^4*b*x^2+3003*a^5)*((b*x^2+a)^2)^(5/2)/x^15/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.22

$$-\frac{b^5}{5\,x^5}-\frac{5\,ab^4}{7\,x^7}-\frac{10\,a^2b^3}{9\,x^9}-\frac{10\,a^3b^2}{11\,x^{11}}-\frac{5\,a^4b}{13\,x^{13}}-\frac{a^5}{15\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/5*b^5/x^5 - 5/7*a*b^4/x^7 - 10/9*a^2*b^3/x^9 - 10/11*a^3*b^2/x^11 - 5/13
*a^4*b/x^13 - 1/15*a^5/x^15
```

Fricas [A]
time = 0.33, size = 59, normalized size = 0.23

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b
^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**16, x)
```

Giac [A]
time = 3.29, size = 107, normalized size = 0.42

$$-\frac{9009\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+32175\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+50050\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+40950\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+17325\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+3003\,a^5\mathrm{sgn}\left(bx^2+a\right)}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(9009*b^5*x^10*sgn(b*x^2 + a) + 32175*a*b^4*x^8*sgn(b*x^2 + a) + 5
0050*a^2*b^3*x^6*sgn(b*x^2 + a) + 40950*a^3*b^2*x^4*sgn(b*x^2 + a) + 17325*
a^4*b*x^2*sgn(b*x^2 + a) + 3003*a^5*sgn(b*x^2 + a))/x^15
```

Mupad [B]
time = 4.21, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{15\,x^{15}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{5\,x^5\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^16,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(15*x^15*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(9*x^9*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))/(11*x^11*(a + b*x^2))
```

3.620 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x}}{11x^{11}(a+bx^2)}$$

```
[Out] -1/17*a^5*((b*x^2+a)^2)^(1/2)/x^17/(b*x^2+a)-1/3*a^4*b*((b*x^2+a)^2)^(1/2)/
x^15/(b*x^2+a)-10/13*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-10/11*a^2*b
^3*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-5/9*a*b^4*((b*x^2+a)^2)^(1/2)/x^9/(b*
x^2+a)-1/7*b^5*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

```
[Out] -1/17*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^17*(a + b*x^2)) - (a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(9*x^9*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*
(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{18}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^{18}}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{18}}+\frac{5a^4b^6}{x^{16}}+\frac{10a^3b^7}{x^{14}}+\frac{10a^2b^8}{x^{12}}+\frac{5ab^9}{x^{10}}+\frac{b^{10}}{x^8}\right)dx}{b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)} - \frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx^2}}{13x^{13}(a+bx}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(9009a^5+51051a^4bx^2+117810a^3b^2x^4+139230a^2b^3x^6+85085ab^4x^8+21879b^5x^{10})}{153153x^{17}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

```
[Out] -1/153153*(Sqrt[(a + b*x^2)^2]*(9009*a^5 + 51051*a^4*b*x^2 + 117810*a^3*b^2
*x^4 + 139230*a^2*b^3*x^6 + 85085*a*b^4*x^8 + 21879*b^5*x^10))/(x^17*(a + b
*x^2))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{17}a^5-\frac{1}{3}b\,a^4x^2-\frac{10}{13}b^2a^3x^4-\frac{10}{11}a^2b^3x^6-\frac{5}{9}b^4a\,x^8-\frac{1}{7}b^5x^{10}\right)}{(b\,x^2+a)x^{17}}$	79
gosper	$-\frac{(21879b^5x^{10}+85085b^4a\,x^8+139230a^2b^3x^6+117810b^2a^3x^4+51051b\,a^4x^2+9009a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{153153x^{17}(b\,x^2+a)^5}$	80
default	$-\frac{(21879b^5x^{10}+85085b^4a\,x^8+139230a^2b^3x^6+117810b^2a^3x^4+51051b\,a^4x^2+9009a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{153153x^{17}(b\,x^2+a)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x,method=_RETURNVERBOSE)
```

```
[Out] -1/153153*(21879*b^5*x^10+85085*a*b^4*x^8+139230*a^2*b^3*x^6+117810*a^3*b^2
*x^4+51051*a^4*b*x^2+9009*a^5)*((b*x^2+a)^2)^(5/2)/x^17/(b*x^2+a)^5
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.22

$$-\frac{b^5}{7\,x^7}-\frac{5\,ab^4}{9\,x^9}-\frac{10\,a^2b^3}{11\,x^{11}}-\frac{10\,a^3b^2}{13\,x^{13}}-\frac{a^4b}{3\,x^{15}}-\frac{a^5}{17\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="maxima")

[Out] -1/7*b^5/x^7 - 5/9*a*b^4/x^9 - 10/11*a^2*b^3/x^11 - 10/13*a^3*b^2/x^13 - 1/3*a^4*b/x^15 - 1/17*a^5/x^17

Fricas [A]
time = 0.36, size = 59, normalized size = 0.23

$$-\frac{21879\, b^5 x^{10} + 85085\, a b^4 x^8 + 139230\, a^2 b^3 x^6 + 117810\, a^3 b^2 x^4 + 51051\, a^4 b x^2 + 9009\, a^5}{153153\, x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="fricas")

[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{18}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**18,x)

[Out] Integral(((a + b*x**2)**2)**(5/2)/x**18, x)

Giac [A]
time = 4.42, size = 107, normalized size = 0.42

$$-\frac{21879\, b^5 x^{10} \mathrm{sgn}(bx^2+a) + 85085\, a b^4 x^8 \mathrm{sgn}(bx^2+a) + 139230\, a^2 b^3 x^6 \mathrm{sgn}(bx^2+a) + 117810\, a^3 b^2 x^4 \mathrm{sgn}(bx^2+a) + 51051\, a^4 b x^2 \mathrm{sgn}(bx^2+a) + 9009\, a^5 \mathrm{sgn}(bx^2+a)}{153153\, x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="giac")

[Out] -1/153153*(21879*b^5*x^10*sgn(b*x^2 + a) + 85085*a*b^4*x^8*sgn(b*x^2 + a) + 139230*a^2*b^3*x^6*sgn(b*x^2 + a) + 117810*a^3*b^2*x^4*sgn(b*x^2 + a) + 51051*a^4*b*x^2*sgn(b*x^2 + a) + 9009*a^5*sgn(b*x^2 + a))/x^17

Mupad [B]
time = 4.31, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{17\,x^{17}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}-\frac{a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^{15}\,\left(b\,x^2+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^18,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(17*x^17*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(3*x^15*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(11*x^11*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
)/(13*x^13*(a + b*x^2))
```

3.621 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{20}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}$$

```
[Out] -1/19*a^5*((b*x^2+a)^2)^(1/2)/x^19/(b*x^2+a)-5/17*a^4*b*((b*x^2+a)^2)^(1/2)
/x^17/(b*x^2+a)-2/3*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-10/13*a^2*b^
3*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-5/11*a*b^4*((b*x^2+a)^2)^(1/2)/x^11/(b
*x^2+a)-1/9*b^5*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1126, 276}

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

```
[Out] -1/19*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^19*(a + b*x^2)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (2*a^3*b^2*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(11*x^11*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x
^9*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{20}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^{20}}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{20}}+\frac{5a^4b^6}{x^{18}}+\frac{10a^3b^7}{x^{16}}+\frac{10a^2b^8}{x^{14}}+\frac{5ab^9}{x^{12}}+\frac{b^{10}}{x^{10}}\right)dx}{b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)} - \frac{2a^3b^2\sqrt{a^2+2abx^2}}{3x^{15}(a+bx}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(21879a^5+122265a^4bx^2+277134a^3b^2x^4+319770a^2b^3x^6+188955ab^4x^8+46189b^5x^{10})}{415701x^{19}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

```
[Out] -1/415701*(Sqrt[(a + b*x^2)^2]*(21879*a^5 + 122265*a^4*b*x^2 + 277134*a^3*b
^2*x^4 + 319770*a^2*b^3*x^6 + 188955*a*b^4*x^8 + 46189*b^5*x^10))/(x^19*(a
+ b*x^2))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{19}a^5-\frac{5}{17}b\,a^4x^2-\frac{2}{3}b^2a^3x^4-\frac{10}{13}a^2b^3x^6-\frac{5}{11}b^4a\,x^8-\frac{1}{9}b^5x^{10}\right)}{(b\,x^2+a)x^{19}}$	79
gosper	$-\frac{(46189b^5x^{10}+188955b^4a\,x^8+319770a^2b^3x^6+277134b^2a^3x^4+122265b\,a^4x^2+21879a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{415701x^{19}(b\,x^2+a)^5}$	80
default	$-\frac{(46189b^5x^{10}+188955b^4a\,x^8+319770a^2b^3x^6+277134b^2a^3x^4+122265b\,a^4x^2+21879a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{415701x^{19}(b\,x^2+a)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x,method=_RETURNVERBOSE)
```

```
[Out] -1/415701*(46189*b^5*x^10+188955*a*b^4*x^8+319770*a^2*b^3*x^6+277134*a^3*b^
2*x^4+122265*a^4*b*x^2+21879*a^5)*((b*x^2+a)^2)^(5/2)/x^19/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.22

$$-\frac{b^5}{9\,x^9}-\frac{5\,ab^4}{11\,x^{11}}-\frac{10\,a^2b^3}{13\,x^{13}}-\frac{2\,a^3b^2}{3\,x^{15}}-\frac{5\,a^4b}{17\,x^{17}}-\frac{a^5}{19\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="maxima")
```

```
[Out] -1/9*b^5/x^9 - 5/11*a*b^4/x^11 - 10/13*a^2*b^3/x^13 - 2/3*a^3*b^2/x^15 - 5/
17*a^4*b/x^17 - 1/19*a^5/x^19
```

Fricas [A]
time = 0.32, size = 59, normalized size = 0.23

$$-\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*
a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{20}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**20,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**20, x)
```

Giac [A]
time = 3.20, size = 107, normalized size = 0.42

$$-\frac{46189\,b^5x^{10}\mathrm{sgn}(bx^2+a)+188955\,ab^4x^8\mathrm{sgn}(bx^2+a)+319770\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+277134\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+122265\,a^4bx^2\mathrm{sgn}(bx^2+a)+21879\,a^5\mathrm{sgn}(bx^2+a)}{415701\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="giac")
```

```
[Out] -1/415701*(46189*b^5*x^10*sgn(b*x^2 + a) + 188955*a*b^4*x^8*sgn(b*x^2 + a)
+ 319770*a^2*b^3*x^6*sgn(b*x^2 + a) + 277134*a^3*b^2*x^4*sgn(b*x^2 + a) + 1
22265*a^4*b*x^2*sgn(b*x^2 + a) + 21879*a^5*sgn(b*x^2 + a))/x^19
```

Mupad [B]
time = 4.27, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{19\,x^{19}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{17\,x^{17}\,\left(b\,x^2+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}-\frac{2\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^{15}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^20,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(19*x^19*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2))/(17*x^17*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(13*x^13*(a + b*x^2)) - (2*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(3*x^15*(a + b*x^2))
```

3.622 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{22}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}$$

```
[Out] -1/21*a^5*((b*x^2+a)^2)^(1/2)/x^21/(b*x^2+a)-5/19*a^4*b*((b*x^2+a)^2)^(1/2)
/x^19/(b*x^2+a)-10/17*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^17/(b*x^2+a)-2/3*a^2*b^
3*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-5/13*a*b^4*((b*x^2+a)^2)^(1/2)/x^13/(b
*x^2+a)-1/11*b^5*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

```
[Out] -1/21*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^21*(a + b*x^2)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (2*a^2*b^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(13*x^13*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*
x^11*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{22}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(ab+b^2x^2\right)^5}{x^{22}}\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{22}}+\frac{5a^4b^6}{x^{20}}+\frac{10a^3b^7}{x^{18}}+\frac{10a^2b^8}{x^{16}}+\frac{5ab^9}{x^{14}}+\frac{b^{10}}{x^{12}}\right)dx}{b^4\left(ab+b^2x^2\right)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx}}{17x^{17}\left(a+b\right.}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(46189a^5+255255a^4bx^2+570570a^3b^2x^4+646646a^2b^3x^6+373065ab^4x^8+88179b^5x^{10}\right)}{969969x^{21}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

```
[Out] -1/969969*(Sqrt[(a + b*x^2)^2]*(46189*a^5 + 255255*a^4*b*x^2 + 570570*a^3*b
^2*x^4 + 646646*a^2*b^3*x^6 + 373065*a*b^4*x^8 + 88179*b^5*x^10))/(x^21*(a
+ b*x^2))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(-\frac{1}{21}a^5-\frac{5}{19}b\,a^4x^2-\frac{10}{17}b^2a^3x^4-\frac{2}{3}a^2b^3x^6-\frac{5}{13}b^4a\,x^8-\frac{1}{11}b^5x^{10}\right)}{\left(b\,x^2+a\right)x^{21}}$	79
gosper	$-\frac{\left(88179b^5x^{10}+373065b^4a\,x^8+646646a^2b^3x^6+570570b^2a^3x^4+255255b\,a^4x^2+46189a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{969969x^{21}\left(b\,x^2+a\right)^5}$	80
default	$-\frac{\left(88179b^5x^{10}+373065b^4a\,x^8+646646a^2b^3x^6+570570b^2a^3x^4+255255b\,a^4x^2+46189a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{969969x^{21}\left(b\,x^2+a\right)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x,method=_RETURNVERBOSE)
```

```
[Out] -1/969969*(88179*b^5*x^10+373065*a*b^4*x^8+646646*a^2*b^3*x^6+570570*a^3*b^
2*x^4+255255*a^4*b*x^2+46189*a^5)*((b*x^2+a)^2)^(5/2)/x^21/(b*x^2+a)^5
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.22

$$-\frac{b^5}{11\,x^{11}}-\frac{5\,ab^4}{13\,x^{13}}-\frac{2\,a^2b^3}{3\,x^{15}}-\frac{10\,a^3b^2}{17\,x^{17}}-\frac{5\,a^4b}{19\,x^{19}}-\frac{a^5}{21\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

```
[Out] -1/11*b^5/x^11 - 5/13*a*b^4/x^13 - 2/3*a^2*b^3/x^15 - 10/17*a^3*b^2/x^17 -
5/19*a^4*b/x^19 - 1/21*a^5/x^21
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.23

$$-\frac{88179\,b^5x^{10}+373065\,ab^4x^8+646646\,a^2b^3x^6+570570\,a^3b^2x^4+255255\,a^4bx^2+46189\,a^5}{969969\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

```
[Out] -1/969969*(88179*b^5*x^10 + 373065*a*b^4*x^8 + 646646*a^2*b^3*x^6 + 570570*
a^3*b^2*x^4 + 255255*a^4*b*x^2 + 46189*a^5)/x^21
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**22,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**22, x)
```

Giac [A]
time = 3.96, size = 107, normalized size = 0.42

$$-\frac{88179\,b^5x^{10}\mathrm{sgn}(bx^2+a)+373065\,ab^4x^8\mathrm{sgn}(bx^2+a)+646646\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+570570\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+255255\,a^4bx^2\mathrm{sgn}(bx^2+a)+46189\,a^5\mathrm{sgn}(bx^2+a)}{969969\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="giac")
```

```
[Out] -1/969969*(88179*b^5*x^10*sgn(b*x^2 + a) + 373065*a*b^4*x^8*sgn(b*x^2 + a)
+ 646646*a^2*b^3*x^6*sgn(b*x^2 + a) + 570570*a^3*b^2*x^4*sgn(b*x^2 + a) + 2
55255*a^4*b*x^2*sgn(b*x^2 + a) + 46189*a^5*sgn(b*x^2 + a))/x^21
```

Mupad [B]
time = 4.34, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{21\,x^{21}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{19\,x^{19}\,\left(b\,x^2+a\right)}-\frac{2\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^{15}\,\left(b\,x^2+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{17\,x^{17}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^22,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(21*x^21*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(1/2))/(19*x^19*(a + b*x^2)) - (2*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(3*x^15*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^
(1/2))/(17*x^17*(a + b*x^2))
```

3.623 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b}}{17x^{17}(a+bx^2)}$$

```
[Out] -1/23*a^5*((b*x^2+a)^2)^(1/2)/x^23/(b*x^2+a)-5/21*a^4*b*((b*x^2+a)^2)^(1/2)
/x^21/(b*x^2+a)-10/19*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^19/(b*x^2+a)-10/17*a^2*
b^3*((b*x^2+a)^2)^(1/2)/x^17/(b*x^2+a)-1/3*a*b^4*((b*x^2+a)^2)^(1/2)/x^15/(
b*x^2+a)-1/13*b^5*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1126, 276}

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

```
[Out] -1/23*(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^23*(a + b*x^2)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (10*a^3*b^2*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (10*a^2*b^3*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(3*x^15*(a + b*x^2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x
^13*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{x^{24}}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{x^{24}}+\frac{5a^4b^6}{x^{22}}+\frac{10a^3b^7}{x^{20}}+\frac{10a^2b^8}{x^{18}}+\frac{5ab^9}{x^{16}}+\frac{b^{10}}{x^{14}}\right)dx}{b^4(ab+b^2x^2)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx}}{19x^{19}(a+b}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(88179a^5+482885a^4bx^2+1067430a^3b^2x^4+1193010a^2b^3x^6+676039ab^4x^8+156009b^5x^{10})}{2028117x^{23}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

```
[Out] -1/2028117*(Sqrt[(a + b*x^2)^2]*(88179*a^5 + 482885*a^4*b*x^2 + 1067430*a^3
*b^2*x^4 + 1193010*a^2*b^3*x^6 + 676039*a*b^4*x^8 + 156009*b^5*x^10))/(x^23
*(a + b*x^2))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{(bx^2+a)^2}\,\left(-\frac{1}{23}a^5-\frac{5}{21}ba^4x^2-\frac{10}{19}b^2a^3x^4-\frac{10}{17}a^2b^3x^6-\frac{1}{3}b^4ax^8-\frac{1}{13}b^5x^{10}\right)}{(bx^2+a)x^{23}}$	79
gosper	$-\frac{(156009b^5x^{10}+676039b^4ax^8+1193010a^2b^3x^6+1067430b^2a^3x^4+482885ba^4x^2+88179a^5)\left((bx^2+a)^2\right)^{\frac{5}{2}}}{2028117x^{23}(bx^2+a)^5}$	80
default	$-\frac{(156009b^5x^{10}+676039b^4ax^8+1193010a^2b^3x^6+1067430b^2a^3x^4+482885ba^4x^2+88179a^5)\left((bx^2+a)^2\right)^{\frac{5}{2}}}{2028117x^{23}(bx^2+a)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2028117*(156009*b^5*x^10+676039*a*b^4*x^8+1193010*a^2*b^3*x^6+1067430*a^
3*b^2*x^4+482885*a^4*b*x^2+88179*a^5)*((b*x^2+a)^2)^(5/2)/x^23/(b*x^2+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.22

$$-\frac{b^5}{13\,x^{13}}-\frac{ab^4}{3\,x^{15}}-\frac{10\,a^2b^3}{17\,x^{17}}-\frac{10\,a^3b^2}{19\,x^{19}}-\frac{5\,a^4b}{21\,x^{21}}-\frac{a^5}{23\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1/13*b^5/x^13 - 1/3*a*b^4/x^15 - 10/17*a^2*b^3/x^17 - 10/19*a^3*b^2/x^19 -
 5/21*a^4*b/x^21 - 1/23*a^5/x^23
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.23

$$-\frac{156009\, b^5 x^{10} + 676039\, a b^4 x^8 + 1193010\, a^2 b^3 x^6 + 1067430\, a^3 b^2 x^4 + 482885\, a^4 b x^2 + 88179\, a^5}{2028117\, x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/2028117*(156009*b^5*x^10 + 676039*a*b^4*x^8 + 1193010*a^2*b^3*x^6 + 1067
430*a^3*b^2*x^4 + 482885*a^4*b*x^2 + 88179*a^5)/x^23
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^2\right)^2\right)^{\frac{5}{2}}}{x^{24}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**24,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**24, x)
```

Giac [A]
time = 5.98, size = 107, normalized size = 0.42

$$-\frac{156009\, b^5 x^{10} \operatorname{sgn}(b x^2 + a) + 676039\, a b^4 x^8 \operatorname{sgn}(b x^2 + a) + 1193010\, a^2 b^3 x^6 \operatorname{sgn}(b x^2 + a) + 1067430\, a^3 b^2 x^4 \operatorname{sgn}(b x^2 + a) + 482885\, a^4 b x^2 \operatorname{sgn}(b x^2 + a) + 88179\, a^5 \operatorname{sgn}(b x^2 + a)}{2028117\, x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="giac")
```

```
[Out] -1/2028117*(156009*b^5*x^10*sgn(b*x^2 + a) + 676039*a*b^4*x^8*sgn(b*x^2 + a
) + 1193010*a^2*b^3*x^6*sgn(b*x^2 + a) + 1067430*a^3*b^2*x^4*sgn(b*x^2 + a)
 + 482885*a^4*b*x^2*sgn(b*x^2 + a) + 88179*a^5*sgn(b*x^2 + a))/x^23
```

Mupad [B]
time = 4.31, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{23\,x^{23}\,\left(b\,x^2+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}-\frac{a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^{15}\,\left(b\,x^2+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{21\,x^{21}\,\left(b\,x^2+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{17\,x^{17}\,\left(b\,x^2+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{19\,x^{19}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^24,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(23*x^23*(a + b*x^2)) - (b^5*(a^2
 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(3*x^15*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(21*x^21*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^
2)^(1/2))/(17*x^17*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(19*x^19*(a + b*x^2))
```

3.624 $\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=127

$$-\frac{ax^2(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^4(a+bx^2)}{4b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2(a+bx^2)\log(a+bx^2)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/2*a*x^2*(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/4*x^4*(b*x^2+a)/b/((b*x^2+a)
^2)^(1/2)+1/2*a^2*(b*x^2+a)*ln(b*x^2+a)/b^3/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$\frac{x^4(a+bx^2)}{4b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax^2(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2(a+bx^2)\log(a+bx^2)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/2*(a*x^2*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^4*(a +
b*x^2))/(4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^2*(a + b*x^2)*Log[a + b*
x^2])/(2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)$$
$$= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{x^2}{ab+b^2x}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \left(-\frac{a}{b^3}+\frac{x}{b^2}+\frac{a^2}{b^3(a+bx)}\right)\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{ax^2(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x^4(a+bx^2)}{4b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^2(a+bx^2)\log(a+b}{2b^3\sqrt{a^2+2abx^2+b^2}}$$

Mathematica [**A**]
time = 0.02, size = 55, normalized size = 0.43

$$\frac{(a+bx^2)\left(bx^2(-2a+bx^2)+2a^2\log(a+bx^2)\right)}{4b^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(b*x^2*(-2*a + b*x^2) + 2*a^2*Log[a + b*x^2]))/(4*b^3*Sqrt[(a
+ b*x^2)^2])
```

Maple [**A**]
time = 0.12, size = 52, normalized size = 0.41

method	result	size
default	$\frac{(b\,x^2+a)\,(b^2x^4-2ab\,x^2+2a^2\ln(b\,x^2+a))}{4\sqrt{(b\,x^2+a)^2}\;b^3}$	52
risch	$\frac{\sqrt{(b\,x^2+a)^2}\;(-b\,x^2+a)^2}{4(b\,x^2+a)b^3}+\frac{\sqrt{(b\,x^2+a)^2}\;a^2\ln(b\,x^2+a)}{2(b\,x^2+a)b^3}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^2+a)*(b^2*x^4-2*a*b*x^2+2*a^2*ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/b^3
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.27

$$\frac{a^2 \log\left(bx^2 + a\right)}{2\, b^3} + \frac{bx^4 - 2\, ax^2}{4\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*a^2*log(b*x^2 + a)/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

Fricas [A]
time = 0.33, size = 33, normalized size = 0.26

$$\frac{b^2 x^4 - 2\, abx^2 + 2\, a^2 \log\left(bx^2 + a\right)}{4\, b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*log(b*x^2 + a))/b^3
```

Sympy [A]
time = 0.05, size = 32, normalized size = 0.25

$$\frac{a^2 \log\left(a + bx^2\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/((b*x**2+a)**2)**(1/2),x)
[Out] a**2*log(a + b*x**2)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b)
```

Giac [A]
time = 4.36, size = 59, normalized size = 0.46

$$\frac{a^2 \log\left(|bx^2 + a|\right) \operatorname{sgn}(bx^2 + a)}{2\, b^3} + \frac{bx^4 \operatorname{sgn}(bx^2 + a) - 2\, ax^2 \operatorname{sgn}(bx^2 + a)}{4\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*a^2*log(abs(b*x^2 + a))*sgn(b*x^2 + a)/b^3 + 1/4*(b*x^4*sgn(b*x^2 + a)
- 2*a*x^2*sgn(b*x^2 + a))/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{{\left(b\, x^2 + a\right)}^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^2)^(1/2),x)
[Out] int(x^5/((a + b*x^2)^2)^(1/2), x)
```

3.625 $\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{a(a+bx^2)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/2*a*(b*x^2+a)*ln(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/2*((b*x^2+a)^2)^(1/2
)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1125, 654, 622, 31}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{a(a+bx^2)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(2*b^2) - (a*(a + b*x^2)*Log[a + b*x^2])/(2
*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} - \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)}{2b}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} - \frac{(a(ab+b^2x^2))\,\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^2\right)}{2b\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} - \frac{a(a+bx^2)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.59

$$\frac{(a+bx^2)\,(bx^2-a\log(a+bx^2))}{2b^2\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] ((a + b*x^2)*(b*x^2 - a*Log[a + b*x^2]))/(2*b^2*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.12, size = 41, normalized size = 0.55

method	result	size
default	$-\frac{(b\,x^2+a)\,(-b\,x^2+a\ln(b\,x^2+a))}{2\sqrt{(b\,x^2+a)^2}\;b^2}$	41
risch	$\frac{\sqrt{(b\,x^2+a)^2}\;x^2}{2(b\,x^2+a)b} - \frac{\sqrt{(b\,x^2+a)^2}\;a\ln(b\,x^2+a)}{2(b\,x^2+a)b^2}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(b*x^2+a)*(-b*x^2+a*ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/b^2
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.31

$$\frac{x^2}{2\,b}-\frac{a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 0.34, size = 22, normalized size = 0.29

$$\frac{bx^2-a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.27

$$-\frac{a\log\left(a+bx^2\right)}{2b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x**2+a)**2)**(1/2),x)
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

Giac [A]
time = 5.99, size = 33, normalized size = 0.44

$$\frac{1}{2}\left(\frac{x^2}{b}-\frac{a\log\left(|bx^2+a|\right)}{b^2}\right)\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(x^2/b - a*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)
```

Mupad [B]
time = 4.52, size = 64, normalized size = 0.85

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^2}-\frac{a\,b\,\ln\left(a\,b+\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{b^2}+b^2\,x^2\right)}{2\,{\left(b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*b^2) - (a*b*log(a*b + ((a + b*x^2)^2)^
(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(3/2))
```

3.626 $\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

[Out] 1/2*(b*x^2+a)*ln(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1121, 622, 31}

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]

[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)\\
&= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.80

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.73

method	result	size
default	$\frac{(b\,x^2+a)\ln(b\,x^2+a)}{2b\sqrt{(b\,x^2+a)^2}}$	32
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\ln(b\,x^2+a)}{2(b\,x^2+a)b}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(b*x^2+a)*ln(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.30

$$\frac{\log(bx^2+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*log(b*x^2 + a)/b
```

Fricas [A]
time = 0.32, size = 13, normalized size = 0.30

$$\frac{\log(bx^2+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.23

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x**2+a)**2)**(1/2),x)
[Out] log(a + b*x**2)/(2*b)
```

Giac [A]
time = 5.40, size = 22, normalized size = 0.50

$$\frac{\log\left(\left|bx^2+a\right|\right)\operatorname{sgn}(bx^2+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))*sgn(b*x^2 + a)/b
```

Mupad [B]
time = 4.42, size = 33, normalized size = 0.75

$$\frac{\ln\left(b^2\,x^2+a\,b\right)\,\operatorname{sign}(2\,b^2\,x^2+2\,a\,b)}{2\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2)^(1/2),x)
[Out] (log(a*b + b^2*x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(1/2))
```

3.627 $\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=80

$$\frac{(a+bx^2)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(a+bx^2)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (b*x^2+a)*ln(x)/a/((b*x^2+a)^2)^(1/2)-1/2*(b*x^2+a)*ln(b*x^2+a)/a/((b*x^2+a
)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.192$, Rules used = {1126, 272, 36, 29, 31}

$$\frac{\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(a+bx^2)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log
[a + b*x^2])/(2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{(ab+b^2x^2)\int \frac{1}{x(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{1}{x(ab+b^2x)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(b(ab+b^2x^2))\,\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{(a+bx^2)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(a+bx^2)\log(a+bx^2)}{2a\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.52

$$\frac{(a+bx^2)\,(2\log(x)-\log(a+bx^2))}{2a\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
[Out] ((a + b*x^2)*(2*Log[x] - Log[a + b*x^2]))/(2*a*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.13, size = 37, normalized size = 0.46

method	result	size
default	$-\frac{(b\,x^2+a)\,(\ln(b\,x^2+a)-2\ln(x))}{2\sqrt{(b\,x^2+a)^2}\;a}$	37
risch	$-\frac{\sqrt{(b\,x^2+a)^2}\;\ln(b\,x^2+a)}{2(b\,x^2+a)a}+\frac{\sqrt{(b\,x^2+a)^2}\;\ln(x)}{(b\,x^2+a)a}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^2+a)*(ln(b*x^2+a)-2*ln(x))/((b*x^2+a)^2)^(1/2)/a
```

Maxima [A]
time = 0.29, size = 23, normalized size = 0.29

$$-\frac{\log \left(bx^2+a\right)}{2\,a}+\frac{\log \left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 + a)/a + 1/2*log(x^2)/a
```

Fricas [A]
time = 0.36, size = 18, normalized size = 0.22

$$-\frac{\log \left(bx^2+a\right)-2\,\log \left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

Sympy [A]
time = 0.09, size = 15, normalized size = 0.19

$$\frac{\log \left(x\right)}{a}-\frac{\log \left(\frac{a}{b}+x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x**2+a)**2)**(1/2),x)
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

Giac [A]
time = 4.73, size = 33, normalized size = 0.41

$$\frac{1}{2}\left(\frac{\log \left(x^2\right)}{a}-\frac{\log \left(\left|bx^2+a\right|\right)}{a}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(log(x^2)/a - log(abs(b*x^2 + a))/a)*sgn(b*x^2 + a)
```

Mupad [B]
time = 4.45, size = 40, normalized size = 0.50

$$-\frac{\ln\left(\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{a^2}+a^2+a\,b\,x^2\right)+\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] -(log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) + a^2 + a*b*x^2) + log(1/x^2))/(2*(
a^2)^(1/2))
```

3.628 $\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=125

$$\frac{-a-bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b(a+bx^2)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b(a+bx^2)\log(a+bx^2)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/2*(-b*x^2-a)/a/x^2/((b*x^2+a)^2)^(1/2)-b*(b*x^2+a)*ln(x)/a^2/((b*x^2+a)^2
)^(1/2)+1/2*b*(b*x^2+a)*ln(b*x^2+a)/a^2/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 46}

$$-\frac{a+bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b\log(x)(a+bx^2)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b(a+bx^2)\log(a+bx^2)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/2*(a + b*x^2)/(a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*(a + b*x^2)*L
og[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*x^2)*Log[a + b*x^2
])/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{1}{x^3(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{1}{x^2(ab+b^2x)}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \left(\frac{1}{abx^2}-\frac{1}{a^2x}+\frac{b}{a^2(a+bx)}\right)\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{a+bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b(a+bx^2)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b(a+bx^2)\log(a+}{2a^2\sqrt{a^2+2abx^2+}}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.43

$$-\frac{(a+bx^2)\left(a+2bx^2\log(x)-bx^2\log\left(a+bx^2\right)\right)}{2a^2x^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/2*((a + b*x^2)*(a + 2*b*x^2*Log[x] - b*x^2*Log[a + b*x^2]))/(a^2*x^2*Sqr
t[(a + b*x^2)^2])
```

Maple [A]
time = 0.14, size = 52, normalized size = 0.42

method	result	size
default	$\frac{(b\,x^2+a)\,(b\ln(b\,x^2+a)x^2-2b\ln(x)x^2-a)}{2\sqrt{(b\,x^2+a)^2}\;a^2x^2}$	52
risch	$-\frac{\sqrt{(b\,x^2+a)^2}}{2(b\,x^2+a)a\,x^2}-\frac{\sqrt{(b\,x^2+a)^2}\;b\ln(x)}{(b\,x^2+a)a^2}+\frac{\sqrt{(b\,x^2+a)^2}\;b\ln(-b\,x^2-a)}{2(b\,x^2+a)a^2}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x^2+a)*(b*ln(b*x^2+a)*x^2-2*b*ln(x)*x^2-a)/((b*x^2+a)^2)^(1/2)/a^2/x
^2
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.26

$$\frac{b\log\left(bx^2+a\right)}{2\,a^2} - \frac{b\log\left(x^2\right)}{2\,a^2} - \frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

Fricas [A]
time = 0.36, size = 33, normalized size = 0.26

$$\frac{bx^2\log(bx^2+a)-2\,bx^2\log(x)-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

Sympy [A]
time = 0.12, size = 31, normalized size = 0.25

$$-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

Giac [A]
time = 4.03, size = 52, normalized size = 0.42

$$-\frac{1}{2}\left(\frac{b\log(x^2)}{a^2}-\frac{b\log(|bx^2+a|)}{a^2}-\frac{bx^2-a}{a^2x^2}\right)\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(x^2)/a^2 - b*log(abs(b*x^2 + a))/a^2 - (b*x^2 - a)/(a^2*x^2))*s
gn(b*x^2 + a)
```

Mupad [B]
time = 4.45, size = 75, normalized size = 0.60

$$\frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^2}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\right)}{2\,{\left(a^2\right)}^{3/2}}-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,a^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] (a*b*atanh((a^2 + a*b*x^2)/((a^2)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)))
)/(2*(a^2)^(3/2)) - (a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*a^2*x^2)
```

3.629 $\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=129

$$-\frac{ax(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -a*x*(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/3*x^3*(b*x^2+a)/b/((b*x^2+a)^2)^(1
/2)+a^(3/2)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/b^(5/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 308, 211}

$$-\frac{ax(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((a*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (x^3*(a + b*x^
2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/2)*(a + b*x^2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{x^4}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(ab+b^2x^2)\int \left(-\frac{a}{b^3}+\frac{x^2}{b^2}+\frac{a^2}{b^2(ab+b^2x^2)}\right)\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{ax(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a^2(ab+b^2x^2))\int \frac{1}{ab+b^2}}{b^2\sqrt{a^2+2abx^2+b^2}}$$
$$= -\frac{ax(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}(a+bx^2)\tan^{-1}\left(\right.}{b^{5/2}\sqrt{a^2+2abx^2+b}}$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.51

$$\frac{(a+bx^2)\left(\sqrt{b}\,x(-3a+bx^2)+3a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] ((a + b*x^2)*(Sqrt[b]*x*(-3*a + b*x^2) + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.14, size = 64, normalized size = 0.50

method	result
default	$-\frac{(b\,x^2+a)\left(-\sqrt{ab}\,b\,x^3+3\sqrt{ab}\,ax-3a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)}{3\sqrt{(b\,x^2+a)^2}\,b^2\sqrt{ab}}$
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,(\frac{1}{3}b\,x^3-ax)}{(b\,x^2+a)b^2}+\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x+a\right)}{2(b\,x^2+a)b^3}-\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,a\ln\left(\sqrt{\right.}{2(b\,x^2+a)b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^2+a)*(-(a*b)^(1/2)*b*x^3+3*(a*b)^(1/2)*a*x-3*a^2*arctan(b*x/(a*b)
^(1/2)))/((b*x^2+a)^2)^(1/2)/b^2/(a*b)^(1/2)
```

Maxima [A]
time = 0.50, size = 37, normalized size = 0.29

$$\frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{bx^3-3\,ax}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*x^3 - 3*a*x)/b^2
```

Fricas [A]
time = 0.34, size = 99, normalized size = 0.77

$$\left[\frac{2\,bx^3+3\,a\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,ax}{6\,b^2},\ \frac{bx^3+3\,a\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,ax}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*x^3 + 3*a*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*a*
x)/b^2]
```

Sympy [A]
time = 0.07, size = 80, normalized size = 0.62

$$-\frac{ax}{b^2}-\frac{\sqrt{-\frac{a^3}{b^5}}\,\log\left(x-\frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2}+\frac{\sqrt{-\frac{a^3}{b^5}}\,\log\left(x+\frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2}+\frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -a*x/b**2 - sqrt(-a**3/b**5)*log(x - b**2*sqrt(-a**3/b**5)/a)/2 + sqrt(-a**
3/b**5)*log(x + b**2*sqrt(-a**3/b**5)/a)/2 + x**3/(3*b)
```

Giac [A]
time = 6.24, size = 64, normalized size = 0.50

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right) \mathrm{sgn}(bx^2+a)}{\sqrt{ab}\, b^2} + \frac{b^2x^3\mathrm{sgn}(bx^2+a) - 3\,abx\mathrm{sgn}(bx^2+a)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3*sgn
(b*x^2 + a) - 3*a*b*x*sgn(b*x^2 + a))/b^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^4/((a + b*x^2)^2)^(1/2), x)
```

3.630 $\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{x(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] x*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)-(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/
2)/b^(3/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 327, 211}

$$\frac{x(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\,(a+bx^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(a + b*x^2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{x^2}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{x(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(a(ab+b^2x^2))\int \frac{1}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{x(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.61

$$\frac{(a+bx^2)\left(\sqrt{b}\,x-\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{b^{3/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x - Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(b^(3/2)*Sq
rt[(a + b*x^2)^2])
```

Maple [A]
time = 0.13, size = 48, normalized size = 0.54

method	result
default	$\frac{(b\,x^2+a)\left(x\sqrt{ab}-a\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)}{\sqrt{(b\,x^2+a)^2}\,b\sqrt{ab}}$
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,x}{(b\,x^2+a)b}+\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{2(b\,x^2+a)b^2}-\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{2(b\,x^2+a)b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)*(x*(a*b)^(1/2)-a*arctan(b*x/(a*b)^(1/2)))/((b*x^2+a)^2)^(1/2)/b/(
a*b)^(1/2)
```

Maxima [A]
time = 0.50, size = 26, normalized size = 0.29

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Fricas [A]
time = 0.36, size = 82, normalized size = 0.92

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},\ -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

Sympy [A]
time = 0.06, size = 56, normalized size = 0.63

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

Giac [A]
time = 4.95, size = 42, normalized size = 0.47

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)\operatorname{sgn}(bx^2+a)}{\sqrt{ab}\,b}+\frac{x\operatorname{sgn}(bx^2+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/(sqrt(a*b)*b) + x*sgn(b*x^2 + a)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^2/((a + b*x^2)^2)^(1/2), x)
```

3.631 $\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=53

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1102, 211}

$$\frac{(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1102

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{(2ab+2b^2x^2)\int \frac{1}{2ab+2b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.83

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[(a + b*x^2)
^2])
```

Maple [A]
time = 0.15, size = 34, normalized size = 0.64

method	result	size
default	$\frac{(b\,x^2+a)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{(b\,x^2+a)^2}\,\sqrt{ab}}$	34
risch	$-\frac{\sqrt{(b\,x^2+a)^2}\,\ln\left(bx+\sqrt{-ab}\right)}{2(b\,x^2+a)\sqrt{-ab}}+\frac{\sqrt{(b\,x^2+a)^2}\,\ln\left(-bx+\sqrt{-ab}\right)}{2(b\,x^2+a)\sqrt{-ab}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 15, normalized size = 0.28

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 0.36, size = 67, normalized size = 1.26

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [A]
time = 0.05, size = 53, normalized size = 1.00

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Giac [A]
time = 3.69, size = 23, normalized size = 0.43

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)\operatorname{sgn}(bx^2+a)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/sqrt(a*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\left(b\,x^2+a\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(1/((a + b*x^2)^2)^(1/2), x)
```

3.632 $\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=92

$$-\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-b*x^2-a)/a/x/((b*x^2+a)^2)^(1/2)-(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))*b^(1
/2)/a^(3/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1126, 331, 211}

$$-\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\,(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((a + b*x^2)/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - (Sqrt[b]*(a + b*x^2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{(ab+b^2x^2)\int \frac{1}{x^2(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(b(ab+b^2x^2))\int \frac{1}{ab+b^2x^2}\,dx}{a\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 0.61

$$-\frac{(a+bx^2)\left(\sqrt{a}+\sqrt{b}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{a^{3/2}x\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(((a + b*x^2)*(Sqrt[a] + Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(a^(3/2)*
x*Sqrt[(a + b*x^2)^2]))
```

Maple [A]
time = 0.13, size = 50, normalized size = 0.54

method	result	size
default	$-\frac{(b\,x^2+a)\left(b\arctan\left(\frac{bx}{\sqrt{ab}}\right)x+\sqrt{ab}\right)}{\sqrt{(b\,x^2+a)^2}\,ax\sqrt{ab}}$	50
risch	$-\frac{\sqrt{(b\,x^2+a)^2}}{(b\,x^2+a)ax}+\frac{\sqrt{(b\,x^2+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^3\_Z^2+b\right)}\_R\ln\left(\left(3\_R^2a^3+2b\right)x+a^2\_R\right)\right)}{2b\,x^2+2a}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^2+a)*(b*arctan(b*x/(a*b)^(1/2))*x+(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a/
x/(a*b)^(1/2)
```

Maxima [A]
time = 0.49, size = 29, normalized size = 0.32

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Fricas [A]
time = 0.35, size = 82, normalized size = 0.89

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax},\,-\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

Sympy [A]
time = 0.07, size = 65, normalized size = 0.71

$$\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

Giac [A]
time = 3.34, size = 37, normalized size = 0.40

$$-\left(\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{1}{ax}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] -(b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*x))*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x^2)^2)^(1/2)),x)
[Out] int(1/(x^2*((a + b*x^2)^2)^(1/2)), x)
```

3.633 $\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=133

$$\frac{-a-bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b(a+bx^2)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/3*(-b*x^2-a)/a/x^3/((b*x^2+a)^2)^(1/2)+b*(b*x^2+a)/a^2/x/((b*x^2+a)^2)^(1
/2)+b^(3/2)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 331, 211}

$$\frac{b(a+bx^2)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/3*(a + b*x^2)/(a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*x^2))/
(a^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{(ab+b^2x^2)\int \frac{1}{x^4(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(b(ab+b^2x^2))\int \frac{1}{x^2(ab+b^2x^2)}\,dx}{a\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b(a+bx^2)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(b^2(ab+b^2x^2))\int}{a^2\sqrt{a^2+2abx^2}}\\ &= -\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b(a+bx^2)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/2}(a+bx^2)\tan}{a^{5/2}\sqrt{a^2+2abx}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 0.53

$$-\frac{(a+bx^2)\left(\sqrt{a}\,(a-3bx^2)-3b^{3/2}x^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{3a^{5/2}x^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
[Out] -1/3*((a + b*x^2)*(Sqrt[a]*(a - 3*b*x^2) - 3*b^(3/2)*x^3*ArcTan[(Sqrt[b]*x)
/Sqrt[a]]))/(a^(5/2)*x^3*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.13, size = 68, normalized size = 0.51

method	result
default	$-\frac{(b\,x^2+a)\left(-3b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3-3b\sqrt{ab}\,x^2+a\sqrt{ab}\right)}{3\sqrt{(b\,x^2+a)^2}\,a^2\sqrt{ab}\,x^3}$
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(\frac{b\,x^2}{a^2}-\frac{1}{3a}\right)}{(b\,x^2+a)x^3}+\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{2(b\,x^2+a)a^3}-\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,b\ln\left(-bx+\right.}{2(b\,x^2+a)a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^2+a)*(-3*b^2*arctan(b*x/(a*b)^(1/2))*x^3-3*b*(a*b)^(1/2)*x^2+a*(a
*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a^2/(a*b)^(1/2)/x^3
```

Maxima [A]
time = 0.53, size = 40, normalized size = 0.30

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, a^2} + \frac{3\, bx^2 - a}{3\, a^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Fricas [A]
time = 0.37, size = 106, normalized size = 0.80

$$\left[\frac{3\, bx^3 \sqrt{-\frac{b}{a}} \log\left(\frac{bx^2+2\, ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) + 6\, bx^2 - 2\, a}{6\, a^2 x^3}, \frac{3\, bx^3 \sqrt{\frac{b}{a}} \arctan\left(x\sqrt{\frac{b}{a}}\right) + 3\, bx^2 - a}{3\, a^2 x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

Sympy [A]
time = 0.10, size = 87, normalized size = 0.65

$$-\frac{\sqrt{-\frac{b^3}{a^5}} \log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2} + x\right)}{2} + \frac{\sqrt{-\frac{b^3}{a^5}} \log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2} + x\right)}{2} + \frac{-a + 3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

Giac [A]
time = 4.67, size = 50, normalized size = 0.38

$$\frac{1}{3}\left(\frac{3\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{a^2x^3}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*b*x^2 - a)/(a^2*x^3))
*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int(1/(x^4*((a + b*x^2)^2)^(1/2)), x)
```

3.634 $\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2(a+bx^2)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a(a+bx^2)\log(}{2b^4\sqrt{a^2+2abx^2}}$$

```
[Out] -3/2*a^2/b^4/((b*x^2+a)^2)^(1/2)+1/4*a^3/b^4/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+
1/2*x^2*(b*x^2+a)/b^3/((b*x^2+a)^2)^(1/2)-3/2*a*(b*x^2+a)*ln(b*x^2+a)/b^4/(
(b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$-\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a(a+bx^2)\log\left(a+bx^2\right)}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2(a+bx^2)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-3*a^2)/(2*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^3/(4*b^4*(a + b*x^2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^2*(a + b*x^2))/(2*b^3*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) - (3*a*(a + b*x^2)*Log[a + b*x^2])/(2*b^4*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{(b^2(ab+b^2x^2))\,\mathrm{Subst}\left(\int \frac{x^3}{(ab+b^2x)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(b^2(ab+b^2x^2))\,\mathrm{Subst}\left(\int \left(\frac{1}{b^6}-\frac{a^3}{b^6(a+bx)^3}+\frac{3a^2}{b^6(a+bx)^2}-\frac{3a}{b^6(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= -\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^3}{4b^4(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2(}{2b^3\sqrt{a^2+}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 81, normalized size = 0.51

$$\frac{-5a^3-4a^2bx^2+4ab^2x^4+2b^3x^6-6a(a+bx^2)^2\log(a+bx^2)}{4b^4(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (-5*a^3 - 4*a^2*b*x^2 + 4*a*b^2*x^4 + 2*b^3*x^6 - 6*a*(a + b*x^2)^2*Log[a +
 b*x^2])/(4*b^4*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.03, size = 103, normalized size = 0.65

method	result	size
default	$-\frac{\left(-2b^3x^6+6\ln(b\,x^2+a)a\,b^2x^4-4a\,b^2x^4+12\ln(b\,x^2+a)a^2b\,x^2+4a^2b\,x^2+6\ln(b\,x^2+a)a^3+5a^3\right)(b\,x^2+a)}{4b^4\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$	103
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,x^2}{2(b\,x^2+a)b^3}+\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{3a^2x^2}{2}-\frac{5a^3}{4b}\right)}{(b\,x^2+a)^3b^3}-\frac{3\sqrt{(b\,x^2+a)^2}\,a\ln(b\,x^2+a)}{2(b\,x^2+a)b^4}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*b^3*x^6+6*ln(b*x^2+a)*a*b^2*x^4-4*a*b^2*x^4+12*ln(b*x^2+a)*a^2*b*x
^2+4*a^2*b*x^2+6*ln(b*x^2+a)*a^3+5*a^3)*(b*x^2+a)/b^4/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 66, normalized size = 0.42

$$-\frac{6\,a^2bx^2+5\,a^3}{4\,(b^6x^4+2\,ab^5x^2+a^2b^4)}+\frac{x^2}{2\,b^3}-\frac{3\,a\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(6*a^2*b*x^2 + 5*a^3)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 1/2*x^2/b^3
- 3/2*a*log(b*x^2 + a)/b^4
```

Fricas [A]
time = 0.36, size = 91, normalized size = 0.58

$$\frac{2\,b^3x^6+4\,ab^2x^4-4\,a^2bx^2-5\,a^3-6\,(ab^2x^4+2\,a^2bx^2+a^3)\log\left(bx^2+a\right)}{4\,(b^6x^4+2\,ab^5x^2+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4 + 2*a^2*b
*x^2 + a^3)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^7}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**7/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 3.68, size = 92, normalized size = 0.58

$$\frac{x^2}{2\,b^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{3\,a\log\left(|bx^2+a|\right)}{2\,b^4\mathrm{sgn}\left(bx^2+a\right)}+\frac{9\,ab^2x^4+12\,a^2bx^2+4\,a^3}{4\,(bx^2+a)^2b^4\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/(b^3*sgn(b*x^2 + a)) - 3/2*a*log(abs(b*x^2 + a))/(b^4*sgn(b*x^2 + a
)) + 1/4*(9*a*b^2*x^4 + 12*a^2*b*x^2 + 4*a^3)/((b*x^2 + a)^2*b^4*sgn(b*x^2
+ a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.635 $\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^2}{4b^3\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] a/b^3/((b*x^2+a)^2)^(1/2)-1/4*a^2/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/2*(b*
x^2+a)*ln(b*x^2+a)/b^3/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$-\frac{a^2}{4b^3\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] a/(b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^2/(4*b^3*(a + b*x^2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[a + b*x^2])/(2*b^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
```

```
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a^2+2abx+b^2x^2)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{(b^2(ab+b^2x^2))\,\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^3}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(b^2(ab+b^2x^2))\,\text{Subst}\left(\int \left(\frac{a^2}{b^5(a+bx)^3}-\frac{2a}{b^5(a+bx)^2}+\frac{1}{b^5(a+bx)}\right)dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^2}{4b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)\log}{2b^3\sqrt{a^2+2a}}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 0.54

$$\frac{a(3a+4bx^2)+2(a+bx^2)^2\log(a+bx^2)}{4b^3(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (a*(3*a + 4*b*x^2) + 2*(a + b*x^2)^2*Log[a + b*x^2])/(4*b^3*(a + b*x^2)*Sqr
t[(a + b*x^2)^2])
```

Maple [A]
time = 0.05, size = 81, normalized size = 0.72

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(\frac{a\,x^2}{b^2}+\frac{3a^2}{4b^3}\right)}{(b\,x^2+a)^3}+\frac{\sqrt{(b\,x^2+a)^2}\,\ln(b\,x^2+a)}{2(b\,x^2+a)b^3}$	73
default	$\frac{(2\ln(b\,x^2+a)b^2x^4+4\ln(b\,x^2+a)ab\,x^2+4ab\,x^2+2a^2\ln(b\,x^2+a)+3a^2)(b\,x^2+a)}{4b^3\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

[Out] 1/4*(2*ln(b*x^2+a)*b^2*x^4+4*ln(b*x^2+a)*a*b*x^2+4*a*b*x^2+2*a^2*ln(b*x^2+a
)+3*a^2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^(3/2)

Maxima [A]
time = 0.29, size = 55, normalized size = 0.49

$$\frac{4\,abx^2+3\,a^2}{4\,(b^5x^4+2\,ab^4x^2+a^2b^3)}+\frac{\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")

[Out] 1/4*(4*a*b*x^2 + 3*a^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + 1/2*log(b*x^2 +
 a)/b^3

Fricas [A]
time = 0.32, size = 69, normalized size = 0.61

$$\frac{4\,abx^2+3\,a^2+2\,(b^2x^4+2\,abx^2+a^2)\log\left(bx^2+a\right)}{4\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")

[Out] 1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^5
*x^4 + 2*a*b^4*x^2 + a^2*b^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)

[Out] Integral(x**5/((a + b*x**2)**2)**(3/2), x)

Giac [A]
time = 4.29, size = 62, normalized size = 0.55

$$\frac{\log\left(|bx^2+a|\right)}{2\,b^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{3\,bx^4+2\,ax^2}{4\,\left(bx^2+a\right)^2b^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")

```
[Out] 1/2*log(abs(b*x^2 + a))/(b^3*sgn(b*x^2 + a)) - 1/4*(3*b*x^4 + 2*a*x^2)/((b*
x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.636 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{x^4}{4a\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/4*x^4/a/(b*x^2+a)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.68, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 654, 621}

$$\frac{a}{4b^2\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/2*1/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a/(4*b^2*(a + b*x^2)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 621

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a^2+2abx+b^2x^2)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a\text{Subst}\left(\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2}}\,dx, x, x^2\right)}{2b}$$
$$= -\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.95

$$\frac{-a-2bx^2}{4b^2(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (-a - 2*b*x^2)/(4*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.02, size = 32, normalized size = 0.78

method	result	size
gosper	$-\frac{(b\,x^2+a)(2b\,x^2+a)}{4b^2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$	32
default	$-\frac{(b\,x^2+a)(2b\,x^2+a)}{4b^2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$	32
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(-\frac{x^2}{2b}-\frac{a}{4b^2}\right)}{(b\,x^2+a)^3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(b*x^2+a)*(2*b*x^2+a)/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 36, normalized size = 0.88

$$-\frac{2\,bx^2+a}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Fricas [A]
time = 0.35, size = 36, normalized size = 0.88

$$-\frac{2\,bx^2+a}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 4.63, size = 32, normalized size = 0.78

$$-\frac{2\,bx^2+a}{4\,(bx^2+a)^2b^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x^2 + a)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

Mupad [B]
time = 4.24, size = 42, normalized size = 1.02

$$-\frac{\left(2\,b\,x^2+a\right)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,b^2\,{\left(b\,x^2+a\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] -((a + 2*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*b^2*(a + b*x^2)^3)
```

3.637 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/4/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1121, 621}

$$-\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/4*1/(b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 621

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^2\right)\\ &= -\frac{1}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^2}{4b\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] -1/4*(a + b*x^2)/(b*((a + b*x^2)^2)^(3/2))
```

Maple [A]
time = 0.02, size = 24, normalized size = 0.63

method	result	size
gosper	$-\frac{b\,x^{2}+a}{4b\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$	24
default	$-\frac{b\,x^{2}+a}{4b\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$	24
risch	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}}{4\left(b\,x^{2}+a\right)^{3}b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(b*x^2+a)/b/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.68

$$-\frac{1}{4\,(b^3x^4+2\,ab^2x^2+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Fricas [A]
time = 0.33, size = 26, normalized size = 0.68

$$-\frac{1}{4\,(b^3x^4+2\,ab^2x^2+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 4.25, size = 24, normalized size = 0.63

$$-\frac{1}{4\left(bx^{2}+a\right)^{2}b\mathrm{sgn}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4/((b*x^2 + a)^2*b*sgn(b*x^2 + a))
```

Mupad [B]
time = 4.34, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}}{4\,b\,\left(b\,x^{2}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] -(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(4*b*(a + b*x^2)^3)
```

3.638 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log(x)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+\right.}{2a^3\sqrt{a^2+2abx^2+\right.}}$$

```
[Out] 1/2/a^2/((b*x^2+a)^2)^(1/2)+1/4/a/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+(b*x^2+a)*l
n(x)/a^3/((b*x^2+a)^2)^(1/2)-1/2*(b*x^2+a)*ln(b*x^2+a)/a^3/((b*x^2+a)^2)^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 46}

$$\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 1/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*(a + b*x^2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - ((a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{1}{x\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx,x$$
$$= \frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx}{a^3\sqrt{a^2+2}}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 0.50

$$\frac{a(3a+2bx^2)+4(a+bx^2)^2\log(x)-2(a+bx^2)^2\log\left(a+bx^2\right)}{4a^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x^2) + 4*(a + b*x^2)^2*Log[x] - 2*(a + b*x^2)^2*Log[a + b*x^2
])/(4*a^3*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.03, size = 107, normalized size = 0.73

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(\frac{b\,x^2}{2a^2}+\frac{3}{4a}\right)}{(b\,x^2+a)^3}+\frac{\sqrt{(b\,x^2+a)^2}\,\ln(x)}{(b\,x^2+a)a^3}-\frac{\sqrt{(b\,x^2+a)^2}\,\ln(b\,x^2+a)}{2(b\,x^2+a)a^3}$	97
default	$-\frac{\left(2\ln(b\,x^2+a)b^2x^4-4\ln(x)b^2x^4+4\ln(b\,x^2+a)ab\,x^2-8\ln(x)ab\,x^2-2ab\,x^2+2a^2\ln(b\,x^2+a)-4a^2\ln(x)-3a^2\right)(b\,x^2+a)}{4a^3\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*ln(b*x^2+a)*b^2*x^4-4*ln(x)*b^2*x^4+4*ln(b*x^2+a)*a*b*x^2-8*ln(x)*a
*b*x^2-2*a*b*x^2+2*a^2*ln(b*x^2+a)-4*a^2*ln(x)-3*a^2)*(b*x^2+a)/a^3/((b*x^2
+a)^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.39

$$\frac{2\,bx^2+3\,a}{4\,(a^2b^2x^4+2\,a^3bx^2+a^4)}-\frac{\log\left(bx^2+a\right)}{2\,a^3}+\frac{\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 + a)/
a^3 + log(x)/a^3
```

Fricas [A]
time = 0.34, size = 90, normalized size = 0.61

$$\frac{2\,abx^2+3\,a^2-2\,(b^2x^4+2\,abx^2+a^2)\log\left(bx^2+a\right)+4\,(b^2x^4+2\,abx^2+a^2)\log\left(x\right)}{4\,(a^3b^2x^4+2\,a^4bx^2+a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a) + 4*(
b^2*x^4 + 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A]
time = 4.63, size = 89, normalized size = 0.61

$$\frac{\log\left(x^2\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{\log\left(|bx^2+a|\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{3\,b^2x^4+8\,abx^2+6\,a^2}{4\,(bx^2+a)^2a^3\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/(a^3*sgn(b*x^2 + a)) - 1/2*log(abs(b*x^2 + a))/(a^3*sgn(b*x^2
+ a)) + 1/4*(3*b^2*x^4 + 8*a*b*x^2 + 6*a^2)/((b*x^2 + a)^2*a^3*sgn(b*x^2 +
a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
[Out] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.639 $\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b(a+bx^2)}{a^4\sqrt{a^2+2abx}}$$

```
[Out] -b/a^3/((b*x^2+a)^2)^(1/2)-1/4*b/a^2/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/2*(-b*
x^2-a)/a^3/x^2/((b*x^2+a)^2)^(1/2)-3*b*(b*x^2+a)*ln(x)/a^4/((b*x^2+a)^2)^(1
/2)+3/2*b*(b*x^2+a)*ln(b*x^2+a)/a^4/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1126, 272, 46}

$$-\frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b\log(x)\left(a+bx^2\right)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3b(a+bx^2)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -(b/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - b/(4*a^2*(a + b*x^2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) - (a + b*x^2)/(2*a^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (3*b*(a + b*x^2)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
3*b*(a + b*x^2)*Log[a + b*x^2])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{1}{x^3(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^2(ab+b^2x^2)\right)\operatorname{Subst}\left(\int \frac{1}{x^2(ab+b^2x)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^2(ab+b^2x^2)\right)\operatorname{Subst}\left(\int \left(\frac{1}{a^3b^3x^2}-\frac{3}{a^4b^2x}+\frac{1}{a^2b(a+bx)^3}+\frac{2}{a^3b(a+bx)^2}+\frac{3}{a^4b(a+bx)}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{2a^3x^2\sqrt{}}$$

Mathematica [A]
time = 0.03, size = 97, normalized size = 0.51

$$\frac{-a(2a^2+9abx^2+6b^2x^4)-12bx^2(a+bx^2)^2\log(x)+6bx^2(a+bx^2)^2\log\left(a+bx^2\right)}{4a^4x^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(a*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)) - 12*b*x^2*(a + b*x^2)^2*Log[x] + 6*b
*x^2*(a + b*x^2)^2*Log[a + b*x^2])/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a + b*x^2)^
2])
```

Maple [A]
time = 0.05, size = 133, normalized size = 0.70

method	result
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(-\frac{3b^2x^4}{2a^3}-\frac{9b\,x^2}{4a^2}-\frac{1}{2a}\right)}{\left(b\,x^2+a\right)^3x^2}-\frac{3\sqrt{\left(b\,x^2+a\right)^2}\,b\ln(x)}{\left(b\,x^2+a\right)a^4}+\frac{3\sqrt{\left(b\,x^2+a\right)^2}\,b\ln\left(-b\,x^2-a\right)}{2\left(b\,x^2+a\right)a^4}$
default	$\frac{\left(6\ln\left(b\,x^2+a\right)b^3x^6-12\ln(x)b^3x^6+12\ln\left(b\,x^2+a\right)a\,b^2x^4-24a\,b^2\ln(x)x^4-6a\,b^2x^4+6\ln\left(b\,x^2+a\right)a^2b\,x^2-12a^2b\ln(x)x^2-9a^2b\,x^2-2a^3\right)(b}{4x^2a^4\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(6*ln(b*x^2+a)*b^3*x^6-12*ln(x)*b^3*x^6+12*ln(b*x^2+a)*a*b^2*x^4-24*a*b
^2*ln(x)*x^4-6*a*b^2*x^4+6*ln(b*x^2+a)*a^2*b*x^2-12*a^2*b*ln(x)*x^2-9*a^2*b
*x^2-2*a^3)*(b*x^2+a)/x^2/a^4/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 75, normalized size = 0.40

$$-\frac{6\,b^{2}x^{4}+9\,abx^{2}+2\,a^{2}}{4\,\left(a^{3}b^{2}x^{6}+2\,a^{4}bx^{4}+a^{5}x^{2}\right)}+\frac{3\,b\log\left(bx^{2}+a\right)}{2\,a^{4}}-\frac{3\,b\log\left(x\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
+ 3/2*b*log(b*x^2 + a)/a^4 - 3*b*log(x)/a^4
```

Fricas [A]
time = 0.34, size = 119, normalized size = 0.63

$$-\frac{6\,ab^{2}x^{4}+9\,a^{2}bx^{2}+2\,a^{3}-6\,\left(b^{3}x^{6}+2\,ab^{2}x^{4}+a^{2}bx^{2}\right)\log\left(bx^{2}+a\right)+12\,\left(b^{3}x^{6}+2\,ab^{2}x^{4}+a^{2}bx^{2}\right)\log\left(x\right)}{4\,\left(a^{4}b^{2}x^{6}+2\,a^{5}bx^{4}+a^{6}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*
x^2)*log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4*b
^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{3}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A]
time = 4.49, size = 122, normalized size = 0.65

$$-\frac{3\,b\log\left(x^{2}\right)}{2\,a^{4}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{3\,b\log\left(\left|bx^{2}+a\right|\right)}{2\,a^{4}\mathrm{sgn}\left(bx^{2}+a\right)}-\frac{9\,b^{3}x^{4}+22\,ab^{2}x^{2}+14\,a^{2}b}{4\,\left(bx^{2}+a\right)^{2}a^{4}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{3\,bx^{2}-a}{2\,a^{4}x^{2}\mathrm{sgn}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -3/2*b*log(x^2)/(a^4*sgn(b*x^2 + a)) + 3/2*b*log(abs(b*x^2 + a))/(a^4*sgn(b
*x^2 + a)) - 1/4*(9*b^3*x^4 + 22*a*b^2*x^2 + 14*a^2*b)/((b*x^2 + a)^2*a^4*s
gn(b*x^2 + a)) + 1/2*(3*b*x^2 - a)/(a^4*x^2*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
[Out] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.640 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -3/8*x/b^2/((b*x^2+a)^2)^(1/2)-1/4*x^3/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+3/8*
(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/b^(5/2)/a^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 294, 211}

$$\frac{3\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^3}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-3*x)/(8*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(4*b*(a + b*x^2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*Sqrt[a]*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2))\int \frac{x^4}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{x^3}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3(ab+b^2x^2))\int \frac{x^2}{(ab+b^2x^2)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3(ab+b^2x}{8b^2\sqrt{a^2+}}$$
$$= -\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3(a+bx}{8\sqrt{a}\,b^{5/2}\sqrt{}}$$

Mathematica [A]
time = 0.02, size = 84, normalized size = 0.66

$$\frac{-\sqrt{a}\sqrt{b}\,x(3a+5bx^2)+3(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{5/2}(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[b]*x*(3*a + 5*b*x^2)) + 3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(8*Sqrt[a]*b^(5/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.04, size = 97, normalized size = 0.76

method	result	siz
default	$-\frac{\left(-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^2x^4+5\sqrt{ab}\,b\,x^3-6\arctan\left(\frac{bx}{\sqrt{ab}}\right)ab\,x^2+3\sqrt{ab}\,ax-3a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)(b\,x^2+a)}{8\sqrt{ab}\,b^2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$	97
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(-\frac{5x^3}{8b}-\frac{3ax}{8b^2}\right)}{(b\,x^2+a)^3} - \frac{3\sqrt{(b\,x^2+a)^2}\,\ln\left(bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,b^2} + \frac{3\sqrt{(b\,x^2+a)^2}\,\ln\left(-bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,b^2}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-3*arctan(b*x/(a*b)^(1/2))*b^2*x^4+5*(a*b)^(1/2)*b*x^3-6*arctan(b*x/(
a*b)^(1/2))*a*b*x^2+3*(a*b)^(1/2)*a*x-3*a^2*arctan(b*x/(a*b)^(1/2)))*(b*x^2
+a)/(a*b)^(1/2)/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.52, size = 59, normalized size = 0.46

$$-\frac{5\,bx^3+3\,ax}{8\,(b^4x^4+2\,ab^3x^2+a^2b^2)}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 3/8*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 0.34, size = 188, normalized size = 1.47

$$\left[-\frac{10\,ab^2x^3+6\,a^2bx+3\,(b^2x^4+2\,abx^2+a^2)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\,(ab^5x^4+2\,a^2b^4x^2+a^3b^3)},-\frac{5\,ab^2x^3+3\,a^2bx-3\,(b^2x^4+2\,abx^2+a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\,(ab^5x^4+2\,a^2b^4x^2+a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(10*a*b^2*x^3 + 6*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^5*x^4 + 2*a^2*b^4*x^2
+ a^3*b^3), -1/8*(5*a*b^2*x^3 + 3*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 3.88, size = 65, normalized size = 0.51

$$\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^2\mathrm{sgn}\,(bx^2+a)}-\frac{5\,bx^3+3\,ax}{8\,(bx^2+a)^2b^2\mathrm{sgn}\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2*sgn(b*x^2 + a)) - 1/8*(5*b*x^3 + 3
*a*x)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.641 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=129

$$\frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/8*x/a/b/((b*x^2+a)^2)^(1/2)-1/4*x/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/8*(b*
x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1126, 294, 205, 211}

$$\frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] x/(8*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(4*b*(a + b*x^2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)
*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2))\int \frac{x^2}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{x}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(ab+b^2x^2)\int \frac{1}{(ab+b^2x^2)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(ab+b^2x^2)}{8ab\sqrt{a^2+2a}}$$
$$= \frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)}{8a^{3/2}b^{3/2}\sqrt{a}}$$

Mathematica [A]
time = 0.02, size = 81, normalized size = 0.63

$$\frac{\sqrt{a}\sqrt{b}\,x(-a+bx^2)+(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-a + b*x^2) + (a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*a^(3/2)*b^(3/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.05, size = 99, normalized size = 0.77

method	result	size

default	$-\frac{\left(-\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^{2}x^{4}-\sqrt{ab}\,b\,x^{3}-2\arctan\left(\frac{bx}{\sqrt{ab}}\right)ab\,x^{2}+\sqrt{ab}\,ax-a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)\left(b\,x^{2}+a\right)}{8\sqrt{ab}\,ba\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$	99
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{x^{3}}{8a}-\frac{x}{8b}\right)}{\left(b\,x^{2}+a\right)^{3}}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(bx+\sqrt{-ab}\right)}{16\left(b\,x^{2}+a\right)\sqrt{-ab}\,ba}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(-bx+\sqrt{-ab}\right)}{16\left(b\,x^{2}+a\right)\sqrt{-ab}\,ba}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-arctan(b*x/(a*b)^(1/2))*b^2*x^4-(a*b)^(1/2)*b*x^3-2*arctan(b*x/(a*b)
^(1/2))*a*b*x^2+(a*b)^(1/2)*a*x-a^2*arctan(b*x/(a*b)^(1/2)))*(b*x^2+a)/(a*b
)^(1/2)/b/a/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.50, size = 62, normalized size = 0.48

$$\frac{bx^{3}-ax}{8\left(ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 1/8*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 0.34, size = 190, normalized size = 1.47

$$\left[\frac{2\,ab^{2}x^{3}-2\,a^{2}bx-\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{16\left(a^{2}b^{4}x^{4}+2\,a^{3}b^{3}x^{2}+a^{4}b^{2}\right)},\frac{ab^{2}x^{3}-a^{2}bx+\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\left(a^{2}b^{4}x^{4}+2\,a^{3}b^{3}x^{2}+a^{4}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*a*b^2*x^3 - 2*a^2*b*x - (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*log
((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 +
a^4*b^2), 1/8*(a*b^2*x^3 - a^2*b*x + (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 4.49, size = 70, normalized size = 0.54

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab\mathrm{sgn}\left(bx^2+a\right)}+\frac{bx^3-ax}{8\left(bx^2+a\right)^2ab\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b*sgn(b*x^2 + a)) + 1/8*(b*x^3 - a*x
)/((b*x^2 + a)^2*a*b*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.642 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{x(a+bx^2)}{4a(a^2+2abx^2+b^2x^4)^{3/2}}+\frac{3x(a+bx^2)^2}{8a^2(a^2+2abx^2+b^2x^4)^{3/2}}+\frac{3(a+bx^2)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,(a^2+2abx^2+b^2x^4)^{3/2}}$$

```
[Out] 1/4*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)+3/8*x*(b*x^2+a)^2/a^2/(b^2*
x^4+2*a*b*x^2+a^2)^(3/2)+3/8*(b*x^2+a)^3*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/
(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1102, 205, 211}

$$\frac{3x(a+bx^2)^2}{8a^2(a^2+2abx^2+b^2x^4)^{3/2}}+\frac{x(a+bx^2)}{4a(a^2+2abx^2+b^2x^4)^{3/2}}+\frac{3(a+bx^2)^3\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,(a^2+2abx^2+b^2x^4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

```
[Out] (x*(a + b*x^2))/(4*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*x*(a + b*x^2)^
2)/(8*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*(a + b*x^2)^3*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1102

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
```

```
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(2ab+2b^2x^2)^3 \int \frac{1}{(2ab+2b^2x^2)^3}\,dx}{(a^2+2abx^2+b^2x^4)^{3/2}}$$

$$= \frac{x(a+bx^2)}{4a\,(a^2+2abx^2+b^2x^4)^{3/2}} + \frac{\left(3(2ab+2b^2x^2)^3\right)\int \frac{1}{(2ab+2b^2x^2)^2}\,dx}{8ab\,(a^2+2abx^2+b^2x^4)^{3/2}}$$

$$= \frac{x(a+bx^2)}{4a\,(a^2+2abx^2+b^2x^4)^{3/2}} + \frac{3x(a+bx^2)^2}{8a^2\,(a^2+2abx^2+b^2x^4)^{3/2}} + \frac{\left(3(2ab+2b^2x^2)^3\right)}{32a^2b^2\,(a^2+2ab}$$

$$= \frac{x(a+bx^2)}{4a\,(a^2+2abx^2+b^2x^4)^{3/2}} + \frac{3x(a+bx^2)^2}{8a^2\,(a^2+2abx^2+b^2x^4)^{3/2}} + \frac{3(a+bx^2)^3\tan}{8a^{5/2}\sqrt{b}\,(a^2+2ab}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 0.61

$$\frac{\sqrt{a}\,\sqrt{b}\,x(5a+3bx^2)+3(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(5*a + 3*b*x^2) + 3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(5/2)*Sqrt[b]*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.04, size = 97, normalized size = 0.72

method	result	size
default	$\frac{\left(3\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^2x^4+3\sqrt{ab}\,b\,x^3+6\arctan\left(\frac{bx}{\sqrt{ab}}\right)ab\,x^2+5\sqrt{ab}\,ax+3a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)(b\,x^2+a)}{8\sqrt{ab}\,a^2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$	97
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(\frac{3b\,x^3}{8a^2}+\frac{5x}{8a}\right)}{(b\,x^2+a)^3}-\frac{3\sqrt{(b\,x^2+a)^2}\,\ln\left(bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,a^2}+\frac{3\sqrt{(b\,x^2+a)^2}\,\ln\left(-bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,a^2}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(3*arctan(b*x/(a*b)^(1/2))*b^2*x^4+3*(a*b)^(1/2)*b*x^3+6*arctan(b*x/(a*
b)^(1/2))*a*b*x^2+5*(a*b)^(1/2)*a*x+3*a^2*arctan(b*x/(a*b)^(1/2)))*(b*x^2+a
)/(a*b)^(1/2)/a^2/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.49, size = 58, normalized size = 0.43

$$\frac{3\,bx^3+5\,ax}{8\,(a^2b^2x^4+2\,a^3bx^2+a^4)}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3/8*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 0.35, size = 188, normalized size = 1.39

$$\left[\frac{6\,ab^2x^3+10\,a^2bx-3\,(b^2x^4+2\,abx^2+a^2)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{16\,(a^3b^3x^4+2\,a^4b^2x^2+a^5b)},\ \frac{3\,ab^2x^3+5\,a^2bx+3\,(b^2x^4+2\,abx^2+a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\,(a^3b^3x^4+2\,a^4b^2x^2+a^5b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*x^3 + 10*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2
 + a^5*b), 1/8*(3*a*b^2*x^3 + 5*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/2), x)
```

Giac [A]
time = 4.18, size = 65, normalized size = 0.48

$$\frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab}\, a^2 \mathrm{sgn}\left(bx^2+a\right)} + \frac{3\, bx^3 + 5\, ax}{8 \left(bx^2+a\right)^2 a^2 \mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*sgn(b*x^2 + a)) + 1/8*(3*b*x^3 + 5
*a*x)/((b*x^2 + a)^2*a^2*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a^2+2\, a\, b\, x^2+b^2\, x^4\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.643 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=169

$$\frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15(a+bx^2)}{8a^3x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15\sqrt{b}\,(a+bx^2)}{8a^{7/2}\sqrt{a^2+}}$$

```
[Out] 5/8/a^2/x/((b*x^2+a)^2)^(1/2)+1/4/a/x/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-15/8*(b
*x^2+a)/a^3/x/((b*x^2+a)^2)^(1/2)-15/8*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))*
b^(1/2)/a^(7/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1126, 296, 331, 211}

$$\frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}-\frac{15\sqrt{b}\,(a+bx^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15(a+bx^2)}{8a^3x\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 5/(8*a^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x*(a + b*x^2)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) - (15*(a + b*x^2))/(8*a^3*x*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (15*Sqrt[b]*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{1}{x^2(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b(ab+b^2x^2)\right)\int \frac{1}{x^2(ab+b^2x^2)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(15(ab}{8a^2}$$

$$= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{8a^3x\sqrt{}}$$

$$= \frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{8a^3x\sqrt{}}$$

Mathematica [A]
time = 0.02, size = 93, normalized size = 0.55

$$\frac{-\sqrt{a}\left(8a^2+25abx^2+15b^2x^4\right)-15\sqrt{b}\,x\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}x\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)) - 15*Sqrt[b]*x*(a + b*x^2)^2*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.03, size = 119, normalized size = 0.70

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(-\frac{15b^2x^4}{8a^3}-\frac{25b\,x^2}{8a^2}-\frac{1}{a}\right)}{(b\,x^2+a)^3x}+\frac{15\sqrt{(b\,x^2+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^7\_Z^2+b\right)}\_R\ln\left(\left(3\_R^2a^7+2b\right)x+a^4\_R\right)\right.}{16(b\,x^2+a)}$
default	$-\frac{\left(15\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^3x^5+15\sqrt{ab}\,b^2x^4+30\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2x^3+25\sqrt{ab}\,ab\,x^2+15\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2bx+8\sqrt{ab}\,a^2\right.}{8x\sqrt{ab}\,a^3\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(15*arctan(b*x/(a*b)^(1/2))*b^3*x^5+15*(a*b)^(1/2)*b^2*x^4+30*arctan(b
*x/(a*b)^(1/2))*a*b^2*x^3+25*(a*b)^(1/2)*a*b*x^2+15*arctan(b*x/(a*b)^(1/2))
*a^2*b*x+8*(a*b)^(1/2)*a^2)*(b*x^2+a)/x/(a*b)^(1/2)/a^3/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.50, size = 71, normalized size = 0.42

$$-\frac{15\,b^2x^4+25\,abx^2+8\,a^2}{8\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}-\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
- 15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.38, size = 202, normalized size = 1.20

$$\left[-\frac{30\,b^2x^4+50\,abx^2-15\,(b^2x^5+2\,abx^3+a^2x)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+16\,a^2}{16\,(a^3b^2x^5+2\,a^4bx^3+a^5x)},\,-\frac{15\,b^2x^4+25\,abx^2+15\,(b^2x^5+2\,abx^3+a^2x)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+8\,a^2}{8\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 16*a^2)/(a^3*b^2*x^5 +
 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 15*(b^2*x^5 + 2*a*b*
```

```
x^3 + a^2*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*
x^3 + a^5*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left((a + bx^2)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A]
time = 6.02, size = 87, normalized size = 0.51

$$-\frac{15\, b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\, a^3 \mathrm{sgn}\left(bx^2 + a\right)} - \frac{7\, b^2 x^3 + 9\, abx}{8\left(bx^2 + a\right)^2 a^3 \mathrm{sgn}\left(bx^2 + a\right)} - \frac{1}{a^3 x \mathrm{sgn}\left(bx^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*sgn(b*x^2 + a)) - 1/8*(7*b^2*x
^3 + 9*a*b*x)/((b*x^2 + a)^2*a^3*sgn(b*x^2 + a)) - 1/(a^3*x*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.644 $\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}} dx$

Optimal. Leaf size=209

$$\frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35\left(a+bx^2\right)}{24a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b}{8a^4x\sqrt{a^2}}$$

```
[Out] 7/8/a^2/x^3/((b*x^2+a)^2)^(1/2)+1/4/a/x^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-35/
24*(b*x^2+a)/a^3/x^3/((b*x^2+a)^2)^(1/2)+35/8*b*(b*x^2+a)/a^4/x/((b*x^2+a)^
2)^(1/2)+35/8*b^(3/2)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/((b*x^2+a
)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1126, 296, 331, 211}

$$\frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b^{3/2}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b\left(a+bx^2\right)}{8a^4x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35\left(a+bx^2\right)}{24a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 7/(8*a^2*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x^3*(a + b*x^2)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*(a + b*x^2))/(24*a^3*x^3*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) + (35*b*(a + b*x^2))/(8*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (35*b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(7b\left(ab+b^2x^2\right)\right)\int \frac{1}{x^4(ab+b^2x^2)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(35(}{8a}\\ &= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{24a^3}\\ &= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{24a^3}\\ &= \frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{24a^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 105, normalized size = 0.50

$$\frac{\sqrt{a}\left(-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6\right)+105b^{3/2}x^3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{24a^{9/2}x^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6) + 105*b^(3/2
)*x^3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(24*a^(9/2)*x^3*(a + b*x^2
)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.04, size = 139, normalized size = 0.67

method	result
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{35 b^{3} x^{6}}{8 a^{4}}+\frac{175 b^{2} x^{4}}{24 a^{3}}+\frac{7 b\,x^{2}}{3 a^{2}}-\frac{1}{3 a}\right)}{\left(b\,x^{2}+a\right)^{3} x^{3}}+\frac{35 \sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(a^{9}\_Z^{2}+b^{3}\right)}\_R\ln\left(\left(3\_R^{2} a^{9}+2 b^{3}\right) x\right.\right.}{16\left(b\,x^{2}+a\right)}$
default	$-\frac{\left(-105\arctan\left(\frac{bx}{\sqrt{ab}}\right) b^{4} x^{7}-105\sqrt{ab}\,b^{3} x^{6}-210\arctan\left(\frac{bx}{\sqrt{ab}}\right) a\,b^{3} x^{5}-175\sqrt{ab}\,a\,b^{2} x^{4}-105\arctan\left(\frac{bx}{\sqrt{ab}}\right) a^{2} b^{2} x^{3}-\right.}{24 x^{3}\sqrt{ab}\,a^{4}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-105*arctan(b*x/(a*b)^(1/2))*b^4*x^7-105*(a*b)^(1/2)*b^3*x^6-210*arc
tan(b*x/(a*b)^(1/2))*a*b^3*x^5-175*(a*b)^(1/2)*a*b^2*x^4-105*arctan(b*x/(a*
b)^(1/2))*a^2*b^2*x^3-56*(a*b)^(1/2)*a^2*b*x^2+8*(a*b)^(1/2)*a^3)*(b*x^2+a)
/x^3/(a*b)^(1/2)/a^4/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.49, size = 86, normalized size = 0.41

$$\frac{105\,b^{3}x^{6}+175\,ab^{2}x^{4}+56\,a^{2}bx^{2}-8\,a^{3}}{24\,\left(a^{4}b^{2}x^{7}+2\,a^{5}bx^{5}+a^{6}x^{3}\right)}+\frac{35\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*(105*b^3*x^6 + 175*a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3)/(a^4*b^2*x^7 + 2*
a^5*b*x^5 + a^6*x^3) + 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 0.33, size = 238, normalized size = 1.14

$$\left[\frac{210\,b^{3}x^{6}+350\,ab^{2}x^{4}+112\,a^{2}bx^{2}-16\,a^{3}+105\,\left(b^{3}x^{7}+2\,ab^{2}x^{5}+a^{2}bx^{3}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{48\,\left(a^{4}b^{2}x^{7}+2\,a^{5}bx^{5}+a^{6}x^{3}\right)},\ \frac{105\,b^{3}x^{6}+175\,ab^{2}x^{4}+56\,a^{2}bx^{2}-8\,a^{3}+105\,\left(b^{3}x^{7}+2\,ab^{2}x^{5}+a^{2}bx^{3}\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{24\,\left(a^{4}b^{2}x^{7}+2\,a^{5}bx^{5}+a^{6}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(210*b^3*x^6 + 350*a*b^2*x^4 + 112*a^2*b*x^2 - 16*a^3 + 105*(b^3*x^7
+ 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b
```

```
*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3), 1/24*(105*b^3*x^6 + 175*
a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3 + 105*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*
sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A]
time = 5.12, size = 101, normalized size = 0.48

$$\frac{35\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4\mathrm{sgn}\left(bx^2+a\right)}+\frac{11\,b^3x^3+13\,ab^2x}{8\left(bx^2+a\right)^2a^4\mathrm{sgn}\left(bx^2+a\right)}+\frac{9\,bx^2-a}{3\,a^4x^3\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*sgn(b*x^2 + a)) + 1/8*(11*b^3
*x^3 + 13*a*b^2*x)/((b*x^2 + a)^2*a^4*sgn(b*x^2 + a)) + 1/3*(9*b*x^2 - a)/(
a^4*x^3*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.645 $\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=238

$$-\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a^4}{6b^6\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+ 2b^6 (\cdots$$

```
[Out] -5*a^2/b^6/((b*x^2+a)^2)^(1/2)+1/8*a^5/b^6/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-
5/6*a^4/b^6/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+5/2*a^3/b^6/(b*x^2+a)/((b*x^2+a
)^2)^(1/2)+1/2*x^2*(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)-5/2*a*(b*x^2+a)*ln(b*x
^2+a)/b^6/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$-\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a^4}{6b^6\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5a^3}{2b^6\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-5*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^5/(8*b^6*(a + b*x^2)^3*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a^4)/(6*b^6*(a + b*x^2)^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (5*a^3)/(2*b^6*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) + (x^2*(a + b*x^2))/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a*
(a + b*x^2)*Log[a + b*x^2])/(2*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{x^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^2\right)\\ &= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \frac{x^5}{(ab+b^2x)^5}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\mathrm{Subst}\left(\int \left(\frac{1}{b^{10}}-\frac{a^5}{b^{10}(a+bx)^5}+\frac{5a^4}{b^{10}(a+bx)^4}-\frac{10a^3}{b^{10}(a+bx)^3}+\frac{10a^2}{b^{10}(a+bx)^2}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{6b^6\left(a+b\right.}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 103, normalized size = 0.43

$$\frac{-77a^5-248a^4bx^2-252a^3b^2x^4-48a^2b^3x^6+48ab^4x^8+12b^5x^{10}-60a\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24b^6\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-77*a^5 - 248*a^4*b*x^2 - 252*a^3*b^2*x^4 - 48*a^2*b^3*x^6 + 48*a*b^4*x^8
+ 12*b^5*x^10 - 60*a*(a + b*x^2)^4*Log[a + b*x^2])/(24*b^6*(a + b*x^2)^3*Sq
rt[(a + b*x^2)^2])
```

Maple [A]
time = 0.03, size = 163, normalized size = 0.68

method	result
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,x^2}{2\left(b\,x^2+a\right)b^5}+\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(-5a^2b^2x^6-\frac{25a^3b\,x^4}{2}-\frac{65a^4x^2}{6}-\frac{77a^5}{24b}\right)}{\left(b\,x^2+a\right)^5b^5}-\frac{5\sqrt{\left(b\,x^2+a\right)^2}\,a\ln\left(b\,x^2+a\right)}{2\left(b\,x^2+a\right)b^6}$
default	$-\frac{\left(-12b^5x^{10}+60\ln\left(b\,x^2+a\right)a\,b^4x^8-48b^4a\,x^8+240\ln\left(b\,x^2+a\right)a^2b^3x^6+48a^2b^3x^6+360\ln\left(b\,x^2+a\right)a^3b^2x^4+252b^2a^3x^4+240\ln\left(b\,x^2+a\right)\right.}{24b^6\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-12*b^5*x^10+60*ln(b*x^2+a)*a*b^4*x^8-48*b^4*a*x^8+240*ln(b*x^2+a)*a
^2*b^3*x^6+48*a^2*b^3*x^6+360*ln(b*x^2+a)*a^3*b^2*x^4+252*b^2*a^3*x^4+240*l
n(b*x^2+a)*a^4*b*x^2+248*b*a^4*x^2+60*ln(b*x^2+a)*a^5+77*a^5)*(b*x^2+a)/b^6
/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.29, size = 110, normalized size = 0.46

$$-\frac{120\,a^2b^3x^6+300\,a^3b^2x^4+260\,a^4bx^2+77\,a^5}{24\,(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6)}+\frac{x^2}{2\,b^5}-\frac{5\,a\log\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(120*a^2*b^3*x^6 + 300*a^3*b^2*x^4 + 260*a^4*b*x^2 + 77*a^5)/(b^10*x^
8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6) + 1/2*x^2/b^5 -
5/2*a*log(b*x^2 + a)/b^6
```

Fricas [A]
time = 0.31, size = 157, normalized size = 0.66

$$\frac{12\,b^5x^{10}+48\,ab^4x^8-48\,a^2b^3x^6-252\,a^3b^2x^4-248\,a^4bx^2-77\,a^5-60\,(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5)\log\left(bx^2+a\right)}{24\,(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 48*a*b^4*x^8 - 48*a^2*b^3*x^6 - 252*a^3*b^2*x^4 - 248*a
^4*b*x^2 - 77*a^5 - 60*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b
*x^2 + a^5)*log(b*x^2 + a))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3
*b^7*x^2 + a^4*b^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{11}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**11/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.43, size = 114, normalized size = 0.48

$$\frac{x^2}{2\,b^5\mathrm{sgn}\left(bx^2+a\right)}-\frac{5\,a\log\left(|bx^2+a|\right)}{2\,b^6\mathrm{sgn}\left(bx^2+a\right)}+\frac{125\,ab^4x^8+380\,a^2b^3x^6+450\,a^3b^2x^4+240\,a^4bx^2+48\,a^5}{24\left(bx^2+a\right)^4b^6\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/(b^5*sgn(b*x^2 + a)) - 5/2*a*log(abs(b*x^2 + a))/(b^6*sgn(b*x^2 + a
)) + 1/24*(125*a*b^4*x^8 + 380*a^2*b^3*x^6 + 450*a^3*b^2*x^4 + 240*a^4*b*x^
2 + 48*a^5)/((b*x^2 + a)^4*b^6*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.646 $\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=196

$$\frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a^3}{3b^5\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{2b^5\left(a\right.}$$

```
[Out] 2*a/b^5/((b*x^2+a)^2)^(1/2)-1/8*a^4/b^5/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+2/3
*a^3/b^5/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-3/2*a^2/b^5/(b*x^2+a)/((b*x^2+a)^2
)^(1/2)+1/2*(b*x^2+a)*ln(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 45}

$$-\frac{3a^2}{2b^5\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a^3}{3b^5\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a)/(b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^4/(8*b^5*(a + b*x^2)^3*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*a^3)/(3*b^5*(a + b*x^2)^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) - (3*a^2)/(2*b^5*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + ((a + b*x^2)*Log[a + b*x^2])/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a^2+2abx+b^2x^2)^{5/2}}\,dx, x, x^2\right)$$
$$= \frac{(b^4(ab+b^2x^2))\,\text{Subst}\left(\int \frac{x^4}{(ab+b^2x)^5}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(b^4(ab+b^2x^2))\,\text{Subst}\left(\int \left(\frac{a^4}{b^9(a+bx)^5}-\frac{4a^3}{b^9(a+bx)^4}+\frac{6a^2}{b^9(a+bx)^3}-\frac{4a}{b^9(a+bx)^2}+\frac{1}{b^9(a+bx}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^4}{8b^5(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{3b^5(a+bx^2}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 0.42

$$\frac{a(25a^3+88a^2bx^2+108ab^2x^4+48b^3x^6)+12(a+bx^2)^4\log(a+bx^2)}{24b^5(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a*(25*a^3 + 88*a^2*b*x^2 + 108*a*b^2*x^4 + 48*b^3*x^6) + 12*(a + b*x^2)^4*
Log[a + b*x^2])/(24*b^5*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.03, size = 141, normalized size = 0.72

method	result
risch	$\frac{\sqrt{(bx^2+a)^2}\left(\frac{2a x^6}{b^2}+\frac{9a^2x^4}{2b^3}+\frac{11a^3x^2}{3b^4}+\frac{25a^4}{24b^5}\right)}{(bx^2+a)^5}+\frac{\sqrt{(bx^2+a)^2}\,\ln(bx^2+a)}{2(bx^2+a)b^5}$
default	$\frac{(12\ln(bx^2+a)b^4x^8+48\ln(bx^2+a)ab^3x^6+48ab^3x^6+72\ln(bx^2+a)a^2b^2x^4+108a^2b^2x^4+48\ln(bx^2+a)a^3bx^2+88a^3bx^2+12\ln(bx^2+a}{24b^5\left((bx^2+a)^2\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(12*ln(b*x^2+a)*b^4*x^8+48*ln(b*x^2+a)*a*b^3*x^6+48*a*b^3*x^6+72*ln(b*
x^2+a)*a^2*b^2*x^4+108*a^2*b^2*x^4+48*ln(b*x^2+a)*a^3*b*x^2+88*a^3*b*x^2+12
*ln(b*x^2+a)*a^4+25*a^4)*(b*x^2+a)/b^5/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 99, normalized size = 0.51

$$\frac{48\,ab^3x^6+108\,a^2b^2x^4+88\,a^3bx^2+25\,a^4}{24\,(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5)}+\frac{\log\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4)/(b^9*x^8 + 4*
a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5) + 1/2*log(b*x^2 + a)/b
^5
```

Fricas [A]
time = 0.33, size = 135, normalized size = 0.69

$$\frac{48\,ab^3x^6+108\,a^2b^2x^4+88\,a^3bx^2+25\,a^4+12\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\log\left(bx^2+a\right)}{24\,(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4 + 12*(b^4*x^8
+ 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^9*x^8
 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**9/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.60, size = 84, normalized size = 0.43

$$\frac{\log\left(|bx^2+a|\right)}{2\,b^5\mathrm{sgn}\left(bx^2+a\right)}-\frac{25\,b^3x^8+52\,ab^2x^6+42\,a^2bx^4+12\,a^3x^2}{24\,(bx^2+a)^4b^4\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/(b^5*sgn(b*x^2 + a)) - 1/24*(25*b^3*x^8 + 52*a*b^2*
x^6 + 42*a^2*b*x^4 + 12*a^3*x^2)/((b*x^2 + a)^4*b^4*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.647 $\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{x^8}{8a\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/8*x^8/a/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 660, 37}

$$\frac{x^8}{8a\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] x^8/(8*a*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a^2+2abx+b^2x^2)^{5/2}}\,dx,x,x^2\right)$$
$$= \frac{(b^4(ab+b^2x^2))\,\text{Subst}\left(\int \frac{x^3}{(ab+b^2x)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{x^8}{8a\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 1.49

$$\frac{-a^3-4a^2bx^2-6ab^2x^4-4b^3x^6}{8b^4\,(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a^3 - 4*a^2*b*x^2 - 6*a*b^2*x^4 - 4*b^3*x^6)/(8*b^4*(a + b*x^2)^3*Sqrt[(a
 + b*x^2)^2])
```

Maple [A]
time = 0.03, size = 54, normalized size = 1.32

method	result	size
gosper	$-\frac{(b\,x^2+a)\,(4b^3x^6+6a\,b^2x^4+4a^2b\,x^2+a^3)}{8b^4\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}$	54
default	$-\frac{(b\,x^2+a)\,(4b^3x^6+6a\,b^2x^4+4a^2b\,x^2+a^3)}{8b^4\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}$	54
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{x^6}{2b}-\frac{3a\,x^4}{4b^2}-\frac{a^2x^2}{2b^3}-\frac{a^3}{8b^4}\right)}{(b\,x^2+a)^5}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(b*x^2+a)*(4*b^3*x^6+6*a*b^2*x^4+4*a^2*b*x^2+a^3)/b^4/((b*x^2+a)^2)^(5
/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.

time = 0.28, size = 80, normalized size = 1.95

$$-\frac{4\,b^{3}x^{6}+6\,ab^{2}x^{4}+4\,a^{2}bx^{2}+a^{3}}{8\,(b^{8}x^{8}+4\,ab^{7}x^{6}+6\,a^{2}b^{6}x^{4}+4\,a^{3}b^{5}x^{2}+a^{4}b^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/(b^8*x^8 + 4*a*b^7*x^6 +
 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(28) = 56$.
time = 0.34, size = 80, normalized size = 1.95

$$-\frac{4\,b^{3}x^{6}+6\,ab^{2}x^{4}+4\,a^{2}bx^{2}+a^{3}}{8\,(b^{8}x^{8}+4\,ab^{7}x^{6}+6\,a^{2}b^{6}x^{4}+4\,a^{3}b^{5}x^{2}+a^{4}b^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/(b^8*x^8 + 4*a*b^7*x^6 +
 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left((a+bx^{2})^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**7/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.94, size = 54, normalized size = 1.32

$$-\frac{4\,b^{3}x^{6}+6\,ab^{2}x^{4}+4\,a^{2}bx^{2}+a^{3}}{8\,(bx^{2}+a)^{4}b^{4}\mathrm{sgn}\,(bx^{2}+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/((b*x^2 + a)^4*b^4*sgn(b
*x^2 + a))
```

Mupad [B]
time = 4.29, size = 144, normalized size = 3.51

$$\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,b^4\,{\left(b\,x^2+a\right)}^5}-\frac{a^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^4\,{\left(b\,x^2+a\right)}^4}-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^4\,{\left(b\,x^2+a\right)}^2}+\frac{3\,a\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,b^4\,{\left(b\,x^2+a\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*b^4*(a + b*x^2)^5) - (a^2*(a^2 +
 b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*b^4*(a + b*x^2)^4) - (a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2)/(2*b^4*(a + b*x^2)^2) + (3*a*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))
/(4*b^4*(a + b*x^2)^3)
```

3.648 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] 1/24*x^6/a^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)+1/8*x^6/a/(b*x^2+a)/(b^2*x^4+2*a
*b*x^2+a^2)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{1123\}$

$$\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] x^6/(24*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + x^6/(8*a*(a + b*x^2)*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 1123

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[2*(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(d*(m + 3)*(2*a + b*x
^2))), x] - Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*d*(m + 3)*
(p + 1))), x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p] && EqQ[m + 4*p + 5, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.68

$$\frac{-a^2-4abx^2-6b^2x^4}{24b^3\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a^2 - 4*a*b*x^2 - 6*b^2*x^4)/(24*b^3*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.03, size = 43, normalized size = 0.58

method	result	size
gosper	$-\frac{(b x^2+a)(6b^2x^4+4ab x^2+a^2)}{24b^3\left((b x^2+a)^2\right)^{\frac{5}{2}}}$	43
default	$-\frac{(b x^2+a)(6b^2x^4+4ab x^2+a^2)}{24b^3\left((b x^2+a)^2\right)^{\frac{5}{2}}}$	43
risch	$\frac{\sqrt{(b x^2+a)^2}\left(-\frac{x^4}{4b}-\frac{a x^2}{6b^2}-\frac{a^2}{24b^3}\right)}{(b x^2+a)^5}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(b*x^2+a)*(6*b^2*x^4+4*a*b*x^2+a^2)/b^3/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.93

$$-\frac{6\,b^2x^4+4\,abx^2+a^2}{24\,(b^7x^8+4\,ab^6x^6+6\,a^2b^5x^4+4\,a^3b^4x^2+a^4b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/(b^7*x^8 + 4*a*b^6*x^6 + 6*a^2*b^5*x^4
+ 4*a^3*b^4*x^2 + a^4*b^3)
```

Fricas [A]
time = 0.33, size = 69, normalized size = 0.93

$$-\frac{6\,b^2x^4+4\,abx^2+a^2}{24\,(b^7x^8+4\,ab^6x^6+6\,a^2b^5x^4+4\,a^3b^4x^2+a^4b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/(b^7*x^8 + 4*a*b^6*x^6 + 6*a^2*b^5*x^4
+ 4*a^3*b^4*x^2 + a^4*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.84, size = 43, normalized size = 0.58

$$-\frac{6\,b^2x^4+4\,abx^2+a^2}{24\,(bx^2+a)^4b^3\mathrm{sgn}\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/((b*x^2 + a)^4*b^3*sgn(b*x^2 + a))
```

Mupad [B]
time = 4.23, size = 53, normalized size = 0.72

$$-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(a^2+4\,a\,b\,x^2+6\,b^2\,x^4\right)}{24\,b^3\,\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] -((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a^2 + 6*b^2*x^4 + 4*a*b*x^2))/(24*b^3*
(a + b*x^2)^5)
```

3.649 $\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{a}{8b^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] -1/6/b^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)+1/8*a/b^2/(b*x^2+a)/(b^2*x^4+2*a*b*x
^2+a^2)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1125, 654, 621}

$$\frac{a}{8b^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}-\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/6*1/(b^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + a/(8*b^2*(a + b*x^2)*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 621

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1125

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist
[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])
```

Rubi steps

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a^2+2abx+b^2x^2)^{5/2}}\,dx, x, x^2\right)$$
$$= -\frac{1}{6b^2(a^2+2abx^2+b^2x^4)^{3/2}} - \frac{a\text{Subst}\left(\int \frac{1}{(a^2+2abx+b^2x^2)^{5/2}}\,dx, x, x^2\right)}{2b}$$
$$= -\frac{1}{6b^2(a^2+2abx^2+b^2x^4)^{3/2}} + \frac{a}{8b^2(a+bx^2)(a^2+2abx^2+b^2x^4)^{3/2}}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.57

$$\frac{-a-4bx^2}{24b^2(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
[Out] (-a - 4*b*x^2)/(24*b^2*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.02, size = 32, normalized size = 0.46

method	result	size
gosper	$-\frac{(b\,x^2+a)(4b\,x^2+a)}{24b^2\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}$	32
default	$-\frac{(b\,x^2+a)(4b\,x^2+a)}{24b^2\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}$	32
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(-\frac{x^2}{6b}-\frac{a}{24b^2}\right)}{(b\,x^2+a)^5}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/24*(b*x^2+a)*(4*b*x^2+a)/b^2/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.84

$$-\frac{4\,bx^2+a}{24\,(b^6x^8+4\,ab^5x^6+6\,a^2b^4x^4+4\,a^3b^3x^2+a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(4*b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^4 + 4*a^3*b^3*x^2
+ a^4*b^2)
```

Fricas [A]
time = 0.35, size = 58, normalized size = 0.84

$$-\frac{4\,bx^2+a}{24\,(b^6x^8+4\,ab^5x^6+6\,a^2b^4x^4+4\,a^3b^3x^2+a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(4*b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^4 + 4*a^3*b^3*x^2
+ a^4*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.60, size = 32, normalized size = 0.46

$$-\frac{4\,bx^2+a}{24\,(bx^2+a)^4b^2\mathrm{sgn}\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*(4*b*x^2 + a)/((b*x^2 + a)^4*b^2*sgn(b*x^2 + a))
```

Mupad [B]
time = 4.26, size = 42, normalized size = 0.61

$$-\frac{(4\,b\,x^2+a)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{24\,b^2\,(b\,x^2+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] -((a + 4*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(24*b^2*(a + b*x^2)^5)
```

3.650 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] -1/8/b/(b*x^2+a)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1121, 621}

$$-\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/8*1/(b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rule 621

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx,x,x^2\right)\\
&= -\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^2}{8b\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
[Out] -1/8*(a + b*x^2)/(b*((a + b*x^2)^2)^(5/2))
```

Maple [A]
time = 0.02, size = 24, normalized size = 0.63

method	result	size
gosper	$-\frac{b\,x^{2}+a}{8b\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$	24
default	$-\frac{b\,x^{2}+a}{8b\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$	24
risch	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}}{8\left(b\,x^{2}+a\right)^{5}b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/8*(b*x^2+a)/b/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.29, size = 48, normalized size = 1.26

$$-\frac{1}{8\,\left(b^{5}x^{8}+4\,ab^{4}x^{6}+6\,a^{2}b^{3}x^{4}+4\,a^{3}b^{2}x^{2}+a^{4}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
[Out] -1/8/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2 + a^4*b)
```

Fricas [A]
time = 0.33, size = 48, normalized size = 1.26

$$-\frac{1}{8\,\left(b^{5}x^{8}+4\,ab^{4}x^{6}+6\,a^{2}b^{3}x^{4}+4\,a^{3}b^{2}x^{2}+a^{4}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
[Out] -1/8/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2 + a^4*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(x/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.59, size = 24, normalized size = 0.63

$$-\frac{1}{8\left(bx^2+a\right)^4 b\operatorname{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
[Out] -1/8/((b*x^2 + a)^4*b*sgn(b*x^2 + a))
```

Mupad [B]
time = 4.27, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,b\,\left(b\,x^2+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] -(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(8*b*(a + b*x^2)^5)
```

3.651 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{1}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{4a^3\,(a+}$$

```
[Out] 1/2/a^4/((b*x^2+a)^2)^(1/2)+1/8/a/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+1/6/a^2/(
b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+1/4/a^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+(b*x^2
+a)*ln(x)/a^5/((b*x^2+a)^2)^(1/2)-1/2*(b*x^2+a)*ln(b*x^2+a)/a^5/((b*x^2+a)^
2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1126, 272, 46}

$$\frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*(a + b*x^2)^3*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) + 1/(6*a^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + 1/(4*a^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2
)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[a + b*x^
2])/(2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{1}{x(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^4(ab+b^2x^2)\right)\text{Subst}\left(\int \frac{1}{x(ab+b^2x)^5}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^4(ab+b^2x^2)\right)\text{Subst}\left(\int \left(\frac{1}{a^5b^5x}-\frac{1}{ab^4(a+bx)^5}-\frac{1}{a^2b^4(a+bx)^4}-\frac{1}{a^3b^4(a+bx)^3}-\frac{1}{a^4\ldots}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{1}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\ldots}{6a^2\left(a+\ldots\right.}$$

Mathematica [A]
time = 0.03, size = 96, normalized size = 0.43

$$\frac{a(25a^3+52a^2bx^2+42ab^2x^4+12b^3x^6)+24(a+bx^2)^4\log(x)-12(a+bx^2)^4\log\left(a+bx^2\right)}{24a^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (a*(25*a^3 + 52*a^2*b*x^2 + 42*a*b^2*x^4 + 12*b^3*x^6) + 24*(a + b*x^2)^4*L
og[x] - 12*(a + b*x^2)^4*Log[a + b*x^2])/(24*a^5*(a + b*x^2)^3*Sqrt[(a + b*
x^2)^2])
```

Maple [A]
time = 0.03, size = 193, normalized size = 0.87

method	result
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(\frac{b^3x^6}{2a^4}+\frac{7b^2x^4}{4a^3}+\frac{13b\,x^2}{6a^2}+\frac{25}{24a}\right)}{\left(b\,x^2+a\right)^5}+\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\ln(x)}{\left(b\,x^2+a\right)a^5}-\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\ln\left(b\,x^2+a\right)}{2\left(b\,x^2+a\right)a^5}$
default	$-\frac{\left(12\ln\left(b\,x^2+a\right)b^4x^8-24\ln(x)b^4x^8+48\ln\left(b\,x^2+a\right)a\,b^3x^6-96\ln(x)a\,b^3x^6-12a\,b^3x^6+72\ln\left(b\,x^2+a\right)a^2b^2x^4-144\ln(x)a^2b^2x^4-42a^2\ldots\right.}{24a^5\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(12*ln(b*x^2+a)*b^4*x^8-24*ln(x)*b^4*x^8+48*ln(b*x^2+a)*a*b^3*x^6-96*
ln(x)*a*b^3*x^6-12*a*b^3*x^6+72*ln(b*x^2+a)*a^2*b^2*x^4-144*ln(x)*a^2*b^2*x
^4-42*a^2*b^2*x^4+48*ln(b*x^2+a)*a^3*b*x^2-96*ln(x)*a^3*b*x^2-52*a^3*b*x^2+
12*ln(b*x^2+a)*a^4-24*a^4*ln(x)-25*a^4)*(b*x^2+a)/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 101, normalized size = 0.45

$$\frac{12\,b^3x^6+42\,ab^2x^4+52\,a^2bx^2+25\,a^3}{24\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}-\frac{\log\left(bx^2+a\right)}{2\,a^5}+\frac{\log\left(x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(12*b^3*x^6 + 42*a*b^2*x^4 + 52*a^2*b*x^2 + 25*a^3)/(a^4*b^4*x^8 + 4*a
^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8) - 1/2*log(b*x^2 + a)/a^5 +
log(x)/a^5
```

Fricas [A]
time = 0.37, size = 178, normalized size = 0.80

$$\frac{12\,ab^3x^6+42\,a^2b^2x^4+52\,a^3bx^2+25\,a^4-12\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\log\left(bx^2+a\right)+24\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\log\left(x\right)}{24\,(a^5b^4x^8+4\,a^6b^3x^6+6\,a^7b^2x^4+4\,a^8bx^2+a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(12*a*b^3*x^6 + 42*a^2*b^2*x^4 + 52*a^3*b*x^2 + 25*a^4 - 12*(b^4*x^8 +
 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(b*x^2 + a) + 24*(b^4*
x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(x))/(a^5*b^4*x^8
 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*b*x^2 + a^9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**2)**2)**(5/2)), x)
```

Giac [A]
time = 4.13, size = 111, normalized size = 0.50

$$\frac{\log\left(x^2\right)}{2\,a^5\mathrm{sgn}\left(bx^2+a\right)}-\frac{\log\left(|bx^2+a|\right)}{2\,a^5\mathrm{sgn}\left(bx^2+a\right)}+\frac{25\,b^4x^8+112\,ab^3x^6+192\,a^2b^2x^4+152\,a^3bx^2+50\,a^4}{24\,(bx^2+a)^4a^5\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/(a^5*sgn(b*x^2 + a)) - 1/2*log(abs(b*x^2 + a))/(a^5*sgn(b*x^2
+ a)) + 1/24*(25*b^4*x^8 + 112*a*b^3*x^6 + 192*a^2*b^2*x^4 + 152*a^3*b*x^2
+ 50*a^4)/((b*x^2 + a)^4*a^5*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

3.652 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{8a^2(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{3a^3(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{4a^4(a}$$

```
[Out] -2*b/a^5/((b*x^2+a)^2)^(1/2)-1/8*b/a^2/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-1/3*
b/a^3/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-3/4*b/a^4/(b*x^2+a)/((b*x^2+a)^2)^(1/
2)+1/2*(-b*x^2-a)/a^5/x^2/((b*x^2+a)^2)^(1/2)-5*b*(b*x^2+a)*ln(x)/a^6/((b*x
^2+a)^2)^(1/2)+5/2*b*(b*x^2+a)*ln(b*x^2+a)/a^6/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$,
Rules used = {1126, 272, 46}

$$-\frac{b}{8a^2(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5b\log(x)(a+bx^2)}{a^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5b(a+bx^2)\log(a+bx^2)}{2a^6\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^5x^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b}{4a^4(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{3a^3(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-2*b)/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(8*a^2*(a + b*x^2)^3*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(3*a^3*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (3*b)/(4*a^4*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a
 + b*x^2)/(2*a^5*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*b*(a + b*x^2)*Lo
g[x])/(a^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*b*(a + b*x^2)*Log[a + b*x^
2])/(2*a^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
```

```
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{1}{x^3(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4(ab+b^2x^2)\right)\mathrm{Subst}\left(\int \frac{1}{x^2(ab+b^2x)^5}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^4(ab+b^2x^2)\right)\mathrm{Subst}\left(\int \left(\frac{1}{a^5b^5x^2}-\frac{5}{a^6b^4x}+\frac{1}{a^2b^3(a+bx)^5}+\frac{2}{a^3b^3(a+bx)^4}+\frac{3}{a^4b^3(a\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{8a^2\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{3a^3\left(a\right.}$$

Mathematica [A]
time = 0.03, size = 119, normalized size = 0.45

$$\frac{-a(12a^4+125a^3bx^2+260a^2b^2x^4+210ab^3x^6+60b^4x^8)-120bx^2(a+bx^2)^4\log(x)+60bx^2(a+bx^2)^4\log\left(a+bx^2\right)}{24a^6x^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x^2 + 260*a^2*b^2*x^4 + 210*a*b^3*x^6 + 60*b^4*x^8
)) - 120*b*x^2*(a + b*x^2)^4*Log[x] + 60*b*x^2*(a + b*x^2)^4*Log[a + b*x^2]
)/(24*a^6*x^2*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.04, size = 219, normalized size = 0.82

method	result
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(-\frac{5b^4x^8}{2a^5}-\frac{35b^3x^6}{4a^4}-\frac{65b^2x^4}{6a^3}-\frac{125b\,x^2}{24a^2}-\frac{1}{2a}\right)}{\left(b\,x^2+a\right)^5x^2}-\frac{5\sqrt{\left(b\,x^2+a\right)^2}\,b\ln(x)}{\left(b\,x^2+a\right)a^6}+\frac{5\sqrt{\left(b\,x^2+a\right)^2}\,b\ln(-b\,x^2-a}{2\left(b\,x^2+a\right)a^6}$
default	$\frac{\left(60\ln\left(b\,x^2+a\right)b^5x^{10}-120\ln(x)b^5x^{10}+240\ln\left(b\,x^2+a\right)a\,b^4x^8-480b^4a\ln(x)x^8-60b^4a\,x^8+360\ln\left(b\,x^2+a\right)a^2b^3x^6-720a^2b^3\ln(x)x^6-2\right.}{24x^2a^6\left((b\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(60*ln(b*x^2+a)*b^5*x^10-120*ln(x)*b^5*x^10+240*ln(b*x^2+a)*a*b^4*x^8-
480*b^4*a*ln(x)*x^8-60*b^4*a*x^8+360*ln(b*x^2+a)*a^2*b^3*x^6-720*a^2*b^3*ln
(x)*x^6-210*a^2*b^3*x^6+240*ln(b*x^2+a)*a^3*b^2*x^4-480*b^2*a^3*ln(x)*x^4-2
60*b^2*a^3*x^4+60*ln(b*x^2+a)*a^4*b*x^2-120*b*a^4*ln(x)*x^2-125*b*a^4*x^2-1
2*a^5)*(b*x^2+a)/x^2/a^6/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.29, size = 119, normalized size = 0.45

$$-\frac{60\,b^4x^8+210\,ab^3x^6+260\,a^2b^2x^4+125\,a^3bx^2+12\,a^4}{24\,(a^5b^4x^{10}+4\,a^6b^3x^8+6\,a^7b^2x^6+4\,a^8bx^4+a^9x^2)}+\frac{5\,b\log\left(bx^2+a\right)}{2\,a^6}-\frac{5\,b\log\left(x\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(60*b^4*x^8 + 210*a*b^3*x^6 + 260*a^2*b^2*x^4 + 125*a^3*b*x^2 + 12*a^
4)/(a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 + 4*a^8*b*x^4 + a^9*x^2) +
 5/2*b*log(b*x^2 + a)/a^6 - 5*b*log(x)/a^6
```

Fricas [A]
time = 0.38, size = 207, normalized size = 0.78

$$-\frac{60\,ab^4x^8+210\,a^2b^3x^6+260\,a^3b^2x^4+125\,a^4bx^2+12\,a^5-60\,(b^5x^{10}+4\,ab^4x^8+6\,a^2b^3x^6+4\,a^3b^2x^4+a^4bx^2)\log\left(bx^2+a\right)+120\,(b^5x^{10}+4\,ab^4x^8+6\,a^2b^3x^6+4\,a^3b^2x^4+a^4bx^2)\log\left(x\right)}{24\,(a^6b^4x^{10}+4\,a^7b^3x^8+6\,a^8b^2x^6+4\,a^9bx^4+a^{10}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(60*a*b^4*x^8 + 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 + 125*a^4*b*x^2 + 1
2*a^5 - 60*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*b*
x^2)*log(b*x^2 + a) + 120*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b
^2*x^4 + a^4*b*x^2)*log(x))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 +
 4*a^9*b*x^4 + a^10*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**2)**2)**(5/2)), x)
```

Giac [A]
time = 3.55, size = 144, normalized size = 0.54

$$-\frac{5\,b\log\left(x^2\right)}{2\,a^6\mathrm{sgn}\left(bx^2+a\right)}+\frac{5\,b\log\left(|bx^2+a|\right)}{2\,a^6\mathrm{sgn}\left(bx^2+a\right)}+\frac{5\,bx^2-a}{2\,a^6x^2\mathrm{sgn}\left(bx^2+a\right)}-\frac{125\,b^5x^8+548\,ab^4x^6+912\,a^2b^3x^4+688\,a^3b^2x^2+202\,a^4b}{24\,(bx^2+a)^4a^6\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -5/2*b*log(x^2)/(a^6*sgn(b*x^2 + a)) + 5/2*b*log(abs(b*x^2 + a))/(a^6*sgn(b
*x^2 + a)) + 1/2*(5*b*x^2 - a)/(a^6*x^2*sgn(b*x^2 + a)) - 1/24*(125*b^5*x^8
 + 548*a*b^4*x^6 + 912*a^2*b^3*x^4 + 688*a^3*b^2*x^2 + 202*a^4*b)/((b*x^2 +
 a)^4*a^6*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

3.653 $\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=211

$$\frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-64b$$

```
[Out] 5/128*x/a/b^3/((b*x^2+a)^2)^(1/2)-1/8*x^5/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)
-5/48*x^3/b^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-5/64*x/b^3/(b*x^2+a)/((b*x^2+
a)^2)^(1/2)+5/128*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(7/2)/((b*x
^2+a)^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1126, 294, 205, 211}

$$-\frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x}{64b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{3/2}b^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (5*x)/(128*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^5/(8*b*(a + b*x^2)^3*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*x^3)/(48*b^2*(a + b*x^2)^2*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) - (5*x)/(64*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(3/2)*b^(7/2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(b^4(ab+b^2x^2))\int \frac{x^6}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^5}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(5b^2(ab+b^2x^2))\int \frac{x^4}{(ab+b^2x^2)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^5}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5x^3}{48b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^5}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5x^3}{48b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^5}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{48b^2(a}$$

$$= \frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^5}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{48b^2(a}$$

Mathematica [A]
time = 0.03, size = 105, normalized size = 0.50

$$\frac{\sqrt{a}\sqrt{b}\,x(-15a^3-55a^2bx^2-73ab^2x^4+15b^3x^6)+15(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{384a^{3/2}b^{7/2}(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

[Out] (Sqrt[a]*Sqrt[b]*x*(-15*a^3 - 55*a^2*b*x^2 - 73*a*b^2*x^4 + 15*b^3*x^6) + 15*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(3/2)*b^(7/2)*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])

Maple [A]
time = 0.04, size = 172, normalized size = 0.82

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(\frac{5x^7}{128a}-\frac{73x^5}{384b}-\frac{55a\,x^3}{384b^2}-\frac{5a^2x}{128b^3}\right)}{(b\,x^2+a)^5}-\frac{5\sqrt{(b\,x^2+a)^2}\,\ln\left(bx+\sqrt{-ab}\right)}{256(b\,x^2+a)\sqrt{-ab}\,b^3a}+\frac{5\sqrt{(b\,x^2+a)^2}\,\ln\left(-bx+\sqrt{\right.}}{256(b\,x^2+a)\sqrt{-ab}\,b^3a}$
default	$-\frac{\left(-15\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^4x^8-15\sqrt{ab}\,b^3x^7-60\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^3x^6+73\sqrt{ab}\,a\,b^2x^5-90\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2b^2x^4+55\sqrt{ab}\right.}{384\sqrt{ab}\,b^3a\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/384*(-15*arctan(b*x/(a*b)^(1/2))*b^4*x^8-15*(a*b)^(1/2)*b^3*x^7-60*arctan(b*x/(a*b)^(1/2))*a*b^3*x^6+73*(a*b)^(1/2)*a*b^2*x^5-90*arctan(b*x/(a*b)^(1/2))*a^2*b^2*x^4+55*(a*b)^(1/2)*a^2*b*x^3-60*arctan(b*x/(a*b)^(1/2))*a^3*b*x^2+15*(a*b)^(1/2)*a^3*x-15*arctan(b*x/(a*b)^(1/2))*a^4)*(b*x^2+a)/(a*b)^(1/2)/b^3/a/((b*x^2+a)^2)^(5/2)

Maxima [A]
time = 0.50, size = 109, normalized size = 0.52

$$\frac{15\,b^3x^7-73\,ab^2x^5-55\,a^2bx^3-15\,a^3x}{384\,(ab^7x^8+4\,a^2b^6x^6+6\,a^3b^5x^4+4\,a^4b^4x^2+a^5b^3)}+\frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/384*(15*b^3*x^7 - 73*a*b^2*x^5 - 55*a^2*b*x^3 - 15*a^3*x)/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3) + 5/128*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3)

Fricas [A]
time = 0.38, size = 324, normalized size = 1.54

$$\left[\frac{30\,ab^4x^7-146\,a^2b^3x^5-110\,a^3b^2x^3-30\,a^4bx-15\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{768\,(a^2b^8x^8+4\,a^3b^7x^6+6\,a^4b^6x^4+4\,a^5b^5x^2+a^6b^4)},\ \frac{15\,ab^4x^7-73\,a^2b^3x^5-55\,a^3b^2x^3-15\,a^4bx+15\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{384\,(a^2b^8x^8+4\,a^3b^7x^6+6\,a^4b^6x^4+4\,a^5b^5x^2+a^6b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")

```
[Out] [1/768*(30*a*b^4*x^7 - 146*a^2*b^3*x^5 - 110*a^3*b^2*x^3 - 30*a^4*b*x - 15*
(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^8*x^8 + 4*a^3*b^7*x^6 + 6
*a^4*b^6*x^4 + 4*a^5*b^5*x^2 + a^6*b^4), 1/384*(15*a*b^4*x^7 - 73*a^2*b^3*x
^5 - 55*a^3*b^2*x^3 - 15*a^4*b*x + 15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^
4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^8*x^8 + 4*a^
3*b^7*x^6 + 6*a^4*b^6*x^4 + 4*a^5*b^5*x^2 + a^6*b^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(x**6/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.40, size = 93, normalized size = 0.44

$$\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\sqrt{ab}\,ab^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{15\,b^3x^7-73\,ab^2x^5-55\,a^2bx^3-15\,a^3x}{384\left(bx^2+a\right)^4ab^3\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
[Out] 5/128*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3*sgn(b*x^2 + a)) + 1/384*(15*b^
3*x^7 - 73*a*b^2*x^5 - 55*a^2*b*x^3 - 15*a^3*x)/((b*x^2 + a)^4*a*b^3*sgn(b*
x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.654 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=212

$$\frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{64}$$

```
[Out] 3/128*x/a^2/b^2/((b*x^2+a)^2)^(1/2)-1/8*x^3/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/
2)-1/16*x/b^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+1/64*x/a/b^2/(b*x^2+a)/((b*x^
2+a)^2)^(1/2)+3/128*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(5/2)/((b
*x^2+a)^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1126, 294, 205, 211}

$$\frac{x}{64ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (3*x)/(128*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(8*b*(a + b*x^2)^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(16*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) + x/(64*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(b^4(ab+b^2x^2))\int \frac{x^4}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3b^2(ab+b^2x^2))\int \frac{x^2}{(ab+b^2x^2)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{16b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x^3}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{16b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{16b^2\,(}$$

$$= \frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{16b^2\,(}$$

Mathematica [A]
time = 0.03, size = 105, normalized size = 0.50

$$\frac{\sqrt{a}\,\sqrt{b}\,x(-3a^3-11a^2bx^2+11ab^2x^4+3b^3x^6)+3(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}\,(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-3*a^3 - 11*a^2*b*x^2 + 11*a*b^2*x^4 + 3*b^3*x^6) + 3*(
a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*(a + b*x^2)^
3*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.04, size = 172, normalized size = 0.81

method	result
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(\frac{3 b\,x^7}{128 a^2}+\frac{11 x^5}{128 a}-\frac{11 x^3}{128 b}-\frac{3 a x}{128 b^2}\right)}{\left(b\,x^2+a\right)^5}-\frac{3\sqrt{\left(b\,x^2+a\right)^2}\,\ln\left(b x+\sqrt{-a b}\right)}{256\left(b\,x^2+a\right)\sqrt{-a b}\,b^2 a^2}+\frac{3\sqrt{\left(b\,x^2+a\right)^2}\,\ln\left(-b x+\sqrt{\cdot}\right.}{256\left(b\,x^2+a\right)\sqrt{-a b}\,b^2 a^2}$
default	$-\frac{\left(-3\arctan\left(\frac{b x}{\sqrt{a b}}\right) b^4 x^8-3\sqrt{a b}\,b^3 x^7-12\arctan\left(\frac{b x}{\sqrt{a b}}\right) a\,b^3 x^6-11\sqrt{a b}\,a\,b^2 x^5-18\arctan\left(\frac{b x}{\sqrt{a b}}\right) a^2 b^2 x^4+11\sqrt{a b}\,\cdots\right.}{128\sqrt{a b}\,b^2 a^2\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(-3*arctan(b*x/(a*b)^(1/2))*b^4*x^8-3*(a*b)^(1/2)*b^3*x^7-12*arctan(
b*x/(a*b)^(1/2))*a*b^3*x^6-11*(a*b)^(1/2)*a*b^2*x^5-18*arctan(b*x/(a*b)^(1/
2))*a^2*b^2*x^4+11*(a*b)^(1/2)*a^2*b*x^3-12*arctan(b*x/(a*b)^(1/2))*a^3*b*x
^2+3*(a*b)^(1/2)*a^3*x-3*arctan(b*x/(a*b)^(1/2))*a^4)*(b*x^2+a)/(a*b)^(1/2)
/b^2/a^2/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.50, size = 111, normalized size = 0.52

$$\frac{3\,b^3x^7+11\,ab^2x^5-11\,a^2bx^3-3\,a^3x}{128\,\left(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2\right)}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/128*(3*b^3*x^7 + 11*a*b^2*x^5 - 11*a^2*b*x^3 - 3*a^3*x)/(a^2*b^6*x^8 + 4*
a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) + 3/128*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

Fricas [A]
time = 0.36, size = 324, normalized size = 1.53

$$\left[\frac{6\,ab^4x^7+22\,a^2b^3x^5-22\,a^3b^2x^3-6\,a^4bx-3\,\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{256\,\left(a^3b^7x^8+4\,a^4b^6x^6+6\,a^5b^5x^4+4\,a^6b^4x^2+a^7b^3\right)},\ \frac{3\,ab^4x^7+11\,a^2b^3x^5-11\,a^3b^2x^3-3\,a^4bx+3\,\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{128\,\left(a^3b^7x^8+4\,a^4b^6x^6+6\,a^5b^5x^4+4\,a^6b^4x^2+a^7b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/256*(6*a*b^4*x^7 + 22*a^2*b^3*x^5 - 22*a^3*b^2*x^3 - 6*a^4*b*x - 3*(b^4*
x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log((b*x^
2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^7*x^8 + 4*a^4*b^6*x^6 + 6*a^5*
b^5*x^4 + 4*a^6*b^4*x^2 + a^7*b^3), 1/128*(3*a*b^4*x^7 + 11*a^2*b^3*x^5 - 1
1*a^3*b^2*x^3 - 3*a^4*b*x + 3*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^
3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^7*x^8 + 4*a^4*b^6*x^
6 + 6*a^5*b^5*x^4 + 4*a^6*b^4*x^2 + a^7*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.17, size = 93, normalized size = 0.44

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\sqrt{ab}\,a^2b^2\mathrm{sgn}\left(bx^2+a\right)}+\frac{3\,b^3x^7+11\,ab^2x^5-11\,a^2bx^3-3\,a^3x}{128\left(bx^2+a\right)^4a^2b^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/128*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2*sgn(b*x^2 + a)) + 1/128*(3*b
^3*x^7 + 11*a*b^2*x^5 - 11*a^2*b*x^3 - 3*a^3*x)/((b*x^2 + a)^4*a^2*b^2*sgn(
b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.655 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{192}$$

```
[Out] 5/128*x/a^3/b/((b*x^2+a)^2)^(1/2)-1/8*x/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+1
/48*x/a/b/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+5/192*x/a^2/b/(b*x^2+a)/((b*x^2+a
)^2)^(1/2)+5/128*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)/((b*x^
2+a)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1126, 294, 205, 211}

$$\frac{5x}{192a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (5*x)/(128*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(8*b*(a + b*x^2)^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + x/(48*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]) + (5*x)/(192*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{x^2}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{1}{(ab+b^2x^2)^4}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48ab\,(a}$$

$$= \frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{48ab\,(a}$$

Mathematica [A]
time = 0.02, size = 105, normalized size = 0.49

$$\frac{\sqrt{a}\,\sqrt{b}\,x(-15a^3+73a^2bx^2+55ab^2x^4+15b^3x^6)+15(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{384a^{7/2}b^{3/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-15*a^3 + 73*a^2*b*x^2 + 55*a*b^2*x^4 + 15*b^3*x^6) + 1
5*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(7/2)*b^(3/2)*(a + b*x^
2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.04, size = 172, normalized size = 0.81

method	result
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{5 b^{2} x^{7}}{128 a^{3}}+\frac{55 b\,x^{5}}{384 a^{2}}+\frac{73 x^{3}}{384 a}-\frac{5 x}{128 b}\right)}{\left(b\,x^{2}+a\right)^{5}}-\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(b x+\sqrt{-a b}\right)}{256\left(b\,x^{2}+a\right)\sqrt{-a b}\,b\,a^{3}}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(-b x+\sqrt{\right.}{256\left(b\,x^{2}+a\right)\sqrt{-a b}\,b\,a^{3}}$
default	$-\frac{\left(-15\arctan\left(\frac{b x}{\sqrt{a b}}\right) b^{4} x^{8}-15\sqrt{a b}\,b^{3} x^{7}-60\arctan\left(\frac{b x}{\sqrt{a b}}\right) a\,b^{3} x^{6}-55\sqrt{a b}\,a\,b^{2} x^{5}-90\arctan\left(\frac{b x}{\sqrt{a b}}\right) a^{2} b^{2} x^{4}-73\sqrt{a b}\right.}{384\sqrt{a b}\,b\,a^{3}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384*(-15*arctan(b*x/(a*b)^(1/2))*b^4*x^8-15*(a*b)^(1/2)*b^3*x^7-60*arcta
n(b*x/(a*b)^(1/2))*a*b^3*x^6-55*(a*b)^(1/2)*a*b^2*x^5-90*arctan(b*x/(a*b)^(
1/2))*a^2*b^2*x^4-73*(a*b)^(1/2)*a^2*b*x^3-60*arctan(b*x/(a*b)^(1/2))*a^3*b
*x^2+15*(a*b)^(1/2)*a^3*x-15*arctan(b*x/(a*b)^(1/2))*a^4)*(b*x^2+a)/(a*b)^(
1/2)/b/a^3/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.49, size = 109, normalized size = 0.51

$$\frac{15\,b^{3}x^{7}+55\,ab^{2}x^{5}+73\,a^{2}bx^{3}-15\,a^{3}x}{384\,(a^{3}b^{5}x^{8}+4\,a^{4}b^{4}x^{6}+6\,a^{5}b^{3}x^{4}+4\,a^{6}b^{2}x^{2}+a^{7}b)}+\frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(15*b^3*x^7 + 55*a*b^2*x^5 + 73*a^2*b*x^3 - 15*a^3*x)/(a^3*b^5*x^8 +
4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) + 5/128*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^3*b)
```

Fricas [A]
time = 0.35, size = 324, normalized size = 1.52

$$\left[\frac{30\,ab^{4}x^{7}+110\,a^{2}b^{3}x^{5}+146\,a^{3}b^{2}x^{3}-30\,a^{4}bx-15\,(b^{4}x^{8}+4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}+4\,a^{3}bx^{2}+a^{4})\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{768\,(a^{4}b^{6}x^{8}+4\,a^{5}b^{5}x^{6}+6\,a^{6}b^{4}x^{4}+4\,a^{7}b^{3}x^{2}+a^{8}b^{2})},\ \frac{15\,ab^{4}x^{7}+55\,a^{2}b^{3}x^{5}+73\,a^{3}b^{2}x^{3}-15\,a^{4}bx+15\,(b^{4}x^{8}+4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}+4\,a^{3}bx^{2}+a^{4})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{384\,(a^{4}b^{6}x^{8}+4\,a^{5}b^{5}x^{6}+6\,a^{6}b^{4}x^{4}+4\,a^{7}b^{3}x^{2}+a^{8}b^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(30*a*b^4*x^7 + 110*a^2*b^3*x^5 + 146*a^3*b^2*x^3 - 30*a^4*b*x - 15*
(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^6*x^8 + 4*a^5*b^5*x^6 + 6
*a^6*b^4*x^4 + 4*a^7*b^3*x^2 + a^8*b^2), 1/384*(15*a*b^4*x^7 + 55*a^2*b^3*x
^5 + 73*a^3*b^2*x^3 - 15*a^4*b*x + 15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^
4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^6*x^8 + 4*a^
5*b^5*x^6 + 6*a^6*b^4*x^4 + 4*a^7*b^3*x^2 + a^8*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
[Out] Integral(x**2/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.26, size = 93, normalized size = 0.44

$$\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\sqrt{ab}\,a^3b\mathrm{sgn}\left(bx^2+a\right)}+\frac{15\,b^3x^7+55\,ab^2x^5+73\,a^2bx^3-15\,a^3x}{384\left(bx^2+a\right)^4a^3b\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
[Out] 5/128*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b*sgn(b*x^2 + a)) + 1/384*(15*b^
3*x^7 + 55*a*b^2*x^5 + 73*a^2*b*x^3 - 15*a^3*x)/((b*x^2 + a)^4*a^3*b*sgn(b*
x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.656 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{x(a+bx^2)}{8a\,(a^2+2abx^2+b^2x^4)^{5/2}}+\frac{7x(a+bx^2)^2}{48a^2\,(a^2+2abx^2+b^2x^4)^{5/2}}+\frac{35x(a+bx^2)^3}{192a^3\,(a^2+2abx^2+b^2x^4)^{5/2}}+\frac{35x(a+bx^2)}{128a^4\,(a^2+2abx^2+}$$

```
[Out] 1/8*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)+7/48*x*(b*x^2+a)^2/a^2/(b^2
*x^4+2*a*b*x^2+a^2)^(5/2)+35/192*x*(b*x^2+a)^3/a^3/(b^2*x^4+2*a*b*x^2+a^2)^
(5/2)+35/128*x*(b*x^2+a)^4/a^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)+35/128*(b*x^2+
a)^5*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/b^(1/2
)
```

Rubi [A]
time = 0.05, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1102, 205, 211}

$$\frac{7x(a+bx^2)^2}{48a^2\,(a^2+2abx^2+b^2x^4)^{5/2}}+\frac{x(a+bx^2)}{8a\,(a^2+2abx^2+b^2x^4)^{5/2}}+\frac{35(a+bx^2)^5\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}\,(a^2+2abx^2+b^2x^4)^{5/2}}+\frac{35x(a+bx^2)^4}{128a^4\,(a^2+2abx^2+b^2x^4)^{5/2}}+\frac{35x(a+bx^2)^3}{192a^3\,(a^2+2abx^2+b^2x^4)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

```
[Out] (x*(a + b*x^2))/(8*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (7*x*(a + b*x^2)^
2)/(48*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(a + b*x^2)^3)/(192*a
^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(a + b*x^2)^4)/(128*a^4*(a^2
+ 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*(a + b*x^2)^5*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(128*a^(9/2)*Sqrt[b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1102

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2
 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{
a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(2ab+2b^2x^2\right)^5 \int \frac{1}{\left(2ab+2b^2x^2\right)^5}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

$$= \frac{x(a+bx^2)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{\left(7\left(2ab+2b^2x^2\right)^5\right)\int \frac{1}{\left(2ab+2b^2x^2\right)^4}\,dx}{16ab\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

$$= \frac{x(a+bx^2)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x(a+bx^2)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{\left(35(2ab+2b^2x^2\right.}{192a^2b^2\left(a^2+\right.}$$

$$= \frac{x(a+bx^2)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x(a+bx^2)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x(a+}{192a^3\left(a^2+2ab\right.}$$

$$= \frac{x(a+bx^2)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x(a+bx^2)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x(a+}{192a^3\left(a^2+2ab\right.}$$

$$= \frac{x(a+bx^2)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{7x(a+bx^2)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}} + \frac{35x(a+}{192a^3\left(a^2+2ab\right.}$$

Mathematica [A]
time = 0.03, size = 105, normalized size = 0.49

$$\frac{\sqrt{a}\,\sqrt{b}\,x\left(279a^3+511a^2bx^2+385ab^2x^4+105b^3x^6\right)+105\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{384a^{9/2}\sqrt{b}\,\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(279*a^3 + 511*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3*x^6)
+ 105*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(9/2)*Sqrt[b]*(a +
b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.04, size = 169, normalized size = 0.79

method	result
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{35 b^{3} x^{7}}{128 a^{4}}+\frac{385 b^{2} x^{5}}{384 a^{3}}+\frac{511 b\,x^{3}}{384 a^{2}}+\frac{93 x}{128 a}\right)}{\left(b\,x^{2}+a\right)^{5}}-\frac{35\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(b x+\sqrt{-a b}\right)}{256\left(b\,x^{2}+a\right)\sqrt{-a b}\,a^{4}}+\frac{35\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(-\right.}{256\left(b\,x^{2}+a\right)\sqrt{-a}}$
default	$\frac{\left(105\arctan\left(\frac{b x}{\sqrt{a b}}\right) b^{4} x^{8}+105\sqrt{a b}\,b^{3} x^{7}+420\arctan\left(\frac{b x}{\sqrt{a b}}\right) a\,b^{3} x^{6}+385\sqrt{a b}\,a\,b^{2} x^{5}+630\arctan\left(\frac{b x}{\sqrt{a b}}\right) a^{2} b^{2} x^{4}+511\sqrt{}\right.}{384\sqrt{a b}\,a^{4}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(105*arctan(b*x/(a*b)^(1/2))*b^4*x^8+105*(a*b)^(1/2)*b^3*x^7+420*arct
an(b*x/(a*b)^(1/2))*a*b^3*x^6+385*(a*b)^(1/2)*a*b^2*x^5+630*arctan(b*x/(a*b
)^(1/2))*a^2*b^2*x^4+511*(a*b)^(1/2)*a^2*b*x^3+420*arctan(b*x/(a*b)^(1/2))*
a^3*b*x^2+279*(a*b)^(1/2)*a^3*x+105*arctan(b*x/(a*b)^(1/2))*a^4)*(b*x^2+a)/
(a*b)^(1/2)/a^4/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.50, size = 102, normalized size = 0.48

$$\frac{105\,b^{3}x^{7}+385\,ab^{2}x^{5}+511\,a^{2}bx^{3}+279\,a^{3}x}{384\left(a^{4}b^{4}x^{8}+4\,a^{5}b^{3}x^{6}+6\,a^{6}b^{2}x^{4}+4\,a^{7}bx^{2}+a^{8}\right)}+\frac{35\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/384*(105*b^3*x^7 + 385*a*b^2*x^5 + 511*a^2*b*x^3 + 279*a^3*x)/(a^4*b^4*x^
8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8) + 35/128*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 0.33, size = 320, normalized size = 1.50

$$\left[\frac{210\,ab^{4}x^{7}+770\,a^{2}b^{3}x^{5}+1022\,a^{3}b^{2}x^{3}+558\,a^{4}bx-105\,(b^{4}x^{8}+4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}+4\,a^{3}bx^{2}+a^{4})\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{768\,(a^{5}b^{5}x^{8}+4\,a^{6}b^{4}x^{6}+6\,a^{7}b^{3}x^{4}+4\,a^{8}b^{2}x^{2}+a^{9}b)},\ \frac{105\,ab^{4}x^{7}+385\,a^{2}b^{3}x^{5}+511\,a^{3}b^{2}x^{3}+279\,a^{4}bx+105\,(b^{4}x^{8}+4\,ab^{3}x^{6}+6\,a^{2}b^{2}x^{4}+4\,a^{3}bx^{2}+a^{4})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{384\,(a^{5}b^{5}x^{8}+4\,a^{6}b^{4}x^{6}+6\,a^{7}b^{3}x^{4}+4\,a^{8}b^{2}x^{2}+a^{9}b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(210*a*b^4*x^7 + 770*a^2*b^3*x^5 + 1022*a^3*b^2*x^3 + 558*a^4*b*x -
105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6
 + 6*a^7*b^3*x^4 + 4*a^8*b^2*x^2 + a^9*b), 1/384*(105*a*b^4*x^7 + 385*a^2*b
```

```
^3*x^5 + 511*a^3*b^2*x^3 + 279*a^4*b*x + 105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2
*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^5*x^8
 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8*b^2*x^2 + a^9*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/2), x)
```

Giac [A]
time = 3.94, size = 87, normalized size = 0.41

$$\frac{35 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128 \sqrt{ab}\, a^4 \mathrm{sgn}\left(bx^2 + a\right)} + \frac{105\, b^3x^7 + 385\, ab^2x^5 + 511\, a^2bx^3 + 279\, a^3x}{384\, (bx^2 + a)^4 a^4 \mathrm{sgn}\left(bx^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 35/128*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*sgn(b*x^2 + a)) + 1/384*(105*b^
3*x^7 + 385*a*b^2*x^5 + 511*a^2*b*x^3 + 279*a^3*x)/((b*x^2 + a)^4*a^4*sgn(b
*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.657 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=251

$$\frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax\,(a+bx^2)^3\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3}{16a^2x\,(a+bx^2)^2\,\sqrt{a^2+2abx^2+b^2x^4}}+$$

```
[Out] 105/128/a^4/x/((b*x^2+a)^2)^(1/2)+1/8/a/x/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+3
/16/a^2/x/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+21/64/a^3/x/(b*x^2+a)/((b*x^2+a)^
2)^(1/2)-315/128*(b*x^2+a)/a^5/x/((b*x^2+a)^2)^(1/2)-315/128*(b*x^2+a)*arct
an(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(11/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1126, 296, 331, 211}

$$\frac{3}{16a^2x\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^2}+\frac{1}{8ax\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^3}-\frac{315\sqrt{b}\,(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{315(a+bx^2)}{128a^5x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21}{64a^3x\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 105/(128*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x*(a + b*x^2)^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 3/(16*a^2*x*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + 21/(64*a^3*x*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (315*(a + b*x^2))/(128*a^5*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (315*Sq
rt[b]*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx &= \frac{\left(b^4\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(9b^3\left(ab+b^2x^2\right)\right)\int \frac{1}{x^2(ab+b^2x^2)^4}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3}{16a^2x\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b}}\\
&= \frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3}{16a^2x\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b}}\\
&= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{16}\\
&= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{16}\\
&= \frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{16}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 115, normalized size = 0.46

$$\frac{-\sqrt{a}\left(128a^4+837a^3bx^2+1533a^2b^2x^4+1155ab^3x^6+315b^4x^8\right)-315\sqrt{b}\,x\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{11/2}x\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-(Sqrt[a]*(128*a^4 + 837*a^3*b*x^2 + 1533*a^2*b^2*x^4 + 1155*a*b^3*x^6 + 3
15*b^4*x^8)) - 315*Sqrt[b]*x*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(12
8*a^(11/2)*x*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.04, size = 191, normalized size = 0.76

method	result
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{315 b^{4} x^{8}}{128 a^{5}}-\frac{1155 b^{3} x^{6}}{128 a^{4}}-\frac{1533 b^{2} x^{4}}{128 a^{3}}-\frac{837 b\,x^{2}}{128 a^{2}}-\frac{1}{a}\right)}{\left(b\,x^{2}+a\right)^{5} x}+\frac{315\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^{2}+b\right)}\_R\ln\left(\left(3\_\right.\right.\right.}{256\left(b\,x^{2}+a\right)}$
default	$-\frac{\left(315\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^{5}x^{9}+315\sqrt{ab}\,b^{4}x^{8}+1260\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^{4}x^{7}+1155\sqrt{ab}\,a\,b^{3}x^{6}+1890\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^{2}b^{3}x^{5}+\right.}{128x\sqrt{ab}\,a^{5}\left(\left(b\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(315*arctan(b*x/(a*b)^(1/2))*b^5*x^9+315*(a*b)^(1/2)*b^4*x^8+1260*ar
ctan(b*x/(a*b)^(1/2))*a*b^4*x^7+1155*(a*b)^(1/2)*a*b^3*x^6+1890*arctan(b*x/
(a*b)^(1/2))*a^2*b^3*x^5+1533*(a*b)^(1/2)*a^2*b^2*x^4+1260*arctan(b*x/(a*b)
^(1/2))*a^3*b^2*x^3+837*(a*b)^(1/2)*a^3*b*x^2+315*arctan(b*x/(a*b)^(1/2))*a
^4*b*x+128*(a*b)^(1/2)*a^4)*(b*x^2+a)/x/(a*b)^(1/2)/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.50, size = 115, normalized size = 0.46

$$-\frac{315\,b^{4}x^{8}+1155\,ab^{3}x^{6}+1533\,a^{2}b^{2}x^{4}+837\,a^{3}bx^{2}+128\,a^{4}}{128\,\left(a^{5}b^{4}x^{9}+4\,a^{6}b^{3}x^{7}+6\,a^{7}b^{2}x^{5}+4\,a^{8}bx^{3}+a^{9}x\right)}-\frac{315\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/128*(315*b^4*x^8 + 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 + 837*a^3*b*x^2 + 1
28*a^4)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)
 - 315/128*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Fricas [A]
time = 0.35, size = 334, normalized size = 1.33

$$\left[-\frac{630\,b^{4}x^{8}+2310\,ab^{3}x^{6}+3066\,a^{2}b^{2}x^{4}+1674\,a^{3}bx^{2}+256\,a^{4}-315\,\left(b^{4}x^{9}+4\,ab^{3}x^{7}+6\,a^{2}b^{2}x^{5}+4\,a^{3}bx^{3}+a^{4}x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{256\,\left(a^{5}b^{4}x^{9}+4\,a^{6}b^{3}x^{7}+6\,a^{7}b^{2}x^{5}+4\,a^{8}bx^{3}+a^{9}x\right)},\ -\frac{315\,b^{4}x^{8}+1155\,ab^{3}x^{6}+1533\,a^{2}b^{2}x^{4}+837\,a^{3}bx^{2}+128\,a^{4}+315\,\left(b^{4}x^{9}+4\,ab^{3}x^{7}+6\,a^{2}b^{2}x^{5}+4\,a^{3}bx^{3}+a^{4}x\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{128\,\left(a^{5}b^{4}x^{9}+4\,a^{6}b^{3}x^{7}+6\,a^{7}b^{2}x^{5}+4\,a^{8}bx^{3}+a^{9}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/256*(630*b^4*x^8 + 2310*a*b^3*x^6 + 3066*a^2*b^2*x^4 + 1674*a^3*b*x^2 +
 256*a^4 - 315*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x
)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^4*x^9
+ 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), -1/128*(315*b^4*x^8
 + 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 + 837*a^3*b*x^2 + 128*a^4 + 315*(b^4*x
^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*sqrt(b/a)*arctan(x*
sqrt(b/a)))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^
9*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(5/2)), x)
```

Giac [A]
time = 3.94, size = 109, normalized size = 0.43

$$-\frac{315\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^5\mathrm{sgn}\left(bx^2+a\right)}-\frac{1}{a^5x\mathrm{sgn}\left(bx^2+a\right)}-\frac{187\,b^4x^7+643\,ab^3x^5+765\,a^2b^2x^3+325\,a^3bx}{128\,\left(bx^2+a\right)^4a^5\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -315/128*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5*sgn(b*x^2 + a)) - 1/(a^5*x*
sgn(b*x^2 + a)) - 1/128*(187*b^4*x^7 + 643*a*b^3*x^5 + 765*a^2*b^2*x^3 + 32
5*a^3*b*x)/((b*x^2 + a)^4*a^5*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

3.658 $\int \frac{1}{x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=291

$$\frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 231/128/a^4/x^3/((b*x^2+a)^2)^(1/2)+1/8/a/x^3/(b*x^2+a)^3/((b*x^2+a)^2)^(1/
2)+11/48/a^2/x^3/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+33/64/a^3/x^3/(b*x^2+a)/((
b*x^2+a)^2)^(1/2)-385/128*(b*x^2+a)/a^5/x^3/((b*x^2+a)^2)^(1/2)+1155/128*b*
(b*x^2+a)/a^6/x/((b*x^2+a)^2)^(1/2)+1155/128*b^(3/2)*(b*x^2+a)*arctan(x*b^(
1/2)/a^(1/2))/a^(13/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1126, 296, 331, 211}

$$\frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155b^{3/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128a^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155b(a+bx^2)}{128a^6x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{385(a+bx^2)}{128a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{33}{64a^3x^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 231/(128*a^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x^3*(a + b*x^2)^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 11/(48*a^2*x^3*(a + b*x^2)^2*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) + 33/(64*a^3*x^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (385*(a + b*x^2))/(128*a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (1155*b*(a + b*x^2))/(128*a^6*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (115
5*b^(3/2)*(a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(13/2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{1}{x^4(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11b^3(ab+b^2x^2)\right)\int \frac{1}{x^4(ab+b^2x^2)^4}\,d}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+}}$$

$$= \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+}}$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= \frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} +$$

Mathematica [A]

time = 0.03, size = 127, normalized size = 0.44

$$\frac{\sqrt{a}\,(-128a^5+1408a^4bx^2+9207a^3b^2x^4+16863a^2b^3x^6+12705ab^4x^8+3465b^5x^{10})+3465b^{3/2}x^3(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{384a^{13/2}x^3\,(a+bx^2)^3\,\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]

[Out] (Sqrt[a]*(-128*a^5 + 1408*a^4*b*x^2 + 9207*a^3*b^2*x^4 + 16863*a^2*b^3*x^6 + 12705*a*b^4*x^8 + 3465*b^5*x^10) + 3465*b^(3/2)*x^3*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(13/2)*x^3*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])

Maple [A]
time = 0.04, size = 211, normalized size = 0.73

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(\frac{1155b^5x^{10}}{128a^6}+\frac{4235b^4x^8}{128a^5}+\frac{5621b^3x^6}{128a^4}+\frac{3069b^2x^4}{128a^3}+\frac{11b\,x^2}{3a^2}-\frac{1}{3a}\right)}{(b\,x^2+a)^5x^3}+\frac{1155\sqrt{(b\,x^2+a)^2}\,\left(\sum_{\_R=\mathrm{RootOf}(a^{13}\_Z^2+b^3)}\right.}{256(b\,x^2+}$
default	$-\frac{\left(-3465\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^6x^{11}-3465\sqrt{ab}\,b^5x^{10}-13860\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^5x^9-12705\sqrt{ab}\,a\,b^4x^8-20790\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/384*(-3465*arctan(b*x/(a*b)^(1/2))*b^6*x^11-3465*(a*b)^(1/2)*b^5*x^10-13860*arctan(b*x/(a*b)^(1/2))*a*b^5*x^9-12705*(a*b)^(1/2)*a*b^4*x^8-20790*arctan(b*x/(a*b)^(1/2))*a^2*b^4*x^7-16863*(a*b)^(1/2)*a^2*b^3*x^6-13860*arctan(b*x/(a*b)^(1/2))*a^3*b^3*x^5-9207*(a*b)^(1/2)*a^3*b^2*x^4-3465*arctan(b*x/(a*b)^(1/2))*a^4*b^2*x^3-1408*(a*b)^(1/2)*a^4*b*x^2+128*(a*b)^(1/2)*a^5)*(b*x^2+a)/x^3/(a*b)^(1/2)/a^6/((b*x^2+a)^2)^(5/2)

Maxima [A]
time = 0.51, size = 130, normalized size = 0.45

$$\frac{3465\,b^5x^{10}+12705\,ab^4x^8+16863\,a^2b^3x^6+9207\,a^3b^2x^4+1408\,a^4bx^2-128\,a^5}{384\,(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3)}+\frac{1155\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/384*(3465*b^5*x^10 + 12705*a*b^4*x^8 + 16863*a^2*b^3*x^6 + 9207*a^3*b^2*x^4 + 1408*a^4*b*x^2 - 128*a^5)/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^

```
7 + 4*a^9*b*x^5 + a^10*x^3) + 1155/128*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^6)
```

Fricas [A]
time = 0.34, size = 370, normalized size = 1.27

$$\left[\frac{6930\,b^{5}x^{10}+25410\,ab^{4}x^{8}+33726\,a^{2}b^{3}x^{6}+18414\,a^{3}b^{2}x^{4}+2816\,a^{4}bx^{2}-256\,a^{5}+3465\,\left(b^{5}x^{11}+4\,ab^{4}x^{9}+6\,a^{2}b^{3}x^{7}+4\,a^{3}b^{2}x^{5}+a^{4}bx^{3}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{768\,\left(a^{6}b^{4}x^{11}+4\,a^{7}b^{3}x^{9}+6\,a^{8}b^{2}x^{7}+4\,a^{9}bx^{5}+a^{10}x^{3}\right)},\ \frac{3465\,b^{5}x^{10}+12705\,ab^{4}x^{8}+16863\,a^{2}b^{3}x^{6}+9207\,a^{3}b^{2}x^{4}+1408\,a^{4}bx^{2}-128\,a^{5}+3465\,\left(b^{5}x^{11}+4\,ab^{4}x^{9}+6\,a^{2}b^{3}x^{7}+4\,a^{3}b^{2}x^{5}+a^{4}bx^{3}\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{384\,\left(a^{6}b^{4}x^{11}+4\,a^{7}b^{3}x^{9}+6\,a^{8}b^{2}x^{7}+4\,a^{9}bx^{5}+a^{10}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(6930*b^5*x^10 + 25410*a*b^4*x^8 + 33726*a^2*b^3*x^6 + 18414*a^3*b^2
*x^4 + 2816*a^4*b*x^2 - 256*a^5 + 3465*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*
x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x
^5 + a^10*x^3), 1/384*(3465*b^5*x^10 + 12705*a*b^4*x^8 + 16863*a^2*b^3*x^6
+ 9207*a^3*b^2*x^4 + 1408*a^4*b*x^2 - 128*a^5 + 3465*(b^5*x^11 + 4*a*b^4*x^
9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)
))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**2)**2)**(5/2)), x)
```

Giac [A]
time = 4.05, size = 123, normalized size = 0.42

$$\frac{1155\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}\,a^{6}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{15\,bx^{2}-a}{3\,a^{6}x^{3}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{1545\,b^{5}x^{7}+5153\,ab^{4}x^{5}+5855\,a^{2}b^{3}x^{3}+2295\,a^{3}b^{2}x}{384\,\left(bx^{2}+a\right)^{4}a^{6}\mathrm{sgn}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1155/128*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6*sgn(b*x^2 + a)) + 1/3*(15
*b*x^2 - a)/(a^6*x^3*sgn(b*x^2 + a)) + 1/384*(1545*b^5*x^7 + 5153*a*b^4*x^5
 + 5855*a^2*b^3*x^3 + 2295*a^3*b^2*x)/((b*x^2 + a)^4*a^6*sgn(b*x^2 + a))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

3.659 $\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=298

$$\frac{3x(a+bx^2)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}}+\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}{5b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

```
[Out] 3/5*x*(b*x^2+a)/b/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)+3/5*3^(3/4)*a^2*(1+b*x^2/a)
^(2/3)*(1-(1+b*x^2/a)^(1/3))*EllipticF((1-(1+b*x^2/a)^(1/3)+3^(1/2))/(1-(1+
b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(1+b*x
^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)/b^2/x
/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)/((-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)
-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1127, 327, 242, 225}

$$\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{5b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}+\frac{3x(a+bx^2)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2))/(5*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)) + (3*3^(3/4)*Sqrt
[2 - Sqrt[3]]*a^2*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1
 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^
2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 -
 Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*(a^2 + 2*a*b*
```

```
x^2 + b^2*x^4)^(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 +
 (b*x^2)/a)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{x^2}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3x(a+bx^2)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(3a\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3x(a+bx^2)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(9a^2\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,d\right.}{10b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3x(a+bx^2)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}} + \frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^2\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right.}{5b^2x\sqrt[3]{a^2+2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.79, size = 64, normalized size = 0.21

$$\frac{3x\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{5b\sqrt[3]{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2,
-((b*x^2)/a)]))/(5*b*((a + b*x^2)^2)^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

```
[Out] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{(a+bx^2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3),x)
[Out] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3), x)
```

3.660 $\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=256

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}\;F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\sqrt[3]{1}}{1-\sqrt{3}-\sqrt[3]{1}}\right.\right.}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

```
[Out] -3^(3/4)*a*(1+b*x^2/a)^(2/3)*(1-(1+b*x^2/a)^(1/3))*EllipticF((1-(1+b*x^2/a)
^(1/3)+3^(1/2))/(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1
/2*2^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3
^(1/2))^2)^(1/2)/b/x/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)/((-1+(1+b*x^2/a)^(1/3))/
(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1103, 242, 225}

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\;F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/3),x]
```

```
[Out] -((3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(
1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3]
 - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a
)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a^2
 + 2*a*b*x^2 + b^2*x^4)^(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[
3] - (1 + (b*x^2)/a)^(1/3))^2)]))
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 1103

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[a^IntPart[
p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2*FracPart[p])), Int
[(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(3a\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1+\frac{bx^2}{a}}\right)}{2bx\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+(1+\cdots}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{b\cdots}{\cdots}}\cdots\right.}}\cdots}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\cdots}{\left(1-\sqrt{3}\cdots\right.}}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.65, size = 48, normalized size = 0.19

$$\frac{x\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/3),x]
```

```
[Out] (x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/((
a + b*x^2)^2)^(1/3)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^2x^4+2ab\,x^2+a^2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

```
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a^2 + 2abx^2 + b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3),x)
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3), x)
```

3.661 $\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=289

$$-\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{2-\sqrt{3}}\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}\,F}{\sqrt[4]{3}\,x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{1+}}{\left(1-\sqrt{3}-\sqrt[3]{1}\right.}}}$$

```
[Out] (-b*x^2-a)/a/x/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)+1/3*(1+b*x^2/a)^(2/3)*(1-(1+b*
x^2/a)^(1/3))*EllipticF((1-(1+b*x^2/a)^(1/3)+3^(1/2))/(1-(1+b*x^2/a)^(1/3)-
3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+
b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)*3^(3/4)/x/(b^2*x^4+2
*a*b*x^2+a^2)^(1/3)/((-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2
)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1127, 331, 242, 225}

$$\frac{\sqrt{2-\sqrt{3}}\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)),x]
```

```
[Out] -((a + b*x^2)/(a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3))) + (Sqrt[2 - Sqrt[3]]
*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a
)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*E
llipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (
b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^
```

```
(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/
3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\int \frac{1}{x^2\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\,dx}{3a\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}} - \frac{\left(\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,d\right.}{2x\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{2-\sqrt{3}}\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{}}{\sqrt[4]{3}\,x\sqrt[3]{a^2+2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.27, size = 51, normalized size = 0.18

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{2/3}{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\sqrt[3]{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)),x]
```

```
[Out] -((((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/2, 2/3, 1/2, -((b*x^2)/a)])/(
x*((a + b*x^2)^2)^(1/3)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
[Out] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt[3]{(a+bx^2)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3)),x)
[Out] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3)), x)
```

3.662 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx$

Optimal. Leaf size=618

$$-\frac{3x(a+bx^2)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{9ax\left(1+\frac{bx^2}{a}\right)^{4/3}}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^2\left(1+\right.}{}$$

[Out] -3/2*x*(b*x^2+a)/b/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)-9/2*a*x*(1+b*x^2/a)^(4/3)/b/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/(1-(1+b*x^2/a)^(1/3)-3^(1/2))-3/2*3^(3/4)*a^2*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*EllipticF((1-(1+b*x^2/a)^(1/3)+3^(1/2))/(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)/b^2/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)*2^(1/2)/((-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)+9/4*3^(1/4)*a^2*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*EllipticE((1-(1+b*x^2/a)^(1/3)+3^(1/2))/(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^2/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/((-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.26, antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1127, 294, 241, 310, 225, 1893}

$$-\frac{3\,3^{3/4}a^2\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\,b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^2\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\frac{9ax\left(\frac{bx^2}{a}+1\right)^{4/3}}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}-\frac{3x(a+bx^2)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3),x]

[Out] (-3*x*(a + b*x^2))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (9*a*x*(1 + (b*x^2)/a)^(4/3))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^2*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)

```
/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + S
qrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7
+ 4*Sqrt[3]])/(4*b^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (
b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (3*3^(3/4)*a^
2*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/
a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 +
(b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*(a^2 + 2*a*b*x^2 + b^2*
x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a
)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
```

```
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}\int \frac{x^2}{\left(1+\frac{bx^2}{a}\right)^{4/3}}\,dx}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= -\frac{3x(a+bx^2)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(3a\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\int \frac{1}{\sqrt[3]{1+\frac{bx^2}{a}}}\,dx}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$= -\frac{3x(a+bx^2)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} + \frac{\left(9a^2\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+\cdots}}\right.}{4b^2x\left(a^2+2abx^2+b^2x^4\right)\cdots}$$

$$= -\frac{3x(a+bx^2)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{\left(9a^2\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}\cdots}{\sqrt{-1+\cdots}}\right.}{4b^2x\left(a^2+2abx^2+b^2x^4\right)\cdots}$$

$$= -\frac{3x(a+bx^2)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{9ax\left(1+\frac{bx^2}{a}\right)^{4/3}\cdots}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\cdots}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.75, size = 64, normalized size = 0.10

$$\frac{3x(a+bx^2)\left(-1+\sqrt[3]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{2b\left((a+bx^2)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)*(-1 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/
2, -((b*x^2)/a)]))/(2*b*((a + b*x^2)^2)^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

```
[Out] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\left(a+bx^2\right)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
[Out] Integral(x**2/((a + b*x**2)**2)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3),x)
[Out] int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3), x)
```

3.663 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx$

Optimal. Leaf size=609

$$\frac{3x(a+bx^2)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}}+\frac{3x\left(1+\frac{bx^2}{a}\right)^{4/3}}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\left(1+\frac{bx^2}{a}\right)}{}$$

```
[Out] 3/2*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)+3/2*x*(1+b*x^2/a)^(4/3)/(b^
2*x^4+2*a*b*x^2+a^2)^(2/3)/(1-(1+b*x^2/a)^(1/3)-3^(1/2))+1/2*3^(3/4)*a*(1+b
*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*EllipticF((1-(1+b*x^2/a)^(1/3)+3^(1/2))
/(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^
2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)/b/x/(b^2*x^4+2*a*b*x^2+a
^2)^(2/3)*2^(1/2)/((-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^
(1/2)-3/4*3^(1/4)*a*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*EllipticE((1-(1
+b*x^2/a)^(1/3)+3^(1/2))/(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(
1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)*
(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/((-1+(1+b*x^2/a
)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1103, 205, 241, 310, 225, 1893}

$$\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\left(\frac{bx^2}{a}+1\right)^{4/3}F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\,bx\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\left(\frac{bx^2}{a}+1\right)^{4/3}E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4bx\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}+\frac{3x\left(\frac{bx^2}{a}+1\right)^{4/3}}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}+\frac{3x(a+bx^2)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2/3),x]
```

```
[Out] (3*x*(a + b*x^2))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) + (3*x*(1 + (b*x^
2)/a)^(4/3))/(2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 + (b*x^
2)/a)^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*a*(1 + (b*x^2)/a)^(4/3)*(1 - (
1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/
```

```
3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3]
- (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqr
t[3]])/(4*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)
^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) + (3^(3/4)*a*(1 + (b*x^2
)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1
 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[Arc
Sin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1
/3))], -7 + 4*Sqrt[3]])/(Sqrt[2]*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt
[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1103

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[a^IntPart[
p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2*FracPart[p])), Int
[(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{4/3}}\,dx}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$= \frac{3x(a+bx^2)}{2a\,(a^2+2abx^2+b^2x^4)^{2/3}} - \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}\int \frac{1}{\sqrt[3]{1+\frac{bx^2}{a}}}\,dx}{2\,(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$= \frac{3x(a+bx^2)}{2a\,(a^2+2abx^2+b^2x^4)^{2/3}} - \frac{\left(3a\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\right.}{4bx\,(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$= \frac{3x(a+bx^2)}{2a\,(a^2+2abx^2+b^2x^4)^{2/3}} + \frac{\left(3a\sqrt{\frac{bx^2}{a}}\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\right.}{4bx\,(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$= \frac{3x(a+bx^2)}{2a\,(a^2+2abx^2+b^2x^4)^{2/3}} + \frac{3x\left(1+\frac{bx^2}{a}\right)^{4/3}}{2\,(a^2+2abx^2+b^2x^4)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 5.35, size = 64, normalized size = 0.11

$$-\frac{x(a+bx^2)\left(-3+\sqrt[3]{1+\frac{bx^2}{a}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)\right)}{2a\left((a+bx^2)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2/3),x]
[Out] -1/2*(x*(a + b*x^2)*(-3 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2,
 3/2, -((b*x^2)/a)]))/(a*((a + b*x^2)^2)^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3),x)
```

```
[Out] int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3), x)
```

3.664 $\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx$

Optimal. Leaf size=649

$$\frac{3(a+bx^2)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{5(a+bx^2)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{5bx\left(1+\frac{bx^2}{a}\right)^{4/3}}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right.}$$

```
[Out] 3/2*(b*x^2+a)/a/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)-5/2*(b*x^2+a)^2/a^2/x/(b^2*
x^4+2*a*b*x^2+a^2)^(2/3)-5/2*b*x*(1+b*x^2/a)^(4/3)/a/(b^2*x^4+2*a*b*x^2+a^2
)^(2/3)/(1-(1+b*x^2/a)^(1/3)-3^(1/2))-5/6*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^
(1/3))*EllipticF((1-(1+b*x^2/a)^(1/3)+3^(1/2))/(1-(1+b*x^2/a)^(1/3)-3^(1/2)
),2*I-I*3^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-(1+b*x^2/a)^(1
/3)-3^(1/2))^2)^(1/2)*3^(3/4)/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)*2^(1/2)/((-1+
(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)+5/4*3^(1/4)*(1+b*
x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*EllipticE((1-(1+b*x^2/a)^(1/3)+3^(1/2))/
(1-(1+b*x^2/a)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2
/a)^(2/3))/(1-(1+b*x^2/a)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))
/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/((-1+(1+b*x^2/a)^(1/3))/(1-(1+b*x^2/a)^(1/
3)-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1127, 296, 331, 241, 310, 225, 1893}

$$-\frac{5\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\sqrt[4]{3}\,x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}+\frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,E\left(\text{ArcSin}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\frac{5(a+bx^2)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}}+\frac{3(a+bx^2)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{5bx\left(\frac{bx^2}{a}+1\right)^{4/3}}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)),x]
```

```
[Out] (3*(a + b*x^2))/(2*a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*(a + b*x^2)^
2)/(2*a^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*b*x*(1 + (b*x^2)/a)^(4/
3))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 - Sqrt[3] - (1 + (b*x^2)/a)^(
```

```
1/3))) + (5*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^
2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 -
Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (b
*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(4
*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1
- Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (5*(1 + (b*x^2)/a)^(4/3)*(1 - (1
+ (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3)
)/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
(1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[
3]])/(Sqrt[2]*3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 +
(b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^{2/3}}\,dx = \frac{\left(1+\frac{bx^2}{a}\right)^{4/3}\int \frac{1}{x^2\left(1+\frac{bx^2}{a}\right)^{4/3}}\,dx}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$= \frac{3(a+bx^2)}{2ax(a^2+2abx^2+b^2x^4)^{2/3}} + \frac{\left(5\left(1+\frac{bx^2}{a}\right)^{4/3}\right)\int \frac{1}{x^2\sqrt[3]{1+\frac{bx^2}{a}}}\,dx}{2(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$= \frac{3(a+bx^2)}{2ax(a^2+2abx^2+b^2x^4)^{2/3}} - \frac{5(a+bx^2)^2}{2a^2x(a^2+2abx^2+b^2x^4)^{2/3}} + \frac{\left(5b\left(1+\frac{bx^2}{a}\right.\right.}{6a(a^2+}$$

$$= \frac{3(a+bx^2)}{2ax(a^2+2abx^2+b^2x^4)^{2/3}} - \frac{5(a+bx^2)^2}{2a^2x(a^2+2abx^2+b^2x^4)^{2/3}} + \frac{\left(5\sqrt{\frac{bx^2}{a}}\,(1\right.}{}$$

$$= \frac{3(a+bx^2)}{2ax(a^2+2abx^2+b^2x^4)^{2/3}} - \frac{5(a+bx^2)^2}{2a^2x(a^2+2abx^2+b^2x^4)^{2/3}} - \frac{\left(5\sqrt{\frac{bx^2}{a}}\,(1\right.}{}$$

$$= \frac{3(a+bx^2)}{2ax(a^2+2abx^2+b^2x^4)^{2/3}} - \frac{5(a+bx^2)^2}{2a^2x(a^2+2abx^2+b^2x^4)^{2/3}} - \frac{}{2a(a^2+2abx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.05, size = 61, normalized size = 0.09

$$-\frac{(a+bx^2)\sqrt[3]{1+\frac{bx^2}{a}}\;{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{1}{2};-\frac{bx^2}{a}\right)}{ax\left((a+bx^2)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)),x]
```

```
[Out] -(((a + b*x^2)*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 4/3, 1/2, -((b
*x^2)/a)])/(a*x*((a + b*x^2)^2)^(2/3)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

```
[Out] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(\left(a + b x^{2}\right)^{2}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(2/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3)),x)
[Out] int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3)), x)
```

3.665 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}$$

```
[Out] 2/7*a^2*(d*x)^(7/2)/d+4/11*a*b*(d*x)^(11/2)/d^3+2/15*b^2*(d*x)^(15/2)/d^5
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(7/2))/(7*d) + (4*a*b*(d*x)^(11/2))/(11*d^3) + (2*b^2*(d*x)^(1
5/2))/(15*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)\,dx &= \int \left(a^2(dx)^{5/2}+\frac{2ab(dx)^{9/2}}{d^2}+\frac{b^2(dx)^{13/2}}{d^4}\right)dx\\ &= \frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.65

$$\frac{2x(dx)^{5/2}\left(165a^2+210abx^2+77b^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (2*x*(d*x)^(5/2)*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155
```

Maple [A]
time = 0.03, size = 42, normalized size = 0.82

method	result	size
gosper	$\frac{2x\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)\left(dx\right)^{\frac{5}{2}}}{1155}$	30
trager	$\frac{2d^{2}x^{3}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)\sqrt{dx}}{1155}$	35
risch	$\frac{2d^{3}x^{4}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)}{1155\sqrt{dx}}$	35
derivativedivides	$\frac{\frac{2b^{2}\left(dx\right)^{\frac{15}{2}}}{15}+\frac{4ab\,d^{2}\left(dx\right)^{\frac{11}{2}}}{11}+\frac{2a^{2}d^{4}\left(dx\right)^{\frac{7}{2}}}{7}}{d^{5}}$	42
default	$\frac{\frac{2b^{2}\left(dx\right)^{\frac{15}{2}}}{15}+\frac{4ab\,d^{2}\left(dx\right)^{\frac{11}{2}}}{11}+\frac{2a^{2}d^{4}\left(dx\right)^{\frac{7}{2}}}{7}}{d^{5}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
[Out] 2/d^5*(1/15*b^2*(d*x)^(15/2)+2/11*a*b*d^2*(d*x)^(11/2)+1/7*a^2*d^4*(d*x)^(7
/2))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.80

$$\frac{2\left(77\left(dx\right)^{\frac{15}{2}}b^{2}+210\left(dx\right)^{\frac{11}{2}}abd^{2}+165\left(dx\right)^{\frac{7}{2}}a^{2}d^{4}\right)}{1155\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] 2/1155*(77*(d*x)^(15/2)*b^2 + 210*(d*x)^(11/2)*a*b*d^2 + 165*(d*x)^(7/2)*a^
2*d^4)/d^5
```

Fricas [A]
time = 0.36, size = 40, normalized size = 0.78

$$\frac{2}{1155}\left(77\,b^{2}d^{2}x^{7}+210\,abd^{2}x^{5}+165\,a^{2}d^{2}x^{3}\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] 2/1155*(77*b^2*d^2*x^7 + 210*a*b*d^2*x^5 + 165*a^2*d^2*x^3)*sqrt(d*x)
```

Sympy [A]
time = 0.42, size = 48, normalized size = 0.94

$$\frac{2a^2x(dx)^{\frac{5}{2}}}{7}+\frac{4abx^3(dx)^{\frac{5}{2}}}{11}+\frac{2b^2x^5(dx)^{\frac{5}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*x*(d*x)**(5/2)/7 + 4*a*b*x**3*(d*x)**(5/2)/11 + 2*b**2*x**5*(d*x)**(
5/2)/15
```

Giac [A]
time = 3.24, size = 48, normalized size = 0.94

$$\frac{2}{15}\sqrt{dx}\,b^2d^2x^7+\frac{4}{11}\sqrt{dx}\,abd^2x^5+\frac{2}{7}\sqrt{dx}\,a^2d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 2/15*sqrt(d*x)*b^2*d^2*x^7 + 4/11*sqrt(d*x)*a*b*d^2*x^5 + 2/7*sqrt(d*x)*a^2
*d^2*x^3
```

Mupad [B]
time = 0.07, size = 40, normalized size = 0.78

$$\frac{\frac{2\,b^2\,{\left(d\,x\right)}^{15/2}}{15}+\frac{2\,a^2\,d^4\,{\left(d\,x\right)}^{7/2}}{7}+\frac{4\,a\,b\,d^2\,{\left(d\,x\right)}^{11/2}}{11}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] ((2*b^2*(d*x)^(15/2))/15 + (2*a^2*d^4*(d*x)^(7/2))/7 + (4*a*b*d^2*(d*x)^(11
/2))/11)/d^5
```

3.666 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}$$

```
[Out] 2/5*a^2*(d*x)^(5/2)/d+4/9*a*b*(d*x)^(9/2)/d^3+2/13*b^2*(d*x)^(13/2)/d^5
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{14\}$

$$\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(5/2))/(5*d) + (4*a*b*(d*x)^(9/2))/(9*d^3) + (2*b^2*(d*x)^(13/
2))/(13*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)\,dx&=\int\left(a^2(dx)^{3/2}+\frac{2ab(dx)^{7/2}}{d^2}+\frac{b^2(dx)^{11/2}}{d^4}\right)dx\\&=\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.65

$$\frac{2}{585}x(dx)^{3/2}\left(117a^2+130abx^2+45b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*(d*x)^(3/2)*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585
```

Maple [A]
time = 0.02, size = 42, normalized size = 0.82

method	result	size
gosper	$\frac{2x\left(45b^{2}x^{4}+130ab\,x^{2}+117a^{2}\right)(dx)^{\frac{3}{2}}}{585}$	30
trager	$\frac{2d\,x^{2}\left(45b^{2}x^{4}+130ab\,x^{2}+117a^{2}\right)\sqrt{dx}}{585}$	33
risch	$\frac{2d^{2}x^{3}\left(45b^{2}x^{4}+130ab\,x^{2}+117a^{2}\right)}{585\sqrt{dx}}$	35
derivativedivides	$\frac{\frac{2b^{2}(dx)^{\frac{13}{2}}}{13}+\frac{4ab\,d^{2}(dx)^{\frac{9}{2}}}{9}+\frac{2a^{2}d^{4}(dx)^{\frac{5}{2}}}{5}}{d^{5}}$	42
default	$\frac{\frac{2b^{2}(dx)^{\frac{13}{2}}}{13}+\frac{4ab\,d^{2}(dx)^{\frac{9}{2}}}{9}+\frac{2a^{2}d^{4}(dx)^{\frac{5}{2}}}{5}}{d^{5}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/13*b^2*(d*x)^(13/2)+2/9*a*b*d^2*(d*x)^(9/2)+1/5*a^2*d^4*(d*x)^(5/2
))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.80

$$\frac{2\left(45\,(dx)^{\frac{13}{2}}\,b^{2}+130\,(dx)^{\frac{9}{2}}\,abd^{2}+117\,(dx)^{\frac{5}{2}}\,a^{2}d^{4}\right)}{585\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 2/585*(45*(d*x)^(13/2)*b^2 + 130*(d*x)^(9/2)*a*b*d^2 + 117*(d*x)^(5/2)*a^2*
d^4)/d^5
```

Fricas [A]
time = 0.34, size = 34, normalized size = 0.67

$$\frac{2}{585}\left(45\,b^{2}dx^{6}+130\,abdx^{4}+117\,a^{2}dx^{2}\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d*x^6 + 130*a*b*d*x^4 + 117*a^2*d*x^2)*sqrt(d*x)
```

Sympy [A]
time = 0.23, size = 48, normalized size = 0.94

$$\frac{2a^2x(dx)^{\frac{3}{2}}}{5} + \frac{4abx^3(dx)^{\frac{3}{2}}}{9} + \frac{2b^2x^5(dx)^{\frac{3}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*x*(d*x)**(3/2)/5 + 4*a*b*x**3*(d*x)**(3/2)/9 + 2*b**2*x**5*(d*x)**(3
/2)/13
```

Giac [A]
time = 3.69, size = 42, normalized size = 0.82

$$\frac{2}{585}\left(45\sqrt{dx}\,b^2x^6 + 130\sqrt{dx}\,abx^4 + 117\sqrt{dx}\,a^2x^2\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 2/585*(45*sqrt(d*x)*b^2*x^6 + 130*sqrt(d*x)*a*b*x^4 + 117*sqrt(d*x)*a^2*x^2
)*d
```

Mupad [B]
time = 4.22, size = 41, normalized size = 0.80

$$\frac{90\,b^2\,(d\,x)^{13/2} + 234\,a^2\,d^4\,(d\,x)^{5/2} + 260\,a\,b\,d^2\,(d\,x)^{9/2}}{585\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (90*b^2*(d*x)^(13/2) + 234*a^2*d^4*(d*x)^(5/2) + 260*a*b*d^2*(d*x)^(9/2))/(
585*d^5)
```

3.667 $\int \sqrt{dx}\,(a^2+2abx^2+b^2x^4)\,dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

```
[Out] 2/3*a^2*(d*x)^(3/2)/d+4/7*a*b*(d*x)^(7/2)/d^3+2/11*b^2*(d*x)^(11/2)/d^5
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.038$, Rules used = $\{14\}$

$$\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(3/2))/(3*d) + (4*a*b*(d*x)^(7/2))/(7*d^3) + (2*b^2*(d*x)^(11/
2))/(11*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\,(a^2+2abx^2+b^2x^4)\,dx &= \int \left(a^2\sqrt{dx}+\frac{2ab(dx)^{5/2}}{d^2}+\frac{b^2(dx)^{9/2}}{d^4}\right)dx\\ &= \frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.65

$$\frac{2}{231}x\sqrt{dx}\,(77a^2+66abx^2+21b^2x^4)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*Sqrt[d*x]*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231
```

Maple [A]
time = 0.02, size = 42, normalized size = 0.82

method	result	size
gosper	$\frac{2x(21b^2x^4+66ab\,x^2+77a^2)\sqrt{dx}}{231}$	30
trager	$\frac{2x(21b^2x^4+66ab\,x^2+77a^2)\sqrt{dx}}{231}$	30
risch	$\frac{2d\,x^2(21b^2x^4+66ab\,x^2+77a^2)}{231\sqrt{dx}}$	33
derivativedivides	$\frac{\frac{2b^2(dx)^{\frac{11}{2}}}{11}+\frac{4ab\,d^2(dx)^{\frac{7}{2}}}{7}+\frac{2a^2d^4(dx)^{\frac{3}{2}}}{3}}{d^5}$	42
default	$\frac{\frac{2b^2(dx)^{\frac{11}{2}}}{11}+\frac{4ab\,d^2(dx)^{\frac{7}{2}}}{7}+\frac{2a^2d^4(dx)^{\frac{3}{2}}}{3}}{d^5}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/11*b^2*(d*x)^(11/2)+2/7*a*b*d^2*(d*x)^(7/2)+1/3*a^2*d^4*(d*x)^(3/2
))
```

Maxima [A]
time = 0.32, size = 41, normalized size = 0.80

$$\frac{2\left(21\,(dx)^{\frac{11}{2}}\,b^2+66\,(dx)^{\frac{7}{2}}\,abd^2+77\,(dx)^{\frac{3}{2}}\,a^2d^4\right)}{231\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/231*(21*(d*x)^(11/2)*b^2 + 66*(d*x)^(7/2)*a*b*d^2 + 77*(d*x)^(3/2)*a^2*d^
4)/d^5
```

Fricas [A]
time = 0.34, size = 29, normalized size = 0.57

$$\frac{2}{231}\left(21\,b^2x^5+66\,abx^3+77\,a^2x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^2*x^5 + 66*a*b*x^3 + 77*a^2*x)*sqrt(d*x)
```

Sympy [A]
time = 0.12, size = 48, normalized size = 0.94

$$\frac{2a^2x\sqrt{dx}}{3} + \frac{4abx^3\sqrt{dx}}{7} + \frac{2b^2x^5\sqrt{dx}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)*(d*x)**(1/2),x)
[Out] 2*a**2*x*sqrt(d*x)/3 + 4*a*b*x**3*sqrt(d*x)/7 + 2*b**2*x**5*sqrt(d*x)/11
```

Giac [A]
time = 4.31, size = 37, normalized size = 0.73

$$\frac{2}{11}\sqrt{dx}\,b^2x^5 + \frac{4}{7}\sqrt{dx}\,abx^3 + \frac{2}{3}\sqrt{dx}\,a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x, algorithm="giac")
[Out] 2/11*sqrt(d*x)*b^2*x^5 + 4/7*sqrt(d*x)*a*b*x^3 + 2/3*sqrt(d*x)*a^2*x
```

Mupad [B]
time = 0.05, size = 41, normalized size = 0.80

$$\frac{42\,b^2\,(d\,x)^{11/2} + 154\,a^2\,d^4\,(d\,x)^{3/2} + 132\,a\,b\,d^2\,(d\,x)^{7/2}}{231\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] (42*b^2*(d*x)^(11/2) + 154*a^2*d^4*(d*x)^(3/2) + 132*a*b*d^2*(d*x)^(7/2))/(
231*d^5)
```

3.668 $\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx$

Optimal. Leaf size=49

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

```
[Out] 4/5*a*b*(d*x)^(5/2)/d^3+2/9*b^2*(d*x)^(9/2)/d^5+2*a^2*(d*x)^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {14}

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
```

```
[Out] (2*a^2*Sqrt[d*x])/d + (4*a*b*(d*x)^(5/2))/(5*d^3) + (2*b^2*(d*x)^(9/2))/(9*
d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx&=\int \left(\frac{a^2}{\sqrt{dx}}+\frac{2ab(dx)^{3/2}}{d^2}+\frac{b^2(dx)^{7/2}}{d^4}\right)dx\\&=\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.67

$$\frac{2x\left(45a^2+18abx^2+5b^2x^4\right)}{45\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
[Out] (2*x*(45*a^2 + 18*a*b*x^2 + 5*b^2*x^4))/(45*Sqrt[d*x])
```

Maple [A]
time = 0.02, size = 41, normalized size = 0.84

method	result	size
gosper	$\frac{2\left(5 b^{2} x^{4}+18 a b \,x^{2}+45 a^{2}\right) x}{45 \sqrt{d x}}$	30
risch	$\frac{2\left(5 b^{2} x^{4}+18 a b \,x^{2}+45 a^{2}\right) x}{45 \sqrt{d x}}$	30
trager	$\frac{\left(\frac{2}{9} b^{2} x^{4}+\frac{4}{5} a b \,x^{2}+2 a^{2}\right) \sqrt{d x}}{d}$	31
derivativedivides	$\frac{\frac{2 b^{2} \left(d x\right)^{\frac{9}{2}}}{9}+\frac{4 a b \,d^{2} \left(d x\right)^{\frac{5}{2}}}{5}+2 a^{2} d^{4} \sqrt{d x}}{d^{5}}$	41
default	$\frac{\frac{2 b^{2} \left(d x\right)^{\frac{9}{2}}}{9}+\frac{4 a b \,d^{2} \left(d x\right)^{\frac{5}{2}}}{5}+2 a^{2} d^{4} \sqrt{d x}}{d^{5}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d^5*(1/9*b^2*(d*x)^(9/2)+2/5*a*b*d^2*(d*x)^(5/2)+a^2*d^4*(d*x)^(1/2))
```

Maxima [A]
time = 0.34, size = 41, normalized size = 0.84

$$\frac{2\left(45\sqrt{dx}\,a^{2}+\frac{5\,(dx)^{\frac{9}{2}}b^{2}}{d^{4}}+\frac{18\,(dx)^{\frac{5}{2}}ab}{d^{2}}\right)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="maxima")
[Out] 2/45*(45*sqrt(d*x)*a^2 + 5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)/d
```

Fricas [A]
time = 0.35, size = 31, normalized size = 0.63

$$\frac{2\left(5\,b^{2}x^{4}+18\,abx^{2}+45\,a^{2}\right)\sqrt{dx}}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*b^2*x^4 + 18*a*b*x^2 + 45*a^2)*sqrt(d*x)/d
```

Sympy [A]
time = 0.22, size = 46, normalized size = 0.94

$$\frac{2a^2x}{\sqrt{dx}} + \frac{4abx^3}{5\sqrt{dx}} + \frac{2b^2x^5}{9\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
[Out] 2*a**2*x/sqrt(d*x) + 4*a*b*x**3/(5*sqrt(d*x)) + 2*b**2*x**5/(9*sqrt(d*x))
```

Giac [A]
time = 4.52, size = 41, normalized size = 0.84

$$\frac{2\left(5\sqrt{dx}\,b^2x^4 + 18\sqrt{dx}\,abx^2 + 45\sqrt{dx}\,a^2\right)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="giac")
[Out] 2/45*(5*sqrt(d*x)*b^2*x^4 + 18*sqrt(d*x)*a*b*x^2 + 45*sqrt(d*x)*a^2)/d
```

Mupad [B]
time = 0.05, size = 41, normalized size = 0.84

$$\frac{10\,b^2\,(d\,x)^{9/2} + 90\,a^2\,d^4\,\sqrt{d\,x} + 36\,a\,b\,d^2\,(d\,x)^{5/2}}{45\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(1/2),x)
[Out] (10*b^2*(d*x)^(9/2) + 90*a^2*d^4*(d*x)^(1/2) + 36*a*b*d^2*(d*x)^(5/2))/(45*
d^5)
```

3.669 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

```
[Out] 4/3*a*b*(d*x)^(3/2)/d^3+2/7*b^2*(d*x)^(7/2)/d^5-2*a^2/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {14}

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^2)/(d*Sqrt[d*x]) + (4*a*b*(d*x)^(3/2))/(3*d^3) + (2*b^2*(d*x)^(7/2))/
(7*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx &= \int \left(\frac{a^2}{(dx)^{3/2}}+\frac{2ab\sqrt{dx}}{d^2}+\frac{b^2(dx)^{5/2}}{d^4}\right)dx\\ &= -\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.67

$$-\frac{2x(21a^2-14abx^2-3b^2x^4)}{21(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
[Out] (-2*x*(21*a^2 - 14*a*b*x^2 - 3*b^2*x^4))/(21*(d*x)^(3/2))
```

Maple [A]
time = 0.03, size = 42, normalized size = 0.86

method	result	size
gosper	$-\frac{2\left(-3 b^{2} x^{4}-14 a b \,x^{2}+21 a^{2}\right) x}{21 \left(d x \right)^{\frac{3}{2}}}$	30
risch	$-\frac{2\left(-3 b^{2} x^{4}-14 a b \,x^{2}+21 a^{2}\right)}{21 d \sqrt{d x}}$	32
trager	$-\frac{2\left(-3 b^{2} x^{4}-14 a b \,x^{2}+21 a^{2}\right) \sqrt{d x}}{21 d^{2} x}$	35
derivativedivides	$\frac{\frac{2 b^{2} \left(d x \right)^{\frac{7}{2}}}{7}+\frac{4 a b \,d^{2} \left(d x \right)^{\frac{3}{2}}}{3}-\frac{2 a^{2} d^{4}}{\sqrt{d x}}}{d^{5}}$	42
default	$\frac{\frac{2 b^{2} \left(d x \right)^{\frac{7}{2}}}{7}+\frac{4 a b \,d^{2} \left(d x \right)^{\frac{3}{2}}}{3}-\frac{2 a^{2} d^{4}}{\sqrt{d x}}}{d^{5}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/d^5*(1/7*b^2*(d*x)^(7/2)+2/3*a*b*d^2*(d*x)^(3/2)-a^2*d^4/(d*x)^(1/2))
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.90

$$-\frac{2 \left(\frac{21 \, a^{2}}{\sqrt{d x}} - \frac{3 \, \left(d x\right)^{\frac{7}{2}} b^{2} + 14 \, \left(d x\right)^{\frac{3}{2}} a b d^{2}}{d^{4}}\right)}{21 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="maxima")
[Out] -2/21*(21*a^2/sqrt(d*x) - (3*(d*x)^(7/2)*b^2 + 14*(d*x)^(3/2)*a*b*d^2)/d^4)
/d
```

Fricas [A]
time = 0.34, size = 34, normalized size = 0.69

$$\frac{2 \, {\left(3 \, b^{2} x^{4} + 14 \, a b x^{2} - 21 \, a^{2}\right)} \sqrt{d x}}{21 \, d^{2} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^2*x^4 + 14*a*b*x^2 - 21*a^2)*sqrt(d*x)/(d^2*x)
```

Sympy [A]
time = 0.23, size = 46, normalized size = 0.94

$$-\frac{2a^2x}{(dx)^{\frac{3}{2}}}+\frac{4abx^3}{3\,(dx)^{\frac{3}{2}}}+\frac{2b^2x^5}{7\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(3/2),x)
```

```
[Out] -2*a**2*x/(d*x)**(3/2) + 4*a*b*x**3/(3*(d*x)**(3/2)) + 2*b**2*x**5/(7*(d*x)
**(3/2))
```

Giac [A]
time = 3.34, size = 51, normalized size = 1.04

$$-\frac{2\left(\frac{21\,a^2}{\sqrt{dx}}-\frac{3\,\sqrt{dx}\,b^2d^{27}x^3+14\,\sqrt{dx}\,abd^{27}x}{d^{28}}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/21*(21*a^2/sqrt(d*x) - (3*sqrt(d*x)*b^2*d^27*x^3 + 14*sqrt(d*x)*a*b*d^27
*x)/d^28)/d
```

Mupad [B]
time = 0.05, size = 31, normalized size = 0.63

$$\frac{-42\,a^2+28\,a\,b\,x^2+6\,b^2\,x^4}{21\,d\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(3/2),x)
```

```
[Out] (6*b^2*x^4 - 42*a^2 + 28*a*b*x^2)/(21*d*(d*x)^(1/2))
```

3.670 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}} \, dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

```
[Out] -2/3*a^2/d/(d*x)^(3/2)+2/5*b^2*(d*x)^(5/2)/d^5+4*a*b*(d*x)^(1/2)/d^3
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{14\}$

$$-\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
```

```
[Out] (-2*a^2)/(3*d*(d*x)^(3/2)) + (4*a*b*Sqrt[d*x])/d^3 + (2*b^2*(d*x)^(5/2))/(5
*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}} \, dx &= \int \left(\frac{a^2}{(dx)^{5/2}}+\frac{2ab}{d^2\sqrt{dx}}+\frac{b^2(dx)^{3/2}}{d^4}\right) dx\\ &= -\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.67

$$-\frac{2x(5a^2-30abx^2-3b^2x^4)}{15(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
[Out] (-2*x*(5*a^2 - 30*a*b*x^2 - 3*b^2*x^4))/(15*(d*x)^(5/2))
```

Maple [A]
time = 0.03, size = 42, normalized size = 0.86

method	result	size
gosper	$-\frac{2\left(-3b^{2}x^{4}-30ab\,x^{2}+5a^{2}\right)x}{15\left(dx\right)^{\frac{5}{2}}}$	30
trager	$-\frac{2\left(-3b^{2}x^{4}-30ab\,x^{2}+5a^{2}\right)\sqrt{dx}}{15d^{3}x^{2}}$	35
risch	$-\frac{2\left(-3b^{2}x^{4}-30ab\,x^{2}+5a^{2}\right)}{15d^{2}x\sqrt{dx}}$	35
derivativedivides	$\frac{\frac{2b^{2}\left(dx\right)^{\frac{5}{2}}}{5}+4ab\,d^{2}\sqrt{dx}-\frac{2a^{2}d^{4}}{3\left(dx\right)^{\frac{3}{2}}}}{d^{5}}$	42
default	$\frac{\frac{2b^{2}\left(dx\right)^{\frac{5}{2}}}{5}+4ab\,d^{2}\sqrt{dx}-\frac{2a^{2}d^{4}}{3\left(dx\right)^{\frac{3}{2}}}}{d^{5}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/d^5*(1/5*b^2*(d*x)^(5/2)+2*a*b*d^2*(d*x)^(1/2)-1/3*a^2*d^4/(d*x)^(3/2))
```

Maxima [A]
time = 0.29, size = 43, normalized size = 0.88

$$-\frac{2\left(\frac{5a^{2}}{\left(dx\right)^{\frac{3}{2}}}-\frac{3\left(\left(dx\right)^{\frac{5}{2}}b^{2}+10\sqrt{dx}\,abd^{2}\right)}{d^{4}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="maxima")
[Out] -2/15*(5*a^2/(d*x)^(3/2) - 3*((d*x)^(5/2)*b^2 + 10*sqrt(d*x)*a*b*d^2)/d^4)/
d
```

Fricas [A]
time = 0.32, size = 34, normalized size = 0.69

$$\frac{2\left(3\,b^{2}x^{4}+30\,abx^{2}-5\,a^{2}\right)\sqrt{dx}}{15\,d^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^4 + 30*a*b*x^2 - 5*a^2)*sqrt(d*x)/(d^3*x^2)
```

Sympy [A]
time = 0.29, size = 46, normalized size = 0.94

$$-\frac{2a^2x}{3\left(dx\right)^{\frac{5}{2}}}+\frac{4abx^3}{\left(dx\right)^{\frac{5}{2}}}+\frac{2b^2x^5}{5\left(dx\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(5/2),x)
[Out] -2*a**2*x/(3*(d*x)**(5/2)) + 4*a*b*x**3/(d*x)**(5/2) + 2*b**2*x**5/(5*(d*x)
**(5/2))
```

Giac [A]
time = 3.61, size = 53, normalized size = 1.08

$$-\frac{2\left(\frac{5\,a^2d}{\sqrt{dx}\,x}-\frac{3\left(\sqrt{dx}\,b^2d^{10}x^2+10\,\sqrt{dx}\,abd^{10}\right)}{d^{10}}\right)}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="giac")
[Out] -2/15*(5*a^2*d/(sqrt(d*x)*x) - 3*(sqrt(d*x)*b^2*d^10*x^2 + 10*sqrt(d*x)*a*b
*d^10)/d^10)/d^3
```

Mupad [B]
time = 4.23, size = 34, normalized size = 0.69

$$\frac{-10\,a^2+60\,a\,b\,x^2+6\,b^2\,x^4}{15\,d^2\,x\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(5/2),x)
[Out] (6*b^2*x^4 - 10*a^2 + 60*a*b*x^2)/(15*d^2*x*(d*x)^(1/2))
```

3.671 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}} \, dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

```
[Out] -2/5*a^2/d/(d*x)^(5/2)+2/3*b^2*(d*x)^(3/2)/d^5-4*a*b/d^3/(d*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{14\}$

$$-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^2)/(5*d*(d*x)^(5/2)) - (4*a*b)/(d^3*Sqrt[d*x]) + (2*b^2*(d*x)^(3/2))/
(3*d^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx &= \int \left(\frac{a^2}{(dx)^{7/2}}+\frac{2ab}{d^2(dx)^{3/2}}+\frac{b^2\sqrt{dx}}{d^4}\right)dx\\ &= -\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 0.78

$$\frac{2\sqrt{dx}\,(-3a^2-30abx^2+5b^2x^4)}{15d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
[Out] (2*Sqrt[d*x]*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*d^4*x^3)
```

Maple [A]
time = 0.03, size = 42, normalized size = 0.86

method	result	size
gosper	$-\frac{2\left(-5b^{2}x^{4}+30ab\,x^{2}+3a^{2}\right)x}{15\left(dx\right)^{\frac{7}{2}}}$	30
trager	$-\frac{2\left(-5b^{2}x^{4}+30ab\,x^{2}+3a^{2}\right)\sqrt{dx}}{15d^{4}x^{3}}$	35
risch	$-\frac{2\left(-5b^{2}x^{4}+30ab\,x^{2}+3a^{2}\right)}{15d^{3}x^{2}\sqrt{dx}}$	35
derivativedivides	$\frac{\frac{2\left(dx\right)^{\frac{3}{2}}b^{2}}{3}-\frac{4ab\,d^{2}}{\sqrt{dx}}-\frac{2a^{2}d^{4}}{5\left(dx\right)^{\frac{5}{2}}}}{d^{5}}$	42
default	$\frac{\frac{2\left(dx\right)^{\frac{3}{2}}b^{2}}{3}-\frac{4ab\,d^{2}}{\sqrt{dx}}-\frac{2a^{2}d^{4}}{5\left(dx\right)^{\frac{5}{2}}}}{d^{5}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/d^5*(1/3*(d*x)^(3/2)*b^2-2*a*b*d^2/(d*x)^(1/2)-1/5*a^2*d^4/(d*x)^(5/2))
```

Maxima [A]
time = 0.31, size = 47, normalized size = 0.96

$$\frac{2\left(\frac{5\,(dx)^{\frac{3}{2}}b^2}{d^4}-\frac{3\,\left(10\,abd^2x^2+a^2d^2\right)}{(dx)^{\frac{5}{2}}d^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="maxima")
[Out] 2/15*(5*(d*x)^(3/2)*b^2/d^4 - 3*(10*a*b*d^2*x^2 + a^2*d^2)/((d*x)^(5/2)*d^2
))/d
```

Fricas [A]
time = 0.36, size = 34, normalized size = 0.69

$$\frac{2\,(5\,b^2x^4-30\,abx^2-3\,a^2)\sqrt{dx}}{15\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*b^2*x^4 - 30*a*b*x^2 - 3*a^2)*sqrt(d*x)/(d^4*x^3)
```

Sympy [A]
time = 0.43, size = 46, normalized size = 0.94

$$-\frac{2a^2x}{5(dx)^{\frac{7}{2}}} - \frac{4abx^3}{(dx)^{\frac{7}{2}}} + \frac{2b^2x^5}{3(dx)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(7/2),x)
[Out] -2*a**2*x/(5*(d*x)**(7/2)) - 4*a*b*x**3/(d*x)**(7/2) + 2*b**2*x**5/(3*(d*x)
**(7/2))
```

Giac [A]
time = 4.48, size = 48, normalized size = 0.98

$$\frac{2\left(5\sqrt{dx}\,b^2x - \frac{3(10\,abd^3x^2+a^2d^3)}{\sqrt{dx}\,d^2x^2}\right)}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="giac")
[Out] 2/15*(5*sqrt(d*x)*b^2*x - 3*(10*a*b*d^3*x^2 + a^2*d^3)/(sqrt(d*x)*d^2*x^2))
/d^4
```

Mupad [B]
time = 0.05, size = 34, normalized size = 0.69

$$-\frac{6\,a^2+60\,a\,b\,x^2-10\,b^2\,x^4}{15\,d^3\,x^2\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(7/2),x)
[Out] -(6*a^2 - 10*b^2*x^4 + 60*a*b*x^2)/(15*d^3*x^2*(d*x)^(1/2))
```

3.672 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{7/2}}{7d}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

```
[Out] 2/7*a^4*(d*x)^(7/2)/d+8/11*a^3*b*(d*x)^(11/2)/d^3+4/5*a^2*b^2*(d*x)^(15/2)/
d^5+8/19*a*b^3*(d*x)^(19/2)/d^7+2/23*b^4*(d*x)^(23/2)/d^9
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {28, 276}

$$\frac{2a^4(dx)^{7/2}}{7d}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(7/2))/(7*d) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (4*a^2*b^2*(d
*x)^(15/2))/(5*d^5) + (8*a*b^3*(d*x)^(19/2))/(19*d^7) + (2*b^4*(d*x)^(23/2)
)/(23*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx &= \frac{\int (dx)^{5/2} \left(ab+b^2x^2\right)^4 \, dx}{b^4}\\
&= \frac{\int \left(a^4b^4(dx)^{5/2}+\frac{4a^3b^5(dx)^{9/2}}{d^2}+\frac{6a^2b^6(dx)^{13/2}}{d^4}+\frac{4ab^7(dx)^{17/2}}{d^6}+\frac{b^8(dx)^{21/2}}{d^8}\right) dx}{b^4}\\
&= \frac{2a^4(dx)^{7/2}}{7d}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23}}{23d^9}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.60

$$\frac{2x(dx)^{5/2}\left(24035a^4+61180a^3bx^2+67298a^2b^2x^4+35420ab^3x^6+7315b^4x^8\right)}{168245}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(24035*a^4 + 61180*a^3*b*x^2 + 67298*a^2*b^2*x^4 + 35420*a
*b^3*x^6 + 7315*b^4*x^8))/168245
```

Maple [A]
time = 0.06, size = 74, normalized size = 0.81

method	result	size
gosper	$\frac{2x\left(7315b^4x^8+35420a\,b^3x^6+67298a^2b^2x^4+61180a^3b\,x^2+24035a^4\right)(dx)^{\frac{5}{2}}}{168245}$	52
trager	$\frac{2d^2x^3\left(7315b^4x^8+35420a\,b^3x^6+67298a^2b^2x^4+61180a^3b\,x^2+24035a^4\right)\sqrt{dx}}{168245}$	57
risch	$\frac{2d^3x^4\left(7315b^4x^8+35420a\,b^3x^6+67298a^2b^2x^4+61180a^3b\,x^2+24035a^4\right)}{168245\sqrt{dx}}$	57
derivativedivides	$\frac{\frac{2b^4(dx)^{\frac{23}{2}}}{23}+\frac{8a\,b^3d^2(dx)^{\frac{19}{2}}}{19}+\frac{4a^2d^4b^2(dx)^{\frac{15}{2}}}{5}+\frac{8a^3d^6b(dx)^{\frac{11}{2}}}{11}+\frac{2a^4d^8(dx)^{\frac{7}{2}}}{7}}{d^9}$	74
default	$\frac{\frac{2b^4(dx)^{\frac{23}{2}}}{23}+\frac{8a\,b^3d^2(dx)^{\frac{19}{2}}}{19}+\frac{4a^2d^4b^2(dx)^{\frac{15}{2}}}{5}+\frac{8a^3d^6b(dx)^{\frac{11}{2}}}{11}+\frac{2a^4d^8(dx)^{\frac{7}{2}}}{7}}{d^9}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^9*(1/23*b^4*(d*x)^(23/2)+4/19*a*b^3*d^2*(d*x)^(19/2)+2/5*a^2*d^4*b^2*(d
*x)^(15/2)+4/11*a^3*d^6*b*(d*x)^(11/2)+1/7*a^4*d^8*(d*x)^(7/2))
```

Maxima [A]
time = 0.30, size = 73, normalized size = 0.80

$$\frac{2\left(7315\,(dx)^{\frac{23}{2}}\,b^4+35420\,(dx)^{\frac{19}{2}}\,ab^3d^2+67298\,(dx)^{\frac{15}{2}}\,a^2b^2d^4+61180\,(dx)^{\frac{11}{2}}\,a^3bd^6+24035\,(dx)^{\frac{7}{2}}\,a^4d^8\right)}{168245\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 2/168245*(7315*(d*x)^(23/2)*b^4 + 35420*(d*x)^(19/2)*a*b^3*d^2 + 67298*(d*x
)^(15/2)*a^2*b^2*d^4 + 61180*(d*x)^(11/2)*a^3*b*d^6 + 24035*(d*x)^(7/2)*a^4
*d^8)/d^9
```

Fricas [A]
time = 0.36, size = 68, normalized size = 0.75

$$\frac{2}{168245}\left(7315\,b^4d^2x^{11}+35420\,ab^3d^2x^9+67298\,a^2b^2d^2x^7+61180\,a^3bd^2x^5+24035\,a^4d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*b^4*d^2*x^11 + 35420*a*b^3*d^2*x^9 + 67298*a^2*b^2*d^2*x^7 +
 61180*a^3*b*d^2*x^5 + 24035*a^4*d^2*x^3)*sqrt(d*x)
```

Sympy [A]
time = 0.70, size = 88, normalized size = 0.97

$$\frac{2a^4x(dx)^{\frac{5}{2}}}{7}+\frac{8a^3bx^3(dx)^{\frac{5}{2}}}{11}+\frac{4a^2b^2x^5(dx)^{\frac{5}{2}}}{5}+\frac{8ab^3x^7(dx)^{\frac{5}{2}}}{19}+\frac{2b^4x^9(dx)^{\frac{5}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*x*(d*x)**(5/2)/7 + 8*a**3*b*x**3*(d*x)**(5/2)/11 + 4*a**2*b**2*x**5*
(d*x)**(5/2)/5 + 8*a*b**3*x**7*(d*x)**(5/2)/19 + 2*b**4*x**9*(d*x)**(5/2)/2
3
```

Giac [A]
time = 3.85, size = 86, normalized size = 0.95

$$\frac{2}{23}\sqrt{dx}\,b^4d^2x^{11}+\frac{8}{19}\sqrt{dx}\,ab^3d^2x^9+\frac{4}{5}\sqrt{dx}\,a^2b^2d^2x^7+\frac{8}{11}\sqrt{dx}\,a^3bd^2x^5+\frac{2}{7}\sqrt{dx}\,a^4d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 2/23*sqrt(d*x)*b^4*d^2*x^11 + 8/19*sqrt(d*x)*a*b^3*d^2*x^9 + 4/5*sqrt(d*x)*
a^2*b^2*d^2*x^7 + 8/11*sqrt(d*x)*a^3*b*d^2*x^5 + 2/7*sqrt(d*x)*a^4*d^2*x^3
```

Mupad [B]
time = 4.20, size = 71, normalized size = 0.78

$$\frac{2\,a^4\,{\left(d\,x\right)}^{7/2}}{7\,d}+\frac{2\,b^4\,{\left(d\,x\right)}^{23/2}}{23\,d^9}+\frac{4\,a^2\,b^2\,{\left(d\,x\right)}^{15/2}}{5\,d^5}+\frac{8\,a^3\,b\,{\left(d\,x\right)}^{11/2}}{11\,d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{19/2}}{19\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*a^4*(d*x)^(7/2))/(7*d) + (2*b^4*(d*x)^(23/2))/(23*d^9) + (4*a^2*b^2*(d*x
)^(15/2))/(5*d^5) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (8*a*b^3*(d*x)^(19/2)
)/(19*d^7)
```

3.673 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

```
[Out] 2/5*a^4*(d*x)^(5/2)/d+8/9*a^3*b*(d*x)^(9/2)/d^3+12/13*a^2*b^2*(d*x)^(13/2)/
d^5+8/17*a*b^3*(d*x)^(17/2)/d^7+2/21*b^4*(d*x)^(21/2)/d^9
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 276}

$$\frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(5/2))/(5*d) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (12*a^2*b^2*(d*
x)^(13/2))/(13*d^5) + (8*a*b^3*(d*x)^(17/2))/(17*d^7) + (2*b^4*(d*x)^(21/2)
)/(21*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2 \, dx &= \frac{\int (dx)^{3/2}\left(ab+b^2x^2\right)^4 \, dx}{b^4}\\
&= \frac{\int \left(a^4b^4(dx)^{3/2}+\frac{4a^3b^5(dx)^{7/2}}{d^2}+\frac{6a^2b^6(dx)^{11/2}}{d^4}+\frac{4ab^7(dx)^{15/2}}{d^6}+\frac{b^8(dx)^{19/2}}{d^8}\right) \, dx}{b^4}\\
&= \frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.60

$$\frac{2x(dx)^{3/2}\left(13923a^4+30940a^3bx^2+32130a^2b^2x^4+16380ab^3x^6+3315b^4x^8\right)}{69615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*(d*x)^(3/2)*(13923*a^4 + 30940*a^3*b*x^2 + 32130*a^2*b^2*x^4 + 16380*a
*b^3*x^6 + 3315*b^4*x^8))/69615
```

Maple [A]
time = 0.06, size = 74, normalized size = 0.81

method	result	size
gosper	$\frac{2x\left(3315b^4x^8+16380a\,b^3x^6+32130a^2b^2x^4+30940a^3b\,x^2+13923a^4\right)(dx)^{\frac{3}{2}}}{69615}$	52
trager	$\frac{2d\,x^2\left(3315b^4x^8+16380a\,b^3x^6+32130a^2b^2x^4+30940a^3b\,x^2+13923a^4\right)\sqrt{dx}}{69615}$	55
risch	$\frac{2d^2x^3\left(3315b^4x^8+16380a\,b^3x^6+32130a^2b^2x^4+30940a^3b\,x^2+13923a^4\right)}{69615\sqrt{dx}}$	57
derivativedivides	$\frac{\frac{2b^4(dx)^{\frac{21}{2}}}{21}+\frac{8a\,b^3d^2(dx)^{\frac{17}{2}}}{17}+\frac{12a^2d^4b^2(dx)^{\frac{13}{2}}}{13}+\frac{8a^3d^6b(dx)^{\frac{9}{2}}}{9}+\frac{2a^4d^8(dx)^{\frac{5}{2}}}{5}}{d^9}$	74
default	$\frac{\frac{2b^4(dx)^{\frac{21}{2}}}{21}+\frac{8a\,b^3d^2(dx)^{\frac{17}{2}}}{17}+\frac{12a^2d^4b^2(dx)^{\frac{13}{2}}}{13}+\frac{8a^3d^6b(dx)^{\frac{9}{2}}}{9}+\frac{2a^4d^8(dx)^{\frac{5}{2}}}{5}}{d^9}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^9*(1/21*b^4*(d*x)^(21/2)+4/17*a*b^3*d^2*(d*x)^(17/2)+6/13*a^2*d^4*b^2*(
d*x)^(13/2)+4/9*a^3*d^6*b*(d*x)^(9/2)+1/5*a^4*d^8*(d*x)^(5/2))
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.80

$$\frac{2\left(3315\,(dx)^{\frac{21}{2}}\,b^4+16380\,(dx)^{\frac{17}{2}}\,ab^3d^2+32130\,(dx)^{\frac{13}{2}}\,a^2b^2d^4+30940\,(dx)^{\frac{9}{2}}\,a^3bd^6+13923\,(dx)^{\frac{5}{2}}\,a^4d^8\right)}{69615\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 2/69615*(3315*(d*x)^(21/2)*b^4 + 16380*(d*x)^(17/2)*a*b^3*d^2 + 32130*(d*x)
^(13/2)*a^2*b^2*d^4 + 30940*(d*x)^(9/2)*a^3*b*d^6 + 13923*(d*x)^(5/2)*a^4*d
^8)/d^9
```

Fricas [A]
time = 0.33, size = 58, normalized size = 0.64

$$\frac{2}{69615}\left(3315\,b^4dx^{10}+16380\,ab^3dx^8+32130\,a^2b^2dx^6+30940\,a^3bdx^4+13923\,a^4dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^4*d*x^10 + 16380*a*b^3*d*x^8 + 32130*a^2*b^2*d*x^6 + 30940*
a^3*b*d*x^4 + 13923*a^4*d*x^2)*sqrt(d*x)
```

Sympy [A]
time = 0.43, size = 88, normalized size = 0.97

$$\frac{2a^4x(dx)^{\frac{3}{2}}}{5}+\frac{8a^3bx^3(dx)^{\frac{3}{2}}}{9}+\frac{12a^2b^2x^5(dx)^{\frac{3}{2}}}{13}+\frac{8ab^3x^7(dx)^{\frac{3}{2}}}{17}+\frac{2b^4x^9(dx)^{\frac{3}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*x*(d*x)**(3/2)/5 + 8*a**3*b*x**3*(d*x)**(3/2)/9 + 12*a**2*b**2*x**5*
(d*x)**(3/2)/13 + 8*a*b**3*x**7*(d*x)**(3/2)/17 + 2*b**4*x**9*(d*x)**(3/2)/
21
```

Giac [A]
time = 4.19, size = 74, normalized size = 0.81

$$\frac{2}{69615}\left(3315\,\sqrt{dx}\,b^4x^{10}+16380\,\sqrt{dx}\,ab^3x^8+32130\,\sqrt{dx}\,a^2b^2x^6+30940\,\sqrt{dx}\,a^3bx^4+13923\,\sqrt{dx}\,a^4x^2\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 2/69615*(3315*sqrt(d*x)*b^4*x^10 + 16380*sqrt(d*x)*a*b^3*x^8 + 32130*sqrt(d
*x)*a^2*b^2*x^6 + 30940*sqrt(d*x)*a^3*b*x^4 + 13923*sqrt(d*x)*a^4*x^2)*d
```

Mupad [B]
time = 0.03, size = 71, normalized size = 0.78

$$\frac{2\,a^4\,{\left(d\,x\right)}^{5/2}}{5\,d}+\frac{2\,b^4\,{\left(d\,x\right)}^{21/2}}{21\,d^9}+\frac{12\,a^2\,b^2\,{\left(d\,x\right)}^{13/2}}{13\,d^5}+\frac{8\,a^3\,b\,{\left(d\,x\right)}^{9/2}}{9\,d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{17/2}}{17\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*a^4*(d*x)^(5/2))/(5*d) + (2*b^4*(d*x)^(21/2))/(21*d^9) + (12*a^2*b^2*(d*
x)^(13/2))/(13*d^5) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (8*a*b^3*(d*x)^(17/2)
)/(17*d^7)
```

3.674 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

```
[Out] 2/3*a^4*(d*x)^(3/2)/d+8/7*a^3*b*(d*x)^(7/2)/d^3+12/11*a^2*b^2*(d*x)^(11/2)/
d^5+8/15*a*b^3*(d*x)^(15/2)/d^7+2/19*b^4*(d*x)^(19/2)/d^9
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 276}

$$\frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(3/2))/(3*d) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (12*a^2*b^2*(d*
x)^(11/2))/(11*d^5) + (8*a*b^3*(d*x)^(15/2))/(15*d^7) + (2*b^4*(d*x)^(19/2)
)/(19*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx &= \frac{\int \sqrt{dx}\left(ab+b^2x^2\right)^4\,dx}{b^4}\\
&= \frac{\int \left(a^4b^4\sqrt{dx}+\frac{4a^3b^5(dx)^{5/2}}{d^2}+\frac{6a^2b^6(dx)^{9/2}}{d^4}+\frac{4ab^7(dx)^{13/2}}{d^6}+\frac{b^8(dx)^{17/2}}{d^8}\right)dx}{b^4}\\
&= \frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.60

$$\frac{2x\sqrt{dx}\,(7315a^4 + 12540a^3bx^2 + 11970a^2b^2x^4 + 5852ab^3x^6 + 1155b^4x^8)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*Sqrt[d*x]*(7315*a^4 + 12540*a^3*b*x^2 + 11970*a^2*b^2*x^4 + 5852*a*b^3
*x^6 + 1155*b^4*x^8))/21945
```

Maple [A]
time = 0.06, size = 74, normalized size = 0.81

method	result	size
gosper	$\frac{2x\left(1155b^4x^8+5852a\,b^3x^6+11970a^2b^2x^4+12540a^3b\,x^2+7315a^4\right)\sqrt{dx}}{21945}$	52
trager	$\frac{2x\left(1155b^4x^8+5852a\,b^3x^6+11970a^2b^2x^4+12540a^3b\,x^2+7315a^4\right)\sqrt{dx}}{21945}$	52
risch	$\frac{2d\,x^2\left(1155b^4x^8+5852a\,b^3x^6+11970a^2b^2x^4+12540a^3b\,x^2+7315a^4\right)}{21945\sqrt{dx}}$	55
derivativedivides	$\frac{\frac{2b^4(dx)^{\frac{19}{2}}}{19}+\frac{8a\,b^3d^2(dx)^{\frac{15}{2}}}{15}+\frac{12a^2d^4b^2(dx)^{\frac{11}{2}}}{11}+\frac{8a^3b\,d^6(dx)^{\frac{7}{2}}}{7}+\frac{2a^4d^8(dx)^{\frac{3}{2}}}{3}}{d^9}$	74
default	$\frac{\frac{2b^4(dx)^{\frac{19}{2}}}{19}+\frac{8a\,b^3d^2(dx)^{\frac{15}{2}}}{15}+\frac{12a^2d^4b^2(dx)^{\frac{11}{2}}}{11}+\frac{8a^3b\,d^6(dx)^{\frac{7}{2}}}{7}+\frac{2a^4d^8(dx)^{\frac{3}{2}}}{3}}{d^9}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^9*(1/19*b^4*(d*x)^(19/2)+4/15*a*b^3*d^2*(d*x)^(15/2)+6/11*a^2*d^4*b^2*(
d*x)^(11/2)+4/7*a^3*b*d^6*(d*x)^(7/2)+1/3*a^4*d^8*(d*x)^(3/2))
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.80

$$\frac{2\left(1155\,(dx)^{\frac{19}{2}}\,b^4 + 5852\,(dx)^{\frac{15}{2}}\,ab^3d^2 + 11970\,(dx)^{\frac{11}{2}}\,a^2b^2d^4 + 12540\,(dx)^{\frac{7}{2}}\,a^3bd^6 + 7315\,(dx)^{\frac{3}{2}}\,a^4d^8\right)}{21945\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21945*(1155*(d*x)^(19/2)*b^4 + 5852*(d*x)^(15/2)*a*b^3*d^2 + 11970*(d*x)^
(11/2)*a^2*b^2*d^4 + 12540*(d*x)^(7/2)*a^3*b*d^6 + 7315*(d*x)^(3/2)*a^4*d^8
)/d^9
```

Fricas [A]
time = 0.33, size = 51, normalized size = 0.56

$$\frac{2}{21945}\left(1155\,b^4x^9+5852\,ab^3x^7+11970\,a^2b^2x^5+12540\,a^3bx^3+7315\,a^4x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^4*x^9 + 5852*a*b^3*x^7 + 11970*a^2*b^2*x^5 + 12540*a^3*b*x^
3 + 7315*a^4*x)*sqrt(d*x)
```

Sympy [A]
time = 0.24, size = 88, normalized size = 0.97

$$\frac{2a^4x\sqrt{dx}}{3}+\frac{8a^3bx^3\sqrt{dx}}{7}+\frac{12a^2b^2x^5\sqrt{dx}}{11}+\frac{8ab^3x^7\sqrt{dx}}{15}+\frac{2b^4x^9\sqrt{dx}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2*(d*x)**(1/2),x)
```

```
[Out] 2*a**4*x*sqrt(d*x)/3 + 8*a**3*b*x**3*sqrt(d*x)/7 + 12*a**2*b**2*x**5*sqrt(d
*x)/11 + 8*a*b**3*x**7*sqrt(d*x)/15 + 2*b**4*x**9*sqrt(d*x)/19
```

Giac [A]
time = 3.45, size = 69, normalized size = 0.76

$$\frac{2}{19}\sqrt{dx}\,b^4x^9+\frac{8}{15}\sqrt{dx}\,ab^3x^7+\frac{12}{11}\sqrt{dx}\,a^2b^2x^5+\frac{8}{7}\sqrt{dx}\,a^3bx^3+\frac{2}{3}\sqrt{dx}\,a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*sqrt(d*x)*b^4*x^9 + 8/15*sqrt(d*x)*a*b^3*x^7 + 12/11*sqrt(d*x)*a^2*b^2
*x^5 + 8/7*sqrt(d*x)*a^3*b*x^3 + 2/3*sqrt(d*x)*a^4*x
```

Mupad [B]
time = 0.03, size = 71, normalized size = 0.78

$$\frac{2\,a^4\,{\left(d\,x\right)}^{3/2}}{3\,d}+\frac{2\,b^4\,{\left(d\,x\right)}^{19/2}}{19\,d^9}+\frac{12\,a^2\,b^2\,{\left(d\,x\right)}^{11/2}}{11\,d^5}+\frac{8\,a^3\,b\,{\left(d\,x\right)}^{7/2}}{7\,d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{15/2}}{15\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*a^4*(d*x)^(3/2))/(3*d) + (2*b^4*(d*x)^(19/2))/(19*d^9) + (12*a^2*b^2*(d*
x)^(11/2))/(11*d^5) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (8*a*b^3*(d*x)^(15/2)
)/(15*d^7)
```

3.675 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}}\,dx$

Optimal. Leaf size=89

$$\frac{2a^4\sqrt{dx}}{d}+\frac{8a^3b(dx)^{5/2}}{5d^3}+\frac{4a^2b^2(dx)^{9/2}}{3d^5}+\frac{8ab^3(dx)^{13/2}}{13d^7}+\frac{2b^4(dx)^{17/2}}{17d^9}$$

```
[Out] 8/5*a^3*b*(d*x)^(5/2)/d^3+4/3*a^2*b^2*(d*x)^(9/2)/d^5+8/13*a*b^3*(d*x)^(13/
2)/d^7+2/17*b^4*(d*x)^(17/2)/d^9+2*a^4*(d*x)^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {28, 276}

$$\frac{2a^4\sqrt{dx}}{d}+\frac{8a^3b(dx)^{5/2}}{5d^3}+\frac{4a^2b^2(dx)^{9/2}}{3d^5}+\frac{8ab^3(dx)^{13/2}}{13d^7}+\frac{2b^4(dx)^{17/2}}{17d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

```
[Out] (2*a^4*Sqrt[d*x])/d + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (4*a^2*b^2*(d*x)^(9/2
))/(3*d^5) + (8*a*b^3*(d*x)^(13/2))/(13*d^7) + (2*b^4*(d*x)^(17/2))/(17*d^9
)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{\sqrt{dx}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{\sqrt{dx}} + \frac{4a^3b^5(dx)^{3/2}}{d^2} + \frac{6a^2b^6(dx)^{7/2}}{d^4} + \frac{4ab^7(dx)^{11/2}}{d^6} + \frac{b^8(dx)^{15/2}}{d^8}\right)dx}{b^4}$$
$$= \frac{2a^4\sqrt{dx}}{d} + \frac{8a^3b(dx)^{5/2}}{5d^3} + \frac{4a^2b^2(dx)^{9/2}}{3d^5} + \frac{8ab^3(dx)^{13/2}}{13d^7} + \frac{2b^4(dx)^{17/2}}{17d^9}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.62

$$\frac{2x(3315a^4+2652a^3bx^2+2210a^2b^2x^4+1020ab^3x^6+195b^4x^8)}{3315\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

```
[Out] (2*x*(3315*a^4 + 2652*a^3*b*x^2 + 2210*a^2*b^2*x^4 + 1020*a*b^3*x^6 + 195*b
^4*x^8))/(3315*Sqrt[d*x])
```

Maple [A]
time = 0.03, size = 73, normalized size = 0.82

method	result	size
gosper	$\frac{2(195b^4x^8+1020a\,b^3x^6+2210a^2b^2x^4+2652a^3b\,x^2+3315a^4)x}{3315\sqrt{dx}}$	52
risch	$\frac{2(195b^4x^8+1020a\,b^3x^6+2210a^2b^2x^4+2652a^3b\,x^2+3315a^4)x}{3315\sqrt{dx}}$	52
trager	$\frac{\left(\frac{2}{17}b^4x^8+\frac{8}{13}a\,b^3x^6+\frac{4}{3}a^2b^2x^4+\frac{8}{5}a^3b\,x^2+2a^4\right)\sqrt{dx}}{d}$	53
derivativedivides	$\frac{\frac{2b^4(dx)^{\frac{17}{2}}}{17}+\frac{8a\,b^3d^2(dx)^{\frac{13}{2}}}{13}+\frac{4a^2d^4b^2(dx)^{\frac{9}{2}}}{3}+\frac{8a^3d^6b(dx)^{\frac{5}{2}}}{5}+2a^4d^8\sqrt{dx}}{d^9}$	73
default	$\frac{\frac{2b^4(dx)^{\frac{17}{2}}}{17}+\frac{8a\,b^3d^2(dx)^{\frac{13}{2}}}{13}+\frac{4a^2d^4b^2(dx)^{\frac{9}{2}}}{3}+\frac{8a^3d^6b(dx)^{\frac{5}{2}}}{5}+2a^4d^8\sqrt{dx}}{d^9}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^9*(1/17*b^4*(d*x)^(17/2)+4/13*a*b^3*d^2*(d*x)^(13/2)+2/3*a^2*d^4*b^2*(d
*x)^(9/2)+4/5*a^3*d^6*b*(d*x)^(5/2)+a^4*d^8*(d*x)^(1/2))
```

Maxima [A]
time = 0.28, size = 90, normalized size = 1.01

$$\frac{2\left(9945\sqrt{dx}\,a^4+\frac{585\,(dx)^{\frac{17}{2}}b^4}{d^8}+\frac{3060\,(dx)^{\frac{13}{2}}ab^3}{d^6}+\frac{4420\,(dx)^{\frac{9}{2}}a^2b^2}{d^4}+442\left(\frac{5\,(dx)^{\frac{9}{2}}b^2}{d^4}+\frac{18\,(dx)^{\frac{5}{2}}ab}{d^2}\right)a^2\right)}{9945\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9945*(9945*sqrt(d*x)*a^4 + 585*(d*x)^(17/2)*b^4/d^8 + 3060*(d*x)^(13/2)*a
*b^3/d^6 + 4420*(d*x)^(9/2)*a^2*b^2/d^4 + 442*(5*(d*x)^(9/2)*b^2/d^4 + 18*(
d*x)^(5/2)*a*b/d^2)*a^2)/d
```

Fricas [A]
time = 0.35, size = 53, normalized size = 0.60

$$\frac{2\,(195\,b^4x^8+1020\,ab^3x^6+2210\,a^2b^2x^4+2652\,a^3bx^2+3315\,a^4)\sqrt{dx}}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*b^4*x^8 + 1020*a*b^3*x^6 + 2210*a^2*b^2*x^4 + 2652*a^3*b*x^2 +
3315*a^4)*sqrt(d*x)/d
```

Sympy [A]
time = 0.34, size = 87, normalized size = 0.98

$$\frac{2a^4x}{\sqrt{dx}}+\frac{8a^3bx^3}{5\sqrt{dx}}+\frac{4a^2b^2x^5}{3\sqrt{dx}}+\frac{8ab^3x^7}{13\sqrt{dx}}+\frac{2b^4x^9}{17\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] 2*a**4*x/sqrt(d*x) + 8*a**3*b*x**3/(5*sqrt(d*x)) + 4*a**2*b**2*x**5/(3*sqrt
(d*x)) + 8*a*b**3*x**7/(13*sqrt(d*x)) + 2*b**4*x**9/(17*sqrt(d*x))
```

Giac [A]
time = 6.40, size = 73, normalized size = 0.82

$$\frac{2\left(195\sqrt{dx}\,b^4x^8+1020\sqrt{dx}\,ab^3x^6+2210\sqrt{dx}\,a^2b^2x^4+2652\sqrt{dx}\,a^3bx^2+3315\sqrt{dx}\,a^4\right)}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3315*(195*sqrt(d*x)*b^4*x^8 + 1020*sqrt(d*x)*a*b^3*x^6 + 2210*sqrt(d*x)*a
^2*b^2*x^4 + 2652*sqrt(d*x)*a^3*b*x^2 + 3315*sqrt(d*x)*a^4)/d
```

Mupad [B]
time = 0.03, size = 71, normalized size = 0.80

$$\frac{2\,a^4\,\sqrt{d\,x}}{d}+\frac{2\,b^4\,{\left(d\,x\right)}^{17/2}}{17\,d^9}+\frac{4\,a^2\,b^2\,{\left(d\,x\right)}^{9/2}}{3\,d^5}+\frac{8\,a^3\,b\,{\left(d\,x\right)}^{5/2}}{5\,d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{13/2}}{13\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(1/2),x)
```

```
[Out] (2*a^4*(d*x)^(1/2))/d + (2*b^4*(d*x)^(17/2))/(17*d^9) + (4*a^2*b^2*(d*x)^(9
/2))/(3*d^5) + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (8*a*b^3*(d*x)^(13/2))/(13*d
^7)
```

3.676 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2a^4}{d\sqrt{dx}}+\frac{8a^3b(dx)^{3/2}}{3d^3}+\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

```
[Out] 8/3*a^3*b*(d*x)^(3/2)/d^3+12/7*a^2*b^2*(d*x)^(7/2)/d^5+8/11*a*b^3*(d*x)^(11
/2)/d^7+2/15*b^4*(d*x)^(15/2)/d^9-2*a^4/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {28, 276}

$$-\frac{2a^4}{d\sqrt{dx}}+\frac{8a^3b(dx)^{3/2}}{3d^3}+\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

```
[Out] (-2*a^4)/(d*Sqrt[d*x]) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (12*a^2*b^2*(d*x)^
(7/2))/(7*d^5) + (8*a*b^3*(d*x)^(11/2))/(11*d^7) + (2*b^4*(d*x)^(15/2))/(15
*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{(dx)^{3/2}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{(dx)^{3/2}} + \frac{4a^3b^5\sqrt{dx}}{d^2} + \frac{6a^2b^6(dx)^{5/2}}{d^4} + \frac{4ab^7(dx)^{9/2}}{d^6} + \frac{b^8(dx)^{13/2}}{d^8}\right)dx}{b^4}$$
$$= -\frac{2a^4}{d\sqrt{dx}} + \frac{8a^3b(dx)^{3/2}}{3d^3} + \frac{12a^2b^2(dx)^{7/2}}{7d^5} + \frac{8ab^3(dx)^{11/2}}{11d^7} + \frac{2b^4(dx)^{15/2}}{15d^9}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.62

$$-\frac{2x(1155a^4 - 1540a^3bx^2 - 990a^2b^2x^4 - 420ab^3x^6 - 77b^4x^8)}{1155(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

```
[Out] (-2*x*(1155*a^4 - 1540*a^3*b*x^2 - 990*a^2*b^2*x^4 - 420*a*b^3*x^6 - 77*b^4
*x^8))/(1155*(d*x)^(3/2))
```

Maple [A]
time = 0.04, size = 74, normalized size = 0.83

method	result	size
gosper	$-\frac{2\left(-77b^4x^8-420a\,b^3x^6-990a^2b^2x^4-1540a^3b\,x^2+1155a^4\right)x}{1155(dx)^{\frac{3}{2}}}$	52
risch	$-\frac{2\left(-77b^4x^8-420a\,b^3x^6-990a^2b^2x^4-1540a^3b\,x^2+1155a^4\right)}{1155d\sqrt{dx}}$	54
trager	$-\frac{2\left(-77b^4x^8-420a\,b^3x^6-990a^2b^2x^4-1540a^3b\,x^2+1155a^4\right)\sqrt{dx}}{1155d^2x}$	57
derivativedivides	$\frac{\frac{2b^4(dx)^{\frac{15}{2}}}{15}+\frac{8a\,b^3d^2(dx)^{\frac{11}{2}}}{11}+\frac{12a^2b^2d^4(dx)^{\frac{7}{2}}}{7}+\frac{8a^3b\,d^6(dx)^{\frac{3}{2}}}{3}-\frac{2a^4d^8}{\sqrt{dx}}}{d^9}$	74
default	$\frac{\frac{2b^4(dx)^{\frac{15}{2}}}{15}+\frac{8a\,b^3d^2(dx)^{\frac{11}{2}}}{11}+\frac{12a^2b^2d^4(dx)^{\frac{7}{2}}}{7}+\frac{8a^3b\,d^6(dx)^{\frac{3}{2}}}{3}-\frac{2a^4d^8}{\sqrt{dx}}}{d^9}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^9*(1/15*b^4*(d*x)^(15/2)+4/11*a*b^3*d^2*(d*x)^(11/2)+6/7*a^2*b^2*d^4*(d
*x)^(7/2)+4/3*a^3*b*d^6*(d*x)^(3/2)-a^4*d^8/(d*x)^(1/2))
```

Maxima [A]
time = 0.27, size = 76, normalized size = 0.85

$$-\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}}-\frac{77\,(dx)^{\frac{15}{2}}b^4+420\,(dx)^{\frac{11}{2}}ab^3d^2+990\,(dx)^{\frac{7}{2}}a^2b^2d^4+1540\,(dx)^{\frac{3}{2}}a^3bd^6}{d^8}\right)}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="maxima")
[Out] -2/1155*(1155*a^4/sqrt(d*x) - (77*(d*x)^(15/2)*b^4 + 420*(d*x)^(11/2)*a*b^3
*d^2 + 990*(d*x)^(7/2)*a^2*b^2*d^4 + 1540*(d*x)^(3/2)*a^3*b*d^6)/d^8)/d
```

Fricas [A]
time = 0.33, size = 56, normalized size = 0.63

$$\frac{2\,(77\,b^4x^8+420\,ab^3x^6+990\,a^2b^2x^4+1540\,a^3bx^2-1155\,a^4)\sqrt{dx}}{1155\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="fricas")
[Out] 2/1155*(77*b^4*x^8 + 420*a*b^3*x^6 + 990*a^2*b^2*x^4 + 1540*a^3*b*x^2 - 115
5*a^4)*sqrt(d*x)/(d^2*x)
```

Sympy [A]
time = 0.35, size = 87, normalized size = 0.98

$$-\frac{2a^4x}{(dx)^{\frac{3}{2}}}+\frac{8a^3bx^3}{3\,(dx)^{\frac{3}{2}}}+\frac{12a^2b^2x^5}{7\,(dx)^{\frac{3}{2}}}+\frac{8ab^3x^7}{11\,(dx)^{\frac{3}{2}}}+\frac{2b^4x^9}{15\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(3/2),x)
[Out] -2*a**4*x/(d*x)**(3/2) + 8*a**3*b*x**3/(3*(d*x)**(3/2)) + 12*a**2*b**2*x**5
/(7*(d*x)**(3/2)) + 8*a*b**3*x**7/(11*(d*x)**(3/2)) + 2*b**4*x**9/(15*(d*x)
**(3/2))
```

Giac [A]
time = 2.89, size = 89, normalized size = 1.00

$$-\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}}-\frac{77\,\sqrt{dx}\,b^4d^{119}x^7+420\,\sqrt{dx}\,ab^3d^{119}x^5+990\,\sqrt{dx}\,a^2b^2d^{119}x^3+1540\,\sqrt{dx}\,a^3bd^{119}x}{d^{120}}\right)}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/1155*(1155*a^4/sqrt(d*x) - (77*sqrt(d*x)*b^4*d^119*x^7 + 420*sqrt(d*x)*a
*b^3*d^119*x^5 + 990*sqrt(d*x)*a^2*b^2*d^119*x^3 + 1540*sqrt(d*x)*a^3*b*d^1
19*x)/d^120)/d
```

Mupad [B]
time = 0.03, size = 71, normalized size = 0.80

$$\frac{2\,b^4\,{\left(d\,x\right)}^{15/2}}{15\,d^9}-\frac{2\,a^4}{d\,\sqrt{d\,x}}+\frac{12\,a^2\,b^2\,{\left(d\,x\right)}^{7/2}}{7\,d^5}+\frac{8\,a^3\,b\,{\left(d\,x\right)}^{3/2}}{3\,d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{11/2}}{11\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(3/2),x)
```

```
[Out] (2*b^4*(d*x)^(15/2))/(15*d^9) - (2*a^4)/(d*(d*x)^(1/2)) + (12*a^2*b^2*(d*x)
^(7/2))/(7*d^5) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (8*a*b^3*(d*x)^(11/2))/(1
1*d^7)
```

3.677 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2a^4}{3d(dx)^{3/2}}+\frac{8a^3b\sqrt{dx}}{d^3}+\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

```
[Out] -2/3*a^4/d/(d*x)^(3/2)+12/5*a^2*b^2*(d*x)^(5/2)/d^5+8/9*a*b^3*(d*x)^(9/2)/d
^7+2/13*b^4*(d*x)^(13/2)/d^9+8*a^3*b*(d*x)^(1/2)/d^3
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {28, 276}

$$-\frac{2a^4}{3d(dx)^{3/2}}+\frac{8a^3b\sqrt{dx}}{d^3}+\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

```
[Out] (-2*a^4)/(3*d*(d*x)^(3/2)) + (8*a^3*b*Sqrt[d*x])/d^3 + (12*a^2*b^2*(d*x)^(5
/2))/(5*d^5) + (8*a*b^3*(d*x)^(9/2))/(9*d^7) + (2*b^4*(d*x)^(13/2))/(13*d^9
)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{5/2}}\,dx = \frac{\int \frac{(ab+b^2x^2)^4}{(dx)^{5/2}}\,dx}{b^4}$$
$$= \frac{\int \left(\frac{a^4b^4}{(dx)^{5/2}} + \frac{4a^3b^5}{d^2\sqrt{dx}} + \frac{6a^2b^6(dx)^{3/2}}{d^4} + \frac{4ab^7(dx)^{7/2}}{d^6} + \frac{b^8(dx)^{11/2}}{d^8}\right)dx}{b^4}$$
$$= -\frac{2a^4}{3d(dx)^{3/2}} + \frac{8a^3b\sqrt{dx}}{d^3} + \frac{12a^2b^2(dx)^{5/2}}{5d^5} + \frac{8ab^3(dx)^{9/2}}{9d^7} + \frac{2b^4(dx)^{13/2}}{13d^9}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.62

$$-\frac{2x(195a^4-2340a^3bx^2-702a^2b^2x^4-260ab^3x^6-45b^4x^8)}{585(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

```
[Out] (-2*x*(195*a^4 - 2340*a^3*b*x^2 - 702*a^2*b^2*x^4 - 260*a*b^3*x^6 - 45*b^4*
x^8))/(585*(d*x)^(5/2))
```

Maple [A]
time = 0.06, size = 74, normalized size = 0.83

method	result	size
gosper	$-\frac{2(-45b^4x^8-260a\,b^3x^6-702a^2b^2x^4-2340a^3b\,x^2+195a^4)x}{585(dx)^{\frac{5}{2}}}$	52
trager	$-\frac{2(-45b^4x^8-260a\,b^3x^6-702a^2b^2x^4-2340a^3b\,x^2+195a^4)\sqrt{dx}}{585d^3x^2}$	57
risch	$-\frac{2(-45b^4x^8-260a\,b^3x^6-702a^2b^2x^4-2340a^3b\,x^2+195a^4)}{585d^2x\sqrt{dx}}$	57
derivativedivides	$\frac{\frac{2b^4(dx)^{\frac{13}{2}}}{13}+\frac{8a\,b^3d^2(dx)^{\frac{9}{2}}}{9}+\frac{12a^2b^2d^4(dx)^{\frac{5}{2}}}{5}+8a^3b\,d^6\sqrt{dx}-\frac{2a^4d^8}{3(dx)^{\frac{3}{2}}}}{d^9}$	74
default	$\frac{\frac{2b^4(dx)^{\frac{13}{2}}}{13}+\frac{8a\,b^3d^2(dx)^{\frac{9}{2}}}{9}+\frac{12a^2b^2d^4(dx)^{\frac{5}{2}}}{5}+8a^3b\,d^6\sqrt{dx}-\frac{2a^4d^8}{3(dx)^{\frac{3}{2}}}}{d^9}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^9*(1/13*b^4*(d*x)^(13/2)+4/9*a*b^3*d^2*(d*x)^(9/2)+6/5*a^2*b^2*d^4*(d*x
)^(5/2)+4*a^3*b*d^6*(d*x)^(1/2)-1/3*a^4*d^8/(d*x)^(3/2))
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.85

$$-\frac{2\left(\frac{195\,a^4}{(dx)^{\frac{3}{2}}}-\frac{45\,(dx)^{\frac{13}{2}}b^4+260\,(dx)^{\frac{9}{2}}ab^3d^2+702\,(dx)^{\frac{5}{2}}a^2b^2d^4+2340\,\sqrt{dx}\,a^3bd^6}{d^8}\right)}{585\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/585*(195*a^4/(d*x)^(3/2) - (45*(d*x)^(13/2)*b^4 + 260*(d*x)^(9/2)*a*b^3*
d^2 + 702*(d*x)^(5/2)*a^2*b^2*d^4 + 2340*sqrt(d*x)*a^3*b*d^6)/d^8)/d
```

Fricas [A]
time = 0.35, size = 56, normalized size = 0.63

$$\frac{2\,(45\,b^4x^8+260\,ab^3x^6+702\,a^2b^2x^4+2340\,a^3bx^2-195\,a^4)\sqrt{dx}}{585\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^4*x^8 + 260*a*b^3*x^6 + 702*a^2*b^2*x^4 + 2340*a^3*b*x^2 - 195*
a^4)*sqrt(d*x)/(d^3*x^2)
```

Sympy [A]
time = 0.39, size = 87, normalized size = 0.98

$$-\frac{2a^4x}{3\,(dx)^{\frac{5}{2}}}+\frac{8a^3bx^3}{(dx)^{\frac{5}{2}}}+\frac{12a^2b^2x^5}{5\,(dx)^{\frac{5}{2}}}+\frac{8ab^3x^7}{9\,(dx)^{\frac{5}{2}}}+\frac{2b^4x^9}{13\,(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(5/2),x)
```

```
[Out] -2*a**4*x/(3*(d*x)**(5/2)) + 8*a**3*b*x**3/(d*x)**(5/2) + 12*a**2*b**2*x**5
/(5*(d*x)**(5/2)) + 8*a*b**3*x**7/(9*(d*x)**(5/2)) + 2*b**4*x**9/(13*(d*x)*
*(5/2))
```

Giac [A]
time = 3.34, size = 92, normalized size = 1.03

$$-\frac{2\left(\frac{195\,a^4d}{\sqrt{dx}\,x}-\frac{45\,\sqrt{dx}\,b^4d^{78}x^6+260\,\sqrt{dx}\,ab^3d^{78}x^4+702\,\sqrt{dx}\,a^2b^2d^{78}x^2+2340\,\sqrt{dx}\,a^3bd^{78}}{d^{78}}\right)}{585\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/585*(195*a^4*d/(sqrt(d*x)*x) - (45*sqrt(d*x)*b^4*d^78*x^6 + 260*sqrt(d*x
)*a*b^3*d^78*x^4 + 702*sqrt(d*x)*a^2*b^2*d^78*x^2 + 2340*sqrt(d*x)*a^3*b*d^
78)/d^78)/d^3
```

Mupad [B]
time = 0.03, size = 71, normalized size = 0.80

$$\frac{2\,b^4\,{\left(d\,x\right)}^{13/2}}{13\,d^9}-\frac{2\,a^4}{3\,d\,{\left(d\,x\right)}^{3/2}}+\frac{12\,a^2\,b^2\,{\left(d\,x\right)}^{5/2}}{5\,d^5}+\frac{8\,a^3\,b\,\sqrt{d\,x}}{d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{9/2}}{9\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(5/2),x)
```

```
[Out] (2*b^4*(d*x)^(13/2))/(13*d^9) - (2*a^4)/(3*d*(d*x)^(3/2)) + (12*a^2*b^2*(d*
x)^(5/2))/(5*d^5) + (8*a^3*b*(d*x)^(1/2))/d^3 + (8*a*b^3*(d*x)^(9/2))/(9*d^
7)
```

3.678 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2a^4}{5d(dx)^{5/2}}-\frac{8a^3b}{d^3\sqrt{dx}}+\frac{4a^2b^2(dx)^{3/2}}{d^5}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

```
[Out] -2/5*a^4/d/(d*x)^(5/2)+4*a^2*b^2*(d*x)^(3/2)/d^5+8/7*a*b^3*(d*x)^(7/2)/d^7+
2/11*b^4*(d*x)^(11/2)/d^9-8*a^3*b/d^3/(d*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {28, 276}

$$-\frac{2a^4}{5d(dx)^{5/2}}-\frac{8a^3b}{d^3\sqrt{dx}}+\frac{4a^2b^2(dx)^{3/2}}{d^5}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

```
[Out] (-2*a^4)/(5*d*(d*x)^(5/2)) - (8*a^3*b)/(d^3*Sqrt[d*x]) + (4*a^2*b^2*(d*x)^(
3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7) + (2*b^4*(d*x)^(11/2))/(11*d^9)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{7/2}}\,dx &= \frac{\int \frac{(ab+b^2x^2)^4}{(dx)^{7/2}}\,dx}{b^4}\\
&= \frac{\int \left(\frac{a^4b^4}{(dx)^{7/2}}+\frac{4a^3b^5}{d^2(dx)^{3/2}}+\frac{6a^2b^6\sqrt{dx}}{d^4}+\frac{4ab^7(dx)^{5/2}}{d^6}+\frac{b^8(dx)^{9/2}}{d^8}\right)dx}{b^4}\\
&= -\frac{2a^4}{5d(dx)^{5/2}}-\frac{8a^3b}{d^3\sqrt{dx}}+\frac{4a^2b^2(dx)^{3/2}}{d^5}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.69

$$-\frac{2\sqrt{dx}\,(77a^4+1540a^3bx^2-770a^2b^2x^4-220ab^3x^6-35b^4x^8)}{385d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[d*x]*(77*a^4 + 1540*a^3*b*x^2 - 770*a^2*b^2*x^4 - 220*a*b^3*x^6 -
35*b^4*x^8))/(385*d^4*x^3)
```

Maple [A]
time = 0.04, size = 74, normalized size = 0.85

method	result	size
gosper	$-\frac{2(-35b^4x^8-220a\,b^3x^6-770a^2b^2x^4+1540a^3b\,x^2+77a^4)x}{385(dx)^{\frac{7}{2}}}$	52
trager	$-\frac{2(-35b^4x^8-220a\,b^3x^6-770a^2b^2x^4+1540a^3b\,x^2+77a^4)\sqrt{dx}}{385d^4x^3}$	57
risch	$-\frac{2(-35b^4x^8-220a\,b^3x^6-770a^2b^2x^4+1540a^3b\,x^2+77a^4)}{385d^3x^2\sqrt{dx}}$	57
derivativedivides	$\frac{\frac{2b^4(dx)^{\frac{11}{2}}}{11}+\frac{8a\,b^3d^2(dx)^{\frac{7}{2}}}{7}+4a^2b^2d^4(dx)^{\frac{3}{2}}-\frac{2a^4d^8}{5(dx)^{\frac{5}{2}}}-\frac{8a^3b\,d^6}{\sqrt{dx}}}{d^9}$	74
default	$\frac{\frac{2b^4(dx)^{\frac{11}{2}}}{11}+\frac{8a\,b^3d^2(dx)^{\frac{7}{2}}}{7}+4a^2b^2d^4(dx)^{\frac{3}{2}}-\frac{2a^4d^8}{5(dx)^{\frac{5}{2}}}-\frac{8a^3b\,d^6}{\sqrt{dx}}}{d^9}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^9*(1/11*b^4*(d*x)^(11/2)+4/7*a*b^3*d^2*(d*x)^(7/2)+2*a^2*b^2*d^4*(d*x)^
(3/2)-1/5*a^4*d^8/(d*x)^(5/2)-4*a^3*b*d^6/(d*x)^(1/2))
```

Maxima [A]
time = 0.28, size = 82, normalized size = 0.94

$$-\frac{2\left(\frac{77\,(20\,a^3bd^2x^2+a^4d^2)}{(dx)^{\frac{5}{2}}d^2}-\frac{5\left(7\,(dx)^{\frac{11}{2}}b^4+44\,(dx)^{\frac{7}{2}}ab^3d^2+154\,(dx)^{\frac{3}{2}}a^2b^2d^4\right)}{d^8}\right)}{385\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/385*(77*(20*a^3*b*d^2*x^2 + a^4*d^2)/((d*x)^(5/2)*d^2) - 5*(7*(d*x)^(11/
2)*b^4 + 44*(d*x)^(7/2)*a*b^3*d^2 + 154*(d*x)^(3/2)*a^2*b^2*d^4)/d^8)/d
```

Fricas [A]
time = 0.34, size = 56, normalized size = 0.64

$$\frac{2\,(35\,b^4x^8+220\,ab^3x^6+770\,a^2b^2x^4-1540\,a^3bx^2-77\,a^4)\sqrt{dx}}{385\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/385*(35*b^4*x^8 + 220*a*b^3*x^6 + 770*a^2*b^2*x^4 - 1540*a^3*b*x^2 - 77*a
^4)*sqrt(d*x)/(d^4*x^3)
```

Sympy [A]
time = 0.48, size = 85, normalized size = 0.98

$$-\frac{2a^4x}{5\,(dx)^{\frac{7}{2}}}-\frac{8a^3bx^3}{(dx)^{\frac{7}{2}}}+\frac{4a^2b^2x^5}{(dx)^{\frac{7}{2}}}+\frac{8ab^3x^7}{7\,(dx)^{\frac{7}{2}}}+\frac{2b^4x^9}{11\,(dx)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(7/2),x)
```

```
[Out] -2*a**4*x/(5*(d*x)**(7/2)) - 8*a**3*b*x**3/(d*x)**(7/2) + 4*a**2*b**2*x**5/
(d*x)**(7/2) + 8*a*b**3*x**7/(7*(d*x)**(7/2)) + 2*b**4*x**9/(11*(d*x)**(7/2
))
```

Giac [A]
time = 3.88, size = 95, normalized size = 1.09

$$-\frac{2\left(\frac{77\,(20\,a^3bd^3x^2+a^4d^3)}{\sqrt{dx}\,d^2x^2}-\frac{5\left(7\,\sqrt{dx}\,b^4d^{55}x^5+44\,\sqrt{dx}\,ab^3d^{55}x^3+154\,\sqrt{dx}\,a^2b^2d^{55}x\right)}{d^{55}}\right)}{385\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/385*(77*(20*a^3*b*d^3*x^2 + a^4*d^3)/(sqrt(d*x)*d^2*x^2) - 5*(7*sqrt(d*x
)*b^4*d^55*x^5 + 44*sqrt(d*x)*a*b^3*d^55*x^3 + 154*sqrt(d*x)*a^2*b^2*d^55*x
)/d^55)/d^4
```

Mupad [B]
time = 0.06, size = 75, normalized size = 0.86

$$\frac{2\,b^4\,{(d\,x)}^{11/2}}{11\,d^9}-\frac{\frac{2\,a^4\,d^2}{5}+8\,b\,a^3\,d^2\,x^2}{d^3\,{(d\,x)}^{5/2}}+\frac{4\,a^2\,b^2\,{(d\,x)}^{3/2}}{d^5}+\frac{8\,a\,b^3\,{(d\,x)}^{7/2}}{7\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(7/2),x)
```

```
[Out] (2*b^4*(d*x)^(11/2))/(11*d^9) - ((2*a^4*d^2)/5 + 8*a^3*b*d^2*x^2)/(d^3*(d*x
)^(5/2)) + (4*a^2*b^2*(d*x)^(3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7)
```

3.679 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=129

$$\frac{2a^6(dx)^{7/2}}{7d}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

```
[Out] 2/7*a^6*(d*x)^(7/2)/d+12/11*a^5*b*(d*x)^(11/2)/d^3+2*a^4*b^2*(d*x)^(15/2)/d
^5+40/19*a^3*b^3*(d*x)^(19/2)/d^7+30/23*a^2*b^4*(d*x)^(23/2)/d^9+4/9*a*b^5*
(d*x)^(27/2)/d^11+2/31*b^6*(d*x)^(31/2)/d^13
```

Rubi [A]
time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {28, 276}

$$\frac{2a^6(dx)^{7/2}}{7d}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(7/2))/(7*d) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (2*a^4*b^2*(
d*x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) + (30*a^2*b^4*(d*x)^(
23/2))/(23*d^9) + (4*a*b^5*(d*x)^(27/2))/(9*d^11) + (2*b^6*(d*x)^(31/2))/(3
1*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3 \, dx &= \frac{\int (dx)^{5/2}\left(ab+b^2x^2\right)^6 \, dx}{b^6}\\ &= \frac{\int \left(a^6b^6(dx)^{5/2}+\frac{6a^5b^7(dx)^{9/2}}{d^2}+\frac{15a^4b^8(dx)^{13/2}}{d^4}+\frac{20a^3b^9(dx)^{17/2}}{d^6}+\frac{15a^2b^{10}(dx)^{21/}}{d^8}\right.}{b^6}\\ &= \frac{2a^6(dx)^{7/2}}{7d}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{30a^2b^4}{2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.60

$$\frac{2x(dx)^{5/2}\left(1341153a^6+5120766a^5bx^2+9388071a^4b^2x^4+9882180a^3b^3x^6+6122655a^2b^4x^8+2086238ab^5x^{10}+302841b^6x^{12}\right)}{9388071}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(1341153*a^6 + 5120766*a^5*b*x^2 + 9388071*a^4*b^2*x^4 + 9
882180*a^3*b^3*x^6 + 6122655*a^2*b^4*x^8 + 2086238*a*b^5*x^10 + 302841*b^6*
x^12))/9388071
```

Maple [A]
time = 0.06, size = 105, normalized size = 0.81

method	result
gosper	$\frac{2x\left(302841b^6x^{12}+2086238a\,b^5x^{10}+6122655a^2b^4x^8+9882180a^3b^3x^6+9388071a^4b^2x^4+5120766a^5b\,x^2+1341153a^6\right)(dx)}{9388071}$
trager	$\frac{2d^2x^3\left(302841b^6x^{12}+2086238a\,b^5x^{10}+6122655a^2b^4x^8+9882180a^3b^3x^6+9388071a^4b^2x^4+5120766a^5b\,x^2+1341153a^6\right)}{9388071}$
risch	$\frac{2d^3x^4\left(302841b^6x^{12}+2086238a\,b^5x^{10}+6122655a^2b^4x^8+9882180a^3b^3x^6+9388071a^4b^2x^4+5120766a^5b\,x^2+1341153a^6\right)}{9388071\sqrt{dx}}$
derivativedivides	$\frac{\frac{2b^6(dx)^{\frac{31}{2}}}{31}+\frac{4a\,b^5d^2(dx)^{\frac{27}{2}}}{9}+\frac{30a^2d^4b^4(dx)^{\frac{23}{2}}}{23}+\frac{40a^3d^6b^3(dx)^{\frac{19}{2}}}{19}+2a^4d^8b^2(dx)^{\frac{15}{2}}+\frac{12a^5d^{10}b(dx)^{\frac{11}{2}}}{11}+\frac{2a^6d^{12}(dx)^{\frac{7}{2}}}{7}}{d^{13}}$
default	$\frac{\frac{2b^6(dx)^{\frac{31}{2}}}{31}+\frac{4a\,b^5d^2(dx)^{\frac{27}{2}}}{9}+\frac{30a^2d^4b^4(dx)^{\frac{23}{2}}}{23}+\frac{40a^3d^6b^3(dx)^{\frac{19}{2}}}{19}+2a^4d^8b^2(dx)^{\frac{15}{2}}+\frac{12a^5d^{10}b(dx)^{\frac{11}{2}}}{11}+\frac{2a^6d^{12}(dx)^{\frac{7}{2}}}{7}}{d^{13}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^13*(1/31*b^6*(d*x)^(31/2)+2/9*a*b^5*d^2*(d*x)^(27/2)+15/23*a^2*d^4*b^4*
(d*x)^(23/2)+20/19*a^3*d^6*b^3*(d*x)^(19/2)+a^4*d^8*b^2*(d*x)^(15/2)+6/11*a
^5*d^10*b*(d*x)^(11/2)+1/7*a^6*d^12*(d*x)^(7/2))
```

Maxima [A]
time = 0.30, size = 105, normalized size = 0.81

$$\frac{2\left(302841\,(dx)^{\frac{31}{2}}\,b^6+2086238\,(dx)^{\frac{27}{2}}\,ab^5d^2+6122655\,(dx)^{\frac{23}{2}}\,a^2b^4d^4+9882180\,(dx)^{\frac{19}{2}}\,a^3b^3d^6+9388071\,(dx)^{\frac{15}{2}}\,a^4b^2d^8+5120766\,(dx)^{\frac{11}{2}}\,a^5bd^{10}+1341153\,(dx)^{\frac{7}{2}}\,a^6d^{12}\right)}{9388071\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2/9388071*(302841*(d*x)^(31/2)*b^6 + 2086238*(d*x)^(27/2)*a*b^5*d^2 + 61226
55*(d*x)^(23/2)*a^2*b^4*d^4 + 9882180*(d*x)^(19/2)*a^3*b^3*d^6 + 9388071*(d
```

```
*x)^(15/2)*a^4*b^2*d^8 + 5120766*(d*x)^(11/2)*a^5*b*d^10 + 1341153*(d*x)^(7
/2)*a^6*d^12)/d^13
```

Fricas [A]

time = 0.34, size = 96, normalized size = 0.74

$$\frac{2}{9388071}\left(302841\,b^{6}d^{2}x^{15}+2086238\,ab^{5}d^{2}x^{13}+6122655\,a^{2}b^{4}d^{2}x^{11}+9882180\,a^{3}b^{3}d^{2}x^{9}+9388071\,a^{4}b^{2}d^{2}x^{7}+5120766\,a^{5}bd^{2}x^{5}+1341153\,a^{6}d^{2}x^{3}\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 2/9388071*(302841*b^6*d^2*x^15 + 2086238*a*b^5*d^2*x^13 + 6122655*a^2*b^4*d
^2*x^11 + 9882180*a^3*b^3*d^2*x^9 + 9388071*a^4*b^2*d^2*x^7 + 5120766*a^5*b
*d^2*x^5 + 1341153*a^6*d^2*x^3)*sqrt(d*x)
```

Sympy [A]

time = 1.07, size = 128, normalized size = 0.99

$$\frac{2a^{6}x(dx)^{\frac{5}{2}}}{7}+\frac{12a^{5}bx^{3}(dx)^{\frac{5}{2}}}{11}+2a^{4}b^{2}x^{5}(dx)^{\frac{5}{2}}+\frac{40a^{3}b^{3}x^{7}(dx)^{\frac{5}{2}}}{19}+\frac{30a^{2}b^{4}x^{9}(dx)^{\frac{5}{2}}}{23}+\frac{4ab^{5}x^{11}(dx)^{\frac{5}{2}}}{9}+\frac{2b^{6}x^{13}(dx)^{\frac{5}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*x*(d*x)**(5/2)/7 + 12*a**5*b*x**3*(d*x)**(5/2)/11 + 2*a**4*b**2*x**5
*(d*x)**(5/2) + 40*a**3*b**3*x**7*(d*x)**(5/2)/19 + 30*a**2*b**4*x**9*(d*x)
**(5/2)/23 + 4*a*b**5*x**11*(d*x)**(5/2)/9 + 2*b**6*x**13*(d*x)**(5/2)/31
```

Giac [A]

time = 4.11, size = 124, normalized size = 0.96

$$\frac{2}{31}\sqrt{dx}\,b^{6}d^{2}x^{15}+\frac{4}{9}\sqrt{dx}\,ab^{5}d^{2}x^{13}+\frac{30}{23}\sqrt{dx}\,a^{2}b^{4}d^{2}x^{11}+\frac{40}{19}\sqrt{dx}\,a^{3}b^{3}d^{2}x^{9}+2\sqrt{dx}\,a^{4}b^{2}d^{2}x^{7}+\frac{12}{11}\sqrt{dx}\,a^{5}bd^{2}x^{5}+\frac{2}{7}\sqrt{dx}\,a^{6}d^{2}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 2/31*sqrt(d*x)*b^6*d^2*x^15 + 4/9*sqrt(d*x)*a*b^5*d^2*x^13 + 30/23*sqrt(d*x
)*a^2*b^4*d^2*x^11 + 40/19*sqrt(d*x)*a^3*b^3*d^2*x^9 + 2*sqrt(d*x)*a^4*b^2*
d^2*x^7 + 12/11*sqrt(d*x)*a^5*b*d^2*x^5 + 2/7*sqrt(d*x)*a^6*d^2*x^3
```

Mupad [B]

time = 0.04, size = 103, normalized size = 0.80

$$\frac{2\,a^6\,{\left(d\,x\right)}^{7/2}}{7\,d}+\frac{2\,b^6\,{\left(d\,x\right)}^{31/2}}{31\,d^{13}}+\frac{2\,a^4\,b^2\,{\left(d\,x\right)}^{15/2}}{d^5}+\frac{40\,a^3\,b^3\,{\left(d\,x\right)}^{19/2}}{19\,d^7}+\frac{30\,a^2\,b^4\,{\left(d\,x\right)}^{23/2}}{23\,d^9}+\frac{12\,a^5\,b\,{\left(d\,x\right)}^{11/2}}{11\,d^3}+\frac{4\,a\,b^5\,{\left(d\,x\right)}^{27/2}}{9\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*a^6*(d*x)^(7/2))/(7*d) + (2*b^6*(d*x)^(31/2))/(31*d^13) + (2*a^4*b^2*(d*
x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) + (30*a^2*b^4*(d*x)^(23
/2))/(23*d^9) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (4*a*b^5*(d*x)^(27/2))/(
9*d^11)
```

3.680 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=131

$$\frac{2a^6(dx)^{5/2}}{5d}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

```
[Out] 2/5*a^6*(d*x)^(5/2)/d+4/3*a^5*b*(d*x)^(9/2)/d^3+30/13*a^4*b^2*(d*x)^(13/2)/
d^5+40/17*a^3*b^3*(d*x)^(17/2)/d^7+10/7*a^2*b^4*(d*x)^(21/2)/d^9+12/25*a*b^
5*(d*x)^(25/2)/d^11+2/29*b^6*(d*x)^(29/2)/d^13
```

Rubi [A]

time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {28, 276}

$$\frac{2a^6(dx)^{5/2}}{5d}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(5/2))/(5*d) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (30*a^4*b^2*(d*
x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7) + (10*a^2*b^4*(d*x
)^(21/2))/(7*d^9) + (12*a*b^5*(d*x)^(25/2))/(25*d^11) + (2*b^6*(d*x)^(29/2)
)/(29*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int (dx)^{3/2}\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\int \left(a^6b^6(dx)^{3/2}+\frac{6a^5b^7(dx)^{7/2}}{d^2}+\frac{15a^4b^8(dx)^{11/2}}{d^4}+\frac{20a^3b^9(dx)^{15/2}}{d^6}+\frac{15a^2b^{10}(dx)^{1}}{d^8}\right.}{b^6}\\ &= \frac{2a^6(dx)^{5/2}}{5d}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+10a^2b\end{aligned}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.59

$$\frac{2x(dx)^{3/2}\left(672945a^6+2243150a^5bx^2+3882375a^4b^2x^4+3958500a^3b^3x^6+2403375a^2b^4x^8+807534ab^5x^{10}+116025b^6x^{12}\right)}{3364725}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*(d*x)^(3/2)*(672945*a^6 + 2243150*a^5*b*x^2 + 3882375*a^4*b^2*x^4 + 39
58500*a^3*b^3*x^6 + 2403375*a^2*b^4*x^8 + 807534*a*b^5*x^10 + 116025*b^6*x^
12))/3364725
```

Maple [A]
time = 0.06, size = 106, normalized size = 0.81

method	result
gosper	$\frac{2x\left(116025b^6x^{12}+807534a\,b^5x^{10}+2403375a^2b^4x^8+3958500a^3b^3x^6+3882375a^4b^2x^4+2243150a^5b\,x^2+672945a^6\right)(dx)^{\frac{3}{2}}}{3364725}$
trager	$\frac{2d\,x^2\left(116025b^6x^{12}+807534a\,b^5x^{10}+2403375a^2b^4x^8+3958500a^3b^3x^6+3882375a^4b^2x^4+2243150a^5b\,x^2+672945a^6\right)\sqrt{dx}}{3364725}$
risch	$\frac{2d^2x^3\left(116025b^6x^{12}+807534a\,b^5x^{10}+2403375a^2b^4x^8+3958500a^3b^3x^6+3882375a^4b^2x^4+2243150a^5b\,x^2+672945a^6\right)}{3364725\sqrt{dx}}$
derivativedivides	$\frac{\frac{2b^6(dx)^{\frac{29}{2}}}{29}+\frac{12a\,b^5d^2(dx)^{\frac{25}{2}}}{25}+\frac{10a^2d^4b^4(dx)^{\frac{21}{2}}}{7}+\frac{40a^3d^6b^3(dx)^{\frac{17}{2}}}{17}+\frac{30a^4d^8b^2(dx)^{\frac{13}{2}}}{13}+\frac{4a^5d^{10}b(dx)^{\frac{9}{2}}}{3}+\frac{2a^6d^{12}(dx)^{\frac{5}{2}}}{5}}{d^{13}}$
default	$\frac{\frac{2b^6(dx)^{\frac{29}{2}}}{29}+\frac{12a\,b^5d^2(dx)^{\frac{25}{2}}}{25}+\frac{10a^2d^4b^4(dx)^{\frac{21}{2}}}{7}+\frac{40a^3d^6b^3(dx)^{\frac{17}{2}}}{17}+\frac{30a^4d^8b^2(dx)^{\frac{13}{2}}}{13}+\frac{4a^5d^{10}b(dx)^{\frac{9}{2}}}{3}+\frac{2a^6d^{12}(dx)^{\frac{5}{2}}}{5}}{d^{13}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^13*(1/29*b^6*(d*x)^(29/2)+6/25*a*b^5*d^2*(d*x)^(25/2)+5/7*a^2*d^4*b^4*(
d*x)^(21/2)+20/17*a^3*d^6*b^3*(d*x)^(17/2)+15/13*a^4*d^8*b^2*(d*x)^(13/2)+2
/3*a^5*d^10*b*(d*x)^(9/2)+1/5*a^6*d^12*(d*x)^(5/2))
```

Maxima [A]
time = 0.30, size = 105, normalized size = 0.80

$$\frac{2\left(116025\,(dx)^{\frac{29}{2}}b^6+807534\,(dx)^{\frac{25}{2}}ab^5d^2+2403375\,(dx)^{\frac{21}{2}}a^2b^4d^4+3958500\,(dx)^{\frac{17}{2}}a^3b^3d^6+3882375\,(dx)^{\frac{13}{2}}a^4b^2d^8+2243150\,(dx)^{\frac{9}{2}}a^5bd^{10}+672945\,(dx)^{\frac{5}{2}}a^6d^{12}\right)}{3364725\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 2/3364725*(116025*(d*x)^(29/2)*b^6 + 807534*(d*x)^(25/2)*a*b^5*d^2 + 240337
5*(d*x)^(21/2)*a^2*b^4*d^4 + 3958500*(d*x)^(17/2)*a^3*b^3*d^6 + 3882375*(d*
```

```
x)^(13/2)*a^4*b^2*d^8 + 2243150*(d*x)^(9/2)*a^5*b*d^10 + 672945*(d*x)^(5/2)
*a^6*d^12)/d^13
```

Fricas [A]
time = 0.33, size = 82, normalized size = 0.63

$$\frac{2}{3364725}\left(116025\,b^{6}dx^{14}+807534\,ab^{5}dx^{12}+2403375\,a^{2}b^{4}dx^{10}+3958500\,a^{3}b^{3}dx^{8}+3882375\,a^{4}b^{2}dx^{6}+2243150\,a^{5}bdx^{4}+672945\,a^{6}dx^{2}\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 2/3364725*(116025*b^6*d*x^14 + 807534*a*b^5*d*x^12 + 2403375*a^2*b^4*d*x^10
 + 3958500*a^3*b^3*d*x^8 + 3882375*a^4*b^2*d*x^6 + 2243150*a^5*b*d*x^4 + 67
2945*a^6*d*x^2)*sqrt(d*x)
```

Sympy [A]
time = 0.69, size = 129, normalized size = 0.98

$$\frac{2a^{6}x(dx)^{\frac{3}{2}}}{5}+\frac{4a^{5}bx^{3}(dx)^{\frac{3}{2}}}{3}+\frac{30a^{4}b^{2}x^{5}(dx)^{\frac{3}{2}}}{13}+\frac{40a^{3}b^{3}x^{7}(dx)^{\frac{3}{2}}}{17}+\frac{10a^{2}b^{4}x^{9}(dx)^{\frac{3}{2}}}{7}+\frac{12ab^{5}x^{11}(dx)^{\frac{3}{2}}}{25}+\frac{2b^{6}x^{13}(dx)^{\frac{3}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*x*(d*x)**(3/2)/5 + 4*a**5*b*x**3*(d*x)**(3/2)/3 + 30*a**4*b**2*x**5*
(d*x)**(3/2)/13 + 40*a**3*b**3*x**7*(d*x)**(3/2)/17 + 10*a**2*b**4*x**9*(d*
x)**(3/2)/7 + 12*a*b**5*x**11*(d*x)**(3/2)/25 + 2*b**6*x**13*(d*x)**(3/2)/2
9
```

Giac [A]
time = 4.29, size = 106, normalized size = 0.81

$$\frac{2}{3364725}\left(116025\,\sqrt{dx}\,b^{6}x^{14}+807534\,\sqrt{dx}\,ab^{5}x^{12}+2403375\,\sqrt{dx}\,a^{2}b^{4}x^{10}+3958500\,\sqrt{dx}\,a^{3}b^{3}x^{8}+3882375\,\sqrt{dx}\,a^{4}b^{2}x^{6}+2243150\,\sqrt{dx}\,a^{5}bx^{4}+672945\,\sqrt{dx}\,a^{6}x^{2}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 2/3364725*(116025*sqrt(d*x)*b^6*x^14 + 807534*sqrt(d*x)*a*b^5*x^12 + 240337
5*sqrt(d*x)*a^2*b^4*x^10 + 3958500*sqrt(d*x)*a^3*b^3*x^8 + 3882375*sqrt(d*x
)*a^4*b^2*x^6 + 2243150*sqrt(d*x)*a^5*b*x^4 + 672945*sqrt(d*x)*a^6*x^2)*d
```

Mupad [B]
time = 0.04, size = 103, normalized size = 0.79

$$\frac{2\,a^{6}\,{\left(d\,x\right)}^{5/2}}{5\,d}+\frac{2\,b^{6}\,{\left(d\,x\right)}^{29/2}}{29\,d^{13}}+\frac{30\,a^{4}\,b^{2}\,{\left(d\,x\right)}^{13/2}}{13\,d^{5}}+\frac{40\,a^{3}\,b^{3}\,{\left(d\,x\right)}^{17/2}}{17\,d^{7}}+\frac{10\,a^{2}\,b^{4}\,{\left(d\,x\right)}^{21/2}}{7\,d^{9}}+\frac{4\,a^{5}\,b\,{\left(d\,x\right)}^{9/2}}{3\,d^{3}}+\frac{12\,a\,b^{5}\,{\left(d\,x\right)}^{25/2}}{25\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*a^6*(d*x)^(5/2))/(5*d) + (2*b^6*(d*x)^(29/2))/(29*d^13) + (30*a^4*b^2*(d
*x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7) + (10*a^2*b^4*(d*
x)^(21/2))/(7*d^9) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (12*a*b^5*(d*x)^(25/2)
)/(25*d^11)
```

3.681 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=131

$$\frac{2a^6(dx)^{3/2}}{3d}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

```
[Out] 2/3*a^6*(d*x)^(3/2)/d+12/7*a^5*b*(d*x)^(7/2)/d^3+30/11*a^4*b^2*(d*x)^(11/2)
/d^5+8/3*a^3*b^3*(d*x)^(15/2)/d^7+30/19*a^2*b^4*(d*x)^(19/2)/d^9+12/23*a*b^
5*(d*x)^(23/2)/d^11+2/27*b^6*(d*x)^(27/2)/d^13
```

Rubi [A]

time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {28, 276}

$$\frac{2a^6(dx)^{3/2}}{3d}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(3/2))/(3*d) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (30*a^4*b^2*(d
*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7) + (30*a^2*b^4*(d*x)
^(19/2))/(19*d^9) + (12*a*b^5*(d*x)^(23/2))/(23*d^11) + (2*b^6*(d*x)^(27/2)
)/(27*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx &= \frac{\int \sqrt{dx}\left(ab+b^2x^2\right)^6\,dx}{b^6}\\ &= \frac{\int \left(a^6b^6\sqrt{dx}+\frac{6a^5b^7(dx)^{5/2}}{d^2}+\frac{15a^4b^8(dx)^{9/2}}{d^4}+\frac{20a^3b^9(dx)^{13/2}}{d^6}+\frac{15a^2b^{10}(dx)^{17/2}}{d^8}+\right.}{b^6}\\ &= \frac{2a^6(dx)^{3/2}}{3d}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^2b^4}{19}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 77, normalized size = 0.59

$$\frac{2x\sqrt{dx}\,(302841a^6+778734a^5bx^2+1238895a^4b^2x^4+1211364a^3b^3x^6+717255a^2b^4x^8+237006ab^5x^{10}+33649b^6x^{12})}{908523}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*Sqrt[d*x]*(302841*a^6 + 778734*a^5*b*x^2 + 1238895*a^4*b^2*x^4 + 12113
64*a^3*b^3*x^6 + 717255*a^2*b^4*x^8 + 237006*a*b^5*x^10 + 33649*b^6*x^12))/
908523
```

Maple [A]
time = 0.08, size = 106, normalized size = 0.81

method	result
gosper	$\frac{2x(33649b^6x^{12}+237006a\,b^5x^{10}+717255a^2b^4x^8+1211364a^3b^3x^6+1238895a^4b^2x^4+778734a^5b\,x^2+302841a^6)\sqrt{dx}}{908523}$
trager	$\frac{2x(33649b^6x^{12}+237006a\,b^5x^{10}+717255a^2b^4x^8+1211364a^3b^3x^6+1238895a^4b^2x^4+778734a^5b\,x^2+302841a^6)\sqrt{dx}}{908523}$
risch	$\frac{2d\,x^2(33649b^6x^{12}+237006a\,b^5x^{10}+717255a^2b^4x^8+1211364a^3b^3x^6+1238895a^4b^2x^4+778734a^5b\,x^2+302841a^6)}{908523\sqrt{dx}}$
derivativedivides	$\frac{\frac{2b^6(dx)^{\frac{27}{2}}}{27}+\frac{12a\,b^5d^2(dx)^{\frac{23}{2}}}{23}+\frac{30a^2d^4b^4(dx)^{\frac{19}{2}}}{19}+\frac{8a^3d^6b^3(dx)^{\frac{15}{2}}}{3}+\frac{30a^4d^8b^2(dx)^{\frac{11}{2}}}{11}+\frac{12a^5d^{10}b(dx)^{\frac{7}{2}}}{7}+\frac{2a^6d^{12}(dx)^{\frac{3}{2}}}{3}}{d^{13}}$
default	$\frac{\frac{2b^6(dx)^{\frac{27}{2}}}{27}+\frac{12a\,b^5d^2(dx)^{\frac{23}{2}}}{23}+\frac{30a^2d^4b^4(dx)^{\frac{19}{2}}}{19}+\frac{8a^3d^6b^3(dx)^{\frac{15}{2}}}{3}+\frac{30a^4d^8b^2(dx)^{\frac{11}{2}}}{11}+\frac{12a^5d^{10}b(dx)^{\frac{7}{2}}}{7}+\frac{2a^6d^{12}(dx)^{\frac{3}{2}}}{3}}{d^{13}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^13*(1/27*b^6*(d*x)^(27/2)+6/23*a*b^5*d^2*(d*x)^(23/2)+15/19*a^2*d^4*b^4
*(d*x)^(19/2)+4/3*a^3*d^6*b^3*(d*x)^(15/2)+15/11*a^4*d^8*b^2*(d*x)^(11/2)+6
/7*a^5*d^10*b*(d*x)^(7/2)+1/3*a^6*d^12*(d*x)^(3/2))
```

Maxima [A]
time = 0.30, size = 105, normalized size = 0.80

$$\frac{2\left(33649\,(dx)^{\frac{27}{2}}\,b^6+237006\,(dx)^{\frac{23}{2}}\,ab^5d^2+717255\,(dx)^{\frac{19}{2}}\,a^2b^4d^4+1211364\,(dx)^{\frac{15}{2}}\,a^3b^3d^6+1238895\,(dx)^{\frac{11}{2}}\,a^4b^2d^8+778734\,(dx)^{\frac{7}{2}}\,a^5bd^{10}+302841\,(dx)^{\frac{3}{2}}\,a^6d^{12}\right)}{908523\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/908523*(33649*(d*x)^(27/2)*b^6 + 237006*(d*x)^(23/2)*a*b^5*d^2 + 717255*(
d*x)^(19/2)*a^2*b^4*d^4 + 1211364*(d*x)^(15/2)*a^3*b^3*d^6 + 1238895*(d*x)^
```

```
(11/2)*a^4*b^2*d^8 + 778734*(d*x)^(7/2)*a^5*b*d^10 + 302841*(d*x)^(3/2)*a^6
*d^12)/d^13
```

Fricas [A]
time = 0.34, size = 73, normalized size = 0.56

$$\frac{2}{908523}\left(33649\,b^6x^{13}+237006\,ab^5x^{11}+717255\,a^2b^4x^9+1211364\,a^3b^3x^7+1238895\,a^4b^2x^5+778734\,a^5bx^3+302841\,a^6x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/908523*(33649*b^6*x^13 + 237006*a*b^5*x^11 + 717255*a^2*b^4*x^9 + 1211364
*a^3*b^3*x^7 + 1238895*a^4*b^2*x^5 + 778734*a^5*b*x^3 + 302841*a^6*x)*sqrt(
d*x)
```

Sympy [A]
time = 0.43, size = 129, normalized size = 0.98

$$\frac{2a^6x\sqrt{dx}}{3}+\frac{12a^5bx^3\sqrt{dx}}{7}+\frac{30a^4b^2x^5\sqrt{dx}}{11}+\frac{8a^3b^3x^7\sqrt{dx}}{3}+\frac{30a^2b^4x^9\sqrt{dx}}{19}+\frac{12ab^5x^{11}\sqrt{dx}}{23}+\frac{2b^6x^{13}\sqrt{dx}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3*(d*x)**(1/2),x)
```

```
[Out] 2*a**6*x*sqrt(d*x)/3 + 12*a**5*b*x**3*sqrt(d*x)/7 + 30*a**4*b**2*x**5*sqrt(
d*x)/11 + 8*a**3*b**3*x**7*sqrt(d*x)/3 + 30*a**2*b**4*x**9*sqrt(d*x)/19 + 1
2*a*b**5*x**11*sqrt(d*x)/23 + 2*b**6*x**13*sqrt(d*x)/27
```

Giac [A]
time = 3.34, size = 101, normalized size = 0.77

$$\frac{2}{27}\sqrt{dx}\,b^6x^{13}+\frac{12}{23}\sqrt{dx}\,ab^5x^{11}+\frac{30}{19}\sqrt{dx}\,a^2b^4x^9+\frac{8}{3}\sqrt{dx}\,a^3b^3x^7+\frac{30}{11}\sqrt{dx}\,a^4b^2x^5+\frac{12}{7}\sqrt{dx}\,a^5bx^3+\frac{2}{3}\sqrt{dx}\,a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(d*x)*b^6*x^13 + 12/23*sqrt(d*x)*a*b^5*x^11 + 30/19*sqrt(d*x)*a^2*
b^4*x^9 + 8/3*sqrt(d*x)*a^3*b^3*x^7 + 30/11*sqrt(d*x)*a^4*b^2*x^5 + 12/7*sq
rt(d*x)*a^5*b*x^3 + 2/3*sqrt(d*x)*a^6*x
```

Mupad [B]
time = 0.04, size = 103, normalized size = 0.79

$$\frac{2\,a^6\,{\left(d\,x\right)}^{3/2}}{3\,d}+\frac{2\,b^6\,{\left(d\,x\right)}^{27/2}}{27\,d^{13}}+\frac{30\,a^4\,b^2\,{\left(d\,x\right)}^{11/2}}{11\,d^5}+\frac{8\,a^3\,b^3\,{\left(d\,x\right)}^{15/2}}{3\,d^7}+\frac{30\,a^2\,b^4\,{\left(d\,x\right)}^{19/2}}{19\,d^9}+\frac{12\,a^5\,b\,{\left(d\,x\right)}^{7/2}}{7\,d^3}+\frac{12\,a\,b^5\,{\left(d\,x\right)}^{23/2}}{23\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*a^6*(d*x)^(3/2))/(3*d) + (2*b^6*(d*x)^(27/2))/(27*d^13) + (30*a^4*b^2*(d
*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7) + (30*a^2*b^4*(d*x)
^(19/2))/(19*d^9) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (12*a*b^5*(d*x)^(23/2)
)/(23*d^11)
```

3.682 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{\sqrt{dx}}\,dx$

Optimal. Leaf size=129

$$\frac{2a^6\sqrt{dx}}{d}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

```
[Out] 12/5*a^5*b*(d*x)^(5/2)/d^3+10/3*a^4*b^2*(d*x)^(9/2)/d^5+40/13*a^3*b^3*(d*x)
^(13/2)/d^7+30/17*a^2*b^4*(d*x)^(17/2)/d^9+4/7*a*b^5*(d*x)^(21/2)/d^11+2/25
*b^6*(d*x)^(25/2)/d^13+2*a^6*(d*x)^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {28, 276}

$$\frac{2a^6\sqrt{dx}}{d}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

```
[Out] (2*a^6*Sqrt[d*x])/d + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (10*a^4*b^2*(d*x)^(9
/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30*a^2*b^4*(d*x)^(17/2
))/(17*d^9) + (4*a*b^5*(d*x)^(21/2))/(7*d^11) + (2*b^6*(d*x)^(25/2))/(25*d^
13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{\sqrt{dx}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{\sqrt{dx}}\,dx}{b^6}$$

$$= \frac{\int \left(\frac{a^6b^6}{\sqrt{dx}} + \frac{6a^5b^7(dx)^{3/2}}{d^2} + \frac{15a^4b^8(dx)^{7/2}}{d^4} + \frac{20a^3b^9(dx)^{11/2}}{d^6} + \frac{15a^2b^{10}(dx)^{15/2}}{d^8} + \frac{6ab^{11}(dx)^{19}}{d^{10}}\right.}{b^6}$$

$$= \frac{2a^6\sqrt{dx}}{d} + \frac{12a^5b(dx)^{5/2}}{5d^3} + \frac{10a^4b^2(dx)^{9/2}}{3d^5} + \frac{40a^3b^3(dx)^{13/2}}{13d^7} + \frac{30a^2b^4(dx)^{17/2}}{17d^9}$$

Mathematica [A]
time = 0.02, size = 77, normalized size = 0.60

$$\frac{2x(116025a^6+139230a^5bx^2+193375a^4b^2x^4+178500a^3b^3x^6+102375a^2b^4x^8+33150ab^5x^{10}+4641b^6x^{12})}{116025\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

```
[Out] (2*x*(116025*a^6 + 139230*a^5*b*x^2 + 193375*a^4*b^2*x^4 + 178500*a^3*b^3*x
^6 + 102375*a^2*b^4*x^8 + 33150*a*b^5*x^10 + 4641*b^6*x^12))/(116025*Sqrt[d
*x])
```

Maple [A]
time = 0.04, size = 105, normalized size = 0.81

method	result	size
gosper	$\frac{2(4641b^6x^{12}+33150a\,b^5x^{10}+102375a^2b^4x^8+178500a^3b^3x^6+193375a^4b^2x^4+139230a^5b\,x^2+116025a^6)x}{116025\sqrt{dx}}$	74
risch	$\frac{2(4641b^6x^{12}+33150a\,b^5x^{10}+102375a^2b^4x^8+178500a^3b^3x^6+193375a^4b^2x^4+139230a^5b\,x^2+116025a^6)x}{116025\sqrt{dx}}$	74
trager	$\frac{(\frac{2}{25}b^6x^{12}+\frac{4}{7}a\,b^5x^{10}+\frac{30}{17}a^2b^4x^8+\frac{40}{13}a^3b^3x^6+\frac{10}{3}a^4b^2x^4+\frac{12}{5}a^5b\,x^2+2a^6)\sqrt{dx}}{d}$	75
derivativedivides	$\frac{\frac{2b^6(dx)^{\frac{25}{2}}}{25}+\frac{4a\,b^5d^2(dx)^{\frac{21}{2}}}{7}+\frac{30a^2d^4b^4(dx)^{\frac{17}{2}}}{17}+\frac{40a^3d^6b^3(dx)^{\frac{13}{2}}}{13}+\frac{10a^4d^8b^2(dx)^{\frac{9}{2}}}{3}+\frac{12a^5d^{10}b(dx)^{\frac{5}{2}}}{5}+2a^6d^{12}\sqrt{dx}}{d^{13}}$	105
default	$\frac{\frac{2b^6(dx)^{\frac{25}{2}}}{25}+\frac{4a\,b^5d^2(dx)^{\frac{21}{2}}}{7}+\frac{30a^2d^4b^4(dx)^{\frac{17}{2}}}{17}+\frac{40a^3d^6b^3(dx)^{\frac{13}{2}}}{13}+\frac{10a^4d^8b^2(dx)^{\frac{9}{2}}}{3}+\frac{12a^5d^{10}b(dx)^{\frac{5}{2}}}{5}+2a^6d^{12}\sqrt{dx}}{d^{13}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^13*(1/25*b^6*(d*x)^(25/2)+2/7*a*b^5*d^2*(d*x)^(21/2)+15/17*a^2*d^4*b^4*
(d*x)^(17/2)+20/13*a^3*d^6*b^3*(d*x)^(13/2)+5/3*a^4*d^8*b^2*(d*x)^(9/2)+6/5
*a^5*d^10*b*(d*x)^(5/2)+a^6*d^12*(d*x)^(1/2))
```

Maxima [A]
time = 0.29, size = 155, normalized size = 1.20

$$\frac{2\left(116025\sqrt{dx}\,a^6+\frac{4641\,(dx)^{\frac{25}{2}}b^6}{d^{12}}+\frac{33150\,(dx)^{\frac{21}{2}}ab^5}{d^{10}}+\frac{81900\,(dx)^{\frac{17}{2}}a^2b^4}{d^8}+\frac{71400\,(dx)^{\frac{13}{2}}a^3b^3}{d^6}+7735\left(\frac{5\,(dx)^{\frac{9}{2}}b^2}{d^4}+\frac{18\,(dx)^{\frac{5}{2}}ab}{d^2}\right)a^4+175\left(\frac{117\,(dx)^{\frac{17}{2}}b^4}{d^8}+\frac{612\,(dx)^{\frac{13}{2}}ab^3}{d^6}+\frac{884\,(dx)^{\frac{9}{2}}a^2b^2}{d^4}\right)a^2\right)}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/116025*(116025*sqrt(d*x)*a^6 + 4641*(d*x)^(25/2)*b^6/d^12 + 33150*(d*x)^(
21/2)*a*b^5/d^10 + 81900*(d*x)^(17/2)*a^2*b^4/d^8 + 71400*(d*x)^(13/2)*a^3*
b^3/d^6 + 7735*(5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)*a^4 + 175*(
117*(d*x)^(17/2)*b^4/d^8 + 612*(d*x)^(13/2)*a*b^3/d^6 + 884*(d*x)^(9/2)*a^2
*b^2/d^4)*a^2)/d
```

Fricas [A]
time = 0.34, size = 75, normalized size = 0.58

$$\frac{2\,(4641\,b^6x^{12}+33150\,ab^5x^{10}+102375\,a^2b^4x^8+178500\,a^3b^3x^6+193375\,a^4b^2x^4+139230\,a^5bx^2+116025\,a^6)\sqrt{dx}}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*b^6*x^12 + 33150*a*b^5*x^10 + 102375*a^2*b^4*x^8 + 178500*a^
3*b^3*x^6 + 193375*a^4*b^2*x^4 + 139230*a^5*b*x^2 + 116025*a^6)*sqrt(d*x)/d
```

Sympy [A]
time = 0.52, size = 128, normalized size = 0.99

$$\frac{2a^6x}{\sqrt{dx}}+\frac{12a^5bx^3}{5\sqrt{dx}}+\frac{10a^4b^2x^5}{3\sqrt{dx}}+\frac{40a^3b^3x^7}{13\sqrt{dx}}+\frac{30a^2b^4x^9}{17\sqrt{dx}}+\frac{4ab^5x^{11}}{7\sqrt{dx}}+\frac{2b^6x^{13}}{25\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
```

```
[Out] 2*a**6*x/sqrt(d*x) + 12*a**5*b*x**3/(5*sqrt(d*x)) + 10*a**4*b**2*x**5/(3*sq
rt(d*x)) + 40*a**3*b**3*x**7/(13*sqrt(d*x)) + 30*a**2*b**4*x**9/(17*sqrt(d*
x)) + 4*a*b**5*x**11/(7*sqrt(d*x)) + 2*b**6*x**13/(25*sqrt(d*x))
```

Giac [A]
time = 3.88, size = 105, normalized size = 0.81

$$\frac{2\left(4641\sqrt{dx}\,b^6x^{12}+33150\sqrt{dx}\,ab^5x^{10}+102375\sqrt{dx}\,a^2b^4x^8+178500\sqrt{dx}\,a^3b^3x^6+193375\sqrt{dx}\,a^4b^2x^4+139230\sqrt{dx}\,a^5bx^2+116025\sqrt{dx}\,a^6\right)}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/116025*(4641*sqrt(d*x)*b^6*x^12 + 33150*sqrt(d*x)*a*b^5*x^10 + 102375*sqr
t(d*x)*a^2*b^4*x^8 + 178500*sqrt(d*x)*a^3*b^3*x^6 + 193375*sqrt(d*x)*a^4*b^
2*x^4 + 139230*sqrt(d*x)*a^5*b*x^2 + 116025*sqrt(d*x)*a^6)/d
```

Mupad [B]
time = 0.04, size = 103, normalized size = 0.80

$$\frac{2\,a^6\,\sqrt{d\,x}}{d}+\frac{2\,b^6\,{\left(d\,x\right)}^{25/2}}{25\,d^{13}}+\frac{10\,a^4\,b^2\,{\left(d\,x\right)}^{9/2}}{3\,d^5}+\frac{40\,a^3\,b^3\,{\left(d\,x\right)}^{13/2}}{13\,d^7}+\frac{30\,a^2\,b^4\,{\left(d\,x\right)}^{17/2}}{17\,d^9}+\frac{12\,a^5\,b\,{\left(d\,x\right)}^{5/2}}{5\,d^3}+\frac{4\,a\,b^5\,{\left(d\,x\right)}^{21/2}}{7\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(1/2),x)
```

```
[Out] (2*a^6*(d*x)^(1/2))/d + (2*b^6*(d*x)^(25/2))/(25*d^13) + (10*a^4*b^2*(d*x)^
(9/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30*a^2*b^4*(d*x)^(17
/2))/(17*d^9) + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (4*a*b^5*(d*x)^(21/2))/(7*
d^11)
```

3.683 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{3/2}} dx$

Optimal. Leaf size=125

$$-\frac{2a^6}{d\sqrt{dx}}+\frac{4a^5b(dx)^{3/2}}{d^3}+\frac{30a^4b^2(dx)^{7/2}}{7d^5}+\frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

```
[Out] 4*a^5*b*(d*x)^(3/2)/d^3+30/7*a^4*b^2*(d*x)^(7/2)/d^5+40/11*a^3*b^3*(d*x)^(1
1/2)/d^7+2*a^2*b^4*(d*x)^(15/2)/d^9+12/19*a*b^5*(d*x)^(19/2)/d^11+2/23*b^6*
(d*x)^(23/2)/d^13-2*a^6/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {28, 276}

$$-\frac{2a^6}{d\sqrt{dx}}+\frac{4a^5b(dx)^{3/2}}{d^3}+\frac{30a^4b^2(dx)^{7/2}}{7d^5}+\frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

```
[Out] (-2*a^6)/(d*Sqrt[d*x]) + (4*a^5*b*(d*x)^(3/2))/d^3 + (30*a^4*b^2*(d*x)^(7/2
))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^2*b^4*(d*x)^(15/2))/
d^9 + (12*a*b^5*(d*x)^(19/2))/(19*d^11) + (2*b^6*(d*x)^(23/2))/(23*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{3/2}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{(dx)^{3/2}}\,dx}{b^6}$$

$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{3/2}} + \frac{6a^5b^7\sqrt{dx}}{d^2} + \frac{15a^4b^8(dx)^{5/2}}{d^4} + \frac{20a^3b^9(dx)^{9/2}}{d^6} + \frac{15a^2b^{10}(dx)^{13/2}}{d^8} + \frac{6ab^{11}(dx)^{17/}}{d^{10}}\right.}{b^6}$$

$$= -\frac{2a^6}{d\sqrt{dx}} + \frac{4a^5b(dx)^{3/2}}{d^3} + \frac{30a^4b^2(dx)^{7/2}}{7d^5} + \frac{40a^3b^3(dx)^{11/2}}{11d^7} + \frac{2a^2b^4(dx)^{15/2}}{d^9} + $$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.62

$$-\frac{2x(33649a^6-67298a^5bx^2-72105a^4b^2x^4-61180a^3b^3x^6-33649a^2b^4x^8-10626ab^5x^{10}-1463b^6x^{12})}{33649(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

```
[Out] (-2*x*(33649*a^6 - 67298*a^5*b*x^2 - 72105*a^4*b^2*x^4 - 61180*a^3*b^3*x^6
- 33649*a^2*b^4*x^8 - 10626*a*b^5*x^10 - 1463*b^6*x^12))/(33649*(d*x)^(3/2)
)
```

Maple [A]
time = 0.04, size = 105, normalized size = 0.84

method	result	size
gosper	$-\frac{2(-1463b^6x^{12}-10626a\,b^5x^{10}-33649a^2b^4x^8-61180a^3b^3x^6-72105a^4b^2x^4-67298a^5b\,x^2+33649a^6)x}{33649(dx)^{\frac{3}{2}}}$	74
risch	$-\frac{2(-1463b^6x^{12}-10626a\,b^5x^{10}-33649a^2b^4x^8-61180a^3b^3x^6-72105a^4b^2x^4-67298a^5b\,x^2+33649a^6)}{33649d\sqrt{dx}}$	76
trager	$-\frac{2(-1463b^6x^{12}-10626a\,b^5x^{10}-33649a^2b^4x^8-61180a^3b^3x^6-72105a^4b^2x^4-67298a^5b\,x^2+33649a^6)\sqrt{dx}}{33649d^2x}$	79
derivativedivides	$\frac{\frac{2b^6(dx)^{\frac{23}{2}}}{23}+\frac{12a\,b^5d^2(dx)^{\frac{19}{2}}}{19}+2a^2b^4d^4(dx)^{\frac{15}{2}}+\frac{40a^3b^3d^6(dx)^{\frac{11}{2}}}{11}+\frac{30a^4b^2d^8(dx)^{\frac{7}{2}}}{7}+4a^5b\,d^{10}(dx)^{\frac{3}{2}}-\frac{2a^6d^{12}}{\sqrt{dx}}}{d^{13}}$	105
default	$\frac{\frac{2b^6(dx)^{\frac{23}{2}}}{23}+\frac{12a\,b^5d^2(dx)^{\frac{19}{2}}}{19}+2a^2b^4d^4(dx)^{\frac{15}{2}}+\frac{40a^3b^3d^6(dx)^{\frac{11}{2}}}{11}+\frac{30a^4b^2d^8(dx)^{\frac{7}{2}}}{7}+4a^5b\,d^{10}(dx)^{\frac{3}{2}}-\frac{2a^6d^{12}}{\sqrt{dx}}}{d^{13}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^13*(1/23*b^6*(d*x)^(23/2)+6/19*a*b^5*d^2*(d*x)^(19/2)+a^2*b^4*d^4*(d*x)
^(15/2)+20/11*a^3*b^3*d^6*(d*x)^(11/2)+15/7*a^4*b^2*d^8*(d*x)^(7/2)+2*a^5*b
*d^10*(d*x)^(3/2)-a^6*d^12/(d*x)^(1/2))
```

Maxima [A]
time = 0.31, size = 108, normalized size = 0.86

$$-\frac{2\left(\frac{33649\,a^6}{\sqrt{dx}}-\frac{1463\,(dx)^{\frac{23}{2}}b^6+10626\,(dx)^{\frac{19}{2}}ab^5d^2+33649\,(dx)^{\frac{15}{2}}a^2b^4d^4+61180\,(dx)^{\frac{11}{2}}a^3b^3d^6+72105\,(dx)^{\frac{7}{2}}a^4b^2d^8+67298\,(dx)^{\frac{3}{2}}a^5bd^{10}}{d^{12}}\right)}{33649\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/33649*(33649*a^6/sqrt(d*x) - (1463*(d*x)^(23/2)*b^6 + 10626*(d*x)^(19/2)
*a*b^5*d^2 + 33649*(d*x)^(15/2)*a^2*b^4*d^4 + 61180*(d*x)^(11/2)*a^3*b^3*d^
6 + 72105*(d*x)^(7/2)*a^4*b^2*d^8 + 67298*(d*x)^(3/2)*a^5*b*d^10)/d^12)/d
```

Fricas [A]
time = 0.34, size = 78, normalized size = 0.62

$$\frac{2\,(1463\,b^6x^{12}+10626\,ab^5x^{10}+33649\,a^2b^4x^8+61180\,a^3b^3x^6+72105\,a^4b^2x^4+67298\,a^5bx^2-33649\,a^6)\sqrt{dx}}{33649\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/33649*(1463*b^6*x^12 + 10626*a*b^5*x^10 + 33649*a^2*b^4*x^8 + 61180*a^3*b
^3*x^6 + 72105*a^4*b^2*x^4 + 67298*a^5*b*x^2 - 33649*a^6)*sqrt(d*x)/(d^2*x)
```

Sympy [A]
time = 0.55, size = 124, normalized size = 0.99

$$-\frac{2a^6x}{(dx)^{\frac{3}{2}}}+\frac{4a^5bx^3}{(dx)^{\frac{3}{2}}}+\frac{30a^4b^2x^5}{7\,(dx)^{\frac{3}{2}}}+\frac{40a^3b^3x^7}{11\,(dx)^{\frac{3}{2}}}+\frac{2a^2b^4x^9}{(dx)^{\frac{3}{2}}}+\frac{12ab^5x^{11}}{19\,(dx)^{\frac{3}{2}}}+\frac{2b^6x^{13}}{23\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(3/2),x)
```

```
[Out] -2*a**6*x/(d*x)**(3/2) + 4*a**5*b*x**3/(d*x)**(3/2) + 30*a**4*b**2*x**5/(7*
(d*x)**(3/2)) + 40*a**3*b**3*x**7/(11*(d*x)**(3/2)) + 2*a**2*b**4*x**9/(d*x
)**(3/2) + 12*a*b**5*x**11/(19*(d*x)**(3/2)) + 2*b**6*x**13/(23*(d*x)**(3/2
))
```

Giac [A]
time = 3.85, size = 127, normalized size = 1.02

$$-\frac{2\left(\frac{33649\,a^6}{\sqrt{dx}}-\frac{1463\,\sqrt{dx}\,b^6d^{275}x^{11}+10626\,\sqrt{dx}\,ab^5d^{275}x^9+33649\,\sqrt{dx}\,a^2b^4d^{275}x^7+61180\,\sqrt{dx}\,a^3b^3d^{275}x^5+72105\,\sqrt{dx}\,a^4b^2d^{275}x^3+67298\,\sqrt{dx}\,a^5bd^{275}x}{d^{276}}\right)}{33649\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/33649*(33649*a^6/sqrt(d*x) - (1463*sqrt(d*x)*b^6*d^275*x^11 + 10626*sqrt
(d*x)*a*b^5*d^275*x^9 + 33649*sqrt(d*x)*a^2*b^4*d^275*x^7 + 61180*sqrt(d*x)
*a^3*b^3*d^275*x^5 + 72105*sqrt(d*x)*a^4*b^2*d^275*x^3 + 67298*sqrt(d*x)*a^
5*b*d^275*x)/d^276)/d
```

Mupad [B]
time = 0.04, size = 103, normalized size = 0.82

$$\frac{2\,b^6\,{\left(d\,x\right)}^{23/2}}{23\,d^{13}}-\frac{2\,a^6}{d\,\sqrt{d\,x}}+\frac{30\,a^4\,b^2\,{\left(d\,x\right)}^{7/2}}{7\,d^5}+\frac{40\,a^3\,b^3\,{\left(d\,x\right)}^{11/2}}{11\,d^7}+\frac{2\,a^2\,b^4\,{\left(d\,x\right)}^{15/2}}{d^9}+\frac{4\,a^5\,b\,{\left(d\,x\right)}^{3/2}}{d^3}+\frac{12\,a\,b^5\,{\left(d\,x\right)}^{19/2}}{19\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(3/2),x)
```

```
[Out] (2*b^6*(d*x)^(23/2))/(23*d^13) - (2*a^6)/(d*(d*x)^(1/2)) + (30*a^4*b^2*(d*x
)^(7/2))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^2*b^4*(d*x)^(1
5/2))/d^9 + (4*a^5*b*(d*x)^(3/2))/d^3 + (12*a*b^5*(d*x)^(19/2))/(19*d^11)
```

3.684 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2a^6}{3d(dx)^{3/2}}+\frac{12a^5b\sqrt{dx}}{d^3}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

```
[Out] -2/3*a^6/d/(d*x)^(3/2)+6*a^4*b^2*(d*x)^(5/2)/d^5+40/9*a^3*b^3*(d*x)^(9/2)/d
^7+30/13*a^2*b^4*(d*x)^(13/2)/d^9+12/17*a*b^5*(d*x)^(17/2)/d^11+2/21*b^6*(d
*x)^(21/2)/d^13+12*a^5*b*(d*x)^(1/2)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {28, 276}

$$-\frac{2a^6}{3d(dx)^{3/2}}+\frac{12a^5b\sqrt{dx}}{d^3}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

```
[Out] (-2*a^6)/(3*d*(d*x)^(3/2)) + (12*a^5*b*Sqrt[d*x])/d^3 + (6*a^4*b^2*(d*x)^(5
/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b^4*(d*x)^(13/2))/(13
*d^9) + (12*a*b^5*(d*x)^(17/2))/(17*d^11) + (2*b^6*(d*x)^(21/2))/(21*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{5/2}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{(dx)^{5/2}}\,dx}{b^6}$$

$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{5/2}} + \frac{6a^5b^7}{d^2\sqrt{dx}} + \frac{15a^4b^8(dx)^{3/2}}{d^4} + \frac{20a^3b^9(dx)^{7/2}}{d^6} + \frac{15a^2b^{10}(dx)^{11/2}}{d^8} + \frac{6ab^{11}(dx)^{15/2}}{d^{10}} \right.}{b^6}$$

$$= -\frac{2a^6}{3d(dx)^{3/2}} + \frac{12a^5b\sqrt{dx}}{d^3} + \frac{6a^4b^2(dx)^{5/2}}{d^5} + \frac{40a^3b^3(dx)^{9/2}}{9d^7} + \frac{30a^2b^4(dx)^{13/2}}{13d^9} +$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.61

$$-\frac{2x(4641a^6-83538a^5bx^2-41769a^4b^2x^4-30940a^3b^3x^6-16065a^2b^4x^8-4914ab^5x^{10}-663b^6x^{12})}{13923(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

```
[Out] (-2*x*(4641*a^6 - 83538*a^5*b*x^2 - 41769*a^4*b^2*x^4 - 30940*a^3*b^3*x^6 -
 16065*a^2*b^4*x^8 - 4914*a*b^5*x^10 - 663*b^6*x^12))/(13923*(d*x)^(5/2))
```

Maple [A]
time = 0.04, size = 106, normalized size = 0.83

method	result	size
gosper	$-\frac{2(-663b^6x^{12}-4914a\,b^5x^{10}-16065a^2b^4x^8-30940a^3b^3x^6-41769a^4b^2x^4-83538a^5b\,x^2+4641a^6)x}{13923(dx)^{\frac{5}{2}}}$	74
trager	$-\frac{2(-663b^6x^{12}-4914a\,b^5x^{10}-16065a^2b^4x^8-30940a^3b^3x^6-41769a^4b^2x^4-83538a^5b\,x^2+4641a^6)\sqrt{dx}}{13923d^3x^2}$	79
risch	$-\frac{2(-663b^6x^{12}-4914a\,b^5x^{10}-16065a^2b^4x^8-30940a^3b^3x^6-41769a^4b^2x^4-83538a^5b\,x^2+4641a^6)}{13923d^2x\sqrt{dx}}$	79
derivativedivides	$\frac{\frac{2b^6(dx)^{\frac{21}{2}}}{21}+\frac{12a\,b^5d^2(dx)^{\frac{17}{2}}}{17}+\frac{30a^2b^4d^4(dx)^{\frac{13}{2}}}{13}+\frac{40a^3b^3d^6(dx)^{\frac{9}{2}}}{9}+6a^4b^2d^8(dx)^{\frac{5}{2}}+12a^5b\,d^{10}\sqrt{dx}-\frac{2a^6d^{12}}{3(dx)^{\frac{3}{2}}}}{d^{13}}$	106
default	$\frac{\frac{2b^6(dx)^{\frac{21}{2}}}{21}+\frac{12a\,b^5d^2(dx)^{\frac{17}{2}}}{17}+\frac{30a^2b^4d^4(dx)^{\frac{13}{2}}}{13}+\frac{40a^3b^3d^6(dx)^{\frac{9}{2}}}{9}+6a^4b^2d^8(dx)^{\frac{5}{2}}+12a^5b\,d^{10}\sqrt{dx}-\frac{2a^6d^{12}}{3(dx)^{\frac{3}{2}}}}{d^{13}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^13*(1/21*b^6*(d*x)^(21/2)+6/17*a*b^5*d^2*(d*x)^(17/2)+15/13*a^2*b^4*d^4
*(d*x)^(13/2)+20/9*a^3*b^3*d^6*(d*x)^(9/2)+3*a^4*b^2*d^8*(d*x)^(5/2)+6*a^5*
b*d^10*(d*x)^(1/2)-1/3*a^6*d^12/(d*x)^(3/2))
```

Maxima [A]
time = 0.29, size = 108, normalized size = 0.85

$$-\frac{2\left(\frac{4641\,a^6}{(dx)^{\frac{3}{2}}}-\frac{663\,(dx)^{\frac{21}{2}}b^6+4914\,(dx)^{\frac{17}{2}}ab^5d^2+16065\,(dx)^{\frac{13}{2}}a^2b^4d^4+30940\,(dx)^{\frac{9}{2}}a^3b^3d^6+41769\,(dx)^{\frac{5}{2}}a^4b^2d^8+83538\,\sqrt{dx}\,a^5bd^{10}}{d^{12}}\right)}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/13923*(4641*a^6/(d*x)^(3/2) - (663*(d*x)^(21/2)*b^6 + 4914*(d*x)^(17/2)*
a*b^5*d^2 + 16065*(d*x)^(13/2)*a^2*b^4*d^4 + 30940*(d*x)^(9/2)*a^3*b^3*d^6
+ 41769*(d*x)^(5/2)*a^4*b^2*d^8 + 83538*sqrt(d*x)*a^5*b*d^10)/d^12)/d
```

Fricas [A]
time = 0.36, size = 78, normalized size = 0.61

$$\frac{2\,(663\,b^6x^{12}+4914\,ab^5x^{10}+16065\,a^2b^4x^8+30940\,a^3b^3x^6+41769\,a^4b^2x^4+83538\,a^5bx^2-4641\,a^6)\sqrt{dx}}{13923\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^6*x^12 + 4914*a*b^5*x^10 + 16065*a^2*b^4*x^8 + 30940*a^3*b^3
*x^6 + 41769*a^4*b^2*x^4 + 83538*a^5*b*x^2 - 4641*a^6)*sqrt(d*x)/(d^3*x^2)
```

Sympy [A]
time = 0.61, size = 126, normalized size = 0.99

$$-\frac{2a^6x}{3\,(dx)^{\frac{5}{2}}}+\frac{12a^5bx^3}{(dx)^{\frac{5}{2}}}+\frac{6a^4b^2x^5}{(dx)^{\frac{5}{2}}}+\frac{40a^3b^3x^7}{9\,(dx)^{\frac{5}{2}}}+\frac{30a^2b^4x^9}{13\,(dx)^{\frac{5}{2}}}+\frac{12ab^5x^{11}}{17\,(dx)^{\frac{5}{2}}}+\frac{2b^6x^{13}}{21\,(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(5/2),x)
```

```
[Out] -2*a**6*x/(3*(d*x)**(5/2)) + 12*a**5*b*x**3/(d*x)**(5/2) + 6*a**4*b**2*x**5
/(d*x)**(5/2) + 40*a**3*b**3*x**7/(9*(d*x)**(5/2)) + 30*a**2*b**4*x**9/(13*
(d*x)**(5/2)) + 12*a*b**5*x**11/(17*(d*x)**(5/2)) + 2*b**6*x**13/(21*(d*x)*
*(5/2))
```

Giac [A]
time = 3.75, size = 130, normalized size = 1.02

$$-\frac{2\left(\frac{4641\,a^6d}{\sqrt{dx}\,x}-\frac{663\,\sqrt{dx}\,b^6d^{210}x^{10}+4914\,\sqrt{dx}\,ab^5d^{210}x^8+16065\,\sqrt{dx}\,a^2b^4d^{210}x^6+30940\,\sqrt{dx}\,a^3b^3d^{210}x^4+41769\,\sqrt{dx}\,a^4b^2d^{210}x^2+83538\,\sqrt{dx}\,a^5bd^{210}}{d^{210}}\right)}{13923\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/13923*(4641*a^6*d/(sqrt(d*x)*x) - (663*sqrt(d*x)*b^6*d^210*x^10 + 4914*s
qrt(d*x)*a*b^5*d^210*x^8 + 16065*sqrt(d*x)*a^2*b^4*d^210*x^6 + 30940*sqrt(d
*x)*a^3*b^3*d^210*x^4 + 41769*sqrt(d*x)*a^4*b^2*d^210*x^2 + 83538*sqrt(d*x)
*a^5*b*d^210)/d^210)/d^3
```

Mupad [B]
time = 0.04, size = 103, normalized size = 0.81

$$\frac{2\,b^6\,{\left(d\,x\right)}^{21/2}}{21\,d^{13}}-\frac{2\,a^6}{3\,d\,{\left(d\,x\right)}^{3/2}}+\frac{6\,a^4\,b^2\,{\left(d\,x\right)}^{5/2}}{d^5}+\frac{40\,a^3\,b^3\,{\left(d\,x\right)}^{9/2}}{9\,d^7}+\frac{30\,a^2\,b^4\,{\left(d\,x\right)}^{13/2}}{13\,d^9}+\frac{12\,a^5\,b\,\sqrt{d\,x}}{d^3}+\frac{12\,a\,b^5\,{\left(d\,x\right)}^{17/2}}{17\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(5/2),x)
```

```
[Out] (2*b^6*(d*x)^(21/2))/(21*d^13) - (2*a^6)/(3*d*(d*x)^(3/2)) + (6*a^4*b^2*(d*
x)^(5/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b^4*(d*x)^(13/2)
)/(13*d^9) + (12*a^5*b*(d*x)^(1/2))/d^3 + (12*a*b^5*(d*x)^(17/2))/(17*d^11)
```

3.685 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2a^6}{5d(dx)^{5/2}}-\frac{12a^5b}{d^3\sqrt{dx}}+\frac{10a^4b^2(dx)^{3/2}}{d^5}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

```
[Out] -2/5*a^6/d/(d*x)^(5/2)+10*a^4*b^2*(d*x)^(3/2)/d^5+40/7*a^3*b^3*(d*x)^(7/2)/
d^7+30/11*a^2*b^4*(d*x)^(11/2)/d^9+4/5*a*b^5*(d*x)^(15/2)/d^11+2/19*b^6*(d*
x)^(19/2)/d^13-12*a^5*b/d^3/(d*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {28, 276}

$$-\frac{2a^6}{5d(dx)^{5/2}}-\frac{12a^5b}{d^3\sqrt{dx}}+\frac{10a^4b^2(dx)^{3/2}}{d^5}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

```
[Out] (-2*a^6)/(5*d*(d*x)^(5/2)) - (12*a^5*b)/(d^3*Sqrt[d*x]) + (10*a^4*b^2*(d*x)
^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d^7) + (30*a^2*b^4*(d*x)^(11/2))/
(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11) + (2*b^6*(d*x)^(19/2))/(19*d^13)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx = \frac{\int \frac{(ab+b^2x^2)^6}{(dx)^{7/2}}\,dx}{b^6}$$
$$= \frac{\int \left(\frac{a^6b^6}{(dx)^{7/2}} + \frac{6a^5b^7}{d^2(dx)^{3/2}} + \frac{15a^4b^8\sqrt{dx}}{d^4} + \frac{20a^3b^9(dx)^{5/2}}{d^6} + \frac{15a^2b^{10}(dx)^{9/2}}{d^8} + \frac{6ab^{11}(dx)^{13/2}}{d^{10}} + \right.}{b^6}$$
$$= -\frac{2a^6}{5d(dx)^{5/2}} - \frac{12a^5b}{d^3\sqrt{dx}} + \frac{10a^4b^2(dx)^{3/2}}{d^5} + \frac{40a^3b^3(dx)^{7/2}}{7d^7} + \frac{30a^2b^4(dx)^{11/2}}{11d^9} +$$

Mathematica [A]
time = 0.03, size = 82, normalized size = 0.65

$$-\frac{2\sqrt{dx}\,(1463a^6+43890a^5bx^2-36575a^4b^2x^4-20900a^3b^3x^6-9975a^2b^4x^8-2926ab^5x^{10}-385b^6x^{12})}{7315d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[d*x]*(1463*a^6 + 43890*a^5*b*x^2 - 36575*a^4*b^2*x^4 - 20900*a^3*b
^3*x^6 - 9975*a^2*b^4*x^8 - 2926*a*b^5*x^10 - 385*b^6*x^12))/(7315*d^4*x^3)
```

Maple [A]
time = 0.05, size = 106, normalized size = 0.83

method	result	size
gosper	$-\frac{2(-385b^6x^{12}-2926a\,b^5x^{10}-9975a^2b^4x^8-20900a^3b^3x^6-36575a^4b^2x^4+43890a^5b\,x^2+1463a^6)x}{7315(dx)^{\frac{7}{2}}}$	74
trager	$-\frac{2(-385b^6x^{12}-2926a\,b^5x^{10}-9975a^2b^4x^8-20900a^3b^3x^6-36575a^4b^2x^4+43890a^5b\,x^2+1463a^6)\sqrt{dx}}{7315d^4x^3}$	79
risch	$-\frac{2(-385b^6x^{12}-2926a\,b^5x^{10}-9975a^2b^4x^8-20900a^3b^3x^6-36575a^4b^2x^4+43890a^5b\,x^2+1463a^6)}{7315d^3x^2\sqrt{dx}}$	79
derivativedivides	$\frac{\frac{2b^6(dx)^{\frac{19}{2}}}{19}+\frac{4a\,b^5d^2(dx)^{\frac{15}{2}}}{5}+\frac{30a^2b^4d^4(dx)^{\frac{11}{2}}}{11}+\frac{40a^3b^3d^6(dx)^{\frac{7}{2}}}{7}+10a^4b^2d^8(dx)^{\frac{3}{2}}-\frac{12a^5b\,d^{10}}{\sqrt{dx}}-\frac{2a^6d^{12}}{5(dx)^{\frac{5}{2}}}}{d^{13}}$	106
default	$\frac{\frac{2b^6(dx)^{\frac{19}{2}}}{19}+\frac{4a\,b^5d^2(dx)^{\frac{15}{2}}}{5}+\frac{30a^2b^4d^4(dx)^{\frac{11}{2}}}{11}+\frac{40a^3b^3d^6(dx)^{\frac{7}{2}}}{7}+10a^4b^2d^8(dx)^{\frac{3}{2}}-\frac{12a^5b\,d^{10}}{\sqrt{dx}}-\frac{2a^6d^{12}}{5(dx)^{\frac{5}{2}}}}{d^{13}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^13*(1/19*b^6*(d*x)^(19/2)+2/5*a*b^5*d^2*(d*x)^(15/2)+15/11*a^2*b^4*d^4*
(d*x)^(11/2)+20/7*a^3*b^3*d^6*(d*x)^(7/2)+5*a^4*b^2*d^8*(d*x)^(3/2)-6*a^5*b
*d^10/(d*x)^(1/2)-1/5*a^6*d^12/(d*x)^(5/2))
```

Maxima [A]
time = 0.27, size = 114, normalized size = 0.90

$$-\frac{2\left(\frac{1463\,(30\,a^5bd^2x^2+a^6d^2)}{(dx)^{\frac{5}{2}}d^2}-\frac{385\,(dx)^{\frac{19}{2}}b^6+2926\,(dx)^{\frac{15}{2}}ab^5d^2+9975\,(dx)^{\frac{11}{2}}a^2b^4d^4+20900\,(dx)^{\frac{7}{2}}a^3b^3d^6+36575\,(dx)^{\frac{3}{2}}a^4b^2d^8}{d^{12}}\right)}{7315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/7315*(1463*(30*a^5*b*d^2*x^2 + a^6*d^2)/((d*x)^(5/2)*d^2) - (385*(d*x)^(
19/2)*b^6 + 2926*(d*x)^(15/2)*a*b^5*d^2 + 9975*(d*x)^(11/2)*a^2*b^4*d^4 + 2
0900*(d*x)^(7/2)*a^3*b^3*d^6 + 36575*(d*x)^(3/2)*a^4*b^2*d^8)/d^12)/d
```

Fricas [A]
time = 0.34, size = 78, normalized size = 0.61

$$\frac{2\,(385\,b^6x^{12}+2926\,ab^5x^{10}+9975\,a^2b^4x^8+20900\,a^3b^3x^6+36575\,a^4b^2x^4-43890\,a^5bx^2-1463\,a^6)\sqrt{dx}}{7315\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^6*x^12 + 2926*a*b^5*x^10 + 9975*a^2*b^4*x^8 + 20900*a^3*b^3*x
^6 + 36575*a^4*b^2*x^4 - 43890*a^5*b*x^2 - 1463*a^6)*sqrt(d*x)/(d^4*x^3)
```

Sympy [A]
time = 0.77, size = 126, normalized size = 0.99

$$-\frac{2a^6x}{5\,(dx)^{\frac{7}{2}}}-\frac{12a^5bx^3}{(dx)^{\frac{7}{2}}}+\frac{10a^4b^2x^5}{(dx)^{\frac{7}{2}}}+\frac{40a^3b^3x^7}{7\,(dx)^{\frac{7}{2}}}+\frac{30a^2b^4x^9}{11\,(dx)^{\frac{7}{2}}}+\frac{4ab^5x^{11}}{5\,(dx)^{\frac{7}{2}}}+\frac{2b^6x^{13}}{19\,(dx)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(7/2),x)
```

```
[Out] -2*a**6*x/(5*(d*x)**(7/2)) - 12*a**5*b*x**3/(d*x)**(7/2) + 10*a**4*b**2*x**
5/(d*x)**(7/2) + 40*a**3*b**3*x**7/(7*(d*x)**(7/2)) + 30*a**2*b**4*x**9/(11
*(d*x)**(7/2)) + 4*a*b**5*x**11/(5*(d*x)**(7/2)) + 2*b**6*x**13/(19*(d*x)**
(7/2))
```

Giac [A]
time = 3.80, size = 133, normalized size = 1.05

$$-\frac{2\left(\frac{1463\,(30\,a^5bd^3x^2+a^6d^3)}{\sqrt{dx}\,d^2x^2}-\frac{385\,\sqrt{dx}\,b^6d^{171}x^9+2926\,\sqrt{dx}\,ab^5d^{171}x^7+9975\,\sqrt{dx}\,a^2b^4d^{171}x^5+20900\,\sqrt{dx}\,a^3b^3d^{171}x^3+36575\,\sqrt{dx}\,a^4b^2d^{171}x}{d^{171}}\right)}{7315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/7315*(1463*(30*a^5*b*d^3*x^2 + a^6*d^3)/(sqrt(d*x)*d^2*x^2) - (385*sqrt(
d*x)*b^6*d^171*x^9 + 2926*sqrt(d*x)*a*b^5*d^171*x^7 + 9975*sqrt(d*x)*a^2*b^
4*d^171*x^5 + 20900*sqrt(d*x)*a^3*b^3*d^171*x^3 + 36575*sqrt(d*x)*a^4*b^2*d
^171*x)/d^171)/d^4
```

Mupad [B]
time = 0.04, size = 107, normalized size = 0.84

$$\frac{2\,b^6\,{\left(d\,x\right)}^{19/2}}{19\,d^{13}}-\frac{\frac{2\,a^6\,d^2}{5}+12\,b\,a^5\,d^2\,x^2}{d^3\,{\left(d\,x\right)}^{5/2}}+\frac{10\,a^4\,b^2\,{\left(d\,x\right)}^{3/2}}{d^5}+\frac{40\,a^3\,b^3\,{\left(d\,x\right)}^{7/2}}{7\,d^7}+\frac{30\,a^2\,b^4\,{\left(d\,x\right)}^{11/2}}{11\,d^9}+\frac{4\,a\,b^5\,{\left(d\,x\right)}^{15/2}}{5\,d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(7/2),x)
```

```
[Out] (2*b^6*(d*x)^(19/2))/(19*d^13) - ((2*a^6*d^2)/5 + 12*a^5*b*d^2*x^2)/(d^3*(d
*x)^(5/2)) + (10*a^4*b^2*(d*x)^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d^7
) + (30*a^2*b^4*(d*x)^(11/2))/(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11)
```

3.686 $\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=316

$$-\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}-\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(1+\sqrt{2}\right.}{4\sqrt{2}\,b^{13/4}}$$

```
[Out] 9/10*d^3*(d*x)^(5/2)/b^2-1/2*d*(d*x)^(9/2)/b/(b*x^2+a)-9/8*a^(5/4)*d^(11/2)
*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(13/4)*2^(1/2)+9/8
*a^(5/4)*d^(11/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(
13/4)*2^(1/2)-9/16*a^(5/4)*d^(11/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^
(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(13/4)*2^(1/2)+9/16*a^(5/4)*d^(11/2)*l
n(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^
(13/4)*2^(1/2)-9/2*a*d^5*(d*x)^(1/2)/b^3
```

Rubi [A]
time = 0.26, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{9a^{5/4}d^{11/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{9a^{5/4}d^{11/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,b^{13/4}}-\frac{9a^{5/4}d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{13/4}}-\frac{9ad^5\sqrt{dx}}{2b^3}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{9d^3(dx)^{5/2}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-9*a*d^5*Sqrt[d*x])/(2*b^3) + (9*d^3*(d*x)^(5/2))/(10*b^2) - (d*(d*x)^(9/2
))/(2*b*(a + b*x^2)) - (9*a^(5/4)*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4))
- (9*a^(5/4)*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*S
qrt[2]*b^(13/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{11/2}}{(ab+b^2x^2)^2}\,dx$$
$$= -\frac{d(dx)^{9/2}}{2b\,(a+bx^2)}+\frac{1}{4}(9d^2)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx$$
$$= \frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\,(a+bx^2)}-\frac{(9ad^4)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{4b}$$
$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\,(a+bx^2)}+\frac{(9a^2d^6)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)}\,dx}{4b^2}$$
$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\,(a+bx^2)}+\frac{(9a^2d^5)\,\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right.}{2b^2}$$
$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\,(a+bx^2)}+\frac{(9a^{3/2}d^4)\,\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x\right.}{4b^2}$$
$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\,(a+bx^2)}-\frac{(9a^{5/4}d^{11/2})\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}}\right.}{8\sqrt{2}\,b^{13/4}}$$
$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\,(a+bx^2)}-\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right.}{8\sqrt{2}\,b^{13/4}}$$
$$= -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2}-\frac{d(dx)^{9/2}}{2b\,(a+bx^2)}-\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{13/4}}$$

Mathematica [A]
time = 0.30, size = 178, normalized size = 0.56

$$\frac{d^5\sqrt{dx}\left(4\sqrt[4]{b}\sqrt{x}\,(-45a^2-36abx^2+4b^2x^4)-45\sqrt{2}\,a^{5/4}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+45\sqrt{2}\,a^{5/4}(a+bx^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{40b^{13/4}\sqrt{x}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^5*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(-45*a^2 - 36*a*b*x^2 + 4*b^2*x^4) - 45*S
qrt[2]*a^(5/4)*(a + b*x^2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x])] + 45*Sqrt[2]*a^(5/4)*(a + b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(40*b^(13/4)*Sqrt[x]*(a + b*x^2))
```

Maple [A]
time = 0.07, size = 207, normalized size = 0.66

method	result
derivativedivides	$2d^3\left(-\frac{-\frac{(dx)^{\frac{5}{2}}b}{5}+2a\,d^2\sqrt{dx}}{b^3}+\frac{a^2d^4\left(-\frac{\sqrt{dx}}{4\left(d^2x^2b+a\,d^2\right)}+\frac{9\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\right.}{}\right.}{b^3}$
default	$2d^3\left(-\frac{-\frac{(dx)^{\frac{5}{2}}b}{5}+2a\,d^2\sqrt{dx}}{b^3}+\frac{a^2d^4\left(-\frac{\sqrt{dx}}{4\left(d^2x^2b+a\,d^2\right)}+\frac{9\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\right.}{}\right.}{b^3}$
risch	$-\frac{2\left(-b\,x^2+10a\right)x\,d^6}{5b^3\sqrt{dx}}+\left(-\frac{a^2d\sqrt{dx}}{2b^3\left(d^2x^2b+a\,d^2\right)}+\frac{9a\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16b^3d}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(-1/b^3*(-1/5*(d*x)^(5/2)*b+2*a*d^2*(d*x)^(1/2))+1/b^3*a^2*d^4*(-1/4*
(d*x)^(1/2)/(b*d^2*x^2+a*d^2)+9/32*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d
```

```
*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^
(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))))
```

Maxima [A]
time = 0.51, size = 300, normalized size = 0.95

$$-\frac{\frac{40\sqrt{dx}\,a^2d^8}{b^4d^2x^2+ab^3d^2}-\frac{45\left(\frac{\sqrt{2}\,d^8\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^8\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^7\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^7\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)a^2}{b^3}-\frac{32\left((dx)^{\frac{5}{2}}bd^4-10\sqrt{dx}\,ad^6\right)}{b^3}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/80*(40*sqrt(d*x)*a^2*d^8/(b^4*d^2*x^2 + a*b^3*d^2) - 45*(sqrt(2)*d^8*log
(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt
(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^7*arctan(1
/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(
a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^7*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a
)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a^2/b^3 - 32*((d*x)^(5/2)*
b*d^4 - 10*sqrt(d*x)*a*d^6)/b^3)/d
```

Fricas [A]
time = 0.36, size = 283, normalized size = 0.90

$$\frac{180\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}(b^4x^2+ab^3)\arctan\left(-\frac{\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{10}d^5-\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{a^2d^{11}x+\sqrt{-\frac{a^5d^{22}}{b^{13}}}\,b^6}\,b^{10}}{a^5d^{22}}\right)+45\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}(b^4x^2+ab^3)\log\left(9\sqrt{dx}\,ad^5+9\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3\right)-45\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}(b^4x^2+ab^3)\log\left(9\sqrt{dx}\,ad^5-9\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3\right)+4\,(4\,b^2d^5x^4-36\,abd^5x^2-45\,a^2d^5)\sqrt{dx}}{40\,(b^4x^2+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/40*(180*(-a^5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*arctan(-((-a^5*d^22/b^13
)^(3/4)*sqrt(d*x)*a*b^10*d^5 - (-a^5*d^22/b^13)^(3/4)*sqrt(a^2*d^11*x + sqr
t(-a^5*d^22/b^13)*b^6)*b^10)/(a^5*d^22)) + 45*(-a^5*d^22/b^13)^(1/4)*(b^4*x
^2 + a*b^3)*log(9*sqrt(d*x)*a*d^5 + 9*(-a^5*d^22/b^13)^(1/4)*b^3) - 45*(-a^
5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*log(9*sqrt(d*x)*a*d^5 - 9*(-a^5*d^22/b
^13)^(1/4)*b^3) + 4*(4*b^2*d^5*x^4 - 36*a*b*d^5*x^2 - 45*a^2*d^5)*sqrt(d*x)
)/(b^4*x^2 + a*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{11}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(11/2)/(a + b*x**2)**2, x)
```

Giac [A]
time = 4.28, size = 297, normalized size = 0.94

$$-\frac{1}{80}\,d^5\left(\frac{40\,\sqrt{dx}\,a^2d^2}{(bd^2x^2+ad^2)b^3}-\frac{90\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{90\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{45\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\,a\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^4}+\frac{45\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\,a\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^4}-\frac{32\left(\sqrt{dx}\,b^8d^{10}x^2-10\,\sqrt{dx}\,ab^7d^{10}\right)}{b^{10}d^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/80*d^5*(40*sqrt(d*x)*a^2*d^2/((b*d^2*x^2 + a*d^2)*b^3) - 90*sqrt(2)*(a*b
^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/b^4 - 90*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^4 - 45*sqrt(2)*(a
*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/b^4 + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*
sqrt(d*x) + sqrt(a*d^2/b))/b^4 - 32*(sqrt(d*x)*b^8*d^10*x^2 - 10*sqrt(d*x)*
a*b^7*d^10)/(b^10*d^10))
```

Mupad [B]
time = 4.27, size = 129, normalized size = 0.41

$$\frac{2\,d^3\,(d\,x)^{5/2}}{5\,b^2}-\frac{9\,(-a)^{5/4}\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,b^{13/4}}-\frac{a^2\,d^7\,\sqrt{d\,x}}{2\,\left(b^4\,d^2\,x^2+a\,b^3\,d^2\right)}-\frac{4\,a\,d^5\,\sqrt{d\,x}}{b^3}+\frac{(-a)^{5/4}\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1\mathrm{i}}{(-a)^{1/4}\,\sqrt{d}}\right)\,9\mathrm{i}}{4\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (2*d^3*(d*x)^(5/2))/(5*b^2) - (9*(-a)^(5/4)*d^(11/2)*atan((b^(1/4)*(d*x)^(1
/2))/((-a)^(1/4)*d^(1/2))))/(4*b^(13/4)) + ((-a)^(5/4)*d^(11/2)*atan((b^(1/
4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*9i)/(4*b^(13/4)) - (a^2*d^7*(d*x)^
(1/2))/(2*(a*b^3*d^2 + b^4*d^2*x^2)) - (4*a*d^5*(d*x)^(1/2))/b^3
```

3.687 $\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=298

$$\frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b\,(a+bx^2)}+\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7a^3 \cdots}{\cdots}$$

```
[Out] 7/6*d^3*(d*x)^(3/2)/b^2-1/2*d*(d*x)^(7/2)/b/(b*x^2+a)+7/8*a^(3/4)*d^(9/2)*a
rctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(11/4)*2^(1/2)-7/8*a
^(3/4)*d^(9/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(11/
4)*2^(1/2)-7/16*a^(3/4)*d^(9/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4
)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(11/4)*2^(1/2)+7/16*a^(3/4)*d^(9/2)*ln(a^(
1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(11/4
)*2^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{7a^{3/4}d^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7a^{3/4}d^{9/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7a^{3/4}d^{9/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{11/4}}+\frac{7a^{3/4}d^{9/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{11/4}}-\frac{d(dx)^{7/2}}{2b\,(a+bx^2)}+\frac{7d^3(dx)^{3/2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (7*d^3*(d*x)^(3/2))/(6*b^2) - (d*(d*x)^(7/2))/(2*b*(a + b*x^2)) + (7*a^(3/4
)*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqr
t[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]
*b^(11/4)) + (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(11/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{9/2}}{(ab+b^2x^2)^2}\,dx$$

$$= -\frac{d(dx)^{7/2}}{2b(a+bx^2)}+\frac{1}{4}(7d^2)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b(a+bx^2)}-\frac{(7ad^4)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4b}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b(a+bx^2)}-\frac{(7ad^3)\,\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{2b}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b(a+bx^2)}+\frac{(7ad^3)\,\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{4b^{3/2}}-\frac{(7ad^3)\cdots}{\cdots}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b(a+bx^2)}-\frac{(7a^{3/4}d^{9/2})\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx,x\cdots\right.}{8\sqrt{2}\,b^{11/4}}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b(a+bx^2)}-\frac{7a^{3/4}d^{9/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,\sqrt{\cdots}\right.}{8\sqrt{2}\,b^{11/4}}$$

$$= \frac{7d^3(dx)^{3/2}}{6b^2}-\frac{d(dx)^{7/2}}{2b(a+bx^2)}+\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{11/4}}-\frac{7a^{3/4}d^{9/2}\tan\cdots}{\cdots}$$

Mathematica [A]

time = 0.28, size = 167, normalized size = 0.56

$$\frac{d^4\sqrt{dx}\left(4b^{3/4}x^{3/2}(7a+4bx^2)+21\sqrt{2}\,a^{3/4}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+21\sqrt{2}\,a^{3/4}(a+bx^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{24b^{11/4}\sqrt{x}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^4*Sqrt[d*x]*(4*b^(3/4)*x^(3/2)*(7*a + 4*b*x^2) + 21*Sqrt[2]*a^(3/4)*(a +
 b*x^2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 2
1*Sqrt[2]*a^(3/4)*(a + b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sq
rt[a] + Sqrt[b]*x)]))/(24*b^(11/4)*Sqrt[x]*(a + b*x^2))
```

Maple [A]

time = 0.09, size = 188, normalized size = 0.63

method	result
derivativedivides	$2d^3\left(\frac{(dx)^{\frac{3}{2}}}{3b^2}-\frac{a\,d^2\left(-\frac{(dx)^{\frac{3}{2}}}{4\left(d^2x^2b+a\,d^2\right)}+\frac{7\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{32b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.}{b^2}\right.$
default	$2d^3\left(\frac{(dx)^{\frac{3}{2}}}{3b^2}-\frac{a\,d^2\left(-\frac{(dx)^{\frac{3}{2}}}{4\left(d^2x^2b+a\,d^2\right)}+\frac{7\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{32b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.}{b^2}\right.$

risch	$\frac{2x^2d^5}{3b^2\sqrt{dx}} + \left(\frac{a(dx)^{\frac{3}{2}}}{2b^2(d^2x^2b+a\,d^2)} - \frac{7a\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16b^3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}} - \frac{7a\sqrt{2}\,\arctan\left(\cdots\right)}{8b^3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}} \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(1/3*(d*x)^(3/2)/b^2-a*d^2/b^2*(-1/4*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+7/
32/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+
2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^
(1/4)*(d*x)^(1/2)-1))))
```

Maxima [A]

time = 0.51, size = 273, normalized size = 0.92

$$\frac{\frac{24\,(dx)^{\frac{3}{2}}ad^6}{b^3d^2x^2+ab^2d^2} - \frac{21\,ad^6\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{b}} + \frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{b}} - \frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b^2} + \frac{32\,(dx)^{\frac{3}{2}}d^4}{b^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/48*(24*(d*x)^(3/2)*a*d^6/(b^3*d^2*x^2 + a*b^2*d^2) - 21*a*d^6*(2*sqrt(2)*
arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sq
rt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sq
rt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d
*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^
(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^2 + 32*(d*x)^(3/2)*d^4/b^2)/d
```

Fricas [A]

time = 0.36, size = 283, normalized size = 0.95

$$\frac{84\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}(b^3x^2+ab^2)\arctan\left(-\frac{\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^3d^{13}-\sqrt{a^4d^{27}x-\sqrt{-\frac{a^3d^{18}}{b^{11}}}\,a^3b^5d^{18}}\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}b^3}{a^3d^{18}}\right) - 21\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}(b^3x^2+ab^2)\log\left(343\sqrt{dx}\,a^2d^{13}+343\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{3}{4}}b^8\right) + 21\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}(b^3x^2+ab^2)\log\left(343\sqrt{dx}\,a^2d^{13}-343\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{3}{4}}b^8\right) + 4\,(4\,bd^4x^3+7\,ad^4x)\sqrt{dx}}{24\,(b^3x^2+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(84*(-a^3*d^18/b^11)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((-a^3*d^18/b^11)
^(1/4)*sqrt(d*x)*a^2*b^3*d^13 - sqrt(a^4*d^27*x - sqrt(-a^3*d^18/b^11)*a^3*
b^5*d^18)*(-a^3*d^18/b^11)^(1/4)*b^3)/(a^3*d^18)) - 21*(-a^3*d^18/b^11)^(1/
4)*(b^3*x^2 + a*b^2)*log(343*sqrt(d*x)*a^2*d^13 + 343*(-a^3*d^18/b^11)^(3/4
)*b^8) + 21*(-a^3*d^18/b^11)^(1/4)*(b^3*x^2 + a*b^2)*log(343*sqrt(d*x)*a^2*
d^13 - 343*(-a^3*d^18/b^11)^(3/4)*b^8) + 4*(4*b*d^4*x^3 + 7*a*d^4*x)*sqrt(d
*x))/(b^3*x^2 + a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{9}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(9/2)/(a + b*x**2)**2, x)
```

Giac [A]
time = 3.84, size = 277, normalized size = 0.93

$$\frac{1}{48}\left(\frac{24\sqrt{dx}\,ad^2x}{(bd^2x^2+ad^2)b^2}+\frac{32\sqrt{dx}\,x}{b^2}-\frac{42\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5d}-\frac{42\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5d}+\frac{21\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^5d}-\frac{21\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^5d}\right)d^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/48*(24*sqrt(d*x)*a*d^2*x/((b*d^2*x^2 + a*d^2)*b^2) + 32*sqrt(d*x)*x/b^2 -
 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) +
 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*d) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^
5*d) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(
d*x) + sqrt(a*d^2/b))/(b^5*d) - 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt
(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^5*d))*d^4
```

Mupad [B]
time = 0.12, size = 112, normalized size = 0.38

$$\frac{2\,d^3\,{(d\,x)}^{3/2}}{3\,b^2}+\frac{7\,{(-a)}^{3/4}\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{4\,b^{11/4}}+\frac{a\,d^5\,{(d\,x)}^{3/2}}{2\,\left(b^3\,d^2\,x^2+a\,b^2\,d^2\right)}+\frac{{(-a)}^{3/4}\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{(-a)}^{1/4}\,\sqrt{d}}\right)\,7{}\mathrm{i}}{4\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (2*d^3*(d*x)^(3/2))/(3*b^2) + (7*(-a)^(3/4)*d^(9/2)*atan((b^(1/4)*(d*x)^(1/
2))/((-a)^(1/4)*d^(1/2))))/(4*b^(11/4)) + ((-a)^(3/4)*d^(9/2)*atan((b^(1/4)
*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*7i)/(4*b^(11/4)) + (a*d^5*(d*x)^(3/2
))/(2*(a*b^2*d^2 + b^3*d^2*x^2))
```

3.688 $\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=298

$$\frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\,(a+bx^2)}+\frac{5\sqrt[4]{a}\,d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,d^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{a}}{}$$

```
[Out] -1/2*d*(d*x)^(5/2)/b/(b*x^2+a)+5/8*a^(1/4)*d^(7/2)*arctan(1-b^(1/4)*2^(1/2)
*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(9/4)*2^(1/2)-5/8*a^(1/4)*d^(7/2)*arctan(1+
b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(9/4)*2^(1/2)+5/16*a^(1/4)*d
^(7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(
1/2))/b^(9/4)*2^(1/2)-5/16*a^(1/4)*d^(7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(9/4)*2^(1/2)+5/2*d^3*(d*x)^(1/
2)/b^2
```

Rubi [A]
time = 0.20, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{5\sqrt[4]{a}\,d^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,d^{7/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{4\sqrt{2}\,b^{9/4}}+\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{9/4}}-\frac{5\sqrt[4]{a}\,d^{7/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,b^{9/4}}-\frac{d(dx)^{5/2}}{2b\,(a+bx^2)}+\frac{5d^3\sqrt{dx}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (5*d^3*Sqrt[d*x])/(2*b^2) - (d*(d*x)^(5/2))/(2*b*(a + b*x^2)) + (5*a^(1/4)*
d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[
2]*b^(9/4)) - (5*a^(1/4)*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(9/4)) + (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(
9/4)) - (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(9/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)} + \frac{1}{4}\left(5d^2\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx$$

$$= \frac{5d^3\sqrt{dx}}{2b^2} - \frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)} - \frac{\left(5ad^4\right)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)}\,dx}{4b}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2} - \frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)} - \frac{\left(5ad^3\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2b}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2} - \frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)} - \frac{\left(5\sqrt{a}\,d^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4b} - \frac{(5\sqrt{}}{}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2} - \frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\left(5\sqrt[4]{a}\,d^{7/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x,\right.}{8\sqrt{2}\,b^{9/4}}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2} - \frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)} + \frac{5\sqrt[4]{a}\,d^{7/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\right.}{8\sqrt{2}\,b^{9/4}}$$

$$= \frac{5d^3\sqrt{dx}}{2b^2} - \frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)} + \frac{5\sqrt[4]{a}\,d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,b^{9/4}} - \frac{5\sqrt[4]{a}\,d^{7/2}\tan}{}$$

Mathematica [A]

time = 0.27, size = 167, normalized size = 0.56

$$\frac{d^3\sqrt{dx}\left(4\sqrt[4]{b}\sqrt{x}\,(5a+4bx^2)+5\sqrt{2}\sqrt[4]{a}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-5\sqrt{2}\sqrt[4]{a}\,(a+bx^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{8b^{9/4}\sqrt{x}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^3*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(5*a + 4*b*x^2) + 5*Sqrt[2]*a^(1/4)*(a +
b*x^2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 5*
Sqrt[2]*a^(1/4)*(a + b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt
[a] + Sqrt[b]*x)]))/(8*b^(9/4)*Sqrt[x]*(a + b*x^2))
```

Maple [A]

time = 0.06, size = 190, normalized size = 0.64

method	result
derivativedivides	$2d^3\left(\frac{\sqrt{dx}}{b^2}-\frac{a\,d^2\left(-\frac{\sqrt{dx}}{4\left(d^2x^2b+a\,d^2\right)}+\frac{5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\left(\cdots\right.}\right.\right.}{32a\,d^2}\cdots\right.}{b^2}\cdots\right.$
default	$2d^3\left(\frac{\sqrt{dx}}{b^2}-\frac{a\,d^2\left(-\frac{\sqrt{dx}}{4\left(d^2x^2b+a\,d^2\right)}+\frac{5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\left(\cdots\right.}\right.\right.}{32a\,d^2}\cdots\right.}{b^2}\cdots\right.$

risch	$\frac{2x\,d^4}{b^2\sqrt{dx}}+\left(\frac{ad\sqrt{dx}}{2b^2(d^2x^2b+a\,d^2)}-\frac{5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16b^2d}-\frac{5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)

[Out] 2*d^3*(1/b^2*(d*x)^(1/2)-1/b^2*a*d^2*(-1/4*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)+5/32*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))))

Maxima [A]
time = 0.49, size = 282, normalized size = 0.95

$$\frac{\frac{8\sqrt{dx}\,ad^6}{b^3d^2x^2+ab^2d^2}+\frac{32\sqrt{dx}\,d^4}{b^2}-\frac{5\left(\frac{\sqrt{2}\,d^6\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^6\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^5\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^5\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)a}{b^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")

[Out] 1/16*(8*sqrt(d*x)*a*d^6/(b^3*d^2*x^2 + a*b^2*d^2) + 32*sqrt(d*x)*d^4/b^2 - 5*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a/b^2)/d

Fricas [A]
time = 0.34, size = 247, normalized size = 0.83

$$-\frac{20\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}(b^3x^2+ab^2)\arctan\left(-\frac{\left(-\frac{ad^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{dx}\,b^7d^3-\sqrt{d^7x+\sqrt{-\frac{ad^{14}}{b^9}}\,b^4}\left(-\frac{ad^{14}}{b^9}\right)^{\frac{3}{4}}b^7}{ad^{14}}\right)+5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}(b^3x^2+ab^2)\log\left(5\sqrt{dx}\,d^3+5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}b^2\right)-5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}(b^3x^2+ab^2)\log\left(5\sqrt{dx}\,d^3-5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}b^2\right)-4\,(4\,bd^3x^2+5\,ad^3)\sqrt{dx}}{8\,(b^3x^2+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")

```
[Out] -1/8*(20*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*arctan(-((-a*d^14/b^9)^(3/4)
*sqrt(d*x)*b^7*d^3 - sqrt(d^7*x + sqrt(-a*d^14/b^9)*b^4)*(-a*d^14/b^9)^(3/4
)*b^7)/(a*d^14)) + 5*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*sqrt(d*x)*
d^3 + 5*(-a*d^14/b^9)^(1/4)*b^2) - 5*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*
log(5*sqrt(d*x)*d^3 - 5*(-a*d^14/b^9)^(1/4)*b^2) - 4*(4*b*d^3*x^2 + 5*a*d^3
)*sqrt(d*x))/(b^3*x^2 + a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**2, x)
```

Giac [A]
time = 3.52, size = 263, normalized size = 0.88

$$\frac{1}{16}\,d^3\left(\frac{8\,\sqrt{dx}\,ad^2}{(bd^2x^2+ad^2)b^2}-\frac{10\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}-\frac{10\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}-\frac{5\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^3}+\frac{5\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^3}+\frac{32\,\sqrt{dx}}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/16*d^3*(8*sqrt(d*x)*a*d^2/((b*d^2*x^2 + a*d^2)*b^2) - 10*sqrt(2)*(a*b^3*d
^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2
/b)^(1/4))/b^3 - 10*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 - 5*sqrt(2)*(a*b^3*d^2)
^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 + 5
*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sq
rt(a*d^2/b))/b^3 + 32*sqrt(d*x)/b^2)
```

Mupad [B]
time = 0.12, size = 112, normalized size = 0.38

$$\frac{2\,d^3\,\sqrt{d\,x}}{b^2}-\frac{5\,{\left(-a\right)}^{1/4}\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{4\,b^{9/4}}+\frac{a\,d^5\,\sqrt{d\,x}}{2\,\left(b^3\,d^2\,x^2+a\,b^2\,d^2\right)}+\frac{{\left(-a\right)}^{1/4}\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,5{}\mathrm{i}}{4\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (2*d^3*(d*x)^(1/2))/b^2 - (5*(-a)^(1/4)*d^(7/2)*atan((b^(1/4)*(d*x)^(1/2))/
((-a)^(1/4)*d^(1/2))))/(4*b^(9/4)) + ((-a)^(1/4)*d^(7/2)*atan((b^(1/4)*(d*x
)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*5i)/(4*b^(9/4)) + (a*d^5*(d*x)^(1/2))/(2*
(a*b^2*d^2 + b^3*d^2*x^2))
```

3.689 $\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=281

$$-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}-\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}+\frac{3d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}+\frac{3d^{5/2}\log\left(\sqrt{a}\,\sqrt{d}\,+\right.}{8}$$

```
[Out] -1/2*d*(d*x)^(3/2)/b/(b*x^2+a)-3/8*d^(5/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(
1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(7/4)*2^(1/2)+3/8*d^(5/2)*arctan(1+b^(1/4)*
2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(7/4)*2^(1/2)+3/16*d^(5/2)*l
n(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^
(1/4)/b^(7/4)*2^(1/2)-3/16*d^(5/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(
1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {28, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}+\frac{3d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}+\frac{3d^{5/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}-\frac{3d^{5/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*(d*(d*x)^(3/2))/(b*(a + b*x^2)) - (3*d^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*d^(5/2)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/
4)*b^(7/4)) + (3*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*d^(5/2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*
Sqrt[2]*a^(1/4)*b^(7/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx = b^2\int \frac{(dx)^{5/2}}{(ab+b^2x^2)^2}\,dx$$
$$= -\frac{d(dx)^{3/2}}{2b(a+bx^2)} + \frac{1}{4}(3d^2)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx$$
$$= -\frac{d(dx)^{3/2}}{2b(a+bx^2)} + \frac{1}{2}(3d)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)$$
$$= -\frac{d(dx)^{3/2}}{2b(a+bx^2)} - \frac{(3d)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4\sqrt{b}} + \frac{(3d)\text{Subst}\left(\int \frac{\sqrt{a}\,d+}{ab+\frac{b}{}}\right.}{4\sqrt{}}$$
$$= -\frac{d(dx)^{3/2}}{2b(a+bx^2)} + \frac{\left(3d^{5/2}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}} + \frac{(3d^{5}}{}$$
$$= -\frac{d(dx)^{3/2}}{2b(a+bx^2)} + \frac{3d^{5/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{7/4}} - \frac{3d^{5/2}\,\text{lo}}{}$$
$$= -\frac{d(dx)^{3/2}}{2b(a+bx^2)} - \frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}} + \frac{3d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}\sqrt[4]{a}\,b^{7/4}}$$

Mathematica [A]
time = 0.26, size = 154, normalized size = 0.55

$$-\frac{(dx)^{5/2}\left(4\sqrt[4]{a}\,b^{3/4}x^{3/2}+3\sqrt{2}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+3\sqrt{2}\,(a+bx^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{8\sqrt[4]{a}\,b^{7/4}x^{5/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/8*((d*x)^(5/2)*(4*a^(1/4)*b^(3/4)*x^(3/2) + 3*Sqrt[2]*(a + b*x^2)*ArcTan
[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(a +
b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(
a^(1/4)*b^(7/4)*x^(5/2)*(a + b*x^2))
```

Maple [A]

time = 0.05, size = 171, normalized size = 0.61

method	result
derivativedivides	$2d^{3}\left(-\frac{(dx)^{\frac{3}{2}}}{4b(d^{2}x^{2}b+a\,d^{2})}+\frac{3\sqrt{2}\,\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+2\,a\right.}{32b^{2}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right.$
default	$2d^{3}\left(-\frac{(dx)^{\frac{3}{2}}}{4b(d^{2}x^{2}b+a\,d^{2})}+\frac{3\sqrt{2}\,\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+2\,a\right.}{32b^{2}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(-1/4/b*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+3/32/b^2/(a*d^2/b)^(1/4)*2^(1/2
)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^
(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.50, size = 256, normalized size = 0.91

$$-\frac{\frac{8\,(dx)^{\frac{3}{2}}d^{4}}{b^{2}d^{2}x^{2}+abd^{2}}-\frac{3\,d^{4}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(d*x)^(3/2)*d^4/(b^2*d^2*x^2 + a*b*d^2) - 3*d^4*(2*sqrt(2)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*s
qrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sq
rt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4))
+ sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(
a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b)/d
```

Fricas [A]
time = 0.35, size = 247, normalized size = 0.88

$$-\frac{4\sqrt{dx}\,d^2x + 12\,(b^2x^2+ab)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\sqrt{dx}\,b^2d^7 - \sqrt{d^{15}x - \sqrt{-\frac{d^{10}}{ab^7}}\,ab^3d^{10}}\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}b^2}{d^{10}}\right) - 3\,(b^2x^2+ab)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\log\left(27\sqrt{dx}\,d^7+27\left(-\frac{d^{10}}{ab^7}\right)^{\frac{3}{4}}ab^5\right) + 3\,(b^2x^2+ab)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\log\left(27\sqrt{dx}\,d^7-27\left(-\frac{d^{10}}{ab^7}\right)^{\frac{3}{4}}ab^5\right)}{8\,(b^2x^2+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*sqrt(d*x)*d^2*x + 12*(b^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4)*arctan(-
((-d^10/(a*b^7))^(1/4)*sqrt(d*x)*b^2*d^7 - sqrt(d^15*x - sqrt(-d^10/(a*b^7)
)*a*b^3*d^10)*(-d^10/(a*b^7))^(1/4)*b^2)/d^10) - 3*(b^2*x^2 + a*b)*(-d^10/(
a*b^7))^(1/4)*log(27*sqrt(d*x)*d^7 + 27*(-d^10/(a*b^7))^(3/4)*a*b^5) + 3*(b
^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4)*log(27*sqrt(d*x)*d^7 - 27*(-d^10/(a*b^7
))^(3/4)*a*b^5))/(b^2*x^2 + a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**2, x)
```

Giac [A]
time = 3.46, size = 277, normalized size = 0.99

$$-\frac{1}{16}\left(\frac{8\sqrt{dx}\,d^2x}{(bd^2x^2+ad^2)b} - \frac{6\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4d} - \frac{6\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4d} + \frac{3\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^4d} - \frac{3\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^4d}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/16*(8*sqrt(d*x)*d^2*x/((b*d^2*x^2 + a*d^2)*b) - 6*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1
/4))/(a*b^4*d) - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4*d) + 3*sqrt(2)*(a*b^3
*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
*b^4*d) - 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqr
t(d*x) + sqrt(a*d^2/b))/(a*b^4*d))*d^2
```

Mupad [B]
time = 4.25, size = 92, normalized size = 0.33

$$\frac{3\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{1/4}\,b^{7/4}}-\frac{3\,d^{5/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{1/4}\,b^{7/4}}-\frac{d^3\,(d\,x)^{3/2}}{2\,b\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (3*d^(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(1/4)*
b^(7/4)) - (3*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4
*(-a)^(1/4)*b^(7/4)) - (d^3*(d*x)^(3/2))/(2*b*(a*d^2 + b*d^2*x^2))
```

3.690 $\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=281

$$-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}-\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{d^{3/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\right.}{8\sqrt{2}}$$

```
[Out] -1/8*d^(3/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/
b^(5/4)*2^(1/2)+1/8*d^(3/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^
(1/2))/a^(3/4)/b^(5/4)*2^(1/2)-1/16*d^(3/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^
(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/16*d^(
3/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/
2))/a^(3/4)/b^(5/4)*2^(1/2)-1/2*d*(d*x)^(1/2)/b/(b*x^2+a)
```

Rubi [A]
time = 0.17, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*(d*Sqrt[d*x])/(b*(a + b*x^2)) - (d^(3/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(
5/4)) - (d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*Log[Sqrt[a]*Sqrt
[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^
(3/4)*b^(5/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{(dx)^{3/2}}{(ab+b^2x^2)^2}\,dx \\
&= -\frac{d\sqrt{dx}}{2b(a+bx^2)}+\frac{1}{4}d^2\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)}\,dx \\
&= -\frac{d\sqrt{dx}}{2b(a+bx^2)}+\frac{1}{2}d\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right) \\
&= -\frac{d\sqrt{dx}}{2b(a+bx^2)}+\frac{\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{4\sqrt{a}}+\frac{\text{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx\right.}{4\sqrt{a}} \\
&= -\frac{d\sqrt{dx}}{2b(a+bx^2)}-\frac{d^{3/2}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{dx}\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{d^{3/2}\text{Sub}}{} \\
&= -\frac{d\sqrt{dx}}{2b(a+bx^2)}-\frac{d^{3/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d^{3/2}\log}{} \\
&= -\frac{d\sqrt{dx}}{2b(a+bx^2)}-\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}\,a^{3/4}b^{5/4}}
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 153, normalized size = 0.54

$$\frac{(dx)^{3/2}\left(-4a^{3/4}\sqrt[4]{b}\sqrt{x}-\sqrt{2}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt{2}\,(a+bx^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{8a^{3/4}b^{5/4}x^{3/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] ((d*x)^(3/2)*(-4*a^(3/4)*b^(1/4)*Sqrt[x] - Sqrt[2]*(a + b*x^2)*ArcTan[(Sqrt
[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*(a + b*x^2)*A
rcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(8*a^(3/4
)*b^(5/4)*x^(3/2)*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 177, normalized size = 0.63

method	result
derivativedivides	$2d^{3}\left(-\frac{\sqrt{dx}}{4b\left(d^{2}x^{2}b+a\,d^{2}\right)}+\frac{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+\right.\right.}{32ba\,d^{2}}\right.$
default	$2d^{3}\left(-\frac{\sqrt{dx}}{4b\left(d^{2}x^{2}b+a\,d^{2}\right)}+\frac{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+\right.\right.}{32ba\,d^{2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(-1/4/b*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)+1/32/b*(a*d^2/b)^(1/4)/a/d^2*2^
(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2
/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]
time = 0.51, size = 265, normalized size = 0.94

$$-\frac{\frac{8\sqrt{dx}\,d^{4}}{b^{2}d^{2}x^{2}+abd^{2}}-\frac{\frac{\sqrt{2}\,d^{4}\log\left(\sqrt{b}\,dx+\sqrt{2}\,\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{4}\log\left(\sqrt{b}\,dx-\sqrt{2}\,\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^{3}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^{3}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}}{b}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*sqrt(d*x)*d^4/(b^2*d^2*x^2 + a*b*d^2) - (sqrt(2)*d^4*log(sqrt(b)*d
*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^
(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1
/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arctan(1/2*sqrt(2)
*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)
*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*
d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/b)/d
```

Fricas [A]
time = 0.36, size = 234, normalized size = 0.83

$$\frac{4\left(b^2x^2+ab\right)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^2b^4d\left(-\frac{d^6}{a^3b^5}\right)^{\frac{3}{4}}-\sqrt{a^2b^2\sqrt{-\frac{d^6}{a^3b^5}}+d^3x}\,a^2b^4\left(-\frac{d^6}{a^3b^5}\right)^{\frac{3}{4}}}{d^6}\right)+\left(b^2x^2+ab\right)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}+\sqrt{dx}\,d\right)-\left(b^2x^2+ab\right)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\log\left(-ab\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}+\sqrt{dx}\,d\right)-4\sqrt{dx}\,d}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*x^2 + a*b)*(-d^6/(a^3*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^2*b^4*d*
(-d^6/(a^3*b^5))^(3/4) - sqrt(a^2*b^2*sqrt(-d^6/(a^3*b^5)) + d^3*x)*a^2*b^4
*(-d^6/(a^3*b^5))^(3/4))/d^6) + (b^2*x^2 + a*b)*(-d^6/(a^3*b^5))^(1/4)*log(
a*b*(-d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) - (b^2*x^2 + a*b)*(-d^6/(a^3*b^5)
)^(1/4)*log(-a*b*(-d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) - 4*sqrt(d*x)*d)/(b^
2*x^2 + a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^{\frac{3}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**2, x)
```

Giac [A]
time = 4.07, size = 261, normalized size = 0.93

$$-\frac{1}{16}d\left(\frac{8\sqrt{dx}\,d^2}{\left(bd^2x^2+ad^2\right)b}-\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^2}-\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^2}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^2}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/16*d*(8*sqrt(d*x)*d^2/((b*d^2*x^2 + a*d^2)*b) - 2*sqrt(2)*(a*b^3*d^2)^(1
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1
/4))/(a*b^2) - 2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^2) - sqrt(2)*(a*b^3*d^2)^
(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^2)
+ sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a*b^2))
```

Mupad [B]

time = 4.35, size = 92, normalized size = 0.33

$$-\frac{d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{3/4}\,b^{5/4}}-\frac{d^{3/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{3/4}\,b^{5/4}}-\frac{d^3\,\sqrt{d\,x}}{2\,b\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] - (d^(3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(3/4)*
b^(5/4)) - (d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(
-a)^(3/4)*b^(5/4)) - (d^3*(d*x)^(1/2))/(2*b*(a*d^2 + b*d^2*x^2))
```

3.691 $\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=283

$$\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}-\frac{\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\sqrt{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\sqrt{d}\,\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\cdots\right.}{8\sqrt{2}\cdots}$$

```
[Out] 1/2*(d*x)^(3/2)/a/d/(b*x^2+a)-1/8*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1
/4)/d^(1/2))*d^(1/2)/a^(5/4)/b^(3/4)*2^(1/2)+1/8*arctan(1+b^(1/4)*2^(1/2)*(
d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(5/4)/b^(3/4)*2^(1/2)+1/16*ln(a^(1/2)
*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(
5/4)/b^(3/4)*2^(1/2)-1/16*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1
/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(5/4)/b^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\sqrt{d}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\sqrt{d}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d*x)^(3/2)/(2*a*d*(a + b*x^2)) - (Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4)) + (Sqrt[d]*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4
)) + (Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/4)*b^(3/4)) - (Sqrt[d]*Log[Sqrt[a]*Sqrt[d]
 + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/
4)*b^(3/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx &= b^2\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx\\
&= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}+\frac{b\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4a}\\
&= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}+\frac{b\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{2ad}\\
&= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}-\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{4ad}+\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}\,d+\sqrt{}}{ab+\frac{b^2}{d}}\right.}{4ac}\\
&= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}+\frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx,x,\sqrt{dx}\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\sqrt{d}\,\text{Sub}}{}\\
&= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}+\frac{\sqrt{d}\,\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{\sqrt{d}\,\log}{}\\
&= \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}-\frac{\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{\sqrt{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}\,a^{5/4}b^{3/4}}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 152, normalized size = 0.54

$$
-\frac{\sqrt{dx}\left(-4\sqrt[4]{a}\,b^{3/4}x^{3/2}+\sqrt{2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt{2}\left(a+bx^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{8a^{5/4}b^{3/4}\sqrt{x}\left(a+bx^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/8*(Sqrt[d*x]*(-4*a^(1/4)*b^(3/4)*x^(3/2) + Sqrt[2]*(a + b*x^2)*ArcTan[(S
qrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*(a + b*x^2
```

```
)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(a^(5/
4)*b^(3/4)*Sqrt[x]*(a + b*x^2))
```

Maple [A]

time = 0.04, size = 180, normalized size = 0.64

method	result
derivativedivides	$2d^3\left(\frac{(dx)^{\frac{3}{2}}}{4a\,d^2(d^2x^2b+a\,d^2)}+\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{ar}\right.}{32a\,d^2b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
default	$2d^3\left(\frac{(dx)^{\frac{3}{2}}}{4a\,d^2(d^2x^2b+a\,d^2)}+\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{ar}\right.}{32a\,d^2b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(1/4*(d*x)^(3/2)/a/d^2/(b*d^2*x^2+a*d^2)+1/32/a/d^2/b/(a*d^2/b)^(1/4)
*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x
+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*
d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
))
```

Maxima [A]

time = 0.50, size = 255, normalized size = 0.90

$$\frac{\frac{8\,(dx)^{\frac{3}{2}}d^2}{abd^2x^2+a^2d^2}+\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/16*(8*(d*x)^(3/2)*d^2/(a*b*d^2*x^2 + a^2*d^2) + d^2*(2*sqrt(2)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)
*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt
```

```
(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(
2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + s
qrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*
d)/((a*d^2)^(1/4)*b^(3/4)))/a)/d
```

Fricas [A]
time = 0.40, size = 232, normalized size = 0.82

$$-\frac{4\left(abx^2+a^2\right)\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,abd\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}-\sqrt{-a^3bd^2\sqrt{-\frac{d^2}{a^5b^3}}+d^3x}\,ab\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}}{d^2}\right)-\left(abx^2+a^2\right)\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a^4b^2\left(-\frac{d^2}{a^5b^3}\right)^{\frac{3}{4}}+\sqrt{dx}\,d\right)+\left(abx^2+a^2\right)\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}\log\left(-a^4b^2\left(-\frac{d^2}{a^5b^3}\right)^{\frac{3}{4}}+\sqrt{dx}\,d\right)-4\sqrt{dx}\,x}{8\left(abx^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*arctan(-(sqrt(d*x)*a*b*d*(-d
^2/(a^5*b^3))^(1/4) - sqrt(-a^3*b*d^2*sqrt(-d^2/(a^5*b^3)) + d^3*x)*a*b*(-d
^2/(a^5*b^3))^(1/4))/d^2) - (a*b*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*log(a^4*
b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x)*d) + (a*b*x^2 + a^2)*(-d^2/(a^5*b^3)
)^(1/4)*log(-a^4*b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x)*d) - 4*sqrt(d*x)*x)
/(a*b*x^2 + a^2)
```

Sympy [A]
time = 2.54, size = 78, normalized size = 0.28

$$\frac{2d^3(dx)^{\frac{3}{2}}}{4a^2d^4+4abd^4x^2}+2d^3\operatorname{RootSum}\left(65536t^4a^5b^3d^{10}+1,\left(t\mapsto t\log\left(4096t^3a^4b^2d^8+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*d**3*(d*x)**(3/2)/(4*a**2*d**4 + 4*a*b*d**4*x**2) + 2*d**3*RootSum(65536*
_t**4*a**5*b**3*d**10 + 1, Lambda(_t, _t*log(4096*_t**3*a**4*b**2*d**8 + sq
rt(d*x))))
```

Giac [A]
time = 3.30, size = 264, normalized size = 0.93

$$\frac{\frac{8\sqrt{dx}\,d^3x}{(bd^2x^2+ad^2)a}+\frac{2\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}+\frac{2\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}-\frac{\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^3}+\frac{\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^3}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/16*(8*sqrt(d*x)*d^3*x/((b*d^2*x^2 + a*d^2)*a) + 2*sqrt(2)*(a*b^3*d^2)^(3/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/
```

```
4))/(a^2*b^3) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3) - sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*
b^3) + sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/(a^2*b^3))/d
```

Mupad [B]
time = 0.11, size = 90, normalized size = 0.32

$$\frac{\sqrt{d}\ \mathrm{atanh}\left(\frac{b^{1/4}\sqrt{d\,x}}{(-a)^{1/4}\sqrt{d}}\right)}{4\,(-a)^{5/4}\,b^{3/4}}-\frac{\sqrt{d}\ \mathrm{atan}\left(\frac{b^{1/4}\sqrt{d\,x}}{(-a)^{1/4}\sqrt{d}}\right)}{4\,(-a)^{5/4}\,b^{3/4}}+\frac{d\,(d\,x)^{3/2}}{2\,a\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
[Out] (d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(5/4)*b
^(3/4)) - (d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a
)^(5/4)*b^(3/4)) + (d*(d*x)^(3/2))/(2*a*(a*d^2 + b*d^2*x^2))
```

3.692 $\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt{dx}}{2ad\,(a+bx^2)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}}-\frac{3\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x-\right.}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}}$$

```
[Out] -3/8*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(1/4)*
2^(1/2)/d^(1/2)+3/8*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a
^(7/4)/b^(1/4)*2^(1/2)/d^(1/2)-3/16*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^
(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/4)/b^(1/4)*2^(1/2)/d^(1/2)+3/16*ln(
a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7
/4)/b^(1/4)*2^(1/2)/d^(1/2)+1/2*(d*x)^(1/2)/a/d/(b*x^2+a)
```

Rubi [A]
time = 0.17, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}}-\frac{3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}}+\frac{3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,\sqrt{d}}+\frac{\sqrt{dx}}{2ad\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] Sqrt[d*x]/(2*a*d*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) + (3*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt
[d]) - (3*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) + (3*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)
*b^(1/4)*Sqrt[d])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)}\,dx = b^2\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^2}\,dx$$
$$= \frac{\sqrt{dx}}{2ad(a+bx^2)} + \frac{(3b)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)}\,dx}{4a}$$
$$= \frac{\sqrt{dx}}{2ad(a+bx^2)} + \frac{(3b)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{2ad}$$
$$= \frac{\sqrt{dx}}{2ad(a+bx^2)} + \frac{(3b)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{4a^{3/2}d^2} + \frac{(3b)\text{Subst}\left(\int\right.}{}$$
$$= \frac{\sqrt{dx}}{2ad(a+bx^2)} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{dx}\right)}{8a^{3/2}\sqrt{b}} + \frac{3\text{Sub}}{}$$
$$= \frac{\sqrt{dx}}{2ad(a+bx^2)} - \frac{3\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}} + \frac{3\log}{}$$
$$= \frac{\sqrt{dx}}{2ad(a+bx^2)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{}}{\sqrt[4]{a}\sqrt{d}}\right.}{4\sqrt{2}\,a^{7/4}\sqrt[4]{b}\sqrt{d}}$$

Mathematica [A]
time = 0.25, size = 154, normalized size = 0.54

$$\frac{\sqrt{x}\left(4a^{3/4}\sqrt[4]{b}\sqrt{x}-3\sqrt{2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+3\sqrt{2}(a+bx^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{8a^{7/4}\sqrt[4]{b}\sqrt{dx}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (Sqrt[x]*(4*a^(3/4)*b^(1/4)*Sqrt[x] - 3*Sqrt[2]*(a + b*x^2)*ArcTan[(Sqrt[a]
 - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(a + b*x^2)*Ar
```

```
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(8*a^(7/4)
*b^(1/4)*Sqrt[d*x]*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 177, normalized size = 0.63

method	result
derivativedivides	$2d^3\left(\frac{\sqrt{dx}}{4a\,d^2(d^2x^2b+a\,d^2)}+\frac{3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{32a^2d^4}\right.$
default	$2d^3\left(\frac{\sqrt{dx}}{4a\,d^2(d^2x^2b+a\,d^2)}+\frac{3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{32a^2d^4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(1/4*(d*x)^(1/2)/a/d^2/(b*d^2*x^2+a*d^2)+3/32/a^2/d^4*(a*d^2/b)^(1/4)
*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x
-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*
d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
))
```

Maxima [A]
time = 0.51, size = 261, normalized size = 0.92

$$\frac{\frac{8\sqrt{dx}\,d^2}{abd^2x^2+a^2d^2}+\frac{3\left(\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(8*sqrt(d*x)*d^2/(a*b*d^2*x^2 + a^2*d^2) + 3*(sqrt(2)*d^2*log(sqrt(b)*
d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b
^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(
1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*
```

```
(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*
d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))
/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/a)/d
```

Fricas [A]
time = 0.37, size = 232, normalized size = 0.82

$$\frac{12\,(abdx^2+a^2d)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^4d^2\sqrt{-\frac{1}{a^7bd^2}}+dx}\,a^5bd\left(-\frac{1}{a^7bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^5bd\left(-\frac{1}{a^7bd^2}\right)^{\frac{3}{4}}\right)+3\,(abdx^2+a^2d)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\log\left(a^2d\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-3\,(abdx^2+a^2d)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\log\left(-a^2d\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4\sqrt{dx}}{8\,(abdx^2+a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(12*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*arctan(sqrt(a^4*d^2*sqrt
(-1/(a^7*b*d^2)) + d*x)*a^5*b*d*(-1/(a^7*b*d^2))^(3/4) - sqrt(d*x)*a^5*b*d*
(-1/(a^7*b*d^2))^(3/4)) + 3*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*log(
a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(d*x)) - 3*(a*b*d*x^2 + a^2*d)*(-1/(a^7*
b*d^2))^(1/4)*log(-a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(d*x)) + 4*sqrt(d*x))
/(a*b*d*x^2 + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**2), x)
```

Giac [A]
time = 3.29, size = 269, normalized size = 0.95

$$\frac{\sqrt{dx}\,d}{2\,(bd^2x^2+ad^2)a}+\frac{3\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2bd}+\frac{3\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2bd}+\frac{3\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^2bd}-\frac{3\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*d/((b*d^2*x^2 + a*d^2)*a) + 3/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a
^2*b*d) + 3/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b*d) + 3/16*sqrt(2)*(a*b^3*d
^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2
```

```
*b*d) - 3/16*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a^2*b*d)
```

Mupad [B]
time = 0.10, size = 90, normalized size = 0.32

$$\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{7/4}\,b^{1/4}\,\sqrt{d}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{7/4}\,b^{1/4}\,\sqrt{d}}+\frac{d\,\sqrt{d\,x}}{2\,a\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (3*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(7/4)*b^(1/4)*
d^(1/2)) + (3*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(7
/4)*b^(1/4)*d^(1/2)) + (d*(d*x)^(1/2))/(2*a*(a*d^2 + b*d^2*x^2))
```

3.693 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx$

Optimal. Leaf size=300

$$-\frac{5}{2a^2d\sqrt{dx}}+\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}+\frac{5\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{9/4}d^{3/2}}-\frac{5\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{9/4}d^{3/2}}-\frac{5\sqrt{}}{}$$

```
[Out] 5/8*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/d
^(3/2)*2^(1/2)-5/8*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(
1/2))/a^(9/4)/d^(3/2)*2^(1/2)-5/16*b^(1/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(9/4)/d^(3/2)*2^(1/2)+5/16*b^(1
/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2
))/a^(9/4)/d^(3/2)*2^(1/2)-5/2/a^2/d/(d*x)^(1/2)+1/2/a/d/(b*x^2+a)/(d*x)^(1
/2)
```

Rubi [A]
time = 0.20, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{5\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{9/4}d^{3/2}}-\frac{5\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,a^{9/4}d^{3/2}}-\frac{5\sqrt[4]{b}\,\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\,\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{9/4}d^{3/2}}-\frac{5}{2a^2d\sqrt{dx}}+\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -5/(2*a^2*d*Sqrt[d*x]) + 1/(2*a*d*Sqrt[d*x]*(a + b*x^2)) + (5*b^(1/4)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(9/4)*d^
(3/2)) - (5*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(4*Sqrt[2]*a^(9/4)*d^(3/2)) - (5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*
Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
 + (5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{1}{2ad\sqrt{dx}\,(a+bx^2)} + \frac{(5b)\int \frac{1}{(dx)^{3/2}(ab+b^2x^2)}\,dx}{4a}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,(a+bx^2)} - \frac{(5b^2)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{4a^2d^2}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,(a+bx^2)} - \frac{(5b^2)\,\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right.}{2a^2d^3}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,(a+bx^2)} + \frac{(5b^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x\right.}{4a^2d^3}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,(a+bx^2)} - \frac{\left(5\sqrt[4]{b}\right)\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{b}}}\right.}{8\sqrt{2}\,a^{9/4}d^{3/2}}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,(a+bx^2)} - \frac{5\sqrt[4]{b}\,\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x-\right.}{8\sqrt{2}\,a^{9/4}d^{3/2}}$$

$$= -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,(a+bx^2)} + \frac{5\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{9/4}d^{3/2}}$$

Mathematica [A]

time = 0.28, size = 165, normalized size = 0.55

$$\frac{x\left(-4\sqrt[4]{a}\,(4a+5bx^2)+5\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+5\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,(a+bx^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{8a^{9/4}(dx)^{3/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (x*(-4*a^(1/4)*(4*a + 5*b*x^2) + 5*Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*x^2)*ArcT
an[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2]*b^(
1/4)*Sqrt[x]*(a + b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a]
 + Sqrt[b]*x)]))/(8*a^(9/4)*(d*x)^(3/2)*(a + b*x^2))
```

Maple [A]

time = 0.07, size = 191, normalized size = 0.64

method	result
derivativedivides	$2d^3\left(-\frac{1}{a^2d^4\sqrt{dx}}-\frac{b\left(\frac{(dx)^{\frac{3}{2}}}{4d^2x^2b+4a\,d^2}+\frac{5\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{32b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^4}\right.$
default	$2d^3\left(-\frac{1}{a^2d^4\sqrt{dx}}-\frac{b\left(\frac{(dx)^{\frac{3}{2}}}{4d^2x^2b+4a\,d^2}+\frac{5\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{32b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^4}\right.$

risch	$-\frac{2}{a^{2} d \sqrt{d x}}+\frac{-\frac{b (d x)^{\frac{3}{2}}}{2 a^{2} \left(d^{2} x^{2} b+a d^{2}\right)}-\frac{5 \sqrt{2}\, \ln \left(\frac{d x-\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{d x}\, \sqrt{2}+\sqrt{\frac{a d^{2}}{b}}}{d x+\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{d x}\, \sqrt{2}+\sqrt{\frac{a d^{2}}{b}}}\right)}{16 a^{2} \left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}-\frac{5 \sqrt{2}\, \arctan \left(\frac{\sqrt{2}\, \sqrt{d x}}{\left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}} \cdots\right.}{8 a^{2} \left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}} \cdots}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(-1/a^2/d^4/(d*x)^(1/2)-b/a^2/d^4*(1/4*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+
5/32/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))
)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b
)^(1/4)*(d*x)^(1/2)-1))))
```

Maxima [A]

time = 0.51, size = 268, normalized size = 0.89

$$-\frac{\frac{8 \left(5 b d^{2} x^{2}+4 a d^{2}\right)}{(d x)^{\frac{5}{2}} a^{2} b+\sqrt{d x}\, a^{3} d^{2}}+\frac{5 b \left(\frac{2 \sqrt{2} \arctan \left(\frac{\sqrt{2} \left(\sqrt{2} \left(a d^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}}+2 \sqrt{d x} \sqrt{b}\right)}{2 \sqrt{\sqrt{a} \sqrt{b} d}}\right)}{\sqrt{\sqrt{a} \sqrt{b} d} \sqrt{b}}+\frac{2 \sqrt{2} \arctan \left(-\frac{\sqrt{2} \left(\sqrt{2} \left(a d^{2}\right)^{\frac{1}{4}} b^{\frac{1}{4}}-2 \sqrt{d x} \sqrt{b}\right)}{2 \sqrt{\sqrt{a} \sqrt{b} d}}\right)}{\sqrt{\sqrt{a} \sqrt{b} d} \sqrt{b}}-\frac{\sqrt{2} \log \left(\sqrt{b} d x+\sqrt{2} \left(a d^{2}\right)^{\frac{1}{4}} \sqrt{d x}\, b^{\frac{1}{4}}+\sqrt{a} d\right)}{\left(a d^{2}\right)^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2} \log \left(\sqrt{b} d x-\sqrt{2} \left(a d^{2}\right)^{\frac{1}{4}} \sqrt{d x}\, b^{\frac{1}{4}}+\sqrt{a} d\right)}{\left(a d^{2}\right)^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{a^{2}}}{16 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(5*b*d^2*x^2 + 4*a*d^2)/((d*x)^(5/2)*a^2*b + sqrt(d*x)*a^3*d^2) +
5*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*s
qrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2
)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((
a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqr
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^2)/d
```

Fricas [A]

time = 0.34, size = 276, normalized size = 0.92

$$\frac{20 \left(a^{2} b d^{2} x^{3}+a^{3} d^{2} x\right) \left(-\frac{b}{a^{9} d^{6}}\right)^{\frac{1}{4}} \arctan \left(-\frac{125 \sqrt{d x}\, a^{2} b d \left(-\frac{b}{a^{9} d^{6}}\right)^{\frac{1}{4}}-\sqrt{-15625 a^{5} b d^{4} \sqrt{-\frac{b}{a^{9} d^{6}}}+15625 b^{2} d x}\, a^{2} d \left(-\frac{b}{a^{9} d^{6}}\right)^{\frac{1}{4}}}{125 b}\right)-5 \left(a^{2} b d^{2} x^{3}+a^{3} d^{2} x\right) \left(-\frac{b}{a^{9} d^{6}}\right)^{\frac{1}{4}} \log \left(125 a^{7} d^{5} \left(-\frac{b}{a^{9} d^{6}}\right)^{\frac{3}{4}}+125 \sqrt{d x}\, b\right)+5 \left(a^{2} b d^{2} x^{3}+a^{3} d^{2} x\right) \left(-\frac{b}{a^{9} d^{6}}\right)^{\frac{1}{4}} \log \left(-125 a^{7} d^{5} \left(-\frac{b}{a^{9} d^{6}}\right)^{\frac{3}{4}}+125 \sqrt{d x}\, b\right)-4 (5 b x^{2}+4 a) \sqrt{d x}}{8 \left(a^{2} b d^{2} x^{3}+a^{3} d^{2} x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/8*(20*(a^2*b*d^2*x^3 + a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*arctan(-1/125*(125
*sqrt(d*x)*a^2*b*d*(-b/(a^9*d^6))^(1/4) - sqrt(-15625*a^5*b*d^4*sqrt(-b/(a^
9*d^6)) + 15625*b^2*d*x)*a^2*d*(-b/(a^9*d^6))^(1/4))/b) - 5*(a^2*b*d^2*x^3
+ a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*log(125*a^7*d^5*(-b/(a^9*d^6))^(3/4) + 12
5*sqrt(d*x)*b) + 5*(a^2*b*d^2*x^3 + a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*log(-12
5*a^7*d^5*(-b/(a^9*d^6))^(3/4) + 125*sqrt(d*x)*b) - 4*(5*b*x^2 + 4*a)*sqrt(
d*x))/(a^2*b*d^2*x^3 + a^3*d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**2), x)
```

Giac [A]
time = 3.08, size = 294, normalized size = 0.98

$$-\frac{\frac{8\,(5\,bd^2x^2+4\,ad^2)}{\left(\sqrt{dx}\,bd^2x^2+\sqrt{dx}\,ad^2\right)a^2}+\frac{10\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2d^2}+\frac{10\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2d^2}-\frac{5\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^2d^2}+\frac{5\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^2d^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/16*(8*(5*b*d^2*x^2 + 4*a*d^2)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)*a
^2) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2*d^2) + 10*sqrt(2)*(a*b^3*d^2)^
(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^3*b^2*d^2) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2*d^2) + 5*sqrt(2)*(a*b^3*d^2)
^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^
2*d^2))/d
```

Mupad [B]
time = 0.12, size = 102, normalized size = 0.34

$$\frac{5\,{(-b)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{9/4}\,d^{3/2}}-\frac{5\,{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{9/4}\,d^{3/2}}-\frac{\frac{2\,d}{a}+\frac{5\,b\,d\,x^2}{2\,a^2}}{b\,{(d\,x)}^{5/2}+a\,d^2\,\sqrt{d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (5*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(4*a^(9/4)
*d^(3/2)) - (5*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))
/(4*a^(9/4)*d^(3/2)) - ((2*d)/a + (5*b*d*x^2)/(2*a^2))/(b*(d*x)^(5/2) + a*d
^2*(d*x)^(1/2))
```

3.694 $\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=300

$$-\frac{7}{6a^2d(dx)^{3/2}}+\frac{1}{2ad(dx)^{3/2}(a+bx^2)}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{11/4}d^{5/2}}-\frac{7b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{11/4}d^{5/2}}$$

```
[Out] -7/6/a^2/d/(d*x)^(3/2)+1/2/a/d/(d*x)^(3/2)/(b*x^2+a)+7/8*b^(3/4)*arctan(1-b
^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/4)/d^(5/2)*2^(1/2)-7/8*b^
(3/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/4)/d^(5/2
)*2^(1/2)+7/16*b^(3/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)
*2^(1/2)*(d*x)^(1/2))/a^(11/4)/d^(5/2)*2^(1/2)-7/16*b^(3/4)*ln(a^(1/2)*d^(1
/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/d^(5/2)
*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$,
Rules used = {28, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{7b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\,a^{11/4}d^{5/2}}-\frac{7b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\,a^{11/4}d^{5/2}}+\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{11/4}d^{5/2}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{11/4}d^{5/2}}-\frac{7}{6a^2d(dx)^{3/2}}+\frac{1}{2ad(dx)^{3/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -7/(6*a^2*d*(d*x)^(3/2)) + 1/(2*a*d*(d*x)^(3/2)*(a + b*x^2)) + (7*b^(3/4)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(11/
4)*d^(5/2)) - (7*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])])/(4*Sqrt[2]*a^(11/4)*d^(5/2)) + (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sq
rt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d
^(5/2)) - (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx &= b^2\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^2}\,dx\\
&= \frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}+\frac{(7b)\int \frac{1}{(dx)^{5/2}(ab+b^2x^2)}\,dx}{4a}\\
&= -\frac{7}{6a^2d(dx)^{3/2}}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}-\frac{\left(7b^2\right)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)}\,dx}{4a^2d^2}\\
&= -\frac{7}{6a^2d(dx)^{3/2}}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}-\frac{\left(7b^2\right)\mathrm{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx,x,\sqrt{\ldots}\right)}{2a^2d^3}\\
&= -\frac{7}{6a^2d(dx)^{3/2}}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}-\frac{\left(7b^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx\ldots\right)}{4a^{5/2}d^4}\\
&= -\frac{7}{6a^2d(dx)^{3/2}}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}+\frac{\left(7b^{3/4}\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\ldots}{\sqrt[4]{b}\ldots}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\ldots}{\ldots}}\ldots\right)}{8\sqrt{2}\,a^{11\ldots}}\\
&= -\frac{7}{6a^2d(dx)^{3/2}}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}+\frac{7b^{3/4}\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\ldots\right.}{8\sqrt{2}\,a^{11/4}\ldots}\\
&= -\frac{7}{6a^2d(dx)^{3/2}}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\ldots\right.}{4\sqrt{2}\,a^{11/4}d^{5/2}}
\end{aligned}$$

Mathematica [A]

time = 0.28, size = 165, normalized size = 0.55

$$\frac{x\left(-4a^{3/4}(4a+7bx^2)+21\sqrt{2}\,b^{3/4}x^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-21\sqrt{2}\,b^{3/4}x^{3/2}(a+bx^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{24a^{11/4}(dx)^{5/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (x*(-4*a^(3/4)*(4*a + 7*b*x^2) + 21*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b*x^2)*Arc
Tan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 21*Sqrt[2]*b
^(3/4)*x^(3/2)*(a + b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[
a] + Sqrt[b]*x)]))/(24*a^(11/4)*(d*x)^(5/2)*(a + b*x^2))
```

Maple [A]

time = 0.07, size = 194, normalized size = 0.65

method	result
derivativedivides	$2d^3\Big(-\frac{1}{3a^2d^4(dx)^{\frac{3}{2}}}-\frac{b\Big(\frac{\sqrt{dx}}{4d^2x^2b+4a\,d^2}+\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\Big(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\Big(\sqrt{2}\cdots}{32a\,d^2}\cdots}{a^2d^4}\cdots$
default	$2d^3\Big(-\frac{1}{3a^2d^4(dx)^{\frac{3}{2}}}-\frac{b\Big(\frac{\sqrt{dx}}{4d^2x^2b+4a\,d^2}+\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\Big(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\Big(\sqrt{2}\cdots}{32a\,d^2}\cdots}{a^2d^4}\cdots$

risch	$-\frac{2}{3a^2x\sqrt{dx}\,d^2}+\frac{-\frac{bd\sqrt{dx}}{2a^2\left(d^2x^2b+a\,d^2\right)}-\frac{7b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16a^3d}-\frac{7b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{}}{}}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(-1/3/a^2/d^4/(d*x)^(3/2)-1/a^2/d^4*b*(1/4*(d*x)^(1/2)/(b*d^2*x^2+a*d
^2)+7/32*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))))
```

Maxima [A]
time = 0.51, size = 275, normalized size = 0.92

$$-\frac{\frac{8\left(7bd^2x^2+4ad^2\right)}{(dx)^{\frac{7}{2}}a^2b+(dx)^{\frac{3}{2}}a^3d^2}+\frac{21\left(\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}\,d}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}\,d}\right)}{a^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -1/48*(8*(7*b*d^2*x^2 + 4*a*d^2)/((d*x)^(7/2)*a^2*b + (d*x)^(3/2)*a^3*d^2)
+ 21*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(
1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*d*x - sqrt(2)
*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) + 2*sqrt(2)*b*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d) + 2*sqrt(2)*b*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt
(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d))/a^2)/d
```

Fricas [A]
time = 0.38, size = 300, normalized size = 1.00

$$-\frac{84\left(a^2bd^3x^4+a^3d^3x^2\right)\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^8bd^7\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^6d^6\sqrt{-\frac{b^3}{a^{11}d^{10}}}+b^2dx}\,a^8d^7\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+21\left(a^2bd^3x^4+a^3d^3x^2\right)\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}\log\left(7a^3d^3\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}+7\sqrt{dx}\,b\right)-21\left(a^2bd^3x^4+a^3d^3x^2\right)\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}\log\left(-7a^3d^3\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}+7\sqrt{dx}\,b\right)+4\left(7bx^2+4a\right)\sqrt{dx}}{24\left(a^2bd^3x^4+a^3d^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/24*(84*(a^2*b*d^3*x^4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10))^(1/4)*arctan(-(s
qrt(d*x)*a^8*b*d^7*(-b^3/(a^11*d^10))^(3/4) - sqrt(a^6*d^6*sqrt(-b^3/(a^11*
```

```
d^10)) + b^2*d*x)*a^8*d^7*(-b^3/(a^11*d^10))^(3/4))/b^3) + 21*(a^2*b*d^3*x^
4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10))^(1/4)*log(7*a^3*d^3*(-b^3/(a^11*d^10))^
(1/4) + 7*sqrt(d*x)*b) - 21*(a^2*b*d^3*x^4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10)
)^(1/4)*log(-7*a^3*d^3*(-b^3/(a^11*d^10))^(1/4) + 7*sqrt(d*x)*b) + 4*(7*b*x
^2 + 4*a)*sqrt(d*x))/(a^2*b*d^3*x^4 + a^3*d^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}}\,(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*(a + b*x**2)**2), x)
```

Giac [A]
time = 3.21, size = 276, normalized size = 0.92

$$-\frac{\sqrt{dx}\,b}{2\,(bd^2x^2+ad^2)a^2d}-\frac{7\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3d^3}-\frac{7\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3d^3}-\frac{7\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^3d^3}+\frac{7\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^3d^3}-\frac{2}{3\,\sqrt{dx}\,a^2d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(d*x)*b/((b*d^2*x^2 + a*d^2)*a^2*d) - 7/8*sqrt(2)*(a*b^3*d^2)^(1/4
)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4
))/(a^3*d^3) - 7/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*d^3) - 7/16*sqrt(2)*(a*
b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a^3*d^3) + 7/16*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*d^3) - 2/3/(sqrt(d*x)*a^2*d^2*x)
```

Mupad [B]
time = 4.40, size = 102, normalized size = 0.34

$$\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{11/4}\,d^{5/2}}-\frac{\frac{2\,d}{3\,a}+\frac{7\,b\,d\,x^2}{6\,a^2}}{b\,{\left(d\,x\right)}^{7/2}+a\,d^2\,{\left(d\,x\right)}^{3/2}}+\frac{7\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{11/4}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] (7*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(4*a^(11/4)
*d^(5/2)) - ((2*d)/(3*a) + (7*b*d*x^2)/(6*a^2))/(b*(d*x)^(7/2) + a*d^2*(d*x
)^(3/2)) + (7*(-b)^(3/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))
/(4*a^(11/4)*d^(5/2))
```

3.695 $\int \frac{1}{(dx)^{7/2}(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=318

$$-\frac{9}{10a^2d(dx)^{5/2}}+\frac{9b}{2a^3d^3\sqrt{dx}}+\frac{1}{2ad(dx)^{5/2}\,(a+bx^2)}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{13/4}d^{7/2}}+\frac{9b^{5/4}\tan^{-1}\left(1+\cdots\right.}{4\sqrt{2}\,a^{13/4}\cdots}$$

```
[Out] -9/10/a^2/d/(d*x)^(5/2)+1/2/a/d/(d*x)^(5/2)/(b*x^2+a)-9/8*b^(5/4)*arctan(1-
b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/d^(7/2)*2^(1/2)+9/8*b
^(5/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/d^(7/
2)*2^(1/2)+9/16*b^(5/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4
)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/d^(7/2)*2^(1/2)-9/16*b^(5/4)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/d^(7/2
)*2^(1/2)+9/2*b/a^3/d^3/(d*x)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{9b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{4\sqrt{2}\,a^{13/4}d^{7/2}}+\frac{9b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{4\sqrt{2}\,a^{13/4}d^{7/2}}+\frac{9b^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{13/4}d^{7/2}}-\frac{9b^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{8\sqrt{2}\,a^{13/4}d^{7/2}}+\frac{9b}{2a^3d^3\sqrt{dx}}-\frac{9}{10a^2d(dx)^{5/2}}+\frac{1}{2ad(dx)^{5/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -9/(10*a^2*d*(d*x)^(5/2)) + (9*b)/(2*a^3*d^3*Sqrt[d*x]) + 1/(2*a*d*(d*x)^(5
/2)*(a + b*x^2)) - (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b
^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(8*Sqrt[2]*a^(13/4)*d^(7/2)) - (9*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(13/4)*
d^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = b^2\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^2}\,dx$$

$$= \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{(9b)\int \frac{1}{(dx)^{7/2}(ab+b^2x^2)}\,dx}{4a}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{(9b^2)\int \frac{1}{(dx)^{3/2}(ab+b^2x^2)}\,dx}{4a^2d^2}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{(9b^3)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,d}{4a^3d^4}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{(9b^3)\,\mathrm{Subst}\left(\int \right.}{2}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{(9b^{5/2})\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{(9b^{5/4})\,\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} + \frac{9b^{5/4}\log\left(\sqrt{a}\right.}{}$$

$$= -\frac{9}{10a^2d(dx)^{5/2}} + \frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)} - \frac{9b^{5/4}\tan^{-1}\left(1\right.}{4\sqrt{2}}$$

Mathematica [A]
time = 0.30, size = 181, normalized size = 0.57

$$-\frac{\sqrt{dx}\left(4\sqrt[4]{a}\left(4a^2-36abx^2-45b^2x^4\right)+45\sqrt{2}\,b^{5/4}x^{5/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+45\sqrt{2}\,b^{5/4}x^{5/2}\left(a+bx^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{40a^{13/4}d^4x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/40*(Sqrt[d*x]*(4*a^(1/4)*(4*a^2 - 36*a*b*x^2 - 45*b^2*x^4) + 45*Sqrt[2]*
b^(5/4)*x^(5/2)*(a + b*x^2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x])] + 45*Sqrt[2]*b^(5/4)*x^(5/2)*(a + b*x^2)*ArcTanh[(Sqrt[2]*
```

a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(a^(13/4)*d^4*x^3*(a + b*
x^2))

Maple [A]
time = 0.07, size = 206, normalized size = 0.65

method	result
derivativedivides	$2d^3\left(-\frac{1}{5a^2d^4(dx)^{\frac{5}{2}}}+\frac{2b}{a^3d^6\sqrt{dx}}+\frac{b^2\left(\frac{(dx)^{\frac{3}{2}}}{4d^2x^2b+4a\,d^2}+\frac{9\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\,a\right.}{32b\left(\frac{a}{}\right.}\right.}{a^3d^6}\right.$
default	$2d^3\left(-\frac{1}{5a^2d^4(dx)^{\frac{5}{2}}}+\frac{2b}{a^3d^6\sqrt{dx}}+\frac{b^2\left(\frac{(dx)^{\frac{3}{2}}}{4d^2x^2b+4a\,d^2}+\frac{9\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\,a\right.}{32b\left(\frac{a}{}\right.}\right.}{a^3d^6}\right.$
risch	$-\frac{2\left(-10b\,x^2+a\right)}{5a^3\sqrt{dx}\,x^2d^3}+\frac{\frac{b^2(dx)^{\frac{3}{2}}}{2a^3\left(d^2x^2b+a\,d^2\right)}+\frac{9b\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{16a^3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+\frac{9b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a\,d^2}{b}\right)}\right.}{8a^3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(-1/5/a^2/d^4/(d*x)^(5/2)+2/a^3/d^6*b/(d*x)^(1/2)+b^2/a^3/d^6*(1/4*(d
*x)^(3/2)/(b*d^2*x^2+a*d^2)+9/32/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1
```

```
/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+
1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))))
```

Maxima [A]
time = 0.50, size = 290, normalized size = 0.91

$$\frac{\frac{8\,(45\,b^2d^4x^4+36\,abd^4x^2-4\,a^2d^4)}{(dx)^{\frac{9}{2}}a^3bd^2+(dx)^{\frac{5}{2}}a^4d^4}+\frac{45\,b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^3d^2}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/80*(8*(45*b^2*d^4*x^4 + 36*a*b*d^4*x^2 - 4*a^2*d^4)/((d*x)^(9/2)*a^3*b*d^
2 + (d*x)^(5/2)*a^4*d^4) + 45*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(s
qrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)
^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a
)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqr
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d
*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^
(3/4)))/(a^3*d^2))/d
```

Fricas [A]
time = 0.35, size = 323, normalized size = 1.02

$$-\frac{180\,(a^3bd^4x^5+a^4d^4x^3)\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{729\sqrt{dx}\,a^3b^4d^3\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-531441\,a^7b^5d^8\sqrt{-\frac{b^5}{a^{13}d^{14}}}+531441\,b^8dx}\,a^3d^3\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}}{729\,b^5}\right)-45\,(a^3bd^4x^5+a^4d^4x^3)\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}\log\left(729\,a^{10}d^{11}\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{3}{4}}+729\sqrt{dx}\,b^4\right)+45\,(a^3bd^4x^5+a^4d^4x^3)\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}\log\left(-729\,a^{10}d^{11}\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{3}{4}}+729\sqrt{dx}\,b^4\right)-4\,(45\,b^2x^4+36\,abx^2-4\,a^2)\sqrt{dx}}{40\,(a^3bd^4x^5+a^4d^4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] -1/40*(180*(a^3*b*d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*arctan(-1
/729*(729*sqrt(d*x)*a^3*b^4*d^3*(-b^5/(a^13*d^14))^(1/4) - sqrt(-531441*a^7
*b^5*d^8*sqrt(-b^5/(a^13*d^14)) + 531441*b^8*d*x)*a^3*d^3*(-b^5/(a^13*d^14)
)^(1/4))/b^5) - 45*(a^3*b*d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*l
og(729*a^10*d^11*(-b^5/(a^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) + 45*(a^3*b*
d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*log(-729*a^10*d^11*(-b^5/(a
^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) - 4*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)
*sqrt(d*x))/(a^3*b*d^4*x^5 + a^4*d^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{7}{2}}\,(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(7/2)*(a + b*x**2)**2), x)
```

Giac [A]
time = 3.95, size = 307, normalized size = 0.97

$$\frac{\sqrt{dx}\,b^2x}{2\left(bd^2x^2+ad^2\right)a^3d^2}+\frac{9\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^4bd^5}+\frac{9\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^4bd^5}-\frac{9\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16a^4bd^5}+\frac{9\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16a^4bd^5}+\frac{2\left(10bd^2x^2-ad^2\right)}{5\sqrt{dx}\,a^3d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*b^2*x/((b*d^2*x^2 + a*d^2)*a^3*d^2) + 9/8*sqrt(2)*(a*b^3*d^2)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^4*b*d^5) + 9/8*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d^5) - 9/16*sq
rt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a^4*b*d^5) + 9/16*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a
*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d^5) + 2/5*(10*b*d^2*x^2 -
a*d^2)/(sqrt(d*x)*a^3*d^5*x^2)
```

Mupad [B]
time = 4.35, size = 113, normalized size = 0.36

$$\frac{\frac{9\,b^2\,d\,x^4}{2\,a^3}-\frac{2\,d}{5\,a}+\frac{18\,b\,d\,x^2}{5\,a^2}}{b\,{\left(d\,x\right)}^{9/2}+a\,d^2\,{\left(d\,x\right)}^{5/2}}-\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{13/4}\,d^{7/2}}+\frac{9\,{\left(-b\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{13/4}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

```
[Out] ((9*b^2*d*x^4)/(2*a^3) - (2*d)/(5*a) + (18*b*d*x^2)/(5*a^2))/(b*(d*x)^(9/2)
 + a*d^2*(d*x)^(5/2)) - (9*(-b)^(5/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4
)*d^(1/2))))/(4*a^(13/4)*d^(7/2)) + (9*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x)^(
1/2))/(a^(1/4)*d^(1/2))))/(4*a^(13/4)*d^(7/2))
```

3.696 $\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=368

$$-\frac{663ad^9\sqrt{dx}}{64b^5}+\frac{663d^7(dx)^{5/2}}{320b^4}-\frac{d(dx)^{17/2}}{6b\,(a+bx^2)^3}-\frac{17d^3(dx)^{13/2}}{48b^2\,(a+bx^2)^2}-\frac{221d^5(dx)^{9/2}}{192b^3\,(a+bx^2)}-\frac{663a^{5/4}d^{19/2}\tan^{-1}\left(1-\sqrt{\right.}}{128\sqrt{2}\,b^{21/4}}$$

```
[Out] 663/320*d^7*(d*x)^(5/2)/b^4-1/6*d*(d*x)^(17/2)/b/(b*x^2+a)^3-17/48*d^3*(d*x
)^(13/2)/b^2/(b*x^2+a)^2-221/192*d^5*(d*x)^(9/2)/b^3/(b*x^2+a)-663/256*a^(5
/4)*d^(19/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(21/4)
*2^(1/2)+663/256*a^(5/4)*d^(19/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1
/4)/d^(1/2))/b^(21/4)*2^(1/2)-663/512*a^(5/4)*d^(19/2)*ln(a^(1/2)*d^(1/2)+x
*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(21/4)*2^(1/2)+663/
512*a^(5/4)*d^(19/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2
^(1/2)*(d*x)^(1/2))/b^(21/4)*2^(1/2)-663/64*a*d^9*(d*x)^(1/2)/b^5
```

Rubi [A]
time = 0.28, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{663a^{5/4}d^{19/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,b^{21/4}}+\frac{663a^{5/4}d^{19/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,b^{21/4}}-\frac{663a^{5/4}d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{21/4}}+\frac{663a^{5/4}d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{21/4}}-\frac{663ad^9\sqrt{dx}}{64b^5}-\frac{221d^5(dx)^{9/2}}{192b^3\,(a+bx^2)}-\frac{17d^3(dx)^{13/2}}{48b^2\,(a+bx^2)^2}-\frac{d(dx)^{17/2}}{6b\,(a+bx^2)^3}+\frac{663d^7(dx)^{5/2}}{320b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-663*a*d^9*Sqrt[d*x])/(64*b^5) + (663*d^7*(d*x)^(5/2))/(320*b^4) - (d*(d*x
)^(17/2))/(6*b*(a + b*x^2)^3) - (17*d^3*(d*x)^(13/2))/(48*b^2*(a + b*x^2)^2
) - (221*d^5*(d*x)^(9/2))/(192*b^3*(a + b*x^2)) - (663*a^(5/4)*d^(19/2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(21/
4)) + (663*a^(5/4)*d^(19/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(128*Sqrt[2]*b^(21/4)) - (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt
[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*
b^(21/4)) + (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(21/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{19/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}+\frac{1}{12}\left(17b^2d^2\right)\int \frac{(dx)^{15/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}-\frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2}+\frac{1}{96}\left(221d^4\right)\int \frac{(dx)^{11/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}-\frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2}-\frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)}+\frac{\left(663d^6\right)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= \frac{663d^7(dx)^{5/2}}{320b^4}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}-\frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2}-\frac{221d^5(dx)^{9/2}}{192b^3\left(a+bx^2\right)}-\frac{(663ad^8}{}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5}+\frac{663d^7(dx)^{5/2}}{320b^4}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}-\frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2}-\frac{221d^5(}{192b^3(a}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5}+\frac{663d^7(dx)^{5/2}}{320b^4}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}-\frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2}-\frac{221d^5(}{192b^3(a}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5}+\frac{663d^7(dx)^{5/2}}{320b^4}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}-\frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2}-\frac{221d^5(}{192b^3(a}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5}+\frac{663d^7(dx)^{5/2}}{320b^4}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}-\frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2}-\frac{221d^5(}{192b^3(a}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5}+\frac{663d^7(dx)^{5/2}}{320b^4}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}-\frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2}-\frac{221d^5(}{192b^3(a}$$

$$= -\frac{663ad^9\sqrt{dx}}{64b^5}+\frac{663d^7(dx)^{5/2}}{320b^4}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}-\frac{17d^3(dx)^{13/2}}{48b^2\left(a+bx^2\right)^2}-\frac{221d^5(}{192b^3(a}$$

Mathematica [A]

time = 0.57, size = 205, normalized size = 0.56

$$\frac{d^9\sqrt{dx}\left(4\sqrt[4]{b}\sqrt{x}\left(-9945a^4-27846a^3bx^2-24973a^2b^2x^4-6528ab^3x^6+384b^4x^8\right)+9945\sqrt{2}\,a^{5/4}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+9945\sqrt{2}\,a^{5/4}\left(a+bx^2\right)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{3840b^{21/4}\sqrt{x}\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^9*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(-9945*a^4 - 27846*a^3*b*x^2 - 24973*a^2*
b^2*x^4 - 6528*a*b^3*x^6 + 384*b^4*x^8) + 9945*Sqrt[2]*a^(5/4)*(a + b*x^2)^
3*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 9945*S
qrt[2]*a^(5/4)*(a + b*x^2)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqr
t[a] + Sqrt[b]*x)]))/(3840*b^(21/4)*Sqrt[x]*(a + b*x^2)^3)
```

Maple [A]
time = 0.10, size = 237, normalized size = 0.64

method	result
derivativedivides	$2d^7\left(-\frac{-\frac{(dx)^{\frac{5}{2}}b}{5}+4a\,d^2\sqrt{dx}}{b^5}+\frac{a^2d^4\left(\frac{-\frac{617b^2(dx)^{\frac{9}{2}}}{384}-\frac{173ab\,d^2(dx)^{\frac{5}{2}}}{64}-\frac{151a^2d^4\sqrt{dx}}{128}}{\left(d^2x^2b+a\,d^2\right)^3}+\frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+}{dx-}\right.\right.}{}\right.}{}\right.$
default	$2d^7\left(-\frac{-\frac{(dx)^{\frac{5}{2}}b}{5}+4a\,d^2\sqrt{dx}}{b^5}+\frac{a^2d^4\left(\frac{-\frac{617b^2(dx)^{\frac{9}{2}}}{384}-\frac{173ab\,d^2(dx)^{\frac{5}{2}}}{64}-\frac{151a^2d^4\sqrt{dx}}{128}}{\left(d^2x^2b+a\,d^2\right)^3}+\frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+}{dx-}\right.\right.}{}\right.}{}\right.$
risch	$-\frac{2(-b\,x^2+20a)x\,d^{10}}{5b^5\sqrt{dx}}+\left(-\frac{617a^2d(dx)^{\frac{9}{2}}}{192b^3(d^2x^2b+a\,d^2)^3}-\frac{173a^3d^3(dx)^{\frac{5}{2}}}{32b^4(d^2x^2b+a\,d^2)^3}-\frac{151a^4d^5\sqrt{dx}}{64b^5(d^2x^2b+a\,d^2)^3}+\frac{663a\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)

[Out] 2*d^7*(-1/b^5*(-1/5*(d*x)^(5/2)*b+4*a*d^2*(d*x)^(1/2))+1/b^5*a^2*d^4*((-617/384*b^2*(d*x)^(9/2)-173/64*a*b*d^2*(d*x)^(5/2)-151/128*a^2*d^4*(d*x)^(1/2))/(b*d^2*x^2+a*d^2)^3+663/1024*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))))

Maxima [A]
time = 0.52, size = 361, normalized size = 0.98

$$-\frac{\frac{40\left(617\,(dx)^{\frac{9}{2}}a^2b^2d^{12}+1038\,(dx)^{\frac{5}{2}}a^3bd^{14}+453\,\sqrt{dx}\,a^4d^{16}\right)}{b^8d^6x^6+3\,ab^7d^6x^4+3\,a^2b^6d^6x^2+a^3b^5d^6}-\frac{9945\left(\frac{\sqrt{2}\,d^{12}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{12}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^{11}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^{11}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}\right)a^2}{b^5}-\frac{3072\left((dx)^{\frac{5}{2}}bd^8-20\sqrt{dx}\,ad^{10}\right)}{b^5}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")

[Out] -1/7680*(40*(617*(d*x)^(9/2)*a^2*b^2*d^12 + 1038*(d*x)^(5/2)*a^3*b*d^14 + 453*sqrt(d*x)*a^4*d^16)/(b^8*d^6*x^6 + 3*a*b^7*d^6*x^4 + 3*a^2*b^6*d^6*x^2 + a^3*b^5*d^6) - 9945*(sqrt(2)*d^12*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^12*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^11*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^11*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a^2/b^5 - 3072*((d*x)^(5/2)*b*d^8 - 20*sqrt(d*x)*a*d^10)/b^5)/d

Fricas [A]
time = 0.40, size = 399, normalized size = 1.08

$$\frac{39780\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)\arctan\left(-\frac{\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{16}d^9-\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{3}{4}}\sqrt{a^2d^{19}x+\sqrt{-\frac{a^5d^{38}}{b^{21}}}\,b^{10}}\,b^{16}}{a^5d^{38}}\right)+9945\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)\log\left(663\sqrt{dx}\,ad^9+663\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}b^5\right)-9945\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)\log\left(663\sqrt{dx}\,ad^9-663\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}b^5\right)+4\,(384\,b^4d^9x^8-6528\,ab^3d^9x^6-24973\,a^2b^2d^9x^4-27846\,a^3bd^9x^2-9945\,a^4d^9)\sqrt{dx}}{3840\,(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")

[Out] 1/3840*(39780*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)*arctan(-((-a^5*d^38/b^21)^(3/4)*sqrt(d*x)*a*b^16*d^9 - (-a^5*d^38/b^21)^(3/4)*sqrt(a^2*d^19*x + sqrt(-a^5*d^38/b^21)*b^10)*b^16)/(a^5*d^38)) + 9945*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)*log(663*sqrt(d*x)*a*d^9 + 663*(-a^5*d^38/b^21)^(1/4)*b^5) - 9945*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)*log(663*sqrt(d*x)*a*d^9 - 663*(-a^5*d^38/b^21)^(1/4)*b^5) + 4*(384*b^4*d^9*x^8

```
- 6528*a*b^3*d^9*x^6 - 24973*a^2*b^2*d^9*x^4 - 27846*a^3*b*d^9*x^2 - 9945*a
^4*d^9)*sqrt(d*x))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{19}{2}}}{(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(19/2)/(a + b*x**2)**4, x)
```

Giac [A]
time = 4.02, size = 336, normalized size = 0.91

$$\frac{1}{7680}d^9\left(\frac{19890\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6}+\frac{19890\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6}+\frac{9945\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\,a\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^6}-\frac{9945\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\,a\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^6}-\frac{40\left(617\sqrt{dx}\,a^2b^2d^6x^4+1038\sqrt{dx}\,a^3bd^6x^2+453\sqrt{dx}\,a^4d^6\right)}{(bd^2x^2+ad^2)^3b^5}+\frac{3072\left(\sqrt{dx}\,b^{16}d^{10}x^2-20\sqrt{dx}\,ab^{15}d^{10}\right)}{b^{20}d^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/7680*d^9*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^6 + 19890*sqrt(2)*(a*b^3*d
^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/b^6 + 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^6 - 9945*sqrt(2)*(a*b^3*d^2)^(1/4)
*a*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^6 - 40*(6
17*sqrt(d*x)*a^2*b^2*d^6*x^4 + 1038*sqrt(d*x)*a^3*b*d^6*x^2 + 453*sqrt(d*x)
*a^4*d^6)/((b*d^2*x^2 + a*d^2)^3*b^5) + 3072*(sqrt(d*x)*b^16*d^10*x^2 - 20*
sqrt(d*x)*a*b^15*d^10)/(b^20*d^10))
```

Mupad [B]
time = 0.13, size = 188, normalized size = 0.51

$$\frac{2\,d^7\,{\left(d\,x\right)}^{5/2}}{5\,b^4}-\frac{\frac{151\,a^4\,d^{15}\,\sqrt{d\,x}}{64}+\frac{617\,a^2\,b^2\,d^{11}\,{\left(d\,x\right)}^{9/2}}{192}+\frac{173\,a^3\,b\,d^{13}\,{\left(d\,x\right)}^{5/2}}{32}}{a^3\,b^5\,d^6+3\,a^2\,b^6\,d^6\,x^2+3\,a\,b^7\,d^6\,x^4+b^8\,d^6\,x^6}-\frac{663\,{\left(-a\right)}^{5/4}\,d^{19/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,b^{21/4}}-\frac{8\,a\,d^9\,\sqrt{d\,x}}{b^5}+\frac{{\left(-a\right)}^{5/4}\,d^{19/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,663{}\mathrm{i}}{128\,b^{21/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (2*d^7*(d*x)^(5/2))/(5*b^4) - ((151*a^4*d^15*(d*x)^(1/2))/64 + (617*a^2*b^2
*d^11*(d*x)^(9/2))/192 + (173*a^3*b*d^13*(d*x)^(5/2))/32)/(a^3*b^5*d^6 + b^
8*d^6*x^6 + 3*a*b^7*d^6*x^4 + 3*a^2*b^6*d^6*x^2) - (663*(-a)^(5/4)*d^(19/2)
*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(21/4)) + ((-a)^(
5/4)*d^(19/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*663i)/(12
8*b^(21/4)) - (8*a*d^9*(d*x)^(1/2))/b^5
```

3.697 $\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2} \, dx$

Optimal. Leaf size=350

$$\frac{385d^7(dx)^{3/2}}{192b^4}-\frac{d(dx)^{15/2}}{6b\,(a+bx^2)^3}-\frac{5d^3(dx)^{11/2}}{16b^2\,(a+bx^2)^2}-\frac{55d^5(dx)^{7/2}}{64b^3\,(a+bx^2)}+\frac{385a^{3/4}d^{17/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{128\sqrt{2}\,b^{19/4}}-\cdots$$

```
[Out] 385/192*d^7*(d*x)^(3/2)/b^4-1/6*d*(d*x)^(15/2)/b/(b*x^2+a)^3-5/16*d^3*(d*x)
^(11/2)/b^2/(b*x^2+a)^2-55/64*d^5*(d*x)^(7/2)/b^3/(b*x^2+a)+385/256*a^(3/4)
*d^(17/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(19/4)*2^
(1/2)-385/256*a^(3/4)*d^(17/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)
/d^(1/2))/b^(19/4)*2^(1/2)-385/512*a^(3/4)*d^(17/2)*ln(a^(1/2)*d^(1/2)+x*b^
(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(19/4)*2^(1/2)+385/512
*a^(3/4)*d^(17/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1
/2)*(d*x)^(1/2))/b^(19/4)*2^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{385a^{3/4}d^{17/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{128\sqrt{2}\,b^{19/4}}-\frac{385a^{3/4}d^{17/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{128\sqrt{2}\,b^{19/4}}-\frac{385a^{3/4}d^{17/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{19/4}}+\frac{385a^{3/4}d^{17/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{19/4}}-\frac{55d^5(dx)^{7/2}}{64b^3\,(a+bx^2)}-\frac{5d^3(dx)^{11/2}}{16b^2\,(a+bx^2)^2}-\frac{d(dx)^{15/2}}{6b\,(a+bx^2)^3}+\frac{385d^7(dx)^{3/2}}{192b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (385*d^7*(d*x)^(3/2))/(192*b^4) - (d*(d*x)^(15/2))/(6*b*(a + b*x^2)^3) - (5
*d^3*(d*x)^(11/2))/(16*b^2*(a + b*x^2)^2) - (55*d^5*(d*x)^(7/2))/(64*b^3*(a
 + b*x^2)) + (385*a^(3/4)*d^(17/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) - (385*a^(3/4)*d^(17/2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) -
(385*a^(3/4)*d^(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(19/4)) + (385*a^(3/4)*d^(17/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(256*Sqrt[2]*b^(19/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx &= b^4 \int \frac{(dx)^{17/2}}{(ab+b^2x^2)^4}\,dx \\
&= -\frac{d(dx)^{15/2}}{6b\,(a+bx^2)^3} + \frac{1}{4}\left(5b^2d^2\right)\int \frac{(dx)^{13/2}}{(ab+b^2x^2)^3}\,dx \\
&= -\frac{d(dx)^{15/2}}{6b\,(a+bx^2)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\,(a+bx^2)^2} + \frac{1}{32}\left(55d^4\right)\int \frac{(dx)^{9/2}}{(ab+b^2x^2)^2}\,dx \\
&= -\frac{d(dx)^{15/2}}{6b\,(a+bx^2)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\,(a+bx^2)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\,(a+bx^2)} + \frac{\left(385d^6\right)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx}{128b^2} \\
&= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\,(a+bx^2)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\,(a+bx^2)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\,(a+bx^2)} - \frac{\left(385ad^8\right)\int}{12} \\
&= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\,(a+bx^2)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\,(a+bx^2)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\,(a+bx^2)} - \frac{\left(385ad^7\right)\mathrm{S}}{} \\
&= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\,(a+bx^2)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\,(a+bx^2)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\,(a+bx^2)} + \frac{\left(385ad^7\right)\mathrm{S}}{} \\
&= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\,(a+bx^2)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\,(a+bx^2)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\,(a+bx^2)} - \frac{\left(385a^{3/4}d^{1}\right.}{} \\
&= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\,(a+bx^2)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\,(a+bx^2)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\,(a+bx^2)} - \frac{385a^{3/4}d^{17}}{} \\
&= \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b\,(a+bx^2)^3} - \frac{5d^3(dx)^{11/2}}{16b^2\,(a+bx^2)^2} - \frac{55d^5(dx)^{7/2}}{64b^3\,(a+bx^2)} + \frac{385a^{3/4}d^{17}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.55, size = 194, normalized size = 0.55

$$
\frac{d^8\sqrt{dx}\left(4b^{3/4}x^{3/2}(385a^3+990a^2bx^2+765ab^2x^4+128b^3x^6)-1155\sqrt{2}\,a^{3/4}(a+bx^2)^3\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+1155\sqrt{2}\,a^{3/4}(a+bx^2)^3\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{768b^{19/4}\sqrt{x}\,(a+bx^2)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^8*Sqrt[d*x]*(4*b^(3/4)*x^(3/2)*(385*a^3 + 990*a^2*b*x^2 + 765*a*b^2*x^4
+ 128*b^3*x^6) - 1155*Sqrt[2]*a^(3/4)*(a + b*x^2)^3*ArcTan[(-Sqrt[a] + Sqrt
[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 1155*Sqrt[2]*a^(3/4)*(a + b*x^2
)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(768
*b^(19/4)*Sqrt[x]*(a + b*x^2)^3)
```

Maple [A]

time = 0.10, size = 218, normalized size = 0.62

method	result
derivativedivides	$2d^7\left(\frac{(dx)^{\frac{3}{2}}}{3b^4}-\frac{a\,d^2\left(\frac{-\frac{127b^2(dx)^{\frac{11}{2}}}{128}-\frac{101ab\,d^2(dx)^{\frac{7}{2}}}{64}-\frac{257a^2d^4(dx)^{\frac{3}{2}}}{384}}{\left(d^2x^2b+a\,d^2\right)^3}+\frac{385\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\cdots}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\cdots}}\right.\right.\cdots}{\cdots}\right.\cdots}{b^4}\cdots\right.$
default	$2d^7\left(\frac{(dx)^{\frac{3}{2}}}{3b^4}-\frac{a\,d^2\left(\frac{-\frac{127b^2(dx)^{\frac{11}{2}}}{128}-\frac{101ab\,d^2(dx)^{\frac{7}{2}}}{64}-\frac{257a^2d^4(dx)^{\frac{3}{2}}}{384}}{\left(d^2x^2b+a\,d^2\right)^3}+\frac{385\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\cdots}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\cdots}}\right.\right.\cdots}{\cdots}\right.\cdots}{b^4}\cdots\right.$
risch	$\frac{2x^2d^9}{3b^4\sqrt{dx}}+\left(\frac{127a(dx)^{\frac{11}{2}}}{64b^2(d^2x^2b+a\,d^2)^3}+\frac{101a^2(dx)^{\frac{7}{2}}d^2}{32b^3(d^2x^2b+a\,d^2)^3}+\frac{257a^3(dx)^{\frac{3}{2}}d^4}{192b^4(d^2x^2b+a\,d^2)^3}-\frac{385a\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\cdots}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\cdots}\right.\cdots}{512b^5\left(\cdots\right.}\cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*(1/3*(d*x)^(3/2)/b^4-a*d^2/b^4*((-127/128*b^2*(d*x)^(11/2)-101/64*a*b
*d^2*(d*x)^(7/2)-257/384*a^2*d^4*(d*x)^(3/2))/(b*d^2*x^2+a*d^2)^3+385/1024/
b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*a
rctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/
4)*(d*x)^(1/2)-1))))
```

Maxima [A]

time = 0.52, size = 334, normalized size = 0.95

$$-\frac{\frac{1155\,ad^{10}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b^4}-\frac{1024\,(dx)^{\frac{3}{2}}d^8}{b^4}-\frac{8\left(381\,(dx)^{\frac{11}{2}}ab^2d^{10}+606\,(dx)^{\frac{7}{2}}a^2bd^{12}+257\,(dx)^{\frac{3}{2}}a^3d^{14}\right)}{b^7d^6x^6+3\,ab^6d^6x^4+3\,a^2b^5d^6x^2+a^3b^4d^6}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(1155*a*d^10*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*
sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4
) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^4 -
 1024*(d*x)^(3/2)*d^8/b^4 - 8*(381*(d*x)^(11/2)*a*b^2*d^10 + 606*(d*x)^(7/2
)*a^2*b*d^12 + 257*(d*x)^(3/2)*a^3*d^14)/(b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 + 3
*a^2*b^5*d^6*x^2 + a^3*b^4*d^6))/d
```

Fricas [A]

time = 0.39, size = 399, normalized size = 1.14

$$\frac{4620\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)\arctan\left(-\frac{\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^5d^{25}-\sqrt{a^4d^{51}x-\sqrt{-\frac{a^3d^{34}}{b^{19}}}\,a^3b^9d^{34}}\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}b^5}{a^3d^{34}}\right)-1155\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)\log\left(57066625\sqrt{dx}\,a^2d^{25}+57066625\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{3}{4}}b^{14}\right)+1155\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)\log\left(57066625\sqrt{dx}\,a^2d^{25}-57066625\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{3}{4}}b^{14}\right)+4\left(128\,b^3d^8x^7+765\,ab^2d^8x^5+990\,a^2bd^8x^3+385\,a^3d^8x\right)\sqrt{dx}}{768\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(4620*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 +
 a^3*b^4)*arctan(-((-a^3*d^34/b^19)^(1/4)*sqrt(d*x)*a^2*b^5*d^25 - sqrt(a^4
*d^51*x - sqrt(-a^3*d^34/b^19)*a^3*b^9*d^34)*(-a^3*d^34/b^19)^(1/4)*b^5)/(a
^3*d^34)) - 1155*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*
x^2 + a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 + 57066625*(-a^3*d^34/b^19)^
(3/4)*b^14) + 1155*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^
5*x^2 + a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 - 57066625*(-a^3*d^34/b^19
)^(3/4)*b^14) + 4*(128*b^3*d^8*x^7 + 765*a*b^2*d^8*x^5 + 990*a^2*b*d^8*x^3
```

```
+ 385*a^3*d^8*x)*sqrt(d*x))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^
4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{17}{2}}}{(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(17/2)/(a + b*x**2)**4, x)
```

Giac [A]
time = 4.15, size = 316, normalized size = 0.90

$$\frac{1}{1536}d^8\left(\frac{1024\sqrt{dx}\,x}{b^4}-\frac{2310\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7d}-\frac{2310\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7d}+\frac{1155\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^7d}-\frac{1155\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^7d}+\frac{8\left(381\sqrt{dx}\,ab^2d^6x^5+606\sqrt{dx}\,a^2bd^6x^3+257\sqrt{dx}\,a^3d^6x\right)}{(bd^2x^2+ad^2)^3b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^8*(1024*sqrt(d*x)*x/b^4 - 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*d)
- 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*d) + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)
*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^7*d) - 115
5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(b^7*d) + 8*(381*sqrt(d*x)*a*b^2*d^6*x^5 + 606*sqrt(d*x)*a^2*
b*d^6*x^3 + 257*sqrt(d*x)*a^3*d^6*x)/((b*d^2*x^2 + a*d^2)^3*b^4))
```

Mupad [B]
time = 4.33, size = 171, normalized size = 0.49

$$\frac{\frac{257\,a^3\,d^{13}\,{(d\,x)}^{3/2}}{192}+\frac{101\,a^2\,b\,d^{11}\,{(d\,x)}^{7/2}}{32}+\frac{127\,a\,b^2\,d^9\,{(d\,x)}^{11/2}}{64}}{a^3\,b^4\,d^6+3\,a^2\,b^5\,d^6\,x^2+3\,a\,b^6\,d^6\,x^4+b^7\,d^6\,x^6}+\frac{2\,d^7\,{(d\,x)}^{3/2}}{3\,b^4}+\frac{385\,{(-a)}^{3/4}\,d^{17/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,b^{19/4}}+\frac{{(-a)}^{3/4}\,d^{17/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{(-a)}^{1/4}\,\sqrt{d}}\right)\,385{}\mathrm{i}}{128\,b^{19/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((257*a^3*d^13*(d*x)^(3/2))/192 + (101*a^2*b*d^11*(d*x)^(7/2))/32 + (127*a*
b^2*d^9*(d*x)^(11/2))/64)/(a^3*b^4*d^6 + b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 + 3*
a^2*b^5*d^6*x^2) + (2*d^7*(d*x)^(3/2))/(3*b^4) + (385*(-a)^(3/4)*d^(17/2)*a
tan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(19/4)) + ((-a)^(3/
4)*d^(17/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*385i)/(128*
b^(19/4))
```

3.698 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=350

$$\frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}+\frac{195\sqrt[4]{a}\,d^{15/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,b^{17/4}}-\frac{195}{}$$

```
[Out] -1/6*d*(d*x)^(13/2)/b/(b*x^2+a)^3-13/48*d^3*(d*x)^(9/2)/b^2/(b*x^2+a)^2-39/
64*d^5*(d*x)^(5/2)/b^3/(b*x^2+a)+195/256*a^(1/4)*d^(15/2)*arctan(1-b^(1/4)*
2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(17/4)*2^(1/2)-195/256*a^(1/4)*d^(15
/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(17/4)*2^(1/2)+
195/512*a^(1/4)*d^(15/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/
4)*2^(1/2)*(d*x)^(1/2))/b^(17/4)*2^(1/2)-195/512*a^(1/4)*d^(15/2)*ln(a^(1/2
)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(17/4)*2
^(1/2)+195/64*d^7*(d*x)^(1/2)/b^4
```

Rubi [A]
time = 0.25, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{195\sqrt[4]{a}\,d^{15/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,b^{17/4}}-\frac{195\sqrt[4]{a}\,d^{15/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,b^{17/4}}+\frac{195\sqrt[4]{a}\,d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{17/4}}-\frac{195\sqrt[4]{a}\,d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,b^{17/4}}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}+\frac{195d^7\sqrt{dx}}{64b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (195*d^7*Sqrt[d*x])/(64*b^4) - (d*(d*x)^(13/2))/(6*b*(a + b*x^2)^3) - (13*d
^3*(d*x)^(9/2))/(48*b^2*(a + b*x^2)^2) - (39*d^5*(d*x)^(5/2))/(64*b^3*(a +
b*x^2)) + (195*a^(1/4)*d^(15/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(
1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) + (19
5*a^(1/4)*d^(15/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(25
6*Sqrt[2]*b^(17/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{(dx)^{15/2}}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3} + \frac{1}{12}\left(13b^2d^2\right)\int \frac{(dx)^{11/2}}{(ab+b^2x^2)^3}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\,(a+bx^2)^2} + \frac{1}{32}\left(39d^4\right)\int \frac{(dx)^{7/2}}{(ab+b^2x^2)^2}\,dx$$

$$= -\frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\,(a+bx^2)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\,(a+bx^2)} + \frac{\left(195d^6\right)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\,(a+bx^2)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\,(a+bx^2)} - \frac{\left(195ad^8\right)\int}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\,(a+bx^2)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\,(a+bx^2)} - \frac{\left(195ad^7\right)\mathrm{S}}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\,(a+bx^2)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\,(a+bx^2)} - \frac{\left(195\sqrt{a}\,d\right.}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\,(a+bx^2)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\,(a+bx^2)} + \frac{\left(195\sqrt[4]{a}\,d\right.}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\,(a+bx^2)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\,(a+bx^2)} + \frac{195\sqrt[4]{a}\,d^{15}}{}$$

$$= \frac{195d^7\sqrt{dx}}{64b^4} - \frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3} - \frac{13d^3(dx)^{9/2}}{48b^2\,(a+bx^2)^2} - \frac{39d^5(dx)^{5/2}}{64b^3\,(a+bx^2)} + \frac{195\sqrt[4]{a}\,d^{15}}{}$$

<u>Mathematica</u> **[A]**
time = 0.47, size = 175, normalized size = 0.50

$$\frac{d^7\sqrt{dx}\left(\frac{4\sqrt[4]{b}\sqrt{x}\,(585a^3+1638a^2bx^2+1469ab^2x^4+384b^3x^6)}{(a+bx^2)^3}+585\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-585\sqrt{2}\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{768b^{17/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^7*Sqrt[d*x]*((4*b^(1/4)*Sqrt[x]*(585*a^3 + 1638*a^2*b*x^2 + 1469*a*b^2*x
^4 + 384*b^3*x^6))/(a + b*x^2)^3 + 585*Sqrt[2]*a^(1/4)*ArcTan[(Sqrt[a] - Sq
rt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 585*Sqrt[2]*a^(1/4)*ArcTanh[(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(768*b^(17/4)*Sqr
t[x])
```

Maple [A]
time = 0.09, size = 220, normalized size = 0.63

method	result
derivativedivides	$2d^7\left(\frac{\sqrt{dx}}{b^4}-\frac{a\,d^2\left(\frac{-\frac{317b^2(dx)^{\frac{9}{2}}}{384}-\frac{81ab\,d^2(dx)^{\frac{5}{2}}}{64}-\frac{67a^2d^4\sqrt{dx}}{128}}{\left(d^2x^2b+a\,d^2\right)^3}+\frac{195\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}}\right.\right.}{}\right.}{b^4}\right.$
default	$2d^7\left(\frac{\sqrt{dx}}{b^4}-\frac{a\,d^2\left(\frac{-\frac{317b^2(dx)^{\frac{9}{2}}}{384}-\frac{81ab\,d^2(dx)^{\frac{5}{2}}}{64}-\frac{67a^2d^4\sqrt{dx}}{128}}{\left(d^2x^2b+a\,d^2\right)^3}+\frac{195\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}}\right.\right.}{}\right.}{b^4}\right.$
risch	$\frac{2x\,d^8}{b^4\sqrt{dx}}+\left(\frac{317ad(dx)^{\frac{9}{2}}}{192b^2(d^2x^2b+a\,d^2)^3}+\frac{81a^2d^3(dx)^{\frac{5}{2}}}{32b^3(d^2x^2b+a\,d^2)^3}+\frac{67a^3d^5\sqrt{dx}}{64b^4(d^2x^2b+a\,d^2)^3}-\frac{195\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d}{b}\right.}{dx-\left(\frac{a\,d}{b}\right.}\right.}{512}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*(1/b^4*(d*x)^(1/2)-1/b^4*a*d^2*((-317/384*b^2*(d*x)^(9/2)-81/64*a*b*d
^2*(d*x)^(5/2)-67/128*a^2*d^4*(d*x)^(1/2))/(b*d^2*x^2+a*d^2)^3+195/1024*(a*
d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*
arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1
/4)*(d*x)^(1/2)-1))))
```

Maxima [A]
time = 0.50, size = 343, normalized size = 0.98

$$\frac{\frac{3072\sqrt{dx}\,d^8}{b^4}+\frac{8\left(317\,(dx)^{\frac{9}{2}}ab^2d^{10}+486\,(dx)^{\frac{5}{2}}a^2bd^{12}+201\sqrt{dx}\,a^3d^{14}\right)}{b^7d^6x^6+3\,ab^6d^6x^4+3\,a^2b^5d^6x^2+a^3b^4d^6}-\frac{585\left(\frac{\sqrt{2}\,d^{10}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{10}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^9\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{a}}+\frac{2\sqrt{2}\,d^9\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{a}}\right)a}{b^4}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(3072*sqrt(d*x)*d^8/b^4 + 8*(317*(d*x)^(9/2)*a*b^2*d^10 + 486*(d*x)^
(5/2)*a^2*b*d^12 + 201*sqrt(d*x)*a^3*d^14)/(b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 +
 3*a^2*b^5*d^6*x^2 + a^3*b^4*d^6) - 585*(sqrt(2)*d^10*log(sqrt(b)*d*x + sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) -
 sqrt(2)*d^10*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + s
qrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^9*arctan(1/2*sqrt(2)*(sqrt(
2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(s
qrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^9*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sq
rt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a/b^4)/d
```

Fricas [A]
time = 0.37, size = 363, normalized size = 1.04

$$-\frac{2340\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)\arctan\left(-\frac{\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{13}d^7-\sqrt{d^{15}x+\sqrt{-\frac{ad^{30}}{b^{17}}}\,b^8}\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{3}{4}}b^{13}}{ad^{30}}\right)+585\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)\log\left(195\sqrt{dx}\,d^7+195\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}b^4\right)-585\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)\log\left(195\sqrt{dx}\,d^7-195\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}b^4\right)-4\left(384\,b^3d^7x^6+1469\,ab^2d^7x^4+1638\,a^2bd^7x^2+585\,a^3d^7\right)\sqrt{dx}}{768\left(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(2340*(-a*d^30/b^17)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 +
a^3*b^4)*arctan(-((-a*d^30/b^17)^(3/4)*sqrt(d*x)*b^13*d^7 - sqrt(d^15*x + s
qrt(-a*d^30/b^17)*b^8)*(-a*d^30/b^17)^(3/4)*b^13)/(a*d^30)) + 585*(-a*d^30/
b^17)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(195*sqrt(
d*x)*d^7 + 195*(-a*d^30/b^17)^(1/4)*b^4) - 585*(-a*d^30/b^17)^(1/4)*(b^7*x^
6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(195*sqrt(d*x)*d^7 - 195*(-a*
d^30/b^17)^(1/4)*b^4) - 4*(384*b^3*d^7*x^6 + 1469*a*b^2*d^7*x^4 + 1638*a^2*
```

```
b*d^7*x^2 + 585*a^3*d^7)*sqrt(d*x))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2
+ a^3*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{15}{2}}}{(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(15/2)/(a + b*x**2)**4, x)
```

Giac [A]
time = 3.17, size = 302, normalized size = 0.86

$$-\frac{1}{1536}\,d^7\left(\frac{1170\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5}+\frac{1170\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5}+\frac{585\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^5}-\frac{585\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^5}-\frac{3072\,\sqrt{dx}}{b^4}-\frac{8\left(317\,\sqrt{dx}\,ab^2d^6x^4+486\,\sqrt{dx}\,a^2bd^6x^2+201\,\sqrt{dx}\,a^3d^6\right)}{(bd^2x^2+ad^2)^3b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/1536*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^5 + 1170*sqrt(2)*(a*b^3*d^2)
^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/b^5 + 585*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1
/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5 - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5 - 3072*sqrt(d*x)/b
^4 - 8*(317*sqrt(d*x)*a*b^2*d^6*x^4 + 486*sqrt(d*x)*a^2*b*d^6*x^2 + 201*sqr
t(d*x)*a^3*d^6)/((b*d^2*x^2 + a*d^2)^3*b^4))
```

Mupad [B]
time = 4.30, size = 171, normalized size = 0.49

$$\frac{\frac{67\,a^3\,d^{13}\,\sqrt{d\,x}}{64}+\frac{81\,a^2\,b\,d^{11}\,{(d\,x)}^{5/2}}{32}+\frac{317\,a\,b^2\,d^9\,{(d\,x)}^{9/2}}{192}}{a^3\,b^4\,d^6+3\,a^2\,b^5\,d^6\,x^2+3\,a\,b^6\,d^6\,x^4+b^7\,d^6\,x^6}+\frac{2\,d^7\,\sqrt{d\,x}}{b^4}-\frac{195\,{(-a)}^{1/4}\,d^{15/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,b^{17/4}}+\frac{{(-a)}^{1/4}\,d^{15/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{(-a)}^{1/4}\,\sqrt{d}}\right)\,195{}\mathrm{i}}{128\,b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((67*a^3*d^13*(d*x)^(1/2))/64 + (81*a^2*b*d^11*(d*x)^(5/2))/32 + (317*a*b^2
*d^9*(d*x)^(9/2))/192)/(a^3*b^4*d^6 + b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 + 3*a^2
*b^5*d^6*x^2) + (2*d^7*(d*x)^(1/2))/b^4 - (195*(-a)^(1/4)*d^(15/2)*atan((b^
(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(17/4)) + ((-a)^(1/4)*d^(1
5/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*195i)/(128*b^(17/4
))
```

3.699 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=333

$$-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}-\frac{77d^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{128\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}}+\frac{77d^{13/2}\tan^{-1}\left(1-\right.}{128\sqrt{2}\,}$$

```
[Out] -1/6*d*(d*x)^(11/2)/b/(b*x^2+a)^3-11/48*d^3*(d*x)^(7/2)/b^2/(b*x^2+a)^2-77/
192*d^5*(d*x)^(3/2)/b^3/(b*x^2+a)-77/256*d^(13/2)*arctan(1-b^(1/4)*2^(1/2)*
(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(15/4)*2^(1/2)+77/256*d^(13/2)*arcta
n(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(15/4)*2^(1/2)+7
7/512*d^(13/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)
*(d*x)^(1/2))/a^(1/4)/b^(15/4)*2^(1/2)-77/512*d^(13/2)*ln(a^(1/2)*d^(1/2)+x
*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(15/4)*2^(1
/2)
```

Rubi [A]
time = 0.23, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{77d^{13/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{128\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}}+\frac{77d^{13/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{128\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}}+\frac{77d^{13/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,\sqrt[4]{a}\,b^{15/4}}-\frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)}-\frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*(d*(d*x)^(11/2))/(b*(a + b*x^2)^3) - (11*d^3*(d*x)^(7/2))/(48*b^2*(a +
 b*x^2)^2) - (77*d^5*(d*x)^(3/2))/(192*b^3*(a + b*x^2)) - (77*d^(13/2)*ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(1/4)
*b^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(128*Sqrt[2]*a^(1/4)*b^(15/4)) + (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d]
 + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(
1/4)*b^(15/4)) - (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/4)*b^(15/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{13/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(11b^2d^2\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} + \frac{1}{96}\left(77d^4\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} + \frac{\left(77d^6\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128b^2}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} + \frac{\left(77d^5\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x}{d^2}}\right.}{64b^2}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} - \frac{\left(77d^5\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d}{ab+}\right.}{128b}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} + \frac{\left(77d^{13/2}\right)\text{Subst}\left(\int -\right.}{2}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} + \frac{77d^{13/2}\log\left(\sqrt{a}\sqrt{d}\right.}{25}$$

$$= -\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3} - \frac{11d^3(dx)^{7/2}}{48b^2\left(a+bx^2\right)^2} - \frac{77d^5(dx)^{3/2}}{192b^3\left(a+bx^2\right)} - \frac{77d^{13/2}\tan^{-1}\left(1-\sqrt{}\right.}{128\sqrt{2}\sqrt[4]{a}\,b}$$

Mathematica [A]

time = 0.47, size = 164, normalized size = 0.49

$$\frac{d^6\sqrt{dx}\left(-\frac{4b^{3/4}x^{3/2}(77a^2+198abx^2+153b^2x^4)}{(a+bx^2)^3}-\frac{231\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}}-\frac{231\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{a}}\right)}{768b^{15/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^6*Sqrt[d*x]*((-4*b^(3/4)*x^(3/2)*(77*a^2 + 198*a*b*x^2 + 153*b^2*x^4))/(
a + b*x^2)^3 - (231*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x])])/a^(1/4) - (231*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4)))/(768*b^(15/4)*Sqrt[x])
```

Maple [A]
time = 0.06, size = 203, normalized size = 0.61

method	result
derivativedivides	$2d^7\left(\frac{-\frac{51(dx)^{\frac{11}{2}}}{128b}-\frac{33d^2a(dx)^{\frac{7}{2}}}{64b^2}-\frac{77d^4a^2(dx)^{\frac{3}{2}}}{384b^3}}{(d^2x^2b+a\,d^2)^3}+\frac{77\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\sqrt{\right.\right.}{1024b^4\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}$
default	$2d^7\left(\frac{-\frac{51(dx)^{\frac{11}{2}}}{128b}-\frac{33d^2a(dx)^{\frac{7}{2}}}{64b^2}-\frac{77d^4a^2(dx)^{\frac{3}{2}}}{384b^3}}{(d^2x^2b+a\,d^2)^3}+\frac{77\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\sqrt{\right.\right.}{1024b^4\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*((-51/128/b*(d*x)^(11/2)-33/64/b^2*d^2*a*(d*x)^(7/2)-77/384/b^3*d^4*a
^2*(d*x)^(3/2))/(b*d^2*x^2+a*d^2)^3+77/1024/b^4/(a*d^2/b)^(1/4)*2^(1/2)*(ln
((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)
*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.50, size = 317, normalized size = 0.95

$$\frac{\frac{231\,d^8\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b^3}-\frac{8\left(153\,(dx)^{\frac{11}{2}}b^2d^8+198\,(dx)^{\frac{7}{2}}abd^{10}+77\,(dx)^{\frac{3}{2}}a^2d^{12}\right)}{b^6d^6x^6+3\,ab^5d^6x^4+3\,a^2b^4d^6x^2+a^3b^3d^6}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(231*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*
sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2
*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(
b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + s
qrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^
2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^3 - 8*(1
53*(d*x)^(11/2)*b^2*d^8 + 198*(d*x)^(7/2)*a*b*d^10 + 77*(d*x)^(3/2)*a^2*d^1
2)/(b^6*d^6*x^6 + 3*a*b^5*d^6*x^4 + 3*a^2*b^4*d^6*x^2 + a^3*b^3*d^6))/d
```

Fricas [A]
time = 0.36, size = 370, normalized size = 1.11

$$-\frac{924\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\sqrt{dx}\,b^4d^{19}-\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{ab^{15}}}\,ab^7d^{26}}\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}b^4}{d^{26}}\right)-231\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\log\left(456533\,\sqrt{dx}\,d^{19}+456533\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{3}{4}}ab^{11}\right)+231\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\log\left(456533\,\sqrt{dx}\,d^{19}-456533\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{3}{4}}ab^{11}\right)+4\,(153\,b^2d^6x^5+198\,abd^6x^3+77\,a^2d^6x)\sqrt{dx}}{768\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(924*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^1
5))^(1/4)*arctan(-((-d^26/(a*b^15))^(1/4)*sqrt(d*x)*b^4*d^19 - sqrt(d^39*x
- sqrt(-d^26/(a*b^15))*a*b^7*d^26)*(-d^26/(a*b^15))^(1/4)*b^4)/d^26) - 231*
(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^15))^(1/4)*lo
g(456533*sqrt(d*x)*d^19 + 456533*(-d^26/(a*b^15))^(3/4)*a*b^11) + 231*(b^6*
x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^15))^(1/4)*log(456
533*sqrt(d*x)*d^19 - 456533*(-d^26/(a*b^15))^(3/4)*a*b^11) + 4*(153*b^2*d^6
*x^5 + 198*a*b*d^6*x^3 + 77*a^2*d^6*x)*sqrt(d*x))/(b^6*x^6 + 3*a*b^5*x^4 +
3*a^2*b^4*x^2 + a^3*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{13}{2}}}{(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)

[Out] Integral((d*x)**(13/2)/(a + b*x**2)**4, x)

Giac [A]
time = 3.42, size = 314, normalized size = 0.94

$$-\frac{1}{1536}d^{6}\left(\frac{8\left(153\sqrt{dx}\,b^{2}d^{6}x^{5}+198\sqrt{dx}\,abd^{6}x^{3}+77\sqrt{dx}\,a^{2}d^{6}x\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{3}b^{3}}-\frac{462\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{ab^{6}d}-\frac{462\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{ab^{6}d}+\frac{231\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{ab^{6}d}-\frac{231\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{ab^{6}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")

[Out] -1/1536*d^6*(8*(153*sqrt(d*x)*b^2*d^6*x^5 + 198*sqrt(d*x)*a*b*d^6*x^3 + 77*sqrt(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*b^3) - 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6*d) - 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6*d) + 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6*d) - 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6*d))

Mupad [B]
time = 0.11, size = 153, normalized size = 0.46

$$\frac{77\,d^{13/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{1/4}\,b^{15/4}}-\frac{\frac{51\,d^{7}\,(d\,x)^{11/2}}{64\,b}+\frac{77\,a^{2}\,d^{11}\,(d\,x)^{3/2}}{192\,b^{3}}+\frac{33\,a\,d^{9}\,(d\,x)^{7/2}}{32\,b^{2}}}{a^{3}\,d^{6}+3\,a^{2}\,b\,d^{6}\,x^{2}+3\,a\,b^{2}\,d^{6}\,x^{4}+b^{3}\,d^{6}\,x^{6}}-\frac{77\,d^{13/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{1/4}\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)

[Out] (77*d^(13/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(1/4)*b^(15/4)) - ((51*d^7*(d*x)^(11/2))/(64*b) + (77*a^2*d^11*(d*x)^(3/2))/(192*b^3) + (33*a*d^9*(d*x)^(7/2))/(32*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*x^4) - (77*d^(13/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(1/4)*b^(15/4))

3.700 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=333

$$-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{15d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{15d^{11/2}\tan^{-1}\left(1+\right.}{128\sqrt{2}\,a^{3}}$$

```
[Out] -1/6*d*(d*x)^(9/2)/b/(b*x^2+a)^3-3/16*d^3*(d*x)^(5/2)/b^2/(b*x^2+a)^2-15/25
6*d^(11/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^
(13/4)*2^(1/2)+15/256*d^(11/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)
/d^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)-15/512*d^(11/2)*ln(a^(1/2)*d^(1/2)+x*b^(
1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)+
15/512*d^(11/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2
)*(d*x)^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)-15/64*d^5*(d*x)^(1/2)/b^3/(b*x^2+a)
```

Rubi [A]
time = 0.22, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{15d^{11/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{15d^{11/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{15d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{15d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)}-\frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*(d*(d*x)^(9/2))/(b*(a + b*x^2)^3) - (3*d^3*(d*x)^(5/2))/(16*b^2*(a + b
*x^2)^2) - (15*d^5*Sqrt[d*x])/(64*b^3*(a + b*x^2)) - (15*d^(11/2)*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(3/4)*b^(1
3/4)) + (15*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(128*Sqrt[2]*a^(3/4)*b^(13/4)) - (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sq
rt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*
b^(13/4)) + (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{(dx)^{11/2}}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b(a+bx^2)^3} + \frac{1}{4}(3b^2d^2)\int \frac{(dx)^{7/2}}{(ab+b^2x^2)^3}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b(a+bx^2)^3} - \frac{3d^3(dx)^{5/2}}{16b^2(a+bx^2)^2} + \frac{1}{32}(15d^4)\int \frac{(dx)^{3/2}}{(ab+b^2x^2)^2}\,dx$$

$$= -\frac{d(dx)^{9/2}}{6b(a+bx^2)^3} - \frac{3d^3(dx)^{5/2}}{16b^2(a+bx^2)^2} - \frac{15d^5\sqrt{dx}}{64b^3(a+bx^2)} + \frac{(15d^6)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)}\,d}{128b^2}$$

$$= -\frac{d(dx)^{9/2}}{6b(a+bx^2)^3} - \frac{3d^3(dx)^{5/2}}{16b^2(a+bx^2)^2} - \frac{15d^5\sqrt{dx}}{64b^3(a+bx^2)} + \frac{(15d^5)\,\mathrm{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\right.}{64b^2}$$

$$= -\frac{d(dx)^{9/2}}{6b(a+bx^2)^3} - \frac{3d^3(dx)^{5/2}}{16b^2(a+bx^2)^2} - \frac{15d^5\sqrt{dx}}{64b^3(a+bx^2)} + \frac{(15d^4)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-}{ab+b}\right.}{128\sqrt{a}}$$

$$= -\frac{d(dx)^{9/2}}{6b(a+bx^2)^3} - \frac{3d^3(dx)^{5/2}}{16b^2(a+bx^2)^2} - \frac{15d^5\sqrt{dx}}{64b^3(a+bx^2)} - \frac{(15d^{11/2})\,\mathrm{Subst}\left(\int \frac{}{-\sqrt{}}\right.}{25}$$

$$= -\frac{d(dx)^{9/2}}{6b(a+bx^2)^3} - \frac{3d^3(dx)^{5/2}}{16b^2(a+bx^2)^2} - \frac{15d^5\sqrt{dx}}{64b^3(a+bx^2)} - \frac{15d^{11/2}\log\left(\sqrt{a}\,\sqrt{d}+\right.}{256}$$

$$= -\frac{d(dx)^{9/2}}{6b(a+bx^2)^3} - \frac{3d^3(dx)^{5/2}}{16b^2(a+bx^2)^2} - \frac{15d^5\sqrt{dx}}{64b^3(a+bx^2)} - \frac{15d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{128\sqrt{2}\,a^{3/4}b^{1}}$$

Mathematica [A]

time = 0.46, size = 164, normalized size = 0.49

$$\frac{d^5\sqrt{dx}\left(-\frac{4\sqrt[4]{b}\,(45a^2+126abx^2+113b^2x^4)}{(a+bx^2)^3}-\frac{45\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{a^{3/4}\sqrt{x}}+\frac{45\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{a^{3/4}\sqrt{x}}\right)}{768b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^5*Sqrt[d*x]*((-4*b^(1/4)*(45*a^2 + 126*a*b*x^2 + 113*b^2*x^4))/(a + b*x^
2)^3 - (45*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])])/(a^(3/4)*Sqrt[x]) + (45*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(3/4)*Sqrt[x])))/(768*b^(13/4))
```

Maple [A]

time = 0.06, size = 209, normalized size = 0.63

method	result
derivativedivides	$2d^7\left(\frac{-\frac{113(dx)^{\frac{9}{2}}}{384b}-\frac{21a\,d^2(dx)^{\frac{5}{2}}}{64b^2}-\frac{15a^2d^4\sqrt{dx}}{128b^3}}{(d^2x^2b+a\,d^2)^3}+\frac{15\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2a\right.}{1024b^3a}\right.$
default	$2d^7\left(\frac{-\frac{113(dx)^{\frac{9}{2}}}{384b}-\frac{21a\,d^2(dx)^{\frac{5}{2}}}{64b^2}-\frac{15a^2d^4\sqrt{dx}}{128b^3}}{(d^2x^2b+a\,d^2)^3}+\frac{15\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2a\right.}{1024b^3a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*((-113/384/b*(d*x)^(9/2)-21/64*a*d^2/b^2*(d*x)^(5/2)-15/128*a^2*d^4/b
^3*(d*x)^(1/2))/(b*d^2*x^2+a*d^2)^3+15/1024/b^3*(a*d^2/b)^(1/4)/a/d^2*2^(1/
2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)
^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.51, size = 326, normalized size = 0.98

$$-\frac{\frac{8\left(113\,(dx)^{\frac{9}{2}}b^2d^8+126\,(dx)^{\frac{5}{2}}abd^{10}+45\,\sqrt{dx}\,a^2d^{12}\right)}{b^6d^6x^6+3\,ab^5d^6x^4+3\,a^2b^4d^6x^2+a^3b^3d^6}-\frac{45\left(\frac{\sqrt{2}\,d^8\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^8\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^7\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^7\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)}{b^3}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(8*(113*(d*x)^(9/2)*b^2*d^8 + 126*(d*x)^(5/2)*a*b*d^10 + 45*sqrt(d*
x)*a^2*d^12)/(b^6*d^6*x^6 + 3*a*b^5*d^6*x^4 + 3*a^2*b^4*d^6*x^2 + a^3*b^3*d
^6) - 45*(sqrt(2)*d^8*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(
1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x - s
qrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4))
 + 2*sqrt(2)*d^7*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt
(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) +
 2*sqrt(2)*d^7*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(
d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/b
^3)/d
```

Fricas [A]
time = 0.35, size = 373, normalized size = 1.12

$$\frac{180\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}\,a^2b^{10}d^5-\sqrt{d^{11}x+\sqrt{-\frac{d^{22}}{a^3b^{13}}}\,a^2b^6}\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{3}{4}}a^2b^{10}}{d^{22}}\right)+45\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}\log\left(15\,\sqrt{dx}\,d^5+15\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}ab^3\right)-45\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}\log\left(15\,\sqrt{dx}\,d^5-15\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}ab^3\right)-4\,(113\,b^2d^5x^4+126\,abd^5x^2+45\,a^2d^5)\sqrt{dx}}{768\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(180*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b^
13))^(1/4)*arctan(-((-d^22/(a^3*b^13))^(3/4)*sqrt(d*x)*a^2*b^10*d^5 - sqrt(
d^11*x + sqrt(-d^22/(a^3*b^13))*a^2*b^6)*(-d^22/(a^3*b^13))^(3/4)*a^2*b^10)
/d^22) + 45*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b
^13))^(1/4)*log(15*sqrt(d*x)*d^5 + 15*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 45*
(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b^13))^(1/4)*
log(15*sqrt(d*x)*d^5 - 15*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 4*(113*b^2*d^5*
x^4 + 126*a*b*d^5*x^2 + 45*a^2*d^5)*sqrt(d*x))/(b^6*x^6 + 3*a*b^5*x^4 + 3*a
^2*b^4*x^2 + a^3*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{11}{2}}}{(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(11/2)/(a + b*x**2)**4, x)
```

Giac [A]
time = 3.46, size = 301, normalized size = 0.90

$$\frac{1}{1536}d^5\left(\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}+\frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^4}-\frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^4}-\frac{8\left(113\sqrt{dx}\,b^2d^6x^4+126\sqrt{dx}\,abd^6x^2+45\sqrt{dx}\,a^2d^6\right)}{\left(bd^2x^2+ad^2\right)^3b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^5*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4) + 90*sqrt(2)*(a*b^3*d^2)^
(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a*b^4) + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^
(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log
(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4) - 8*(113*
sqrt(d*x)*b^2*d^6*x^4 + 126*sqrt(d*x)*a*b*d^6*x^2 + 45*sqrt(d*x)*a^2*d^6)/(
(b*d^2*x^2 + a*d^2)^3*b^3))
```

Mupad [B]
time = 4.29, size = 153, normalized size = 0.46

$$-\frac{\frac{113\,d^7\,{\left(d\,x\right)}^{9/2}}{192\,b}+\frac{15\,a^2\,d^{11}\,\sqrt{d\,x}}{64\,b^3}+\frac{21\,a\,d^9\,{\left(d\,x\right)}^{5/2}}{32\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}-\frac{15\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{3/4}\,b^{13/4}}-\frac{15\,d^{11/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{3/4}\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] - ((113*d^7*(d*x)^(9/2))/(192*b) + (15*a^2*d^11*(d*x)^(1/2))/(64*b^3) + (21
*a*d^9*(d*x)^(5/2))/(32*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*
a*b^2*d^6*x^4) - (15*d^(11/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(128*(-a)^(3/4)*b^(13/4)) - (15*d^(11/2)*atanh((b^(1/4)*(d*x)^(1/2))/(
(-a)^(1/4)*d^(1/2))))/(128*(-a)^(3/4)*b^(13/4))
```

3.701 $\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=336

$$-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}-\frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2}+\frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)}-\frac{7d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{7d^{9/2}\tan^{-1}\left(1+\sqrt{\ldots}\right.}{128\sqrt{2}\,a^{5/4}\ldots}$$

```
[Out] -1/6*d*(d*x)^(7/2)/b/(b*x^2+a)^3-7/48*d^3*(d*x)^(3/2)/b^2/(b*x^2+a)^2+7/64*
d^3*(d*x)^(3/2)/a/b^2/(b*x^2+a)-7/256*d^(9/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x
)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)+7/256*d^(9/2)*arctan(1+b^
(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)+7/512*d
^(9/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(
1/2))/a^(5/4)/b^(11/4)*2^(1/2)-7/512*d^(9/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d
^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{7d^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{7d^{9/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{7d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{7d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)}-\frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*(d*(d*x)^(7/2))/(b*(a + b*x^2)^3) - (7*d^3*(d*x)^(3/2))/(48*b^2*(a + b
*x^2)^2) + (7*d^3*(d*x)^(3/2))/(64*a*b^2*(a + b*x^2)) - (7*d^(9/2)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(5/4)*b^(
11/4)) + (7*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(128*Sqrt[2]*a^(5/4)*b^(11/4)) + (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[
b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(
11/4)) - (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} + \frac{1}{12}\left(7b^2d^2\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^3}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{1}{32}\left(7d^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{\left(7d^4\right)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128ab}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{\left(7d^3\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,d\cdots\right.}{64ab}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} - \frac{\left(7d^3\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{\cdots}}{ab+\frac{b^2\cdots}{d^2}}\cdots\right.}{128ab^{3\cdots}}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{\left(7d^{9/2}\right)\text{Subst}\left(\int \frac{\cdots}{-\frac{\sqrt{a}\cdots}{\sqrt{b}\cdots}}\cdots\right.}{256\cdots}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} + \frac{7d^{9/2}\log\left(\sqrt{a}\sqrt{d}+\sqrt{\cdots}\right.}{256\sqrt{\cdots}}$$

$$= -\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3} - \frac{7d^3(dx)^{3/2}}{48b^2\left(a+bx^2\right)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2\left(a+bx^2\right)} - \frac{7d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a}\cdots}\right.}{128\sqrt{2}\,a^{5/4}b^{11/\cdots}}$$

Mathematica [A]
time = 0.44, size = 164, normalized size = 0.49

$$\frac{d^4\sqrt{dx}\left(\frac{4\sqrt[4]{a}\,b^{3/4}x^{3/2}\left(-7a^2-18abx^2+21b^2x^4\right)}{\left(a+bx^2\right)^3}-21\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-21\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{768a^{5/4}b^{11/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^4*Sqrt[d*x]*((4*a^(1/4)*b^(3/4)*x^(3/2)*(-7*a^2 - 18*a*b*x^2 + 21*b^2*x^
4))/(a + b*x^2)^3 - 21*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4
```

```
)*b^(1/4)*Sqrt[x])] - 21*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/
(Sqrt[a] + Sqrt[b]*x)]))/(768*a^(5/4)*b^(11/4)*Sqrt[x])
```

Maple [A]

time = 0.06, size = 206, normalized size = 0.61

method	result
derivativedivides	$2d^{7}\left(\frac{\frac{7(dx)^{\frac{11}{2}}}{128a\,d^{2}}-\frac{3(dx)^{\frac{7}{2}}}{64b}-\frac{7a\,d^{2}(dx)^{\frac{3}{2}}}{384b^{2}}}{\left(d^{2}x^{2}b+a\,d^{2}\right)^{3}}+\frac{7\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\ldots}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\ldots\right.\right.}{1024a\,d^{2}b^{3}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right.$
default	$2d^{7}\left(\frac{\frac{7(dx)^{\frac{11}{2}}}{128a\,d^{2}}-\frac{3(dx)^{\frac{7}{2}}}{64b}-\frac{7a\,d^{2}(dx)^{\frac{3}{2}}}{384b^{2}}}{\left(d^{2}x^{2}b+a\,d^{2}\right)^{3}}+\frac{7\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\ldots}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\ldots\right.\right.}{1024a\,d^{2}b^{3}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*((7/128/a/d^2*(d*x)^(11/2)-3/64/b*(d*x)^(7/2)-7/384*a*d^2/b^2*(d*x)^(
3/2))/(b*d^2*x^2+a*d^2)^3+7/1024/a/d^2/b^3/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x
-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x
)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.52, size = 323, normalized size = 0.96

$$\frac{\frac{21\,d^{6}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^{2}\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^{2}\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{ab^{2}}+\frac{8\left(21\left(dx\right)^{\frac{11}{2}}b^{2}d^{6}-18\left(dx\right)^{\frac{7}{2}}abd^{8}-7\left(dx\right)^{\frac{3}{2}}a^{2}d^{10}\right)}{ab^{5}d^{6}x^{6}+3\,a^{2}b^{4}d^{6}x^{4}+3\,a^{3}b^{3}d^{6}x^{2}+a^{4}b^{2}d^{6}}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
 + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b
```

```
)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sq
rt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2
)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a*b^2) + 8
*(21*(d*x)^(11/2)*b^2*d^6 - 18*(d*x)^(7/2)*a*b*d^8 - 7*(d*x)^(3/2)*a^2*d^10
)/(a*b^5*d^6*x^6 + 3*a^2*b^4*d^6*x^4 + 3*a^3*b^3*d^6*x^2 + a^4*b^2*d^6))/d
```

Fricas [A]
time = 0.35, size = 390, normalized size = 1.16

$$-\frac{84\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\sqrt{dx}\,ab^3d^{13}-\sqrt{d^{27}x-\sqrt{-\frac{d^{18}}{a^5b^{11}}}\,a^3b^5d^{18}}\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}ab^3}{d^{18}}\right)-21\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\log\left(343\sqrt{dx}\,d^{13}+343\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{3}{4}}a^4b^8\right)+21\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\log\left(343\sqrt{dx}\,d^{13}-343\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{3}{4}}a^4b^8\right)-4\left(21b^2d^4x^5-18abd^4x^3-7a^2d^4x\right)\sqrt{dx}}{768\left(ab^5x^6+3a^2b^4x^4+3a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(84*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^18/(a^
5*b^11))^(1/4)*arctan(-((-d^18/(a^5*b^11))^(1/4)*sqrt(d*x)*a*b^3*d^13 - sqr
t(d^27*x - sqrt(-d^18/(a^5*b^11))*a^3*b^5*d^18)*(-d^18/(a^5*b^11))^(1/4)*a*
b^3)/d^18) - 21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^1
8/(a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 + 343*(-d^18/(a^5*b^11))^(3/4)*a
^4*b^8) + 21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^18/(
a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 - 343*(-d^18/(a^5*b^11))^(3/4)*a^4*
b^8) - 4*(21*b^2*d^4*x^5 - 18*a*b*d^4*x^3 - 7*a^2*d^4*x)*sqrt(d*x))/(a*b^5*
x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^{\frac{9}{2}}}{\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(9/2)/(a + b*x**2)**4, x)
```

Giac [A]
time = 3.96, size = 317, normalized size = 0.94

$$\frac{1}{1536}d^4\left(\frac{8\left(21\sqrt{dx}\,b^2d^6x^5-18\sqrt{dx}\,abd^6x^3-7\sqrt{dx}\,a^2d^6x\right)}{\left(bd^2x^2+ad^2\right)^3ab^2}+\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5d}+\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5d}-\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^5d}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^5d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^4*(8*(21*sqrt(d*x)*b^2*d^6*x^5 - 18*sqrt(d*x)*a*b*d^6*x^3 - 7*sqrt
(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*a*b^2) + 42*sqrt(2)*(a*b^3*d^2)^(3/
```

```
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/
4))/(a^2*b^5*d) + 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5*d) - 21*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^2*b^5*d) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5*d))
```

Mupad [B]
time = 4.26, size = 150, normalized size = 0.45

$$\frac{7\,d^{9/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{5/4}\,b^{11/4}}-\frac{7\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{5/4}\,b^{11/4}}-\frac{\frac{3\,d^7\,(d\,x)^{7/2}}{32\,b}-\frac{7\,d^5\,(d\,x)^{11/2}}{64\,a}+\frac{7\,a\,d^9\,(d\,x)^{3/2}}{192\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (7*d^(9/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(5/
4)*b^(11/4)) - (7*d^(9/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))
/(128*(-a)^(5/4)*b^(11/4)) - ((3*d^7*(d*x)^(7/2))/(32*b) - (7*d^5*(d*x)^(11
/2))/(64*a) + (7*a*d^9*(d*x)^(3/2))/(192*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a
^2*b*d^6*x^2 + 3*a*b^2*d^6*x^4)
```

3.702 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal. Leaf size=336

$$-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2}+\frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)}-\frac{5d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{5d^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\right.}{128\sqrt{2}\,a^{7/4}}$$

```
[Out] -1/6*d*(d*x)^(5/2)/b/(b*x^2+a)^3-5/256*d^(7/2)*arctan(1-b^(1/4)*2^(1/2)*(d*
x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)+5/256*d^(7/2)*arctan(1+b^
(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)-5/512*d^
(7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1
/2))/a^(7/4)/b^(9/4)*2^(1/2)+5/512*d^(7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)-5/48*d^3*
(d*x)^(1/2)/b^2/(b*x^2+a)^2+5/192*d^3*(d*x)^(1/2)/a/b^2/(b*x^2+a)
```

Rubi [A]

time = 0.23, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{5d^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{5d^{7/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{5d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{5d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)}-\frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*(d*(d*x)^(5/2))/(b*(a + b*x^2)^3) - (5*d^3*Sqrt[d*x])/(48*b^2*(a + b*x
^2)^2) + (5*d^3*Sqrt[d*x])/(192*a*b^2*(a + b*x^2)) - (5*d^(7/2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(7/4)*b^(9/4
)) + (5*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(128*Sqrt[2]*a^(7/4)*b^(9/4)) - (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sq
rt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4))
 + (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{(dx)^{7/2}}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{d(dx)^{5/2}}{6b\,(a+bx^2)^3} + \frac{1}{12}\left(5b^2d^2\right)\int \frac{(dx)^{3/2}}{(ab+b^2x^2)^3}\,dx$$

$$= -\frac{d(dx)^{5/2}}{6b\,(a+bx^2)^3} - \frac{5d^3\sqrt{dx}}{48b^2\,(a+bx^2)^2} + \frac{1}{96}\left(5d^4\right)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^2}\,dx$$

$$= -\frac{d(dx)^{5/2}}{6b\,(a+bx^2)^3} - \frac{5d^3\sqrt{dx}}{48b^2\,(a+bx^2)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\,(a+bx^2)} + \frac{\left(5d^4\right)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)}}{128ab}$$

$$= -\frac{d(dx)^{5/2}}{6b\,(a+bx^2)^3} - \frac{5d^3\sqrt{dx}}{48b^2\,(a+bx^2)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\,(a+bx^2)} + \frac{\left(5d^3\right)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x}{d^2}}\right.}{64ab}$$

$$= -\frac{d(dx)^{5/2}}{6b\,(a+bx^2)^3} - \frac{5d^3\sqrt{dx}}{48b^2\,(a+bx^2)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\,(a+bx^2)} + \frac{\left(5d^2\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d}{ab+}\right.}{128a}$$

$$= -\frac{d(dx)^{5/2}}{6b\,(a+bx^2)^3} - \frac{5d^3\sqrt{dx}}{48b^2\,(a+bx^2)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\,(a+bx^2)} - \frac{\left(5d^{7/2}\right)\text{Subst}\left(\int \right.}{}$$

$$= -\frac{d(dx)^{5/2}}{6b\,(a+bx^2)^3} - \frac{5d^3\sqrt{dx}}{48b^2\,(a+bx^2)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\,(a+bx^2)} - \frac{5d^{7/2}\log\left(\sqrt{a}\sqrt{d}+\right.}{25}$$

$$= -\frac{d(dx)^{5/2}}{6b\,(a+bx^2)^3} - \frac{5d^3\sqrt{dx}}{48b^2\,(a+bx^2)^2} + \frac{5d^3\sqrt{dx}}{192ab^2\,(a+bx^2)} - \frac{5d^{7/2}\tan^{-1}\left(1-\sqrt{2}\right.}{128\sqrt{2}\,a^{7/4}}$$

Mathematica [A]

time = 0.43, size = 164, normalized size = 0.49

$$\frac{d^3\sqrt{dx}\left(\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}\,(-15a^2-42abx^2+5b^2x^4)}{(a+bx^2)^3} - 15\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 15\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{768a^{7/4}b^{9/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^3*Sqrt[d*x]*((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-15*a^2 - 42*a*b*x^2 + 5*b^2*x^
4))/(a + b*x^2)^3 - 15*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4
```

```
)*b^(1/4)*Sqrt[x])] + 15*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/
(Sqrt[a] + Sqrt[b]*x)]))/(768*a^(7/4)*b^(9/4)*Sqrt[x])
```

Maple [A]
time = 0.06, size = 206, normalized size = 0.61

method	result
derivativedivides	$2d^{7}\left(\frac{\frac{5(dx)^{\frac{9}{2}}}{384a\,d^{2}}-\frac{7(dx)^{\frac{5}{2}}}{64b}-\frac{5a\,d^{2}\sqrt{dx}}{128b^{2}}}{\left(d^{2}x^{2}b+a\,d^{2}\right)^{3}}+\frac{5\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\left(\sqrt{\right.\right.}{1024a^{2}d^{4}b^{2}}\right.$
default	$2d^{7}\left(\frac{\frac{5(dx)^{\frac{9}{2}}}{384a\,d^{2}}-\frac{7(dx)^{\frac{5}{2}}}{64b}-\frac{5a\,d^{2}\sqrt{dx}}{128b^{2}}}{\left(d^{2}x^{2}b+a\,d^{2}\right)^{3}}+\frac{5\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\left(\sqrt{\right.\right.}{1024a^{2}d^{4}b^{2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*((5/384/a/d^2*(d*x)^(9/2)-7/64/b*(d*x)^(5/2)-5/128*a*d^2/b^2*(d*x)^(1
/2))/(b*d^2*x^2+a*d^2)^3+5/1024/a^2/d^4/b^2*(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*
x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*
x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]
time = 0.50, size = 332, normalized size = 0.99

$$\frac{8\left(5\,(dx)^{\frac{9}{2}}b^{2}d^{6}-42\,(dx)^{\frac{5}{2}}abd^{8}-15\,\sqrt{dx}\,a^{2}d^{10}\right)}{ab^{5}d^{6}x^{6}+3\,a^{2}b^{4}d^{6}x^{4}+3\,a^{3}b^{3}d^{6}x^{2}+a^{4}b^{2}d^{6}}+\frac{15\left(\frac{\sqrt{2}\,d^{6}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{6}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^{5}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^{5}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)}{ab^{2}}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(5*(d*x)^(9/2)*b^2*d^6 - 42*(d*x)^(5/2)*a*b*d^8 - 15*sqrt(d*x)*a^
2*d^10)/(a*b^5*d^6*x^6 + 3*a^2*b^4*d^6*x^4 + 3*a^3*b^3*d^6*x^2 + a^4*b^2*d^
6) + 15*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1
/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sq
```

```
rt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4))
+ 2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(
d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) +
2*sqrt(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a
*b^2))/d
```

Fricas [A]
time = 0.35, size = 389, normalized size = 1.16

$$\frac{60\,(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2)\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^5b^7d^3\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{3}{4}}-\sqrt{a^4b^4\sqrt{-\frac{d^{14}}{a^7b^9}}+d^7x}\,a^5b^7\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{3}{4}}}{d^{14}}\right)+15\,(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2)\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}\log\left(5\,a^2b^2\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}+5\,\sqrt{dx}\,d^3\right)-15\,(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2)\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}\log\left(-5\,a^2b^2\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}+5\,\sqrt{dx}\,d^3\right)+4\,(5\,b^2d^3x^4-42\,abd^3x^2-15\,a^2d^3)\sqrt{dx}}{768\,(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(60*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7
*b^9))^(1/4)*arctan(-(sqrt(d*x)*a^5*b^7*d^3*(-d^14/(a^7*b^9))^(3/4) - sqrt(
a^4*b^4*sqrt(-d^14/(a^7*b^9)) + d^7*x)*a^5*b^7*(-d^14/(a^7*b^9))^(3/4))/d^1
4) + 15*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7*b
^9))^(1/4)*log(5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^3) - 15*(a
*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7*b^9))^(1/4)
*log(-5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^3) + 4*(5*b^2*d^3*x
^4 - 42*a*b*d^3*x^2 - 15*a^2*d^3)*sqrt(d*x))/(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3
*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**4, x)
```

Giac [A]
time = 3.29, size = 304, normalized size = 0.90

$$\frac{1}{1536}\,d^3\left(\frac{30\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}+\frac{30\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}+\frac{15\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^3}-\frac{15\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^3}+\frac{8\left(5\,\sqrt{dx}\,b^2d^6x^4-42\,\sqrt{dx}\,abd^6x^2-15\,\sqrt{dx}\,a^2d^6\right)}{(bd^2x^2+ad^2)^3ab^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^3*(30*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3) + 30*sqrt(2)*(a*b^3*d^2
)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a^2*b^3) + 15*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3) - 15*sqrt(2)*(a*b^3*d^2)^(1/
4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3) +
 8*(5*sqrt(d*x)*b^2*d^6*x^4 - 42*sqrt(d*x)*a*b*d^6*x^2 - 15*sqrt(d*x)*a^2*d
^6)/((b*d^2*x^2 + a*d^2)^3*a*b^2))
```

Mupad [B]
time = 4.26, size = 150, normalized size = 0.45

$$\frac{5\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{7/4}\,b^{9/4}}-\frac{\frac{7\,d^7\,{\left(d\,x\right)}^{5/2}}{32\,b}-\frac{5\,d^5\,{\left(d\,x\right)}^{9/2}}{192\,a}+\frac{5\,a\,d^9\,\sqrt{d\,x}}{64\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}+\frac{5\,d^{7/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{7/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (5*d^(7/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(7/4
)*b^(9/4)) - ((7*d^7*(d*x)^(5/2))/(32*b) - (5*d^5*(d*x)^(9/2))/(192*a) + (5
*a*d^9*(d*x)^(1/2))/(64*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*
a*b^2*d^6*x^4) + (5*d^(7/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2)
)))/(128*(-a)^(7/4)*b^(9/4))
```

3.703 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$-\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3}+\frac{d(dx)^{3/2}}{16ab\,(a+bx^2)^2}+\frac{5d(dx)^{3/2}}{64a^2b\,(a+bx^2)}-\frac{5d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{128\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{5d^{5/2}\tan^{-1}\left(1+\sqrt{\ }\right.}{128\sqrt{2}\,a^{9/}}$$

```
[Out] -1/6*d*(d*x)^(3/2)/b/(b*x^2+a)^3+1/16*d*(d*x)^(3/2)/a/b/(b*x^2+a)^2+5/64*d*
(d*x)^(3/2)/a^2/b/(b*x^2+a)-5/256*d^(5/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1
/2)/a^(1/4)/d^(1/2))/a^(9/4)/b^(7/4)*2^(1/2)+5/256*d^(5/2)*arctan(1+b^(1/4)
*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/b^(7/4)*2^(1/2)+5/512*d^(5/2)
*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/
a^(9/4)/b^(7/4)*2^(1/2)-5/512*d^(5/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+
a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(9/4)/b^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{5d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{128\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{5d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{5d^{5/2}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{5d^{5/2}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{5d(dx)^{3/2}}{64a^2b\,(a+bx^2)}+\frac{d(dx)^{3/2}}{16ab\,(a+bx^2)^2}-\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*(d*(d*x)^(3/2))/(b*(a + b*x^2)^3) + (d*(d*x)^(3/2))/(16*a*b*(a + b*x^2
)^2) + (5*d*(d*x)^(3/2))/(64*a^2*b*(a + b*x^2)) - (5*d^(5/2)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(9/4)*b^(7/4))
+ (5*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(12
8*Sqrt[2]*a^(9/4)*b^(7/4)) + (5*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[
d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4)) -
(5*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{(dx)^{5/2}}{(ab+b^2x^2)^4}\,dx$$
$$= -\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3} + \frac{1}{4}(b^2d^2)\int \frac{\sqrt{dx}}{(ab+b^2x^2)^3}\,dx$$
$$= -\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3} + \frac{d(dx)^{3/2}}{16ab\,(a+bx^2)^2} + \frac{(5bd^2)\int \frac{\sqrt{dx}}{(ab+b^2x^2)^2}\,dx}{32a}$$
$$= -\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3} + \frac{d(dx)^{3/2}}{16ab\,(a+bx^2)^2} + \frac{5d(dx)^{3/2}}{64a^2b\,(a+bx^2)} + \frac{(5d^2)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128a^2}$$
$$= -\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3} + \frac{d(dx)^{3/2}}{16ab\,(a+bx^2)^2} + \frac{5d(dx)^{3/2}}{64a^2b\,(a+bx^2)} + \frac{(5d)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx\right.}{64a^2}$$
$$= -\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3} + \frac{d(dx)^{3/2}}{16ab\,(a+bx^2)^2} + \frac{5d(dx)^{3/2}}{64a^2b\,(a+bx^2)} - \frac{(5d)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}}{ab+\frac{b^2x^4}{d^2}}\right.}{128a^2\sqrt{}}$$
$$= -\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3} + \frac{d(dx)^{3/2}}{16ab\,(a+bx^2)^2} + \frac{5d(dx)^{3/2}}{64a^2b\,(a+bx^2)} + \frac{(5d^{5/2})\text{ Subst}\left(\int \frac{}{-\frac{\sqrt{a}}{\sqrt{b}}}\right.}{25}$$
$$= -\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3} + \frac{d(dx)^{3/2}}{16ab\,(a+bx^2)^2} + \frac{5d(dx)^{3/2}}{64a^2b\,(a+bx^2)} + \frac{5d^{5/2}\log\left(\sqrt{a}\sqrt{d}+\right.}{256}$$
$$= -\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3} + \frac{d(dx)^{3/2}}{16ab\,(a+bx^2)^2} + \frac{5d(dx)^{3/2}}{64a^2b\,(a+bx^2)} - \frac{5d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{a}}\right.}{128\sqrt{2}\,a^{9/4}b^{7/}}$$

Mathematica [A]
time = 0.39, size = 161, normalized size = 0.48

$$\frac{(dx)^{5/2}\left(\frac{4\sqrt[4]{a}\,b^{3/4}x^{3/2}(-5a^2+42abx^2+15b^2x^4)}{(a+bx^2)^3} - 15\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 15\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{768a^{9/4}b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((d*x)^(5/2)*((4*a^(1/4)*b^(3/4)*x^(3/2)*(-5*a^2 + 42*a*b*x^2 + 15*b^2*x^4)
)/(a + b*x^2)^3 - 15*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*
```

```
b^(1/4)*Sqrt[x])] - 15*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(S
qrt[a] + Sqrt[b]*x)]))/(768*a^(9/4)*b^(7/4)*x^(5/2))
```

Maple [A]

time = 0.08, size = 206, normalized size = 0.61

method	result
derivativedivides	$2d^7\left(\frac{\frac{5b(dx)^{\frac{11}{2}}}{128a^2d^4}+\frac{7(dx)^{\frac{7}{2}}}{64a\,d^2}-\frac{5(dx)^{\frac{3}{2}}}{384b}}{(d^2x^2b+a\,d^2)^3}+\frac{5\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{1024a^2d^4b^2\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
default	$2d^7\left(\frac{\frac{5b(dx)^{\frac{11}{2}}}{128a^2d^4}+\frac{7(dx)^{\frac{7}{2}}}{64a\,d^2}-\frac{5(dx)^{\frac{3}{2}}}{384b}}{(d^2x^2b+a\,d^2)^3}+\frac{5\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{1024a^2d^4b^2\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*((5/128/a^2/d^4*b*(d*x)^(11/2)+7/64/a/d^2*(d*x)^(7/2)-5/384/b*(d*x)^(
3/2))/(b*d^2*x^2+a*d^2)^3+5/1024/a^2/d^4/b^2/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d
*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d
*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.51, size = 323, normalized size = 0.96

$$\frac{\frac{15\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^2b}+\frac{8\left(15\,(dx)^{\frac{11}{2}}b^2d^4+42\,(dx)^{\frac{7}{2}}abd^6-5\,(dx)^{\frac{3}{2}}a^2d^8\right)}{a^2b^4d^6x^6+3\,a^3b^3d^6x^4+3\,a^4b^2d^6x^2+a^5bd^6}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(15*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
 + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b
```

```
)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sq
rt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2
)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^2*b) + 8
*(15*(d*x)^(11/2)*b^2*d^4 + 42*(d*x)^(7/2)*a*b*d^6 - 5*(d*x)^(3/2)*a^2*d^8)
/(a^2*b^4*d^6*x^6 + 3*a^3*b^3*d^6*x^4 + 3*a^4*b^2*d^6*x^2 + a^5*b*d^6))/d
```

Fricas [A]
time = 0.36, size = 396, normalized size = 1.18

$$-\frac{60\,(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b)\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,\sqrt{dx}\,a^2b^2d^7\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}-\sqrt{-15625\,a^5b^3d^{10}\sqrt{-\frac{d^{10}}{a^9b^7}}+15625\,d^{15}x}\,a^2b^2\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}}{125\,d^{10}}\right)-15\,(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b)\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}\log\left(125\,a^7b^5\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{3}{4}}+125\,\sqrt{dx}\,d^7\right)+15\,(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b)\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}\log\left(-125\,a^7b^5\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{3}{4}}+125\,\sqrt{dx}\,d^7\right)-4\,(15\,b^2d^2x^5+42\,abd^2x^3-5\,a^2d^2x)\sqrt{dx}}{768\,(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(60*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^10/(a^
9*b^7))^(1/4)*arctan(-1/125*(125*sqrt(d*x)*a^2*b^2*d^7*(-d^10/(a^9*b^7))^(1
/4) - sqrt(-15625*a^5*b^3*d^10*sqrt(-d^10/(a^9*b^7)) + 15625*d^15*x)*a^2*b^
2*(-d^10/(a^9*b^7))^(1/4))/d^10) - 15*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*
b^2*x^2 + a^5*b)*(-d^10/(a^9*b^7))^(1/4)*log(125*a^7*b^5*(-d^10/(a^9*b^7))^
(3/4) + 125*sqrt(d*x)*d^7) + 15*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^
2 + a^5*b)*(-d^10/(a^9*b^7))^(1/4)*log(-125*a^7*b^5*(-d^10/(a^9*b^7))^(3/4)
 + 125*sqrt(d*x)*d^7) - 4*(15*b^2*d^2*x^5 + 42*a*b*d^2*x^3 - 5*a^2*d^2*x)*s
qrt(d*x))/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**4, x)
```

Giac [A]
time = 3.67, size = 317, normalized size = 0.95

$$\frac{1}{1536}\,d^2\left(\frac{8\left(15\,\sqrt{dx}\,b^2d^6x^5+42\,\sqrt{dx}\,abd^6x^3-5\,\sqrt{dx}\,a^2d^6x\right)}{(bd^2x^2+ad^2)^3a^2b}+\frac{30\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4d}+\frac{30\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4d}-\frac{15\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^4d}+\frac{15\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^4d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^2*(8*(15*sqrt(d*x)*b^2*d^6*x^5 + 42*sqrt(d*x)*a*b*d^6*x^3 - 5*sqrt
(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*a^2*b) + 30*sqrt(2)*(a*b^3*d^2)^(3/
```

```
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/
4))/(a^3*b^4*d) + 30*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4*d) - 15*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^3*b^4*d) + 15*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4*d))
```

Mupad [B]
time = 4.23, size = 149, normalized size = 0.44

$$\frac{\frac{7\,d^5\,(d\,x)^{7/2}}{32\,a}-\frac{5\,d^7\,(d\,x)^{3/2}}{192\,b}+\frac{5\,b\,d^3\,(d\,x)^{11/2}}{64\,a^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}+\frac{5\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{9/4}\,b^{7/4}}-\frac{5\,d^{5/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{9/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((7*d^5*(d*x)^(7/2))/(32*a) - (5*d^7*(d*x)^(3/2))/(192*b) + (5*b*d^3*(d*x)^
(11/2))/(64*a^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*x^
4) + (5*d^(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)
^(9/4)*b^(7/4)) - (5*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(128*(-a)^(9/4)*b^(7/4))
```

3.704 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}+\frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2}+\frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)}-\frac{7d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{7d^{3/2}\tan^{-1}\left(1+\sqrt{\ }\right.}{128\sqrt{2}\,a^{11/}}$$

```
[Out] -7/256*d^(3/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/
4)/b^(5/4)*2^(1/2)+7/256*d^(3/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/
4)/d^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)-7/512*d^(3/2)*ln(a^(1/2)*d^(1/2)+x*b^(
1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+
7/512*d^(3/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*
(d*x)^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)-1/6*d*(d*x)^(1/2)/b/(b*x^2+a)^3+1/48*
d*(d*x)^(1/2)/a/b/(b*x^2+a)^2+7/192*d*(d*x)^(1/2)/a^2/b/(b*x^2+a)
```

Rubi [A]
time = 0.23, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{7d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{7d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{7d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{7d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)}+\frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -1/6*(d*Sqrt[d*x])/(b*(a + b*x^2)^3) + (d*Sqrt[d*x])/(48*a*b*(a + b*x^2)^2)
 + (7*d*Sqrt[d*x])/(192*a^2*b*(a + b*x^2)) - (7*d^(3/2)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) + (7
*d^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sq
rt[2]*a^(11/4)*b^(5/4)) - (7*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*
x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4)) + (7
*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = b^4\int \frac{(dx)^{3/2}}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{d\sqrt{dx}}{6b(a+bx^2)^3} + \frac{1}{12}(b^2d^2)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^3}\,dx$$

$$= -\frac{d\sqrt{dx}}{6b(a+bx^2)^3} + \frac{d\sqrt{dx}}{48ab(a+bx^2)^2} + \frac{(7bd^2)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^2}\,dx}{96a}$$

$$= -\frac{d\sqrt{dx}}{6b(a+bx^2)^3} + \frac{d\sqrt{dx}}{48ab(a+bx^2)^2} + \frac{7d\sqrt{dx}}{192a^2b(a+bx^2)} + \frac{(7d^2)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)}\ldots}{128a^2}$$

$$= -\frac{d\sqrt{dx}}{6b(a+bx^2)^3} + \frac{d\sqrt{dx}}{48ab(a+bx^2)^2} + \frac{7d\sqrt{dx}}{192a^2b(a+bx^2)} + \frac{(7d)\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\ldots\right.}{64a^2}$$

$$= -\frac{d\sqrt{dx}}{6b(a+bx^2)^3} + \frac{d\sqrt{dx}}{48ab(a+bx^2)^2} + \frac{7d\sqrt{dx}}{192a^2b(a+bx^2)} + \frac{7\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\ldots}{ab+\frac{b^2x^4}{d^2}}\ldots\right.}{128a^5\ldots}$$

$$= -\frac{d\sqrt{dx}}{6b(a+bx^2)^3} + \frac{d\sqrt{dx}}{48ab(a+bx^2)^2} + \frac{7d\sqrt{dx}}{192a^2b(a+bx^2)} - \frac{(7d^{3/2})\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{d\sqrt{dx}}{6b(a+bx^2)^3} + \frac{d\sqrt{dx}}{48ab(a+bx^2)^2} + \frac{7d\sqrt{dx}}{192a^2b(a+bx^2)} - \frac{7d^{3/2}\log\left(\sqrt{a}\sqrt{d}+\ldots\right.}{25\ldots}$$

$$= -\frac{d\sqrt{dx}}{6b(a+bx^2)^3} + \frac{d\sqrt{dx}}{48ab(a+bx^2)^2} + \frac{7d\sqrt{dx}}{192a^2b(a+bx^2)} - \frac{7d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\ldots}{\ldots}\right.}{128\sqrt{2}\,a^{11/4}\ldots}$$

Mathematica [A]
time = 0.38, size = 161, normalized size = 0.48

$$\frac{(dx)^{3/2}\left(\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}\,(-21a^2+18abx^2+7b^2x^4)}{(a+bx^2)^3} - 21\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 21\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{768a^{11/4}b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((d*x)^(3/2)*((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-21*a^2 + 18*a*b*x^2 + 7*b^2*x^4)
)/(a + b*x^2)^3 - 21*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*
```

```
b^(1/4)*Sqrt[x])] + 21*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(S
qrt[a] + Sqrt[b]*x)]))/(768*a^(11/4)*b^(5/4)*x^(3/2))
```

Maple [A]
time = 0.08, size = 206, normalized size = 0.61

method	result
derivativedivides	$2d^7\left(\frac{\frac{7b(dx)^{\frac{9}{2}}}{384a^2d^4}+\frac{3(dx)^{\frac{5}{2}}}{64a\,d^2}-\frac{7\sqrt{dx}}{128b}}{(d^2x^2b+a\,d^2)^3}+\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a\,d^2}{b}\right.}\right.\right.}{1024a^3d^6b}\right.$
default	$2d^7\left(\frac{\frac{7b(dx)^{\frac{9}{2}}}{384a^2d^4}+\frac{3(dx)^{\frac{5}{2}}}{64a\,d^2}-\frac{7\sqrt{dx}}{128b}}{(d^2x^2b+a\,d^2)^3}+\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a\,d^2}{b}\right.}\right.\right.}{1024a^3d^6b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*((7/384/a^2/d^4*b*(d*x)^(9/2)+3/64/a/d^2*(d*x)^(5/2)-7/128/b*(d*x)^(1
/2))/(b*d^2*x^2+a*d^2)^3+7/1024/a^3/d^6/b*(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x+
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)
^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]
time = 0.53, size = 332, normalized size = 0.99

$$\frac{\frac{8\left(7\,(dx)^{\frac{9}{2}}b^2d^4+18\,(dx)^{\frac{5}{2}}abd^6-21\sqrt{dx}\,a^2d^8\right)}{a^2b^4d^6x^6+3\,a^3b^3d^6x^4+3\,a^4b^2d^6x^2+a^5bd^6}+\frac{21\left(\frac{\sqrt{2}\,d^4\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^4\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)}{a^2b}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(7*(d*x)^(9/2)*b^2*d^4 + 18*(d*x)^(5/2)*a*b*d^6 - 21*sqrt(d*x)*a^
2*d^8)/(a^2*b^4*d^6*x^6 + 3*a^3*b^3*d^6*x^4 + 3*a^4*b^2*d^6*x^2 + a^5*b*d^6
) + 21*(sqrt(2)*d^4*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/
4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x - sqr
```

```
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) +
 2*sqrt(2)*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2
*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*
x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^
2*b))/d
```

Fricas [A]
time = 0.35, size = 373, normalized size = 1.11

$$\frac{84\,(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b)\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^8b^4d\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{3}{4}}-\sqrt{a^6b^2\sqrt{-\frac{d^6}{a^{11}b^5}}+d^3x}\,a^8b^4\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{3}{4}}}{d^6}\right)+21\,(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b)\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}\log\left(7\,a^3b\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}+7\,\sqrt{dx}\,d\right)-21\,(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b)\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}\log\left(-7\,a^3b\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}+7\,\sqrt{dx}\,d\right)+4\,(7\,b^2dx^4+18\,abdx^2-21\,a^2d)\sqrt{dx}}{768\,(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(84*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11
*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^8*b^4*d*(-d^6/(a^11*b^5))^(3/4) - sqrt(a^
6*b^2*sqrt(-d^6/(a^11*b^5)) + d^3*x)*a^8*b^4*(-d^6/(a^11*b^5))^(3/4))/d^6)
+ 21*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11*b^5)
)^(1/4)*log(7*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) - 21*(a^2*b^4*
x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11*b^5))^(1/4)*log(-7
*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) + 4*(7*b^2*d*x^4 + 18*a*b*d
*x^2 - 21*a^2*d)*sqrt(d*x))/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 +
a^5*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**4, x)
```

Giac [A]
time = 4.21, size = 302, normalized size = 0.90

$$\frac{1}{1536}\,d\left(\frac{42\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2}+\frac{42\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2}+\frac{21\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^2}-\frac{21\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^2}+\frac{8\left(7\,\sqrt{dx}\,b^2d^6x^4+18\,\sqrt{dx}\,abd^6x^2-21\,\sqrt{dx}\,a^2d^6\right)}{(bd^2x^2+ad^2)^3a^2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d*(42*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2) + 42*sqrt(2)*(a*b^3*d^2)^
(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)
^(1/4))/(a^3*b^2) + 21*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b
)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2) - 21*sqrt(2)*(a*b^3*d^2)^(1/4)
*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2) + 8
*(7*sqrt(d*x)*b^2*d^6*x^4 + 18*sqrt(d*x)*a*b*d^6*x^2 - 21*sqrt(d*x)*a^2*d^6
)/((b*d^2*x^2 + a*d^2)^3*a^2*b))
```

Mupad [B]
time = 4.27, size = 149, normalized size = 0.44

$$\frac{\frac{3\,d^5\,{\left(d\,x\right)}^{5/2}}{32\,a}-\frac{7\,d^7\,\sqrt{d\,x}}{64\,b}+\frac{7\,b\,d^3\,{\left(d\,x\right)}^{9/2}}{192\,a^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}-\frac{7\,d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{11/4}\,b^{5/4}}-\frac{7\,d^{3/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{11/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((3*d^5*(d*x)^(5/2))/(32*a) - (7*d^7*(d*x)^(1/2))/(64*b) + (7*b*d^3*(d*x)^(
9/2))/(192*a^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*x^4
) - (7*d^(3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^
(11/4)*b^(5/4)) - (7*d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(128*(-a)^(11/4)*b^(5/4))
```

3.705 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}+\frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2}+\frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)}-\frac{15\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{15\sqrt{d}\,\tan^{-1}\left(1-\right.}{128\sqrt{2}}$$

```
[Out] 1/6*(d*x)^(3/2)/a/d/(b*x^2+a)^3+3/16*(d*x)^(3/2)/a^2/d/(b*x^2+a)^2+15/64*(d
*x)^(3/2)/a^3/d/(b*x^2+a)-15/256*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/
4)/d^(1/2))*d^(1/2)/a^(13/4)/b^(3/4)*2^(1/2)+15/256*arctan(1+b^(1/4)*2^(1/2
)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(13/4)/b^(3/4)*2^(1/2)+15/512*ln(a
^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/
2)/a^(13/4)/b^(3/4)*2^(1/2)-15/512*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(
1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(13/4)/b^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{15\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{15\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{15\sqrt{d}\,\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\,\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)}+\frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d*x)^(3/2)/(6*a*d*(a + b*x^2)^3) + (3*(d*x)^(3/2))/(16*a^2*d*(a + b*x^2)^2
) + (15*(d*x)^(3/2))/(64*a^3*d*(a + b*x^2)) - (15*Sqrt[d]*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(13/4)*b^(3/4)) +
(15*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128
*Sqrt[2]*a^(13/4)*b^(3/4)) + (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
- (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{\left(3b^3\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^3}\,dx}{4a}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{\left(15b^2\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^2}\,dx}{32a^2}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{(15b)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{128a^3}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{(15b)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x}{d^2}}\right.}{64a^3d}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} - \frac{\left(15\sqrt{b}\right)\text{Subst}\left(\int \right.}{12}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{\left(15\sqrt{d}\right)\text{Subst}\left(\int \right.}{}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} + \frac{15\sqrt{d}\,\log\left(\sqrt{a}\,\sqrt{d}\right.}{2}$$

$$= \frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3} + \frac{3(dx)^{3/2}}{16a^2d\left(a+bx^2\right)^2} + \frac{15(dx)^{3/2}}{64a^3d\left(a+bx^2\right)} - \frac{15\sqrt{d}\,\tan^{-1}\left(1-\right.}{128\sqrt{2}\,a^{13/}}$$

Mathematica [A]

time = 0.27, size = 161, normalized size = 0.48

$$\frac{\sqrt{dx}\left(\frac{4\sqrt[4]{a}\,x^{3/2}(113a^2+126abx^2+45b^2x^4)}{(a+bx^2)^3}-\frac{45\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}}-\frac{45\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{b^{3/4}}\right)}{768a^{13/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (Sqrt[d*x]*((4*a^(1/4)*x^(3/2)*(113*a^2 + 126*a*b*x^2 + 45*b^2*x^4))/(a + b
*x^2)^3 - (45*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x])])/b^(3/4) - (45*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
/(Sqrt[a] + Sqrt[b]*x)])/b^(3/4)))/(768*a^(13/4)*Sqrt[x])
```

Maple [A]

time = 0.06, size = 212, normalized size = 0.63

method	result
derivativedivides	$2d^7\left(\frac{\frac{15b^2(dx)^{\frac{11}{2}}}{128a^3d^6}+\frac{21b(dx)^{\frac{7}{2}}}{64a^2d^4}+\frac{113(dx)^{\frac{3}{2}}}{384a\,d^2}}{(d^2x^2b+a\,d^2)^3}+\frac{15\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{a\,d^2}{b}\right)\cdots}\right.\cdots}{1024a^3d^6b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\cdots$
default	$2d^7\left(\frac{\frac{15b^2(dx)^{\frac{11}{2}}}{128a^3d^6}+\frac{21b(dx)^{\frac{7}{2}}}{64a^2d^4}+\frac{113(dx)^{\frac{3}{2}}}{384a\,d^2}}{(d^2x^2b+a\,d^2)^3}+\frac{15\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{a\,d^2}{b}\right)\cdots}\right.\cdots}{1024a^3d^6b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*((15/128/a^3/d^6*b^2*(d*x)^(11/2)+21/64/a^2*b/d^4*(d*x)^(7/2)+113/384
/a/d^2*(d*x)^(3/2))/(b*d^2*x^2+a*d^2)^3+15/1024/a^3/d^6/b/(a*d^2/b)^(1/4)*2
^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^
2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.50, size = 317, normalized size = 0.95

$$\frac{\frac{8\left(45\,(dx)^{\frac{11}{2}}b^2d^2+126\,(dx)^{\frac{7}{2}}abd^4+113\,(dx)^{\frac{3}{2}}a^2d^6\right)}{a^3b^3d^6x^6+3\,a^4b^2d^6x^4+3\,a^5bd^6x^2+a^6d^6}+\frac{45\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^3}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(45*(d*x)^(11/2)*b^2*d^2 + 126*(d*x)^(7/2)*a*b*d^4 + 113*(d*x)^(3
/2)*a^2*d^6)/(a^3*b^3*d^6*x^6 + 3*a^4*b^2*d^6*x^4 + 3*a^5*b*d^6*x^2 + a^6*d
^6) + 45*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) +
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqr
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sq
rt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
 - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt
(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^
(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^3)/d
```

Fricas [A]
time = 0.36, size = 359, normalized size = 1.07

$$-\frac{180\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{3375\sqrt{dx}\,a^3bd\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}-\sqrt{-11390625\,a^7bd^2\sqrt{-\frac{d^2}{a^{13}b^3}}+11390625\,d^3x}\,a^3b\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}}{3375\,d^2}\right)-45\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}\log\left(3375\,a^{10}b^2\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{3}{4}}+3375\sqrt{dx}\,d\right)+45\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}\log\left(-3375\,a^{10}b^2\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{3}{4}}+3375\sqrt{dx}\,d\right)-4\left(45\,b^2x^5+126\,abx^3+113\,a^2x\right)\sqrt{dx}}{768\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(180*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*(-d^2/(a^13*b
^3))^(1/4)*arctan(-1/3375*(3375*sqrt(d*x)*a^3*b*d*(-d^2/(a^13*b^3))^(1/4) -
 sqrt(-11390625*a^7*b*d^2*sqrt(-d^2/(a^13*b^3)) + 11390625*d^3*x)*a^3*b*(-d
^2/(a^13*b^3))^(1/4))/d^2) - 45*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2
+ a^6)*(-d^2/(a^13*b^3))^(1/4)*log(3375*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) +
3375*sqrt(d*x)*d) + 45*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*(-
d^2/(a^13*b^3))^(1/4)*log(-3375*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) + 3375*sqr
t(d*x)*d) - 4*(45*b^2*x^5 + 126*a*b*x^3 + 113*a^2*x)*sqrt(d*x))/(a^3*b^3*x^
6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

Sympy [A]
time = 11.49, size = 252, normalized size = 0.75

$$\frac{226a^2d^{11}(dx)^{\frac{3}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}x^6}+\frac{252abd^9(dx)^{\frac{7}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}x^6}+\frac{90b^2d^7(dx)^{\frac{11}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}x^6}+2d^7\operatorname{RootSum}\left(68719476736t^4a^{13}b^3d^{26}+50625,\left(t\mapsto t\log\left(\frac{134217728t^3a^{10}b^2d^{20}}{3375}+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 226*a**2*d**11*(d*x)**(3/2)/(384*a**6*d**12 + 1152*a**5*b*d**12*x**2 + 1152
*a**4*b**2*d**12*x**4 + 384*a**3*b**3*d**12*x**6) + 252*a*b*d**9*(d*x)**(7/
2)/(384*a**6*d**12 + 1152*a**5*b*d**12*x**2 + 1152*a**4*b**2*d**12*x**4 + 3
84*a**3*b**3*d**12*x**6) + 90*b**2*d**7*(d*x)**(11/2)/(384*a**6*d**12 + 115
2*a**5*b*d**12*x**2 + 1152*a**4*b**2*d**12*x**4 + 384*a**3*b**3*d**12*x**6)
 + 2*d**7*RootSum(68719476736*_t**4*a**13*b**3*d**26 + 50625, Lambda(_t, _t
*log(134217728*_t**3*a**10*b**2*d**20/3375 + sqrt(d*x))))
```

Giac [A]
time = 3.23, size = 302, normalized size = 0.90

$$\frac{\frac{90 \sqrt{2} \left(a b^{3} d^{2}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} + 2 \sqrt{d x}\right)}{2 \left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4} b^{3}} + \frac{90 \sqrt{2} \left(a b^{3} d^{2}\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} - 2 \sqrt{d x}\right)}{2 \left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4} b^{3}} - \frac{45 \sqrt{2} \left(a b^{3} d^{2}\right)^{\frac{3}{4}} \log\left(d x + \sqrt{2} \left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{d x} + \sqrt{\frac{a d^{2}}{b}}\right)}{a^{4} b^{3}} + \frac{45 \sqrt{2} \left(a b^{3} d^{2}\right)^{\frac{3}{4}} \log\left(d x - \sqrt{2} \left(\frac{a d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{d x} + \sqrt{\frac{a d^{2}}{b}}\right)}{a^{4} b^{3}} + \frac{8 \left(45 \sqrt{d x} b^{2} d^{7} x^{5} + 126 \sqrt{d x} a b d^{7} x^{3} + 113 \sqrt{d x} a^{2} d^{7} x\right)}{\left(b d^{2} x^{2} + a d^{2}\right)^{3} a^{3}}}{1536 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*(90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3) + 90*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^4*b^3) - 45*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^
(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) + 45*sqrt(2)*(a*b^3*d^2)^(3/4)*l
og(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) + 8*(
45*sqrt(d*x)*b^2*d^7*x^5 + 126*sqrt(d*x)*a*b*d^7*x^3 + 113*sqrt(d*x)*a^2*d^
7*x)/((b*d^2*x^2 + a*d^2)^3*a^3))/d
```

Mupad [B]
time = 0.10, size = 150, normalized size = 0.45

$$\frac{\frac{113\,d^5\,{\left(d\,x\right)}^{3/2}}{192\,a}+\frac{21\,b\,d^3\,{\left(d\,x\right)}^{7/2}}{32\,a^2}+\frac{15\,b^2\,d\,{\left(d\,x\right)}^{11/2}}{64\,a^3}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}-\frac{15\,\sqrt{d}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{13/4}\,b^{3/4}}+\frac{15\,\sqrt{d}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{13/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] ((113*d^5*(d*x)^(3/2))/(192*a) + (21*b*d^3*(d*x)^(7/2))/(32*a^2) + (15*b^2*
d*(d*x)^(11/2))/(64*a^3))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^
2*d^6*x^4) - (15*d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/
(128*(-a)^(13/4)*b^(3/4)) + (15*d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(
1/4)*d^(1/2))))/(128*(-a)^(13/4)*b^(3/4))
```

3.706 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=335

$$\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}+\frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2}+\frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)}-\frac{77\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\tan^{-1}\left(1+\frac{\sqrt{2}\ldots}{\sqrt[4]{a}\ldots}\right.}{128\sqrt{2}\,a^{15/4}\sqrt[4]{b}\ldots}$$

```
[Out] -77/256*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(15/4)/b^(1
/4)*2^(1/2)/d^(1/2)+77/256*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(
1/2))/a^(15/4)/b^(1/4)*2^(1/2)/d^(1/2)-77/512*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*
d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)/d^(1/
2)+77/512*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x
)^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)/d^(1/2)+1/6*(d*x)^(1/2)/a/d/(b*x^2+a)^3+1
1/48*(d*x)^(1/2)/a^2/d/(b*x^2+a)^2+77/192*(d*x)^(1/2)/a^3/d/(b*x^2+a)
```

Rubi [A]
time = 0.22, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{77\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}-\frac{77\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)}+\frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] Sqrt[d*x]/(6*a*d*(a + b*x^2)^3) + (11*Sqrt[d*x])/(48*a^2*d*(a + b*x^2)^2) +
 (77*Sqrt[d*x])/(192*a^3*d*(a + b*x^2)) - (77*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (77*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(
15/4)*b^(1/4)*Sqrt[d]) - (77*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (7
7*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\,\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{(11b^3)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^3}\,dx}{12a}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{(77b^2)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^2}\,dx}{96a^2}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\int \frac{}{\sqrt{dx}}}{128}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\mathrm{Subst}\Big(\int}{}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{(77b)\mathrm{Subst}\Big(\int}{}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} + \frac{77\mathrm{Subst}\Big(\int}{}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} - \frac{77\log\left(\sqrt{a}\right.}{}$$

$$= \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3} + \frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2} + \frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)} - \frac{77\tan^{-1}\left(1-\right.}{128\sqrt{2}\,a}$$

Mathematica [A]

time = 0.26, size = 161, normalized size = 0.48

$$\frac{\sqrt{x}\left(\frac{4a^{3/4}\sqrt{x}\,(153a^2+198abx^2+77b^2x^4)}{(a+bx^2)^3}-\frac{231\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt[4]{b}}+\frac{231\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{b}}\right)}{768a^{15/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (Sqrt[x]*((4*a^(3/4)*Sqrt[x]*(153*a^2 + 198*a*b*x^2 + 77*b^2*x^4))/(a + b*x
^2)^3 - (231*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x])])/b^(1/4) + (231*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4)))/(768*a^(15/4)*Sqrt[d*x])
```

Maple [A]
time = 0.06, size = 209, normalized size = 0.62

method	result
derivativedivides	$2d^7\left(\frac{\frac{77b^2(dx)^{\frac{9}{2}}}{384a^3d^6}+\frac{33b(dx)^{\frac{5}{2}}}{64a^2d^4}+\frac{51\sqrt{dx}}{128a\,d^2}}{(d^2x^2b+a\,d^2)^3}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\right.\right.}{1024a^4d^8}\right.$
default	$2d^7\left(\frac{\frac{77b^2(dx)^{\frac{9}{2}}}{384a^3d^6}+\frac{33b(dx)^{\frac{5}{2}}}{64a^2d^4}+\frac{51\sqrt{dx}}{128a\,d^2}}{(d^2x^2b+a\,d^2)^3}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\right.\right.}{1024a^4d^8}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*((77/384/a^3/d^6*b^2*(d*x)^(9/2)+33/64/a^2/d^4*b*(d*x)^(5/2)+51/128/a
/d^2*(d*x)^(1/2))/(b*d^2*x^2+a*d^2)^3+77/1024/a^4/d^8*(a*d^2/b)^(1/4)*2^(1/
2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)
^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.51, size = 322, normalized size = 0.96

$$\frac{\frac{8\left(77\,(dx)^{\frac{9}{2}}b^2d^2+198\,(dx)^{\frac{5}{2}}abd^4+153\sqrt{dx}\,a^2d^6\right)}{a^3b^3d^6x^6+3\,a^4b^2d^6x^4+3\,a^5bd^6x^2+a^6d^6}+\frac{231\left(\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)}{a^3}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/1536*(8*(77*(d*x)^(9/2)*b^2*d^2 + 198*(d*x)^(5/2)*a*b*d^4 + 153*sqrt(d*x)
*a^2*d^6)/(a^3*b^3*d^6*x^6 + 3*a^4*b^2*d^6*x^4 + 3*a^5*b*d^6*x^2 + a^6*d^6)
 + 231*(sqrt(2)*d^2*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/
4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) +
 2*sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x
)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*s
qrt(2)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*s
qrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/a^3)/d
```

Fricas [A]
time = 0.37, size = 357, normalized size = 1.07

$$\frac{924\,(a^3b^3dx^6+3\,a^4b^2dx^4+3\,a^5bdx^2+a^6d)\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^8d^2\sqrt{-\frac{1}{a^{15}bd^2}}+dx}\,a^{11}bd\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^{11}bd\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{3}{4}}\right)+231\,(a^3b^3dx^6+3\,a^4b^2dx^4+3\,a^5bdx^2+a^6d)\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}\log\left(a^4d\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-231\,(a^3b^3dx^6+3\,a^4b^2dx^4+3\,a^5bdx^2+a^6d)\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}\log\left(-a^4d\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4\,(77\,b^2x^4+198\,abx^2+153\,a^2)\sqrt{dx}}{768\,(a^3b^3dx^6+3\,a^4b^2dx^4+3\,a^5bdx^2+a^6d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(924*(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a
^15*b*d^2))^(1/4)*arctan(sqrt(a^8*d^2*sqrt(-1/(a^15*b*d^2)) + d*x)*a^11*b*d
*(-1/(a^15*b*d^2))^(3/4) - sqrt(d*x)*a^11*b*d*(-1/(a^15*b*d^2))^(3/4)) + 23
1*(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a^15*b*d^2
))^(1/4)*log(a^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) - 231*(a^3*b^3*d*x^
6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a^15*b*d^2))^(1/4)*log(-a
^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) + 4*(77*b^2*x^4 + 198*a*b*x^2 + 1
53*a^2)*sqrt(d*x))/(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(a+bx^2\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**4), x)
```

Giac [A]
time = 3.95, size = 308, normalized size = 0.92

$$\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^4bd}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^4bd}+\frac{77\sqrt{dx}\,b^2d^5x^4+198\sqrt{dx}\,abd^5x^2+153\sqrt{dx}\,a^2d^5}{192\left(bd^2x^2+ad^2\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 77/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d) + 77/256*sqrt(2)*(a*b^3*d^2)^(1
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^4*b*d) + 77/512*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d) - 77/512*sqrt(2)*(a*b^3*d^2)
^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*
d) + 1/192*(77*sqrt(d*x)*b^2*d^5*x^4 + 198*sqrt(d*x)*a*b*d^5*x^2 + 153*sqrt
(d*x)*a^2*d^5)/((b*d^2*x^2 + a*d^2)^3*a^3)
```

Mupad [B]
time = 4.28, size = 150, normalized size = 0.45

$$\frac{\frac{51\,d^5\,\sqrt{d\,x}}{64\,a}+\frac{33\,b\,d^3\,{\left(d\,x\right)}^{5/2}}{32\,a^2}+\frac{77\,b^2\,d\,{\left(d\,x\right)}^{9/2}}{192\,a^3}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}+\frac{77\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{15/4}\,b^{1/4}\,\sqrt{d}}+\frac{77\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{15/4}\,b^{1/4}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((51*d^5*(d*x)^(1/2))/(64*a) + (33*b*d^3*(d*x)^(5/2))/(32*a^2) + (77*b^2*d*
(d*x)^(9/2))/(192*a^3))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*
d^6*x^4) + (77*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^
(15/4)*b^(1/4)*d^(1/2)) + (77*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/
2))))/(128*(-a)^(15/4)*b^(1/4)*d^(1/2))
```

3.707 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=352

$$-\frac{195}{64a^4d\sqrt{dx}}+\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}+\frac{13}{48a^2d\sqrt{dx}\,(a+bx^2)^2}+\frac{39}{64a^3d\sqrt{dx}\,(a+bx^2)}+\frac{195\sqrt[4]{b}\,\tan^{-1}\left(1-\right.}{128\sqrt{2}\,a^{17/4}}$$

```
[Out] 195/256*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(17
/4)/d^(3/2)*2^(1/2)-195/256*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^
(1/4)/d^(1/2))/a^(17/4)/d^(3/2)*2^(1/2)-195/512*b^(1/4)*ln(a^(1/2)*d^(1/2)+
x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(17/4)/d^(3/2)*2^(
1/2)+195/512*b^(1/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2
^(1/2)*(d*x)^(1/2))/a^(17/4)/d^(3/2)*2^(1/2)-195/64/a^4/d/(d*x)^(1/2)+1/6/a
/d/(b*x^2+a)^3/(d*x)^(1/2)+13/48/a^2/d/(b*x^2+a)^2/(d*x)^(1/2)+39/64/a^3/d/
(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{195\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{17/4}d^{3/2}}-\frac{195\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{17/4}d^{3/2}}-\frac{195\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{17/4}d^{3/2}}-\frac{195}{64a^4d\sqrt{dx}}+\frac{39}{64a^3d\sqrt{dx}\,(a+bx^2)}+\frac{13}{48a^2d\sqrt{dx}\,(a+bx^2)^2}+\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -195/(64*a^4*d*Sqrt[d*x]) + 1/(6*a*d*Sqrt[d*x]*(a + b*x^2)^3) + 13/(48*a^2*
d*Sqrt[d*x]*(a + b*x^2)^2) + 39/(64*a^3*d*Sqrt[d*x]*(a + b*x^2)) + (195*b^(
1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2
]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sq
rt[2]*a^(17/4)*d^(3/2)) + (195*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d
]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(17/4)*d^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ad\sqrt{dx}\,\left(a+bx^2\right)^3} + \frac{(13b^3)\int \frac{1}{(dx)^{3/2}(ab+b^2x^2)^3}\,dx}{12a}$$

$$= \frac{1}{6ad\sqrt{dx}\,\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\,\left(a+bx^2\right)^2} + \frac{(39b^2)\int \frac{1}{(dx)^{3/2}(ab+b^2x^2)^2}}{32a^2}$$

$$= \frac{1}{6ad\sqrt{dx}\,\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\,\left(a+bx^2\right)^2} + \frac{39}{64a^3d\sqrt{dx}\,\left(a+bx^2\right)} -$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\,\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\,\left(a+bx^2\right)^2} + \frac{}{64a^3d}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\,\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\,\left(a+bx^2\right)^2} + \frac{}{64a^3d}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\,\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\,\left(a+bx^2\right)^2} + \frac{}{64a^3d}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\,\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\,\left(a+bx^2\right)^2} + \frac{}{64a^3d}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\,\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\,\left(a+bx^2\right)^2} + \frac{}{64a^3d}$$

$$= -\frac{195}{64a^4d\sqrt{dx}} + \frac{1}{6ad\sqrt{dx}\,\left(a+bx^2\right)^3} + \frac{13}{48a^2d\sqrt{dx}\,\left(a+bx^2\right)^2} + \frac{}{64a^3d}$$

Mathematica [A]
time = 0.49, size = 173, normalized size = 0.49

$$\frac{x\left(-\frac{4\sqrt[4]{a}\,\left(384a^3+1469a^2bx^2+1638ab^2x^4+585b^3x^6\right)}{\left(a+bx^2\right)^3}+585\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+585\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{768a^{17/4}(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (x*((-4*a^(1/4)*(384*a^3 + 1469*a^2*b*x^2 + 1638*a*b^2*x^4 + 585*b^3*x^6))/
(a + b*x^2)^3 + 585*Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 585*Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTanh[(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(768*a^(17/4)*(d*x)^
(3/2))
```

Maple [A]
time = 0.10, size = 221, normalized size = 0.63

method	result
derivativedivides	$2d^7\left(-\frac{1}{a^4 d^8\sqrt{dx}} - \frac{b\left(\frac{\frac{67b^2(dx)^{\frac{11}{2}}}{128}+\frac{81abd^2(dx)^{\frac{7}{2}}}{64}+\frac{317a^2d^4(dx)^{\frac{3}{2}}}{384}}{\left(d^2x^2b+ad^2\right)^3} + \frac{195\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\ldots}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\ldots}}\right)\ldots\right.}{\ldots}\right.}{a^4d^8}\ldots\right.$
default	$2d^7\left(-\frac{1}{a^4 d^8\sqrt{dx}} - \frac{b\left(\frac{\frac{67b^2(dx)^{\frac{11}{2}}}{128}+\frac{81abd^2(dx)^{\frac{7}{2}}}{64}+\frac{317a^2d^4(dx)^{\frac{3}{2}}}{384}}{\left(d^2x^2b+ad^2\right)^3} + \frac{195\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\ldots}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\ldots}}\right)\ldots\right.}{\ldots}\right.}{a^4d^8}\ldots\right.$
risch	$-\frac{2}{a^4 d\sqrt{dx}} + \frac{-\frac{67b^3(dx)^{\frac{11}{2}}}{64a^4\left(d^2x^2b+ad^2\right)^3}-\frac{81b^2(dx)^{\frac{7}{2}}d^2}{32a^3\left(d^2x^2b+ad^2\right)^3}-\frac{317b(dx)^{\frac{3}{2}}d^4}{192a^2\left(d^2x^2b+ad^2\right)^3}-\frac{195\sqrt{2}\ln\left(\frac{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\ldots}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\ldots}\right)\ldots}{512a^4\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\ldots}\ldots}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*(-1/a^4/d^8/(d*x)^(1/2)-b/a^4/d^8*((67/128*b^2*(d*x)^(11/2)+81/64*a*b
*d^2*(d*x)^(7/2)+317/384*a^2*d^4*(d*x)^(3/2))/(b*d^2*x^2+a*d^2)^3+195/1024/
b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*a
rctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/
4)*(d*x)^(1/2)-1))))
```

Maxima [A]
time = 0.51, size = 328, normalized size = 0.93

$$-\frac{\frac{8\,(585\,b^3d^6x^6+1638\,ab^2d^6x^4+1469\,a^2bd^6x^2+384\,a^3d^6)}{(dx)^{\frac{13}{2}}a^4b^3+3\,(dx)^{\frac{9}{2}}a^5b^2d^2+3\,(dx)^{\frac{5}{2}}a^6bd^4+\sqrt{dx}\,a^7d^6}+\frac{585\,b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac14}b^{\frac14}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac14}b^{\frac14}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac14}\sqrt{dx}\,b^{\frac14}+\sqrt{a}\,d\right)}{(ad^2)^{\frac14}b^{\frac34}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac14}\sqrt{dx}\,b^{\frac14}+\sqrt{a}\,d\right)}{(ad^2)^{\frac14}b^{\frac34}}\right)}{a^4}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(8*(585*b^3*d^6*x^6 + 1638*a*b^2*d^6*x^4 + 1469*a^2*b*d^6*x^2 + 384
*a^3*d^6)/((d*x)^(13/2)*a^4*b^3 + 3*(d*x)^(9/2)*a^5*b^2*d^2 + 3*(d*x)^(5/2)
*a^6*b*d^4 + sqrt(d*x)*a^7*d^6) + 585*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt
(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(
sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(
sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/
4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqr
t(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1
/4)*b^(3/4)))/a^4)/d
```

Fricas [A]
time = 0.39, size = 410, normalized size = 1.16

$$\frac{2340\,(a^4b^3d^2x^7+3\,a^5b^2d^2x^5+3\,a^6bd^2x^3+a^7d^2x)\left(-\frac{b}{a^{17}d^6}\right)^{\frac14}\arctan\left(-\frac{7414875\sqrt{dx}\,a^4bd\left(-\frac{b}{a^{17}d^6}\right)^{\frac14}-\sqrt{-54980371265625\,a^9bd^4\sqrt{-\frac{b}{a^{17}d^6}}+54980371265625\,b^2dx}\,a^4d\left(-\frac{b}{a^{17}d^6}\right)^{\frac14}}{7414875\,b}\right)-585\,(a^4b^3d^2x^7+3\,a^5b^2d^2x^5+3\,a^6bd^2x^3+a^7d^2x)\left(-\frac{b}{a^{17}d^6}\right)^{\frac14}\log\left(7414875\,a^{13}d^5\left(-\frac{b}{a^{17}d^6}\right)^{\frac34}+7414875\sqrt{dx}\,b\right)+585\,(a^4b^3d^2x^7+3\,a^5b^2d^2x^5+3\,a^6bd^2x^3+a^7d^2x)\left(-\frac{b}{a^{17}d^6}\right)^{\frac14}\log\left(-7414875\,a^{13}d^5\left(-\frac{b}{a^{17}d^6}\right)^{\frac34}+7414875\sqrt{dx}\,b\right)-4\,(585\,b^3x^6+1638\,ab^2x^4+1469\,a^2bx^2+384\,a^3)\sqrt{dx}}{768\,(a^4b^3d^2x^7+3\,a^5b^2d^2x^5+3\,a^6bd^2x^3+a^7d^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(2340*(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^
2*x)*(-b/(a^17*d^6))^(1/4)*arctan(-1/7414875*(7414875*sqrt(d*x)*a^4*b*d*(-b
/(a^17*d^6))^(1/4) - sqrt(-54980371265625*a^9*b*d^4*sqrt(-b/(a^17*d^6)) + 5
4980371265625*b^2*d*x)*a^4*d*(-b/(a^17*d^6))^(1/4))/b) - 585*(a^4*b^3*d^2*x
^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)*(-b/(a^17*d^6))^(1/4)
*log(7414875*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 7414875*sqrt(d*x)*b) + 585*(a
^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)*(-b/(a^17
*d^6))^(1/4)*log(-7414875*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 7414875*sqrt(d*x
```

```
)*b) - 4*(585*b^3*x^6 + 1638*a*b^2*x^4 + 1469*a^2*b*x^2 + 384*a^3)*sqrt(d*x
))/(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\,(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**4), x)
```

Giac [A]
time = 4.81, size = 327, normalized size = 0.93

$$-\frac{\frac{3072}{\sqrt{dx}\,a^4}+\frac{8\left(201\sqrt{dx}\,b^3d^5x^5+486\sqrt{dx}\,ab^2d^5x^3+317\sqrt{dx}\,a^2bd^5x\right)}{(bd^2x^2+ad^2)^3a^4}+\frac{1170\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2d^2}+\frac{1170\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2d^2}-\frac{585\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^2d^2}+\frac{585\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^2d^2}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -1/1536*(3072/(sqrt(d*x)*a^4) + 8*(201*sqrt(d*x)*b^3*d^5*x^5 + 486*sqrt(d*x
)*a*b^2*d^5*x^3 + 317*sqrt(d*x)*a^2*b*d^5*x)/((b*d^2*x^2 + a*d^2)^3*a^4) +
1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
+ 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2*d^2) + 1170*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^5*b^2*d^2) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2*d^2) + 585*sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*
b^2*d^2))/d
```

Mupad [B]
time = 0.14, size = 166, normalized size = 0.47

$$\frac{195\,{(-b)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{17/4}\,d^{3/2}}-\frac{195\,{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{17/4}\,d^{3/2}}-\frac{\frac{2\,d^5}{a}+\frac{1469\,b\,d^5\,x^2}{192\,a^2}+\frac{273\,b^2\,d^5\,x^4}{32\,a^3}+\frac{195\,b^3\,d^5\,x^6}{64\,a^4}}{b^3\,{(d\,x)}^{13/2}+a^3\,d^6\,\sqrt{d\,x}+3\,a^2\,b\,d^4\,{(d\,x)}^{5/2}+3\,a\,b^2\,d^2\,{(d\,x)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] (195*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^(
17/4)*d^(3/2)) - (195*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(
1/2))))/(128*a^(17/4)*d^(3/2)) - ((2*d^5)/a + (1469*b*d^5*x^2)/(192*a^2) +
(273*b^2*d^5*x^4)/(32*a^3) + (195*b^3*d^5*x^6)/(64*a^4))/(b^3*(d*x)^(13/2)
+ a^3*d^6*(d*x)^(1/2) + 3*a^2*b*d^4*(d*x)^(5/2) + 3*a*b^2*d^2*(d*x)^(9/2))
```

3.708 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal. Leaf size=352

$$-\frac{385}{192a^4d(dx)^{3/2}}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}+\frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2}+\frac{55}{64a^3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{385b^{3/4}\tan^{-1}\left(1\right.}{128\sqrt{2}}$$

```
[Out] -385/192/a^4/d/(d*x)^(3/2)+1/6/a/d/(d*x)^(3/2)/(b*x^2+a)^3+5/16/a^2/d/(d*x)
^(3/2)/(b*x^2+a)^2+55/64/a^3/d/(d*x)^(3/2)/(b*x^2+a)+385/256*b^(3/4)*arctan
(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(19/4)/d^(5/2)*2^(1/2)-38
5/256*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(19/4
)/d^(5/2)*2^(1/2)+385/512*b^(3/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1
/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(19/4)/d^(5/2)*2^(1/2)-385/512*b^(3/4)*l
n(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^
(19/4)/d^(5/2)*2^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{385b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\,a^{19/4}d^{5/2}}-\frac{385b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\,a^{19/4}d^{5/2}}+\frac{385b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{19/4}d^{5/2}}-\frac{385b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{256\sqrt{2}\,a^{19/4}d^{5/2}}-\frac{385}{192a^4d(dx)^{3/2}}+\frac{55}{64a^3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -385/(192*a^4*d*(d*x)^(3/2)) + 1/(6*a*d*(d*x)^(3/2)*(a + b*x^2)^3) + 5/(16*
a^2*d*(d*x)^(3/2)*(a + b*x^2)^2) + 55/(64*a^3*d*(d*x)^(3/2)*(a + b*x^2)) +
(385*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(12
8*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(19/4)*d^(5/2)) + (385*b^(3/4)*Lo
g[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(256*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(19/4)*d^(
5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2}\, dx = b^4 \int \frac{1}{(dx)^{5/2} \left(ab+b^2x^2\right)^4}\, dx$$

$$= \frac{1}{6ad(dx)^{3/2} \left(a+bx^2\right)^3} + \frac{(5b^3) \int \frac{1}{(dx)^{5/2}(ab+b^2x^2)^3}\, dx}{4a}$$

$$= \frac{1}{6ad(dx)^{3/2} \left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2} \left(a+bx^2\right)^2} + \frac{(55b^2) \int \frac{1}{(dx)^{5/2}(ab+}}{32a^2}$$

$$= \frac{1}{6ad(dx)^{3/2} \left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2} \left(a+bx^2\right)^2} + \frac{55}{64a^3d(dx)^{3/2} (a+}$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2} \left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2} \left(a+bx^2\right)^2} +$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2} \left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2} \left(a+bx^2\right)^2} +$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2} \left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2} \left(a+bx^2\right)^2} +$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2} \left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2} \left(a+bx^2\right)^2} +$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2} \left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2} \left(a+bx^2\right)^2} +$$

$$= -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2} \left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2} \left(a+bx^2\right)^2} +$$

Mathematica [A]
time = 0.46, size = 173, normalized size = 0.49

$$\frac{x\left(-\frac{4a^{3/4}(128a^3+765a^2bx^2+990ab^2x^4+385b^3x^6)}{(a+bx^2)^3}+1155\sqrt{2}\, b^{3/4}x^{3/2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\, x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-1155\sqrt{2}\, b^{3/4}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\, x}\right)\right)}{768a^{19/4}(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (x*((-4*a^(3/4)*(128*a^3 + 765*a^2*b*x^2 + 990*a*b^2*x^4 + 385*b^3*x^6))/(a
 + b*x^2)^3 + 1155*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 1155*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTanh[(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(768*a^(19/4)*(d*x)^
(5/2))
```

Maple [A]
time = 0.11, size = 224, normalized size = 0.64

method	result
derivativedivides	$2d^7\left(-\frac{1}{3a^4d^8(dx)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{257b^2(dx)^{\frac{9}{2}}}{384}+\frac{101ab\,d^2(dx)^{\frac{5}{2}}}{64}+\frac{127a^2d^4\sqrt{dx}}{128}}{\left(d^2x^2b+a\,d^2\right)^3}+\frac{385\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}\right.\right.}{}\right.}{a^4d^8}\right.$
default	$2d^7\left(-\frac{1}{3a^4d^8(dx)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{257b^2(dx)^{\frac{9}{2}}}{384}+\frac{101ab\,d^2(dx)^{\frac{5}{2}}}{64}+\frac{127a^2d^4\sqrt{dx}}{128}}{\left(d^2x^2b+a\,d^2\right)^3}+\frac{385\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}\right.\right.}{}\right.}{a^4d^8}\right.$
risch	$-\frac{2}{3a^4x\sqrt{dx}\,d^2}+\frac{-\frac{257b^3d(dx)^{\frac{9}{2}}}{192a^4\left(d^2x^2b+a\,d^2\right)^3}-\frac{101b^2d^3(dx)^{\frac{5}{2}}}{32a^3\left(d^2x^2b+a\,d^2\right)^3}-\frac{127b\,d^5\sqrt{dx}}{64a^2\left(d^2x^2b+a\,d^2\right)^3}-\frac{385b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right.}{dx-\left(\frac{a\,d^2}{b}\right.}\right.}{512a^5}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*(-1/3/a^4/d^8/(d*x)^(3/2)-1/a^4/d^8*b*((257/384*b^2*(d*x)^(9/2)+101/6
4*a*b*d^2*(d*x)^(5/2)+127/128*a^2*d^4*(d*x)^(1/2))/(b*d^2*x^2+a*d^2)^3+385/
1024*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d
^2/b)^(1/4)*(d*x)^(1/2)-1))))
```

Maxima [**A**]
time = 0.51, size = 335, normalized size = 0.95

$$-\frac{\frac{8\,(385\,b^3d^6x^6+990\,ab^2d^6x^4+765\,a^2bd^6x^2+128\,a^3d^6)}{(dx)^{\frac{15}{2}}a^4b^3+3\,(dx)^{\frac{11}{2}}a^5b^2d^2+3\,(dx)^{\frac{7}{2}}a^6bd^4+(dx)^{\frac{3}{2}}a^7d^6}+\frac{1155\left(\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}\,d}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}\,d}\right)}{a^4}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/1536*(8*(385*b^3*d^6*x^6 + 990*a*b^2*d^6*x^4 + 765*a^2*b*d^6*x^2 + 128*a
^3*d^6)/((d*x)^(15/2)*a^4*b^3 + 3*(d*x)^(11/2)*a^5*b^2*d^2 + 3*(d*x)^(7/2)*
a^6*b*d^4 + (d*x)^(3/2)*a^7*d^6) + 1155*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x +
sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2
)*b^(3/4)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(
a)*d)/(a*d^2)^(3/4) + 2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)
*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt
(b)*d)*sqrt(a)*d) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*
b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(
b)*d)*sqrt(a)*d))/a^4)/d
```

Fricas [**A**]
time = 0.38, size = 434, normalized size = 1.23

$$-\frac{4620\,(a^4b^3d^3x^8+3\,a^5b^2d^3x^6+3\,a^6bd^3x^4+a^7d^3x^2)\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{14}bd^7\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^{10}d^6\sqrt{-\frac{b^3}{a^{19}d^{10}}}+b^2dx}\,a^{14}d^7\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+1155\,(a^4b^3d^3x^8+3\,a^5b^2d^3x^6+3\,a^6bd^3x^4+a^7d^3x^2)\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}\log\left(385\,a^5d^3\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}+385\sqrt{dx}\,b\right)-1155\,(a^4b^3d^3x^8+3\,a^5b^2d^3x^6+3\,a^6bd^3x^4+a^7d^3x^2)\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}\log\left(-385\,a^5d^3\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}}+385\sqrt{dx}\,b\right)+4\,(385\,b^3x^6+990\,ab^2x^4+765\,a^2bx^2+128\,a^3)\sqrt{dx}}{768\,(a^4b^3d^3x^8+3\,a^5b^2d^3x^6+3\,a^6bd^3x^4+a^7d^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(4620*(a^4*b^3*d^3*x^8 + 3*a^5*b^2*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d
^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*arctan(-(sqrt(d*x)*a^14*b*d^7*(-b^3/(a^19*
d^10))^(3/4) - sqrt(a^10*d^6*sqrt(-b^3/(a^19*d^10)) + b^2*d*x)*a^14*d^7*(-b
^3/(a^19*d^10))^(3/4))/b^3) + 1155*(a^4*b^3*d^3*x^8 + 3*a^5*b^2*d^3*x^6 + 3
*a^6*b*d^3*x^4 + a^7*d^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*log(385*a^5*d^3*(-b^
3/(a^19*d^10))^(1/4) + 385*sqrt(d*x)*b) - 1155*(a^4*b^3*d^3*x^8 + 3*a^5*b^2
*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*log(-385
*a^5*d^3*(-b^3/(a^19*d^10))^(1/4) + 385*sqrt(d*x)*b) + 4*(385*b^3*x^6 + 990
```

```
*a*b^2*x^4 + 765*a^2*b*x^2 + 128*a^3)*sqrt(d*x))/(a^4*b^3*d^3*x^8 + 3*a^5*b
^2*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}}\,(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(5/2)*(a + b*x**2)**4), x)
```

Giac [A]
time = 6.98, size = 308, normalized size = 0.88

$$-\frac{385\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^5d^3}-\frac{385\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^5d^3}-\frac{385\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^5d^3}+\frac{385\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^5d^3}-\frac{385\,b^3d^6x^6+990\,ab^2d^6x^4+765\,a^2bd^6x^2+128\,a^3d^6}{192\left(\sqrt{dx}\,bd^2x^2+\sqrt{dx}\,ad^2\right)^3a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] -385/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*d^3) - 385/256*sqrt(2)*(a*b^3*d^2)
^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/(a^5*d^3) - 385/512*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a
*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*d^3) + 385/512*sqrt(2)*(a*b^3
*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
^5*d^3) - 1/192*(385*b^3*d^6*x^6 + 990*a*b^2*d^6*x^4 + 765*a^2*b*d^6*x^2 +
128*a^3*d^6)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)^3*a^4*d)
```

Mupad [B]
time = 4.25, size = 166, normalized size = 0.47

$$\frac{385\,{(-b)}^{3/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{19/4}\,d^{5/2}}-\frac{\frac{2\,d^5}{3\,a}+\frac{255\,b\,d^5\,x^2}{64\,a^2}+\frac{165\,b^2\,d^5\,x^4}{32\,a^3}+\frac{385\,b^3\,d^5\,x^6}{192\,a^4}}{b^3\,{(d\,x)}^{15/2}+a^3\,d^6\,{(d\,x)}^{3/2}+3\,a^2\,b\,d^4\,{(d\,x)}^{7/2}+3\,a\,b^2\,d^2\,{(d\,x)}^{11/2}}+\frac{385\,{(-b)}^{3/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{19/4}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] (385*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^(1
9/4)*d^(5/2)) - ((2*d^5)/(3*a) + (255*b*d^5*x^2)/(64*a^2) + (165*b^2*d^5*x^
4)/(32*a^3) + (385*b^3*d^5*x^6)/(192*a^4))/(b^3*(d*x)^(15/2) + a^3*d^6*(d*x
)^(3/2) + 3*a^2*b*d^4*(d*x)^(7/2) + 3*a*b^2*d^2*(d*x)^(11/2)) + (385*(-b)^(
3/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^(19/4)*d^(5/
2))
```

3.709 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2} \, dx$

Optimal. Leaf size=370

$$-\frac{663}{320a^4d(dx)^{5/2}}+\frac{663b}{64a^5d^3\sqrt{dx}}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}+\frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2}+\frac{221}{192a^3d(dx)^{5/2}\left(a+bx^2\right)}-$$

```
[Out] -663/320/a^4/d/(d*x)^(5/2)+1/6/a/d/(d*x)^(5/2)/(b*x^2+a)^3+17/48/a^2/d/(d*x
)^(5/2)/(b*x^2+a)^2+221/192/a^3/d/(d*x)^(5/2)/(b*x^2+a)-663/256*b^(5/4)*arc
tan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(21/4)/d^(7/2)*2^(1/2)
+663/256*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(2
1/4)/d^(7/2)*2^(1/2)+663/512*b^(5/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a
^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(21/4)/d^(7/2)*2^(1/2)-663/512*b^(5/4
)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))
/a^(21/4)/d^(7/2)*2^(1/2)+663/64*b/a^5/d^3/(d*x)^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{663b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{21/4}d^{7/2}}+\frac{663b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{21/4}d^{7/2}}+\frac{663b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{21/4}d^{7/2}}-\frac{663b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{21/4}d^{7/2}}+\frac{663b}{64a^5d^3\sqrt{dx}}-\frac{663}{320a^4d(dx)^{5/2}}+\frac{221}{192a^3d(dx)^{5/2}\left(a+bx^2\right)}+\frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -663/(320*a^4*d*(d*x)^(5/2)) + (663*b)/(64*a^5*d^3*Sqrt[d*x]) + 1/(6*a*d*(d
*x)^(5/2)*(a + b*x^2)^3) + 17/(48*a^2*d*(d*x)^(5/2)*(a + b*x^2)^2) + 221/(1
92*a^3*d*(d*x)^(5/2)*(a + b*x^2)) - (663*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(21/4)*d^(7/2)) + (663*b^(5
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]
*a^(21/4)*d^(7/2)) + (663*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2)) - (663*
b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = b^4\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^4}\,dx$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{(17b^3)\int \frac{1}{(dx)^{7/2}(ab+b^2x^2)^3}\,dx}{12a}$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{(221b^2)\int \frac{1}{(dx)^{7/2}(ab+\cdots}}{96a^2}$$

$$= \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{221}{192a^3d(dx)^{5/2}(a+\cdots}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \cdots$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{1\cdots}{48a^2d(dx)^{5\cdots}}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{1\cdots}{48a^2d(dx)^{5\cdots}}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{1\cdots}{48a^2d(dx)^{5\cdots}}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{1\cdots}{48a^2d(dx)^{5\cdots}}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{1\cdots}{48a^2d(dx)^{5\cdots}}$$

$$= -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{1\cdots}{48a^2d(dx)^{5\cdots}}$$

Mathematica [A]
time = 0.49, size = 189, normalized size = 0.51

$$\frac{\sqrt{dx}\left(\frac{4\sqrt[4]{a}\left(-384a^4+6528a^3bx^2+24973a^2b^2x^4+27846ab^3x^6+9945b^4x^8\right)}{(a+bx^2)^3} - 9945\sqrt{2}\,b^{5/4}x^{5/2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right) - 9945\sqrt{2}\,b^{5/4}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{3840a^{21/4}d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (Sqrt[d*x]*((4*a^(1/4)*(-384*a^4 + 6528*a^3*b*x^2 + 24973*a^2*b^2*x^4 + 278
46*a*b^3*x^6 + 9945*b^4*x^8))/(a + b*x^2)^3 - 9945*Sqrt[2]*b^(5/4)*x^(5/2)*
ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 9945*Sqrt
[2]*b^(5/4)*x^(5/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sq
rt[b]*x)]))/(3840*a^(21/4)*d^4*x^3)
```

Maple [A]
time = 0.11, size = 236, normalized size = 0.64

method	result
derivativedivides	$2d^7\left(-\frac{1}{5a^4d^8(dx)^{\frac{5}{2}}}+\frac{4b}{a^5d^{10}\sqrt{dx}}+\frac{b^2\left(\frac{\frac{151b^2(dx)^{\frac{11}{2}}}{128}+\frac{173ab\,d^2(dx)^{\frac{7}{2}}}{64}+\frac{617a^2d^4(dx)^{\frac{3}{2}}}{384}}{\left(d^2x^2b+a\,d^2\right)^3}+\frac{663\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\cdots}{dx+\left(\frac{a\,d^2}{b}\cdots}\right)\cdots\right)}{\cdots}\right)}{\cdots}\right)$
default	$2d^7\left(-\frac{1}{5a^4d^8(dx)^{\frac{5}{2}}}+\frac{4b}{a^5d^{10}\sqrt{dx}}+\frac{b^2\left(\frac{\frac{151b^2(dx)^{\frac{11}{2}}}{128}+\frac{173ab\,d^2(dx)^{\frac{7}{2}}}{64}+\frac{617a^2d^4(dx)^{\frac{3}{2}}}{384}}{\left(d^2x^2b+a\,d^2\right)^3}+\frac{663\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\cdots}{dx+\left(\frac{a\,d^2}{b}\cdots}\right)\cdots\right)}{\cdots}\right)}{\cdots}\right)$
risch	$-\frac{2\left(-20b\,x^2+a\right)}{5a^5\sqrt{dx}\,x^2d^3}+\frac{\frac{151b^4(dx)^{\frac{11}{2}}}{64a^5\left(d^2x^2b+a\,d^2\right)^3}+\frac{173b^3(dx)^{\frac{7}{2}}d^2}{32a^4\left(d^2x^2b+a\,d^2\right)^3}+\frac{617b^2(dx)^{\frac{3}{2}}d^4}{192a^3\left(d^2x^2b+a\,d^2\right)^3}+\frac{663b\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{d\cdots}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{d\cdots}}\right)\cdots}{512a^5\left(\frac{a\,d^2}{b}\cdots\right.}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^7*(-1/5/a^4/d^8/(d*x)^(5/2)+4/a^5/d^10*b/(d*x)^(1/2)+b^2/a^5/d^10*((151
/128*b^2*(d*x)^(11/2)+173/64*a*b*d^2*(d*x)^(7/2)+617/384*a^2*d^4*(d*x)^(3/2
))/(b*d^2*x^2+a*d^2)^3+663/1024/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1
)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))))
```

Maxima [A]

time = 0.51, size = 350, normalized size = 0.95

$$\frac{\frac{8\left(9945\,b^4d^8x^8+27846\,ab^3d^8x^6+24973\,a^2b^2d^8x^4+6528\,a^3bd^8x^2-384\,a^4d^8\right)}{(dx)^{\frac{17}{2}}a^5b^3d^2+3\,(dx)^{\frac{13}{2}}a^6b^2d^4+3\,(dx)^{\frac{9}{2}}a^7bd^6+(dx)^{\frac{5}{2}}a^8d^8}+\frac{9945\,b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac14}b^{\frac14}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac14}b^{\frac14}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac14}\sqrt{dx}\,b^{\frac14}+\sqrt{a}\,d\right)}{(ad^2)^{\frac14}b^{\frac34}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac14}\sqrt{dx}\,b^{\frac14}+\sqrt{a}\,d\right)}{(ad^2)^{\frac14}b^{\frac34}}\right)}{a^5d^2}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/7680*(8*(9945*b^4*d^8*x^8 + 27846*a*b^3*d^8*x^6 + 24973*a^2*b^2*d^8*x^4 +
 6528*a^3*b*d^8*x^2 - 384*a^4*d^8)/((d*x)^(17/2)*a^5*b^3*d^2 + 3*(d*x)^(13/
2)*a^6*b^2*d^4 + 3*(d*x)^(9/2)*a^7*b*d^6 + (d*x)^(5/2)*a^8*d^8) + 9945*b^2*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*
sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(
b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*lo
g(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^
2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*
x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^5*d^2))/d
```

Fricas [A]

time = 0.37, size = 457, normalized size = 1.24

$$-\frac{39780\left(a^5b^3d^4x^9+3a^6b^2d^4x^7+3a^7bd^4x^5+a^8d^4x^3\right)\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac14}\arctan\left(-\frac{291434247\sqrt{dx}\,a^5b^4d^3\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac14}-\sqrt{-84933920324457009\,a^{11}b^5d^8\sqrt{-\frac{b^5}{a^{21}d^{14}}}+84933920324457009\,b^8dx}\,a^5d^3\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac14}}{291434247\,b^5}\right)-9945\left(a^5b^3d^4x^9+3a^6b^2d^4x^7+3a^7bd^4x^5+a^8d^4x^3\right)\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac14}\log\left(291434247\,a^{16}d^{11}\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac34}+291434247\sqrt{dx}\,b^4\right)+9945\left(a^5b^3d^4x^9+3a^6b^2d^4x^7+3a^7bd^4x^5+a^8d^4x^3\right)\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac14}\log\left(-291434247\,a^{16}d^{11}\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac34}+291434247\sqrt{dx}\,b^4\right)-4\left(9945\,b^4x^8+27846\,ab^3x^6+24973\,a^2b^2x^4+6528\,a^3bx^2-384\,a^4\right)\sqrt{dx}}{3840\left(a^5b^3d^4x^9+3a^6b^2d^4x^7+3a^7bd^4x^5+a^8d^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3840*(39780*(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5 + a^8
*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*arctan(-1/291434247*(291434247*sqrt(d*x)
*a^5*b^4*d^3*(-b^5/(a^21*d^14))^(1/4) - sqrt(-84933920324457009*a^11*b^5*d^
8*sqrt(-b^5/(a^21*d^14)) + 84933920324457009*b^8*d*x)*a^5*d^3*(-b^5/(a^21*d
^14))^(1/4))/b^5) - 9945*(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4
*x^5 + a^8*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*log(291434247*a^16*d^11*(-b^5/
(a^21*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4) + 9945*(a^5*b^3*d^4*x^9 + 3*a
^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5 + a^8*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*lo
```

```
g(-291434247*a^16*d^11*(-b^5/(a^21*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4)
- 4*(9945*b^4*x^8 + 27846*a*b^3*x^6 + 24973*a^2*b^2*x^4 + 6528*a^3*b*x^2 -
384*a^4)*sqrt(d*x))/(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5
+ a^8*d^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{7}{2}} (a+bx^2)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral(1/((d*x)**(7/2)*(a + b*x**2)**4), x)
```

Giac [A]
time = 5.54, size = 349, normalized size = 0.94

$$\frac{663\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^6bd^5}+\frac{663\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^6bd^5}-\frac{663\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^6bd^5}+\frac{663\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^6bd^5}+\frac{453\sqrt{dx}\,b^4d^5x^5+1038\sqrt{dx}\,ab^3d^5x^3+617\sqrt{dx}\,a^2b^2d^5x}{192\,(bd^2x^2+ad^2)^3a^5d^3}+\frac{2\,(20\,bd^2x^2-ad^2)}{5\sqrt{dx}\,a^5d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] 663/256*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d^5) + 663/256*sqrt(2)*(a*b^3*d^2
)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a^6*b*d^5) - 663/512*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b*d^5) + 663/512*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^6*b*d^5) + 1/192*(453*sqrt(d*x)*b^4*d^5*x^5 + 1038*sqrt(d*x)*a*b^3*d^
5*x^3 + 617*sqrt(d*x)*a^2*b^2*d^5*x)/((b*d^2*x^2 + a*d^2)^3*a^5*d^3) + 2/5*
(20*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^5*d^5*x^2)
```

Mupad [B]
time = 4.33, size = 179, normalized size = 0.48

$$\frac{\frac{34\,b\,d^5\,x^2}{5\,a^2}-\frac{2\,d^5}{5\,a}+\frac{24973\,b^2\,d^5\,x^4}{960\,a^3}+\frac{4641\,b^3\,d^5\,x^6}{160\,a^4}+\frac{663\,b^4\,d^5\,x^8}{64\,a^5}}{b^3\,{(d\,x)}^{17/2}+a^3\,d^6\,{(d\,x)}^{5/2}+3\,a^2\,b\,d^4\,{(d\,x)}^{9/2}+3\,a\,b^2\,d^2\,{(d\,x)}^{13/2}}-\frac{663\,{(-b)}^{5/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{21/4}\,d^{7/2}}+\frac{663\,{(-b)}^{5/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{21/4}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

```
[Out] ((34*b*d^5*x^2)/(5*a^2) - (2*d^5)/(5*a) + (24973*b^2*d^5*x^4)/(960*a^3) + (
4641*b^3*d^5*x^6)/(160*a^4) + (663*b^4*d^5*x^8)/(64*a^5))/(b^3*(d*x)^(17/2)
 + a^3*d^6*(d*x)^(5/2) + 3*a^2*b*d^4*(d*x)^(9/2) + 3*a*b^2*d^2*(d*x)^(13/2)
) - (663*(-b)^(5/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*
a^(21/4)*d^(7/2)) + (663*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)
*d^(1/2))))/(128*a^(21/4)*d^(7/2))
```

3.710 $\int \frac{(dx)^{27/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=420

$$-\frac{69615ad^{13}\sqrt{dx}}{4096b^7}+\frac{13923d^{11}(dx)^{5/2}}{4096b^6}-\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5}-\frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4}-\frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3}-\frac{595d^7(dx)^{13/2}}{1024b^4\left(a+bx^2\right)^2}-$$

```
[Out] 13923/4096*d^11*(d*x)^(5/2)/b^6-1/10*d*(d*x)^(25/2)/b/(b*x^2+a)^5-5/32*d^3*
(d*x)^(21/2)/b^2/(b*x^2+a)^4-35/128*d^5*(d*x)^(17/2)/b^3/(b*x^2+a)^3-595/10
24*d^7*(d*x)^(13/2)/b^4/(b*x^2+a)^2-7735/4096*d^9*(d*x)^(9/2)/b^5/(b*x^2+a)
-69615/16384*a^(5/4)*d^(27/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/
d^(1/2))/b^(29/4)*2^(1/2)+69615/16384*a^(5/4)*d^(27/2)*arctan(1+b^(1/4)*2^(
1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(29/4)*2^(1/2)-69615/32768*a^(5/4)*d^(2
7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/
2))/b^(29/4)*2^(1/2)+69615/32768*a^(5/4)*d^(27/2)*ln(a^(1/2)*d^(1/2)+x*b^(1
/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(29/4)*2^(1/2)-69615/409
6*a*d^13*(d*x)^(1/2)/b^7
```

Rubi [A]
time = 0.35, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{69615a^{5/4}d^{27/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,b^{29/4}}-\frac{69615a^{5/4}d^{27/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{29/4}}-\frac{69615ad^{13}\sqrt{dx}}{4096b^7}-\frac{7735d^9(dx)^{9/2}}{4096b^5\left(a+bx^2\right)}-\frac{595d^7(dx)^{13/2}}{1024b^4\left(a+bx^2\right)^2}-\frac{35d^5(dx)^{17/2}}{128b^3\left(a+bx^2\right)^3}-\frac{5d^3(dx)^{21/2}}{32b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5}+\frac{13923d^{11}(dx)^{5/2}}{4096b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-69615*a*d^13*Sqrt[d*x])/(4096*b^7) + (13923*d^11*(d*x)^(5/2))/(4096*b^6)
- (d*(d*x)^(25/2))/(10*b*(a + b*x^2)^5) - (5*d^3*(d*x)^(21/2))/(32*b^2*(a +
 b*x^2)^4) - (35*d^5*(d*x)^(17/2))/(128*b^3*(a + b*x^2)^3) - (595*d^7*(d*x)
^(13/2))/(1024*b^4*(a + b*x^2)^2) - (7735*d^9*(d*x)^(9/2))/(4096*b^5*(a + b
*x^2)) - (69615*a^(5/4)*d^(27/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(27/2)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29/4))
- (69615*a^(5/4)*d^(27/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(2
7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(16384*Sqrt[2]*b^(29/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{27/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{(dx)^{27/2}}{(ab+b^2x^2)^6}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} + \frac{1}{4}\left(5b^4d^2\right)\int \frac{(dx)^{23/2}}{(ab+b^2x^2)^5}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\,(a+bx^2)^4} + \frac{1}{64}\left(105b^2d^4\right)\int \frac{(dx)^{19/2}}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\,(a+bx^2)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\,(a+bx^2)^3} + \frac{1}{256}\left(595d^6\right)\int \frac{(d}{(ab+}$$

$$= -\frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\,(a+bx^2)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\,(a+bx^2)^3} - \frac{595d^7(dx)^{13/2}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\,(a+bx^2)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\,(a+bx^2)^3} - \frac{595d^7(dx)^{13/2}}{1024b^4\,(a+bx^2)^2} -$$

$$= \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\,(a+bx^2)^4} - \frac{35d^5(dx)^{17/2}}{128b^3\,(a+bx^2)^3} - \frac{5}{10}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\,(a+bx^2)^4} - \frac{}{1}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\,(a+bx^2)^4} - \frac{}{1}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\,(a+bx^2)^4} - \frac{}{1}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\,(a+bx^2)^4} - \frac{}{1}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\,(a+bx^2)^4} - \frac{}{1}$$

$$= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7} + \frac{13923d^{11}(dx)^{5/2}}{4096b^6} - \frac{d(dx)^{25/2}}{10b\,(a+bx^2)^5} - \frac{5d^3(dx)^{21/2}}{32b^2\,(a+bx^2)^4} - \frac{}{1}$$

Mathematica [A]

time = 0.93, size = 227, normalized size = 0.54

$$\frac{d^{13}\sqrt{dx}\left(4\sqrt[4]{b}\,\sqrt{x}\,(-348075a^6-1670760a^5bx^2-3171350a^4b^2x^4-2951200a^3b^3x^6-1317575a^2b^4x^8-204800ab^5x^{10}+8192b^6x^{12})+348075\sqrt{2}\,a^{5/4}(a+bx^2)^5\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+348075\sqrt{2}\,a^{5/4}(a+bx^2)^5\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{81920b^{29/4}\sqrt{x}\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^13*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(-348075*a^6 - 1670760*a^5*b*x^2 - 31713
50*a^4*b^2*x^4 - 2951200*a^3*b^3*x^6 - 1317575*a^2*b^4*x^8 - 204800*a*b^5*x
^10 + 8192*b^6*x^12) + 348075*Sqrt[2]*a^(5/4)*(a + b*x^2)^5*ArcTan[(-Sqrt[a
] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 348075*Sqrt[2]*a^(5/4)*
(a + b*x^2)^5*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*
x)]))/(81920*b^(29/4)*Sqrt[x]*(a + b*x^2)^5)
```

Maple [A]

time = 0.15, size = 269, normalized size = 0.64

method	result
derivativedivides	$2d^{11}\left(-\frac{-\frac{(dx)^{\frac{5}{2}}b}{5}+6a\,d^2\sqrt{dx}}{b^7}+\frac{a^2d^4\left(\frac{-\frac{20463a^4d^8\sqrt{dx}}{8192}-\frac{56269a^3d^6b(dx)^{\frac{5}{2}}}{5120}-\frac{75471a^2d^4b^2(dx)^{\frac{9}{2}}}{4096}-\frac{3597a\,b^3d^2(dx)^{\frac{1}{\cdots}}}{256}\cdots}{\left(d^2x^2b+a\,d^2\right)^5}\cdots\right.}{\cdots}\right.$
default	$2d^{11}\left(-\frac{-\frac{(dx)^{\frac{5}{2}}b}{5}+6a\,d^2\sqrt{dx}}{b^7}+\frac{a^2d^4\left(\frac{-\frac{20463a^4d^8\sqrt{dx}}{8192}-\frac{56269a^3d^6b(dx)^{\frac{5}{2}}}{5120}-\frac{75471a^2d^4b^2(dx)^{\frac{9}{2}}}{4096}-\frac{3597a\,b^3d^2(dx)^{\frac{1}{\cdots}}}{256}\cdots}{\left(d^2x^2b+a\,d^2\right)^5}\cdots\right.}{\cdots}\right.$

risch	$-\frac{2(-b x^2+30a) x d^{14}}{5b^7\sqrt{dx}}+\left(-\frac{20463a^6d^9\sqrt{dx}}{4096b^7(d^2x^2b+a\,d^2)^5}-\frac{56269a^5d^7(dx)^{\frac{5}{2}}}{2560b^6(d^2x^2b+a\,d^2)^5}-\frac{75471a^4d^5(dx)^{\frac{9}{2}}}{2048b^5(d^2x^2b+a\,d^2)^5}-\frac{3597a}{128b^4(d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*(-1/b^7*(-1/5*(d*x)^(5/2)*b+6*a*d^2*(d*x)^(1/2))+1/b^7*a^2*d^4*((-20
463/8192*a^4*d^8*(d*x)^(1/2)-56269/5120*a^3*d^6*b*(d*x)^(5/2)-75471/4096*a^
2*d^4*b^2*(d*x)^(9/2)-3597/256*a*b^3*d^2*(d*x)^(13/2)-34139/8192*b^4*(d*x)^
(17/2))/(b*d^2*x^2+a*d^2)^5+69615/65536*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((
d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(
d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))))
```

Maxima [A]
time = 0.53, size = 421, normalized size = 1.00

$$-\frac{\frac{8\left(170695\,(dx)^{\frac{17}{2}}a^2b^4d^{16}+575520\,(dx)^{\frac{13}{2}}a^3b^3d^{18}+754710\,(dx)^{\frac{9}{2}}a^4b^2d^{20}+450152\,(dx)^{\frac{5}{2}}a^5bd^{22}+102315\,\sqrt{dx}\,a^6d^{24}\right)}{b^{12}d^{10}x^{10}+5\,ab^{11}d^{10}x^8+10\,a^2b^{10}d^{10}x^6+10\,a^3b^9d^{10}x^4+5\,a^4b^8d^{10}x^2+a^5b^7d^{10}}-\frac{348075\left(\frac{\sqrt{2}\,d^{16}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{16}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^{15}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^{15}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}\right)a^2}{b^7}-\frac{65536\left((dx)^{\frac{5}{2}}bd^{12}-30\sqrt{dx}\,ad^{14}\right)}{b^7}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/163840*(8*(170695*(d*x)^(17/2)*a^2*b^4*d^16 + 575520*(d*x)^(13/2)*a^3*b^
3*d^18 + 754710*(d*x)^(9/2)*a^4*b^2*d^20 + 450152*(d*x)^(5/2)*a^5*b*d^22 +
102315*sqrt(d*x)*a^6*d^24)/(b^12*d^10*x^10 + 5*a*b^11*d^10*x^8 + 10*a^2*b^1
0*d^10*x^6 + 10*a^3*b^9*d^10*x^4 + 5*a^4*b^8*d^10*x^2 + a^5*b^7*d^10) - 348
075*(sqrt(2)*d^16*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^16*log(sqrt(b)*d*x - sqrt
(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) +
2*sqrt(2)*d^15*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2
*sqrt(2)*d^15*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a^
2/b^7 - 65536*((d*x)^(5/2)*b*d^12 - 30*sqrt(d*x)*a*d^14)/b^7)/d
```

Fricas [A]
time = 0.35, size = 515, normalized size = 1.23

$$\frac{1392300\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7)\arctan\left(-\frac{\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{22}d^{13}-\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{3}{4}}\sqrt{a^2d^{27}x+\sqrt{-\frac{a^5d^{54}}{b^{29}}}\,b^{14}}\,b^{22}}{a^5d^{54}}\right)+348075\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7)\log\left(69615\sqrt{dx}\,ad^{13}+69615\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}b^7\right)-348075\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7)\log\left(69615\sqrt{dx}\,ad^{13}-69615\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}b^7\right)+4\,(8192\,b^6d^{13}x^{12}-204800\,ab^5d^{13}x^{10}-1317575\,a^2b^4d^{13}x^8-2951200\,a^3b^3d^{13}x^6-3171350\,a^4b^2d^{13}x^4-1670760\,a^5bd^{13}x^2-348075\,a^6d^{13})\sqrt{dx}}{81920\,(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(1392300*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*
b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)*arctan(-((-a^5*d^54/b^
29)^(3/4)*sqrt(d*x)*a*b^22*d^13 - (-a^5*d^54/b^29)^(3/4)*sqrt(a^2*d^27*x +
sqrt(-a^5*d^54/b^29)*b^14)*b^22)/(a^5*d^54)) + 348075*(-a^5*d^54/b^29)^(1/4
)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*
x^2 + a^5*b^7)*log(69615*sqrt(d*x)*a*d^13 + 69615*(-a^5*d^54/b^29)^(1/4)*b^
7) - 348075*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*
x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)*log(69615*sqrt(d*x)*a*d^13
- 69615*(-a^5*d^54/b^29)^(1/4)*b^7) + 4*(8192*b^6*d^13*x^12 - 204800*a*b^5*
d^13*x^10 - 1317575*a^2*b^4*d^13*x^8 - 2951200*a^3*b^3*d^13*x^6 - 3171350*a
^4*b^2*d^13*x^4 - 1670760*a^5*b*d^13*x^2 - 348075*a^6*d^13)*sqrt(d*x))/(b^1
2*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 +
a^5*b^7)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(27/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.56, size = 374, normalized size = 0.89

$$\frac{1}{163840}d^{13}\left(\frac{696150\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8}+\frac{696150\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8}+\frac{348075\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^8}-\frac{348075\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^8}-\frac{8\left(170695\sqrt{dx}a^2b^4d^{10}x^8+575520\sqrt{dx}a^3b^3d^{10}x^6+754710\sqrt{dx}a^4b^2d^{10}x^4+450152\sqrt{dx}a^5bd^{10}x^2+102315\sqrt{dx}a^6d^{10}\right)}{\left(bd^2x^2+ad^2\right)^5b^7}+\frac{65536\left(\sqrt{dx}b^{24}d^{10}x^2-30\sqrt{dx}ab^{23}d^{10}\right)}{b^{30}d^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^13*(696150*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(
2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^8 + 696150*sqrt(2)*(a*
b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x)
)/(a*d^2/b)^(1/4))/b^8 + 348075*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^8 - 348075*sqrt(2)*(a*b^3*d
^2)^(1/4)*a*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^
8 - 8*(170695*sqrt(d*x)*a^2*b^4*d^10*x^8 + 575520*sqrt(d*x)*a^3*b^3*d^10*x^
6 + 754710*sqrt(d*x)*a^4*b^2*d^10*x^4 + 450152*sqrt(d*x)*a^5*b*d^10*x^2 + 1
02315*sqrt(d*x)*a^6*d^10)/((b*d^2*x^2 + a*d^2)^5*b^7) + 65536*(sqrt(d*x)*b^
24*d^10*x^2 - 30*sqrt(d*x)*a*b^23*d^10)/(b^30*d^10))
```

Mupad [B]
time = 4.40, size = 248, normalized size = 0.59

$$\frac{2\,d^{11}\,{\left(d\,x\right)}^{5/2}}{5\,b^6}-\frac{\frac{20463\,a^6\,d^{23}\,\sqrt{d\,x}}{4096}+\frac{75471\,a^4\,b^2\,d^{19}\,{\left(d\,x\right)}^{9/2}}{2048}+\frac{3597\,a^3\,b^3\,d^{17}\,{\left(d\,x\right)}^{13/2}}{128}+\frac{34139\,a^2\,b^4\,d^{15}\,{\left(d\,x\right)}^{17/2}}{4096}+\frac{56269\,a^5\,b\,d^{21}\,{\left(d\,x\right)}^{5/2}}{2560}}{a^5\,b^7\,d^{10}+5\,a^4\,b^8\,d^{10}\,x^2+10\,a^3\,b^9\,d^{10}\,x^4+10\,a^2\,b^{10}\,d^{10}\,x^6+5\,a\,b^{11}\,d^{10}\,x^8+b^{12}\,d^{10}\,x^{10}}-\frac{69615\,{\left(-a\right)}^{5/4}\,d^{27/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,b^{29/4}}-\frac{12\,a\,d^{13}\,\sqrt{d\,x}}{b^7}+\frac{{\left(-a\right)}^{5/4}\,d^{27/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,69615{}\mathrm{i}}{8192\,b^{29/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(27/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (2*d^11*(d*x)^(5/2))/(5*b^6) - ((20463*a^6*d^23*(d*x)^(1/2))/4096 + (75471*
a^4*b^2*d^19*(d*x)^(9/2))/2048 + (3597*a^3*b^3*d^17*(d*x)^(13/2))/128 + (34
139*a^2*b^4*d^15*(d*x)^(17/2))/4096 + (56269*a^5*b*d^21*(d*x)^(5/2))/2560)/
(a^5*b^7*d^10 + b^12*d^10*x^10 + 5*a*b^11*d^10*x^8 + 5*a^4*b^8*d^10*x^2 + 1
0*a^3*b^9*d^10*x^4 + 10*a^2*b^10*d^10*x^6) - (69615*(-a)^(5/4)*d^(27/2)*ata
n((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(29/4)) + ((-a)^(5/4
)*d^(27/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*69615i)/(819
2*b^(29/4)) - (12*a*d^13*(d*x)^(1/2))/b^7
```

3.711 $\int \frac{(dx)^{25/2}}{\left(a^2+2abx^2+b^2x^4\right)^3} \, dx$

Optimal. Leaf size=402

$$\frac{33649d^{11}(dx)^{3/2}}{12288b^6}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}-\frac{437d^7(dx)^{11/2}}{1024b^4\left(a+bx^2\right)^2}-\frac{4807d^9(dx)^{7/2}}{4096b^5\left(a+bx^2\right)}$$

```
[Out] 33649/12288*d^11*(d*x)^(3/2)/b^6-1/10*d*(d*x)^(23/2)/b/(b*x^2+a)^5-23/160*d
^3*(d*x)^(19/2)/b^2/(b*x^2+a)^4-437/1920*d^5*(d*x)^(15/2)/b^3/(b*x^2+a)^3-4
37/1024*d^7*(d*x)^(11/2)/b^4/(b*x^2+a)^2-4807/4096*d^9*(d*x)^(7/2)/b^5/(b*x
^2+a)+33649/16384*a^(3/4)*d^(25/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(
1/4)/d^(1/2))/b^(27/4)*2^(1/2)-33649/16384*a^(3/4)*d^(25/2)*arctan(1+b^(1/4
)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(27/4)*2^(1/2)-33649/32768*a^(3/4)
*d^(25/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x
)^(1/2))/b^(27/4)*2^(1/2)+33649/32768*a^(3/4)*d^(25/2)*ln(a^(1/2)*d^(1/2)+x
*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(27/4)*2^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{33649a^{3/4}d^{25/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,b^{27/4}}-\frac{33649a^{3/4}d^{25/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,b^{27/4}}-\frac{33649a^{3/4}d^{25/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}\,b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}\,b^{27/4}}-\frac{4807d^9(dx)^{7/2}}{4096b^5\left(a+bx^2\right)}-\frac{437d^7(dx)^{11/2}}{1024b^4\left(a+bx^2\right)^2}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}+\frac{33649d^{11}(dx)^{3/2}}{12288b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (33649*d^11*(d*x)^(3/2))/(12288*b^6) - (d*(d*x)^(23/2))/(10*b*(a + b*x^2)^5
) - (23*d^3*(d*x)^(19/2))/(160*b^2*(a + b*x^2)^4) - (437*d^5*(d*x)^(15/2))/
(1920*b^3*(a + b*x^2)^3) - (437*d^7*(d*x)^(11/2))/(1024*b^4*(a + b*x^2)^2)
- (4807*d^9*(d*x)^(7/2))/(4096*b^5*(a + b*x^2)) + (33649*a^(3/4)*d^(25/2)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(
27/4)) - (33649*a^(3/4)*d^(25/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(27/4)) - (33649*a^(3/4)*d^(25/2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(1638
4*Sqrt[2]*b^(27/4)) + (33649*a^(3/4)*d^(25/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(27/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{25/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{25/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}+\frac{1}{20}\left(23b^4d^2\right)\int \frac{(dx)^{21/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}+\frac{1}{320}\left(437b^2d^4\right)\int \frac{(dx)^{17/2}}{\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}+\frac{1}{256}\left(437d^6\right)\int \frac{}{(a}$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}-\frac{437d^7(dx)^{11/2}}{1024b^4\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}-\frac{437d^7(dx)^{11/2}}{1024b^4\left(a+bx^2\right)^2}$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}-$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}-$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}-$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}-$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}-$$

$$= \frac{33649d^{11}(dx)^{3/2}}{12288b^6}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}-\frac{23d^3(dx)^{19/2}}{160b^2\left(a+bx^2\right)^4}-\frac{437d^5(dx)^{15/2}}{1920b^3\left(a+bx^2\right)^3}-$$

Mathematica [A]

time = 0.89, size = 216, normalized size = 0.54

$$\frac{d^{12}\sqrt{dx}\left(4b^{3/4}x^{3/2}(168245a^5+769120a^4bx^2+1367810a^3b^2x^4+1157176a^2b^3x^6+437345ab^4x^8+40960b^5x^{10})-504735\sqrt{2}\,a^{3/4}(a+bx^2)^5\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+504735\sqrt{2}\,a^{3/4}(a+bx^2)^5\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{245760b^{27/4}\sqrt{x}\,\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^12*Sqrt[d*x]*(4*b^(3/4)*x^(3/2)*(168245*a^5 + 769120*a^4*b*x^2 + 1367810
*a^3*b^2*x^4 + 1157176*a^2*b^3*x^6 + 437345*a*b^4*x^8 + 40960*b^5*x^10) - 5
04735*Sqrt[2]*a^(3/4)*(a + b*x^2)^5*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])] + 504735*Sqrt[2]*a^(3/4)*(a + b*x^2)^5*ArcTanh[(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(245760*b^(27/4)*S
qrt[x]*(a + b*x^2)^5)
```

Maple [A]
time = 0.14, size = 250, normalized size = 0.62

method	result
derivativedivides	$2d^{11}\left(\frac{(dx)^{\frac{3}{2}}}{3b^{6}}-\frac{a\,d^{2}\left(\frac{-\frac{25457a^{4}d^{8}(dx)^{\frac{3}{2}}}{24576}-\frac{3527a^{3}b\,d^{6}(dx)^{\frac{7}{2}}}{768}-\frac{95821a^{2}d^{4}b^{2}(dx)^{\frac{11}{2}}}{12288}-\frac{31149a\,b^{3}d^{2}(dx)^{\frac{15}{2}}}{5120}-\frac{15503b^{4}(dx)^{\frac{19}{2}}}{8192}}{\left(d^{2}x^{2}b+a\,d^{2}\right)^{5}}+\cdots\right.}{\cdots}\right.$
default	$2d^{11}\left(\frac{(dx)^{\frac{3}{2}}}{3b^{6}}-\frac{a\,d^{2}\left(\frac{-\frac{25457a^{4}d^{8}(dx)^{\frac{3}{2}}}{24576}-\frac{3527a^{3}b\,d^{6}(dx)^{\frac{7}{2}}}{768}-\frac{95821a^{2}d^{4}b^{2}(dx)^{\frac{11}{2}}}{12288}-\frac{31149a\,b^{3}d^{2}(dx)^{\frac{15}{2}}}{5120}-\frac{15503b^{4}(dx)^{\frac{19}{2}}}{8192}}{\left(d^{2}x^{2}b+a\,d^{2}\right)^{5}}+\cdots\right.}{\cdots}\right.$

risch	$\frac{2x^2d^{13}}{3b^6\sqrt{dx}}+\left(\frac{25457a^5d^8(dx)^{\frac{3}{2}}}{12288b^6(d^2x^2b+a\,d^2)^5}+\frac{3527a^4d^6(dx)^{\frac{7}{2}}}{384b^5(d^2x^2b+a\,d^2)^5}+\frac{95821a^3d^4(dx)^{\frac{11}{2}}}{6144b^4(d^2x^2b+a\,d^2)^5}+\frac{31149a^2d^2(dx)^{\frac{15}{2}}}{2560b^3(d^2x^2b+a\,d^2)^5}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*(1/3*(d*x)^(3/2)/b^6-a*d^2/b^6*((-25457/24576*a^4*d^8*(d*x)^(3/2)-35
27/768*a^3*b*d^6*(d*x)^(7/2)-95821/12288*a^2*d^4*b^2*(d*x)^(11/2)-31149/512
0*a*b^3*d^2*(d*x)^(15/2)-15503/8192*b^4*(d*x)^(19/2))/(b*d^2*x^2+a*d^2)^5+3
3649/65536/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a
*d^2/b)^(1/4)*(d*x)^(1/2)-1))))
```

Maxima [A]
time = 0.51, size = 394, normalized size = 0.98

$$-\frac{\frac{504735\,ad^{14}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b^6}-\frac{327680\,(dx)^{\frac{3}{2}}d^{12}}{b^6}-\frac{8\left(232545\,(dx)^{\frac{19}{2}}ab^4d^{14}+747576\,(dx)^{\frac{15}{2}}a^2b^3d^{16}+958210\,(dx)^{\frac{11}{2}}a^3b^2d^{18}+564320\,(dx)^{\frac{7}{2}}a^4bd^{20}+127285\,(dx)^{\frac{3}{2}}a^5d^{22}\right)}{b^{11}d^{10}x^{10}+5\,ab^{10}d^{10}x^8+10\,a^2b^9d^{10}x^6+10\,a^3b^8d^{10}x^4+5\,a^4b^7d^{10}x^2+a^5b^6d^{10}}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/491520*(504735*a*d^14*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/
4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sq
rt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^
(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)
*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^
(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt
(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b
^6 - 327680*(d*x)^(3/2)*d^12/b^6 - 8*(232545*(d*x)^(19/2)*a*b^4*d^14 + 7475
76*(d*x)^(15/2)*a^2*b^3*d^16 + 958210*(d*x)^(11/2)*a^3*b^2*d^18 + 564320*(d
*x)^(7/2)*a^4*b*d^20 + 127285*(d*x)^(3/2)*a^5*d^22)/(b^11*d^10*x^10 + 5*a*b
^10*d^10*x^8 + 10*a^2*b^9*d^10*x^6 + 10*a^3*b^8*d^10*x^4 + 5*a^4*b^7*d^10*x
^2 + a^5*b^6*d^10))/d
```

Fricas [A]
time = 0.37, size = 515, normalized size = 1.28

$$\frac{2018940\left(-\frac{a^5d^{50}}{b^{27}}\right)^{\frac{1}{4}}(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)\arctan\left(-\frac{\left(-\frac{a^5d^{50}}{b^{27}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^7d^{37}-\sqrt{a^4d^{75}x-\sqrt{-\frac{a^5d^{50}}{b^{27}}}\,a^3b^{13}d^{50}}\left(-\frac{a^5d^{50}}{b^{27}}\right)^{\frac{1}{4}}b^7}{a^2d^{50}}\right)-504735\left(-\frac{a^5d^{50}}{b^{27}}\right)^{\frac{1}{4}}(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)\log\left(38099255258449\sqrt{dx}\,a^2d^{37}+38099255258449\left(-\frac{a^5d^{50}}{b^{27}}\right)^{\frac{3}{4}}b^{20}\right)+504735\left(-\frac{a^5d^{50}}{b^{27}}\right)^{\frac{1}{4}}(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)\log\left(38099255258449\sqrt{dx}\,a^2d^{37}-38099255258449\left(-\frac{a^5d^{50}}{b^{27}}\right)^{\frac{3}{4}}b^{20}\right)+4\,(40960\,b^5d^{12}x^{11}+437345\,ab^4d^{12}x^9+1157176\,a^2b^3d^{12}x^7+1367810\,a^3b^2d^{12}x^5+769120\,a^4bd^{12}x^3+168245\,a^5d^{12}x)\sqrt{dx}}{245760\,(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(2018940*(-a^3*d^50/b^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2
*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*arctan(-((-a^3*d^50/b^
27)^(1/4)*sqrt(d*x)*a^2*b^7*d^37 - sqrt(a^4*d^75*x - sqrt(-a^3*d^50/b^27)*a
^3*b^13*d^50)*(-a^3*d^50/b^27)^(1/4)*b^7)/(a^3*d^50)) - 504735*(-a^3*d^50/b
^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*
a^4*b^7*x^2 + a^5*b^6)*log(38099255258449*sqrt(d*x)*a^2*d^37 + 380992552584
49*(-a^3*d^50/b^27)^(3/4)*b^20) + 504735*(-a^3*d^50/b^27)^(1/4)*(b^11*x^10
+ 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
*log(38099255258449*sqrt(d*x)*a^2*d^37 - 38099255258449*(-a^3*d^50/b^27)^(3
/4)*b^20) + 4*(40960*b^5*d^12*x^11 + 437345*a*b^4*d^12*x^9 + 1157176*a^2*b^
3*d^12*x^7 + 1367810*a^3*b^2*d^12*x^5 + 769120*a^4*b*d^12*x^3 + 168245*a^5*
d^12*x)*sqrt(d*x))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*
x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(25/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.72, size = 354, normalized size = 0.88

$$\frac{1}{491520}d^{12}\left(\frac{327680\sqrt{dx}\,x}{b^6}-\frac{1009470\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^9d}-\frac{1009470\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^9d}+\frac{504735\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^9d}-\frac{504735\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^9d}+\frac{8\left(232545\sqrt{dx}\,ab^4d^{10}x^9+747576\sqrt{dx}\,a^2b^3d^{10}x^7+958210\sqrt{dx}\,a^3b^2d^{10}x^5+564320\sqrt{dx}\,a^4bd^{10}x^3+127285\sqrt{dx}\,a^5d^{10}x\right)}{(bd^2x^2+ad^2)^5b^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^12*(327680*sqrt(d*x)*x/b^6 - 1009470*sqrt(2)*(a*b^3*d^2)^(3/4)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/
(b^9*d) - 1009470*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^9*d) + 504735*sqrt(2)*(a*b
^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(b^9*d) - 504735*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4
)*sqrt(d*x) + sqrt(a*d^2/b))/(b^9*d) + 8*(232545*sqrt(d*x)*a*b^4*d^10*x^9 +
 747576*sqrt(d*x)*a^2*b^3*d^10*x^7 + 958210*sqrt(d*x)*a^3*b^2*d^10*x^5 + 56
4320*sqrt(d*x)*a^4*b*d^10*x^3 + 127285*sqrt(d*x)*a^5*d^10*x)/((b*d^2*x^2 +
a*d^2)^5*b^6))
```

Mupad [B]

time = 0.24, size = 231, normalized size = 0.57

$$\frac{\frac{25457\,a^5\,d^{21}\,{\left(d\,x\right)}^{3/2}}{12288}+\frac{95821\,a^3\,b^2\,d^{17}\,{\left(d\,x\right)}^{11/2}}{6144}+\frac{31149\,a^2\,b^3\,d^{15}\,{\left(d\,x\right)}^{15/2}}{2560}+\frac{3527\,a^4\,b\,d^{19}\,{\left(d\,x\right)}^{7/2}}{384}+\frac{15503\,a\,b^4\,d^{13}\,{\left(d\,x\right)}^{19/2}}{4096}}{a^5\,b^6\,d^{10}+5\,a^4\,b^7\,d^{10}\,x^2+10\,a^3\,b^8\,d^{10}\,x^4+10\,a^2\,b^9\,d^{10}\,x^6+5\,a\,b^{10}\,d^{10}\,x^8+b^{11}\,d^{10}\,x^{10}}+\frac{2\,d^{11}\,{\left(d\,x\right)}^{3/2}}{3\,b^6}+\frac{33649\,{\left(-a\right)}^{3/4}\,d^{25/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,b^{27/4}}+\frac{{\left(-a\right)}^{3/4}\,d^{25/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,33649{}\mathrm{i}}{8192\,b^{27/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(25/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((25457*a^5*d^21*(d*x)^(3/2))/12288 + (95821*a^3*b^2*d^17*(d*x)^(11/2))/614
4 + (31149*a^2*b^3*d^15*(d*x)^(15/2))/2560 + (3527*a^4*b*d^19*(d*x)^(7/2))/
384 + (15503*a*b^4*d^13*(d*x)^(19/2))/4096)/(a^5*b^6*d^10 + b^11*d^10*x^10
+ 5*a*b^10*d^10*x^8 + 5*a^4*b^7*d^10*x^2 + 10*a^3*b^8*d^10*x^4 + 10*a^2*b^9
*d^10*x^6) + (2*d^11*(d*x)^(3/2))/(3*b^6) + (33649*(-a)^(3/4)*d^(25/2)*atan
((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(27/4)) + ((-a)^(3/4)
*d^(25/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*33649i)/(8192
*b^(27/4))
```

3.712 $\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=402

$$\frac{13923d^{11}\sqrt{dx}}{4096b^6}-\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5}-\frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4}-\frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3}-\frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2}-\frac{13923d^9(dx)^{5/2}}{20480b^5\left(a+bx^2\right)}+$$

```
[Out] -1/10*d*(d*x)^(21/2)/b/(b*x^2+a)^5-21/160*d^3*(d*x)^(17/2)/b^2/(b*x^2+a)^4-
119/640*d^5*(d*x)^(13/2)/b^3/(b*x^2+a)^3-1547/5120*d^7*(d*x)^(9/2)/b^4/(b*x
^2+a)^2-13923/20480*d^9*(d*x)^(5/2)/b^5/(b*x^2+a)+13923/16384*a^(1/4)*d^(23
/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(25/4)*2^(1/2)-
13923/16384*a^(1/4)*d^(23/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d
^(1/2))/b^(25/4)*2^(1/2)+13923/32768*a^(1/4)*d^(23/2)*ln(a^(1/2)*d^(1/2)+x*
b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(25/4)*2^(1/2)-13923
/32768*a^(1/4)*d^(23/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4
)*2^(1/2)*(d*x)^(1/2))/b^(25/4)*2^(1/2)+13923/4096*d^11*(d*x)^(1/2)/b^6
```

Rubi [A]

time = 0.31, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{13923\sqrt[4]{a}\,d^{23/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,b^{25/4}}-\frac{13923\sqrt[4]{a}\,d^{23/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,b^{25/4}}+\frac{13923\sqrt[4]{a}\,d^{23/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{25/4}}-\frac{13923\sqrt[4]{a}\,d^{23/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,b^{25/4}}-\frac{13923d^9(dx)^{5/2}}{20480b^5\left(a+bx^2\right)}-\frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2}-\frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3}-\frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5}+\frac{13923d^{11}\sqrt{dx}}{4096b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (13923*d^11*Sqrt[d*x])/(4096*b^6) - (d*(d*x)^(21/2))/(10*b*(a + b*x^2)^5) -
 (21*d^3*(d*x)^(17/2))/(160*b^2*(a + b*x^2)^4) - (119*d^5*(d*x)^(13/2))/(64
0*b^3*(a + b*x^2)^3) - (1547*d^7*(d*x)^(9/2))/(5120*b^4*(a + b*x^2)^2) - (1
3923*d^9*(d*x)^(5/2))/(20480*b^5*(a + b*x^2)) + (13923*a^(1/4)*d^(23/2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25
/4)) - (13923*a^(1/4)*d^(23/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25/4)) + (13923*a^(1/4)*d^(23/2)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*
Sqrt[2]*b^(25/4)) - (13923*a^(1/4)*d^(23/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*S
qrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(25/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6 \int \frac{(dx)^{23/2}}{(ab+b^2x^2)^6}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\,(a+bx^2)^5} + \frac{1}{20}\left(21b^4d^2\right)\int \frac{(dx)^{19/2}}{(ab+b^2x^2)^5}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\,(a+bx^2)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\,(a+bx^2)^4} + \frac{1}{320}\left(357b^2d^4\right)\int \frac{(dx)^{15/2}}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{d(dx)^{21/2}}{10b\,(a+bx^2)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\,(a+bx^2)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\,(a+bx^2)^3} + \frac{\left(1547d^6\right)\int \frac{(dx)^{11/2}}{(ab+b^2x^2)^3}}{1280}$$

$$= -\frac{d(dx)^{21/2}}{10b\,(a+bx^2)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\,(a+bx^2)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\,(a+bx^2)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\,(a+bx^2)^2}$$

$$= -\frac{d(dx)^{21/2}}{10b\,(a+bx^2)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\,(a+bx^2)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\,(a+bx^2)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4\,(a+bx^2)^2}$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\,(a+bx^2)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\,(a+bx^2)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\,(a+bx^2)^3} -$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\,(a+bx^2)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\,(a+bx^2)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\,(a+bx^2)^3} -$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\,(a+bx^2)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\,(a+bx^2)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\,(a+bx^2)^3} -$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\,(a+bx^2)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\,(a+bx^2)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\,(a+bx^2)^3} -$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\,(a+bx^2)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\,(a+bx^2)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\,(a+bx^2)^3} -$$

$$= \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b\,(a+bx^2)^5} - \frac{21d^3(dx)^{17/2}}{160b^2\,(a+bx^2)^4} - \frac{119d^5(dx)^{13/2}}{640b^3\,(a+bx^2)^3} -$$

Mathematica [A]

time = 0.84, size = 216, normalized size = 0.54

$$\frac{d^{11}\sqrt{dx}\left(4\sqrt[4]{b}\,\sqrt{x}\,(69615a^5+334152a^4bx^2+634270a^3b^2x^4+590240a^2b^3x^6+263515ab^4x^8+40960b^5x^{10})-69615\sqrt{2}\,\sqrt[4]{a}\,(a+bx^2)^5\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-69615\sqrt{2}\,\sqrt[4]{a}\,(a+bx^2)^5\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{81920b^{25/4}\sqrt{x}\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] (d^11*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(69615*a^5 + 334152*a^4*b*x^2 + 634270*a^3*b^2*x^4 + 590240*a^2*b^3*x^6 + 263515*a*b^4*x^8 + 40960*b^5*x^10) - 69615*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 69615*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(81920*b^(25/4)*Sqrt[x]*(a + b*x^2)^5)

Maple [A]
time = 0.12, size = 252, normalized size = 0.63

method	result
derivativedivides	$2d^{11}\left(\frac{\sqrt{dx}}{b^6}-\frac{a\,d^2\left(\frac{-\frac{5731a^4d^8\sqrt{dx}}{8192}-\frac{16169a^3d^6b(dx)^{\frac{5}{2}}}{5120}-\frac{22467a^2d^4b^2(dx)^{\frac{9}{2}}}{4096}-\frac{1129a\,b^3d^2(dx)^{\frac{13}{2}}}{256}-\frac{11743b^4(dx)^{\frac{17}{2}}}{8192}}{\left(d^2x^2b+a\,d^2\right)^5}+\right.}{}\right.$
default	$2d^{11}\left(\frac{\sqrt{dx}}{b^6}-\frac{a\,d^2\left(\frac{-\frac{5731a^4d^8\sqrt{dx}}{8192}-\frac{16169a^3d^6b(dx)^{\frac{5}{2}}}{5120}-\frac{22467a^2d^4b^2(dx)^{\frac{9}{2}}}{4096}-\frac{1129a\,b^3d^2(dx)^{\frac{13}{2}}}{256}-\frac{11743b^4(dx)^{\frac{17}{2}}}{8192}}{\left(d^2x^2b+a\,d^2\right)^5}+\right.}{}\right.$

risch	$\frac{2x\,d^{12}}{b^6\sqrt{dx}} + \left(\frac{5731a^5d^9\sqrt{dx}}{4096b^6(d^2x^2b+a\,d^2)^5} + \frac{16169a^4d^7(dx)^{\frac{5}{2}}}{2560b^5(d^2x^2b+a\,d^2)^5} + \frac{22467a^3d^5(dx)^{\frac{9}{2}}}{2048b^4(d^2x^2b+a\,d^2)^5} + \frac{1129a^2d^3(dx)^{\frac{13}{2}}}{128b^3(d^2x^2b+a\,d^2)^5} + \dots\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)

[Out] 2*d^11*(1/b^6*(d*x)^(1/2)-1/b^6*a*d^2*((-5731/8192*a^4*d^8*(d*x)^(1/2)-16169/5120*a^3*d^6*b*(d*x)^(5/2)-22467/4096*a^2*d^4*b^2*(d*x)^(9/2)-1129/256*a*b^3*d^2*(d*x)^(13/2)-11743/8192*b^4*(d*x)^(17/2))/(b*d^2*x^2+a*d^2)^5+13923/65536*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))))

Maxima [A]

time = 0.51, size = 403, normalized size = 1.00

$$\frac{\frac{327680\sqrt{dx}\,d^{12}}{b^6} + \frac{8\left(58715\,(dx)^{\frac{17}{2}}ab^4d^{14}+180640\,(dx)^{\frac{13}{2}}a^2b^3d^{16}+224670\,(dx)^{\frac{9}{2}}a^3b^2d^{18}+129352\,(dx)^{\frac{5}{2}}a^4bd^{20}+28655\sqrt{dx}\,a^5d^{22}\right)}{b^{11}d^{10}x^{10}+5\,ab^{10}d^{10}x^8+10\,a^2b^9d^{10}x^6+10\,a^3b^8d^{10}x^4+5\,a^4b^7d^{10}x^2+a^5b^6d^{10}} - \frac{69615\left(\frac{\sqrt{2}\,d^{14}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}\,d^{14}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{2\sqrt{2}\,d^{13}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}} + \frac{2\sqrt{2}\,d^{13}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}\right)a}{b^6}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")

[Out] 1/163840*(327680*sqrt(d*x)*d^12/b^6 + 8*(58715*(d*x)^(17/2)*a*b^4*d^14 + 180640*(d*x)^(13/2)*a^2*b^3*d^16 + 224670*(d*x)^(9/2)*a^3*b^2*d^18 + 129352*(d*x)^(5/2)*a^4*b*d^20 + 28655*sqrt(d*x)*a^5*d^22)/(b^11*d^10*x^10 + 5*a*b^10*d^10*x^8 + 10*a^2*b^9*d^10*x^6 + 10*a^3*b^8*d^10*x^4 + 5*a^4*b^7*d^10*x^2 + a^5*b^6*d^10) - 69615*(sqrt(2)*d^14*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^14*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^13*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^13*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a/b^6)/d

Fricas [A]

time = 0.36, size = 479, normalized size = 1.19

$$-\frac{278460\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)\arctan\left(-\frac{\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{19}d^{11}-\sqrt{d^{23}x+\sqrt{-\frac{ad^{46}}{b^{25}}}\,b^{12}}\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{3}{4}}b^{19}}{ad^{46}}\right)+69615\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)\log\left(13923\sqrt{dx}\,d^{11}+13923\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}b^6\right)-69615\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)\log\left(13923\sqrt{dx}\,d^{11}-13923\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}b^6\right)-4\left(40960\,b^5d^{11}x^{10}+263515\,ab^4d^{11}x^8+590240\,a^2b^3d^{11}x^6+634270\,a^3b^2d^{11}x^4+334152\,a^4bd^{11}x^2+69615\,a^5d^{11}\right)\sqrt{dx}}{81920\left(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(278460*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^
9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*arctan(-((-a*d^46/b^25)^(
3/4)*sqrt(d*x)*b^19*d^11 - sqrt(d^23*x + sqrt(-a*d^46/b^25)*b^12)*(-a*d^46/
b^25)^(3/4)*b^19)/(a*d^46)) + 69615*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b
^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*log(13
923*sqrt(d*x)*d^11 + 13923*(-a*d^46/b^25)^(1/4)*b^6) - 69615*(-a*d^46/b^25)
^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*
b^7*x^2 + a^5*b^6)*log(13923*sqrt(d*x)*d^11 - 13923*(-a*d^46/b^25)^(1/4)*b^
6) - 4*(40960*b^5*d^11*x^10 + 263515*a*b^4*d^11*x^8 + 590240*a^2*b^3*d^11*x
^6 + 634270*a^3*b^2*d^11*x^4 + 334152*a^4*b*d^11*x^2 + 69615*a^5*d^11)*sqrt
(d*x))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*
b^7*x^2 + a^5*b^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.36, size = 340, normalized size = 0.85

$$-\frac{1}{163840}d^{11}\left(\frac{139230\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}+\frac{139230\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}+\frac{69615\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^7}-\frac{69615\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^7}-\frac{327680\sqrt{dx}}{b^6}-\frac{8\left(58715\sqrt{dx}\,ab^4d^{10}x^8+180640\sqrt{dx}\,a^2b^3d^{10}x^6+224670\sqrt{dx}\,a^3b^2d^{10}x^4+129352\sqrt{dx}\,a^4bd^{10}x^2+28655\sqrt{dx}\,a^5d^{10}\right)}{(bd^2x^2+ad^2)^5b^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/163840*d^11*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^7 + 139230*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/b^7 + 69615*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*
d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7 - 69615*sqrt(2)*(a*b^3*d^2)^(1/
4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7 - 32768
0*sqrt(d*x)/b^6 - 8*(58715*sqrt(d*x)*a*b^4*d^10*x^8 + 180640*sqrt(d*x)*a^2*
b^3*d^10*x^6 + 224670*sqrt(d*x)*a^3*b^2*d^10*x^4 + 129352*sqrt(d*x)*a^4*b*d
^10*x^2 + 28655*sqrt(d*x)*a^5*d^10)/((b*d^2*x^2 + a*d^2)^5*b^6))
```

Mupad [B]
time = 4.36, size = 231, normalized size = 0.57

$$\frac{\frac{5731\,a^5\,d^{21}\,\sqrt{d\,x}}{4096}+\frac{22467\,a^3\,b^2\,d^{17}\,{\left(d\,x\right)}^{9/2}}{2048}+\frac{1129\,a^2\,b^3\,d^{15}\,{\left(d\,x\right)}^{13/2}}{128}+\frac{16169\,a^4\,b\,d^{19}\,{\left(d\,x\right)}^{5/2}}{2560}+\frac{11743\,a\,b^4\,d^{13}\,{\left(d\,x\right)}^{17/2}}{4096}}{a^5\,b^6\,d^{10}+5\,a^4\,b^7\,d^{10}\,x^2+10\,a^3\,b^8\,d^{10}\,x^4+10\,a^2\,b^9\,d^{10}\,x^6+5\,a\,b^{10}\,d^{10}\,x^8+b^{11}\,d^{10}\,x^{10}}+\frac{2\,d^{11}\,\sqrt{d\,x}}{b^6}-\frac{13923\,{\left(-a\right)}^{1/4}\,d^{23/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,b^{25/4}}+\frac{{\left(-a\right)}^{1/4}\,d^{23/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)\,13923{}\mathrm{i}}{8192\,b^{25/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((5731*a^5*d^21*(d*x)^(1/2))/4096 + (22467*a^3*b^2*d^17*(d*x)^(9/2))/2048 +
 (1129*a^2*b^3*d^15*(d*x)^(13/2))/128 + (16169*a^4*b*d^19*(d*x)^(5/2))/2560
 + (11743*a*b^4*d^13*(d*x)^(17/2))/4096)/(a^5*b^6*d^10 + b^11*d^10*x^10 + 5
*a*b^10*d^10*x^8 + 5*a^4*b^7*d^10*x^2 + 10*a^3*b^8*d^10*x^4 + 10*a^2*b^9*d^
10*x^6) + (2*d^11*(d*x)^(1/2))/b^6 - (13923*(-a)^(1/4)*d^(23/2)*atan((b^(1/
4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(25/4)) + ((-a)^(1/4)*d^(23/
2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*13923i)/(8192*b^(25/
4))
```

3.713 $\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal. Leaf size=385

$$-\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5}-\frac{19d^3(dx)^{15/2}}{160b^2\,(a+bx^2)^4}-\frac{19d^5(dx)^{11/2}}{128b^3\,(a+bx^2)^3}-\frac{209d^7(dx)^{7/2}}{1024b^4\,(a+bx^2)^2}-\frac{1463d^9(dx)^{3/2}}{4096b^5\,(a+bx^2)}-\frac{4389d^{21/2}\tan^{-1}}{8192}$$

```
[Out] -1/10*d*(d*x)^(19/2)/b/(b*x^2+a)^5-19/160*d^3*(d*x)^(15/2)/b^2/(b*x^2+a)^4-
19/128*d^5*(d*x)^(11/2)/b^3/(b*x^2+a)^3-209/1024*d^7*(d*x)^(7/2)/b^4/(b*x^2
+a)^2-1463/4096*d^9*(d*x)^(3/2)/b^5/(b*x^2+a)-4389/16384*d^(21/2)*arctan(1-
b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(23/4)*2^(1/2)+4389/
16384*d^(21/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4
)/b^(23/4)*2^(1/2)+4389/32768*d^(21/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)
-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(23/4)*2^(1/2)-4389/32768*d
^(21/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^
(1/2))/a^(1/4)/b^(23/4)*2^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{4389d^{21/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\sqrt[4]{a}\,b^{23/4}}+\frac{4389d^{21/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\sqrt[4]{a}\,b^{23/4}}+\frac{4389d^{21/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\sqrt[4]{a}\,b^{23/4}}-\frac{4389d^{21/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\sqrt[4]{a}\,b^{23/4}}-\frac{1463d^9(dx)^{3/2}}{4096b^5\,(a+bx^2)}-\frac{209d^7(dx)^{7/2}}{1024b^4\,(a+bx^2)^2}-\frac{19d^5(dx)^{11/2}}{128b^3\,(a+bx^2)^3}-\frac{19d^3(dx)^{15/2}}{160b^2\,(a+bx^2)^4}-\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(d*(d*x)^(19/2))/(b*(a + b*x^2)^5) - (19*d^3*(d*x)^(15/2))/(160*b^2*(
a + b*x^2)^4) - (19*d^5*(d*x)^(11/2))/(128*b^3*(a + b*x^2)^3) - (209*d^7*(d
*x)^(7/2))/(1024*b^4*(a + b*x^2)^2) - (1463*d^9*(d*x)^(3/2))/(4096*b^5*(a +
 b*x^2)) - (4389*d^(21/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) + (4389*d^(21/2)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4))
+ (4389*d^(21/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1/4)*b^(23/4)) - (4389*d^(21/2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(1
6384*Sqrt[2]*a^(1/4)*b^(23/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{(dx)^{21/2}}{(ab+b^2x^2)^6}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5} + \frac{1}{20}\left(19b^4d^2\right)\int \frac{(dx)^{17/2}}{(ab+b^2x^2)^5}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\,(a+bx^2)^4} + \frac{1}{64}\left(57b^2d^4\right)\int \frac{(dx)^{13/2}}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\,(a+bx^2)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\,(a+bx^2)^3} + \frac{1}{256}\left(209d^6\right)\int \frac{(}{(ab}$$

$$= -\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\,(a+bx^2)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\,(a+bx^2)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\,(a+bx^2)^2}$$

$$= -\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\,(a+bx^2)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\,(a+bx^2)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\,(a+bx^2)^2}$$

$$= -\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\,(a+bx^2)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\,(a+bx^2)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\,(a+bx^2)^2}$$

$$= -\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\,(a+bx^2)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\,(a+bx^2)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\,(a+bx^2)^2}$$

$$= -\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\,(a+bx^2)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\,(a+bx^2)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\,(a+bx^2)^2}$$

$$= -\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\,(a+bx^2)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\,(a+bx^2)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\,(a+bx^2)^2}$$

$$= -\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\,(a+bx^2)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\,(a+bx^2)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\,(a+bx^2)^2}$$

Mathematica [A]
time = 0.83, size = 205, normalized size = 0.53

$$\frac{d^9(dx)^{3/2}\left(-4\sqrt[4]{a}\,b^{3/4}x^{3/2}(7315a^4+33440a^3bx^2+59470a^2b^2x^4+50312ab^3x^6+19015b^4x^8)+21945\sqrt{2}\,(a+bx^2)^5\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-21945\sqrt{2}\,(a+bx^2)^5\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{81920\sqrt[4]{a}\,b^{23/4}x^{3/2}\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^9*(d*x)^(3/2)*(-4*a^(1/4)*b^(3/4)*x^(3/2)*(7315*a^4 + 33440*a^3*b*x^2 +
59470*a^2*b^2*x^4 + 50312*a*b^3*x^6 + 19015*b^4*x^8) + 21945*Sqrt[2]*(a + b
*x^2)^5*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] -
21945*Sqrt[2]*(a + b*x^2)^5*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt
[a] + Sqrt[b]*x)]))/(81920*a^(1/4)*b^(23/4)*x^(3/2)*(a + b*x^2)^5)
```

Maple [A]
time = 0.08, size = 235, normalized size = 0.61

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{1463d^8a^4(dx)^{\frac{3}{2}}}{8192b^5}-\frac{209d^6a^3(dx)^{\frac{7}{2}}}{256b^4}-\frac{5947a^2d^4(dx)^{\frac{11}{2}}}{4096b^3}-\frac{6289a\,d^2(dx)^{\frac{15}{2}}}{5120b^2}-\frac{3803(dx)^{\frac{19}{2}}}{8192b}}{(d^2x^2b+a\,d^2)^5}+\frac{4389\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)}{dx+\left(\frac{a\,d^2}{b}\right)}\right)\right)}{}\right)$
default	$2d^{11}\left(\frac{-\frac{1463d^8a^4(dx)^{\frac{3}{2}}}{8192b^5}-\frac{209d^6a^3(dx)^{\frac{7}{2}}}{256b^4}-\frac{5947a^2d^4(dx)^{\frac{11}{2}}}{4096b^3}-\frac{6289a\,d^2(dx)^{\frac{15}{2}}}{5120b^2}-\frac{3803(dx)^{\frac{19}{2}}}{8192b}}{(d^2x^2b+a\,d^2)^5}+\frac{4389\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)}{dx+\left(\frac{a\,d^2}{b}\right)}\right)\right)}{}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((-1463/8192/b^5*d^8*a^4*(d*x)^(3/2)-209/256/b^4*d^6*a^3*(d*x)^(7/2)
-5947/4096*a^2*d^4/b^3*(d*x)^(11/2)-6289/5120*a*d^2/b^2*(d*x)^(15/2)-3803/8
192/b*(d*x)^(19/2))/(b*d^2*x^2+a*d^2)^5+4389/65536/b^6/(a*d^2/b)^(1/4)*2^(1
/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b
)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]
time = 0.51, size = 377, normalized size = 0.98

$$\frac{\frac{21945\,d^{12}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(a d^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(a d^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(a d^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(a d^2)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(a d^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(a d^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b^5}-\frac{8\left(19015\,(dx)^{\frac{19}{2}}b^4d^{12}+50312\,(dx)^{\frac{15}{2}}ab^3d^{14}+59470\,(dx)^{\frac{11}{2}}a^2b^2d^{16}+33440\,(dx)^{\frac{7}{2}}a^3bd^{18}+7315\,(dx)^{\frac{3}{2}}a^4d^{20}\right)}{b^{10}d^{10}x^{10}+5\,ab^9d^{10}x^8+10\,a^2b^8d^{10}x^6+10\,a^3b^7d^{10}x^4+5\,a^4b^6d^{10}x^2+a^5b^5d^{10}}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(21945*d^12*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*
sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4
) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^5 -
 8*(19015*(d*x)^(19/2)*b^4*d^12 + 50312*(d*x)^(15/2)*a*b^3*d^14 + 59470*(d*
x)^(11/2)*a^2*b^2*d^16 + 33440*(d*x)^(7/2)*a^3*b*d^18 + 7315*(d*x)^(3/2)*a^
4*d^20)/(b^10*d^10*x^10 + 5*a*b^9*d^10*x^8 + 10*a^2*b^8*d^10*x^6 + 10*a^3*b
^7*d^10*x^4 + 5*a^4*b^6*d^10*x^2 + a^5*b^5*d^10))/d
```

Fricas [A]
time = 0.37, size = 486, normalized size = 1.26

$$-\frac{87780\,(b^{10}x^{10}+5\,ab^{9}x^{8}+10\,a^{2}b^{8}x^{6}+10\,a^{3}b^{7}x^{4}+5\,a^{4}b^{6}x^{2}+a^{5}b^{5})\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac14}\arctan\left(-\frac{\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac14}\sqrt{dx}\,b^{6}d^{31}-\sqrt{d^{63}x-\sqrt{-\frac{d^{42}}{ab^{23}}}\,ab^{11}d^{42}}\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac14}b^{6}}{d^{42}}\right)-21945\,(b^{10}x^{10}+5\,ab^{9}x^{8}+10\,a^{2}b^{8}x^{6}+10\,a^{3}b^{7}x^{4}+5\,a^{4}b^{6}x^{2}+a^{5}b^{5})\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac14}\log\left(84546715869\,\sqrt{dx}\,d^{31}+84546715869\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac34}ab^{17}\right)+21945\,(b^{10}x^{10}+5\,ab^{9}x^{8}+10\,a^{2}b^{8}x^{6}+10\,a^{3}b^{7}x^{4}+5\,a^{4}b^{6}x^{2}+a^{5}b^{5})\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac14}\log\left(84546715869\,\sqrt{dx}\,d^{31}-84546715869\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac34}ab^{17}\right)+4\,(19015\,b^{4}d^{10}x^{9}+50312\,ab^{3}d^{10}x^{7}+59470\,a^{2}b^{2}d^{10}x^{5}+33440\,a^{3}bd^{10}x^{3}+7315\,a^{4}d^{10}x)\sqrt{dx}}{81920\,(b^{10}x^{10}+5\,ab^{9}x^{8}+10\,a^{2}b^{8}x^{6}+10\,a^{3}b^{7}x^{4}+5\,a^{4}b^{6}x^{2}+a^{5}b^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(87780*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4
+ 5*a^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4)*arctan(-((-d^42/(a*b^23))
^(1/4)*sqrt(d*x)*b^6*d^31 - sqrt(d^63*x - sqrt(-d^42/(a*b^23))*a*b^11*d^42)
*(-d^42/(a*b^23))^(1/4)*b^6)/d^42) - 21945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^
2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4
)*log(84546715869*sqrt(d*x)*d^31 + 84546715869*(-d^42/(a*b^23))^(3/4)*a*b^1
7) + 21945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a
^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4)*log(84546715869*sqrt(d*x)*d^31
 - 84546715869*(-d^42/(a*b^23))^(3/4)*a*b^17) + 4*(19015*b^4*d^10*x^9 + 503
12*a*b^3*d^10*x^7 + 59470*a^2*b^2*d^10*x^5 + 33440*a^3*b*d^10*x^3 + 7315*a^
4*d^10*x)*sqrt(d*x))/(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7
*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.30, size = 352, normalized size = 0.91

$$\frac{1}{163840}\,d^{10}\left(\frac{43890\,\sqrt{2}\,(ab^{3}d^{2})^{\frac34}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac14}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac14}}\right)}{ab^{8}d}+\frac{43890\,\sqrt{2}\,(ab^{3}d^{2})^{\frac34}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac14}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac14}}\right)}{ab^{8}d}-\frac{21945\,\sqrt{2}\,(ab^{3}d^{2})^{\frac34}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac14}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{ab^{8}d}+\frac{21945\,\sqrt{2}\,(ab^{3}d^{2})^{\frac34}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac14}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{ab^{8}d}-\frac{8\left(19015\,\sqrt{dx}\,b^{4}d^{10}x^{9}+50312\,\sqrt{dx}\,ab^{3}d^{10}x^{7}+59470\,\sqrt{dx}\,a^{2}b^{2}d^{10}x^{5}+33440\,\sqrt{dx}\,a^{3}bd^{10}x^{3}+7315\,\sqrt{dx}\,a^{4}d^{10}x\right)}{(bd^{2}x^{2}+ad^{2})^{5}b^{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^10*(43890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^8*d) + 43890*sqrt(2)*(
a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x)
)/(a*d^2/b)^(1/4))/(a*b^8*d) - 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^8*d) + 21945*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a*b^8*d) - 8*(19015*sqrt(d*x)*b^4*d^10*x^9 + 50312*sqrt(d*x)*a*b^3*d^1
0*x^7 + 59470*sqrt(d*x)*a^2*b^2*d^10*x^5 + 33440*sqrt(d*x)*a^3*b*d^10*x^3 +
 7315*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*b^5))
```

Mupad [B]
time = 0.21, size = 213, normalized size = 0.55

$$\frac{4389\,d^{21/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{1/4}\,b^{23/4}}-\frac{\frac{3803\,d^{11}\,{\left(d\,x\right)}^{19/2}}{4096\,b}+\frac{5947\,a^2\,d^{15}\,{\left(d\,x\right)}^{11/2}}{2048\,b^3}+\frac{209\,a^3\,d^{17}\,{\left(d\,x\right)}^{7/2}}{128\,b^4}+\frac{1463\,a^4\,d^{19}\,{\left(d\,x\right)}^{3/2}}{4096\,b^5}+\frac{6289\,a\,d^{13}\,{\left(d\,x\right)}^{15/2}}{2560\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{4389\,d^{21/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{1/4}\,b^{23/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (4389*d^(21/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)
^(1/4)*b^(23/4)) - ((3803*d^11*(d*x)^(19/2))/(4096*b) + (5947*a^2*d^15*(d*x
)^(11/2))/(2048*b^3) + (209*a^3*d^17*(d*x)^(7/2))/(128*b^4) + (1463*a^4*d^1
9*(d*x)^(3/2))/(4096*b^5) + (6289*a*d^13*(d*x)^(15/2))/(2560*b^2))/(a^5*d^1
0 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x
^4 + 10*a^2*b^3*d^10*x^6) - (4389*d^(21/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a
)^(1/4)*d^(1/2))))/(8192*(-a)^(1/4)*b^(23/4))
```

3.714 $\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal. Leaf size=385

$$-\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5}-\frac{17d^3(dx)^{13/2}}{160b^2\,(a+bx^2)^4}-\frac{221d^5(dx)^{9/2}}{1920b^3\,(a+bx^2)^3}-\frac{663d^7(dx)^{5/2}}{5120b^4\,(a+bx^2)^2}-\frac{663d^9\sqrt{dx}}{4096b^5\,(a+bx^2)}-\frac{663d^{19/2}\tan}{819}$$

```
[Out] -1/10*d*(d*x)^(17/2)/b/(b*x^2+a)^5-17/160*d^3*(d*x)^(13/2)/b^2/(b*x^2+a)^4-
221/1920*d^5*(d*x)^(9/2)/b^3/(b*x^2+a)^3-663/5120*d^7*(d*x)^(5/2)/b^4/(b*x^
2+a)^2-663/16384*d^(19/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1
/2))/a^(3/4)/b^(21/4)*2^(1/2)+663/16384*d^(19/2)*arctan(1+b^(1/4)*2^(1/2)*(
d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^(21/4)*2^(1/2)-663/32768*d^(19/2)*ln(
a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(3
/4)/b^(21/4)*2^(1/2)+663/32768*d^(19/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(3/4)/b^(21/4)*2^(1/2)-663/4096*d^
9*(d*x)^(1/2)/b^5/(b*x^2+a)
```

Rubi [A]

time = 0.28, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{663d^{19/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{3/4}b^{21/4}}+\frac{663d^{19/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{3/4}b^{21/4}}-\frac{663d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{3/4}b^{21/4}}+\frac{663d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{3/4}b^{21/4}}-\frac{663d^9\sqrt{dx}}{4096b^5\,(a+bx^2)}-\frac{663d^7(dx)^{5/2}}{5120b^4\,(a+bx^2)^2}-\frac{221d^5(dx)^{9/2}}{1920b^3\,(a+bx^2)^3}-\frac{17d^3(dx)^{13/2}}{160b^2\,(a+bx^2)^4}-\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(d*(d*x)^(17/2))/(b*(a + b*x^2)^5) - (17*d^3*(d*x)^(13/2))/(160*b^2*(
a + b*x^2)^4) - (221*d^5*(d*x)^(9/2))/(1920*b^3*(a + b*x^2)^3) - (663*d^7*(
d*x)^(5/2))/(5120*b^4*(a + b*x^2)^2) - (663*d^9*Sqrt[d*x])/(4096*b^5*(a + b
*x^2)) - (663*d^(19/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) - (6
63*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2)*Log[Sqrt[a]
*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*S
qrt[2]*a^(3/4)*b^(21/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{(dx)^{19/2}}{(ab+b^2x^2)^6}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5} + \frac{1}{20}\left(17b^4d^2\right)\int \frac{(dx)^{15/2}}{(ab+b^2x^2)^5}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\,(a+bx^2)^4} + \frac{1}{320}\left(221b^2d^4\right)\int \frac{(dx)^{11/2}}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\,(a+bx^2)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\,(a+bx^2)^3} + \frac{(663d^6)\int \frac{(dx)^{7/2}}{(ab+b^2x^2)^3}}{1280}$$

$$= -\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\,(a+bx^2)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\,(a+bx^2)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\,(a+bx^2)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\,(a+bx^2)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\,(a+bx^2)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\,(a+bx^2)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\,(a+bx^2)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\,(a+bx^2)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\,(a+bx^2)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\,(a+bx^2)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\,(a+bx^2)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\,(a+bx^2)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\,(a+bx^2)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\,(a+bx^2)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\,(a+bx^2)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\,(a+bx^2)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\,(a+bx^2)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\,(a+bx^2)^2} -$$

$$= -\frac{d(dx)^{17/2}}{10b\,(a+bx^2)^5} - \frac{17d^3(dx)^{13/2}}{160b^2\,(a+bx^2)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3\,(a+bx^2)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4\,(a+bx^2)^2} -$$

Mathematica [A]
time = 0.66, size = 205, normalized size = 0.53

$$\frac{d^9\sqrt{dx}\left(-4a^{3/4}\sqrt[4]{b}\,\sqrt{x}\,(9945a^4+47736a^3bx^2+90610a^2b^2x^4+84320ab^3x^6+37645b^4x^8)+9945\sqrt{2}\,(a+bx^2)^5\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+9945\sqrt{2}\,(a+bx^2)^5\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{245760a^{3/4}b^{21/4}\sqrt{x}\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^9*Sqrt[d*x]*(-4*a^(3/4)*b^(1/4)*Sqrt[x]*(9945*a^4 + 47736*a^3*b*x^2 + 90
610*a^2*b^2*x^4 + 84320*a*b^3*x^6 + 37645*b^4*x^8) + 9945*Sqrt[2]*(a + b*x^
2)^5*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 994
5*Sqrt[2]*(a + b*x^2)^5*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a]
+ Sqrt[b]*x)]))/(245760*a^(3/4)*b^(21/4)*Sqrt[x]*(a + b*x^2)^5)
```

Maple [A]

time = 0.09, size = 241, normalized size = 0.63

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{663a^4d^8\sqrt{dx}}{8192b^5}-\frac{1989a^3d^6(dx)^{\frac{5}{2}}}{5120b^4}-\frac{9061a^2d^4(dx)^{\frac{9}{2}}}{12288b^3}-\frac{527a\,d^2(dx)^{\frac{13}{2}}}{768b^2}-\frac{7529(dx)^{\frac{17}{2}}}{24576b}}{(d^2x^2b+a\,d^2)^5}+\frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{663a^4d^8\sqrt{dx}}{8192b^5}-\frac{1989a^3d^6(dx)^{\frac{5}{2}}}{5120b^4}-\frac{9061a^2d^4(dx)^{\frac{9}{2}}}{12288b^3}-\frac{527a\,d^2(dx)^{\frac{13}{2}}}{768b^2}-\frac{7529(dx)^{\frac{17}{2}}}{24576b}}{(d^2x^2b+a\,d^2)^5}+\frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((-663/8192*a^4*d^8/b^5*(d*x)^(1/2)-1989/5120*a^3*d^6/b^4*(d*x)^(5/2
)-9061/12288*a^2*d^4/b^3*(d*x)^(9/2)-527/768*a*d^2/b^2*(d*x)^(13/2)-7529/24
576/b*(d*x)^(17/2))/(b*d^2*x^2+a*d^2)^5+663/65536/b^5*(a*d^2/b)^(1/4)/a/d^2
*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x
-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*
d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
))
```

Maxima [A]

time = 0.51, size = 386, normalized size = 1.00

$$-\frac{\frac{8\left(37645\,(dx)^{\frac{17}{2}}b^4d^{12}+84320\,(dx)^{\frac{13}{2}}ab^3d^{14}+90610\,(dx)^{\frac{9}{2}}a^2b^2d^{16}+47736\,(dx)^{\frac{5}{2}}a^3bd^{18}+9945\,\sqrt{dx}\,a^4d^{20}\right)}{b^{10}d^{10}x^{10}+5\,ab^9d^{10}x^8+10\,a^2b^8d^{10}x^6+10\,a^3b^7d^{10}x^4+5\,a^4b^6d^{10}x^2+a^5b^5d^{10}}-\frac{9945\left(\frac{\sqrt{2}\,d^{12}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{12}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^{11}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^{11}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}\right)}{b^5}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/491520*(8*(37645*(d*x)^(17/2)*b^4*d^12 + 84320*(d*x)^(13/2)*a*b^3*d^14 +
 90610*(d*x)^(9/2)*a^2*b^2*d^16 + 47736*(d*x)^(5/2)*a^3*b*d^18 + 9945*sqrt(
d*x)*a^4*d^20)/(b^10*d^10*x^10 + 5*a*b^9*d^10*x^8 + 10*a^2*b^8*d^10*x^6 + 1
0*a^3*b^7*d^10*x^4 + 5*a^4*b^6*d^10*x^2 + a^5*b^5*d^10) - 9945*(sqrt(2)*d^1
2*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((
a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^12*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4
)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^11*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^11*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/b^5)/d
```

Fricas [A]
time = 0.39, size = 489, normalized size = 1.27

$$\frac{39780\,(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5)\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac14}\arctan\left(-\frac{\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac34}\sqrt{dx}\,a^2b^{16}d^9-\sqrt{d^{19}x+\sqrt{-\frac{d^{38}}{a^3b^{21}}}\,a^2b^{10}}\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac34}a^2b^{16}}{d^{38}}\right)+9945\,(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5)\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac14}\log\left(663\,\sqrt{dx}\,d^9+663\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac14}ab^5\right)-9945\,(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5)\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac14}\log\left(663\,\sqrt{dx}\,d^9-663\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac14}ab^5\right)-4\,(37645\,b^4d^9x^8+84320\,ab^3d^9x^6+90610\,a^2b^2d^9x^4+47736\,a^3bd^9x^2+9945\,a^4d^9)\sqrt{dx}}{245760\,(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(39780*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4
+ 5*a^4*b^6*x^2 + a^5*b^5)*(-d^38/(a^3*b^21))^(1/4)*arctan(-((-d^38/(a^3*b^
21))^(3/4)*sqrt(d*x)*a^2*b^16*d^9 - sqrt(d^19*x + sqrt(-d^38/(a^3*b^21))*a^
2*b^10)*(-d^38/(a^3*b^21))^(3/4)*a^2*b^16)/d^38) + 9945*(b^10*x^10 + 5*a*b^
9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^38/(
a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9 + 663*(-d^38/(a^3*b^21))^(1/4)*a*b^5
) - 9945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4
*b^6*x^2 + a^5*b^5)*(-d^38/(a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9 - 663*(-
d^38/(a^3*b^21))^(1/4)*a*b^5) - 4*(37645*b^4*d^9*x^8 + 84320*a*b^3*d^9*x^6
+ 90610*a^2*b^2*d^9*x^4 + 47736*a^3*b*d^9*x^2 + 9945*a^4*d^9)*sqrt(d*x))/(b
^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 +
a^5*b^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.36, size = 339, normalized size = 0.88

$$\frac{1}{491520}\,d^9\left(\frac{19890\,\sqrt{2}\,(ab^3d^2)^{\frac14}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac14}}\right)}{ab^6}+\frac{19890\,\sqrt{2}\,(ab^3d^2)^{\frac14}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac14}}\right)}{ab^6}+\frac{9945\,\sqrt{2}\,(ab^3d^2)^{\frac14}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^6}-\frac{9945\,\sqrt{2}\,(ab^3d^2)^{\frac14}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^6}-\frac{8\left(37645\,\sqrt{dx}\,b^4d^{10}x^8+84320\,\sqrt{dx}\,ab^3d^{10}x^6+90610\,\sqrt{dx}\,a^2b^2d^{10}x^4+47736\,\sqrt{dx}\,a^3bd^{10}x^2+9945\,\sqrt{dx}\,a^4d^{10}\right)}{(bd^2x^2+ad^2)^5b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^9*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6) + 19890*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a*b^6) + 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6) - 9945*sqrt(2)*(a*b^3*d^
2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^
6) - 8*(37645*sqrt(d*x)*b^4*d^10*x^8 + 84320*sqrt(d*x)*a*b^3*d^10*x^6 + 906
10*sqrt(d*x)*a^2*b^2*d^10*x^4 + 47736*sqrt(d*x)*a^3*b*d^10*x^2 + 9945*sqrt(
d*x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*b^5))
```

Mupad [B]
time = 4.27, size = 213, normalized size = 0.55

$$-\frac{\frac{7529\,d^{11}\,{\left(d\,x\right)}^{17/2}}{12288\,b}+\frac{9061\,a^2\,d^{15}\,{\left(d\,x\right)}^{9/2}}{6144\,b^3}+\frac{1989\,a^3\,d^{17}\,{\left(d\,x\right)}^{5/2}}{2560\,b^4}+\frac{663\,a^4\,d^{19}\,\sqrt{d\,x}}{4096\,b^5}+\frac{527\,a\,d^{13}\,{\left(d\,x\right)}^{13/2}}{384\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{663\,d^{19/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{3/4}\,b^{21/4}}-\frac{663\,d^{19/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{3/4}\,b^{21/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] - ((7529*d^11*(d*x)^(17/2))/(12288*b) + (9061*a^2*d^15*(d*x)^(9/2))/(6144*b
^3) + (1989*a^3*d^17*(d*x)^(5/2))/(2560*b^4) + (663*a^4*d^19*(d*x)^(1/2))/(
4096*b^5) + (527*a*d^13*(d*x)^(13/2))/(384*b^2))/(a^5*d^10 + b^5*d^10*x^10
+ 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^
10*x^6) - (663*d^(19/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(
8192*(-a)^(3/4)*b^(21/4)) - (663*d^(19/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)
^(1/4)*d^(1/2))))/(8192*(-a)^(3/4)*b^(21/4))
```

3.715 $\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=388

$$-\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5}-\frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4}-\frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3}-\frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2}+\frac{231d^7(dx)^{3/2}}{4096ab^4\left(a+bx^2\right)}-\frac{231d^{17/2}\tan^{-1}}{8192}$$

```
[Out] -1/10*d*(d*x)^(15/2)/b/(b*x^2+a)^5-3/32*d^3*(d*x)^(11/2)/b^2/(b*x^2+a)^4-11
/128*d^5*(d*x)^(7/2)/b^3/(b*x^2+a)^3-77/1024*d^7*(d*x)^(3/2)/b^4/(b*x^2+a)^
2+231/4096*d^7*(d*x)^(3/2)/a/b^4/(b*x^2+a)-231/16384*d^(17/2)*arctan(1-b^(1
/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(19/4)*2^(1/2)+231/16384
*d^(17/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(
19/4)*2^(1/2)+231/32768*d^(17/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/
4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(19/4)*2^(1/2)-231/32768*d^(17/2)
*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/
a^(5/4)/b^(19/4)*2^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{231d^{17/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{5/4}b^{19/4}}+\frac{231d^{17/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{5/4}b^{19/4}}+\frac{231d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{5/4}b^{19/4}}-\frac{231d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{5/4}b^{19/4}}+\frac{231d^7(dx)^{3/2}}{4096ab^4\left(a+bx^2\right)}-\frac{77d^7(dx)^{3/2}}{1024b^4\left(a+bx^2\right)^2}-\frac{11d^5(dx)^{7/2}}{128b^3\left(a+bx^2\right)^3}-\frac{3d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(d*(d*x)^(15/2))/(b*(a + b*x^2)^5) - (3*d^3*(d*x)^(11/2))/(32*b^2*(a
+ b*x^2)^4) - (11*d^5*(d*x)^(7/2))/(128*b^3*(a + b*x^2)^3) - (77*d^7*(d*x)^
(3/2))/(1024*b^4*(a + b*x^2)^2) + (231*d^7*(d*x)^(3/2))/(4096*a*b^4*(a + b*
x^2)) - (231*d^(17/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (231*d^(17/2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (23
1*d^(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(5/4)*b^(19/4)) - (231*d^(17/2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sq
rt[2]*a^(5/4)*b^(19/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6 \int \frac{(dx)^{17/2}}{(ab+b^2x^2)^6}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\,(a+bx^2)^5} + \frac{1}{4}(3b^4d^2) \int \frac{(dx)^{13/2}}{(ab+b^2x^2)^5}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\,(a+bx^2)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^4} + \frac{1}{64}(33b^2d^4) \int \frac{(dx)^{9/2}}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{d(dx)^{15/2}}{10b\,(a+bx^2)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\,(a+bx^2)^3} + \frac{1}{256}(77d^6) \int \frac{(dx}{(ab+}$$

$$= -\frac{d(dx)^{15/2}}{10b\,(a+bx^2)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\,(a+bx^2)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{15/2}}{10b\,(a+bx^2)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\,(a+bx^2)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{15/2}}{10b\,(a+bx^2)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\,(a+bx^2)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{15/2}}{10b\,(a+bx^2)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\,(a+bx^2)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{15/2}}{10b\,(a+bx^2)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\,(a+bx^2)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{15/2}}{10b\,(a+bx^2)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\,(a+bx^2)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{15/2}}{10b\,(a+bx^2)^5} - \frac{3d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^4} - \frac{11d^5(dx)^{7/2}}{128b^3\,(a+bx^2)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4\,(a+bx^2)^2} +$$

Mathematica [A]

time = 0.74, size = 205, normalized size = 0.53

$$\frac{d^8\sqrt{dx}\left(4\sqrt[4]{a}\,b^{3/4}x^{3/2}(-385a^4-1760a^3bx^2-3130a^2b^2x^4-2648ab^3x^6+1155b^4x^8)+1155\sqrt{2}\,(a+bx^2)^5\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-1155\sqrt{2}\,(a+bx^2)^5\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{81920a^{5/4}b^{19/4}\sqrt{x}\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^8*Sqrt[d*x]*(4*a^(1/4)*b^(3/4)*x^(3/2)*(-385*a^4 - 1760*a^3*b*x^2 - 3130
*a^2*b^2*x^4 - 2648*a*b^3*x^6 + 1155*b^4*x^8) + 1155*Sqrt[2]*(a + b*x^2)^5*
ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 1155*Sqr
t[2]*(a + b*x^2)^5*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqr
t[b]*x)]))/(81920*a^(5/4)*b^(19/4)*Sqrt[x]*(a + b*x^2)^5)
```

Maple [A]

time = 0.10, size = 238, normalized size = 0.61

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{77a^3d^6(dx)^{\frac{3}{2}}}{8192b^4}-\frac{11a^2d^4(dx)^{\frac{7}{2}}}{256b^3}-\frac{313a\,d^2(dx)^{\frac{11}{2}}}{4096b^2}-\frac{331(dx)^{\frac{15}{2}}}{5120b}+\frac{231(dx)^{\frac{19}{2}}}{8192a\,d^2}}{(d^2x^2b+a\,d^2)^5}+\frac{231\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\cdots}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\cdots}\right.\right.}{\cdots}\right.$
default	$2d^{11}\left(\frac{-\frac{77a^3d^6(dx)^{\frac{3}{2}}}{8192b^4}-\frac{11a^2d^4(dx)^{\frac{7}{2}}}{256b^3}-\frac{313a\,d^2(dx)^{\frac{11}{2}}}{4096b^2}-\frac{331(dx)^{\frac{15}{2}}}{5120b}+\frac{231(dx)^{\frac{19}{2}}}{8192a\,d^2}}{(d^2x^2b+a\,d^2)^5}+\frac{231\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\cdots}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\cdots}\right.\right.}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((-77/8192*a^3*d^6/b^4*(d*x)^(3/2)-11/256/b^3*a^2*d^4*(d*x)^(7/2)-31
3/4096/b^2*a*d^2*(d*x)^(11/2)-331/5120/b*(d*x)^(15/2)+231/8192/a/d^2*(d*x)^
(19/2))/(b*d^2*x^2+a*d^2)^5+231/65536/a/d^2/b^5/(a*d^2/b)^(1/4)*2^(1/2)*(ln
((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)
*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.52, size = 383, normalized size = 0.99

$$\frac{\frac{1155\,d^{10}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{ab^4}+\frac{8\left(1155\,(dx)^{\frac{19}{2}}b^4d^{10}-2648\,(dx)^{\frac{15}{2}}ab^3d^{12}-3130\,(dx)^{\frac{11}{2}}a^2b^2d^{14}-1760\,(dx)^{\frac{7}{2}}a^3bd^{16}-385\,(dx)^{\frac{3}{2}}a^4d^{18}\right)}{ab^9d^{10}x^{10}+5\,a^2b^8d^{10}x^8+10\,a^3b^7d^{10}x^6+10\,a^4b^6d^{10}x^4+5\,a^5b^5d^{10}x^2+a^6b^4d^{10}}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

[Out] 1/163840*(1155*d^10*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a*b^4) + 8*(1155*(d*x)^(19/2)*b^4*d^10 - 2648*(d*x)^(15/2)*a*b^3*d^12 - 3130*(d*x)^(11/2)*a^2*b^2*d^14 - 1760*(d*x)^(7/2)*a^3*b*d^16 - 385*(d*x)^(3/2)*a^4*d^18)/(a*b^9*d^10*x^10 + 5*a^2*b^8*d^10*x^8 + 10*a^3*b^7*d^10*x^6 + 10*a^4*b^6*d^10*x^4 + 5*a^5*b^5*d^10*x^2 + a^6*b^4*d^10))/d

Fricas [**A**]
time = 0.35, size = 506, normalized size = 1.30

$$-\frac{4620\,{\left(ab^{9}x^{10}+5\,a^{2}b^{8}x^{8}+10\,a^{3}b^{7}x^{6}+10\,a^{4}b^{6}x^{4}+5\,a^{5}b^{5}x^{2}+a^{6}b^{4}\right)}\left(-\frac{d^{34}}{a^{5}b^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{34}}{a^{5}b^{19}}\right)^{\frac{1}{4}}\sqrt{dx}\,ab^{5}d^{25}-\sqrt{d^{51}x-\sqrt{-\frac{d^{34}}{a^{5}b^{19}}}\,a^{3}b^{9}d^{34}}\left(-\frac{d^{34}}{a^{5}b^{19}}\right)^{\frac{1}{4}}ab^{5}}{d^{34}}\right)-1155\,{\left(ab^{9}x^{10}+5\,a^{2}b^{8}x^{8}+10\,a^{3}b^{7}x^{6}+10\,a^{4}b^{6}x^{4}+5\,a^{5}b^{5}x^{2}+a^{6}b^{4}\right)}\left(-\frac{d^{34}}{a^{5}b^{19}}\right)^{\frac{1}{4}}\log\left(12326391\,\sqrt{dx}\,d^{25}+12326391\,\left(-\frac{d^{34}}{a^{5}b^{19}}\right)^{\frac{3}{4}}a^{4}b^{14}\right)+1155\,{\left(ab^{9}x^{10}+5\,a^{2}b^{8}x^{8}+10\,a^{3}b^{7}x^{6}+10\,a^{4}b^{6}x^{4}+5\,a^{5}b^{5}x^{2}+a^{6}b^{4}\right)}\left(-\frac{d^{34}}{a^{5}b^{19}}\right)^{\frac{1}{4}}\log\left(12326391\,\sqrt{dx}\,d^{25}-12326391\,\left(-\frac{d^{34}}{a^{5}b^{19}}\right)^{\frac{3}{4}}a^{4}b^{14}\right)-4\,{\left(1155\,b^{4}d^{8}x^{9}-2648\,ab^{3}d^{8}x^{7}-3130\,a^{2}b^{2}d^{8}x^{5}-1760\,a^{3}bd^{8}x^{3}-385\,a^{4}d^{8}x\right)}\sqrt{dx}}{81920\,{\left(ab^{9}x^{10}+5\,a^{2}b^{8}x^{8}+10\,a^{3}b^{7}x^{6}+10\,a^{4}b^{6}x^{4}+5\,a^{5}b^{5}x^{2}+a^{6}b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")

[Out] -1/81920*(4620*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*arctan(-((-d^34/(a^5*b^19))^(1/4)*sqrt(d*x)*a*b^5*d^25 - sqrt(d^51*x - sqrt(-d^34/(a^5*b^19))*a^3*b^9*d^34)*(-d^34/(a^5*b^19))^(1/4)*a*b^5)/d^34) - 1155*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*log(12326391*sqrt(d*x)*d^25 + 12326391*(-d^34/(a^5*b^19))^(3/4)*a^4*b^14) + 1155*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*log(12326391*sqrt(d*x)*d^25 - 12326391*(-d^34/(a^5*b^19))^(3/4)*a^4*b^14) - 4*(1155*b^4*d^8*x^9 - 2648*a*b^3*d^8*x^7 - 3130*a^2*b^2*d^8*x^5 - 1760*a^3*b*d^8*x^3 - 385*a^4*d^8*x)*sqrt(d*x))/(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)

[Out] Timed out

Giac [**A**]
time = 11.85, size = 355, normalized size = 0.91

$$\frac{1}{163840}\,d^{8}\left(\frac{2310\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{7}d}+\frac{2310\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}b^{7}d}-\frac{1155\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{7}d}+\frac{1155\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{2}b^{7}d}+\frac{8\left(1155\,\sqrt{dx}\,b^{4}d^{10}x^{9}-2648\,\sqrt{dx}\,ab^{3}d^{10}x^{7}-3130\,\sqrt{dx}\,a^{2}b^{2}d^{10}x^{5}-1760\,\sqrt{dx}\,a^{3}bd^{10}x^{3}-385\,\sqrt{dx}\,a^{4}d^{10}x\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{5}ab^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^8*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^7*d) + 2310*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^2*b^7*d) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^7*d) + 1155*sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^2*b^7*d) + 8*(1155*sqrt(d*x)*b^4*d^10*x^9 - 2648*sqrt(d*x)*a*b^3*d^
10*x^7 - 3130*sqrt(d*x)*a^2*b^2*d^10*x^5 - 1760*sqrt(d*x)*a^3*b*d^10*x^3 -
385*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a*b^4))
```

Mupad [B]
time = 4.29, size = 210, normalized size = 0.54

$$\frac{231\,d^{17/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{5/4}\,b^{19/4}}-\frac{231\,d^{17/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{5/4}\,b^{19/4}}-\frac{\frac{331\,d^{11}\,{(d\,x)}^{15/2}}{2560\,b}-\frac{231\,d^9\,{(d\,x)}^{19/2}}{4096\,a}+\frac{11\,a^2\,d^{15}\,{(d\,x)}^{7/2}}{128\,b^3}+\frac{77\,a^3\,d^{17}\,{(d\,x)}^{3/2}}{4096\,b^4}+\frac{313\,a\,d^{13}\,{(d\,x)}^{11/2}}{2048\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (231*d^(17/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)
^(5/4)*b^(19/4)) - (231*d^(17/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(
1/2))))/(8192*(-a)^(5/4)*b^(19/4)) - ((331*d^11*(d*x)^(15/2))/(2560*b) - (2
31*d^9*(d*x)^(19/2))/(4096*a) + (11*a^2*d^15*(d*x)^(7/2))/(128*b^3) + (77*a
^3*d^17*(d*x)^(3/2))/(4096*b^4) + (313*a*d^13*(d*x)^(11/2))/(2048*b^2))/(a^
5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d
^10*x^4 + 10*a^2*b^3*d^10*x^6)
```

3.716 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=388

$$-\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5}-\frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4}-\frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3}-\frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2}+\frac{39d^7\sqrt{dx}}{4096ab^4\left(a+bx^2\right)}-\frac{117d^{15/2}\tan}{819}$$

```
[Out] -1/10*d*(d*x)^(13/2)/b/(b*x^2+a)^5-13/160*d^3*(d*x)^(9/2)/b^2/(b*x^2+a)^4-3
9/640*d^5*(d*x)^(5/2)/b^3/(b*x^2+a)^3-117/16384*d^(15/2)*arctan(1-b^(1/4)*2
^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(17/4)*2^(1/2)+117/16384*d^(1
5/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(17/4)
*2^(1/2)-117/32768*d^(15/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^
(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/4)/b^(17/4)*2^(1/2)+117/32768*d^(15/2)*ln(a
^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/
4)/b^(17/4)*2^(1/2)-39/1024*d^7*(d*x)^(1/2)/b^4/(b*x^2+a)^2+39/4096*d^7*(d*
x)^(1/2)/a/b^4/(b*x^2+a)
```

Rubi [A]
time = 0.29, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{117d^{15/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{7/4}b^{17/4}}+\frac{117d^{15/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{7/4}b^{17/4}}-\frac{117d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{7/4}b^{17/4}}+\frac{117d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{7/4}b^{17/4}}+\frac{39d^7\sqrt{dx}}{4096ab^4\left(a+bx^2\right)}-\frac{39d^7\sqrt{dx}}{1024b^4\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{640b^3\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{160b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(d*(d*x)^(13/2))/(b*(a + b*x^2)^5) - (13*d^3*(d*x)^(9/2))/(160*b^2*(a
 + b*x^2)^4) - (39*d^5*(d*x)^(5/2))/(640*b^3*(a + b*x^2)^3) - (39*d^7*Sqrt[
d*x])/(1024*b^4*(a + b*x^2)^2) + (39*d^7*Sqrt[d*x])/(4096*a*b^4*(a + b*x^2)
) - (117*d^(15/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(8192*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(7/4)*b^(17/4)) - (117*d^
(15/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(16384*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*Log[Sqrt[a]*Sqrt
[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2
]*a^(7/4)*b^(17/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{(dx)^{15/2}}{(ab+b^2x^2)^6}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\,(a+bx^2)^5} + \frac{1}{20}\left(13b^4d^2\right)\int \frac{(dx)^{11/2}}{(ab+b^2x^2)^5}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\,(a+bx^2)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\,(a+bx^2)^4} + \frac{1}{320}\left(117b^2d^4\right)\int \frac{(dx)^{7/2}}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{d(dx)^{13/2}}{10b\,(a+bx^2)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\,(a+bx^2)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\,(a+bx^2)^3} + \frac{1}{256}\left(39d^6\right)\int \frac{(dx}{(ab+}$$

$$= -\frac{d(dx)^{13/2}}{10b\,(a+bx^2)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\,(a+bx^2)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\,(a+bx^2)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{13/2}}{10b\,(a+bx^2)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\,(a+bx^2)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\,(a+bx^2)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{13/2}}{10b\,(a+bx^2)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\,(a+bx^2)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\,(a+bx^2)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{13/2}}{10b\,(a+bx^2)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\,(a+bx^2)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\,(a+bx^2)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{13/2}}{10b\,(a+bx^2)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\,(a+bx^2)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\,(a+bx^2)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{13/2}}{10b\,(a+bx^2)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\,(a+bx^2)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\,(a+bx^2)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\,(a+bx^2)^2} +$$

$$= -\frac{d(dx)^{13/2}}{10b\,(a+bx^2)^5} - \frac{13d^3(dx)^{9/2}}{160b^2\,(a+bx^2)^4} - \frac{39d^5(dx)^{5/2}}{640b^3\,(a+bx^2)^3} - \frac{39d^7\sqrt{dx}}{1024b^4\,(a+bx^2)^2} +$$

Mathematica [A]

time = 0.70, size = 186, normalized size = 0.48

$$\frac{d^7\sqrt{dx}\left(-\frac{4a^{3/4}\sqrt[4]{b}\,(585a^4+2808a^3bx^2+5330a^2b^2x^4+4960ab^3x^6-195b^4x^8)}{(a+bx^2)^5} - \frac{585\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt{x}} + \frac{585\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{x}}\right)}{81920a^{7/4}b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^7*Sqrt[d*x]*((-4*a^(3/4)*b^(1/4)*(585*a^4 + 2808*a^3*b*x^2 + 5330*a^2*b^
2*x^4 + 4960*a*b^3*x^6 - 195*b^4*x^8))/(a + b*x^2)^5 - (585*Sqrt[2]*ArcTan[
(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/Sqrt[x] + (585*Sq
rt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/Sqr
t[x]))/(81920*a^(7/4)*b^(17/4))
```

Maple [A]
time = 0.08, size = 238, normalized size = 0.61

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{117a^3d^6\sqrt{dx}}{8192b^4}-\frac{351a^2d^4(dx)^{\frac{5}{2}}}{5120b^3}-\frac{533a\,d^2(dx)^{\frac{9}{2}}}{4096b^2}-\frac{31(dx)^{\frac{13}{2}}}{256b}+\frac{39(dx)^{\frac{17}{2}}}{8192a\,d^2}}{(d^2x^2b+a\,d^2)^5}+\frac{117\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right.}{dx-\left(\frac{a\,d^2}{b}\right.}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{117a^3d^6\sqrt{dx}}{8192b^4}-\frac{351a^2d^4(dx)^{\frac{5}{2}}}{5120b^3}-\frac{533a\,d^2(dx)^{\frac{9}{2}}}{4096b^2}-\frac{31(dx)^{\frac{13}{2}}}{256b}+\frac{39(dx)^{\frac{17}{2}}}{8192a\,d^2}}{(d^2x^2b+a\,d^2)^5}+\frac{117\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right.}{dx-\left(\frac{a\,d^2}{b}\right.}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((-117/8192*a^3*d^6/b^4*(d*x)^(1/2)-351/5120*a^2*d^4/b^3*(d*x)^(5/2)
-533/4096/b^2*a*d^2*(d*x)^(9/2)-31/256/b*(d*x)^(13/2)+39/8192/a/d^2*(d*x)^(
17/2))/(b*d^2*x^2+a*d^2)^5+117/65536/a^2/d^4/b^4*(a*d^2/b)^(1/4)*2^(1/2)*(l
n((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4
)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]
time = 0.51, size = 392, normalized size = 1.01

$$\frac{\frac{8\left(195\,(dx)^{\frac{17}{2}}b^4d^{10}-4960\,(dx)^{\frac{13}{2}}ab^3d^{12}-5330\,(dx)^{\frac{9}{2}}a^2b^2d^{14}-2808\,(dx)^{\frac{5}{2}}a^3bd^{16}-585\,\sqrt{dx}\,a^4d^{18}\right)}{ab^9d^{10}x^{10}+5\,a^2b^8d^{10}x^8+10\,a^3b^7d^{10}x^6+10\,a^4b^6d^{10}x^4+5\,a^5b^5d^{10}x^2+a^6b^4d^{10}}+\frac{585\left(\frac{\sqrt{2}\,d^{10}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{10}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^9\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^9\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)}{ab^4}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(195*(d*x)^(17/2)*b^4*d^10 - 4960*(d*x)^(13/2)*a*b^3*d^12 - 533
0*(d*x)^(9/2)*a^2*b^2*d^14 - 2808*(d*x)^(5/2)*a^3*b*d^16 - 585*sqrt(d*x)*a^
4*d^18)/(a*b^9*d^10*x^10 + 5*a^2*b^8*d^10*x^8 + 10*a^3*b^7*d^10*x^6 + 10*a^
4*b^6*d^10*x^4 + 5*a^5*b^5*d^10*x^2 + a^6*b^4*d^10) + 585*(sqrt(2)*d^10*log
(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(3/4)*b^(1/4)) - sqrt(2)*d^10*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqr
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^9*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^9*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(
a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a*b^4))/d
```

Fricas [A]
time = 0.35, size = 505, normalized size = 1.30

$$\frac{2340\,(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4)\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac14}\arctan\left(-\frac{\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac34}\sqrt{dx}\,a^5b^{13}d^7-\sqrt{d^{15}x+\sqrt{-\frac{d^{30}}{a^7b^{17}}}\,a^4b^8}\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac34}a^5b^{13}}{d^{30}}\right)+585\,(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4)\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac14}\log\left(117\sqrt{dx}\,d^7+117\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac14}a^2b^4\right)-585\,(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4)\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac14}\log\left(117\sqrt{dx}\,d^7-117\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac14}a^2b^4\right)+4\,(195\,b^4d^7x^8-4960\,ab^3d^7x^6-5330\,a^2b^2d^7x^4-2808\,a^3bd^7x^2-585\,a^4d^7)\sqrt{dx}}{81920\,(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(2340*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4
 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*arctan(-((-d^30/(a^7*b
^17))^(3/4)*sqrt(d*x)*a^5*b^13*d^7 - sqrt(d^15*x + sqrt(-d^30/(a^7*b^17))*a
^4*b^8)*(-d^30/(a^7*b^17))^(3/4)*a^5*b^13)/d^30) + 585*(a*b^9*x^10 + 5*a^2*
b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30
/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 + 117*(-d^30/(a^7*b^17))^(1/4)*a^2
*b^4) - 585*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 +
 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 -
117*(-d^30/(a^7*b^17))^(1/4)*a^2*b^4) + 4*(195*b^4*d^7*x^8 - 4960*a*b^3*d^7
*x^6 - 5330*a^2*b^2*d^7*x^4 - 2808*a^3*b*d^7*x^2 - 585*a^4*d^7)*sqrt(d*x))/
(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x
^2 + a^6*b^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 14.30, size = 342, normalized size = 0.88

$$\frac{1}{163840}d^7\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}+\frac{585\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^5}-\frac{585\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^5}+\frac{8\left(195\sqrt{dx}\,b^4d^{10}x^8-4960\sqrt{dx}\,ab^3d^{10}x^6-5330\sqrt{dx}\,a^2b^2d^{10}x^4-2808\sqrt{dx}\,a^3bd^{10}x^2-585\sqrt{dx}\,a^4d^{10}\right)}{\left(bd^2x^2+ad^2\right)^5ab^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5) + 1170*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^2*b^5) + 585*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5) - 585*sqrt(2)*(a*b^3*
d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
2*b^5) + 8*(195*sqrt(d*x)*b^4*d^10*x^8 - 4960*sqrt(d*x)*a*b^3*d^10*x^6 - 53
30*sqrt(d*x)*a^2*b^2*d^10*x^4 - 2808*sqrt(d*x)*a^3*b*d^10*x^2 - 585*sqrt(d*
x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a*b^4))
```

Mupad [B]
time = 0.13, size = 210, normalized size = 0.54

$$\frac{117\,d^{15/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{7/4}\,b^{17/4}}-\frac{\frac{31\,d^{11}\,{\left(d\,x\right)}^{13/2}}{128\,b}-\frac{39\,d^9\,{\left(d\,x\right)}^{17/2}}{4096\,a}+\frac{351\,a^2\,d^{15}\,{\left(d\,x\right)}^{5/2}}{2560\,b^3}+\frac{117\,a^3\,d^{17}\,\sqrt{d\,x}}{4096\,b^4}+\frac{533\,a\,d^{13}\,{\left(d\,x\right)}^{9/2}}{2048\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\frac{117\,d^{15/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{7/4}\,b^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (117*d^(15/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^
(7/4)*b^(17/4)) - ((31*d^11*(d*x)^(13/2))/(128*b) - (39*d^9*(d*x)^(17/2))/(
4096*a) + (351*a^2*d^15*(d*x)^(5/2))/(2560*b^3) + (117*a^3*d^17*(d*x)^(1/2)
)/(4096*b^4) + (533*a*d^13*(d*x)^(9/2))/(2048*b^2))/(a^5*d^10 + b^5*d^10*x^
10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3
*d^10*x^6) + (117*d^(15/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))
))/(8192*(-a)^(7/4)*b^(17/4))
```

3.717 $\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=391

$$-\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5}-\frac{11d^3(dx)^{7/2}}{160b^2\,(a+bx^2)^4}-\frac{77d^5(dx)^{3/2}}{1920b^3\,(a+bx^2)^3}+\frac{77d^5(dx)^{3/2}}{5120ab^3\,(a+bx^2)^2}+\frac{77d^5(dx)^{3/2}}{4096a^2b^3\,(a+bx^2)}-\frac{77d^{13/2}\tan}{8}$$

```
[Out] -1/10*d*(d*x)^(11/2)/b/(b*x^2+a)^5-11/160*d^3*(d*x)^(7/2)/b^2/(b*x^2+a)^4-7
7/1920*d^5*(d*x)^(3/2)/b^3/(b*x^2+a)^3+77/5120*d^5*(d*x)^(3/2)/a/b^3/(b*x^2
+a)^2+77/4096*d^5*(d*x)^(3/2)/a^2/b^3/(b*x^2+a)-77/16384*d^(13/2)*arctan(1-
b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/b^(15/4)*2^(1/2)+77/16
384*d^(13/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/
b^(15/4)*2^(1/2)+77/32768*d^(13/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(
1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(9/4)/b^(15/4)*2^(1/2)-77/32768*d^(13/2
)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))
/a^(9/4)/b^(15/4)*2^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{77d^{13/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{9/4}b^{15/4}}+\frac{77d^{13/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{9/4}b^{15/4}}+\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{9/4}b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{9/4}b^{15/4}}+\frac{77d^5(dx)^{3/2}}{4096a^2b^3\,(a+bx^2)}+\frac{77d^5(dx)^{3/2}}{5120ab^3\,(a+bx^2)^2}-\frac{77d^5(dx)^{3/2}}{1920b^3\,(a+bx^2)^3}-\frac{11d^3(dx)^{7/2}}{160b^2\,(a+bx^2)^4}-\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(d*(d*x)^(11/2))/(b*(a + b*x^2)^5) - (11*d^3*(d*x)^(7/2))/(160*b^2*(a
 + b*x^2)^4) - (77*d^5*(d*x)^(3/2))/(1920*b^3*(a + b*x^2)^3) + (77*d^5*(d*x
)^(3/2))/(5120*a*b^3*(a + b*x^2)^2) + (77*d^5*(d*x)^(3/2))/(4096*a^2*b^3*(a
 + b*x^2)) - (77*d^(13/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) +
(77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(9/4)*b^(15/4)) - (77*d^(13/2)*Log[Sqrt[a]
*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*S
qrt[2]*a^(9/4)*b^(15/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{(dx)^{13/2}}{(ab+b^2x^2)^6}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5} + \frac{1}{20}\left(11b^4d^2\right)\int \frac{(dx)^{9/2}}{(ab+b^2x^2)^5}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\,(a+bx^2)^4} + \frac{1}{320}\left(77b^2d^4\right)\int \frac{(dx)^{5/2}}{(ab+b^2x^2)^4}\,dx$$

$$= -\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\,(a+bx^2)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\,(a+bx^2)^3} + \frac{(77d^6)\int \frac{\sqrt{dx}}{(ab+b^2x^2)^3}}{1280}$$

$$= -\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\,(a+bx^2)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\,(a+bx^2)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\,(a+bx^2)}$$

$$= -\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\,(a+bx^2)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\,(a+bx^2)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\,(a+bx^2)}$$

$$= -\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\,(a+bx^2)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\,(a+bx^2)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\,(a+bx^2)}$$

$$= -\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\,(a+bx^2)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\,(a+bx^2)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\,(a+bx^2)}$$

$$= -\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\,(a+bx^2)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\,(a+bx^2)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\,(a+bx^2)}$$

$$= -\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\,(a+bx^2)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\,(a+bx^2)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\,(a+bx^2)}$$

$$= -\frac{d(dx)^{11/2}}{10b\,(a+bx^2)^5} - \frac{11d^3(dx)^{7/2}}{160b^2\,(a+bx^2)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3\,(a+bx^2)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3\,(a+bx^2)}$$

Mathematica [A]
time = 0.63, size = 186, normalized size = 0.48

$$\frac{d^6\sqrt{dx}\left(\frac{4\sqrt[4]{a}\,b^{3/4}x^{3/2}\left(-385a^4-1760a^3bx^2-3130a^2b^2x^4+5544ab^3x^6+1155b^4x^8\right)}{(a+bx^2)^5} - 1155\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right) - 1155\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{245760a^{9/4}b^{15/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^6*Sqrt[d*x]*((4*a^(1/4)*b^(3/4)*x^(3/2)*(-385*a^4 - 1760*a^3*b*x^2 - 313
0*a^2*b^2*x^4 + 5544*a*b^3*x^6 + 1155*b^4*x^8))/(a + b*x^2)^5 - 1155*Sqrt[2
]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 1155*Sq
rt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(2
45760*a^(9/4)*b^(15/4)*Sqrt[x])
```

Maple [A]

time = 0.09, size = 236, normalized size = 0.60

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{77d^4a^2(dx)^{\frac{3}{2}}}{24576b^3}-\frac{11d^2a(dx)^{\frac{7}{2}}}{768b^2}-\frac{313(dx)^{\frac{11}{2}}}{12288b}+\frac{231(dx)^{\frac{15}{2}}}{5120a\,d^2}+\frac{77b(dx)^{\frac{19}{2}}}{8192a^2d^4}}{(d^2x^2b+a\,d^2)^5}+\frac{77\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{77d^4a^2(dx)^{\frac{3}{2}}}{24576b^3}-\frac{11d^2a(dx)^{\frac{7}{2}}}{768b^2}-\frac{313(dx)^{\frac{11}{2}}}{12288b}+\frac{231(dx)^{\frac{15}{2}}}{5120a\,d^2}+\frac{77b(dx)^{\frac{19}{2}}}{8192a^2d^4}}{(d^2x^2b+a\,d^2)^5}+\frac{77\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((-77/24576/b^3*d^4*a^2*(d*x)^(3/2)-11/768/b^2*d^2*a*(d*x)^(7/2)-313
/12288/b*(d*x)^(11/2)+231/5120/a/d^2*(d*x)^(15/2)+77/8192/a^2/d^4*b*(d*x)^(
19/2))/(b*d^2*x^2+a*d^2)^5+77/65536/a^2/d^4/b^4/(a*d^2/b)^(1/4)*2^(1/2)*(ln
((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)
*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.50, size = 385, normalized size = 0.98

$$\frac{\frac{1155\,d^8\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^2b^3}+\frac{8\left(1155\,(dx)^{\frac{19}{2}}b^4d^8+5544\,(dx)^{\frac{15}{2}}ab^3d^{10}-3130\,(dx)^{\frac{11}{2}}a^2b^2d^{12}-1760\,(dx)^{\frac{7}{2}}a^3bd^{14}-385\,(dx)^{\frac{3}{2}}a^4d^{16}\right)}{a^2b^8d^{10}x^{10}+5\,a^3b^7d^{10}x^8+10\,a^4b^6d^{10}x^6+10\,a^5b^5d^{10}x^4+5\,a^6b^4d^{10}x^2+a^7b^3d^{10}}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/491520*(1155*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(
1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*
d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
- 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sq
rt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a
*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^2*b^
3) + 8*(1155*(d*x)^(19/2)*b^4*d^8 + 5544*(d*x)^(15/2)*a*b^3*d^10 - 3130*(d*
x)^(11/2)*a^2*b^2*d^12 - 1760*(d*x)^(7/2)*a^3*b*d^14 - 385*(d*x)^(3/2)*a^4*
d^16)/(a^2*b^8*d^10*x^10 + 5*a^3*b^7*d^10*x^8 + 10*a^4*b^6*d^10*x^6 + 10*a^
5*b^5*d^10*x^4 + 5*a^6*b^4*d^10*x^2 + a^7*b^3*d^10))/d
```

Fricas [A]
time = 0.37, size = 518, normalized size = 1.32

$$-\frac{4620\,(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3)\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac14}\arctan\left(-\frac{\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac14}\sqrt{dx}\,a^2b^4d^{19}-\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{a^9b^{15}}}\,a^5b^7d^{26}}\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac14}a^2b^4}{d^{26}}\right)-1155\,(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3)\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac14}\log\left(456533\sqrt{dx}\,d^{19}+456533\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac34}a^7b^{11}\right)+1155\,(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3)\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac14}\log\left(456533\sqrt{dx}\,d^{19}-456533\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac34}a^7b^{11}\right)-4\,(1155\,b^4d^6x^9+5544\,ab^3d^6x^7-3130\,a^2b^2d^6x^5-1760\,a^3bd^6x^3-385\,a^4d^6x)\sqrt{dx}}{245760\,(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(4620*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5
*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*arctan(-((-d^26/(a
^9*b^15))^(1/4)*sqrt(d*x)*a^2*b^4*d^19 - sqrt(d^39*x - sqrt(-d^26/(a^9*b^15
))*a^5*b^7*d^26)*(-d^26/(a^9*b^15))^(1/4)*a^2*b^4)/d^26) - 1155*(a^2*b^8*x^
10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*
b^3)*(-d^26/(a^9*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 + 456533*(-d^26/(a^
9*b^15))^(3/4)*a^7*b^11) + 1155*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*
x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*lo
g(456533*sqrt(d*x)*d^19 - 456533*(-d^26/(a^9*b^15))^(3/4)*a^7*b^11) - 4*(11
55*b^4*d^6*x^9 + 5544*a*b^3*d^6*x^7 - 3130*a^2*b^2*d^6*x^5 - 1760*a^3*b*d^6
*x^3 - 385*a^4*d^6*x)*sqrt(d*x))/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6
*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{13}{2}}}{(a+bx^2)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(13/2)/(a + b*x**2)**6, x)
```

Giac [A]
time = 10.90, size = 355, normalized size = 0.91

$$\frac{1}{491520}\,d^6\left(\frac{2310\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^6d}+\frac{2310\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^6d}-\frac{1155\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^6d}+\frac{1155\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^6d}+\frac{8\left(1155\,\sqrt{dx}\,b^4d^{10}x^9+5544\,\sqrt{dx}\,ab^3d^{10}x^7-3130\,\sqrt{dx}\,a^2b^2d^{10}x^5-1760\,\sqrt{dx}\,a^3bd^{10}x^3-385\,\sqrt{dx}\,a^4d^{10}x\right)}{(bd^2x^2+ad^2)^5a^2b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^6*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^6*d) + 2310*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^3*b^6*d) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^6*d) + 1155*sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^3*b^6*d) + 8*(1155*sqrt(d*x)*b^4*d^10*x^9 + 5544*sqrt(d*x)*a*b^3*d^
10*x^7 - 3130*sqrt(d*x)*a^2*b^2*d^10*x^5 - 1760*sqrt(d*x)*a^3*b*d^10*x^3 -
385*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))
```

Mupad [B]
time = 4.32, size = 208, normalized size = 0.53

$$\frac{77\,d^{13/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{9/4}\,b^{15/4}}-\frac{\frac{313\,d^{11}\,{(d\,x)}^{11/2}}{6144\,b}-\frac{231\,d^9\,{(d\,x)}^{15/2}}{2560\,a}+\frac{77\,a^2\,d^{15}\,{(d\,x)}^{3/2}}{12288\,b^3}+\frac{11\,a\,d^{13}\,{(d\,x)}^{7/2}}{384\,b^2}-\frac{77\,b\,d^7\,{(d\,x)}^{19/2}}{4096\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{77\,d^{13/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{9/4}\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (77*d^(13/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(
9/4)*b^(15/4)) - ((313*d^11*(d*x)^(11/2))/(6144*b) - (231*d^9*(d*x)^(15/2))
/(2560*a) + (77*a^2*d^15*(d*x)^(3/2))/(12288*b^3) + (11*a*d^13*(d*x)^(7/2))
/(384*b^2) - (77*b*d^7*(d*x)^(19/2))/(4096*a^2))/(a^5*d^10 + b^5*d^10*x^10
+ 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^
10*x^6) - (77*d^(13/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(
8192*(-a)^(9/4)*b^(15/4))
```

3.718 $\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=391

$$-\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5}-\frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4}-\frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3}+\frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2}+\frac{21d^5\sqrt{dx}}{4096a^2b^3\left(a+bx^2\right)}-\frac{63d^{11/2}\tan}{81}$$

```
[Out] -1/10*d*(d*x)^(9/2)/b/(b*x^2+a)^5-9/160*d^3*(d*x)^(5/2)/b^2/(b*x^2+a)^4-63/
16384*d^(11/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/
4)/b^(13/4)*2^(1/2)+63/16384*d^(11/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/
a^(1/4)/d^(1/2))/a^(11/4)/b^(13/4)*2^(1/2)-63/32768*d^(11/2)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(13/
4)*2^(1/2)+63/32768*d^(11/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b
^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(13/4)*2^(1/2)-3/128*d^5*(d*x)^(1/2)
/b^3/(b*x^2+a)^3+3/1024*d^5*(d*x)^(1/2)/a/b^3/(b*x^2+a)^2+21/4096*d^5*(d*x)
^(1/2)/a^2/b^3/(b*x^2+a)
```

Rubi [A]
time = 0.28, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{63d^{11/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{11/4}b^{13/4}}+\frac{63d^{11/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{11/4}b^{13/4}}-\frac{63d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{11/4}b^{13/4}}+\frac{63d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{11/4}b^{13/4}}+\frac{21d^5\sqrt{dx}}{4096a^2b^3\left(a+bx^2\right)}+\frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2}-\frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3}-\frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(d*(d*x)^(9/2))/(b*(a + b*x^2)^5) - (9*d^3*(d*x)^(5/2))/(160*b^2*(a +
 b*x^2)^4) - (3*d^5*Sqrt[d*x])/(128*b^3*(a + b*x^2)^3) + (3*d^5*Sqrt[d*x])/
(1024*a*b^3*(a + b*x^2)^2) + (21*d^5*Sqrt[d*x])/(4096*a^2*b^3*(a + b*x^2))
- (63*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
8192*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(11/4)*b^(13/4)) - (63*d^(11
/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(16384*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*Log[Sqrt[a]*Sqrt[d]
 + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a
^(11/4)*b^(13/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx &= b^6\int \frac{(dx)^{11/2}}{(ab+b^2x^2)^6}\,dx \\
&= -\frac{d(dx)^{9/2}}{10b\,(a+bx^2)^5}+\frac{1}{20}\left(9b^4d^2\right)\int \frac{(dx)^{7/2}}{(ab+b^2x^2)^5}\,dx \\
&= -\frac{d(dx)^{9/2}}{10b\,(a+bx^2)^5}-\frac{9d^3(dx)^{5/2}}{160b^2\,(a+bx^2)^4}+\frac{1}{64}\left(9b^2d^4\right)\int \frac{(dx)^{3/2}}{(ab+b^2x^2)^4}\,dx \\
&= -\frac{d(dx)^{9/2}}{10b\,(a+bx^2)^5}-\frac{9d^3(dx)^{5/2}}{160b^2\,(a+bx^2)^4}-\frac{3d^5\sqrt{dx}}{128b^3\,(a+bx^2)^3}+\frac{1}{256}\left(3d^6\right)\int \frac{}{\sqrt{dx}\,(}
\\
&= -\frac{d(dx)^{9/2}}{10b\,(a+bx^2)^5}-\frac{9d^3(dx)^{5/2}}{160b^2\,(a+bx^2)^4}-\frac{3d^5\sqrt{dx}}{128b^3\,(a+bx^2)^3}+\frac{3d^5\sqrt{dx}}{1024ab^3\,(a+bx^2)^2}
\\
&= -\frac{d(dx)^{9/2}}{10b\,(a+bx^2)^5}-\frac{9d^3(dx)^{5/2}}{160b^2\,(a+bx^2)^4}-\frac{3d^5\sqrt{dx}}{128b^3\,(a+bx^2)^3}+\frac{3d^5\sqrt{dx}}{1024ab^3\,(a+bx^2)^2}
\\
&= -\frac{d(dx)^{9/2}}{10b\,(a+bx^2)^5}-\frac{9d^3(dx)^{5/2}}{160b^2\,(a+bx^2)^4}-\frac{3d^5\sqrt{dx}}{128b^3\,(a+bx^2)^3}+\frac{3d^5\sqrt{dx}}{1024ab^3\,(a+bx^2)^2}
\\
&= -\frac{d(dx)^{9/2}}{10b\,(a+bx^2)^5}-\frac{9d^3(dx)^{5/2}}{160b^2\,(a+bx^2)^4}-\frac{3d^5\sqrt{dx}}{128b^3\,(a+bx^2)^3}+\frac{3d^5\sqrt{dx}}{1024ab^3\,(a+bx^2)^2}
\\
&= -\frac{d(dx)^{9/2}}{10b\,(a+bx^2)^5}-\frac{9d^3(dx)^{5/2}}{160b^2\,(a+bx^2)^4}-\frac{3d^5\sqrt{dx}}{128b^3\,(a+bx^2)^3}+\frac{3d^5\sqrt{dx}}{1024ab^3\,(a+bx^2)^2}
\\
&= -\frac{d(dx)^{9/2}}{10b\,(a+bx^2)^5}-\frac{9d^3(dx)^{5/2}}{160b^2\,(a+bx^2)^4}-\frac{3d^5\sqrt{dx}}{128b^3\,(a+bx^2)^3}+\frac{3d^5\sqrt{dx}}{1024ab^3\,(a+bx^2)^2}
\\
&= -\frac{d(dx)^{9/2}}{10b\,(a+bx^2)^5}-\frac{9d^3(dx)^{5/2}}{160b^2\,(a+bx^2)^4}-\frac{3d^5\sqrt{dx}}{128b^3\,(a+bx^2)^3}+\frac{3d^5\sqrt{dx}}{1024ab^3\,(a+bx^2)^2}
\end{aligned}
$$

Mathematica [A]
time = 0.62, size = 186, normalized size = 0.48

$$
\frac{d^5\sqrt{dx}\left(\frac{4a^{3/4}\sqrt[4]{b}\,\sqrt{x}\,(-315a^4-1512a^3bx^2-2870a^2b^2x^4+480ab^3x^6+105b^4x^8)}{(a+bx^2)^5}-315\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+315\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{81920a^{11/4}b^{13/4}\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^5*Sqrt[d*x]*((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-315*a^4 - 1512*a^3*b*x^2 - 287
0*a^2*b^2*x^4 + 480*a*b^3*x^6 + 105*b^4*x^8))/(a + b*x^2)^5 - 315*Sqrt[2]*A
rcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 315*Sqrt[2
]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(81920
*a^(11/4)*b^(13/4)*Sqrt[x])
```

Maple [A]
time = 0.08, size = 236, normalized size = 0.60

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{63a^2d^4\sqrt{dx}}{8192b^3}-\frac{189a\,d^2(dx)^{\frac{5}{2}}}{5120b^2}-\frac{287(dx)^{\frac{9}{2}}}{4096b}+\frac{3(dx)^{\frac{13}{2}}}{256a\,d^2}+\frac{21b(dx)^{\frac{17}{2}}}{8192a^2d^4}}{(d^2x^2b+a\,d^2)^5}+\frac{63\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{d}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{d}}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{63a^2d^4\sqrt{dx}}{8192b^3}-\frac{189a\,d^2(dx)^{\frac{5}{2}}}{5120b^2}-\frac{287(dx)^{\frac{9}{2}}}{4096b}+\frac{3(dx)^{\frac{13}{2}}}{256a\,d^2}+\frac{21b(dx)^{\frac{17}{2}}}{8192a^2d^4}}{(d^2x^2b+a\,d^2)^5}+\frac{63\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{d}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{d}}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((-63/8192*a^2*d^4/b^3*(d*x)^(1/2)-189/5120*a*d^2/b^2*(d*x)^(5/2)-28
7/4096/b*(d*x)^(9/2)+3/256/a/d^2*(d*x)^(13/2)+21/8192/a^2/d^4*b*(d*x)^(17/2
))/(b*d^2*x^2+a*d^2)^5+63/65536/a^3/d^6/b^3*(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*
x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*
x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]
time = 0.51, size = 394, normalized size = 1.01

$$\frac{\frac{8\left(105\,(dx)^{\frac{17}{2}}b^4d^8+480\,(dx)^{\frac{13}{2}}ab^3d^{10}-2870\,(dx)^{\frac{9}{2}}a^2b^2d^{12}-1512\,(dx)^{\frac{5}{2}}a^3bd^{14}-315\,\sqrt{dx}\,a^4d^{16}\right)}{a^2b^8d^{10}x^{10}+5\,a^3b^7d^{10}x^8+10\,a^4b^6d^{10}x^6+10\,a^5b^5d^{10}x^4+5\,a^6b^4d^{10}x^2+a^7b^3d^{10}}+\frac{315\left(\frac{\sqrt{2}\,d^8\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^8\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^7\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^7\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}\right)}{a^2b^3}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(105*(d*x)^(17/2)*b^4*d^8 + 480*(d*x)^(13/2)*a*b^3*d^10 - 2870*
(d*x)^(9/2)*a^2*b^2*d^12 - 1512*(d*x)^(5/2)*a^3*b*d^14 - 315*sqrt(d*x)*a^4*
d^16)/(a^2*b^8*d^10*x^10 + 5*a^3*b^7*d^10*x^8 + 10*a^4*b^6*d^10*x^6 + 10*a^
5*b^5*d^10*x^4 + 5*a^6*b^4*d^10*x^2 + a^7*b^3*d^10) + 315*(sqrt(2)*d^8*log(
sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)
^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(
d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^7*arctan(1/
2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a
)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^7*arctan(-1/2
*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)
*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^2*b^3))/d
```

Fricas [A]

time = 0.37, size = 513, normalized size = 1.31

$$\frac{1260\,(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3)\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac14}\arctan\left(-\frac{\sqrt{dx}\,a^8b^{10}d^5\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac34}-\sqrt{a^6b^6\sqrt{-\frac{d^{22}}{a^{11}b^{13}}}+d^{11}x}\,a^8b^{10}\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac34}}{d^{22}}\right)+315\,(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3)\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac14}\log\left(63\,a^3b^3\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac14}+63\,\sqrt{dx}\,d^5\right)-315\,(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3)\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac14}\log\left(-63\,a^3b^3\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac14}+63\,\sqrt{dx}\,d^5\right)+4\,(105\,b^4d^5x^8+480\,ab^3d^5x^6-2870\,a^2b^2d^5x^4-1512\,a^3bd^5x^2-315\,a^4d^5)\sqrt{dx}}{81920\,(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(1260*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x
^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^22/(a^11*b^13))^(1/4)*arctan(-(sqrt(d*x)*
a^8*b^10*d^5*(-d^22/(a^11*b^13))^(3/4) - sqrt(a^6*b^6*sqrt(-d^22/(a^11*b^13
)) + d^11*x)*a^8*b^10*(-d^22/(a^11*b^13))^(3/4))/d^22) + 315*(a^2*b^8*x^10
+ 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3
)*(-d^22/(a^11*b^13))^(1/4)*log(63*a^3*b^3*(-d^22/(a^11*b^13))^(1/4) + 63*s
qrt(d*x)*d^5) - 315*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5
*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^22/(a^11*b^13))^(1/4)*log(-63*a^3*b
^3*(-d^22/(a^11*b^13))^(1/4) + 63*sqrt(d*x)*d^5) + 4*(105*b^4*d^5*x^8 + 480
*a*b^3*d^5*x^6 - 2870*a^2*b^2*d^5*x^4 - 1512*a^3*b*d^5*x^2 - 315*a^4*d^5)*s
qrt(d*x))/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 +
 5*a^6*b^4*x^2 + a^7*b^3)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{11}{2}}}{(a+bx^2)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(11/2)/(a + b*x**2)**6, x)
```

Giac [A]

time = 18.07, size = 342, normalized size = 0.87

$$\frac{1}{163840}\,d^5\left(\frac{630\,\sqrt{2}\,(ab^3d^2)^{\frac14}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac14}}\right)}{a^3b^4}+\frac{630\,\sqrt{2}\,(ab^3d^2)^{\frac14}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac14}}\right)}{a^3b^4}+\frac{315\,\sqrt{2}\,(ab^3d^2)^{\frac14}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^4}-\frac{315\,\sqrt{2}\,(ab^3d^2)^{\frac14}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^4}+\frac{8\left(105\,\sqrt{dx}\,b^4d^{10}x^8+480\,\sqrt{dx}\,ab^3d^{10}x^6-2870\,\sqrt{dx}\,a^2b^2d^{10}x^4-1512\,\sqrt{dx}\,a^3bd^{10}x^2-315\,\sqrt{dx}\,a^4d^{10}\right)}{(bd^2x^2+ad^2)^5a^2b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^5*(630*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4) + 630*sqrt(2)*(a*b^3
*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^3*b^4) + 315*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(
a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4) - 315*sqrt(2)*(a*b^3*d^
2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*
b^4) + 8*(105*sqrt(d*x)*b^4*d^10*x^8 + 480*sqrt(d*x)*a*b^3*d^10*x^6 - 2870*
sqrt(d*x)*a^2*b^2*d^10*x^4 - 1512*sqrt(d*x)*a^3*b*d^10*x^2 - 315*sqrt(d*x)*
a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))
```

Mupad [B]

time = 4.23, size = 208, normalized size = 0.53

$$-\frac{\frac{287\,d^{11}\,{\left(d\,x\right)}^{9/2}}{2048\,b}-\frac{3\,d^9\,{\left(d\,x\right)}^{13/2}}{128\,a}+\frac{63\,a^2\,d^{15}\,\sqrt{d\,x}}{4096\,b^3}+\frac{189\,a\,d^{13}\,{\left(d\,x\right)}^{5/2}}{2560\,b^2}-\frac{21\,b\,d^7\,{\left(d\,x\right)}^{17/2}}{4096\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{63\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{11/4}\,b^{13/4}}-\frac{63\,d^{11/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{11/4}\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] - ((287*d^11*(d*x)^(9/2))/(2048*b) - (3*d^9*(d*x)^(13/2))/(128*a) + (63*a^2
*d^15*(d*x)^(1/2))/(4096*b^3) + (189*a*d^13*(d*x)^(5/2))/(2560*b^2) - (21*b
*d^7*(d*x)^(17/2))/(4096*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2
 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (63*d^(1
1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(11/4)*b^
(13/4)) - (63*d^(11/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(
8192*(-a)^(11/4)*b^(13/4))
```

3.719 $\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=394

$$-\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}-\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}+\frac{7d^3(dx)^{3/2}}{640ab^2\,(a+bx^2)^3}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\,(a+bx^2)^2}+\frac{63d^3(dx)^{3/2}}{4096a^3b^2\,(a+bx^2)}-\frac{63d^{9/2}\tan}{8}$$

```
[Out] -1/10*d*(d*x)^(7/2)/b/(b*x^2+a)^5-7/160*d^3*(d*x)^(3/2)/b^2/(b*x^2+a)^4+7/6
40*d^3*(d*x)^(3/2)/a/b^2/(b*x^2+a)^3+63/5120*d^3*(d*x)^(3/2)/a^2/b^2/(b*x^2
+a)^2+63/4096*d^3*(d*x)^(3/2)/a^3/b^2/(b*x^2+a)-63/16384*d^(9/2)*arctan(1-b
^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/b^(11/4)*2^(1/2)+63/16
384*d^(9/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/
b^(11/4)*2^(1/2)+63/32768*d^(9/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1
/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/b^(11/4)*2^(1/2)-63/32768*d^(9/2)
*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/
a^(13/4)/b^(11/4)*2^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{63d^{9/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{13/4}b^{11/4}}+\frac{63d^{9/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{13/4}b^{11/4}}+\frac{63d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{13/4}b^{11/4}}-\frac{63d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{13/4}b^{11/4}}+\frac{63d^3(dx)^{3/2}}{4096a^3b^2\,(a+bx^2)}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\,(a+bx^2)^2}+\frac{7d^3(dx)^{3/2}}{640ab^2\,(a+bx^2)^3}-\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}-\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(d*(d*x)^(7/2))/(b*(a + b*x^2)^5) - (7*d^3*(d*x)^(3/2))/(160*b^2*(a +
 b*x^2)^4) + (7*d^3*(d*x)^(3/2))/(640*a*b^2*(a + b*x^2)^3) + (63*d^3*(d*x)^
(3/2))/(5120*a^2*b^2*(a + b*x^2)^2) + (63*d^3*(d*x)^(3/2))/(4096*a^3*b^2*(a
 + b*x^2)) - (63*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) + (63*d^(9/2)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) +
(63*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(13/4)*b^(11/4)) - (63*d^(9/2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sq
rt[2]*a^(13/4)*b^(11/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx &= b^6\int \frac{(dx)^{9/2}}{(ab+b^2x^2)^6}\,dx \\
&= -\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}+\frac{1}{20}\left(7b^4d^2\right)\int \frac{(dx)^{5/2}}{(ab+b^2x^2)^5}\,dx \\
&= -\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}-\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}+\frac{1}{320}\left(21b^2d^4\right)\int \frac{\sqrt{dx}}{(ab+b^2x^2)^4}\,dx \\
&= -\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}-\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}+\frac{7d^3(dx)^{3/2}}{640ab^2\,(a+bx^2)^3}+\frac{(63bd^4)\int \frac{\sqrt{dx}}{(ab+b^2x^2)}}{1280a} \\
&= -\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}-\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}+\frac{7d^3(dx)^{3/2}}{640ab^2\,(a+bx^2)^3}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\,(a+bx^2} \\
&= -\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}-\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}+\frac{7d^3(dx)^{3/2}}{640ab^2\,(a+bx^2)^3}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\,(a+bx^2} \\
&= -\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}-\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}+\frac{7d^3(dx)^{3/2}}{640ab^2\,(a+bx^2)^3}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\,(a+bx^2} \\
&= -\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}-\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}+\frac{7d^3(dx)^{3/2}}{640ab^2\,(a+bx^2)^3}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\,(a+bx^2} \\
&= -\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}-\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}+\frac{7d^3(dx)^{3/2}}{640ab^2\,(a+bx^2)^3}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\,(a+bx^2} \\
&= -\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}-\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}+\frac{7d^3(dx)^{3/2}}{640ab^2\,(a+bx^2)^3}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\,(a+bx^2} \\
&= -\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}-\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}+\frac{7d^3(dx)^{3/2}}{640ab^2\,(a+bx^2)^3}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\,(a+bx^2}
\end{aligned}
$$

Mathematica [A]
time = 0.57, size = 186, normalized size = 0.47

$$
\frac{d^4\sqrt{dx}\left(\frac{4\sqrt[4]{a}\,b^{3/4}x^{3/2}\left(-105a^4-480a^3bx^2+2870a^2b^2x^4+1512ab^3x^6+315b^4x^8\right)}{(a+bx^2)^5}-315\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-315\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{81920a^{13/4}b^{11/4}\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^4*Sqrt[d*x]*((4*a^(1/4)*b^(3/4)*x^(3/2)*(-105*a^4 - 480*a^3*b*x^2 + 2870
*a^2*b^2*x^4 + 1512*a*b^3*x^6 + 315*b^4*x^8))/(a + b*x^2)^5 - 315*Sqrt[2]*A
rcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 315*Sqrt[2
]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(81920
*a^(13/4)*b^(11/4)*Sqrt[x])
```

Maple [A]

time = 0.08, size = 236, normalized size = 0.60

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{21 a\,d^{2} (dx)^{\frac{3}{2}}}{8192 b^{2}}-\frac{3 (dx)^{\frac{7}{2}}}{256 b}+\frac{287 (dx)^{\frac{11}{2}}}{4096 a\,d^{2}}+\frac{189 b (dx)^{\frac{15}{2}}}{5120 a^{2} d^{4}}+\frac{63 b^{2} (dx)^{\frac{19}{2}}}{8192 a^{3} d^{6}}}{\left(d^{2} x^{2} b+a\,d^{2}\right)^{5}}+\frac{63\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\quad}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\quad}}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{21 a\,d^{2} (dx)^{\frac{3}{2}}}{8192 b^{2}}-\frac{3 (dx)^{\frac{7}{2}}}{256 b}+\frac{287 (dx)^{\frac{11}{2}}}{4096 a\,d^{2}}+\frac{189 b (dx)^{\frac{15}{2}}}{5120 a^{2} d^{4}}+\frac{63 b^{2} (dx)^{\frac{19}{2}}}{8192 a^{3} d^{6}}}{\left(d^{2} x^{2} b+a\,d^{2}\right)^{5}}+\frac{63\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\quad}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\quad}}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((-21/8192*a*d^2/b^2*(d*x)^(3/2)-3/256/b*(d*x)^(7/2)+287/4096/a/d^2*
(d*x)^(11/2)+189/5120/a^2*b/d^4*(d*x)^(15/2)+63/8192/a^3/d^6*b^2*(d*x)^(19/
2))/(b*d^2*x^2+a*d^2)^5+63/65536/a^3/d^6/b^3/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d
*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d
*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.51, size = 385, normalized size = 0.98

$$\frac{\frac{315\,d^{6}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(a d^{2})^{\frac{1}{4}} b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(a d^{2})^{\frac{1}{4}} b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(a d^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(a d^{2})^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(a d^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(a d^{2})^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{a^{3} b^{2}}+\frac{8\left(315\,(dx)^{\frac{19}{2}} b^{4} d^{6}+1512\,(dx)^{\frac{15}{2}} a b^{3} d^{8}+2870\,(dx)^{\frac{11}{2}} a^{2} b^{2} d^{10}-480\,(dx)^{\frac{7}{2}} a^{3} b d^{12}-105\,(dx)^{\frac{3}{2}} a^{4} d^{14}\right)}{a^{3} b^{7} d^{10} x^{10}+5\,a^{4} b^{6} d^{10} x^{8}+10\,a^{5} b^{5} d^{10} x^{6}+10\,a^{6} b^{4} d^{10} x^{4}+5\,a^{7} b^{3} d^{10} x^{2}+a^{8} b^{2} d^{10}}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(315*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1
/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d
)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) -
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqr
t(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) +
 sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*
d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^3*b^2
) + 8*(315*(d*x)^(19/2)*b^4*d^6 + 1512*(d*x)^(15/2)*a*b^3*d^8 + 2870*(d*x)^
(11/2)*a^2*b^2*d^10 - 480*(d*x)^(7/2)*a^3*b*d^12 - 105*(d*x)^(3/2)*a^4*d^14
)/(a^3*b^7*d^10*x^10 + 5*a^4*b^6*d^10*x^8 + 10*a^5*b^5*d^10*x^6 + 10*a^6*b^
4*d^10*x^4 + 5*a^7*b^3*d^10*x^2 + a^8*b^2*d^10))/d
```

Fricas [**A**]
time = 0.37, size = 520, normalized size = 1.32

$$-\frac{1260\left(a^{3}b^{7}x^{10}+5a^{4}b^{6}x^{8}+10a^{5}b^{5}x^{6}+10a^{6}b^{4}x^{4}+5a^{7}b^{3}x^{2}+a^{8}b^{2}\right)\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{250047\sqrt{dx}\,a^{3}b^{3}d^{13}\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}-\sqrt{-62523502209\,a^{7}b^{5}d^{18}\sqrt{-\frac{d^{18}}{a^{13}b^{11}}}+62523502209\,d^{27}x}\,a^{3}b^{3}\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}}{250047\,d^{18}}\right)-315\left(a^{3}b^{7}x^{10}+5a^{4}b^{6}x^{8}+10a^{5}b^{5}x^{6}+10a^{6}b^{4}x^{4}+5a^{7}b^{3}x^{2}+a^{8}b^{2}\right)\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}\log\left(250047\,a^{10}b^{8}\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{3}{4}}+250047\sqrt{dx}\,d^{13}\right)+315\left(a^{3}b^{7}x^{10}+5a^{4}b^{6}x^{8}+10a^{5}b^{5}x^{6}+10a^{6}b^{4}x^{4}+5a^{7}b^{3}x^{2}+a^{8}b^{2}\right)\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{1}{4}}\log\left(-250047\,a^{10}b^{8}\left(-\frac{d^{18}}{a^{13}b^{11}}\right)^{\frac{3}{4}}+250047\sqrt{dx}\,d^{13}\right)-4\left(315\,b^{4}d^{4}x^{9}+1512\,ab^{3}d^{4}x^{7}+2870\,a^{2}b^{2}d^{4}x^{5}-480\,a^{3}bd^{4}x^{3}-105\,a^{4}d^{4}x\right)\sqrt{dx}}{81920\left(a^{3}b^{7}x^{10}+5a^{4}b^{6}x^{8}+10a^{5}b^{5}x^{6}+10a^{6}b^{4}x^{4}+5a^{7}b^{3}x^{2}+a^{8}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(1260*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*
x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*arctan(-1/250047*(
250047*sqrt(d*x)*a^3*b^3*d^13*(-d^18/(a^13*b^11))^(1/4) - sqrt(-62523502209
*a^7*b^5*d^18*sqrt(-d^18/(a^13*b^11)) + 62523502209*d^27*x)*a^3*b^3*(-d^18/
(a^13*b^11))^(1/4))/d^18) - 315*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*
x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*l
og(250047*a^10*b^8*(-d^18/(a^13*b^11))^(3/4) + 250047*sqrt(d*x)*d^13) + 315
*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^
3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*log(-250047*a^10*b^8*(-d^18/(a^1
3*b^11))^(3/4) + 250047*sqrt(d*x)*d^13) - 4*(315*b^4*d^4*x^9 + 1512*a*b^3*d
^4*x^7 + 2870*a^2*b^2*d^4*x^5 - 480*a^3*b*d^4*x^3 - 105*a^4*d^4*x)*sqrt(d*x
))/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*
b^3*x^2 + a^8*b^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{9}{2}}}{(a+bx^2)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(9/2)/(a + b*x**2)**6, x)
```

Giac [A]
time = 15.02, size = 355, normalized size = 0.90

$$\frac{1}{163840}\,d^4\left(\frac{630\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^5d}+\frac{630\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^5d}-\frac{315\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^5d}+\frac{315\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^5d}+\frac{8\left(315\,\sqrt{dx}\,b^4d^{10}x^9+1512\,\sqrt{dx}\,ab^3d^{10}x^7+2870\,\sqrt{dx}\,a^2b^2d^{10}x^5-480\,\sqrt{dx}\,a^3bd^{10}x^3-105\,\sqrt{dx}\,a^4d^{10}x\right)}{\left(bd^2x^2+ad^2\right)^5a^3b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^4*(630*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^5*d) + 630*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^4*b^5*d) - 315*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^5*d) + 315*sqrt(2)*(a*
b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a^4*b^5*d) + 8*(315*sqrt(d*x)*b^4*d^10*x^9 + 1512*sqrt(d*x)*a*b^3*d^10*x^
7 + 2870*sqrt(d*x)*a^2*b^2*d^10*x^5 - 480*sqrt(d*x)*a^3*b*d^10*x^3 - 105*sq
rt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^3*b^2))
```

Mupad [B]
time = 0.12, size = 207, normalized size = 0.53

$$\frac{\frac{287\,d^9\,{\left(d\,x\right)}^{11/2}}{2048\,a}-\frac{3\,d^{11}\,{\left(d\,x\right)}^{7/2}}{128\,b}+\frac{63\,b^2\,d^5\,{\left(d\,x\right)}^{19/2}}{4096\,a^3}-\frac{21\,a\,d^{13}\,{\left(d\,x\right)}^{3/2}}{4096\,b^2}+\frac{189\,b\,d^7\,{\left(d\,x\right)}^{15/2}}{2560\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{63\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{13/4}\,b^{11/4}}+\frac{63\,d^{9/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{13/4}\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((287*d^9*(d*x)^(11/2))/(2048*a) - (3*d^11*(d*x)^(7/2))/(128*b) + (63*b^2*d
^5*(d*x)^(19/2))/(4096*a^3) - (21*a*d^13*(d*x)^(3/2))/(4096*b^2) + (189*b*d
^7*(d*x)^(15/2))/(2560*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 +
 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (63*d^(9/2
)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(13/4)*b^(11
/4)) + (63*d^(9/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192
*(-a)^(13/4)*b^(11/4))
```

3.720 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=394

$$-\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}+\frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}+\frac{77d^3\sqrt{dx}}{12288a^3b^2\left(a+bx^2\right)}-\frac{77d^{7/2}\,\mathrm{ta}}{}$$

```
[Out] -1/10*d*(d*x)^(5/2)/b/(b*x^2+a)^5-77/16384*d^(7/2)*arctan(1-b^(1/4)*2^(1/2)
*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(15/4)/b^(9/4)*2^(1/2)+77/16384*d^(7/2)*arc
tan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(15/4)/b^(9/4)*2^(1/2)
-77/32768*d^(7/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1
/2)*(d*x)^(1/2))/a^(15/4)/b^(9/4)*2^(1/2)+77/32768*d^(7/2)*ln(a^(1/2)*d^(1/
2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/b^(9/4)*
2^(1/2)-1/32*d^3*(d*x)^(1/2)/b^2/(b*x^2+a)^4+1/384*d^3*(d*x)^(1/2)/a/b^2/(b
*x^2+a)^3+11/3072*d^3*(d*x)^(1/2)/a^2/b^2/(b*x^2+a)^2+77/12288*d^3*(d*x)^(1
/2)/a^3/b^2/(b*x^2+a)
```

Rubi [A]
time = 0.29, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{77d^{7/2}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{15/4}b^{9/4}}+\frac{77d^{7/2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{15/4}b^{9/4}}-\frac{77d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{15/4}b^{9/4}}+\frac{77d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{15/4}b^{9/4}}+\frac{77d^3\sqrt{dx}}{12288a^3b^2\left(a+bx^2\right)}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}+\frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}-\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(d*(d*x)^(5/2))/(b*(a + b*x^2)^5) - (d^3*Sqrt[d*x])/(32*b^2*(a + b*x^
2)^4) + (d^3*Sqrt[d*x])/(384*a*b^2*(a + b*x^2)^3) + (11*d^3*Sqrt[d*x])/(307
2*a^2*b^2*(a + b*x^2)^2) + (77*d^3*Sqrt[d*x])/(12288*a^3*b^2*(a + b*x^2)) -
 (77*d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(81
92*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(15/4)*b^(9/4)) - (77*d^(7/2)*Lo
g[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(16384*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[
b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(15/4)*
b^(9/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{7/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}+\frac{1}{4}\left(b^4d^2\right)\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}+\frac{1}{64}\left(b^2d^4\right)\int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)^4}\,dx$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}+\frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3}+\frac{\left(11bd^4\right)\int \frac{1}{\sqrt{dx}\,(ab+b}}{768a}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}+\frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}+\frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}+\frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}+\frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}+\frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}+\frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}$$

$$= -\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}-\frac{d^3\sqrt{dx}}{32b^2\left(a+bx^2\right)^4}+\frac{d^3\sqrt{dx}}{384ab^2\left(a+bx^2\right)^3}+\frac{11d^3\sqrt{dx}}{3072a^2b^2\left(a+bx^2\right)^2}$$

Mathematica [A]
time = 0.55, size = 186, normalized size = 0.47

$$\frac{d^3\sqrt{dx}\left(\frac{4a^{3/4}\sqrt[4]{b}\,\sqrt{x}\,\left(-1155a^4-5544a^3bx^2+3130a^2b^2x^4+1760ab^3x^6+385b^4x^8\right)}{\left(a+bx^2\right)^5}-1155\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+1155\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{245760a^{15/4}b^{9/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^3*Sqrt[d*x]*((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-1155*a^4 - 5544*a^3*b*x^2 + 31
30*a^2*b^2*x^4 + 1760*a*b^3*x^6 + 385*b^4*x^8))/(a + b*x^2)^5 - 1155*Sqrt[2
]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 1155*Sq
rt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(2
45760*a^(15/4)*b^(9/4)*Sqrt[x])
```

Maple [A]

time = 0.08, size = 236, normalized size = 0.60

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{77a\,d^2\sqrt{dx}}{8192b^2}-\frac{231(dx)^{\frac{5}{2}}}{5120b}+\frac{313(dx)^{\frac{9}{2}}}{12288a\,d^2}+\frac{11b(dx)^{\frac{13}{2}}}{768a^2d^4}+\frac{77b^2(dx)^{\frac{17}{2}}}{24576a^3d^6}}{(d^2x^2b+a\,d^2)^5}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{77a\,d^2\sqrt{dx}}{8192b^2}-\frac{231(dx)^{\frac{5}{2}}}{5120b}+\frac{313(dx)^{\frac{9}{2}}}{12288a\,d^2}+\frac{11b(dx)^{\frac{13}{2}}}{768a^2d^4}+\frac{77b^2(dx)^{\frac{17}{2}}}{24576a^3d^6}}{(d^2x^2b+a\,d^2)^5}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((-77/8192*a*d^2/b^2*(d*x)^(1/2)-231/5120/b*(d*x)^(5/2)+313/12288/a/
d^2*(d*x)^(9/2)+11/768/a^2/d^4*b*(d*x)^(13/2)+77/24576/a^3/d^6*b^2*(d*x)^(1
7/2))/(b*d^2*x^2+a*d^2)^5+77/65536/a^4/d^8/b^2*(a*d^2/b)^(1/4)*2^(1/2)*(ln(
(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*
(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.51, size = 394, normalized size = 1.00

$$\frac{\frac{8\left(385\,(dx)^{\frac{17}{2}}b^4d^6+1760\,(dx)^{\frac{13}{2}}ab^3d^8+3130\,(dx)^{\frac{9}{2}}a^2b^2d^{10}-5544\,(dx)^{\frac{5}{2}}a^3bd^{12}-1155\,\sqrt{dx}\,a^4d^{14}\right)}{a^3b^7d^{10}x^{10}+5\,a^4b^6d^{10}x^8+10\,a^5b^5d^{10}x^6+10\,a^6b^4d^{10}x^4+5\,a^7b^3d^{10}x^2+a^8b^2d^{10}}+\frac{1155\left(\frac{\sqrt{2}\,d^6\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^6\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^5\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^5\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)}{a^3b^2}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/491520*(8*(385*(d*x)^(17/2)*b^4*d^6 + 1760*(d*x)^(13/2)*a*b^3*d^8 + 3130*
(d*x)^(9/2)*a^2*b^2*d^10 - 5544*(d*x)^(5/2)*a^3*b*d^12 - 1155*sqrt(d*x)*a^4
*d^14)/(a^3*b^7*d^10*x^10 + 5*a^4*b^6*d^10*x^8 + 10*a^5*b^5*d^10*x^6 + 10*a
^6*b^4*d^10*x^4 + 5*a^7*b^3*d^10*x^2 + a^8*b^2*d^10) + 1155*(sqrt(2)*d^6*lo
g(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^
2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqr
t(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^5*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^5*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(
a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^3*b^2))/d
```

Fricas [A]
time = 0.37, size = 513, normalized size = 1.30

$$\frac{4620\,(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2)\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac14}\arctan\left(-\frac{\sqrt{dx}\,a^{11}b^7d^3\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac34}-\sqrt{a^8b^4\sqrt{-\frac{d^{14}}{a^{15}b^9}}+d^7x}\,a^{11}b^7\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac34}}{d^{14}}\right)+1155\,(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2)\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac14}\log\left(77\,a^4b^2\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac14}+77\,\sqrt{dx}\,d^3\right)-1155\,(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2)\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac14}\log\left(-77\,a^4b^2\left(-\frac{d^{14}}{a^{15}b^9}\right)^{\frac14}+77\,\sqrt{dx}\,d^3\right)+4\,(385\,b^4d^3x^8+1760\,ab^3d^3x^6+3130\,a^2b^2d^3x^4-5544\,a^3bd^3x^2-1155\,a^4d^3)\sqrt{dx}}{245760\,(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(4620*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*
x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^14/(a^15*b^9))^(1/4)*arctan(-(sqrt(d*x)*
a^11*b^7*d^3*(-d^14/(a^15*b^9))^(3/4) - sqrt(a^8*b^4*sqrt(-d^14/(a^15*b^9))
 + d^7*x)*a^11*b^7*(-d^14/(a^15*b^9))^(3/4))/d^14) + 1155*(a^3*b^7*x^10 + 5
*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(
-d^14/(a^15*b^9))^(1/4)*log(77*a^4*b^2*(-d^14/(a^15*b^9))^(1/4) + 77*sqrt(d
*x)*d^3) - 1155*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4
*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^14/(a^15*b^9))^(1/4)*log(-77*a^4*b^2*(-
d^14/(a^15*b^9))^(1/4) + 77*sqrt(d*x)*d^3) + 4*(385*b^4*d^3*x^8 + 1760*a*b^
3*d^3*x^6 + 3130*a^2*b^2*d^3*x^4 - 5544*a^3*b*d^3*x^2 - 1155*a^4*d^3)*sqrt(
d*x))/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a
^7*b^3*x^2 + a^8*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{(a+bx^2)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**6, x)
```

Giac [A]
time = 16.39, size = 342, normalized size = 0.87

$$\frac{1}{491520}\,d^{3}\left(\frac{2310\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4}b^{3}}+\frac{2310\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4}b^{3}}+\frac{1155\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{4}b^{3}}-\frac{1155\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{a^{4}b^{3}}+\frac{8\left(385\,\sqrt{dx}\,b^{4}d^{10}x^{8}+1760\,\sqrt{dx}\,ab^{3}d^{10}x^{6}+3130\,\sqrt{dx}\,a^{2}b^{2}d^{10}x^{4}-5544\,\sqrt{dx}\,a^{3}bd^{10}x^{2}-1155\,\sqrt{dx}\,a^{4}d^{10}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{3}b^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^3*(2310*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3) + 2310*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^4*b^3) + 1155*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) - 1155*sqrt(2)*(a*b^
3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a^4*b^3) + 8*(385*sqrt(d*x)*b^4*d^10*x^8 + 1760*sqrt(d*x)*a*b^3*d^10*x^6 +
3130*sqrt(d*x)*a^2*b^2*d^10*x^4 - 5544*sqrt(d*x)*a^3*b*d^10*x^2 - 1155*sqrt
(d*x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a^3*b^2))
```

Mupad [B]
time = 4.27, size = 207, normalized size = 0.53

$$\frac{\frac{313\,d^9\,{\left(d\,x\right)}^{9/2}}{6144\,a}-\frac{231\,d^{11}\,{\left(d\,x\right)}^{5/2}}{2560\,b}+\frac{77\,b^2\,d^5\,{\left(d\,x\right)}^{17/2}}{12288\,a^3}-\frac{77\,a\,d^{13}\,\sqrt{d\,x}}{4096\,b^2}+\frac{11\,b\,d^7\,{\left(d\,x\right)}^{13/2}}{384\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\frac{77\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{15/4}\,b^{9/4}}+\frac{77\,d^{7/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{15/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((313*d^9*(d*x)^(9/2))/(6144*a) - (231*d^11*(d*x)^(5/2))/(2560*b) + (77*b^2
*d^5*(d*x)^(17/2))/(12288*a^3) - (77*a*d^13*(d*x)^(1/2))/(4096*b^2) + (11*b
*d^7*(d*x)^(13/2))/(384*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2
+ 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) + (77*d^(7/
2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(15/4)*b^(9
/4)) + (77*d^(7/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192
*(-a)^(15/4)*b^(9/4))
```

3.721 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=389

$$-\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5}+\frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4}+\frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3}+\frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2}+\frac{117d(dx)^{3/2}}{4096a^4b\left(a+bx^2\right)}-\frac{117d^{5/2}\tan\ldots}{81\ldots}$$

```
[Out] -1/10*d*(d*x)^(3/2)/b/(b*x^2+a)^5+3/160*d*(d*x)^(3/2)/a/b/(b*x^2+a)^4+13/64
0*d*(d*x)^(3/2)/a^2/b/(b*x^2+a)^3+117/5120*d*(d*x)^(3/2)/a^3/b/(b*x^2+a)^2+
117/4096*d*(d*x)^(3/2)/a^4/b/(b*x^2+a)-117/16384*d^(5/2)*arctan(1-b^(1/4)*2
^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(17/4)/b^(7/4)*2^(1/2)+117/16384*d^(5
/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(17/4)/b^(7/4)*
2^(1/2)+117/32768*d^(5/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1
/4)*2^(1/2)*(d*x)^(1/2))/a^(17/4)/b^(7/4)*2^(1/2)-117/32768*d^(5/2)*ln(a^(1
/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(17/4)
/b^(7/4)*2^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{117d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{17/4}b^{7/4}}+\frac{117d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{17/4}b^{7/4}}+\frac{117d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{17/4}b^{7/4}}-\frac{117d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{17/4}b^{7/4}}+\frac{117d(dx)^{3/2}}{4096a^4b\left(a+bx^2\right)}+\frac{117d(dx)^{3/2}}{5120a^3b\left(a+bx^2\right)^2}+\frac{13d(dx)^{3/2}}{640a^2b\left(a+bx^2\right)^3}+\frac{3d(dx)^{3/2}}{160ab\left(a+bx^2\right)^4}-\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(d*(d*x)^(3/2))/(b*(a + b*x^2)^5) + (3*d*(d*x)^(3/2))/(160*a*b*(a + b
*x^2)^4) + (13*d*(d*x)^(3/2))/(640*a^2*b*(a + b*x^2)^3) + (117*d*(d*x)^(3/2
))/(5120*a^3*b*(a + b*x^2)^2) + (117*d*(d*x)^(3/2))/(4096*a^4*b*(a + b*x^2)
) - (117*d^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])
/(8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5
/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(16384*Sqrt[2]*a^(17/4)*b^(7/4)) - (117*d^(5/2)*Log[Sqrt[a]*Sqrt[d]
+ Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^
(17/4)*b^(7/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = b^6\int \frac{(dx)^{5/2}}{(ab+b^2x^2)^6}\,dx$$

$$= -\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5} + \frac{1}{20}\left(3b^4d^2\right)\int \frac{\sqrt{dx}}{(ab+b^2x^2)^5}\,dx$$

$$= -\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5} + \frac{3d(dx)^{3/2}}{160ab\,(a+bx^2)^4} + \frac{(39b^3d^2)\int \frac{\sqrt{dx}}{(ab+b^2x^2)^4}\,dx}{320a}$$

$$= -\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5} + \frac{3d(dx)^{3/2}}{160ab\,(a+bx^2)^4} + \frac{13d(dx)^{3/2}}{640a^2b\,(a+bx^2)^3} + \frac{(117b^2d^2)\int \frac{\sqrt{d}}{(ab+b^2}}{1280a^2}$$

$$= -\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5} + \frac{3d(dx)^{3/2}}{160ab\,(a+bx^2)^4} + \frac{13d(dx)^{3/2}}{640a^2b\,(a+bx^2)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\,(a+bx^2)}$$

$$= -\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5} + \frac{3d(dx)^{3/2}}{160ab\,(a+bx^2)^4} + \frac{13d(dx)^{3/2}}{640a^2b\,(a+bx^2)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\,(a+bx^2)}$$

$$= -\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5} + \frac{3d(dx)^{3/2}}{160ab\,(a+bx^2)^4} + \frac{13d(dx)^{3/2}}{640a^2b\,(a+bx^2)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\,(a+bx^2)}$$

$$= -\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5} + \frac{3d(dx)^{3/2}}{160ab\,(a+bx^2)^4} + \frac{13d(dx)^{3/2}}{640a^2b\,(a+bx^2)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\,(a+bx^2)}$$

$$= -\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5} + \frac{3d(dx)^{3/2}}{160ab\,(a+bx^2)^4} + \frac{13d(dx)^{3/2}}{640a^2b\,(a+bx^2)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\,(a+bx^2)}$$

$$= -\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5} + \frac{3d(dx)^{3/2}}{160ab\,(a+bx^2)^4} + \frac{13d(dx)^{3/2}}{640a^2b\,(a+bx^2)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\,(a+bx^2)}$$

$$= -\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5} + \frac{3d(dx)^{3/2}}{160ab\,(a+bx^2)^4} + \frac{13d(dx)^{3/2}}{640a^2b\,(a+bx^2)^3} + \frac{117d(dx)^{3/2}}{5120a^3b\,(a+bx^2)}$$

<u>Mathematica</u> [A]
time = 0.47, size = 183, normalized size = 0.47

$$\frac{(dx)^{5/2}\left(\frac{4\sqrt[4]{a}\,b^{3/4}x^{3/2}(-195a^4+4960a^3bx^2+5330a^2b^2x^4+2808ab^3x^6+585b^4x^8)}{(a+bx^2)^5} - 585\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right) - 585\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{81920a^{17/4}b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((d*x)^(5/2)*((4*a^(1/4)*b^(3/4)*x^(3/2)*(-195*a^4 + 4960*a^3*b*x^2 + 5330*
a^2*b^2*x^4 + 2808*a*b^3*x^6 + 585*b^4*x^8))/(a + b*x^2)^5 - 585*Sqrt[2]*Ar
cTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 585*Sqrt[2]
*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(81920*
a^(17/4)*b^(7/4)*x^(5/2))
```

Maple [A]

time = 0.08, size = 238, normalized size = 0.61

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{39(dx)^{\frac{3}{2}}}{8192b}+\frac{31(dx)^{\frac{7}{2}}}{256a\,d^2}+\frac{533b(dx)^{\frac{11}{2}}}{4096a^2d^4}+\frac{351b^2(dx)^{\frac{15}{2}}}{5120a^3d^6}+\frac{117b^3(dx)^{\frac{19}{2}}}{8192a^4d^8}}{(d^2x^2b+a\,d^2)^5}+\frac{117\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{}}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{39(dx)^{\frac{3}{2}}}{8192b}+\frac{31(dx)^{\frac{7}{2}}}{256a\,d^2}+\frac{533b(dx)^{\frac{11}{2}}}{4096a^2d^4}+\frac{351b^2(dx)^{\frac{15}{2}}}{5120a^3d^6}+\frac{117b^3(dx)^{\frac{19}{2}}}{8192a^4d^8}}{(d^2x^2b+a\,d^2)^5}+\frac{117\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{}}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((-39/8192/b*(d*x)^(3/2)+31/256/a/d^2*(d*x)^(7/2)+533/4096/a^2/d^4*b
*(d*x)^(11/2)+351/5120/a^3*b^2/d^6*(d*x)^(15/2)+117/8192/a^4/d^8*b^3*(d*x)^
(19/2))/(b*d^2*x^2+a*d^2)^5+117/65536/a^4/d^8/b^2/(a*d^2/b)^(1/4)*2^(1/2)*(
ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/
4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.51, size = 383, normalized size = 0.98

$$\frac{\frac{8\left(585\,(dx)^{\frac{19}{2}}b^4d^4+2808\,(dx)^{\frac{15}{2}}ab^3d^6+5330\,(dx)^{\frac{11}{2}}a^2b^2d^8+4960\,(dx)^{\frac{7}{2}}a^3bd^{10}-195\,(dx)^{\frac{3}{2}}a^4d^{12}\right)}{a^4b^6d^{10}x^{10}+5\,a^5b^5d^{10}x^8+10\,a^6b^4d^{10}x^6+10\,a^7b^3d^{10}x^4+5\,a^8b^2d^{10}x^2+a^9bd^{10}}+\frac{585\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^4b}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(585*(d*x)^(19/2)*b^4*d^4 + 2808*(d*x)^(15/2)*a*b^3*d^6 + 5330*
(d*x)^(11/2)*a^2*b^2*d^8 + 4960*(d*x)^(7/2)*a^3*b*d^10 - 195*(d*x)^(3/2)*a^
4*d^12)/(a^4*b^6*d^10*x^10 + 5*a^5*b^5*d^10*x^8 + 10*a^6*b^4*d^10*x^6 + 10*
a^7*b^3*d^10*x^4 + 5*a^8*b^2*d^10*x^2 + a^9*b*d^10) + 585*d^4*(2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt
(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt
(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x
 + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3
/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) +
sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^4*b))/d
```

Fricas [**A**]
time = 0.35, size = 512, normalized size = 1.32

$$-\frac{2340\,\left(a^{4} b^{6} x^{10} + 5\,a^{5} b^{5} x^{8} + 10\,a^{6} b^{4} x^{6} + 10\,a^{7} b^{3} x^{4} + 5\,a^{8} b^{2} x^{2} + a^{9} b\right) \left(-\frac{d^{10}}{a^{17} b^{7}}\right)^{\frac{1}{4}} \arctan\left(-\frac{1601613\,\sqrt{d x}\,a^{4} b^{2} d^{7} \left(-\frac{d^{10}}{a^{17} b^{7}}\right)^{\frac{1}{4}} - \sqrt{-2565164201769\,a^{9} b^{3} d^{10} \sqrt{-\frac{d^{10}}{a^{17} b^{7}}} + 2565164201769\,d^{15} x}\,a^{4} b^{2} \left(-\frac{d^{10}}{a^{17} b^{7}}\right)^{\frac{1}{4}}}{1601613\,d^{10}}\right) - 585\,\left(a^{4} b^{6} x^{10} + 5\,a^{5} b^{5} x^{8} + 10\,a^{6} b^{4} x^{6} + 10\,a^{7} b^{3} x^{4} + 5\,a^{8} b^{2} x^{2} + a^{9} b\right) \left(-\frac{d^{10}}{a^{17} b^{7}}\right)^{\frac{1}{4}} \log\left(1601613\,a^{13} b^{5} \left(-\frac{d^{10}}{a^{17} b^{7}}\right)^{\frac{3}{4}} + 1601613\,\sqrt{d x}\,d^{7}\right) + 585\,\left(a^{4} b^{6} x^{10} + 5\,a^{5} b^{5} x^{8} + 10\,a^{6} b^{4} x^{6} + 10\,a^{7} b^{3} x^{4} + 5\,a^{8} b^{2} x^{2} + a^{9} b\right) \left(-\frac{d^{10}}{a^{17} b^{7}}\right)^{\frac{1}{4}} \log\left(-1601613\,a^{13} b^{5} \left(-\frac{d^{10}}{a^{17} b^{7}}\right)^{\frac{3}{4}} + 1601613\,\sqrt{d x}\,d^{7}\right) - 4\,\left(585\,b^{4} d^{2} x^{9} + 2808\,a b^{3} d^{2} x^{7} + 5330\,a^{2} b^{2} d^{2} x^{5} + 4960\,a^{3} b d^{2} x^{3} - 195\,a^{4} d^{2} x\right) \sqrt{d x}}{81920\,\left(a^{4} b^{6} x^{10} + 5\,a^{5} b^{5} x^{8} + 10\,a^{6} b^{4} x^{6} + 10\,a^{7} b^{3} x^{4} + 5\,a^{8} b^{2} x^{2} + a^{9} b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(2340*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*
x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*arctan(-1/1601613*(16
01613*sqrt(d*x)*a^4*b^2*d^7*(-d^10/(a^17*b^7))^(1/4) - sqrt(-2565164201769*
a^9*b^3*d^10*sqrt(-d^10/(a^17*b^7)) + 2565164201769*d^15*x)*a^4*b^2*(-d^10/
(a^17*b^7))^(1/4))/d^10) - 585*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x
^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*log(1
601613*a^13*b^5*(-d^10/(a^17*b^7))^(3/4) + 1601613*sqrt(d*x)*d^7) + 585*(a^
4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^
2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*log(-1601613*a^13*b^5*(-d^10/(a^17*b^7)
)^(3/4) + 1601613*sqrt(d*x)*d^7) - 4*(585*b^4*d^2*x^9 + 2808*a*b^3*d^2*x^7
+ 5330*a^2*b^2*d^2*x^5 + 4960*a^3*b*d^2*x^3 - 195*a^4*d^2*x)*sqrt(d*x))/(a^
4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^
2 + a^9*b)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left(a + bx^2\right)^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**6, x)
```

Giac [A]
time = 7.90, size = 355, normalized size = 0.91

$$\frac{1}{163840}\,d^2\left(\frac{1170\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^4d}+\frac{1170\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^4d}-\frac{585\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^4d}+\frac{585\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^4d}+\frac{8\left(585\sqrt{dx}\,b^4d^{10}x^9+2808\sqrt{dx}\,ab^3d^{10}x^7+5330\sqrt{dx}\,a^2b^2d^{10}x^5+4960\sqrt{dx}\,a^3bd^{10}x^3-195\sqrt{dx}\,a^4d^{10}x\right)}{(bd^2x^2+ad^2)^5a^4b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^2*(1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^4*d) + 1170*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^5*b^4*d) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqr
t(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^4*d) + 585*sqrt(2)*(
a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^5*b^4*d) + 8*(585*sqrt(d*x)*b^4*d^10*x^9 + 2808*sqrt(d*x)*a*b^3*d^10*
x^7 + 5330*sqrt(d*x)*a^2*b^2*d^10*x^5 + 4960*sqrt(d*x)*a^3*b*d^10*x^3 - 195
*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^4*b))
```

Mupad [B]
time = 4.28, size = 209, normalized size = 0.54

$$\frac{\frac{31\,d^9\,(d\,x)^{7/2}}{128\,a}-\frac{39\,d^{11}\,(d\,x)^{3/2}}{4096\,b}+\frac{351\,b^2\,d^5\,(d\,x)^{15/2}}{2560\,a^3}+\frac{117\,b^3\,d^3\,(d\,x)^{19/2}}{4096\,a^4}+\frac{533\,b\,d^7\,(d\,x)^{11/2}}{2048\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\frac{117\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{17/4}\,b^{7/4}}-\frac{117\,d^{5/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{17/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((31*d^9*(d*x)^(7/2))/(128*a) - (39*d^11*(d*x)^(3/2))/(4096*b) + (351*b^2*d
^5*(d*x)^(15/2))/(2560*a^3) + (117*b^3*d^3*(d*x)^(19/2))/(4096*a^4) + (533*
b*d^7*(d*x)^(11/2))/(2048*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^
2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) + (117*d^
(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(17/4)*b
^(7/4)) - (117*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(
8192*(-a)^(17/4)*b^(7/4))
```

3.722 $\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=389

$$-\frac{d\sqrt{dx}}{10b\,(a+bx^2)^5}+\frac{d\sqrt{dx}}{160ab\,(a+bx^2)^4}+\frac{d\sqrt{dx}}{128a^2b\,(a+bx^2)^3}+\frac{11d\sqrt{dx}}{1024a^3b\,(a+bx^2)^2}+\frac{77d\sqrt{dx}}{4096a^4b\,(a+bx^2)}-\frac{231d^{3/2}\,\mathrm{ta}}{8}$$

```
[Out] -231/16384*d^(3/2)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^
(19/4)/b^(5/4)*2^(1/2)+231/16384*d^(3/2)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/
2)/a^(1/4)/d^(1/2))/a^(19/4)/b^(5/4)*2^(1/2)-231/32768*d^(3/2)*ln(a^(1/2)*d
^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(19/4)/b^(5
/4)*2^(1/2)+231/32768*d^(3/2)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*
b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(19/4)/b^(5/4)*2^(1/2)-1/10*d*(d*x)^(1/2)/b/
(b*x^2+a)^5+1/160*d*(d*x)^(1/2)/a/b/(b*x^2+a)^4+1/128*d*(d*x)^(1/2)/a^2/b/(
b*x^2+a)^3+11/1024*d*(d*x)^(1/2)/a^3/b/(b*x^2+a)^2+77/4096*d*(d*x)^(1/2)/a^
4/b/(b*x^2+a)
```

Rubi [A]
time = 0.29, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{231d^{3/2}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{19/4}b^{5/4}}+\frac{231d^{3/2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{19/4}b^{5/4}}-\frac{231d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{19/4}b^{5/4}}+\frac{231d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{19/4}b^{5/4}}+\frac{77d\sqrt{dx}}{4096a^4b\,(a+bx^2)}+\frac{11d\sqrt{dx}}{1024a^3b\,(a+bx^2)^2}+\frac{d\sqrt{dx}}{128a^2b\,(a+bx^2)^3}+\frac{d\sqrt{dx}}{160ab\,(a+bx^2)^4}-\frac{d\sqrt{dx}}{10b\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/10*(d*Sqrt[d*x])/(b*(a + b*x^2)^5) + (d*Sqrt[d*x])/(160*a*b*(a + b*x^2)^
4) + (d*Sqrt[d*x])/(128*a^2*b*(a + b*x^2)^3) + (11*d*Sqrt[d*x])/(1024*a^3*b
*(a + b*x^2)^2) + (77*d*Sqrt[d*x])/(4096*a^4*b*(a + b*x^2)) - (231*d^(3/2)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^
(19/4)*b^(5/4)) + (231*d^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(19/4)*b^(5/4)) - (231*d^(3/2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqr
t[2]*a^(19/4)*b^(5/4)) + (231*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(19/4)*b^(5/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{(dx)^{3/2}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}+\frac{1}{20}\left(b^4d^2\right)\int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)^5}\,dx$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}+\frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4}+\frac{\left(3b^3d^2\right)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^4}\,dx}{64a}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}+\frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4}+\frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3}+\frac{\left(11b^2d^2\right)\int \frac{1}{\sqrt{dx}\,(ab}}{256a^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}+\frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4}+\frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3}+\frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}+\frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4}+\frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3}+\frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}+\frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4}+\frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3}+\frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}+\frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4}+\frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3}+\frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}+\frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4}+\frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3}+\frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}+\frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4}+\frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3}+\frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

$$= -\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}+\frac{d\sqrt{dx}}{160ab\left(a+bx^2\right)^4}+\frac{d\sqrt{dx}}{128a^2b\left(a+bx^2\right)^3}+\frac{11d\sqrt{dx}}{1024a^3b\left(a+bx^2\right)^2}$$

Mathematica [A]

time = 0.46, size = 183, normalized size = 0.47

$$\frac{(dx)^{3/2}\left(\frac{4a^{3/4}\sqrt[4]{b}\,\sqrt{x}\,\left(-1155a^4+2648a^3bx^2+3130a^2b^2x^4+1760ab^3x^6+385b^4x^8\right)}{\left(a+bx^2\right)^5}-1155\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+1155\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{81920a^{19/4}b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((d*x)^(3/2)*((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-1155*a^4 + 2648*a^3*b*x^2 + 3130
*a^2*b^2*x^4 + 1760*a*b^3*x^6 + 385*b^4*x^8))/(a + b*x^2)^5 - 1155*Sqrt[2]*
ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 1155*Sqrt
[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(819
20*a^(19/4)*b^(5/4)*x^(3/2))
```

Maple [A]

time = 0.08, size = 238, normalized size = 0.61

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{231\sqrt{dx}}{8192b}+\frac{331(dx)^{\frac{5}{2}}}{5120a\,d^{2}}+\frac{313b(dx)^{\frac{9}{2}}}{4096a^{2}d^{4}}+\frac{11b^{2}(dx)^{\frac{13}{2}}}{256a^{3}d^{6}}+\frac{77b^{3}(dx)^{\frac{17}{2}}}{8192a^{4}d^{8}}}{\left(d^{2}x^{2}b+a\,d^{2}\right)^{5}}+\frac{231\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\ldots}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\ldots}\right.\right.}{\ldots}\right.$
default	$2d^{11}\left(\frac{-\frac{231\sqrt{dx}}{8192b}+\frac{331(dx)^{\frac{5}{2}}}{5120a\,d^{2}}+\frac{313b(dx)^{\frac{9}{2}}}{4096a^{2}d^{4}}+\frac{11b^{2}(dx)^{\frac{13}{2}}}{256a^{3}d^{6}}+\frac{77b^{3}(dx)^{\frac{17}{2}}}{8192a^{4}d^{8}}}{\left(d^{2}x^{2}b+a\,d^{2}\right)^{5}}+\frac{231\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\ldots}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\ldots}\right.\right.}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((-231/8192/b*(d*x)^(1/2)+331/5120/a/d^2*(d*x)^(5/2)+313/4096/a^2/d^
4*b*(d*x)^(9/2)+11/256/a^3/d^6*b^2*(d*x)^(13/2)+77/8192/a^4/d^8*b^3*(d*x)^(
17/2))/(b*d^2*x^2+a*d^2)^5+231/65536/a^5/d^10/b*(a*d^2/b)^(1/4)*2^(1/2)*(ln
((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)
*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Maxima [A]

time = 0.50, size = 392, normalized size = 1.01

$$\frac{8\left(385\,(dx)^{\frac{17}{2}}b^{4}d^{4}+1760\,(dx)^{\frac{13}{2}}ab^{3}d^{6}+3130\,(dx)^{\frac{9}{2}}a^{2}b^{2}d^{8}+2648\,(dx)^{\frac{5}{2}}a^{3}bd^{10}-1155\sqrt{dx}\,a^{4}d^{12}\right)}{a^{4}b^{6}d^{10}x^{10}+5\,a^{5}b^{5}d^{10}x^{8}+10\,a^{6}b^{4}d^{10}x^{6}+10\,a^{7}b^{3}d^{10}x^{4}+5\,a^{8}b^{2}d^{10}x^{2}+a^{9}bd^{10}}+\frac{1155\left(\frac{\sqrt{2}\,d^{4}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^{4}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^{3}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{a}}+\frac{2\sqrt{2}\,d^{3}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{a}}\right)}{a^{4}b}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(385*(d*x)^(17/2)*b^4*d^4 + 1760*(d*x)^(13/2)*a*b^3*d^6 + 3130*
(d*x)^(9/2)*a^2*b^2*d^8 + 2648*(d*x)^(5/2)*a^3*b*d^10 - 1155*sqrt(d*x)*a^4*
d^12)/(a^4*b^6*d^10*x^10 + 5*a^5*b^5*d^10*x^8 + 10*a^6*b^4*d^10*x^6 + 10*a^
7*b^3*d^10*x^4 + 5*a^8*b^2*d^10*x^2 + a^9*b*d^10) + 1155*(sqrt(2)*d^4*log(s
qrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^
(3/4)*b^(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d
*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arctan(1/2
*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)
*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arctan(-1/2*
sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*
sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^4*b))/d
```

Fricas [A]
time = 0.37, size = 485, normalized size = 1.25

$$\frac{4620\,(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b)\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{14}b^4d\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{3}{4}}-\sqrt{a^{10}b^2\sqrt{-\frac{d^6}{a^{19}b^5}}+d^3x}\,a^{14}b^4\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{3}{4}}}{d^6}\right)+1155\,(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b)\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{1}{4}}\log\left(231\,a^5b\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{1}{4}}+231\,\sqrt{dx}\,d\right)-1155\,(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b)\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{1}{4}}\log\left(-231\,a^5b\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{1}{4}}+231\,\sqrt{dx}\,d\right)+4\,(385\,b^4dx^8+1760\,ab^3dx^6+3130\,a^2b^2dx^4+2648\,a^3bdx^2-1155\,a^4d)\sqrt{dx}}{81920\,(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(4620*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x
^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^6/(a^19*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^14
*b^4*d*(-d^6/(a^19*b^5))^(3/4) - sqrt(a^10*b^2*sqrt(-d^6/(a^19*b^5)) + d^3*
x)*a^14*b^4*(-d^6/(a^19*b^5))^(3/4))/d^6) + 1155*(a^4*b^6*x^10 + 5*a^5*b^5*
x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^6/(a^19*
b^5))^(1/4)*log(231*a^5*b*(-d^6/(a^19*b^5))^(1/4) + 231*sqrt(d*x)*d) - 1155
*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^
2*x^2 + a^9*b)*(-d^6/(a^19*b^5))^(1/4)*log(-231*a^5*b*(-d^6/(a^19*b^5))^(1/
4) + 231*sqrt(d*x)*d) + 4*(385*b^4*d*x^8 + 1760*a*b^3*d*x^6 + 3130*a^2*b^2*
d*x^4 + 2648*a^3*b*d*x^2 - 1155*a^4*d)*sqrt(d*x))/(a^4*b^6*x^10 + 5*a^5*b^5
*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{(a+bx^2)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**6, x)
```

Giac [A]
time = 7.48, size = 340, normalized size = 0.87

$$\frac{1}{163840}\,d\left(\frac{2310\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2}+\frac{2310\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2}+\frac{1155\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^2}-\frac{1155\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^2}+\frac{8\left(385\,\sqrt{dx}\,b^4d^{10}x^8+1760\,\sqrt{dx}\,ab^3d^{10}x^6+3130\,\sqrt{dx}\,a^2b^2d^{10}x^4+2648\,\sqrt{dx}\,a^3bd^{10}x^2-1155\,\sqrt{dx}\,a^4d^{10}\right)}{(bd^2x^2+ad^2)^5a^4b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d*(2310*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2) + 2310*sqrt(2)*(a*b^3
*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^5*b^2) + 1155*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2) - 1155*sqrt(2)*(a*b^3*
d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
5*b^2) + 8*(385*sqrt(d*x)*b^4*d^10*x^8 + 1760*sqrt(d*x)*a*b^3*d^10*x^6 + 31
30*sqrt(d*x)*a^2*b^2*d^10*x^4 + 2648*sqrt(d*x)*a^3*b*d^10*x^2 - 1155*sqrt(d
*x)*a^4*d^10)/((b*d^2*x^2 + a*d^2)^5*a^4*b))
```

Mupad [B]
time = 0.13, size = 209, normalized size = 0.54

$$\frac{\frac{331\,d^9\,(d\,x)^{5/2}}{2560\,a}-\frac{231\,d^{11}\,\sqrt{d\,x}}{4096\,b}+\frac{11\,b^2\,d^5\,(d\,x)^{13/2}}{128\,a^3}+\frac{77\,b^3\,d^3\,(d\,x)^{17/2}}{4096\,a^4}+\frac{313\,b\,d^7\,(d\,x)^{9/2}}{2048\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{231\,d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{19/4}\,b^{5/4}}-\frac{231\,d^{3/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{19/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((331*d^9*(d*x)^(5/2))/(2560*a) - (231*d^11*(d*x)^(1/2))/(4096*b) + (11*b^2
*d^5*(d*x)^(13/2))/(128*a^3) + (77*b^3*d^3*(d*x)^(17/2))/(4096*a^4) + (313*
b*d^7*(d*x)^(9/2))/(2048*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2
 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (231*d^(
3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(19/4)*b^
(5/4)) - (231*d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8
192*(-a)^(19/4)*b^(5/4))
```

3.723 $\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=387

$$\frac{(dx)^{3/2}}{10ad\,(a+bx^2)^5}+\frac{17(dx)^{3/2}}{160a^2d\,(a+bx^2)^4}+\frac{221(dx)^{3/2}}{1920a^3d\,(a+bx^2)^3}+\frac{663(dx)^{3/2}}{5120a^4d\,(a+bx^2)^2}+\frac{663(dx)^{3/2}}{4096a^5d\,(a+bx^2)}-\frac{663\sqrt{d}\ \text{t}}{}$$

```
[Out] 1/10*(d*x)^(3/2)/a/d/(b*x^2+a)^5+17/160*(d*x)^(3/2)/a^2/d/(b*x^2+a)^4+221/1
920*(d*x)^(3/2)/a^3/d/(b*x^2+a)^3+663/5120*(d*x)^(3/2)/a^4/d/(b*x^2+a)^2+66
3/4096*(d*x)^(3/2)/a^5/d/(b*x^2+a)-663/16384*arctan(1-b^(1/4)*2^(1/2)*(d*x)
^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(21/4)/b^(3/4)*2^(1/2)+663/16384*arctan(1
+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(21/4)/b^(3/4)*2^(1
/2)+663/32768*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*
(d*x)^(1/2))*d^(1/2)/a^(21/4)/b^(3/4)*2^(1/2)-663/32768*ln(a^(1/2)*d^(1/2)+
x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(21/4)/b^(
3/4)*2^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{663\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{21/4}b^{3/4}}+\frac{663\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{21/4}b^{3/4}}+\frac{663\sqrt{d}\,\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\,\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{21/4}b^{3/4}}+\frac{663(dx)^{3/2}}{4096a^5d\,(a+bx^2)}+\frac{663(dx)^{3/2}}{5120a^4d\,(a+bx^2)^2}+\frac{221(dx)^{3/2}}{1920a^3d\,(a+bx^2)^3}+\frac{17(dx)^{3/2}}{160a^2d\,(a+bx^2)^4}+\frac{(dx)^{3/2}}{10ad\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d*x)^(3/2)/(10*a*d*(a + b*x^2)^5) + (17*(d*x)^(3/2))/(160*a^2*d*(a + b*x^2
)^4) + (221*(d*x)^(3/2))/(1920*a^3*d*(a + b*x^2)^3) + (663*(d*x)^(3/2))/(51
20*a^4*d*(a + b*x^2)^2) + (663*(d*x)^(3/2))/(4096*a^5*d*(a + b*x^2)) - (663
*Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*S
qrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
16384*Sqrt[2]*a^(21/4)*b^(3/4)) - (663*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(21/4)*b
^(3/4))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{\left(17b^5\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{\left(221b^4\right)\int \frac{\sqrt{dx}}{\left(ab+b^2x^2\right)^4}\,dx}{320a^2}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{\left(663b^3\right)\int \frac{\sqrt{d}}{(ab+b^2}}{1280a^3}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\,(a+b}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\,(a+b}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\,(a+b}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\,(a+b}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\,(a+b}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\,(a+b}$$

$$= \frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5} + \frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4} + \frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3} + \frac{663(dx)^{3/2}}{5120a^4d\,(a+b}$$

Mathematica [A]

time = 0.34, size = 183, normalized size = 0.47

$$\frac{\sqrt{dx}\left(\frac{4\sqrt[4]{a}\,x^{3/2}(37645a^4+84320a^3bx^2+90610a^2b^2x^4+47736ab^3x^6+9945b^4x^8)}{(a+bx^2)^5}-\frac{9945\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}}-\frac{9945\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{b^{3/4}}\right)}{245760a^{21/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (Sqrt[d*x]*((4*a^(1/4)*x^(3/2)*(37645*a^4 + 84320*a^3*b*x^2 + 90610*a^2*b^2
*x^4 + 47736*a*b^3*x^6 + 9945*b^4*x^8))/(a + b*x^2)^5 - (9945*Sqrt[2]*ArcTa
n[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) - (9945
*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/
b^(3/4)))/(245760*a^(21/4)*Sqrt[x])
```

Maple [A]

time = 0.08, size = 244, normalized size = 0.63

method	result
derivativedivides	$2d^{11}\left(\frac{\frac{7529(dx)^{\frac{3}{2}}}{24576a\,d^2}+\frac{527b(dx)^{\frac{7}{2}}}{768a^2d^4}+\frac{9061b^2(dx)^{\frac{11}{2}}}{12288a^3d^6}+\frac{1989b^3(dx)^{\frac{15}{2}}}{5120a^4d^8}+\frac{663b^4(dx)^{\frac{19}{2}}}{8192a^5d^{10}}}{(d^2x^2b+a\,d^2)^5}+\frac{663\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}\cdots}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}\cdots}\right)\cdots\right.}{\cdots}\right.$
default	$2d^{11}\left(\frac{\frac{7529(dx)^{\frac{3}{2}}}{24576a\,d^2}+\frac{527b(dx)^{\frac{7}{2}}}{768a^2d^4}+\frac{9061b^2(dx)^{\frac{11}{2}}}{12288a^3d^6}+\frac{1989b^3(dx)^{\frac{15}{2}}}{5120a^4d^8}+\frac{663b^4(dx)^{\frac{19}{2}}}{8192a^5d^{10}}}{(d^2x^2b+a\,d^2)^5}+\frac{663\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}\cdots}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}\cdots}\right)\cdots\right.}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((7529/24576/a/d^2*(d*x)^(3/2)+527/768/a^2*b/d^4*(d*x)^(7/2)+9061/12
288/a^3/d^6*b^2*(d*x)^(11/2)+1989/5120/a^4*b^3/d^8*(d*x)^(15/2)+663/8192/a^
5/d^10*b^4*(d*x)^(19/2))/(b*d^2*x^2+a*d^2)^5+663/65536/a^5/d^10/b/(a*d^2/b)
^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
)/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/
2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1
/2)-1)))
```

Maxima [A]

time = 0.52, size = 377, normalized size = 0.97

$$\frac{8\left(9945\,(dx)^{\frac{19}{2}}b^{4}d^{2}+47736\,(dx)^{\frac{15}{2}}ab^{3}d^{4}+90610\,(dx)^{\frac{11}{2}}a^{2}b^{2}d^{6}+84320\,(dx)^{\frac{7}{2}}a^{3}bd^{8}+37645\,(dx)^{\frac{3}{2}}a^{4}d^{10}\right)}{a^{5}b^{5}d^{10}x^{10}+5\,a^{6}b^{4}d^{10}x^{8}+10\,a^{7}b^{3}d^{10}x^{6}+10\,a^{8}b^{2}d^{10}x^{4}+5\,a^{9}bd^{10}x^{2}+a^{10}d^{10}}+\frac{9945\,d^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^{5}}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/491520*(8*(9945*(d*x)^(19/2)*b^4*d^2 + 47736*(d*x)^(15/2)*a*b^3*d^4 + 906
10*(d*x)^(11/2)*a^2*b^2*d^6 + 84320*(d*x)^(7/2)*a^3*b*d^8 + 37645*(d*x)^(3/
2)*a^4*d^10)/(a^5*b^5*d^10*x^10 + 5*a^6*b^4*d^10*x^8 + 10*a^7*b^3*d^10*x^6
+ 10*a^8*b^2*d^10*x^4 + 5*a^9*b*d^10*x^2 + a^10*d^10) + 9945*d^2*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/s
qrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(s
qrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*
d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b
^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^5)/d
```

Fricas [A]

time = 0.38, size = 469, normalized size = 1.21

$$-\frac{39780\left(a^{5}b^{5}x^{10}+5a^{6}b^{4}x^{8}+10a^{7}b^{3}x^{6}+10a^{8}b^{2}x^{4}+5a^{9}bx^{2}+a^{10}\right)\left(-\frac{d^{2}}{a^{21}b^{3}}\right)^{\frac{1}{4}}\arctan\left(-\frac{291434247\sqrt{dx}\,a^{5}bd\left(-\frac{d^{2}}{a^{21}b^{3}}\right)^{\frac{1}{4}}-\sqrt{-84933920324457009\,a^{11}bd^{2}\sqrt{-\frac{d^{2}}{a^{21}b^{3}}}+84933920324457009\,d^{3}x}\,a^{5}b\left(-\frac{d^{2}}{a^{21}b^{3}}\right)^{\frac{1}{4}}}{291434247\,d^{2}}\right)-9945\left(a^{5}b^{5}x^{10}+5a^{6}b^{4}x^{8}+10a^{7}b^{3}x^{6}+10a^{8}b^{2}x^{4}+5a^{9}bx^{2}+a^{10}\right)\left(-\frac{d^{2}}{a^{21}b^{3}}\right)^{\frac{1}{4}}\log\left(291434247\,a^{16}b^{2}\left(-\frac{d^{2}}{a^{21}b^{3}}\right)^{\frac{3}{4}}+291434247\sqrt{dx}\,d\right)+9945\left(a^{5}b^{5}x^{10}+5a^{6}b^{4}x^{8}+10a^{7}b^{3}x^{6}+10a^{8}b^{2}x^{4}+5a^{9}bx^{2}+a^{10}\right)\left(-\frac{d^{2}}{a^{21}b^{3}}\right)^{\frac{1}{4}}\log\left(-291434247\,a^{16}b^{2}\left(-\frac{d^{2}}{a^{21}b^{3}}\right)^{\frac{3}{4}}+291434247\sqrt{dx}\,d\right)-4\left(9945b^{4}x^{9}+47736ab^{3}x^{7}+90610a^{2}b^{2}x^{5}+84320a^{3}bx^{3}+37645a^{4}x\right)\sqrt{dx}}{245760\left(a^{5}b^{5}x^{10}+5a^{6}b^{4}x^{8}+10a^{7}b^{3}x^{6}+10a^{8}b^{2}x^{4}+5a^{9}bx^{2}+a^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(39780*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^
2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3))^(1/4)*arctan(-1/291434247*(29
1434247*sqrt(d*x)*a^5*b*d*(-d^2/(a^21*b^3))^(1/4) - sqrt(-84933920324457009
*a^11*b*d^2*sqrt(-d^2/(a^21*b^3)) + 84933920324457009*d^3*x)*a^5*b*(-d^2/(a
^21*b^3))^(1/4))/d^2) - 9945*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6
 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3))^(1/4)*log(2914342
47*a^16*b^2*(-d^2/(a^21*b^3))^(3/4) + 291434247*sqrt(d*x)*d) + 9945*(a^5*b^
5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^
10)*(-d^2/(a^21*b^3))^(1/4)*log(-291434247*a^16*b^2*(-d^2/(a^21*b^3))^(3/4)
 + 291434247*sqrt(d*x)*d) - 4*(9945*b^4*x^9 + 47736*a*b^3*x^7 + 90610*a^2*b
^2*x^5 + 84320*a^3*b*x^3 + 37645*a^4*x)*sqrt(d*x))/(a^5*b^5*x^10 + 5*a^6*b^
4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)
```

Sympy [A]

time = 33.31, size = 547, normalized size = 1.41

$$\frac{75290a^{4}d^{19}(dx)^{\frac{3}{2}}}{122880a^{10}d^{20}+614400a^{9}bd^{20}x^{2}+1228800a^{8}b^{2}d^{20}x^{4}+1228800a^{7}b^{3}d^{20}x^{6}+614400a^{6}b^{4}d^{20}x^{8}+122880a^{5}b^{5}d^{20}x^{10}}+\frac{168640a^{3}bd^{17}(dx)^{\frac{7}{2}}}{122880a^{10}d^{20}+614400a^{9}bd^{20}x^{2}+1228800a^{8}b^{2}d^{20}x^{4}+1228800a^{7}b^{3}d^{20}x^{6}+614400a^{6}b^{4}d^{20}x^{8}+122880a^{5}b^{5}d^{20}x^{10}}+\frac{181220a^{2}b^{2}d^{15}(dx)^{\frac{11}{2}}}{122880a^{10}d^{20}+614400a^{9}bd^{20}x^{2}+1228800a^{8}b^{2}d^{20}x^{4}+1228800a^{7}b^{3}d^{20}x^{6}+614400a^{6}b^{4}d^{20}x^{8}+122880a^{5}b^{5}d^{20}x^{10}}+\frac{95472ab^{3}d^{13}(dx)^{\frac{15}{2}}}{122880a^{10}d^{20}+614400a^{9}bd^{20}x^{2}+1228800a^{8}b^{2}d^{20}x^{4}+1228800a^{7}b^{3}d^{20}x^{6}+614400a^{6}b^{4}d^{20}x^{8}+122880a^{5}b^{5}d^{20}x^{10}}+\frac{19890b^{4}d^{11}(dx)^{\frac{19}{2}}}{122880a^{10}d^{20}+614400a^{9}bd^{20}x^{2}+1228800a^{8}b^{2}d^{20}x^{4}+1228800a^{7}b^{3}d^{20}x^{6}+614400a^{6}b^{4}d^{20}x^{8}+122880a^{5}b^{5}d^{20}x^{10}}+2d^{11}\operatorname{RootSum}\left(1152921504606846976t^{4}a^{21}b^{3}d^{42}+193220905761,\left(t\mapsto t\log\left(\frac{35184372088832t^{3}a^{16}b^{2}d^{32}}{291434247}+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 75290*a**4*d**19*(d*x)**(3/2)/(122880*a**10*d**20 + 614400*a**9*b*d**20*x**
2 + 1228800*a**8*b**2*d**20*x**4 + 1228800*a**7*b**3*d**20*x**6 + 614400*a*
*6*b**4*d**20*x**8 + 122880*a**5*b**5*d**20*x**10) + 168640*a**3*b*d**17*(d
*x)**(7/2)/(122880*a**10*d**20 + 614400*a**9*b*d**20*x**2 + 1228800*a**8*b*
*2*d**20*x**4 + 1228800*a**7*b**3*d**20*x**6 + 614400*a**6*b**4*d**20*x**8
+ 122880*a**5*b**5*d**20*x**10) + 181220*a**2*b**2*d**15*(d*x)**(11/2)/(122
880*a**10*d**20 + 614400*a**9*b*d**20*x**2 + 1228800*a**8*b**2*d**20*x**4 +
 1228800*a**7*b**3*d**20*x**6 + 614400*a**6*b**4*d**20*x**8 + 122880*a**5*b
**5*d**20*x**10) + 95472*a*b**3*d**13*(d*x)**(15/2)/(122880*a**10*d**20 + 6
14400*a**9*b*d**20*x**2 + 1228800*a**8*b**2*d**20*x**4 + 1228800*a**7*b**3*
d**20*x**6 + 614400*a**6*b**4*d**20*x**8 + 122880*a**5*b**5*d**20*x**10) +
19890*b**4*d**11*(d*x)**(19/2)/(122880*a**10*d**20 + 614400*a**9*b*d**20*x*
*2 + 1228800*a**8*b**2*d**20*x**4 + 1228800*a**7*b**3*d**20*x**6 + 614400*a
**6*b**4*d**20*x**8 + 122880*a**5*b**5*d**20*x**10) + 2*d**11*RootSum(11529
21504606846976*_t**4*a**21*b**3*d**42 + 193220905761, Lambda(_t, _t*log(351
84372088832*_t**3*a**16*b**2*d**32/291434247 + sqrt(d*x))))
```

Giac [A]
time = 7.66, size = 340, normalized size = 0.88

$$\frac{\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^3}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^3}-\frac{9945\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^6b^3}+\frac{9945\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^6b^3}+\frac{8\left(9945\sqrt{dx}\,b^4d^{11}x^9+47736\sqrt{dx}\,ab^3d^{11}x^7+90610\sqrt{dx}\,a^2b^2d^{11}x^5+84320\sqrt{dx}\,a^3bd^{11}x^3+37645\sqrt{dx}\,a^4d^{11}x\right)}{\left(bd^2x^2+ad^2\right)^5a^5}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*(19890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b^3) + 19890*sqrt(2)*(a*b^3
*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^6*b^3) - 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^3) + 9945*sqrt(2)*(a*b^3*
d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
6*b^3) + 8*(9945*sqrt(d*x)*b^4*d^11*x^9 + 47736*sqrt(d*x)*a*b^3*d^11*x^7 +
90610*sqrt(d*x)*a^2*b^2*d^11*x^5 + 84320*sqrt(d*x)*a^3*b*d^11*x^3 + 37645*s
qrt(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a^5))/d
```

Mupad [B]
time = 4.25, size = 210, normalized size = 0.54

$$\frac{\frac{7529\,d^9\,{\left(d\,x\right)}^{3/2}}{12288\,a}+\frac{9061\,b^2\,d^5\,{\left(d\,x\right)}^{11/2}}{6144\,a^3}+\frac{1989\,b^3\,d^3\,{\left(d\,x\right)}^{15/2}}{2560\,a^4}+\frac{527\,b\,d^7\,{\left(d\,x\right)}^{7/2}}{384\,a^2}+\frac{663\,b^4\,d\,{\left(d\,x\right)}^{19/2}}{4096\,a^5}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}-\frac{663\,\sqrt{d}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{21/4}\,b^{3/4}}+\frac{663\,\sqrt{d}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{8192\,{\left(-a\right)}^{21/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] ((7529*d^9*(d*x)^(3/2))/(12288*a) + (9061*b^2*d^5*(d*x)^(11/2))/(6144*a^3)
+ (1989*b^3*d^3*(d*x)^(15/2))/(2560*a^4) + (527*b*d^7*(d*x)^(7/2))/(384*a^2
) + (663*b^4*d*(d*x)^(19/2))/(4096*a^5))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*
b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6)
- (663*d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)
^(21/4)*b^(3/4)) + (663*d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(
1/2))))/(8192*(-a)^(21/4)*b^(3/4))
```

3.724 $\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=387

$$\frac{\sqrt{dx}}{10ad\,(a+bx^2)^5}+\frac{19\sqrt{dx}}{160a^2d\,(a+bx^2)^4}+\frac{19\sqrt{dx}}{128a^3d\,(a+bx^2)^3}+\frac{209\sqrt{dx}}{1024a^4d\,(a+bx^2)^2}+\frac{1463\sqrt{dx}}{4096a^5d\,(a+bx^2)}-\frac{4389\tan^{-}}{8192}$$

```
[Out] -4389/16384*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(23/4)/
b^(1/4)*2^(1/2)/d^(1/2)+4389/16384*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(
1/4)/d^(1/2))/a^(23/4)/b^(1/4)*2^(1/2)/d^(1/2)-4389/32768*ln(a^(1/2)*d^(1/2
)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(23/4)/b^(1/4)*2
^(1/2)/d^(1/2)+4389/32768*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1
/4)*2^(1/2)*(d*x)^(1/2))/a^(23/4)/b^(1/4)*2^(1/2)/d^(1/2)+1/10*(d*x)^(1/2)/
a/d/(b*x^2+a)^5+19/160*(d*x)^(1/2)/a^2/d/(b*x^2+a)^4+19/128*(d*x)^(1/2)/a^3
/d/(b*x^2+a)^3+209/1024*(d*x)^(1/2)/a^4/d/(b*x^2+a)^2+1463/4096*(d*x)^(1/2)
/a^5/d/(b*x^2+a)
```

Rubi [A]
time = 0.29, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {28, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{4389\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{4389\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{23/4}\sqrt[4]{b}\sqrt{d}}-\frac{4389\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{4389\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{1463\sqrt{dx}}{4096a^5d\,(a+bx^2)}+\frac{209\sqrt{dx}}{1024a^4d\,(a+bx^2)^2}+\frac{19\sqrt{dx}}{128a^3d\,(a+bx^2)^3}+\frac{19\sqrt{dx}}{160a^2d\,(a+bx^2)^4}+\frac{\sqrt{dx}}{10ad\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] Sqrt[d*x]/(10*a*d*(a + b*x^2)^5) + (19*Sqrt[d*x])/(160*a^2*d*(a + b*x^2)^4)
 + (19*Sqrt[d*x])/(128*a^3*d*(a + b*x^2)^3) + (209*Sqrt[d*x])/(1024*a^4*d*(
a + b*x^2)^2) + (1463*Sqrt[d*x])/(4096*a^5*d*(a + b*x^2)) - (4389*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(23/4)*b^
(1/4)*Sqrt[d]) + (4389*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(8192*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d]) - (4389*Log[Sqrt[a]*Sqrt[d]
+ Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^
(23/4)*b^(1/4)*Sqrt[d]) + (4389*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\,\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6 \int \frac{1}{\sqrt{dx}\,\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{(19b^5)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^5}\,dx}{20a}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{(57b^4)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^4}\,dx}{64a^2}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{(209b^3)\int \cdots}{\cdots}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{\cdots}}{1024a^4d\,(\cdots}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{\cdots}}{1024a^4d\,(\cdots}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{\cdots}}{1024a^4d\,(\cdots}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{\cdots}}{1024a^4d\,(\cdots}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{\cdots}}{1024a^4d\,(\cdots}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{\cdots}}{1024a^4d\,(\cdots}$$

$$= \frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5} + \frac{19\sqrt{dx}}{160a^2d\left(a+bx^2\right)^4} + \frac{19\sqrt{dx}}{128a^3d\left(a+bx^2\right)^3} + \frac{209\sqrt{\cdots}}{1024a^4d\,(\cdots}$$

Mathematica [A]

time = 0.33, size = 183, normalized size = 0.47

$$\frac{\sqrt{x}\left(\frac{4a^{3/4}\sqrt{x}\,(19015a^4+50312a^3bx^2+59470a^2b^2x^4+33440ab^3x^6+7315b^4x^8)}{(a+bx^2)^5}-\frac{21945\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}}+\frac{21945\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{b}}\right)}{81920a^{23/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (Sqrt[x]*((4*a^(3/4)*Sqrt[x]*(19015*a^4 + 50312*a^3*b*x^2 + 59470*a^2*b^2*x
^4 + 33440*a*b^3*x^6 + 7315*b^4*x^8))/(a + b*x^2)^5 - (21945*Sqrt[2]*ArcTan
[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (21945
*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/
b^(1/4)))/(81920*a^(23/4)*Sqrt[d*x])
```

Maple [A]

time = 0.08, size = 241, normalized size = 0.62

method	result
derivativedivides	$2d^{11}\left(\frac{\frac{3803\sqrt{dx}}{8192a\,d^2}+\frac{6289b(dx)^{\frac{5}{2}}}{5120a^2d^4}+\frac{5947b^2(dx)^{\frac{9}{2}}}{4096a^3d^6}+\frac{209b^3(dx)^{\frac{13}{2}}}{256a^4d^8}+\frac{1463b^4(dx)^{\frac{17}{2}}}{8192a^5d^{10}}}{(d^2x^2b+a\,d^2)^5}+\frac{4389\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{\frac{3803\sqrt{dx}}{8192a\,d^2}+\frac{6289b(dx)^{\frac{5}{2}}}{5120a^2d^4}+\frac{5947b^2(dx)^{\frac{9}{2}}}{4096a^3d^6}+\frac{209b^3(dx)^{\frac{13}{2}}}{256a^4d^8}+\frac{1463b^4(dx)^{\frac{17}{2}}}{8192a^5d^{10}}}{(d^2x^2b+a\,d^2)^5}+\frac{4389\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*((3803/8192/a/d^2*(d*x)^(1/2)+6289/5120/a^2/d^4*b*(d*x)^(5/2)+5947/4
096/a^3/d^6*b^2*(d*x)^(9/2)+209/256/a^4/d^8*b^3*(d*x)^(13/2)+1463/8192/a^5/
d^10*b^4*(d*x)^(17/2))/(b*d^2*x^2+a*d^2)^5+4389/65536/a^6/d^12*(a*d^2/b)^(1
/4)*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(
d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)
-1)))
```

Maxima [A]
time = 0.52, size = 382, normalized size = 0.99

$$\frac{\frac{8\left(7315\,(dx)^{\frac{17}{2}}b^4d^2+33440\,(dx)^{\frac{13}{2}}ab^3d^4+59470\,(dx)^{\frac{9}{2}}a^2b^2d^6+50312\,(dx)^{\frac{5}{2}}a^3bd^8+19015\,\sqrt{dx}\,a^4d^{10}\right)}{a^5b^5d^{10}x^{10}+5\,a^6b^4d^{10}x^8+10\,a^7b^3d^{10}x^6+10\,a^8b^2d^{10}x^4+5\,a^9bd^{10}x^2+a^{10}d^{10}}+\frac{21945\left(\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\,\sqrt{a}}\right)}{a^5}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/163840*(8*(7315*(d*x)^(17/2)*b^4*d^2 + 33440*(d*x)^(13/2)*a*b^3*d^4 + 594
70*(d*x)^(9/2)*a^2*b^2*d^6 + 50312*(d*x)^(5/2)*a^3*b*d^8 + 19015*sqrt(d*x)*
a^4*d^10)/(a^5*b^5*d^10*x^10 + 5*a^6*b^4*d^10*x^8 + 10*a^7*b^3*d^10*x^6 + 1
0*a^8*b^2*d^10*x^4 + 5*a^9*b*d^10*x^2 + a^10*d^10) + 21945*(sqrt(2)*d^2*log
(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt
(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d*arctan(1/2
*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)
*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sq
rt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/a^5)/d
```

Fricas [A]
time = 0.38, size = 475, normalized size = 1.23

$$\frac{87780\left(a^5b^5dx^{10}+5\,a^6b^4dx^8+10\,a^7b^3dx^6+10\,a^8b^2dx^4+5\,a^9bdx^2+a^{10}d\right)\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^{12}d^2\sqrt{-\frac{1}{a^{23}bd^2}}+dx}\,a^{17}bd\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^{17}bd\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{3}{4}}\right)+21945\left(a^5b^5dx^{10}+5\,a^6b^4dx^8+10\,a^7b^3dx^6+10\,a^8b^2dx^4+5\,a^9bdx^2+a^{10}d\right)\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}\log\left(a^6d\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-21945\left(a^5b^5dx^{10}+5\,a^6b^4dx^8+10\,a^7b^3dx^6+10\,a^8b^2dx^4+5\,a^9bdx^2+a^{10}d\right)\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}\log\left(-a^6d\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4\left(7315\,b^4x^8+33440\,ab^3x^6+59470\,a^2b^2x^4+50312\,a^3bx^2+19015\,a^4\right)\sqrt{dx}}{81920\left(a^5b^5dx^{10}+5\,a^6b^4dx^8+10\,a^7b^3dx^6+10\,a^8b^2dx^4+5\,a^9bdx^2+a^{10}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/81920*(87780*(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^
8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)*(-1/(a^23*b*d^2))^(1/4)*arctan(sqrt(a
^12*d^2*sqrt(-1/(a^23*b*d^2)) + d*x)*a^17*b*d*(-1/(a^23*b*d^2))^(3/4) - sqr
t(d*x)*a^17*b*d*(-1/(a^23*b*d^2))^(3/4)) + 21945*(a^5*b^5*d*x^10 + 5*a^6*b^
4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)*(-1
/(a^23*b*d^2))^(1/4)*log(a^6*d*(-1/(a^23*b*d^2))^(1/4) + sqrt(d*x)) - 21945
*(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 +
5*a^9*b*d*x^2 + a^10*d)*(-1/(a^23*b*d^2))^(1/4)*log(-a^6*d*(-1/(a^23*b*d^2)
)^(1/4) + sqrt(d*x)) + 4*(7315*b^4*x^8 + 33440*a*b^3*x^6 + 59470*a^2*b^2*x^
4 + 50312*a^3*b*x^2 + 19015*a^4)*sqrt(d*x))/(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x
^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(a+bx^2\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**6), x)
```

Giac [A]
time = 4.90, size = 346, normalized size = 0.89

$$\frac{4389\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6bd}+\frac{4389\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6bd}+\frac{4389\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^6bd}-\frac{4389\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^6bd}+\frac{7315\,\sqrt{dx}\,b^4d^9x^8+33440\,\sqrt{dx}\,ab^3d^9x^6+59470\,\sqrt{dx}\,a^2b^2d^9x^4+50312\,\sqrt{dx}\,a^3bd^9x^2+19015\,\sqrt{dx}\,a^4d^9}{20480\,(bd^2x^2+ad^2)^5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4389/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d) + 4389/16384*sqrt(2)*(a*b^3
*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/(a^6*b*d) + 4389/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b*d) - 4389/32768*sqr
t(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^6*b*d) + 1/20480*(7315*sqrt(d*x)*b^4*d^9*x^8 + 33440*sqrt(d*x)*
a*b^3*d^9*x^6 + 59470*sqrt(d*x)*a^2*b^2*d^9*x^4 + 50312*sqrt(d*x)*a^3*b*d^9
*x^2 + 19015*sqrt(d*x)*a^4*d^9)/((b*d^2*x^2 + a*d^2)^5*a^5)
```

Mupad [B]
time = 4.29, size = 210, normalized size = 0.54

$$\frac{\frac{3803\,d^9\,\sqrt{d\,x}}{4096\,a}+\frac{5947\,b^2\,d^5\,{(d\,x)}^{9/2}}{2048\,a^3}+\frac{209\,b^3\,d^3\,{(d\,x)}^{13/2}}{128\,a^4}+\frac{6289\,b\,d^7\,{(d\,x)}^{5/2}}{2560\,a^2}+\frac{1463\,b^4\,d\,{(d\,x)}^{17/2}}{4096\,a^5}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}+\frac{4389\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{23/4}\,b^{1/4}\,\sqrt{d}}+\frac{4389\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{23/4}\,b^{1/4}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((3803*d^9*(d*x)^(1/2))/(4096*a) + (5947*b^2*d^5*(d*x)^(9/2))/(2048*a^3) +
(209*b^3*d^3*(d*x)^(13/2))/(128*a^4) + (6289*b*d^7*(d*x)^(5/2))/(2560*a^2)
+ (1463*b^4*d*(d*x)^(17/2))/(4096*a^5))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b
*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) +
 (4389*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(23/4)*
b^(1/4)*d^(1/2)) + (4389*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))
/(8192*(-a)^(23/4)*b^(1/4)*d^(1/2))
```

3.725 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=404

$$-\frac{13923}{4096a^6d\sqrt{dx}}+\frac{1}{10ad\sqrt{dx}\,(a+bx^2)^5}+\frac{21}{160a^2d\sqrt{dx}\,(a+bx^2)^4}+\frac{119}{640a^3d\sqrt{dx}\,(a+bx^2)^3}+\frac{1547}{5120a^4d\sqrt{dx}\,(}$$

```
[Out] 13923/16384*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a
^(25/4)/d^(3/2)*2^(1/2)-13923/16384*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^
(1/2)/a^(1/4)/d^(1/2))/a^(25/4)/d^(3/2)*2^(1/2)-13923/32768*b^(1/4)*ln(a^(1
/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(25/4)
/d^(3/2)*2^(1/2)+13923/32768*b^(1/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a
^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(25/4)/d^(3/2)*2^(1/2)-13923/4096/a^6
/d/(d*x)^(1/2)+1/10/a/d/(b*x^2+a)^5/(d*x)^(1/2)+21/160/a^2/d/(b*x^2+a)^4/(d
*x)^(1/2)+119/640/a^3/d/(b*x^2+a)^3/(d*x)^(1/2)+1547/5120/a^4/d/(b*x^2+a)^2
/(d*x)^(1/2)+13923/20480/a^5/d/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{13923\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\,a^{25/4}d^{3/2}}-\frac{13923\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\,a^{25/4}d^{3/2}}-\frac{13923\sqrt[4]{b}\,\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\,\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{16384\sqrt{2}\,a^{25/4}d^{3/2}}-\frac{13923}{4096a^6d\sqrt{dx}}+\frac{13923}{20480a^5d\sqrt{dx}\,(a+bx^2)}+\frac{1547}{5120a^4d\sqrt{dx}\,(a+bx^2)^2}+\frac{119}{640a^3d\sqrt{dx}\,(a+bx^2)^3}+\frac{21}{160a^2d\sqrt{dx}\,(a+bx^2)^4}+\frac{1}{10ad\sqrt{dx}\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -13923/(4096*a^6*d*Sqrt[d*x]) + 1/(10*a*d*Sqrt[d*x]*(a + b*x^2)^5) + 21/(16
0*a^2*d*Sqrt[d*x]*(a + b*x^2)^4) + 119/(640*a^3*d*Sqrt[d*x]*(a + b*x^2)^3)
+ 1547/(5120*a^4*d*Sqrt[d*x]*(a + b*x^2)^2) + 13923/(20480*a^5*d*Sqrt[d*x]*
(a + b*x^2)) + (13923*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/2)) - (13923*b^(1/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/
2)) - (13923*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(25/4)*d^(3/2)) + (13923*b^(1/4)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(16384*Sqrt[2]*a^(25/4)*d^(3/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad\sqrt{dx}\,\left(a+bx^2\right)^5} + \frac{\left(21b^5\right)\int \frac{1}{(dx)^{3/2}\left(ab+b^2x^2\right)^5}\,dx}{20a}$$

$$= \frac{1}{10ad\sqrt{dx}\,\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\,\left(a+bx^2\right)^4} + \frac{\left(357b^4\right)\int \frac{1}{(dx)^{3/2}(ab+b}}{320a^2}$$

$$= \frac{1}{10ad\sqrt{dx}\,\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\,\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\,(a+bx}$$

$$= \frac{1}{10ad\sqrt{dx}\,\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\,\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\,(a+bx}$$

$$= \frac{1}{10ad\sqrt{dx}\,\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\,\left(a+bx^2\right)^4} + \frac{119}{640a^3d\sqrt{dx}\,(a+bx}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\,\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\,\left(a+bx^2\right)^4} + \frac{}{6}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\,\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\,\left(a+bx^2\right)^4} + \frac{}{6}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\,\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\,\left(a+bx^2\right)^4} + \frac{}{6}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\,\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\,\left(a+bx^2\right)^4} + \frac{}{6}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\,\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\,\left(a+bx^2\right)^4} + \frac{}{6}$$

$$= -\frac{13923}{4096a^6d\sqrt{dx}} + \frac{1}{10ad\sqrt{dx}\,\left(a+bx^2\right)^5} + \frac{21}{160a^2d\sqrt{dx}\,\left(a+bx^2\right)^4} + \frac{}{6}$$

Mathematica [A]

time = 0.75, size = 195, normalized size = 0.48

$$\frac{x\left(-\frac{4\sqrt[4]{a}\,(40960a^5+263515a^4bx^2+590240a^3b^2x^4+634270a^2b^3x^6+334152ab^4x^8+69615b^5x^{10})}{(a+bx^2)^5}+69615\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+69615\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{81920a^{25/4}(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (x*((-4*a^(1/4)*(40960*a^5 + 263515*a^4*b*x^2 + 590240*a^3*b^2*x^4 + 634270
*a^2*b^3*x^6 + 334152*a*b^4*x^8 + 69615*b^5*x^10))/(a + b*x^2)^5 + 69615*Sq
rt[2]*b^(1/4)*Sqrt[x]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x])] + 69615*Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(81920*a^(25/4)*(d*x)^(3/2))
```

Maple [A]

time = 0.16, size = 253, normalized size = 0.63

method	result
derivativedivides	$2d^{11}\left(-\frac{1}{a^6 d^{12}\sqrt{dx}}-\frac{b\left(\frac{\frac{11743a^4d^8(dx)^{\frac{3}{2}}}{8192}+\frac{1129a^3b\,d^6(dx)^{\frac{7}{2}}}{256}+\frac{22467a^2d^4b^2(dx)^{\frac{11}{2}}}{4096}+\frac{16169a\,b^3d^2(dx)^{\frac{15}{2}}}{5120}+\frac{5731b^4(dx)}{8192}\right.}{\left(d^2x^2b+a\,d^2\right)^5}\right.}{}\right.$
default	$2d^{11}\left(-\frac{1}{a^6 d^{12}\sqrt{dx}}-\frac{b\left(\frac{\frac{11743a^4d^8(dx)^{\frac{3}{2}}}{8192}+\frac{1129a^3b\,d^6(dx)^{\frac{7}{2}}}{256}+\frac{22467a^2d^4b^2(dx)^{\frac{11}{2}}}{4096}+\frac{16169a\,b^3d^2(dx)^{\frac{15}{2}}}{5120}+\frac{5731b^4(dx)}{8192}\right.}{\left(d^2x^2b+a\,d^2\right)^5}\right.}{}\right.$

risch	$-\frac{2}{a^{6} d \sqrt{dx}}+\frac{-\frac{11743 b\,d^{8} (dx)^{\frac{3}{2}}}{4096 a^{2} \left(d^{2} x^{2} b+a\,d^{2}\right)^{5}}-\frac{1129 b^{2} d^{6} (dx)^{\frac{7}{2}}}{128 a^{3} \left(d^{2} x^{2} b+a\,d^{2}\right)^{5}}-\frac{22467 b^{3} d^{4} (dx)^{\frac{11}{2}}}{2048 a^{4} \left(d^{2} x^{2} b+a\,d^{2}\right)^{5}}-\frac{16169 b^{4} d^{2} (dx)^{\frac{15}{2}}}{2560 a^{5} \left(d^{2} x^{2} b+a\,d^{2}\right)^{5}}-\frac{5731 b^{5}}{4096 a^{6} \left(d^{2}\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*(-1/a^6/d^12/(d*x)^(1/2)-b/a^6/d^12*((11743/8192*a^4*d^8*(d*x)^(3/2)
+1129/256*a^3*b*d^6*(d*x)^(7/2)+22467/4096*a^2*d^4*b^2*(d*x)^(11/2)+16169/5
120*a*b^3*d^2*(d*x)^(15/2)+5731/8192*b^4*(d*x)^(19/2))/(b*d^2*x^2+a*d^2)^5+
13923/65536/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(
a*d^2/b)^(1/4)*(d*x)^(1/2)-1))))
```

Maxima [A]

time = 0.51, size = 388, normalized size = 0.96

$$-\frac{\frac{8\,(69615\,b^{5}d^{10}x^{10}+334152\,ab^{4}d^{10}x^{8}+634270\,a^{2}b^{3}d^{10}x^{6}+590240\,a^{3}b^{2}d^{10}x^{4}+263515\,a^{4}bd^{10}x^{2}+40960\,a^{5}d^{10})}{(dx)^{\frac{21}{2}}a^{6}b^{5}+5\,(dx)^{\frac{17}{2}}a^{7}b^{4}d^{2}+10\,(dx)^{\frac{13}{2}}a^{8}b^{3}d^{4}+10\,(dx)^{\frac{9}{2}}a^{9}b^{2}d^{6}+5\,(dx)^{\frac{5}{2}}a^{10}bd^{8}+\sqrt{dx}\,a^{11}d^{10}}+\frac{69615\,b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}\,d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^{6}}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/163840*(8*(69615*b^5*d^10*x^10 + 334152*a*b^4*d^10*x^8 + 634270*a^2*b^3*
d^10*x^6 + 590240*a^3*b^2*d^10*x^4 + 263515*a^4*b*d^10*x^2 + 40960*a^5*d^10
)/((d*x)^(21/2)*a^6*b^5 + 5*(d*x)^(17/2)*a^7*b^4*d^2 + 10*(d*x)^(13/2)*a^8*
b^3*d^4 + 10*(d*x)^(9/2)*a^9*b^2*d^6 + 5*(d*x)^(5/2)*a^10*b*d^8 + sqrt(d*x)
*a^11*d^10) + 69615*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*
b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(
b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/
4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)
*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/
4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)
*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^6)
/d
```

Fricas [A]

time = 0.37, size = 544, normalized size = 1.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(278460*(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7
+ 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)
*arctan(-1/2698972561467*(2698972561467*sqrt(d*x)*a^6*b*d*(-b/(a^25*d^6))^(
1/4) - sqrt(-7284452887551739093192089*a^13*b*d^4*sqrt(-b/(a^25*d^6)) + 728
4452887551739093192089*b^2*d*x)*a^6*d*(-b/(a^25*d^6))^(1/4))/b) - 69615*(a^
6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^
5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)*log(2698972561467*
a^19*d^5*(-b/(a^25*d^6))^(3/4) + 2698972561467*sqrt(d*x)*b) + 69615*(a^6*b^
5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^5 +
5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/4)*log(-2698972561467*a^1
9*d^5*(-b/(a^25*d^6))^(3/4) + 2698972561467*sqrt(d*x)*b) - 4*(69615*b^5*x^1
0 + 334152*a*b^4*x^8 + 634270*a^2*b^3*x^6 + 590240*a^3*b^2*x^4 + 263515*a^4
*b*x^2 + 40960*a^5)*sqrt(d*x))/(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a
^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\,(a+bx^2)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**6), x)
```

Giac [A]
time = 3.83, size = 365, normalized size = 0.90

$$-\frac{\frac{327680}{\sqrt{dx}\,a^6}+\frac{139230\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^7b^2d^2}+\frac{139230\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^7b^2d^2}-\frac{69615\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^7b^2d^2}+\frac{69615\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^7b^2d^2}+\frac{8\left(28655\sqrt{dx}\,b^5d^9x^9+129352\sqrt{dx}\,ab^4d^9x^7+224670\sqrt{dx}\,a^2b^3d^9x^5+180640\sqrt{dx}\,a^3b^2d^9x^3+58715\sqrt{dx}\,a^4bd^9x\right)}{(bd^2x^2+ad^2)^5a^6}}{163840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] -1/163840*(327680/(sqrt(d*x)*a^6) + 139230*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*
b^2*d^2) + 139230*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b^2*d^2) - 69615*sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^7*b^2*d^2) + 69615*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*b^2*d^2) + 8*(28655*sqrt(d*x)*b
^5*d^9*x^9 + 129352*sqrt(d*x)*a*b^4*d^9*x^7 + 224670*sqrt(d*x)*a^2*b^3*d^9*
```

```
x^5 + 180640*sqrt(d*x)*a^3*b^2*d^9*x^3 + 58715*sqrt(d*x)*a^4*b*d^9*x)/((b*d
^2*x^2 + a*d^2)^5*a^6))/d
```

Mupad [B]

time = 0.21, size = 226, normalized size = 0.56

$$\frac{13923\,{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{25/4}\,d^{3/2}}-\frac{13923\,{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{25/4}\,d^{3/2}}-\frac{\frac{2\,d^9}{a}+\frac{52703\,b\,d^9\,x^2}{4096\,a^2}+\frac{3689\,b^2\,d^9\,x^4}{128\,a^3}+\frac{63427\,b^3\,d^9\,x^6}{2048\,a^4}+\frac{41769\,b^4\,d^9\,x^8}{2560\,a^5}+\frac{13923\,b^5\,d^9\,x^{10}}{4096\,a^6}}{b^5\,{\left(d\,x\right)}^{21/2}+a^5\,d^{10}\,\sqrt{d\,x}+10\,a^3\,b^2\,d^6\,{\left(d\,x\right)}^{9/2}+10\,a^2\,b^3\,d^4\,{\left(d\,x\right)}^{13/2}+5\,a^4\,b\,d^8\,{\left(d\,x\right)}^{5/2}+5\,a\,b^4\,d^2\,{\left(d\,x\right)}^{17/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] (13923*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*
a^(25/4)*d^(3/2)) - (13923*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4
)*d^(1/2))))/(8192*a^(25/4)*d^(3/2)) - ((2*d^9)/a + (52703*b*d^9*x^2)/(4096
*a^2) + (3689*b^2*d^9*x^4)/(128*a^3) + (63427*b^3*d^9*x^6)/(2048*a^4) + (41
769*b^4*d^9*x^8)/(2560*a^5) + (13923*b^5*d^9*x^10)/(4096*a^6))/(b^5*(d*x)^(
21/2) + a^5*d^10*(d*x)^(1/2) + 10*a^3*b^2*d^6*(d*x)^(9/2) + 10*a^2*b^3*d^4*
(d*x)^(13/2) + 5*a^4*b*d^8*(d*x)^(5/2) + 5*a*b^4*d^2*(d*x)^(17/2))
```

3.726 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=404

$$-\frac{33649}{12288a^6d(dx)^{3/2}}+\frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5}+\frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4}+\frac{437}{1920a^3d(dx)^{3/2}\left(a+bx^2\right)^3}+\frac{}{1024a^4}$$

```
[Out] -33649/12288/a^6/d/(d*x)^(3/2)+1/10/a/d/(d*x)^(3/2)/(b*x^2+a)^5+23/160/a^2/
d/(d*x)^(3/2)/(b*x^2+a)^4+437/1920/a^3/d/(d*x)^(3/2)/(b*x^2+a)^3+437/1024/a
^4/d/(d*x)^(3/2)/(b*x^2+a)^2+4807/4096/a^5/d/(d*x)^(3/2)/(b*x^2+a)+33649/16
384*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(27/4)/
d^(5/2)*2^(1/2)-33649/16384*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^
(1/4)/d^(1/2))/a^(27/4)/d^(5/2)*2^(1/2)+33649/32768*b^(3/4)*ln(a^(1/2)*d^(1
/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(27/4)/d^(5/2)
*2^(1/2)-33649/32768*b^(3/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b
^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(27/4)/d^(5/2)*2^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {28, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{33649b^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{27/4}d^{5/2}}+\frac{33649b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{27/4}d^{5/2}}-\frac{33649}{12288a^6d(dx)^{3/2}}+\frac{4807}{4096a^5d(dx)^{3/2}\left(a+bx^2\right)}+\frac{437}{1024a^4d(dx)^{3/2}\left(a+bx^2\right)^2}+\frac{437}{1920a^3d(dx)^{3/2}\left(a+bx^2\right)^3}+\frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4}+\frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -33649/(12288*a^6*d*(d*x)^(3/2)) + 1/(10*a*d*(d*x)^(3/2)*(a + b*x^2)^5) + 2
3/(160*a^2*d*(d*x)^(3/2)*(a + b*x^2)^4) + 437/(1920*a^3*d*(d*x)^(3/2)*(a +
b*x^2)^3) + 437/(1024*a^4*d*(d*x)^(3/2)*(a + b*x^2)^2) + 4807/(4096*a^5*d*(
d*x)^(3/2)*(a + b*x^2)) + (33649*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(27/4)*d^(5/2)) - (33649*b^(3/4)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(
27/4)*d^(5/2)) + (33649*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(5/2)) - (3364
9*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(5/2))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{5/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{(23b^5)\int \frac{1}{(dx)^{5/2}(ab+b^2x^2)^5}\,dx}{20a}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{(437b^4)\int \frac{1}{(dx)^{5/2}(a}}{320a^2}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\,(a}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\,(a}$$

$$= \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} + \frac{437}{1920a^3d(dx)^{3/2}\,(a}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)}$$

$$= -\frac{33649}{12288a^6d(dx)^{3/2}} + \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)}$$

Mathematica [A]

time = 0.73, size = 195, normalized size = 0.48

$$\frac{x\left(-\frac{4a^{3/4}\left(40960a^5+437345a^4bx^2+1157176a^3b^2x^4+1367810a^2b^3x^6+769120ab^4x^8+168245b^5x^{10}\right)}{(a+bx^2)^5}+504735\sqrt{2}\,b^{3/4}x^{3/2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-504735\sqrt{2}\,b^{3/4}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{245760a^{27/4}(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (x*((-4*a^(3/4)*(40960*a^5 + 437345*a^4*b*x^2 + 1157176*a^3*b^2*x^4 + 13678
10*a^2*b^3*x^6 + 769120*a*b^4*x^8 + 168245*b^5*x^10))/(a + b*x^2)^5 + 50473
5*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x])] - 504735*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(245760*a^(27/4)*(d*x)^(5/2))
```

Maple [A]

time = 0.14, size = 256, normalized size = 0.63

method	result
derivativedivides	$2d^{11}\left(-\frac{b\left(\frac{\frac{15503a^4d^8\sqrt{dx}}{8192}+\frac{31149a^3d^6b(dx)^{\frac{5}{2}}}{5120}+\frac{95821a^2d^4b^2(dx)^{\frac{9}{2}}}{12288}+\frac{3527a\,b^3d^2(dx)^{\frac{13}{2}}}{768}+\frac{25457b^4(dx)^{\frac{17}{2}}}{24576}}{\left(d^2x^2b+a\,d^2\right)^5}+\frac{33649\left(\frac{a\,d^2}{b}\right.}{}\right.}{}\right.$
default	$2d^{11}\left(-\frac{b\left(\frac{\frac{15503a^4d^8\sqrt{dx}}{8192}+\frac{31149a^3d^6b(dx)^{\frac{5}{2}}}{5120}+\frac{95821a^2d^4b^2(dx)^{\frac{9}{2}}}{12288}+\frac{3527a\,b^3d^2(dx)^{\frac{13}{2}}}{768}+\frac{25457b^4(dx)^{\frac{17}{2}}}{24576}}{\left(d^2x^2b+a\,d^2\right)^5}+\frac{33649\left(\frac{a\,d^2}{b}\right.}{}\right.}{}\right.$
risch	$-\frac{2}{3a^6x\sqrt{dx}\;d^2}+\frac{-\frac{15503b\,d^9\sqrt{dx}}{4096a^2\left(d^2x^2b+a\,d^2\right)^5}-\frac{31149b^2d^7(dx)^{\frac{5}{2}}}{2560a^3\left(d^2x^2b+a\,d^2\right)^5}-\frac{95821b^3d^5(dx)^{\frac{9}{2}}}{6144a^4\left(d^2x^2b+a\,d^2\right)^5}-\frac{3527b^4d^3(dx)^{\frac{13}{2}}}{384a^5\left(d^2x^2b+a\,d^2\right)^5}-\frac{}{1228}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^11*(-1/a^6/d^12*b*((15503/8192*a^4*d^8*(d*x)^(1/2)+31149/5120*a^3*d^6*b
*(d*x)^(5/2)+95821/12288*a^2*d^4*b^2*(d*x)^(9/2)+3527/768*a*b^3*d^2*(d*x)^(
13/2)+25457/24576*b^4*(d*x)^(17/2))/(b*d^2*x^2+a*d^2)^5+33649/65536*(a*d^2/
b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/
b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arct
an(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*
(d*x)^(1/2)-1)))-1/3/a^6/d^12/(d*x)^(3/2))
```

Maxima [A]
time = 0.52, size = 395, normalized size = 0.98

$$-\frac{\frac{8\,(168245\,b^5d^{10}x^{10}+769120\,ab^4d^{10}x^8+1367810\,a^2b^3d^{10}x^6+1157176\,a^3b^2d^{10}x^4+437345\,a^4bd^{10}x^2+40960\,a^5d^{10})}{(dx)^{\frac{23}{2}}a^6b^5+5\,(dx)^{\frac{19}{2}}a^7b^4d^2+10\,(dx)^{\frac{15}{2}}a^8b^3d^4+10\,(dx)^{\frac{11}{2}}a^9b^2d^6+5\,(dx)^{\frac{7}{2}}a^{10}bd^8+(dx)^{\frac{3}{2}}a^{11}d^{10}}+\frac{504735\left(\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}\,d}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}\,d}\right)}{a^6}}{491520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/491520*(8*(168245*b^5*d^10*x^10 + 769120*a*b^4*d^10*x^8 + 1367810*a^2*b^
3*d^10*x^6 + 1157176*a^3*b^2*d^10*x^4 + 437345*a^4*b*d^10*x^2 + 40960*a^5*d
^10)/((d*x)^(23/2)*a^6*b^5 + 5*(d*x)^(19/2)*a^7*b^4*d^2 + 10*(d*x)^(15/2)*a
^8*b^3*d^4 + 10*(d*x)^(11/2)*a^9*b^2*d^6 + 5*(d*x)^(7/2)*a^10*b*d^8 + (d*x)
^(3/2)*a^11*d^10) + 504735*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*(a*d^
2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/4)*log
(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)
^(3/4) + 2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a
)*d) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*s
qrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)
*d))/a^6)/d
```

Fricas [A]
time = 0.37, size = 568, normalized size = 1.41

$$-\frac{2018940\,(a^6b^5d^3x^{12}+5\,a^7b^4d^3x^{10}+10\,a^8b^3d^3x^8+10\,a^9b^2d^3x^6+5\,a^{10}bd^3x^4+a^{11}d^3x^2)\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{20}bd^7\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^{14}d^6\sqrt{-\frac{b^3}{a^{27}d^{10}}}+b^2dx}\,a^{20}d^7\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+504735\,(a^6b^5d^3x^{12}+5\,a^7b^4d^3x^{10}+10\,a^8b^3d^3x^8+10\,a^9b^2d^3x^6+5\,a^{10}bd^3x^4+a^{11}d^3x^2)\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{1}{4}}\log\left(33649\,a^7d^8\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{3}{4}}+33649\sqrt{dx}\,b\right)-504735\,(a^6b^5d^3x^{12}+5\,a^7b^4d^3x^{10}+10\,a^8b^3d^3x^8+10\,a^9b^2d^3x^6+5\,a^{10}bd^3x^4+a^{11}d^3x^2)\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{1}{4}}\log\left(-33649\,a^7d^8\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{3}{4}}+33649\sqrt{dx}\,b\right)+4\,(168245\,b^5x^{10}+769120\,ab^4x^8+1367810\,a^2b^3x^6+1157176\,a^3b^2x^4+437345\,a^4bx^2+40960\,a^5)\sqrt{dx}}{245760\,(a^6b^5d^3x^{12}+5\,a^7b^4d^3x^{10}+10\,a^8b^3d^3x^8+10\,a^9b^2d^3x^6+5\,a^{10}bd^3x^4+a^{11}d^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(2018940*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*
x^8 + 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^1
0))^(1/4)*arctan(-(sqrt(d*x)*a^20*b*d^7*(-b^3/(a^27*d^10))^(3/4) - sqrt(a^1
4*d^6*sqrt(-b^3/(a^27*d^10)) + b^2*d*x)*a^20*d^7*(-b^3/(a^27*d^10))^(3/4))/
b^3) + 504735*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*x^8 +
```

```
 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^10))^(
1/4)*log(33649*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(d*x)*b) - 5047
35*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*x^8 + 10*a^9*b^2
*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^10))^(1/4)*log(-3
3649*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(d*x)*b) + 4*(168245*b^5*
x^10 + 769120*a*b^4*x^8 + 1367810*a^2*b^3*x^6 + 1157176*a^3*b^2*x^4 + 43734
5*a^4*b*x^2 + 40960*a^5)*sqrt(d*x))/(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10
+ 10*a^8*b^3*d^3*x^8 + 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left(a+bx^{2}\right)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
[Out] Integral(1/((d*x)**(5/2)*(a + b*x**2)**6), x)
```

Giac [A]
time = 3.20, size = 356, normalized size = 0.88

$$-\frac{33649\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}-\frac{33649\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}-\frac{33649\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^7d^3}+\frac{33649\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^7d^3}-\frac{2}{3\sqrt{dx}\,a^6d^2x}-\frac{127285\sqrt{dx}\,b^5d^8x^8+564320\sqrt{dx}\,ab^4d^8x^6+958210\sqrt{dx}\,a^2b^3d^8x^4+747576\sqrt{dx}\,a^3b^2d^8x^2+232545\sqrt{dx}\,a^4bd^8}{61440\,(bd^2x^2+ad^2)^5a^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
[Out] -33649/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*d^3) - 33649/16384*sqrt(2)*(a*
b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/(a^7*d^3) - 33649/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x
+ sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*d^3) + 33649/3276
8*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a^7*d^3) - 2/3/(sqrt(d*x)*a^6*d^2*x) - 1/61440*(127285*sqrt(
d*x)*b^5*d^8*x^8 + 564320*sqrt(d*x)*a*b^4*d^8*x^6 + 958210*sqrt(d*x)*a^2*b^
3*d^8*x^4 + 747576*sqrt(d*x)*a^3*b^2*d^8*x^2 + 232545*sqrt(d*x)*a^4*b*d^8)/
((b*d^2*x^2 + a*d^2)^5*a^6*d)
```

Mupad [B]
time = 4.46, size = 226, normalized size = 0.56

$$\frac{33649\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{27/4}\,d^{5/2}}-\frac{\frac{2\,d^9}{3\,a}+\frac{87469\,b\,d^9\,x^2}{12288\,a^2}+\frac{144647\,b^2\,d^9\,x^4}{7680\,a^3}+\frac{136781\,b^3\,d^9\,x^6}{6144\,a^4}+\frac{4807\,b^4\,d^9\,x^8}{384\,a^5}+\frac{33649\,b^5\,d^9\,x^{10}}{12288\,a^6}}{b^5\,{\left(d\,x\right)}^{23/2}+a^5\,d^{10}\,{\left(d\,x\right)}^{3/2}+10\,a^3\,b^2\,d^6\,{\left(d\,x\right)}^{11/2}+10\,a^2\,b^3\,d^4\,{\left(d\,x\right)}^{15/2}+5\,a^4\,b\,d^8\,{\left(d\,x\right)}^{7/2}+5\,a\,b^4\,d^2\,{\left(d\,x\right)}^{19/2}}+\frac{33649\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{27/4}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] (33649*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*a
^(27/4)*d^(5/2)) - ((2*d^9)/(3*a) + (87469*b*d^9*x^2)/(12288*a^2) + (144647
*b^2*d^9*x^4)/(7680*a^3) + (136781*b^3*d^9*x^6)/(6144*a^4) + (4807*b^4*d^9*
x^8)/(384*a^5) + (33649*b^5*d^9*x^10)/(12288*a^6))/(b^5*(d*x)^(23/2) + a^5*
d^10*(d*x)^(3/2) + 10*a^3*b^2*d^6*(d*x)^(11/2) + 10*a^2*b^3*d^4*(d*x)^(15/2
) + 5*a^4*b*d^8*(d*x)^(7/2) + 5*a*b^4*d^2*(d*x)^(19/2)) + (33649*(-b)^(3/4)
*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*a^(27/4)*d^(5/2))
```

3.727 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=422

$$-\frac{13923}{4096a^6d(dx)^{5/2}}+\frac{69615b}{4096a^7d^3\sqrt{dx}}+\frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5}+\frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4}+\frac{35}{128a^3d(dx)^{5/2}\left(a+b\right.}$$

```
[Out] -13923/4096/a^6/d/(d*x)^(5/2)+1/10/a/d/(d*x)^(5/2)/(b*x^2+a)^5+5/32/a^2/d/(
d*x)^(5/2)/(b*x^2+a)^4+35/128/a^3/d/(d*x)^(5/2)/(b*x^2+a)^3+595/1024/a^4/d/
(d*x)^(5/2)/(b*x^2+a)^2+7735/4096/a^5/d/(d*x)^(5/2)/(b*x^2+a)-69615/16384*b
^(5/4)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(29/4)/d^(7/
2)*2^(1/2)+69615/16384*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)
/d^(1/2))/a^(29/4)/d^(7/2)*2^(1/2)+69615/32768*b^(5/4)*ln(a^(1/2)*d^(1/2)+x
*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(29/4)/d^(7/2)*2^(1
/2)-69615/32768*b^(5/4)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4
)*2^(1/2)*(d*x)^(1/2))/a^(29/4)/d^(7/2)*2^(1/2)+69615/4096*b/a^7/d^3/(d*x)^
(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {28, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{69615b^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{29/4}d^{7/2}}+\frac{69615b^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{29/4}d^{7/2}}+\frac{69615b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{29/4}d^{7/2}}-\frac{69615b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{29/4}d^{7/2}}+\frac{69615b}{4096a^7d^3\sqrt{dx}}-\frac{13923}{4096a^6d(dx)^{5/2}}+\frac{7735}{4096a^5d(dx)^{5/2}\left(a+bx^2\right)}+\frac{595}{1024a^4d(dx)^{5/2}\left(a+bx^2\right)^2}+\frac{35}{128a^3d(dx)^{5/2}\left(a+bx^2\right)^3}+\frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4}+\frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -13923/(4096*a^6*d*(d*x)^(5/2)) + (69615*b)/(4096*a^7*d^3*Sqrt[d*x]) + 1/(1
0*a*d*(d*x)^(5/2)*(a + b*x^2)^5) + 5/(32*a^2*d*(d*x)^(5/2)*(a + b*x^2)^4) +
 35/(128*a^3*d*(d*x)^(5/2)*(a + b*x^2)^3) + 595/(1024*a^4*d*(d*x)^(5/2)*(a
+ b*x^2)^2) + 7735/(4096*a^5*d*(d*x)^(5/2)*(a + b*x^2)) - (69615*b^(5/4)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(2
9/4)*d^(7/2)) + (69615*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(29/4)*d^(7/2)) + (69615*b^(5/4)*Log[Sqrt[a]
*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*S
qrt[2]*a^(29/4)*d^(7/2)) - (69615*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqr
t[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(29/4)*d^(7/2
))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
```

```
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = b^6\int \frac{1}{(dx)^{7/2}\left(ab+b^2x^2\right)^6}\,dx$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{(5b^5)\int \frac{1}{(dx)^{7/2}(ab+b^2x^2)^5}\,dx}{4a}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{(105b^4)\int \frac{1}{(dx)^{7/2}(ab-}}{64a^2}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}(a+}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}(a+}$$

$$= \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} + \frac{35}{128a^3d(dx)^{5/2}(a+}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{}{32a^2d(}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{}{32a^2d(}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{}{32a^2d(}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{}{32a^2d(}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{}{32a^2d(}$$

$$= -\frac{13923}{4096a^6d(dx)^{5/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{}{32a^2d(}$$

Mathematica [A]

time = 0.74, size = 211, normalized size = 0.50

$$\frac{\sqrt{dx}\left(\frac{4\sqrt[4]{a}\,(-8192a^6+204800a^5bx^2+1317575a^4b^2x^4+2951200a^3b^3x^6+3171350a^2b^4x^8+1670760ab^5x^{10}+348075b^6x^{12})}{(a+bx^2)^5}-348075\sqrt{2}\,b^{5/4}x^{5/2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-348075\sqrt{2}\,b^{5/4}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{81920a^{29/4}d^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (Sqrt[d*x]*((4*a^(1/4)*(-8192*a^6 + 204800*a^5*b*x^2 + 1317575*a^4*b^2*x^4
+ 2951200*a^3*b^3*x^6 + 3171350*a^2*b^4*x^8 + 1670760*a*b^5*x^10 + 348075*b
^6*x^12))/(a + b*x^2)^5 - 348075*Sqrt[2]*b^(5/4)*x^(5/2)*ArcTan[(Sqrt[a] -
Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 348075*Sqrt[2]*b^(5/4)*x^(5
/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(819
20*a^(29/4)*d^4*x^3)
```

Maple [A]

time = 0.16, size = 268, normalized size = 0.64

method	result
derivativedivides	$2d^{11}\left(\frac{b^2\left(\frac{\frac{34139a^4d^8(dx)^{\frac{3}{2}}}{8192}+\frac{3597a^3b\,d^6(dx)^{\frac{7}{2}}}{256}+\frac{75471a^2d^4b^2(dx)^{\frac{11}{2}}}{4096}+\frac{56269a\,b^3d^2(dx)^{\frac{15}{2}}}{5120}+\frac{20463b^4(dx)^{\frac{19}{2}}}{8192}}{(d^2x^2b+a\,d^2)^5}+\frac{69615\sqrt{2}\,(1}{}\right.}{a^7d^{14}}\right.$
default	$2d^{11}\left(\frac{b^2\left(\frac{\frac{34139a^4d^8(dx)^{\frac{3}{2}}}{8192}+\frac{3597a^3b\,d^6(dx)^{\frac{7}{2}}}{256}+\frac{75471a^2d^4b^2(dx)^{\frac{11}{2}}}{4096}+\frac{56269a\,b^3d^2(dx)^{\frac{15}{2}}}{5120}+\frac{20463b^4(dx)^{\frac{19}{2}}}{8192}}{(d^2x^2b+a\,d^2)^5}+\frac{69615\sqrt{2}\,(1}{}\right.}{a^7d^{14}}\right.$

risch	$-\frac{2(-30 b x^2+a)}{5 a^7 \sqrt{dx}\, x^2 d^3}+\frac{\frac{34139 b^2 d^8 (dx)^{\frac{3}{2}}}{4096 a^3 \left(d^2 x^2 b+a d^2\right)^5}+\frac{3597 b^3 d^6 (dx)^{\frac{7}{2}}}{128 a^4 \left(d^2 x^2 b+a d^2\right)^5}+\frac{75471 b^4 d^4 (dx)^{\frac{11}{2}}}{2048 a^5 \left(d^2 x^2 b+a d^2\right)^5}+\frac{56269 b^5 d^2 (dx)^{\frac{15}{2}}}{2560 a^6 \left(d^2 x^2 b+a d^2\right)^5}+\frac{204\ldots}{4096 a^7 \ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)

[Out] 2*d^11*(b^2/a^7/d^14*((34139/8192*a^4*d^8*(d*x)^(3/2)+3597/256*a^3*b*d^6*(d*x)^(7/2)+75471/4096*a^2*d^4*b^2*(d*x)^(11/2)+56269/5120*a*b^3*d^2*(d*x)^(15/2)+20463/8192*b^4*(d*x)^(19/2))/(b*d^2*x^2+a*d^2)^5+69615/65536/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))-1/5/a^6/d^12/(d*x)^(5/2)+6/a^7/d^14*b/(d*x)^(1/2))

Maxima [A]
time = 0.53, size = 410, normalized size = 0.97

$$\frac{\frac{8\left(348075 b^6 d^{12} x^{12}+1670760 a b^5 d^{12} x^{10}+3171350 a^2 b^4 d^{12} x^8+2951200 a^3 b^3 d^{12} x^6+1317575 a^4 b^2 d^{12} x^4+204800 a^5 b d^{12} x^2-8192 a^6 d^{12}\right)}{(dx)^{\frac{25}{2}} a^7 b^5 d^2+5 (dx)^{\frac{21}{2}} a^8 b^4 d^4+10 (dx)^{\frac{17}{2}} a^9 b^3 d^6+10 (dx)^{\frac{13}{2}} a^{10} b^2 d^8+5 (dx)^{\frac{9}{2}} a^{11} b d^{10}+(dx)^{\frac{5}{2}} a^{12} d^{12}}+\frac{348075 b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac14} b^{\frac14}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac14} b^{\frac14}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}(ad^2)^{\frac14}\sqrt{dx}b^{\frac14}+\sqrt{a}d\right)}{(ad^2)^{\frac14} b^{\frac34}}+\frac{\sqrt{2}\log\left(\sqrt{b}dx-\sqrt{2}(ad^2)^{\frac14}\sqrt{dx}b^{\frac14}+\sqrt{a}d\right)}{(ad^2)^{\frac14} b^{\frac34}}\right)}{a^7 d^2}}{163840\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")

[Out] 1/163840*(8*(348075*b^6*d^12*x^12 + 1670760*a*b^5*d^12*x^10 + 3171350*a^2*b^4*d^12*x^8 + 2951200*a^3*b^3*d^12*x^6 + 1317575*a^4*b^2*d^12*x^4 + 204800*a^5*b*d^12*x^2 - 8192*a^6*d^12)/((d*x)^(25/2)*a^7*b^5*d^2 + 5*(d*x)^(21/2)*a^8*b^4*d^4 + 10*(d*x)^(17/2)*a^9*b^3*d^6 + 10*(d*x)^(13/2)*a^10*b^2*d^8 + 5*(d*x)^(9/2)*a^11*b*d^10 + (d*x)^(5/2)*a^12*d^12) + 348075*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^7*d^2))/d

Fricas [A]
time = 0.37, size = 591, normalized size = 1.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(1392300*(a^7*b^5*d^4*x^13 + 5*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x
^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4*x^5 + a^12*d^4*x^3)*(-b^5/(a^29*d^1
4))^(1/4)*arctan(-1/337371570183375*(337371570183375*sqrt(d*x)*a^7*b^4*d^3*
(-b^5/(a^29*d^14))^(1/4) - sqrt(-113819576367995923331126390625*a^15*b^5*d^
8*sqrt(-b^5/(a^29*d^14)) + 113819576367995923331126390625*b^8*d*x)*a^7*d^3*
(-b^5/(a^29*d^14))^(1/4))/b^5) - 348075*(a^7*b^5*d^4*x^13 + 5*a^8*b^4*d^4*x
^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4*x^5 + a^12*d^
4*x^3)*(-b^5/(a^29*d^14))^(1/4)*log(337371570183375*a^22*d^11*(-b^5/(a^29*d
^14))^(3/4) + 337371570183375*sqrt(d*x)*b^4) + 348075*(a^7*b^5*d^4*x^13 + 5
*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4
*x^5 + a^12*d^4*x^3)*(-b^5/(a^29*d^14))^(1/4)*log(-337371570183375*a^22*d^1
1*(-b^5/(a^29*d^14))^(3/4) + 337371570183375*sqrt(d*x)*b^4) - 4*(348075*b^6
*x^12 + 1670760*a*b^5*x^10 + 3171350*a^2*b^4*x^8 + 2951200*a^3*b^3*x^6 + 13
17575*a^4*b^2*x^4 + 204800*a^5*b*x^2 - 8192*a^6)*sqrt(d*x))/(a^7*b^5*d^4*x^
13 + 5*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11
*b*d^4*x^5 + a^12*d^4*x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{7}{2}}\,(a+bx^2)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral(1/((d*x)**(7/2)*(a + b*x**2)**6), x)
```

Giac [**A**]
time = 3.42, size = 362, normalized size = 0.86

$$\frac{69615\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^8bd^5}+\frac{69615\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^8bd^5}-\frac{69615\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^8bd^5}+\frac{69615\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^8bd^5}+\frac{348075\,b^6d^{12}x^{12}+1670760\,ab^5d^{12}x^{10}+3171350\,a^2b^4d^{12}x^8+2951200\,a^3b^3d^{12}x^6+1317575\,a^4b^2d^{12}x^4+204800\,a^5bd^{12}x^2-8192\,a^6d^{12}}{20480\left(\sqrt{dx}\,bd^2x^2+\sqrt{dx}\,ad^2\right)^5a^7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] 69615/16384*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^8*b*d^5) + 69615/16384*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^8*b*d^5) - 69615/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d
*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^8*b*d^5) + 69615
/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
```

```
) + sqrt(a*d^2/b))/(a^8*b*d^5) + 1/20480*(348075*b^6*d^12*x^12 + 1670760*a*
b^5*d^12*x^10 + 3171350*a^2*b^4*d^12*x^8 + 2951200*a^3*b^3*d^12*x^6 + 13175
75*a^4*b^2*d^12*x^4 + 204800*a^5*b*d^12*x^2 - 8192*a^6*d^12)/((sqrt(d*x)*b*
d^2*x^2 + sqrt(d*x)*a*d^2)^5*a^7*d^3)
```

Mupad [B]
time = 0.27, size = 239, normalized size = 0.57

$$\frac{\frac{10\,b\,d^9\,x^2}{a^2}-\frac{2\,d^9}{5\,a}+\frac{263515\,b^2\,d^9\,x^4}{4096\,a^3}+\frac{18445\,b^3\,d^9\,x^6}{128\,a^4}+\frac{317135\,b^4\,d^9\,x^8}{2048\,a^5}+\frac{41769\,b^5\,d^9\,x^{10}}{512\,a^6}+\frac{69615\,b^6\,d^9\,x^{12}}{4096\,a^7}}{b^5\,{\left(d\,x\right)}^{25/2}+a^5\,d^{10}\,{\left(d\,x\right)}^{5/2}+10\,a^3\,b^2\,d^6\,{\left(d\,x\right)}^{13/2}+10\,a^2\,b^3\,d^4\,{\left(d\,x\right)}^{17/2}+5\,a^4\,b\,d^8\,{\left(d\,x\right)}^{9/2}+5\,a\,b^4\,d^2\,{\left(d\,x\right)}^{21/2}}-\frac{69615\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{29/4}\,d^{7/2}}+\frac{69615\,{\left(-b\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{29/4}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

```
[Out] ((10*b*d^9*x^2)/a^2 - (2*d^9)/(5*a) + (263515*b^2*d^9*x^4)/(4096*a^3) + (18
445*b^3*d^9*x^6)/(128*a^4) + (317135*b^4*d^9*x^8)/(2048*a^5) + (41769*b^5*d
^9*x^10)/(512*a^6) + (69615*b^6*d^9*x^12)/(4096*a^7))/(b^5*(d*x)^(25/2) + a
^5*d^10*(d*x)^(5/2) + 10*a^3*b^2*d^6*(d*x)^(13/2) + 10*a^2*b^3*d^4*(d*x)^(1
7/2) + 5*a^4*b*d^8*(d*x)^(9/2) + 5*a*b^4*d^2*(d*x)^(21/2)) - (69615*(-b)^(5
/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*a^(29/4)*d^(7/2
)) + (69615*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(
8192*a^(29/4)*d^(7/2))
```

3.728 $\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}+\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}$$

```
[Out] 2/7*a*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/11*b*(d*x)^(11/2)*((b*x
^2+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1126, 14}

$$\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (2*b*
(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{5/2}\left(ab+b^2x^2\right)\,dx}{ab+b^2x^2}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^{5/2}+\frac{b^2(dx)^{9/2}}{d^2}\right)\,dx}{ab+b^2x^2}\\
&= \frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}+\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.47

$$\frac{2x(dx)^{5/2}\sqrt{(a+bx^2)^2}\,(11a+7bx^2)}{77\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(11*a + 7*b*x^2))/(77*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 41, normalized size = 0.44

method	result	size
gosper	$\frac{2x(7b\,x^2+11a)(dx)^{\frac{5}{2}}\sqrt{(b\,x^2+a)^2}}{77(b\,x^2+a)}$	39
default	$\frac{2\sqrt{(b\,x^2+a)^2}\,(dx)^{\frac{7}{2}}(7b\,x^2+11a)}{77d(b\,x^2+a)}$	41
risch	$\frac{2d^3\sqrt{(b\,x^2+a)^2}\,x^4(7b\,x^2+11a)}{77(b\,x^2+a)\sqrt{dx}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/77*((b*x^2+a)^2)^(1/2)*(d*x)^(7/2)/d*(7*b*x^2+11*a)/(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 25, normalized size = 0.27

$$\frac{2\left(7\,(dx)^{\frac{11}{2}}\,b+11\,(dx)^{\frac{7}{2}}\,ad^2\right)}{77\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 2/77*(7*(d*x)^(11/2)*b + 11*(d*x)^(7/2)*a*d^2)/d^3
```

Fricas [A]
time = 0.32, size = 26, normalized size = 0.28

$$\frac{2}{77}\left(7\,bd^2x^5+11\,ad^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b*d^2*x^5 + 11*a*d^2*x^3)*sqrt(d*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**A**]
time = 4.81, size = 45, normalized size = 0.48

$$\frac{2}{11}\sqrt{dx}\,bd^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}\,ad^2x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/11*sqrt(d*x)*b*d^2*x^5*sgn(b*x^2 + a) + 2/7*sqrt(d*x)*a*d^2*x^3*sgn(b*x^2
 + a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{5/2}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(5/2)*((a + b*x^2)^2)^(1/2), x)
```

3.729 $\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}+\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\,(a+bx^2)}$$

```
[Out] 2/5*a*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/9*b*(d*x)^(9/2)*((b*x^2
+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A]

time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1126, 14}

$$\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\,(a+bx^2)}+\frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (2*b*
(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{3/2}\left(ab+b^2x^2\right)\,dx}{ab+b^2x^2}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^{3/2}+\frac{b^2(dx)^{7/2}}{d^2}\right)\,dx}{ab+b^2x^2}\\ &= \frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}+\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\,(a+bx^2)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.47

$$\frac{2x(dx)^{3/2}\sqrt{(a+bx^2)^2}\,(9a+5bx^2)}{45\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(9*a + 5*b*x^2))/(45*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 41, normalized size = 0.44

method	result	size
gosper	$\frac{2x(5b\,x^2+9a)(dx)^{\frac{3}{2}}\sqrt{(b\,x^2+a)^2}}{45(b\,x^2+a)}$	39
default	$\frac{2\sqrt{(b\,x^2+a)^2}\,(dx)^{\frac{5}{2}}(5b\,x^2+9a)}{45d(b\,x^2+a)}$	41
risch	$\frac{2d^2\sqrt{(b\,x^2+a)^2}\,x^3(5b\,x^2+9a)}{45(b\,x^2+a)\sqrt{dx}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*((b*x^2+a)^2)^(1/2)*(d*x)^(5/2)/d*(5*b*x^2+9*a)/(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 25, normalized size = 0.27

$$\frac{2\left(5\,(dx)^{\frac{9}{2}}\,b+9\,(dx)^{\frac{5}{2}}\,ad^2\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(5*(d*x)^(9/2)*b + 9*(d*x)^(5/2)*a*d^2)/d^3
```

Fricas [A]
time = 0.35, size = 22, normalized size = 0.24

$$\frac{2}{45}\left(5\,bdx^4+9\,adx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b*d*x^4 + 9*a*d*x^2)*sqrt(d*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.56, size = 42, normalized size = 0.45

$$\frac{2}{45}\left(5\sqrt{dx}\,bx^4\mathrm{sgn}\left(bx^2+a\right)+9\sqrt{dx}\,ax^2\mathrm{sgn}\left(bx^2+a\right)\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(5*sqrt(d*x)*b*x^4*sgn(b*x^2 + a) + 9*sqrt(d*x)*a*x^2*sgn(b*x^2 + a))*
d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(3/2)*((a + b*x^2)^2)^(1/2), x)
```

3.730 $\int \sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(a+bx^2)}+\frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\,(a+bx^2)}$$

```
[Out] 2/3*a*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/7*b*(d*x)^(7/2)*((b*x^2
+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1126, 14}

$$\frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\,(a+bx^2)}+\frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (2*b*
(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\,(ab+b^2x^2)\,dx}{ab+b^2x^2}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab\sqrt{dx}+\frac{b^2(dx)^{5/2}}{d^2}\right)dx}{ab+b^2x^2}\\ &= \frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(a+bx^2)}+\frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\,(a+bx^2)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.47

$$\frac{2x\sqrt{dx}\sqrt{(a+bx^2)^2}\,(7a+3bx^2)}{21\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (2*x*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(7*a + 3*b*x^2))/(21*(a + b*x^2))
```

Maple [A]
time = 0.01, size = 41, normalized size = 0.44

method	result	size
gosper	$\frac{2x(3b\,x^2+7a)\sqrt{dx}\,\sqrt{(b\,x^2+a)^2}}{21(b\,x^2+a)}$	39
default	$\frac{2\sqrt{(b\,x^2+a)^2}\,(dx)^{\frac{3}{2}}(3b\,x^2+7a)}{21d(b\,x^2+a)}$	41
risch	$\frac{2d\sqrt{(b\,x^2+a)^2}\,(3b\,x^2+7a)x^2}{21(b\,x^2+a)\sqrt{dx}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/21*((b*x^2+a)^2)^(1/2)*(d*x)^(3/2)/d*(3*b*x^2+7*a)/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.27

$$\frac{2\left(3\,(dx)^{\frac{7}{2}}\,b+7\,(dx)^{\frac{3}{2}}\,ad^2\right)}{21\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 2/21*(3*(d*x)^(7/2)*b + 7*(d*x)^(3/2)*a*d^2)/d^3
```

Fricas [A]
time = 0.33, size = 18, normalized size = 0.19

$$\frac{2}{21}\,(3\,bx^3+7\,ax)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b*x^3 + 7*a*x)*sqrt(d*x)
```

Sympy [A]
time = 39.24, size = 27, normalized size = 0.29

$$\frac{2a(dx)^{\frac{3}{2}}}{3d}+\frac{2b(dx)^{\frac{7}{2}}}{7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] 2*a*(d*x)**(3/2)/(3*d) + 2*b*(d*x)**(7/2)/(7*d**3)
```

Giac [A]
time = 4.58, size = 37, normalized size = 0.40

$$\frac{2}{7}\sqrt{dx}\,bx^3\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\sqrt{dx}\,ax\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*sqrt(d*x)*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt(d*x)*a*x*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

3.731 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=91

$$\frac{2a\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\,(a+bx^2)}+\frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\,(a+bx^2)}$$

```
[Out] 2/5*b*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+2*a*(d*x)^(1/2)*((b*x^2
+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 14}

$$\frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\,(a+bx^2)}+\frac{2a\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (2*b*(d*x
)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{\sqrt{dx}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{\sqrt{dx}}+\frac{b^2(dx)^{3/2}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= \frac{2a\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\,(a+bx^2)} + \frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\,(a+bx^2)}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.47

$$\frac{2x\sqrt{(a+bx^2)^2}\,(5a+bx^2)}{5\sqrt{dx}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(5*a + b*x^2))/(5*Sqrt[d*x]*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 40, normalized size = 0.44

method	result	size
gosper	$\frac{2x(b\,x^2+5a)\sqrt{(b\,x^2+a)^2}}{5(b\,x^2+a)\sqrt{dx}}$	38
risch	$\frac{2x(b\,x^2+5a)\sqrt{(b\,x^2+a)^2}}{5(b\,x^2+a)\sqrt{dx}}$	38
default	$\frac{2\sqrt{(b\,x^2+a)^2}\,\sqrt{dx}\,(b\,x^2+5a)}{5d(b\,x^2+a)}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*((b*x^2+a)^2)^(1/2)*(d*x)^(1/2)/d*(b*x^2+5*a)/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.26

$$\frac{2\left(5\,\sqrt{dx}\,a+\frac{(dx)^{\frac{5}{2}}b}{d^2}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="maxima")
[Out] 2/5*(5*sqrt(d*x)*a + (d*x)^(5/2)*b/d^2)/d
```

Fricas [A]
time = 0.35, size = 19, normalized size = 0.21

$$\frac{2\left(bx^2+5\,a\right)\sqrt{dx}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="fricas")
[Out] 2/5*(b*x^2 + 5*a)*sqrt(d*x)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\left(a+bx^2\right)^2}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(1/2),x)
[Out] Integral(sqrt((a + b*x**2)**2)/sqrt(d*x), x)
```

Giac [A]
time = 4.27, size = 40, normalized size = 0.44

$$\frac{2\left(\sqrt{dx}\,bx^2\mathrm{sgn}(bx^2+a)+5\,\sqrt{dx}\,a\mathrm{sgn}(bx^2+a)\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="giac")
[Out] 2/5*(sqrt(d*x)*b*x^2*sgn(b*x^2 + a) + 5*sqrt(d*x)*a*sgn(b*x^2 + a))/d
```

Mupad [B]
time = 4.36, size = 47, normalized size = 0.52

$$\frac{\left(\frac{2\,x^3}{5}+\frac{2\,a\,x}{b}\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(1/2),x)
[Out] (((2*x^3)/5 + (2*a*x)/b)*((a + b*x^2)^2)^(1/2))/(x^2*(d*x)^(1/2) + (a*(d*x)
^(1/2))/b)
```

3.732 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}} \, dx$

Optimal. Leaf size=91

$$-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)}+\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)}$$

```
[Out] 2/3*b*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)-2*a*((b*x^2+a)^2)^(1/2)
/d/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 14}

$$\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (2*b*(d*
x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{3/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{3/2}}+\frac{b^2\sqrt{dx}}{d^2}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)} + \frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.48

$$-\frac{2x(3a-bx^2)\sqrt{(a+bx^2)^2}}{3(dx)^{3/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
[Out] (-2*x*(3*a - b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(3/2)*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 41, normalized size = 0.45

method	result	size
gosper	$-\frac{2x(-b\,x^2+3a)\sqrt{(b\,x^2+a)^2}}{3(b\,x^2+a)(dx)^{\frac{3}{2}}}$	39
default	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(-b\,x^2+3a)}{3d(b\,x^2+a)\sqrt{dx}}$	41
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(-b\,x^2+3a)}{3d(b\,x^2+a)\sqrt{dx}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3*((b*x^2+a)^2)^(1/2)/d*(-b*x^2+3*a)/(b*x^2+a)/(d*x)^(1/2)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.27

$$-\frac{2\left(\frac{3\,a}{\sqrt{dx}}-\frac{(dx)^{\frac{3}{2}}\,b}{d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="maxima")
[Out] -2/3*(3*a/sqrt(d*x) - (d*x)^(3/2)*b/d^2)/d
```

Fricas [A]
time = 0.35, size = 22, normalized size = 0.24

$$\frac{2\,(bx^2-3\,a)\sqrt{dx}}{3\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="fricas")
[Out] 2/3*(b*x^2 - 3*a)*sqrt(d*x)/(d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(a+bx^2)^2}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(3/2),x)
[Out] Integral(sqrt((a + b*x**2)**2)/(d*x)**(3/2), x)
```

Giac [A]
time = 4.27, size = 41, normalized size = 0.45

$$\frac{2\left(\frac{\sqrt{dx}\,bx\mathrm{sgn}(bx^2+a)}{d}-\frac{3\,a\mathrm{sgn}(bx^2+a)}{\sqrt{dx}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="giac")
[Out] 2/3*(sqrt(d*x)*b*x*sgn(b*x^2 + a)/d - 3*a*sgn(b*x^2 + a)/sqrt(d*x))/d
```

Mupad [B]
time = 4.35, size = 52, normalized size = 0.57

$$\frac{\left(\frac{2\,x^2}{3\,d}-\frac{2\,a}{b\,d}\right)\,\sqrt{(b\,x^2+a)^2}}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(3/2),x)
[Out] (((2*x^2)/(3*d) - (2*a)/(b*d))*((a + b*x^2)^2)^(1/2))/(x^2*(d*x)^(1/2) + (a
*(d*x)^(1/2))/b)
```

3.733 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\,(a+bx^2)}+\frac{2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\,(a+bx^2)}$$

```
[Out] -2/3*a*((b*x^2+a)^2)^(1/2)/d/(d*x)^(3/2)/(b*x^2+a)+2*b*(d*x)^(1/2)*((b*x^2+
a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1126, 14}

$$\frac{2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\,(a+bx^2)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (2*b
*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{5/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{5/2}}+\frac{b^2}{d^2\sqrt{dx}}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\,(a+bx^2)} + \frac{2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\,(a+bx^2)}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.46

$$-\frac{2x(a-3bx^2)\sqrt{(a+bx^2)^2}}{3(dx)^{5/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
[Out] (-2*x*(a - 3*b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(5/2)*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 39, normalized size = 0.43

method	result	size
gosper	$-\frac{2x(-3b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{3(b\,x^2+a)(dx)^{\frac{5}{2}}}$	37
default	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(-3b\,x^2+a)}{3d(b\,x^2+a)(dx)^{\frac{3}{2}}}$	39
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(-3b\,x^2+a)}{3d^2(b\,x^2+a)x\sqrt{dx}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3*((b*x^2+a)^2)^(1/2)/d*(-3*b*x^2+a)/(b*x^2+a)/(d*x)^(3/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.26

$$-\frac{2\left(\frac{a}{(dx)^{\frac{3}{2}}}-\frac{3\sqrt{dx}\,b}{d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(a/(d*x)^(3/2) - 3*sqrt(d*x)*b/d^2)/d
```

Fricas [A]
time = 0.34, size = 23, normalized size = 0.25

$$\frac{2\,(3\,bx^2 - a)\sqrt{dx}}{3\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b*x^2 - a)*sqrt(d*x)/(d^3*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.46, size = 42, normalized size = 0.46

$$\frac{2\left(3\sqrt{dx}\,b\mathrm{sgn}(bx^2+a) - \frac{ad\mathrm{sgn}(bx^2+a)}{\sqrt{dx}\,x}\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(d*x)*b*sgn(b*x^2 + a) - a*d*sgn(b*x^2 + a)/(sqrt(d*x)*x))/d^3
```

Mupad [B]
time = 4.38, size = 53, normalized size = 0.58

$$\frac{\left(\frac{2\,x^2}{d^2} - \frac{2\,a}{3\,b\,d^2}\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x^3\,\sqrt{d\,x} + \frac{a\,x\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(5/2),x)
```

```
[Out] (((2*x^2)/d^2 - (2*a)/(3*b*d^2))*((a + b*x^2)^2)^(1/2))/(x^3*(d*x)^(1/2) +
(a*x*(d*x)^(1/2))/b)
```

3.734 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\,(a+bx^2)}-\frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,(a+bx^2)}$$

```
[Out] -2/5*a*((b*x^2+a)^2)^(1/2)/d/(d*x)^(5/2)/(b*x^2+a)-2*b*((b*x^2+a)^2)^(1/2)/
d^3/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{1126, 14\}$

$$-\frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,(a+bx^2)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (2*b
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{ab+b^2x^2}{(dx)^{7/2}}\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{ab}{(dx)^{7/2}}+\frac{b^2}{d^2(dx)^{3/2}}\right)dx}{ab+b^2x^2}$$
$$= -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\,(a+bx^2)} - \frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,(a+bx^2)}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.46

$$-\frac{2x\sqrt{(a+bx^2)^2}\,(a+5bx^2)}{5(dx)^{7/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
[Out] (-2*x*Sqrt[(a + b*x^2)^2]*(a + 5*b*x^2))/(5*(d*x)^(7/2)*(a + b*x^2))
```

Maple [A]
time = 0.02, size = 39, normalized size = 0.43

method	result	size
gosper	$-\frac{2x(5b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{5(b\,x^2+a)(dx)^{\frac{7}{2}}}$	37
default	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(5b\,x^2+a)}{5d(b\,x^2+a)(dx)^{\frac{5}{2}}}$	39
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(5b\,x^2+a)}{5d^3(b\,x^2+a)x^2\sqrt{dx}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5*((b*x^2+a)^2)^(1/2)/d*(5*b*x^2+a)/(b*x^2+a)/(d*x)^(5/2)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.27

$$-\frac{2\,(5\,bd^2x^2+ad^2)}{5\,(dx)^{\frac{5}{2}}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*b*d^2*x^2 + a*d^2)/((d*x)^(5/2)*d^3)
```

Fricas [A]
time = 0.37, size = 21, normalized size = 0.23

$$-\frac{2\,(5\,bx^2+a)\sqrt{dx}}{5\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(5*b*x^2 + a)*sqrt(d*x)/(d^4*x^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [A]
time = 3.62, size = 44, normalized size = 0.48

$$-\frac{2\,(5\,bd^3x^2\mathrm{sgn}(bx^2+a)+ad^3\mathrm{sgn}(bx^2+a))}{5\,\sqrt{dx}\,d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/5*(5*b*d^3*x^2*sgn(b*x^2 + a) + a*d^3*sgn(b*x^2 + a))/(sqrt(d*x)*d^6*x^2
)
```

Mupad [B]
time = 4.32, size = 56, normalized size = 0.62

$$-\frac{\left(\frac{2\,x^2}{d^3}+\frac{2\,a}{5\,b\,d^3}\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x^4\,\sqrt{d\,x}+\frac{a\,x^2\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2)^(1/2)/(d*x)^(7/2),x)
```

```
[Out] -(((2*x^2)/d^3 + (2*a)/(5*b*d^3))*((a + b*x^2)^2)^(1/2))/(x^4*(d*x)^(1/2) +
 (a*x^2*(d*x)^(1/2))/b)
```

3.735 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=195

$$\frac{2a^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2ab^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{2b^3(\ldots}{}$$

```
[Out] 2/7*a^3*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+6/11*a^2*b*(d*x)^(11/2)
*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+2/5*a*b^2*(d*x)^(15/2)*((b*x^2+a)^2)^(1/
2)/d^5/(b*x^2+a)+2/19*b^3*(d*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Rubi [A]

time = 0.04, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{2ab^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2a^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (6*
a^2*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2)) +
(2*a*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^5*(a + b*x^2))
+ (2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (dx)^{5/2} \left(ab+b^2x^2\right)^3 dx}{b^2 (ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3(dx)^{5/2} + \frac{3a^2b^4(dx)^{9/2}}{d^2} + \frac{3ab^5(dx)^{13/2}}{d^4} + \frac{b^6(d}{}\right.}{b^2 (ab+b^2x^2)}$$
$$= \frac{2a^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d (a+bx^2)} + \frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3 (a+bx^2)}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.34

$$\frac{2x(dx)^{5/2}\sqrt{(a+bx^2)^2}\,(1045a^3+1995a^2bx^2+1463ab^2x^4+385b^3x^6)}{7315\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(1045*a^3 + 1995*a^2*b*x^2 + 1463*a*b^
2*x^4 + 385*b^3*x^6))/(7315*(a + b*x^2))
```

Maple [A]
time = 0.06, size = 63, normalized size = 0.32

method	result	size
gosper	$\frac{2x\left(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3\right)(dx)^{\frac{5}{2}}\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{7315\left(b\,x^2+a\right)^3}$	61
default	$\frac{2\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}(dx)^{\frac{7}{2}}\left(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3\right)}{7315d\left(b\,x^2+a\right)^3}$	63
risch	$\frac{2d^3\sqrt{\left(b\,x^2+a\right)^2}\;x^4\left(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3\right)}{7315\left(b\,x^2+a\right)\sqrt{dx}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7315*((b*x^2+a)^2)^(3/2)*(d*x)^(7/2)/d*(385*b^3*x^6+1463*a*b^2*x^4+1995*a
^2*b*x^2+1045*a^3)/(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 83, normalized size = 0.43

$$\frac{2}{285}\left(15\,b^3d^{\frac{5}{2}}x^3+19\,ab^2d^{\frac{5}{2}}x\right)x^{\frac{13}{2}}+\frac{4}{165}\left(11\,ab^2d^{\frac{5}{2}}x^3+15\,a^2bd^{\frac{5}{2}}x\right)x^{\frac{9}{2}}+\frac{2}{77}\left(7\,a^2bd^{\frac{5}{2}}x^3+11\,a^3d^{\frac{5}{2}}x\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] 2/285*(15*b^3*d^(5/2)*x^3 + 19*a*b^2*d^(5/2)*x)*x^(13/2) + 4/165*(11*a*b^2*
d^(5/2)*x^3 + 15*a^2*b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^2*b*d^(5/2)*x^3 + 11*
a^3*d^(5/2)*x)*x^(5/2)
```

Fricas [A]
time = 0.33, size = 54, normalized size = 0.28

$$\frac{2}{7315}\left(385\,b^3d^2x^9+1463\,ab^2d^2x^7+1995\,a^2bd^2x^5+1045\,a^3d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] 2/7315*(385*b^3*d^2*x^9 + 1463*a*b^2*d^2*x^7 + 1995*a^2*b*d^2*x^5 + 1045*a^
3*d^2*x^3)*sqrt(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(dx)^{\frac{5}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral((d*x)**(5/2)*((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 3.36, size = 99, normalized size = 0.51

$$\frac{2}{19}\sqrt{dx}\,b^3d^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{5}\sqrt{dx}\,ab^2d^2x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{6}{11}\sqrt{dx}\,a^2bd^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}\,a^3d^2x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] 2/19*sqrt(d*x)*b^3*d^2*x^9*sgn(b*x^2 + a) + 2/5*sqrt(d*x)*a*b^2*d^2*x^7*sgn
(b*x^2 + a) + 6/11*sqrt(d*x)*a^2*b*d^2*x^5*sgn(b*x^2 + a) + 2/7*sqrt(d*x)*a
^3*d^2*x^3*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(d\,x\right)}^{5/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.736 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=195

$$\frac{2a^3(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)}+\frac{6ab^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\,(a+bx^2)}+\frac{2b^3}{}$$

```
[Out] 2/5*a^3*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/3*a^2*b*(d*x)^(9/2)*(
(b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+6/13*a*b^2*(d*x)^(13/2)*((b*x^2+a)^2)^(1/2
)/d^5/(b*x^2+a)+2/17*b^3*(d*x)^(17/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{6ab^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\,(a+bx^2)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)}+\frac{2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\,(a+bx^2)}+\frac{2a^3(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (2*
a^2*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2)) + (6
*a*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^5*(a + b*x^2)) +
 (2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (dx)^{3/2} \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3(dx)^{3/2} + \frac{3a^2b^4(dx)^{7/2}}{d^2} + \frac{3ab^5(dx)^{11/2}}{d^4} + \frac{b^6(dx}{d}\right.}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{2a^3(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)} + \frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)} +$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.34

$$\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(663a^3+1105a^2bx^2+765ab^2x^4+195b^3x^6\right)}{3315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(663*a^3 + 1105*a^2*b*x^2 + 765*a*b^2*
x^4 + 195*b^3*x^6))/(3315*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 63, normalized size = 0.32

method	result	size
gosper	$\frac{2x\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)(dx)^{\frac{3}{2}}\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{3315\left(b\,x^2+a\right)^3}$	61
default	$\frac{2\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}(dx)^{\frac{5}{2}}\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)}{3315d\left(b\,x^2+a\right)^3}$	63
risch	$\frac{2d^2\sqrt{\left(b\,x^2+a\right)^2}\,x^3\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)}{3315\left(b\,x^2+a\right)\sqrt{dx}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3315*((b*x^2+a)^2)^(3/2)*(d*x)^(5/2)/d*(195*b^3*x^6+765*a*b^2*x^4+1105*a^
2*b*x^2+663*a^3)/(b*x^2+a)^3
```

Maxima [A]
time = 0.30, size = 83, normalized size = 0.43

$$\frac{2}{221}\left(13\,b^3d^{\frac{3}{2}}x^3+17\,ab^2d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+\frac{4}{117}\left(9\,ab^2d^{\frac{3}{2}}x^3+13\,a^2bd^{\frac{3}{2}}x\right)x^{\frac{7}{2}}+\frac{2}{45}\left(5\,a^2bd^{\frac{3}{2}}x^3+9\,a^3d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] 2/221*(13*b^3*d^(3/2)*x^3 + 17*a*b^2*d^(3/2)*x)*x^(11/2) + 4/117*(9*a*b^2*d
^(3/2)*x^3 + 13*a^2*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^2*b*d^(3/2)*x^3 + 9*a^
3*d^(3/2)*x)*x^(3/2)
```

Fricas [A]
time = 0.35, size = 46, normalized size = 0.24

$$\frac{2}{3315}\left(195\,b^3dx^8+765\,ab^2dx^6+1105\,a^2bdx^4+663\,a^3dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
[Out] 2/3315*(195*b^3*d*x^8 + 765*a*b^2*d*x^6 + 1105*a^2*b*d*x^4 + 663*a^3*d*x^2)
*sqrt(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral((d*x)**(3/2)*((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 2.95, size = 90, normalized size = 0.46

$$\frac{2}{3315}\left(195\sqrt{dx}\,b^3x^8\mathrm{sgn}\left(bx^2+a\right)+765\sqrt{dx}\,ab^2x^6\mathrm{sgn}\left(bx^2+a\right)+1105\sqrt{dx}\,a^2bx^4\mathrm{sgn}\left(bx^2+a\right)+663\sqrt{dx}\,a^3x^2\mathrm{sgn}\left(bx^2+a\right)\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
[Out] 2/3315*(195*sqrt(d*x)*b^3*x^8*sgn(b*x^2 + a) + 765*sqrt(d*x)*a*b^2*x^6*sgn(
b*x^2 + a) + 1105*sqrt(d*x)*a^2*b*x^4*sgn(b*x^2 + a) + 663*sqrt(d*x)*a^3*x^
2*sgn(b*x^2 + a))*d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,x\right)}^{3/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.737 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=195

$$\frac{2a^3(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(a+bx^2)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\,(a+bx^2)}+\frac{6ab^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\,(a+bx^2)}+\frac{2b^3(d$$

```
[Out] 2/3*a^3*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+6/7*a^2*b*(d*x)^(7/2)*(
(b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+6/11*a*b^2*(d*x)^(11/2)*((b*x^2+a)^2)^(1/2
)/d^5/(b*x^2+a)+2/15*b^3*(d*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1126, 276}

$$\frac{6ab^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\,(a+bx^2)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\,(a+bx^2)}+\frac{2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^7\,(a+bx^2)}+\frac{2a^3(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (6*
a^2*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2)) + (6
*a*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^5*(a + b*x^2)) +
 (2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^7*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\,(ab+b^2x^2)^3\,dx}{b^2\,(ab+b^2x^2)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^3b^3\sqrt{dx}+\frac{3a^2b^4(dx)^{5/2}}{d^2}+\frac{3ab^5(dx)^{9/2}}{d^4}+\frac{b^6(dx)^{13}}{d^6}\right.}{b^2\,(ab+b^2x^2)}$$

$$= \frac{2a^3(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(a+bx^2)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\,(a+bx^2)}+$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.34

$$\frac{2x\sqrt{dx}\,\sqrt{(a+bx^2)^2}\,(385a^3+495a^2bx^2+315ab^2x^4+77b^3x^6)}{1155\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (2*x*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(385*a^3 + 495*a^2*b*x^2 + 315*a*b^2*x^4
 + 77*b^3*x^6))/(1155*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 63, normalized size = 0.32

method	result	size
gosper	$\frac{2x\left(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\sqrt{dx}}{1155\left(b\,x^2+a\right)^3}$	61
default	$\frac{2\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}(dx)^{\frac{3}{2}}\left(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3\right)}{1155d\left(b\,x^2+a\right)^3}$	63
risch	$\frac{2d\sqrt{\left(b\,x^2+a\right)^2}\,x^2\left(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3\right)}{1155\left(b\,x^2+a\right)\sqrt{dx}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/1155*((b*x^2+a)^2)^(3/2)*(d*x)^(3/2)/d*(77*b^3*x^6+315*a*b^2*x^4+495*a^2*
b*x^2+385*a^3)/(b*x^2+a)^3
```

Maxima [A]
time = 0.29, size = 83, normalized size = 0.43

$$\frac{2}{165}\left(11\,b^3\sqrt{d}\,x^3+15\,ab^2\sqrt{d}\,x\right)x^{\frac{9}{2}}+\frac{4}{77}\left(7\,ab^2\sqrt{d}\,x^3+11\,a^2b\sqrt{d}\,x\right)x^{\frac{5}{2}}+\frac{2}{21}\left(3\,a^2b\sqrt{d}\,x^3+7\,a^3\sqrt{d}\,x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/165*(11*b^3*sqrt(d)*x^3 + 15*a*b^2*sqrt(d)*x)*x^(9/2) + 4/77*(7*a*b^2*sqr
t(d)*x^3 + 11*a^2*b*sqrt(d)*x)*x^(5/2) + 2/21*(3*a^2*b*sqrt(d)*x^3 + 7*a^3*
sqrt(d)*x)*sqrt(x)
```

Fricas [A]
time = 0.33, size = 40, normalized size = 0.21

$$\frac{2}{1155}\left(77\,b^3x^7+315\,ab^2x^5+495\,a^2bx^3+385\,a^3x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*x^7 + 315*a*b^2*x^5 + 495*a^2*b*x^3 + 385*a^3*x)*sqrt(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{dx}\,\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)*(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 2.85, size = 85, normalized size = 0.44

$$\frac{2}{15}\sqrt{dx}\,b^3x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{6}{11}\sqrt{dx}\,ab^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{6}{7}\sqrt{dx}\,a^2bx^3\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\sqrt{dx}\,a^3x\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*sqrt(d*x)*b^3*x^7*sgn(b*x^2 + a) + 6/11*sqrt(d*x)*a*b^2*x^5*sgn(b*x^2
+ a) + 6/7*sqrt(d*x)*a^2*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt(d*x)*a^3*x*sgn(b*x
^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{d\,x}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.738 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=193

$$\frac{2a^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\,(a+bx^2)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\,(a+bx^2)}+\frac{2ab^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\,(a+bx^2)}+\frac{2b^3(d$$

```
[Out] 6/5*a^2*b*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+2/3*a*b^2*(d*x)^(9/
2)*((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+2/13*b^3*(d*x)^(13/2)*((b*x^2+a)^2)^(1
/2)/d^7/(b*x^2+a)+2*a^3*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A]

time = 0.04, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{2ab^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\,(a+bx^2)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\,(a+bx^2)}+\frac{2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\,(a+bx^2)}+\frac{2a^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*a^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (6*a^2*
b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*x^2)) + (2*a*b
^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2)) + (2*b^
3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{\sqrt{dx}}\,dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{\sqrt{dx}}+\frac{3a^2b^4(dx)^{3/2}}{d^2}+\frac{3ab^5(dx)^{7/2}}{d^4}+\frac{b^6(dx)^{11/2}}{d^6}\right)dx}{b^2(ab+b^2x^2)}$$
$$= \frac{2a^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d(a+bx^2)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3(a+bx^2)}+\frac{2ab^2(dx$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.34

$$\frac{2x\sqrt{(a+bx^2)^2}\,(195a^3+117a^2bx^2+65ab^2x^4+15b^3x^6)}{195\sqrt{dx}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(195*a^3 + 117*a^2*b*x^2 + 65*a*b^2*x^4 + 15*b^3*x
^6))/(195*Sqrt[d*x]*(a + b*x^2))
```

Maple [A]
time = 0.01, size = 63, normalized size = 0.33

method	result	size
gosper	$\frac{2x(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{195(b\,x^2+a)^3\sqrt{dx}}$	61
risch	$\frac{2\sqrt{(b\,x^2+a)^2}\,(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3)x}{195(b\,x^2+a)\sqrt{dx}}$	61
default	$\frac{2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}\sqrt{dx}\,(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3)}{195d(b\,x^2+a)^3}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/195*((b*x^2+a)^2)^(3/2)*(d*x)^(1/2)/d*(15*b^3*x^6+65*a*b^2*x^4+117*a^2*b*
x^2+195*a^3)/(b*x^2+a)^3
```

Maxima [A]
time = 0.29, size = 87, normalized size = 0.45

$$\frac{2\left(5\left(9\,b^3\sqrt{d}\,x^3+13\,ab^2\sqrt{d}\,x\right)x^{\frac{7}{2}}+26\left(5\,ab^2\sqrt{d}\,x^3+9\,a^2b\sqrt{d}\,x\right)x^{\frac{3}{2}}+\frac{117\left(a^2b\sqrt{d}\,x^3+5\,a^3\sqrt{d}\,x\right)}{\sqrt{x}}\right)}{585\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/585*(5*(9*b^3*sqrt(d)*x^3 + 13*a*b^2*sqrt(d)*x)*x^(7/2) + 26*(5*a*b^2*sqr
t(d)*x^3 + 9*a^2*b*sqrt(d)*x)*x^(3/2) + 117*(a^2*b*sqrt(d)*x^3 + 5*a^3*sqrt
(d)*x)/sqrt(x))/d
```

Fricas [A]
time = 0.35, size = 42, normalized size = 0.22

$$\frac{2\,(15\,b^3x^6+65\,ab^2x^4+117\,a^2bx^2+195\,a^3)\sqrt{dx}}{195\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/195*(15*b^3*x^6 + 65*a*b^2*x^4 + 117*a^2*b*x^2 + 195*a^3)*sqrt(d*x)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/sqrt(d*x), x)
```

Giac [A]
time = 3.08, size = 89, normalized size = 0.46

$$\frac{2\left(15\sqrt{dx}\,b^3x^6\mathrm{sgn}(bx^2+a)+65\sqrt{dx}\,ab^2x^4\mathrm{sgn}(bx^2+a)+117\sqrt{dx}\,a^2bx^2\mathrm{sgn}(bx^2+a)+195\sqrt{dx}\,a^3\mathrm{sgn}(bx^2+a)\right)}{195\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/195*(15*sqrt(d*x)*b^3*x^6*sgn(b*x^2 + a) + 65*sqrt(d*x)*a*b^2*x^4*sgn(b*x
^2 + a) + 117*sqrt(d*x)*a^2*b*x^2*sgn(b*x^2 + a) + 195*sqrt(d*x)*a^3*sgn(b*
x^2 + a))/d
```

Mupad [B]
time = 4.50, size = 76, normalized size = 0.39

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{6\,a^2\,x^3}{5}+\frac{2\,b^2\,x^7}{13}+\frac{2\,a^3\,x}{b}+\frac{2\,a\,b\,x^5}{3}\right)}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((6*a^2*x^3)/5 + (2*b^2*x^7)/13 + (2*a^3
*x)/b + (2*a*b*x^5)/3))/(x^2*(d*x)^(1/2) + (a*(d*x)^(1/2))/b)
```

3.739 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\left(a+bx^2\right)}+\frac{2b^3(dx)^{11}}{}$$

```
[Out] 2*a^2*b*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+6/7*a*b^2*(d*x)^(7/2)
*((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+2/11*b^3*(d*x)^(11/2)*((b*x^2+a)^2)^(1/2
)/d^7/(b*x^2+a)-2*a^3*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (2*a^2
*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (6*a*b^
2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^5*(a + b*x^2)) + (2*b^3
*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{(dx)^{3/2}}\,dx}{b^2(ab+b^2x^2)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{(dx)^{3/2}}+\frac{3a^2b^4\sqrt{dx}}{d^2}+\frac{3ab^5(dx)^{5/2}}{d^4}+\frac{b^6(dx)^{9/2}}{d^6}\right)dx}{b^2(ab+b^2x^2)}$$

$$= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(a+bx^2)}+\frac{6ab^2(dx)^{7/2}}{7c}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.35

$$-\frac{2x\left((a+bx^2)^2\right)^{3/2}(77a^3-77a^2bx^2-33ab^2x^4-7b^3x^6)}{77(dx)^{3/2}(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*x*((a + b*x^2)^2)^(3/2)*(77*a^3 - 77*a^2*b*x^2 - 33*a*b^2*x^4 - 7*b^3*x
^6))/(77*(d*x)^(3/2)*(a + b*x^2)^3)
```

Maple [A]
time = 0.02, size = 63, normalized size = 0.33

method	result	size
gosper	$-\frac{2x(-7b^3x^6-33a\,b^2x^4-77a^2b\,x^2+77a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{77(b\,x^2+a)^3(dx)^{\frac{3}{2}}}$	61
default	$-\frac{2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}(-7b^3x^6-33a\,b^2x^4-77a^2b\,x^2+77a^3)}{77d(b\,x^2+a)^3\sqrt{dx}}$	63
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(-7b^3x^6-33a\,b^2x^4-77a^2b\,x^2+77a^3)}{77d(b\,x^2+a)\sqrt{dx}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*((b*x^2+a)^2)^(3/2)/d*(-7*b^3*x^6-33*a*b^2*x^4-77*a^2*b*x^2+77*a^3)/(
b*x^2+a)^3/(d*x)^(1/2)
```

Maxima [A]
time = 0.28, size = 87, normalized size = 0.46

$$\frac{2\left(3\left(7\,b^3\sqrt{d}\,x^3+11\,ab^2\sqrt{d}\,x\right)x^{\frac{5}{2}}+22\left(3\,ab^2\sqrt{d}\,x^3+7\,a^2b\sqrt{d}\,x\right)\sqrt{x}+\frac{77\left(a^2b\sqrt{d}\,x^3-3\,a^3\sqrt{d}\,x\right)}{x^{\frac{3}{2}}}\right)}{231\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/231*(3*(7*b^3*sqrt(d)*x^3 + 11*a*b^2*sqrt(d)*x)*x^(5/2) + 22*(3*a*b^2*sqr
t(d)*x^3 + 7*a^2*b*sqrt(d)*x)*sqrt(x) + 77*(a^2*b*sqrt(d)*x^3 - 3*a^3*sqrt(
d)*x)/x^(3/2))/d^2
```

Fricas [A]
time = 0.34, size = 45, normalized size = 0.24

$$\frac{2\,(7\,b^3x^6+33\,ab^2x^4+77\,a^2bx^2-77\,a^3)\sqrt{dx}}{77\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)*sqrt(d*x)/(d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(3/2), x)
```

Giac [A]
time = 3.74, size = 102, normalized size = 0.53

$$-\frac{2\left(\frac{77\,a^3\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}}-\frac{7\,\sqrt{dx}\,b^3d^{65}x^5\mathrm{sgn}\left(bx^2+a\right)+33\,\sqrt{dx}\,ab^2d^{65}x^3\mathrm{sgn}\left(bx^2+a\right)+77\,\sqrt{dx}\,a^2bd^{65}x\mathrm{sgn}\left(bx^2+a\right)}{d^{66}}\right)}{77\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/77*(77*a^3*sgn(b*x^2 + a)/sqrt(d*x) - (7*sqrt(d*x)*b^3*d^65*x^5*sgn(b*x^
2 + a) + 33*sqrt(d*x)*a*b^2*d^65*x^3*sgn(b*x^2 + a) + 77*sqrt(d*x)*a^2*b*d^
65*x*sgn(b*x^2 + a))/d^66)/d
```

Mupad [B]
time = 4.53, size = 87, normalized size = 0.46

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{2\,a^2\,x^2}{d}-\frac{2\,a^3}{b\,d}+\frac{2\,b^2\,x^6}{11\,d}+\frac{6\,a\,b\,x^4}{7\,d}\right)}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(3/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*a^2*x^2)/d - (2*a^3)/(b*d) + (2*b^2*
x^6)/(11*d) + (6*a*b*x^4)/(7*d)))/(x^2*(d*x)^(1/2) + (a*(d*x)^(1/2))/b)
```

3.740 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{2b^3(dx)^{9/2}\ldots}{9d\ldots}$$

```
[Out] -2/3*a^3*((b*x^2+a)^2)^(1/2)/d/(d*x)^(3/2)/(b*x^2+a)+6/5*a*b^2*(d*x)^(5/2)*
((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+2/9*b^3*(d*x)^(9/2)*((b*x^2+a)^2)^(1/2)/d
^7/(b*x^2+a)+6*a^2*b*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A]

time = 0.04, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (6
*a^2*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (6*a*
b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^5*(a + b*x^2)) + (2*b
^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{(dx)^{5/2}}\,dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{(dx)^{5/2}}+\frac{3a^2b^4}{d^2\sqrt{dx}}+\frac{3ab^5(dx)^{3/2}}{d^4}+\frac{b^6(dx)^{7/2}}{d^6}\right)dx}{b^2(ab+b^2x^2)}$$
$$= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}(a+bx^2)}+\frac{6a^2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(a+bx^2)}+\frac{6ab^2(dx)^{5/2}\sqrt{\ }}{5d^5}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.34

$$-\frac{2x\left((a+bx^2)^2\right)^{3/2}(15a^3-135a^2bx^2-27ab^2x^4-5b^3x^6)}{45(dx)^{5/2}(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```

```
[Out] (-2*x*((a + b*x^2)^2)^(3/2)*(15*a^3 - 135*a^2*b*x^2 - 27*a*b^2*x^4 - 5*b^3*
x^6))/(45*(d*x)^(5/2)*(a + b*x^2)^3)
```

Maple [A]
time = 0.04, size = 63, normalized size = 0.33

method	result	size
gosper	$-\frac{2x(-5b^3x^6-27a\,b^2x^4-135a^2b\,x^2+15a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{45(b\,x^2+a)^3(dx)^{\frac{5}{2}}}$	61
default	$-\frac{2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}(-5b^3x^6-27a\,b^2x^4-135a^2b\,x^2+15a^3)}{45d(b\,x^2+a)^3(dx)^{\frac{3}{2}}}$	63
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(-5b^3x^6-27a\,b^2x^4-135a^2b\,x^2+15a^3)}{45d^2(b\,x^2+a)x\sqrt{dx}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*((b*x^2+a)^2)^(3/2)/d*(-5*b^3*x^6-27*a*b^2*x^4-135*a^2*b*x^2+15*a^3)/
(b*x^2+a)^3/(d*x)^(3/2)
```

Maxima [A]
time = 0.30, size = 86, normalized size = 0.45

$$\frac{2\left(\left(5\,b^3\sqrt{d}\,x^3+9\,ab^2\sqrt{d}\,x\right)x^{\frac{3}{2}}+\frac{18\left(ab^2\sqrt{d}\,x^3+5\,a^2b\sqrt{d}\,x\right)}{\sqrt{x}}+\frac{15\left(3\,a^2b\sqrt{d}\,x^3-a^3\sqrt{d}\,x\right)}{x^{\frac{5}{2}}}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45*((5*b^3*sqrt(d)*x^3 + 9*a*b^2*sqrt(d)*x)*x^(3/2) + 18*(a*b^2*sqrt(d)*x
^3 + 5*a^2*b*sqrt(d)*x)/sqrt(x) + 15*(3*a^2*b*sqrt(d)*x^3 - a^3*sqrt(d)*x)/
x^(5/2))/d^3
```

Fricas [A]
time = 0.36, size = 45, normalized size = 0.23

$$\frac{2\left(5\,b^3x^6+27\,ab^2x^4+135\,a^2bx^2-15\,a^3\right)\sqrt{dx}}{45\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)*sqrt(d*x)/(d^3*x^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(5/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(5/2), x)
```

Giac [A]
time = 3.39, size = 105, normalized size = 0.54

$$-\frac{2\left(\frac{15\,a^3d\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}\,x}-\frac{5\,\sqrt{dx}\,b^3d^{36}x^4\mathrm{sgn}\left(bx^2+a\right)+27\,\sqrt{dx}\,ab^2d^{36}x^2\mathrm{sgn}\left(bx^2+a\right)+135\,\sqrt{dx}\,a^2bd^{36}\mathrm{sgn}\left(bx^2+a\right)}{d^{36}}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/45*(15*a^3*d*sgn(b*x^2 + a)/(sqrt(d*x)*x) - (5*sqrt(d*x)*b^3*d^36*x^4*sg
n(b*x^2 + a) + 27*sqrt(d*x)*a*b^2*d^36*x^2*sgn(b*x^2 + a) + 135*sqrt(d*x)*a
^2*b*d^36*sgn(b*x^2 + a))/d^36)/d^3
```

Mupad [B]
time = 4.49, size = 88, normalized size = 0.46

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{6\,a^2\,x^2}{d^2}-\frac{2\,a^3}{3\,b\,d^2}+\frac{2\,b^2\,x^6}{9\,d^2}+\frac{6\,a\,b\,x^4}{5\,d^2}\right)}{x^3\,\sqrt{d\,x}+\frac{a\,x\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(5/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((6*a^2*x^2)/d^2 - (2*a^3)/(3*b*d^2) + (
2*b^2*x^6)/(9*d^2) + (6*a*b*x^4)/(5*d^2)))/(x^3*(d*x)^(1/2) + (a*x*(d*x)^(1
/2))/b)
```

3.741 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\,(a+bx^2)}-\frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,\,(a+bx^2)}+\frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\,(a+bx^2)}+\frac{2b^3(dx)^{7/2}\sqrt{a^2+}}{7d^7\,(a-}$$

```
[Out] -2/5*a^3*((b*x^2+a)^2)^(1/2)/d/(d*x)^(5/2)/(b*x^2+a)+2*a*b^2*(d*x)^(3/2)*((
b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+2/7*b^3*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d^7
/(b*x^2+a)-6*a^2*b*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\,(a+bx^2)}-\frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,\,(a+bx^2)}+\frac{2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\,(a+bx^2)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (6
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2)) + (2*a*
b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(a + b*x^2)) + (2*b^3
*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^3}{(dx)^{7/2}}\,dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3b^3}{(dx)^{7/2}}+\frac{3a^2b^4}{d^2(dx)^{3/2}}+\frac{3ab^5\sqrt{dx}}{d^4}+\frac{b^6(dx)^{5/2}}{d^6}\right)dx}{b^2(ab+b^2x^2)}$$
$$= -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}(a+bx^2)} - \frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,(a+bx^2)} + \frac{2ab^2(dx)^{3/2}\sqrt{a^2+}}{d^5(a+}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.35

$$-\frac{2x\left((a+bx^2)^2\right)^{3/2}(7a^3+105a^2bx^2-35ab^2x^4-5b^3x^6)}{35(dx)^{7/2}(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*x*((a + b*x^2)^2)^(3/2)*(7*a^3 + 105*a^2*b*x^2 - 35*a*b^2*x^4 - 5*b^3*x
^6))/(35*(d*x)^(7/2)*(a + b*x^2)^3)
```

Maple [A]
time = 0.02, size = 63, normalized size = 0.33

method	result	size
gosper	$-\frac{2x(-5b^3x^6-35a\,b^2x^4+105a^2b\,x^2+7a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{35(b\,x^2+a)^3(dx)^{\frac{7}{2}}}$	61
default	$-\frac{2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}(-5b^3x^6-35a\,b^2x^4+105a^2b\,x^2+7a^3)}{35d(b\,x^2+a)^3(dx)^{\frac{5}{2}}}$	63
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(-5b^3x^6-35a\,b^2x^4+105a^2b\,x^2+7a^3)}{35d^3(b\,x^2+a)x^2\sqrt{dx}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*((b*x^2+a)^2)^(3/2)/d*(-5*b^3*x^6-35*a*b^2*x^4+105*a^2*b*x^2+7*a^3)/(
b*x^2+a)^3/(d*x)^(5/2)
```

Maxima [A]
time = 0.30, size = 86, normalized size = 0.45

$$\frac{2\left(5\left(3\,b^{3}\sqrt{d}\,x^{3}+7\,ab^{2}\sqrt{d}\,x\right)\sqrt{x}+\frac{70\left(ab^{2}\sqrt{d}\,x^{3}-3\,a^{2}b\sqrt{d}\,x\right)}{x^{\frac{3}{2}}}-\frac{21\left(5\,a^{2}b\sqrt{d}\,x^{3}+a^{3}\sqrt{d}\,x\right)}{x^{\frac{7}{2}}}\right)}{105\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="maxima")
[Out] 2/105*(5*(3*b^3*sqrt(d)*x^3 + 7*a*b^2*sqrt(d)*x)*sqrt(x) + 70*(a*b^2*sqrt(d
)*x^3 - 3*a^2*b*sqrt(d)*x)/x^(3/2) - 21*(5*a^2*b*sqrt(d)*x^3 + a^3*sqrt(d)*
x)/x^(7/2))/d^4
```

Fricas [A]
time = 0.33, size = 45, normalized size = 0.24

$$\frac{2\,(5\,b^{3}x^{6}+35\,ab^{2}x^{4}-105\,a^{2}bx^{2}-7\,a^{3})\sqrt{dx}}{35\,d^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="fricas")
[Out] 2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)*sqrt(d*x)/(d^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^{2})^{2}\right)^{\frac{3}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(7/2),x)
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(7/2), x)
```

Giac [A]
time = 3.78, size = 107, normalized size = 0.56

$$-\frac{2\left(\frac{7\,(15\,a^{2}bd^{3}x^{2}\mathrm{sgn}(bx^{2}+a)+a^{3}d^{3}\mathrm{sgn}(bx^{2}+a))}{\sqrt{dx}\,d^{2}x^{2}}-\frac{5\left(\sqrt{dx}\,b^{3}d^{21}x^{3}\mathrm{sgn}(bx^{2}+a)+7\,\sqrt{dx}\,ab^{2}d^{21}x\mathrm{sgn}(bx^{2}+a)\right)}{d^{21}}\right)}{35\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/35*(7*(15*a^2*b*d^3*x^2*sgn(b*x^2 + a) + a^3*d^3*sgn(b*x^2 + a))/(sqrt(d
*x)*d^2*x^2) - 5*(sqrt(d*x)*b^3*d^21*x^3*sgn(b*x^2 + a) + 7*sqrt(d*x)*a*b^2
*d^21*x*sgn(b*x^2 + a))/d^21)/d^4
```

Mupad [B]
time = 4.53, size = 91, normalized size = 0.48

$$-\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{2\,a^3}{5\,b\,d^3}+\frac{6\,a^2\,x^2}{d^3}-\frac{2\,b^2\,x^6}{7\,d^3}-\frac{2\,a\,b\,x^4}{d^3}\right)}{x^4\,\sqrt{d\,x}+\frac{a\,x^2\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(7/2),x)
```

```
[Out] -((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*a^3)/(5*b*d^3) + (6*a^2*x^2)/d^3 -
(2*b^2*x^6)/(7*d^3) - (2*a*b*x^4)/d^3))/(x^4*(d*x)^(1/2) + (a*x^2*(d*x)^(1/
2))/b)
```

3.742 $\int (dx)^{5/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=297

$$\frac{2a^5(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\,(a+bx^2)}+\frac{10a^4b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\,(a+bx^2)}+\frac{4a^3b^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\,(a+bx^2)}+$$

```
[Out] 2/7*a^5*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+10/11*a^4*b*(d*x)^(11/2
)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+4/3*a^3*b^2*(d*x)^(15/2)*((b*x^2+a)^2)^
(1/2)/d^5/(b*x^2+a)+20/19*a^2*b^3*(d*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x
^2+a)+10/23*a*b^4*(d*x)^(23/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/27*b^5*(
d*x)^(27/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Rubi [A]
time = 0.06, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{2b^5(dx)^{27/2}\sqrt{a^2+2abx^2+b^2x^4}}{27d^{11}\,(a+bx^2)}+\frac{10ab^4(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^9\,(a+bx^2)}+\frac{20a^2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\,(a+bx^2)}+\frac{2a^5(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\,(a+bx^2)}+\frac{10a^4b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\,(a+bx^2)}+\frac{4a^3b^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a^5*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(a + b*x^2)) +
 (4*a^3*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2
)) + (20*a^2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a +
 b*x^2)) + (10*a*b^4*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^9*
(a + b*x^2)) + (2*b^5*(d*x)^(27/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(27*d^1
1*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (dx)^{5/2} \left(ab+b^2x^2\right)^5 dx}{b^4 \left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^5b^5(dx)^{5/2} + \frac{5a^4b^6(dx)^{9/2}}{d^2} + \frac{10a^3b^7(dx)^{13/2}}{d^4} + \frac{10a}{} \right.}{b^4 \left(ab+b^2x^2\right)}$$
$$= \frac{2a^5(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)} + \frac{10a^4b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 0.30

$$\frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}\left(129789a^5+412965a^4bx^2+605682a^3b^2x^4+478170a^2b^3x^6+197505ab^4x^8+33649b^5x^{10}\right)}{908523\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(129789*a^5 + 412965*a^4*b*x^2 + 60568
2*a^3*b^2*x^4 + 478170*a^2*b^3*x^6 + 197505*a*b^4*x^8 + 33649*b^5*x^10))/(9
08523*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 85, normalized size = 0.29

method	result	size
gosper	$\frac{2x\left(33649b^5x^{10}+197505b^4a\,x^8+478170a^2b^3x^6+605682b^2a^3x^4+412965b\,a^4x^2+129789a^5\right)(dx)^{\frac{5}{2}}\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{908523\left(b\,x^2+a\right)^5}$	83
default	$\frac{2\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}(dx)^{\frac{7}{2}}\left(33649b^5x^{10}+197505b^4a\,x^8+478170a^2b^3x^6+605682b^2a^3x^4+412965b\,a^4x^2+129789a^5\right)}{908523d\left(b\,x^2+a\right)^5}$	85
risch	$\frac{2d^3\sqrt{\left(b\,x^2+a\right)^2}\,x^4\left(33649b^5x^{10}+197505b^4a\,x^8+478170a^2b^3x^6+605682b^2a^3x^4+412965b\,a^4x^2+129789a^5\right)}{908523\left(b\,x^2+a\right)\sqrt{dx}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/908523*((b*x^2+a)^2)^(5/2)*(d*x)^(7/2)/d*(33649*b^5*x^10+197505*a*b^4*x^8
+478170*a^2*b^3*x^6+605682*a^3*b^2*x^4+412965*a^4*b*x^2+129789*a^5)/(b*x^2+
a)^5
```

Maxima [A]
time = 0.28, size = 147, normalized size = 0.49

$$\frac{2}{621}\left(23\,b^5d^{\frac{5}{2}}x^3+27\,ab^4d^{\frac{5}{2}}x\right)x^{\frac{21}{2}}+\frac{8}{437}\left(19\,ab^4d^{\frac{5}{2}}x^3+23\,a^2b^3d^{\frac{5}{2}}x\right)x^{\frac{17}{2}}+\frac{4}{95}\left(15\,a^2b^3d^{\frac{5}{2}}x^3+19\,a^3b^2d^{\frac{5}{2}}x\right)x^{\frac{13}{2}}+\frac{8}{165}\left(11\,a^3b^2d^{\frac{5}{2}}x^3+15\,a^4bd^{\frac{5}{2}}x\right)x^{\frac{9}{2}}+\frac{2}{77}\left(7\,a^4bd^{\frac{5}{2}}x^3+11\,a^5d^{\frac{5}{2}}x\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/621*(23*b^5*d^(5/2)*x^3 + 27*a*b^4*d^(5/2)*x)*x^(21/2) + 8/437*(19*a*b^4*
d^(5/2)*x^3 + 23*a^2*b^3*d^(5/2)*x)*x^(17/2) + 4/95*(15*a^2*b^3*d^(5/2)*x^3
 + 19*a^3*b^2*d^(5/2)*x)*x^(13/2) + 8/165*(11*a^3*b^2*d^(5/2)*x^3 + 15*a^4*
b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^4*b*d^(5/2)*x^3 + 11*a^5*d^(5/2)*x)*x^(5/2
)
```

Fricas [A]
time = 0.35, size = 82, normalized size = 0.28

$$\frac{2}{908523}\left(33649\,b^5d^2x^{13}+197505\,ab^4d^2x^{11}+478170\,a^2b^3d^2x^9+605682\,a^3b^2d^2x^7+412965\,a^4bd^2x^5+129789\,a^5d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/908523*(33649*b^5*d^2*x^13 + 197505*a*b^4*d^2*x^11 + 478170*a^2*b^3*d^2*x
^9 + 605682*a^3*b^2*d^2*x^7 + 412965*a^4*b*d^2*x^5 + 129789*a^5*d^2*x^3)*sq
rt(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{5}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(5/2)*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.01, size = 153, normalized size = 0.52

$$\frac{2}{27}\sqrt{dx}\,b^5d^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{23}\sqrt{dx}\,ab^4d^2x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{20}{19}\sqrt{dx}\,a^2b^3d^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{4}{3}\sqrt{dx}\,a^3b^2d^2x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{11}\sqrt{dx}\,a^4bd^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}\,a^5d^2x^3\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(d*x)*b^5*d^2*x^13*sgn(b*x^2 + a) + 10/23*sqrt(d*x)*a*b^4*d^2*x^11
*sgn(b*x^2 + a) + 20/19*sqrt(d*x)*a^2*b^3*d^2*x^9*sgn(b*x^2 + a) + 4/3*sqrt
(d*x)*a^3*b^2*d^2*x^7*sgn(b*x^2 + a) + 10/11*sqrt(d*x)*a^4*b*d^2*x^5*sgn(b*
x^2 + a) + 2/7*sqrt(d*x)*a^5*d^2*x^3*sgn(b*x^2 + a)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^{5/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.743 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=297

$$\frac{2a^5(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}+\frac{10a^4b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\,(a+bx^2)}+\frac{20a^3b^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\,(a+bx^2)}+$$

```
[Out] 2/5*a^5*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+10/9*a^4*b*(d*x)^(9/2)*
((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/13*a^3*b^2*(d*x)^(13/2)*((b*x^2+a)^2)^
(1/2)/d^5/(b*x^2+a)+20/17*a^2*b^3*(d*x)^(17/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x
^2+a)+10/21*a*b^4*(d*x)^(21/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/25*b^5*(
d*x)^(25/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{2b^5(dx)^{25/2}\sqrt{a^2+2abx^2+b^2x^4}}{25d^{11}\,(a+bx^2)}+\frac{10ab^4(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^9\,(a+bx^2)}+\frac{20a^2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\,(a+bx^2)}+\frac{2a^5(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}+\frac{10a^4b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\,(a+bx^2)}+\frac{20a^3b^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a^5*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a + b*x^2)) + (
20*a^3*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^5*(a + b*x^2
)) + (20*a^2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a +
 b*x^2)) + (10*a*b^4*(d*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^9*
(a + b*x^2)) + (2*b^5*(d*x)^(25/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(25*d^1
1*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{3/2}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^5b^5(dx)^{3/2}+\frac{5a^4b^6(dx)^{7/2}}{d^2}+\frac{10a^3b^7(dx)^{11/2}}{d^4}+\frac{10a}{}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{2a^5(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.30

$$\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(69615a^5+193375a^4bx^2+267750a^3b^2x^4+204750a^2b^3x^6+82875ab^4x^8+13923b^5x^{10}\right)}{348075\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(69615*a^5 + 193375*a^4*b*x^2 + 267750
*a^3*b^2*x^4 + 204750*a^2*b^3*x^6 + 82875*a*b^4*x^8 + 13923*b^5*x^10))/(348
075*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 85, normalized size = 0.29

method	result	size
gosper	$\frac{2x\left(13923b^5x^{10}+82875b^4a\,x^8+204750a^2b^3x^6+267750b^2a^3x^4+193375b\,a^4x^2+69615a^5\right)(dx)^{\frac{3}{2}}\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}}{348075\left(b\,x^2+a\right)^5}$	83
default	$\frac{2\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}(dx)^{\frac{5}{2}}\left(13923b^5x^{10}+82875b^4a\,x^8+204750a^2b^3x^6+267750b^2a^3x^4+193375b\,a^4x^2+69615a^5\right)}{348075d\left(b\,x^2+a\right)^5}$	85
risch	$\frac{2d^2\sqrt{\left(b\,x^2+a\right)^2}\,x^3\left(13923b^5x^{10}+82875b^4a\,x^8+204750a^2b^3x^6+267750b^2a^3x^4+193375b\,a^4x^2+69615a^5\right)}{348075\left(b\,x^2+a\right)\sqrt{dx}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/348075*((b*x^2+a)^2)^(5/2)*(d*x)^(5/2)/d*(13923*b^5*x^10+82875*a*b^4*x^8+
204750*a^2*b^3*x^6+267750*a^3*b^2*x^4+193375*a^4*b*x^2+69615*a^5)/(b*x^2+a)
^5
```

Maxima [A]
time = 0.30, size = 147, normalized size = 0.49

$$\frac{2}{525}\left(21\,b^5d^{\frac{3}{2}}x^3+25\,ab^4d^{\frac{3}{2}}x\right)x^{\frac{19}{2}}+\frac{8}{357}\left(17\,ab^4d^{\frac{3}{2}}x^3+21\,a^2b^3d^{\frac{3}{2}}x\right)x^{\frac{15}{2}}+\frac{12}{221}\left(13\,a^2b^3d^{\frac{3}{2}}x^3+17\,a^3b^2d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+\frac{8}{117}\left(9\,a^3b^2d^{\frac{3}{2}}x^3+13\,a^4bd^{\frac{3}{2}}x\right)x^{\frac{7}{2}}+\frac{2}{45}\left(5\,a^4bd^{\frac{3}{2}}x^3+9\,a^5d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/525*(21*b^5*d^(3/2)*x^3 + 25*a*b^4*d^(3/2)*x)*x^(19/2) + 8/357*(17*a*b^4*
d^(3/2)*x^3 + 21*a^2*b^3*d^(3/2)*x)*x^(15/2) + 12/221*(13*a^2*b^3*d^(3/2)*x
^3 + 17*a^3*b^2*d^(3/2)*x)*x^(11/2) + 8/117*(9*a^3*b^2*d^(3/2)*x^3 + 13*a^4
*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^4*b*d^(3/2)*x^3 + 9*a^5*d^(3/2)*x)*x^(3/2
)
```

Fricas [A]
time = 0.33, size = 70, normalized size = 0.24

$$\frac{2}{348075}\left(13923\,b^5dx^{12}+82875\,ab^4dx^{10}+204750\,a^2b^3dx^8+267750\,a^3b^2dx^6+193375\,a^4bdx^4+69615\,a^5dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^5*d*x^12 + 82875*a*b^4*d*x^10 + 204750*a^2*b^3*d*x^8 + 26
7750*a^3*b^2*d*x^6 + 193375*a^4*b*d*x^4 + 69615*a^5*d*x^2)*sqrt(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(3/2)*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.96, size = 138, normalized size = 0.46

$$\frac{2}{348075}\left(13923\,\sqrt{dx}\,b^5x^{12}\mathrm{sgn}\left(bx^2+a\right)+82875\,\sqrt{dx}\,ab^4x^{10}\mathrm{sgn}\left(bx^2+a\right)+204750\,\sqrt{dx}\,a^2b^3x^8\mathrm{sgn}\left(bx^2+a\right)+267750\,\sqrt{dx}\,a^3b^2x^6\mathrm{sgn}\left(bx^2+a\right)+193375\,\sqrt{dx}\,a^4bx^4\mathrm{sgn}\left(bx^2+a\right)+69615\,\sqrt{dx}\,a^5x^2\mathrm{sgn}\left(bx^2+a\right)\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/348075*(13923*sqrt(d*x)*b^5*x^12*sgn(b*x^2 + a) + 82875*sqrt(d*x)*a*b^4*x
^10*sgn(b*x^2 + a) + 204750*sqrt(d*x)*a^2*b^3*x^8*sgn(b*x^2 + a) + 267750*s
```

```
qrt(d*x)*a^3*b^2*x^6*sgn(b*x^2 + a) + 193375*sqrt(d*x)*a^4*b*x^4*sgn(b*x^2
+ a) + 69615*sqrt(d*x)*a^5*x^2*sgn(b*x^2 + a))*d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^{3/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.744 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=297

$$\frac{2a^5(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}+$$

```
[Out] 2/3*a^5*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+10/7*a^4*b*(d*x)^(7/2)*
((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/11*a^3*b^2*(d*x)^(11/2)*((b*x^2+a)^2)^
(1/2)/d^5/(b*x^2+a)+4/3*a^2*b^3*(d*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2
+a)+10/19*a*b^4*(d*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/23*b^5*(d*
x)^(23/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{2b^5(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^9\left(a+bx^2\right)}+\frac{4a^2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^7\left(a+bx^2\right)}+\frac{2a^5(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a^5*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (10
*a^4*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a + b*x^2)) + (
20*a^3*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^5*(a + b*x^2
)) + (4*a^2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^7*(a + b
*x^2)) + (10*a*b^4*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^9*(a
 + b*x^2)) + (2*b^5*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^11*
(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^5b^5\sqrt{dx}+\frac{5a^4b^6(dx)^{5/2}}{d^2}+\frac{10a^3b^7(dx)^{9/2}}{d^4}+\frac{10a^2b^8(\ldots}{d^6}\right.}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{2a^5(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\ldots$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.30

$$\frac{2x\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}\left(33649a^5+72105a^4bx^2+91770a^3b^2x^4+67298a^2b^3x^6+26565ab^4x^8+4389b^5x^{10}\right)}{100947\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(33649*a^5 + 72105*a^4*b*x^2 + 91770*a^3
*b^2*x^4 + 67298*a^2*b^3*x^6 + 26565*a*b^4*x^8 + 4389*b^5*x^10))/(100947*(a
 + b*x^2))
```

Maple [A]
time = 0.04, size = 85, normalized size = 0.29

method	result	size
gosper	$\frac{2x\left(4389b^5x^{10}+26565b^4a\,x^8+67298a^2b^3x^6+91770b^2a^3x^4+72105b\,a^4x^2+33649a^5\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}\sqrt{dx}}{100947\left(b\,x^2+a\right)^5}$	83
default	$\frac{2\left(\left(b\,x^2+a\right)^2\right)^{\frac{5}{2}}(dx)^{\frac{3}{2}}\left(4389b^5x^{10}+26565b^4a\,x^8+67298a^2b^3x^6+91770b^2a^3x^4+72105b\,a^4x^2+33649a^5\right)}{100947d\left(b\,x^2+a\right)^5}$	85
risch	$\frac{2d\sqrt{\left(b\,x^2+a\right)^2}\,x^2\left(4389b^5x^{10}+26565b^4a\,x^8+67298a^2b^3x^6+91770b^2a^3x^4+72105b\,a^4x^2+33649a^5\right)}{100947\left(b\,x^2+a\right)\sqrt{dx}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/100947*((b*x^2+a)^2)^(5/2)*(d*x)^(3/2)/d*(4389*b^5*x^10+26565*a*b^4*x^8+6
7298*a^2*b^3*x^6+91770*a^3*b^2*x^4+72105*a^4*b*x^2+33649*a^5)/(b*x^2+a)^5
```

Maxima [A]
time = 0.29, size = 147, normalized size = 0.49

$$\frac{2}{437}\left(19\,b^5\sqrt{d}\,x^3+23\,ab^4\sqrt{d}\,x\right)x^{\frac{17}{2}}+\frac{8}{285}\left(15\,ab^4\sqrt{d}\,x^3+19\,a^2b^3\sqrt{d}\,x\right)x^{\frac{13}{2}}+\frac{4}{55}\left(11\,a^2b^3\sqrt{d}\,x^3+15\,a^3b^2\sqrt{d}\,x\right)x^{\frac{9}{2}}+\frac{8}{77}\left(7\,a^3b^2\sqrt{d}\,x^3+11\,a^4b\sqrt{d}\,x\right)x^{\frac{5}{2}}+\frac{2}{21}\left(3\,a^4b\sqrt{d}\,x^3+7\,a^5\sqrt{d}\,x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/437*(19*b^5*sqrt(d)*x^3 + 23*a*b^4*sqrt(d)*x)*x^(17/2) + 8/285*(15*a*b^4*
sqrt(d)*x^3 + 19*a^2*b^3*sqrt(d)*x)*x^(13/2) + 4/55*(11*a^2*b^3*sqrt(d)*x^3
 + 15*a^3*b^2*sqrt(d)*x)*x^(9/2) + 8/77*(7*a^3*b^2*sqrt(d)*x^3 + 11*a^4*b*s
qrt(d)*x)*x^(5/2) + 2/21*(3*a^4*b*sqrt(d)*x^3 + 7*a^5*sqrt(d)*x)*sqrt(x)
```

Fricas [A]
time = 0.33, size = 62, normalized size = 0.21

$$\frac{2}{100947}\left(4389\,b^5x^{11}+26565\,ab^4x^9+67298\,a^2b^3x^7+91770\,a^3b^2x^5+72105\,a^4bx^3+33649\,a^5x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/100947*(4389*b^5*x^11 + 26565*a*b^4*x^9 + 67298*a^2*b^3*x^7 + 91770*a^3*b
^2*x^5 + 72105*a^4*b*x^3 + 33649*a^5*x)*sqrt(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{dx}\,\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)*(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 4.28, size = 133, normalized size = 0.45

$$\frac{2}{23}\sqrt{dx}\,b^5x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{19}\sqrt{dx}\,ab^4x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{4}{3}\sqrt{dx}\,a^2b^3x^7\mathrm{sgn}\left(bx^2+a\right)+\frac{20}{11}\sqrt{dx}\,a^3b^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{7}\sqrt{dx}\,a^4bx^3\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\sqrt{dx}\,a^5x\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/23*sqrt(d*x)*b^5*x^11*sgn(b*x^2 + a) + 10/19*sqrt(d*x)*a*b^4*x^9*sgn(b*x^
2 + a) + 4/3*sqrt(d*x)*a^2*b^3*x^7*sgn(b*x^2 + a) + 20/11*sqrt(d*x)*a^3*b^2
*x^5*sgn(b*x^2 + a) + 10/7*sqrt(d*x)*a^4*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt(d*
x)*a^5*x*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\sqrt{d\,x}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.745 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=293

$$\frac{2a^5\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\,(a+bx^2)}+\frac{2a^4b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\,(a+bx^2)}+\frac{20a^3b^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^5\,(a+bx^2)}+20a$$

```
[Out] 2*a^4*b*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/9*a^3*b^2*(d*x)^(9
/2)*((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+20/13*a^2*b^3*(d*x)^(13/2)*((b*x^2+a)
^2)^(1/2)/d^7/(b*x^2+a)+10/17*a*b^4*(d*x)^(17/2)*((b*x^2+a)^2)^(1/2)/d^9/(b
*x^2+a)+2/21*b^5*(d*x)^(21/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)+2*a^5*(d*x
)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{2b^5(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^{11}\,(a+bx^2)}+\frac{10ab^4(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^9\,(a+bx^2)}+\frac{20a^2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\,(a+bx^2)}+\frac{2a^5\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d\,(a+bx^2)}+\frac{2a^4b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\,(a+bx^2)}+\frac{20a^3b^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^5\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```

```
[Out] (2*a^5*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) + (2*a^4*
b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (20*a^3*
b^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^5*(a + b*x^2)) + (20*
a^2*b^3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2))
+ (10*a*b^4*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^9*(a + b*x^
2)) + (2*b^5*(d*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^11*(a + b*
x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{\sqrt{dx}}\,dx}{b^4(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{\sqrt{dx}} + \frac{5a^4b^6(dx)^{3/2}}{d^2} + \frac{10a^3b^7(dx)^{7/2}}{d^4} + \frac{10a^2b^8(dx)^{11/2}}{d^6} + \frac{5\right.}{}}{b^4(ab+b^2x^2)}$$
$$= \frac{2a^5\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d(a+bx^2)} + \frac{2a^4b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(a+bx^2)} + \frac{20a^3b^2(}{}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.30

$$\frac{2x\sqrt{(a+bx^2)^2}\,(13923a^5+13923a^4bx^2+15470a^3b^2x^4+10710a^2b^3x^6+4095ab^4x^8+663b^5x^{10})}{13923\sqrt{dx}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(13923*a^5 + 13923*a^4*b*x^2 + 15470*a^3*b^2*x^4 +
 10710*a^2*b^3*x^6 + 4095*a*b^4*x^8 + 663*b^5*x^10))/(13923*Sqrt[d*x]*(a +
b*x^2))
```

Maple [A]
time = 0.02, size = 85, normalized size = 0.29

method	result	size
gosper	$\frac{2x(663b^5x^{10}+4095b^4a\,x^8+10710a^2b^3x^6+15470b^2a^3x^4+13923b\,a^4x^2+13923a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{13923(b\,x^2+a)^5\sqrt{dx}}$	83
risch	$\frac{2\sqrt{(b\,x^2+a)^2}\,(663b^5x^{10}+4095b^4a\,x^8+10710a^2b^3x^6+15470b^2a^3x^4+13923b\,a^4x^2+13923a^5)x}{13923(b\,x^2+a)\sqrt{dx}}$	83
default	$\frac{2\left((b\,x^2+a)^2\right)^{\frac{5}{2}}\sqrt{dx}\,(663b^5x^{10}+4095b^4a\,x^8+10710a^2b^3x^6+15470b^2a^3x^4+13923b\,a^4x^2+13923a^5)}{13923d(b\,x^2+a)^5}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13923*((b*x^2+a)^2)^(5/2)*(d*x)^(1/2)/d*(663*b^5*x^10+4095*a*b^4*x^8+1071
0*a^2*b^3*x^6+15470*a^3*b^2*x^4+13923*a^4*b*x^2+13923*a^5)/(b*x^2+a)^5
```

Maxima [A]
time = 0.29, size = 151, normalized size = 0.52

$$\frac{2\left(195\left(17\,b^5\sqrt{d}\,x^3+21\,ab^4\sqrt{d}\,x\right)x^{\frac{15}{2}}+1260\left(13\,ab^4\sqrt{d}\,x^3+17\,a^2b^3\sqrt{d}\,x\right)x^{\frac{11}{2}}+3570\left(9\,a^2b^3\sqrt{d}\,x^3+13\,a^3b^2\sqrt{d}\,x\right)x^{\frac{7}{2}}+6188\left(5\,a^3b^2\sqrt{d}\,x^3+9\,a^4b\sqrt{d}\,x\right)x^{\frac{3}{2}}+\frac{13923\left(a^4b\sqrt{d}\,x^3+5\,a^5\sqrt{d}\,x\right)}{\sqrt{x}}\right)}{69615\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/69615*(195*(17*b^5*sqrt(d)*x^3 + 21*a*b^4*sqrt(d)*x)*x^(15/2) + 1260*(13*
a*b^4*sqrt(d)*x^3 + 17*a^2*b^3*sqrt(d)*x)*x^(11/2) + 3570*(9*a^2*b^3*sqrt(d
)*x^3 + 13*a^3*b^2*sqrt(d)*x)*x^(7/2) + 6188*(5*a^3*b^2*sqrt(d)*x^3 + 9*a^4
*b*sqrt(d)*x)*x^(3/2) + 13923*(a^4*b*sqrt(d)*x^3 + 5*a^5*sqrt(d)*x)/sqrt(x)
)/d
```

Fricas [A]
time = 0.37, size = 64, normalized size = 0.22

$$\frac{2\left(663\,b^5x^{10}+4095\,ab^4x^8+10710\,a^2b^3x^6+15470\,a^3b^2x^4+13923\,a^4bx^2+13923\,a^5\right)\sqrt{dx}}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^5*x^10 + 4095*a*b^4*x^8 + 10710*a^2*b^3*x^6 + 15470*a^3*b^2*
x^4 + 13923*a^4*b*x^2 + 13923*a^5)*sqrt(d*x)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/sqrt(d*x), x)
```

Giac [A]
time = 4.00, size = 137, normalized size = 0.47

$$\frac{2\left(663\,\sqrt{dx}\,b^5x^{10}\mathrm{sgn}\left(bx^2+a\right)+4095\,\sqrt{dx}\,ab^4x^8\mathrm{sgn}\left(bx^2+a\right)+10710\,\sqrt{dx}\,a^2b^3x^6\mathrm{sgn}\left(bx^2+a\right)+15470\,\sqrt{dx}\,a^3b^2x^4\mathrm{sgn}\left(bx^2+a\right)+13923\,\sqrt{dx}\,a^4bx^2\mathrm{sgn}\left(bx^2+a\right)+13923\,\sqrt{dx}\,a^5\mathrm{sgn}\left(bx^2+a\right)\right)}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/13923*(663*sqrt(d*x)*b^5*x^10*sgn(b*x^2 + a) + 4095*sqrt(d*x)*a*b^4*x^8*s
gn(b*x^2 + a) + 10710*sqrt(d*x)*a^2*b^3*x^6*sgn(b*x^2 + a) + 15470*sqrt(d*x
)*a^3*b^2*x^4*sgn(b*x^2 + a) + 13923*sqrt(d*x)*a^4*b*x^2*sgn(b*x^2 + a) + 1
3923*sqrt(d*x)*a^5*sgn(b*x^2 + a))/d
```

Mupad [B]
time = 4.57, size = 112, normalized size = 0.38

$$\frac{2\,x\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(5731\,a^4+8192\,a^3\,b\,x^2+7278\,a^2\,b^2\,x^4+3432\,a\,b^3\,x^6+663\,b^4\,x^8\right)}{13923\,\sqrt{d\,x}}+\frac{16384\,a^5\,x\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13923\,\sqrt{d\,x}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(1/2),x)
```

```
[Out] (2*x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(5731*a^4 + 663*b^4*x^8 + 8192*a^3*b
*x^2 + 3432*a*b^3*x^6 + 7278*a^2*b^2*x^4))/(13923*(d*x)^(1/2)) + (16384*a^5
*x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13923*(d*x)^(1/2)*(a + b*x^2))
```

3.746 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=295

$$-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)}+\frac{10a^4b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)}+\frac{20a^3b^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\,(a+bx^2)}+\frac{20a^2b^3}{}$$

```
[Out] 10/3*a^4*b*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/7*a^3*b^2*(d*x)
^(7/2)*((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+20/11*a^2*b^3*(d*x)^(11/2)*((b*x^2
+a)^2)^(1/2)/d^7/(b*x^2+a)+2/3*a*b^4*(d*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^9/(
b*x^2+a)+2/19*b^5*(d*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)-2*a^5*((b
*x^2+a)^2)^(1/2)/d/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{2b^5(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^{11}\,(a+bx^2)}+\frac{2ab^4(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^9\,(a+bx^2)}+\frac{20a^2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\,(a+bx^2)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)}+\frac{10a^4b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)}+\frac{20a^3b^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)) + (10*a^
4*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b*x^2)) + (20*
a^3*b^2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^5*(a + b*x^2)) +
(20*a^2*b^3*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^
2)) + (2*a*b^4*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^9*(a + b*
x^2)) + (2*b^5*(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^11*(a +
b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{(dx)^{3/2}}\,dx}{b^4(ab+b^2x^2)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{3/2}} + \frac{5a^4b^6\sqrt{dx}}{d^2} + \frac{10a^3b^7(dx)^{5/2}}{d^4} + \frac{10a^2b^8(dx)^{9/2}}{d^6} + \frac{5a}{}\right.}{b^4(ab+b^2x^2)}$$

$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)} + \frac{10a^4b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3(a+bx^2)} + \frac{20a^3b^2(dx}{}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.30

$$-\frac{2x\left((a+bx^2)^2\right)^{5/2}(4389a^5-7315a^4bx^2-6270a^3b^2x^4-3990a^2b^3x^6-1463ab^4x^8-231b^5x^{10})}{4389(dx)^{3/2}(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*x*((a + b*x^2)^2)^(5/2)*(4389*a^5 - 7315*a^4*b*x^2 - 6270*a^3*b^2*x^4 -
 3990*a^2*b^3*x^6 - 1463*a*b^4*x^8 - 231*b^5*x^10))/(4389*(d*x)^(3/2)*(a +
b*x^2)^5)
```

Maple [A]
time = 0.02, size = 85, normalized size = 0.29

method	result	size
gosper	$-\frac{2x(-231b^5x^{10}-1463b^4a\,x^8-3990a^2b^3x^6-6270b^2a^3x^4-7315b\,a^4x^2+4389a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{4389(b\,x^2+a)^5(dx)^{\frac{3}{2}}}$	83
default	$-\frac{2\left((b\,x^2+a)^2\right)^{\frac{5}{2}}(-231b^5x^{10}-1463b^4a\,x^8-3990a^2b^3x^6-6270b^2a^3x^4-7315b\,a^4x^2+4389a^5)}{4389d(b\,x^2+a)^5\sqrt{dx}}$	85
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(-231b^5x^{10}-1463b^4a\,x^8-3990a^2b^3x^6-6270b^2a^3x^4-7315b\,a^4x^2+4389a^5)}{4389d(b\,x^2+a)\sqrt{dx}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/4389*((b*x^2+a)^2)^(5/2)/d*(-231*b^5*x^10-1463*a*b^4*x^8-3990*a^2*b^3*x^
6-6270*a^3*b^2*x^4-7315*a^4*b*x^2+4389*a^5)/(b*x^2+a)^5/(d*x)^(1/2)
```

Maxima [A]
time = 0.29, size = 151, normalized size = 0.51

$$\frac{2\left(77\left(15\,b^5\sqrt{d}\,x^3+19\,ab^4\sqrt{d}\,x\right)x^{\frac{13}{2}}+532\left(11\,ab^4\sqrt{d}\,x^3+15\,a^2b^3\sqrt{d}\,x\right)x^{\frac{9}{2}}+1710\left(7\,a^2b^3\sqrt{d}\,x^3+11\,a^3b^2\sqrt{d}\,x\right)x^{\frac{5}{2}}+4180\left(3\,a^3b^2\sqrt{d}\,x^3+7\,a^4b\sqrt{d}\,x\right)\sqrt{x}+\frac{7315\left(a^4b\sqrt{d}\,x^3-3\,a^5\sqrt{d}\,x\right)}{x^{\frac{3}{2}}}\right)}{21945\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21945*(77*(15*b^5*sqrt(d)*x^3 + 19*a*b^4*sqrt(d)*x)*x^(13/2) + 532*(11*a*
b^4*sqrt(d)*x^3 + 15*a^2*b^3*sqrt(d)*x)*x^(9/2) + 1710*(7*a^2*b^3*sqrt(d)*x
^3 + 11*a^3*b^2*sqrt(d)*x)*x^(5/2) + 4180*(3*a^3*b^2*sqrt(d)*x^3 + 7*a^4*b*
sqrt(d)*x)*sqrt(x) + 7315*(a^4*b*sqrt(d)*x^3 - 3*a^5*sqrt(d)*x)/x^(3/2))/d^
2
```

Fricas [A]
time = 0.34, size = 67, normalized size = 0.23

$$\frac{2\left(231\,b^5x^{10}+1463\,ab^4x^8+3990\,a^2b^3x^6+6270\,a^3b^2x^4+7315\,a^4bx^2-4389\,a^5\right)\sqrt{dx}}{4389\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/4389*(231*b^5*x^10 + 1463*a*b^4*x^8 + 3990*a^2*b^3*x^6 + 6270*a^3*b^2*x^4
 + 7315*a^4*b*x^2 - 4389*a^5)*sqrt(d*x)/(d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{\left(dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(3/2), x)
```

Giac [A]
time = 3.98, size = 156, normalized size = 0.53

$$-\frac{2\left(\frac{4389\,a^5\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}}-\frac{231\,\sqrt{dx}\,b^5d^{189}x^9\mathrm{sgn}\left(bx^2+a\right)+1463\,\sqrt{dx}\,ab^4d^{189}x^7\mathrm{sgn}\left(bx^2+a\right)+3990\,\sqrt{dx}\,a^2b^3d^{189}x^5\mathrm{sgn}\left(bx^2+a\right)+6270\,\sqrt{dx}\,a^3b^2d^{189}x^3\mathrm{sgn}\left(bx^2+a\right)+7315\,\sqrt{dx}\,a^4bd^{189}x\mathrm{sgn}\left(bx^2+a\right)}{d^{190}}\right)}{4389\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/4389*(4389*a^5*sgn(b*x^2 + a)/sqrt(d*x) - (231*sqrt(d*x)*b^5*d^189*x^9*s
gn(b*x^2 + a) + 1463*sqrt(d*x)*a*b^4*d^189*x^7*sgn(b*x^2 + a) + 3990*sqrt(d
*x)*a^2*b^3*d^189*x^5*sgn(b*x^2 + a) + 6270*sqrt(d*x)*a^3*b^2*d^189*x^3*sgn
(b*x^2 + a) + 7315*sqrt(d*x)*a^4*b*d^189*x*sgn(b*x^2 + a))/d^190)/d
```

Mupad [B]
time = 4.54, size = 116, normalized size = 0.39

$$\frac{2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(3803\,a^4+3512\,a^3\,b\,x^2+2758\,a^2\,b^2\,x^4+1232\,a\,b^3\,x^6+231\,b^4\,x^8\right)}{4389\,d\,\sqrt{d\,x}}-\frac{16384\,a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4389\,d\,\sqrt{d\,x}\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(3/2),x)
```

```
[Out] (2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(3803*a^4 + 231*b^4*x^8 + 3512*a^3*b*x
^2 + 1232*a*b^3*x^6 + 2758*a^2*b^2*x^4))/(4389*d*(d*x)^(1/2)) - (16384*a^5*
(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4389*d*(d*x)^(1/2)*(a + b*x^2))
```

3.747 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=293

$$-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\,(a+bx^2)}+\frac{10a^4b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\,(a+bx^2)}+\frac{4a^3b^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\,(a+bx^2)}+\frac{20a^2b^3(d$$

```
[Out] -2/3*a^5*((b*x^2+a)^2)^(1/2)/d/(d*x)^(3/2)/(b*x^2+a)+4*a^3*b^2*(d*x)^(5/2)*
((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+20/9*a^2*b^3*(d*x)^(9/2)*((b*x^2+a)^2)^(1
/2)/d^7/(b*x^2+a)+10/13*a*b^4*(d*x)^(13/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a
)+2/17*b^5*(d*x)^(17/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)+10*a^4*b*(d*x)^(
1/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A]
time = 0.06, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1126, 276}

$$\frac{2b^5(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^{11}\,(a+bx^2)}+\frac{10ab^4(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^9\,(a+bx^2)}+\frac{20a^2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\,(a+bx^2)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\,(a+bx^2)}+\frac{10a^4b\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}{d^3\,(a+bx^2)}+\frac{4a^3b^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^2)) + (1
0*a^4*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) + (4*a
^3*b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(a + b*x^2)) + (20
*a^2*b^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2)) +
 (10*a*b^4*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^9*(a + b*x^2
)) + (2*b^5*(d*x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^11*(a + b*x
^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{(dx)^{5/2}}\,dx}{b^4(ab+b^2x^2)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{5/2}}+\frac{5a^4b^6}{d^2\sqrt{dx}}+\frac{10a^3b^7(dx)^{3/2}}{d^4}+\frac{10a^2b^8(dx)^{7/2}}{d^6}+\frac{5ab^9(\cdots}{\cdots}\right.}{b^4(ab+b^2x^2)}$$

$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}(a+bx^2)}+\frac{10a^4b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(a+bx^2)}+\frac{4a^3b^2(dx)^{5/\cdots}}{a\cdots}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.30

$$-\frac{2x\left((a+bx^2)^2\right)^{5/2}(663a^5-9945a^4bx^2-3978a^3b^2x^4-2210a^2b^3x^6-765ab^4x^8-117b^5x^{10})}{1989(dx)^{5/2}(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```

```
[Out] (-2*x*((a + b*x^2)^2)^(5/2)*(663*a^5 - 9945*a^4*b*x^2 - 3978*a^3*b^2*x^4 -
2210*a^2*b^3*x^6 - 765*a*b^4*x^8 - 117*b^5*x^10))/(1989*(d*x)^(5/2)*(a + b*
x^2)^5)
```

Maple [A]
time = 0.02, size = 85, normalized size = 0.29

method	result	size
gosper	$-\frac{2x(-117b^5x^{10}-765b^4a\,x^8-2210a^2b^3x^6-3978b^2a^3x^4-9945b\,a^4x^2+663a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{1989(b\,x^2+a)^5(dx)^{\frac{5}{2}}}$	83
default	$-\frac{2\left((b\,x^2+a)^2\right)^{\frac{5}{2}}(-117b^5x^{10}-765b^4a\,x^8-2210a^2b^3x^6-3978b^2a^3x^4-9945b\,a^4x^2+663a^5)}{1989d(b\,x^2+a)^5(dx)^{\frac{3}{2}}}$	85
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(-117b^5x^{10}-765b^4a\,x^8-2210a^2b^3x^6-3978b^2a^3x^4-9945b\,a^4x^2+663a^5)}{1989d^2(b\,x^2+a)x\sqrt{dx}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1989*((b*x^2+a)^2)^(5/2)/d*(-117*b^5*x^10-765*a*b^4*x^8-2210*a^2*b^3*x^6
-3978*a^3*b^2*x^4-9945*a^4*b*x^2+663*a^5)/(b*x^2+a)^5/(d*x)^(3/2)
```

Maxima [A]

time = 0.29, size = 151, normalized size = 0.52

$$\frac{2\left(45\left(13\,b^5\sqrt{d}\,x^3+17\,ab^4\sqrt{d}\,x\right)x^{\frac{11}{2}}+340\left(9\,ab^4\sqrt{d}\,x^3+13\,a^2b^3\sqrt{d}\,x\right)x^{\frac{7}{2}}+1326\left(5\,a^2b^3\sqrt{d}\,x^3+9\,a^3b^2\sqrt{d}\,x\right)x^{\frac{3}{2}}+\frac{7956\left(a^3b^2\sqrt{d}\,x^3+5\,a^4b\sqrt{d}\,x\right)}{\sqrt{x}}+\frac{3315\left(3\,a^4b\sqrt{d}\,x^3-a^5\sqrt{d}\,x\right)}{x^{\frac{5}{2}}}\right)}{9945\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9945*(45*(13*b^5*sqrt(d)*x^3 + 17*a*b^4*sqrt(d)*x)*x^(11/2) + 340*(9*a*b^
4*sqrt(d)*x^3 + 13*a^2*b^3*sqrt(d)*x)*x^(7/2) + 1326*(5*a^2*b^3*sqrt(d)*x^3
 + 9*a^3*b^2*sqrt(d)*x)*x^(3/2) + 7956*(a^3*b^2*sqrt(d)*x^3 + 5*a^4*b*sqrt(
d)*x)/sqrt(x) + 3315*(3*a^4*b*sqrt(d)*x^3 - a^5*sqrt(d)*x)/x^(5/2))/d^3
```

Fricas [A]

time = 0.36, size = 67, normalized size = 0.23

$$\frac{2\,(117\,b^5x^{10}+765\,ab^4x^8+2210\,a^2b^3x^6+3978\,a^3b^2x^4+9945\,a^4bx^2-663\,a^5)\sqrt{dx}}{1989\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*b^5*x^10 + 765*a*b^4*x^8 + 2210*a^2*b^3*x^6 + 3978*a^3*b^2*x^4
+ 9945*a^4*b*x^2 - 663*a^5)*sqrt(d*x)/(d^3*x^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(5/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(5/2), x)
```

Giac [A]

time = 4.09, size = 159, normalized size = 0.54

$$-\frac{2\left(\frac{663\,a^5d\mathrm{sgn}(bx^2+a)}{\sqrt{dx}\,x}-\frac{117\,\sqrt{dx}\,b^5d^{136}x^8\mathrm{sgn}(bx^2+a)+765\,\sqrt{dx}\,ab^4d^{136}x^6\mathrm{sgn}(bx^2+a)+2210\,\sqrt{dx}\,a^2b^3d^{136}x^4\mathrm{sgn}(bx^2+a)+3978\,\sqrt{dx}\,a^3b^2d^{136}x^2\mathrm{sgn}(bx^2+a)+9945\,\sqrt{dx}\,a^4bd^{136}\mathrm{sgn}(bx^2+a)}{d^{136}}\right)}{1989\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/1989*(663*a^5*d*sgn(b*x^2 + a)/(sqrt(d*x)*x) - (117*sqrt(d*x)*b^5*d^136*
x^8*sgn(b*x^2 + a) + 765*sqrt(d*x)*a*b^4*d^136*x^6*sgn(b*x^2 + a) + 2210*sq
rt(d*x)*a^2*b^3*d^136*x^4*sgn(b*x^2 + a) + 3978*sqrt(d*x)*a^3*b^2*d^136*x^2
*sgn(b*x^2 + a) + 9945*sqrt(d*x)*a^4*b*d^136*sgn(b*x^2 + a))/d^136)/d^3
```

Mupad [B]
time = 4.56, size = 116, normalized size = 0.40

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{10\,a^4\,x^2}{d^2}-\frac{2\,a^5}{3\,b\,d^2}+\frac{2\,b^4\,x^{10}}{17\,d^2}+\frac{4\,a^3\,b\,x^4}{d^2}+\frac{10\,a\,b^3\,x^8}{13\,d^2}+\frac{20\,a^2\,b^2\,x^6}{9\,d^2}\right)}{x^3\,\sqrt{d\,x}+\frac{a\,x\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(5/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((10*a^4*x^2)/d^2 - (2*a^5)/(3*b*d^2) +
(2*b^4*x^10)/(17*d^2) + (4*a^3*b*x^4)/d^2 + (10*a*b^3*x^8)/(13*d^2) + (20*a
^2*b^2*x^6)/(9*d^2)))/(x^3*(d*x)^(1/2) + (a*x*(d*x)^(1/2))/b)
```

3.748 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=295

$$-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}-\frac{10a^4b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{7/2}}{7a}$$

```
[Out] -2/5*a^5*((b*x^2+a)^2)^(1/2)/d/(d*x)^(5/2)/(b*x^2+a)+20/3*a^3*b^2*(d*x)^(3/
2)*((b*x^2+a)^2)^(1/2)/d^5/(b*x^2+a)+20/7*a^2*b^3*(d*x)^(7/2)*((b*x^2+a)^2)
^(1/2)/d^7/(b*x^2+a)+10/11*a*b^4*(d*x)^(11/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^
2+a)+2/15*b^5*(d*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)-10*a^4*b*((b*
x^2+a)^2)^(1/2)/d^3/(b*x^2+a)/(d*x)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{1126, 276\}$

$$\frac{2b^5(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}-\frac{10a^4b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^2)) - (1
0*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b*x^2)) + (20*
a^3*b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^5*(a + b*x^2)) +
(20*a^2*b^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2)
) + (10*a*b^4*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^9*(a + b*
x^2)) + (2*b^5*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^11*(a +
b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)^5}{(dx)^{7/2}}\,dx}{b^4\,(ab+b^2x^2)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5b^5}{(dx)^{7/2}}+\frac{5a^4b^6}{d^2(dx)^{3/2}}+\frac{10a^3b^7\sqrt{dx}}{d^4}+\frac{10a^2b^8(dx)^{5/2}}{d^6}+\frac{5ab^9(}{d}\right.}{b^4\,(ab+b^2x^2)}$$

$$= -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\,(a+bx^2)} - \frac{10a^4b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,(a+bx^2)} + \frac{20a^3b^2(dx)^{3/2}\sqrt{a}}{3d^5\,(a}$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 0.30

$$-\frac{2x\left((a+bx^2)^2\right)^{5/2}(231a^5+5775a^4bx^2-3850a^3b^2x^4-1650a^2b^3x^6-525ab^4x^8-77b^5x^{10})}{1155(dx)^{7/2}\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*x*((a + b*x^2)^2)^(5/2)*(231*a^5 + 5775*a^4*b*x^2 - 3850*a^3*b^2*x^4 -
1650*a^2*b^3*x^6 - 525*a*b^4*x^8 - 77*b^5*x^10))/(1155*(d*x)^(7/2)*(a + b*x
^2)^5)
```

Maple [A]
time = 0.04, size = 85, normalized size = 0.29

method	result	size
gosper	$-\frac{2x(-77b^5x^{10}-525b^4a\,x^8-1650a^2b^3x^6-3850b^2a^3x^4+5775b\,a^4x^2+231a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{1155(b\,x^2+a)^5(dx)^{\frac{7}{2}}}$	83
default	$-\frac{2\left((b\,x^2+a)^2\right)^{\frac{5}{2}}(-77b^5x^{10}-525b^4a\,x^8-1650a^2b^3x^6-3850b^2a^3x^4+5775b\,a^4x^2+231a^5)}{1155d(b\,x^2+a)^5(dx)^{\frac{5}{2}}}$	85
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(-77b^5x^{10}-525b^4a\,x^8-1650a^2b^3x^6-3850b^2a^3x^4+5775b\,a^4x^2+231a^5)}{1155d^3(b\,x^2+a)x^2\sqrt{dx}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1155*((b*x^2+a)^2)^(5/2)/d*(-77*b^5*x^10-525*a*b^4*x^8-1650*a^2*b^3*x^6-
3850*a^3*b^2*x^4+5775*a^4*b*x^2+231*a^5)/(b*x^2+a)^5/(d*x)^(5/2)
```

Maxima [A]
time = 0.30, size = 150, normalized size = 0.51

$$\frac{2\left(7\left(11\,b^{5}\sqrt{d}\,x^{3}+15\,ab^{4}\sqrt{d}\,x\right)x^{\frac{9}{2}}+60\left(7\,ab^{4}\sqrt{d}\,x^{3}+11\,a^{2}b^{3}\sqrt{d}\,x\right)x^{\frac{5}{2}}+330\left(3\,a^{2}b^{3}\sqrt{d}\,x^{3}+7\,a^{3}b^{2}\sqrt{d}\,x\right)\sqrt{x}+\frac{1540\left(a^{3}b^{2}\sqrt{d}\,x^{3}-3\,a^{4}b\sqrt{d}\,x\right)}{x^{\frac{3}{2}}}-\frac{231\left(5\,a^{4}b\sqrt{d}\,x^{3}+a^{5}\sqrt{d}\,x\right)}{x^{\frac{7}{2}}}\right)}{1155\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(7*(11*b^5*sqrt(d)*x^3 + 15*a*b^4*sqrt(d)*x)*x^(9/2) + 60*(7*a*b^4*s
qrt(d)*x^3 + 11*a^2*b^3*sqrt(d)*x)*x^(5/2) + 330*(3*a^2*b^3*sqrt(d)*x^3 + 7
*a^3*b^2*sqrt(d)*x)*sqrt(x) + 1540*(a^3*b^2*sqrt(d)*x^3 - 3*a^4*b*sqrt(d)*x
)/x^(3/2) - 231*(5*a^4*b*sqrt(d)*x^3 + a^5*sqrt(d)*x)/x^(7/2))/d^4
```

Fricas [A]
time = 0.36, size = 67, normalized size = 0.23

$$\frac{2\,(77\,b^{5}x^{10}+525\,ab^{4}x^{8}+1650\,a^{2}b^{3}x^{6}+3850\,a^{3}b^{2}x^{4}-5775\,a^{4}bx^{2}-231\,a^{5})\sqrt{dx}}{1155\,d^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^5*x^10 + 525*a*b^4*x^8 + 1650*a^2*b^3*x^6 + 3850*a^3*b^2*x^4 -
 5775*a^4*b*x^2 - 231*a^5)*sqrt(d*x)/(d^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(7/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(7/2), x)
```

Giac [A]
time = 4.53, size = 162, normalized size = 0.55

$$-\frac{2\left(\frac{231\,(25\,a^{4}bd^{3}x^{2}\mathrm{sgn}\,(bx^{2}+a)+a^{5}d^{3}\mathrm{sgn}\,(bx^{2}+a))}{\sqrt{dx}\,d^{2}x^{2}}-\frac{77\,\sqrt{dx}\,b^{5}d^{105}x^{7}\mathrm{sgn}\,(bx^{2}+a)+525\,\sqrt{dx}\,ab^{4}d^{105}x^{5}\mathrm{sgn}\,(bx^{2}+a)+1650\,\sqrt{dx}\,a^{2}b^{3}d^{105}x^{3}\mathrm{sgn}\,(bx^{2}+a)+3850\,\sqrt{dx}\,a^{3}b^{2}d^{105}x\mathrm{sgn}\,(bx^{2}+a)}{d^{105}}\right)}{1155\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/1155*(231*(25*a^4*b*d^3*x^2*sgn(b*x^2 + a) + a^5*d^3*sgn(b*x^2 + a))/(sq
rt(d*x)*d^2*x^2) - (77*sqrt(d*x)*b^5*d^105*x^7*sgn(b*x^2 + a) + 525*sqrt(d*
x)*a*b^4*d^105*x^5*sgn(b*x^2 + a) + 1650*sqrt(d*x)*a^2*b^3*d^105*x^3*sgn(b*
x^2 + a) + 3850*sqrt(d*x)*a^3*b^2*d^105*x*sgn(b*x^2 + a))/d^105)/d^4
```

Mupad [B]
time = 4.72, size = 118, normalized size = 0.40

$$\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{2\,b^4\,x^{10}}{15\,d^3}-\frac{10\,a^4\,x^2}{d^3}-\frac{2\,a^5}{5\,b\,d^3}+\frac{20\,a^3\,b\,x^4}{3\,d^3}+\frac{10\,a\,b^3\,x^8}{11\,d^3}+\frac{20\,a^2\,b^2\,x^6}{7\,d^3}\right)}{x^4\,\sqrt{d\,x}+\frac{a\,x^2\,\sqrt{d\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(7/2),x)
```

```
[Out] ((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*b^4*x^10)/(15*d^3) - (10*a^4*x^2)/d^
3 - (2*a^5)/(5*b*d^3) + (20*a^3*b*x^4)/(3*d^3) + (10*a*b^3*x^8)/(11*d^3) +
(20*a^2*b^2*x^6)/(7*d^3)))/(x^4*(d*x)^(1/2) + (a*x^2*(d*x)^(1/2))/b)
```

3.749 $\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=457

$$-\frac{2ad^3\sqrt{dx}\,(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2d(dx)^{5/2}\,(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}}{}$$

```
[Out] 2/5*d*(d*x)^(5/2)*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)-1/2*a^(5/4)*d^(7/2)*(b*x^
2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(9/4)*2^(1/2)/
((b*x^2+a)^2)^(1/2)+1/2*a^(5/4)*d^(7/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*
(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/4*a^(5/4
)*d^(7/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^
(1/2)*(d*x)^(1/2))/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1/4*a^(5/4)*d^(7/2)*
(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x
)^(1/2))/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-2*a*d^3*(b*x^2+a)*(d*x)^(1/2)/
b^2/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{2ad^3\sqrt{dx}\,(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2d(dx)^{5/2}\,(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (-2*a*d^3*Sqrt[d*x]*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2
*d*(d*x)^(5/2)*(a + b*x^2))/(5*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4
)*d^(7/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqr
t[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4)*d^(7/2)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{(dx)^{7/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{5/2}(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(ad^2(ab+b^2x^2))\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2ad^3\sqrt{dx}(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a^2d^4(ab+b^2x^2))\int \frac{}{\sqrt{}}}{b^2\sqrt{a^2+2abx^2}}$$

$$= -\frac{2ad^3\sqrt{dx}(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(2a^2d^3(ab+b^2x^2))\,\mathrm{Sub}}{b^2\sqrt{a^2+}}$$

$$= -\frac{2ad^3\sqrt{dx}(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a^{3/2}d^2(ab+b^2x^2))\,\mathrm{Su}}{b^2\sqrt{a}}$$

$$= -\frac{2ad^3\sqrt{dx}(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(a^{5/4}d^{7/2}(ab+b^2x^2))\,\mathrm{S}}{2\sqrt{}}$$

$$= -\frac{2ad^3\sqrt{dx}(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{5/4}d^{7/2}(a+bx^2)\log\Big(}{2\sqrt{2}}$$

$$= -\frac{2ad^3\sqrt{dx}(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{5/4}d^{7/2}(a+bx^2)\tan^{-}}{\sqrt{2}\,b^{9/4}\sqrt{a^2+}}$$

Mathematica [A]

time = 0.17, size = 163, normalized size = 0.36

$$\frac{d^3\sqrt{dx}\,(a+bx^2)\left(4\sqrt[4]{b}\,\sqrt{x}\,(-5a+bx^2)-5\sqrt{2}\,a^{5/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+5\sqrt{2}\,a^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{10b^{9/4}\sqrt{x}\,\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (d^3*Sqrt[d*x]*(a + b*x^2)*(4*b^(1/4)*Sqrt[x]*(-5*a + b*x^2) - 5*Sqrt[2]*a^
(5/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*S
qrt[2]*a^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]
*x)]))/(10*b^(9/4)*Sqrt[x]*Sqrt[(a + b*x^2)^2])
```

Maple [A]

time = 0.15, size = 239, normalized size = 0.52

method	result
default	$\frac{(bx^2+a)d\left(5ad^2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+10ad^2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{20\sqrt{(bx^2+a)^2}\,b^2}$
risch	$-\frac{2(-bx^2+5a)xd^4\sqrt{(bx^2+a)^2}}{5b^2\sqrt{dx}\,(bx^2+a)}+\frac{\left(\frac{a\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)}{4b^2d}+\frac{a\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\right.}{2b^2d}\right.}{bx^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(b*x^2+a)*d*(5*a*d^2*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2)))+10*a*d^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))+10*a*d^2*(a*d^2/b)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))+8*(d*x)^(5/2)*b
-40*a*d^2*(d*x)^(1/2))/((b*x^2+a)^2)^(1/2)/b^2
```

Maxima [A]

time = 0.50, size = 266, normalized size = 0.58

$$\frac{\frac{5\left(\frac{\sqrt{2}\,d^6\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^6\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d^5\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d^5\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}\right)a^2}{b^2}+\frac{8\left((dx)^{\frac{5}{2}}bd^2-5\sqrt{dx}\,ad^4\right)}{b^2}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*(5*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1
/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sq
rt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4))
+ 2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(
d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) +
2*sqrt(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a^
2/b^2 + 8*((d*x)^(5/2)*b*d^2 - 5*sqrt(d*x)*a*d^4)/b^2)/d
```

Fricas [A]
time = 0.36, size = 223, normalized size = 0.49

$$\frac{20\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\arctan\left(-\frac{\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^7d^3-\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{3}{4}}\sqrt{a^2d^7x+\sqrt{-\frac{a^5d^{14}}{b^9}}\,b^4}\,b^7}{a^5d^{14}}\right)+5\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\log\left(\sqrt{dx}\,ad^3+\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\right)-5\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\log\left(\sqrt{dx}\,ad^3-\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\right)+4\left(bd^3x^2-5\,ad^3\right)\sqrt{dx}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(20*(-a^5*d^14/b^9)^(1/4)*b^2*arctan(-((-a^5*d^14/b^9)^(3/4)*sqrt(d*x)
*a*b^7*d^3 - (-a^5*d^14/b^9)^(3/4)*sqrt(a^2*d^7*x + sqrt(-a^5*d^14/b^9)*b^4
)*b^7)/(a^5*d^14)) + 5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 + (-a^
5*d^14/b^9)^(1/4)*b^2) - 5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 -
(-a^5*d^14/b^9)^(1/4)*b^2) + 4*(b*d^3*x^2 - 5*a*d^3)*sqrt(d*x))/b^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [A]
time = 5.10, size = 273, normalized size = 0.60

$$\frac{1}{20}\,d^3\left(\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}+\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^3}-\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^3}+\frac{8\left(\sqrt{dx}\,b^4d^{10}x^2-5\sqrt{dx}\,ab^3d^{10}\right)}{b^5d^{10}}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*d^3*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 + 10*sqrt(2)*(a*b^3*d^2)^(1/4
)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/b^3 + 5*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)
*sqrt(d*x) + sqrt(a*d^2/b))/b^3 - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x - s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 + 8*(sqrt(d*x)*b^4*d^
10*x^2 - 5*sqrt(d*x)*a*b^3*d^10)/(b^5*d^10))*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{7/2}}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(7/2)/((a + b*x^2)^2)^(1/2), x)
```

3.750 $\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=412

$$\frac{2d(dx)^{3/2}\,(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{3/4}d^{5/2}(a+bx^2)\tan^{-1}\left(1+\frac{\sqrt{2}\,\cdots}{\sqrt[4]{\cdots}}\right.}{\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2\cdots}}$$

```
[Out] 2/3*d*(d*x)^(3/2)*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)+1/2*a^(3/4)*d^(5/2)*(b*x^
2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(7/4)*2^(1/2)/
((b*x^2+a)^2)^(1/2)-1/2*a^(3/4)*d^(5/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*
(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/4*a^(3/4
)*d^(5/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^
(1/2)*(d*x)^(1/2))/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1/4*a^(3/4)*d^(5/2)*
(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x
)^(1/2))/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$,
Rules used = {1126, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{2d(dx)^{3/2}\,(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{3/4}d^{5/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{3/4}d^{5/2}(a+bx^2)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}(a+bx^2)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*d*(d*x)^(3/2)*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3
/4)*d^(5/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2
)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/4)*d^(5/
2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{(dx)^{5/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(ad^2(ab+b^2x^2))\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(2ad(ab+b^2x^2))\,\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(ad(ab+b^2x^2))\,\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right.}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(a^{3/4}d^{5/2}(ab+b^2x^2))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{}}{\sqrt[4]{b}}}\right.}{2\sqrt{2}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{3/4}d^{5/2}(a+bx^2)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\right.}{2\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d(dx)^{3/2}(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^{3/4}d^{5/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{3}}{}$$

Mathematica [A]
time = 0.15, size = 151, normalized size = 0.37

$$\frac{(dx)^{5/2}(a+bx^2)\left(4b^{3/4}x^{3/2}+3\sqrt{2}\,a^{3/4}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+3\sqrt{2}\,a^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{6b^{7/4}x^{5/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(5/2)*(a + b*x^2)*(4*b^(3/4)*x^(3/2) + 3*Sqrt[2]*a^(3/4)*ArcTan[(Sqr
t[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*a^(3/4)*Ar
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(6*b^(7/4)
*x^(5/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.15, size = 221, normalized size = 0.54

method	result
default	$\frac{\left(b\,x^{2}+a\right)d\left(8\left(dx\right)^{\frac{3}{2}}b\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}-3a\,d^{2}\sqrt{2}\,\ln\left(-\frac{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)-6a\,d^{2}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right.\right.}{12\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{2}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}$
risch	$\frac{2x^{2}d^{3}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{3b\sqrt{dx}\,\left(b\,x^{2}+a\right)}+\frac{\left(-\frac{a\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{4b^{2}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-\frac{a\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-\frac{a\sqrt{2}\,\mathrm{arcta}}{2}\right.}{b\,x^{2}+a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(b*x^2+a)*d*(8*(d*x)^(3/2)*b*(a*d^2/b)^(1/4)-3*a*d^2*2^(1/2)*ln(-((a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))-6*a*d^2*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-6*a*d^2*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)))/((b*x^2+a)^2)^(1/2)/b^2/(a*d^2/
b)^(1/4)
```

Maxima [A]
time = 0.51, size = 241, normalized size = 0.58

$$-\frac{\frac{3\,ad^{4}\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{b}-\frac{8\,(dx)^{\frac{3}{2}}d^{2}}{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
 + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b
)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sq
rt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2
)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b - 8*(d*x)
^(3/2)*d^2/b)/d
```

Fricas [**A**]
time = 0.36, size = 219, normalized size = 0.53

$$\frac{4\,\sqrt{dx}\,d^2x+12\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^2d^7-\sqrt{a^4d^{15}x-\sqrt{-\frac{a^3d^{10}}{b^7}}\,a^3b^3d^{10}}\,\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b^2}{a^3d^{10}}\right)-3\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}\,a^2d^7+\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{3}{4}}b^5\right)+3\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}\,a^2d^7-\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{3}{4}}b^5\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(d*x)*d^2*x + 12*(-a^3*d^10/b^7)^(1/4)*b*arctan(-((-a^3*d^10/b^7
)^(1/4)*sqrt(d*x)*a^2*b^2*d^7 - sqrt(a^4*d^15*x - sqrt(-a^3*d^10/b^7)*a^3*b
^3*d^10)*(-a^3*d^10/b^7)^(1/4)*b^2)/(a^3*d^10)) - 3*(-a^3*d^10/b^7)^(1/4)*b
*log(sqrt(d*x)*a^2*d^7 + (-a^3*d^10/b^7)^(3/4)*b^5) + 3*(-a^3*d^10/b^7)^(1/
4)*b*log(sqrt(d*x)*a^2*d^7 - (-a^3*d^10/b^7)^(3/4)*b^5))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.97, size = 254, normalized size = 0.62

$$\frac{1}{12}\,d^2\left(\frac{8\,\sqrt{dx}\,x}{b}-\frac{6\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4d}-\frac{6\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4d}+\frac{3\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^4d}-\frac{3\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^4d}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*d^2*(8*sqrt(d*x)*x/b - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*d) - 6*sqrt(2
```

```
)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d
*x))/(a*d^2/b)^(1/4))/(b^4*d) + 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*d) - 3*sqrt(2)*(a*b^3*d^
2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*
d))*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{5/2}}{\sqrt{{(b\,x^2+a)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(5/2)/((a + b*x^2)^2)^(1/2), x)
```

3.751 $\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=410

$$\frac{2d\sqrt{dx}\,(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{a}\,d^{3/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}\,d^{3/2}(a+bx^2)\tan^{-1}\left(1+\frac{\sqrt{2}\,\ldots}{\sqrt[4]{\ldots}}\right.}{\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^{\ldots}}}$$

```
[Out] 1/2*a^(1/4)*d^(3/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/
d^(1/2))/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/2*a^(1/4)*d^(3/2)*(b*x^2+a)*
arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(5/4)*2^(1/2)/((b*x
^2+a)^2)^(1/2)+1/4*a^(1/4)*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d
^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(
1/2)-1/4*a^(1/4)*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(
1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+2*d*(
b*x^2+a)*(d*x)^(1/2)/b/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{a}\,d^{3/2}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}\,d^{3/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2d\sqrt{dx}\,(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{a}\,d^{3/2}(a+bx^2)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}\,d^{3/2}(a+bx^2)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*d*Sqrt[d*x]*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(1/4)*
d^(3/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3/2)*(a
 + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[
2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(1/4)*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3/2)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{(dx)^{3/2}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\,(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(ad^2(ab+b^2x^2))\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)}\,dx}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\,(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(2ad(ab+b^2x^2))\,\text{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\,(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\sqrt{a}\,(ab+b^2x^2)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right.}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\,(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt[4]{a}\,d^{3/2}(ab+b^2x^2)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}}{\sqrt[4]{b}}+2}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt{d}}{\sqrt[4]{b}}}\right.}{2\sqrt{2}\,b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\,(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{a}\,d^{3/2}(a+bx^2)\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x-\sqrt{2}\right.}{2\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{2d\sqrt{dx}\,(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{a}\,d^{3/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{a}}{}$$

Mathematica [A]
time = 0.14, size = 150, normalized size = 0.37

$$\frac{(dx)^{3/2}(a+bx^2)\left(4\sqrt[4]{b}\,\sqrt{x}+\sqrt{2}\,\sqrt[4]{a}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-\sqrt{2}\,\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{2b^{5/4}x^{3/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(4*b^(1/4)*Sqrt[x] + Sqrt[2]*a^(1/4)*ArcTan[(Sqrt[
a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - Sqrt[2]*a^(1/4)*ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(2*b^(5/4)*x^(
3/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.14, size = 214, normalized size = 0.52

method	result
default	$-\frac{(b x^2+a) d \left(\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\, \sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\, \sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+2 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+2 \left(a \right.\right.}{4 \sqrt{(b x^2+a)^2}\, b}$
risch	$\frac{2 x d^2 \sqrt{(b x^2+a)^2}}{b \sqrt{dx}\, (b x^2+a)}+\frac{\left(-\frac{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\, \sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\, \sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)}{4 b d}-\frac{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, \sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}+1\right.}{2 b d}\right.}{b x^2+a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b*x^2+a)*d*((a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2
/b)^(1/2)))+2*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))+2*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-8*(d*x)^(1/2))/((b*x^2+a)^2)^(1/2)/b
```

Maxima [A]
time = 0.51, size = 250, normalized size = 0.61

$$\frac{\frac{8 \sqrt{dx}\, d^2}{b}-\frac{\left(\frac{\sqrt{2}\, d^4 \log\left(\sqrt{b}\, dx+\sqrt{2}\, (a d^2)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{(a d^2)^{\frac{3}{4}} b^{\frac{1}{4}}}-\frac{\sqrt{2}\, d^4 \log\left(\sqrt{b}\, dx-\sqrt{2}\, (a d^2)^{\frac{1}{4}} \sqrt{dx}\, b^{\frac{1}{4}}+\sqrt{a}\, d\right)}{(a d^2)^{\frac{3}{4}} b^{\frac{1}{4}}}+\frac{2 \sqrt{2}\, d^3 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\, (a d^2)^{\frac{1}{4}} b^{\frac{1}{4}}+2 \sqrt{dx}\, \sqrt{b}\right)}{2 \sqrt{\sqrt{a}\, \sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\, \sqrt{b}\, d}\, \sqrt{a}}+\frac{2 \sqrt{2}\, d^3 \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\, (a d^2)^{\frac{1}{4}} b^{\frac{1}{4}}-2 \sqrt{dx}\, \sqrt{b}\right)}{2 \sqrt{\sqrt{a}\, \sqrt{b}\, d}}\right)}{\sqrt{\sqrt{a}\, \sqrt{b}\, d}\, \sqrt{a}}\right) a}{b}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*sqrt(d*x)*d^2/b - (sqrt(2)*d^4*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/
4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^4*log
(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*
b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(
b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(a)))*a/b)/d
```

Fricas [A]
time = 0.37, size = 170, normalized size = 0.41

$$-\frac{4\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\left(-\frac{ad^6}{b^5}\right)^{\frac{3}{4}}\sqrt{dx}\,b^4d-\sqrt{d^3x+\sqrt{-\frac{ad^6}{b^5}}\,b^2}\left(-\frac{ad^6}{b^5}\right)^{\frac{3}{4}}b^4}{ad^6}\right)+\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}\,d+\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\right)-\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}\,d-\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\right)-4\sqrt{dx}\,d}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(-a*d^6/b^5)^(1/4)*b*arctan(-((-a*d^6/b^5)^(3/4)*sqrt(d*x)*b^4*d -
sqrt(d^3*x + sqrt(-a*d^6/b^5)*b^2)*(-a*d^6/b^5)^(3/4)*b^4)/(a*d^6)) + (-a*d
^6/b^5)^(1/4)*b*log(sqrt(d*x)*d + (-a*d^6/b^5)^(1/4)*b) - (-a*d^6/b^5)^(1/4
)*b*log(sqrt(d*x)*d - (-a*d^6/b^5)^(1/4)*b) - 4*sqrt(d*x)*d)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^{\frac{3}{2}}}{\sqrt{(a+bx^2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral((d*x)**(3/2)/sqrt((a + b*x**2)**2), x)
```

Giac [A]
time = 3.27, size = 238, normalized size = 0.58

$$-\frac{1}{4}d\left(\frac{2\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{2\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^2}-\frac{\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^2}-\frac{8\sqrt{dx}}{b}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^2 + 2*sqrt(2)*(a*b^3*d^2)^(1/4)*arct
an(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^
2 + sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/b^2 - sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^2 - 8*sqrt(d*x)/b)*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{3/2}}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(3/2)/((a + b*x^2)^2)^(1/2), x)
```

3.752 $\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=368

$$-\frac{\sqrt{d}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{d}\,(a+bx^2)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{d}\,(a+bx^2)\log}{2\sqrt{}}$$

```
[Out] -1/2*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2
)/a^(1/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1/2*(b*x^2+a)*arctan(1+b^(1/4
)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(1/4)/b^(3/4)*2^(1/2)/((b*
x^2+a)^2)^(1/2)+1/4*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(1/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2)^
(1/2)-1/4*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^
(1/2)*(d*x)^(1/2))*d^(1/2)/a^(1/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1126, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{d}\,(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{d}\,(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{d}\,(a+bx^2)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\,(a+bx^2)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((Sqrt[d]*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (Sqrt[d
]*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (Sqrt[d]*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[
d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{(ab+b^2x^2)\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{(2(ab+b^2x^2))\,\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}} \\
&= -\frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{\sqrt{b}\,d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{\sqrt{c}}{}\right.}{\sqrt{b}\,d\sqrt{a^2+2}} \\
&= \frac{\left(\sqrt{d}\,(ab+b^2x^2)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt{d}\right.}{} \\
&= \frac{\sqrt{d}\,(a+bx^2)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{d}\,(a+bx}{} \\
&= -\frac{\sqrt{d}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{d}\,(a+bx^2)\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{a^2+2abx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 123, normalized size = 0.33

$$
-\frac{\sqrt{dx}\,(a+bx^2)\left(\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}\sqrt{x}\sqrt{(a+bx^2)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((Sqrt[d*x]*(a + b*x^2)*(ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x])] + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[
b]*x)]))/(Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[x]*Sqrt[(a + b*x^2)^2]))
```

Maple [A]
time = 0.13, size = 183, normalized size = 0.50

method	result
default	$\frac{(b\,x^2+a)d\sqrt{2}\,\left(\ln\left(-\frac{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)\right)}{4\sqrt{(b\,x^2+a)^2}\,b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)*d/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln(-((a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(
1/4))/(a*d^2/b)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4)))
```

Maxima [A]
time = 0.51, size = 216, normalized size = 0.59

$$\frac{1}{4}\,d\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\,\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}\,b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\,\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}\,b^{\frac{3}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt
(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) +
 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)
*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt
(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/
((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*s
qrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))
```

Fricas [A]
time = 0.36, size = 173, normalized size = 0.47

$$-2\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,bd\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}-\sqrt{-abd^2\sqrt{-\frac{d^2}{ab^3}}+d^3x}\,b\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}}{d^2}\right)+\frac{1}{2}\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{dx}\,d\right)-\frac{1}{2}\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{dx}\,d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(-d^2/(a*b^3))^(1/4)*arctan(-(sqrt(d*x)*b*d*(-d^2/(a*b^3))^(1/4) - sqrt(
-a*b*d^2*sqrt(-d^2/(a*b^3)) + d^3*x)*b*(-d^2/(a*b^3))^(1/4))/d^2) + 1/2*(-d
^2/(a*b^3))^(1/4)*log(a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*d) - 1/2*(-d^2
/(a*b^3))^(1/4)*log(-a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*d)
```

Sympy [A]
time = 15.63, size = 41, normalized size = 0.11

$$2d\,\mathrm{RootSum}\left(256t^4ab^3d^2+1,\,\left(t\mapsto t\log\left(64t^3ab^2d^2+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] 2*d*RootSum(256*_t**4*a*b**3*d**2 + 1, Lambda(_t, _t*log(64*_t**3*a*b**2*d*
*2 + sqrt(d*x))))
```

Giac [A]
time = 4.32, size = 242, normalized size = 0.66

$$\frac{\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^3}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^3}\right)\mathrm{sgn}\left(bx^2+a\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(
a*b^3) - sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a*b^3) + sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3))*sgn(b*x^2 + a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x}}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d*x)^(1/2)/((a + b*x^2)^2)^(1/2), x)
```

3.753 $\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=368

$$-\frac{(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(a+bx^2)\log\left(\sqrt{a}\sqrt{d}+\cdots\right.}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\cdots}$$

```
[Out] -1/2*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4
)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)^2)^(1/2)+1/2*(b*x^2+a)*arctan(1+b^(1/4
)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*
x^2+a)^2)^(1/2)-1/4*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)^2)^
(1/2)+1/4*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^
(1/2)*(d*x)^(1/2))/a^(3/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1126, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(((a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + ((a + b
*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a
^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(2(ab+b^2x^2))\,\mathrm{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(ab+b^2x^2)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{b}\,x^2}{ab+\frac{b^2x^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{\sqrt{a}\,d^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(ab+b^2x^2)\,\mathrm{Subst}\left(\int \right.}{\sqrt{a}\,d^2\sqrt{a^2}}$$

$$= \frac{(ab+b^2x^2)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{dx}\right)}{2\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(ab+b^2x^2}{}$$

$$= -\frac{(a+bx^2)\log\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)}{}$$

$$= -\frac{(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}\sqrt{}}\right.}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2}}$$

Mathematica [A]
time = 0.11, size = 125, normalized size = 0.34

$$-\frac{\sqrt{x}\,(a+bx^2)\left(\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{\sqrt{2}\,a^{3/4}\sqrt[4]{b}\sqrt{dx}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((Sqrt[x]*(a + b*x^2)*(ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])] - ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]
*x)]))/(Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]))
```

Maple [A]
time = 0.13, size = 182, normalized size = 0.49

method	result

default	$\frac{(b x^2+a)\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d}\cdots}{\left(\frac{a}{\cdots}\right.}\right.\right.}{4\sqrt{(b x^2+a)^2}\, da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/d*(a*d^2/b)^(1/4)/a*2^(1/2)*(ln((d*x+(a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1
/4))/(a*d^2/b)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2
/b)^(1/4)))
```

Maxima [A]
time = 0.50, size = 226, normalized size = 0.61

$$\frac{\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,d^2\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}+\frac{2\sqrt{2}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*d^2*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt(2
)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*
sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*s
qrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt
(2)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt
(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/d
```

Fricas [A]
time = 0.38, size = 165, normalized size = 0.45

$$2\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2d^2\sqrt{-\frac{1}{a^3bd^2}}+dx}\;a^2bd\left(-\frac{1}{a^3bd^2}\right)^{\frac{3}{4}}-\sqrt{dx}\;a^2bd\left(-\frac{1}{a^3bd^2}\right)^{\frac{3}{4}}\right)+\frac{1}{2}\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(ad\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-\frac{1}{2}\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(-ad\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(a^3*b*d^2))^(1/4)*arctan(sqrt(a^2*d^2*sqrt(-1/(a^3*b*d^2)) + d*x)*a^
2*b*d*(-1/(a^3*b*d^2))^(3/4) - sqrt(d*x)*a^2*b*d*(-1/(a^3*b*d^2))^(3/4)) +
1/2*(-1/(a^3*b*d^2))^(1/4)*log(a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x)) - 1/
2*(-1/(a^3*b*d^2))^(1/4)*log(-a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\sqrt{(a+bx^2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*sqrt((a + b*x**2)**2)), x)
```

Giac [A]
time = 3.69, size = 251, normalized size = 0.68

$$\frac{1}{4}\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{abd}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{abd}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{abd}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{abd}\right)\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b*d) + 2*sqrt(2)*(a*b^3*d^2)^(1/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(
a*b*d) + sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a*b*d) - sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b*d))*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d\,x}\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int(1/((d*x)^(1/2)*((a + b*x^2)^2)^(1/2)), x)
```

3.754 $\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=412

$$-\frac{2(a+bx^2)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{b}\,(a+bx^2)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{}}\right)}{\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2-}}$$

```
[Out] 1/2*b^(1/4)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))
/a^(5/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/2*b^(1/4)*(b*x^2+a)*arctan(1
+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/d^(3/2)*2^(1/2)/((b*x
^2+a)^2)^(1/2)-1/4*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a
^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(
1/2)+1/4*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(
1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-2*(b*
x^2+a)/a/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{2(a+bx^2)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\,(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{b}\,(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{b}\,(a+bx^2)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}\,(a+bx^2)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(a*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sq
rt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(5
/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*S
qrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(2*Sqrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{1}{(dx)^{3/2}(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2(a+bx^2)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(b(ab+b^2x^2))\int \frac{\sqrt{dx}}{ab+b^2x^2}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2(a+bx^2)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(2b(ab+b^2x^2))\,\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2x^4}{d^2}}\,dx \ldots\right.}{ad^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2(a+bx^2)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(\sqrt{b}\,(ab+b^2x^2)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,d-\ldots}{ab+\ldots}\right.}{ad^3\sqrt{a^2+2abx^2+b^2 \ldots}}$$

$$= -\frac{2(a+bx^2)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}\ldots}{\sqrt[4]{b}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\ldots}{\ldots}}\right.}{2\sqrt{2}\,a^{5/4}b^{3/4}d^{3/2}\sqrt{a^2+\ldots}}$$

$$= -\frac{2(a+bx^2)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{b}\,(a+bx^2)\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\ldots\right.}{2\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2\ldots}}$$

$$= -\frac{2(a+bx^2)}{ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{b}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\ldots}{\sqrt[4]{a}\,\sqrt{\ldots}}\right.}{\sqrt{2}\,a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x\ldots}}$$

Mathematica [A]
time = 0.15, size = 150, normalized size = 0.36

$$\frac{x(a+bx^2)\left(-4\sqrt[4]{a}+\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{2a^{5/4}(dx)^{3/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2)*(-4*a^(1/4) + Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTan[(Sqrt[a] - Sqrt
[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(2*a^(5/4)*(d*x
)^(3/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.14, size = 224, normalized size = 0.54

method	result
default	$-\frac{(bx^2+a)\left(\sqrt{2}\,\sqrt{dx}\,\ln\left(-\frac{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+2\sqrt{2}\,\sqrt{dx}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)+2\sqrt{2}\cdots}{4d\sqrt{(bx^2+a)^2}\,a\sqrt{dx}\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}$
risch	$-\frac{2\sqrt{(bx^2+a)^2}}{a\sqrt{dx}\,d(bx^2+a)}+\frac{\left(-\frac{\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)}{4a\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}+1\right)}{2a\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\cdots\right)}{2a\left(\cdots\right)}\cdots\right)}{d(bx^2+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b*x^2+a)/d*(2^(1/2)*(d*x)^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))+2*2^(1/2)*(d*x)^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))+2*2^(1/2)*(d*x)^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))+8*(a*d^2/b)^(1/4))/((b*x^2+a)^2)^(1/2)/a/(d*x)^(
1/2)/(a*d^2/b)^(1/4)
```

Maxima [A]
time = 0.49, size = 234, normalized size = 0.57

$$-\frac{\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a}+\frac{8}{\sqrt{dx}\,a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sq
rt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*
x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sq
rt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d
)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)
*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a + 8/(sqrt(d*x)*a
))/d
```

Fricas [A]
time = 0.36, size = 198, normalized size = 0.48

$$\frac{4\,ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,abd\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}-\sqrt{-a^3bd^4\sqrt{-\frac{b}{a^5d^6}}+b^2dx}\,ad\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}}{b}\right)-ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\log\left(a^4d^5\left(-\frac{b}{a^5d^6}\right)^{\frac{3}{4}}+\sqrt{dx}\,b\right)+ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\log\left(-a^4d^5\left(-\frac{b}{a^5d^6}\right)^{\frac{3}{4}}+\sqrt{dx}\,b\right)-4\sqrt{dx}}{2\,ad^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*d^2*x*(-b/(a^5*d^6))^(1/4)*arctan(-(sqrt(d*x)*a*b*d*(-b/(a^5*d^6))
^(1/4) - sqrt(-a^3*b*d^4*sqrt(-b/(a^5*d^6)) + b^2*d*x)*a*d*(-b/(a^5*d^6))^(
1/4))/b) - a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(a^4*d^5*(-b/(a^5*d^6))^(3/4) +
sqrt(d*x)*b) + a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(-a^4*d^5*(-b/(a^5*d^6))^(3/
4) + sqrt(d*x)*b) - 4*sqrt(d*x))/(a*d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\sqrt{(a+bx^2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*sqrt((a + b*x**2)**2)), x)
```

Giac [A]
time = 3.08, size = 264, normalized size = 0.64

$$-\frac{\left(\frac{8}{\sqrt{dx}\,a}+\frac{2\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2}+\frac{2\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2}-\frac{\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^2d^2}+\frac{\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^2d^2}\right)\operatorname{sgn}(bx^2+a)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(8/(sqrt(d*x)*a) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*d^2) + 2*sqrt
(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt
(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*d^2) - sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x +
 sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*d^2) + sqrt(2)
*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^2*b^2*d^2))*sgn(b*x^2 + a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{3/2}\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int(1/((d*x)^(3/2)*((a + b*x^2)^2)^(1/2)), x)
```

3.755 $\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=414

$$-\frac{2(a+bx^2)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}(a+bx^2)\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx}}$$

```
[Out] -2/3*(b*x^2+a)/a/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+1/2*b^(3/4)*(b*x^2+a)*ar
ctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/d^(5/2)*2^(1/2)
/((b*x^2+a)^2)^(1/2)-1/2*b^(3/4)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(
1/2)/a^(1/4)/d^(1/2))/a^(7/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1/4*b^(3/
4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(
d*x)^(1/2))/a^(7/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/4*b^(3/4)*(b*x^2+
a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2)
)/a^(7/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{2(a+bx^2)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}(a+bx^2)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}(a+bx^2)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(3*a*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(
3/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])
/(Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)*(a +
b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*
a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/4)*(a + b*x^2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)*(a +
 b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{1}{(dx)^{5/2}(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2(a+bx^2)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(b(ab+b^2x^2))\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2(a+bx^2)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(2b(ab+b^2x^2))\,\mathrm{Subst}\left(\int \frac{1}{ab+\frac{b^2x^4}{d^2}} \cdots\right)}{ad^3\sqrt{a^2+2abx^2+b^2x \cdots}}$$

$$= -\frac{2(a+bx^2)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(b(ab+b^2x^2))\,\mathrm{Subst}\left(\int \frac{\sqrt{a}\,d-\sqrt{\cdots}}{ab+\frac{b^2x \cdots}{d^2}} \cdots\right)}{a^{3/2}d^4\sqrt{a^2+2abx^2+\cdots}}$$

$$= -\frac{2(a+bx^2)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(ab+b^2x^2)\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}}{-\frac{\sqrt{a}\,d}{\sqrt{b}}-\frac{\sqrt{2}\cdots}{\cdots}} \cdots\right)}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}\,d^{5/2}\sqrt{a^2 \cdots}}$$

$$= -\frac{2(a+bx^2)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/4}(a+bx^2)\log\left(\sqrt{a}\,\sqrt{d}+\sqrt{b}\cdots\right)}{2\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+\cdots}}$$

$$= -\frac{2(a+bx^2)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/4}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right)}{\sqrt{2}\,a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b\cdots}}$$

Mathematica [A]
time = 0.16, size = 152, normalized size = 0.37

$$\frac{x(a+bx^2)\left(-4a^{3/4}+3\sqrt{2}\,b^{3/4}x^{3/2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-3\sqrt{2}\,b^{3/4}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{6a^{7/4}(dx)^{5/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2)*(-4*a^(3/4) + 3*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTan[(Sqrt[a] - Sq
rt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*b^(3/4)*x^(3/2)*Arc
Tanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(6*a^(7/4)*
(d*x)^(5/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.14, size = 239, normalized size = 0.58

method	result
default	$-\frac{(b x^2+a)\left(3b\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,(dx)^{\frac{3}{2}}\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+6b\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,(dx)^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(a\cdots\right.}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right.\cdots\right.}{12 d^3\sqrt{\left(b x^2+a\right)^2}\,a^2 (dx)^{\frac{3}{2}}}$
risch	$-\frac{2\sqrt{\left(b x^2+a\right)^2}}{3 a x\sqrt{dx}\,d^2(b x^2+a)}+\frac{\left(-\frac{b\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)}{4a^2 d}-\frac{b\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}\cdots}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right.\cdots}{2a^2 d}\cdots\right.}{d^2(b x^2+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(b*x^2+a)/d^3*(3*b*(a*d^2/b)^(1/4)*2^(1/2)*(d*x)^(3/2)*ln((d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+6*b*(a*d^2/b)^(1/4)*2^(1/2)*(d*x)^(3/2)*arct
an((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))+6*b*(a*d^2/b)^(1/
4)*2^(1/2)*(d*x)^(3/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))+8*a*d^2)/((b*x^2+a)^2)^(1/2)/a^2/(d*x)^(3/2)
```

Maxima [A]
time = 0.51, size = 242, normalized size = 0.58

$$-\frac{\frac{3\left(\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^2)^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^2)^{\frac{3}{4}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}\,d}+\frac{2\sqrt{2}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\,\sqrt{b}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{a}\,d}\right)}{a}+\frac{8}{(dx)^{\frac{3}{2}}a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)
*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*d*x - sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) + 2*sqrt(2)
*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))
/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d) + 2*sqrt(2)*b
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/
sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d))/a + 8/((d*x)^
(3/2)*a))/d
```

Fricas [A]
time = 0.36, size = 227, normalized size = 0.55

$$-\frac{12\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^5bd^7\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^4d^6\sqrt{-\frac{b^3}{a^7d^{10}}}+b^2dx}\,a^5d^7\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+3\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\log\left(a^2d^3\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}\,b\right)-3\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\log\left(-a^2d^3\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}\,b\right)+4\sqrt{dx}}{6\,ad^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*d^3*x^2*(-b^3/(a^7*d^10))^(1/4)*arctan(-(sqrt(d*x)*a^5*b*d^7*(-b
^3/(a^7*d^10))^(3/4) - sqrt(a^4*d^6*sqrt(-b^3/(a^7*d^10)) + b^2*d*x)*a^5*d^
7*(-b^3/(a^7*d^10))^(3/4))/b^3) + 3*a*d^3*x^2*(-b^3/(a^7*d^10))^(1/4)*log(a
^2*d^3*(-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b) - 3*a*d^3*x^2*(-b^3/(a^7*d^10
))^(1/4)*log(-a^2*d^3*(-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b) + 4*sqrt(d*x))
/(a*d^3*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.91, size = 256, normalized size = 0.62

$$-\frac{1}{12}\left(\frac{6\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3}+\frac{6\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3}+\frac{3\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2d^3}-\frac{3\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2d^3}+\frac{8}{\sqrt{dx}\,ad^2x}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*d^3) + 6*sqrt(2)*(a*b^3*d^2)^(1/4)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4
))/(a^2*d^3) + 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4
)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*d^3) - 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*
x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*d^3) + 8/(sqrt(
d*x)*a*d^2*x))*sgn(b*x^2 + a)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{5/2}\,\sqrt{(b\,x^2+a)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int(1/((d*x)^(5/2)*((a + b*x^2)^2)^(1/2)), x)
```

3.756 $\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=459

$$-\frac{2(a+bx^2)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2b(a+bx^2)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right.}{\sqrt{2}\,a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -2/5*(b*x^2+a)/a/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)-1/2*b^(5/4)*(b*x^2+a)*ar
ctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/d^(7/2)*2^(1/2)
/((b*x^2+a)^2)^(1/2)+1/2*b^(5/4)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(
1/2)/a^(1/4)/d^(1/2))/a^(9/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1/4*b^(5/
4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(
d*x)^(1/2))/a^(9/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1/4*b^(5/4)*(b*x^2+
a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2)
)/a^(9/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+2*b*(b*x^2+a)/a^2/d^3/(d*x)^(
1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$,
Rules used = {1126, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{2b(a+bx^2)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2(a+bx^2)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{\sqrt{2}\,a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{\sqrt{2}\,a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}(a+bx^2)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}(a+bx^2)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{2\sqrt{2}\,a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(5*a*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*b
*(a + b*x^2))/(a^2*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(5/4
)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(S
qrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x
^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(
9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x^2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*
Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(5/4)*(a + b*
x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(2*Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{1}{(dx)^{7/2}(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2(a+bx^2)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(b(ab+b^2x^2))\int \frac{1}{(dx)^{3/2}(ab+b^2x^2)}\,dx}{ad^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2(a+bx^2)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b(a+bx^2)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= -\frac{2(a+bx^2)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b(a+bx^2)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= -\frac{2(a+bx^2)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b(a+bx^2)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} -$$

$$= -\frac{2(a+bx^2)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b(a+bx^2)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= -\frac{2(a+bx^2)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b(a+bx^2)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} +$$

$$= -\frac{2(a+bx^2)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b(a+bx^2)}{a^2d^3\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} -$$

Mathematica [A]

time = 0.17, size = 160, normalized size = 0.35

$$-\frac{x(a+bx^2)\left(4\sqrt[4]{a}\,(a-5bx^2)+5\sqrt{2}\,b^{5/4}x^{5/2}\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+5\sqrt{2}\,b^{5/4}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{10a^{9/4}(dx)^{7/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/10*(x*(a + b*x^2)*(4*a^(1/4)*(a - 5*b*x^2) + 5*Sqrt[2]*b^(5/4)*x^(5/2)*A
rcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2]*
b^(5/4)*x^(5/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b
]*x)]))/(a^(9/4)*(d*x)^(7/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.13, size = 251, normalized size = 0.55

method	result
risch	$-\frac{2(-5bx^2+a)\sqrt{(bx^2+a)^2}}{5a^2\sqrt{dx}\,x^2d^3(bx^2+a)}+\frac{\left(\frac{b\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)}{4a^2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}+\frac{b\sqrt{\cdots}}{\cdots}\cdots\right)}{d^3(bx^2+a)\cdots}$
default	$-\frac{(bx^2+a)\left(-5b\sqrt{2}\,(dx)^{\frac{5}{2}}\ln\left(-\frac{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)-10b\sqrt{2}\,(dx)^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-10b\cdots\right)}{20d^3\sqrt{(bx^2+a)^2}\,a^2(dx)^{\frac{5}{2}}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(b*x^2+a)*(-5*b*2^(1/2)*(d*x)^(5/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^
2/b)^(1/2)))-10*b*2^(1/2)*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))-10*b*2^(1/2)*(d*x)^(5/2)*arctan((2^(1/2)*(d*x)^(1/
2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-40*b*d^2*x^2*(a*d^2/b)^(1/4)+8*a*d^2*(
a*d^2/b)^(1/4))/d^3/((b*x^2+a)^2)^(1/2)/a^2/(d*x)^(5/2)/(a*d^2/b)^(1/4)
```

Maxima [A]

time = 0.50, size = 259, normalized size = 0.56

$$\frac{\frac{5\,b^{2}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(ad^{2})^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\,\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}\,d}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}\,d}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,dx+\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,dx-\sqrt{2}\,(ad^{2})^{\frac{1}{4}}\sqrt{dx}\,b^{\frac{1}{4}}+\sqrt{a}\,d\right)}{(ad^{2})^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{a^{2}d^{2}}+\frac{8\,(5\,bd^{2}x^{2}-ad^{2})}{(dx)^{\frac{5}{2}}a^{2}d^{2}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/20*(5*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) +
2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt
(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqr
t(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
- sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(
a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(
1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^2*d^2) + 8*
(5*b*d^2*x^2 - a*d^2)/((d*x)^(5/2)*a^2*d^2))/d
```

Fricas [A]
time = 0.35, size = 253, normalized size = 0.55

$$-\frac{20\,a^{2}d^{4}x^{3}\left(-\frac{b^{5}}{a^{9}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{2}b^{4}d^{3}\left(-\frac{b^{5}}{a^{9}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-a^{5}b^{5}d^{8}\sqrt{-\frac{b^{5}}{a^{9}d^{14}}}+b^{8}dx}\,a^{2}d^{3}\left(-\frac{b^{5}}{a^{9}d^{14}}\right)^{\frac{1}{4}}}{b^{5}}\right)-5\,a^{2}d^{4}x^{3}\left(-\frac{b^{5}}{a^{9}d^{14}}\right)^{\frac{1}{4}}\log\left(a^{7}d^{11}\left(-\frac{b^{5}}{a^{9}d^{14}}\right)^{\frac{3}{4}}+\sqrt{dx}\,b^{4}\right)+5\,a^{2}d^{4}x^{3}\left(-\frac{b^{5}}{a^{9}d^{14}}\right)^{\frac{1}{4}}\log\left(-a^{7}d^{11}\left(-\frac{b^{5}}{a^{9}d^{14}}\right)^{\frac{3}{4}}+\sqrt{dx}\,b^{4}\right)-4\,(5\,bx^{2}-a)\sqrt{dx}}{10\,a^{2}d^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*a^2*d^4*x^3*(-b^5/(a^9*d^14))^(1/4)*arctan(-(sqrt(d*x)*a^2*b^4*d^
3*(-b^5/(a^9*d^14))^(1/4) - sqrt(-a^5*b^5*d^8*sqrt(-b^5/(a^9*d^14)) + b^8*d
*x)*a^2*d^3*(-b^5/(a^9*d^14))^(1/4))/b^5) - 5*a^2*d^4*x^3*(-b^5/(a^9*d^14))
^(1/4)*log(a^7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d*x)*b^4) + 5*a^2*d^4*x^
3*(-b^5/(a^9*d^14))^(1/4)*log(-a^7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d*x)
*b^4) - 4*(5*b*x^2 - a)*sqrt(d*x))/(a^2*d^4*x^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [A]
time = 4.05, size = 284, normalized size = 0.62

$$\frac{1}{20}\left(\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3bd^5}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3bd^5}-\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3bd^5}+\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3bd^5}+\frac{8\left(5\,bd^2x^2-ad^2\right)}{\sqrt{dx}\,a^2d^5x^2}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*(10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d^5) + 10*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^3*b*d^5) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d^5) + 5*sqrt(2)*(a*b^3*d^2)^(3/4)
*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d^5) +
 8*(5*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^2*d^5*x^2))*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d\,x\right)}^{7/2}\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int(1/((d*x)^(7/2)*((a + b*x^2)^2)^(1/2)), x)
```

3.757 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=551

$$-\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117d^5(dx)^{5/2}}{80b^3\sqrt{a^2+2ab}$$

```
[Out] -13/16*d^3*(d*x)^(9/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(13/2)/b/(b*x^2+
a)/((b*x^2+a)^2)^(1/2)+117/80*d^5*(d*x)^(5/2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^(
1/2)-117/64*a^(5/4)*d^(15/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)
/a^(1/4)/d^(1/2))/b^(17/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+117/64*a^(5/4)*d^(15
/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(17/4
)*2^(1/2)/((b*x^2+a)^2)^(1/2)-117/128*a^(5/4)*d^(15/2)*(b*x^2+a)*ln(a^(1/2)
*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(17/4)*2^
(1/2)/((b*x^2+a)^2)^(1/2)+117/128*a^(5/4)*d^(15/2)*(b*x^2+a)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(17/4)*2^(1/2
)/((b*x^2+a)^2)^(1/2)-117/16*a*d^7*(b*x^2+a)*(d*x)^(1/2)/b^4/((b*x^2+a)^2)^
(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117d^5(dx)^{5/2}\left(a+bx^2\right)}{80b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-13*d^3*(d*x)^(9/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(
13/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a*d^7*Sqrt[
d*x]*(a + b*x^2))/(16*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*d^5*(d*x)
^(5/2)*(a + b*x^2))/(80*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)
*d^(15/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*a^(5/4)*
d^(15/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)*d
^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (117*a^(5/4)*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{15/2}}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}} \\
&= -\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{11/2}}{(ab+b^2x^2)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}} \\
&= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(117d^4(ab\cdots}{32b^2\sqrt{\cdots}} \\
&= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{117d^5(d\cdots}{80b^3\sqrt{a^2\cdots}} \\
&= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\cdots}{16b^4\sqrt{a^2\cdots}} \\
&= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\cdots}{16b^4\sqrt{a^2\cdots}} \\
&= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\cdots}{16b^4\sqrt{a^2\cdots}} \\
&= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\cdots}{16b^4\sqrt{a^2\cdots}} \\
&= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\cdots}{16b^4\sqrt{a^2\cdots}} \\
&= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\cdots}{16b^4\sqrt{a^2\cdots}}
\end{aligned}
$$

Mathematica [A]
time = 0.37, size = 206, normalized size = 0.37

$$
\frac{d^7\sqrt{dx}\left(4\sqrt[4]{b}\sqrt{x}\left(-585a^3-1053a^2bx^2-416ab^2x^4+32b^3x^6\right)-585\sqrt{2}\,a^{5/4}\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+585\sqrt{2}\,a^{5/4}\left(a+bx^2\right)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{320b^{17/4}\sqrt{x}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (d^7*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(-585*a^3 - 1053*a^2*b*x^2 - 416*a*b^2*x^
4 + 32*b^3*x^6) - 585*Sqrt[2]*a^(5/4)*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[
b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 585*Sqrt[2]*a^(5/4)*(a + b*x^2)^
2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(320*b
^(17/4)*Sqrt[x]*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(355) = 710$.
time = 0.07, size = 737, normalized size = 1.34

method	result
risch	$-\frac{2(-b x^2+15a) x d^8 \sqrt{(b x^2+a)^2}}{5 b^4 \sqrt{dx}\,(b x^2+a)}+\frac{\left(-\frac{25 a^2 d (dx)^{\frac{5}{2}}}{16 b^3 (d^2x^2b+a d^2)^2}-\frac{21 a^3 d^3 \sqrt{dx}}{16 b^4 (d^2x^2b+a d^2)^2}+\frac{117 a \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{d}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{d}}\right.}{128 b^4 d}\right.}{}$
default	$-\frac{\left(-256 (dx)^{\frac{5}{2}} b^3 x^4-585 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\, \sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\, \sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right) a b^2 d^2 x^4-1170 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \arctan\left(\frac{\sqrt{2}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/640*(-256*(d*x)^(5/2)*b^3*x^4-585*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^2*d^2*x^4-1170*(a*d^2/b)^(1/4)*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^2*d^2*x^4-
1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(
a*d^2/b)^(1/4))*a*b^2*d^2*x^4-512*(d*x)^(5/2)*a*b^2*x^2+3840*(d*x)^(1/2)*a*
b^2*d^2*x^4-1170*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/
b)^(1/2)))*a^2*b*d^2*x^2-2340*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)
^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b*d^2*x^2-2340*(a*d^2/b)^(1/4)
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*
b*d^2*x^2+744*(d*x)^(5/2)*a^2*b+7680*(d*x)^(1/2)*a^2*b*d^2*x^2-585*(a*d^2/b
)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
)/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*d^2-1170*(
a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*a^3*d^2-1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-
```

```
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*d^2+4680*(d*x)^(1/2)*a^3*d^2)*d^5*(b*
x^2+a)/b^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*d^(15/2)*x^(5/2)/(a*b^4*x^2 + a^2*b^3 + (b^5*x^2 + a*b^4)*x^2) - 2
*a*d^(15/2)*integrate(x^(3/2)/(b^4*x^2 + a*b^3), x) + d^(15/2)*integrate(x^
(7/2)/(b^3*x^2 + a*b^2), x) + 21/128*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(s
qrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^
(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + s
qrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b
^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
 sqrt(a))/b^(1/4))*d^(15/2)/b^4 - 1/16*(17*a^2*b*d^(15/2)*x^(5/2) + 21*a^3*
d^(15/2)*sqrt(x))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Fricas [A]
time = 0.34, size = 341, normalized size = 0.62

$$\frac{2340\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}(b^6x^4+2ab^5x^2+a^2b^4)\arctan\left(-\frac{\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{13}d^7-\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{3}{4}}\sqrt{a^2d^{15}x+\sqrt{-\frac{a^5d^{30}}{b^{17}}}\,b^8}\,b^{13}}{a^5d^{30}}\right)+585\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}(b^6x^4+2ab^5x^2+a^2b^4)\log\left(117\sqrt{dx}\,ad^7+117\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}b^4\right)-585\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}(b^6x^4+2ab^5x^2+a^2b^4)\log\left(117\sqrt{dx}\,ad^7-117\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}b^4\right)+4(32b^3d^7x^6-416ab^2d^7x^4-1053a^2bd^7x^2-585a^3d^7)\sqrt{dx}}{320(b^6x^4+2ab^5x^2+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/320*(2340*(-a^5*d^30/b^17)^(1/4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*arctan
(-((-a^5*d^30/b^17)^(3/4)*sqrt(d*x)*a*b^13*d^7 - (-a^5*d^30/b^17)^(3/4)*sqr
t(a^2*d^15*x + sqrt(-a^5*d^30/b^17)*b^8)*b^13)/(a^5*d^30)) + 585*(-a^5*d^30
/b^17)^(1/4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*log(117*sqrt(d*x)*a*d^7 + 11
7*(-a^5*d^30/b^17)^(1/4)*b^4) - 585*(-a^5*d^30/b^17)^(1/4)*(b^6*x^4 + 2*a*b
^5*x^2 + a^2*b^4)*log(117*sqrt(d*x)*a*d^7 - 117*(-a^5*d^30/b^17)^(1/4)*b^4)
 + 4*(32*b^3*d^7*x^6 - 416*a*b^2*d^7*x^4 - 1053*a^2*b*d^7*x^2 - 585*a^3*d^7
)*sqrt(d*x))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5986 deep
```

Giac [A]
time = 3.57, size = 377, normalized size = 0.68

$$\frac{1}{640}\,d^{7}\left(\frac{1170\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{5}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{1170\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{5}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{585\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{5}\mathrm{sgn}\left(bx^{2}+a\right)}-\frac{585\,\sqrt{2}\,\left(ab^{3}d^{2}\right)^{\frac{1}{4}}a\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{b^{5}\mathrm{sgn}\left(bx^{2}+a\right)}-\frac{40\left(25\,\sqrt{dx}\,a^{2}bd^{4}x^{2}+21\,\sqrt{dx}\,a^{3}d^{4}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{2}b^{4}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{256\left(\sqrt{dx}\,b^{12}d^{10}x^{2}-15\,\sqrt{dx}\,ab^{11}d^{10}\right)}{b^{15}d^{10}\mathrm{sgn}\left(bx^{2}+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/640*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*sgn(b*x^2 + a)) + 1170*sq
rt(2)*(a*b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*
sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*sgn(b*x^2 + a)) + 585*sqrt(2)*(a*b^3*d^2)^
(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^5*s
gn(b*x^2 + a)) - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x - sqrt(2)*(a*d^2/b
)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^5*sgn(b*x^2 + a)) - 40*(25*sqrt(d*x)*
a^2*b*d^4*x^2 + 21*sqrt(d*x)*a^3*d^4)/((b*d^2*x^2 + a*d^2)^2*b^4*sgn(b*x^2
+ a)) + 256*(sqrt(d*x)*b^12*d^10*x^2 - 15*sqrt(d*x)*a*b^11*d^10)/(b^15*d^10
*sgn(b*x^2 + a)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x\right)}^{15/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.758 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=504

$$-\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77a^{3/4}d^{13/2}(a+}{32\sqrt{2}\,b}$$

```
[Out] -11/16*d^3*(d*x)^(7/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(11/2)/b/(b*x^2+
a)/((b*x^2+a)^2)^(1/2)+77/48*d^5*(d*x)^(3/2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^(1
/2)+77/64*a^(3/4)*d^(13/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a
^(1/4)/d^(1/2))/b^(15/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-77/64*a^(3/4)*d^(13/2)
*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(15/4)*2
^(1/2)/((b*x^2+a)^2)^(1/2)-77/128*a^(3/4)*d^(13/2)*(b*x^2+a)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(15/4)*2^(1/2
)/((b*x^2+a)^2)^(1/2)+77/128*a^(3/4)*d^(13/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+
x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(15/4)*2^(1/2)/((b
*x^2+a)^2)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 504, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-11*d^3*(d*x)^(7/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(
11/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^5*(d*x)^(3
/2)*(a + b*x^2))/(48*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*a^(3/4)*d^(
13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(32*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13
/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(32*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13/2
)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7
7*a^(3/4)*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2))\int \frac{(dx)^{13/2}}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(11d^2(ab+b^2x^2))\int \frac{(dx)^{9/2}}{(ab+b^2x^2)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(77d^4(ab}{32b^2\sqrt{}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(a}{48b^3\sqrt{a^2}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(a}{48b^3\sqrt{a^2}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(a}{48b^3\sqrt{a^2}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(a}{48b^3\sqrt{a^2}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(a}{48b^3\sqrt{a^2}}$$

$$= -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77d^5(a}{48b^3\sqrt{a^2}}$$

Mathematica [A]
time = 0.38, size = 195, normalized size = 0.39

$$\frac{d^6\sqrt{dx}\left(4b^{3/4}x^{3/2}(77a^2+121abx^2+32b^2x^4)+231\sqrt{2}\,a^{3/4}(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+231\sqrt{2}\,a^{3/4}(a+bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{192b^{15/4}\sqrt{x}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (d^6*Sqrt[d*x]*(4*b^(3/4)*x^(3/2)*(77*a^2 + 121*a*b*x^2 + 32*b^2*x^4) + 231
*Sqrt[2]*a^(3/4)*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4
```

```
)*b^(1/4)*Sqrt[x])] + 231*Sqrt[2]*a^(3/4)*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(192*b^(15/4)*Sqrt[x]*(a +
b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(323) = 646$.

time = 0.07, size = 679, normalized size = 1.35

method	result
risch	$\frac{2x^2d^7\sqrt{(bx^2+a)^2}}{3b^3\sqrt{dx}\,(bx^2+a)}+\frac{\left(\frac{19a(dx)^{\frac{7}{2}}}{16b^2\left(d^2x^2b+a\,d^2\right)^2}+\frac{15a^2(dx)^{\frac{3}{2}}d^2}{16b^3\left(d^2x^2b+a\,d^2\right)^2}-\frac{77a\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{128b^4\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-\frac{77a\sqrt{\cdots}}{\cdots}\cdots}{b\,x^2+a}$
default	$\frac{\left(256\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}(dx)^{\frac{3}{2}}b^3d^2x^4-231\sqrt{2}\,\ln\left(-\frac{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)a\,b^2d^4x^4-462\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d}{b}\cdots\right.}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(256*(a*d^2/b)^(1/4)*(d*x)^(3/2)*b^3*d^2*x^4-231*2^(1/2)*ln(-((a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^2*d^4*x^4-462*2^(1/2)*arctan((2^(1/2)
*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^2*d^4*x^4-462*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^2*d^4*x^4+4
56*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a*b^2+512*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a*b^2*
d^2*x^2-462*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^
(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b*d^4
*x^2-924*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*a^2*b*d^4*x^2-924*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/
(a*d^2/b)^(1/4))*a^2*b*d^4*x^2+616*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^2*b*d^2-23
1*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*
x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*d^4-462*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*d^4-462*
2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*d
^4)*d^3*(b*x^2+a)/(a*d^2/b)^(1/4)/b^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*d^(13/2)*x^(3/2)/(a*b^4*x^2 + a^2*b^3 + (b^5*x^2 + a*b^4)*x^2) - 2
*a*d^(13/2)*integrate(sqrt(x)/(b^4*x^2 + a*b^3), x) + d^(13/2)*integrate(x^
(5/2)/(b^3*x^2 + a*b^2), x) + 19/128*a*d^(13/2)*(2*sqrt(2)*arctan(1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/
4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b
))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqr
t(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^3 + 1/16*(19*a*b*d^(13/2)*x^(7/2) +
 23*a^2*d^(13/2)*x^(3/2))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Fricas [A]
time = 0.37, size = 341, normalized size = 0.68

$$\frac{924 \left(-\frac{a^3 d^{26}}{b^{15}}\right)^{\frac{1}{4}} (b^5x^4+2\,ab^4x^2+a^2b^3) \arctan\left(-\frac{\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}\sqrt{dx}\,a^2b^4d^{19}-\sqrt{a^4d^{39}x-\sqrt{-\frac{a^3d^{26}}{b^{15}}}\,a^3b^7d^{26}}\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}b^4}{a^3d^{26}}\right) - 231\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}(b^5x^4+2\,ab^4x^2+a^2b^3)\log\left(456533\sqrt{dx}\,a^2d^{19}+456533\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{3}{4}}b^{11}\right) + 231\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}(b^5x^4+2\,ab^4x^2+a^2b^3)\log\left(456533\sqrt{dx}\,a^2d^{19}-456533\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{3}{4}}b^{11}\right) + 4\,(32\,b^2d^6x^5+121\,abd^6x^3+77\,a^2d^6x)\sqrt{dx}}{192\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/192*(924*(-a^3*d^26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*arctan(
-((-a^3*d^26/b^15)^(1/4)*sqrt(d*x)*a^2*b^4*d^19 - sqrt(a^4*d^39*x - sqrt(-a
^3*d^26/b^15)*a^3*b^7*d^26)*(-a^3*d^26/b^15)^(1/4)*b^4)/(a^3*d^26)) - 231*(
-a^3*d^26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(456533*sqrt(d*x
)*a^2*d^19 + 456533*(-a^3*d^26/b^15)^(3/4)*b^11) + 231*(-a^3*d^26/b^15)^(1/
4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(456533*sqrt(d*x)*a^2*d^19 - 456533
*(-a^3*d^26/b^15)^(3/4)*b^11) + 4*(32*b^2*d^6*x^5 + 121*a*b*d^6*x^3 + 77*a^
2*d^6*x)*sqrt(d*x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4062 deep
```

Giac [A]
time = 3.13, size = 357, normalized size = 0.71

$$\frac{1}{384}\,d^6\left(\frac{256\,\sqrt{dx}\,x}{b^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{24\left(19\,\sqrt{dx}\,abd^4x^3+15\,\sqrt{dx}\,a^2d^4x\right)}{\left(bd^2x^2+ad^2\right)^2b^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{462\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6d\mathrm{sgn}\left(bx^2+a\right)}-\frac{462\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6d\mathrm{sgn}\left(bx^2+a\right)}+\frac{231\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^6d\mathrm{sgn}\left(bx^2+a\right)}-\frac{231\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^6d\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*d^6*(256*sqrt(d*x)*x/(b^3*sgn(b*x^2 + a)) + 24*(19*sqrt(d*x)*a*b*d^4*
x^3 + 15*sqrt(d*x)*a^2*d^4*x)/((b*d^2*x^2 + a*d^2)^2*b^3*sgn(b*x^2 + a)) -
462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) +
 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*d*sgn(b*x^2 + a)) - 462*sqrt(2)*(a*b^3*
d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/(b^6*d*sgn(b*x^2 + a)) + 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x
 + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*d*sgn(b*x^2 + a)
) - 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*
x) + sqrt(a*d^2/b))/(b^6*d*sgn(b*x^2 + a)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{13/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.759 $\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=504

$$-\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}\,d^{11/2}(a\ \cdots}{32\sqrt{2}\ \cdots}$$

```
[Out] -9/16*d^3*(d*x)^(5/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(9/2)/b/(b*x^2+a)
/((b*x^2+a)^2)^(1/2)+45/64*a^(1/4)*d^(11/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1
/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-45/64
*a^(1/4)*d^(11/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^
(1/2))/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+45/128*a^(1/4)*d^(11/2)*(b*x^2+
a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2)
)/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-45/128*a^(1/4)*d^(11/2)*(b*x^2+a)*ln
(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(
13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+45/16*d^5*(b*x^2+a)*(d*x)^(1/2)/b^3/((b*x
^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 504, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{45\sqrt[4]{a}\,d^{11/2}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{a}\,d^{11/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}\,d^{11/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{a}\,d^{11/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-9*d^3*(d*x)^(5/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(9
/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^5*Sqrt[d*x]*
(a + b*x^2))/(16*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/2
)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(3
2*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^(1/4)*d^(11/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*
Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/2)*(a
 + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^
(1/4)*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

& (LtQ[a, 0] || LtQ[b, 0])

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2))\int \frac{(dx)^{11/2}}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(9d^2(ab+b^2x^2))\int \frac{(dx)^{7/2}}{(ab+b^2x^2)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(45d^4(ab}{32b^2\sqrt{a}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{}}{16b^3\sqrt{a^2}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{}}{16b^3\sqrt{a^2}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{}}{16b^3\sqrt{a^2}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{}}{16b^3\sqrt{a^2}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{}}{16b^3\sqrt{a^2}}$$

$$= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{}}{16b^3\sqrt{a^2}}$$

Mathematica [A]
time = 0.36, size = 195, normalized size = 0.39

$$\frac{d^5\sqrt{dx}\left(4\sqrt[4]{b}\,\sqrt{x}\,(45a^2+81abx^2+32b^2x^4)+45\sqrt{2}\,\sqrt[4]{a}\,(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-45\sqrt{2}\,\sqrt[4]{a}\,(a+bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{64b^{13/4}\sqrt{x}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (d^5*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(45*a^2 + 81*a*b*x^2 + 32*b^2*x^4) + 45*S
qrt[2]*a^(1/4)*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*
```

```
b^(1/4)*Sqrt[x])] - 45*Sqrt[2]*a^(1/4)*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(64*b^(13/4)*Sqrt[x]*(a + b*x^
2)*Sqrt[(a + b*x^2)^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(323) = 646$.
time = 0.07, size = 696, normalized size = 1.38

method	result
risch	$\frac{2 x d^{6} \sqrt{\left(b\,x^{2}+a\right)^{2}}}{b^{3} \sqrt{dx}\,\left(b\,x^{2}+a\right)}+\frac{\left(\frac{17 a d (dx)^{\frac{5}{2}}}{16 b^{2}\left(d^{2}x^{2}b+a\,d^{2}\right)^{2}}+\frac{13 a^{2} d^{3} \sqrt{dx}}{16 b^{3}\left(d^{2}x^{2}b+a\,d^{2}\right)^{2}}-\frac{45\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{128 b^{3} d} \cdots\right.}{\cdots}$
default	$\frac{\left(-45\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right) b^{2} d^{2} x^{4}-90\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right) b^{2} d^{2} \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(-45*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))*b^2*d^2*x^4-90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*d^2*x^4-90*(a*d^2/b)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*d^2*x^4+256*(d
*x)^(1/2)*b^2*d^2*x^4-90*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2)))*a*b*d^2*x^2-180*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*
(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^2*x^2-180*(a*d^2/b)^(1/
4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*
b*d^2*x^2+136*(d*x)^(5/2)*a*b+512*(d*x)^(1/2)*a*b*d^2*x^2-45*(a*d^2/b)^(1/4
)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x
-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*d^2-90*(a*d^2/b)
^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*a^2*d^2-90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^
(1/4))/(a*d^2/b)^(1/4))*a^2*d^2+360*(d*x)^(1/2)*a^2*d^2)*d^3*(b*x^2+a)/b^3/
((b*x^2+a)^2)^(3/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*d^(11/2)*x^(5/2)/(a*b^3*x^2 + a^2*b^2 + (b^4*x^2 + a*b^3)*x^2) + d^(1
1/2)*integrate(x^(3/2)/(b^3*x^2 + a*b^2), x) - 13/128*(2*sqrt(2)*sqrt(a)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqrt(2)*(s
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqr
t(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(
b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt
(x) + sqrt(b)*x + sqrt(a))/b^(1/4))*d^(11/2)/b^3 + 1/16*(9*a*b*d^(11/2)*x^(
5/2) + 13*a^2*d^(11/2)*sqrt(x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Fricas [A]
time = 0.37, size = 305, normalized size = 0.61

$$-\frac{180\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}(b^5x^4+2\,ab^4x^2+a^2b^3)\arctan\left(-\frac{\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{3}{4}}\sqrt{dx}\,b^{10}d^5-\sqrt{d^{11}x+\sqrt{-\frac{ad^{22}}{b^{13}}}\,b^6}\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{3}{4}}b^{10}}{ad^{22}}\right)+45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}(b^5x^4+2\,ab^4x^2+a^2b^3)\log\left(45\sqrt{dx}\,d^5+45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3\right)-45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}(b^5x^4+2\,ab^4x^2+a^2b^3)\log\left(45\sqrt{dx}\,d^5-45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3\right)-4\,(32\,b^2d^5x^4+81\,abd^5x^2+45\,a^2d^5)\sqrt{dx}}{64\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(180*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*arctan(-(
(-a*d^22/b^13)^(3/4)*sqrt(d*x)*b^10*d^5 - sqrt(d^11*x + sqrt(-a*d^22/b^13)*
b^6)*(-a*d^22/b^13)^(3/4)*b^10)/(a*d^22)) + 45*(-a*d^22/b^13)^(1/4)*(b^5*x^
4 + 2*a*b^4*x^2 + a^2*b^3)*log(45*sqrt(d*x)*d^5 + 45*(-a*d^22/b^13)^(1/4)*b
^3) - 45*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(45*sqrt
(d*x)*d^5 - 45*(-a*d^22/b^13)^(1/4)*b^3) - 4*(32*b^2*d^5*x^4 + 81*a*b*d^5*x
^2 + 45*a^2*d^5)*sqrt(d*x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{11}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(11/2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]

time = 3.62, size = 343, normalized size = 0.68

$$-\frac{1}{128}\,d^5\left(\frac{90\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4\mathrm{sgn}\left(bx^2+a\right)}+\frac{90\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4\mathrm{sgn}\left(bx^2+a\right)}+\frac{45\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^4\mathrm{sgn}\left(bx^2+a\right)}-\frac{45\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^4\mathrm{sgn}\left(bx^2+a\right)}-\frac{256\,\sqrt{dx}}{b^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{8\left(17\,\sqrt{dx}\,abd^4x^2+13\,\sqrt{dx}\,a^2d^4\right)}{(bd^2x^2+ad^2)^2b^3\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*d^5*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*sgn(b*x^2 + a)) + 90*sqrt(2)
*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*
x))/(a*d^2/b)^(1/4))/(b^4*sgn(b*x^2 + a)) + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*lo
g(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*sgn(b*x^2 +
 a)) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(
d*x) + sqrt(a*d^2/b))/(b^4*sgn(b*x^2 + a)) - 256*sqrt(d*x)/(b^3*sgn(b*x^2 +
 a)) - 8*(17*sqrt(d*x)*a*b*d^4*x^2 + 13*sqrt(d*x)*a^2*d^4)/((b*d^2*x^2 + a*
d^2)^2*b^3*sgn(b*x^2 + a)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{11/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.760 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} \, dx$

Optimal. Leaf size=458

$$-\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21d^{9/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -7/16*d^3*(d*x)^(3/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(7/2)/b/(b*x^2+a)
/((b*x^2+a)^2)^(1/2)-21/64*d^(9/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)
^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(11/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+21/64*
d^(9/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(
1/4)/b^(11/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+21/128*d^(9/2)*(b*x^2+a)*ln(a^(1/
2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b
^(11/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-21/128*d^(9/2)*(b*x^2+a)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(11/4
)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{21d^{9/2}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21d^{9/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\sqrt[4]{a}\,b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-7*d^3*(d*x)^(3/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(7
/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*d^(9/2)*(a + b
*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2
]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*d^(9
/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{9/2}}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(7d^2\left(ab+b^2x^2\right)\right)\int \frac{(dx)^{5/2}}{\left(ab+b^2x^2\right)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^4(ab \cdots}{32b^2\sqrt{a \cdots}}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^3(ab \cdots}{\cdots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(21d^3(ab \cdots}{\cdots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21d^{9/2}(a \cdots}{\cdots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{21d^{9/2}(a \cdots}{\cdots}$$

$$= -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21d^{9/2}(a \cdots}{32\sqrt{2} \cdots}$$

Mathematica [A]
time = 0.35, size = 184, normalized size = 0.40

$$-\frac{d^4\sqrt{dx}\left(4\sqrt[4]{a}\,b^{3/4}x^{3/2}(7a+11bx^2)+21\sqrt{2}\,(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+21\sqrt{2}\,(a+bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{64\sqrt[4]{a}\,b^{11/4}\sqrt{x}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/64*(d^4*Sqrt[d*x]*(4*a^(1/4)*b^(3/4)*x^(3/2)*(7*a + 11*b*x^2) + 21*Sqrt[
2]*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])] + 21*Sqrt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/
(Sqrt[a] + Sqrt[b]*x)]))/(a^(1/4)*b^(11/4)*Sqrt[x]*(a + b*x^2)*Sqrt[(a + b*
x^2)^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 611 vs. 2(292) = 584.
time = 0.04, size = 612, normalized size = 1.34

method	result
default	$-\frac{\left(-21\sqrt{2}\,\ln\left(-\frac{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)b^2d^4x^4-42\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)b^2d^4x^4-42\sqrt{2}\,\text{a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(-21*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^
(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^2*d^4*x
^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))
*b^2*d^4*x^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2
/b)^(1/4))*b^2*d^4*x^4+88*(d*x)^(7/2)*(a*d^2/b)^(1/4)*b^2-42*2^(1/2)*ln(-((
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b*d^4*x^2-84*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^4*x^2-84*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^4*x^2+56*
(d*x)^(3/2)*(a*d^2/b)^(1/4)*a*b*d^2-21*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a*d^2/b)^(1/2)))*a^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^
(1/4))/(a*d^2/b)^(1/4))*a^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^4)*d*(b*x^2+a)/(a*d^2/b)^(1/4)/b^3/((b*
x^2+a)^2)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*d^(9/2)*x^(3/2)/(a*b^3*x^2 + a^2*b^2 + (b^4*x^2 + a*b^3)*x^2) + d^(9/
2)*integrate(sqrt(x)/(b^3*x^2 + a*b^2), x) - 11/128*d^(9/2)*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^2 - 1/16*(11*b*d^(9/2)*
x^(7/2) + 15*a*d^(9/2)*x^(3/2))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Fricas [A]
time = 0.40, size = 312, normalized size = 0.68

$$-\frac{84\,(b^4x^4+2\,ab^3x^2+a^2b^2)\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\sqrt{dx}\,b^3d^{13}-\sqrt{d^{27}x-\sqrt{-\frac{d^{18}}{ab^{11}}}\,ab^5d^{18}}\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}b^3}{d^{18}}\right)-21\,(b^4x^4+2\,ab^3x^2+a^2b^2)\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\log\left(9261\,\sqrt{dx}\,d^{13}+9261\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{3}{4}}ab^8\right)+21\,(b^4x^4+2\,ab^3x^2+a^2b^2)\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\log\left(9261\,\sqrt{dx}\,d^{13}-9261\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{3}{4}}ab^8\right)+4\,(11\,bd^4x^3+7\,ad^4x)\sqrt{dx}}{64\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(84*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^18/(a*b^11))^(1/4)*arctan(-
((-d^18/(a*b^11))^(1/4)*sqrt(d*x)*b^3*d^13 - sqrt(d^27*x - sqrt(-d^18/(a*b^
11))*a*b^5*d^18)*(-d^18/(a*b^11))^(1/4)*b^3)/d^18) - 21*(b^4*x^4 + 2*a*b^3*
x^2 + a^2*b^2)*(-d^18/(a*b^11))^(1/4)*log(9261*sqrt(d*x)*d^13 + 9261*(-d^18
/(a*b^11))^(3/4)*a*b^8) + 21*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^18/(a*b^
11))^(1/4)*log(9261*sqrt(d*x)*d^13 - 9261*(-d^18/(a*b^11))^(3/4)*a*b^8) + 4
*(11*b*d^4*x^3 + 7*a*d^4*x)*sqrt(d*x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^{\frac{9}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(9/2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 4.26, size = 345, normalized size = 0.75

$$-\frac{1}{128}\,d^4\left(\frac{8\left(11\,\sqrt{dx}\,bd^4x^3+7\,\sqrt{dx}\,ad^4x\right)}{\left(bd^2x^2+ad^2\right)^2b^2\mathrm{sgn}\left(bx^2+a\right)}-\frac{42\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5d\,\mathrm{sgn}\left(bx^2+a\right)}-\frac{42\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5d\,\mathrm{sgn}\left(bx^2+a\right)}+\frac{21\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^5d\,\mathrm{sgn}\left(bx^2+a\right)}-\frac{21\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^5d\,\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*d^4*(8*(11*sqrt(d*x)*b*d^4*x^3 + 7*sqrt(d*x)*a*d^4*x)/((b*d^2*x^2 +
a*d^2)^2*b^2*sgn(b*x^2 + a)) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt
(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*d*sgn(b
*x^2 + a)) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*d*sgn(b*x^2 + a)) + 21*s
qrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/(a*b^5*d*sgn(b*x^2 + a)) - 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5*d*sgn(b*x^2 + a
)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x\right)}^{9/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.761 $\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=458

$$-\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{4b\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-$$

```
[Out] -1/4*d*(d*x)^(5/2)/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-5/64*d^(7/2)*(b*x^2+a)*a
rctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^(9/4)*2^(1/2
)/((b*x^2+a)^2)^(1/2)+5/64*d^(7/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)
^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-5/128*d
^(7/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/
2)*(d*x)^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+5/128*d^(7/2)*(
b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)
^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-5/16*d^3*(d*x)^(1/2)/b^
2/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{4b\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^{7/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{dx}}{\sqrt[4]{a}\,\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}(a+bx^2)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^{7/2}(a+bx^2)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{dx}+\sqrt{a}\,\sqrt{d}+\sqrt{b}\,\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*d^3*Sqrt[d*x])/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(5/2
))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^
2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^(7/2)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(3/4
)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*
Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^(7/2)*(a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{(dx)^{7/2}}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5d^2(ab+b^2x^2)\right)\int \frac{(dx)^{3/2}}{(ab+b^2x^2)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(5d^4(ab+}{32b^2}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(5d^3(ab+}{}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(5d^2(ab+}{}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(5d^{7/2}(ab}{}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^{7/2}(a+}{}$$

$$= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^{7/2}(a+}{32\sqrt{2}}$$

Mathematica [**A**]

time = 0.35, size = 184, normalized size = 0.40

$$-\frac{d^3\sqrt{dx}\left(4a^{3/4}\sqrt[4]{b}\,\sqrt{x}\,(5a+9bx^2)+5\sqrt{2}\,(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-5\sqrt{2}\,(a+bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{64a^{3/4}b^{9/4}\sqrt{x}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/64*(d^3*Sqrt[d*x]*(4*a^(3/4)*b^(1/4)*Sqrt[x]*(5*a + 9*b*x^2) + 5*Sqrt[2]
*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
])] - 5*Sqrt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sq
rt[a] + Sqrt[b]*x)]))/(a^(3/4)*b^(9/4)*Sqrt[x]*(a + b*x^2)*Sqrt[(a + b*x^2)
^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(292) = 584.
time = 0.04, size = 666, normalized size = 1.45

method	result
default	$\frac{\left(5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)b^2d^2x^4+10\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)b^2d^2x^4\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(5*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
))*b^2*d^2*x^4+10*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*d^2*x^4+10*(a*d^2/b)^(1/4)*2^(1/2)*arctan(
(2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*d^2*x^4+10*(a*d^
2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b*d^2*x^2
+20*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a
*d^2/b)^(1/4))*a*b*d^2*x^2+20*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)
^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^2*x^2+5*(a*d^2/b)^(1/4)*2^(1
/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*d^2+10*(a*d^2/b)^(1/4)
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*
d^2+10*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*a^2*d^2-72*(d*x)^(5/2)*a*b-40*(d*x)^(1/2)*a^2*d^2)*d*(b*x
^2+a)/a/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.53, size = 279, normalized size = 0.61

$$-\frac{d^{\frac{7}{2}}x^{\frac{5}{2}}}{2\left(ab^2x^2+a^2b+(b^3x^2+ab^2)x^2\right)}+\frac{5\,d^3\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,b^2}-\frac{bd^{\frac{7}{2}}x^{\frac{5}{2}}+5\,ad^{\frac{7}{2}}\sqrt{x}}{16\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*d^(7/2)*x^(5/2)/(a*b^2*x^2 + a^2*b + (b^3*x^2 + a*b^2)*x^2) + 5/128*d^
3*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(
2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*sqrt(d)*
log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)
) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
(a))/(a^(3/4)*b^(1/4)))/b^2 - 1/16*(b*d^(7/2)*x^(5/2) + 5*a*d^(7/2)*sqrt(x)
)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Fricas [A]
time = 0.39, size = 315, normalized size = 0.69

$$\frac{20\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{3}{4}}\sqrt{dx}\,a^2b^7d^3-\sqrt{d^7x+\sqrt{-\frac{d^{14}}{a^3b^9}}\,a^2b^4}\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{3}{4}}a^2b^7}{d^{14}}\right)+5\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}\log\left(5\sqrt{dx}\,d^3+5\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}ab^2\right)-5\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}\log\left(5\sqrt{dx}\,d^3-5\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}ab^2\right)-4\left(9\,bd^3x^2+5\,ad^3\right)\sqrt{dx}}{64\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*arctan(-
((-d^14/(a^3*b^9))^(3/4)*sqrt(d*x)*a^2*b^7*d^3 - sqrt(d^7*x + sqrt(-d^14/(a
^3*b^9))*a^2*b^4)*(-d^14/(a^3*b^9))^(3/4)*a^2*b^7)/d^14) + 5*(b^4*x^4 + 2*a
*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*log(5*sqrt(d*x)*d^3 + 5*(-d^14/
(a^3*b^9))^(1/4)*a*b^2) - 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b
^9))^(1/4)*log(5*sqrt(d*x)*d^3 - 5*(-d^14/(a^3*b^9))^(1/4)*a*b^2) - 4*(9*b*
d^3*x^2 + 5*a*d^3)*sqrt(d*x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx)^{\frac{7}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(7/2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 4.78, size = 332, normalized size = 0.72

$$\frac{1}{128}\,d^{3}\left(\frac{10\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{ab^{3}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{10\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{ab^{3}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{5\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{ab^{3}\mathrm{sgn}\left(bx^{2}+a\right)}-\frac{5\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{ab^{3}\mathrm{sgn}\left(bx^{2}+a\right)}-\frac{8\left(9\,\sqrt{dx}\,bd^{4}x^{2}+5\,\sqrt{dx}\,ad^{4}\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{2}b^{2}\mathrm{sgn}\left(bx^{2}+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d^3*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/
b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*sgn(b*x^2 + a)) + 10*sqrt(2
)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d
*x))/(a*d^2/b)^(1/4))/(a*b^3*sgn(b*x^2 + a)) + 5*sqrt(2)*(a*b^3*d^2)^(1/4)*
log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3*sgn(b*x
^2 + a)) - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a*b^3*sgn(b*x^2 + a)) - 8*(9*sqrt(d*x)*b*d^4*x^2
+ 5*sqrt(d*x)*a*d^4)/((b*d^2*x^2 + a*d^2)^2*b^2*sgn(b*x^2 + a)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{7/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.762 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} \, dx$

Optimal. Leaf size=459

$$\frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\cdots$$

```
[Out] 3/16*d*(d*x)^(3/2)/a/b/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(3/2)/b/(b*x^2+a)/((
b*x^2+a)^2)^(1/2)-3/64*d^(5/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/
2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+3/64*d^(5/2
)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b
^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+3/128*d^(5/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/
2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(7/4)*2
^(1/2)/((b*x^2+a)^2)^(1/2)-3/128*d^(5/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(
1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(7/4)*2^(1/2)/(
(b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{5/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{5/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (3*d*(d*x)^(3/2))/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^(3/2)
)/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(5/2)*(a + b*x^2
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^
(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(5/4)
*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*S
qrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(5/2)*(a + b
*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d*x]])/(64*Sqrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2))\int \frac{(dx)^{5/2}}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{4b\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3d^2(ab+b^2x^2))\int \frac{\sqrt{dx}}{(ab+b^2x^2)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3d^2(ab+}{32ab\sqrt{a}}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3d(ab+}{}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(3d(ab+}{}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3d^{5/2}(ab}{}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3d^{5/2}(a+}{}$$

$$= \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{5/2}(a+}{32\sqrt{2}}$$

Mathematica [A]
time = 0.34, size = 179, normalized size = 0.39

$$-\frac{(dx)^{5/2}\left(4\sqrt[4]{a}\,b^{3/4}x^{3/2}(a-3bx^2)+3\sqrt{2}\,(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+3\sqrt{2}\,(a+bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{64a^{5/4}b^{7/4}x^{5/2}\,(a+bx^2)\,\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/64*((d*x)^(5/2)*(4*a^(1/4)*b^(3/4)*x^(3/2)*(a - 3*b*x^2) + 3*Sqrt[2]*(a
+ b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])]
```

```
+ 3*Sqrt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a
] + Sqrt[b]*x)]))/(a^(5/4)*b^(7/4)*x^(5/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(293) = 586$.
time = 0.04, size = 617, normalized size = 1.34

method	result
default	$\left(3\sqrt{2}\,\ln\left(-\frac{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)b^2d^4x^4+6\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)b^2d^4x^4+6\sqrt{2}\,\arctan\left(\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(3*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/
2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^2*d^4*x^4+
6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2
*d^4*x^4+6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*b^2*d^4*x^4+24*(d*x)^(7/2)*(a*d^2/b)^(1/4)*b^2+6*2^(1/2)*ln(-((a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b*d^4*x^2+12*2^(1/2)*arctan((2^(1/2)*(d
*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^4*x^2+12*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^4*x^2-8*(d*x)^(
3/2)*(a*d^2/b)^(1/4)*a*b*d^2+3*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2)))*a^2*d^4+6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a
*d^2/b)^(1/4))*a^2*d^4+6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*a^2*d^4)/d*(b*x^2+a)/(a*d^2/b)^(1/4)/b^2/a/((b*x^2+a)^2
)^(3/2)
```

Maxima [A]
time = 0.53, size = 272, normalized size = 0.59

$$-\frac{d^{\frac{5}{2}}x^{\frac{3}{2}}}{2\,(ab^2x^2+a^2b+(b^3x^2+ab^2)x^2)}+\frac{3\,d^{\frac{5}{2}}\left(\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,ab}+\frac{3\,bd^{\frac{5}{2}}x^{\frac{7}{2}}+7\,ad^{\frac{5}{2}}x^{\frac{3}{2}}}{16\,(ab^3x^4+2\,a^2b^2x^2+a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*d^(5/2)*x^(3/2)/(a*b^2*x^2 + a^2*b + (b^3*x^2 + a*b^2)*x^2) + 3/128*d^
(5/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*a
```

```
rctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b
) + 1/16*(3*b*d^(5/2)*x^(7/2) + 7*a*d^(5/2)*x^(3/2))/(a*b^3*x^4 + 2*a^2*b^2
*x^2 + a^3*b)
```

Fricas [A]
time = 0.35, size = 326, normalized size = 0.71

$$-\frac{12\,(ab^3x^4+2\,a^2b^2x^2+a^3b)\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{27\,\sqrt{dx}\,ab^2d^7\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}-\sqrt{-729\,a^3b^3d^{10}\sqrt{-\frac{d^{10}}{a^5b^7}}+729\,d^{15}x}\,ab^2\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}}{27\,d^{10}}\right)-3\,(ab^3x^4+2\,a^2b^2x^2+a^3b)\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}\log\left(27\,a^4b^5\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{3}{4}}+27\,\sqrt{dx}\,d^7\right)+3\,(ab^3x^4+2\,a^2b^2x^2+a^3b)\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}\log\left(-27\,a^4b^5\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{3}{4}}+27\,\sqrt{dx}\,d^7\right)-4\,(3\,bd^2x^3-ad^2x)\sqrt{dx}}{64\,(ab^3x^4+2\,a^2b^2x^2+a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*arcta
n(-1/27*(27*sqrt(d*x)*a*b^2*d^7*(-d^10/(a^5*b^7))^(1/4) - sqrt(-729*a^3*b^3
*d^10*sqrt(-d^10/(a^5*b^7)) + 729*d^15*x)*a*b^2*(-d^10/(a^5*b^7))^(1/4))/d^
10) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(27*
a^4*b^5*(-d^10/(a^5*b^7))^(3/4) + 27*sqrt(d*x)*d^7) + 3*(a*b^3*x^4 + 2*a^2*
b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(-27*a^4*b^5*(-d^10/(a^5*b^7))^
(3/4) + 27*sqrt(d*x)*d^7) - 4*(3*b*d^2*x^3 - a*d^2*x)*sqrt(d*x))/(a*b^3*x^4
 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(5/2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 3.55, size = 348, normalized size = 0.76

$$\frac{1}{128}\,d^2\left(\frac{8\left(3\,\sqrt{dx}\,bd^4x^3-\sqrt{dx}\,ad^4x\right)}{(bd^2x^2+ad^2)^2ab\mathrm{sgn}\,(bx^2+a)}+\frac{6\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4d\mathrm{sgn}\,(bx^2+a)}+\frac{6\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4d\mathrm{sgn}\,(bx^2+a)}-\frac{3\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^4d\mathrm{sgn}\,(bx^2+a)}+\frac{3\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^4d\mathrm{sgn}\,(bx^2+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d^2*(8*(3*sqrt(d*x)*b*d^4*x^3 - sqrt(d*x)*a*d^4*x)/((b*d^2*x^2 + a*d^
2)^2*a*b*sgn(b*x^2 + a)) + 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*d*sgn(b*x^
2 + a)) + 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*d*sgn(b*x^2 + a)) - 3*sqrt
(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*
d^2/b))/(a^2*b^4*d*sgn(b*x^2 + a)) + 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x -
sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4*d*sgn(b*x^2 + a
)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{5/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.763 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=459

$$\frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{3/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+$$

```
[Out] -3/64*d^(3/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))/a^(7/4)/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+3/64*d^(3/2)*(b*x^2+a)*arcta
n(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)/((
b*x^2+a)^2)^(1/2)-3/128*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1
/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)/((b*x^2+a)
^2)^(1/2)+3/128*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1
/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(1/2
)+1/16*d*(d*x)^(1/2)/a/b/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(1/2)/b/(b*x^2+a)/
((b*x^2+a)^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{3/2}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{3/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{3/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{3/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (d*Sqrt[d*x])/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[d*x])/(4*b
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*
b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/2)*(a + b*x^2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*b^(5/
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]
*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(64*Sqrt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2))\int \frac{(dx)^{3/2}}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(d^2(ab+b^2x^2))\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^2}\,dx}{8\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3d^2(ab+b}{32ab\sqrt{}}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3d(ab+b^2}{1}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3(ab+b^2x}{}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(3d^{3/2}(ab+}{}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{3/2}(a+b}{}$$

$$= \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{3/2}(a+b}{32\sqrt{2}\,a}$$

Mathematica [A]
time = 0.33, size = 180, normalized size = 0.39

$$-\frac{(dx)^{3/2}\left(-4a^{3/4}\sqrt[4]{b}\sqrt{x}\,(-3a+bx^2)+3\sqrt{2}\,(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-3\sqrt{2}\,(a+bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{64a^{7/4}b^{5/4}x^{3/2}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/64*((d*x)^(3/2)*(-4*a^(3/4)*b^(1/4)*Sqrt[x]*(-3*a + b*x^2) + 3*Sqrt[2]*(
a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
] - 3*Sqrt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt
```

```
[a] + Sqrt[b]*x)]))/(a^(7/4)*b^(5/4)*x^(3/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(293) = 586$.

time = 0.04, size = 668, normalized size = 1.46

method	result
default	$\frac{\left(3\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)b^{2}d^{2}x^{4}+6\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)b^{2}d^{2}x^{4}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(3*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
))*b^2*d^2*x^4+6*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2
/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*d^2*x^4+6*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*d^2*x^4+6*(a*d^2/b
)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
)/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b*d^2*x^2+12
*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*a*b*d^2*x^2+12*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^2*x^2+3*(a*d^2/b)^(1/4)*2^(1/2)
*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*d^2+6*(a*d^2/b)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2+
6*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d
^2/b)^(1/4))*a^2*d^2+8*(d*x)^(5/2)*a*b-24*(d*x)^(1/2)*a^2*d^2)/d*(b*x^2+a)/
b/a^2/((b*x^2+a)^2)^(3/2)
```

Maxima [A]

time = 0.53, size = 281, normalized size = 0.61

$$\frac{d^{\frac{3}{2}}x^{\frac{5}{2}}}{2\,(a^{2}bx^{2}+a^{3}+(ab^{2}x^{2}+a^{2}b)x^{2})}-\frac{7\,bd^{\frac{3}{2}}x^{\frac{5}{2}}+3\,ad^{\frac{3}{2}}\sqrt{x}}{16\,(ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b)}+\frac{3\,d\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*d^(3/2)*x^(5/2)/(a^2*b*x^2 + a^3 + (a*b^2*x^2 + a^2*b)*x^2) - 1/16*(7*b
*d^(3/2)*x^(5/2) + 3*a*d^(3/2)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

```
 + 3/128*d*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) +
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
+ 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)
*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4
)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

Fricas [A]
time = 0.35, size = 308, normalized size = 0.67

$$\frac{12\,(ab^3x^4+2\,a^2b^2x^2+a^3b)\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^5b^4d\left(-\frac{d^6}{a^7b^5}\right)^{\frac{3}{4}}-\sqrt{a^4b^2\sqrt{-\frac{d^6}{a^7b^5}}+d^3x}\,a^5b^4\left(-\frac{d^6}{a^7b^5}\right)^{\frac{3}{4}}}{d^6}\right)+3\,(ab^3x^4+2\,a^2b^2x^2+a^3b)\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}\log\left(3\,a^2b\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}+3\,\sqrt{dx}\,d\right)-3\,(ab^3x^4+2\,a^2b^2x^2+a^3b)\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}\log\left(-3\,a^2b\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}+3\,\sqrt{dx}\,d\right)+4\,(bdx^2-3\,ad)\sqrt{dx}}{64\,(ab^3x^4+2\,a^2b^2x^2+a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(1/4)*arctan(
-(sqrt(d*x)*a^5*b^4*d*(-d^6/(a^7*b^5))^(3/4) - sqrt(a^4*b^2*sqrt(-d^6/(a^7*
b^5)) + d^3*x)*a^5*b^4*(-d^6/(a^7*b^5))^(3/4))/d^6) + 3*(a*b^3*x^4 + 2*a^2*
b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(1/4)*log(3*a^2*b*(-d^6/(a^7*b^5))^(1/4)
+ 3*sqrt(d*x)*d) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(
1/4)*log(-3*a^2*b*(-d^6/(a^7*b^5))^(1/4) + 3*sqrt(d*x)*d) + 4*(b*d*x^2 - 3*
a*d)*sqrt(d*x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 4.22, size = 332, normalized size = 0.72

$$\frac{1}{128}\,d\left(\frac{6\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2\mathrm{sgn}\left(bx^2+a\right)}+\frac{6\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2\mathrm{sgn}\left(bx^2+a\right)}+\frac{3\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^2\mathrm{sgn}\left(bx^2+a\right)}-\frac{3\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^2\mathrm{sgn}\left(bx^2+a\right)}+\frac{8\left(\sqrt{dx}\,bd^4x^2-3\,\sqrt{dx}\,ad^4\right)}{(bd^2x^2+ad^2)^2ab\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d*(6*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*sgn(b*x^2 + a)) + 6*sqrt(2)*
(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x
))/(a*d^2/b)^(1/4))/(a^2*b^2*sgn(b*x^2 + a)) + 3*sqrt(2)*(a*b^3*d^2)^(1/4)*
log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*sgn(b
*x^2 + a)) - 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*
sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*sgn(b*x^2 + a)) + 8*(sqrt(d*x)*b*d^4*x^
2 - 3*sqrt(d*x)*a*d^4)/((b*d^2*x^2 + a*d^2)^2*a*b*sgn(b*x^2 + a)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{3/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.764 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=460

$$\frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 5/16*(d*x)^(3/2)/a^2/d/((b*x^2+a)^2)^(1/2)+1/4*(d*x)^(3/2)/a/d/(b*x^2+a)/((
b*x^2+a)^2)^(1/2)-5/64*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/
4)/d^(1/2))*d^(1/2)/a^(9/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+5/64*(b*x^2
+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(9/4)/b
^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+5/128*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(
1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(9/4)/b^(3/4)*2
^(1/2)/((b*x^2+a)^2)^(1/2)-5/128*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(9/4)/b^(3/4)*2^(1/2)/(
(b*x^2+a)^2)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5\sqrt{d}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (5*(d*x)^(3/2))/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x)^(3/2)/(4
*a*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*Sqrt[d]*(a + b*x^2)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9
/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9/4)*b
^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a
]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqr
t[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*Sqrt[d]*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(64*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2))\int \frac{\sqrt{dx}}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(5b(ab+b^2x^2))\int \frac{\sqrt{dx}}{(ab+b^2x^2)^2}\,dx}{8a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(5(ab+}{32a^2\sqrt{a}}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(5(ab+}{}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(5(ab+}{}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5\sqrt{d}\,(a\right.}{}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{d}\,(a}{}$$

$$= \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5\sqrt{d}\,(a}{32\sqrt{}}$$

Mathematica [A]

time = 0.24, size = 181, normalized size = 0.39

$$\frac{\sqrt{dx}\left(4\sqrt[4]{a}\,b^{3/4}x^{3/2}(9a+5bx^2)-5\sqrt{2}\,(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-5\sqrt{2}\,(a+bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{64a^{9/4}b^{3/4}\sqrt{x}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[d*x]*(4*a^(1/4)*b^(3/4)*x^(3/2)*(9*a + 5*b*x^2) - 5*Sqrt[2]*(a + b*x^
2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 5*Sq
rt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sq
rt[b]*x)]))/(64*a^(9/4)*b^(3/4)*Sqrt[x]*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(294) = 588$.
time = 0.04, size = 617, normalized size = 1.34

method	result
default	$\frac{\left(5\sqrt{2}\,\ln\left(-\frac{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)b^2d^2x^4+10\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)b^2d^2x^4+10\sqrt{2}\,\text{arctan}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(5*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/
2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^2*d^2*x^4+
10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^
2*d^2*x^4+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)
^(1/4))*b^2*d^2*x^4+40*(a*d^2/b)^(1/4)*(d*x)^(3/2)*b^2*x^2+10*2^(1/2)*ln(-(
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b*d^2*x^2+20*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^2*x^2+20*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^2*x^2+72
*(d*x)^(3/2)*a*b*(a*d^2/b)^(1/4)+5*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2)))*a^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*a^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2)/d*(b*x^2+a)/(a*d^2/b)^(1/4)/b/a^2/((b*x^
2+a)^2)^(3/2)
```

Maxima [A]
time = 0.52, size = 265, normalized size = 0.58

$$\frac{\sqrt{d}\,x^{\frac{3}{2}}}{2\,(a^2bx^2+a^3+(ab^2x^2+a^2b)x^2)}+\frac{5\,b\sqrt{d}\,x^{\frac{7}{2}}+a\sqrt{d}\,x^{\frac{3}{2}}}{16\,(a^2b^2x^4+2\,a^3bx^2+a^4)}+\frac{5\,\sqrt{d}\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d)*x^(3/2)/(a^2*b*x^2 + a^3 + (a*b^2*x^2 + a^2*b)*x^2) + 1/16*(5*b
*sqrt(d)*x^(7/2) + a*sqrt(d)*x^(3/2))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 5
/128*sqrt(d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
))/a^2
```

Fricas [A]
time = 0.34, size = 304, normalized size = 0.66

$$-\frac{20\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{125\,\sqrt{dx}\,a^2bd\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}-\sqrt{-15625\,a^5bd^2\sqrt{-\frac{d^2}{a^9b^3}}+15625\,d^3x}\,a^2b\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}}{125\,d^2}\right)-5\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}\log\left(125\,a^7b^2\left(-\frac{d^2}{a^9b^3}\right)^{\frac{3}{4}}+125\,\sqrt{dx}\,d\right)+5\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}\log\left(-125\,a^7b^2\left(-\frac{d^2}{a^9b^3}\right)^{\frac{3}{4}}+125\,\sqrt{dx}\,d\right)-4\,(5\,bx^3+9\,ax)\sqrt{dx}}{64\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(20*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*arctan(-
1/125*(125*sqrt(d*x)*a^2*b*d*(-d^2/(a^9*b^3))^(1/4) - sqrt(-15625*a^5*b*d^2
*sqrt(-d^2/(a^9*b^3)) + 15625*d^3*x)*a^2*b*(-d^2/(a^9*b^3))^(1/4))/d^2) - 5
*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*log(125*a^7*b^2*(
-d^2/(a^9*b^3))^(3/4) + 125*sqrt(d*x)*d) + 5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a
^4)*(-d^2/(a^9*b^3))^(1/4)*log(-125*a^7*b^2*(-d^2/(a^9*b^3))^(3/4) + 125*sq
rt(d*x)*d) - 4*(5*b*x^3 + 9*a*x)*sqrt(d*x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^
4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(sqrt(d*x)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 4.58, size = 333, normalized size = 0.72

$$\frac{\frac{8\left(5\sqrt{dx}\,bd^5x^3+9\sqrt{dx}\,ad^5x\right)}{\left(bd^2x^2+ad^2\right)^2a^2\mathrm{sgn}\left(bx^2+a\right)}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^3\mathrm{sgn}\left(bx^2+a\right)}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*(8*(5*sqrt(d*x)*b*d^5*x^3 + 9*sqrt(d*x)*a*d^5*x)/((b*d^2*x^2 + a*d^2)
^2*a^2*sgn(b*x^2 + a)) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*x^2 +
 a)) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*x^2 + a)) - 5*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a^3*b^3*sgn(b*x^2 + a)) + 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^3*sgn(b*x^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.765 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=460

$$\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+$$

```
[Out] -21/64*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1
1/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)^2)^(1/2)+21/64*(b*x^2+a)*arctan(1+b
^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)/d^(1/2
)/((b*x^2+a)^2)^(1/2)-21/128*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)
-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*
x^2+a)^2)^(1/2)+21/128*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/
4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)
^2)^(1/2)+7/16*(d*x)^(1/2)/a^2/d/((b*x^2+a)^2)^(1/2)+1/4*(d*x)^(1/2)/a/d/(b
*x^2+a)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21(a+bx^2)\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21(a+bx^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (7*Sqrt[d*x])/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d*x]/(4*a*d
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4
)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*(a + b*x^2)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4)*S
qrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^
(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(64*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2))\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(7b(ab+b^2x^2))\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^2}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(21}{}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(21}{}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(21}{}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(21}{}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21}{}$$

$$= \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21}{32}$$

Mathematica [A]

time = 0.23, size = 181, normalized size = 0.39

$$\frac{\sqrt{x}\left(4a^{3/4}\sqrt[4]{b}\,\sqrt{x}\,(11a+7bx^2)-21\sqrt{2}\,(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+21\sqrt{2}\,(a+bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{64a^{11/4}\sqrt[4]{b}\,\sqrt{dx}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[x]*(4*a^(3/4)*b^(1/4)*Sqrt[x]*(11*a + 7*b*x^2) - 21*Sqrt[2]*(a + b*x^
2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 21*S
qrt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + S
qrt[b]*x)]))/(64*a^(11/4)*b^(1/4)*Sqrt[d*x]*(a + b*x^2)*Sqrt[(a + b*x^2)^2]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(294) = 588$.
time = 0.04, size = 638, normalized size = 1.39

method	result
default	$\frac{\left(21\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)b^2x^4+42\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)b^2x^4+42\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(21*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
)))*b^2*x^4+42*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*b^2*x^4+42*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2
)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*x^4+42*(a*d^2/b)^(1/4)*
2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b*x^2+84*(a*d^2/b)^(
1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
a*b*x^2+84*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1
/4))/(a*d^2/b)^(1/4))*a*b*x^2+21*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2)))*a^2+42*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2+42*(a*d^2/b)^(1/4)*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2+56*(d*x)
^(1/2)*a*b*x^2+88*(d*x)^(1/2)*a^2)/d*(b*x^2+a)/a^3/((b*x^2+a)^2)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*b*sqrt(d)*x^(5/2)/(a^3*b*d*x^2 + a^4*d + (a^2*b^2*d*x^2 + a^3*b*d)*x^2
) + 1/16*(15*b*x^(5/2) + 11*a*sqrt(x))/(a^2*b^2*sqrt(d)*x^4 + 2*a^3*b*sqrt(
d)*x^2 + a^4*sqrt(d)) - 11/128*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(
2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(b))) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))) + sqrt(2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*d) + integrate(1/
((a^2*b*sqrt(d)*x^2 + a^3*sqrt(d))*sqrt(x)), x)
```

Fricas [A]
time = 0.35, size = 298, normalized size = 0.65

$$\frac{84\left(a^{2}b^{2}dx^{4}+2\,a^{3}bdx^{2}+a^{4}d\right)\left(-\frac{1}{a^{11}bd^{2}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^{6}d^{2}\sqrt{-\frac{1}{a^{11}bd^{2}}}+dx}\,a^{8}bd\left(-\frac{1}{a^{11}bd^{2}}\right)^{\frac{3}{4}}-\sqrt{dx}\,a^{8}bd\left(-\frac{1}{a^{11}bd^{2}}\right)^{\frac{3}{4}}\right)+21\left(a^{2}b^{2}dx^{4}+2\,a^{3}bdx^{2}+a^{4}d\right)\left(-\frac{1}{a^{11}bd^{2}}\right)^{\frac{1}{4}}\log\left(a^{3}d\left(-\frac{1}{a^{11}bd^{2}}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-21\left(a^{2}b^{2}dx^{4}+2\,a^{3}bdx^{2}+a^{4}d\right)\left(-\frac{1}{a^{11}bd^{2}}\right)^{\frac{1}{4}}\log\left(-a^{3}d\left(-\frac{1}{a^{11}bd^{2}}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4\left(7\,bx^{2}+11\,a\right)\sqrt{dx}}{64\left(a^{2}b^{2}dx^{4}+2\,a^{3}bdx^{2}+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(84*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2))^(1/4)*ar
ctan(sqrt(a^6*d^2*sqrt(-1/(a^11*b*d^2)) + d*x)*a^8*b*d*(-1/(a^11*b*d^2))^(3
/4) - sqrt(d*x)*a^8*b*d*(-1/(a^11*b*d^2))^(3/4)) + 21*(a^2*b^2*d*x^4 + 2*a^
3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2))^(1/4)*log(a^3*d*(-1/(a^11*b*d^2))^(1/4
) + sqrt(d*x)) - 21*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2
))^(1/4)*log(-a^3*d*(-1/(a^11*b*d^2))^(1/4) + sqrt(d*x)) + 4*(7*b*x^2 + 11*
a)*sqrt(d*x))/(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A]
time = 3.82, size = 339, normalized size = 0.74

$$\frac{7\sqrt{dx}\,bd^3x^2+11\sqrt{dx}\,ad^3}{16\left(bd^2x^2+ad^2\right)^2a^2\mathrm{sgn}\left(bx^2+a\right)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3bd\mathrm{sgn}\left(bx^2+a\right)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3bd\mathrm{sgn}\left(bx^2+a\right)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^3bd\mathrm{sgn}\left(bx^2+a\right)}-\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^3bd\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(7*sqrt(d*x)*b*d^3*x^2 + 11*sqrt(d*x)*a*d^3)/((b*d^2*x^2 + a*d^2)^2*a^
2*sgn(b*x^2 + a)) + 21/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d*sgn(b*x^2 + a
)) + 21/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d*sgn(b*x^2 + a)) + 21/128*sq
rt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a^3*b*d*sgn(b*x^2 + a)) - 21/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d
*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d*sgn(b*x^2
+ a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d\,x}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.766 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=506

$$\frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad\sqrt{dx}\,\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\left(a+bx^2\right)}{16a^3d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 45/64*b^(1/4)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))/a^(13/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-45/64*b^(1/4)*(b*x^2+a)*arc
tan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/d^(3/2)*2^(1/2)
/((b*x^2+a)^2)^(1/2)-45/128*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*
d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/d^(3/2)*2^(1/2)/((b*x
^2+a)^2)^(1/2)+45/128*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^
2)^(1/2)+9/16/a^2/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+1/4/a/d/(b*x^2+a)/(d*x)
^(1/2)/((b*x^2+a)^2)^(1/2)-45/16*(b*x^2+a)/a^3/d/(d*x)^(1/2)/((b*x^2+a)^2)^
(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}+\frac{45\sqrt[4]{b}\,(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{b}\,(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{b}\,(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{b}\,(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45(a+bx^2)}{16a^3d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 9/(16*a^2*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*Sqrt[d*x]
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*(a + b*x^2))/(16*a^3*d*
Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*b^(1/4)*(a + b*x^2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d
^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d^(3
/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[
2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*b^(1/4)*(a + b*x
^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(64*Sqrt[2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{1}{(dx)^{3/2}(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad\sqrt{dx}\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(9b(ab+b^2x^2)\right)\int \frac{}{(dx)^{3/2}(}}{8a\sqrt{a^2+2abx^2+}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\,(a+bx^2)\sqrt{a^2+2abx^2}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\,(a+bx^2)\sqrt{a^2+2abx^2}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\,(a+bx^2)\sqrt{a^2+2abx^2}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\,(a+bx^2)\sqrt{a^2+2abx^2}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\,(a+bx^2)\sqrt{a^2+2abx^2}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\,(a+bx^2)\sqrt{a^2+2abx^2}}$$

$$= \frac{9}{16a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad\sqrt{dx}\,(a+bx^2)\sqrt{a^2+2abx^2}}$$

Mathematica [A]
time = 0.38, size = 193, normalized size = 0.38

$$\frac{x\left(-4\sqrt[4]{a}\,(32a^2+81abx^2+45b^2x^4)+45\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+45\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,(a+bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{64a^{13/4}(dx)^{3/2}\,(a+bx^2)\,\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
[Out] (x*(-4*a^(1/4)*(32*a^2 + 81*a*b*x^2 + 45*b^2*x^4) + 45*Sqrt[2]*b^(1/4)*Sqrt
[x]*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
```

```
t[x])] + 45*Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(64*a^(13/4)*(d*x)^(3/2)*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.07, size = 645, normalized size = 1.27

method	result
risch	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}}{a^{3}\sqrt{dx}\;d\left(b\,x^{2}+a\right)}+\frac{\left(-\frac{13 b^{2} \left(dx\right)^{\frac{7}{2}}}{16 a^{3} \left(d^{2} x^{2} b+a\,d^{2}\right)^{2}}-\frac{17 b \left(dx\right)^{\frac{3}{2}} d^{2}}{16 a^{2} \left(d^{2} x^{2} b+a\,d^{2}\right)^{2}}-\frac{45\sqrt{2}\;\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\;\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\;\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)}{128 a^{3}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-\frac{45\sqrt{2}}{}$
default	$-\frac{\left(45\sqrt{2}\;\ln\left(-\frac{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\;\sqrt{2}-dx-\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\;\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)\sqrt{dx}\;b^{2}x^{4}+90\sqrt{2}\;\arctan\left(\frac{\sqrt{2}\;\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)\sqrt{dx}\;b^{2}x^{4}+90}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128/d*(45*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^(1
/2)*b^2*x^4+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*(d*x)^(1/2)*b^2*x^4+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2
/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(1/2)*b^2*x^4+360*(a*d^2/b)^(1/4)*b^2*x^4
+90*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(
d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^(1/2)*a*b*x
^2+180*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*(d*x)^(1/2)*a*b*x^2+180*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/
4))/(a*d^2/b)^(1/4))*(d*x)^(1/2)*a*b*x^2+648*(a*d^2/b)^(1/4)*a*b*x^2+45*2^(
1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^(1/2)*a^2+90*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(1/2
)*a^2+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*(d*x)^(1/2)*a^2+256*(a*d^2/b)^(1/4)*a^2)*(b*x^2+a)/(a*d^2/b)^(1/4)/(d*x
)^(1/2)/a^3/((b*x^2+a)^2)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*b*x^(3/2)/(a^3*b*d^(3/2)*x^2 + a^4*d^(3/2) + (a^2*b^2*d^(3/2)*x^2 + a^
3*b*d^(3/2))*x^2) - 1/16*(13*b^2*x^(7/2) + 9*a*b*x^(3/2))/(a^3*b^2*d^(3/2)*
x^4 + 2*a^4*b*d^(3/2)*x^2 + a^5*d^(3/2)) - 13/128*b*(2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a
^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sq
rt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
 sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^3*d^(3/2)) + integrate(1/((a^2
*b*d^(3/2)*x^2 + a^3*d^(3/2))*x^(3/2)), x)
```

Fricas [A]
time = 0.37, size = 343, normalized size = 0.68

$$\frac{180\,(a^3b^2d^2x^5+2\,a^4bd^2x^3+a^5d^2x)\left(-\frac{b}{a^{13}d^6}\right)^{\frac14}\arctan\left(-\frac{91125\,\sqrt{dx}\,a^3bd\left(-\frac{b}{a^{13}d^6}\right)^{\frac14}-\sqrt{-8303765625\,a^7bd^4\sqrt{-\frac{b}{a^{13}d^6}}+8303765625\,b^2dx}\,a^3d\left(-\frac{b}{a^{13}d^6}\right)^{\frac14}}{91125\,b}\right)-45\,(a^3b^2d^2x^5+2\,a^4bd^2x^3+a^5d^2x)\left(-\frac{b}{a^{13}d^6}\right)^{\frac14}\log\left(91125\,a^{10}d^5\left(-\frac{b}{a^{13}d^6}\right)^{\frac34}+91125\,\sqrt{dx}\,b\right)+45\,(a^3b^2d^2x^5+2\,a^4bd^2x^3+a^5d^2x)\left(-\frac{b}{a^{13}d^6}\right)^{\frac14}\log\left(-91125\,a^{10}d^5\left(-\frac{b}{a^{13}d^6}\right)^{\frac34}+91125\,\sqrt{dx}\,b\right)-4\,(45\,b^2x^4+81\,abx^2+32\,a^2)\sqrt{dx}}{64\,(a^3b^2d^2x^5+2\,a^4bd^2x^3+a^5d^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(180*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(-b/(a^13*d^6))^(
1/4)*arctan(-1/91125*(91125*sqrt(d*x)*a^3*b*d*(-b/(a^13*d^6))^(1/4) - sqrt(
-8303765625*a^7*b*d^4*sqrt(-b/(a^13*d^6)) + 8303765625*b^2*d*x)*a^3*d*(-b/(
a^13*d^6))^(1/4))/b) - 45*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(
-b/(a^13*d^6))^(1/4)*log(91125*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(
d*x)*b) + 45*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(-b/(a^13*d^6)
)^(1/4)*log(-91125*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(d*x)*b) - 4*
(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2)*sqrt(d*x))/(a^3*b^2*d^2*x^5 + 2*a^4*b*d^
2*x^3 + a^5*d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A]
time = 3.93, size = 368, normalized size = 0.73

$$-\frac{\frac{256}{\sqrt{dx}\,a^3\operatorname{sgn}\left(bx^2+a\right)}+\frac{8\left(13\sqrt{dx}\,b^2d^3x^3+17\sqrt{dx}\,abd^3x\right)}{\left(bd^2x^2+ad^2\right)^2a^3\operatorname{sgn}\left(bx^2+a\right)}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2d^2\operatorname{sgn}\left(bx^2+a\right)}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2d^2\operatorname{sgn}\left(bx^2+a\right)}-\frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^2d^2\operatorname{sgn}\left(bx^2+a\right)}+\frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^2d^2\operatorname{sgn}\left(bx^2+a\right)}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*(256/(sqrt(d*x)*a^3*sgn(b*x^2 + a)) + 8*(13*sqrt(d*x)*b^2*d^3*x^3 +
17*sqrt(d*x)*a*b*d^3*x)/((b*d^2*x^2 + a*d^2)^2*a^3*sgn(b*x^2 + a)) + 90*sqr
t(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt
(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*d^2*sgn(b*x^2 + a)) + 90*sqrt(2)*(a*b^3*d^
2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2
/b)^(1/4))/(a^4*b^2*d^2*sgn(b*x^2 + a)) - 45*sqrt(2)*(a*b^3*d^2)^(3/4)*log(
d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^2*d^2*sgn(b
*x^2 + a)) + 45*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)
*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^2*d^2*sgn(b*x^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d\,x\right)}^{3/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.767 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=506

$$\frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77\left(a+bx^2\right)}{48a^3d(dx)^{3/2}\sqrt{a^2+2abx^2+\cdots}}$$

```
[Out] 11/16/a^2/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+1/4/a/d/(d*x)^(3/2)/(b*x^2+a)/(
(b*x^2+a)^2)^(1/2)-77/48*(b*x^2+a)/a^3/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+77
/64*b^(3/4)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))
/a^(15/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-77/64*b^(3/4)*(b*x^2+a)*arcta
n(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(15/4)/d^(5/2)*2^(1/2)/(
(b*x^2+a)^2)^(1/2)+77/128*b^(3/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^
(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/d^(5/2)*2^(1/2)/((b*x^2
+a)^2)^(1/2)-77/128*b^(3/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+
a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)
^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}+\frac{77b^{3/4}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77b^{3/4}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77b^{3/4}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77b^{3/4}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77(a+bx^2)}{48a^3d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 11/(16*a^2*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*(d*x)^
(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*(a + b*x^2))/(48*a
^3*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(
15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(15/
4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(6
4*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(64*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{1}{(dx)^{5/2}(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(11b(ab+b^2x^2)\right)\int \frac{}{(dx)^{5/2}}}{8a\sqrt{a^2+2abx^2+}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx}}$$

$$= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx}}$$

<u>Mathematica</u> [A]
time = 0.38, size = 193, normalized size = 0.38

$$\frac{x\left(-4a^{3/4}(32a^2+121abx^2+77b^2x^4)+231\sqrt{2}\,b^{3/4}x^{3/2}(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-231\sqrt{2}\,b^{3/4}x^{3/2}(a+bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{192a^{15/4}(dx)^{5/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (x*(-4*a^(3/4)*(32*a^2 + 121*a*b*x^2 + 77*b^2*x^4) + 231*Sqrt[2]*b^(3/4)*x^
(3/2)*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*S
```

```
qrt[x])] - 231*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(192*a^(15/4)*(d*x)^(5/2)*(a +
 b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(325) = 650$.

time = 0.07, size = 707, normalized size = 1.40

method	result
risch	$-\frac{2\sqrt{(b x^2+a)^2}}{3a^3x\sqrt{dx}\,d^2(b x^2+a)}+\frac{\left(-\frac{15b^2d(dx)^{\frac{5}{2}}}{16a^3\left(d^2x^2b+a\,d^2\right)^2}-\frac{19b\,d^3\sqrt{dx}}{16a^2\left(d^2x^2b+a\,d^2\right)^2}-\frac{77b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\cdots}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\cdots}}\right.}{128a^4d}\right.}{\cdots}$
default	$-\frac{\left(231\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)(dx)^{\frac{3}{2}}b^3x^4+462\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384/d^3*(231*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2)))*(d*x)^(3/2)*b^3*x^4+462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*b^3*x^4+462*(a*d^2
/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1
/4))*(d*x)^(3/2)*b^3*x^4+462*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a*d^2/b)^(1/2)))*(d*x)^(3/2)*a*b^2*x^2+924*(a*d^2/b)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*a*b
^2*x^2+924*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1
/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*a*b^2*x^2+231*(a*d^2/b)^(1/4)*2^(1/2)*ln(
(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^(3/2)*a^2*b+462*(a*d^2/b)^(
1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
(d*x)^(3/2)*a^2*b+462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(
a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*a^2*b+616*a*b^2*d^2*x^4+968*a^
2*b*d^2*x^2+256*a^3*d^2)*(b*x^2+a)/(d*x)^(3/2)/a^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/2*b^2*x^(5/2)/(a^4*b*d^(5/2)*x^2 + a^5*d^(5/2) + (a^3*b^2*d^(5/2)*x^2 + a
^4*b*d^(5/2))*x^2) - 2*b*integrate(1/((a^3*b*d^(5/2)*x^2 + a^4*d^(5/2))*sqr
t(x)), x) - 1/16*(23*b^2*x^(5/2) + 19*a*b*sqrt(x))/(a^3*b^2*d^(5/2)*x^4 + 2
*a^4*b*d^(5/2)*x^2 + a^5*d^(5/2)) + 19/128*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/a^(3/4))/(a^3*d^(5/2)) + integrate(1/((a^2*b*d^(5/2
)*x^2 + a^3*d^(5/2))*x^(5/2)), x)
```

Fricas [A]
time = 0.37, size = 367, normalized size = 0.73

$$-\frac{924\,(a^3b^2d^3x^6+2\,a^4bd^3x^4+a^5d^3x^2)\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{11}bd^7\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^8d^6\sqrt{-\frac{b^3}{a^{15}d^{10}}}+b^2dx}\,a^{11}d^7\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+231\,(a^3b^2d^3x^6+2\,a^4bd^3x^4+a^5d^3x^2)\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}\log\left(77\,a^4d^3\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}+77\,\sqrt{dx}\,b\right)-231\,(a^3b^2d^3x^6+2\,a^4bd^3x^4+a^5d^3x^2)\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}\log\left(-77\,a^4d^3\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}+77\,\sqrt{dx}\,b\right)+4\,(77\,b^2x^4+121\,abx^2+32\,a^2)\sqrt{dx}}{192\,(a^3b^2d^3x^6+2\,a^4bd^3x^4+a^5d^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/192*(924*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d
^10))^(1/4)*arctan(-(sqrt(d*x)*a^11*b*d^7*(-b^3/(a^15*d^10))^(3/4) - sqrt(a
^8*d^6*sqrt(-b^3/(a^15*d^10)) + b^2*d*x)*a^11*d^7*(-b^3/(a^15*d^10))^(3/4))
/b^3) + 231*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d
^10))^(1/4)*log(77*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) - 231
*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d^10))^(1/4)
*log(-77*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) + 4*(77*b^2*x^4
 + 121*a*b*x^2 + 32*a^2)*sqrt(d*x))/(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^
5*d^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A]
time = 3.71, size = 359, normalized size = 0.71

$$-\frac{15\,\sqrt{dx}\,b^2d^2x^2+19\,\sqrt{dx}\,abd^2}{16\,(bd^2x^2+ad^2)^2a^3d\mathrm{sgn}\,(bx^2+a)}-\frac{77\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4d^3\mathrm{sgn}\,(bx^2+a)}-\frac{77\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4d^3\mathrm{sgn}\,(bx^2+a)}-\frac{77\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^4d^3\mathrm{sgn}\,(bx^2+a)}+\frac{77\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^4d^3\mathrm{sgn}\,(bx^2+a)}-\frac{2}{3\,\sqrt{dx}\,a^3d^2x\mathrm{sgn}\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*(15*sqrt(d*x)*b^2*d^2*x^2 + 19*sqrt(d*x)*a*b*d^2)/((b*d^2*x^2 + a*d^2
)^2*a^3*d*sgn(b*x^2 + a)) - 77/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt
(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*d^3*sgn(b
*x^2 + a)) - 77/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*d^3*sgn(b*x^2 + a)) - 7
7/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(a^4*d^3*sgn(b*x^2 + a)) + 77/128*sqrt(2)*(a*b^3*d^2)^(1/
4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*d^3*sg
n(b*x^2 + a)) - 2/3/(sqrt(d*x)*a^3*d^2*x*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d\,x\right)}^{5/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.768 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=553

$$\frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117(a+bx^2)}{80a^3d(dx)^{5/2}\sqrt{a^2+2abx^2+}$$

```
[Out] 13/16/a^2/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)+1/4/a/d/(d*x)^(5/2)/(b*x^2+a)/(
(b*x^2+a)^2)^(1/2)-117/80*(b*x^2+a)/a^3/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)-1
17/64*b^(5/4)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))/a^(17/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+117/64*b^(5/4)*(b*x^2+a)*ar
ctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(17/4)/d^(7/2)*2^(1/2
)/((b*x^2+a)^2)^(1/2)+117/128*b^(5/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2
)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(17/4)/d^(7/2)*2^(1/2)/((b
*x^2+a)^2)^(1/2)-117/128*b^(5/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(17/4)/d^(7/2)*2^(1/2)/((b*x^2+
a)^2)^(1/2)+117/16*b*(b*x^2+a)/a^4/d^3/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117b^{5/4}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\,a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\,a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117b^{5/4}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{64\sqrt{2}\,a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b(a+bx^2)}{16a^4d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117(a+bx^2)}{80a^3d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 13/(16*a^2*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d*(d*x)^
(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*(a + b*x^2))/(80*
a^3*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b*(a + b*x^2))/(1
6*a^4*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^(5/4)*(a + b*
x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]
*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a
^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^2)
*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x
]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^
(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{1}{(dx)^{7/2}(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b(ab+b^2x^2)\right)\int \frac{}{(dx)^{7}}}{8a\sqrt{a^2+2abx^2}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2a}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2a}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2a}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2a}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2a}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2a}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2a}}$$

$$= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2a}}$$

Mathematica [A]
time = 0.38, size = 204, normalized size = 0.37

$$\frac{x\left(4\sqrt[4]{a}\left(-32a^3+416a^2bx^2+1053ab^2x^4+585b^3x^6\right)-585\sqrt{2}\,b^{5/4}x^{5/2}(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-585\sqrt{2}\,b^{5/4}x^{5/2}(a+bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{320a^{17/4}(dx)^{7/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (x*(4*a^(1/4)*(-32*a^3 + 416*a^2*b*x^2 + 1053*a*b^2*x^4 + 585*b^3*x^6) - 58
5*Sqrt[2]*b^(5/4)*x^(5/2)*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 585*Sqrt[2]*b^(5/4)*x^(5/2)*(a + b*x^2)^2*Ar
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(320*a^(17
/4)*(d*x)^(7/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.07, size = 687, normalized size = 1.24

method	result
risch	$-\frac{2(-15bx^2+a)\sqrt{(bx^2+a)^2}}{5a^4\sqrt{dx}\,x^2d^3(bx^2+a)}+\frac{\left(\frac{21b^3(dx)^{\frac{7}{2}}}{16a^4\left(d^2x^2b+a\,d^2\right)^2}+\frac{25b^2(dx)^{\frac{3}{2}}d^2}{16a^3\left(d^2x^2b+a\,d^2\right)^2}+\frac{117b\sqrt{2}\,\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{128a^4\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.}{d}$
default	$-\frac{\left(-585\sqrt{2}\,\ln\left(-\frac{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)(dx)^{\frac{5}{2}}b^3x^4-1170\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)(dx)^{\frac{5}{2}}b^3x^4-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/640/d^3*(-585*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^
2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x
)^(5/2)*b^3*x^4-1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(
a*d^2/b)^(1/4))*(d*x)^(5/2)*b^3*x^4-1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2
)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*b^3*x^4-4680*(a*d^2/b)^(1/4
)*b^3*d^2*x^6-1170*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*
d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d
*x)^(5/2)*a*b^2*x^2-2340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*a*b^2*x^2-2340*2^(1/2)*arctan((2^(1/2)*(d*x
)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*a*b^2*x^2-8424*(a*d^2
/b)^(1/4)*a*b^2*d^2*x^4-585*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*(d*x)^(5/2)*a^2*b-1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*a^2*b-1170*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*a^2*b-3328*(a*d^2/b)
^(1/4)*a^2*b*d^2*x^2+256*(a*d^2/b)^(1/4)*a^3*d^2)*(b*x^2+a)/(a*d^2/b)^(1/4)
/(d*x)^(5/2)/a^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/2*b^2*x^(3/2)/(a^4*b*d^(7/2)*x^2 + a^5*d^(7/2) + (a^3*b^2*d^(7/2)*x^2 + a
^4*b*d^(7/2))*x^2) - 2*b*integrate(1/((a^3*b*d^(7/2)*x^2 + a^4*d^(7/2))*x^(
3/2)), x) + 1/16*(21*b^3*x^(7/2) + 17*a*b^2*x^(3/2))/(a^4*b^2*d^(7/2)*x^4 +
 2*a^5*b*d^(7/2)*x^2 + a^6*d^(7/2)) + 21/128*b^2*(2*sqrt(2)*arctan(1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1
/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(
b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sq
rt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^4*d^(7/2)) + integrate(1/((a^2*b*
d^(7/2)*x^2 + a^3*d^(7/2))*x^(7/2)), x)
```

Fricas [A]
time = 0.37, size = 390, normalized size = 0.71

$$-\frac{2340\,(a^4b^2d^4x^7+2\,a^5bd^4x^5+a^6d^4x^3)\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{1601613\,\sqrt{dx}\,a^4b^4d^3\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-2565164201769\,a^9b^5d^8\sqrt{-\frac{b^5}{a^{17}d^{14}}}+2565164201769\,b^8dx}\,a^4d^3\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}}{1601613\,b^5}\right)-585\,(a^4b^2d^4x^7+2\,a^5bd^4x^5+a^6d^4x^3)\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}\log\left(1601613\,a^{13}d^{11}\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{3}{4}}+1601613\,\sqrt{dx}\,b^4\right)+585\,(a^4b^2d^4x^7+2\,a^5bd^4x^5+a^6d^4x^3)\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}\log\left(-1601613\,a^{13}d^{11}\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{3}{4}}+1601613\,\sqrt{dx}\,b^4\right)-4\,(585\,b^3x^6+1053\,ab^2x^4+416\,a^2bx^2-32\,a^3)\sqrt{dx}}{320\,(a^4b^2d^4x^7+2\,a^5bd^4x^5+a^6d^4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/320*(2340*(a^4*b^2*d^4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*
d^14))^(1/4)*arctan(-1/1601613*(1601613*sqrt(d*x)*a^4*b^4*d^3*(-b^5/(a^17*d
^14))^(1/4) - sqrt(-2565164201769*a^9*b^5*d^8*sqrt(-b^5/(a^17*d^14)) + 2565
164201769*b^8*d*x)*a^4*d^3*(-b^5/(a^17*d^14))^(1/4))/b^5) - 585*(a^4*b^2*d^
4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*d^14))^(1/4)*log(1601613
*a^13*d^11*(-b^5/(a^17*d^14))^(3/4) + 1601613*sqrt(d*x)*b^4) + 585*(a^4*b^2
*d^4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*d^14))^(1/4)*log(-160
1613*a^13*d^11*(-b^5/(a^17*d^14))^(3/4) + 1601613*sqrt(d*x)*b^4) - 4*(585*b
^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^3)*sqrt(d*x))/(a^4*b^2*d^4*x
^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{7}{2}}\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(7/2)*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A]
time = 4.08, size = 390, normalized size = 0.71

$$\frac{21\sqrt{dx}\,b^3d^3x^3+25\sqrt{dx}\,ab^2d^3x}{16\left(bd^2x^2+ad^2\right)^2a^4d^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5bd^5\mathrm{sgn}\left(bx^2+a\right)}+\frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5bd^5\mathrm{sgn}\left(bx^2+a\right)}-\frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^5bd^5\mathrm{sgn}\left(bx^2+a\right)}+\frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^5bd^5\mathrm{sgn}\left(bx^2+a\right)}+\frac{2\left(15\,bd^2x^2-ad^2\right)}{5\sqrt{dx}\,a^4d^5x^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(21*sqrt(d*x)*b^3*d^3*x^3 + 25*sqrt(d*x)*a*b^2*d^3*x)/((b*d^2*x^2 + a*
d^2)^2*a^4*d^3*sgn(b*x^2 + a)) + 117/64*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d
^5*sgn(b*x^2 + a)) + 117/64*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d^5*sgn(b*x^
2 + a)) - 117/128*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d^5*sgn(b*x^2 + a)) + 117/128*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a^5*b*d^5*sgn(b*x^2 + a)) + 2/5*(15*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^4*
d^5*x^2*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d\,x\right)}^{7/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.769 $\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^{5/2}} dx$

Optimal. Leaf size=647

$$-\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-$$

```
[Out] -1547/1024*d^7*(d*x)^(9/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(21/2)/b/(b*
x^2+a)^3/((b*x^2+a)^2)^(1/2)-7/32*d^3*(d*x)^(17/2)/b^2/(b*x^2+a)^2/((b*x^2+
a)^2)^(1/2)-119/256*d^5*(d*x)^(13/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1392
3/5120*d^9*(d*x)^(5/2)*(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)-13923/4096*a^(5/4)
*d^(23/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b
^(25/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+13923/4096*a^(5/4)*d^(23/2)*(b*x^2+a)*a
rctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(25/4)*2^(1/2)/((b*x
^2+a)^2)^(1/2)-13923/8192*a^(5/4)*d^(23/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b
^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(25/4)*2^(1/2)/((b*x^
2+a)^2)^(1/2)+13923/8192*a^(5/4)*d^(23/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^
(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(25/4)*2^(1/2)/((b*x^2
+a)^2)^(1/2)-13923/1024*a*d^11*(b*x^2+a)*(d*x)^(1/2)/b^6/((b*x^2+a)^2)^(1/2
)
```

Rubi [A]
time = 0.33, antiderivative size = 647, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923ad^{11}\sqrt{dx}\left(a+bx^2\right)}{1024b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923d^9(dx)^{5/2}\left(a+bx^2\right)}{5120b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{119d^5(dx)^{13/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923a^{5/4}d^{23/2}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923a^{5/4}d^{23/2}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923a^{5/4}d^{23/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923a^{5/4}d^{23/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-1547*d^7*(d*x)^(9/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*
x)^(21/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7*d^3*(d*
x)^(17/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (119*d^
5*(d*x)^(13/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13
923*a*d^11*Sqrt[d*x]*(a + b*x^2))/(1024*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (13923*d^9*(d*x)^(5/2)*(a + b*x^2))/(5120*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(25/
```

```
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(4096*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{(dx)^{23/2}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21b^2d^2(ab+b^2x^2)\right)\int \frac{(dx)^{19/2}}{(ab+b^2x^2)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} \ldots$$

$$= -\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} \ldots$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\,(a \ldots}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\,(a \ldots}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\,(a \ldots}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\,(a \ldots}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\,(a \ldots}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\,(a \ldots}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\,(a \ldots}$$

$$= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\ldots}{32b^2\,(a \ldots}$$

Mathematica [A]

time = 0.73, size = 229, normalized size = 0.35

$$\frac{d^{11}\sqrt{dx}\left(4\sqrt[4]{b}\,\sqrt{x}\,(-69615a^5-264537a^4bx^2-369733a^3b^2x^4-220507a^2b^3x^6-43008ab^4x^8+2048b^5x^{10})+69615\sqrt{2}\,a^{5/4}(a+bx^2)^4\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+69615\sqrt{2}\,a^{5/4}(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{20480b^{25/4}\sqrt{x}\,(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (d^11*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(-69615*a^5 - 264537*a^4*b*x^2 - 369733*
a^3*b^2*x^4 - 220507*a^2*b^3*x^6 - 43008*a*b^4*x^8 + 2048*b^5*x^10) + 69615
*Sqrt[2]*a^(5/4)*(a + b*x^2)^4*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x])] + 69615*Sqrt[2]*a^(5/4)*(a + b*x^2)^4*ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(20480*b^(25/4)*Sqrt[x]*
(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1286 vs. $2(421) = 842$.
time = 0.09, size = 1287, normalized size = 1.99

method	result
risch	$-\frac{2(-b\,x^2+25a)x\,d^{12}\sqrt{(b\,x^2+a)^2}}{5b^6\sqrt{dx}\,(b\,x^2+a)}+\frac{\left(-\frac{3683a^5d^7\sqrt{dx}}{1024b^6\left(d^2x^2b+a\,d^2\right)^4}-\frac{12357a^4d^5(dx)^{\frac{5}{2}}}{1024b^5\left(d^2x^2b+a\,d^2\right)^4}-\frac{14145a^3d^3(dx)^{\frac{9}{2}}}{1024b^4\left(d^2x^2b+a\,d^2\right)^4}-\frac{5599a^2d(dx}{1024b^3\left(d^2x^2b-\right.}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/40960*(-65536*(d*x)^(5/2)*a*b^4*d^4*x^6+409600*(d*x)^(1/2)*a*b^4*d^6*x^8
-98304*(d*x)^(5/2)*a^2*b^3*d^4*x^4+1638400*(d*x)^(1/2)*a^2*b^3*d^6*x^6-6553
6*(d*x)^(5/2)*a^3*b^2*d^4*x^2+2457600*(d*x)^(1/2)*a^3*b^2*d^6*x^4+1638400*(
d*x)^(1/2)*a^4*b*d^6*x^2-69615*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2)))*a^5*d^6-139230*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^5*d^6-139230*(a*d^2/b)
^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*a^5*d^6-16384*(d*x)^(5/2)*b^5*d^4*x^8+565800*(d*x)^(9/2)*a^3*b^2*d^2+4778
96*(d*x)^(5/2)*a^4*b*d^4-139230*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^4*d^6*x^8-139230*(a*d^2/b)^(
1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
a*b^4*d^6*x^8-278460*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^
```

```
(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2)))*a^2*b^3*d^6*x^6-556920*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/
2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^3*d^6*x^6-556920*(a*
d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)
^(1/4))*a^2*b^3*d^6*x^6-417690*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2)))*a^3*b^2*d^6*x^4-835380*(a*d^2/b)^(1/4)*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b^2*d^6*x^4-
835380*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*a^3*b^2*d^6*x^4-278460*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*b*d^6*x^2-556920*(a*d^2/b)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*b*d^
6*x^2-556920*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^
(1/4))/(a*d^2/b)^(1/4))*a^4*b*d^6*x^2-69615*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x
+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^4*d^6*x^8+223960*(d*x)^(13/2)*a^2
*b^3+556920*(d*x)^(1/2)*a^5*d^6)*d^5*(b*x^2+a)/b^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -4*a*d^(23/2)*integrate(x^(3/2)/(b^6*x^2 + a*b^5), x) + d^(23/2)*integrate(
x^(7/2)/(b^5*x^2 + a*b^4), x) + 3683/8192*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/s
qrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/
4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)
) + sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(
a))/b^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/b^(1/4))*d^(23/2)/b^6 - 1/3072*(6925*a^2*b^3*d^(23/2)*x^(13/
2) + 23395*a^3*b^2*d^(23/2)*x^(9/2) + 27135*a^4*b*d^(23/2)*x^(5/2) + 11049*
a^5*d^(23/2)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x
^2 + a^4*b^6) - 1/192*((617*a^2*b^4*d^(23/2)*x^5 + 1386*a^3*b^3*d^(23/2)*x^
3 + 801*a^4*b^2*d^(23/2)*x)*x^(11/2) + 2*(519*a^3*b^3*d^(23/2)*x^5 + 1182*a
^4*b^2*d^(23/2)*x^3 + 695*a^5*b*d^(23/2)*x)*x^(7/2) + (453*a^4*b^2*d^(23/2)
*x^5 + 1042*a^5*b*d^(23/2)*x^3 + 621*a^6*d^(23/2)*x)*x^(3/2))/(a^3*b^8*x^6
+ 3*a^4*b^7*x^4 + 3*a^5*b^6*x^2 + a^6*b^5 + (b^11*x^6 + 3*a*b^10*x^4 + 3*a^
2*b^9*x^2 + a^3*b^8)*x^6 + 3*(a*b^10*x^6 + 3*a^2*b^9*x^4 + 3*a^3*b^8*x^2 +
a^4*b^7)*x^4 + 3*(a^2*b^9*x^6 + 3*a^3*b^8*x^4 + 3*a^4*b^7*x^2 + a^5*b^6)*x^
2)
```

Fricas [A]
time = 0.37, size = 457, normalized size = 0.71

$$\frac{278460\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6\right)\arctan\left(-\frac{\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{3}{4}}\sqrt{dx}\,ab^{19}d^{11}-\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{3}{4}}\sqrt{a^2d^{23}x+\sqrt{-\frac{a^5d^{46}}{b^{25}}}\,b^{12}}\,b^{19}}{a^5d^{46}}\right)+69615\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6\right)\log\left(13923\sqrt{dx}\,ad^{11}+13923\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}}b^6\right)-69615\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}}\left(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6\right)\log\left(13923\sqrt{dx}\,ad^{11}-13923\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}}b^6\right)+4\left(2048\,b^5d^{11}x^{10}-43008\,ab^4d^{11}x^8-220507\,a^2b^3d^{11}x^6-369733\,a^3b^2d^{11}x^4-264537\,a^4bd^{11}x^2-69615\,a^5d^{11}\right)\sqrt{dx}}{20480\left(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(278460*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*
x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*arctan(-((-a^5*d^46/b^25)^(3/4)*sqrt(d*x)*a*
b^19*d^11 - (-a^5*d^46/b^25)^(3/4)*sqrt(a^2*d^23*x + sqrt(-a^5*d^46/b^25)*b
^12)*b^19)/(a^5*d^46)) + 69615*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x
^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*log(13923*sqrt(d*x)*a*d^11 +
13923*(-a^5*d^46/b^25)^(1/4)*b^6) - 69615*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8
+ 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*log(13923*sqrt(d*x
)*a*d^11 - 13923*(-a^5*d^46/b^25)^(1/4)*b^6) + 4*(2048*b^5*d^11*x^10 - 4300
8*a*b^4*d^11*x^8 - 220507*a^2*b^3*d^11*x^6 - 369733*a^3*b^2*d^11*x^4 - 2645
37*a^4*b*d^11*x^2 - 69615*a^5*d^11)*sqrt(d*x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*
a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.42, size = 415, normalized size = 0.64

$$\frac{1}{40960}\,d^{11}\left(\frac{139230\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7\mathrm{sgn}\left(bx^2+a\right)}+\frac{139230\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7\mathrm{sgn}\left(bx^2+a\right)}+\frac{69615\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^7\mathrm{sgn}\left(bx^2+a\right)}-\frac{69615\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}a\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^7\mathrm{sgn}\left(bx^2+a\right)}-\frac{40\left(5599\sqrt{dx}\,a^2b^3d^8x^6+14145\sqrt{dx}\,a^3b^2d^8x^4+12357\sqrt{dx}\,a^4bd^8x^2+3683\sqrt{dx}\,a^5d^8\right)}{\left(bd^2x^2+ad^2\right)^4b^6\mathrm{sgn}\left(bx^2+a\right)}+\frac{16384\left(\sqrt{dx}\,b^{20}d^{10}x^2-25\sqrt{dx}\,ab^{19}d^{10}\right)}{b^{25}d^{10}\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/40960*d^11*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*sgn(b*x^2 + a)) + 13
9230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*sgn(b*x^2 + a)) + 69615*sqrt(2)*(a*
b^3*d^2)^(1/4)*a*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(b^7*sgn(b*x^2 + a)) - 69615*sqrt(2)*(a*b^3*d^2)^(1/4)*a*log(d*x - sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^7*sgn(b*x^2 + a)) - 40*(55
```

```
99*sqrt(d*x)*a^2*b^3*d^8*x^6 + 14145*sqrt(d*x)*a^3*b^2*d^8*x^4 + 12357*sqrt
(d*x)*a^4*b*d^8*x^2 + 3683*sqrt(d*x)*a^5*d^8)/((b*d^2*x^2 + a*d^2)^4*b^6*sg
n(b*x^2 + a)) + 16384*(sqrt(d*x)*b^20*d^10*x^2 - 25*sqrt(d*x)*a*b^19*d^10)/
(b^25*d^10*sgn(b*x^2 + a)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{23/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.770 $\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=600

$$-\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-$$

```
[Out] -1045/1024*d^7*(d*x)^(7/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(19/2)/b/(b*
x^2+a)^3/((b*x^2+a)^2)^(1/2)-19/96*d^3*(d*x)^(15/2)/b^2/(b*x^2+a)^2/((b*x^2
+a)^2)^(1/2)-95/256*d^5*(d*x)^(11/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+7315
/3072*d^9*(d*x)^(3/2)*(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)+7315/4096*a^(3/4)*d
^(21/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(
23/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-7315/4096*a^(3/4)*d^(21/2)*(b*x^2+a)*arct
an(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(23/4)*2^(1/2)/((b*x^2+
a)^2)^(1/2)-7315/8192*a^(3/4)*d^(21/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/
2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(23/4)*2^(1/2)/((b*x^2+a)
^2)^(1/2)+7315/8192*a^(3/4)*d^(21/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)
*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(23/4)*2^(1/2)/((b*x^2+a)^2
)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315d^9(dx)^{3/2}\left(a+bx^2\right)}{3072b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{95d^5(dx)^{11/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315a^{3/4}d^{21/2}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\,b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315a^{3/4}d^{21/2}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\,b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315a^{3/4}d^{21/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\,b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315a^{3/4}d^{21/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\,b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-1045*d^7*(d*x)^(7/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*
x)^(19/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (19*d^3*(d
*x)^(15/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (95*d^
5*(d*x)^(11/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (73
15*d^9*(d*x)^(3/2)*(a + b*x^2))/(3072*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x
])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(23/4)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) + (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*S
```

```
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt
[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{(dx)^{21/2}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(19b^2d^2(ab+b^2x^2)\right)\int \frac{(dx)^{17/2}}{(ab+b^2x^2)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a\cdots\right.}$$

$$= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\cdots}{96b^2\left(a\cdots\right.}$$

Mathematica [A]
time = 0.67, size = 218, normalized size = 0.36

$$\frac{d^9(dx)^{3/2}\left(4b^{3/4}x^{3/2}(7315a^4+26125a^3bx^2+33345a^2b^2x^4+16967ab^3x^6+2048b^4x^8)-21945\sqrt{2}\,a^{3/4}(a+bx^2)^4\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+21945\sqrt{2}\,a^{3/4}(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{12288b^{23/4}x^{3/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (d^9*(d*x)^(3/2)*(4*b^(3/4)*x^(3/2)*(7315*a^4 + 26125*a^3*b*x^2 + 33345*a^2
*b^2*x^4 + 16967*a*b^3*x^6 + 2048*b^4*x^8) - 21945*Sqrt[2]*a^(3/4)*(a + b*x
^2)^4*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 21
945*Sqrt[2]*a^(3/4)*(a + b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
/(Sqrt[a] + Sqrt[b]*x)]))/(12288*b^(23/4)*x^(3/2)*(a + b*x^2)^3*Sqrt[(a + b
*x^2)^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1170 vs. $2(389) = 778$.
time = 0.08, size = 1171, normalized size = 1.95

method	result
risch	$\frac{2x^2d^{11}\sqrt{\left(b\,x^2+a\right)^2}}{3b^5\sqrt{dx}\,\left(b\,x^2+a\right)} + \frac{\left(\frac{5267a^4d^6(dx)^{\frac{3}{2}}}{3072b^5\left(d^2x^2b+a\,d^2\right)^4}+\frac{17933a^3d^4(dx)^{\frac{7}{2}}}{3072b^4\left(d^2x^2b+a\,d^2\right)^4}+\frac{7019a^2d^2(dx)^{\frac{11}{2}}}{1024b^3\left(d^2x^2b+a\,d^2\right)^4}+\frac{2925a(dx)^{\frac{15}{2}}}{1024b^2\left(d^2x^2b+a\,d^2\right)^4}-\frac{7315a\sqrt{\ldots}}{\ldots}\right.}{\ldots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24576*(16384*(d*x)^(3/2)*(a*d^2/b)^(1/4)*b^5*d^6*x^8-21945*2^(1/2)*ln(-((
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^4*d^8*x^8-43890*2^(1/2)*arctan(
(2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^4*d^8*x^8-43890*
2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^4
*d^8*x^8+70200*(d*x)^(15/2)*(a*d^2/b)^(1/4)*a*b^4+65536*(d*x)^(3/2)*(a*d^2/
b)^(1/4)*a*b^4*d^6*x^6-87780*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2)))*a^2*b^3*d^8*x^6-175560*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^3*d^8*x^6-175560*2^(1/2)*arctan((2^(1/2)*(d
*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^3*d^8*x^6+168456*(d*x)^(1
1/2)*(a*d^2/b)^(1/4)*a^2*b^3*d^2+98304*(d*x)^(3/2)*(a*d^2/b)^(1/4)*a^2*b^3*
d^6*x^4-131670*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/
b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b^
2*d^8*x^4-263340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*a^3*b^2*d^8*x^4-263340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b^2*d^8*x^4+143464*(d*x)^(7/2)*(a*d^2/b)^
```

```
(1/4)*a^3*b^2*d^4+65536*(d*x)^(3/2)*(a*d^2/b)^(1/4)*a^3*b^2*d^6*x^2-87780*2
^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*b*d^8*x^2-175560*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*b*
d^8*x^2-175560*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*a^4*b*d^8*x^2+58520*(d*x)^(3/2)*(a*d^2/b)^(1/4)*a^4*b*d^6-21945*2
^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^5*d^8-43890*2^(1/2)*
arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^5*d^8-43890
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^5*
d^8)*d^3*(b*x^2+a)/(a*d^2/b)^(1/4)/b^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -4*a*d^(21/2)*integrate(sqrt(x)/(b^6*x^2 + a*b^5), x) + d^(21/2)*integrate(
x^(5/2)/(b^5*x^2 + a*b^4), x) + 2925/8192*a*d^(21/2)*(2*sqrt(2)*arctan(1/2*
sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*
a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*s
qrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^5 + 1/3072*(8775*a*b^3*d^(21/2
)*x^(15/2) + 29649*a^2*b^2*d^(21/2)*x^(11/2) + 34285*a^3*b*d^(21/2)*x^(7/2)
 + 13795*a^4*d^(21/2)*x^(3/2))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a
^3*b^6*x^2 + a^4*b^5) - 1/192*((537*a^2*b^4*d^(21/2)*x^5 + 1210*a^3*b^3*d^(
21/2)*x^3 + 705*a^4*b^2*d^(21/2)*x)*x^(9/2) + 2*(443*a^3*b^3*d^(21/2)*x^5 +
 1014*a^4*b^2*d^(21/2)*x^3 + 603*a^5*b*d^(21/2)*x)*x^(5/2) + (381*a^4*b^2*d
^(21/2)*x^5 + 882*a^5*b*d^(21/2)*x^3 + 533*a^6*d^(21/2)*x)*sqrt(x))/(a^3*b^
8*x^6 + 3*a^4*b^7*x^4 + 3*a^5*b^6*x^2 + a^6*b^5 + (b^11*x^6 + 3*a*b^10*x^4
+ 3*a^2*b^9*x^2 + a^3*b^8)*x^6 + 3*(a*b^10*x^6 + 3*a^2*b^9*x^4 + 3*a^3*b^8*
x^2 + a^4*b^7)*x^4 + 3*(a^2*b^9*x^6 + 3*a^3*b^8*x^4 + 3*a^4*b^7*x^2 + a^5*b
^6)*x^2)
```

Fricas [A]
time = 0.36, size = 457, normalized size = 0.76

$$\frac{87780 \left(-\frac{a^3 d^{42}}{b^{23}}\right)^{\frac{1}{4}} (b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5) \arctan\left(-\frac{\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}} \sqrt{dx}\, a^2b^6d^{31} - \sqrt{a^4d^{63}x - \sqrt{-\frac{a^3d^{42}}{b^{23}}}\, a^3b^{11}d^{42}}\, \left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}} b^6}{a^3d^{42}}\right) - 21945 \left(-\frac{a^3 d^{42}}{b^{23}}\right)^{\frac{1}{4}} (b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5) \log\left(391419980875 \sqrt{dx}\, a^2d^{31} + 391419980875 \left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{3}{4}} b^{17}\right) + 21945 \left(-\frac{a^3 d^{42}}{b^{23}}\right)^{\frac{1}{4}} (b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5) \log\left(391419980875 \sqrt{dx}\, a^2d^{31} - 391419980875 \left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{3}{4}} b^{17}\right) + 4\,(2048\,b^4d^{10}x^9 + 16967\,ab^3d^{10}x^7 + 33345\,a^2b^2d^{10}x^5 + 26125\,a^3bd^{10}x^3 + 7315\,a^4d^{10}x)\sqrt{dx}}{12288\,(b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")

[Out] 1/12288*(87780*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*arctan(-((-a^3*d^42/b^23)^(1/4)*sqrt(d*x)*a^2*b^6*d^31 - sqrt(a^4*d^63*x - sqrt(-a^3*d^42/b^23)*a^3*b^11*d^42)*(-a^3*d^42/b^23)^(1/4)*b^6)/(a^3*d^42)) - 21945*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*log(391419980875*sqrt(d*x)*a^2*d^31 + 391419980875*(-a^3*d^42/b^23)^(3/4)*b^17) + 21945*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*log(391419980875*sqrt(d*x)*a^2*d^31 - 391419980875*(-a^3*d^42/b^23)^(3/4)*b^17) + 4*(2048*b^4*d^10*x^9 + 16967*a*b^3*d^10*x^7 + 33345*a^2*b^2*d^10*x^5 + 26125*a^3*b*d^10*x^3 + 7315*a^4*d^10*x)*sqrt(d*x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)

[Out] Timed out

Giac [**A**]
time = 4.14, size = 395, normalized size = 0.66

$$\frac{1}{24576}d^{10}\left(\frac{16384\sqrt{dx}\,x}{b^5\mathrm{sgn}\left(bx^2+a\right)}-\frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8d\mathrm{sgn}\left(bx^2+a\right)}-\frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8d\mathrm{sgn}\left(bx^2+a\right)}+\frac{21945\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^8d\mathrm{sgn}\left(bx^2+a\right)}-\frac{21945\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^8d\mathrm{sgn}\left(bx^2+a\right)}+\frac{8\left(8775\sqrt{dx}\,ab^3d^8x^7+21057\sqrt{dx}\,a^2b^2d^8x^5+17933\sqrt{dx}\,a^3bd^8x^3+5267\sqrt{dx}\,a^4d^8x\right)}{\left(bd^2x^2+ad^2\right)^4b^5\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")

[Out] 1/24576*d^10*(16384*sqrt(d*x)*x/(b^5*sgn(b*x^2 + a)) - 43890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^8*d*sgn(b*x^2 + a)) - 43890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^8*d*sgn(b*x^2 + a)) + 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^8*d*sgn(b*x^2 + a)) - 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^8*d*sgn(b*x^2 + a)) + 8*(8775*sqrt(d*x)*a*b^3*d^8*x^7 + 21057*sqrt(d*x)*a^2*b^2*d^8*x^5 + 17933*sqrt(d*x)*a^3*b*d^8*x^3 + 5267*sqrt(d*x)*a^4*d^8*x)/((b*d^2*x^2 + a*d^2)^4*b^5*sgn(b*x^2 + a)))

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x\right)}^{21/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.771 $\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=600

$$-\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-$$

```
[Out] -663/1024*d^7*(d*x)^(5/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(17/2)/b/(b*x
^2+a)^3/((b*x^2+a)^2)^(1/2)-17/96*d^3*(d*x)^(13/2)/b^2/(b*x^2+a)^2/((b*x^2+
a)^2)^(1/2)-221/768*d^5*(d*x)^(9/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+3315/
4096*a^(1/4)*d^(19/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4
)/d^(1/2))/b^(21/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-3315/4096*a^(1/4)*d^(19/2)*
(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/b^(21/4)*2^
(1/2)/((b*x^2+a)^2)^(1/2)+3315/8192*a^(1/4)*d^(19/2)*(b*x^2+a)*ln(a^(1/2)*d
^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(21/4)*2^(1
/2)/((b*x^2+a)^2)^(1/2)-3315/8192*a^(1/4)*d^(19/2)*(b*x^2+a)*ln(a^(1/2)*d^(
1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/b^(21/4)*2^(1/2
)/((b*x^2+a)^2)^(1/2)+3315/1024*d^9*(b*x^2+a)*(d*x)^(1/2)/b^5/((b*x^2+a)^2)
^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1126, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{3315\sqrt[4]{a}\,d^{19/2}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\,b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{a}\,d^{19/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\,b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{a}\,d^{19/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\,b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{a}\,d^{19/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\,b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315d^9\sqrt{dx}\,(a+bx^2)}{1024b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{221d^5(dx)^{9/2}}{768b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-663*d^7*(d*x)^(5/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(17/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (17*d^3*(d*
x)^(13/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (221*d^
5*(d*x)^(9/2))/(768*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (331
5*d^9*Sqrt[d*x]*(a + b*x^2))/(1024*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a
^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(21/4)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) - (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt
```

```
[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]
*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{(dx)^{19/2}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(17b^2d^2(ab+b^2x^2)\right)\int \frac{(dx)^{15/2}}{(ab+b^2x^2)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

$$= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

Mathematica [A]

time = 0.63, size = 218, normalized size = 0.36

$$\frac{d^9\sqrt{dx}\left(4\sqrt[4]{b}\sqrt{x}\,(9945a^4+37791a^3bx^2+52819a^2b^2x^4+31501ab^3x^6+6144b^4x^8)-9945\sqrt{2}\sqrt[4]{a}\,(a+bx^2)^4\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-9945\sqrt{2}\sqrt[4]{a}\,(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{12288b^{21/4}\sqrt{x}\,(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (d^9*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(9945*a^4 + 37791*a^3*b*x^2 + 52819*a^2*b
^2*x^4 + 31501*a*b^3*x^6 + 6144*b^4*x^8) - 9945*Sqrt[2]*a^(1/4)*(a + b*x^2)
^4*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 9945*
Sqrt[2]*a^(1/4)*(a + b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sq
rt[a] + Sqrt[b]*x)]))/(12288*b^(21/4)*Sqrt[x]*(a + b*x^2)^3*Sqrt[(a + b*x^2
)^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. $2(389) = 778$.
time = 0.09, size = 1202, normalized size = 2.00

method	result
risch	$\frac{2x\,d^{10}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{b^{5}\sqrt{dx}\,\left(b\,x^{2}+a\right)}+\frac{\left(\frac{1267a^{4}d^{7}\sqrt{dx}}{1024b^{5}\left(d^{2}x^{2}b+a\,d^{2}\right)^{4}}+\frac{4405a^{3}d^{5}(dx)^{\frac{5}{2}}}{1024b^{4}\left(d^{2}x^{2}b+a\,d^{2}\right)^{4}}+\frac{15955a^{2}d^{3}(dx)^{\frac{9}{2}}}{3072b^{3}\left(d^{2}x^{2}b+a\,d^{2}\right)^{4}}+\frac{6925ad(dx)^{\frac{13}{2}}}{3072b^{2}\left(d^{2}x^{2}b+a\,d^{2}\right)^{4}}-\frac{3315\left(\frac{a\,d}{b}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24576*(-9945*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))*b^4*d^6*x^8-19890*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8-19890*(a*d^2/b)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*
x^8+49152*(d*x)^(1/2)*b^4*d^6*x^8-39780*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^6*x^6-79560*(a*d^2/b)^(1/4)*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*
x^6-79560*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/
4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+55400*(d*x)^(13/2)*a*b^3+196608*(d*x)^(1
/2)*a*b^3*d^6*x^6-59670*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a*d^2/b)^(1/2)))*a^2*b^2*d^6*x^4-119340*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^
(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4-119340*
(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2
/b)^(1/4))*a^2*b^2*d^6*x^4+127640*(d*x)^(9/2)*a^2*b^2*d^2+294912*(d*x)^(1/2
```

```
)*a^2*b^2*d^6*x^4-39780*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a*d^2/b)^(1/2)))*a^3*b*d^6*x^2-79560*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/
2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^6*x^2-79560*(a*d^2
/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1
/4))*a^3*b*d^6*x^2+105720*(d*x)^(5/2)*a^3*b*d^4+196608*(d*x)^(1/2)*a^3*b*d^
6*x^2-9945*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))*a^4*d^6-19890*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-19890*(a*d^2/b)^(1/4)*2^(1/2)*arctan(
(2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6+79560*(d*x)^
(1/2)*a^4*d^6)*d^3*(b*x^2+a)/b^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] d^(19/2)*integrate(x^(3/2)/(b^5*x^2 + a*b^4), x) - 1267/8192*(2*sqrt(2)*sqr
t(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/s
qrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))*d^(19/2)/b^5 + 1/3072*(1853*a*b^
3*d^(19/2)*x^(13/2) + 6515*a^2*b^2*d^(19/2)*x^(9/2) + 8079*a^3*b*d^(19/2)*x
^(5/2) + 3801*a^4*d^(19/2)*sqrt(x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4
+ 4*a^3*b^6*x^2 + a^4*b^5) + 1/192*((317*a*b^4*d^(19/2)*x^5 + 738*a^2*b^3*d
^(19/2)*x^3 + 453*a^3*b^2*d^(19/2)*x)*x^(11/2) + 2*(243*a^2*b^3*d^(19/2)*x^
5 + 582*a^3*b^2*d^(19/2)*x^3 + 371*a^4*b*d^(19/2)*x)*x^(7/2) + (201*a^3*b^2
*d^(19/2)*x^5 + 490*a^4*b*d^(19/2)*x^3 + 321*a^5*d^(19/2)*x)*x^(3/2))/(a^3*
b^7*x^6 + 3*a^4*b^6*x^4 + 3*a^5*b^5*x^2 + a^6*b^4 + (b^10*x^6 + 3*a*b^9*x^4
 + 3*a^2*b^8*x^2 + a^3*b^7)*x^6 + 3*(a*b^9*x^6 + 3*a^2*b^8*x^4 + 3*a^3*b^7*
x^2 + a^4*b^6)*x^4 + 3*(a^2*b^8*x^6 + 3*a^3*b^7*x^4 + 3*a^4*b^6*x^2 + a^5*b
^5)*x^2)
```

Fricas [A]
time = 0.38, size = 421, normalized size = 0.70

$$-\frac{39780 \left(-\frac{a d^{38}}{b^{21}}\right)^{\frac{1}{4}} (b^9 x^8 + 4 a b^8 x^6 + 6 a^2 b^7 x^4 + 4 a^3 b^6 x^2 + a^4 b^5) \arctan\left(-\frac{\left(-\frac{a d^{38}}{b^{21}}\right)^{\frac{3}{4}} \sqrt{dx}\, b^{16} d^9 - \sqrt{d^{19} x + \sqrt{-\frac{a d^{38}}{b^{21}}}\, b^{10}} \left(-\frac{a d^{38}}{b^{21}}\right)^{\frac{3}{4}} b^{16}}{a d^{38}}\right) + 9945 \left(-\frac{a d^{38}}{b^{21}}\right)^{\frac{1}{4}} (b^9 x^8 + 4 a b^8 x^6 + 6 a^2 b^7 x^4 + 4 a^3 b^6 x^2 + a^4 b^5) \log\left(3315 \sqrt{dx}\, d^9 + 3315 \left(-\frac{a d^{38}}{b^{21}}\right)^{\frac{1}{4}} b^5\right) - 9945 \left(-\frac{a d^{38}}{b^{21}}\right)^{\frac{1}{4}} (b^9 x^8 + 4 a b^8 x^6 + 6 a^2 b^7 x^4 + 4 a^3 b^6 x^2 + a^4 b^5) \log\left(3315 \sqrt{dx}\, d^9 - 3315 \left(-\frac{a d^{38}}{b^{21}}\right)^{\frac{1}{4}} b^5\right) - 4 (6144 b^4 d^9 x^8 + 31501 a b^3 d^9 x^6 + 52819 a^2 b^2 d^9 x^4 + 37791 a^3 b d^9 x^2 + 9945 a^4 d^9) \sqrt{dx}}{12288 (b^9 x^8 + 4 a b^8 x^6 + 6 a^2 b^7 x^4 + 4 a^3 b^6 x^2 + a^4 b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(39780*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4
 + 4*a^3*b^6*x^2 + a^4*b^5)*arctan(-((-a*d^38/b^21)^(3/4)*sqrt(d*x)*b^16*d^
9 - sqrt(d^19*x + sqrt(-a*d^38/b^21)*b^10)*(-a*d^38/b^21)^(3/4)*b^16)/(a*d^
38)) + 9945*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4
*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d*x)*d^9 + 3315*(-a*d^38/b^21)^(1/4)*
b^5) - 9945*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4
*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d*x)*d^9 - 3315*(-a*d^38/b^21)^(1/4)*
b^5) - 4*(6144*b^4*d^9*x^8 + 31501*a*b^3*d^9*x^6 + 52819*a^2*b^2*d^9*x^4 +
37791*a^3*b*d^9*x^2 + 9945*a^4*d^9)*sqrt(d*x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a
^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.54, size = 381, normalized size = 0.64

$$-\frac{1}{24576}\,d^9\left(\frac{19890\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\mathrm{sgn}\left(bx^2+a\right)}+\frac{19890\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\mathrm{sgn}\left(bx^2+a\right)}+\frac{9945\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^6\mathrm{sgn}\left(bx^2+a\right)}-\frac{9945\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{b^6\mathrm{sgn}\left(bx^2+a\right)}-\frac{49152\,\sqrt{dx}}{b^5\mathrm{sgn}\left(bx^2+a\right)}-\frac{8\,\left(6925\,\sqrt{dx}\,ab^3d^8x^6+15955\,\sqrt{dx}\,a^2b^2d^8x^4+13215\,\sqrt{dx}\,a^3bd^8x^2+3801\,\sqrt{dx}\,a^4d^8\right)}{\left(bd^2x^2+ad^2\right)^4b^5\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*d^9*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sgn(b*x^2 + a)) + 19890
*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2
*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sgn(b*x^2 + a)) + 9945*sqrt(2)*(a*b^3*d^2
)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*s
gn(b*x^2 + a)) - 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*sgn(b*x^2 + a)) - 49152*sqrt(d*x)/(b
^5*sgn(b*x^2 + a)) - 8*(6925*sqrt(d*x)*a*b^3*d^8*x^6 + 15955*sqrt(d*x)*a^2*
b^2*d^8*x^4 + 13215*sqrt(d*x)*a^3*b*d^8*x^2 + 3801*sqrt(d*x)*a^4*d^8)/((b*d
^2*x^2 + a*d^2)^4*b^5*sgn(b*x^2 + a)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x\right)}^{19/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.772 $\int \frac{(dx)^{17/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}} \, dx$

Optimal. Leaf size=554

$$-\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-$$

```
[Out] -385/1024*d^7*(d*x)^(3/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(15/2)/b/(b*x
^2+a)^3/((b*x^2+a)^2)^(1/2)-5/32*d^3*(d*x)^(11/2)/b^2/(b*x^2+a)^2/((b*x^2+a
)^2)^(1/2)-55/256*d^5*(d*x)^(7/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-1155/40
96*d^(17/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))
/a^(1/4)/b^(19/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+1155/4096*d^(17/2)*(b*x^2+a)*
arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(1/4)/b^(19/4)*2^(1
/2)/((b*x^2+a)^2)^(1/2)+1155/8192*d^(17/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b
^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(19/4)*2^(1/2
)/((b*x^2+a)^2)^(1/2)-1155/8192*d^(17/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(
1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(1/4)/b^(19/4)*2^(1/2)/
((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{1155d^{17/2}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\sqrt[4]{a}\,b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155d^{17/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\sqrt[4]{a}\,b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3(dx)^{11/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{15/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155d^{17/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\sqrt[4]{a}\,b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155d^{17/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\sqrt[4]{a}\,b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{55d^5(dx)^{7/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-385*d^7*(d*x)^(3/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(15/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^3*(d*x
)^(11/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (55*d^5*
(d*x)^(7/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*
d^(17/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (115
5*d^(17/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1
155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) - (1155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(1
9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(b^4(ab+b^2x^2))\int \frac{(dx)^{17/2}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{15/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(15b^2d^2(ab+b^2x^2))\int \frac{(dx)^{13/2}}{(ab+b^2x^2)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{15/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{15/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(a}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(a}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(a}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(a}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(a}$$

$$= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{15/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(a}$$

Mathematica [A]
time = 0.55, size = 207, normalized size = 0.37

$$\frac{d^8\sqrt{dx}\left(-4\sqrt[4]{a}\,b^{3/4}x^{3/2}(385a^3+1375a^2bx^2+1755ab^2x^4+893b^3x^6)+1155\sqrt{2}\,(a+bx^2)^4\tan^{-1}\left(\frac{-\sqrt{a}+\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-1155\sqrt{2}\,(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{4096\sqrt[4]{a}\,b^{19/4}\sqrt{x}\,(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (d^8*Sqrt[d*x]*(-4*a^(1/4)*b^(3/4)*x^(3/2)*(385*a^3 + 1375*a^2*b*x^2 + 1755
*a*b^2*x^4 + 893*b^3*x^6) + 1155*Sqrt[2]*(a + b*x^2)^4*ArcTan[(-Sqrt[a] + S
qrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 1155*Sqrt[2]*(a + b*x^2)^4*A
rcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(4096*a^(
1/4)*b^(19/4)*Sqrt[x]*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(358) = 716$.

time = 0.05, size = 1046, normalized size = 1.89

method	result	size
default	Expression too large to display	1046

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8192*(-1155*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/
b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^4*d^
8*x^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*b^4*d^8*x^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*b^4*d^8*x^8+7144*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4-4620*2^
(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^8*x^6-9240*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^8
*x^6-9240*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1
/4))*a*b^3*d^8*x^6+14040*(a*d^2/b)^(1/4)*(d*x)^(11/2)*a*b^3*d^2-6930*2^(1/2
)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^8*x^4-13860*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^
8*x^4-13860*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^
(1/4))*a^2*b^2*d^8*x^4+11000*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4-4620*2
^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^8*x^2-9240*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^
8*x^2-9240*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*a^3*b*d^8*x^2+3080*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6-1155*2^(1/2)
*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8-2310*2^(1/2)*arctan(
(2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8-2310*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)*d*(
b*x^2+a)/(a*d^2/b)^(1/4)/b^5/((b*x^2+a)^2)^(5/2)
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] d^(17/2)*integrate(sqrt(x)/(b^5*x^2 + a*b^4), x) - 893/8192*d^(17/2)*(2*sqr
t(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt
(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^4 - 1/3072*(26
79*b^3*d^(17/2)*x^(15/2) + 9441*a*b^2*d^(17/2)*x^(11/2) + 11645*a^2*b*d^(17
/2)*x^(7/2) + 5267*a^3*d^(17/2)*x^(3/2))/(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6
*x^4 + 4*a^3*b^5*x^2 + a^4*b^4) + 1/192*((261*a*b^4*d^(17/2)*x^5 + 610*a^2*
b^3*d^(17/2)*x^3 + 381*a^3*b^2*d^(17/2)*x)*x^(9/2) + 2*(191*a^2*b^3*d^(17/2
)*x^5 + 462*a^3*b^2*d^(17/2)*x^3 + 303*a^4*b*d^(17/2)*x)*x^(5/2) + (153*a^3
*b^2*d^(17/2)*x^5 + 378*a^4*b*d^(17/2)*x^3 + 257*a^5*d^(17/2)*x)*sqrt(x))/(
a^3*b^7*x^6 + 3*a^4*b^6*x^4 + 3*a^5*b^5*x^2 + a^6*b^4 + (b^10*x^6 + 3*a*b^9
*x^4 + 3*a^2*b^8*x^2 + a^3*b^7)*x^6 + 3*(a*b^9*x^6 + 3*a^2*b^8*x^4 + 3*a^3*
b^7*x^2 + a^4*b^6)*x^4 + 3*(a^2*b^8*x^6 + 3*a^3*b^7*x^4 + 3*a^4*b^6*x^2 + a
^5*b^5)*x^2)
```

Fricas [A]
time = 0.37, size = 428, normalized size = 0.77

$$-\frac{4620\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\sqrt{dx}\,b^5d^{25}-\sqrt{d^{51}x-\sqrt{-\frac{d^{34}}{ab^{19}}}\,ab^9d^{34}}\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}b^5}{d^{34}}\right)-1155\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\log\left(1540798875\,\sqrt{dx}\,d^{25}+1540798875\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{3}{4}}ab^{14}\right)+1155\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\log\left(1540798875\,\sqrt{dx}\,d^{25}-1540798875\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{3}{4}}ab^{14}\right)+4\,(893\,b^3d^8x^7+1755\,ab^2d^8x^5+1375\,a^2bd^8x^3+385\,a^3d^8x)\sqrt{dx}}{4096\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4096*(4620*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*
b^4)*(-d^34/(a*b^19))^(1/4)*arctan(-((-d^34/(a*b^19))^(1/4)*sqrt(d*x)*b^5*d
^25 - sqrt(d^51*x - sqrt(-d^34/(a*b^19))*a*b^9*d^34)*(-d^34/(a*b^19))^(1/4)
*b^5)/d^34) - 1155*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 +
 a^4*b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*x)*d^25 + 1540798875
*(-d^34/(a*b^19))^(3/4)*a*b^14) + 1155*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x
^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*
x)*d^25 - 1540798875*(-d^34/(a*b^19))^(3/4)*a*b^14) + 4*(893*b^3*d^8*x^7 +
1755*a*b^2*d^8*x^5 + 1375*a^2*b*d^8*x^3 + 385*a^3*d^8*x)*sqrt(d*x))/(b^8*x^
8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: SystemError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8438 deep
```

Giac [A]

time = 3.09, size = 383, normalized size = 0.69

$$\frac{1}{8192}d^{8}\left(\frac{2310\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{ab^{7}d\operatorname{sgn}\left(bx^{2}+a\right)}+\frac{2310\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{ab^{7}d\operatorname{sgn}\left(bx^{2}+a\right)}-\frac{1155\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{ab^{7}d\operatorname{sgn}\left(bx^{2}+a\right)}+\frac{1155\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{ab^{7}d\operatorname{sgn}\left(bx^{2}+a\right)}-\frac{8\left(893\sqrt{dx}\,b^{3}d^{8}x^{7}+1755\sqrt{dx}\,ab^{2}d^{8}x^{5}+1375\sqrt{dx}\,a^{2}bd^{8}x^{3}+385\sqrt{dx}\,a^{3}d^{8}x\right)}{\left(bd^{2}x^{2}+ad^{2}\right)^{4}b^{4}\operatorname{sgn}\left(bx^{2}+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d^8*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^7*d*sgn(b*x^2 + a)) + 2310
*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2
*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^7*d*sgn(b*x^2 + a)) - 1155*sqrt(2)*(a*b^3
*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
*b^7*d*sgn(b*x^2 + a)) + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(
a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^7*d*sgn(b*x^2 + a)) - 8*(893
*sqrt(d*x)*b^3*d^8*x^7 + 1755*sqrt(d*x)*a*b^2*d^8*x^5 + 1375*sqrt(d*x)*a^2*
b*d^8*x^3 + 385*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^2 + a*d^2)^4*b^4*sgn(b*x^2 +
 a)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{17/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.773 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=554

$$-\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-25$$

```
[Out] -1/8*d*(d*x)^(13/2)/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-13/96*d^3*(d*x)^(9/2)
/b^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-39/256*d^5*(d*x)^(5/2)/b^3/(b*x^2+a)/(
(b*x^2+a)^2)^(1/2)-195/4096*d^(15/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*
x)^(1/2)/a^(1/4)/d^(1/2))/a^(3/4)/b^(17/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+195/
4096*d^(15/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))/a^(3/4)/b^(17/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-195/8192*d^(15/2)*(b*x^2+a)
*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/
a^(3/4)/b^(17/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+195/8192*d^(15/2)*(b*x^2+a)*ln
(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(
3/4)/b^(17/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-195/1024*d^7*(d*x)^(1/2)/b^4/((b*
x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{39d^5(dx)^{5/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^{15/2}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195d^{15/2}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^{15/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195d^{15/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-195*d^7*Sqrt[d*x])/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(13/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13*d^3*(d*x)
^(9/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (39*d^5*(d
*x)^(5/2))/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d^(
15/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*d^
(15/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d
^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]) + (195*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[
d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{(dx)^{15/2}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b^2d^2(ab+b^2x^2)\right)\int \frac{(dx)^{11/2}}{(ab+b^2x^2)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)^{9/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

$$= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

Mathematica [A]
time = 0.58, size = 201, normalized size = 0.36

$$\frac{(dx)^{15/2}\left(a+bx^2\right)\left(-4\sqrt[4]{b}\sqrt{x}\left(585a^3+2223a^2bx^2+3107ab^2x^4+1853b^3x^6\right)-\frac{585\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}}+\frac{585\sqrt{2}\left(a+bx^2\right)^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{3/4}}\right)}{12288b^{17/4}x^{15/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(15/2)*(a + b*x^2)*(-4*b^(1/4)*Sqrt[x]*(585*a^3 + 2223*a^2*b*x^2 + 3
107*a*b^2*x^4 + 1853*b^3*x^6) - (585*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a]
- Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (585*Sqrt[2]*(a
+ b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]
)/a^(3/4)))/(12288*b^(17/4)*x^(15/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1133 vs. $2(358) = 716$.

time = 0.05, size = 1134, normalized size = 2.05

method	result	size
default	Expression too large to display	1134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24576*(585*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*b^4*d^6*x^8+1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+1170*(a*d^2/b)^(1/4)*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+
2340*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a
*b^3*d^6*x^6+4680*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+4680*(a*d^2/b)^(1/4)*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+35
10*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2
*b^2*d^6*x^4+7020*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4+7020*(a*d^2/b)^(1/4)*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^
4-14824*(d*x)^(13/2)*a*b^3+2340*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^6*x^2+4680*(a*d^2/b)^(1/4)*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^6*x^2+4680*
(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2
/b)^(1/4))*a^3*b*d^6*x^2-24856*(d*x)^(9/2)*a^2*b^2*d^2+585*(a*d^2/b)^(1/4)*
2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^6+1170*(a*d^2/b)
^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*a^4*d^6+1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-17784*(d*x)^(5/2)*a^3*b*d^4-4680*(d*x)^(1
/2)*a^4*d^6)*d*(b*x^2+a)/a/b^4/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.56, size = 583, normalized size = 1.05

$$\frac{195\,d^7\left(\frac{2\,\sqrt{2}\,\sqrt{d}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,\sqrt{d}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,\sqrt{d}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,\sqrt{d}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{8192\,b^4}-\frac{15\,b^3d^{\frac{15}{2}}x^{\frac{13}{2}}+65\,ab^2d^{\frac{15}{2}}x^{\frac{9}{2}}+117\,a^2bd^{\frac{15}{2}}x^{\frac{5}{2}}+195\,a^3d^{\frac{15}{2}}\sqrt{x}}{1024\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)}-\frac{\left(113\,b^4d^{\frac{15}{2}}x^5+282\,ab^3d^{\frac{15}{2}}x^3+201\,a^2b^2d^{\frac{15}{2}}x\right)x^{\frac{11}{2}}+2\left(63\,ab^3d^{\frac{15}{2}}x^5+174\,a^2b^2d^{\frac{15}{2}}x^3+143\,a^3bd^{\frac{15}{2}}x\right)x^{\frac{7}{2}}+\left(45\,a^2b^2d^{\frac{15}{2}}x^5+130\,a^3bd^{\frac{15}{2}}x^3+117\,a^4d^{\frac{15}{2}}x\right)x^{\frac{3}{2}}}{192\,(a^3b^6x^6+3\,a^4b^5x^4+3\,a^5b^4x^2+a^6b^3+(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)x^6+3\,(ab^8x^6+3\,a^2b^7x^4+3\,a^3b^6x^2+a^4b^5)x^4+3\,(a^2b^7x^6+3\,a^3b^6x^4+3\,a^4b^5x^2+a^5b^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 195/8192*d^7*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(
2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3
/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(
b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^4 - 1/1024*(15*b^3*d^(15/2)*x^(13/2) +
 65*a*b^2*d^(15/2)*x^(9/2) + 117*a^2*b*d^(15/2)*x^(5/2) + 195*a^3*d^(15/2)*
sqrt(x))/(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
- 1/192*((113*b^4*d^(15/2)*x^5 + 282*a*b^3*d^(15/2)*x^3 + 201*a^2*b^2*d^(15
/2)*x)*x^(11/2) + 2*(63*a*b^3*d^(15/2)*x^5 + 174*a^2*b^2*d^(15/2)*x^3 + 143
*a^3*b*d^(15/2)*x)*x^(7/2) + (45*a^2*b^2*d^(15/2)*x^5 + 130*a^3*b*d^(15/2)*
x^3 + 117*a^4*d^(15/2)*x)*x^(3/2))/(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4
*x^2 + a^6*b^3 + (b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)*x^6 + 3*
(a*b^8*x^6 + 3*a^2*b^7*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^4 + 3*(a^2*b^7*x^6
+ 3*a^3*b^6*x^4 + 3*a^4*b^5*x^2 + a^5*b^4)*x^2)
```

Fricas [A]
time = 0.36, size = 431, normalized size = 0.78

$$\frac{2340\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{3}{4}}\sqrt{dx}\,a^2b^{13}d^7-\sqrt{d^{15}x+\sqrt{-\frac{d^{30}}{a^3b^{17}}}\,a^2b^8}\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{3}{4}}a^2b^{13}}{d^{30}}\right)+585\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}\log\left(195\,\sqrt{dx}\,d^7+195\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}ab^4\right)-585\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}\log\left(195\,\sqrt{dx}\,d^7-195\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}ab^4\right)-4\,(1853\,b^3d^7x^6+3107\,ab^2d^7x^4+2223\,a^2bd^7x^2+585\,a^3d^7)\sqrt{dx}}{12288\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(2340*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*
b^4)*(-d^30/(a^3*b^17))^(1/4)*arctan(-((-d^30/(a^3*b^17))^(3/4)*sqrt(d*x)*a
^2*b^13*d^7 - sqrt(d^15*x + sqrt(-d^30/(a^3*b^17))*a^2*b^8)*(-d^30/(a^3*b^1
7))^(3/4)*a^2*b^13)/d^30) + 585*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*
a^3*b^5*x^2 + a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*log(195*sqrt(d*x)*d^7 + 195
*(-d^30/(a^3*b^17))^(1/4)*a*b^4) - 585*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x
^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*log(195*sqrt(d*x)*d^
7 - 195*(-d^30/(a^3*b^17))^(1/4)*a*b^4) - 4*(1853*b^3*d^7*x^6 + 3107*a*b^2*
d^7*x^4 + 2223*a^2*b*d^7*x^2 + 585*a^3*d^7)*sqrt(d*x))/(b^8*x^8 + 4*a*b^7*x
^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5986 deep
```

Giac [A]
time = 3.78, size = 370, normalized size = 0.67

$$\frac{1}{24576}\,d^7\left(\frac{1170\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5\mathrm{sgn}\left(bx^2+a\right)}+\frac{1170\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5\mathrm{sgn}\left(bx^2+a\right)}+\frac{585\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^5\mathrm{sgn}\left(bx^2+a\right)}-\frac{585\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{ab^5\mathrm{sgn}\left(bx^2+a\right)}-\frac{8\left(1853\,\sqrt{dx}\,b^3d^8x^6+3107\,\sqrt{dx}\,ab^2d^8x^4+2223\,\sqrt{dx}\,a^2bd^8x^2+585\,\sqrt{dx}\,a^3d^8\right)}{(bd^2x^2+ad^2)^4b^4\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^7*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sgn(b*x^2 + a)) + 1170*
sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*
sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sgn(b*x^2 + a)) + 585*sqrt(2)*(a*b^3*d^2
)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5
*sgn(b*x^2 + a)) - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b
)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5*sgn(b*x^2 + a)) - 8*(1853*sqrt(d*
x)*b^3*d^8*x^6 + 3107*sqrt(d*x)*a*b^2*d^8*x^4 + 2223*sqrt(d*x)*a^2*b*d^8*x^
2 + 585*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 + a*d^2)^4*b^4*sgn(b*x^2 + a)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{15/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.774 $\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-$$

```
[Out] 77/1024*d^5*(d*x)^(3/2)/a/b^3/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(11/2)/b/(b*x
^2+a)^3/((b*x^2+a)^2)^(1/2)-11/96*d^3*(d*x)^(7/2)/b^2/(b*x^2+a)^2/((b*x^2+a
)^2)^(1/2)-77/768*d^5*(d*x)^(3/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-77/4096
*d^(13/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a
^(5/4)/b^(15/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+77/4096*d^(13/2)*(b*x^2+a)*arct
an(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(5/4)/b^(15/4)*2^(1/2)/
((b*x^2+a)^2)^(1/2)+77/8192*d^(13/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)
*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(15/4)*2^(1/2)/((b*
x^2+a)^2)^(1/2)-77/8192*d^(13/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(
1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(5/4)/b^(15/4)*2^(1/2)/((b*x^2+
a)^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{11d^3(dx)^{7/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^5(dx)^{3/2}}{768b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{13/2}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{13/2}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{13/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{13/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (77*d^5*(d*x)^(3/2))/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(11/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (11*d^3*(d*
x)^(7/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^5*
(d*x)^(3/2))/(768*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^
(13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(13/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (77*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(b^4(ab+b^2x^2))\int \frac{(dx)^{13/2}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(11b^2d^2(ab+b^2x^2))\int \frac{(dx)^{9/2}}{(ab+b^2x^2)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)^{7/2}}{96b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{11/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)^{7/2}}{96b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

$$= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{11/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(a}$$

Mathematica [A]
time = 0.57, size = 201, normalized size = 0.36

$$\frac{(dx)^{13/2}\,(a+bx^2)\left(-4\sqrt[4]{a}\,b^{3/4}x^{3/2}(77a^3+275a^2bx^2+351ab^2x^4-231b^3x^6)-231\sqrt{2}\,(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-231\sqrt{2}\,(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{12288a^{5/4}b^{15/4}x^{13/2}\left((a+bx^2)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(13/2)*(a + b*x^2)*(-4*a^(1/4)*b^(3/4)*x^(3/2)*(77*a^3 + 275*a^2*b*x
^2 + 351*a*b^2*x^4 - 231*b^3*x^6) - 231*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[
a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 231*Sqrt[2]*(a + b*x^2
)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(122
88*a^(5/4)*b^(15/4)*x^(13/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(361) = 722$.
time = 0.05, size = 1051, normalized size = 1.89

method	result	size
default	Expression too large to display	1051

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24576*(231*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^4*d^8*
x^8+462*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*b^4*d^8*x^8+462*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*b^4*d^8*x^8+1848*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+924*2^(1/2)
*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^8*x^6+1848*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^8*x^6+
1848*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
a*b^3*d^8*x^6-2808*(a*d^2/b)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+1386*2^(1/2)*ln(-
((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^8*x^4+2772*2^(1/2)*arct
an((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^8*x^4+2
772*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a
^2*b^2*d^8*x^4-2200*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+924*2^(1/2)*ln(
-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^8*x^2+1848*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^8*x^2+1848
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*
b*d^8*x^2-616*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+231*2^(1/2)*ln(-((a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8+462*2^(1/2)*arctan((2^(1/2)*(d*x
)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8+462*2^(1/2)*arctan((2^(1/
2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)/d*(b*x^2+a)/(a*d^
2/b)^(1/4)/b^4/a/((b*x^2+a)^2)^(5/2)
```

Maple [A]

time = 0.56, size = 577, normalized size = 1.04

$$\frac{77\,d^{\frac{13}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8192\,ab^{3}}+\frac{77\,b^{3}d^{\frac{13}{2}}x^{\frac{15}{2}}+315\,ab^{2}d^{\frac{13}{2}}x^{\frac{11}{2}}+495\,a^{2}bd^{\frac{13}{2}}x^{\frac{7}{2}}+385\,a^{3}d^{\frac{13}{2}}x^{\frac{3}{2}}}{1024\left(ab^{7}x^{8}+4\,a^{2}b^{6}x^{6}+6\,a^{3}b^{5}x^{4}+4\,a^{4}b^{4}x^{2}+a^{5}b^{3}\right)}-\frac{\left(81\,b^{4}d^{\frac{13}{2}}x^{5}+202\,ab^{3}d^{\frac{13}{2}}x^{3}+153\,a^{2}b^{2}d^{\frac{13}{2}}x\right)x^{\frac{9}{2}}+2\left(35\,ab^{3}d^{\frac{13}{2}}x^{5}+102\,a^{2}b^{2}d^{\frac{13}{2}}x^{3}+99\,a^{3}bd^{\frac{13}{2}}x\right)x^{\frac{5}{2}}+\left(21\,a^{2}b^{2}d^{\frac{13}{2}}x^{5}+66\,a^{3}bd^{\frac{13}{2}}x^{3}+77\,a^{4}d^{\frac{13}{2}}x\right)\sqrt{x}}{192\left(a^{3}b^{6}x^{6}+3\,a^{4}b^{5}x^{4}+3\,a^{5}b^{4}x^{2}+a^{6}b^{3}+\left(b^{9}x^{6}+3\,ab^{8}x^{4}+3\,a^{2}b^{7}x^{2}+a^{3}b^{6}\right)x^{6}+3\left(ab^{8}x^{6}+3\,a^{2}b^{7}x^{4}+3\,a^{3}b^{6}x^{2}+a^{4}b^{5}\right)x^{4}+3\left(a^{2}b^{7}x^{6}+3\,a^{3}b^{6}x^{4}+3\,a^{4}b^{5}x^{2}+a^{5}b^{4}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 77/8192*d^(13/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(
2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(
3/4)))/(a*b^3) + 1/1024*(77*b^3*d^(13/2)*x^(15/2) + 315*a*b^2*d^(13/2)*x^(1
1/2) + 495*a^2*b*d^(13/2)*x^(7/2) + 385*a^3*d^(13/2)*x^(3/2))/(a*b^7*x^8 +
4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3) - 1/192*((81*b^4*d
^(13/2)*x^5 + 202*a*b^3*d^(13/2)*x^3 + 153*a^2*b^2*d^(13/2)*x)*x^(9/2) + 2*
(35*a*b^3*d^(13/2)*x^5 + 102*a^2*b^2*d^(13/2)*x^3 + 99*a^3*b*d^(13/2)*x)*x^
(5/2) + (21*a^2*b^2*d^(13/2)*x^5 + 66*a^3*b*d^(13/2)*x^3 + 77*a^4*d^(13/2)*
x)*sqrt(x))/(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b^3 + (b^9*x
^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)*x^6 + 3*(a*b^8*x^6 + 3*a^2*b^7*
x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^4 + 3*(a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4
*b^5*x^2 + a^5*b^4)*x^2)
```

Fricas [A]
time = 0.40, size = 448, normalized size = 0.80

$$-\frac{924\left(ab^{7}x^{8}+4\,a^{2}b^{6}x^{6}+6\,a^{3}b^{5}x^{4}+4\,a^{4}b^{4}x^{2}+a^{5}b^{3}\right)\left(-\frac{d^{26}}{a^{5}b^{15}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(-\frac{d^{26}}{a^{5}b^{15}}\right)^{\frac{1}{4}}\sqrt{dx}\,ab^{4}d^{19}-\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{a^{5}b^{15}}}\,a^{3}b^{7}d^{26}}\left(-\frac{d^{26}}{a^{5}b^{15}}\right)^{\frac{1}{4}}ab^{4}}{d^{26}}\right)-231\left(ab^{7}x^{8}+4\,a^{2}b^{6}x^{6}+6\,a^{3}b^{5}x^{4}+4\,a^{4}b^{4}x^{2}+a^{5}b^{3}\right)\left(-\frac{d^{26}}{a^{5}b^{15}}\right)^{\frac{1}{4}}\log\left(456533\sqrt{dx}\,d^{19}+456533\left(-\frac{d^{26}}{a^{5}b^{15}}\right)^{\frac{3}{4}}a^{4}b^{11}\right)+231\left(ab^{7}x^{8}+4\,a^{2}b^{6}x^{6}+6\,a^{3}b^{5}x^{4}+4\,a^{4}b^{4}x^{2}+a^{5}b^{3}\right)\left(-\frac{d^{26}}{a^{5}b^{15}}\right)^{\frac{1}{4}}\log\left(456533\sqrt{dx}\,d^{19}-456533\left(-\frac{d^{26}}{a^{5}b^{15}}\right)^{\frac{3}{4}}a^{4}b^{11}\right)-4\left(231\,b^{3}d^{6}x^{7}-351\,ab^{2}d^{6}x^{5}-275\,a^{2}bd^{6}x^{3}-77\,a^{3}d^{6}x\right)\sqrt{dx}}{12288\left(ab^{7}x^{8}+4\,a^{2}b^{6}x^{6}+6\,a^{3}b^{5}x^{4}+4\,a^{4}b^{4}x^{2}+a^{5}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(924*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 +
a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*arctan(-((-d^26/(a^5*b^15))^(1/4)*sqrt(d*
x)*a*b^4*d^19 - sqrt(d^39*x - sqrt(-d^26/(a^5*b^15))*a^3*b^7*d^26)*(-d^26/(
a^5*b^15))^(1/4)*a*b^4)/d^26) - 231*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*
x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*log(456533*sqrt(d*x
)*d^19 + 456533*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11) + 231*(a*b^7*x^8 + 4*a^2
*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4
)*log(456533*sqrt(d*x)*d^19 - 456533*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11) - 4
*(231*b^3*d^6*x^7 - 351*a*b^2*d^6*x^5 - 275*a^2*b*d^6*x^3 - 77*a^3*d^6*x)*s
qrt(d*x))/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*
b^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4062 deep
```

Giac [A]
time = 3.76, size = 386, normalized size = 0.69

$$\frac{1}{24576}\,d^6\left(\frac{462\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^6d\mathrm{sgn}\left(bx^2+a\right)}+\frac{462\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^6d\mathrm{sgn}\left(bx^2+a\right)}-\frac{231\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^6d\mathrm{sgn}\left(bx^2+a\right)}+\frac{231\,\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^6d\mathrm{sgn}\left(bx^2+a\right)}+\frac{8\left(231\,\sqrt{dx}\,b^3d^8x^7-351\,\sqrt{dx}\,ab^2d^8x^5-275\,\sqrt{dx}\,a^2bd^8x^3-77\,\sqrt{dx}\,a^3d^8x\right)}{(bd^2x^2+ad^2)^4ab^3\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^6*(462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^6*d*sgn(b*x^2 + a)) + 46
2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) -
2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^6*d*sgn(b*x^2 + a)) - 231*sqrt(2)*(a*b
^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(a^2*b^6*d*sgn(b*x^2 + a)) + 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^6*d*sgn(b*x^2 + a)) + 8
*(231*sqrt(d*x)*b^3*d^8*x^7 - 351*sqrt(d*x)*a*b^2*d^8*x^5 - 275*sqrt(d*x)*a
^2*b*d^8*x^3 - 77*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^2 + a*d^2)^4*a*b^3*sgn(b*x
^2 + a)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,x\right)}^{13/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.775 $\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^3(dx)^{5/2}}{32b^2\,(a+bx^2)^2\,\sqrt{a^2+2abx^2+b^2x^4}}-25$$

```
[Out] -1/8*d*(d*x)^(9/2)/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-3/32*d^3*(d*x)^(5/2)/b
^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-45/4096*d^(11/2)*(b*x^2+a)*arctan(1-b^(1
/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)/((b*x^2+a
)^2)^(1/2)+45/4096*d^(11/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/
a^(1/4)/d^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)-45/8192*d^(11
/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*
(d*x)^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+45/8192*d^(11/2)*
(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x
)^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+15/1024*d^5*(d*x)^(1/
2)/a/b^3/((b*x^2+a)^2)^(1/2)-15/256*d^5*(d*x)^(1/2)/b^3/(b*x^2+a)/((b*x^2+a
)^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3d^3(dx)^{5/2}}{32b^2\,(a+bx^2)^2\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15d^5\sqrt{dx}}{256b^3\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{11/2}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\,a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{11/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\,a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{11/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{11/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (15*d^5*Sqrt[d*x])/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(9/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^3*(d*x)^(
5/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (15*d^5*Sqrt
[d*x])/(256*b^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(11/2)
*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + (45*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(b^4(ab+b^2x^2))\int \frac{(dx)^{11/2}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(9b^2d^2(ab+b^2x^2))\int \frac{(dx)^{7/2}}{(ab+b^2x^2)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx)^{5/2}}{32b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx)^{5/2}}{32b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(}$$

$$= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{32b^2\,(}$$

Mathematica [A]

time = 0.58, size = 202, normalized size = 0.36

$$\frac{d(dx)^{9/2}\,(a+bx^2)\left(-4a^{3/4}\sqrt[4]{b}\,\sqrt{x}\,(45a^3+171a^2bx^2+239ab^2x^4-15b^3x^6)-45\sqrt{2}\,(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+45\sqrt{2}\,(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{4096a^{7/4}b^{13/4}x^{9/2}\left((a+bx^2)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (d*(d*x)^(9/2)*(a + b*x^2)*(-4*a^(3/4)*b^(1/4)*Sqrt[x]*(45*a^3 + 171*a^2*b*
x^2 + 239*a*b^2*x^4 - 15*b^3*x^6) - 45*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a
] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 45*Sqrt[2]*(a + b*x^2)^
4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(4096*
a^(7/4)*b^(13/4)*x^(9/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(361) = 722$.
time = 0.05, size = 1136, normalized size = 2.04

method	result	size
default	Expression too large to display	1136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8192*(45*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))*b^4*d^6*x^8+90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+90*(a*d^2/b)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+180*(a
*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^
6*x^6+360*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/
4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+360*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+120*(d*x)^(
13/2)*a*b^3+270*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2)))*a^2*b^2*d^6*x^4+540*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)
^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4+540*(a*d^2/b)^(1/4
)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2
*b^2*d^6*x^4-1912*(d*x)^(9/2)*a^2*b^2*d^2+180*(a*d^2/b)^(1/4)*2^(1/2)*ln((d
*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^6*x^2+360*(a*d^2/b)^(1/4)*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*
d^6*x^2+360*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(
1/4))/(a*d^2/b)^(1/4))*a^3*b*d^6*x^2-1368*(d*x)^(5/2)*a^3*b*d^4+45*(a*d^2/b
)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
)/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^6+90*(a*
d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)
^(1/4))*a^4*d^6+90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-360*(d*x)^(1/2)*a^4*d^6)/d*(b*x^2+a)/
b^3/a^2/((b*x^2+a)^2)^(5/2)
```

Maxima [A]

time = 0.55, size = 595, normalized size = 1.07

$$\frac{45 \, d^{5} {\left(\frac{2 \, \sqrt{2} \sqrt{d} \arctan\left(\frac{\sqrt{2} {\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} + 2 \, \sqrt{b} \sqrt{x}\right)}}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}} + \frac{2 \, \sqrt{2} \sqrt{d} \arctan\left(-\frac{\sqrt{2} {\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} - 2 \, \sqrt{b} \sqrt{x}\right)}}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}} + \frac{\sqrt{2} \sqrt{d} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{1}{4}}} - \frac{\sqrt{2} \sqrt{d} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{1}{4}}}\right)}}{8192 \, a b^{3}} - \frac{35 \, b^{3} d^{\frac{11}{2}} x^{\frac{13}{2}} + 173 \, a b^{2} d^{\frac{11}{2}} x^{\frac{9}{2}} + 657 \, a^{2} b d^{\frac{11}{2}} x^{\frac{5}{2}} + 135 \, a^{3} d^{\frac{11}{2}} \sqrt{x}}{3072 \, {\left(a b^{7} x^{8} + 4 \, a^{2} b^{6} x^{6} + 6 \, a^{3} b^{5} x^{4} + 4 \, a^{4} b^{4} x^{2} + a^{5} b^{3}\right)}} + \frac{{\left(5 \, b^{4} d^{\frac{11}{2}} x^{5} + 18 \, a b^{3} d^{\frac{11}{2}} x^{3} + 45 \, a^{2} b^{2} d^{\frac{11}{2}} x\right)} x^{\frac{11}{2}} - 2 \, {\left(21 \, a b^{3} d^{\frac{11}{2}} x^{5} + 42 \, a^{2} b^{2} d^{\frac{11}{2}} x^{3} - 11 \, a^{3} b d^{\frac{11}{2}} x\right)} x^{\frac{7}{2}} - {\left(15 \, a^{2} b^{2} d^{\frac{11}{2}} x^{5} + 38 \, a^{3} b d^{\frac{11}{2}} x^{3} - 9 \, a^{4} d^{\frac{11}{2}} x\right)} x^{\frac{3}{2}}}{192 \, {\left(a^{4} b^{5} x^{6} + 3 \, a^{5} b^{4} x^{4} + 3 \, a^{6} b^{3} x^{2} + a^{7} b^{2} + {\left(a b^{8} x^{6} + 3 \, a^{2} b^{7} x^{4} + 3 \, a^{3} b^{6} x^{2} + a^{4} b^{5}\right)} x^{6} + 3 \, {\left(a^{2} b^{7} x^{6} + 3 \, a^{3} b^{6} x^{4} + 3 \, a^{4} b^{5} x^{2} + a^{5} b^{4}\right)} x^{4} + 3 \, {\left(a^{3} b^{6} x^{6} + 3 \, a^{4} b^{5} x^{4} + 3 \, a^{5} b^{4} x^{2} + a^{6} b^{3}\right)} x^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 45/8192*d^5*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^3) - 1/3072*(35*b^3*d^(11/2)*x^(13/2
) + 173*a*b^2*d^(11/2)*x^(9/2) + 657*a^2*b*d^(11/2)*x^(5/2) + 135*a^3*d^(11
/2)*sqrt(x))/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a
^5*b^3) + 1/192*((5*b^4*d^(11/2)*x^5 + 18*a*b^3*d^(11/2)*x^3 + 45*a^2*b^2*d
^(11/2)*x)*x^(11/2) - 2*(21*a*b^3*d^(11/2)*x^5 + 42*a^2*b^2*d^(11/2)*x^3 -
11*a^3*b*d^(11/2)*x)*x^(7/2) - (15*a^2*b^2*d^(11/2)*x^5 + 38*a^3*b*d^(11/2)
*x^3 - 9*a^4*d^(11/2)*x)*x^(3/2))/(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*
x^2 + a^7*b^2 + (a*b^8*x^6 + 3*a^2*b^7*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^6 +
 3*(a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4*b^5*x^2 + a^5*b^4)*x^4 + 3*(a^3*b^6
*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b^3)*x^2)
```

Fricas [A]

time = 0.36, size = 447, normalized size = 0.80

$$\frac{180 \, {\left(a b^{7} x^{8} + 4 \, a^{2} b^{6} x^{6} + 6 \, a^{3} b^{5} x^{4} + 4 \, a^{4} b^{4} x^{2} + a^{5} b^{3}\right)} \left(-\frac{d^{22}}{a^{7} b^{13}}\right)^{\frac{1}{4}} \arctan\left(-\frac{\left(-\frac{d^{22}}{a^{7} b^{13}}\right)^{\frac{3}{4}} \sqrt{d x} a^{5} b^{10} d^{5} - \sqrt{d^{11} x + \sqrt{-\frac{d^{22}}{a^{7} b^{13}}} a^{4} b^{6}} \left(-\frac{d^{22}}{a^{7} b^{13}}\right)^{\frac{3}{4}} a^{5} b^{10}}{d^{22}}\right) + 45 \, {\left(a b^{7} x^{8} + 4 \, a^{2} b^{6} x^{6} + 6 \, a^{3} b^{5} x^{4} + 4 \, a^{4} b^{4} x^{2} + a^{5} b^{3}\right)} \left(-\frac{d^{22}}{a^{7} b^{13}}\right)^{\frac{1}{4}} \log\left(45 \, \sqrt{d x} d^{5} + 45 \, \left(-\frac{d^{22}}{a^{7} b^{13}}\right)^{\frac{1}{4}} a^{2} b^{3}\right) - 45 \, {\left(a b^{7} x^{8} + 4 \, a^{2} b^{6} x^{6} + 6 \, a^{3} b^{5} x^{4} + 4 \, a^{4} b^{4} x^{2} + a^{5} b^{3}\right)} \left(-\frac{d^{22}}{a^{7} b^{13}}\right)^{\frac{1}{4}} \log\left(45 \, \sqrt{d x} d^{5} - 45 \, \left(-\frac{d^{22}}{a^{7} b^{13}}\right)^{\frac{1}{4}} a^{2} b^{3}\right) + 4 \, {\left(15 \, b^{3} d^{5} x^{6} - 239 \, a b^{2} d^{5} x^{4} - 171 \, a^{2} b d^{5} x^{2} - 45 \, a^{3} d^{5}\right)} \sqrt{d x}}{4096 \, {\left(a b^{7} x^{8} + 4 \, a^{2} b^{6} x^{6} + 6 \, a^{3} b^{5} x^{4} + 4 \, a^{4} b^{4} x^{2} + a^{5} b^{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4096*(180*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^
5*b^3)*(-d^22/(a^7*b^13))^(1/4)*arctan(-((-d^22/(a^7*b^13))^(3/4)*sqrt(d*x)
*a^5*b^10*d^5 - sqrt(d^11*x + sqrt(-d^22/(a^7*b^13))*a^4*b^6)*(-d^22/(a^7*b
^13))^(3/4)*a^5*b^10)/d^22) + 45*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4
 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*log(45*sqrt(d*x)*d^5 +
 45*(-d^22/(a^7*b^13))^(1/4)*a^2*b^3) - 45*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a
^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*log(45*sqrt(
d*x)*d^5 - 45*(-d^22/(a^7*b^13))^(1/4)*a^2*b^3) + 4*(15*b^3*d^5*x^6 - 239*a
*b^2*d^5*x^4 - 171*a^2*b*d^5*x^2 - 45*a^3*d^5)*sqrt(d*x))/(a*b^7*x^8 + 4*a^
2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{11}{2}}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(11/2)/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 2.68, size = 373, normalized size = 0.67

$$\frac{1}{8192}\,d^5\left(\frac{90\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4\mathrm{sgn}\,(bx^2+a)}+\frac{90\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4\mathrm{sgn}\,(bx^2+a)}+\frac{45\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^4\mathrm{sgn}\,(bx^2+a)}-\frac{45\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^4\mathrm{sgn}\,(bx^2+a)}+\frac{8\left(15\,\sqrt{dx}\,b^3d^8x^6-239\,\sqrt{dx}\,ab^2d^8x^4-171\,\sqrt{dx}\,a^2bd^8x^2-45\,\sqrt{dx}\,a^3d^8\right)}{(bd^2x^2+ad^2)^4ab^3\mathrm{sgn}\,(bx^2+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d^5*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(b*x^2 + a)) + 90*sqr
t(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqr
t(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(b*x^2 + a)) + 45*sqrt(2)*(a*b^3*d^2)^
(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4
*sgn(b*x^2 + a)) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4*sgn(b*x^2 + a)) + 8*(15*sqrt(d*x
)*b^3*d^8*x^6 - 239*sqrt(d*x)*a*b^2*d^8*x^4 - 171*sqrt(d*x)*a^2*b*d^8*x^2 -
 45*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 + a*d^2)^4*a*b^3*sgn(b*x^2 + a)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{11/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.776 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=560

$$\frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+$$

```
[Out] 35/1024*d^3*(d*x)^(3/2)/a^2/b^2/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(7/2)/b/(b*
x^2+a)^3/((b*x^2+a)^2)^(1/2)-7/96*d^3*(d*x)^(3/2)/b^2/(b*x^2+a)^2/((b*x^2+a
)^2)^(1/2)+7/256*d^3*(d*x)^(3/2)/a/b^2/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-35/409
6*d^(9/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a
^(9/4)/b^(11/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+35/4096*d^(9/2)*(b*x^2+a)*arcta
n(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(9/4)/b^(11/4)*2^(1/2)/(
(b*x^2+a)^2)^(1/2)+35/8192*d^(9/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d
^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(9/4)/b^(11/4)*2^(1/2)/((b*x^
2+a)^2)^(1/2)-35/8192*d^(9/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(9/4)/b^(11/4)*2^(1/2)/((b*x^2+a)^
2)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7d^3(dx)^{3/2}}{256ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{3/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{9/2}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{9/2}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{9/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{9/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (35*d^3*(d*x)^(3/2))/(1024*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d
*x)^(7/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7*d^3*(d*
x)^(3/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7*d^3*(
d*x)^(3/2))/(256*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d
^(9/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^
(9/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(
9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) - (35*d^(9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(b^4(ab+b^2x^2))\int \frac{(dx)^{9/2}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(7b^2d^2(ab+b^2x^2))\int \frac{(dx)^{5/2}}{(ab+b^2x^2)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{3/2}}{96b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{7/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{3/2}}{96b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(}$$

$$= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2\,(}$$

Mathematica [A]
time = 0.49, size = 201, normalized size = 0.36

$$\frac{(dx)^{9/2}(a+bx^2)\left(4\sqrt[4]{a}\,b^{3/4}x^{3/2}(-35a^3-125a^2bx^2+399ab^2x^4+105b^3x^6)-105\sqrt{2}\,(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-105\sqrt{2}\,(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{12288a^{9/4}b^{11/4}x^{9/2}\left((a+bx^2)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(9/2)*(a + b*x^2)*(4*a^(1/4)*b^(3/4)*x^(3/2)*(-35*a^3 - 125*a^2*b*x^
2 + 399*a*b^2*x^4 + 105*b^3*x^6) - 105*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a
] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 105*Sqrt[2]*(a + b*x^2)
^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(1228
8*a^(9/4)*b^(11/4)*x^(9/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(364) = 728$.
time = 0.05, size = 1051, normalized size = 1.88

method	result	size
default	Expression too large to display	1051

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24576*(105*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^4*d^8*
x^8+210*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*b^4*d^8*x^8+210*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*b^4*d^8*x^8+840*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+420*2^(1/2)*
ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^8*x^6+840*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^8*x^6+84
0*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b
^3*d^8*x^6+3192*(a*d^2/b)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+630*2^(1/2)*ln(-((a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^8*x^4+1260*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^8*x^4+1260*
2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b
^2*d^8*x^4-1000*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+420*2^(1/2)*ln(-((a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^8*x^2+840*2^(1/2)*arctan((2^
(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^8*x^2+840*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^8*
x^2-280*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+105*2^(1/2)*ln(-((a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8+210*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2
)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8+210*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)/d^3*(b*x^2+a)/(a*d^2/b)
^(1/4)/b^3/a^2/((b*x^2+a)^2)^(5/2)
```

Maxima [A]

time = 0.56, size = 584, normalized size = 1.04

$$\frac{35\,d^{\frac{9}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8192\,a^2b^2}+\frac{105\,b^3d^{\frac{9}{2}}x^{\frac{15}{2}}+447\,ab^2d^{\frac{9}{2}}x^{\frac{11}{2}}+803\,a^2bd^{\frac{9}{2}}x^{\frac{7}{2}}+77\,a^3d^{\frac{9}{2}}x^{\frac{3}{2}}}{3072\,(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2)}-\frac{\left(3\,b^4d^{\frac{9}{2}}x^5+14\,ab^3d^{\frac{9}{2}}x^3-21\,a^2b^2d^{\frac{9}{2}}x\right)x^{\frac{9}{2}}+2\left(25\,ab^3d^{\frac{9}{2}}x^5+66\,a^2b^2d^{\frac{9}{2}}x^3+9\,a^3bd^{\frac{9}{2}}x\right)x^{\frac{5}{2}}+\left(15\,a^2b^2d^{\frac{9}{2}}x^5+54\,a^3bd^{\frac{9}{2}}x^3+7\,a^4d^{\frac{9}{2}}x\right)\sqrt{x}}{192\,(a^4b^5x^6+3\,a^5b^4x^4+3\,a^6b^3x^2+a^7b^2+(ab^8x^6+3\,a^2b^7x^4+3\,a^3b^6x^2+a^4b^5)x^6+3\,(a^2b^7x^6+3\,a^3b^6x^4+3\,a^4b^5x^2+a^5b^4)x^4+3\,(a^3b^6x^6+3\,a^4b^5x^4+3\,a^5b^4x^2+a^6b^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 35/8192*d^(9/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2
)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2
)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3
/4)))/(a^2*b^2) + 1/3072*(105*b^3*d^(9/2)*x^(15/2) + 447*a*b^2*d^(9/2)*x^(1
1/2) + 803*a^2*b*d^(9/2)*x^(7/2) + 77*a^3*d^(9/2)*x^(3/2))/(a^2*b^6*x^8 + 4
*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) - 1/192*((3*b^4*d^(
9/2)*x^5 + 14*a*b^3*d^(9/2)*x^3 - 21*a^2*b^2*d^(9/2)*x)*x^(9/2) + 2*(25*a*b
^3*d^(9/2)*x^5 + 66*a^2*b^2*d^(9/2)*x^3 + 9*a^3*b*d^(9/2)*x)*x^(5/2) + (15*
a^2*b^2*d^(9/2)*x^5 + 54*a^3*b*d^(9/2)*x^3 + 7*a^4*d^(9/2)*x)*sqrt(x))/(a^4
*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2 + (a*b^8*x^6 + 3*a^2*b^7
*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^6 + 3*(a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^
4*b^5*x^2 + a^5*b^4)*x^4 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 +
 a^6*b^3)*x^2)
```

<u>Fricas</u> [A]
time = 0.35, size = 462, normalized size = 0.82

$$-\frac{420\,(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2)\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}\arctan\left(-\frac{42875\sqrt{dx}\,a^2b^3d^{13}\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}-\sqrt{-1838265625\,a^5b^5d^{18}\sqrt{-\frac{d^{18}}{a^9b^{11}}}+1838265625\,d^{27}x}\,a^2b^3\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}}{42875\,d^{18}}\right)-105\,(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2)\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}\log\left(42875\,a^7b^8\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{3}{4}}+42875\sqrt{dx}\,d^{13}\right)+105\,(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2)\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}\log\left(-42875\,a^7b^8\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{3}{4}}+42875\sqrt{dx}\,d^{13}\right)-4\,(105\,b^3d^4x^7+399\,ab^2d^4x^5-125\,a^2bd^4x^3-35\,a^3d^4x)\sqrt{dx}}{12288\,(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(420*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2
+ a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*arctan(-1/42875*(42875*sqrt(d*x)*a^2*b^
3*d^13*(-d^18/(a^9*b^11))^(1/4) - sqrt(-1838265625*a^5*b^5*d^18*sqrt(-d^18/
(a^9*b^11)) + 1838265625*d^27*x)*a^2*b^3*(-d^18/(a^9*b^11))^(1/4))/d^18) -
105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
*(-d^18/(a^9*b^11))^(1/4)*log(42875*a^7*b^8*(-d^18/(a^9*b^11))^(3/4) + 4287
5*sqrt(d*x)*d^13) + 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^
5*b^3*x^2 + a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*log(-42875*a^7*b^8*(-d^18/(a^
9*b^11))^(3/4) + 42875*sqrt(d*x)*d^13) - 4*(105*b^3*d^4*x^7 + 399*a*b^2*d^4
*x^5 - 125*a^2*b*d^4*x^3 - 35*a^3*d^4*x)*sqrt(d*x))/(a^2*b^6*x^8 + 4*a^3*b^
5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{9}{2}}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(9/2)/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 2.85, size = 386, normalized size = 0.69

$$\frac{1}{24576}d^4\left(\frac{210\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^5d\,\mathrm{sgn}(bx^2+a)}+\frac{210\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^5d\,\mathrm{sgn}(bx^2+a)}-\frac{105\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^5d\,\mathrm{sgn}(bx^2+a)}+\frac{105\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^5d\,\mathrm{sgn}(bx^2+a)}+\frac{8\left(105\sqrt{dx}\,b^3d^8x^7+399\sqrt{dx}\,ab^2d^8x^5-125\sqrt{dx}\,a^2bd^8x^3-35\sqrt{dx}\,a^3d^8x\right)}{(bd^2x^2+ad^2)^4a^2b^2\,\mathrm{sgn}(bx^2+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^4*(210*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^5*d*sgn(b*x^2 + a)) + 21
0*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) -
2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^5*d*sgn(b*x^2 + a)) - 105*sqrt(2)*(a*b
^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(a^3*b^5*d*sgn(b*x^2 + a)) + 105*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^5*d*sgn(b*x^2 + a)) + 8
*(105*sqrt(d*x)*b^3*d^8*x^7 + 399*sqrt(d*x)*a*b^2*d^8*x^5 - 125*sqrt(d*x)*a
^2*b*d^8*x^3 - 35*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^2 + a*d^2)^4*a^2*b^2*sgn(b
*x^2 + a)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{9/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.777 $\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{5/2}} dx$

Optimal. Leaf size=560

$$\frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{}{7}$$

```
[Out] -1/8*d*(d*x)^(5/2)/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-35/4096*d^(7/2)*(b*x^2
+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/4)/b^(9/4)*
2^(1/2)/((b*x^2+a)^2)^(1/2)+35/4096*d^(7/2)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1
/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(11/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/
2)-35/8192*d^(7/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b
^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+35
/8192*d^(7/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4
)*2^(1/2)*(d*x)^(1/2))/a^(11/4)/b^(9/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+35/3072
*d^3*(d*x)^(1/2)/a^2/b^2/((b*x^2+a)^2)^(1/2)-5/96*d^3*(d*x)^(1/2)/b^2/(b*x^
2+a)^2/((b*x^2+a)^2)^(1/2)+5/768*d^3*(d*x)^(1/2)/a/b^2/(b*x^2+a)/((b*x^2+a)
^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^3\sqrt{dx}}{768ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{7/2}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{7/2}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (35*d^3*Sqrt[d*x])/(3072*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x
)^(5/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^3*Sqrt[
d*x])/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*d^3*Sqrt[
d*x])/(768*a*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(7/2)*
(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(204
8*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)*(
a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (35*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{(dx)^{7/2}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(5b^2d^2(ab+b^2x^2)\right)\int \frac{(dx)^{3/2}}{(ab+b^2x^2)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

$$= \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{96b^2}$$

Mathematica [A]

time = 0.47, size = 201, normalized size = 0.36

$$\frac{(dx)^{7/2}\left(a+bx^2\right)\left(4a^{3/4}\sqrt[4]{b}\sqrt{x}\left(-105a^3-399a^2bx^2+125ab^2x^4+35b^3x^6\right)-105\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+105\sqrt{2}\left(a+bx^2\right)^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)\right)}{12288a^{11/4}b^{9/4}x^{7/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(7/2)*(a + b*x^2)*(4*a^(3/4)*b^(1/4)*Sqrt[x]*(-105*a^3 - 399*a^2*b*x
^2 + 125*a*b^2*x^4 + 35*b^3*x^6) - 105*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a
] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 105*Sqrt[2]*(a + b*x^2)
^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(1228
8*a^(11/4)*b^(9/4)*x^(7/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(364) = 728$.
time = 0.05, size = 1136, normalized size = 2.03

method	result	size
default	Expression too large to display	1136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24576*(105*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*b^4*d^6*x^8+210*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+210*(a*d^2/b)^(1/4)*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+42
0*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^
2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^
3*d^6*x^6+840*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+840*(a*d^2/b)^(1/4)*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+280*(d*
x)^(13/2)*a*b^3+630*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2)))*a^2*b^2*d^6*x^4+1260*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*
(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4+1260*(a*d^2/b
)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*a^2*b^2*d^6*x^4+1000*(d*x)^(9/2)*a^2*b^2*d^2+420*(a*d^2/b)^(1/4)*2^(1/2)
*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^6*x^2+840*(a*d^2/b)^(
1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
a^3*b*d^6*x^2+840*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^6*x^2-3192*(d*x)^(5/2)*a^3*b*d^4+105*(
a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^6
+210*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(
a*d^2/b)^(1/4))*a^4*d^6+210*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(
1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-840*(d*x)^(1/2)*a^4*d^6)/d^3
*(b*x^2+a)/b^2/a^3/((b*x^2+a)^2)^(5/2)
```

Maxima [A]

time = 0.56, size = 597, normalized size = 1.07

$$-\frac{77\,b^{3}d^{\frac{7}{2}}x^{\frac{13}{2}}+803\,ab^{2}d^{\frac{7}{2}}x^{\frac{9}{2}}+447\,a^{2}bd^{\frac{7}{2}}x^{\frac{5}{2}}+105\,a^{3}d^{\frac{7}{2}}\sqrt{x}}{3072\,(a^{2}b^{6}x^{8}+4\,a^{3}b^{5}x^{6}+6\,a^{4}b^{4}x^{4}+4\,a^{5}b^{3}x^{2}+a^{6}b^{2})}+\frac{\left(7\,b^{4}d^{\frac{7}{2}}x^{5}+54\,ab^{3}d^{\frac{7}{2}}x^{3}+15\,a^{2}b^{2}d^{\frac{7}{2}}x\right)x^{\frac{11}{2}}+2\left(9\,ab^{3}d^{\frac{7}{2}}x^{5}+66\,a^{2}b^{2}d^{\frac{7}{2}}x^{3}+25\,a^{3}bd^{\frac{7}{2}}x\right)x^{\frac{7}{2}}-\left(21\,a^{2}b^{2}d^{\frac{7}{2}}x^{5}-14\,a^{3}bd^{\frac{7}{2}}x^{3}-3\,a^{4}d^{\frac{7}{2}}x\right)x^{\frac{3}{2}}}{192\,(a^{5}b^{4}x^{6}+3\,a^{6}b^{3}x^{4}+3\,a^{7}b^{2}x^{2}+a^{8}b+(a^{2}b^{7}x^{6}+3\,a^{3}b^{6}x^{4}+3\,a^{4}b^{5}x^{2}+a^{5}b^{4})x^{6}+3\,(a^{3}b^{6}x^{6}+3\,a^{4}b^{5}x^{4}+3\,a^{5}b^{4}x^{2}+a^{6}b^{3})x^{4}+3\,(a^{4}b^{5}x^{6}+3\,a^{5}b^{4}x^{4}+3\,a^{6}b^{3}x^{2}+a^{7}b^{2})x^{2})}+\frac{35\,d^{3}\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}-\frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}\right)}{8192\,a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3072*(77*b^3*d^(7/2)*x^(13/2) + 803*a*b^2*d^(7/2)*x^(9/2) + 447*a^2*b*d^
(7/2)*x^(5/2) + 105*a^3*d^(7/2)*sqrt(x))/(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a
^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) + 1/192*((7*b^4*d^(7/2)*x^5 + 54*a*b^
3*d^(7/2)*x^3 + 15*a^2*b^2*d^(7/2)*x)*x^(11/2) + 2*(9*a*b^3*d^(7/2)*x^5 + 6
6*a^2*b^2*d^(7/2)*x^3 + 25*a^3*b*d^(7/2)*x)*x^(7/2) - (21*a^2*b^2*d^(7/2)*x
^5 - 14*a^3*b*d^(7/2)*x^3 - 3*a^4*d^(7/2)*x)*x^(3/2))/(a^5*b^4*x^6 + 3*a^6*
b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b + (a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4*b^5*
x^2 + a^5*b^4)*x^6 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b
^3)*x^4 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2)*x^2) +
35/8192*d^3*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*b^2)
```

Fricas [A]

time = 0.34, size = 455, normalized size = 0.81

$$\frac{420\,(a^{2}b^{6}x^{8}+4\,a^{3}b^{5}x^{6}+6\,a^{4}b^{4}x^{4}+4\,a^{5}b^{3}x^{2}+a^{6}b^{2})\left(-\frac{d^{14}}{a^{11}b^{9}}\right)^{\frac14}\arctan\left(-\frac{\sqrt{dx}\,a^{8}b^{7}d^{3}\left(-\frac{d^{14}}{a^{11}b^{9}}\right)^{\frac34}-\sqrt{a^{6}b^{4}\sqrt{-\frac{d^{14}}{a^{11}b^{9}}}+d^{7}x}\,a^{8}b^{7}\left(-\frac{d^{14}}{a^{11}b^{9}}\right)^{\frac34}}{d^{14}}\right)+105\,(a^{2}b^{6}x^{8}+4\,a^{3}b^{5}x^{6}+6\,a^{4}b^{4}x^{4}+4\,a^{5}b^{3}x^{2}+a^{6}b^{2})\left(-\frac{d^{14}}{a^{11}b^{9}}\right)^{\frac14}\log\left(35\,a^{3}b^{2}\left(-\frac{d^{14}}{a^{11}b^{9}}\right)^{\frac14}+35\sqrt{dx}\,d^{3}\right)-105\,(a^{2}b^{6}x^{8}+4\,a^{3}b^{5}x^{6}+6\,a^{4}b^{4}x^{4}+4\,a^{5}b^{3}x^{2}+a^{6}b^{2})\left(-\frac{d^{14}}{a^{11}b^{9}}\right)^{\frac14}\log\left(-35\,a^{3}b^{2}\left(-\frac{d^{14}}{a^{11}b^{9}}\right)^{\frac14}+35\sqrt{dx}\,d^{3}\right)+4\,(35\,b^{3}d^{3}x^{6}+125\,ab^{2}d^{3}x^{4}-399\,a^{2}bd^{3}x^{2}-105\,a^{3}d^{3})\sqrt{dx}}{12288\,(a^{2}b^{6}x^{8}+4\,a^{3}b^{5}x^{6}+6\,a^{4}b^{4}x^{4}+4\,a^{5}b^{3}x^{2}+a^{6}b^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(420*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 +
 a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*arctan(-(sqrt(d*x)*a^8*b^7*d^3*(-d^14/(a
^11*b^9))^(3/4) - sqrt(a^6*b^4*sqrt(-d^14/(a^11*b^9)) + d^7*x)*a^8*b^7*(-d^
14/(a^11*b^9))^(3/4))/d^14) + 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*
x^4 + 4*a^5*b^3*x^2 + a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(35*a^3*b^2*(-d^
14/(a^11*b^9))^(1/4) + 35*sqrt(d*x)*d^3) - 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6
 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(-3
5*a^3*b^2*(-d^14/(a^11*b^9))^(1/4) + 35*sqrt(d*x)*d^3) + 4*(35*b^3*d^3*x^6
+ 125*a*b^2*d^3*x^4 - 399*a^2*b*d^3*x^2 - 105*a^3*d^3)*sqrt(d*x))/(a^2*b^6*
x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{7}{2}}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(7/2)/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 3.57, size = 373, normalized size = 0.67

$$\frac{1}{24576}\,d^3\left(\frac{210\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{210\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{105\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{105\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^3b^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{8\left(35\,\sqrt{dx}\,b^3d^8x^6+125\,\sqrt{dx}\,ab^2d^8x^4-399\,\sqrt{dx}\,a^2bd^8x^2-105\,\sqrt{dx}\,a^3d^8\right)}{(bd^2x^2+ad^2)^4a^2b^2\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^3*(210*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*x^2 + a)) + 210*
sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*
sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*x^2 + a)) + 105*sqrt(2)*(a*b^3*d
^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3
*b^3*sgn(b*x^2 + a)) - 105*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^3*sgn(b*x^2 + a)) + 8*(35*sqr
t(d*x)*b^3*d^8*x^6 + 125*sqrt(d*x)*a*b^2*d^8*x^4 - 399*sqrt(d*x)*a^2*b*d^8*
x^2 - 105*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 + a*d^2)^4*a^2*b^2*sgn(b*x^2 + a))
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{7/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.778 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d(dx)^{3/2}}{32ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+$$

```
[Out] 45/1024*d*(d*x)^(3/2)/a^3/b/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(3/2)/b/(b*x^2+
a)^3/((b*x^2+a)^2)^(1/2)+1/32*d*(d*x)^(3/2)/a/b/(b*x^2+a)^2/((b*x^2+a)^2)^(
1/2)+9/256*d*(d*x)^(3/2)/a^2/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-45/4096*d^(5/2
)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/
b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+45/4096*d^(5/2)*(b*x^2+a)*arctan(1+b^(1
/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(13/4)/b^(7/4)*2^(1/2)/((b*x^2+a
)^2)^(1/2)+45/8192*d^(5/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a
^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^
(1/2)-45/8192*d^(5/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4
)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(13/4)/b^(7/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{9d(dx)^{3/2}}{256a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d(dx)^{3/2}}{32ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{5/2}(a+bx^2)\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{5/2}(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{5/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{5/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (45*d*(d*x)^(3/2))/(1024*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(d*x)^
(3/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(d*x)^(3/2)
)/(32*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (9*d*(d*x)^(3/2)
)/(256*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(5/2)*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a +
 b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sq
rt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a +
b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (45*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{(dx)^{5/2}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(3b^2d^2(ab+b^2x^2)\right)\int \frac{\sqrt{dx}}{(ab+b^2x^2)^4}\,dx}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/2}}{32ab\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d(dx)^{3/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/2}}{32ab\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32ab\,(a}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32ab\,(a}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32ab\,(a}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32ab\,(a}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32ab\,(a}$$

$$= \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{32ab\,(a}$$

Mathematica [A]
time = 0.46, size = 201, normalized size = 0.36

$$\frac{(dx)^{5/2}\,(a+bx^2)\left(4\sqrt[4]{a}\,b^{3/4}x^{3/2}(-15a^3+239a^2bx^2+171ab^2x^4+45b^3x^6)-45\sqrt{2}\,(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-45\sqrt{2}\,(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{4096a^{13/4}b^{7/4}x^{5/2}\left((a+bx^2)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(5/2)*(a + b*x^2)*(4*a^(1/4)*b^(3/4)*x^(3/2)*(-15*a^3 + 239*a^2*b*x^
2 + 171*a*b^2*x^4 + 45*b^3*x^6) - 45*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a]
- Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 45*Sqrt[2]*(a + b*x^2)^4*
ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(4096*a^
(13/4)*b^(7/4)*x^(5/2)*((a + b*x^2)^2)^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(361) = 722$.
time = 0.05, size = 1051, normalized size = 1.89

method	result	size
default	Expression too large to display	1051

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8192*(45*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(
1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^4*d^8*x^
8+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
b^4*d^8*x^8+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*b^4*d^8*x^8+360*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+180*2^(1/2)*ln(-
((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^8*x^6+360*2^(1/2)*arctan(
(2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^8*x^6+360*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d
^8*x^6+1368*(a*d^2/b)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+270*2^(1/2)*ln(-((a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^8*x^4+540*2^(1/2)*arctan((2^(1/
2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^8*x^4+540*2^(1/2
)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^8
*x^4+1912*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+180*2^(1/2)*ln(-((a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^8*x^2+360*2^(1/2)*arctan((2^(1/2)*
(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^8*x^2+360*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^8*x^2-12
0*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+45*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2)))*a^4*d^8+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2
/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)/d^5*(b*x^2+a)/(a*d^2/b)^(1/4)/b^
2/a^3/((b*x^2+a)^2)^(5/2)
```

Maxima [**A**]

time = 0.55, size = 582, normalized size = 1.04

$$\frac{135\,b^3d^{\frac{5}{2}}x^{\frac{15}{2}}+657\,ab^2d^{\frac{5}{2}}x^{\frac{11}{2}}+173\,a^2bd^{\frac{5}{2}}x^{\frac{7}{2}}+35\,a^3d^{\frac{5}{2}}x^{\frac{3}{2}}}{3072\,(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b)}-\frac{\left(9\,b^4d^{\frac{5}{2}}x^5-38\,ab^3d^{\frac{5}{2}}x^3-15\,a^2b^2d^{\frac{5}{2}}x\right)x^{\frac{9}{2}}+2\left(11\,ab^3d^{\frac{5}{2}}x^5-42\,a^2b^2d^{\frac{5}{2}}x^3-21\,a^3bd^{\frac{5}{2}}x\right)x^{\frac{5}{2}}+\left(45\,a^2b^2d^{\frac{5}{2}}x^5+18\,a^3bd^{\frac{5}{2}}x^3+5\,a^4d^{\frac{5}{2}}x\right)\sqrt{x}}{192\,(a^5b^4x^6+3\,a^6b^3x^4+3\,a^7b^2x^2+a^8b+(a^2b^7x^6+3\,a^3b^6x^4+3\,a^4b^5x^2+a^5b^4)x^6+3\,(a^3b^6x^6+3\,a^4b^5x^4+3\,a^5b^4x^2+a^6b^3)x^4+3\,(a^4b^5x^6+3\,a^5b^4x^4+3\,a^6b^3x^2+a^7b^2)x^2)}+\frac{45\,d^{\frac{5}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8192\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3072*(135*b^3*d^(5/2)*x^(15/2) + 657*a*b^2*d^(5/2)*x^(11/2) + 173*a^2*b*d
^(5/2)*x^(7/2) + 35*a^3*d^(5/2)*x^(3/2))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a
^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) - 1/192*((9*b^4*d^(5/2)*x^5 - 38*a*b^3*
d^(5/2)*x^3 - 15*a^2*b^2*d^(5/2)*x)*x^(9/2) + 2*(11*a*b^3*d^(5/2)*x^5 - 42*
a^2*b^2*d^(5/2)*x^3 - 21*a^3*b*d^(5/2)*x)*x^(5/2) + (45*a^2*b^2*d^(5/2)*x^5
 + 18*a^3*b*d^(5/2)*x^3 + 5*a^4*d^(5/2)*x)*sqrt(x))/(a^5*b^4*x^6 + 3*a^6*b^
3*x^4 + 3*a^7*b^2*x^2 + a^8*b + (a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4*b^5*x^
2 + a^5*b^4)*x^6 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b^3
)*x^4 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2)*x^2) + 45
/8192*d^(5/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sq
rt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/(a^3*b)
```

Fricas [A]
time = 0.36, size = 454, normalized size = 0.82

$$-\frac{180\,(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b)\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}\arctan\left(-\frac{91125\sqrt{dx}\,a^3b^2d^7\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}-\sqrt{-8303765625\,a^7b^3d^{10}\sqrt{-\frac{d^{10}}{a^{13}b^7}}+8303765625\,d^{15}x}\,a^3b^2\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}}{91125\,d^{10}}\right)-45\,(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b)\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}\log\left(91125\,a^{10}b^5\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{3}{4}}+91125\sqrt{dx}\,d^7\right)+45\,(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b)\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}\log\left(-91125\,a^{10}b^5\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{3}{4}}+91125\sqrt{dx}\,d^7\right)-4\,(45\,b^3d^2x^7+171\,ab^2d^2x^5+239\,a^2bd^2x^3-15\,a^3d^2x)\sqrt{dx}}{4096\,(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4096*(180*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 +
 a^7*b)*(-d^10/(a^13*b^7))^(1/4)*arctan(-1/91125*(91125*sqrt(d*x)*a^3*b^2*d
^7*(-d^10/(a^13*b^7))^(1/4) - sqrt(-8303765625*a^7*b^3*d^10*sqrt(-d^10/(a^1
3*b^7)) + 8303765625*d^15*x)*a^3*b^2*(-d^10/(a^13*b^7))^(1/4))/d^10) - 45*(
a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)*(-d^10
/(a^13*b^7))^(1/4)*log(91125*a^10*b^5*(-d^10/(a^13*b^7))^(3/4) + 91125*sqrt
(d*x)*d^7) + 45*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^
2 + a^7*b)*(-d^10/(a^13*b^7))^(1/4)*log(-91125*a^10*b^5*(-d^10/(a^13*b^7))^
(3/4) + 91125*sqrt(d*x)*d^7) - 4*(45*b^3*d^2*x^7 + 171*a*b^2*d^2*x^5 + 239*
a^2*b*d^2*x^3 - 15*a^3*d^2*x)*sqrt(d*x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a
^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{5}{2}}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(5/2)/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]

time = 4.52, size = 386, normalized size = 0.69

$$\frac{1}{8192}d^2\left(\frac{90\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^4d\,\mathrm{sgn}\left(bx^2+a\right)}+\frac{90\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^4d\,\mathrm{sgn}\left(bx^2+a\right)}-\frac{45\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^4d\,\mathrm{sgn}\left(bx^2+a\right)}+\frac{45\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^4d\,\mathrm{sgn}\left(bx^2+a\right)}+\frac{8\left(45\sqrt{dx}\,b^3d^8x^7+171\sqrt{dx}\,ab^2d^8x^5+239\sqrt{dx}\,a^2bd^8x^3-15\sqrt{dx}\,a^3d^8x\right)}{\left(bd^2x^2+ad^2\right)^4a^3b\,\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d^2*(90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^4*d*sgn(b*x^2 + a)) + 90*s
qrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*s
qrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^4*d*sgn(b*x^2 + a)) - 45*sqrt(2)*(a*b^3*d
^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4
*b^4*d*sgn(b*x^2 + a)) + 45*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*
d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^4*d*sgn(b*x^2 + a)) + 8*(45*
sqrt(d*x)*b^3*d^8*x^7 + 171*sqrt(d*x)*a*b^2*d^8*x^5 + 239*sqrt(d*x)*a^2*b*d
^8*x^3 - 15*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^2 + a*d^2)^4*a^3*b*sgn(b*x^2 + a
)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^{5/2}}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.779 $\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{76}{}$$

```
[Out] -77/4096*d^(3/2)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(
1/2))/a^(15/4)/b^(5/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+77/4096*d^(3/2)*(b*x^2+a
)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(15/4)/b^(5/4)*2^
(1/2)/((b*x^2+a)^2)^(1/2)-77/8192*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^
(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)
/((b*x^2+a)^2)^(1/2)+77/8192*d^(3/2)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)
*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)/((b*
x^2+a)^2)^(1/2)+77/3072*d*(d*x)^(1/2)/a^3/b/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)
^(1/2)/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+1/96*d*(d*x)^(1/2)/a/b/(b*x^2+a)^2
/((b*x^2+a)^2)^(1/2)+11/768*d*(d*x)^(1/2)/a^2/b/(b*x^2+a)/((b*x^2+a)^2)^(1/
2)
```

Rubi [A]
time = 0.28, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{11d\sqrt{dx}}{768a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{3/2}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (77*d*Sqrt[d*x])/(3072*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[d*x
])/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*Sqrt[d*x])/(96*
a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (11*d*Sqrt[d*x])/(768*
a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*
a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(3/2)*(a + b*x^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a
^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]
])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^
(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{(dx)^{3/2}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2d^2(ab+b^2x^2)\right)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^4}\,d}{16\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96ab\,(}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96ab\,(}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96ab\,(}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96ab\,(}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96ab\,(}$$

$$= \frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96ab\,(}$$

Mathematica [A]
time = 0.44, size = 201, normalized size = 0.36

$$\frac{(dx)^{3/2}\left(a+bx^2\right)\left(4a^{3/4}\sqrt[4]{b}\sqrt{x}\left(-231a^3+351a^2bx^2+275ab^2x^4+77b^3x^6\right)-231\sqrt{2}\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+231\sqrt{2}\left(a+bx^2\right)^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{12288a^{15/4}b^{5/4}x^{3/2}\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(4*a^(3/4)*b^(1/4)*Sqrt[x]*(-231*a^3 + 351*a^2*b*x
^2 + 275*a*b^2*x^4 + 77*b^3*x^6) - 231*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a
] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 231*Sqrt[2]*(a + b*x^2)
^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(1228
8*a^(15/4)*b^(5/4)*x^(3/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(361) = 722$.
time = 0.05, size = 1136, normalized size = 2.04

method	result	size
default	Expression too large to display	1136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24576*(231*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*b^4*d^6*x^8+462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+462*(a*d^2/b)^(1/4)*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+92
4*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^
2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^
3*d^6*x^6+1848*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+1848*(a*d^2/b)^(1/4)*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+616*(
d*x)^(13/2)*a*b^3+1386*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2)))*a^2*b^2*d^6*x^4+2772*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/
2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4+2772*(a*d^
2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*a^2*b^2*d^6*x^4+2200*(d*x)^(9/2)*a^2*b^2*d^2+924*(a*d^2/b)^(1/4)*2^(1
/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^6*x^2+1848*(a*d^2/
b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*a^3*b*d^6*x^2+1848*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^6*x^2+2808*(d*x)^(5/2)*a^3*b*d^4+
231*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^
4*d^6+462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/
4))/(a*d^2/b)^(1/4))*a^4*d^6+462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d
*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-1848*(d*x)^(1/2)*a^4*d^
6)/d^5*(b*x^2+a)/b/a^4/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.55, size = 586, normalized size = 1.05

$$-\frac{385\,b^{3}d^{\frac{3}{2}}x^{\frac{13}{2}}+495\,ab^{2}d^{\frac{3}{2}}x^{\frac{9}{2}}+315\,a^{2}bd^{\frac{3}{2}}x^{\frac{5}{2}}+77\,a^{3}d^{\frac{3}{2}}\sqrt{x}}{1024\,(a^{3}b^{5}x^{8}+4\,a^{4}b^{4}x^{6}+6\,a^{5}b^{3}x^{4}+4\,a^{6}b^{2}x^{2}+a^{7}b)}+\frac{\left(77\,b^{4}d^{\frac{3}{2}}x^{5}+66\,ab^{3}d^{\frac{3}{2}}x^{3}+21\,a^{2}b^{2}d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+2\left(99\,ab^{3}d^{\frac{3}{2}}x^{5}+102\,a^{2}b^{2}d^{\frac{3}{2}}x^{3}+35\,a^{3}bd^{\frac{3}{2}}x\right)x^{\frac{7}{2}}+\left(153\,a^{2}b^{2}d^{\frac{3}{2}}x^{5}+202\,a^{3}bd^{\frac{3}{2}}x^{3}+81\,a^{4}d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}}{192\,(a^{6}b^{3}x^{6}+3\,a^{7}b^{2}x^{4}+3\,a^{8}bx^{2}+a^{9}+(a^{3}b^{6}x^{6}+3\,a^{4}b^{5}x^{4}+3\,a^{5}b^{4}x^{2}+a^{6}b^{3})x^{6}+3\,(a^{4}b^{5}x^{6}+3\,a^{5}b^{4}x^{4}+3\,a^{6}b^{3}x^{2}+a^{7}b^{2})x^{4}+3\,(a^{5}b^{4}x^{6}+3\,a^{6}b^{3}x^{4}+3\,a^{7}b^{2}x^{2}+a^{8}b)x^{2})}+\frac{77\,d\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{8192\,a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/1024*(385*b^3*d^(3/2)*x^(13/2) + 495*a*b^2*d^(3/2)*x^(9/2) + 315*a^2*b*d
^(3/2)*x^(5/2) + 77*a^3*d^(3/2)*sqrt(x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a
^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) + 1/192*((77*b^4*d^(3/2)*x^5 + 66*a*b^3
*d^(3/2)*x^3 + 21*a^2*b^2*d^(3/2)*x)*x^(11/2) + 2*(99*a*b^3*d^(3/2)*x^5 + 1
02*a^2*b^2*d^(3/2)*x^3 + 35*a^3*b*d^(3/2)*x)*x^(7/2) + (153*a^2*b^2*d^(3/2)
*x^5 + 202*a^3*b*d^(3/2)*x^3 + 81*a^4*d^(3/2)*x)*x^(3/2))/(a^6*b^3*x^6 + 3*
a^7*b^2*x^4 + 3*a^8*b*x^2 + a^9 + (a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*
x^2 + a^6*b^3)*x^6 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b
^2)*x^4 + 3*(a^5*b^4*x^6 + 3*a^6*b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b)*x^2) + 77
/8192*d*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2
*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*sq
rt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b
^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(3/4)*b^(1/4)))/(a^3*b)
```

Fricas [A]
time = 0.36, size = 429, normalized size = 0.77

$$\frac{924\,(a^{3}b^{5}x^{8}+4\,a^{4}b^{4}x^{6}+6\,a^{5}b^{3}x^{4}+4\,a^{6}b^{2}x^{2}+a^{7}b)\left(-\frac{d^{6}}{a^{15}b^{5}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{11}b^{4}d\left(-\frac{d^{6}}{a^{15}b^{5}}\right)^{\frac{3}{4}}-\sqrt{a^{8}b^{2}\sqrt{-\frac{d^{6}}{a^{15}b^{5}}}+d^{3}x}\,a^{11}b^{4}\left(-\frac{d^{6}}{a^{15}b^{5}}\right)^{\frac{3}{4}}}{d^{6}}\right)+231\,(a^{3}b^{5}x^{8}+4\,a^{4}b^{4}x^{6}+6\,a^{5}b^{3}x^{4}+4\,a^{6}b^{2}x^{2}+a^{7}b)\left(-\frac{d^{6}}{a^{15}b^{5}}\right)^{\frac{1}{4}}\log\left(77\,a^{4}b\left(-\frac{d^{6}}{a^{15}b^{5}}\right)^{\frac{1}{4}}+77\sqrt{dx}\,d\right)-231\,(a^{3}b^{5}x^{8}+4\,a^{4}b^{4}x^{6}+6\,a^{5}b^{3}x^{4}+4\,a^{6}b^{2}x^{2}+a^{7}b)\left(-\frac{d^{6}}{a^{15}b^{5}}\right)^{\frac{1}{4}}\log\left(-77\,a^{4}b\left(-\frac{d^{6}}{a^{15}b^{5}}\right)^{\frac{1}{4}}+77\sqrt{dx}\,d\right)+4\,(77\,b^{3}dx^{6}+275\,ab^{2}dx^{4}+351\,a^{2}bdx^{2}-231\,a^{3}d)\sqrt{dx}}{12288\,(a^{3}b^{5}x^{8}+4\,a^{4}b^{4}x^{6}+6\,a^{5}b^{3}x^{4}+4\,a^{6}b^{2}x^{2}+a^{7}b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(924*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 +
 a^7*b)*(-d^6/(a^15*b^5))^(1/4)*arctan(-(sqrt(d*x)*a^11*b^4*d*(-d^6/(a^15*b
^5))^(3/4) - sqrt(a^8*b^2*sqrt(-d^6/(a^15*b^5)) + d^3*x)*a^11*b^4*(-d^6/(a^
15*b^5))^(3/4))/d^6) + 231*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4
*a^6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(77*a^4*b*(-d^6/(a^15*b^5)
)^(1/4) + 77*sqrt(d*x)*d) - 231*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^
4 + 4*a^6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(-77*a^4*b*(-d^6/(a^1
5*b^5))^(1/4) + 77*sqrt(d*x)*d) + 4*(77*b^3*d*x^6 + 275*a*b^2*d*x^4 + 351*a
^2*b*d*x^2 - 231*a^3*d)*sqrt(d*x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3
*x^4 + 4*a^6*b^2*x^2 + a^7*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(3/2)/((a + b*x**2)**2)**(5/2), x)
```

Giac [A]
time = 2.91, size = 371, normalized size = 0.67

$$\frac{1}{24576}\,d\left(\frac{462\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2\mathrm{sgn}\left(bx^2+a\right)}+\frac{462\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2\mathrm{sgn}\left(bx^2+a\right)}+\frac{231\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^2\mathrm{sgn}\left(bx^2+a\right)}-\frac{231\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^4b^2\mathrm{sgn}\left(bx^2+a\right)}+\frac{8\left(77\,\sqrt{dx}\,b^3d^8x^6+275\,\sqrt{dx}\,ab^2d^8x^4+351\,\sqrt{dx}\,a^2bd^8x^2-231\,\sqrt{dx}\,a^3d^8\right)}{(bd^2x^2+ad^2)^4a^3b\mathrm{sgn}\left(bx^2+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d*(462*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*sgn(b*x^2 + a)) + 462*sq
rt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sq
rt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*sgn(b*x^2 + a)) + 231*sqrt(2)*(a*b^3*d^2
)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b
^2*sgn(b*x^2 + a)) - 231*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^2*sgn(b*x^2 + a)) + 8*(77*sqrt(
d*x)*b^3*d^8*x^6 + 275*sqrt(d*x)*a*b^2*d^8*x^4 + 351*sqrt(d*x)*a^2*b*d^8*x^
2 - 231*sqrt(d*x)*a^3*d^8)/((b*d^2*x^2 + a*d^2)^4*a^3*b*sgn(b*x^2 + a)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{(d\,x)}^{3/2}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.780 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=556

$$\frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 195/1024*(d*x)^(3/2)/a^4/d/((b*x^2+a)^2)^(1/2)+1/8*(d*x)^(3/2)/a/d/(b*x^2+a
)^3/((b*x^2+a)^2)^(1/2)+13/96*(d*x)^(3/2)/a^2/d/(b*x^2+a)^2/((b*x^2+a)^2)^(
1/2)+39/256*(d*x)^(3/2)/a^3/d/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-195/4096*(b*x^2
+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(17/4)/
b^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1/2)+195/4096*(b*x^2+a)*arctan(1+b^(1/4)*2^(
1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*d^(1/2)/a^(17/4)/b^(3/4)*2^(1/2)/((b*x^2+
a)^2)^(1/2)+195/8192*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)
*b^(1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(17/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2
)^(1/2)-195/8192*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(
1/4)*2^(1/2)*(d*x)^(1/2))*d^(1/2)/a^(17/4)/b^(3/4)*2^(1/2)/((b*x^2+a)^2)^(1
/2)
```

Rubi [A]
time = 0.29, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{13(dx)^{3/2}}{96a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195\sqrt{d}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195\sqrt{d}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{39(dx)^{3/2}}{256a^3d\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (195*(d*x)^(3/2))/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x)^(3/2
)/(8*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13*(d*x)^(3/2))/
(96*a^2*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (39*(d*x)^(3/2))
/(256*a^3*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*Sqrt[d]*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a
+ b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a
+ b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (195*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
```

```
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{\sqrt{dx}}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(13b^3(ab+b^2x^2)\right)\int \frac{\sqrt{dx}}{(ab+b^2x^2)^4}\,dx}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(dx)^{3/2}}{96a^2d\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(dx)^{3/2}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(dx)^{3/2}}{96a^2d\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a^2d}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a^2d}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a^2d}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a^2d}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a^2d}$$

$$= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{}{96a^2d}$$

Mathematica [A]
time = 0.30, size = 201, normalized size = 0.36

$$\frac{\sqrt{dx}\,(a+bx^2)\left(4\sqrt[4]{a}\,x^{3/2}(1853a^3+3107a^2bx^2+2223ab^2x^4+585b^3x^6)-\frac{585\sqrt{2}\,(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{b^{3/4}}-\frac{585\sqrt{2}\,(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{b^{3/4}}\right)}{12288a^{17/4}\sqrt{x}\,\left((a+bx^2)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[d*x]*(a + b*x^2)*(4*a^(1/4)*x^(3/2)*(1853*a^3 + 3107*a^2*b*x^2 + 2223
*a*b^2*x^4 + 585*b^3*x^6) - (585*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a] - Sq
rt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) - (585*Sqrt[2]*(a + b*
x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^
(3/4)))/(12288*a^(17/4)*Sqrt[x]*((a + b*x^2)^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(360) = 720$.
time = 0.05, size = 1051, normalized size = 1.89

method	result	size
default	Expression too large to display	1051

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24576*(585*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^4*d^8*
x^8+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*b^4*d^8*x^8+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(
a*d^2/b)^(1/4))*b^4*d^8*x^8+4680*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+2340*2^(1
/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^8*x^6+4680*2^(1/2
)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^8*x
^6+4680*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*a*b^3*d^8*x^6+17784*(a*d^2/b)^(1/4)*(d*x)^(11/2)*a*b^3*d^2+3510*2^(1/2)*
ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^8*x^4+7020*2^(1/2)*
arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^8*x
^4+7020*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*a^2*b^2*d^8*x^4+24856*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+2340*2^(1/
2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^8*x^2+4680*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^8*x^
2+4680*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*a^3*b*d^8*x^2+14824*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+585*2^(1/2)*ln(
-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8+1170*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8+1170*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)/d^7*(b*
x^2+a)/(a*d^2/b)^(1/4)/b/a^4/((b*x^2+a)^2)^(5/2)
```

Maxima [A]
time = 0.54, size = 569, normalized size = 1.02

$$\frac{195\,b^3\sqrt{d}\,x^{\frac{15}{2}}+117\,ab^2\sqrt{d}\,x^{\frac{11}{2}}+65\,a^2b\sqrt{d}\,x^{\frac{7}{2}}+15\,a^3\sqrt{d}\,x^{\frac{3}{2}}}{1024\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}+\frac{\left(117\,b^4\sqrt{d}\,x^5+130\,ab^3\sqrt{d}\,x^3+45\,a^2b^2\sqrt{d}\,x\right)x^{\frac{9}{2}}+2\left(143\,ab^3\sqrt{d}\,x^5+174\,a^2b^2\sqrt{d}\,x^3+63\,a^3b\sqrt{d}\,x\right)x^{\frac{5}{2}}+\left(201\,a^2b^2\sqrt{d}\,x^5+282\,a^3b\sqrt{d}\,x^3+113\,a^4\sqrt{d}\,x\right)\sqrt{x}}{192\,(a^6b^3x^6+3\,a^7b^2x^4+3\,a^8bx^2+a^9+(a^3b^6x^6+3\,a^4b^5x^4+3\,a^5b^4x^2+a^6b^3)x^6+3\,(a^4b^5x^6+3\,a^5b^4x^4+3\,a^6b^3x^2+a^7b^2)x^4+3\,(a^5b^4x^6+3\,a^6b^3x^4+3\,a^7b^2x^2+a^8b)x^2)}+\frac{195\,\sqrt{d}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8192\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/1024*(195*b^3*sqrt(d)*x^(15/2) + 117*a*b^2*sqrt(d)*x^(11/2) + 65*a^2*b*sq
rt(d)*x^(7/2) + 15*a^3*sqrt(d)*x^(3/2))/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^
6*b^2*x^4 + 4*a^7*b*x^2 + a^8) + 1/192*((117*b^4*sqrt(d)*x^5 + 130*a*b^3*sq
rt(d)*x^3 + 45*a^2*b^2*sqrt(d)*x)*x^(9/2) + 2*(143*a*b^3*sqrt(d)*x^5 + 174*
a^2*b^2*sqrt(d)*x^3 + 63*a^3*b*sqrt(d)*x)*x^(5/2) + (201*a^2*b^2*sqrt(d)*x^
5 + 282*a^3*b*sqrt(d)*x^3 + 113*a^4*sqrt(d)*x)*sqrt(x))/(a^6*b^3*x^6 + 3*a^
7*b^2*x^4 + 3*a^8*b*x^2 + a^9 + (a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^
2 + a^6*b^3)*x^6 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2
)*x^4 + 3*(a^5*b^4*x^6 + 3*a^6*b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b)*x^2) + 195/
8192*sqrt(d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
))/a^4
```

Fricas [A]
time = 0.36, size = 414, normalized size = 0.74

$$-\frac{2340\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{7414875\,\sqrt{dx}\,a^4bd\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}-\sqrt{-54980371265625\,a^9bd^2\sqrt{-\frac{d^2}{a^{17}b^3}}+54980371265625\,d^3x}\,a^4b\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}}{7414875\,d^2}\right)-585\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}\log\left(7414875\,a^{13}b^2\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{3}{4}}+7414875\,\sqrt{dx}\,d\right)+585\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}\log\left(-7414875\,a^{13}b^2\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{3}{4}}+7414875\,\sqrt{dx}\,d\right)-4\,(585\,b^3x^7+2223\,ab^2x^5+3107\,a^2bx^3+1853\,a^3x)\sqrt{dx}}{12288\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(2340*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 +
 a^8)*(-d^2/(a^17*b^3))^(1/4)*arctan(-1/7414875*(7414875*sqrt(d*x)*a^4*b*d*
(-d^2/(a^17*b^3))^(1/4) - sqrt(-54980371265625*a^9*b*d^2*sqrt(-d^2/(a^17*b^
3)) + 54980371265625*d^3*x)*a^4*b*(-d^2/(a^17*b^3))^(1/4))/d^2) - 585*(a^4*
b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)*(-d^2/(a^17*b^
3))^(1/4)*log(7414875*a^13*b^2*(-d^2/(a^17*b^3))^(3/4) + 7414875*sqrt(d*x)*
d) + 585*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)*
(-d^2/(a^17*b^3))^(1/4)*log(-7414875*a^13*b^2*(-d^2/(a^17*b^3))^(3/4) + 741
4875*sqrt(d*x)*d) - 4*(585*b^3*x^7 + 2223*a*b^2*x^5 + 3107*a^2*b*x^3 + 1853
*a^3*x)*sqrt(d*x))/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x
^2 + a^8)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(sqrt(d*x)/((a + b*x**2)**2)**(5/2), x)
```

Giac [**A**]
time = 3.88, size = 371, normalized size = 0.67

$$\frac{\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^3\mathrm{sgn}(bx^2+a)}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^3\mathrm{sgn}(bx^2+a)}-\frac{585\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^3\mathrm{sgn}(bx^2+a)}+\frac{585\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^5b^3\mathrm{sgn}(bx^2+a)}+\frac{8\left(585\sqrt{dx}\,b^3d^9x^7+2223\sqrt{dx}\,ab^2d^9x^5+3107\sqrt{dx}\,a^2bd^9x^3+1853\sqrt{dx}\,a^3d^9x\right)}{\left(bd^2x^2+ad^2\right)^4a^4\mathrm{sgn}(bx^2+a)}}{24576\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*(1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/
b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^3*sgn(b*x^2 + a)) + 1170*sq
rt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sq
rt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^3*sgn(b*x^2 + a)) - 585*sqrt(2)*(a*b^3*d^2
)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b
^3*sgn(b*x^2 + a)) + 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^3*sgn(b*x^2 + a)) + 8*(585*sqrt
(d*x)*b^3*d^9*x^7 + 2223*sqrt(d*x)*a*b^2*d^9*x^5 + 3107*sqrt(d*x)*a^2*b*d^9
*x^3 + 1853*sqrt(d*x)*a^3*d^9*x)/((b*d^2*x^2 + a*d^2)^4*a^4*sgn(b*x^2 + a))
)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.781 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=556

$$\frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+$$

```
[Out] -1155/4096*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/
a^(19/4)/b^(1/4)*2^(1/2)/d^(1/2)/((b*x^2+a)^2)^(1/2)+1155/4096*(b*x^2+a)*ar
ctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(19/4)/b^(1/4)*2^(1/2
)/d^(1/2)/((b*x^2+a)^2)^(1/2)-1155/8192*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1
/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(19/4)/b^(1/4)*2^(1/2)/d
^(1/2)/((b*x^2+a)^2)^(1/2)+1155/8192*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)
*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(19/4)/b^(1/4)*2^(1/2)/d^(1
/2)/((b*x^2+a)^2)^(1/2)+385/1024*(d*x)^(1/2)/a^4/d/((b*x^2+a)^2)^(1/2)+1/8*
(d*x)^(1/2)/a/d/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+5/32*(d*x)^(1/2)/a^2/d/(b*x
^2+a)^2/((b*x^2+a)^2)^(1/2)+55/256*(d*x)^(1/2)/a^3/d/(b*x^2+a)/((b*x^2+a)^2
)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 556, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1126, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{55\sqrt{dx}}{256a^3d\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (385*Sqrt[d*x])/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d*x]/(8
*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d*x])/(32*a^2
*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (55*Sqrt[d*x])/(256*a^3
*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a + b*x^2)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)*
b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*(a + b*x^2)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)
*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4
096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (11
55*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\,\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{1}{\sqrt{dx}\,(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(15b^3(ab+b^2x^2)\right)\int \frac{1}{\sqrt{dx}\,(ab+b^2}}{16a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\sqrt{dx}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{dx}}{32a^2d\,(a+bx^2)^2\sqrt{a^2+2abx^2}}$$

$$= \frac{\sqrt{dx}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{dx}}{32a^2d\,(a+bx^2)^2\sqrt{a^2+2abx^2}}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A]
time = 0.29, size = 201, normalized size = 0.36

$$\frac{\sqrt{x}\,(a+bx^2)\left(4a^{3/4}\sqrt{x}\,(893a^3+1755a^2bx^2+1375ab^2x^4+385b^3x^6)-\frac{1155\sqrt{2}\,(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt[4]{b}}+\frac{1155\sqrt{2}\,(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt[4]{b}}\right)}{4096a^{19/4}\sqrt{dx}\,\left((a+bx^2)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]

[Out] (Sqrt[x]*(a + b*x^2)*(4*a^(3/4)*Sqrt[x]*(893*a^3 + 1755*a^2*b*x^2 + 1375*a*b^2*x^4 + 385*b^3*x^6) - (1155*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (1155*Sqrt[2]*(a + b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4)))/(4096*a^(19/4)*Sqrt[d*x]*((a + b*x^2)^2)^(5/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. $2(360) = 720$.
time = 0.05, size = 1133, normalized size = 2.04

method	result	size
default	Expression too large to display	1133

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8192*(1155*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^4*d^6*x^8+2310*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+2310*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+4620*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^6*x^6+9240*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+9240*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+6930*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^6*x^4+13860*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4+13860*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4+3080*(d*x)^(13/2)*a*b^3+4620*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^6*x^2+9240*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^6*x^2+9240*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^6*x^2+11000*(d*x)^(9/2)*a^2*b^2*d^2+1155*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^6+2310*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6+2310*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2

```
/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6+14040*(d*x)^(5/2)*a^3*b*d^4+7144*(d*x)^
(1/2)*a^4*d^6)/d^7*(b*x^2+a)/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3072*(5267*b^3*x^(13/2) + 11645*a*b^2*x^(9/2) + 9441*a^2*b*x^(5/2) + 2679
*a^3*sqrt(x))/(a^4*b^4*sqrt(d)*x^8 + 4*a^5*b^3*sqrt(d)*x^6 + 6*a^6*b^2*sqrt
(d)*x^4 + 4*a^7*b*sqrt(d)*x^2 + a^8*sqrt(d)) - 1/192*((257*b^5*sqrt(d)*x^5
+ 378*a*b^4*sqrt(d)*x^3 + 153*a^2*b^3*sqrt(d)*x)*x^(11/2) + 2*(303*a*b^4*sq
rt(d)*x^5 + 462*a^2*b^3*sqrt(d)*x^3 + 191*a^3*b^2*sqrt(d)*x)*x^(7/2) + (381
*a^2*b^3*sqrt(d)*x^5 + 610*a^3*b^2*sqrt(d)*x^3 + 261*a^4*b*sqrt(d)*x)*x^(3/
2))/(a^7*b^3*d*x^6 + 3*a^8*b^2*d*x^4 + 3*a^9*b*d*x^2 + a^10*d + (a^4*b^6*d*
x^6 + 3*a^5*b^5*d*x^4 + 3*a^6*b^4*d*x^2 + a^7*b^3*d)*x^6 + 3*(a^5*b^5*d*x^6
 + 3*a^6*b^4*d*x^4 + 3*a^7*b^3*d*x^2 + a^8*b^2*d)*x^4 + 3*(a^6*b^4*d*x^6 +
3*a^7*b^3*d*x^4 + 3*a^8*b^2*d*x^2 + a^9*b*d)*x^2) - 893/8192*(2*sqrt(2)*sqr
t(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*sqrt(d)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*sqrt(d)*log(sqrt(2)*a^(1
/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*sqrt
(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^
(1/4)))/(a^4*d) + integrate(1/((a^4*b*sqrt(d)*x^2 + a^5*sqrt(d))*sqrt(x)),
x)
```

Fricas [A]
time = 0.35, size = 416, normalized size = 0.75

$$\frac{4620\,(a^4b^4dx^8+4\,a^5b^3dx^6+6\,a^6b^2dx^4+4\,a^7bdx^2+a^8d)\left(-\frac{1}{a^{19}bd^2}\right)^{\frac14}\arctan\left(\sqrt{a^{10}d^2\sqrt{-\frac{1}{a^{19}bd^2}}+dx}\,a^{14}bd\left(-\frac{1}{a^{19}bd^2}\right)^{\frac34}-\sqrt{dx}\,a^{14}bd\left(-\frac{1}{a^{19}bd^2}\right)^{\frac34}\right)+1155\,(a^4b^4dx^8+4\,a^5b^3dx^6+6\,a^6b^2dx^4+4\,a^7bdx^2+a^8d)\left(-\frac{1}{a^{19}bd^2}\right)^{\frac14}\log\left(a^5d\left(-\frac{1}{a^{19}bd^2}\right)^{\frac14}+\sqrt{dx}\right)-1155\,(a^4b^4dx^8+4\,a^5b^3dx^6+6\,a^6b^2dx^4+4\,a^7bdx^2+a^8d)\left(-\frac{1}{a^{19}bd^2}\right)^{\frac14}\log\left(-a^5d\left(-\frac{1}{a^{19}bd^2}\right)^{\frac14}+\sqrt{dx}\right)+4\,(385\,b^3x^6+1375\,ab^2x^4+1755\,a^2bx^2+893\,a^3)\sqrt{dx}}{4096\,(a^4b^4dx^8+4\,a^5b^3dx^6+6\,a^6b^2dx^4+4\,a^7bdx^2+a^8d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/4096*(4620*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4 + 4*a^7*b*d
*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*arctan(sqrt(a^10*d^2*sqrt(-1/(a^19*b*
d^2)) + d*x)*a^14*b*d*(-1/(a^19*b*d^2))^(3/4) - sqrt(d*x)*a^14*b*d*(-1/(a^1
9*b*d^2))^(3/4)) + 1155*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4
+ 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*log(a^5*d*(-1/(a^19*b*d^2)
)^(1/4) + sqrt(d*x)) - 1155*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*
```

```
x^4 + 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*log(-a^5*d*(-1/(a^19*b
*d^2))^(1/4) + sqrt(d*x)) + 4*(385*b^3*x^6 + 1375*a*b^2*x^4 + 1755*a^2*b*x^
2 + 893*a^3)*sqrt(d*x))/(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4
+ 4*a^7*b*d*x^2 + a^8*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(5/2)), x)
```

Giac [A]
time = 4.11, size = 377, normalized size = 0.68

$$\frac{1155\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\mathrm{sgn}\left(bx^2+a\right)}+\frac{1155\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\mathrm{sgn}\left(bx^2+a\right)}+\frac{1155\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^5bd\mathrm{sgn}\left(bx^2+a\right)}-\frac{1155\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^5bd\mathrm{sgn}\left(bx^2+a\right)}+\frac{385\sqrt{dx}\,b^3d^7x^6+1375\sqrt{dx}\,ab^2d^7x^4+1755\sqrt{dx}\,a^2bd^7x^2+893\sqrt{dx}\,a^3d^7}{1024\left(bd^2x^2+ad^2\right)^4a^4\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1155/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d*sgn(b*x^2 + a)) + 1155/4096*s
qrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*s
qrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d*sgn(b*x^2 + a)) + 1155/8192*sqrt(2)*(a*
b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a^5*b*d*sgn(b*x^2 + a)) - 1155/8192*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d*sgn(b*x^2 + a))
+ 1/1024*(385*sqrt(d*x)*b^3*d^7*x^6 + 1375*sqrt(d*x)*a*b^2*d^7*x^4 + 1755*s
qrt(d*x)*a^2*b*d^7*x^2 + 893*sqrt(d*x)*a^3*d^7)/((b*d^2*x^2 + a*d^2)^4*a^4*
sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d\,x}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

```
[Out] int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

3.782 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=602

$$\frac{663}{1024a^4d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ad\sqrt{dx}\,\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{17}{96a^2d\sqrt{dx}\,\left(a+bx^2\right)^2\sqrt{a^2}}$$

```
[Out] 3315/4096*b^(1/4)*(b*x^2+a)*arctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^
(1/2))/a^(21/4)/d^(3/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-3315/4096*b^(1/4)*(b*x^
2+a)*arctan(1+b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(21/4)/d^(3/2)
*2^(1/2)/((b*x^2+a)^2)^(1/2)-3315/8192*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)
+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(21/4)/d^(3/2)*2^
(1/2)/((b*x^2+a)^2)^(1/2)+3315/8192*b^(1/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*
b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(21/4)/d^(3/2)*2^(1/
2)/((b*x^2+a)^2)^(1/2)+663/1024/a^4/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+1/8/a
/d/(b*x^2+a)^3/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+17/96/a^2/d/(b*x^2+a)^2/(d*x
)^(1/2)/((b*x^2+a)^2)^(1/2)+221/768/a^3/d/(b*x^2+a)/(d*x)^(1/2)/((b*x^2+a)^
2)^(1/2)-3315/1024*(b*x^2+a)/a^5/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{17}{96a^2d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^2}+\frac{1}{8ad\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^3}+\frac{3315\sqrt[4]{b}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\,a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{b}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\,a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{b}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{b}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\left(a+bx^2\right)}{1024a^5d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{663}{1024a^4d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{221}{768a^3d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 663/(1024*a^4*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*Sqrt[
d*x]*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 17/(96*a^2*d*Sqrt[d*x
]*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 221/(768*a^3*d*Sqrt[d*x]
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*(a + b*x^2))/(1024*a^
5*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3315*b^(1/4)*(a + b*x^2)*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^
(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b*x^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a
^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (331
5*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{1}{(dx)^{3/2}(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad\sqrt{dx}\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(17b^3(ab+b^2x^2)\right)\int \frac{\cdots}{(dx)^{3/\cdots}}}{16a\sqrt{a^2+2abx^2+\cdots}}$$

$$= \frac{1}{8ad\sqrt{dx}\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{17}{96a^2d\sqrt{dx}\,(a+bx^2)^2\sqrt{\cdots}}$$

$$= \frac{1}{8ad\sqrt{dx}\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{17}{96a^2d\sqrt{dx}\,(a+bx^2)^2\sqrt{\cdots}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\,(a+bx^2)^3\sqrt{a^2+2ab\cdots}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\,(a+bx^2)^3\sqrt{a^2+2ab\cdots}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\,(a+bx^2)^3\sqrt{a^2+2ab\cdots}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\,(a+bx^2)^3\sqrt{a^2+2ab\cdots}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\,(a+bx^2)^3\sqrt{a^2+2ab\cdots}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\,(a+bx^2)^3\sqrt{a^2+2ab\cdots}}$$

$$= \frac{663}{1024a^4d\sqrt{dx}\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad\sqrt{dx}\,(a+bx^2)^3\sqrt{a^2+2ab\cdots}}$$

Mathematica [A]
time = 0.61, size = 213, normalized size = 0.35

$$\frac{x(a+bx^2)\left(-4\sqrt[4]{a}\,(6144a^4+31501a^3bx^2+52819a^2b^2x^4+37791ab^3x^6+9945b^4x^8)+9945\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)+9945\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}\,(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{12288a^{21/4}(dx)^{3/2}\left((a+bx^2)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (x*(a + b*x^2)*(-4*a^(1/4)*(6144*a^4 + 31501*a^3*b*x^2 + 52819*a^2*b^2*x^4
+ 37791*a*b^3*x^6 + 9945*b^4*x^8) + 9945*Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*x^2
)^4*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 9945*
Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])/(Sqrt[a] + Sqrt[b]*x)]))/(12288*a^(21/4)*(d*x)^(3/2)*((a + b*x^2)^2)^(
5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1080 vs. $2(391) = 782$.
time = 0.08, size = 1081, normalized size = 1.80

method	result
risch	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}}{a^{5}\sqrt{dx}\;d\left(b\,x^{2}+a\right)}+\frac{\left(-\frac{6925b\,d^{6}(dx)^{\frac{3}{2}}}{3072a^{2}\left(d^{2}x^{2}b+a\,d^{2}\right)^{4}}-\frac{15955b^{2}d^{4}(dx)^{\frac{7}{2}}}{3072a^{3}\left(d^{2}x^{2}b+a\,d^{2}\right)^{4}}-\frac{4405b^{3}d^{2}(dx)^{\frac{11}{2}}}{1024a^{4}\left(d^{2}x^{2}b+a\,d^{2}\right)^{4}}-\frac{1267b^{4}(dx)^{\frac{15}{2}}}{1024a^{5}\left(d^{2}x^{2}b+a\,d^{2}\right)^{4}}-\frac{3315\sqrt{2}\ldots}{\ldots}\right.}{\ldots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24576/d*(9945*(d*x)^(1/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2)))*b^4*x^8+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*x^8+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*x^8+39780*(d*x)^(1/2)
*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x
+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*x^6+79560*(d*x
)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*a*b^3*x^6+79560*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*x^6+79560*(a*d^2/b)^(1/4)*b^4*x^8+59670*(d*
x)^(1/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/
2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*x^4+
119340*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*a^2*b^2*x^4+119340*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*x^4+302328*(a*d^2/b)^(1/4)*
a*b^3*x^6+39780*(d*x)^(1/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
```

```
1/2)))*a^3*b*x^2+79560*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*x^2+79560*(d*x)^(1/2)*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*x^2+422552*(a*d^
2/b)^(1/4)*a^2*b^2*x^4+9945*(d*x)^(1/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2)))*a^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2
)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4+19890*(d*x)^(1/2)*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4+252008*(a*d^2/b)^(
1/4)*a^3*b*x^2+49152*(a*d^2/b)^(1/4)*a^4)*(b*x^2+a)/(d*x)^(1/2)/(a*d^2/b)^(
1/4)/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/3072*(3801*b^4*x^(15/2) + 8079*a*b^3*x^(11/2) + 6515*a^2*b^2*x^(7/2) + 1
853*a^3*b*x^(3/2))/(a^5*b^4*d^(3/2)*x^8 + 4*a^6*b^3*d^(3/2)*x^6 + 6*a^7*b^2
*d^(3/2)*x^4 + 4*a^8*b*d^(3/2)*x^2 + a^9*d^(3/2)) - 1/192*((321*b^5*sqrt(d)
*x^5 + 490*a*b^4*sqrt(d)*x^3 + 201*a^2*b^3*sqrt(d)*x)*x^(9/2) + 2*(371*a*b^
4*sqrt(d)*x^5 + 582*a^2*b^3*sqrt(d)*x^3 + 243*a^3*b^2*sqrt(d)*x)*x^(5/2) +
(453*a^2*b^3*sqrt(d)*x^5 + 738*a^3*b^2*sqrt(d)*x^3 + 317*a^4*b*sqrt(d)*x)*s
qrt(x))/(a^7*b^3*d^2*x^6 + 3*a^8*b^2*d^2*x^4 + 3*a^9*b*d^2*x^2 + a^10*d^2 +
 (a^4*b^6*d^2*x^6 + 3*a^5*b^5*d^2*x^4 + 3*a^6*b^4*d^2*x^2 + a^7*b^3*d^2)*x^
6 + 3*(a^5*b^5*d^2*x^6 + 3*a^6*b^4*d^2*x^4 + 3*a^7*b^3*d^2*x^2 + a^8*b^2*d^
2)*x^4 + 3*(a^6*b^4*d^2*x^6 + 3*a^7*b^3*d^2*x^4 + 3*a^8*b^2*d^2*x^2 + a^9*b
*d^2)*x^2) - 1267/8192*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(
1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
 + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(
1/4)*b^(3/4)))/(a^5*d^(3/2)) + integrate(1/((a^4*b*d^(3/2)*x^2 + a^5*d^(3/2
))*x^(3/2)), x)
```

Fricas [A]
time = 0.36, size = 477, normalized size = 0.79

$$\frac{39780\,(a^5b^4d^2x^9+4\,a^6b^3d^2x^7+6\,a^7b^2d^2x^5+4\,a^8bd^2x^3+a^9d^2x)\left(-\frac{b}{a^{21}d^6}\right)^{\frac14}\arctan\left(-\frac{36429280875\,\sqrt{dx}\,a^5bd\left(-\frac{b}{a^{21}d^6}\right)^{\frac14}-\sqrt{-1327092505069640765625\,a^{11}bd^4\sqrt{-\frac{b}{a^{21}d^6}}+1327092505069640765625\,b^2dx}\,a^5d\left(-\frac{b}{a^{21}d^6}\right)^{\frac14}}{36429280875\,b}\right)-9945\,(a^5b^4d^2x^9+4\,a^6b^3d^2x^7+6\,a^7b^2d^2x^5+4\,a^8bd^2x^3+a^9d^2x)\left(-\frac{b}{a^{21}d^6}\right)^{\frac14}\log\left(36429280875\,a^{16}d^5\left(-\frac{b}{a^{21}d^6}\right)^{\frac34}+36429280875\,\sqrt{dx}\,b\right)+9945\,(a^5b^4d^2x^9+4\,a^6b^3d^2x^7+6\,a^7b^2d^2x^5+4\,a^8bd^2x^3+a^9d^2x)\left(-\frac{b}{a^{21}d^6}\right)^{\frac14}\log\left(-36429280875\,a^{16}d^5\left(-\frac{b}{a^{21}d^6}\right)^{\frac34}+36429280875\,\sqrt{dx}\,b\right)-4\,(9945\,b^4x^8+37791\,ab^3x^6+52819\,a^2b^2x^4+31501\,a^3bx^2+6144\,a^4)\sqrt{dx}}{12288\,(a^5b^4d^2x^9+4\,a^6b^3d^2x^7+6\,a^7b^2d^2x^5+4\,a^8bd^2x^3+a^9d^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/12288*(39780*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4
*a^8*b*d^2*x^3 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*arctan(-1/36429280875*(36
429280875*sqrt(d*x)*a^5*b*d*(-b/(a^21*d^6))^(1/4) - sqrt(-1327092505069640
765625*a^11*b*d^4*sqrt(-b/(a^21*d^6)) + 1327092505069640765625*b^2*d*x)*a^5*
d*(-b/(a^21*d^6))^(1/4))/b) - 9945*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6
*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*log(3
6429280875*a^16*d^5*(-b/(a^21*d^6))^(3/4) + 36429280875*sqrt(d*x)*b) + 9945
*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3
 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*log(-36429280875*a^16*d^5*(-b/(a^21*d^6
))^(3/4) + 36429280875*sqrt(d*x)*b) - 4*(9945*b^4*x^8 + 37791*a*b^3*x^6 + 5
2819*a^2*b^2*x^4 + 31501*a^3*b*x^2 + 6144*a^4)*sqrt(d*x))/(a^5*b^4*d^2*x^9
+ 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(5/2)), x)
```

Giac [**A**]
time = 3.90, size = 406, normalized size = 0.67

$$-\frac{\frac{49152}{\sqrt{dx}\,a^5\operatorname{sgn}(bx^2+a)}+\frac{19890\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^2d^2\operatorname{sgn}(bx^2+a)}+\frac{19890\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^2d^2\operatorname{sgn}(bx^2+a)}-\frac{9945\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^6b^2d^2\operatorname{sgn}(bx^2+a)}+\frac{9945\sqrt{2}\,(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^6b^2d^2\operatorname{sgn}(bx^2+a)}+\frac{8\left(3801\sqrt{dx}\,b^4d^7x^7+13215\sqrt{dx}\,ab^3d^7x^5+15955\sqrt{dx}\,a^2b^2d^7x^3+6925\sqrt{dx}\,a^3bd^7x\right)}{(bd^2x^2+ad^2)^4a^5\operatorname{sgn}(bx^2+a)}}{24576\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*(49152/(sqrt(d*x)*a^5*sgn(b*x^2 + a)) + 19890*sqrt(2)*(a*b^3*d^2)^
(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^
(1/4))/(a^6*b^2*d^2*sgn(b*x^2 + a)) + 19890*sqrt(2)*(a*b^3*d^2)^(3/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^
6*b^2*d^2*sgn(b*x^2 + a)) - 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^2*d^2*sgn(b*x^2 + a)) +
 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(a^6*b^2*d^2*sgn(b*x^2 + a)) + 8*(3801*sqrt(d*x)*b^4*d^7*
x^7 + 13215*sqrt(d*x)*a*b^3*d^7*x^5 + 15955*sqrt(d*x)*a^2*b^2*d^7*x^3 + 692
5*sqrt(d*x)*a^3*b*d^7*x)/((b*d^2*x^2 + a*d^2)^4*a^5*sgn(b*x^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{3/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

3.783 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=602

$$\frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{19}{96a^2d(dx)^{3/2}\left(a+bx^2\right)^2}$$

```
[Out] 1045/1024/a^4/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+1/8/a/d/(d*x)^(3/2)/(b*x^2+
a)^3/((b*x^2+a)^2)^(1/2)+19/96/a^2/d/(d*x)^(3/2)/(b*x^2+a)^2/((b*x^2+a)^2)^
(1/2)+95/256/a^3/d/(d*x)^(3/2)/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-7315/3072*(b*x
^2+a)/a^5/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+7315/4096*b^(3/4)*(b*x^2+a)*arc
tan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(23/4)/d^(5/2)*2^(1/2)
/((b*x^2+a)^2)^(1/2)-7315/4096*b^(3/4)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2)*(
d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(23/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+73
15/8192*b^(3/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*b^(1
/4)*2^(1/2)*(d*x)^(1/2))/a^(23/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-7315/
8192*b^(3/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^(1/4)
*2^(1/2)*(d*x)^(1/2))/a^(23/4)/d^(5/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{19}{96a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^2}+\frac{1}{8ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)^3}+\frac{7315b^{3/4}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\,a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315b^{3/4}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\,a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}\,x\right)}{4096\sqrt{2}\,a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315\left(a+bx^2\right)}{3072a^5d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{95}{256a^3d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1045/(1024*a^4*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*(d
*x)^(3/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 19/(96*a^2*d*(d*
x)^(3/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 95/(256*a^3*d*(d*
x)^(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*(a + b*x^2))/
(3072*a^5*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*(a
 + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*
Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*b^(3/4)*(
a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048
*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*
(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (7315*b^(3/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{1}{(dx)^{5/2}(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(19b^3(ab+b^2x^2)\right)\int \frac{}{(dx)}}{16a\sqrt{a^2+2abx^2}}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{19}{96a^2d(dx)^{3/2}\left(a+bx^2\right)^2}$$

$$= \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{19}{96a^2d(dx)^{3/2}\left(a+bx^2\right)^2}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

Mathematica [A]
time = 0.58, size = 213, normalized size = 0.35

$$\frac{x(a+bx^2)\left(-4a^{3/4}(2048a^4+16967a^3bx^2+33345a^2b^2x^4+26125ab^3x^6+7315b^4x^8)+21945\sqrt{2}\,b^{3/4}x^{3/2}(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-21945\sqrt{2}\,b^{3/4}x^{3/2}(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{12288a^{23/4}(dx)^{5/2}\left((a+bx^2)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (x*(a + b*x^2)*(-4*a^(3/4)*(2048*a^4 + 16967*a^3*b*x^2 + 33345*a^2*b^2*x^4
+ 26125*a*b^3*x^6 + 7315*b^4*x^8) + 21945*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b*x^
2)^4*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 2194
5*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(12288*a^(23/4)*(d*x)^(5/2)*((a + b*x^2)^2)
^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1182 vs. $2(391) = 782$.
time = 0.08, size = 1183, normalized size = 1.97

method	result
risch	$-\frac{2\sqrt{(bx^2+a)^2}}{3a^5x\sqrt{dx}\,d^2(bx^2+a)}+\frac{\left(-\frac{2925b\,d^7\sqrt{dx}}{1024a^2\left(d^2x^2b+a\,d^2\right)^4}-\frac{7019b^2d^5(dx)^{\frac{5}{2}}}{1024a^3\left(d^2x^2b+a\,d^2\right)^4}-\frac{17933b^3d^3(dx)^{\frac{9}{2}}}{3072a^4\left(d^2x^2b+a\,d^2\right)^4}-\frac{5267b^4d(dx)^{\frac{13}{2}}}{3072a^5\left(d^2x^2b+a\,d^2\right)^4}-\cdots\right.}{\cdots}$ 7315b
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24576/d^3*(21945*(d*x)^(3/2)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2)))*(a*d^2/b)^(1/4)*b^5*x^8+43890*(d*x)^(3/2)*2^(1/2)*arctan((2^(1/2)
*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(a*d^2/b)^(1/4)*b^5*x^8+4389
0*(d*x)^(3/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*(a*d^2/b)^(1/4)*b^5*x^8+87780*(d*x)^(3/2)*2^(1/2)*ln((d*x+(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2)))*(a*d^2/b)^(1/4)*a*b^4*x^6+175560*(d*x)^(3/2)*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(a*d^2
/b)^(1/4)*a*b^4*x^6+175560*(d*x)^(3/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(a*d^2/b)^(1/4)*a*b^4*x^6+131670*(d*x)^(3
/2)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d
*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(a*d^2/b)^(1/4)*a^
2*b^3*x^4+263340*(d*x)^(3/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^
(1/4))/(a*d^2/b)^(1/4))*(a*d^2/b)^(1/4)*a^2*b^3*x^4+263340*(d*x)^(3/2)*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(a*d^2/b)
^(1/4)*a^2*b^3*x^4+58520*a*b^4*d^2*x^8+87780*(d*x)^(3/2)*2^(1/2)*ln((d*x+(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(a*d^2/b)^(1/4)*a^3*b^2*x^2+175560*(d*x)
```

```
^(3/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*(a*d^2/b)^(1/4)*a^3*b^2*x^2+175560*(d*x)^(3/2)*2^(1/2)*arctan((2^(1/2)*(d
*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(a*d^2/b)^(1/4)*a^3*b^2*x^2+209
000*a^2*b^3*d^2*x^6+21945*(d*x)^(3/2)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2)))*(a*d^2/b)^(1/4)*a^4*b+43890*(d*x)^(3/2)*2^(1/2)*arctan((2^(
1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(a*d^2/b)^(1/4)*a^4*b+43
890*(d*x)^(3/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2
/b)^(1/4))*(a*d^2/b)^(1/4)*a^4*b+266760*a^3*b^2*d^2*x^4+135736*a^4*b*d^2*x^
2+16384*a^5*d^2)*(b*x^2+a)/(d*x)^(3/2)/a^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -4*b*integrate(1/((a^5*b*d^(5/2)*x^2 + a^6*d^(5/2))*sqrt(x)), x) - 1/3072*(
13795*b^4*x^(13/2) + 34285*a*b^3*x^(9/2) + 29649*a^2*b^2*x^(5/2) + 8775*a^3
*b*sqrt(x))/(a^5*b^4*d^(5/2)*x^8 + 4*a^6*b^3*d^(5/2)*x^6 + 6*a^7*b^2*d^(5/2
)*x^4 + 4*a^8*b*d^(5/2)*x^2 + a^9*d^(5/2)) + 1/192*((533*b^6*x^5 + 882*a*b^
5*x^3 + 381*a^2*b^4*x)*x^(11/2) + 2*(603*a*b^5*x^5 + 1014*a^2*b^4*x^3 + 443
*a^3*b^3*x)*x^(7/2) + (705*a^2*b^4*x^5 + 1210*a^3*b^3*x^3 + 537*a^4*b^2*x)*
x^(3/2))/(a^8*b^3*d^(5/2)*x^6 + 3*a^9*b^2*d^(5/2)*x^4 + 3*a^10*b*d^(5/2)*x^
2 + a^11*d^(5/2) + (a^5*b^6*d^(5/2)*x^6 + 3*a^6*b^5*d^(5/2)*x^4 + 3*a^7*b^4
*d^(5/2)*x^2 + a^8*b^3*d^(5/2))*x^6 + 3*(a^6*b^5*d^(5/2)*x^6 + 3*a^7*b^4*d^
(5/2)*x^4 + 3*a^8*b^3*d^(5/2)*x^2 + a^9*b^2*d^(5/2))*x^4 + 3*(a^7*b^4*d^(5/
2)*x^6 + 3*a^8*b^3*d^(5/2)*x^4 + 3*a^9*b^2*d^(5/2)*x^2 + a^10*b*d^(5/2))*x^
2) + 2925/8192*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)
*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(
2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3
/4))/(a^5*d^(5/2)) + integrate(1/((a^4*b*d^(5/2)*x^2 + a^5*d^(5/2))*x^(5/2)
), x)
```

Fricas [A]
time = 0.36, size = 501, normalized size = 0.83

$$-\frac{87780\,(a^5b^4d^3x^{10}+4\,a^6b^3d^3x^8+6\,a^7b^2d^3x^6+4\,a^8bd^3x^4+a^9d^3x^2)\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}\,a^{17}bd^7\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{3}{4}}-\sqrt{a^{12}d^6\sqrt{-\frac{b^3}{a^{23}d^{10}}}+b^2dx}\,a^{17}d^7\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{3}{4}}}{b^3}\right)+21945\,(a^5b^4d^3x^{10}+4\,a^6b^3d^3x^8+6\,a^7b^2d^3x^6+4\,a^8bd^3x^4+a^9d^3x^2)\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}\log\left(7315\,a^6d^3\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}+7315\,\sqrt{dx}\,b\right)-21945\,(a^5b^4d^3x^{10}+4\,a^6b^3d^3x^8+6\,a^7b^2d^3x^6+4\,a^8bd^3x^4+a^9d^3x^2)\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}\log\left(-7315\,a^6d^3\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}+7315\,\sqrt{dx}\,b\right)+4\,(7315\,b^4x^8+26125\,ab^3x^6+33345\,a^2b^2x^4+16967\,a^3bx^2+2048\,a^4)\sqrt{dx}}{12288\,(a^5b^4d^3x^{10}+4\,a^6b^3d^3x^8+6\,a^7b^2d^3x^6+4\,a^8bd^3x^4+a^9d^3x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/12288*(87780*(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 +
 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)*(-b^3/(a^23*d^10))^(1/4)*arctan(-(sqrt(d*x)
*a^17*b*d^7*(-b^3/(a^23*d^10))^(3/4) - sqrt(a^12*d^6*sqrt(-b^3/(a^23*d^10))
 + b^2*d*x)*a^17*d^7*(-b^3/(a^23*d^10))^(3/4))/b^3) + 21945*(a^5*b^4*d^3*x^
10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)
*(-b^3/(a^23*d^10))^(1/4)*log(7315*a^6*d^3*(-b^3/(a^23*d^10))^(1/4) + 7315*
sqrt(d*x)*b) - 21945*(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*
x^6 + 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)*(-b^3/(a^23*d^10))^(1/4)*log(-7315*a^6
*d^3*(-b^3/(a^23*d^10))^(1/4) + 7315*sqrt(d*x)*b) + 4*(7315*b^4*x^8 + 26125
*a*b^3*x^6 + 33345*a^2*b^2*x^4 + 16967*a^3*b*x^2 + 2048*a^4)*sqrt(d*x))/(a^
5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*b*d^3*x^4 +
a^9*d^3*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*((a + b*x**2)**2)**(5/2)), x)
```

Giac [**A**]
time = 4.47, size = 397, normalized size = 0.66

$$-\frac{7315\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^6d^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{7315\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^6d^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{7315\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^6d^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{7315\,\sqrt{2}\,(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^6d^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{2}{3\,\sqrt{dx}\,a^5d^2x\mathrm{sgn}\left(bx^2+a\right)}-\frac{5267\,\sqrt{dx}\,b^4d^6x^6+17933\,\sqrt{dx}\,ab^3d^6x^4+21057\,\sqrt{dx}\,a^2b^2d^6x^2+8775\,\sqrt{dx}\,a^3bd^6}{3072\,(bd^2x^2+ad^2)^4a^5d\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -7315/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*d^3*sgn(b*x^2 + a)) - 7315/4096*
sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*
sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*d^3*sgn(b*x^2 + a)) - 7315/8192*sqrt(2)*(a
*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(a^6*d^3*sgn(b*x^2 + a)) + 7315/8192*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x -
sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*d^3*sgn(b*x^2 + a))
 - 2/3/(sqrt(d*x)*a^5*d^2*x*sgn(b*x^2 + a)) - 1/3072*(5267*sqrt(d*x)*b^4*d^
6*x^6 + 17933*sqrt(d*x)*a*b^3*d^6*x^4 + 21057*sqrt(d*x)*a^2*b^2*d^6*x^2 + 8
775*sqrt(d*x)*a^3*b*d^6)/((b*d^2*x^2 + a*d^2)^4*a^5*d*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{5/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

3.784 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=649

$$\frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2}$$

```
[Out] 1547/1024/a^4/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)+1/8/a/d/(d*x)^(5/2)/(b*x^2+
a)^3/((b*x^2+a)^2)^(1/2)+7/32/a^2/d/(d*x)^(5/2)/(b*x^2+a)^2/((b*x^2+a)^2)^(
1/2)+119/256/a^3/d/(d*x)^(5/2)/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-13923/5120*(b*
x^2+a)/a^5/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)-13923/4096*b^(5/4)*(b*x^2+a)*a
rctan(1-b^(1/4)*2^(1/2)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(25/4)/d^(7/2)*2^(1/
2)/((b*x^2+a)^2)^(1/2)+13923/4096*b^(5/4)*(b*x^2+a)*arctan(1+b^(1/4)*2^(1/2
)*(d*x)^(1/2)/a^(1/4)/d^(1/2))/a^(25/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)
+13923/8192*b^(5/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*(d*x)^(1/2))/a^(25/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)-1
3923/8192*b^(5/4)*(b*x^2+a)*ln(a^(1/2)*d^(1/2)+x*b^(1/2)*d^(1/2)+a^(1/4)*b^
(1/4)*2^(1/2)*(d*x)^(1/2))/a^(25/4)/d^(7/2)*2^(1/2)/((b*x^2+a)^2)^(1/2)+139
23/1024*b*(b*x^2+a)/a^6/d^3/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1126, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923b^{5/4}\left(a+bx^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}\left(a+bx^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b\left(a+bx^2\right)}{1024a^6d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923\left(a+bx^2\right)}{5120a^5d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{119}{256a^3d(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1547/(1024*a^4*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*d*(d
*x)^(5/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 7/(32*a^2*d*(d*x
)^(5/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 119/(256*a^3*d*(d*
x)^(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*(a + b*x^2))
/(5120*a^5*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*b*(a + b
*x^2))/(1024*a^6*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*b^
(5/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*
b^(5/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1392
3*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (13923*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sq
```

```
rt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^2)\right)\int \frac{1}{(dx)^{7/2}(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(21b^3(ab+b^2x^2)\right)\int \frac{}{(dx)}}{16a\sqrt{a^2+2abx^2}}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2}$$

$$= \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

$$= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2}}$$

Mathematica [A]

time = 0.59, size = 224, normalized size = 0.35

$$\frac{x(a+bx^2)\left(4\sqrt[4]{a}\left(-2048a^5+43008a^4bx^2+220507a^3b^2x^4+369733a^2b^3x^6+264537ab^4x^8+69615b^5x^{10}\right)-69615\sqrt{2}\,b^{5/4}x^{5/2}(a+bx^2)^4\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)-69615\sqrt{2}\,b^{5/4}x^{5/2}(a+bx^2)^4\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)\right)}{20480a^{25/4}(dx)^{7/2}\left((a+bx^2)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (x*(a + b*x^2)*(4*a^(1/4)*(-2048*a^5 + 43008*a^4*b*x^2 + 220507*a^3*b^2*x^4
 + 369733*a^2*b^3*x^6 + 264537*a*b^4*x^8 + 69615*b^5*x^10) - 69615*Sqrt[2]*
b^(5/4)*x^(5/2)*(a + b*x^2)^4*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x])] - 69615*Sqrt[2]*b^(5/4)*x^(5/2)*(a + b*x^2)^4*ArcTanh[(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(20480*a^(25/4)*(d
*x)^(7/2)*((a + b*x^2)^2)^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. $2(423) = 846$.
time = 0.08, size = 1129, normalized size = 1.74

method	result
risch	$-\frac{2(-25b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{5a^6\sqrt{dx}\;x^2d^3(b\,x^2+a)}+\frac{\left(\frac{5599b^2d^6(dx)^{\frac{3}{2}}}{1024a^3\left(d^2x^2b+a\,d^2\right)^4}+\frac{14145b^3d^4(dx)^{\frac{7}{2}}}{1024a^4\left(d^2x^2b+a\,d^2\right)^4}+\frac{12357b^4d^2(dx)^{\frac{11}{2}}}{1024a^5\left(d^2x^2b+a\,d^2\right)^4}+\frac{3683b^5(dx)^{\frac{15}{2}}}{1024a^6\left(d^2x^2b+a\,d^2\right)}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40960/d^3*(69615*(d*x)^(5/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2)))*b^5*x^8+139230*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a
*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^5*x^8+139230*(d*x)^(5/2)*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^5*x^8+278460*(d*x)^
(5/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))
/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^4*x^6+55692
0*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*a*b^4*x^6+556920*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(
a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^4*x^6+556920*(a*d^2/b)^(1/4)*b^5*d^2*x
^10+417690*(d*x)^(5/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x
-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))
)*a^2*b^3*x^4+835380*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2
```

```
/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^3*x^4+835380*(d*x)^(5/2)*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^3*x^4+2116296*(
a*d^2/b)^(1/4)*a*b^4*d^2*x^8+278460*(d*x)^(5/2)*2^(1/2)*ln(-((a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b^2*x^2+556920*(d*x)^(5/2)*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b^2*x^2+556920*(d*
x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*a^3*b^2*x^2+2957864*(a*d^2/b)^(1/4)*a^2*b^3*d^2*x^6+69615*(d*x)^(5/2)*2
^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*b+139230*(d*x)^(5/
2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^
4*b+139230*(d*x)^(5/2)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*a^4*b+1764056*(a*d^2/b)^(1/4)*a^3*b^2*d^2*x^4+344064*(a*d
^2/b)^(1/4)*a^4*b*d^2*x^2-16384*(a*d^2/b)^(1/4)*a^5*d^2)*(b*x^2+a)/(d*x)^(5
/2)/(a*d^2/b)^(1/4)/a^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -4*b*integrate(1/((a^5*b*d^(7/2)*x^2 + a^6*d^(7/2))*x^(3/2)), x) + 1/3072*(
11049*b^5*x^(15/2) + 27135*a*b^4*x^(11/2) + 23395*a^2*b^3*x^(7/2) + 6925*a^
3*b^2*x^(3/2))/(a^6*b^4*d^(7/2)*x^8 + 4*a^7*b^3*d^(7/2)*x^6 + 6*a^8*b^2*d^(
7/2)*x^4 + 4*a^9*b*d^(7/2)*x^2 + a^10*d^(7/2)) + 1/192*((621*b^6*x^5 + 1042
*a*b^5*x^3 + 453*a^2*b^4*x)*x^(9/2) + 2*(695*a*b^5*x^5 + 1182*a^2*b^4*x^3 +
 519*a^3*b^3*x)*x^(5/2) + (801*a^2*b^4*x^5 + 1386*a^3*b^3*x^3 + 617*a^4*b^2
*x)*sqrt(x))/(a^8*b^3*d^(7/2)*x^6 + 3*a^9*b^2*d^(7/2)*x^4 + 3*a^10*b*d^(7/2
)*x^2 + a^11*d^(7/2) + (a^5*b^6*d^(7/2)*x^6 + 3*a^6*b^5*d^(7/2)*x^4 + 3*a^7
*b^4*d^(7/2)*x^2 + a^8*b^3*d^(7/2))*x^6 + 3*(a^6*b^5*d^(7/2)*x^6 + 3*a^7*b^
4*d^(7/2)*x^4 + 3*a^8*b^3*d^(7/2)*x^2 + a^9*b^2*d^(7/2))*x^4 + 3*(a^7*b^4*d
^(7/2)*x^6 + 3*a^8*b^3*d^(7/2)*x^4 + 3*a^9*b^2*d^(7/2)*x^2 + a^10*b*d^(7/2)
)*x^2) + 3683/8192*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(
b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log
(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) +
 sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/
4)*b^(3/4)))/(a^6*d^(7/2)) + integrate(1/((a^4*b*d^(7/2)*x^2 + a^5*d^(7/2))
*x^(7/2)), x)
```

Fricas [A]

time = 0.36, size = 524, normalized size = 0.81

$$-\frac{278460\left(a^{6}b^{4}d^{4}x^{11}+4a^{7}b^{3}d^{4}x^{9}+6a^{8}b^{2}d^{4}x^{7}+4a^{9}bd^{4}x^{5}+a^{10}d^{4}x^{3}\right)\left(-\frac{b^{5}}{a^{25}d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{2698972561467\sqrt{dx}\,a^{6}b^{4}d^{3}\left(-\frac{b^{5}}{a^{25}d^{14}}\right)^{\frac{1}{4}}-\sqrt{-7284452887551739093192089\,a^{13}b^{5}d^{8}\sqrt{-\frac{b^{5}}{a^{25}d^{14}}}+7284452887551739093192089\,b^{8}dx}\;a^{6}d^{3}\left(-\frac{b^{5}}{a^{25}d^{14}}\right)^{\frac{1}{4}}}{2698972561467\,b^{5}}\right)-69615\left(a^{6}b^{4}d^{4}x^{11}+4a^{7}b^{3}d^{4}x^{9}+6a^{8}b^{2}d^{4}x^{7}+4a^{9}bd^{4}x^{5}+a^{10}d^{4}x^{3}\right)\left(-\frac{b^{5}}{a^{25}d^{14}}\right)^{\frac{1}{4}}\log\left(2698972561467\,a^{19}d^{11}\left(-\frac{b^{5}}{a^{25}d^{14}}\right)^{\frac{3}{4}}+2698972561467\sqrt{dx}\,b^{4}\right)+69615\left(a^{6}b^{4}d^{4}x^{11}+4a^{7}b^{3}d^{4}x^{9}+6a^{8}b^{2}d^{4}x^{7}+4a^{9}bd^{4}x^{5}+a^{10}d^{4}x^{3}\right)\left(-\frac{b^{5}}{a^{25}d^{14}}\right)^{\frac{1}{4}}\log\left(-2698972561467\,a^{19}d^{11}\left(-\frac{b^{5}}{a^{25}d^{14}}\right)^{\frac{3}{4}}+2698972561467\sqrt{dx}\,b^{4}\right)-4\left(69615b^{5}x^{10}+264537ab^{4}x^{8}+369733a^{2}b^{3}x^{6}+220507a^{3}b^{2}x^{4}+43008a^{4}bx^{2}-2048a^{5}\right)\sqrt{dx}}{20480\left(a^{6}b^{4}d^{4}x^{11}+4a^{7}b^{3}d^{4}x^{9}+6a^{8}b^{2}d^{4}x^{7}+4a^{9}bd^{4}x^{5}+a^{10}d^{4}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] -1/20480*(278460*(a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7
+ 4*a^9*b*d^4*x^5 + a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*arctan(-1/269897
2561467*(2698972561467*sqrt(d*x)*a^6*b^4*d^3*(-b^5/(a^25*d^14))^(1/4) - sqr
t(-7284452887551739093192089*a^13*b^5*d^8*sqrt(-b^5/(a^25*d^14)) + 72844528
87551739093192089*b^8*d*x)*a^6*d^3*(-b^5/(a^25*d^14))^(1/4))/b^5) - 69615*(
a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*d^4*x^5
+ a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*log(2698972561467*a^19*d^11*(-b^5/
(a^25*d^14))^(3/4) + 2698972561467*sqrt(d*x)*b^4) + 69615*(a^6*b^4*d^4*x^11
 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*d^4*x^5 + a^10*d^4*x^3)*
(-b^5/(a^25*d^14))^(1/4)*log(-2698972561467*a^19*d^11*(-b^5/(a^25*d^14))^(3
/4) + 2698972561467*sqrt(d*x)*b^4) - 4*(69615*b^5*x^10 + 264537*a*b^4*x^8 +
 369733*a^2*b^3*x^6 + 220507*a^3*b^2*x^4 + 43008*a^4*b*x^2 - 2048*a^5)*sqrt
(d*x))/(a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*
d^4*x^5 + a^10*d^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{7}{2}}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/((d*x)**(7/2)*((a + b*x**2)**2)**(5/2)), x)
```

Giac [A]
time = 4.40, size = 428, normalized size = 0.66

$$\frac{13923\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^{7}bd^{5}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{13923\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^{7}bd^{5}\mathrm{sgn}\left(bx^{2}+a\right)}-\frac{13923\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{8192\,a^{7}bd^{5}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{13923\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{8192\,a^{7}bd^{5}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{3683\sqrt{dx}\,b^{5}d^{7}x^{7}+12357\sqrt{dx}\,ab^{4}d^{7}x^{5}+14145\sqrt{dx}\,a^{2}b^{3}d^{7}x^{3}+5599\sqrt{dx}\,a^{3}b^{2}d^{7}x}{1024\left(bd^{2}x^{2}+ad^{2}\right)^{4}a^{6}d^{3}\mathrm{sgn}\left(bx^{2}+a\right)}+\frac{2\left(25\,bd^{2}x^{2}-ad^{2}\right)}{5\sqrt{dx}\,a^{6}d^{5}x^{2}\mathrm{sgn}\left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 13923/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b*d^5*sgn(b*x^2 + a)) + 13923/40
96*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) -
```

```
 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b*d^5*sgn(b*x^2 + a)) - 13923/8192*sqrt
(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*
d^2/b))/(a^7*b*d^5*sgn(b*x^2 + a)) + 13923/8192*sqrt(2)*(a*b^3*d^2)^(3/4)*l
og(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*b*d^5*sgn(
b*x^2 + a)) + 1/1024*(3683*sqrt(d*x)*b^5*d^7*x^7 + 12357*sqrt(d*x)*a*b^4*d^
7*x^5 + 14145*sqrt(d*x)*a^2*b^3*d^7*x^3 + 5599*sqrt(d*x)*a^3*b^2*d^7*x)/((b
*d^2*x^2 + a*d^2)^4*a^6*d^3*sgn(b*x^2 + a)) + 2/5*(25*b*d^2*x^2 - a*d^2)/(s
qrt(d*x)*a^6*d^5*x^2*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(d\,x)^{7/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
[Out] int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

3.785 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=150

$$\frac{a^6(dx)^{1+m}}{d(1+m)}+\frac{6a^5b(dx)^{3+m}}{d^3(3+m)}+\frac{15a^4b^2(dx)^{5+m}}{d^5(5+m)}+\frac{20a^3b^3(dx)^{7+m}}{d^7(7+m)}+\frac{15a^2b^4(dx)^{9+m}}{d^9(9+m)}+\frac{6ab^5(dx)^{11+m}}{d^{11}(11+m)}+\frac{b^6(dx)^{13+m}}{d^{13}(13+m)}$$

```
[Out] a^6*(d*x)^(1+m)/d/(1+m)+6*a^5*b*(d*x)^(3+m)/d^3/(3+m)+15*a^4*b^2*(d*x)^(5+m
)/d^5/(5+m)+20*a^3*b^3*(d*x)^(7+m)/d^7/(7+m)+15*a^2*b^4*(d*x)^(9+m)/d^9/(9+
m)+6*a*b^5*(d*x)^(11+m)/d^11/(11+m)+b^6*(d*x)^(13+m)/d^13/(13+m)
```

Rubi [A]
time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {28, 276}

$$\frac{a^6(dx)^{m+1}}{d(m+1)}+\frac{6a^5b(dx)^{m+3}}{d^3(m+3)}+\frac{15a^4b^2(dx)^{m+5}}{d^5(m+5)}+\frac{20a^3b^3(dx)^{m+7}}{d^7(m+7)}+\frac{15a^2b^4(dx)^{m+9}}{d^9(m+9)}+\frac{6ab^5(dx)^{m+11}}{d^{11}(m+11)}+\frac{b^6(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*(d*x)^(1 + m))/(d*(1 + m)) + (6*a^5*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
15*a^4*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (20*a^3*b^3*(d*x)^(7 + m))/(d^7*(
7 + m)) + (15*a^2*b^4*(d*x)^(9 + m))/(d^9*(9 + m)) + (6*a*b^5*(d*x)^(11 + m
))/(d^11*(11 + m)) + (b^6*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{\int (dx)^m \left(ab+b^2x^2\right)^6 \, dx}{b^6}$$
$$= \frac{\int \left(a^6b^6(dx)^m + \frac{6a^5b^7(dx)^{2+m}}{d^2} + \frac{15a^4b^8(dx)^{4+m}}{d^4} + \frac{20a^3b^9(dx)^{6+m}}{d^6} + \frac{15a^2b^{10}(dx)^{8+m}}{d^8}\right.}{b^6}$$
$$= \frac{a^6(dx)^{1+m}}{d(1+m)} + \frac{6a^5b(dx)^{3+m}}{d^3(3+m)} + \frac{15a^4b^2(dx)^{5+m}}{d^5(5+m)} + \frac{20a^3b^3(dx)^{7+m}}{d^7(7+m)} + \frac{15a^2b^4}{d^9(9}$$

Mathematica [A]
time = 0.18, size = 105, normalized size = 0.70

$$x(dx)^m\left(\frac{a^6}{1+m}+\frac{6a^5bx^2}{3+m}+\frac{15a^4b^2x^4}{5+m}+\frac{20a^3b^3x^6}{7+m}+\frac{15a^2b^4x^8}{9+m}+\frac{6ab^5x^{10}}{11+m}+\frac{b^6x^{12}}{13+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x*(d*x)^m*(a^6/(1 + m) + (6*a^5*b*x^2)/(3 + m) + (15*a^4*b^2*x^4)/(5 + m) +
 (20*a^3*b^3*x^6)/(7 + m) + (15*a^2*b^4*x^8)/(9 + m) + (6*a*b^5*x^10)/(11 +
 m) + (b^6*x^12)/(13 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(150) = 300.
time = 0.02, size = 602, normalized size = 4.01

method	result
gosper	$\frac{(dx)^m\left(b^6m^6x^{12}+36b^6m^5x^{12}+6a\,b^5m^6x^{10}+505b^6m^4x^{12}+228a\,b^5m^5x^{10}+3480b^6m^3x^{12}+15a^2b^4m^6x^8+3330a\,b^5m^4x^{10}+12139b^6m^2x\right.}{}$
risch	$\frac{(dx)^m\left(b^6m^6x^{12}+36b^6m^5x^{12}+6a\,b^5m^6x^{10}+505b^6m^4x^{12}+228a\,b^5m^5x^{10}+3480b^6m^3x^{12}+15a^2b^4m^6x^8+3330a\,b^5m^4x^{10}+12139b^6m^2x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (d*x)^m*(b^6*m^6*x^12+36*b^6*m^5*x^12+6*a*b^5*m^6*x^10+505*b^6*m^4*x^12+228
*a*b^5*m^5*x^10+3480*b^6*m^3*x^12+15*a^2*b^4*m^6*x^8+3330*a*b^5*m^4*x^10+12
139*b^6*m^2*x^12+600*a^2*b^4*m^5*x^8+23640*a*b^5*m^3*x^10+19524*b^6*m*x^12+
20*a^3*b^3*m^6*x^6+9195*a^2*b^4*m^4*x^8+84234*a*b^5*m^2*x^10+10395*b^6*x^12
+840*a^3*b^3*m^5*x^6+67920*a^2*b^4*m^3*x^8+137412*a*b^5*m*x^10+15*a^4*b^2*m
^6*x^4+13580*a^3*b^3*m^4*x^6+249405*a^2*b^4*m^2*x^8+73710*a*b^5*x^10+660*a^
4*b^2*m^5*x^4+105840*a^3*b^3*m^3*x^6+415320*a^2*b^4*m*x^8+6*a^5*b*m^6*x^2+1
1295*a^4*b^2*m^4*x^4+406700*a^3*b^3*m^2*x^6+225225*a^2*b^4*x^8+276*a^5*b*m^
```

```
5*x^2+94200*a^4*b^2*m^3*x^4+699720*a^3*b^3*m*x^6+a^6*m^6+5010*a^5*b*m^4*x^2
+389685*a^4*b^2*m^2*x^4+386100*a^3*b^3*x^6+48*a^6*m^5+45240*a^5*b*m^3*x^2+7
11540*a^4*b^2*m*x^4+925*a^6*m^4+208554*a^5*b*m^2*x^2+405405*a^4*b^2*x^4+912
0*a^6*m^3+438324*a^5*b*m*x^2+48259*a^6*m^2+270270*a^5*b*x^2+129072*a^6*m+13
5135*a^6)*x/(13+m)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [A]
time = 0.29, size = 144, normalized size = 0.96

$$\frac{b^6 d^m x^{13} x^m}{m+13}+\frac{6\,ab^5 d^m x^{11} x^m}{m+11}+\frac{15\,a^2 b^4 d^m x^9 x^m}{m+9}+\frac{20\,a^3 b^3 d^m x^7 x^m}{m+7}+\frac{15\,a^4 b^2 d^m x^5 x^m}{m+5}+\frac{6\,a^5 b d^m x^3 x^m}{m+3}+\frac{(dx)^{m+1}\,a^6}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] b^6*d^m*x^13*x^m/(m + 13) + 6*a*b^5*d^m*x^11*x^m/(m + 11) + 15*a^2*b^4*d^m*
x^9*x^m/(m + 9) + 20*a^3*b^3*d^m*x^7*x^m/(m + 7) + 15*a^4*b^2*d^m*x^5*x^m/(
m + 5) + 6*a^5*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^6/(d*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(150) = 300.
time = 0.34, size = 507, normalized size = 3.38

$$\frac{\left(\left(b^6m^6+36\,b^6m^5+505\,b^6m^4+3480\,b^6m^3+12139\,b^6m^2+19524\,b^6m+10395\,b^6\right)x^{13}+6\left(ab^5m^6+38\,ab^5m^5+555\,ab^5m^4+3940\,ab^5m^3+14039\,ab^5m^2+22902\,ab^5m+12285\,ab^5\right)x^{11}+15\left(a^2b^4m^6+40\,a^2b^4m^5+613\,a^2b^4m^4+4528\,a^2b^4m^3+16627\,a^2b^4m^2+27688\,a^2b^4m+15015\,a^2b^4\right)x^9+20\left(a^3b^3m^6+42\,a^3b^3m^5+679\,a^3b^3m^4+5292\,a^3b^3m^3+20335\,a^3b^3m^2+34986\,a^3b^3m+19305\,a^3b^3\right)x^7+15\left(a^4b^2m^6+44\,a^4b^2m^5+753\,a^4b^2m^4+6280\,a^4b^2m^3+25979\,a^4b^2m^2+47436\,a^4b^2m+27027\,a^4b^2\right)x^5+6\left(a^5bm^6+46\,a^5bm^5+835\,a^5bm^4+7540\,a^5bm^3+34759\,a^5bm^2+73054\,a^5bm+45045\,a^5b\right)x^3+\left(a^6m^6+48\,a^6m^5+925\,a^6m^4+9120\,a^6m^3+48259\,a^6m^2+129072\,a^6m+135135\,a^6\right)x\right)(dx)^m}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] ((b^6*m^6 + 36*b^6*m^5 + 505*b^6*m^4 + 3480*b^6*m^3 + 12139*b^6*m^2 + 19524
*b^6*m + 10395*b^6)*x^13 + 6*(a*b^5*m^6 + 38*a*b^5*m^5 + 555*a*b^5*m^4 + 39
40*a*b^5*m^3 + 14039*a*b^5*m^2 + 22902*a*b^5*m + 12285*a*b^5)*x^11 + 15*(a^
2*b^4*m^6 + 40*a^2*b^4*m^5 + 613*a^2*b^4*m^4 + 4528*a^2*b^4*m^3 + 16627*a^2
*b^4*m^2 + 27688*a^2*b^4*m + 15015*a^2*b^4)*x^9 + 20*(a^3*b^3*m^6 + 42*a^3*
b^3*m^5 + 679*a^3*b^3*m^4 + 5292*a^3*b^3*m^3 + 20335*a^3*b^3*m^2 + 34986*a^
3*b^3*m + 19305*a^3*b^3)*x^7 + 15*(a^4*b^2*m^6 + 44*a^4*b^2*m^5 + 753*a^4*b
^2*m^4 + 6280*a^4*b^2*m^3 + 25979*a^4*b^2*m^2 + 47436*a^4*b^2*m + 27027*a^4
*b^2)*x^5 + 6*(a^5*b*m^6 + 46*a^5*b*m^5 + 835*a^5*b*m^4 + 7540*a^5*b*m^3 +
34759*a^5*b*m^2 + 73054*a^5*b*m + 45045*a^5*b)*x^3 + (a^6*m^6 + 48*a^6*m^5
+ 925*a^6*m^4 + 9120*a^6*m^3 + 48259*a^6*m^2 + 129072*a^6*m + 135135*a^6)*x
)*(d*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 26
4207*m + 135135)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3104 vs. 2(138) = 276.
time = 0.95, size = 3104, normalized size = 20.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**3,x)

[Out] Piecewise(((-a**6/(12*x**12) - 3*a**5*b/(5*x**10) - 15*a**4*b**2/(8*x**8) - 10*a**3*b**3/(3*x**6) - 15*a**2*b**4/(4*x**4) - 3*a*b**5/x**2 + b**6*log(x))/d**13, Eq(m, -13)), ((-a**6/(10*x**10) - 3*a**5*b/(4*x**8) - 5*a**4*b**2/(2*x**6) - 5*a**3*b**3/x**4 - 15*a**2*b**4/(2*x**2) + 6*a*b**5*log(x) + b**6*x**2/2)/d**11, Eq(m, -11)), ((-a**6/(8*x**8) - a**5*b/x**6 - 15*a**4*b**2/(4*x**4) - 10*a**3*b**3/x**2 + 15*a**2*b**4*log(x) + 3*a*b**5*x**2 + b**6*x**4/4)/d**9, Eq(m, -9)), ((-a**6/(6*x**6) - 3*a**5*b/(2*x**4) - 15*a**4*b**2/(2*x**2) + 20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5*x**4/2 + b**6*x**6/6)/d**7, Eq(m, -7)), ((-a**6/(4*x**4) - 3*a**5*b/x**2 + 15*a**4*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a*b**5*x**6 + b**6*x**8/8)/d**5, Eq(m, -5)), ((-a**6/(2*x**2) + 6*a**5*b*log(x) + 15*a**4*b**2*x**2/2 + 5*a**3*b**3*x**4 + 5*a**2*b**4*x**6/2 + 3*a*b**5*x**8/4 + b**6*x**10/10)/d**3, Eq(m, -3)), ((a**6*log(x) + 3*a**5*b*x**2 + 15*a**4*b**2*x**4/4 + 10*a**3*b**3*x**6/3 + 15*a**2*b**4*x**8/8 + 3*a*b**5*x**10/5 + b**6*x**12/12)/d, Eq(m, -1)), (a**6*m**6*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48*a**6*m**5*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 925*a**6*m**4*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9120*a**6*m**3*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48259*a**6*m**2*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 129072*a**6*m*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**6*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 6*a**5*b*m**6*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 276*a**5*b*m**5*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 5010*a**5*b*m**4*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 45240*a**5*b*m**3*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 208554*a**5*b*m**2*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 438324*a**5*b*m*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 270270*a**5*b*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 15*a**4*b**2*m**6*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 660*a**4*b**2*m**5*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 11295*a**4*b**2*m**4*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m

```
 + 135135) + 94200*a**4*b**2*m**3*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5
+ 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 389685*a**4*
b**2*m**2*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**
3 + 177331*m**2 + 264207*m + 135135) + 711540*a**4*b**2*m*x**5*(d*x)**m/(m*
*7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
+ 135135) + 405405*a**4*b**2*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 100
45*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 20*a**3*b**3*m**6
*x**7*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17733
1*m**2 + 264207*m + 135135) + 840*a**3*b**3*m**5*x**7*(d*x)**m/(m**7 + 49*m
**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
 + 13580*a**3*b**3*m**4*x**7*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m*
*4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 105840*a**3*b**3*m**3*
x**7*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331
*m**2 + 264207*m + 135135) + 406700*a**3*b**3*m**2*x**7*(d*x)**m/(m**7 + 49
*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13513
5) + 699720*a**3*b**3*m*x**7*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m*
*4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 386100*a**3*b**3*x**7*
(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
 + 264207*m + 135135) + 15*a**2*b**4*m**6*x**9*(d*x)**m/(m**7 + 49*m**6 + 9
73*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 600*
a**2*b**4*m**5*x**9*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 5737
9*m**3 + 177331*m**2 + 264207*m + 135135) + 9195*a**2*b**4*m**4*x**9*(d*x)*
*m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264
207*m + 135135) + 67920*a**2*b**4*m**3*x**9*(d*x)**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 249405*
a**2*b**4*m**2*x**9*(d*x)**m/(m**7 + 49*m**6 + ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(150) = 300.
time = 4.43, size = 847, normalized size = 5.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^6*m^6*x^13 + 36*(d*x)^m*b^6*m^5*x^13 + 6*(d*x)^m*a*b^5*m^6*x^11
+ 505*(d*x)^m*b^6*m^4*x^13 + 228*(d*x)^m*a*b^5*m^5*x^11 + 3480*(d*x)^m*b^6*
m^3*x^13 + 15*(d*x)^m*a^2*b^4*m^6*x^9 + 3330*(d*x)^m*a*b^5*m^4*x^11 + 12139
*(d*x)^m*b^6*m^2*x^13 + 600*(d*x)^m*a^2*b^4*m^5*x^9 + 23640*(d*x)^m*a*b^5*m
^3*x^11 + 19524*(d*x)^m*b^6*m*x^13 + 20*(d*x)^m*a^3*b^3*m^6*x^7 + 9195*(d*x
)^m*a^2*b^4*m^4*x^9 + 84234*(d*x)^m*a*b^5*m^2*x^11 + 10395*(d*x)^m*b^6*x^13
 + 840*(d*x)^m*a^3*b^3*m^5*x^7 + 67920*(d*x)^m*a^2*b^4*m^3*x^9 + 137412*(d*
x)^m*a*b^5*m*x^11 + 15*(d*x)^m*a^4*b^2*m^6*x^5 + 13580*(d*x)^m*a^3*b^3*m^4*
x^7 + 249405*(d*x)^m*a^2*b^4*m^2*x^9 + 73710*(d*x)^m*a*b^5*x^11 + 660*(d*x)
```

```
^m*a^4*b^2*m^5*x^5 + 105840*(d*x)^m*a^3*b^3*m^3*x^7 + 415320*(d*x)^m*a^2*b^
4*m*x^9 + 6*(d*x)^m*a^5*b*m^6*x^3 + 11295*(d*x)^m*a^4*b^2*m^4*x^5 + 406700*
(d*x)^m*a^3*b^3*m^2*x^7 + 225225*(d*x)^m*a^2*b^4*x^9 + 276*(d*x)^m*a^5*b*m^
5*x^3 + 94200*(d*x)^m*a^4*b^2*m^3*x^5 + 699720*(d*x)^m*a^3*b^3*m*x^7 + (d*x
)^m*a^6*m^6*x + 5010*(d*x)^m*a^5*b*m^4*x^3 + 389685*(d*x)^m*a^4*b^2*m^2*x^5
 + 386100*(d*x)^m*a^3*b^3*x^7 + 48*(d*x)^m*a^6*m^5*x + 45240*(d*x)^m*a^5*b*
m^3*x^3 + 711540*(d*x)^m*a^4*b^2*m*x^5 + 925*(d*x)^m*a^6*m^4*x + 208554*(d*
x)^m*a^5*b*m^2*x^3 + 405405*(d*x)^m*a^4*b^2*x^5 + 9120*(d*x)^m*a^6*m^3*x +
438324*(d*x)^m*a^5*b*m*x^3 + 48259*(d*x)^m*a^6*m^2*x + 270270*(d*x)^m*a^5*b
*x^3 + 129072*(d*x)^m*a^6*m*x + 135135*(d*x)^m*a^6*x)/(m^7 + 49*m^6 + 973*m
^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

Mupad [B]

time = 4.58, size = 540, normalized size = 3.60

$$\frac{a^6\,x\,{\left(d\,x\right)}^m\,\left(m^6+48\,m^5+925\,m^4+9120\,m^3+48259\,m^2+129072\,m+135135\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{b^6\,x^{13}\,{\left(d\,x\right)}^m\,\left(m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{6\,a\,b^5\,x^{11}\,{\left(d\,x\right)}^m\,\left(m^6+38\,m^5+555\,m^4+3940\,m^3+14039\,m^2+22902\,m+12285\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{6\,a^5\,b\,x^3\,{\left(d\,x\right)}^m\,\left(m^6+46\,m^5+835\,m^4+7540\,m^3+34759\,m^2+73054\,m+45045\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{15\,a^2\,b^4\,x^9\,{\left(d\,x\right)}^m\,\left(m^6+40\,m^5+613\,m^4+4528\,m^3+16627\,m^2+27688\,m+15015\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{20\,a^3\,b^3\,x^7\,{\left(d\,x\right)}^m\,\left(m^6+42\,m^5+679\,m^4+5292\,m^3+20335\,m^2+34986\,m+19305\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{15\,a^4\,b^2\,x^5\,{\left(d\,x\right)}^m\,\left(m^6+44\,m^5+753\,m^4+6280\,m^3+25979\,m^2+47436\,m+27027\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] (a^6*x*(d*x)^m*(129072*m + 48259*m^2 + 9120*m^3 + 925*m^4 + 48*m^5 + m^6 +
135135))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6
+ m^7 + 135135) + (b^6*x^13*(d*x)^m*(19524*m + 12139*m^2 + 3480*m^3 + 505*m
^4 + 36*m^5 + m^6 + 10395))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4
+ 973*m^5 + 49*m^6 + m^7 + 135135) + (6*a*b^5*x^11*(d*x)^m*(22902*m + 14039
*m^2 + 3940*m^3 + 555*m^4 + 38*m^5 + m^6 + 12285))/(264207*m + 177331*m^2 +
 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (6*a^5*b*x^3*(d
*x)^m*(73054*m + 34759*m^2 + 7540*m^3 + 835*m^4 + 46*m^5 + m^6 + 45045))/(2
64207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135
135) + (15*a^2*b^4*x^9*(d*x)^m*(27688*m + 16627*m^2 + 4528*m^3 + 613*m^4 +
40*m^5 + m^6 + 15015))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973
*m^5 + 49*m^6 + m^7 + 135135) + (20*a^3*b^3*x^7*(d*x)^m*(34986*m + 20335*m^
2 + 5292*m^3 + 679*m^4 + 42*m^5 + m^6 + 19305))/(264207*m + 177331*m^2 + 57
379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (15*a^4*b^2*x^5*(d
*x)^m*(47436*m + 25979*m^2 + 6280*m^3 + 753*m^4 + 44*m^5 + m^6 + 27027))/(2
64207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135
135)
```

3.786 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=104

$$\frac{a^4(dx)^{1+m}}{d(1+m)}+\frac{4a^3b(dx)^{3+m}}{d^3(3+m)}+\frac{6a^2b^2(dx)^{5+m}}{d^5(5+m)}+\frac{4ab^3(dx)^{7+m}}{d^7(7+m)}+\frac{b^4(dx)^{9+m}}{d^9(9+m)}$$

```
[Out] a^4*(d*x)^(1+m)/d/(1+m)+4*a^3*b*(d*x)^(3+m)/d^3/(3+m)+6*a^2*b^2*(d*x)^(5+m)
/d^5/(5+m)+4*a*b^3*(d*x)^(7+m)/d^7/(7+m)+b^4*(d*x)^(9+m)/d^9/(9+m)
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {28, 276}

$$\frac{a^4(dx)^{m+1}}{d(m+1)}+\frac{4a^3b(dx)^{m+3}}{d^3(m+3)}+\frac{6a^2b^2(dx)^{m+5}}{d^5(m+5)}+\frac{4ab^3(dx)^{m+7}}{d^7(m+7)}+\frac{b^4(dx)^{m+9}}{d^9(m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*(d*x)^(1 + m))/(d*(1 + m)) + (4*a^3*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
6*a^2*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (4*a*b^3*(d*x)^(7 + m))/(d^7*(7 +
m)) + (b^4*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2\,dx &= \frac{\int (dx)^m \left(ab+b^2x^2\right)^4\,dx}{b^4}\\ &= \frac{\int \left(a^4b^4(dx)^m+\frac{4a^3b^5(dx)^{2+m}}{d^2}+\frac{6a^2b^6(dx)^{4+m}}{d^4}+\frac{4ab^7(dx)^{6+m}}{d^6}+\frac{b^8(dx)^{8+m}}{d^8}\right) dx}{b^4}\\ &= \frac{a^4(dx)^{1+m}}{d(1+m)}+\frac{4a^3b(dx)^{3+m}}{d^3(3+m)}+\frac{6a^2b^2(dx)^{5+m}}{d^5(5+m)}+\frac{4ab^3(dx)^{7+m}}{d^7(7+m)}+\frac{b^4(dx)^{9+m}}{d^9(9+m)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 73, normalized size = 0.70

$$x(dx)^m\left(\frac{a^4}{1+m}+\frac{4a^3bx^2}{3+m}+\frac{6a^2b^2x^4}{5+m}+\frac{4ab^3x^6}{7+m}+\frac{b^4x^8}{9+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x*(d*x)^m*(a^4/(1 + m) + (4*a^3*b*x^2)/(3 + m) + (6*a^2*b^2*x^4)/(5 + m) +
(4*a*b^3*x^6)/(7 + m) + (b^4*x^8)/(9 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(104) = 208$.
time = 0.01, size = 292, normalized size = 2.81

method	result
gosper	$\frac{(dx)^m\left(b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176m\,x^8b^4+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4x^8+120a^2b^2m^3x^4+\right.}{}$
risch	$\frac{(dx)^m\left(b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176m\,x^8b^4+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4x^8+120a^2b^2m^3x^4+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (d*x)^m*(b^4*m^4*x^8+16*b^4*m^3*x^8+4*a*b^3*m^4*x^6+86*b^4*m^2*x^8+72*a*b^3
*m^3*x^6+176*b^4*m*x^8+6*a^2*b^2*m^4*x^4+416*a*b^3*m^2*x^6+105*b^4*x^8+120*
a^2*b^2*m^3*x^4+888*a*b^3*m*x^6+4*a^3*b*m^4*x^2+780*a^2*b^2*m^2*x^4+540*a*b
^3*x^6+88*a^3*b*m^3*x^2+1800*a^2*b^2*m*x^4+a^4*m^4+656*a^3*b*m^2*x^2+1134*a
^2*b^2*x^4+24*a^4*m^3+1832*a^3*b*m*x^2+206*a^4*m^2+1260*a^3*b*x^2+744*a^4*m
+945*a^4)*x/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [A]
time = 0.29, size = 100, normalized size = 0.96

$$\frac{b^4d^mx^9x^m}{m+9}+\frac{4\,ab^3d^mx^7x^m}{m+7}+\frac{6\,a^2b^2d^mx^5x^m}{m+5}+\frac{4\,a^3bd^mx^3x^m}{m+3}+\frac{(dx)^{m+1}\,a^4}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] b^4*d^m*x^9*x^m/(m + 9) + 4*a*b^3*d^m*x^7*x^m/(m + 7) + 6*a^2*b^2*d^m*x^5*x
^m/(m + 5) + 4*a^3*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^4/(d*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(104) = 208$.
time = 0.35, size = 253, normalized size = 2.43

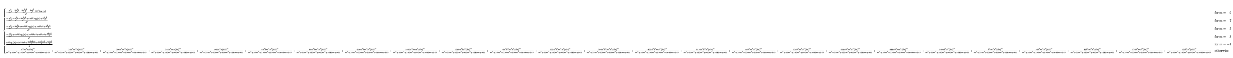
$$\frac{\left(\left(b^4m^4+16\,b^4m^3+86\,b^4m^2+176\,b^4m+105\,b^4\right)x^9+4\left(ab^3m^4+18\,ab^3m^3+104\,ab^3m^2+222\,ab^3m+135\,ab^3\right)x^7+6\left(a^2b^2m^4+20\,a^2b^2m^3+130\,a^2b^2m^2+300\,a^2b^2m+189\,a^2b^2\right)x^5+4\left(a^3bm^4+22\,a^3bm^3+164\,a^3bm^2+458\,a^3bm+315\,a^3b\right)x^3+\left(a^4m^4+24\,a^4m^3+206\,a^4m^2+744\,a^4m+945\,a^4\right)x\right)(dx)^m}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] ((b^4*m^4 + 16*b^4*m^3 + 86*b^4*m^2 + 176*b^4*m + 105*b^4)*x^9 + 4*(a*b^3*m
^4 + 18*a*b^3*m^3 + 104*a*b^3*m^2 + 222*a*b^3*m + 135*a*b^3)*x^7 + 6*(a^2*b
^2*m^4 + 20*a^2*b^2*m^3 + 130*a^2*b^2*m^2 + 300*a^2*b^2*m + 189*a^2*b^2)*x^
5 + 4*(a^3*b*m^4 + 22*a^3*b*m^3 + 164*a^3*b*m^2 + 458*a^3*b*m + 315*a^3*b)*
x^3 + (a^4*m^4 + 24*a^4*m^3 + 206*a^4*m^2 + 744*a^4*m + 945*a^4)*x)*(d*x)^m
/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1278 vs. $2(94) = 188$.
time = 0.51, size = 1278, normalized size = 12.29



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Piecewise(((-a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x**4) - 2*a
*b**3/x**2 + b**4*log(x))/d**9, Eq(m, -9)), ((-a**4/(6*x**6) - a**3*b/x**4
- 3*a**2*b**2/x**2 + 4*a*b**3*log(x) + b**4*x**2/2)/d**7, Eq(m, -7)), ((-a*
*4/(4*x**4) - 2*a**3*b/x**2 + 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**
4/4)/d**5, Eq(m, -5)), ((-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**
2 + a*b**3*x**4 + b**4*x**6/6)/d**3, Eq(m, -3)), ((a**4*log(x) + 2*a**3*b*x
**2 + 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/3 + b**4*x**8/8)/d, Eq(m, -1)), (a
**4*m**4*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
 24*a**4*m**3*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 206*a**4*m**2*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 744*a**4*m*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 945*a**4*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
1689*m + 945) + 4*a**3*b*m**4*x**3*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 95
0*m**2 + 1689*m + 945) + 88*a**3*b*m**3*x**3*(d*x)**m/(m**5 + 25*m**4 + 230
*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*m**2*x**3*(d*x)**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1832*a**3*b*m*x**3*(d*x)**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1260*a**3*b*x**3*(d*
x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 6*a**2*b**2*m
**4*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1
20*a**2*b**2*m**3*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 168
9*m + 945) + 780*a**2*b**2*m**2*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 +
950*m**2 + 1689*m + 945) + 1800*a**2*b**2*m*x**5*(d*x)**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 1134*a**2*b**2*x**5*(d*x)**m/(m**5 +
 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a*b**3*m**4*x**7*(d*x)**
m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 72*a*b**3*m**3*x*
*7*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 416*a*b
```

```
**3*m**2*x**7*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945
) + 888*a*b**3*m*x**7*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 540*a*b**3*x**7*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
+ 1689*m + 945) + b**4*m**4*x**9*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*
m**2 + 1689*m + 945) + 16*b**4*m**3*x**9*(d*x)**m/(m**5 + 25*m**4 + 230*m**
3 + 950*m**2 + 1689*m + 945) + 86*b**4*m**2*x**9*(d*x)**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 176*b**4*m*x**9*(d*x)**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*b**4*x**9*(d*x)**m/(m**5 +
 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(104) = 208.
time = 2.58, size = 415, normalized size = 3.99

$$\frac{(dx)^m b^4m^4x^9 + 16\,(dx)^m b^4m^3x^9 + 4\,(dx)^m ab^3m^4x^7 + 86\,(dx)^m b^4m^2x^9 + 72\,(dx)^m ab^3m^3x^7 + 176\,(dx)^m b^4mx^9 + 6\,(dx)^m a^2b^2m^4x^5 + 416\,(dx)^m ab^3m^2x^7 + 105\,(dx)^m b^4x^9 + 120\,(dx)^m a^2b^2m^3x^5 + 888\,(dx)^m ab^3mx^7 + 4\,(dx)^m a^3bm^4x^3 + 780\,(dx)^m a^2b^2m^2x^5 + 540\,(dx)^m ab^3x^7 + 88\,(dx)^m a^3bm^3x^3 + 1800\,(dx)^m a^2b^2mx^5 + (dx)^m a^4m^4x + 656\,(dx)^m a^3bm^2x^3 + 1134\,(dx)^m a^2b^2x^5 + 24\,(dx)^m a^4m^3x + 1832\,(dx)^m a^3bmx^3 + 206\,(dx)^m a^4m^2x + 1260\,(dx)^m a^3bx^3 + 744\,(dx)^m a^4mx + 945\,(dx)^m a^4x}{m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^4*m^4*x^9 + 16*(d*x)^m*b^4*m^3*x^9 + 4*(d*x)^m*a*b^3*m^4*x^7 + 8
6*(d*x)^m*b^4*m^2*x^9 + 72*(d*x)^m*a*b^3*m^3*x^7 + 176*(d*x)^m*b^4*m*x^9 +
6*(d*x)^m*a^2*b^2*m^4*x^5 + 416*(d*x)^m*a*b^3*m^2*x^7 + 105*(d*x)^m*b^4*x^9
 + 120*(d*x)^m*a^2*b^2*m^3*x^5 + 888*(d*x)^m*a*b^3*m*x^7 + 4*(d*x)^m*a^3*b*
m^4*x^3 + 780*(d*x)^m*a^2*b^2*m^2*x^5 + 540*(d*x)^m*a*b^3*x^7 + 88*(d*x)^m*
a^3*b*m^3*x^3 + 1800*(d*x)^m*a^2*b^2*m*x^5 + (d*x)^m*a^4*m^4*x + 656*(d*x)^
m*a^3*b*m^2*x^3 + 1134*(d*x)^m*a^2*b^2*x^5 + 24*(d*x)^m*a^4*m^3*x + 1832*(d
*x)^m*a^3*b*m*x^3 + 206*(d*x)^m*a^4*m^2*x + 1260*(d*x)^m*a^3*b*x^3 + 744*(d
*x)^m*a^4*m*x + 945*(d*x)^m*a^4*x)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689
*m + 945)
```

Mupad [B]
time = 4.51, size = 263, normalized size = 2.53

$$(d\,x)^m\,\left(\frac{b^4\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{a^4\,x\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{4\,a\,b^3\,x^7\,\left(m^4+18\,m^3+104\,m^2+222\,m+135\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{4\,a^3\,b\,x^3\,\left(m^4+22\,m^3+164\,m^2+458\,m+315\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{6\,a^2\,b^2\,x^5\,\left(m^4+20\,m^3+130\,m^2+300\,m+189\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

```
[Out] (d*x)^m*((b^4*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2
+ 230*m^3 + 25*m^4 + m^5 + 945) + (a^4*x*(744*m + 206*m^2 + 24*m^3 + m^4 +
945))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (4*a*b^3*x^7*(222
*m + 104*m^2 + 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 +
m^5 + 945) + (4*a^3*b*x^3*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m +
 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (6*a^2*b^2*x^5*(300*m + 130*m^2
+ 20*m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))
```

3.787 $\int (dx)^m (a^2+2abx^2+b^2x^4)\,dx$

Optimal. Leaf size=58

$$\frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{b^2(dx)^{5+m}}{d^5(5+m)}$$

```
[Out] a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*(d*x)^(3+m)/d^3/(3+m)+b^2*(d*x)^(5+m)/d^5/(5+
m)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {14}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{b^2(dx)^{m+5}}{d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (b^
2*(d*x)^(5 + m))/(d^5*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a^2+2abx^2+b^2x^4\right)\,dx &= \int \left(a^2(dx)^m+\frac{2ab(dx)^{2+m}}{d^2}+\frac{b^2(dx)^{4+m}}{d^4}\right)\,dx\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{b^2(dx)^{5+m}}{d^5(5+m)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 0.71

$$x(dx)^m\left(\frac{a^2}{1+m}+\frac{2abx^2}{3+m}+\frac{b^2x^4}{5+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

Maple [A]
time = 0.02, size = 57, normalized size = 0.98

method	result	size
norman	$\frac{a^{2} x \,{\mathrm e}^{m \ln \left(d x \right)}}{1+m}+\frac{b^{2} x^{5} {\mathrm e}^{m \ln \left(d x \right)}}{5+m}+\frac{2 a b \,x^{3} {\mathrm e}^{m \ln \left(d x \right)}}{3+m}$	57
gosper	$\frac{\left(d x \right)^{m} \left(b^{2} m^{2} x^{4}+4 b^{2} m \,x^{4}+2 a b \,m^{2} x^{2}+3 b^{2} x^{4}+12 a b m \,x^{2}+a^{2} m^{2}+10 a b \,x^{2}+8 a^{2} m +15 a^{2}\right) x}{\left(5+m \right) \left(3+m \right) \left(1+m \right)}$	94
risch	$\frac{\left(d x \right)^{m} \left(b^{2} m^{2} x^{4}+4 b^{2} m \,x^{4}+2 a b \,m^{2} x^{2}+3 b^{2} x^{4}+12 a b m \,x^{2}+a^{2} m^{2}+10 a b \,x^{2}+8 a^{2} m +15 a^{2}\right) x}{\left(5+m \right) \left(3+m \right) \left(1+m \right)}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/(1+m)*x*exp(m*ln(d*x))+b^2/(5+m)*x^5*exp(m*ln(d*x))+2*a*b/(3+m)*x^3*exp
(m*ln(d*x))
```

Maxima [A]
time = 0.29, size = 56, normalized size = 0.97

$$\frac{b^{2} d^{m} x^{5} x^{m}}{m+5}+\frac{2 \, a b d^{m} x^{3} x^{m}}{m+3}+\frac{\left(d x\right)^{m+1} a^{2}}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^5*x^m/(m + 5) + 2*a*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^2/(d*
(m + 1))
```

Fricas [A]
time = 0.36, size = 87, normalized size = 1.50

$$\frac{((b^{2} m^{2} + 4 \, b^{2} m + 3 \, b^{2})x^{5} + 2 \, (abm^{2} + 6 \, abm + 5 \, ab)x^{3} + (a^{2} m^{2} + 8 \, a^{2} m + 15 \, a^{2})x)(dx)^{m}}{m^{3} + 9 \, m^{2} + 23 \, m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^2
*m^2 + 8*a^2*m + 15*a^2)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(49) = 98$.

time = 0.24, size = 330, normalized size = 5.69

$$\begin{cases} \frac{-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log{(x)}}{d^5} & \text{for } m=-5 \\ \frac{-\frac{a^2}{2x^2}+2ab\log{(x)}+\frac{b^2x^2}{2}}{d^3} & \text{for } m=-3 \\ \frac{a^2\log{(x)}+abx^2+\frac{b^2x^4}{4}}{d} & \text{for } m=-1 \\ \frac{a^2m^2x(dx)^m}{m^3+9m^2+23m+15}+\frac{8a^2mx(dx)^m}{m^3+9m^2+23m+15}+\frac{15a^2x(dx)^m}{m^3+9m^2+23m+15}+\frac{2abm^2x^3(dx)^m}{m^3+9m^2+23m+15}+\frac{12abmx^3(dx)^m}{m^3+9m^2+23m+15}+\frac{10abx^3(dx)^m}{m^3+9m^2+23m+15}+\frac{b^2m^2x^5(dx)^m}{m^3+9m^2+23m+15}+\frac{4b^2mx^5(dx)^m}{m^3+9m^2+23m+15}+\frac{3b^2x^5(dx)^m}{m^3+9m^2+23m+15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Piecewise(((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x))/d**5, Eq(m, -5)), ((-a
**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2)/d**3, Eq(m, -3)), ((a**2*log(x)
+ a*b*x**2 + b**2*x**4/4)/d, Eq(m, -1)), (a**2*m**2*x*(d*x)**m/(m**3 + 9*m*
*2 + 23*m + 15) + 8*a**2*m*x*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a**2
*x*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 2*a*b*m**2*x**3*(d*x)**m/(m**3 +
9*m**2 + 23*m + 15) + 12*a*b*m*x**3*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) +
10*a*b*x**3*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + b**2*m**2*x**5*(d*x)**m/
(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*m*x**5*(d*x)**m/(m**3 + 9*m**2 + 23*m
+ 15) + 3*b**2*x**5*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(58) = 116.
time = 3.03, size = 135, normalized size = 2.33

$$\frac{(dx)^m b^2m^2x^5+4(dx)^m b^2mx^5+2(dx)^m abm^2x^3+3(dx)^m b^2x^5+12(dx)^m abmx^3+(dx)^m a^2m^2x+10(dx)^m abx^3+8(dx)^m a^2mx+15(dx)^m a^2x}{m^3+9m^2+23m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^2*m^2*x^5 + 4*(d*x)^m*b^2*m*x^5 + 2*(d*x)^m*a*b*m^2*x^3 + 3*(d*x
)^m*b^2*x^5 + 12*(d*x)^m*a*b*m*x^3 + (d*x)^m*a^2*m^2*x + 10*(d*x)^m*a*b*x^3
 + 8*(d*x)^m*a^2*m*x + 15*(d*x)^m*a^2*x)/(m^3 + 9*m^2 + 23*m + 15)
```

Mupad [B]
time = 4.27, size = 95, normalized size = 1.64

$$(d\,x)^m\,\left(\frac{a^2\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15}+\frac{b^2\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15}+\frac{2\,a\,b\,x^3\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] (d*x)^m*((a^2*x*(8*m + m^2 + 15))/(23*m + 9*m^2 + m^3 + 15) + (b^2*x^5*(4*m
 + m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (2*a*b*x^3*(6*m + m^2 + 5))/(23*m
+ 9*m^2 + m^3 + 15))
```

3.788 $\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=44

$$\frac{(dx)^{1+m} {}_2F_1\left(2, \frac{1+m}{2}; \frac{3+m}{2}; -\frac{bx^2}{a}\right)}{a^2d(1+m)}$$

```
[Out] (d*x)^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/d/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {28, 371}

$$\frac{(dx)^{m+1} {}_2F_1\left(2, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a^2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^2*d*(1 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4} dx &= b^2 \int \frac{(dx)^m}{(ab+b^2x^2)^2} dx\\ &= \frac{(dx)^{1+m} {}_2F_1\left(2, \frac{1+m}{2}; \frac{3+m}{2}; -\frac{bx^2}{a}\right)}{a^2d(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.95

$$\frac{x(dx)^m\,{}_2F_1\left(2,\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
[Out] (x*(d*x)^m*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^2*(1 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b^2x^4+2ab\,x^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x)
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x)^m}{a^2+2\,a\,b\,x^2+b^2\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2), x)
```

3.789 $\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=44

$$\frac{(dx)^{1+m}\,{}_2F_1\left(4,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^4d(1+m)}$$

```
[Out] (d*x)^(1+m)*hypergeom([4, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^4/d/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {28, 371}

$$\frac{(dx)^{m+1}\,{}_2F_1\left(4,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^4d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[4, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^4*d*(1 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^2}\,dx &= b^4\int \frac{(dx)^m}{(ab+b^2x^2)^4}\,dx\\ &= \frac{(dx)^{1+m}\,{}_2F_1\left(4,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^4d(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.95

$$\frac{x(dx)^m\,{}_2F_1\left(4,\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a^4(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (x*(d*x)^m*Hypergeometric2F1[4, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^4*(1 + m))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b^2x^4+2ab\,x^2+a^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
[Out] Integral((d*x)**m/(a + b*x**2)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x)^m}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^2, x)
```

3.790 $\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=44

$$\frac{(dx)^{1+m}\,{}_2F_1\left(6,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^6d(1+m)}$$

```
[Out] (d*x)^(1+m)*hypergeom([6, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^6/d/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {28, 371}

$$\frac{(dx)^{m+1}\,{}_2F_1\left(6,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^6d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[6, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^6*d*(1 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^3}\,dx &= b^6\int \frac{(dx)^m}{(ab+b^2x^2)^6}\,dx\\ &= \frac{(dx)^{1+m}\,{}_2F_1\left(6,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^6d(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.95

$$\frac{x(dx)^m\,{}_2F_1\left(6,\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a^6(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (x*(d*x)^m*Hypergeometric2F1[6, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(a
^6*(1 + m))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b^2x^4+2ab\,x^2+a^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^6*x^12 + 6*a*b^5*x^10 + 15*a^2*b^4*x^8 + 20*a^3*b^3*x^6
 + 15*a^4*b^2*x^4 + 6*a^5*b*x^2 + a^6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a+bx^2)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2)**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x)^m}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^3, x)
```

3.791 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=313

$$\frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)}+\frac{5a^4b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)}+\frac{10a^3b^2(dx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^5(5+m)\left(a+bx^2\right)}+$$

```
[Out] a^5*(d*x)^(1+m)*((b*x^2+a)^2)^(1/2)/d/(1+m)/(b*x^2+a)+5*a^4*b*(d*x)^(3+m)*(
(b*x^2+a)^2)^(1/2)/d^3/(3+m)/(b*x^2+a)+10*a^3*b^2*(d*x)^(5+m)*((b*x^2+a)^2)
^(1/2)/d^5/(5+m)/(b*x^2+a)+10*a^2*b^3*(d*x)^(7+m)*((b*x^2+a)^2)^(1/2)/d^7/(
7+m)/(b*x^2+a)+5*a*b^4*(d*x)^(9+m)*((b*x^2+a)^2)^(1/2)/d^9/(9+m)/(b*x^2+a)+
b^5*(d*x)^(11+m)*((b*x^2+a)^2)^(1/2)/d^11/(11+m)/(b*x^2+a)
```

Rubi [A]
time = 0.08, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1126, 276}

$$\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+11}}{d^{11}(m+11)\left(a+bx^2\right)}+\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+9}}{d^9(m+9)\left(a+bx^2\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^2\right)}+\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+1}}{d(m+1)\left(a+bx^2\right)}+\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\left(a+bx^2\right)}+\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+5}}{d^5(m+5)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2))
 + (5*a^4*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a
+ b*x^2)) + (10*a^3*b^2*(d*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5
*(5 + m)*(a + b*x^2)) + (10*a^2*b^3*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(d^7*(7 + m)*(a + b*x^2)) + (5*a*b^4*(d*x)^(9 + m)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(d^9*(9 + m)*(a + b*x^2)) + (b^5*(d*x)^(11 + m)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(d^11*(11 + m)*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m \left(ab+b^2x^2\right)^5\,dx}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a^5b^5(dx)^m+\frac{5a^4b^6(dx)^{2+m}}{d^2}+\frac{10a^3b^7(dx)^{4+m}}{d^4}+\frac{10a^2\cdots}{}\right.}{b^4\left(ab+b^2x^2\right)}$$
$$= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)}+\frac{5a^4b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)}+\cdots$$

Mathematica [A]
time = 0.12, size = 111, normalized size = 0.35

$$\frac{x(dx)^m\left(\left(a+bx^2\right)^2\right)^{5/2}\left(\frac{a^5}{1+m}+\frac{5a^4bx^2}{3+m}+\frac{10a^3b^2x^4}{5+m}+\frac{10a^2b^3x^6}{7+m}+\frac{5ab^4x^8}{9+m}+\frac{b^5x^{10}}{11+m}\right)}{\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^2)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^2)/(3 + m) + (10
*a^3*b^2*x^4)/(5 + m) + (10*a^2*b^3*x^6)/(7 + m) + (5*a*b^4*x^8)/(9 + m) +
(b^5*x^10)/(11 + m)))/(a + b*x^2)^5
```

Maple [A]
time = 0.03, size = 453, normalized size = 1.45

method	result
gosper	$\frac{x\left(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8+1689m\,x^{10}b^5+290a^2b\cdots\right.}{}$
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8+168\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^5*m^5*x^10+25*b^5*m^4*x^10+5*a*b^4*m^5*x^8+230*b^5*m^3*x^10+135*a*b^4*
m^4*x^8+950*b^5*m^2*x^10+10*a^2*b^3*m^5*x^6+1310*a*b^4*m^3*x^8+1689*b^5*m*x
^10+290*a^2*b^3*m^4*x^6+5610*a*b^4*m^2*x^8+945*b^5*x^10+10*a^3*b^2*m^5*x^4+
3020*a^2*b^3*m^3*x^6+10205*a*b^4*m*x^8+310*a^3*b^2*m^4*x^4+13660*a^2*b^3*m^
2*x^6+5775*a*b^4*x^8+5*a^4*b*m^5*x^2+3500*a^3*b^2*m^3*x^4+25770*a^2*b^3*m*x
^6+165*a^4*b*m^4*x^2+17300*a^3*b^2*m^2*x^4+14850*a^2*b^3*x^6+a^5*m^5+2030*a
```

```
^4*b*m^3*x^2+34890*a^3*b^2*m*x^4+35*a^5*m^4+11310*a^4*b*m^2*x^2+20790*a^3*b
^2*x^4+470*a^5*m^3+26765*a^4*b*m*x^2+3010*a^5*m^2+17325*a^4*b*x^2+9129*a^5*
m+10395*a^5)*(d*x)^m*((b*x^2+a)^2)^(5/2)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+
m)/(b*x^2+a)^5
```

Maxima [A]
time = 0.30, size = 243, normalized size = 0.78

$$\frac{\left(\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)b^5d^mx^{11}+5\,\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)ab^4d^mx^9+10\,\left(m^5+29\,m^4+302\,m^3+1366\,m^2+2577\,m+1485\right)a^2b^3d^mx^7+10\,\left(m^5+31\,m^4+350\,m^3+1730\,m^2+3489\,m+2079\right)a^3b^2d^mx^5+5\,\left(m^5+33\,m^4+406\,m^3+2262\,m^2+5353\,m+3465\right)a^4bd^mx^3+\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)a^5d^mx\right)x^m}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*d^m*x^11 + 5*(m^5 +
27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*d^m*x^9 + 10*(m^5 + 29*m
^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*d^m*x^7 + 10*(m^5 + 31*m^4
 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*d^m*x^5 + 5*(m^5 + 33*m^4 +
406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*d^m*x^3 + (m^5 + 35*m^4 + 470*m^3
 + 3010*m^2 + 9129*m + 10395)*a^5*d^m*x)*x^m/(m^6 + 36*m^5 + 505*m^4 + 3480
*m^3 + 12139*m^2 + 19524*m + 10395)
```

Fricas [A]
time = 0.38, size = 369, normalized size = 1.18

$$\frac{\left(\left(b^5m^5+25\,b^5m^4+230\,b^5m^3+950\,b^5m^2+1689\,b^5m+945\,b^5\right)x^{11}+5\,\left(ab^4m^5+27\,ab^4m^4+262\,ab^4m^3+1122\,ab^4m^2+2041\,ab^4m+1155\,ab^4\right)x^9+10\,\left(a^2b^3m^5+29\,a^2b^3m^4+302\,a^2b^3m^3+1366\,a^2b^3m^2+2577\,a^2b^3m+1485\,a^2b^3\right)x^7+10\,\left(a^3b^2m^5+31\,a^3b^2m^4+350\,a^3b^2m^3+1730\,a^3b^2m^2+3489\,a^3b^2m+2079\,a^3b^2\right)x^5+5\,\left(a^4bm^5+33\,a^4bm^4+406\,a^4bm^3+2262\,a^4bm^2+5353\,a^4bm+3465\,a^4b\right)x^3+\left(a^5m^5+35\,a^5m^4+470\,a^5m^3+3010\,a^5m^2+9129\,a^5m+10395\,a^5\right)x\right)(dx)^m}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 25*b^5*m^4 + 230*b^5*m^3 + 950*b^5*m^2 + 1689*b^5*m + 945*b^5)*
x^11 + 5*(a*b^4*m^5 + 27*a*b^4*m^4 + 262*a*b^4*m^3 + 1122*a*b^4*m^2 + 2041*
a*b^4*m + 1155*a*b^4)*x^9 + 10*(a^2*b^3*m^5 + 29*a^2*b^3*m^4 + 302*a^2*b^3*
m^3 + 1366*a^2*b^3*m^2 + 2577*a^2*b^3*m + 1485*a^2*b^3)*x^7 + 10*(a^3*b^2*m
^5 + 31*a^3*b^2*m^4 + 350*a^3*b^2*m^3 + 1730*a^3*b^2*m^2 + 3489*a^3*b^2*m +
 2079*a^3*b^2)*x^5 + 5*(a^4*b*m^5 + 33*a^4*b*m^4 + 406*a^4*b*m^3 + 2262*a^4
*b*m^2 + 5353*a^4*b*m + 3465*a^4*b)*x^3 + (a^5*m^5 + 35*a^5*m^4 + 470*a^5*m
^3 + 3010*a^5*m^2 + 9129*a^5*m + 10395*a^5)*x)*(d*x)^m/(m^6 + 36*m^5 + 505*
m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 900 vs. 2(247) = 494.
time = 4.45, size = 900, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^5*m^5*x^11*sgn(b*x^2 + a) + 25*(d*x)^m*b^5*m^4*x^11*sgn(b*x^2 +
a) + 5*(d*x)^m*a*b^4*m^5*x^9*sgn(b*x^2 + a) + 230*(d*x)^m*b^5*m^3*x^11*sgn(
b*x^2 + a) + 135*(d*x)^m*a*b^4*m^4*x^9*sgn(b*x^2 + a) + 950*(d*x)^m*b^5*m^2
*x^11*sgn(b*x^2 + a) + 10*(d*x)^m*a^2*b^3*m^5*x^7*sgn(b*x^2 + a) + 1310*(d*
x)^m*a*b^4*m^3*x^9*sgn(b*x^2 + a) + 1689*(d*x)^m*b^5*m*x^11*sgn(b*x^2 + a)
+ 290*(d*x)^m*a^2*b^3*m^4*x^7*sgn(b*x^2 + a) + 5610*(d*x)^m*a*b^4*m^2*x^9*s
gn(b*x^2 + a) + 945*(d*x)^m*b^5*x^11*sgn(b*x^2 + a) + 10*(d*x)^m*a^3*b^2*m^
5*x^5*sgn(b*x^2 + a) + 3020*(d*x)^m*a^2*b^3*m^3*x^7*sgn(b*x^2 + a) + 10205*
(d*x)^m*a*b^4*m*x^9*sgn(b*x^2 + a) + 310*(d*x)^m*a^3*b^2*m^4*x^5*sgn(b*x^2
+ a) + 13660*(d*x)^m*a^2*b^3*m^2*x^7*sgn(b*x^2 + a) + 5775*(d*x)^m*a*b^4*x^
9*sgn(b*x^2 + a) + 5*(d*x)^m*a^4*b*m^5*x^3*sgn(b*x^2 + a) + 3500*(d*x)^m*a^
3*b^2*m^3*x^5*sgn(b*x^2 + a) + 25770*(d*x)^m*a^2*b^3*m*x^7*sgn(b*x^2 + a) +
 165*(d*x)^m*a^4*b*m^4*x^3*sgn(b*x^2 + a) + 17300*(d*x)^m*a^3*b^2*m^2*x^5*s
gn(b*x^2 + a) + 14850*(d*x)^m*a^2*b^3*x^7*sgn(b*x^2 + a) + (d*x)^m*a^5*m^5*
x*sgn(b*x^2 + a) + 2030*(d*x)^m*a^4*b*m^3*x^3*sgn(b*x^2 + a) + 34890*(d*x)^
m*a^3*b^2*m*x^5*sgn(b*x^2 + a) + 35*(d*x)^m*a^5*m^4*x*sgn(b*x^2 + a) + 1131
0*(d*x)^m*a^4*b*m^2*x^3*sgn(b*x^2 + a) + 20790*(d*x)^m*a^3*b^2*x^5*sgn(b*x^
2 + a) + 470*(d*x)^m*a^5*m^3*x*sgn(b*x^2 + a) + 26765*(d*x)^m*a^4*b*m*x^3*s
gn(b*x^2 + a) + 3010*(d*x)^m*a^5*m^2*x*sgn(b*x^2 + a) + 17325*(d*x)^m*a^4*b
*x^3*sgn(b*x^2 + a) + 9129*(d*x)^m*a^5*m*x*sgn(b*x^2 + a) + 10395*(d*x)^m*a
^5*x*sgn(b*x^2 + a))/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524
*m + 10395)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.792 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=205

$$\frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\,(a+bx^2)}+\frac{3a^2b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\,(a+bx^2)}+\frac{3ab^2(dx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^5(5+m)\,(a+bx^2)}+\frac{b^3}{}$$

```
[Out] a^3*(d*x)^(1+m)*((b*x^2+a)^2)^(1/2)/d/(1+m)/(b*x^2+a)+3*a^2*b*(d*x)^(3+m)*(
(b*x^2+a)^2)^(1/2)/d^3/(3+m)/(b*x^2+a)+3*a*b^2*(d*x)^(5+m)*((b*x^2+a)^2)^(1
/2)/d^5/(5+m)/(b*x^2+a)+b^3*(d*x)^(7+m)*((b*x^2+a)^2)^(1/2)/d^7/(7+m)/(b*x^
2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1126, 276}

$$\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+5}}{d^5(m+5)\,(a+bx^2)}+\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\,(a+bx^2)}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+7}}{d^7(m+7)\,(a+bx^2)}+\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+1}}{d(m+1)\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2))
 + (3*a^2*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a
+ b*x^2)) + (3*a*b^2*(d*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(5
 + m)*(a + b*x^2)) + (b^3*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d
^7*(7 + m)*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (dx)^m \left(ab+b^2x^2\right)^3 dx}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3(dx)^m + \frac{3a^2b^4(dx)^{2+m}}{d^2} + \frac{3ab^5(dx)^{4+m}}{d^4} + \frac{b^6(dx)^6}{d^6}\right.}{b^2\left(ab+b^2x^2\right)}$$

$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)} + \frac{3a^2b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)} +$$

Mathematica [A]
time = 0.07, size = 131, normalized size = 0.64

$$\frac{x(dx)^m\sqrt{(a+bx^2)^2}\left(a^3(105+71m+15m^2+m^3)+3a^2b(35+47m+13m^2+m^3)x^2+3ab^2(21+31m+11m^2+m^3)x^4+b^3(15+23m+9m^2+m^3)x^6\right)}{(1+m)(3+m)(5+m)(7+m)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^2)^2]*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^2*b*(
35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^4 + b^
3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(a + b*x
^2))
```

Maple [A]
time = 0.01, size = 199, normalized size = 0.97

method	result
gosper	$\frac{x\left(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23m\,x^6b^3+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93m\,x^4a\,b^2+39a^2b\,m^2x^2+63a\,b^2x^4+a^3m^3+141m\,x\right.}{(7+m)(5+m)(3+m)(1+m)\left(b\,x^2+a\right)^3}$
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23m\,x^6b^3+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93m\,x^4a\,b^2+39a^2b\,m^2x^2+63a\,b^2x\right.}{\left(b\,x^2+a\right)(7+m)(5+m)(3+m)(1+m)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^2*x^4+
15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x^4+a^3
*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)*(d*x)^m*((b
*x^2+a)^2)^(3/2)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 119, normalized size = 0.58

$$\frac{((m^3+9\,m^2+23\,m+15)b^3d^mx^7+3\,(m^3+11\,m^2+31\,m+21)ab^2d^mx^5+3\,(m^3+13\,m^2+47\,m+35)a^2bd^mx^3+(m^3+15\,m^2+71\,m+105)a^3d^mx)x^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*d^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a*b^2
*d^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*d^m*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*a^3*d^m*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Fricas [A]
time = 0.36, size = 159, normalized size = 0.78

$$\frac{\left(\left(b^3m^3+9\,b^3m^2+23\,b^3m+15\,b^3\right)x^7+3\,\left(ab^2m^3+11\,ab^2m^2+31\,ab^2m+21\,ab^2\right)x^5+3\,\left(a^2bm^3+13\,a^2bm^2+47\,a^2bm+35\,a^2b\right)x^3+\left(a^3m^3+15\,a^3m^2+71\,a^3m+105\,a^3\right)x\right)(dx)^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 9*b^3*m^2 + 23*b^3*m + 15*b^3)*x^7 + 3*(a*b^2*m^3 + 11*a*b^2*m^
2 + 31*a*b^2*m + 21*a*b^2)*x^5 + 3*(a^2*b*m^3 + 13*a^2*b*m^2 + 47*a^2*b*m +
 35*a^2*b)*x^3 + (a^3*m^3 + 15*a^3*m^2 + 71*a^3*m + 105*a^3)*x)*(d*x)^m/(m^
4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(161) = 322.
time = 2.73, size = 384, normalized size = 1.87

$$\frac{(dx)^m b^3m^3x^7\mathrm{sgn}(bx^2+a)+9\,(dx)^m b^3m^2x^7\mathrm{sgn}(bx^2+a)+3\,(dx)^m ab^2m^3x^5\mathrm{sgn}(bx^2+a)+23\,(dx)^m b^3mx^7\mathrm{sgn}(bx^2+a)+33\,(dx)^m ab^2m^2x^5\mathrm{sgn}(bx^2+a)+15\,(dx)^m b^3x^7\mathrm{sgn}(bx^2+a)+3\,(dx)^m a^2bm^3x^3\mathrm{sgn}(bx^2+a)+93\,(dx)^m ab^2mx^5\mathrm{sgn}(bx^2+a)+39\,(dx)^m a^2bm^2x^3\mathrm{sgn}(bx^2+a)+63\,(dx)^m ab^2x^5\mathrm{sgn}(bx^2+a)+(dx)^m a^3m^3x\mathrm{sgn}(bx^2+a)+141\,(dx)^m a^2bmx^3\mathrm{sgn}(bx^2+a)+15\,(dx)^m a^3m^2x\mathrm{sgn}(bx^2+a)+105\,(dx)^m a^2bx^3\mathrm{sgn}(bx^2+a)+71\,(dx)^m a^3mx\mathrm{sgn}(bx^2+a)+105\,(dx)^m a^3x\mathrm{sgn}(bx^2+a)}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^3*m^3*x^7*sgn(b*x^2 + a) + 9*(d*x)^m*b^3*m^2*x^7*sgn(b*x^2 + a)
+ 3*(d*x)^m*a*b^2*m^3*x^5*sgn(b*x^2 + a) + 23*(d*x)^m*b^3*m*x^7*sgn(b*x^2 +
 a) + 33*(d*x)^m*a*b^2*m^2*x^5*sgn(b*x^2 + a) + 15*(d*x)^m*b^3*x^7*sgn(b*x^
2 + a) + 3*(d*x)^m*a^2*b*m^3*x^3*sgn(b*x^2 + a) + 93*(d*x)^m*a*b^2*m*x^5*sg
n(b*x^2 + a) + 39*(d*x)^m*a^2*b*m^2*x^3*sgn(b*x^2 + a) + 63*(d*x)^m*a*b^2*x
^5*sgn(b*x^2 + a) + (d*x)^m*a^3*m^3*x*sgn(b*x^2 + a) + 141*(d*x)^m*a^2*b*m*
```

```
x^3*sgn(b*x^2 + a) + 15*(d*x)^m*a^3*m^2*x*sgn(b*x^2 + a) + 105*(d*x)^m*a^2*
b*x^3*sgn(b*x^2 + a) + 71*(d*x)^m*a^3*m*x*sgn(b*x^2 + a) + 105*(d*x)^m*a^3*
x*sgn(b*x^2 + a))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.793 $\int (dx)^m \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=97

$$\frac{a(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\,(a+bx^2)}+\frac{b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\,(a+bx^2)}$$

```
[Out] a*(d*x)^(1+m)*((b*x^2+a)^2)^(1/2)/d/(1+m)/(b*x^2+a)+b*(d*x)^(3+m)*((b*x^2+a
)^2)^(1/2)/d^3/(3+m)/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1126, 14}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+3}}{d^3(m+3)\,(a+bx^2)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(dx)^{m+1}}{d(m+1)\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a + b*x^2)) +
 (b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(3 + m)*(a + b*x^2)
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (dx)^m\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m\left(ab+b^2x^2\right)\,dx}{ab+b^2x^2}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab(dx)^m+\frac{b^2(dx)^{2+m}}{d^2}\right)\,dx}{ab+b^2x^2}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)}+\frac{b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.55

$$\frac{x(dx)^m\sqrt{\left(a+bx^2\right)^2}\left(a(3+m)+b(1+m)x^2\right)}{(1+m)(3+m)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^2)^2]*(a*(3 + m) + b*(1 + m)*x^2))/((1 + m)*(3 + m
)*(a + b*x^2))
```

Maple [A]
time = 0.01, size = 56, normalized size = 0.58

method	result	size
gosper	$\frac{x\left(bm\,x^2+b\,x^2+am+3a\right)(dx)^m\sqrt{\left(b\,x^2+a\right)^2}}{(3+m)(1+m)\left(b\,x^2+a\right)}$	56
risch	$\frac{x\left(bm\,x^2+b\,x^2+am+3a\right)(dx)^m\sqrt{\left(b\,x^2+a\right)^2}}{(3+m)(1+m)\left(b\,x^2+a\right)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*m*x^2+b*x^2+a*m+3*a)*(d*x)^m*((b*x^2+a)^2)^(1/2)/(3+m)/(1+m)/(b*x^2+a)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.36

$$\frac{\left(bd^m(m+1)x^3+ad^m(m+3)x\right)x^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*d^m*(m + 1)*x^3 + a*d^m*(m + 3)*x)*x^m/(m^2 + 4*m + 3)
```

Fricas [A]
time = 0.35, size = 35, normalized size = 0.36

$$\frac{((bm+b)x^3+(am+3\,a)x)(dx)^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^3 + (a*m + 3*a)*x)*(d*x)^m/(m^2 + 4*m + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**2)**2), x)
```

Giac [A]
time = 3.57, size = 83, normalized size = 0.86

$$\frac{(dx)^m\,bmx^3\mathrm{sgn}(bx^2+a)+(dx)^m\,bx^3\mathrm{sgn}(bx^2+a)+(dx)^m\,amx\mathrm{sgn}(bx^2+a)+3\,(dx)^m\,ax\mathrm{sgn}(bx^2+a)}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b*m*x^3*sgn(b*x^2 + a) + (d*x)^m*b*x^3*sgn(b*x^2 + a) + (d*x)^m*a*
m*x*sgn(b*x^2 + a) + 3*(d*x)^m*a*x*sgn(b*x^2 + a))/(m^2 + 4*m + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

3.794 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=73

$$\frac{(dx)^{1+m}\,(a+bx^2)\;{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (d*x)^(1+m)*(b*x^2+a)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/d/(1
+m)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1126, 371\}$

$$\frac{(a+bx^2)\,(dx)^{m+1}\;{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(ab+b^2x^2)\int \frac{(dx)^m}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(dx)^{1+m}\,(a+bx^2)\;{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.85

$$\frac{x(dx)^m\,(a+bx^2)\;{}_2F_1\left(1,\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*
x^2)/a)])/(a*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b^2x^4+2ab\,x^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fricas")
[Out] integral((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{(a+bx^2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
[Out] Integral((d*x)**m/sqrt((a + b*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

3.795 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{(dx)^{1+m}\,(a+bx^2)\;{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (d*x)^(1+m)*(b*x^2+a)*hypergeom([3, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/d/
(1+m)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1126, 371}

$$\frac{(a+bx^2)\,(dx)^{m+1}\;{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2))\int \frac{(dx)^m}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(dx)^{1+m}\,(a+bx^2)\;{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.82

$$\frac{x(dx)^m\,(a+bx^2)\;{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^3(1+m)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m/(b^4*x^8 + 4*a*b^3*x^6 + 6
*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**2)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.796 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{(dx)^{1+m}\left(a+bx^2\right){}_2F_1\left(5,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (d*x)^(1+m)*(b*x^2+a)*hypergeom([5, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^5/d/
(1+m)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1126, 371}

$$\frac{\left(a+bx^2\right)(dx)^{m+1}{}_2F_1\left(5,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1126

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*Fra
cPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m
, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(b^4(ab+b^2x^2))\int \frac{(dx)^m}{(ab+b^2x^2)^5}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{(dx)^{1+m}(a+bx^2)\,{}_2F_1\left(5,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.82

$$\frac{x(dx)^m(a+bx^2)\,{}_2F_1\left(5,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{a^5(1+m)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m/(b^6*x^12 + 6*a*b^5*x^10 +
 15*a^2*b^4*x^8 + 20*a^3*b^3*x^6 + 15*a^4*b^2*x^4 + 6*a^5*b*x^2 + a^6), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**2)**2)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.797 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=74

$$\frac{(dx)^{1+m}\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(1,\frac{1}{2}(3+m+4p);\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ad(1+m)}$$

```
[Out] (d*x)^(1+m)*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([1, 3/2+1/2*m+2*p
],[3/2+1/2*m],-b*x^2/a)/a/d/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1127, 371}

$$\frac{(dx)^{m+1}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{m+1}{2},-2p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[(1 + m)/2, -
2*p, (3 + m)/2, -((b*x^2)/a)])/(d*(1 + m)*(1 + (b*x^2)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p \, dx &= \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int (dx)^m \left(1+\frac{bx^2}{a}\right)^{2p} dx\\ &= \frac{(dx)^{1+m}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{1+m}{2},-2p;\frac{3+m}{2};-\frac{bx^2}{a}\right)}{d(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.89

$$\frac{x(dx)^m\left((a+bx^2)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}{}_2F_1\left(\frac{1+m}{2},-2p;1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] (x*(d*x)^m*((a + b*x^2)^2)^p*Hypergeometric2F1[(1 + m)/2, -2*p, 1 + (1 + m)
/2, -((b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^(2*p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m\left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
[Out] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\left(\left(a+bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

3.798 $\int x^7(a^2 + 2abx^2 + b^2x^4)^p \, dx$

Optimal. Leaf size=174

$$-\frac{a^3(a+bx^2)(a^2+2abx^2+b^2x^4)^p}{2b^4(1+2p)}+\frac{3a^2(a+bx^2)^2(a^2+2abx^2+b^2x^4)^p}{4b^4(1+p)}-\frac{3a(a+bx^2)^3(a^2+2abx^2+b^2x^4)^p}{2b^4(3+2p)}$$

```
[Out] -1/2*a^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^4/(1+2*p)+3/4*a^2*(b*x^2+a)^
2*(b^2*x^4+2*a*b*x^2+a^2)^p/b^4/(1+p)-3/2*a*(b*x^2+a)^3*(b^2*x^4+2*a*b*x^2+
a^2)^p/b^4/(3+2*p)+1/4*(b*x^2+a)^4*(b^2*x^4+2*a*b*x^2+a^2)^p/b^4/(2+p)
```

Rubi [A]
time = 0.08, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1127, 272, 45}

$$\frac{(a+bx^2)^4(a^2+2abx^2+b^2x^4)^p}{4b^4(p+2)}-\frac{3a(a+bx^2)^3(a^2+2abx^2+b^2x^4)^p}{2b^4(2p+3)}+\frac{3a^2(a+bx^2)^2(a^2+2abx^2+b^2x^4)^p}{4b^4(p+1)}-\frac{a^3(a+bx^2)(a^2+2abx^2+b^2x^4)^p}{2b^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -1/2*(a^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^4*(1 + 2*p)) + (3*a
^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(1 + p)) - (3*a*(a +
 b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4*(3 + 2*p)) + ((a + b*x^2)^4
*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(2 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int x^7\left(a^2+2abx^2+b^2x^4\right)^p\,dx &= \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^7\left(1+\frac{bx^2}{a}\right)^{2p}\,dx\\
&= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^3\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x\right.\\
&= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^3\left(1+\frac{bx}{a}\right)^{2p}}{b^3}+\frac{3a}{}\right.\right.\\
&= -\frac{a^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(1+2p)}+\frac{3a^2(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(1+p)}-
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 110, normalized size = 0.63

$$
\frac{(a+bx^2)\left((a+bx^2)^2\right)^p\left(-3a^3+3a^2b(1+2p)x^2-3ab^2(1+3p+2p^2)\,x^4+b^3(3+11p+12p^2+4p^3)\,x^6\right)}{4b^4(1+p)(2+p)(1+2p)(3+2p)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^2 - 3*a*b^2*(1
 + 3*p + 2*p^2)*x^4 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^6))/(4*b^4*(1 + p)*
(2 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A]
time = 0.04, size = 150, normalized size = 0.86

method	result
gosper	$-\frac{(b^2x^4+2ab\,x^2+a^2)^p(-4b^3p^3x^6-12b^3p^2x^6-11b^3p\,x^6+6a\,b^2p^2x^4-3b^3x^6+9a\,b^2p\,x^4+3a\,b^2x^4-6a^2bp\,x^2-3a^2b\,x^2+3a^3)(b\,x^2+a)}{4b^4(4p^4+20p^3+35p^2+25p+6)}$
risch	$-\frac{(-4b^4p^3x^8-12b^4p^2x^8-4a\,b^3p^3x^6-11b^4p\,x^8-6a\,b^3p^2x^6-3b^4x^8-2ap\,x^6b^3+6a^2b^2p^2x^4+3a^2p\,x^4b^2-6a^3p\,x^2b+3a^4)\left((b\,x^2+a)^2\right)^p}{4(3+2p)(2+p)(1+p)(1+2p)b^4}$
norman	$\frac{x^8\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4p+8}-\frac{3a^4\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4b^4(4p^4+20p^3+35p^2+25p+6)}+\frac{ap\,x^6\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2b(2p^2+7p+6)}-\frac{3a^2p\,x^4\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4b^2(2p^3+9p^2+13p+6)}+\frac{3}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-4*b^3*p^3*x^6-12*b^3*p^2*x^6-11*b^3*p*x^6+
6*a*b^2*p^2*x^4-3*b^3*x^6+9*a*b^2*p*x^4+3*a*b^2*x^4-6*a^2*b*p*x^2-3*a^2*b*x
^2+3*a^3)*(b*x^2+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

Maxima [A]
time = 0.30, size = 115, normalized size = 0.66

$$\frac{\left((4\,p^3+12\,p^2+11\,p+3)b^4x^8+2\,(2\,p^3+3\,p^2+p)ab^3x^6-3\,(2\,p^2+p)a^2b^2x^4+6\,a^3bpx^2-3\,a^4\right)(bx^2+a)^{2\,p}}{4\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^6
- 3*(2*p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b*x^2 + a)^(2*p)/((4*
p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

Fricas [A]
time = 0.35, size = 163, normalized size = 0.94

$$\frac{\left((4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4)x^8+6\,a^3bpx^2+2\,(2\,ab^3p^3+3\,ab^3p^2+ab^3p)x^6-3\,(2\,a^2b^2p^2+a^2b^2p)x^4-3\,a^4\right)(b^2x^4+2\,abx^2+a^2)^p}{4\,(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^8 + 6*a^3*b*p*x^2 + 2*(2
*a*b^3*p^3 + 3*a*b^3*p^2 + a*b^3*p)*x^6 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*x^4
 - 3*a^4)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^4*p^
2 + 25*b^4*p + 6*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^8(a^2)^p}{8} & \text{for } b=0 \\ \frac{6a^3\log\left(x-\sqrt{-\frac{a}{b}}\right)}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{6a^3\log\left(x+\sqrt{-\frac{a}{b}}\right)}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{11a^3}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{18a^2bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{18a^2bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{27a^2bx^2}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{18ab^2x^4\log\left(x-\sqrt{-\frac{a}{b}}\right)}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{18ab^2x^4\log\left(x+\sqrt{-\frac{a}{b}}\right)}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{18ab^2x^4}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{6b^3x^6\log\left(x-\sqrt{-\frac{a}{b}}\right)}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{6b^3x^6\log\left(x+\sqrt{-\frac{a}{b}}\right)}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6} & \text{for } p=-2 \\ \int\frac{x^7}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx & \text{for } p=-\frac{3}{2} \\ \frac{6a^3\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4ab^4+4b^5x^2}+\frac{6a^3\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4ab^4+4b^5x^2}+\frac{6a^3}{4ab^4+4b^5x^2}+\frac{6a^2bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4ab^4+4b^5x^2}+\frac{6a^2bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4ab^4+4b^5x^2}-\frac{3ab^2x^4}{4ab^4+4b^5x^2}+\frac{b^3x^6}{4ab^4+4b^5x^2} & \text{for } p=-1 \\ \int\frac{x^7}{\sqrt{(a+bx^2)^2}}\,dx & \text{for } p=-\frac{1}{2} \\ -\frac{3a^4(a^2+2abx^2+b^2x^4)^p}{16b^4p^4+80b^4p^3+140b^4p^2+100b^4p+24b^4}+\frac{6a^3bpx^2(a^2+2abx^2+b^2x^4)^p}{16b^4p^4+80b^4p^3+140b^4p^2+100b^4p+24b^4}-\frac{6a^2b^2p^2x^4(a^2+2abx^2+b^2x^4)^p}{16b^4p^4+80b^4p^3+140b^4p^2+100b^4p+24b^4}-\frac{3a^2b^2px^4(a^2+2abx^2+b^2x^4)^p}{16b^4p^4+80b^4p^3+140b^4p^2+100b^4p+24b^4}+\frac{4ab^3p^3x^6(a^2+2abx^2+b^2x^4)^p}{16b^4p^4+80b^4p^3+140b^4p^2+100b^4p+24b^4}+\frac{6ab^3p^2x^6(a^2+2abx^2+b^2x^4)^p}{16b^4p^4+80b^4p^3+140b^4p^2+100b^4p+24b^4}+\frac{2ab^3px^6(a^2+2abx^2+b^2x^4)^p}{16b^4p^4+80b^4p^3+140b^4p^2+100b^4p+24b^4}+\frac{4b^4p^3x^8(a^2+2abx^2+b^2x^4)^p}{16b^4p^4+80b^4p^3+140b^4p^2+100b^4p+24b^4}+\frac{12b^4p^2x^8(a^2+2abx^2+b^2x^4)^p}{16b^4p^4+80b^4p^3+140b^4p^2+100b^4p+24b^4}+\frac{11b^4px^8(a^2+2abx^2+b^2x^4)^p}{16b^4p^4+80b^4p^3+140b^4p^2+100b^4p+24b^4}+\frac{3b^4x^8(a^2+2abx^2+b^2x^4)^p}{16b^4p^4+80b^4p^3+140b^4p^2+100b^4p+24b^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**8*(a**2)**p/8, Eq(b, 0)), (6*a**3*log(x - sqrt(-a/b))/(12*a**
3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*log(x
+ sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*
x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**
7*x**6) + 18*a**2*b*x**2*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x
**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(x + sqrt(-a/b))/(
12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2
*b*x**2/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6)
+ 18*a*b**2*x**4*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36
```

```
*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(x + sqrt(-a/b))/(12*a**3*
b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4/
(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3
*x**6*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**
4 + 12*b**7*x**6) + 6*b**3*x**6*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2
*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -2)), (Integral(x**7/((a
 + b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (6*a**3*log(x - sqrt(-a/b))/(4*a*b
**4 + 4*b**5*x**2) + 6*a**3*log(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) +
6*a**3/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(x - sqrt(-a/b))/(4*a*b*
*4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(x + sqrt(-a/b))/(4*a*b**4 + 4*b**5*x*
*2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b**5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5
*x**2), Eq(p, -1)), (Integral(x**7/sqrt((a + b*x**2)**2), x), Eq(p, -1/2)),
 (-3*a**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 +
 140*b**4*p**2 + 100*b**4*p + 24*b**4) + 6*a**3*b*p*x**2*(a**2 + 2*a*b*x**2
 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p
+ 24*b**4) - 6*a**2*b**2*p**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b
**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4) - 3*a**2*b*
*2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 +
 140*b**4*p**2 + 100*b**4*p + 24*b**4) + 4*a*b**3*p**3*x**6*(a**2 + 2*a*b*x
**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4
*p + 24*b**4) + 6*a*b**3*p**2*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b
**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4) + 2*a*b**3*
p*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 14
0*b**4*p**2 + 100*b**4*p + 24*b**4) + 4*b**4*p**3*x**8*(a**2 + 2*a*b*x**2 +
 b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p +
24*b**4) + 12*b**4*p**2*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p*
*4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4) + 11*b**4*p*x**8*
(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*
p**2 + 100*b**4*p + 24*b**4) + 3*b**4*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)*
*p/(16*b**4*p**4 + 80*b**4*p**3 + 140*b**4*p**2 + 100*b**4*p + 24*b**4), Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(166) = 332.
time = 3.77, size = 375, normalized size = 2.16

$$\frac{4 \left(b^2x^4+2\,abx^2+a^2\right)^p b^4p^3x^8+12 \left(b^2x^4+2\,abx^2+a^2\right)^p b^4p^2x^8+4 \left(b^2x^4+2\,abx^2+a^2\right)^p ab^3p^3x^6+11 \left(b^2x^4+2\,abx^2+a^2\right)^p b^4px^8+6 \left(b^2x^4+2\,abx^2+a^2\right)^p ab^3p^2x^6+3 \left(b^2x^4+2\,abx^2+a^2\right)^p b^4x^8+2 \left(b^2x^4+2\,abx^2+a^2\right)^p ab^3px^6-6 \left(b^2x^4+2\,abx^2+a^2\right)^p a^2b^2p^2x^4-3 \left(b^2x^4+2\,abx^2+a^2\right)^p a^2b^2px^4+6 \left(b^2x^4+2\,abx^2+a^2\right)^p a^3bpx^2-3 \left(b^2x^4+2\,abx^2+a^2\right)^p a^4}{4 \left(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^3*x^8 + 12*(b^2*x^4 + 2*a*b*x^2
+ a^2)^p*b^4*p^2*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^3*x^6 + 11*(
b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p*x^8 + 6*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*
b^3*p^2*x^6 + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 2*(b^2*x^4 + 2*a*b*
```

```
x^2 + a^2)^p*a*b^3*p*x^6 - 6*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2*x^4
- 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 + 6*(b^2*x^4 + 2*a*b*x^2 +
a^2)^p*a^3*b*p*x^2 - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^4)/(4*b^4*p^4 + 20*b
^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

Mupad [B]
time = 4.40, size = 206, normalized size = 1.18

$$(a^2+2\,a\,b\,x^2+b^2\,x^4)^p\,\left(\frac{x^8\,\left(p^3+3\,p^2+\frac{11\,p}{4}+\frac{3}{4}\right)}{4\,p^4+20\,p^3+35\,p^2+25\,p+6}-\frac{3\,a^4}{4\,b^4\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}+\frac{3\,a^3\,p\,x^2}{2\,b^3\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}+\frac{a\,p\,x^6\,\left(2\,p^2+3\,p+1\right)}{2\,b\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}-\frac{3\,a^2\,p\,x^4\,\left(2\,p+1\right)}{4\,b^2\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^8*((11*p)/4 + 3*p^2 + p^3 + 3/4))/(25*p +
 35*p^2 + 20*p^3 + 4*p^4 + 6) - (3*a^4)/(4*b^4*(25*p + 35*p^2 + 20*p^3 + 4*
p^4 + 6)) + (3*a^3*p*x^2)/(2*b^3*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) + (a
*p*x^6*(3*p + 2*p^2 + 1))/(2*b*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) - (3*a
^2*p*x^4*(2*p + 1))/(4*b^2*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)))
```

3.799 $\int x^5(a^2+2abx^2+b^2x^4)^p\,dx$

Optimal. Leaf size=130

$$\frac{a^2(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(1+2p)}-\frac{a(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(1+p)}+\frac{(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(3+2p)}$$

```
[Out] 1/2*a^2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(1+2*p)-1/2*a*(b*x^2+a)^2*(
b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(1+p)+1/2*(b*x^2+a)^3*(b^2*x^4+2*a*b*x^2+a^2)^
p/b^3/(3+2*p)
```

Rubi [A]
time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {1127, 272, 45}

$$\frac{(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+3)}-\frac{a(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(p+1)}+\frac{a^2(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + 2*p)) - (a*(a +
 b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + p)) + ((a + b*x^2)^3*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(3 + 2*p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^5\left(1+\frac{bx^2}{a}\right)^{2p}\,dx$$
$$=\frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^2\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^2\right.$$
$$=\frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(1+\frac{bx}{a}\right)^{2p}}{b^2}-\frac{2a^2(1\ldots}{\ldots}\right.\right.$$
$$=\frac{a^2(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(1+2p)}-\frac{a(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(1+p)}+\frac{(a+\ldots}{\ldots}$$

Mathematica [A]
time = 0.05, size = 77, normalized size = 0.59

$$\frac{(a+bx^2)\left((a+bx^2)^2\right)^p\left(a^2-ab(1+2p)x^2+b^2\left(1+3p+2p^2\right)x^4\right)}{2b^3(1+p)(1+2p)(3+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(a^2 - a*b*(1 + 2*p)*x^2 + b^2*(1 + 3*p + 2*
p^2)*x^4))/(2*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A]
time = 0.03, size = 96, normalized size = 0.74

method	result	size
gosper	$\frac{(b\,x^2+a)\left(2b^2p^2x^4+3b^2p\,x^4+b^2x^4-2abp\,x^2-ab\,x^2+a^2\right)\left(b^2x^4+2ab\,x^2+a^2\right)^p}{2b^3\left(4p^3+12p^2+11p+3\right)}$	96
risch	$\frac{\left(2b^3p^2x^6+3b^3p\,x^6+2a\,b^2p^2x^4+b^3x^6+a\,b^2p\,x^4-2a^2bp\,x^2+a^3\right)\left(\left(b\,x^2+a\right)^2\right)^p}{2(1+p)(3+2p)(1+2p)b^3}$	98
norman	$\frac{x^6\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4p+6}+\frac{a^3\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2b^3\left(4p^3+12p^2+11p+3\right)}+\frac{ap\,x^4\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2b\left(2p^2+5p+3\right)}-\frac{p\,a^2x^2\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{b^2\left(4p^3+12p^2+11p+3\right)}$	178

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x^2+a)*(2*b^2*p^2*x^4+3*b^2*p*x^4+b^2*x^4-2*a*b*p*x^2-a*b*x^2+a^2)*(
b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.61

$$\frac{((2\,p^2+3\,p+1)b^3x^6+(2\,p^2+p)ab^2x^4-2\,a^2bpx^2+a^3)(bx^2+a)^{2\,p}}{2\,(4\,p^3+12\,p^2+11\,p+3)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a^
3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Fricas [A]
time = 0.36, size = 108, normalized size = 0.83

$$\frac{((2\,b^3p^2+3\,b^3p+b^3)x^6-2\,a^2bpx^2+(2\,ab^2p^2+ab^2p)x^4+a^3)(b^2x^4+2\,abx^2+a^2)^p}{2\,(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*((2*b^3*p^2 + 3*b^3*p + b^3)*x^6 - 2*a^2*b*p*x^2 + (2*a*b^2*p^2 + a*b^2
*p)*x^4 + a^3)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b
^3*p + 3*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^6(a^2)^p}{6} & \text{for}\ b=0 \\ \int \frac{x^5}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx & \text{for}\ p=-\frac{3}{2} \\ -\frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2a^2}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}-\frac{2abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2}+\frac{b^2x^4}{2ab^3+2b^4x^2} & \text{for}\ p=-1 \\ \int \frac{x^5}{\sqrt{(a+bx^2)^2}}\,dx & \text{for}\ p=-\frac{1}{2} \\ \frac{a^3(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3}-\frac{2a^2bpx^2(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3}+\frac{2ab^2p^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3}+\frac{ab^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3}+\frac{2b^3p^2x^6(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3}+\frac{3b^3px^6(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3}+\frac{b^3x^6(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**6*(a**2)**p/6, Eq(b, 0)), (Integral(x**5/((a + b*x**2)**2)**(
3/2), x), Eq(p, -3/2)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**
2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3
 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b
**3 + 2*b**4*x**2), Eq(p, -1)), (Integral(x**5/sqrt((a + b*x**2)**2), x), E
q(p, -1/2)), (a**3*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**
3*p**2 + 22*b**3*p + 6*b**3) - 2*a**2*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x*
```

```
*4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3) + 2*a*b**2*p**2*x*
*4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3
*p + 6*b**3) + a*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**
3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3) + 2*b**3*p**2*x**6*(a**2 + 2*a*b*x**
2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3) + 3*b**
3*p*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 2
2*b**3*p + 6*b**3) + b**3*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p
**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3), True))
```

Giac [A]
time = 2.57, size = 235, normalized size = 1.81

$$\frac{2\,(b^2x^4+2\,abx^2+a^2)^p b^3p^2x^6+3\,(b^2x^4+2\,abx^2+a^2)^p b^3px^6+2\,(b^2x^4+2\,abx^2+a^2)^p ab^2p^2x^4+(b^2x^4+2\,abx^2+a^2)^p b^3x^6+(b^2x^4+2\,abx^2+a^2)^p ab^2px^4-2\,(b^2x^4+2\,abx^2+a^2)^p a^2bpx^2+(b^2x^4+2\,abx^2+a^2)^p a^3}{2\,(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p^2*x^6 + 3*(b^2*x^4 + 2*a*b*x^2 +
 a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p^2*x^4 + (b^2*x^
4 + 2*a*b*x^2 + a^2)^p*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x^4
- 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)
^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Mupad [B]
time = 4.27, size = 137, normalized size = 1.05

$$(a^2+2\,a\,b\,x^2+b^2\,x^4)^p\,\left(\frac{x^6\,\left(p^2+\frac{3\,p}{2}+\frac{1}{2}\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{a^3}{2\,b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{a^2\,p\,x^2}{b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)}+\frac{a\,p\,x^4\,\left(2\,p+1\right)}{2\,b\,\left(4\,p^3+12\,p^2+11\,p+3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^6*((3*p)/2 + p^2 + 1/2))/(11*p + 12*p^2 +
 4*p^3 + 3) + a^3/(2*b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (a^2*p*x^2)/(b^2*(1
1*p + 12*p^2 + 4*p^3 + 3)) + (a*p*x^4*(2*p + 1))/(2*b*(11*p + 12*p^2 + 4*p^
3 + 3)))
```

3.800 $\int x^3(a^2 + 2abx^2 + b^2x^4)^p \, dx$

Optimal. Leaf size=84

$$-\frac{a(a+bx^2)(a^2+2abx^2+b^2x^4)^p}{2b^2(1+2p)}+\frac{(a+bx^2)^2(a^2+2abx^2+b^2x^4)^p}{4b^2(1+p)}$$

```
[Out] -1/2*a*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^2/(1+2*p)+1/4*(b*x^2+a)^2*(b^2
*x^4+2*a*b*x^2+a^2)^p/b^2/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1127, 272, 45}

$$\frac{(a+bx^2)^2(a^2+2abx^2+b^2x^4)^p}{4b^2(p+1)}-\frac{a(a+bx^2)(a^2+2abx^2+b^2x^4)^p}{2b^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -1/2*(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^2*(1 + 2*p)) + ((a +
b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int x^3 (a^2 + 2abx^2 + b^2x^4)^p \, dx = \left(\left(1 + \frac{bx^2}{a}\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \right) \int x^3 \left(1 + \frac{bx^2}{a}\right)^{2p} dx$$

$$= \frac{1}{2} \left(\left(1 + \frac{bx^2}{a}\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \right) \text{Subst}\left(\int x \left(1 + \frac{bx}{a}\right)^{2p} dx, x, x^2 \right)$$

$$= \frac{1}{2} \left(\left(1 + \frac{bx^2}{a}\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \right) \text{Subst}\left(\int \left(-\frac{a\left(1 + \frac{bx}{a}\right)^{2p}}{b} + \frac{a\left(1 -\right.}{} \right.\right.$$

$$= -\frac{a(a + bx^2)(a^2 + 2abx^2 + b^2x^4)^p}{2b^2(1 + 2p)} + \frac{(a + bx^2)^2 (a^2 + 2abx^2 + b^2x^4)^p}{4b^2(1 + p)}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.61

$$\frac{(a + bx^2)\left((a + bx^2)^2\right)^p (-a + b(1 + 2p)x^2)}{4b^2(1 + p)(1 + 2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(-a + b*(1 + 2*p)*x^2))/(4*b^2*(1 + p)*(1 + 2*p))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.69

method	result	size
risch	$-\frac{(-2b^2p\,x^4-b^2x^4-2abp\,x^2+a^2)\left((b\,x^2+a)^2\right)^p}{4b^2(1+p)(1+2p)}$	58
gosper	$-\frac{(b^2x^4+2ab\,x^2+a^2)^p(-2x^2pb-b\,x^2+a)(b\,x^2+a)}{4b^2(2p^2+3p+1)}$	60
norman	$\frac{x^4 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4p+4}-\frac{a^2 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4b^2(2p^2+3p+1)}+\frac{pa\,x^2 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2b(2p^2+3p+1)}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
[Out] -1/4*(-2*b^2*p*x^4-b^2*x^4-2*a*b*p*x^2+a^2)/b^2/(1+p)/(1+2*p)*((b*x^2+a)^2)^p
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.64

$$\frac{(b^2(2p+1)x^4+2abpx^2-a^2)(bx^2+a)^{2p}}{4(2p^2+3p+1)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p
 + 1)*b^2)
```

Fricas [A]
time = 0.34, size = 70, normalized size = 0.83

$$\frac{(2abpx^2+(2b^2p+b^2)x^4-a^2)(b^2x^4+2abx^2+a^2)^p}{4(2b^2p^2+3b^2p+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*p*x^2 + (2*b^2*p + b^2)*x^4 - a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p
/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^4(a^2)^p}{4} & \text{for } b=0 \\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for } p=-1 \\ \int\frac{x^3}{\sqrt{(a+bx^2)^2}}\,dx & \text{for } p=-\frac{1}{2} \\ -\frac{a^2(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+12b^2p+4b^2}+\frac{2abpx^2(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+12b^2p+4b^2}+\frac{2b^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+12b^2p+4b^2}+\frac{b^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+12b^2p+4b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**4*(a**2)**p/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 +
2*b**3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2
 + 2*b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x
**2*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -1)), (Integral(x**
3/sqrt((a + b*x**2)**2), x), Eq(p, -1/2)), (-a**2*(a**2 + 2*a*b*x**2 + b**2
*x**4)**p/(8*b**2*p**2 + 12*b**2*p + 4*b**2) + 2*a*b*p*x**2*(a**2 + 2*a*b*x
**2 + b**2*x**4)**p/(8*b**2*p**2 + 12*b**2*p + 4*b**2) + 2*b**2*p*x**4*(a**
```

```
2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 12*b**2*p + 4*b**2) + b**2*x*
*4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 12*b**2*p + 4*b**2), T
rue))
```

Giac [A]
time = 3.21, size = 132, normalized size = 1.57

$$\frac{2\,(b^2x^4+2\,abx^2+a^2)^p b^2px^4+(b^2x^4+2\,abx^2+a^2)^p b^2x^4+2\,(b^2x^4+2\,abx^2+a^2)^p abpx^2-(b^2x^4+2\,abx^2+a^2)^p a^2}{4\,(2\,b^2p^2+3\,b^2p+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^2*p*x^4 + (b^2*x^4 + 2*a*b*x^2 + a^2
)^p*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b*p*x^2 - (b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Mupad [B]
time = 4.26, size = 85, normalized size = 1.01

$$\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,\left(\frac{x^4\,\left(2\,p+1\right)}{4\,\left(2\,p^2+3\,p+1\right)}-\frac{a^2}{4\,b^2\,\left(2\,p^2+3\,p+1\right)}+\frac{a\,p\,x^2}{2\,b\,\left(2\,p^2+3\,p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^4*(2*p + 1))/(4*(3*p + 2*p^2 + 1)) - a^2/
(4*b^2*(3*p + 2*p^2 + 1)) + (a*p*x^2)/(2*b*(3*p + 2*p^2 + 1)))
```

3.801 $\int x(a^2+2abx^2+b^2x^4)^p\,dx$

Optimal. Leaf size=41

$$\frac{(a+bx^2)\,(a^2+2abx^2+b^2x^4)^p}{2b(1+2p)}$$

```
[Out] 1/2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b/(1+2*p)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1121, 623}

$$\frac{(a+bx^2)\,(a^2+2abx^2+b^2x^4)^p}{2b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b*(1 + 2*p))
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\int x\left(a^2+2abx^2+b^2x^4\right)^p\,dx &= \frac{1}{2}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)\\
&= \frac{(a+bx^2)\,(a^2+2abx^2+b^2x^4)^p}{2b(1+2p)}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 0.71

$$\frac{(a+bx^2)\left((a+bx^2)^2\right)^p}{2b+4bp}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p)/(2*b + 4*b*p)
```

Maple [A]
time = 0.02, size = 31, normalized size = 0.76

method	result	size
risch	$\frac{\left(b\,x^{2}+a\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{p}}{2b\left(1+2p\right)}$	31
gosper	$\frac{\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{p}}{2b\left(1+2p\right)}$	40
norman	$\frac{x^{2}{\mathrm e}^{p\ln\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}}{4p+2}+\frac{a\,{\mathrm e}^{p\ln\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}}{2b\left(1+2p\right)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
[Out] 1/2*(b*x^2+a)/b/(1+2*p)*((b*x^2+a)^2)^p
```

Maxima [A]
time = 0.28, size = 30, normalized size = 0.73

$$\frac{(bx^2+a)(bx^2+a)^{2p}}{2\,b(2\,p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)/(b*(2*p + 1))
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.90

$$\frac{(bx^2+a)(b^2x^4+2\,abx^2+a^2)^p}{2\,(2\,bp+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
[Out] 1/2*(b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b*p + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^2}{2\sqrt{a^2}} & \text{for } b = 0 \wedge p = -\frac{1}{2} \\ \frac{x^2(a^2)^p}{2} & \text{for } b = 0 \\ \int \frac{x}{\sqrt{(a+bx^2)^2}}\,dx & \text{for } p = -\frac{1}{2} \\ \frac{a(a^2+2abx^2+b^2x^4)^p}{4bp+2b} + \frac{bx^2(a^2+2abx^2+b^2x^4)^p}{4bp+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**2/(2*sqrt(a**2)), Eq(b, 0) & Eq(p, -1/2)), (x**2*(a**2)**p/2,
 Eq(b, 0)), (Integral(x/sqrt((a + b*x**2)**2), x), Eq(p, -1/2)), (a*(a**2 +
 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 2*b) + b*x**2*(a**2 + 2*a*b*x**2 + b**
2*x**4)**p/(4*b*p + 2*b), True))
```

Giac [A]
time = 3.50, size = 58, normalized size = 1.41

$$\frac{(b^2x^4 + 2\,abx^2 + a^2)^p bx^2 + (b^2x^4 + 2\,abx^2 + a^2)^p a}{2\,(2\,bp + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*((b^2*x^4 + 2*a*b*x^2 + a^2)^p*b*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a)
/(2*b*p + b)
```

Mupad [B]
time = 4.67, size = 46, normalized size = 1.12

$$\left(\frac{x^2}{2\,(2\,p+1)} + \frac{a}{2\,b\,(2\,p+1)}\right)\,\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (x^2/(2*(2*p + 1)) + a/(2*b*(2*p + 1)))*(a^2 + b^2*x^4 + 2*a*b*x^2)^p
```

3.802 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x}\,dx$

Optimal. Leaf size=63

$$-\frac{(a+bx^2)\,(a^2+2abx^2+b^2x^4)^p\;{}_2F_1\left(1,1+2p;2(1+p);1+\frac{bx^2}{a}\right)}{2a(1+2p)}$$

```
[Out] -1/2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([1, 1+2*p],[2+2*p],1+b*x
^2/a)/a/(1+2*p)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1127, 272, 67}

$$-\frac{(a+bx^2)\,(a^2+2abx^2+b^2x^4)^p\;{}_2F_1\left(1,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x,x]
```

```
[Out] -1/2*((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1, 1 + 2*
p, 2*(1 + p), 1 + (b*x^2)/a])/(a*(1 + 2*p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x}\,dx$$
$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}\,dx, x, x^2\right)$$
$$= -\frac{(a+bx^2)(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(1,1+2p;2(1+p);1+\frac{bx^2}{a}\right)}{2a(1+2p)}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.86

$$-\frac{(a+bx^2)\left((a+bx^2)^2\right)^p\,{}_2F_1\left(1,1+2p;2+2p;1+\frac{bx^2}{a}\right)}{2a(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x,x]
[Out] -1/2*((a + b*x^2)*((a + b*x^2)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p,
1 + (b*x^2)/a])/(a*(1 + 2*p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x,x)
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{2}\right)^{2}\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x,x)
[Out] Integral(((a + b*x**2)**2)**p/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x, x)
```

3.803 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}\,dx$

Optimal. Leaf size=64

$$\frac{b(a+bx^2)\,(a^2+2abx^2+b^2x^4)^p\ {}_2F_1\left(2,1+2p;2(1+p);1+\frac{bx^2}{a}\right)}{2a^2(1+2p)}$$

```
[Out] 1/2*b*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([2, 1+2*p],[2+2*p],1+b*
x^2/a)/a^2/(1+2*p)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1127, 272, 67}

$$\frac{b(a+bx^2)\,(a^2+2abx^2+b^2x^4)^p\ {}_2F_1\left(2,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^3,x]
```

```
[Out] (b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 + (b*x^2)/a])/(2*a^2*(1 + 2*p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x^3}\,dx$$
$$= \frac{1}{2}\left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^2}\,dx, x, x^2\right)$$
$$= \frac{b(a+bx^2)\,(a^2+2abx^2+b^2x^4)^p\;{}_2F_1\left(2,1+2p;2(1+p);1+\frac{bx^2}{a}\right)}{2a^2(1+2p)}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.86

$$\frac{b(a+bx^2)\left((a+bx^2)^2\right)^p\;{}_2F_1\left(2,1+2p;2+2p;1+\frac{bx^2}{a}\right)}{2a^2(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^3,x]
```

```
[Out] (b*(a + b*x^2)*((a + b*x^2)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, 1 +
 (b*x^2)/a])/(2*a^2*(1 + 2*p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**3,x)
[Out] Integral(((a + b*x**2)**2)**p/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^3,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^3, x)
```

3.804 $\int x^4(a^2+2abx^2+b^2x^4)^p \, dx$

Optimal. Leaf size=60

$$\frac{1}{5}x^5\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/5*x^5*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([5/2, -2*p],[7/2],-b*x^2/a)/((1
+b*x^2/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1127, 371}

$$\frac{1}{5}x^5\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/2, -2*p, 7/2, -((b*x
^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a^2+2abx^2+b^2x^4\right)^p \, dx &= \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^4\left(1+\frac{bx^2}{a}\right)^{2p} dx\\ &= \frac{1}{5}x^5\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.85

$$\frac{1}{5}x^5\left(\left(a+bx^2\right)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}{}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] (x^5*((a + b*x^2)^2)^p*Hypergeometric2F1[5/2, -2*p, 7/2, -((b*x^2)/a)])/(5*
(1 + (b*x^2)/a)^(2*p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4\left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
[Out] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(\left(a+bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^4\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

3.805 $\int x^2(a^2+2abx^2+b^2x^4)^p\,dx$

Optimal. Leaf size=60

$$\frac{1}{3}x^3\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] 1/3*x^3*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([3/2, -2*p],[5/2],-b*x^2/a)/((1
+b*x^2/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1127, 371}

$$\frac{1}{3}x^3\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/2, -2*p, 5/2, -((b*x
^2)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^2+b^2x^4\right)^p\,dx &= \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int x^2\left(1+\frac{bx^2}{a}\right)^{2p}\,dx\\ &= \frac{1}{3}x^3\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 0.85

$$\frac{1}{3}x^3\left(\left(a+bx^2\right)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}{}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^3*((a + b*x^2)^2)^p*Hypergeometric2F1[3/2, -2*p, 5/2, -((b*x^2)/a)])/(3*
(1 + (b*x^2)/a)^(2*p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2\left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\left(a+bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
[Out] Integral(x**2*((a + b*x**2)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
[Out] int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

3.806 $\int (a^2+2abx^2+b^2x^4)^p \, dx$

Optimal. Leaf size=55

$$x\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

[Out] x*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([1/2, -2*p],[3/2],-b*x^2/a)/((1+b*x^2/a)^(2*p))

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1103, 251}

$$x\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]

[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/2, -2*p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(2*p)

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1103

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[a^IntPart[
p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2*FracPart[p])), Int
[(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^2+b^2x^4\right)^p \, dx &= \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \left(1+\frac{bx^2}{a}\right)^{2p} dx\\ &= x\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.84

$$x\left(\left(a+bx^2\right)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}{}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]

[Out] (x*((a + b*x^2)^2)^p*Hypergeometric2F1[1/2, -2*p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(2*p)

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*x^4+2*a*b*x^2+a^2)^p,x)

[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")

[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")

[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

3.807 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

```
[Out] -(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-1/2, -2*p],[1/2],-b*x^2/a)/x/((1+b*x
^2/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1127, 371}

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^2,x]
```

```
[Out] -(((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, -((b*x^
2)/a)])/(x*(1 + (b*x^2)/a)^(2*p)))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x^2}\,dx$$
$$= -\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.84

$$-\frac{\left((a+bx^2)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^2,x]
[Out] -((((a + b*x^2)^2)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, -((b*x^2)/a)])/(x*(
1 + (b*x^2)/a)^(2*p)))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x)
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^2)^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^2,x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^2, x)
```

3.808 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^4}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

```
[Out] -1/3*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-3/2, -2*p],[-1/2],-b*x^2/a)/x^3/
((1+b*x^2/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1127, 371}

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^4,x]
```

```
[Out] -1/3*((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, -((
b*x^2)/a)])/(x^3*(1 + (b*x^2)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^4}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{x^4}\,dx$$
$$= -\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.85

$$-\frac{\left((a+bx^2)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}{}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^4,x]
[Out] -1/3*(((a + b*x^2)^2)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, -((b*x^2)/a)])/
(x^3*(1 + (b*x^2)/a)^(2*p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x)
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="fricas")
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^2)^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**4,x)
[Out] Integral(((a + b*x**2)**2)**p/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a^2+2\,a\,b\,x^2+b^2\,x^4)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^4,x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^4, x)
```

3.809 $\int (dx)^{3/2} (a^2 + 2abx^2 + b^2x^4)^p \, dx$

Optimal. Leaf size=67

$$\frac{2(dx)^{5/2} \left(1 + \frac{bx^2}{a}\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \,{}_2F_1\left(\frac{5}{4}, -2p; \frac{9}{4}; -\frac{bx^2}{a}\right)}{5d}$$

```
[Out] 2/5*(d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([5/4, -2*p],[9/4],-b*x^
2/a)/d/((1+b*x^2/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1127, 371}

$$\frac{2(dx)^{5/2} \left(\frac{bx^2}{a} + 1\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \,{}_2F_1\left(\frac{5}{4}, -2p; \frac{9}{4}; -\frac{bx^2}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/4, -2*p, 9
/4, -((b*x^2)/a)])/(5*d*(1 + (b*x^2)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2} (a^2 + 2abx^2 + b^2x^4)^p \, dx &= \left(\left(1 + \frac{bx^2}{a}\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p\right) \int (dx)^{3/2} \left(1 + \frac{bx^2}{a}\right)^{2p} dx \\ &= \frac{2(dx)^{5/2} \left(1 + \frac{bx^2}{a}\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \,{}_2F_1\left(\frac{5}{4}, -2p; \frac{9}{4}; -\frac{bx^2}{a}\right)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 56, normalized size = 0.84

$$\frac{2}{5}x(dx)^{3/2}\left(\left(a+bx^2\right)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}{}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*x*(d*x)^(3/2)*((a + b*x^2)^2)^p*Hypergeometric2F1[5/4, -2*p, 9/4, -((b*x
^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*d*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral((d*x)**(3/2)*((a + b*x**2)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,x\right)}^{3/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

3.810 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{2(dx)^{3/2}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3d}$$

```
[Out] 2/3*(d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([3/4, -2*p],[7/4],-b*x^
2/a)/d/((1+b*x^2/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1127, 371}

$$\frac{2(dx)^{3/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/4, -2*p, 7
/4, -((b*x^2)/a)])/(3*d*(1 + (b*x^2)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p\,dx &= \left(\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\int \sqrt{dx}\left(1+\frac{bx^2}{a}\right)^{2p}\,dx\\ &= \frac{2(dx)^{3/2}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 56, normalized size = 0.84

$$\frac{2}{3}x\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}{}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*x*Sqrt[d*x]*((a + b*x^2)^2)^p*Hypergeometric2F1[3/4, -2*p, 7/4, -((b*x^2
)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\left(b^2x^4+2ab\,x^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\left(\left(a+bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

3.811 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{\sqrt{dx}}\,dx$

Optimal. Leaf size=65

$$\frac{2\sqrt{dx}\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\ {}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{d}$$

```
[Out] 2*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([1/4, -2*p],[5/4],-b*x^2/a)*(d*x)^(1/
2)/d/((1+b*x^2/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1127, 371}

$$\frac{2\sqrt{dx}\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\ {}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/4, -2*p, 5/4
, -((b*x^2)/a)])/(d*(1 + (b*x^2)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{\sqrt{dx}}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{\sqrt{dx}}\,dx$$
$$= \frac{2\sqrt{dx}\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\ {}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{d}$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 0.83

$$\frac{2x\left((a+bx^2)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}\ {}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/Sqrt[d*x],x]
[Out] (2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[1/4, -2*p, 5/4, -((b*x^2)/a)])/(Sq
rt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x)
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^2)^2\right)^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(1/2),x)
[Out] Integral(((a + b*x**2)**2)**p/sqrt(d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a^2+2\,a\,b\,x^2+b^2\,x^4)^p}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(1/2),x)
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(1/2), x)
```

3.812 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\ {}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

```
[Out] -2*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-1/4, -2*p],[3/4],-b*x^2/a)/d/((1+b
*x^2/a)^(2*p))/(d*x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1127, 371}

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\ {}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(3/2),x]
```

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/4, -2*p, 3/4, -((b*x
^2)/a)])/(d*Sqrt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{3/2}}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{(dx)^{3/2}}\,dx$$
$$= -\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

Mathematica [A]
time = 0.13, size = 54, normalized size = 0.83

$$-\frac{2x\left((a+bx^2)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(3/2),x]
[Out] (-2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[-1/4, -2*p, 3/4, -((b*x^2)/a)])/(
(d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x)
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^p}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(3/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/(d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p}{{\left(d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(3/2),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(3/2), x)
```

3.813 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\ {}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

```
[Out] -2/3*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-3/4, -2*p],[1/4],-b*x^2/a)/d/(d*
x)^(3/2)/((1+b*x^2/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1127, 371}

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\ {}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(5/2),x]
```

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/4, -2*p, 1/4, -((b*x
^2)/a)])/(3*d*(d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1127

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/(1 + 2*c*(x^2/b))^(2
*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/b))^(2*p), x], x] /; FreeQ[{a, b,
 c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{5/2}}\,dx = \left(\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\right)\int \frac{\left(1+\frac{bx^2}{a}\right)^{2p}}{(dx)^{5/2}}\,dx$$
$$= -\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

Mathematica [A]
time = 0.14, size = 56, normalized size = 0.84

$$-\frac{2x\left((a+bx^2)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}{}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(5/2),x]
[Out] (-2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[-3/4, -2*p, 1/4, -((b*x^2)/a)])/(
3*(d*x)^(5/2)*(1 + (b*x^2)/a)^(2*p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x)
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="maxima")
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d^3*x^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^p}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(5/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/(d*x)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p}{{\left(d\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(5/2),x)
```

```
[Out] int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(5/2), x)
```

3.814 $\int x^2(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.062$, Rules used = $\{14\}$

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2+cx^4\right)\,dx&=\int \left(ax^2+bx^4+cx^6\right)\,dx\\&=\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Maple [A]
time = 0.01, size = 20, normalized size = 0.80

method	result	size
gosper	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
default	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
norman	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
risch	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^{7}+\frac{1}{5}\,bx^{5}+\frac{1}{3}\,ax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A]
time = 0.34, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^{7}+\frac{1}{5}\,bx^{5}+\frac{1}{3}\,ax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.76

$$\frac{ax^{3}}{3}+\frac{bx^{5}}{5}+\frac{cx^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

Giac [A]
time = 3.90, size = 19, normalized size = 0.76

$$\frac{1}{7}cx^7 + \frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^7}{7} + \frac{b\,x^5}{5} + \frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

3.815 $\int x(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

```
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.071$, Rules used = $\{14\}$

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2+cx^4\right)\,dx&=\int \left(ax+bx^3+cx^5\right)\,dx\\&=\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Maple [A]
time = 0.01, size = 20, normalized size = 0.80

method	result	size
gosper	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	20
default	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	20
norman	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	20
risch	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}+\frac{1}{6}c\,x^{6}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^{6}+\frac{1}{4}\,bx^{4}+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Fricas [A]
time = 0.36, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^{6}+\frac{1}{4}\,bx^{4}+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.76

$$\frac{ax^{2}}{2}+\frac{bx^{4}}{4}+\frac{cx^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a),x)
```

```
[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

Giac [A]
time = 3.55, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^6}{6} + \frac{b\,x^4}{4} + \frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4),x)
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

3.816 $\int (a+bx^2+cx^4) \, dx$

Optimal. Leaf size=20

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

```
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^2 + c*x^4,x]
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi steps

$$\int (a+bx^2+cx^4) \, dx = ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^2 + c*x^4,x]
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.85

method	result	size
gosper	$ax+\frac{1}{3}b\,x^3+\frac{1}{5}c\,x^5$	17
default	$ax+\frac{1}{3}b\,x^3+\frac{1}{5}c\,x^5$	17

norman	$ax+\frac{1}{3}b\,x^3+\frac{1}{5}c\,x^5$	17
risch	$ax+\frac{1}{3}b\,x^3+\frac{1}{5}c\,x^5$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^4+b*x^2+a,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^5+\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="maxima")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Fricas [A]
time = 0.34, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^5+\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="fricas")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.75

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**2+a,x)
```

```
[Out] a*x + b*x**3/3 + c*x**5/5
```

Giac [A]
time = 3.44, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^5+\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^2+a,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Mupad [B]
time = 0.02, size = 16, normalized size = 0.80

$$\frac{c\,x^5}{5}+\frac{b\,x^3}{3}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x^2 + c*x^4,x)
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

3.817 $\int \frac{a+bx^2+cx^4}{x} dx$

Optimal. Leaf size=21

$$\frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)$$

```
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{14\}$

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x}\,dx &= \int \left(\frac{a}{x}+bx+cx^3\right)dx\\ &= \frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.86

method	result	size
default	$\frac{b\,x^{2}}{2}+\frac{c\,x^{4}}{4}+a\ln\left(x\right)$	18
norman	$\frac{b\,x^{2}}{2}+\frac{c\,x^{4}}{4}+a\ln\left(x\right)$	18
risch	$\frac{c\,x^{4}}{4}+\frac{b\,x^{2}}{2}+\frac{b^{2}}{4c}+a\ln\left(x\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Maxima [A]
time = 0.30, size = 20, normalized size = 0.95

$$\frac{1}{4}\,cx^{4}+\frac{1}{2}\,bx^{2}+\frac{1}{2}\,a\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

Fricas [A]
time = 0.35, size = 17, normalized size = 0.81

$$\frac{1}{4}\,cx^{4}+\frac{1}{2}\,bx^{2}+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Sympy [A]
time = 0.02, size = 17, normalized size = 0.81

$$a\log\left(x\right)+\frac{bx^{2}}{2}+\frac{cx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x,x)
```

```
[Out] a*log(x) + b*x**2/2 + c*x**4/4
```

Giac [A]
time = 3.99, size = 20, normalized size = 0.95

$$\frac{1}{4}\,cx^4 + \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

Mupad [B]
time = 0.02, size = 17, normalized size = 0.81

$$\frac{b\,x^2}{2} + \frac{c\,x^4}{4} + a\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x,x)
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*log(x)
```

3.818 $\int \frac{a+bx^2+cx^4}{x^2} \, dx$

Optimal. Leaf size=18

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

```
[Out] -a/x+b*x+1/3*c*x^3
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{14\}$

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^2,x]
```

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2+cx^4}{x^2} \, dx &= \int \left(b+\frac{a}{x^2}+cx^2\right) \, dx \\ &= -\frac{a}{x}+bx+\frac{cx^3}{3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^2,x]
```

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.94

method	result	size
default	$-\frac{a}{x}+bx+\frac{c\,x^{3}}{3}$	17
risch	$-\frac{a}{x}+bx+\frac{c\,x^{3}}{3}$	17
norman	$\frac{\frac{1}{3}c\,x^{4}+b\,x^{2}-a}{x}$	20
gosper	$-\frac{-c\,x^{4}-3b\,x^{2}+3a}{3x}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
[Out] -a/x+b*x+1/3*c*x^3
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.89

$$\frac{1}{3}\,cx^{3}+bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
[Out] 1/3*c*x^3 + b*x - a/x
```

Fricas [A]
time = 0.35, size = 20, normalized size = 1.11

$$\frac{cx^{4}+3\,bx^{2}-3\,a}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
[Out] 1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.67

$$-\frac{a}{x}+bx+\frac{cx^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a/x + b*x + c*x**3/3
```

Giac [A]
time = 4.17, size = 16, normalized size = 0.89

$$\frac{1}{3}cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + b*x - a/x
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.89

$$b\,x - \frac{a}{x} + \frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^2,x)
```

```
[Out] b*x - a/x + (c*x^3)/3
```

3.819 $\int \frac{a+bx^2+cx^4}{x^3} \, dx$

Optimal. Leaf size=21

$$-\frac{a}{2x^2}+\frac{cx^2}{2}+b\log(x)$$

```
[Out] -1/2*a/x^2+1/2*c*x^2+b*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^3,x]
```

```
[Out] -1/2*a/x^2 + (c*x^2)/2 + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^3} \, dx &= \int \left(\frac{a}{x^3}+\frac{b}{x}+cx\right) \, dx\\ &= -\frac{a}{2x^2}+\frac{cx^2}{2}+b\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{a}{2x^2}+\frac{cx^2}{2}+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^3,x]
```

```
[Out] -1/2*a/x^2 + (c*x^2)/2 + b*Log[x]
```

Maple [A]
time = 0.01, size = 18, normalized size = 0.86

method	result	size
default	$-\frac{a}{2x^2}+\frac{c\,x^2}{2}+b\ln\left(x\right)$	18
risch	$-\frac{a}{2x^2}+\frac{c\,x^2}{2}+b\ln\left(x\right)$	18
norman	$\frac{-\frac{a}{2}+\frac{c\,x^4}{2}}{x^2}+b\ln\left(x\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a/x^2+1/2*c*x^2+b*ln(x)
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.95

$$\frac{1}{2}\,cx^2+\frac{1}{2}\,b\log\left(x^2\right)-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
[Out] 1/2*c*x^2 + 1/2*b*log(x^2) - 1/2*a/x^2
```

Fricas [A]
time = 0.38, size = 22, normalized size = 1.05

$$\frac{cx^4+2\,bx^2\log\left(x\right)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
[Out] 1/2*(c*x^4 + 2*b*x^2*log(x) - a)/x^2
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.81

$$-\frac{a}{2x^2}+b\log\left(x\right)+\frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**3,x)
[Out] -a/(2*x**2) + b*log(x) + c*x**2/2
```

Giac [A]
time = 6.09, size = 26, normalized size = 1.24

$$\frac{1}{2}\,cx^2 + \frac{1}{2}\,b\log\left(x^2\right) - \frac{bx^2 + a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
[Out] 1/2*c*x^2 + 1/2*b*log(x^2) - 1/2*(b*x^2 + a)/x^2
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.81

$$\frac{c\,x^2}{2} - \frac{a}{2\,x^2} + b\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^3,x)
[Out] (c*x^2)/2 - a/(2*x^2) + b*log(x)
```

3.820 $\int \frac{a+bx^2+cx^4}{x^4} dx$

Optimal. Leaf size=18

$$-\frac{a}{3x^3} - \frac{b}{x} + cx$$

```
[Out] -1/3*a/x^3-b/x+c*x
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{14\}$

$$-\frac{a}{3x^3} - \frac{b}{x} + cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/x + c*x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^4} dx = \int \left(c + \frac{a}{x^4} + \frac{b}{x^2}\right) dx$$
$$= -\frac{a}{3x^3} - \frac{b}{x} + cx$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-\frac{a}{3x^3} - \frac{b}{x} + cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/x + c*x
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.94

method	result	size
default	$-\frac{a}{3x^3}-\frac{b}{x}+cx$	17
risch	$cx+\frac{-b\,x^2-\frac{a}{3}}{x^3}$	19
gosper	$-\frac{-3c\,x^4+3b\,x^2+a}{3x^3}$	20
norman	$\frac{c\,x^4-b\,x^2-\frac{1}{3}a}{x^3}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a/x^3-b/x+c*x
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.94

$$cx-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")
[Out] c*x - 1/3*(3*b*x^2 + a)/x^3
```

Fricas [A]
time = 0.36, size = 21, normalized size = 1.17

$$\frac{3\,cx^4-3\,bx^2-a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*c*x^4 - 3*b*x^2 - a)/x^3
```

Sympy [A]
time = 0.04, size = 17, normalized size = 0.94

$$cx+\frac{-a-3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] c*x + (-a - 3*b*x**2)/(3*x**3)
```

Giac [A]
time = 3.64, size = 17, normalized size = 0.94

$$cx - \frac{3\,bx^2 + a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="giac")
```

```
[Out] c*x - 1/3*(3*b*x^2 + a)/x^3
```

Mupad [B]
time = 0.02, size = 18, normalized size = 1.00

$$c\,x - \frac{b\,x^2 + \frac{a}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^4,x)
```

```
[Out] c*x - (a/3 + b*x^2)/x^3
```

3.821 $\int \frac{a+bx^2+cx^4}{x^5} dx$

Optimal. Leaf size=21

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

```
[Out] -1/4*a/x^4-1/2*b/x^2+c*ln(x)
```

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {14}

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(2*x^2) + c*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^5} dx &= \int \left(\frac{a}{x^5} + \frac{b}{x^3} + \frac{c}{x}\right) dx \\ &= -\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)\end{aligned}$$

Mathematica [A]

time = 0.00, size = 21, normalized size = 1.00

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(2*x^2) + c*Log[x]
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.86

method	result	size
default	$-\frac{a}{4x^4}-\frac{b}{2x^2}+c\ln\left(x\right)$	18
norman	$\frac{-\frac{b\,x^2}{2}-\frac{a}{4}}{x^4}+c\ln\left(x\right)$	20
risch	$\frac{-\frac{b\,x^2}{2}-\frac{a}{4}}{x^4}+c\ln\left(x\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a/x^4-1/2*b/x^2+c*ln(x)
```

Maxima [A]
time = 0.28, size = 21, normalized size = 1.00

$$\frac{1}{2}\,c\log\left(x^2\right)-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")
[Out] 1/2*c*log(x^2) - 1/4*(2*b*x^2 + a)/x^4
```

Fricas [A]
time = 0.36, size = 23, normalized size = 1.10

$$\frac{4\,cx^4\log\left(x\right)-2\,bx^2-a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")
[Out] 1/4*(4*c*x^4*log(x) - 2*b*x^2 - a)/x^4
```

Sympy [A]
time = 0.09, size = 19, normalized size = 0.90

$$c\log\left(x\right)+\frac{-a-2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**5,x)
[Out] c*log(x) + (-a - 2*b*x**2)/(4*x**4)
```

Giac [A]
time = 3.05, size = 27, normalized size = 1.29

$$\frac{1}{2}\,c\log\left(x^2\right) - \frac{3\,cx^4 + 2\,bx^2 + a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="giac")
[Out] 1/2*c*log(x^2) - 1/4*(3*c*x^4 + 2*b*x^2 + a)/x^4
```

Mupad [B]
time = 0.04, size = 20, normalized size = 0.95

$$c\,\ln\left(x\right) - \frac{\frac{b\,x^2}{2} + \frac{a}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^5,x)
[Out] c*log(x) - (a/4 + (b*x^2)/2)/x^4
```

3.822 $\int \frac{a+bx^2+cx^4}{x^6}\,dx$

Optimal. Leaf size=23

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

```
[Out] -1/5*a/x^5-1/3*b/x^3-c/x
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{14\}$

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -1/5*a/x^5 - b/(3*x^3) - c/x
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^6}\,dx &= \int \left(\frac{a}{x^6}+\frac{b}{x^4}+\frac{c}{x^2}\right)dx\\ &= -\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -1/5*a/x^5 - b/(3*x^3) - c/x
```

Maple [A]
time = 0.01, size = 20, normalized size = 0.87

method	result	size
default	$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$	20
norman	$\frac{-c\,x^4-\frac{1}{3}b\,x^2-\frac{1}{5}a}{x^5}$	21
risch	$\frac{-c\,x^4-\frac{1}{3}b\,x^2-\frac{1}{5}a}{x^5}$	21
gosper	$-\frac{15c\,x^4+5b\,x^2+3a}{15x^5}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/x^5-1/3*b/x^3-c/x
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.91

$$-\frac{15\,cx^4+5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

Fricas [A]
time = 0.32, size = 21, normalized size = 0.91

$$-\frac{15\,cx^4+5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

Sympy [A]
time = 0.11, size = 22, normalized size = 0.96

$$\frac{-3a-5bx^2-15cx^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] (-3*a - 5*b*x**2 - 15*c*x**4)/(15*x**5)
```

Giac [A]
time = 4.18, size = 21, normalized size = 0.91

$$-\frac{15\,cx^4+5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.87

$$-\frac{c\,x^4+\frac{b\,x^2}{3}+\frac{a}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^6,x)
```

```
[Out] -(a/5 + (b*x^2)/3 + c*x^4)/x^5
```

3.823 $\int \frac{a+bx^2+cx^4}{x^7} \, dx$

Optimal. Leaf size=25

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

```
[Out] -1/6*a/x^6-1/4*b/x^4-1/2*c/x^2
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{14\}$

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -1/6*a/x^6 - b/(4*x^4) - c/(2*x^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^7} \, dx &= \int \left(\frac{a}{x^7}+\frac{b}{x^5}+\frac{c}{x^3}\right) \, dx\\ &= -\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -1/6*a/x^6 - b/(4*x^4) - c/(2*x^2)
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.80

method	result	size
default	$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$	20
norman	$\frac{-\frac{1}{2}c\,x^4-\frac{1}{4}b\,x^2-\frac{1}{6}a}{x^6}$	21
risch	$\frac{-\frac{1}{2}c\,x^4-\frac{1}{4}b\,x^2-\frac{1}{6}a}{x^6}$	21
gosper	$-\frac{6c\,x^4+3b\,x^2+2a}{12x^6}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/x^6-1/4*b/x^4-1/2*c/x^2
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.84

$$-\frac{6\,cx^4+3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

Fricas [A]
time = 0.36, size = 21, normalized size = 0.84

$$-\frac{6\,cx^4+3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

Sympy [A]
time = 0.15, size = 22, normalized size = 0.88

$$\frac{-2a-3bx^2-6cx^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] (-2*a - 3*b*x**2 - 6*c*x**4)/(12*x**6)
```

Giac [A]
time = 3.90, size = 21, normalized size = 0.84

$$-\frac{6\,cx^4+3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.84

$$-\frac{\frac{c\,x^4}{2}+\frac{b\,x^2}{4}+\frac{a}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^7,x)
```

```
[Out] -(a/6 + (b*x^2)/4 + (c*x^4)/2)/x^6
```

3.824 $\int \frac{a+bx^2+cx^4}{x^8} \, dx$

Optimal. Leaf size=25

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

```
[Out] -1/7*a/x^7-1/5*b/x^5-1/3*c/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -1/7*a/x^7 - b/(5*x^5) - c/(3*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2+cx^4}{x^8} \, dx &= \int \left(\frac{a}{x^8}+\frac{b}{x^6}+\frac{c}{x^4}\right) \, dx \\ &= -\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -1/7*a/x^7 - b/(5*x^5) - c/(3*x^3)
```

Maple [A]
time = 0.01, size = 20, normalized size = 0.80

method	result	size
default	$-\frac{a}{7x^{7}}-\frac{b}{5x^{5}}-\frac{c}{3x^{3}}$	20
norman	$\frac{-\frac{1}{3}c\,x^{4}-\frac{1}{5}b\,x^{2}-\frac{1}{7}a}{x^{7}}$	21
risch	$\frac{-\frac{1}{3}c\,x^{4}-\frac{1}{5}b\,x^{2}-\frac{1}{7}a}{x^{7}}$	21
gosper	$-\frac{35c\,x^{4}+21b\,x^{2}+15a}{105x^{7}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/7*a/x^7-1/5*b/x^5-1/3*c/x^3
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.84

$$-\frac{35\,cx^{4}+21\,bx^{2}+15\,a}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="maxima")
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

Fricas [A]
time = 0.35, size = 21, normalized size = 0.84

$$-\frac{35\,cx^{4}+21\,bx^{2}+15\,a}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="fricas")
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

Sympy [A]
time = 0.15, size = 22, normalized size = 0.88

$$\frac{-15a-21bx^{2}-35cx^{4}}{105x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**8,x)
```

```
[Out] (-15*a - 21*b*x**2 - 35*c*x**4)/(105*x**7)
```

Giac [A]
time = 3.88, size = 21, normalized size = 0.84

$$-\frac{35\,cx^4+21\,bx^2+15\,a}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.84

$$-\frac{\frac{c\,x^4}{3}+\frac{b\,x^2}{5}+\frac{a}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^8,x)
```

```
[Out] -(a/7 + (b*x^2)/5 + (c*x^4)/3)/x^7
```

3.825 $\int x^2(a+bx^2+cx^4)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.056$, Rules used = {1122}

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2+cx^4\right)^2\,dx&=\int\left(a^2x^2+2abx^4+\left(b^2+2ac\right)x^6+2bcx^8+c^2x^{10}\right)dx\\&=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^2,x]
```

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*x^11)/11

Maple [A]
time = 0.04, size = 45, normalized size = 0.83

method	result	size
default	$\frac{a^2x^3}{3}+\frac{2ab\,x^5}{5}+\frac{(2ac+b^2)x^7}{7}+\frac{2bc\,x^9}{9}+\frac{c^2x^{11}}{11}$	45
norman	$\frac{c^2x^{11}}{11}+\frac{2bc\,x^9}{9}+\left(\frac{2ac}{7}+\frac{b^2}{7}\right)x^7+\frac{2ab\,x^5}{5}+\frac{a^2x^3}{3}$	46
gosper	$\frac{1}{11}c^2x^{11}+\frac{2}{9}bc\,x^9+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{2}{5}ab\,x^5+\frac{1}{3}a^2x^3$	47
risch	$\frac{1}{11}c^2x^{11}+\frac{2}{9}bc\,x^9+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{2}{5}ab\,x^5+\frac{1}{3}a^2x^3$	47

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11

Maxima [A]
time = 0.28, size = 44, normalized size = 0.81

$$\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\,\left(b^2+2\,ac\right)x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*(b^2 + 2*a*c)*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3

Fricas [A]
time = 0.36, size = 44, normalized size = 0.81

$$\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\,\left(b^2+2\,ac\right)x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*(b^2 + 2*a*c)*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3

Sympy [A]
time = 0.01, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{2bcx^9}{9}+\frac{c^2x^{11}}{11}+x^7\cdot\left(\frac{2ac}{7}+\frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*(2*a*c/7 +
 b**2/7)
```

Giac [A]
time = 3.10, size = 46, normalized size = 0.85

$$\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\,b^2x^7+\frac{2}{7}\,acx^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b*x^5 + 1/3
*a^2*x^3
```

Mupad [B]
time = 0.03, size = 45, normalized size = 0.83

$$x^7\left(\frac{b^2}{7}+\frac{2\,a\,c}{7}\right)+\frac{a^2\,x^3}{3}+\frac{c^2\,x^{11}}{11}+\frac{2\,a\,b\,x^5}{5}+\frac{2\,b\,c\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^7*((2*a*c)/7 + b^2/7) + (a^2*x^3)/3 + (c^2*x^11)/11 + (2*a*b*x^5)/5 + (2*
b*c*x^9)/9
```

3.826 $\int x(a+bx^2+cx^4)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {1121, 625}

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10
)/10
```

Rule 625

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2+cx^4\right)^2\,dx &= \frac{1}{2}\text{Subst}\left(\int\left(a+bx+cx^2\right)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(a^2+2abx+b^2\left(1+\frac{2ac}{b^2}\right)x^2+2bcx^3+c^2x^4\right)\,dx,x,x^2\right)\\ &= \frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 0.89

$$\frac{1}{60}x^2\left(30a^2+30abx^2+10\left(b^2+2ac\right)x^4+15bcx^6+6c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8))
/60
```

Maple [A]
time = 0.04, size = 45, normalized size = 0.83

method	result	size
default	$\frac{a^2x^2}{2}+\frac{ab\,x^4}{2}+\frac{(2ac+b^2)x^6}{6}+\frac{bc\,x^8}{4}+\frac{c^2x^{10}}{10}$	45
norman	$\frac{c^2x^{10}}{10}+\frac{bc\,x^8}{4}+\left(\frac{ac}{3}+\frac{b^2}{6}\right)x^6+\frac{ab\,x^4}{2}+\frac{a^2x^2}{2}$	46
gosper	$\frac{1}{10}c^2x^{10}+\frac{1}{4}bc\,x^8+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	47
risch	$\frac{1}{10}c^2x^{10}+\frac{1}{4}bc\,x^8+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Maxima [A]
time = 0.29, size = 44, normalized size = 0.81

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,\left(b^2+2\,ac\right)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

Fricas [A]
time = 0.35, size = 44, normalized size = 0.81

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,\left(b^2+2\,ac\right)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

Sympy [**A**]
time = 0.01, size = 46, normalized size = 0.85

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}+x^6\left(\frac{ac}{3}+\frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2/
6)
```

Giac [**A**]
time = 4.16, size = 46, normalized size = 0.85

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,b^2x^6+\frac{1}{3}\,acx^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/2
*a^2*x^2
```

Mupad [**B**]
time = 0.02, size = 45, normalized size = 0.83

$$x^6\left(\frac{b^2}{6}+\frac{a\,c}{3}\right)+\frac{a^2\,x^2}{2}+\frac{c^2\,x^{10}}{10}+\frac{a\,b\,x^4}{2}+\frac{b\,c\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^6*((a*c)/3 + b^2/6) + (a^2*x^2)/2 + (c^2*x^10)/10 + (a*b*x^4)/2 + (b*c*x^
8)/4
```

3.827 $\int \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=49

$$a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1104\}$

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2+2abx^2+b^2\left(1+\frac{2ac}{b^2}\right)x^4+2bcx^6+c^2x^8\right) dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 49, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Maple [A]
time = 0.01, size = 42, normalized size = 0.86

method	result	size
default	$a^2x+\frac{2ab\,x^3}{3}+\frac{(2ac+b^2)x^5}{5}+\frac{2bc\,x^7}{7}+\frac{c^2x^9}{9}$	42
norman	$\frac{c^2x^9}{9}+\frac{2bc\,x^7}{7}+\left(\frac{2ac}{5}+\frac{b^2}{5}\right)x^5+\frac{2ab\,x^3}{3}+a^2x$	43
gosper	$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{2}{3}ab\,x^3+a^2x$	44
risch	$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{2}{3}ab\,x^3+a^2x$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A]
time = 0.29, size = 45, normalized size = 0.92

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5+a^2x+\frac{2}{15}\,\left(3\,cx^5+5\,bx^3\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)*
a
```

Fricas [A]
time = 0.35, size = 41, normalized size = 0.84

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,\left(b^2+2\,ac\right)x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A]
time = 0.01, size = 48, normalized size = 0.98

$$a^2x+\frac{2abx^3}{3}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}+x^5\cdot\left(\frac{2ac}{5}+\frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

Giac [A]
time = 5.00, size = 43, normalized size = 0.88

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5+\frac{2}{5}\,acx^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B]
time = 0.02, size = 42, normalized size = 0.86

$$a^2\,x+x^5\,\left(\frac{b^2}{5}+\frac{2\,a\,c}{5}\right)+\frac{c^2\,x^9}{9}+\frac{2\,a\,b\,x^3}{3}+\frac{2\,b\,c\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2,x)
```

```
[Out] a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7)
/7
```

3.828 $\int \frac{(a+bx^2+cx^4)^2}{x} \, dx$

Optimal. Leaf size=47

$$abx^2+\frac{1}{4}\left(b^2+2ac\right)x^4+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}+a^2\log(x)$$

```
[Out] a*b*x^2+1/4*(2*a*c+b^2)*x^4+1/3*b*c*x^6+1/8*c^2*x^8+a^2*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1128, 712}

$$a^2\log(x)+\frac{1}{4}x^4\left(2ac+b^2\right)+abx^2+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x,x]
```

```
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2*Log[x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2+cx^4\right)^2}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+\left(b^2+2ac\right)x+2bcx^2+c^2x^3\right)\,dx,x,x^2\right)\\
&= abx^2+\frac{1}{4}\left(b^2+2ac\right)x^4+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}+a^2\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.00

$$abx^2+\frac{1}{4}\left(b^2+2ac\right)x^4+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}+a^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x,x]
```

```
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2*Log[x]
```

Maple [A]
time = 0.05, size = 44, normalized size = 0.94

method	result	size
norman	$\left(\frac{ac}{2}+\frac{b^2}{4}\right)x^4+ab\,x^2+\frac{c^2x^8}{8}+\frac{bc\,x^6}{3}+a^2\ln\left(x\right)$	43
default	$\frac{c^2x^8}{8}+\frac{bc\,x^6}{3}+\frac{ac\,x^4}{2}+\frac{b^2x^4}{4}+ab\,x^2+a^2\ln\left(x\right)$	44
risch	$\frac{ac\,x^4}{2}+ab\,x^2+\frac{a\,b^2}{2c}+\frac{bc\,x^6}{3}+\frac{b^2x^4}{4}-\frac{b^4}{24c^2}+\frac{c^2x^8}{8}+a^2\ln\left(x\right)$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c^2*x^8+1/3*b*c*x^6+1/2*a*c*x^4+1/4*b^2*x^4+a*b*x^2+a^2*ln(x)
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.94

$$\frac{1}{8}\,c^2x^8+\frac{1}{3}\,bcx^6+\frac{1}{4}\,\left(b^2+2\,ac\right)x^4+abx^2+\frac{1}{2}\,a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + 1/2*a^2*log(x
^2)
```

Fricas [A]
time = 0.37, size = 41, normalized size = 0.87

$$\frac{1}{8}\,c^2x^8+\frac{1}{3}\,bcx^6+\frac{1}{4}\,\left(b^2+2\,ac\right)x^4+abx^2+a^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + a^2*log(x)
```

Sympy [A]
time = 0.04, size = 42, normalized size = 0.89

$$a^2 \log(x) + abx^2 + \frac{bcx^6}{3} + \frac{c^2x^8}{8} + x^4\left(\frac{ac}{2} + \frac{b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x,x)
[Out] a**2*log(x) + a*b*x**2 + b*c*x**6/3 + c**2*x**8/8 + x**4*(a*c/2 + b**2/4)
```

Giac [A]
time = 3.84, size = 46, normalized size = 0.98

$$\frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}b^2x^4 + \frac{1}{2}acx^4 + abx^2 + \frac{1}{2}a^2\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="giac")
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4 + 1/2*a*c*x^4 + a*b*x^2 + 1/2*a^2*l
og(x^2)
```

Mupad [B]
time = 0.02, size = 42, normalized size = 0.89

$$a^2 \ln(x) + x^4\left(\frac{b^2}{4} + \frac{a\,c}{2}\right) + \frac{c^2\,x^8}{8} + a\,b\,x^2 + \frac{b\,c\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x,x)
[Out] a^2*log(x) + x^4*((a*c)/2 + b^2/4) + (c^2*x^8)/8 + a*b*x^2 + (b*c*x^6)/3
```

3.829 $\int \frac{(a+bx^2+cx^4)^2}{x^2} dx$

Optimal. Leaf size=48

$$-\frac{a^2}{x}+2abx+\frac{1}{3}\left(b^2+2ac\right)x^3+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

```
[Out] -a^2/x+2*a*b*x+1/3*(2*a*c+b^2)*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1122}

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx &= \int \left(2ab+\frac{a^2}{x^2}+\left(b^2+2ac\right)x^2+2bcx^4+c^2x^6\right)dx\\ &= -\frac{a^2}{x}+2abx+\frac{1}{3}\left(b^2+2ac\right)x^3+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.00

$$-\frac{a^2}{x}+2abx+\frac{1}{3}\left(b^2+2ac\right)x^3+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Maple [A]
time = 0.03, size = 45, normalized size = 0.94

method	result	size
default	$\frac{c^2x^7}{7}+\frac{2bc\,x^5}{5}+\frac{2ac\,x^3}{3}+\frac{b^2x^3}{3}+2abx-\frac{a^2}{x}$	45
risch	$\frac{c^2x^7}{7}+\frac{2bc\,x^5}{5}+\frac{2ac\,x^3}{3}+\frac{b^2x^3}{3}+2abx-\frac{a^2}{x}$	45
norman	$\frac{\frac{c^2x^8}{7}+\frac{2bc\,x^6}{5}+\left(\frac{2ac}{3}+\frac{b^2}{3}\right)x^4+2ab\,x^2-a^2}{x}$	47
gosper	$-\frac{-15c^2x^8-42bc\,x^6-70ac\,x^4-35b^2x^4-210ab\,x^2+105a^2}{105x}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)
[Out] 1/7*c^2*x^7+2/5*b*c*x^5+2/3*a*c*x^3+1/3*b^2*x^3+2*a*b*x-a^2/x
```

Maxima [A]
time = 0.30, size = 42, normalized size = 0.88

$$\frac{1}{7}\,c^2x^7+\frac{2}{5}\,bcx^5+\frac{1}{3}\,\left(b^2+2\,ac\right)x^3+2\,abx-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*(b^2 + 2*a*c)*x^3 + 2*a*b*x - a^2/x
```

Fricas [A]
time = 0.35, size = 46, normalized size = 0.96

$$\frac{15\,c^2x^8+42\,bcx^6+35\,(b^2+2\,ac)x^4+210\,abx^2-105\,a^2}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")
[Out] 1/105*(15*c^2*x^8 + 42*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 210*a*b*x^2 - 105*a
^2)/x
```

Sympy [A]
time = 0.04, size = 44, normalized size = 0.92

$$-\frac{a^2}{x}+2abx+\frac{2bcx^5}{5}+\frac{c^2x^7}{7}+x^3\cdot\left(\frac{2ac}{3}+\frac{b^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + 2*b*c*x**5/5 + c**2*x**7/7 + x**3*(2*a*c/3 + b**2/3)
```

Giac [A]
time = 3.06, size = 44, normalized size = 0.92

$$\frac{1}{7}c^2x^7 + \frac{2}{5}bcx^5 + \frac{1}{3}b^2x^3 + \frac{2}{3}acx^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3 + 2/3*a*c*x^3 + 2*a*b*x - a^2/x
```

Mupad [B]
time = 0.02, size = 43, normalized size = 0.90

$$x^3\left(\frac{b^2}{3} + \frac{2\,a\,c}{3}\right) - \frac{a^2}{x} + \frac{c^2\,x^7}{7} + 2\,a\,b\,x + \frac{2\,b\,c\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^2,x)
```

```
[Out] x^3*((2*a*c)/3 + b^2/3) - a^2/x + (c^2*x^7)/7 + 2*a*b*x + (2*b*c*x^5)/5
```

3.830 $\int \frac{(a+bx^2+cx^4)^2}{x^3} dx$

Optimal. Leaf size=51

$$-\frac{a^2}{2x^2}+\frac{1}{2}\left(b^2+2ac\right)x^2+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}+2ab\log(x)$$

```
[Out] -1/2*a^2/x^2+1/2*(2*a*c+b^2)*x^2+1/2*b*c*x^4+1/6*c^2*x^6+2*a*b*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1128, 712}

$$-\frac{a^2}{2x^2}+\frac{1}{2}x^2\left(2ac+b^2\right)+2ab\log(x)+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + ((b^2 + 2*a*c)*x^2)/2 + (b*c*x^4)/2 + (c^2*x^6)/6 + 2*a*b*Lo
g[x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(b^2\left(1+\frac{2ac}{b^2}\right)+\frac{a^2}{x^2}+\frac{2ab}{x}+2bcx+c^2x^2\right)\,dx, x, x^2\right)\\
&= -\frac{a^2}{2x^2}+\frac{1}{2}\left(b^2+2ac\right)x^2+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}+2ab\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.90

$$\frac{1}{6}\left(-\frac{3a^2}{x^2}+3\left(b^2+2ac\right)x^2+3bcx^4+c^2x^6+12ab\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] ((-3*a^2)/x^2 + 3*(b^2 + 2*a*c)*x^2 + 3*b*c*x^4 + c^2*x^6 + 12*a*b*Log[x])/
6
```

Maple [A]
time = 0.02, size = 45, normalized size = 0.88

method	result	size
default	$\frac{c^{2} x^{6}}{6}+\frac{b c \,x^{4}}{2}+a c \,x^{2}+\frac{b^{2} x^{2}}{2}-\frac{a^{2}}{2 x^{2}}+2 a b \ln \left(x \right)$	45
risch	$\frac{c^{2} x^{6}}{6}+\frac{b c \,x^{4}}{2}+a c \,x^{2}+\frac{b^{2} x^{2}}{2}-\frac{a^{2}}{2 x^{2}}+2 a b \ln \left(x \right)$	45
norman	$\frac{\left(a c +\frac{b^{2}}{2}\right) x^{4}-\frac{a^{2}}{2}+\frac{c^{2} x^{8}}{6}+\frac{b c \,x^{6}}{2}}{x^{2}}+2 a b \ln \left(x \right)$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c^2*x^6+1/2*b*c*x^4+a*c*x^2+1/2*b^2*x^2-1/2*a^2/x^2+2*a*b*ln(x)
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.86

$$\frac{1}{6}\,c^2x^6+\frac{1}{2}\,bcx^4+\frac{1}{2}\,(b^2+2\,ac)x^2+ab\log\left(x^2\right)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*(b^2 + 2*a*c)*x^2 + a*b*log(x^2) - 1/2*a^2/
x^2
```

Fricas [A]
time = 0.33, size = 47, normalized size = 0.92

$$\frac{c^2x^8+3\,bcx^6+3\,(b^2+2\,ac)x^4+12\,abx^2\log\left(x\right)-3\,a^2}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(c^2*x^8 + 3*b*c*x^6 + 3*(b^2 + 2*a*c)*x^4 + 12*a*b*x^2*log(x) - 3*a^2)
/x^2
```

Sympy [A]
time = 0.05, size = 44, normalized size = 0.86

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{bcx^4}{2} + \frac{c^2x^6}{6} + x^2\left(ac + \frac{b^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b*c*x**4/2 + c**2*x**6/6 + x**2*(a*c + b**2
/2)
```

Giac [A]
time = 3.87, size = 53, normalized size = 1.04

$$\frac{1}{6}c^2x^6 + \frac{1}{2}bcx^4 + \frac{1}{2}b^2x^2 + acx^2 + ab\log(x^2) - \frac{2abx^2 + a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2 + a*c*x^2 + a*b*log(x^2) - 1/2*(2*a
*b*x^2 + a^2)/x^2
```

Mupad [B]
time = 0.03, size = 43, normalized size = 0.84

$$x^2\left(\frac{b^2}{2} + a\,c\right) - \frac{a^2}{2\,x^2} + \frac{c^2\,x^6}{6} + 2\,a\,b\,\ln(x) + \frac{b\,c\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^3,x)
```

```
[Out] x^2*(a*c + b^2/2) - a^2/(2*x^2) + (c^2*x^6)/6 + 2*a*b*log(x) + (b*c*x^4)/2
```

3.831 $\int \frac{(a+bx^2+cx^4)^2}{x^4} dx$

Optimal. Leaf size=47

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+\left(b^2+2ac\right)x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

```
[Out] -1/3*a^2/x^3-2*a*b/x+(2*a*c+b^2)*x+2/3*b*c*x^3+1/5*c^2*x^5
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1122}

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^4}\,dx &= \int \left(b^2\left(1+\frac{2ac}{b^2}\right)+\frac{a^2}{x^4}+\frac{2ab}{x^2}+2bcx^2+c^2x^4\right)\,dx\\ &= -\frac{a^2}{3x^3}-\frac{2ab}{x}+\left(b^2+2ac\right)x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.00

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+\left(b^2+2ac\right)x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Maple [A]
time = 0.02, size = 42, normalized size = 0.89

method	result	size
default	$\frac{c^2x^5}{5}+\frac{2bc\,x^3}{3}+2acx+b^2x-\frac{2ab}{x}-\frac{a^2}{3x^3}$	42
risch	$\frac{c^2x^5}{5}+\frac{2bc\,x^3}{3}+2acx+b^2x+\frac{-2ab\,x^2-\frac{1}{3}a^2}{x^3}$	44
norman	$\frac{\frac{c^2x^8}{5}+\frac{2bc\,x^6}{3}+(2ac+b^2)x^4-2ab\,x^2-\frac{a^2}{3}}{x^3}$	45
gosper	$-\frac{-3c^2x^8-10bc\,x^6-30ac\,x^4-15b^2x^4+30ab\,x^2+5a^2}{15x^3}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^4,x,method=_RETURNVERBOSE)
[Out] 1/5*c^2*x^5+2/3*b*c*x^3+2*a*c*x+b^2*x-2*a*b/x-1/3*a^2/x^3
```

Maxima [A]
time = 0.28, size = 42, normalized size = 0.89

$$\frac{1}{5}\,c^2x^5+\frac{2}{3}\,bcx^3+\left(b^2+2\,ac\right)x-\frac{6\,abx^2+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + (b^2 + 2*a*c)*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Fricas [A]
time = 0.33, size = 46, normalized size = 0.98

$$\frac{3\,c^2x^8+10\,bcx^6+15\,(b^2+2\,ac)x^4-30\,abx^2-5\,a^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")
[Out] 1/15*(3*c^2*x^8 + 10*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 - 30*a*b*x^2 - 5*a^2)/x
^3
```

Sympy [A]
time = 0.06, size = 46, normalized size = 0.98

$$\frac{2bcx^3}{3}+\frac{c^2x^5}{5}+x\left(2ac+b^2\right)+\frac{-a^2-6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] 2*b*c*x**3/3 + c**2*x**5/5 + x*(2*a*c + b**2) + (-a**2 - 6*a*b*x**2)/(3*x**
3)
```

Giac [A]
time = 3.74, size = 42, normalized size = 0.89

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x + 2\,acx - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x + 2*a*c*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Mupad [B]
time = 0.04, size = 44, normalized size = 0.94

$$x\,\left(b^2+2\,a\,c\right)-\frac{\frac{a^2}{3}+2\,b\,a\,x^2}{x^3}+\frac{c^2\,x^5}{5}+\frac{2\,b\,c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^4,x)
```

```
[Out] x*(2*a*c + b^2) - (a^2/3 + 2*a*b*x^2)/x^3 + (c^2*x^5)/5 + (2*b*c*x^3)/3
```

3.832 $\int \frac{(a+bx^2+cx^4)^2}{x^5} dx$

Optimal. Leaf size=45

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}+\left(b^2+2ac\right)\log(x)$$

```
[Out] -1/4*a^2/x^4-a*b/x^2+b*c*x^2+1/4*c^2*x^4+(2*a*c+b^2)*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {1128, 712}

$$-\frac{a^2}{4x^4}+\log(x)\left(2ac+b^2\right)-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (a*b)/x^2 + b*c*x^2 + (c^2*x^4)/4 + (b^2 + 2*a*c)*Log[x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2+cx^4\right)^2}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(2bc+\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2+2ac}{x}+c^2x\right)\,dx,x,x^2\right)\\
&= -\frac{a^2}{4x^4}-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}+\left(b^2+2ac\right)\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.91

$$\frac{(-a+cx^4)(a+4bx^2+cx^4)}{4x^4}+(b^2+2ac)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] ((-a + c*x^4)*(a + 4*b*x^2 + c*x^4))/(4*x^4) + (b^2 + 2*a*c)*Log[x]
```

Maple [A]
time = 0.03, size = 42, normalized size = 0.93

method	result	size
default	$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+bc\,x^2+\frac{c^2x^4}{4}+(2ac+b^2)\ln(x)$	42
norman	$\frac{bc\,x^6-\frac{1}{4}a^2+\frac{1}{4}c^2x^8-ab\,x^2}{x^4}+(2ac+b^2)\ln(x)$	44
risch	$\frac{c^2x^4}{4}+bc\,x^2+b^2+\frac{-\frac{1}{4}a^2-ab\,x^2}{x^4}+2\ln(x)\,ac+b^2\ln(x)$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^2/x^4-a*b/x^2+b*c*x^2+1/4*c^2*x^4+(2*a*c+b^2)*ln(x)
```

Maxima [A]
time = 0.28, size = 45, normalized size = 1.00

$$\frac{1}{4}c^2x^4+bcx^2+\frac{1}{2}(b^2+2\,ac)\log(x^2)-\frac{4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/
x^4
```

Fricas [A]
time = 0.38, size = 47, normalized size = 1.04

$$\frac{c^2x^8+4\,bcx^6+4\,(b^2+2\,ac)x^4\log(x)-4\,abx^2-a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(c^2*x^8 + 4*b*c*x^6 + 4*(b^2 + 2*a*c)*x^4*log(x) - 4*a*b*x^2 - a^2)/x^
4
```

Sympy [A]
time = 0.17, size = 44, normalized size = 0.98

$$bcx^2 + \frac{c^2x^4}{4} + (2ac + b^2)\log(x) + \frac{-a^2 - 4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] b*c*x**2 + c**2*x**4/4 + (2*a*c + b**2)*log(x) + (-a**2 - 4*a*b*x**2)/(4*x*
*4)
```

Giac [A]
time = 3.71, size = 60, normalized size = 1.33

$$\frac{1}{4}c^2x^4 + bcx^2 + \frac{1}{2}(b^2 + 2ac)\log(x^2) - \frac{3b^2x^4 + 6acx^4 + 4abx^2 + a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*log(x^2) - 1/4*(3*b^2*x^4 + 6*a*c
*x^4 + 4*a*b*x^2 + a^2)/x^4
```

Mupad [B]
time = 0.04, size = 43, normalized size = 0.96

$$\ln(x)\,(b^2 + 2\,a\,c) - \frac{\frac{a^2}{4} + b\,a\,x^2}{x^4} + \frac{c^2\,x^4}{4} + b\,c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^5,x)
```

```
[Out] log(x)*(2*a*c + b^2) - (a^2/4 + a*b*x^2)/x^4 + (c^2*x^4)/4 + b*c*x^2
```

3.833 $\int \frac{(a+bx^2+cx^4)^2}{x^6}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2+2ac}{x}+2bcx+\frac{c^2x^3}{3}$$

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3+(-2*a*c-b^2)/x+2*b*c*x+1/3*c^2*x^3
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1122}

$$-\frac{a^2}{5x^5}-\frac{2ac+b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) - (b^2 + 2*a*c)/x + 2*b*c*x + (c^2*x^3)/3
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{x^6}\,dx &= \int \left(2bc+\frac{a^2}{x^6}+\frac{2ab}{x^4}+\frac{b^2+2ac}{x^2}+c^2x^2\right)dx\\ &= -\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2+2ac}{x}+2bcx+\frac{c^2x^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 1.02

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}+\frac{-b^2-2ac}{x}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^6,x]
```

[Out] -1/5*a^2/x^5 - (2*a*b)/(3*x^3) + (-b^2 - 2*a*c)/x + 2*b*c*x + (c^2*x^3)/3

Maple [A]
time = 0.02, size = 43, normalized size = 0.90

method	result	size
default	$\frac{c^2x^3}{3}+2bcx-\frac{a^2}{5x^5}-\frac{2ac+b^2}{x}-\frac{2ab}{3x^3}$	43
risch	$\frac{c^2x^3}{3}+2bcx+\frac{(-2ac-b^2)x^4-\frac{2ab\,x^2}{3}-\frac{a^2}{5}}{x^5}$	46
norman	$\frac{\frac{c^2x^8}{3}+2bc\,x^6+(-2ac-b^2)x^4-\frac{2ab\,x^2}{3}-\frac{a^2}{5}}{x^5}$	47
gosper	$-\frac{-5c^2x^8-30bc\,x^6+30ac\,x^4+15b^2x^4+10ab\,x^2+3a^2}{15x^5}$	49

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^2/x^6,x,method=_RETURNVERBOSE)

[Out] 1/3*c^2*x^3+2*b*c*x-1/5*a^2/x^5-(2*a*c+b^2)/x-2/3*a*b/x^3

Maxima [A]
time = 0.28, size = 45, normalized size = 0.94

$$\frac{1}{3}\,c^2x^3+2\,bcx-\frac{15\,(b^2+2\,ac)x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")

[Out] 1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*(b^2 + 2*a*c)*x^4 + 10*a*b*x^2 + 3*a^2)/x^
5

Fricas [A]
time = 0.36, size = 46, normalized size = 0.96

$$\frac{5\,c^2x^8+30\,bcx^6-15\,(b^2+2\,ac)x^4-10\,abx^2-3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")

[Out] 1/15*(5*c^2*x^8 + 30*b*c*x^6 - 15*(b^2 + 2*a*c)*x^4 - 10*a*b*x^2 - 3*a^2)/x
^5

Sympy [A]
time = 0.20, size = 48, normalized size = 1.00

$$2bcx+\frac{c^2x^3}{3}+\frac{-3a^2-10abx^2+x^4(-30ac-15b^2)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] 2*b*c*x + c**2*x**3/3 + (-3*a**2 - 10*a*b*x**2 + x**4*(-30*a*c - 15*b**2))/
(15*x**5)
```

Giac [A]
time = 3.13, size = 47, normalized size = 0.98

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{15\,b^2x^4 + 30\,acx^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*b^2*x^4 + 30*a*c*x^4 + 10*a*b*x^2 + 3*a^2)
/x^5
```

Mupad [B]
time = 0.04, size = 44, normalized size = 0.92

$$\frac{c^2\,x^3}{3} - \frac{x^4\,\left(b^2 + 2\,a\,c\right) + \frac{a^2}{5} + \frac{2\,a\,b\,x^2}{3}}{x^5} + 2\,b\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^6,x)
```

```
[Out] (c^2*x^3)/3 - (x^4*(2*a*c + b^2) + a^2/5 + (2*a*b*x^2)/3)/x^5 + 2*b*c*x
```

3.834 $\int \frac{(a+bx^2+cx^4)^2}{x^7}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2+2ac}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)$$

```
[Out] -1/6*a^2/x^6-1/2*a*b/x^4+1/2*(-2*a*c-b^2)/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1128, 712}

$$-\frac{a^2}{6x^6}-\frac{2ac+b^2}{2x^2}-\frac{ab}{2x^4}+2bc\log(x)+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -1/6*a^2/x^6 - (a*b)/(2*x^4) - (b^2 + 2*a*c)/(2*x^2) + (c^2*x^2)/2 + 2*b*c*
Log[x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2+cx^4)^2}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^4}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(c^2+\frac{a^2}{x^4}+\frac{2ab}{x^3}+\frac{b^2+2ac}{x^2}+\frac{2bc}{x}\right)dx, x, x^2\right)\\
&= -\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2+2ac}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.98

$$-\frac{a^2+3abx^2+3b^2x^4+6acx^4-3c^2x^8-12bcx^6\log(x)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -1/6*(a^2 + 3*a*b*x^2 + 3*b^2*x^4 + 6*a*c*x^4 - 3*c^2*x^8 - 12*b*c*x^6*Log[
x])/x^6
```

Maple [A]
time = 0.03, size = 44, normalized size = 0.86

method	result	size
default	$\frac{c^2x^2}{2}-\frac{ab}{2x^4}-\frac{2ac+b^2}{2x^2}+2bc\ln\left(x\right)-\frac{a^2}{6x^6}$	44
norman	$\frac{\left(-ac-\frac{b^2}{2}\right)x^4-\frac{a^2}{6}+\frac{c^2x^8}{2}-\frac{ab\,x^2}{2}}{x^6}+2bc\ln\left(x\right)$	47
risch	$\frac{c^2x^2}{2}+\frac{\left(-ac-\frac{b^2}{2}\right)x^4-\frac{ab\,x^2}{2}-\frac{a^2}{6}}{x^6}+2bc\ln\left(x\right)$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c^2*x^2-1/2*a*b/x^4-1/2*(2*a*c+b^2)/x^2+2*b*c*ln(x)-1/6*a^2/x^6
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.88

$$\frac{1}{2}\,c^2x^2+bc\log\left(x^2\right)-\frac{3\,(b^2+2\,ac)x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/6*(3*(b^2 + 2*a*c)*x^4 + 3*a*b*x^2 + a^2)/x^
6
```

Fricas [A]
time = 0.35, size = 48, normalized size = 0.94

$$\frac{3\,c^2x^8+12\,bcx^6\log\left(x\right)-3\,(b^2+2\,ac)x^4-3\,abx^2-a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(3*c^2*x^8 + 12*b*c*x^6*log(x) - 3*(b^2 + 2*a*c)*x^4 - 3*a*b*x^2 - a^2)
/x^6
```

Sympy [A]
time = 0.37, size = 48, normalized size = 0.94

$$2bc\log(x) + \frac{c^2x^2}{2} + \frac{-a^2 - 3abx^2 + x^4(-6ac - 3b^2)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] 2*b*c*log(x) + c**2*x**2/2 + (-a**2 - 3*a*b*x**2 + x**4*(-6*a*c - 3*b**2))/
(6*x**6)
```

Giac [A]
time = 3.51, size = 54, normalized size = 1.06

$$\frac{1}{2}\,c^2x^2 + bc\log\left(x^2\right) - \frac{11\,bcx^6 + 3\,b^2x^4 + 6\,acx^4 + 3\,abx^2 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/6*(11*b*c*x^6 + 3*b^2*x^4 + 6*a*c*x^4 + 3*a*
b*x^2 + a^2)/x^6
```

Mupad [B]
time = 4.14, size = 46, normalized size = 0.90

$$\frac{c^2\,x^2}{2} - \frac{\frac{a^2}{6} + x^4\,\left(\frac{b^2}{2} + a\,c\right) + \frac{a\,b\,x^2}{2}}{x^6} + 2\,b\,c\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^7,x)
```

```
[Out] (c^2*x^2)/2 - (a^2/6 + x^4*(a*c + b^2/2) + (a*b*x^2)/2)/x^6 + 2*b*c*log(x)
```

3.835 $\int \frac{(a+bx^2+cx^4)^2}{x^8} dx$

Optimal. Leaf size=47

$$-\frac{a^2}{7x^7} - \frac{2ab}{5x^5} - \frac{b^2+2ac}{3x^3} - \frac{2bc}{x} + c^2x$$

```
[Out] -1/7*a^2/x^7-2/5*a*b/x^5+1/3*(-2*a*c-b^2)/x^3-2*b*c/x+c^2*x
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1122}

$$-\frac{a^2}{7x^7} - \frac{2ac+b^2}{3x^3} - \frac{2ab}{5x^5} - \frac{2bc}{x} + c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) - (b^2 + 2*a*c)/(3*x^3) - (2*b*c)/x + c^2*x
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2+cx^4)^2}{x^8} dx &= \int \left(c^2 + \frac{a^2}{x^8} + \frac{2ab}{x^6} + \frac{b^2+2ac}{x^4} + \frac{2bc}{x^2}\right) dx \\
&= -\frac{a^2}{7x^7} - \frac{2ab}{5x^5} - \frac{b^2+2ac}{3x^3} - \frac{2bc}{x} + c^2x
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 1.04

$$-\frac{a^2}{7x^7} - \frac{2ab}{5x^5} + \frac{-b^2-2ac}{3x^3} - \frac{2bc}{x} + c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^8,x]
```

[Out] -1/7*a^2/x^7 - (2*a*b)/(5*x^5) + (-b^2 - 2*a*c)/(3*x^3) - (2*b*c)/x + c^2*x

Maple [A]
time = 0.02, size = 42, normalized size = 0.89

method	result	size
default	$c^{2}x-\frac{2ab}{5x^{5}}-\frac{2bc}{x}-\frac{2ac+b^{2}}{3x^{3}}-\frac{a^{2}}{7x^{7}}$	42
risch	$c^{2}x+\frac{-2bc\,x^{6}+\left(-\frac{2ac}{3}-\frac{b^{2}}{3}\right)x^{4}-\frac{2ab\,x^{2}}{5}-\frac{a^{2}}{7}}{x^{7}}$	45
norman	$\frac{c^{2}x^{8}-2bc\,x^{6}+\left(-\frac{2ac}{3}-\frac{b^{2}}{3}\right)x^{4}-\frac{2ab\,x^{2}}{5}-\frac{a^{2}}{7}}{x^{7}}$	46
gosper	$-\frac{-105c^{2}x^{8}+210bc\,x^{6}+70ac\,x^{4}+35b^{2}x^{4}+42ab\,x^{2}+15a^{2}}{105x^{7}}$	49

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^2/x^8,x,method=_RETURNVERBOSE)

[Out] c^2*x-2/5*a*b/x^5-2*b*c/x-1/3*(2*a*c+b^2)/x^3-1/7*a^2/x^7

Maxima [A]
time = 0.28, size = 44, normalized size = 0.94

$$c^{2}x-\frac{210\,bcx^{6}+35\,(b^{2}+2\,ac)x^{4}+42\,abx^{2}+15\,a^{2}}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="maxima")

[Out] c^2*x - 1/105*(210*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 42*a*b*x^2 + 15*a^2)/x^
7

Fricas [A]
time = 0.35, size = 46, normalized size = 0.98

$$\frac{105\,c^{2}x^{8}-210\,bcx^{6}-35\,(b^{2}+2\,ac)x^{4}-42\,abx^{2}-15\,a^{2}}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="fricas")

[Out] 1/105*(105*c^2*x^8 - 210*b*c*x^6 - 35*(b^2 + 2*a*c)*x^4 - 42*a*b*x^2 - 15*a
^2)/x^7

Sympy [A]
time = 0.36, size = 46, normalized size = 0.98

$$c^{2}x+\frac{-15a^{2}-42abx^{2}-210bcx^{6}+x^{4}(-70ac-35b^{2})}{105x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**8,x)
```

```
[Out] c**2*x + (-15*a**2 - 42*a*b*x**2 - 210*b*c*x**6 + x**4*(-70*a*c - 35*b**2))
/(105*x**7)
```

Giac [A]
time = 4.39, size = 46, normalized size = 0.98

$$c^2x - \frac{210\,bcx^6 + 35\,b^2x^4 + 70\,acx^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="giac")
```

```
[Out] c^2*x - 1/105*(210*b*c*x^6 + 35*b^2*x^4 + 70*a*c*x^4 + 42*a*b*x^2 + 15*a^2)
/x^7
```

Mupad [B]
time = 4.17, size = 45, normalized size = 0.96

$$c^2\,x - \frac{\frac{a^2}{7} + x^4\,\left(\frac{b^2}{3} + \frac{2\,a\,c}{3}\right) + \frac{2\,a\,b\,x^2}{5} + 2\,b\,c\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^8,x)
```

```
[Out] c^2*x - (a^2/7 + x^4*((2*a*c)/3 + b^2/3) + (2*a*b*x^2)/5 + 2*b*c*x^6)/x^7
```

3.836 $\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2+2ac}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

```
[Out] -1/8*a^2/x^8-1/3*a*b/x^6+1/4*(-2*a*c-b^2)/x^4-b*c/x^2+c^2*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1128, 712}

$$-\frac{a^2}{8x^8}-\frac{2ac+b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -1/8*a^2/x^8 - (a*b)/(3*x^6) - (b^2 + 2*a*c)/(4*x^4) - (b*c)/x^2 + c^2*Log[
x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^5}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2+2ac}{x^3}+\frac{2bc}{x^2}+\frac{c^2}{x}\right)dx,x,x^2\right)\\
&= -\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2+2ac}{4x^4}-\frac{bc}{x^2}+c^2\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.04

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}+\frac{-b^2-2ac}{4x^4}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -1/8*a^2/x^8 - (a*b)/(3*x^6) + (-b^2 - 2*a*c)/(4*x^4) - (b*c)/x^2 + c^2*Log
[x]
```

Maple [A]
time = 0.02, size = 43, normalized size = 0.90

method	result	size
default	$-\frac{2ac+b^2}{4x^4}-\frac{bc}{x^2}-\frac{a^2}{8x^8}+c^2\ln(x)-\frac{ab}{3x^6}$	43
norman	$\frac{\left(-\frac{ac}{2}-\frac{b^2}{4}\right)x^4-\frac{a^2}{8}-\frac{ab\,x^2}{3}-bc\,x^6}{x^8}+c^2\ln(x)$	46
risch	$\frac{\left(-\frac{ac}{2}-\frac{b^2}{4}\right)x^4-\frac{a^2}{8}-\frac{ab\,x^2}{3}-bc\,x^6}{x^8}+c^2\ln(x)$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*a*c+b^2)/x^4-b*c/x^2-1/8*a^2/x^8+c^2*ln(x)-1/3*a*b/x^6
```

Maxima [A]
time = 0.30, size = 48, normalized size = 1.00

$$\frac{1}{2}c^2\log\left(x^2\right)-\frac{24\,bcx^6+6\,(b^2+2\,ac)x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2) - 1/24*(24*b*c*x^6 + 6*(b^2 + 2*a*c)*x^4 + 8*a*b*x^2 + 3*a
^2)/x^8
```

Fricas [A]
time = 0.37, size = 48, normalized size = 1.00

$$\frac{24\,c^2x^8\log(x)-24\,bcx^6-6\,(b^2+2\,ac)x^4-8\,abx^2-3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(24*c^2*x^8*log(x) - 24*b*c*x^6 - 6*(b^2 + 2*a*c)*x^4 - 8*a*b*x^2 - 3*
a^2)/x^8
```

Sympy [A]
time = 0.69, size = 48, normalized size = 1.00

$$c^2\log(x) + \frac{-3a^2 - 8abx^2 - 24bcx^6 + x^4(-12ac - 6b^2)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**9,x)
```

```
[Out] c**2*log(x) + (-3*a**2 - 8*a*b*x**2 - 24*b*c*x**6 + x**4*(-12*a*c - 6*b**2)
)/(24*x**8)
```

Giac [A]
time = 4.18, size = 58, normalized size = 1.21

$$\frac{1}{2}c^2\log\left(x^2\right) - \frac{25\,c^2x^8 + 24\,bcx^6 + 6\,b^2x^4 + 12\,acx^4 + 8\,abx^2 + 3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2) - 1/24*(25*c^2*x^8 + 24*b*c*x^6 + 6*b^2*x^4 + 12*a*c*x^4 +
 8*a*b*x^2 + 3*a^2)/x^8
```

Mupad [B]
time = 4.18, size = 45, normalized size = 0.94

$$c^2\ln(x) - \frac{\frac{a^2}{8} + x^4\left(\frac{b^2}{4} + \frac{a\,c}{2}\right) + \frac{a\,b\,x^2}{3} + b\,c\,x^6}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^9,x)
```

```
[Out] c^2*log(x) - (a^2/8 + x^4*((a*c)/2 + b^2/4) + (a*b*x^2)/3 + b*c*x^6)/x^8
```

3.837 $\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx$

Optimal. Leaf size=52

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2+2ac}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

```
[Out] -1/9*a^2/x^9-2/7*a*b/x^7+1/5*(-2*a*c-b^2)/x^5-2/3*b*c/x^3-c^2/x
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1122}

$$-\frac{a^2}{9x^9}-\frac{2ac+b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -1/9*a^2/x^9 - (2*a*b)/(7*x^7) - (b^2 + 2*a*c)/(5*x^5) - (2*b*c)/(3*x^3) - c^2/x
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^8}+\frac{b^2+2ac}{x^6}+\frac{2bc}{x^4}+\frac{c^2}{x^2}\right)dx\\ &= -\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2+2ac}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.96

$$-\frac{35a^2+90abx^2+63b^2x^4+126acx^4+210bcx^6+315c^2x^8}{315x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -1/315*(35*a^2 + 90*a*b*x^2 + 63*b^2*x^4 + 126*a*c*x^4 + 210*b*c*x^6 + 315*
c^2*x^8)/x^9
```

Maple [A]
time = 0.01, size = 45, normalized size = 0.87

method	result	size
default	$-\frac{2ac+b^2}{5x^5}-\frac{c^2}{x}-\frac{2bc}{3x^3}-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}$	45
norman	$\frac{-c^2x^8-\frac{2bc\,x^6}{3}+\left(-\frac{2ac}{5}-\frac{b^2}{5}\right)x^4-\frac{2ab\,x^2}{7}-\frac{a^2}{9}}{x^9}$	47
risch	$\frac{-c^2x^8-\frac{2bc\,x^6}{3}+\left(-\frac{2ac}{5}-\frac{b^2}{5}\right)x^4-\frac{2ab\,x^2}{7}-\frac{a^2}{9}}{x^9}$	47
gosper	$-\frac{315c^2x^8+210bc\,x^6+126ac\,x^4+63b^2x^4+90ab\,x^2+35a^2}{315x^9}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(2*a*c+b^2)/x^5-c^2/x-2/3*b*c/x^3-1/9*a^2/x^9-2/7*a*b/x^7
```

Maxima [A]
time = 0.29, size = 46, normalized size = 0.88

$$-\frac{315\,c^2x^8+210\,bcx^6+63\,(b^2+2\,ac)x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b*x^2 + 35*
a^2)/x^9
```

Fricas [A]
time = 0.36, size = 46, normalized size = 0.88

$$-\frac{315\,c^2x^8+210\,bcx^6+63\,(b^2+2\,ac)x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b*x^2 + 35*
a^2)/x^9
```

Sympy [A]
time = 0.75, size = 49, normalized size = 0.94

$$\frac{-35a^2 - 90abx^2 - 210bcx^6 - 315c^2x^8 + x^4(-126ac - 63b^2)}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**10,x)
```

```
[Out] (-35*a**2 - 90*a*b*x**2 - 210*b*c*x**6 - 315*c**2*x**8 + x**4*(-126*a*c - 6
3*b**2))/(315*x**9)
```

Giac [A]
time = 4.62, size = 48, normalized size = 0.92

$$-\frac{315\,c^2x^8 + 210\,bcx^6 + 63\,b^2x^4 + 126\,acx^4 + 90\,abx^2 + 35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*b^2*x^4 + 126*a*c*x^4 + 90*a*b*x^2 +
 35*a^2)/x^9
```

Mupad [B]
time = 0.03, size = 46, normalized size = 0.88

$$-\frac{\frac{a^2}{9} + x^4\left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + c^2\,x^8 + \frac{2\,a\,b\,x^2}{7} + \frac{2\,b\,c\,x^6}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^10,x)
```

```
[Out] -(a^2/9 + x^4*((2*a*c)/5 + b^2/5) + c^2*x^8 + (2*a*b*x^2)/7 + (2*b*c*x^6)/3
)/x^9
```

3.838 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{11}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^2}{10x^{10}}-\frac{ab}{4x^8}-\frac{b^2+2ac}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

```
[Out] -1/10*a^2/x^10-1/4*a*b/x^8+1/6*(-2*a*c-b^2)/x^6-1/2*b*c/x^4-1/2*c^2/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1128, 712}

$$-\frac{a^2}{10x^{10}}-\frac{2ac+b^2}{6x^6}-\frac{ab}{4x^8}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -1/10*a^2/x^10 - (a*b)/(4*x^8) - (b^2 + 2*a*c)/(6*x^6) - (b*c)/(2*x^4) - c^
2/(2*x^2)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{11}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^2}{x^6}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2+2ac}{x^4}+\frac{2bc}{x^3}+\frac{c^2}{x^2}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{10x^{10}}-\frac{ab}{4x^8}-\frac{b^2+2ac}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.98

$$-\frac{6a^2+5a(3bx^2+4cx^4)+10x^4(b^2+3bcx^2+3c^2x^4)}{60x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -1/60*(6*a^2 + 5*a*(3*b*x^2 + 4*c*x^4) + 10*x^4*(b^2 + 3*b*c*x^2 + 3*c^2*x^
4))/x^10
```

Maple [A]
time = 0.02, size = 45, normalized size = 0.83

method	result	size
default	$-\frac{bc}{2x^4}-\frac{c^2}{2x^2}-\frac{ab}{4x^8}-\frac{a^2}{10x^{10}}-\frac{2ac+b^2}{6x^6}$	45
norman	$\frac{-\frac{c^2x^8}{2}-\frac{bc\,x^6}{2}+\left(-\frac{ac}{3}-\frac{b^2}{6}\right)x^4-\frac{ab\,x^2}{4}-\frac{a^2}{10}}{x^{10}}$	47
risch	$\frac{-\frac{c^2x^8}{2}-\frac{bc\,x^6}{2}+\left(-\frac{ac}{3}-\frac{b^2}{6}\right)x^4-\frac{ab\,x^2}{4}-\frac{a^2}{10}}{x^{10}}$	47
gosper	$-\frac{30c^2x^8+30bc\,x^6+20ac\,x^4+10b^2x^4+15ab\,x^2+6a^2}{60x^{10}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*c/x^4-1/2*c^2/x^2-1/4*a*b/x^8-1/10*a^2/x^10-1/6*(2*a*c+b^2)/x^6
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.85

$$-\frac{30\,c^2x^8+30\,bcx^6+10\,(b^2+2\,ac)x^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^2 + 6*a^2)
/x^10
```

Fricas [A]
time = 0.32, size = 46, normalized size = 0.85

$$-\frac{30\,c^2x^8+30\,bcx^6+10\,(b^2+2\,ac)x^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^2 + 6*a^2)
/x^10
```

Sympy [A]
time = 1.05, size = 49, normalized size = 0.91

$$\frac{-6a^2 - 15abx^2 - 30bcx^6 - 30c^2x^8 + x^4(-20ac - 10b^2)}{60x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**11,x)
```

```
[Out] (-6*a**2 - 15*a*b*x**2 - 30*b*c*x**6 - 30*c**2*x**8 + x**4*(-20*a*c - 10*b*
*2))/(60*x**10)
```

Giac [A]
time = 3.85, size = 48, normalized size = 0.89

$$-\frac{30\,c^2x^8 + 30\,bcx^6 + 10\,b^2x^4 + 20\,acx^4 + 15\,abx^2 + 6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*b^2*x^4 + 20*a*c*x^4 + 15*a*b*x^2 + 6*a
^2)/x^10
```

Mupad [B]
time = 4.12, size = 47, normalized size = 0.87

$$-\frac{\frac{a^2}{10} + x^4\left(\frac{b^2}{6} + \frac{a\,c}{3}\right) + \frac{c^2\,x^8}{2} + \frac{a\,b\,x^2}{4} + \frac{b\,c\,x^6}{2}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^11,x)
```

```
[Out] -(a^2/10 + x^4*((a*c)/3 + b^2/6) + (c^2*x^8)/2 + (a*b*x^2)/4 + (b*c*x^6)/2)
/x^10
```

3.839 $\int \frac{(a+bx^2+cx^4)^2}{x^{12}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^2}{11x^{11}}-\frac{2ab}{9x^9}-\frac{b^2+2ac}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

```
[Out] -1/11*a^2/x^11-2/9*a*b/x^9+1/7*(-2*a*c-b^2)/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1122}

$$-\frac{a^2}{11x^{11}}-\frac{2ac+b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -1/11*a^2/x^11 - (2*a*b)/(9*x^9) - (b^2 + 2*a*c)/(7*x^7) - (2*b*c)/(5*x^5)
- c^2/(3*x^3)
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{x^{12}}\,dx &= \int \left(\frac{a^2}{x^{12}}+\frac{2ab}{x^{10}}+\frac{b^2+2ac}{x^8}+\frac{2bc}{x^6}+\frac{c^2}{x^4}\right)dx\\ &= -\frac{a^2}{11x^{11}}-\frac{2ab}{9x^9}-\frac{b^2+2ac}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 1.04

$$-\frac{a^2}{11x^{11}}-\frac{2ab}{9x^9}+\frac{-b^2-2ac}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -1/11*a^2/x^11 - (2*a*b)/(9*x^9) + (-b^2 - 2*a*c)/(7*x^7) - (2*b*c)/(5*x^5)
 - c^2/(3*x^3)
```

Maple [A]
time = 0.02, size = 45, normalized size = 0.83

method	result	size
default	$-\frac{a^2}{11x^{11}}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}-\frac{2ab}{9x^9}-\frac{2ac+b^2}{7x^7}$	45
norman	$\frac{-\frac{c^2x^8}{3}-\frac{2bc\,x^6}{5}+\left(-\frac{2ac}{7}-\frac{b^2}{7}\right)x^4-\frac{2ab\,x^2}{9}-\frac{a^2}{11}}{x^{11}}$	47
risch	$\frac{-\frac{c^2x^8}{3}-\frac{2bc\,x^6}{5}+\left(-\frac{2ac}{7}-\frac{b^2}{7}\right)x^4-\frac{2ab\,x^2}{9}-\frac{a^2}{11}}{x^{11}}$	47
gosper	$-\frac{1155c^2x^8+1386bc\,x^6+990ac\,x^4+495b^2x^4+770ab\,x^2+315a^2}{3465x^{11}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*a^2/x^11-2/5*b*c/x^5-1/3*c^2/x^3-2/9*a*b/x^9-1/7*(2*a*c+b^2)/x^7
```

Maxima [A]
time = 0.31, size = 46, normalized size = 0.85

$$-\frac{1155\,c^2x^8+1386\,bcx^6+495\,(b^2+2\,ac)x^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2
+ 315*a^2)/x^11
```

Fricas [A]
time = 0.36, size = 46, normalized size = 0.85

$$-\frac{1155\,c^2x^8+1386\,bcx^6+495\,(b^2+2\,ac)x^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2
+ 315*a^2)/x^11
```

Sympy [A]
time = 1.21, size = 49, normalized size = 0.91

$$\frac{-315a^2 - 770abx^2 - 1386bcx^6 - 1155c^2x^8 + x^4(-990ac - 495b^2)}{3465x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**12,x)
```

```
[Out] (-315*a**2 - 770*a*b*x**2 - 1386*b*c*x**6 - 1155*c**2*x**8 + x**4*(-990*a*c
 - 495*b**2))/(3465*x**11)
```

Giac [A]
time = 4.94, size = 48, normalized size = 0.89

$$-\frac{1155\,c^2x^8 + 1386\,bcx^6 + 495\,b^2x^4 + 990\,acx^4 + 770\,abx^2 + 315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*b^2*x^4 + 990*a*c*x^4 + 770*a*b*
x^2 + 315*a^2)/x^11
```

Mupad [B]
time = 4.16, size = 47, normalized size = 0.87

$$-\frac{\frac{a^2}{11} + x^4\left(\frac{b^2}{7} + \frac{2\,a\,c}{7}\right) + \frac{c^2\,x^8}{3} + \frac{2\,a\,b\,x^2}{9} + \frac{2\,b\,c\,x^6}{5}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^12,x)
```

```
[Out] -(a^2/11 + x^4*((2*a*c)/7 + b^2/7) + (c^2*x^8)/3 + (2*a*b*x^2)/9 + (2*b*c*x
^6)/5)/x^11
```

3.840 $\int \frac{(a+bx^2+cx^4)^2}{x^{13}} dx$

Optimal. Leaf size=54

$$-\frac{a^2}{12x^{12}}-\frac{ab}{5x^{10}}-\frac{b^2+2ac}{8x^8}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

```
[Out] -1/12*a^2/x^12-1/5*a*b/x^10+1/8*(-2*a*c-b^2)/x^8-1/3*b*c/x^6-1/4*c^2/x^4
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1128, 712}

$$-\frac{a^2}{12x^{12}}-\frac{2ac+b^2}{8x^8}-\frac{ab}{5x^{10}}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^13,x]
```

```
[Out] -1/12*a^2/x^12 - (a*b)/(5*x^10) - (b^2 + 2*a*c)/(8*x^8) - (b*c)/(3*x^6) - c
^2/(4*x^4)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2+cx^4)^2}{x^{13}} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^2}{x^7} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2+2ac}{x^5}+\frac{2bc}{x^4}+\frac{c^2}{x^3}\right) dx, x, x^2\right) \\
&= -\frac{a^2}{12x^{12}}-\frac{ab}{5x^{10}}-\frac{b^2+2ac}{8x^8}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.93

$$-\frac{10a^2+24abx^2+15b^2x^4+30acx^4+40bcx^6+30c^2x^8}{120x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^13,x]
```

```
[Out] -1/120*(10*a^2 + 24*a*b*x^2 + 15*b^2*x^4 + 30*a*c*x^4 + 40*b*c*x^6 + 30*c^2
*x^8)/x^12
```

Maple [A]
time = 0.02, size = 45, normalized size = 0.83

method	result	size
default	$-\frac{c^2}{4x^4}-\frac{a^2}{12x^{12}}-\frac{2ac+b^2}{8x^8}-\frac{ab}{5x^{10}}-\frac{bc}{3x^6}$	45
norman	$\frac{-\frac{c^2x^8}{4}-\frac{bc\,x^6}{3}+\left(-\frac{ac}{4}-\frac{b^2}{8}\right)x^4-\frac{ab\,x^2}{5}-\frac{a^2}{12}}{x^{12}}$	47
risch	$\frac{-\frac{c^2x^8}{4}-\frac{bc\,x^6}{3}+\left(-\frac{ac}{4}-\frac{b^2}{8}\right)x^4-\frac{ab\,x^2}{5}-\frac{a^2}{12}}{x^{12}}$	47
gosper	$-\frac{30c^2x^8+40bc\,x^6+30ac\,x^4+15b^2x^4+24ab\,x^2+10a^2}{120x^{12}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*c^2/x^4-1/12*a^2/x^12-1/8*(2*a*c+b^2)/x^8-1/5*a*b/x^10-1/3*b*c/x^6
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.85

$$-\frac{30\,c^2x^8+40\,bcx^6+15\,(b^2+2\,ac)x^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="maxima")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x^2 + 10*a^
2)/x^12
```

Fricas [A]
time = 0.33, size = 46, normalized size = 0.85

$$-\frac{30\,c^2x^8+40\,bcx^6+15\,(b^2+2\,ac)x^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x^2 + 10*a^
2)/x^12
```

Sympy [A]
time = 1.48, size = 49, normalized size = 0.91

$$\frac{-10a^2 - 24abx^2 - 40bcx^6 - 30c^2x^8 + x^4(-30ac - 15b^2)}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**13,x)
```

```
[Out] (-10*a**2 - 24*a*b*x**2 - 40*b*c*x**6 - 30*c**2*x**8 + x**4*(-30*a*c - 15*b
**2))/(120*x**12)
```

Giac [A]
time = 4.01, size = 48, normalized size = 0.89

$$-\frac{30\,c^2x^8 + 40\,bcx^6 + 15\,b^2x^4 + 30\,acx^4 + 24\,abx^2 + 10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="giac")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*b^2*x^4 + 30*a*c*x^4 + 24*a*b*x^2 + 10
*a^2)/x^12
```

Mupad [B]
time = 4.16, size = 47, normalized size = 0.87

$$-\frac{\frac{a^2}{12} + x^4\left(\frac{b^2}{8} + \frac{a\,c}{4}\right) + \frac{c^2\,x^8}{4} + \frac{a\,b\,x^2}{5} + \frac{b\,c\,x^6}{3}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^13,x)
```

```
[Out] -(a^2/12 + x^4*((a*c)/4 + b^2/8) + (c^2*x^8)/4 + (a*b*x^2)/5 + (b*c*x^6)/3)
/x^12
```

3.841 $\int x^2(a+bx^2+cx^4)^3\,dx$

Optimal. Leaf size=89

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}a(b^2+ac)\,x^7+\frac{1}{9}b(b^2+6ac)\,x^9+\frac{3}{11}c(b^2+ac)\,x^{11}+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

```
[Out] 1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*(a*c+b^2)*x^7+1/9*b*(6*a*c+b^2)*x^9+3/11*c*
(a*c+b^2)*x^11+3/13*b*c^2*x^13+1/15*c^3*x^15
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1122}

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{11}cx^{11}(ac+b^2)+\frac{1}{9}bx^9(6ac+b^2)+\frac{3}{7}ax^7(ac+b^2)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2 + 6*a*c)*
x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13 + (c^3*x^15)/15
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2+cx^4\right)^3\,dx&=\int\left(a^3x^2+3a^2bx^4+3a\left(b^2+ac\right)x^6+b\left(b^2+6ac\right)x^8+3c\left(b^2+ac\right)x^{10}+3bc^2x\right.\\&=\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}a\left(b^2+ac\right)x^7+\frac{1}{9}b\left(b^2+6ac\right)x^9+\frac{3}{11}c\left(b^2+ac\right)x^{11}+\frac{3}{13}bc\end{aligned}$$

Mathematica [A]
time = 0.01, size = 89, normalized size = 1.00

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}a(b^2+ac)\,x^7+\frac{1}{9}b(b^2+6ac)\,x^9+\frac{3}{11}c(b^2+ac)\,x^{11}+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2 + 6*a*c)*
x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13 + (c^3*x^15)/15
```

Maple [A]
time = 0.05, size = 111, normalized size = 1.25

method	result
norman	$\frac{a^3x^3}{3}+\frac{3a^2b\,x^5}{5}+\left(\frac{3}{7}a^2c+\frac{3}{7}a\,b^2\right)x^7+\left(\frac{2}{3}abc+\frac{1}{9}b^3\right)x^9+\left(\frac{3}{11}c^2a+\frac{3}{11}b^2c\right)x^{11}+\frac{3b\,c^2x^{13}}{13}+\frac{c^3x^{15}}{15}$
gosper	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}x^7a^2c+\frac{3}{7}a\,b^2x^7+\frac{2}{3}x^9abc+\frac{1}{9}b^3x^9+\frac{3}{11}x^{11}c^2a+\frac{3}{11}x^{11}b^2c+\frac{3}{13}b\,c^2x^{13}+\frac{1}{15}c^3x^1$
risch	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}x^7a^2c+\frac{3}{7}a\,b^2x^7+\frac{2}{3}x^9abc+\frac{1}{9}b^3x^9+\frac{3}{11}x^{11}c^2a+\frac{3}{11}x^{11}b^2c+\frac{3}{13}b\,c^2x^{13}+\frac{1}{15}c^3x^1$
default	$\frac{c^3x^{15}}{15}+\frac{3b\,c^2x^{13}}{13}+\frac{\left(c^2a+2b^2c+c\left(2ac+b^2\right)\right)x^{11}}{11}+\frac{\left(4abc+b\left(2ac+b^2\right)\right)x^9}{9}+\frac{\left(a\left(2ac+b^2\right)+2a\,b^2+a^2c\right)x^7}{7}+\frac{3a^2b\,x^5}{5}+\frac{a^3x^3}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*c^3*x^15+3/13*b*c^2*x^13+1/11*(c^2*a+2*b^2*c+c*(2*a*c+b^2))*x^11+1/9*(
4*a*b*c+b*(2*a*c+b^2))*x^9+1/7*(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*x^7+3/5*a^2*b*
x^5+1/3*a^3*x^3
```

Maxima [A]
time = 0.28, size = 81, normalized size = 0.91

$$\frac{1}{15}\,c^3x^{15}+\frac{3}{13}\,bc^2x^{13}+\frac{3}{11}\,(b^2c+ac^2)x^{11}+\frac{1}{9}\,(b^3+6\,abc)x^9+\frac{3}{5}\,a^2bx^5+\frac{3}{7}\,(ab^2+a^2c)x^7+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*(b^2*c + a*c^2)*x^11 + 1/9*(b^3 + 6*
a*b*c)*x^9 + 3/5*a^2*b*x^5 + 3/7*(a*b^2 + a^2*c)*x^7 + 1/3*a^3*x^3
```

Fricas [A]
time = 0.32, size = 81, normalized size = 0.91

$$\frac{1}{15}\,c^3x^{15}+\frac{3}{13}\,bc^2x^{13}+\frac{3}{11}\,(b^2c+ac^2)x^{11}+\frac{1}{9}\,(b^3+6\,abc)x^9+\frac{3}{5}\,a^2bx^5+\frac{3}{7}\,(ab^2+a^2c)x^7+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*(b^2*c + a*c^2)*x^11 + 1/9*(b^3 + 6*
a*b*c)*x^9 + 3/5*a^2*b*x^5 + 3/7*(a*b^2 + a^2*c)*x^7 + 1/3*a^3*x^3
```

Sympy [A]
time = 0.01, size = 97, normalized size = 1.09

$$\frac{a^3x^3}{3}+\frac{3a^2bx^5}{5}+\frac{3bc^2x^{13}}{13}+\frac{c^3x^{15}}{15}+x^{11}\cdot\left(\frac{3ac^2}{11}+\frac{3b^2c}{11}\right)+x^9\cdot\left(\frac{2abc}{3}+\frac{b^3}{9}\right)+x^7\cdot\left(\frac{3a^2c}{7}+\frac{3ab^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*b*c**2*x**13/13 + c**3*x**15/15 + x**11*(
3*a*c**2/11 + 3*b**2*c/11) + x**9*(2*a*b*c/3 + b**3/9) + x**7*(3*a**2*c/7 +
 3*a*b**2/7)
```

Giac [A]
time = 5.11, size = 87, normalized size = 0.98

$$\frac{1}{15}\,c^3x^{15}+\frac{3}{13}\,bc^2x^{13}+\frac{3}{11}\,b^2cx^{11}+\frac{3}{11}\,ac^2x^{11}+\frac{1}{9}\,b^3x^9+\frac{2}{3}\,abcx^9+\frac{3}{7}\,ab^2x^7+\frac{3}{7}\,a^2cx^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*b^2*c*x^11 + 3/11*a*c^2*x^11 + 1/9*b
^3*x^9 + 2/3*a*b*c*x^9 + 3/7*a*b^2*x^7 + 3/7*a^2*c*x^7 + 3/5*a^2*b*x^5 + 1/
3*a^3*x^3
```

Mupad [B]
time = 0.03, size = 76, normalized size = 0.85

$$x^9\left(\frac{b^3}{9}+\frac{2\,a\,c\,b}{3}\right)+\frac{a^3\,x^3}{3}+\frac{c^3\,x^{15}}{15}+\frac{3\,a^2\,b\,x^5}{5}+\frac{3\,b\,c^2\,x^{13}}{13}+\frac{3\,a\,x^7\,\left(b^2+a\,c\right)}{7}+\frac{3\,c\,x^{11}\,\left(b^2+a\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^9*(b^3/9 + (2*a*b*c)/3) + (a^3*x^3)/3 + (c^3*x^15)/15 + (3*a^2*b*x^5)/5 +
 (3*b*c^2*x^13)/13 + (3*a*x^7*(a*c + b^2))/7 + (3*c*x^11*(a*c + b^2))/11
```

3.842 $\int x(a+bx^2+cx^4)^3\,dx$

Optimal. Leaf size=89

$$\frac{a^3x^2}{2}+\frac{3}{4}a^2bx^4+\frac{1}{2}a\left(b^2+ac\right)x^6+\frac{1}{8}b\left(b^2+6ac\right)x^8+\frac{3}{10}c\left(b^2+ac\right)x^{10}+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

```
[Out] 1/2*a^3*x^2+3/4*a^2*b*x^4+1/2*a*(a*c+b^2)*x^6+1/8*b*(6*a*c+b^2)*x^8+3/10*c*
(a*c+b^2)*x^10+1/4*b*c^2*x^12+1/14*c^3*x^14
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1121, 625}

$$\frac{a^3x^2}{2}+\frac{3}{4}a^2bx^4+\frac{3}{10}cx^{10}\left(ac+b^2\right)+\frac{1}{8}bx^8\left(6ac+b^2\right)+\frac{1}{2}ax^6\left(ac+b^2\right)+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + (3*a^2*b*x^4)/4 + (a*(b^2 + a*c)*x^6)/2 + (b*(b^2 + 6*a*c)*x^
8)/8 + (3*c*(b^2 + a*c)*x^10)/10 + (b*c^2*x^12)/4 + (c^3*x^14)/14
```

Rule 625

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2+cx^4\right)^3\,dx&=\frac{1}{2}\text{Subst}\left(\int\left(a+bx+cx^2\right)^3\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^3+3a^2bx+3ab^2\left(1+\frac{ac}{b^2}\right)x^2+b^3\left(1+\frac{6ac}{b^2}\right)x^3+3b^2c\left(1+\frac{ac}{b^2}\right)x\right.\right.\\&=\frac{a^3x^2}{2}+\frac{3}{4}a^2bx^4+\frac{1}{2}a\left(b^2+ac\right)x^6+\frac{1}{8}b\left(b^2+6ac\right)x^8+\frac{3}{10}c\left(b^2+ac\right)x^{10}+\frac{1}{4}bc^2x^{1}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 79, normalized size = 0.89

$$\frac{1}{280}x^2\left(140a^3+210a^2bx^2+140a\left(b^2+ac\right)x^4+35b\left(b^2+6ac\right)x^6+84c\left(b^2+ac\right)x^8+70bc^2x^{10}+20c^3x^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(140*a^3 + 210*a^2*b*x^2 + 140*a*(b^2 + a*c)*x^4 + 35*b*(b^2 + 6*a*c)*
x^6 + 84*c*(b^2 + a*c)*x^8 + 70*b*c^2*x^10 + 20*c^3*x^12))/280
```

Maple [A]
time = 0.04, size = 111, normalized size = 1.25

method	result
norman	$\frac{a^3x^2}{2}+\frac{3a^2b\,x^4}{4}+\left(\frac{1}{2}a^2c+\frac{1}{2}a\,b^2\right)x^6+\left(\frac{3}{4}abc+\frac{1}{8}b^3\right)x^8+\left(\frac{3}{10}c^2a+\frac{3}{10}b^2c\right)x^{10}+\frac{b\,c^2x^{12}}{4}+\frac{c^3x^{14}}{14}$
gosper	$\frac{1}{2}a^3x^2+\frac{3}{4}a^2b\,x^4+\frac{1}{2}x^6a^2c+\frac{1}{2}a\,b^2x^6+\frac{3}{4}x^8abc+\frac{1}{8}b^3x^8+\frac{3}{10}x^{10}c^2a+\frac{3}{10}x^{10}b^2c+\frac{1}{4}b\,c^2x^{12}+\frac{1}{14}c^3x$
risch	$\frac{1}{2}a^3x^2+\frac{3}{4}a^2b\,x^4+\frac{1}{2}x^6a^2c+\frac{1}{2}a\,b^2x^6+\frac{3}{4}x^8abc+\frac{1}{8}b^3x^8+\frac{3}{10}x^{10}c^2a+\frac{3}{10}x^{10}b^2c+\frac{1}{4}b\,c^2x^{12}+\frac{1}{14}c^3x$
default	$\frac{c^3x^{14}}{14}+\frac{b\,c^2x^{12}}{4}+\frac{\left(c^2a+2b^2c+c\left(2ac+b^2\right)\right)x^{10}}{10}+\frac{\left(4abc+b\left(2ac+b^2\right)\right)x^8}{8}+\frac{\left(a\left(2ac+b^2\right)+2a\,b^2+a^2c\right)x^6}{6}+\frac{3a^2b\,x^4}{4}+\frac{a^3x^2}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*c^3*x^14+1/4*b*c^2*x^12+1/10*(c^2*a+2*b^2*c+c*(2*a*c+b^2))*x^10+1/8*(4
*a*b*c+b*(2*a*c+b^2))*x^8+1/6*(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*x^6+3/4*a^2*b*x
^4+1/2*a^3*x^2
```

Maxima [A]
time = 0.27, size = 81, normalized size = 0.91

$$\frac{1}{14}\,c^3x^{14}+\frac{1}{4}\,bc^2x^{12}+\frac{3}{10}\,\left(b^2c+ac^2\right)x^{10}+\frac{1}{8}\,\left(b^3+6\,abc\right)x^8+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,\left(ab^2+a^2c\right)x^6+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*(b^2*c + a*c^2)*x^10 + 1/8*(b^3 + 6*a
*b*c)*x^8 + 3/4*a^2*b*x^4 + 1/2*(a*b^2 + a^2*c)*x^6 + 1/2*a^3*x^2
```

Fricas [A]
time = 0.35, size = 81, normalized size = 0.91

$$\frac{1}{14}\,c^3x^{14}+\frac{1}{4}\,bc^2x^{12}+\frac{3}{10}\,\left(b^2c+ac^2\right)x^{10}+\frac{1}{8}\,\left(b^3+6\,abc\right)x^8+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,\left(ab^2+a^2c\right)x^6+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*(b^2*c + a*c^2)*x^10 + 1/8*(b^3 + 6*a
*b*c)*x^8 + 3/4*a^2*b*x^4 + 1/2*(a*b^2 + a^2*c)*x^6 + 1/2*a^3*x^2
```

Sympy [A]
time = 0.02, size = 92, normalized size = 1.03

$$\frac{a^3x^2}{2}+\frac{3a^2bx^4}{4}+\frac{bc^2x^{12}}{4}+\frac{c^3x^{14}}{14}+x^{10}\cdot\left(\frac{3ac^2}{10}+\frac{3b^2c}{10}\right)+x^8\cdot\left(\frac{3abc}{4}+\frac{b^3}{8}\right)+x^6\left(\frac{a^2c}{2}+\frac{ab^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x**2/2 + 3*a**2*b*x**4/4 + b*c**2*x**12/4 + c**3*x**14/14 + x**10*(3*a
*c**2/10 + 3*b**2*c/10) + x**8*(3*a*b*c/4 + b**3/8) + x**6*(a**2*c/2 + a*b*
*2/2)
```

Giac [A]
time = 3.45, size = 87, normalized size = 0.98

$$\frac{1}{14}\,c^3x^{14}+\frac{1}{4}\,bc^2x^{12}+\frac{3}{10}\,b^2cx^{10}+\frac{3}{10}\,ac^2x^{10}+\frac{1}{8}\,b^3x^8+\frac{3}{4}\,abcx^8+\frac{1}{2}\,ab^2x^6+\frac{1}{2}\,a^2cx^6+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*b^2*c*x^10 + 3/10*a*c^2*x^10 + 1/8*b^
3*x^8 + 3/4*a*b*c*x^8 + 1/2*a*b^2*x^6 + 1/2*a^2*c*x^6 + 3/4*a^2*b*x^4 + 1/2
*a^3*x^2
```

Mupad [B]
time = 0.03, size = 76, normalized size = 0.85

$$x^8\left(\frac{b^3}{8}+\frac{3\,a\,c\,b}{4}\right)+\frac{a^3\,x^2}{2}+\frac{c^3\,x^{14}}{14}+\frac{3\,a^2\,b\,x^4}{4}+\frac{b\,c^2\,x^{12}}{4}+\frac{a\,x^6\,\left(b^2+a\,c\right)}{2}+\frac{3\,c\,x^{10}\,\left(b^2+a\,c\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^8*(b^3/8 + (3*a*b*c)/4) + (a^3*x^2)/2 + (c^3*x^14)/14 + (3*a^2*b*x^4)/4 +
 (b*c^2*x^12)/4 + (a*x^6*(a*c + b^2))/2 + (3*c*x^10*(a*c + b^2))/10
```

3.843 $\int (a+bx^2+cx^4)^3 \, dx$

Optimal. Leaf size=81

$$a^3x+a^2bx^3+\frac{3}{5}a\left(b^2+ac\right)x^5+\frac{1}{7}b\left(b^2+6ac\right)x^7+\frac{1}{3}c\left(b^2+ac\right)x^9+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

```
[Out] a^3*x+a^2*b*x^3+3/5*a*(a*c+b^2)*x^5+1/7*b*(6*a*c+b^2)*x^7+1/3*c*(a*c+b^2)*x
^9+3/11*b*c^2*x^11+1/13*c^3*x^13
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1104}

$$a^3x+a^2bx^3+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c*
(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2+cx^4\right)^3 \, dx &= \int \left(a^3+3a^2bx^2+3ab^2\left(1+\frac{ac}{b^2}\right)x^4+b^3\left(1+\frac{6ac}{b^2}\right)x^6+3b^2c\left(1+\frac{ac}{b^2}\right)x^8+3bc^2\right.\\ &= a^3x+a^2bx^3+\frac{3}{5}a\left(b^2+ac\right)x^5+\frac{1}{7}b\left(b^2+6ac\right)x^7+\frac{1}{3}c\left(b^2+ac\right)x^9+\frac{3}{11}bc^2x^{11}+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 1.00

$$a^3x+a^2bx^3+\frac{3}{5}a\left(b^2+ac\right)x^5+\frac{1}{7}b\left(b^2+6ac\right)x^7+\frac{1}{3}c\left(b^2+ac\right)x^9+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c*
(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Maple [A]
time = 0.01, size = 107, normalized size = 1.32

method	result	s
norman	$\frac{c^3x^{13}}{13}+\frac{3b\,c^2x^{11}}{11}+\left(\frac{1}{3}c^2a+\frac{1}{3}b^2c\right)x^9+\left(\frac{6}{7}abc+\frac{1}{7}b^3\right)x^7+\left(\frac{3}{5}a^2c+\frac{3}{5}a\,b^2\right)x^5+a^2b\,x^3+a^3x$	8
gosper	$\frac{1}{13}c^3x^{13}+\frac{3}{11}b\,c^2x^{11}+\frac{1}{3}x^9c^2a+\frac{1}{3}x^9b^2c+\frac{6}{7}x^7abc+\frac{1}{7}b^3x^7+\frac{3}{5}x^5a^2c+\frac{3}{5}a\,b^2x^5+a^2b\,x^3+a^3x$	8
risch	$\frac{1}{13}c^3x^{13}+\frac{3}{11}b\,c^2x^{11}+\frac{1}{3}x^9c^2a+\frac{1}{3}x^9b^2c+\frac{6}{7}x^7abc+\frac{1}{7}b^3x^7+\frac{3}{5}x^5a^2c+\frac{3}{5}a\,b^2x^5+a^2b\,x^3+a^3x$	8
default	$\frac{c^3x^{13}}{13}+\frac{3b\,c^2x^{11}}{11}+\frac{(c^2a+2b^2c+c(2ac+b^2))x^9}{9}+\frac{(4abc+b(2ac+b^2))x^7}{7}+\frac{(a(2ac+b^2)+2a\,b^2+a^2c)x^5}{5}+a^2b\,x^3+a^3x$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c^3*x^13+3/11*b*c^2*x^11+1/9*(c^2*a+2*b^2*c+c*(2*a*c+b^2))*x^9+1/7*(4*
a*b*c+b*(2*a*c+b^2))*x^7+1/5*(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*x^5+a^2*b*x^3+a^
3*x
```

Maxima [A]
time = 0.27, size = 85, normalized size = 1.05

$$\frac{1}{13}\,c^3x^{13}+\frac{3}{11}\,bc^2x^{11}+\frac{1}{3}\,b^2cx^9+\frac{1}{7}\,b^3x^7+a^3x+\frac{1}{5}\,\left(3\,cx^5+5\,bx^3\right)a^2+\frac{1}{105}\,\left(35\,c^2x^9+90\,bcx^7+63\,b^2x^5\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/7*b^3*x^7 + a^3*x + 1/5
*(3*c*x^5 + 5*b*x^3)*a^2 + 1/105*(35*c^2*x^9 + 90*b*c*x^7 + 63*b^2*x^5)*a
```

Fricas [A]
time = 0.34, size = 77, normalized size = 0.95

$$\frac{1}{13}\,c^3x^{13}+\frac{3}{11}\,bc^2x^{11}+\frac{1}{3}\,\left(b^2c+ac^2\right)x^9+\frac{1}{7}\,\left(b^3+6\,abc\right)x^7+a^2bx^3+\frac{3}{5}\,\left(ab^2+a^2c\right)x^5+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*(b^2*c + a*c^2)*x^9 + 1/7*(b^3 + 6*a*
b*c)*x^7 + a^2*b*x^3 + 3/5*(a*b^2 + a^2*c)*x^5 + a^3*x
```

Sympy [A]
time = 0.01, size = 87, normalized size = 1.07

$$a^3x+a^2bx^3+\frac{3bc^2x^{11}}{11}+\frac{c^3x^{13}}{13}+x^9\left(\frac{ac^2}{3}+\frac{b^2c}{3}\right)+x^7\cdot\left(\frac{6abc}{7}+\frac{b^3}{7}\right)+x^5\cdot\left(\frac{3a^2c}{5}+\frac{3ab^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*b*x**3 + 3*b*c**2*x**11/11 + c**3*x**13/13 + x**9*(a*c**2/3 +
 b**2*c/3) + x**7*(6*a*b*c/7 + b**3/7) + x**5*(3*a**2*c/5 + 3*a*b**2/5)
```

Giac [A]
time = 5.07, size = 83, normalized size = 1.02

$$\frac{1}{13}\,c^3x^{13}+\frac{3}{11}\,bc^2x^{11}+\frac{1}{3}\,b^2cx^9+\frac{1}{3}\,ac^2x^9+\frac{1}{7}\,b^3x^7+\frac{6}{7}\,abcx^7+\frac{3}{5}\,ab^2x^5+\frac{3}{5}\,a^2cx^5+a^2bx^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/3*a*c^2*x^9 + 1/7*b^3*x
^7 + 6/7*a*b*c*x^7 + 3/5*a*b^2*x^5 + 3/5*a^2*c*x^5 + a^2*b*x^3 + a^3*x
```

Mupad [B]
time = 0.03, size = 72, normalized size = 0.89

$$x^7\left(\frac{b^3}{7}+\frac{6\,a\,c\,b}{7}\right)+a^3\,x+\frac{c^3\,x^{13}}{13}+a^2\,b\,x^3+\frac{3\,b\,c^2\,x^{11}}{11}+\frac{3\,a\,x^5\,\left(b^2+a\,c\right)}{5}+\frac{c\,x^9\,\left(b^2+a\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^7*(b^3/7 + (6*a*b*c)/7) + a^3*x + (c^3*x^13)/13 + a^2*b*x^3 + (3*b*c^2*x^
11)/11 + (3*a*x^5*(a*c + b^2))/5 + (c*x^9*(a*c + b^2))/3
```

3.844 $\int \frac{(a+bx^2+cx^4)^3}{x} \, dx$

Optimal. Leaf size=85

$$\frac{3}{2}a^2bx^2+\frac{3}{4}a\left(b^2+ac\right)x^4+\frac{1}{6}b\left(b^2+6ac\right)x^6+\frac{3}{8}c\left(b^2+ac\right)x^8+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}+a^3\log(x)$$

```
[Out] 3/2*a^2*b*x^2+3/4*a*(a*c+b^2)*x^4+1/6*b*(6*a*c+b^2)*x^6+3/8*c*(a*c+b^2)*x^8
+3/10*b*c^2*x^10+1/12*c^3*x^12+a^3*ln(x)
```

Rubi [A]

time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1128, 712}

$$a^3\log(x)+\frac{3}{2}a^2bx^2+\frac{3}{8}cx^8\left(ac+b^2\right)+\frac{1}{6}bx^6\left(6ac+b^2\right)+\frac{3}{4}ax^4\left(ac+b^2\right)+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)/6 + (3*c*
(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12 + a^3*Log[x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^3}{x}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(3a^2b+\frac{a^3}{x}+3a\left(b^2+ac\right)x+b\left(b^2+6ac\right)x^2+3c\left(b^2+ac\right)x^3+3bc^2x\right.\right.$$

$$= \frac{3}{2}a^2bx^2+\frac{3}{4}a\left(b^2+ac\right)x^4+\frac{1}{6}b\left(b^2+6ac\right)x^6+\frac{3}{8}c\left(b^2+ac\right)x^8+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}$$

Mathematica [A]
time = 0.02, size = 85, normalized size = 1.00

$$\frac{3}{2}a^2bx^2 + \frac{3}{4}a(b^2+ac)\,x^4 + \frac{1}{6}b(b^2+6ac)\,x^6 + \frac{3}{8}c(b^2+ac)\,x^8 + \frac{3}{10}bc^2x^{10} + \frac{c^3x^{12}}{12} + a^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)/6 + (3*c*
(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12 + a^3*Log[x]
```

Maple [A]
time = 0.04, size = 85, normalized size = 1.00

method	result
norman	$\left(\frac{3}{4}a^2c+\frac{3}{4}a\,b^2\right)x^4+\left(\frac{3}{8}c^2a+\frac{3}{8}b^2c\right)x^8+\left(abc+\frac{1}{6}b^3\right)x^6+\frac{c^3x^{12}}{12}+\frac{3a^2b\,x^2}{2}+\frac{3b\,c^2x^{10}}{10}+a^3\ln\left(x\right)$
default	$\frac{c^3x^{12}}{12}+\frac{3b\,c^2x^{10}}{10}+\frac{3a\,c^2x^8}{8}+\frac{3b^2c\,x^8}{8}+abc\,x^6+\frac{b^3x^6}{6}+\frac{3a^2c\,x^4}{4}+\frac{3a\,b^2x^4}{4}+\frac{3a^2b\,x^2}{2}+a^3\ln\left(x\right)$
risch	$\frac{3a^2b\,x^2}{2}+abc\,x^6+\frac{3a\,b^2x^4}{4}+\frac{3a\,c^2x^8}{8}-\frac{a\,b^4}{8c^2}+\frac{b^6}{120c^3}+\frac{c^3x^{12}}{12}+\frac{3a^2c\,x^4}{4}+\frac{3a^2b^2}{4c}+\frac{3b\,c^2x^{10}}{10}+\frac{3b^2c\,x^8}{8}+\frac{b^3x^6}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*c^3*x^12+3/10*b*c^2*x^10+3/8*a*c^2*x^8+3/8*b^2*c*x^8+a*b*c*x^6+1/6*b^3
*x^6+3/4*a^2*c*x^4+3/4*a*b^2*x^4+3/2*a^2*b*x^2+a^3*ln(x)
```

Maxima [A]
time = 0.28, size = 82, normalized size = 0.96

$$\frac{1}{12}\,c^3x^{12} + \frac{3}{10}\,bc^2x^{10} + \frac{3}{8}\,(b^2c+ac^2)x^8 + \frac{1}{6}\,(b^3+6\,abc)x^6 + \frac{3}{2}\,a^2bx^2 + \frac{3}{4}\,(ab^2+a^2c)x^4 + \frac{1}{2}\,a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(b^3 + 6*a*
b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + 1/2*a^3*log(x^2)
```

Fricas [A]
time = 0.33, size = 79, normalized size = 0.93

$$\frac{1}{12}\,c^3x^{12} + \frac{3}{10}\,bc^2x^{10} + \frac{3}{8}\,(b^2c+ac^2)x^8 + \frac{1}{6}\,(b^3+6\,abc)x^6 + \frac{3}{2}\,a^2bx^2 + \frac{3}{4}\,(ab^2+a^2c)x^4 + a^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(b^3 + 6*a*
b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + a^3*log(x)
```

Sympy [A]
time = 0.06, size = 92, normalized size = 1.08

$$a^3 \log (x) + \frac{3a^2bx^2}{2} + \frac{3bc^2x^{10}}{10} + \frac{c^3x^{12}}{12} + x^8 \cdot \left(\frac{3ac^2}{8} + \frac{3b^2c}{8}\right) + x^6 \left(abc + \frac{b^3}{6}\right) + x^4 \cdot \left(\frac{3a^2c}{4} + \frac{3ab^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] a**3*log(x) + 3*a**2*b*x**2/2 + 3*b*c**2*x**10/10 + c**3*x**12/12 + x**8*(3
*a*c**2/8 + 3*b**2*c/8) + x**6*(a*b*c + b**3/6) + x**4*(3*a**2*c/4 + 3*a*b*
*2/4)
```

Giac [A]
time = 5.01, size = 87, normalized size = 1.02

$$\frac{1}{12}\,c^3x^{12} + \frac{3}{10}\,bc^2x^{10} + \frac{3}{8}\,b^2cx^8 + \frac{3}{8}\,ac^2x^8 + \frac{1}{6}\,b^3x^6 + abcx^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{4}\,a^2cx^4 + \frac{3}{2}\,a^2bx^2 + \frac{1}{2}\,a^3 \log \left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*b^2*c*x^8 + 3/8*a*c^2*x^8 + 1/6*b^3*x
^6 + a*b*c*x^6 + 3/4*a*b^2*x^4 + 3/4*a^2*c*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*lo
g(x^2)
```

Mupad [B]
time = 0.03, size = 73, normalized size = 0.86

$$a^3\,\ln (x) + x^6\,\left(\frac{b^3}{6} + a\,c\,b\right) + \frac{c^3\,x^{12}}{12} + \frac{3\,a^2\,b\,x^2}{2} + \frac{3\,b\,c^2\,x^{10}}{10} + \frac{3\,a\,x^4\,\left(b^2 + a\,c\right)}{4} + \frac{3\,c\,x^8\,\left(b^2 + a\,c\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x,x)
```

```
[Out] a^3*log(x) + x^6*(b^3/6 + a*b*c) + (c^3*x^12)/12 + (3*a^2*b*x^2)/2 + (3*b*c
^2*x^10)/10 + (3*a*x^4*(a*c + b^2))/4 + (3*c*x^8*(a*c + b^2))/8
```

3.845 $\int \frac{(a+bx^2+cx^4)^3}{x^2} dx$

Optimal. Leaf size=80

$$-\frac{a^3}{x}+3a^2bx+a\left(b^2+ac\right)x^3+\frac{1}{5}b\left(b^2+6ac\right)x^5+\frac{3}{7}c\left(b^2+ac\right)x^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

```
[Out] -a^3/x+3*a^2*b*x+a*(a*c+b^2)*x^3+1/5*b*(6*a*c+b^2)*x^5+3/7*c*(a*c+b^2)*x^7+
1/3*b*c^2*x^9+1/11*c^3*x^11
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1122}

$$-\frac{a^3}{x}+3a^2bx+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/5 + (3*c*(
b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x^2}\,dx=\int \left(3a^2b+\frac{a^3}{x^2}+3a\left(b^2+ac\right)x^2+b\left(b^2+6ac\right)x^4+3c\left(b^2+ac\right)x^6+3bc^2x^8+c^3x$$

$$=-\frac{a^3}{x}+3a^2bx+a\left(b^2+ac\right)x^3+\frac{1}{5}b\left(b^2+6ac\right)x^5+\frac{3}{7}c\left(b^2+ac\right)x^7+\frac{1}{3}bc^2x^9+\frac{c^3x}{1}$$

Mathematica [A]
time = 0.02, size = 80, normalized size = 1.00

$$-\frac{a^3}{x}+3a^2bx+a\left(b^2+ac\right)x^3+\frac{1}{5}b\left(b^2+6ac\right)x^5+\frac{3}{7}c\left(b^2+ac\right)x^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^2,x]
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/5 + (3*c*(
b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Maple [A]
time = 0.04, size = 84, normalized size = 1.05

method	result	size
default	$\frac{c^3x^{11}}{11}+\frac{b\,c^2x^9}{3}+\frac{3a\,c^2x^7}{7}+\frac{3b^2c\,x^7}{7}+\frac{6abc\,x^5}{5}+\frac{b^3x^5}{5}+a^2c\,x^3+a\,b^2x^3+3a^2bx-\frac{a^3}{x}$	84
norman	$\frac{\frac{c^3x^{12}}{11}+\frac{b\,c^2x^{10}}{3}+\left(\frac{3}{7}c^2a+\frac{3}{7}b^2c\right)x^8+\left(\frac{6}{5}abc+\frac{1}{5}b^3\right)x^6+\left(a^2c+a\,b^2\right)x^4+3a^2b\,x^2-a^3}{x}$	84
risch	$\frac{c^3x^{11}}{11}+\frac{b\,c^2x^9}{3}+\frac{3a\,c^2x^7}{7}+\frac{3b^2c\,x^7}{7}+\frac{6abc\,x^5}{5}+\frac{b^3x^5}{5}+a^2c\,x^3+a\,b^2x^3+3a^2bx-\frac{a^3}{x}$	84
gosper	$-\frac{-105c^3x^{12}-385b\,c^2x^{10}-495a\,c^2x^8-495b^2c\,x^8-1386abc\,x^6-231b^3x^6-1155a^2c\,x^4-1155a\,b^2x^4-3465a^2b\,x^2+1155a^3}{1155x}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
[Out] 1/11*c^3*x^11+1/3*b*c^2*x^9+3/7*a*c^2*x^7+3/7*b^2*c*x^7+6/5*a*b*c*x^5+1/5*b
^3*x^5+a^2*c*x^3+a*b^2*x^3+3*a^2*b*x-a^3/x
```

Maxima [A]
time = 0.27, size = 78, normalized size = 0.98

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,\left(b^2c+ac^2\right)x^7+\frac{1}{5}\,\left(b^3+6\,abc\right)x^5+3\,a^2bx+\left(ab^2+a^2c\right)x^3-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*(b^2*c + a*c^2)*x^7 + 1/5*(b^3 + 6*a*b*
c)*x^5 + 3*a^2*b*x + (a*b^2 + a^2*c)*x^3 - a^3/x
```

Fricas [A]
time = 0.33, size = 83, normalized size = 1.04

$$\frac{105\,c^3x^{12}+385\,bc^2x^{10}+495\,(b^2c+ac^2)x^8+231\,(b^3+6\,abc)x^6+3465\,a^2bx^2+1155\,(ab^2+a^2c)x^4-1155\,a^3}{1155\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")
[Out] 1/1155*(105*c^3*x^12 + 385*b*c^2*x^10 + 495*(b^2*c + a*c^2)*x^8 + 231*(b^3
+ 6*a*b*c)*x^6 + 3465*a^2*b*x^2 + 1155*(a*b^2 + a^2*c)*x^4 - 1155*a^3)/x
```

Sympy [A]
time = 0.06, size = 82, normalized size = 1.02

$$-\frac{a^3}{x} + 3a^2bx + \frac{bc^2x^9}{3} + \frac{c^3x^{11}}{11} + x^7 \cdot \left(\frac{3ac^2}{7} + \frac{3b^2c}{7}\right) + x^5 \cdot \left(\frac{6abc}{5} + \frac{b^3}{5}\right) + x^3\left(a^2c + ab^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*x + b*c**2*x**9/3 + c**3*x**11/11 + x**7*(3*a*c**2/7 + 3
*b**2*c/7) + x**5*(6*a*b*c/5 + b**3/5) + x**3*(a**2*c + a*b**2)
```

Giac [A]
time = 3.87, size = 83, normalized size = 1.04

$$\frac{1}{11}c^3x^{11} + \frac{1}{3}bc^2x^9 + \frac{3}{7}b^2cx^7 + \frac{3}{7}ac^2x^7 + \frac{1}{5}b^3x^5 + \frac{6}{5}abcx^5 + ab^2x^3 + a^2cx^3 + 3a^2bx - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 3/7*a*c^2*x^7 + 1/5*b^3*x^5
 + 6/5*a*b*c*x^5 + a*b^2*x^3 + a^2*c*x^3 + 3*a^2*b*x - a^3/x
```

Mupad [B]
time = 0.03, size = 73, normalized size = 0.91

$$x^5\left(\frac{b^3}{5} + \frac{6\,a\,c\,b}{5}\right) - \frac{a^3}{x} + \frac{c^3\,x^{11}}{11} + \frac{b\,c^2\,x^9}{3} + a\,x^3\,\left(b^2 + a\,c\right) + \frac{3\,c\,x^7\,\left(b^2 + a\,c\right)}{7} + 3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^2,x)
```

```
[Out] x^5*(b^3/5 + (6*a*b*c)/5) - a^3/x + (c^3*x^11)/11 + (b*c^2*x^9)/3 + a*x^3*(
a*c + b^2) + (3*c*x^7*(a*c + b^2))/7 + 3*a^2*b*x
```

3.846 $\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx$

Optimal. Leaf size=86

$$-\frac{a^3}{2x^2}+\frac{3}{2}a\left(b^2+ac\right)x^2+\frac{1}{4}b\left(b^2+6ac\right)x^4+\frac{1}{2}c\left(b^2+ac\right)x^6+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}+3a^2b\log(x)$$

```
[Out] -1/2*a^3/x^2+3/2*a*(a*c+b^2)*x^2+1/4*b*(6*a*c+b^2)*x^4+1/2*c*(a*c+b^2)*x^6+
3/8*b*c^2*x^8+1/10*c^3*x^10+3*a^2*b*ln(x)
```

Rubi [A]

time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1128, 712}

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{1}{2}cx^6\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+\frac{3}{2}ax^2\left(ac+b^2\right)+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] -1/2*a^3/x^2 + (3*a*(b^2 + a*c)*x^2)/2 + (b*(b^2 + 6*a*c)*x^4)/4 + (c*(b^2
+ a*c)*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10 + 3*a^2*b*Log[x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^3}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(3a\left(b^2+ac\right)+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b\left(b^2+6ac\right)x+3c\left(b^2+ac\right)x^2+3bc^2x^3\right.\right.\\ &= -\frac{a^3}{2x^2}+\frac{3}{2}a\left(b^2+ac\right)x^2+\frac{1}{4}b\left(b^2+6ac\right)x^4+\frac{1}{2}c\left(b^2+ac\right)x^6+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}+3a\end{aligned}$$

Mathematica [A]
time = 0.02, size = 78, normalized size = 0.91

$$\frac{1}{40}\left(-\frac{20a^3}{x^2}+60a\left(b^2+ac\right)x^2+10b\left(b^2+6ac\right)x^4+20c\left(b^2+ac\right)x^6+15bc^2x^8+4c^3x^{10}+120a^2b\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] ((-20*a^3)/x^2 + 60*a*(b^2 + a*c)*x^2 + 10*b*(b^2 + 6*a*c)*x^4 + 20*c*(b^2
+ a*c)*x^6 + 15*b*c^2*x^8 + 4*c^3*x^10 + 120*a^2*b*Log[x])/40
```

Maple [A]
time = 0.02, size = 87, normalized size = 1.01

method	result	size
norman	$\frac{\left(\frac{3}{2}a^2c+\frac{3}{2}a\,b^2\right)x^4+\left(\frac{1}{2}c^2a+\frac{1}{2}b^2c\right)x^8+\left(\frac{3}{2}abc+\frac{1}{4}b^3\right)x^6-\frac{a^3}{2}+\frac{c^3x^{12}}{10}+\frac{3b\,c^2x^{10}}{8}}{x^2}+3a^2b\ln\left(x\right)$	86
default	$\frac{c^3x^{10}}{10}+\frac{3b\,c^2x^8}{8}+\frac{a\,c^2x^6}{2}+\frac{b^2c\,x^6}{2}+\frac{3abc\,x^4}{2}+\frac{b^3x^4}{4}+\frac{3a^2c\,x^2}{2}+\frac{3a\,b^2x^2}{2}-\frac{a^3}{2x^2}+3a^2b\ln\left(x\right)$	87
risch	$\frac{c^3x^{10}}{10}+\frac{3b\,c^2x^8}{8}+\frac{a\,c^2x^6}{2}+\frac{b^2c\,x^6}{2}+\frac{3abc\,x^4}{2}+\frac{b^3x^4}{4}+\frac{3a^2c\,x^2}{2}+\frac{3a\,b^2x^2}{2}-\frac{a^3}{2x^2}+3a^2b\ln\left(x\right)$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c^3*x^10+3/8*b*c^2*x^8+1/2*a*c^2*x^6+1/2*b^2*c*x^6+3/2*a*b*c*x^4+1/4*b
^3*x^4+3/2*a^2*c*x^2+3/2*a*b^2*x^2-1/2*a^3/x^2+3*a^2*b*ln(x)
```

Maxima [A]
time = 0.27, size = 82, normalized size = 0.95

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,\left(b^2c+ac^2\right)x^6+\frac{1}{4}\,\left(b^3+6\,abc\right)x^4+\frac{3}{2}\,a^2b\log\left(x^2\right)+\frac{3}{2}\,\left(ab^2+a^2c\right)x^2-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*(b^2*c + a*c^2)*x^6 + 1/4*(b^3 + 6*a*b*
c)*x^4 + 3/2*a^2*b*log(x^2) + 3/2*(a*b^2 + a^2*c)*x^2 - 1/2*a^3/x^2
```

Fricas [A]
time = 0.37, size = 85, normalized size = 0.99

$$\frac{4\,c^3x^{12}+15\,bc^2x^{10}+20\,\left(b^2c+ac^2\right)x^8+10\,\left(b^3+6\,abc\right)x^6+120\,a^2bx^2\log\left(x\right)+60\,\left(ab^2+a^2c\right)x^4-20\,a^3}{40\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/40*(4*c^3*x^12 + 15*b*c^2*x^10 + 20*(b^2*c + a*c^2)*x^8 + 10*(b^3 + 6*a*b
*c)*x^6 + 120*a^2*b*x^2*log(x) + 60*(a*b^2 + a^2*c)*x^4 - 20*a^3)/x^2
```

Sympy [A]
time = 0.08, size = 92, normalized size = 1.07

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3bc^2x^8}{8}+\frac{c^3x^{10}}{10}+x^6\left(\frac{ac^2}{2}+\frac{b^2c}{2}\right)+x^4\cdot\left(\frac{3abc}{2}+\frac{b^3}{4}\right)+x^2\cdot\left(\frac{3a^2c}{2}+\frac{3ab^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*b*c**2*x**8/8 + c**3*x**10/10 + x**6*(
a*c**2/2 + b**2*c/2) + x**4*(3*a*b*c/2 + b**3/4) + x**2*(3*a**2*c/2 + 3*a*b
**2/2)
```

Giac [A]
time = 4.14, size = 98, normalized size = 1.14

$$\frac{1}{10}c^3x^{10}+\frac{3}{8}bc^2x^8+\frac{1}{2}b^2cx^6+\frac{1}{2}ac^2x^6+\frac{1}{4}b^3x^4+\frac{3}{2}abcx^4+\frac{3}{2}ab^2x^2+\frac{3}{2}a^2cx^2+\frac{3}{2}a^2b\log\left(x^2\right)-\frac{3\,a^2bx^2+a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/2*a*c^2*x^6 + 1/4*b^3*x^4
 + 3/2*a*b*c*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*c*x^2 + 3/2*a^2*b*log(x^2) - 1/2
*(3*a^2*b*x^2 + a^3)/x^2
```

Mupad [B]
time = 0.04, size = 75, normalized size = 0.87

$$x^4\left(\frac{b^3}{4}+\frac{3\,a\,c\,b}{2}\right)-\frac{a^3}{2\,x^2}+\frac{c^3\,x^{10}}{10}+\frac{3\,b\,c^2\,x^8}{8}+3\,a^2\,b\,\ln(x)+\frac{3\,a\,x^2\,(b^2+a\,c)}{2}+\frac{c\,x^6\,(b^2+a\,c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^3,x)
```

```
[Out] x^4*(b^3/4 + (3*a*b*c)/2) - a^3/(2*x^2) + (c^3*x^10)/10 + (3*b*c^2*x^8)/8 +
 3*a^2*b*log(x) + (3*a*x^2*(a*c + b^2))/2 + (c*x^6*(a*c + b^2))/2
```

3.847 $\int \frac{(a+bx^2+cx^4)^3}{x^4}\,dx$

Optimal. Leaf size=83

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3a\left(b^2+ac\right)x+\frac{1}{3}b\left(b^2+6ac\right)x^3+\frac{3}{5}c\left(b^2+ac\right)x^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

```
[Out] -1/3*a^3/x^3-3*a^2*b/x+3*a*(a*c+b^2)*x+1/3*b*(6*a*c+b^2)*x^3+3/5*c*(a*c+b^2
)*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1122}

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^3)/3 +
(3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^3}{x^4}\,dx &= \int \left(3a\left(b^2+ac\right)+\frac{a^3}{x^4}+\frac{3a^2b}{x^2}+b\left(b^2+6ac\right)x^2+3c\left(b^2+ac\right)x^4+3bc^2x^6+c^3x^8\right)dx\\ &= -\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3a\left(b^2+ac\right)x+\frac{1}{3}b\left(b^2+6ac\right)x^3+\frac{3}{5}c\left(b^2+ac\right)x^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 1.00

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3a\left(b^2+ac\right)x+\frac{1}{3}b\left(b^2+6ac\right)x^3+\frac{3}{5}c\left(b^2+ac\right)x^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^3)/3 +
(3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Maple [A]
time = 0.02, size = 84, normalized size = 1.01

method	result	size
default	$\frac{c^3x^9}{9}+\frac{3b\,c^2x^7}{7}+\frac{3a\,c^2x^5}{5}+\frac{3b^2c\,x^5}{5}+2abc\,x^3+\frac{b^3x^3}{3}+3a^2cx+3a\,b^2x-\frac{3a^2b}{x}-\frac{a^3}{3x^3}$	84
norman	$\frac{\frac{c^3x^{12}}{9}+\frac{3b\,c^2x^{10}}{7}+\left(\frac{3}{5}c^2a+\frac{3}{5}b^2c\right)x^8+\left(2abc+\frac{1}{3}b^3\right)x^6+\left(3a^2c+3a\,b^2\right)x^4-3a^2b\,x^2-\frac{a^3}{3}}{x^3}$	86
risch	$\frac{c^3x^9}{9}+\frac{3b\,c^2x^7}{7}+\frac{3a\,c^2x^5}{5}+\frac{3b^2c\,x^5}{5}+2abc\,x^3+\frac{b^3x^3}{3}+3a^2cx+3a\,b^2x+\frac{-3a^2b\,x^2-\frac{1}{3}a^3}{x^3}$	86
gosper	$-\frac{-35c^3x^{12}-135b\,c^2x^{10}-189a\,c^2x^8-189b^2c\,x^8-630abc\,x^6-105b^3x^6-945a^2c\,x^4-945a\,b^2x^4+945a^2b\,x^2+105a^3}{315x^3}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c^3*x^9+3/7*b*c^2*x^7+3/5*a*c^2*x^5+3/5*b^2*c*x^5+2*a*b*c*x^3+1/3*b^3*x
^3+3*a^2*c*x+3*a*b^2*x-3*a^2*b/x-1/3*a^3/x^3
```

Maxima [A]
time = 0.27, size = 80, normalized size = 0.96

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,\left(b^2c+ac^2\right)x^5+\frac{1}{3}\,\left(b^3+6\,abc\right)x^3+3\,\left(ab^2+a^2c\right)x-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*(b^2*c + a*c^2)*x^5 + 1/3*(b^3 + 6*a*b*c)
*x^3 + 3*(a*b^2 + a^2*c)*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

Fricas [A]
time = 0.33, size = 83, normalized size = 1.00

$$\frac{35\,c^3x^{12}+135\,bc^2x^{10}+189\,(b^2c+ac^2)x^8+105\,(b^3+6\,abc)x^6-945\,a^2bx^2+945\,(ab^2+a^2c)x^4-105\,a^3}{315\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/315*(35*c^3*x^12 + 135*b*c^2*x^10 + 189*(b^2*c + a*c^2)*x^8 + 105*(b^3 +
6*a*b*c)*x^6 - 945*a^2*b*x^2 + 945*(a*b^2 + a^2*c)*x^4 - 105*a^3)/x^3
```

Sympy [A]
time = 0.08, size = 90, normalized size = 1.08

$$\frac{3bc^2x^7}{7}+\frac{c^3x^9}{9}+x^5\cdot\left(\frac{3ac^2}{5}+\frac{3b^2c}{5}\right)+x^3\cdot\left(2abc+\frac{b^3}{3}\right)+x\left(3a^2c+3ab^2\right)+\frac{-a^3-9a^2bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**4,x)
```

```
[Out] 3*b*c**2*x**7/7 + c**3*x**9/9 + x**5*(3*a*c**2/5 + 3*b**2*c/5) + x**3*(2*a*
b*c + b**3/3) + x*(3*a**2*c + 3*a*b**2) + (-a**3 - 9*a**2*b*x**2)/(3*x**3)
```

Giac [A]
time = 4.17, size = 84, normalized size = 1.01

$$\frac{1}{9}c^3x^9+\frac{3}{7}bc^2x^7+\frac{3}{5}b^2cx^5+\frac{3}{5}ac^2x^5+\frac{1}{3}b^3x^3+2\,abcx^3+3\,ab^2x+3\,a^2cx-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 3/5*a*c^2*x^5 + 1/3*b^3*x^3 +
 2*a*b*c*x^3 + 3*a*b^2*x + 3*a^2*c*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

Mupad [B]
time = 0.03, size = 77, normalized size = 0.93

$$x^3\left(\frac{b^3}{3}+2\,a\,c\,b\right)-\frac{\frac{a^3}{3}+3\,b\,a^2\,x^2}{x^3}+\frac{c^3\,x^9}{9}+\frac{3\,b\,c^2\,x^7}{7}+3\,a\,x\,\left(b^2+a\,c\right)+\frac{3\,c\,x^5\,\left(b^2+a\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^4,x)
```

```
[Out] x^3*(b^3/3 + 2*a*b*c) - (a^3/3 + 3*a^2*b*x^2)/x^3 + (c^3*x^9)/9 + (3*b*c^2*
x^7)/7 + 3*a*x*(a*c + b^2) + (3*c*x^5*(a*c + b^2))/5
```

3.848 $\int \frac{x^7}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=100

$$-\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

```
[Out] -1/2*b*x^2/c^2+1/4*x^4/c+1/4*(-a*c+b^2)*ln(c*x^4+b*x^2+a)/c^3+1/2*b*(-3*a*c
+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1128, 715, 648, 632, 212, 642}

$$\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*(b*x^2)/c^2 + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+(b^2-ac)\,x}{c^2\,(a+bx+cx^2)}\right)\,dx, x, x^2\right)\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\text{Subst}\left(\int \frac{ab+(b^2-ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c^2}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}-\frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^3}+\frac{(b^2-ac)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\right.}{4c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, \right.}{2c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.93

$$
\frac{cx^2(-2b+cx^2)-\frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A]
time = 0.06, size = 105, normalized size = 1.05

method	result
default	$-\frac{-\frac{1}{2}c\,x^4+b\,x^2}{2c^2}+\frac{\frac{\left(-ac+b^2\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(ab-\frac{\left(-ac+b^2\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c^2}$
risch	$\frac{x^4}{4c}-\frac{b\,x^2}{2c^2}+\frac{b^2}{4c^3}-\frac{\ln\left(\left(\left(12a^2b\,c^2-7a\,b^3c+b^5+\sqrt{-b^2\left(4ac-b^2\right)\left(3ac-b^2\right)^2}\,b\right)x^2+2\sqrt{-b^2\left(4ac-b^2\right)\left(3ac-\right.}\right.\right.}{c(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^2*(-1/2*c*x^4+b*x^2)+1/2/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^4+b*x^2+a)+2*(
a*b-1/2*(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 313, normalized size = 3.13

$$\left[\frac{(b^2c^2-4ac^3)x^4-2(b^3c-4abc^2)x^2-(b^3-3abc)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)+(b^4-5ab^2c+4a^2c^2)\log(cx^4+bx^2+a)}{4(b^2c^3-4ac^4)},\ \frac{(b^2c^2-4ac^3)x^4-2(b^3c-4abc^2)x^2+2(b^3-3abc)\sqrt{-b^2+4ac}\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+(b^4-5ab^2c+4a^2c^2)\log(cx^4+bx^2+a)}{4(b^2c^3-4ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 - (b^3 - 3*a*b*c)
*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)
*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2
*(b^3*c - 4*a*b*c^2)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2
)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(92) = 184$.

time = 1.81, size = 391, normalized size = 3.91

$$-\frac{bx^2}{2c^2}+\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)}{3abc-b^3}\right)+\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)}{3abc-b^3}\right)+\frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c -
b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(-
b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)
/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*
a*c - b**2)) - (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c +
 b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(
x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/
(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(b*sqrt(-4*a
*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)))
/(3*a*b*c - b**3)) + x**4/(4*c)
```

Giac [A]

time = 4.96, size = 92, normalized size = 0.92

$$\frac{cx^4-2\,bx^2}{4\,c^2}+\frac{(b^2-ac)\log\left(cx^4+bx^2+a\right)}{4\,c^3}-\frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/c^3 - 1/
2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*c^3)
```

Mupad [B]

time = 4.40, size = 842, normalized size = 8.42

$$\frac{x^4}{4\,c}-\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{2\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}-\frac{b\,x^2}{2\,c^2}+\frac{b\,\mathrm{atan}\left(\frac{2\,c^4\,\left(4\,a\,c-b^2\right)\,\left(\frac{\frac{b\,\left(3\,a\,c-b^2\right)\,\left(\frac{8\,a^2\,c^4-8\,a\,b^2\,c^3}{c^4}-\frac{8\,a\,c^2\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{16\,a\,c^4-4\,b^2\,c^3}\right)}{8\,c^3\,\sqrt{4\,a\,c-b^2}}-\frac{a\,b\,\left(3\,a\,c-b^2\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{c\,\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}}{a}-x^2\,\left(\frac{\frac{b\,\left(\frac{6\,b^3\,c^3-10\,a\,b\,c^4}{c^4}+\frac{4\,b\,c^2\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{16\,a\,c^4-4\,b^2\,c^3}\right)\,\left(3\,a\,c-b^2\right)}{8\,c^3\,\sqrt{4\,a\,c-b^2}}+\frac{b^2\,\left(3\,a\,c-b^2\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{2\,c\,\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}}{a}+\frac{b\,\left(\frac{2\,a^2\,b\,c^2-3\,a\,b^3\,c+b^5}{c^4}+\frac{\left(\frac{6\,b^3\,c^3-10\,a\,b\,c^4}{c^4}+\frac{4\,b\,c^2\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{16\,a\,c^4-4\,b^2\,c^3}\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{2\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}-\frac{b^3\,{\left(3\,a\,c-b^2\right)}^2}{2\,c^4\,\left(4\,a\,c-b^2\right)}\right)}{2\,a\,\sqrt{4\,a\,c-b^2}}\right)+\frac{b\,\left(\frac{\left(\frac{8\,a^2\,c^4-8\,a\,b^2\,c^3}{c^4}-\frac{8\,a\,c^2\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{16\,a\,c^4-4\,b^2\,c^3}\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{2\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}-\frac{a^3\,c^2-2\,a^2\,b^2\,c+a\,b^4}{c^4}+\frac{a\,b^2\,{\left(3\,a\,c-b^2\right)}^2}{c^4\,\left(4\,a\,c-b^2\right)}\right)}{2\,a\,\sqrt{4\,a\,c-b^2}}\right)}{9\,a^2\,b^2\,c^2-6\,a\,b^4\,c+b^6}\right)\,\left(3\,a\,c-b^2\right)}{2\,c^3\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(a + b*x^2 + c*x^4),x)

[Out] x^4/(4*c) - (log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b*x^2)/(2*c^2) + (b*atan((2*c^4*(4*a*c - b^2)*(((b*(3*a*c - b^2)*((8*a^2*c^4 - 8*a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*b*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a - x^2*(((b*((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(3*a*c - b^2))/(8*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((b^5 + 2*a^2*b*c^2 - 3*a*b^3*c)/c^4 + (((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b^3*(3*a*c - b^2)^2)/(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*a^2*c^4 - 8*a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (a*b^4 + a^3*c^2 - 2*a^2*b^2*c)/c^4 + (a*b^2*(3*a*c - b^2)^2)/(c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^6 + 9*a^2*b^2*c^2 - 6*a*b^4*c))*(3*a*c - b^2))/(2*c^3*(4*a*c - b^2)^(1/2))

3.849 $\int \frac{x^5}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=81

$$\frac{x^2}{2c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] 1/2*x^2/c-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/2*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/
(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1128, 717, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2c}+\frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx,x,x^2\right)}{2c}\\
&= \frac{x^2}{2c}-\frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}\\
&= \frac{x^2}{2c}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^2}\\
&= \frac{x^2}{2c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 0.96

$$\frac{2cx^2+\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A]
time = 0.04, size = 83, normalized size = 1.02

method	result
default	$\frac{x^{2}}{2c}+\frac{-\frac{b\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(-a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2c}$
risch	$\frac{x^{2}}{2c}-\frac{\ln\left(\left(\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}+\sqrt{-\left(4ac-b^{2}\right)\left(2ac-b^{2}\right)^{2}}\,b\right)x^{2}+2\sqrt{-\left(4ac-b^{2}\right)\left(2ac-b^{2}\right)^{2}}\,a\right)ab}{c\left(4ac-b^{2}\right)}+\frac{\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c+1/2/c*(-1/2*b/c*ln(c*x^4+b*x^2+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.34, size = 254, normalized size = 3.14

$$\left[\frac{2\,(b^{2}c-4\,ac^{2})x^{2}-(b^{2}-2\,ac)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)-(b^{3}-4\,abc)\log\left(cx^{4}+bx^{2}+a\right)}{4\,(b^{2}c^{2}-4\,ac^{3})},\ \frac{2\,(b^{2}c-4\,ac^{2})x^{2}-2\,(b^{2}-2\,ac)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-(b^{3}-4\,abc)\log\left(cx^{4}+bx^{2}+a\right)}{4\,(b^{2}c^{2}-4\,ac^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b
```

```
*x^2 + a)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(71) = 142$.
time = 1.15, size = 316, normalized size = 3.90

$$\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*
log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(4*c**2) + sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (-a*b
- 8*a*c**2*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c
 - b**2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

Giac [A]
time = 4.70, size = 75, normalized size = 0.93

$$\frac{x^2}{2\,c}-\frac{b\log\left(cx^4+bx^2+a\right)}{4\,c^2}+\frac{(b^2-2\,ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/4*b*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]
time = 4.75, size = 655, normalized size = 8.09

$$\frac{x^2}{2\,c}+\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,b^3-8\,a\,b\,c\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}+\frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\,\left(\frac{\frac{\left(8\,a\,b+\frac{8\,a\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\,\left(2\,a\,c-b^2\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}+\frac{a\,\left(2\,b^3-8\,a\,b\,c\right)\,\left(2\,a\,c-b^2\right)}{\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}}{a}-x^2\,\left(\frac{\frac{\left(2\,a\,c-b^2\right)\,\left(\frac{4\,a\,c^3-6\,b^2\,c^2}{c^2}-\frac{4\,b\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}-\frac{b\,\left(2\,b^3-8\,a\,b\,c\right)\,\left(2\,a\,c-b^2\right)}{2\,\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}}{a}+\frac{b\,\left(\frac{\left(2\,b^3-8\,a\,b\,c\right)\,\left(\frac{4\,a\,c^3-6\,b^2\,c^2}{c^2}-\frac{4\,b\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}-\frac{b^3-a\,b\,c}{c^2}+\frac{b\,{\left(2\,a\,c-b^2\right)}^2}{2\,c^2\,\left(4\,a\,c-b^2\right)}\right)}{2\,a\,\sqrt{4\,a\,c-b^2}}\right)+\frac{b\,\left(\frac{a\,b^2}{c^2}+\frac{\left(2\,b^3-8\,a\,b\,c\right)\,\left(8\,a\,b+\frac{8\,a\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}-\frac{a\,{\left(2\,a\,c-b^2\right)}^2}{c^2\,\left(4\,a\,c-b^2\right)}\right)}{2\,a\,\sqrt{4\,a\,c-b^2}}\right)}{4\,a^2\,c^2-4\,a\,b^2\,c+b^4}\right)\,\left(2\,a\,c-b^2\right)}{2\,c^2\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^2
*c^2)) + (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c))
/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*(2
*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2))
)/a - x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8
*a*b*c))/(16*a*c^3 - 4*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2*b^3 -
 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a
+ (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8*a
*b*c))/(16*a*c^3 - 4*b^2*c^2))))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3 - a*b*c)/
c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))
) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c
))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*a*c - b^2)^2
)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a^2*c^2 - 4*a*
b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))
```

3.850 $\int \frac{x^3}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=63

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1128, 648, 632, 212, 642}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(4*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c}\\ &= \frac{\log\left(a+bx^2+cx^4\right)}{4c} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c}\\ &= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^2+cx^4\right)}{4c}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.98

$$\frac{-\frac{2b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^2 + c*x^4])/(4*c)
```

Maple [A]
time = 0.03, size = 60, normalized size = 0.95

method	result

default	$\frac{\ln(c\,x^4+b\,x^2+a)}{4c}-\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2c\sqrt{4ac-b^2}}$
risch	$\frac{\ln\left(\left(\left(-4abc+b^3+\sqrt{-b^2\,(4ac-b^2)}\,b\right)x^2+2\sqrt{-b^2\,(4ac-b^2)}\,a\right)a}{4ac-b^2}-\frac{\ln\left(\left(\left(-4abc+b^3+\sqrt{-b^2\,(4ac-b^2)}\,b\right)x^2+\right.\right.}{4c(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c-1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.36, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(b^2-4\,ac)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c-4\,ac^2)},\frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(b^2-4\,ac)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c-4\,ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4 + b*x^2
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.
time = 0.52, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(-b*
sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c +
b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b**2)/
(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

Giac [A]
time = 4.11, size = 59, normalized size = 0.94

$$-\frac{b \arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c}+\frac{\log\left(cx^4+bx^2+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
4*log(c*x^4 + b*x^2 + a)/c
```

Mupad [B]
time = 4.26, size = 118, normalized size = 1.87

$$\frac{4\,a\,c\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b^2\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^2}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4),x)
```

```
[Out] (4*a*c*log(a + b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a + b*x^2 +
c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^2)/(4
*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

3.851 $\int \frac{x}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1121, 632, 212}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.08

$$\frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4),x]
[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Maple [A]
time = 0.02, size = 36, normalized size = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	36
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^2}\right)x^2-2a\right)}{2\sqrt{-4ac+b^2}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^2}\right)x^2+2a\right)}{2\sqrt{-4ac+b^2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.35, size = 129, normalized size = 3.58

$$\left[\frac{\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(34) = 68$.
time = 0.26, size = 131, normalized size = 3.64

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2
```

Giac [A]
time = 3.55, size = 35, normalized size = 0.97

$$\frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B]
time = 4.27, size = 41, normalized size = 1.14

$$\frac{\operatorname{atan}\left(\frac{2\,a\,c\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4),x)
[Out] atan((a*b + 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

3.852 $\int \frac{1}{x(a+bx^2+cx^4)} dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}$$

```
[Out] ln(x)/a-1/4*ln(c*x^4+b*x^2+a)/a+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2
))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1128, 719, 29, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^2+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a}+\frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{\log(x)}{a}-\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}\\
&= \frac{\log(x)}{a}-\frac{\log(a+bx^2+cx^4)}{4a}+\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log(a+bx^2+cx^4)}{4a}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 113, normalized size = 1.64

$$\frac{4\sqrt{b^2-4ac}\,\log(x)-\left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.03, size = 65, normalized size = 0.94

method	result	size
default	$-\frac{\frac{\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2}+\frac{b\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a}+\frac{\ln\left(x\right)}{a}$	65
risch	$\frac{\ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{2}c-a\,b^{2}\right)\_Z^{2}+\left(4ac-b^{2}\right)\_Z+c\right)}\_R\ln\left(\left(\left(10ac-3b^{2}\right)\_R+5c\right)x^{2}-ab\_R+2b\right)\right)}{2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*(1/2*ln(c*x^4+b*x^2+a)+b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2)))+ln(x)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^{2}-4\,ac}\,b\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+\left(2\,cx^{2}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)-\left(b^{2}-4\,ac\right)\log\left(cx^{4}+bx^{2}+a\right)+4\left(b^{2}-4\,ac\right)\log\left(x\right)}{4\left(ab^{2}-4\,a^{2}c\right)},\ \frac{2\sqrt{-b^{2}+4\,ac}\,b\arctan\left(-\frac{\left(2\,cx^{2}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-\left(b^{2}-4\,ac\right)\log\left(cx^{4}+bx^{2}+a\right)+4\left(b^{2}-4\,ac\right)\log\left(x\right)}{4\left(ab^{2}-4\,a^{2}c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.
time = 8.32, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\,(4ac-b^2)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4a\,(4ac-b^2)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(-b*s
qrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) +
 (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(b*sqr
t(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) + l
og(x)/a
```

Giac [A]
time = 3.71, size = 68, normalized size = 0.99

$$-\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a}-\frac{\log\left(cx^4+bx^2+a\right)}{4\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
4*log(c*x^4 + b*x^2 + a)/a + 1/2*log(x^2)/a
```

Mupad [B]
time = 4.94, size = 1014, normalized size = 14.70

$$\frac{\ln(x)}{a}+\frac{\ln\left(cx^4+bx^2+a\right)\left(8ac-2b^2\right)}{2\left(4ab^2-16a^2c\right)}+\frac{b\,\mathrm{atan}\left([illegible]\right)}{2a\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] log(x)/a + (log(a + b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c)
) + (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3 -
 ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*
(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*
c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(12*b^
3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*(
(b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(12*b^3*c
^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(4*a*c - b
^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a
*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^
2)^(3/2))/(b^2*c^2) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*(((8*a*c - 2
*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(
4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))
/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(8*a*c - 2*b^2))/
(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*c - 6*b^2)) - (2*
(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2)/(16*a^2*(4*a*c -
 b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 - 16*a^2*c)^2*(4*a*c
 - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^
2))/(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))
/(b^2*c^4*(25*a*c - 6*b^2))))/(2*a*(4*a*c - b^2)^(1/2))
```

3.853 $\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2ax^2}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}$$

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/2*(-2*a*c+b^2)*arctanh
((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1128, 723, 814, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*1/(a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
2*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a
^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx,x,x^2\right)$$
$$= -\frac{1}{2ax^2} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx,x,x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx,x,x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2}$$
$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2} + \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\right.}{4a^2}$$
$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^2+cx^4)}{4a^2} - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,\right.}{2a^2}$$
$$= -\frac{1}{2ax^2} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^2+cx^4)}{4a^2}$$

Mathematica [A]
time = 0.08, size = 135, normalized size = 1.52

$$\frac{-\frac{2a}{x^2} - 4b\log(x) + \frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A]
time = 0.04, size = 85, normalized size = 0.96

method	result
default	$-\frac{-\frac{b\ln\left(cx^4+bx^2+a\right)}{2}+\frac{2\left(ac-\frac{b^2}{2}\right)\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a^2}-\frac{1}{2ax^2}-\frac{b\ln(x)}{a^2}$

risch	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(-4abc+b^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(10a^3c-3a^2b^2\right)\_R^2-4\_Rabc+2c^2\right)x^2-a^3b\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*(-1/2*b*ln(c*x^4+b*x^2+a)+2*(a*c-1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2/a/x^2-b*ln(x)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 293, normalized size = 3.29

$$\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(b^3-4\,abc)x^2\log\left(cx^4+bx^2+a\right)+4\,(b^3-4\,abc)x^2\log\left(x\right)+2\,ab^2-8\,a^2c}{4\,(a^2b^2-4\,a^3c)x^2},\ -\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\,x^2\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)x^2\log\left(cx^4+bx^2+a\right)+4\,(b^3-4\,abc)x^2\log\left(x\right)+2\,ab^2-8\,a^2c}{4\,(a^2b^2-4\,a^3c)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2
 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.73, size = 94, normalized size = 1.06

$$\frac{b\log\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*a
rctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*x
^2 - a)/(a^2*x^2)
```

Mupad [B]
time = 5.89, size = 2033, normalized size = 22.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(c^5/a^3 + ((2*b^3 - 8*a*
b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((((2
*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*
b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b
^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^
2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a
^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2
)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2))))/(8*a^3*c
^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((2*b^3 - 8*a*b*c)*(((2*a*c - b^2)*(
(20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^
3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) +
((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4
*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((
40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)/(64*a^9*(4*a*c - b^2)^(3/2)
) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)
```

```
^(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2
)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3) - (2*a^3*(4*a*c - b^2)*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)
*(((2*b^3 - 8*a*b*c)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2
*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2
)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)^(1/
2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((
a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c
^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a
^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(2*a*c - b^2)^3
)/(16*a^5*(4*a*c - b^2)^(3/2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2
*c^4 + b^4*c^2 - 4*a*b^2*c^3)) + (2*a^3*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^
2 - 9*a*b^2*c)*((b*c^4)/a^3 - ((2*b^3 - 8*a*b*c)*((a^2*c^4 - 4*a*b^2*c^3)/a
^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(
2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*
(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a
^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))
/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2
*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)
) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2)^2)/(8*a^3*(4*a*c - b^2)*(16*a^
3*c - 4*a^2*b^2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3)))*(2*a*c - b^2))/(2*a^2*(4*a*c - b^2)^(1/2)) - (b*log(x))/
a^2 - (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2))
 - 1/(2*a*x^2)
```

3.854 $\int \frac{1}{x^5(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=114

$$-\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4a^3}$$

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+(-a*c+b^2)*ln(x)/a^3-1/4*(-a*c+b^2)*ln(c*x^4+b*x^2
+a)/a^3+1/2*b*(-3*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*
a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1128, 723, 814, 648, 632, 212, 642}

$$\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}-\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b
^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx+cx^2)}\,dx, x, x^2\right)$$
$$= -\frac{1}{4ax^4} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x^2(a+bx+cx^2)}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{4ax^4} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax^2} + \frac{b^2-ac}{a^2x} + \frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)}\right)dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{(b^2-ac)\log(x)}{a^3} + \frac{\text{Subst}\left(\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^3}$$
$$= -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^3}$$
$$= -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx^2+cx^4)}{4a^3} + \frac{(b(b^2-3a}{}$$
$$= -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2}{}$$

Mathematica [A]
time = 0.15, size = 188, normalized size = 1.65

$$\frac{-\frac{a^2}{x^4} + \frac{2ab}{x^2} + 4(b^2-ac)\log(x) - \frac{\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(b^3-3abc-b^2\sqrt{b^2-4ac}+ac\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(a^2/x^4) + (2*a*b)/x^2 + 4*(b^2 - a*c)*Log[x] - ((b^3 - 3*a*b*c + b^2*Sq
rt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/Sqrt[b^2 - 4*a*c] + ((b^3 - 3*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a
^3)
```

Maple [A]
time = 0.04, size = 134, normalized size = 1.18

method	result
default	$\frac{\frac{(c^2a-b^2c)\ln(cx^4+bx^2+a)}{2c} + \frac{2\left(2abc-b^3-\frac{(c^2a-b^2c)b}{2c}\right)\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a^3} - \frac{1}{4a\,x^4} + \frac{(-ac+b^2)\ln(x)}{a^3} + \frac{b}{2a^2x^2}$

risch	$\frac{\frac{b\,x^2}{2a^2}-\frac{1}{4a}}{x^4}-\frac{\ln(x)c}{a^2}+\frac{\ln(x)b^2}{a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^4-b^2a^3\right)\_Z^2+\left(-4a^2c^2+5a\,b^2c-b^4\right)\_Z+c^3\right)}\_R\ln\left(\left(\left(10c\,a^5-3a^4b^2\right)\_R^2+(-\right.\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/2/a^3*(1/2*(a*c^2-b^2*c)/c*ln(c*x^4+b*x^2+a)+2*(2*a*b*c-b^3-1/2*(a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/4/a/x^4+(-a*c+b^2)*ln(x)/a^3+1/2*b/a^2/x^2

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more deta

Fricas [A]
time = 0.39, size = 374, normalized size = 3.28

$$\left[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}\,x^4\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(b^4-5\,ab^2c+4\,a^2c^2)x^4\log\left(cx^4+bx^2+a\right)-4\,(b^4-5\,ab^2c+4\,a^2c^2)x^4\log\left(x\right)+a^2b^2-4\,a^3c-2\,(ab^3-4\,a^2bc)x^2}{4\,(a^3b^2-4\,a^4c)x^4},\ \frac{2\,(b^3-3\,abc)\sqrt{-b^2+4\,ac}\,x^4\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^4-5\,ab^2c+4\,a^2c^2)x^4\log\left(cx^4+bx^2+a\right)+4\,(b^4-5\,ab^2c+4\,a^2c^2)x^4\log\left(x\right)-a^2b^2+4\,a^3c+2\,(ab^3-4\,a^2bc)x^2}{4\,(a^3b^2-4\,a^4c)x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] [-1/4*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^4*log(c*x^4 + b*x^2 + a) - 4*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^4*log(x) + a^2*b^2 - 4*a^3*c - 2*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2 - 4*a^4*c)*x^4), 1/4*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^4*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^4*log(c*x^4 + b*x^2 + a) + 4*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^4*log(x) - a^2*b^2 + 4*a^3*c + 2*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2 - 4*a^4*c)*x^4)]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.13, size = 126, normalized size = 1.11

$$-\frac{(b^2-ac)\log(cx^4+bx^2+a)}{4a^3}+\frac{(b^2-ac)\log(x^2)}{2a^3}-\frac{(b^3-3abc)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}\,a^3}-\frac{3b^2x^4-3acx^4-2abx^2+a^2}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2 - a*c)*log(x^2)/a^3
- 1/2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
 4*a*c)*a^3) - 1/4*(3*b^2*x^4 - 3*a*c*x^4 - 2*a*b*x^2 + a^2)/(a^3*x^4)
```

Mupad [B]
time = 6.37, size = 2451, normalized size = 21.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*
a^3*b^2)) - (1/(4*a) - (b*x^2)/(2*a^2))/x^4 - (log(x)*(a*c - b^2))/a^3 + (b
*atan((2*a^6*(4*a*c - b^2)*((((b*(3*a*c - b^2)*((4*a^4*b^4*c^2 - 8*a^5*b^2*
c^3)/a^6 - (2*a*b^2*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a^4*c - 4*a^3
*b^2)))/(4*a^3*(4*a*c - b^2)^(1/2)) - (b^3*c^2*(3*a*c - b^2)*(2*b^4 + 8*a^2
*c^2 - 10*a*b^2*c))/(2*a^2*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(2*
b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)) + (b^5*c^2*(3*a*c
 - b^2)^3)/(16*a^8*(4*a*c - b^2)^(3/2)) + (b*(3*a*c - b^2)*((4*a^2*b^4*c^3
- 5*a^3*b^2*c^4)/a^6 + (((4*a^4*b^4*c^2 - 8*a^5*b^2*c^3)/a^6 - (2*a*b^2*c^2
*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a^4*c - 4*a^3*b^2))*(2*b^4 + 8*a^2*c
^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2))))/(4*a^3*(4*a*c - b^2)^(1/2)))
*(3*b^6 - 10*a^3*c^3 + 27*a^2*b^2*c^2 - 18*a*b^4*c))/(c^2*(b^6*c^2 - 6*a*b^
4*c^3 + 9*a^2*b^2*c^4)*(6*b^6 - 25*a^3*c^3 + 54*a^2*b^2*c^2 - 36*a*b^4*c))
- (16*a^9*x^2*((3*b*(b^4 + 3*a^2*c^2 - 4*a*b^2*c)*((((5*a^3*b*c^5 - 6*a^2*b
^3*c^4)/a^6 - (((10*a^5*b*c^4 + 2*a^4*b^3*c^3)/a^6 + ((40*a^7*b*c^3 - 12*a^
6*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*a^6*(16*a^4*c - 4*a^3*b^2))
)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)))*(2*b^4 + 8*
a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)) - (b^3*c^5)/a^6 + (b*(3*a
*c - b^2)*((b*((10*a^5*b*c^4 + 2*a^4*b^3*c^3)/a^6 + ((40*a^7*b*c^3 - 12*a^6
```

```
*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))
*(3*a*c - b^2))/(4*a^3*(4*a*c - b^2)^(1/2)) + (b*(40*a^7*b*c^3 - 12*a^6*b^3
*c^2)*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(8*a^9*(4*a*c - b^2)^
(1/2)*(16*a^4*c - 4*a^3*b^2))))/(4*a^3*(4*a*c - b^2)^(1/2)) + (b^2*(40*a^7*
b*c^3 - 12*a^6*b^3*c^2)*(3*a*c - b^2)^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(
32*a^12*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2))))/(8*a^3*c^2*(6*b^6 - 25*a^3*
c^3 + 54*a^2*b^2*c^2 - 36*a*b^4*c)) + (((b^3*(40*a^7*b*c^3 - 12*a^6*b^3*c^2
)*(3*a*c - b^2)^3)/(64*a^15*(4*a*c - b^2)^(3/2)) - (((b*((10*a^5*b*c^4 + 2*
a^4*b^3*c^3)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10
*a*b^2*c))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(3*a*c - b^2))/(4*a^3*(4*a*c - b
^2)^(1/2)) + (b*(40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(3*a*c - b^2)*(2*b^4 + 8*a^
2*c^2 - 10*a*b^2*c))/(8*a^9*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(2
*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)) + (b*((5*a^3*b*c
^5 - 6*a^2*b^3*c^4)/a^6 - (((10*a^5*b*c^4 + 2*a^4*b^3*c^3)/a^6 + ((40*a^7*b
*c^3 - 12*a^6*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*a^6*(16*a^4*c -
 4*a^3*b^2)))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)))
*(3*a*c - b^2))/(4*a^3*(4*a*c - b^2)^(1/2)))*(3*b^6 - 10*a^3*c^3 + 27*a^2*b
^2*c^2 - 18*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(6*b^6 - 25*a^3*c^3 +
54*a^2*b^2*c^2 - 36*a*b^4*c)))*(4*a*c - b^2)^(3/2))/(b^6*c^2 - 6*a*b^4*c^3
+ 9*a^2*b^2*c^4) + (6*a^6*b*(4*a*c - b^2)^(3/2)*(b^4 + 3*a^2*c^2 - 4*a*b^2*
c)*((b^4*c^4 - a*b^2*c^5)/a^6 + (((4*a^2*b^4*c^3 - 5*a^3*b^2*c^4)/a^6 + (((
4*a^4*b^4*c^2 - 8*a^5*b^2*c^3)/a^6 - (2*a*b^2*c^2*(2*b^4 + 8*a^2*c^2 - 10*a
*b^2*c))/(16*a^4*c - 4*a^3*b^2))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a
^4*c - 4*a^3*b^2)))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*
b^2)) - (b*((b*(3*a*c - b^2)*((4*a^4*b^4*c^2 - 8*a^5*b^2*c^3)/a^6 - (2*a*b^
2*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a^4*c - 4*a^3*b^2)))/(4*a^3*(4*
a*c - b^2)^(1/2)) - (b^3*c^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c)
)/(2*a^2*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(3*a*c - b^2))/(4*a^3
*(4*a*c - b^2)^(1/2)) + (b^4*c^2*(3*a*c - b^2)^2*(2*b^4 + 8*a^2*c^2 - 10*a*
b^2*c))/(8*a^5*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2))))/(c^2*(b^6*c^2 - 6*a*
b^4*c^3 + 9*a^2*b^2*c^4)*(6*b^6 - 25*a^3*c^3 + 54*a^2*b^2*c^2 - 36*a*b^4*c)
))*(3*a*c - b^2))/(2*a^3*(4*a*c - b^2)^(1/2))
```

3.855 $\int \frac{x^6}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=203

$$-\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -b*x/c^2+1/3*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))
*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1136, 1293, 1180, 211}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
```

```
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{a+bx^2+cx^4}\,dx &= \frac{x^3}{3c}-\frac{\int \frac{x^2(3a+3bx^2)}{a+bx^2+cx^4}\,dx}{3c}\\
&= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\int \frac{3ab+3(b^2-ac)x^2}{a+bx^2+cx^4}\,dx}{3c^2}\\
&= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}+\frac{\left(b^2-ac+\frac{b(b^2-3}{\sqrt{b^2-}}\right.}{}\\
&= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2-ac+\right.}{}
\end{aligned}$$

Mathematica [A]

time = 0.11, size = 250, normalized size = 1.23

$$-\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3 - 3
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.04, size = 217, normalized size = 1.07

method	result
risch	$\frac{x^3}{3c} - \frac{bx}{c^2} + \frac{\sum_{\_R=\text{RootOf}\left(c\_Z^4+\_Z^2b+a\right)} \frac{\left(\left(-ac+b^2\right)\_R^2+ab\right)\ln\left(x-\_R\right)}{2\_R^3c+\_Rb}}{2c^2}$
default	$-\frac{-\frac{1}{3}cx^3+bx}{c^2} + \frac{-\frac{\left(-ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}+3abc-b^3\right)\sqrt{2}\,\text{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\cdots}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(-1/3*c*x^3+b*x)+4/c*(-1/8*(-a*c*(-4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^2)
^(1/2)+3*a*b*c-b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-a*c*(-
4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^2)^(1/2)-3*a*b*c+b^3)/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x^4 + b*x^2
 + a), x)/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. $2(167) = 334$.
time = 0.41, size = 1564, normalized size = 7.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b
^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c
^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c
^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(
1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sq
rt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^
2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sq
rt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^
2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*
a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
```

Sympy [A]
time = 11.34, size = 194, normalized size = 0.96

$$- \frac{bx}{c^2} + \operatorname{RootSum}\left(t^4 \cdot \left(256a^2c^7 - 128ab^2c^6 + 16b^4c^5\right) + t^2\left(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7\right) + a^5, \left(t \mapsto t\log\left(x + \frac{-64t^3a^2c^7 + 48t^3ab^2c^6 - 8t^3b^4c^5 + 14ta^3bc^3 - 28ta^2b^3c^2 + 14tab^5c - 2tb^7}{a^4c^2 - 3a^3b^2c + a^2b^4}\right)\right)\right) + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5)
+ _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a**
```

```
5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8*_t
**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c -
 2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2457 vs. $2(167) = 334$.
time = 4.84, size = 2457, normalized size = 12.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 -
(2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 -
4*a*c)*a^2*c^4)*c^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*
c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 -
2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan(2*sqr
t(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b
^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^
2) + 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
```

```
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c
^5 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b
^2 - 4*a*c)*a^2*c^4)*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2
*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c
^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan(
2*sqrt(1/2)*x/sqrt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*
a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^
7)*c^2) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

Mupad [B]
time = 5.01, size = 2500, normalized size = 12.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3/(3*c) - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16
*a*b*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3
*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7
 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*
a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a
^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c -
3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2)*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b
```

```
*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b
^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*
b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2)*1i)/((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a
^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*
c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3
 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
 + (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^
7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
 b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6
*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(
a^4*c - a^3*b^2))/c^3))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^
3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2
*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2
)*2i - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*
c^6)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 +
 a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a
*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2
*c^2 - 6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3
 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((
b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
```

```
)/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7
+ b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 -
6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((
((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3...
```

3.856 $\int \frac{x^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=179

$$\frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arc
tan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c
+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1136, 1180, 211}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a+bx^2+cx^4}\,dx &= \frac{x}{c}-\frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c}\\ &= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4}}\\ &= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b}}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 202, normalized size = 1.13

$$\frac{x}{c}-\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.03, size = 169, normalized size = 0.94

method	result

risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)} \frac{\left(-\_R^2 b-a\right)\ln\left(x-\_R\right)}{2\_R^3 c+\_R b}}{2c}$
default	$\frac{x}{c}-\frac{\left(-b\sqrt{-4ac+b^2}-2ac+b^2\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-b\sqrt{-4ac+b^2}+2ac-b^2\right)}{2c\sqrt{-4ac+b}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] x/c-1/2*(-b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2*(-b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(143) = 286.
time = 0.37, size = 1059, normalized size = 5.92

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -

```
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [A]
time = 1.22, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{2}c^{5}-128ab^{2}c^{4}+16b^{4}c^{3}\right)+t^{2}\cdot\left(48a^{2}bc^{2}-28ab^{3}c+4b^{5}\right)+a^{3},\left(t\mapsto t\log\left(x+\frac{32t^{3}abc^{4}-8t^{3}b^{3}c^{3}-4ta^{2}c^{2}+8tab^{2}c-2tb^{4}}{a^{2}c-ab^{2}}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2109 vs. 2(143) = 286.
time = 5.98, size = 2109, normalized size = 11.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*
```

```
b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a
*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*
c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)
)/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*
b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^
2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3
- 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^
2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*
c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)
```

Mupad [B]
time = 0.65, size = 3026, normalized size = 16.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4),x)
```

```
[Out] x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x
*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4
*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4
*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c
))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2
*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c -
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2
*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))
```

```
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4
*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4
)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2
*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*
b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

3.857 $\int \frac{x^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/
2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1144, 211}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{x^2}{a+bx^2+cx^4}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int$$

$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-}}$$

Mathematica [A]
time = 0.06, size = 165, normalized size = 1.10

$$\frac{\left(-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.03, size = 149, normalized size = 0.99

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^3c+\_Rb}\right)}{2}$
default	$4c\left(-\frac{\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\mathrm{arctan}}{8c\sqrt{-4ac+b^2}\sqrt{(b}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/8/(-4*a*c+b^2)^(1/2)/c*(-b+(-4*a*c+b^2)^(1/2))*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)+1/8*(b+(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(115) = 230.
time = 0.35, size = 559, normalized size = 3.73

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}(b^2c-4ac^2)\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}(b^2c-4ac^2)\sqrt{-\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}(b^2c-4ac^2)\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}(b^2c-4ac^2)\sqrt{-\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/
2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c -
4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqr
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2
*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4
*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(
b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*s
qrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a
*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

Sympy [A]
time = 0.43, size = 75, normalized size = 0.50

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\left(-16abc+4b^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c-2tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(-16*a*
b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c - 2*
_t*b + x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(115) = 230.
time = 4.76, size = 503, normalized size = 3.35

$$-\frac{\left(2b^2c^2-8ac^3-\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^2+4\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ac+2\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,bc-\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,c^2-2\left(b^2-4ac\right)c^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{c}}}\right)}{2\left(b^4-8ab^2c-2b^3c+16a^2c^2+8abc^2+b^2c^2-4ac^3\right)|c|}-\frac{\left(2b^2c^2-8ac^3-\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^2+4\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,ac+2\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,bc-\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,c^2-2\left(b^2-4ac\right)c^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{c}}}\right)}{2\left(b^4-8ab^2c-2b^3c+16a^2c^2+8abc^2+b^2c^2-4ac^3\right)|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a
*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*
b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c)) - 1/2
*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*
c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*b^2*
c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c))
```

Mupad [B]
time = 4.46, size = 416, normalized size = 2.77

$$-2\,\mathrm{atanh}\left(\frac{\left(x\left(4ac^2-2b^2c\right)+\frac{x\left(8b^3c^2-32abc^3\right)\left(b^3+\sqrt{-\left(4ac-b^2\right)^3}-4abc\right)}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}\right)\sqrt{-\frac{b^3+\sqrt{-\left(4ac-b^2\right)^3}-4abc}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}}}{ac}\right)\sqrt{-\frac{b^3+\sqrt{-\left(4ac-b^2\right)^3}-4abc}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}}-2\,\mathrm{atanh}\left(\frac{\left(x\left(4ac^2-2b^2c\right)-\frac{x\left(8b^3c^2-32abc^3\right)\left(\sqrt{-\left(4ac-b^2\right)^3}-b^3+4abc\right)}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}\right)\sqrt{\frac{\sqrt{-\left(4ac-b^2\right)^3}-b^3+4abc}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}}}{ac}\right)\sqrt{\frac{\sqrt{-\left(4ac-b^2\right)^3}-b^3+4abc}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4),x)
```

```
[Out] - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-
(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2
*c^2)))^(1/2))/(a*c))*(-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*
c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) - (
x*(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*
b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(((-(4*a*c - b^2
)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

3.858 $\int \frac{1}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*
a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*2^(1/2)*c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1107, 211}

$$\frac{\sqrt{2}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.05, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-1),x]
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [A]
time = 0.03, size = 117, normalized size = 0.78

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3 c+\_R b}\right)}{2}$	38
default	$4c\left(-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/4/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(114) = 228.
time = 0.36, size = 613, normalized size = 4.09

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

Sympy [A]
time = 0.63, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac-2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1024 vs. $2(114) = 228$.

time = 4.23, size = 1024, normalized size = 6.83

$$\frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^4-8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ab^2c-2\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^3c-2b^4c+16\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,a^2c^2+8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,abc^2+\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^2c^2+16ab^2c^2-2b^3c^2-4\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ac^3-32a^2c^3+8abc^3+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^3-4\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,abc-2\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^2c+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,bc^2+2(b^2-4ac)b^2c-8(b^2-4ac)ac^2+2(b^2-4ac)bc^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{c}}}\right)}{4(ab^4-8a^2b^2c-2ab^3c+16a^3c^2+8a^2bc^2+ab^2c^2-4a^2c^3)|c|}+\frac{\left(\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^4-8\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,ab^2c-2\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^3c+2b^4c+16\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,a^2c^2+8\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,abc^2+\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^2c^2-16ab^2c^2-2b^3c^2-4\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,ac^3+32a^2c^3+8abc^3+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^3-4\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,abc-2\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^2c+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,bc^2-2(b^2-4ac)b^2c+8(b^2-4ac)ac^2+2(b^2-4ac)bc^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{c}}}\right)}{4(ab^4-8a^2b^2c-2ab^3c+16a^3c^2+8a^2bc^2+ab^2c^2-4a^2c^3)|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^
2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs
(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 -
4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4
 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c
^3)*abs(c))
```

Mupad [B]

time = 4.61, size = 763, normalized size = 5.09

$$-\mathrm{atan}\left(\frac{b^4\,x\,1{}\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1{}\mathrm{i}+a^2\,c^2\,x\,16{}\mathrm{i}-a\,b^2\,c\,x\,8{}\mathrm{i}}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2{}\mathrm{i}-\mathrm{atan}\left(\frac{b^4\,x\,1{}\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1{}\mathrm{i}+a^2\,c^2\,x\,16{}\mathrm{i}-a\,b^2\,c\,x\,8{}\mathrm{i}}{4\,a\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*
a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
- 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2
) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(b^3 + (
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 +
 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i
)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(((b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

3.859 $\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=174

$$-\frac{1}{ax}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x
*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*a*c+b^2)^(1
/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1137, 1180, 211}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx = -\frac{1}{ax}+\frac{\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx}{a}$$

$$= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$= -\frac{1}{ax}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-}}\right.}{\sqrt{}}$$

Mathematica [A]
time = 0.27, size = 191, normalized size = 1.10

$$-\frac{\frac{2}{x}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/a
```

Maple [A]
time = 0.04, size = 159, normalized size = 0.91

method	result
default	$\frac{4c\left(-\frac{\left(-b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{}{\sqrt{(b}}\right.}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-\right.}}}\right.}{a}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^5c^2-8a^4b^2c+a^3b^4\right)\_Z^4+\left(12a^2b\,c^2-7a\,b^3c+b^5\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(40a^5c^2-22a^4b^2c+3a^3b^4\right)\_R^4+(25a\right.\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*c*(-1/8*(-b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)+1/8*(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/a/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. $2(137) = 274$.
time = 0.37, size = 1116, normalized size = 6.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2
*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a
^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))
)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
```

```
2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 -
 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5
 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^
3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
- 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*s
qrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqr
t(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*
c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c
)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x)
```

Sympy [**A**]
time = 1.90, size = 148, normalized size = 0.85

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{5}c^{2}-128a^{4}b^{2}c+16a^{3}b^{4}\right)+t^{2}\cdot\left(48a^{2}bc^{2}-28ab^{3}c+4b^{5}\right)+c^{3},\left(t\mapsto t\log\left(x+\frac{-64t^{3}a^{5}c^{2}+48t^{3}a^{4}b^{2}c-8t^{3}a^{3}b^{4}-10ta^{2}bc^{2}+10tab^{3}c-2tb^{5}}{ac^{3}-b^{2}c^{2}}\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t**
3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c**2
+ 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. 2(137) = 274.
time = 5.04, size = 1839, normalized size = 10.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*b^2*c
```

```
^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*
(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 -
 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^
2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*
x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*
a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(
c)) + 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*
b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2
 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 -
2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(
1/2)*x/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c
 - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)
*abs(c)) - 1/(a*x)
```

Mupad [B]

time = 4.85, size = 2997, normalized size = 17.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)),x)

[Out] - atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^
3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b
^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b
*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2
*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5
+ b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - a
tan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*
b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c
^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
```

```
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a
^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^
3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^
3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*
(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(a
*x)
```

3.860 $\int \frac{1}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=196

$$-\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/3/a/x^3+b/a^2/x+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*c^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(
1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1137, 1295, 1180, 211}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(a^2*x) + (Sqrt[c]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b
 + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx^2+cx^4)}\,dx &= -\frac{1}{3ax^3}+\frac{\int \frac{-3b-3cx^2}{x^2(a+bx^2+cx^4)}\,dx}{3a}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}-\frac{\int \frac{-3(b^2-ac)-3bcx^2}{a+bx^2+cx^4}\,dx}{3a^2}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2}+\frac{\left(c\left(b+\frac{}{\sqrt{}}\right.\right.}{}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\,(b}{}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 216, normalized size = 1.10

$$\frac{-\frac{2a}{x^3}+\frac{6b}{x}+\frac{3\sqrt{2}\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^3 + (6*b)/x + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-b^2 + 2*a*c + b*Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

Maple [A]
time = 0.04, size = 179, normalized size = 0.91

method	result
default	$\frac{4c\left(-\frac{\left(b\sqrt{-4ac+b^2}-2ac+b^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(b\sqrt{-4ac+b^2}+2ac-b^2\right)\sqrt{2}\,\operatorname{arctan}}{8\sqrt{-4ac+b^2}\sqrt{(b+}}\right.}{a^2}$
risch	$\frac{\frac{b\,x^2}{a^2}-\frac{1}{3a}}{x^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16c^2a^7-8a^6b^2c+a^5b^4\right)\_Z^4+\left(-20a^3b\,c^3+25a^2b^3c^2-9a\,b^5c+b^7\right)\_Z^2+c^5\right)}\_R\ln\left(\left(\left(40c^2a^7-22a^6b^2c+3a^5b^4\right.\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a^2*c*(-1/8*(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))+1/8*(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)))-1/3/a/x^3+b/a^2/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((b*c*x^2 + b^2 - a*c)/(c*x^4 + b*x^2 + a), x)/a^2 + 1/3*(3*b*x^2
- a)/(a^2*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. $2(160) = 320$.
time = 0.38, size = 1622, normalized size = 8.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 -
 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3*a*b^2*c^4
+ a^2*c^5)*x + sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3
 + 4*a^4*c^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*sqrt(-
(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^
2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 +
 (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3
*a*b^2*c^4 + a^2*c^5)*x - sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*
a^3*b^2*c^3 + 4*a^4*c^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*
c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c
)))/(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*
a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(
b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x + sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4 + (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^
2 - 4*a^11*c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2
 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*
c))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x - sqrt(1/2)*(b^8 - 8*a*b^6*c
+ 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4 + (a^5*b^5 - 7*a^6*b^3*c + 12
*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2
- 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 6*b*x^2 + 2*a)/(a^2*x^3)
```

Sympy [A]
time = 71.43, size = 211, normalized size = 1.08

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{7}c^{2}-128a^{6}b^{2}c+16a^{5}b^{4}\right)+t^{2}\left(-80a^{3}bc^{3}+100a^{2}b^{3}c^{2}-36ab^{5}c+4b^{7}\right)+c^{5},\left(t\mapsto t\log\left(x+\frac{-96t^{3}a^{7}bc^{2}+56t^{3}a^{6}b^{3}c-8t^{3}a^{5}b^{5}-4ta^{4}c^{4}+32ta^{3}b^{2}c^{3}-40ta^{2}b^{4}c^{2}+16tab^{6}c-2tb^{8}}{a^{2}c^{5}-3ab^{2}c^{4}+b^{4}c^{3}}\right)\right)\right)+\frac{-a+3bx^{2}}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**7*c**2 - 128*a**6*b**2*c + 16*a**5*b**4) + _t**2*(-80
*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + c**5, Lambda(_t
, _t*log(x + (-96*_t**3*a**7*b*c**2 + 56*_t**3*a**6*b**3*c - 8*_t**3*a**5*b
**5 - 4*_t*a**4*c**4 + 32*_t*a**3*b**2*c**3 - 40*_t*a**2*b**4*c**2 + 16*_t*
a*b**6*c - 2*_t*b**8)/(a**2*c**5 - 3*a*b**2*c**4 + b**4*c**3)))) + (-a + 3*
b*x**2)/(3*a**2*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. $2(160) = 320$.
time = 4.65, size = 1640, normalized size = 8.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c
 - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a^2*b^2*c
^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*
a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c
 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 -
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 3*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)
*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*
a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b +
sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c +
16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/4*(sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c +
24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^4*c^2 - 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 5*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 48*a^2*b^2*c^3 + 14*a*b^3
```

```
*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 - 24*
a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 -
 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 12*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c + 10*(
b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3
- 6*(b^2 - 4*a*c)*a*b*c^3)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b - sqrt(a^4*b^2
- 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 +
8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/3*(3*b*x^2 - a)/(a^2*x^3
)
```

Mupad [B]
time = 0.79, size = 2500, normalized size = 12.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - (1/(3*a) - (b*x^2)/a^2)/x^3 - atan(((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b
^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6
*b^2*c)))^(1/2)*(16*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3
) - x*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7
*c^2 - 8*a^6*b^2*c)))^(1/2)*1i - (((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20
*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c
- 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*
c)))^(1/2)*(16*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^
4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3) + x
*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)^(1/2
) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
+ 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2
- 8*a^6*b^2*c)))^(1/2)*1i)/((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*
b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a
*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^
```

```
(1/2)*(16*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3) - x*(4*a
^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b
^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a
*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a
^6*b^2*c)))^(1/2) + (((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 -
25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(1
6*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4*(-(4*a*c - b^2)^3)^(1
/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2
) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^
2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3) + x*(4*a^8*c^5 +
 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*
a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c -
 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c
)))^(1/2) - 2*a^6*b*c^5))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c
^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/
2)*2i - atan((((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^
2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a
*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*a^1
0*c^4 + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)
*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^
2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)) - x*(4*a^8*c^5 + 2*a
^6*b^4*c^3 - 8*a^7*b^2*c^4))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3
*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))
^(1/2)*1i - ((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*
b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*a^10*
c^4 + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(
-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)) + x*(4*a^8*c^5 + 2*a^6
*b^4*c^3 - 8*a^7*b^2*c^4))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(
1/2)*1i)/(((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^
3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*a^10*c^
4 + 4*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + x*(32*a^11...
```

3.861 $\int \frac{x^7}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=132

$$-\frac{bx^2}{2c(b^2-4ac)}+\frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}}+\frac{\log(a+bx^2+cx^4)}{4c^2}$$

```
[Out] -1/2*b*x^2/c/(-4*a*c+b^2)+1/2*x^4*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+
1/2*b*(-6*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)
^(3/2)+1/4*ln(c*x^4+b*x^2+a)/c^2
```

Rubi [A]
time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1128, 752, 787, 648, 632, 212, 642}

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}}-\frac{bx^2}{2c(b^2-4ac)}+\frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\log(a+bx^2+cx^4)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(b*x^2)/(c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]
)/(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 + c*x^4]/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$
$$= \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}$$
$$= -\frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{-ab+(-b^2+4ac)x}{a+bx+cx^2}\,dx, x, x^2\right.}{2c(b^2-4ac)}$$
$$= -\frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} - (b$$
$$= -\frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\log(a+bx^2+cx^4)}{4c^2} + (b(b^2-6a$$
$$= -\frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right.}{2c^2(b^2-4ac)^{3/2}}$$

Mathematica [A]
time = 0.12, size = 121, normalized size = 0.92

$$\frac{\frac{2(-2a^2c+b^3x^2+ab(b-3cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{2b(b^2-6ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + \log(a+bx^2+cx^4)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2
 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A]
time = 0.07, size = 179, normalized size = 1.36

method	result
default	$\frac{\frac{b(3ac-b^2)x^2}{c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{(4ac-b^2)c^2}}{2c\,x^4+2b\,x^2+2a} + \frac{\frac{(4ac-b^2)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\left(-ab-\frac{(4ac-b^2)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c(4ac-b^2)}$

risch	$\frac{\frac{b\left(3ac-b^2\right)x^2}{2c^2\left(4ac-b^2\right)}+\frac{a\left(2ac-b^2\right)}{2\left(4ac-b^2\right)c^2}}{c\,x^4+b\,x^2+a}+\frac{4\ln\left(\left(\left(-24a^2b\,c^2+10a\,b^3c-b^5+\sqrt{-b^2\left(4ac-b^2\right)\left(6ac-b^2\right)^2}\,b\right)x^2+2\sqrt{-b^2\left(4ac-\right.}\right.\right.}{\left(4ac-b^2\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^
4+b*x^2+a)+1/2/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^4+b*x^2+a)+2*(-a*b-1
/2*(4*a*c-b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(120) = 240$.
time = 0.35, size = 663, normalized size = 5.02

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\,(b^5-7\,ab^3c+12\,a^2bc^2)x^2+((b^3c-6\,abc^2)x^4+ab^3-6\,a^2bc+(b^4-6\,ab^2c)x^2)\sqrt{b^2-4\,ac}\,\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+(2cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)\log\left(cx^4+bx^2+a\right)}{4\,(ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4+(b^4c^3-8\,ab^2c^4+16\,a^2c^5)x^4+(b^5c^2-8\,ab^3c^3+16\,a^2bc^4)x^2)},\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\,(b^5-7\,ab^3c+12\,a^2bc^2)x^2+2\,((b^3c-6\,abc^2)x^4+ab^3-6\,a^2bc+(b^4-6\,ab^2c)x^2)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)\log\left(cx^4+bx^2+a\right)}{4\,(ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4+(b^4c^3-8\,ab^2c^4+16\,a^2c^5)x^4+(b^5c^2-8\,ab^3c^3+16\,a^2bc^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c
^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*
x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2
+ b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a
^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3
*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (
b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^
```

```
2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 +
 b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.99, size = 152, normalized size = 1.15

$$-\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}}-\frac{b^2cx^4-4\,ac^2x^4-b^3x^2+2\,abcx^2-ab^2}{4\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)}+\frac{\log{(cx^4+bx^2+a)}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4
*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*a*
b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x^4
+ b*x^2 + a)/c^2
```

Mupad [**B**]
time = 5.10, size = 1336, normalized size = 10.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)) + (b*x^2*(3*a*c - b^2))/(2*c^2*(4*
a*c - b^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3
*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4
*c^3 - 192*a^2*b^2*c^4)) + (b*atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4
*a*c - b^2)^3)*(x^2*(((b*((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((
8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)
)/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*
b^2*c^4)))*(6*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(3/2)) + (b*(8*b^3*c^4 - 32*
```

```
a*b*c^5)*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))
/(16*c^2*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 +
 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - (b*((b^3 - 5*a*b*c)/
(4*a*c^3 - b^2*c^2) + (((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((8*
b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/
(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^
2*c^4)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c
^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (b^2*((b^3*c^4)/2 - 2*a
*b*c^5)*(6*a*c - b^2)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b^2*c^2))))/(2*a*(
4*a*c - b^2)^(3/2))) - ((b*(6*a*c - b^2)*(8*a + (8*a*c^2*(2*b^6 - 128*a^3*c
^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3
- 192*a^2*b^2*c^4)))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a*b*(6*a*c - b^2)*(2*b^
6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/((4*a*c - b^2)^(3/2)*(256*a
^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) +
(b*(a/c^2 + ((8*a + (8*a*c^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b
^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*b^6 -
 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 +
48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (a*b^2*(6*a*c - b^2)^2)/(c^2*(4*a*c - b^
2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(b^6 + 36*a^2*b^2*c^2 - 12*a*b^4*c))*(6
*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(3/2))
```

3.862 $\int \frac{x^5}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=78

$$\frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*a*arctanh((2*c*x^2+b)/(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1128, 736, 632, 212}

$$\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
```

```
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^2}\,dx,x,x^2\right)\\ &= \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{a\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{b^2-4ac}\\ &= \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{(2a)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{b^2-4ac}\\ &= \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 1.19

$$\frac{b^2x^2+a(b-2cx^2)}{2c(-b^2+4ac)(a+bx^2+cx^4)}+\frac{2a\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^2,x]
[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A]
time = 0.04, size = 104, normalized size = 1.33

method	result
default	$\frac{-\frac{(2ac-b^2)x^2}{c(4ac-b^2)}+\frac{ab}{c(4ac-b^2)}}{2c\,x^4+2b\,x^2+2a}+\frac{2a\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$

risch	$\frac{-\frac{(2ac-b^2)x^2}{2c(4ac-b^2)}+\frac{ab}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\frac{a\ln\left(\left((-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^2+8a^2c-2ab^2\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{a\ln\left(\left((-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)x^2-8a^2c+2a\right.}{(-4ac+b^2)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2*a/
(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(72) = 144$.
time = 0.35, size = 407, normalized size = 5.22

$$\left[-\frac{ab^3-4a^2bc+(b^4-6ab^2c+8a^2c^2)x^2+2(ac^2x^4+abcx^2+a^2c)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)}{2(ab^4c-8a^2b^2c^2+16a^3c^3+(b^4c^2-8ab^2c^3+16a^2c^4)x^4+(b^5c-8ab^3c^2+16a^2bc^3)x^2)},-\frac{ab^3-4a^2bc+(b^4-6ab^2c+8a^2c^2)x^2-4(ac^2x^4+abcx^2+a^2c)\sqrt{-b^2+4ac}\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{2(ab^4c-8a^2b^2c^2+16a^3c^3+(b^4c^2-8ab^2c^3+16a^2c^4)x^4+(b^5c-8ab^3c^2+16a^2bc^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^4
 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c - 8
*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a
*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*c - 8*a
^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(70) = 140$.
time = 0.83, size = 282, normalized size = 3.62

$$-a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right)+a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right)+\frac{ab+x^2(-2ac+b^2)}{8a^2c^2-2ab^2c+x^4\cdot(8ac^3-2b^2c^2)+x^2\cdot(8abc^2-2b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*b**4*sqrt(-1/(4*a*c
- b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a
**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)*
*3) + a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) + (a*b + x**2*(-2*a
*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**
2*(8*a*b*c**2 - 2*b**3*c))
```

Giac [A]
time = 4.18, size = 96, normalized size = 1.23

$$-\frac{2\,a\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{b^2x^2-2\,acx^2+ab}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c
^2))
```

Mupad [B]
time = 0.18, size = 187, normalized size = 2.40

$$-\frac{\frac{x^2\,(2\,a\,c-b^2)}{2\,c\,(4\,a\,c-b^2)}-\frac{a\,b}{2\,c\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a}-\frac{2\,a\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}-\frac{x^2\,\left(\frac{4\,a\,c^2}{\left(4\,a\,c-b^2\right)^{7/2}}+\frac{4\,a\,\left(b^3\,c^2-4\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{13/2}}\right)\,(4\,a\,c-b^2)^4}{8\,a^2\,c^2}\right)}{(4\,a\,c-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b)/(2*c*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) - (2*a*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*((4
*a*c^2)/(4*a*c - b^2)^(7/2) + (4*a*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(
4*a*c - b^2)^(13/2))*(4*a*c - b^2)^4)/(8*a^2*c^2)))/(4*a*c - b^2)^(3/2)
```

3.863 $\int \frac{x^3}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=75

$$\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*arctanh((2*c*x^2+b)/(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]

time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1128, 652, 632, 212}

$$\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, x^2\right)\\ &= \frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}\\ &= \frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\ &= \frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 1.05

$$\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(b + 2*c*x^
2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A]
time = 0.04, size = 77, normalized size = 1.03

method	result
default	$\frac{-b\,x^2-2a}{2(4ac-b^2)(c\,x^4+b\,x^2+a)} - \frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{b\,x^2}{2\left(4ac-b^2\right)}-\frac{a}{4ac-b^2}}{c\,x^4+b\,x^2+a} + \frac{b\ln\left(\left(-(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^2+8a^2c-2a\,b^2\right)}{2(-4ac+b^2)^{\frac{3}{2}}} - \frac{b\ln\left(\left(-(-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)x^2-8a^2c+2a\,b^2\right)}{2(-4ac+b^2)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(69) = 138$.
time = 0.36, size = 360, normalized size = 4.80

$$\left[\frac{2\,ab^{2}-8\,a^{2}c+(b^{3}-4\,abc)x^{2}-(bcx^{4}+b^{2}x^{2}+ab)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)}{2\,(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3})x^{4}+(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2})x^{2})},\ \frac{2\,ab^{2}-8\,a^{2}c+(b^{3}-4\,abc)x^{2}-2\,(bcx^{4}+b^{2}x^{2}+ab)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{2\,(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3})x^{4}+(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2})x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 +
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*
x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - 2*(b*c*x^4 + b^2*x^2 +
 a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(63) = 126$.
time = 0.72, size = 269, normalized size = 3.59

$$\frac{b\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}\log\left(x^{2}+\frac{-16a^{2}bc^{2}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+8ab^{3}c\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}-b^{5}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+b^{2}}{2bc}\right)}{2}-\frac{b\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}\log\left(x^{2}+\frac{16a^{2}bc^{2}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}-8ab^{3}c\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+b^{5}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+b^{2}}{2bc}\right)}{2}+\frac{-2a-bx^{2}}{8a^{2}c-2ab^{2}+x^{4}\cdot(8ac^{2}-2b^{2}c)+x^{2}\cdot(8abc-2b^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b*
*2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a*
*2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3
) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 + (-2*a - b*x**2)/(8
*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

Giac [A]
time = 3.90, size = 82, normalized size = 1.09

$$\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{bx^2+2\,a}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

Mupad [B]
time = 4.57, size = 178, normalized size = 2.37

$$\frac{b\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}-\frac{x^2\,\left(4\,a\,c-b^2\right)^4\,\left(\frac{b^2\,c^2}{a\,\left(4\,a\,c-b^2\right)^{7/2}}+\frac{b^2\,\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,\left(4\,a\,c-b^2\right)^{13/2}}\right)}{2\,b^2\,c^2}\right)}{(4\,a\,c-b^2)^{3/2}}-\frac{\frac{a}{4\,a\,c-b^2}+\frac{b\,x^2}{2\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (b*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((b^2*c^
2)/(a*(4*a*c - b^2)^(7/2)) + (b^2*(2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c))/
(2*a*(4*a*c - b^2)^(13/2))))/(2*b^2*c^2)))/(4*a*c - b^2)^(3/2) - (a/(4*a*c
- b^2) + (b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

3.864 $\int \frac{x}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=74

$$-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*(-2*c*x^2-b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*c*arctanh((2*c*x^2+b)/(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1121, 628, 632, 212}

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^2+cx^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx, x, x^2\right)\\ &= -\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{c\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac}\\ &= -\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{(2c)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\ &= -\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 79, normalized size = 1.07

$$-\frac{\frac{b+2cx^2}{a+bx^2+cx^4} + \frac{4c\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b
^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c)
```

Maple [A]
time = 0.04, size = 75, normalized size = 1.01

method	result	size
default	$\frac{2c\,x^2+b}{2(4ac-b^2)(c\,x^4+b\,x^2+a)}+\frac{2c\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	75
risch	$\frac{\frac{c\,x^2}{4ac-b^2}+\frac{b}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{c\ln\left(\left((-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)x^2+8a^2c-2a\,b^2\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{c\ln\left(\left((-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)x^2-8a^2c+2a\,b^2\right)}{(-4ac+b^2)^{\frac{3}{2}}}$	151

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(68) = 136$.
time = 0.36, size = 361, normalized size = 4.88

$$\left[-\frac{b^{3}-4\,abc+2\,(b^{2}c-4\,ac^{2})x^{2}+2\,(c^{2}x^{4}+bcx^{2}+ac)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac-(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)}{2\,(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3})x^{4}+(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2})x^{2})},-\frac{b^{3}-4\,abc+2\,(b^{2}c-4\,ac^{2})x^{2}-4\,(c^{2}x^{4}+bcx^{2}+ac)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{2\,(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3})x^{4}+(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2})x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*c
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*x^2), -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 + b*c*x
^2 + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(66) = 132$.
time = 0.69, size = 267, normalized size = 3.61

$$-c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}\log\left(x^{2}+\frac{-16a^{2}c^{3}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+8ab^{2}c^{2}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}-b^{4}c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+bc}{2c^{2}}\right)+c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}\log\left(x^{2}+\frac{16a^{2}c^{3}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}-8ab^{2}c^{2}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+b^{4}c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+bc}{2c^{2}}\right)+\frac{b+2cx^{2}}{8a^{2}c-2ab^{2}+x^{4}\cdot\left(8ac^{2}-2b^{2}c\right)+x^{2}\cdot\left(8abc-2b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(-1/(4*a*c
- b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*
a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**3) + b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x**2)/
(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)
)
```

Giac [A]
time = 4.93, size = 82, normalized size = 1.11

$$-\frac{2\,c\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,cx^2+b}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

Mupad [B]
time = 4.31, size = 172, normalized size = 2.32

$$\frac{\frac{b}{2\,(4\,a\,c-b^2)}+\frac{c\,x^2}{4\,a\,c-b^2}}{c\,x^4+b\,x^2+a}-\frac{2\,c\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}-\frac{x^2\,(4\,a\,c-b^2)^4\left(\frac{4\,c^4}{a\,\left(4\,a\,c-b^2\right)^{7/2}}+\frac{4\,c^2\,\left(b^3\,c^2-4\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{a\,\left(4\,a\,c-b^2\right)^{13/2}}\right)}{8\,c^4}\right)}{(4\,a\,c-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (b/(2*(4*a*c - b^2)) + (c*x^2)/(4*a*c - b^2))/(a + b*x^2 + c*x^4) - (2*c*at
an((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((4*c^4)/(a*(
4*a*c - b^2)^(7/2)) + (4*c^2*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(a*(4*a
*c - b^2)^(13/2))))/(8*c^4)))/(4*a*c - b^2)^(3/2)
```

3.865 $\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=122

$$\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}$$

```
[Out] 1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*b*(-6*a*c+b^2)*a
rctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/4
*ln(c*x^4+b*x^2+a)/a^2
```

Rubi [A]
time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1128, 754, 814, 648, 632, 212, 642}

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-bcx}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax} + \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right)\,dx, x, \right.}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} - \frac{(b(b^2 -}{}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{(b(b^2-6ac))\,\text{Su}}{}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(x)}{a^2} -$$

Mathematica [A]
time = 0.22, size = 207, normalized size = 1.70

$$\frac{\frac{2a\left(b^2-2ac+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + 4\log(x) - \frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(b^3-6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log[
x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log
[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c -
b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [A]
time = 0.06, size = 185, normalized size = 1.52

method	result

default	$-\frac{\frac{\frac{abc\,x^2}{4ac-b^2}-\frac{a\left(2ac-b^2\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(4c^2a-b^2c\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(5abc-b^3-\frac{\left(4c^2a-b^2c\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}+\frac{\ln(x)}{a^2}$
risch	$\frac{-\frac{bc\,x^2}{2a\left(4ac-b^2\right)}+\frac{2ac-b^2}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\ln(x)}{a^2}+\Bigg(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3-48a^4b^2c^2+12a^3b^4c-b^6a^2\right)\_Z^2+\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)\_Z+16a\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*((a*b*c/(4*a*c-b^2)*x^2-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)
+1/(4*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^4+b*x^2+a)+2*(5*a*b*c-b^3-1/2*
(4*a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)
)))+ln(x)/a^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(112) = 224$.
time = 0.42, size = 813, normalized size = 6.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 +
 ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt
(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt
(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^
2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
```

```
8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^
4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a*b^4 - 12
*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b
*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*
a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.56, size = 166, normalized size = 1.36

$$-\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}}+\frac{b^2cx^4-4\,ac^2x^4+b^3x^2-2\,abcx^2+3\,ab^2-8\,a^2c}{4\,(cx^4+bx^2+a)(a^2b^2-4\,a^3c)}-\frac{\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4
*a^3*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*a*
b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) - 1/
4*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

Mupad [**B**]
time = 8.29, size = 2500, normalized size = 20.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(2*a*(4*a*c - b^2)) - (b*c*x^2)/(2*a*(4*a*c - b
^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3*c^3 +
96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c +
192*a^4*b^2*c^2)) + (b*atan((x^2*((((((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3 +
36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*
a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*
b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*
b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2
*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*
c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056
*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*
a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*
b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4
*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (b*((
6*a*b^5*c^4 + 80*a^3*b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^
4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)
 - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 1
2*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*
(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5
*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*
c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2
*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)^3*(
2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688
*a^6*b^3*c^5))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c)
)/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 7
2*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((6*a*b^5*c^4 + 80*a^3*
b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/
(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*
c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a
^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c +
 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2
*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*
a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^
3*b^4*c + 192*a^4*b^2*c^2)) - (b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2) - (b*(6*a*c - b^2)*((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3
+ 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(256
0*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^
6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a
```

```
^2*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^
2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 10
56*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^
3*b^4*c + 192*a^4*b^2*c^2))))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b
^2)^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 +
 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(
32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))))/(8*a^3*c^2
*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16
*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b
^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2)))/(b^6*c^2 -
 12*a*b^4*c^3 + 36*a^2*b^2*c^4) + (((b*((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 + 80*a
^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5
*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4
*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6
*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) - (((b*((4*a^2*b^6*c^2 - 36*a^3*b^
4*c^3 + 80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*
c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 -...
```

3.866 $\int \frac{1}{x^3(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=162

$$-\frac{b^2-3ac}{a^2(b^2-4ac)x^2}+\frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)}-\frac{(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}}-\frac{2b\log(x)}{a^3}$$

```
[Out] (3*a*c-b^2)/a^2/(-4*a*c+b^2)/x^2+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2
/(c*x^4+b*x^2+a)-(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+1/2*b*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A]
time = 0.16, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$,
Rules used = {1128, 754, 814, 648, 632, 212, 642}

$$\frac{b\log(a+bx^2+cx^4)}{2a^3}-\frac{2b\log(x)}{a^3}-\frac{b^2-3ac}{a^2x^2(b^2-4ac)}-\frac{(6a^2c^2-6ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx^2}{2ax^2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^
2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x]
)/a^3 + (b*Log[a + b*x^2 + c*x^4])/(2*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^2+cx^4)^2} \, dx = \frac{1}{2} \text{Subst}\left(\int \frac{1}{x^2 (a+bx+cx^2)^2} \, dx, x, x^2 \right)$$

$$= \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)\, x^2 (a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{-2(b^2-3ac)-2bcx}{x^2(a+bx+cx^2)} \, dx, x, x^2 \right)}{2a(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)\, x^2 (a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \left(\frac{2(-b^2+3ac)}{ax^2} - \frac{2b(-b^2+4ac)}{a^2x} + \frac{2(-b^4+5ab \ldots}{\ldots} \right.\right.}{2a(b^2-4ac)\ldots}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)\, x^2} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)\, x^2 (a+bx^2+cx^4)} - \frac{2b\log(x)}{a^3} - \frac{\text{Subst}\left(\int \frac{-b^4 \ldots}{\ldots} \right.}{\ldots}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)\, x^2} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)\, x^2 (a+bx^2+cx^4)} - \frac{2b\log(x)}{a^3} + \frac{b\text{Subst}\left(\int \frac{b \ldots}{a+\ldots} \right.}{\ldots}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)\, x^2} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)\, x^2 (a+bx^2+cx^4)} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx \ldots}{2a^3 \ldots}$$

$$= -\frac{b^2-3ac}{a^2(b^2-4ac)\, x^2} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)\, x^2 (a+bx^2+cx^4)} - \frac{(b^4-6ab^2c+6a^2c^2)\tan\ldots}{a^3(b^2-4a\ldots}$$

Mathematica [A]
time = 0.18, size = 248, normalized size = 1.53

$$\frac{-\frac{a}{x^2} - \frac{a(b^3-3abc+b^2cx^2-2ac^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)} - 4b\log(x) + \frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{\left(-b^4+6ab^2c-6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 -
4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
b^2 - 4*a*c)^(3/2))/(2*a^3)
```

Maple [A]
time = 0.06, size = 213, normalized size = 1.31

method	result

default	$\frac{-\frac{\frac{ac(2ac-b^2)x^2}{4ac-b^2}+\frac{ab(3ac-b^2)}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\frac{(-4abc^2+b^3c)\ln(cx^4+bx^2+a)}{c}+\frac{4\left(3a^2c^2-5ab^2c+b^4-\frac{(-4abc^2+b^3c)b}{2c}\right)\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^3}-\cdots$
risch	$\frac{-\frac{c(3ac-b^2)x^4}{a^2(4ac-b^2)}-\frac{b(7ac-2b^2)x^2}{2(4ac-b^2)a^2}-\frac{1}{2a}}{x^2(cx^4+bx^2+a)}-\frac{2b\ln(x)}{a^3}+\left(\sum_{\_R=\mathrm{RootOf}((64a^6c^3-48a^5b^2c^2+12b^4a^4c-b^6a^3)\_Z^2+(-64a^3bc^3+48a^2b^3c\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x^2+a*b*(3*a*c-b^2)/(4*a*c-b^2))/(c*
x^4+b*x^2+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c)/c*ln(c*x^4+b*x^2+a)+2*(3
*a^2*c^2-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))-1/2/a^2/x^2-2*b*ln(x)/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(154) = 308$.
time = 0.48, size = 1007, normalized size = 6.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12
*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6*a
*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 -
6*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
+ b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)
```

```
*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4
*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 +
 (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^
2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*
b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2
+ a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*
a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*
b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b
*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.72, size = 182, normalized size = 1.12

$$\frac{(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}}-\frac{2\,b^2cx^4-6\,ac^2x^4+2\,b^3x^2-7\,abcx^2+ab^2-4\,a^2c}{2\,(cx^6+bx^4+ax^2)(a^2b^2-4\,a^3c)}+\frac{b\log\left(cx^4+bx^2+a\right)}{2\,a^3}-\frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^2*c*x^4 - 6*a*c^2*x^4 + 2*b
^3*x^2 - 7*a*b*c*x^2 + a*b^2 - 4*a^2*c)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 -
 4*a^3*c)) + 1/2*b*log(c*x^4 + b*x^2 + a)/a^3 - b*log(x^2)/a^3
```

Mupad [**B**]
time = 8.81, size = 2500, normalized size = 15.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)^2),x)

[Out] (log(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))
/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (1/(2*a) - (x
^2*(2*b^3 - 7*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(3*a*c - b^2))/(a^2*(4
*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) - (2*b*log(x))/a^3 + (atan(((2*a^9*b^
6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4
*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 3*a^3*c^3
 + 36*a^2*b^2*c^2 - 21*a*b^4*c)*((4*(2*b^5*c^4 - 12*a*b^3*c^5 + 18*a^2*b*c^
6))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) + (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3
+ 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) -
(((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6
*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9
*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 1
6*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^
3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2)) + (((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c
^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3
 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^
6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2))
- ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^
2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(a^3*(4*a*c - b^2)
^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^
2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^
2 - 6*a*b^2*c)^2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*a^6
*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4
- 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b^8
*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)) - (x^2*((((4*(54*a^
3*c^8 - 2*b^6*c^5 + 18*a*b^4*c^6 - 54*a^2*b^2*c^7))/(a^6*b^6 - 64*a^9*c^3 -
 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(276*a^5*b*c^7 - 6*a^2*b^7*c^4 + 65*
a^3*b^5*c^5 - 233*a^4*b^3*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2) - (((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*
c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6
 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a
^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3
- 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 1
2*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^
7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (((((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^
```

```
5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a
*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4
- 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)
*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2
- 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*
(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*
b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(a^3*(4*a*c
- b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^
6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^
2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7 -
 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^
2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(2*a^6*(4*a*c - b^
2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a
^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c
^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b
^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((((4*(480*a^8*c^7 - a^4*b^8
*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9
*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3
*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a
^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a
^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 +
 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 -
6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2...
```

3.867 $\int \frac{x^8}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=331

$$\frac{(3b^2-10ac)\,x}{2c^2\,(b^2-4ac)}-\frac{bx^3}{2c\,(b^2-4ac)}+\frac{x^5(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{\left(3b^3-13abc-\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\cdots\right.}{2\sqrt{2}\,c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-\cdots}}}$$

```
[Out] 1/2*(-10*a*c+3*b^2)*x/c^2/(-4*a*c+b^2)-1/2*b*x^3/c/(-4*a*c+b^2)+1/2*x^5*(b*
x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*c+(-20*a^2*c^2+19*a*b^2*c-3*b^4)/(-4*a*
c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4
*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*c+(20
*a^2*c^2-19*a*b^2*c+3*b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1134, 1293, 1180, 211}

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{5/2}\,(b^2-4ac)\,\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(3b^2-10ac)}{2c^2\,(b^2-4ac)}-\frac{bx^3}{2c\,(b^2-4ac)}+\frac{x^5(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) +
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*
b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
```

```
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^8}{(a+bx^2+cx^4)^2}\,dx = \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{x^4(10a+3bx^2)}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{x^2(9ab+3(3b^2-10ac)x^2)}{a+bx^2+cx^4}\,dx}{6c(b^2-4ac)}$$

$$= \frac{(3b^2-10ac)x}{2c^2(b^2-4ac)} - \frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{3a(3b^2-10ac)+3b(3b \cdots}{a+bx^2+cx^4 \cdots}}{6c^2(b^2-4 \cdots}$$

$$= \frac{(3b^2-10ac)x}{2c^2(b^2-4ac)} - \frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3-13abc-\cdots\right.}{\cdots}$$

$$= \frac{(3b^2-10ac)x}{2c^2(b^2-4ac)} - \frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3-13abc-\cdots\right.}{2\sqrt{2}\cdots}$$

Mathematica [A]

time = 0.42, size = 327, normalized size = 0.99

$$\frac{4\sqrt{c}\,x-\frac{2\sqrt{c}\,x\left(2a^2c-b^3x^2-ab(b-3cx^2)\right)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\sqrt{2}\left(-3b^4+19ab^2c-20a^2c^2+3b^3\sqrt{b^2-4ac}-13abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(3b^4-19ab^2c+20a^2c^2+3b^3\sqrt{b^2-4ac}-13abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2
 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^2
 + 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[b^2 -
 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c
^(5/2))
```

Maple [A]

time = 0.07, size = 319, normalized size = 0.96

method	result
risch	$\frac{x}{c^2}+\frac{\frac{b(3ac-b^2)x^3}{8ac-2b^2}+\frac{a(2ac-b^2)x}{8ac-2b^2}}{c^2(cx^4+bx^2+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4c+\_Z^2b+a)}\frac{\left(-\frac{b(13ac-3b^2)\_R^2}{4ac-b^2}-\frac{(10ac-3b^2)a}{4ac-b^2}\right)\ln(x-\_R)}{2\_R^3c+\_Rb}}{4c^2}$
default	$\frac{x}{c^2}-\frac{-\frac{b(3ac-b^2)x^3}{2(4ac-b^2)}-\frac{a(2ac-b^2)x}{2(4ac-b^2)}}{cx^4+bx^2+a}+\frac{2c\left(-\frac{\left(13\sqrt{-4ac+b^2}\,abc-3\sqrt{-4ac+b^2}\,b^3+20a^2c^2-19ab^2c+3b^4\right)\sqrt{2}\,\mathrm{arctanh}\left(\cdots\right)}{8c\sqrt{-4ac+b^2}\sqrt{(-b+\sqrt{-4ac+b^2})c\cdots}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^2-1/c^2*((-1/2*b*(3*a*c-b^2)/(4*a*c-b^2)*x^3-1/2*a*(2*a*c-b^2)/(4*a*c-b
^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(13*(-4*a*c+b^2)^(1/2)*a*b*c-3
*(-4*a*c+b^2)^(1/2)*b^3+20*a^2*c^2-19*a*b^2*c+3*b^4)/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))+1/8*(13*(-4*a*c+b^2)^(1/2)*a*b*c-3*(-4*a*c+b^2)^(1/2)*
b^3-20*a^2*c^2+19*a*b^2*c-3*b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b
^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*integrate(-(3*a*b^
2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4
*a*c^3) + x/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2856 vs. 2(285) = 570.
time = 0.59, size = 2856, normalized size = 8.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/2)*(a*b^2*
c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt
(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*
b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^
2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64
*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5
*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^
3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 +
336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^
6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385
*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b
^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*s
qrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12
*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051
*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*
a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818
```

```
*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6
 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c +
385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^1
3)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(
a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2
)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 -
 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 25
00*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8
818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b^7*
c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c
^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b
^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2
)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*
x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c
^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 -
 2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2
- 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b
^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 -
 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10
 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^
5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^
2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a
^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(3*
a*b^2 - 10*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^
3*c^2 - 4*a*b*c^3)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3339 vs. $2(285) = 570$.
time = 5.30, size = 3339, normalized size = 10.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b*c*x^3 + a*b^2*x - 2*a^2*c*x)/((c*x^4 + b*x^2 + a)*(b^2
*c^2 - 4*a*c^3)) + x/c^2 - 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8
 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 19
2*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50
*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 +
13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b
^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 + 2*(
3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^5*c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 160*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2
```

```
- 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^6 + 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7
- 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^2*c^5 -
80*(b^2 - 4*a*c)*a^3*c^6)*abs(-b^2*c^2 + 4*a*c^3))*arctan(2*sqrt(1/2)*x/sqr
t((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^
2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4
*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c
^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)*abs
(c)) - 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 +
 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*
c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5
*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c
^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 +
464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 +
 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7
+ 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 -
320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 1
60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 -
96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 +
80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*
(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*
b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2
*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 +
25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2
+ 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 + 2*(3*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 6*a*b
^6*c^4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 ...
```

Mupad [B]
time = 1.57, size = 2500, normalized size = 7.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2 + c*x^4)^2,x)
```

[Out] ((b*x^3*(3*a*c - b^2))/(2*(4*a*c - b^2)) + (a*x*(2*a*c - b^2))/(2*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^

2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 128
0*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 -
192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 2
40*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))
^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*
a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a
^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7 + 48*a*b^8*c^3 -
736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b
^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 128...

3.868 $\int \frac{x^6}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=271

$$-\frac{bx}{2c\left(b^2-4ac\right)}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\cdots$$

```
[Out] -1/2*b*x/c/(-4*a*c+b^2)+1/2*x^3*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/
4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-6*a*c-b*(-8*a
*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-
6*a*c+b*(-8*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1134, 1293, 1180, 211}

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx}{2c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(b*x)/(c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
```

```
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x^3(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^2(6a+bx^2)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{ab+(b^2-6ac)x^2}{a+bx^2+cx^4}\,dx}{2c\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-6ac-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\int \frac{\ }{\frac{b}{2}-}}{4c\left(b^2-4ac\right)}$$

$$= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-6ac-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b}}$$

Mathematica [A]
time = 0.33, size = 282, normalized size = 1.04

$$\frac{-\frac{2\sqrt{c}\,x(b^2x^2+a(b-2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^3+8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^3-8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
(4*c^(3/2))
```

Maple [A]
time = 0.05, size = 279, normalized size = 1.03

method	result
risch	$\frac{-\frac{\left(2ac-b^2\right)x^3}{2c\left(4ac-b^2\right)}+\frac{abx}{2c\left(4ac-b^2\right)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c+\_Z^2b+a\right)}\frac{\left(\frac{\left(6ac-b^2\right)\_R^2}{4ac-b^2}-\frac{ab}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2\_R^3c+\_Rb}}{4c}$
default	$\frac{-\frac{\left(2ac-b^2\right)x^3}{2c\left(4ac-b^2\right)}+\frac{abx}{2c\left(4ac-b^2\right)}}{cx^4+bx^2+a}+\frac{-\frac{\left(6ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}-8abc+b^3\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)}}\right)}{4c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2*a*b/c/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a
)+2/(4*a*c-b^2)*(-1/8*(6*a*c*(-4*a*c+b^2)^(1/2)-b^2*(-4*a*c+b^2)^(1/2)-8*a*
b*c+b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(6*a*c*(-4*a*c+b^2)
^(1/2)-b^2*(-4*a*c+b^2)^(1/2)+8*a*b*c-b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2
*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2 - 6*a*c)*x^2 + a*b)/
(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2257 vs. $2(227) = 454$.

time = 0.42, size = 2257, normalized size = 8.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60
*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*
a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 8
8*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) +
sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c
^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x
- 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c
^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((
b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 6
4*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c
^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a
*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*
a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^
4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a
*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88
*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^
6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c +
```

```
 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + s
qrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^
2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x -
 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^
3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4
 + 48*a^2*b^2*c^5 - 64*a^3*c^6))))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a
^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2736 vs. $2(227) = 454$.
time = 7.37, size = 2736, normalized size = 10.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*x^3 - 2*a*c*x^3 + a*b*x)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
- 1/16*(2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*
```

c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24
*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a
*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
- 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*
c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2 - 2*(sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*
c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a
^2*b*c^4)*abs(b^2*c - 4*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^
2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3
)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a
^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2
*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) + 1/16*(2*b^8*c^4 - 32*
a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^
5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*
a*c^3)*(b^2*c - 4*a*c^2)^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c

```
)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a
^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b
*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(b^2*c - 4
*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b
*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)
))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3
*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*
abs(b^2*c - 4*a*c^2)*abs(c))
```

Mupad [B]
time = 6.00, size = 2500, normalized size = 9.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^3*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x)/(2*c*(4*a*c - b^2)))/(
a + b*x^2 + c*x^4) - atan(((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c
^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16
*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*
c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c
^3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2
*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b
^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(1
6*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a
^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9
*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^
10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b
```

```
^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)))^(1/2)*1i)/((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5
*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*
c^3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^
2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a
*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*
b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(
16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*
a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^
9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b
^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*
b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*
(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
 - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3
- 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6
144*a^5*b^2*c^8)))^(1/2) + (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c
^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16
*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))^(1/2) + (5*a^2*b^4 + 216*a^4*c^2 - 66*a^3*b^2*c)/(4*(b^6*c
 - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))...
```

3.869 $\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=237

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\mathrm{t}}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)}$$

```
[Out] 1/2*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(3/2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1134, 1180, 211}

$$\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 +
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
```

```
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{x(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.27, size = 235, normalized size = 0.99

$$\frac{1}{4}\left(\frac{2(2ax+bx^3)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/4
```

Maple [A]
time = 0.04, size = 230, normalized size = 0.97

method	result
risch	$\frac{-\frac{b\,x^{3}}{2\left(4ac-b^{2}\right)}-\frac{ax}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}c+\_Z^{2}b+a\right)}\frac{\left(-\frac{b\,\_R^{2}}{4ac-b^{2}}+\frac{2a}{4ac-b^{2}}\right)\ln\left(x-\_R\right)}{2\,\_R^{3}c+\_R b}\right)}{4}$
default	$\frac{-\frac{b\,x^{3}}{2\left(4ac-b^{2}\right)}-\frac{ax}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{2c\left(-\frac{\left(-b\sqrt{-4ac+b^{2}}+4ac+b^{2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,c\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\left(-b\sqrt{-4ac \ldots}\right.}{\ldots}\right.}{4ac-b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-
1/8*(-b*(-4*a*c+b^2)^(1/2)+4*a*c+b^2)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))+1/8*(-b*(-4*a*c+b^2)^(1/2)-4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1668 vs. 2(193) = 386.
time = 0.40, size = 1668, normalized size = 7.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*
x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)*((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b
*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b
^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*
c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2
+ 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^
2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a
*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c
^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3
+ 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4))) + 4*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [A]
time = 7.35, size = 296, normalized size = 1.25

$$\frac{-2ax - bx^3}{8a^2c - 2ab^2 + x^4 \cdot (8ac^2 - 2b^2c) + x^2 \cdot (8abc - 2b^3)} + \operatorname{RootSum}\left(t^4 \cdot (1048576a^6c^7 - 1572864a^5b^2c^6 + 983040a^4b^4c^5 - 327680a^3b^6c^4 + 61440a^2b^8c^3 - 6144ab^{10}c^2 + 256b^{12}c) + t^2(-12288a^4bc^4 + 8192a^3b^3c^3 - 1536a^2b^5c^2 + 16b^9) + 16a^3c^2 + 24a^2b^2c + 9ab^4, \left(t \mapsto t\log\left(x + \frac{16384t^3a^3bc^4 - 12288t^3a^2b^3c^3 + 3072t^3ab^5c^2 - 256t^3b^7c + 64ta^2c^2 - 128tab^2c - 4tb^4}{4ac + 3b^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-2*a*x - b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*
(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7 - 1572864*a**5*b**2*
c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b**6*c**4 + 61440*a**2*b**8*c**3
 - 6144*a*b**10*c**2 + 256*b**12*c) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3
*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_t**3*a
**2*b**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a**2*c**2
 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2132 vs. $2(193) = 386$.
time = 4.15, size = 2132, normalized size = 9.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) + 1/16*(2*b^7*c^2 -
 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*(b^2 - 4*a*c)^2 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*
c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2
 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arctan(2*s
qrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*
c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a
*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^
3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) - 1/16*(2*b^
7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
```

```
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 -
 (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 -
 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2 + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2
*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 -
 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arc
tan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 -
 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^
2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4
- 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c))
```

Mupad [B]
time = 5.91, size = 2500, normalized size = 10.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4
)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^
2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(
b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3
- 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*((
(-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b
^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(b^4*c + 8
*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b
^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i - (((2048*a^4*c^5 - 32*a
*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^
```

2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i)/((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (4*a^2*b*c^2 + 3*a*b^3*c)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*2i - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)

```
*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^
4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^1
0*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^
2*c^6)))^(1/2) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240
*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*...
```

3.870 $\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=221

$$-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right.}{\sqrt{2}\,(b^2-4a}$$

```
[Out] -1/2*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*arctan(x*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1133, 1180, 211}

$$\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(2*b
- Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*
b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
```

```
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(1+\frac{2b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.29, size = 222, normalized size = 1.00

$$\frac{-bx-2cx^3}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(-2*b +
 Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
 + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.07, size = 271, normalized size = 1.23

method	result
risch	$\frac{\frac{c\,x^3}{4ac-b^2}+\frac{bx}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 c+\_Z^2 b+a\right)}\frac{\left(\frac{2c\_R^2}{4ac-b^2}-\frac{b}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2\_R^3 c+\_R b}\right)}{4}$
default	$16c^2\left(\frac{\frac{\sqrt{-4ac+b^2}\,x}{8c\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}+\frac{\left(b+\frac{\sqrt{-4ac+b^2}}{2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{4(4ac-b^2)c\sqrt{-4ac+b^2}}+\frac{\frac{\sqrt{-\cdots}}{8c\left(x^2+\frac{b}{2c}-\cdots\right.}}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*(1/8/c*(-4*a*c+b^2)^(1/2)*x/(x
^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/4*(b+1/2*(-4*a*c+b^2)^(1/2))*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)))+1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*(1/8/c*(-4*a*c+b^2)^(1/2)
*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))-1/4*(-b+1/2*(-4*a*c+b^2)^(1/2))*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*
c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1680 vs. $2(180) = 360$.
time = 0.41, size = 1680, normalized size = 7.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c
^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*
c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6
*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3
*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*
((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
+ 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256
*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a
^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2
- 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2
)*x - 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c
+ 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^
2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + 2*b*x)/((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1969 vs. $2(180) = 360$.
time = 5.34, size = 1969, normalized size = 8.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) + 1/8*(4*b^6*c^2 -
 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*
c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)
^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c
^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3
- 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/
((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^
4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4
*a*c)*abs(c)) + 1/8*(4*b^6*c^2 - 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*
c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
```

```
- 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*
(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2 + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*
c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/
sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c -
 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3
*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4*a*c)*abs(c))
```

Mupad [B]
time = 1.35, size = 2500, normalized size = 11.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1
/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 +
 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a
^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*
b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)
^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*
b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2
*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*
b^2*c^5)))^(1/2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2
*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*
a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^
3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^
5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(
((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*
b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 128
0*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4
- 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) -
b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*
```

a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*1i)/((4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*2i + atan(((((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)...

3.871 $\int \frac{1}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=252

$$\frac{x(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(b^2-12\ldots\right.}{2\ldots}$$

```
[Out] 1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/
2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)
^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1106, 1180, 211}

$$\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(-2ac+b^2+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{x(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{b^2-2ac-2(b^2-4ac)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\,(b^2-4ac)}$$

$$= \frac{x(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+ \cdots}}{4a\,(b^2-4ac)^{3/2}}$$

$$= \frac{x(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\sqrt{c}\,\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{b-\sqrt{b^2\cdots}}}\right.}{2\sqrt{2}\,a\,(b^2-4ac)^{3/2}\,\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.28, size = 243, normalized size = 0.96

$$\frac{\frac{2x(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\,\sqrt{c}\,\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\,\sqrt{c}\,\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [A]
time = 0.06, size = 320, normalized size = 1.27

method	result
risch	$\frac{-\frac{b c x^{3}}{2 a \left(4 a c-b^{2}\right)}+\frac{\left(2 a c-b^{2}\right) x}{2 a \left(4 a c-b^{2}\right)}}{c x^{4}+b x^{2}+a}+\frac{\sum_{\_R =\operatorname{RootOf}\left(\_Z^{4} c+\_Z^{2} b+a\right)}\frac{\left(-\frac{b c \_R^{2}}{4 a c-b^{2}}+\frac{6 a c-b^{2}}{4 a c-b^{2}}\right) \ln \left(x-\_R \right)}{2 \_R^{3} c+\_R b}}{4 a}$
default	$16c^{2}\left(-\frac{\frac{\left(b \sqrt{-4 a c+b^{2}}+4 a c-b^{2}\right) x}{16 a \,c^{2} \left(x^{2}+\frac{\sqrt{-4 a c+b^{2}}}{2 c}+\frac{b}{2 c}\right)}+\frac{\left(b \sqrt{-4 a c+b^{2}}+12 a c-b^{2}\right) \sqrt{2}\, \arctan \left(\frac{c x \sqrt{2}}{\sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}\right)}{16 a c \sqrt{\left(b+\sqrt{-4 a c+b^{2}}\right) c}}}{4 \sqrt{-4 a c+b^{2}}\, \left(4 a c-b^{2}\right)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(-1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*(1/16/a/c^2*(b*(-4*a*c+b^2)^(1/
2)+4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/16*(b*(-4*a*c+b^2)
^(1/2)+12*a*c-b^2)/a/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^
2)*(-1/16/a/c^2*(4*a*c-b^2-b*(-4*a*c+b^2)^(1/2))*x/(x^2+1/2*b/c-1/2/c*(-4*a
*c+b^2)^(1/2))-1/16*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))/a/c*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. $2(206) = 412$.
time = 0.46, size = 2309, normalized size = 9.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*

```
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. $2(206) = 412$.
time = 4.28, size = 2682, normalized size = 10.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1
/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*
b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^
3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c
)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
```

*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) + 1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c))

Mupad [B]
time = 6.00, size = 2500, normalized size = 9.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^2,x)
[Out] ((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 +
 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4
*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
 - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 38
40*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840
*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*
b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
- b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 384
0*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2
 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2
)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4
*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 409
6*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b
^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a
^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-
(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1
504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c
^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^
5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^
5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11
+ b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
```

3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*
b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16
*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 24
0*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(
1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4
- 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^
8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(10
24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2
*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*
b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27
*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 2
4*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5...

3.872 $\int \frac{1}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\left(3b^3-16abc+\left(3b^2-10ac\right)\sqrt{b^2-4ac}\right)\tan^{-1}\Bigg(}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4a}}}$$

```
[Out] 1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2
)/x/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c
+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-
4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A]
time = 0.91, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1135, 1295, 1180, 211}

$$-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a^2\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{3b^2-10ac}{2a^2x\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(3*b^2 - 10*a*c)/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*
(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2
- 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*
c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
```

```
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-3b^2+10ac-3bcx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}\\ &= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{\int \frac{-b\left(3b^2-13ac\right)-c\left(3b^2-10ac\right)x^2}{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)}\\ &= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}-\frac{\left(c\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-}}\right.\right.}{}\\ &= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\left(3b^2-10ac+\frac{3}{\sqrt{b^2-}}\right.}{2\sqrt{2}}\end{aligned}$$

Mathematica [A]

time = 0.39, size = 302, normalized size = 0.98

$$\frac{-\frac{4}{x}-\frac{2x(b^3-3abc+b^2cx^2-2ac^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4
*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2
]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a
*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [A]
time = 0.06, size = 294, normalized size = 0.95

method	result
default	$-\frac{\frac{c(2ac-b^2)x^3}{8ac-2b^2}+\frac{b(3ac-b^2)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(-\frac{\left(10ac\sqrt{-4ac+b^2}-3b^2\sqrt{-4ac+b^2}+16abc-3b^3\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{(-b+\sqrt{-4a}}}\right)}{8\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{a}$
risch	$\frac{-\frac{c(10ac-3b^2)x^4}{2a^2(4ac-b^2)}-\frac{b(11ac-3b^2)x^2}{2(4ac-b^2)a^2}-\frac{1}{a}}{x(c\,x^4+b\,x^2+a)}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4096a^{11}c^6-6144a^{10}b^2c^5+3840a^9b^4c^4-1280a^8b^6c^3+240a^7b^8c^2-24a^6b^{10}c+a^5b^{12}\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((1/2*c*(2*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*b*(3*a*c-b^2)/(4*a*c-b^2)*x)
/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(10*a*c*(-4*a*c+b^2)^(1/2)-3*b^2*(-4
*a*c+b^2)^(1/2)+16*a*b*c-3*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/
8*(10*a*c*(-4*a*c+b^2)^(1/2)-3*b^2*(-4*a*c+b^2)^(1/2)-16*a*b*c+3*b^3)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-1/a^2/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*x^2
)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a
^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*x^2)/(c*
x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2912 vs. $2(260) = 520$.
time = 0.56, size = 2912, normalized size = 9.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c
)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3
+ (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
+ 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c
 + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^
5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*
a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
- 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c
^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*
b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^
5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7
*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-
(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sq
rt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 144
08*a^4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*
c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*
```

```
c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/
2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*
a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b
^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^
7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*
c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7
*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (3*a^5*b^10
 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1
280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^
6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 -
4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a
^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 -
1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sqrt(1/2)*(27*b^1
1 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
- 5200*a^5*b*c^5 + (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*
b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3
*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^
3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4*a^3*c^2
)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3087 vs. $2(260) = 520$.
time = 5.70, size = 3087, normalized size = 10.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c*x^4 - 10*a*c^2*x^4 + 3*b^3*x^2 - 11*a*b*c*x^2 + 2*a*b^2 - 8*a
^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*(6*a^4*b^8*c^2 - 8
0*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 5
6*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + (6*b^4*c^2 -
44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^
2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*
a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2 + 2*(3*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c
- 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 5
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 -
304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 41
6*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a^2*b^5*c - 50*(b^2 - 4*a*c)*a^3*b^3*c^2 + 10
4*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a^3*c))*arctan(2*sqrt(1/2)*x/sqr
t((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*
c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b
^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8
*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*ab
```

```
s(c)) + 1/16*(6*a^4*b^8*c^2 - 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^
2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8
 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c +
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 17
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 5
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 25
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 1
28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 +
28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 -
64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 -
6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a
*c)*a^6*b^2*c^4 + (6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*
b^2 - 4*a^3*c)^2 - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 3
7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 74*a^3*
b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50
*(b^2 - 4*a*c)*a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a
^3*c))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a...
```

Mupad [B]
time = 6.72, size = 2500, normalized size = 8.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - atan((((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8
*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(85
```

1968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i - ((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -

25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + ((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)...

3.873 $\int \frac{x^{11}}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=209

$$-\frac{b(b^2-7ac)\,x^2}{2c^2\,(b^2-4ac)^2}+\frac{x^8(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{x^4(a(b^2-16ac)+b(b^2-10ac)\,x^2)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{b(b^4-10ab^2c+30a^2c}{2c^3\,(b^2}$$

```
[Out] -1/2*b*(-7*a*c+b^2)*x^2/c^2/(-4*a*c+b^2)^2+1/4*x^8*(b*x^2+2*a)/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)^2+1/4*x^4*(a*(-16*a*c+b^2)+b*(-10*a*c+b^2)*x^2)/c/(-4*a*c+
b^2)^2/(c*x^4+b*x^2+a)+1/2*b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x^2+b
)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(5/2)+1/4*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A]
time = 0.25, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1128, 752, 832, 787, 648, 632, 212, 642}

$$\frac{b(30a^2c^2-10ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2-4ac)^{5/2}}-\frac{bx^2(b^2-7ac)}{2c^2\,(b^2-4ac)^2}+\frac{x^4(bx^2(b^2-10ac)+a(b^2-16ac))}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{x^8(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{\log(a+bx^2+cx^4)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/2*(b*(b^2 - 7*a*c)*x^2)/(c^2*(b^2 - 4*a*c)^2) + (x^8*(2*a + b*x^2))/(4*(
b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^4*(a*(b^2 - 16*a*c) + b*(b^2 - 10*
a*c)*x^2))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c
 + 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*
c)^(5/2)) + Log[a + b*x^2 + c*x^4]/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
```

```
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^{11}}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^5}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= \frac{x^8(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\text{Subst}\left(\int \frac{x^3(8a+bx)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4(b^2-4ac)}$$

$$= \frac{x^8(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^4(a(b^2-16ac)+b(b^2-10ac)x^2)}{4c(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x(2a}{}\right.}{}$$

$$= -\frac{b(b^2-7ac)x^2}{2c^2(b^2-4ac)^2} + \frac{x^8(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^4(a(b^2-16ac)+b(b^2-10ac)}{4c(b^2-4ac)^2(a+bx^2+cx^4}$$

$$= -\frac{b(b^2-7ac)x^2}{2c^2(b^2-4ac)^2} + \frac{x^8(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^4(a(b^2-16ac)+b(b^2-10ac)}{4c(b^2-4ac)^2(a+bx^2+cx^4}$$

$$= -\frac{b(b^2-7ac)x^2}{2c^2(b^2-4ac)^2} + \frac{x^8(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^4(a(b^2-16ac)+b(b^2-10ac)}{4c(b^2-4ac)^2(a+bx^2+cx^4}$$

$$= -\frac{b(b^2-7ac)x^2}{2c^2(b^2-4ac)^2} + \frac{x^8(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^4(a(b^2-16ac)+b(b^2-10ac)}{4c(b^2-4ac)^2(a+bx^2+cx^4}$$

Mathematica [A]
time = 0.21, size = 244, normalized size = 1.17

$$\frac{\frac{-b^6+11ab^4c-39a^2b^2c^2+32a^3c^3+4b^5cx^2-30ab^3c^2x^2+50a^2bc^3x^2}{(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{2a^3c^2+b^5x^2+ab^3(b-5cx^2)+a^2bc(-4b+5cx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{2bc(b^4-10ab^2c+30a^2c^2)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}} + c\log(a+bx^2+cx^4)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x^2 - 30*a*b^3*
c^2*x^2 + 50*a^2*b*c^3*x^2)/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (2*a^3*
c^2 + b^5*x^2 + a*b^3*(b - 5*c*x^2) + a^2*b*c*(-4*b + 5*c*x^2))/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[
(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + b*x^2 +
 c*x^4])/(4*c^4)
```

Maple [A]
time = 0.10, size = 367, normalized size = 1.76

method	result
default	$\frac{\frac{b\left(25a^2c^2-15a\,b^2c+2b^4\right)x^6}{c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(32a^3c^3+11a^2b^2c^2-19a\,b^4c+3b^6\right)x^4}{2c^3\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{ab\left(31a^2c^2-22a\,b^2c+3b^4\right)x^2}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^3}+\frac{3a^2\left(8a^2c^2-7a\,b^2c+b^4\right)}{2c^3\left(16a^2c^2-8a\,b^2c+b^4\right)}}{2(c\,x^4+b\,x^2+a)^2}+\frac{\left(16a^2c^2-8a\,b\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1
/2*(32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^4+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^
2+3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+
b*x^2+a)^2+1/2/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*a^2*c^2-8*a*b^2*c+b^
4)/c*ln(c*x^4+b*x^2+a)+2*(-7*a^2*b*c+a*b^3-1/2*(16*a^2*c^2-8*a*b^2*c+b^4)*b
/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(195) = 390$.
time = 0.41, size = 1631, normalized size = 7.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
- 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^6 + (3*b^8 - 31*a*b^6*c
+ 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(3*a*b^7 - 34*a^2*
b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x^2 + ((b^5*c^2 - 10*a*b^3*c^3 + 3
```

```
0*a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*
b^4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*
b*c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*
c))/(c*x^4 + b*x^2 + a)) + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*
c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^
6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b
^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*
a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3
*c^5 - 64*a^4*b*c^6)*x^2), 1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2
- 96*a^5*c^3 + 2*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*
x^6 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*
x^4 + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x^2 + 2*
((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*
a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*
c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b
^2*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b
^2 - 4*a*c)) + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8
+ a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^
5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4
*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3
*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*c^3 - 12*a^3
*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b
^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*
a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6
- 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^
4*b*c^6)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.80, size = 306, normalized size = 1.46

$$-\frac{\left(b^{5}-10\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(b^{4}c^{3}-8\,ab^{2}c^{4}+16\,a^{2}c^{5}\right)\sqrt{-b^{2}+4\,ac}}-\frac{3\,b^{4}c^{2}x^{8}-24\,ab^{2}c^{3}x^{8}+48\,a^{2}c^{4}x^{8}-2\,b^{5}cx^{6}+12\,ab^{3}c^{2}x^{6}-4\,a^{2}bc^{3}x^{6}-3\,b^{6}x^{4}+20\,ab^{4}cx^{4}-22\,a^{2}b^{2}c^{2}x^{4}+32\,a^{3}c^{3}x^{4}-6\,ab^{5}x^{2}+40\,a^{2}b^{3}cx^{2}-28\,a^{3}bc^{2}x^{2}-3\,a^{2}b^{4}+18\,a^{3}b^{2}c}{8\,\left(b^{4}c^{3}-8\,ab^{2}c^{4}+16\,a^{2}c^{5}\right)\left(cx^{4}+bx^{2}+a\right)^{2}}+\frac{\log\left(cx^{4}+bx^{2}+a\right)}{4\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a
*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) - 1/8*(3*b^4
*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8 - 2*b^5*c*x^6 + 12*a*b^3*c^2*x
^6 - 4*a^2*b*c^3*x^6 - 3*b^6*x^4 + 20*a*b^4*c*x^4 - 22*a^2*b^2*c^2*x^4 + 32
*a^3*c^3*x^4 - 6*a*b^5*x^2 + 40*a^2*b^3*c*x^2 - 28*a^3*b*c^2*x^2 - 3*a^2*b^
4 + 18*a^3*b^2*c)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2 + a)
^2) + 1/4*log(c*x^4 + b*x^2 + a)/c^3
```

Mupad [B]
time = 7.30, size = 2588, normalized size = 12.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^4*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(4*c^3*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) + (x^2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2))/(2*c^
3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*(a*b^4 + 8*a^3*c^2 - 7*a^2*b^2*c))
/(4*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^6*(2*b^4 + 25*a^2*c^2 - 15*a
*b^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 +
 c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (log((a/c^4 + ((c^3*(-(b^2*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(1/2) - 1)*((8*a)/c + (2*(c^3
*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(1/2) - 1
)*(2*a + b*x^2))/c + (2*b*x^2*(3*b^4 + 62*a^2*c^2 - 26*a*b^2*c))/(c*(4*a*c
- b^2)^2)))/(4*c^3) + (x^2*(b^5 + 23*a^2*b*c^2 - 9*a*b^3*c))/(c^4*(4*a*c -
b^2)^2))*(a/c^4 - ((c^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a
*c - b^2)^5))^(1/2) + 1)*((8*a)/c - (2*(c^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a
*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(1/2) + 1)*(2*a + b*x^2))/c + (2*b*x^2*(3
*b^4 + 62*a^2*c^2 - 26*a*b^2*c))/(c*(4*a*c - b^2)^2)))/(4*c^3) + (x^2*(b^5
+ 23*a^2*b*c^2 - 9*a*b^3*c))/(c^4*(4*a*c - b^2)^2)))*(2*b^10 - 2048*a^5*c^5
 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*
(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*
c^6 - 5120*a^4*b^2*c^7)) - (b*atan(((x^2*(((b*((6*b^5*c^3 - 52*a*b^3*c^4 +
124*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + ((8*b^5*c^6 - 64*a*b^
3*c^7 + 128*a^2*b*c^8)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*
b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^
2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a
^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(8*c^3*(4
*a*c - b^2)^(5/2)) + (b*(8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(b^4 + 3
0*a^2*c^2 - 10*a*b^2*c)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3
*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(16*c^3*(4*a*c - b^2)^(5/2)*(16*
```

a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))/(a*(4*a*c - b^2)^2) - (b*((b^5 + 23*a^2*b*c^2 - 9*a*b^3*c)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + (((6*b^5*c^3 - 52*a*b^3*c^4 + 124*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) - (b^2*((b^5*c^6)/2 - 4*a*b^3*c^7 + 8*a^2*b*c^8)*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5))))/(2*a*(4*a*c - b^2)^(5/2))) + ((b*((8*a)/c + (8*a*c^2*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(8*c^3*(4*a*c - b^2)^(5/2)) + (a*b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(c*(4*a*c - b^2)^(5/2)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))/(a*(4*a*c - b^2)^2) - (b*(a/c^4 + (((8*a)/c + (8*a*c^2*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) - (a*b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^4*(4*a*c - b^2)^5)))/(2*a*(4*a*c - b^2)^(5/2)))*(32*a^2*c^6*(4*a*c - b^2)^5 + 2*b^4*c^4*(4*a*c - b^2)^5 - 16*a*b^2*c^5*(4*a*c - b^2)^5))/(b^10 + 160*a^2*b^6*c^2 - 600*a^3*b^4*c^3 + 900*a^4*b^2*c^4 - 20*a*b^8*c))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(2*c^3*(4*a*c - b^2)^(5/2))

3.874 $\int \frac{x^9}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=121

$$\frac{x^6(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3ax^2(2a+bx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] 1/4*x^6*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/2*a*x^2*(b*x^2+2*a)/(-
4*a*c+b^2)^2/(c*x^4+b*x^2+a)-6*a^2*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(5/2)
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1128, 736, 632, 212}

$$-\frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{3ax^2(2a+bx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{x^6(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^6*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*a*x^2*(2*a
 + b*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*a^2*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
```

```
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{(a+bx^2+cx^4)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx+cx^2)^3}\,dx, x, x^2\right)\\
&= \frac{x^6(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{(3a)\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{2\,(b^2-4ac)}\\
&= \frac{x^6(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{3ax^2(2a+bx^2)}{2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{a+bx}\right.}{(b^2-4a}\\
&= \frac{x^6(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{3ax^2(2a+bx^2)}{2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{(6a^2)\,\text{Subst}\left(\int \frac{}{b^2-4}\right.}{(b^2-}\\
&= \frac{x^6(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{3ax^2(2a+bx^2)}{2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{6a^2\tanh^{-1}\left(\frac{b+2}{\sqrt{b^2-}}\right.}{(b^2-4ac)^{5/}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 194, normalized size = 1.60

$$\frac{1}{4}\left(\frac{b^5-8ab^3c+22a^2bc^2-2b^4cx^2+16ab^2c^2x^2-20a^2c^3x^2}{c^3\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{b^4x^2+ab^2(b-4cx^2)+a^2c(-3b+2cx^2)}{c^3\,(-b^2+4ac)\,(a+bx^2+cx^4)^2}+\frac{24a^2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x^2 + 16*a*b^2*c^2*x^2 - 20*a^2*
c^3*x^2)/(c^3*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^4*x^2 + a*b^2*(b -
4*c*x^2) + a^2*c*(-3*b + 2*c*x^2))/(c^3*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^
2) + (24*a^2*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
)/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(113) = 226$.
time = 0.08, size = 267, normalized size = 2.21

method	result
default	$\frac{-\frac{(10a^2c^2-8a\,b^2c+b^4)x^6}{c(16a^2c^2-8a\,b^2c+b^4)}+\frac{b(2a^2c^2+8a\,b^2c-b^4)x^4}{2c^2(16a^2c^2-8a\,b^2c+b^4)}-\frac{a(6a^2c^2-10a\,b^2c+b^4)x^2}{(16a^2c^2-8a\,b^2c+b^4)c^2}+\frac{a^2b(10ac-b^2)}{2c^2(16a^2c^2-8a\,b^2c+b^4)}}{2(c\,x^4+b\,x^2+a)^2}+\frac{6a^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-}}\right.}{(16a^2c^2-8a\,b^2c+b^4)\sqrt{4ac}}$
risch	$\frac{-\frac{(10a^2c^2-8a\,b^2c+b^4)x^6}{2c(16a^2c^2-8a\,b^2c+b^4)}+\frac{b(2a^2c^2+8a\,b^2c-b^4)x^4}{4c^2(16a^2c^2-8a\,b^2c+b^4)}-\frac{a(6a^2c^2-10a\,b^2c+b^4)x^2}{2(16a^2c^2-8a\,b^2c+b^4)c^2}+\frac{a^2b(10ac-b^2)}{4c^2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}-\frac{3a^2\ln\left(\left((-4ac+b^2)^{\frac{5}{2}}-16a\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*b*(
2*a^2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-a*(6*a^2*c^2-10
*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2+1/2*a^2*b*(10*a*c-b^2)/c^2
/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+6*a^2/(16*a^2*c^2-8*a*b^2*c+
b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(113) = 226$.
time = 0.36, size = 973, normalized size = 8.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 42
*a^2*b^2*c^3 - 40*a^3*c^4)*x^6 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3
```

```
*b*c^3)*x^4 + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x^2 -
12*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^3*b*c^2*x^2 + a^4*c^2 + (a^2*b^2*c^
2 + 2*a^3*c^3)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*
a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a^2*b^6*c^2 -
 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 4
8*a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^
5 - 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b
^2*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4
- 64*a^4*b*c^5)*x^2), -1/4*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*
c - 12*a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^6 + (b^7 - 12*a*b^5*c + 3
0*a^2*b^3*c^2 + 8*a^3*b*c^3)*x^4 + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2
 - 24*a^4*c^3)*x^2 + 24*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^3*b*c^2*x^2 +
a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 4
8*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*
a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x
^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^
6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x^2
)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(112) = 224$.
time = 2.40, size = 554, normalized size = 4.58

$$-3a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-36a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+3a^2b}{6a^2c}\right)+3a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+36a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}-3a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+3a^2b}{6a^2c}\right)+\frac{10a^3bc-a^2b^3+x^6(-20a^2c^3+16ab^2c^2-2b^4c)+x^4\cdot(2a^2bc^2+8ab^3c-b^5)+x^2(-12a^3c^2+20a^2b^2c-2ab^4)}{64a^4c^4-32a^3b^2c^3+4a^2b^4c^2+x^8\cdot(64a^2c^6-32ab^2c^5+4b^4c^4)+x^6\cdot(128a^2bc^5-64ab^3c^4+8b^5c^3)+x^4\cdot(128a^3c^5-24ab^4c^3+4b^6c^2)+x^2\cdot(128a^3bc^4-64a^2b^3c^3+8ab^5c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**5*c**3*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**3*b**
4*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 3
*a**2*b)/(6*a**2*c)) + 3*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a*
*5*c**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**4*b**2*c**2*sqrt(-1/(4*a*c - b*
*2)**5) + 36*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a**2*b**6*sqrt(-1/(
4*a*c - b**2)**5) + 3*a**2*b)/(6*a**2*c)) + (10*a**3*b*c - a**2*b**3 + x**6
*(-20*a**2*c**3 + 16*a*b**2*c**2 - 2*b**4*c) + x**4*(2*a**2*b*c**2 + 8*a*b*
*3*c - b**5) + x**2*(-12*a**3*c**2 + 20*a**2*b**2*c - 2*a*b**4))/(64*a**4*c
**4 - 32*a**3*b**2*c**3 + 4*a**2*b**4*c**2 + x**8*(64*a**2*c**6 - 32*a*b**2
*c**5 + 4*b**4*c**4) + x**6*(128*a**2*b*c**5 - 64*a*b**3*c**4 + 8*b**5*c**3
) + x**4*(128*a**3*c**5 - 24*a*b**4*c**3 + 4*b**6*c**2) + x**2*(128*a**3*b*
c**4 - 64*a**2*b**3*c**3 + 8*a*b**5*c**2))
```

Giac [A]

time = 5.76, size = 212, normalized size = 1.75

$$\frac{6\,a^2\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{2\,b^4cx^6-16\,ab^2c^2x^6+20\,a^2c^3x^6+b^5x^4-8\,ab^3cx^4-2\,a^2bc^2x^4+2\,ab^4x^2-20\,a^2b^2cx^2+12\,a^3c^2x^2+a^2b^3-10\,a^3bc}{4\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)(cx^4+bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 6*a^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*b^4*c*x^6 - 16*a*b^2*c^2*x^6 + 20*a^2*c^3*
x^6 + b^5*x^4 - 8*a*b^3*c*x^4 - 2*a^2*b*c^2*x^4 + 2*a*b^4*x^2 - 20*a^2*b^2*
c*x^2 + 12*a^3*c^2*x^2 + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B]
time = 4.53, size = 444, normalized size = 3.67

$$\frac{6\,a^2\,\mathrm{atan}\left(\frac{\left(x^2\,\left(\frac{36\,a^3\,c^2}{{\left(4\,a\,c-b^2\right)}^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{36\,a^3\,b\,\left(16\,a^2\,b\,c^4-8\,a\,b^3\,c^3+b^5\,c^2\right)}{{\left(4\,a\,c-b^2\right)}^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{72\,a^4\,b\,c^2}{{\left(4\,a\,c-b^2\right)}^{15/2}}\right)\,\left(b^4\,{\left(4\,a\,c-b^2\right)}^5+16\,a^2\,c^2\,{\left(4\,a\,c-b^2\right)}^5-8\,a\,b^2\,c\,{\left(4\,a\,c-b^2\right)}^5\right)}{72\,a^4\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{\frac{x^6\,\left(10\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(b^3-10\,a\,b\,c\right)}{4\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^4\,\left(2\,a^2\,b\,c^2+8\,a\,b^3\,c-b^5\right)}{4\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x^2\,\left(6\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (6*a^2*atan(((x^2*((36*a^3*c^2)/((4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (36*a^3*b*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4))/((4*a*c - b^2
)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (72*a^4*b*c^2)/(4*a*c - b^2)^(1
5/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c
- b^2)^5))/(72*a^4*c^2)))/(4*a*c - b^2)^(5/2) - ((x^6*(b^4 + 10*a^2*c^2 - 8
*a*b^2*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(b^3 - 10*a*b*c))/(4
*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^4*(2*a^2*b*c^2 - b^5 + 8*a*b^3*c)
)/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x^2*(b^4 + 6*a^2*c^2 - 10*a*b
^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c
^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

3.875 $\int \frac{x^7}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=119

$$-\frac{x^6(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3bx^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] -1/4*x^6*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/4*b*x^2*(b*x^2+2*a)/(
-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3*a*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1128, 742, 736, 632, 212}

$$\frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3bx^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x^6(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x^6*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*b*x^2*(
2*a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*a*b*ArcTanh[(b +
 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
```

```
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Dist[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c))), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{(3b)\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4(b^2-4ac)}$$

$$= -\frac{x^6(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3bx^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(3ab)\text{Subst}(\int \ldots}{2(b^2 \ldots}$$

$$= -\frac{x^6(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3bx^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(3ab)\text{Subst}(\int \ldots}{(\ldots}$$

$$= -\frac{x^6(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3bx^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3ab\tanh^{-1}\left(\ldots\right)}{(b^2-4a\ldots}$$

Mathematica [A]
time = 0.13, size = 137, normalized size = 1.15

$$-\frac{8a^3c+b^4x^4+abx^2(2b^2+bcx^2+6c^2x^4)+a^2(b^2+10bcx^2+16c^2x^4)}{4c(b^2-4ac)^2(a+bx^2+cx^4)^2} - \frac{3ab\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(8*a^3*c + b^4*x^4 + a*b*x^2*(2*b^2 + b*c*x^2 + 6*c^2*x^4) + a^2*(b^2
+ 10*b*c*x^2 + 16*c^2*x^4))/(c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) - (3*
a*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(111) = 222$.
time = 0.06, size = 230, normalized size = 1.93

method	result
default	$\frac{-\frac{3abc\,x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{\left(16a^{2}c^{2}+a\,b^{2}c+b^{4}\right)x^{4}}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5ac+b^{2}\right)ab\,x^{2}}{c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a^{2}\left(8ac+b^{2}\right)}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}-\frac{3ab\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}$
risch	$\frac{-\frac{3abc\,x^{6}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(16a^{2}c^{2}+a\,b^{2}c+b^{4}\right)x^{4}}{4c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5ac+b^{2}\right)ab\,x^{2}}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a^{2}\left(8ac+b^{2}\right)}{4c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}-\frac{3ba\ln\left(\left(-\left(-4ac+b^{2}\right)^{\frac{5}{2}}-16a^{2}b\,c^{2}\right.\right.}{2(-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^2-1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-3
*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(111) = 222$.
time = 0.39, size = 892, normalized size = 7.50

$$\left[-\frac{6\,(ab^3c^2-4\,a^2bc^3)x^6+a^2b^4+4\,a^3b^2c-32\,a^4c^2+(b^6-3\,ab^4c+12\,a^2b^2c^2-64\,a^3c^3)x^4+2\,(ab^5+a^2b^3c-20\,a^3bc^2)x^2-6\,(abc^3x^8+2\,ab^2c^2x^6+2\,a^2b^2cx^2+a^3bc+(ab^3c+2\,a^2bc^2)x^4)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{4\,(a^2b^6c-12\,a^3b^4c^2+48\,a^4b^2c^3-64\,a^5c^4+(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6)x^8+2\,(b^7c^2-12\,ab^5c^3+48\,a^2b^3c^4-64\,a^3bc^5)x^6+(b^8c-10\,ab^6c^2+24\,a^2b^4c^3+32\,a^3b^2c^4-128\,a^4c^5)x^4+2\,(ab^7c-12\,a^2b^5c^2+48\,a^3b^3c^3-64\,a^4bc^4)x^2)},\ -\frac{6\,(ab^3c^2-4\,a^2bc^3)x^6+a^2b^4+4\,a^3b^2c-32\,a^4c^2+(b^6-3\,ab^4c+12\,a^2b^2c^2-64\,a^3c^3)x^4+2\,(ab^5+a^2b^3c-20\,a^3bc^2)x^2-12\,(abc^3x^8+2\,ab^2c^2x^6+2\,a^2b^2cx^2+a^3bc+(ab^3c+2\,a^2bc^2)x^4)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{4\,(a^2b^6c-12\,a^3b^4c^2+48\,a^4b^2c^3-64\,a^5c^4+(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6)x^8+2\,(b^7c^2-12\,ab^5c^3+48\,a^2b^3c^4-64\,a^3bc^5)x^6+(b^8c-10\,ab^6c^2+24\,a^2b^4c^3+32\,a^3b^2c^4-128\,a^4c^5)x^4+2\,(ab^7c-12\,a^2b^5c^2+48\,a^3b^3c^3-64\,a^4bc^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^6 + a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2
 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^4 + 2*(a*b^5 + a^2*b^3
*c - 20*a^3*b*c^2)*x^2 - 6*(a*b*c^3*x^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*x^2
 + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4
+ 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
 + a)))/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^
3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5
*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*
b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 +
 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2), -1/4*(6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^6
 + a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 -
 64*a^3*c^3)*x^4 + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x^2 - 12*(a*b*c^3*x
^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*x^2 + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*
x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)))/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3
- 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5*c
^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^
4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 4
8*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(112) = 224$.
time = 1.80, size = 524, normalized size = 4.40

$$\frac{3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3ab^7\sqrt{-\frac{1}{(4ac-b^2)^5}}+3ab^2}{6abc}\right)}{2}-\frac{3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}-3ab^7\sqrt{-\frac{1}{(4ac-b^2)^5}}+3ab^2}{6abc}\right)}{2}+\frac{-8a^3c-a^2b^2-6abc^2x^6+x^4(-16a^2c^2-ab^2c-b^4)+x^2(-10a^2bc-2ab^3)}{64a^4c^3-32a^3b^2c^2+4a^2b^4c+x^8\cdot(64a^2c^5-32ab^2c^4+4b^4c^3)+x^6\cdot(128a^2bc^4-64ab^3c^3+8b^5c^2)+x^4\cdot(128a^3c^4-24ab^4c^2+4b^6c)+x^2\cdot(128a^3bc^3-64a^2b^3c^2+8ab^5c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**4*b*c**3*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**
5*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*
b**2)/(6*a*b*c))/2 - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**4*
b*c**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**
2)**5) + 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*
c - b**2)**5) + 3*a*b**2)/(6*a*b*c))/2 + (-8*a**3*c - a**2*b**2 - 6*a*b*c**
2*x**6 + x**4*(-16*a**2*c**2 - a*b**2*c - b**4) + x**2*(-10*a**2*b*c - 2*a*
b**3))/(64*a**4*c**3 - 32*a**3*b**2*c**2 + 4*a**2*b**4*c + x**8*(64*a**2*c*
*5 - 32*a*b**2*c**4 + 4*b**4*c**3) + x**6*(128*a**2*b*c**4 - 64*a*b**3*c**3
 + 8*b**5*c**2) + x**4*(128*a**3*c**4 - 24*a*b**4*c**2 + 4*b**6*c) + x**2*(
128*a**3*b*c**3 - 64*a**2*b**3*c**2 + 8*a*b**5*c))
```

Giac [A]
time = 4.48, size = 171, normalized size = 1.44

$$-\frac{3\,ab\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,abc^2x^6+b^4x^4+ab^2cx^4+16\,a^2c^2x^4+2\,ab^3x^2+10\,a^2bcx^2+a^2b^2+8\,a^3c}{4\,\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)\left(cx^4+bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*a*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*
c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*a*b*c^2*x^6 + b^4*x^4 + a*b^2*c*x^4 + 16*
a^2*c^2*x^4 + 2*a*b^3*x^2 + 10*a^2*b*c*x^2 + a^2*b^2 + 8*a^3*c)/((b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B]
time = 4.44, size = 423, normalized size = 3.55

$$-\frac{\frac{x^2\,\left(5\,c\,a^2\,b+a\,b^3\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^4\,\left(16\,a^2\,c^2+a\,b^2\,c+b^4\right)}{4\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,\left(8\,c\,a^2+a\,b^2\right)}{4\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,a\,b\,c\,x^6}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}-\frac{3\,a\,b\,\mathrm{atan}\left(\frac{\left(x^2\,\left(\frac{9\,a\,b^2\,c^2}{{\left(4\,a\,c-b^2\right)}^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,a\,b^3\,\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,{\left(4\,a\,c-b^2\right)}^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{18\,a^2\,b^3\,c^2}{{\left(4\,a\,c-b^2\right)}^{15/2}}\right)\,\left(b^4\,{\left(4\,a\,c-b^2\right)}^5+16\,a^2\,c^2\,{\left(4\,a\,c-b^2\right)}^5-8\,a\,b^2\,c\,{\left(4\,a\,c-b^2\right)}^5\right)}{18\,a^2\,b^2\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((x^2*(a*b^3 + 5*a^2*b*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(b
^4 + 16*a^2*c^2 + a*b^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*(a*b^
2 + 8*a^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*b*c*x^6)/(2*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2
+ 2*b*c*x^6) - (3*a*b*atan(((x^2*((9*a*b^2*c^2)/((4*a*c - b^2)^(9/2)*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) + (9*a*b^3*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c
^4))/(2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (18*a^2*b^3
*c^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)
^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*a^2*b^2*c^2)))/(4*a*c - b^2)^(5/2)
```

3.876 $\int \frac{x^5}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=130

$$\frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3ab+(b^2+2ac)x^2}{2(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{(b^2+2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] 1/4*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/2*(3*a*b+(2*a*c+b^2)*x
^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-(2*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A]
time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1128, 752, 652, 632, 212}

$$-\frac{(2ac+b^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x^2(2ac+b^2)+3ab}{2(b^2-4ac)^2(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*a*b + (b^2
 + 2*a*c)*x^2)/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2 + 2*a*c)*Arc
Tanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
```

```
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a+bx^2+cx^4)^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^3}\,dx, x, x^2\right)\\
&= \frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\text{Subst}\left(\int \frac{2a-2bx}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4(b^2-4ac)}\\
&= \frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3ab+(b^2+2ac)x^2}{2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(b^2+2ac)\,\text{Subst}\left(\int\right.}{2(b^2-}\\
&= \frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3ab+(b^2+2ac)x^2}{2(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(b^2+2ac)\,\text{Subst}\left(\int\right.}{(b}\\
&= \frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3ab+(b^2+2ac)x^2}{2(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(b^2+2ac)\tanh^{-1}\left(\right.}{(b^2-4a}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 145, normalized size = 1.12

$$\frac{1}{4}\left(\frac{(b^2+2ac)(b+2cx^2)}{c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{b^2x^2+a(b-2cx^2)}{c(-b^2+4ac)(a+bx^2+cx^4)^2} + \frac{4(b^2+2ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2 + 2*a*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b
^2*x^2 + a*(b - 2*c*x^2))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(b^
2 + 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/
4
```

Maple [A]
time = 0.07, size = 216, normalized size = 1.66

method	result
default	$\frac{\frac{c\left(2ac+b^{2}\right)x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3b\left(2ac+b^{2}\right)x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(2ac-5b^{2}\right)x^{2}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3a^{2}b}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{\left(2ac+b^{2}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}$
risch	$\frac{\frac{c\left(2ac+b^{2}\right)x^{6}}{32a^{2}c^{2}-16a\,b^{2}c+2b^{4}}+\frac{3b\left(2ac+b^{2}\right)x^{4}}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(2ac-5b^{2}\right)x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{3a^{2}b}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}-\frac{\ln\left(\left(\left(-4ac+b^{2}\right)^{\frac{5}{2}}-16a^{2}b\,c^{2}+8a\,b^{3}c-\right.\right.}{\left(-4ac+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3/2*b*(2*a*c+b^2)/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^4-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+3*a^
2*b/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+(2*a*c+b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(122) = 244.
time = 0.38, size = 907, normalized size = 6.98

$$\left[\frac{2\,(b^{4}c-2\,ab^{2}c^{2}-8\,a^{2}c^{3})x^{6}+6\,a^{2}b^{3}-24\,a^{3}bc+3\,(b^{5}-2\,ab^{3}c-8\,a^{2}bc^{2})x^{4}+2\,(5\,ab^{4}-22\,a^{2}b^{2}c+8\,a^{3}c^{2})x^{2}+2\,((b^{2}c^{2}+2\,ac^{3})x^{8}+2\,(b^{3}c+2\,abc^{2})x^{6}+(b^{4}+4\,ab^{2}c+4\,a^{2}c^{2})x^{4}+a^{2}b^{2}+2\,a^{3}c+2\,(ab^{3}+2\,a^{2}bc)x^{2})\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac-(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)}{4\,((b^{6}c^{2}-12\,ab^{4}c^{3}+48\,a^{2}b^{2}c^{4}-64\,a^{3}c^{5})x^{8}+a^{2}b^{6}-12\,a^{3}b^{4}c+48\,a^{4}b^{2}c^{2}-64\,a^{5}c^{3}+2\,(b^{7}c-12\,ab^{5}c^{2}+48\,a^{2}b^{3}c^{3}-64\,a^{3}bc^{4})x^{6}+(b^{8}-10\,ab^{6}c+24\,a^{2}b^{4}c^{2}+32\,a^{3}b^{2}c^{3}-128\,a^{4}c^{4})x^{4}+2\,(ab^{7}-12\,a^{2}b^{5}c+48\,a^{3}b^{3}c^{2}-64\,a^{4}bc^{3})x^{2})},\ \frac{2\,(b^{4}c-2\,ab^{2}c^{2}-8\,a^{2}c^{3})x^{6}+6\,a^{2}b^{3}-24\,a^{3}bc+3\,(b^{5}-2\,ab^{3}c-8\,a^{2}bc^{2})x^{4}+2\,(5\,ab^{4}-22\,a^{2}b^{2}c+8\,a^{3}c^{2})x^{2}-4\,((b^{2}c^{2}+2\,ac^{3})x^{8}+2\,(b^{3}c+2\,abc^{2})x^{6}+(b^{4}+4\,ab^{2}c+4\,a^{2}c^{2})x^{4}+a^{2}b^{2}+2\,a^{3}c+2\,(ab^{3}+2\,a^{2}bc)x^{2})\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{4\,((b^{6}c^{2}-12\,ab^{4}c^{3}+48\,a^{2}b^{2}c^{4}-64\,a^{3}c^{5})x^{8}+a^{2}b^{6}-12\,a^{3}b^{4}c+48\,a^{4}b^{2}c^{2}-64\,a^{5}c^{3}+2\,(b^{7}c-12\,ab^{5}c^{2}+48\,a^{2}b^{3}c^{3}-64\,a^{3}bc^{4})x^{6}+(b^{8}-10\,ab^{6}c+24\,a^{2}b^{4}c^{2}+32\,a^{3}b^{2}c^{3}-128\,a^{4}c^{4})x^{4}+2\,(ab^{7}-12\,a^{2}b^{5}c+48\,a^{3}b^{3}c^{2}-64\,a^{4}bc^{3})x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] [1/4*(2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^6 + 6*a^2*b^3 - 24*a^3*b*c + 3*
(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^4 + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2
)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^8 + 2*(b^3*c + 2*a*b*c^2)*x^6 + (b^4 + 4*a
*b^2*c + 4*a^2*c^2)*x^4 + a^2*b^2 + 2*a^3*c + 2*(a*b^3 + 2*a^2*b*c)*x^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sq
rt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 -
10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7
- 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), 1/4*(2*(b^4*c - 2*a*
b^2*c^2 - 8*a^2*c^3)*x^6 + 6*a^2*b^3 - 24*a^3*b*c + 3*(b^5 - 2*a*b^3*c - 8*
a^2*b*c^2)*x^4 + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x^2 - 4*((b^2*c^2 +
2*a*c^3)*x^8 + 2*(b^3*c + 2*a*b*c^2)*x^6 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x
^4 + a^2*b^2 + 2*a^3*c + 2*(a*b^3 + 2*a^2*b*c)*x^2)*sqrt(-b^2 + 4*a*c)*arct
an(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4
)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*
x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(119) = 238$.
time = 2.54, size = 580, normalized size = 4.46

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)\log\left(x^2+\frac{-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+2abc+b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+b^3}{4ac^2+2b^2c}\right)}{2}+\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)\log\left(x^2+\frac{64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+2abc-b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+b^3}{4ac^2+2b^2c}\right)}{2}+\frac{6a^2b+x^6\cdot(4ac^2+2b^2c)+x^4\cdot(6abc+3b^3)+x^2(-4a^2c+10ab^2)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\cdot(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6\cdot(128a^2bc^3-64ab^3c^2+8b^5c)+x^4\cdot(128a^3c^3-24ab^4c+4b^6)+x^2\cdot(128a^3bc^2-64a^2b^3c+8ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**4+b*x**2+a)**3,x)

[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x**2 + (-64*a**3*c**3*sqrt(-
1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b*
*2)**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b*
*2) + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*a
*c**2 + 2*b**2*c))/2 + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x**2 +
(64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**
2*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c -
b**2)**5)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c
+ b**2) + b**3)/(4*a*c**2 + 2*b**2*c))/2 + (6*a**2*b + x**6*(4*a*c**2 + 2*
b**2*c) + x**4*(6*a*b*c + 3*b**3) + x**2*(-4*a**2*c + 10*a*b**2))/(64*a**4*
c**2 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 +
4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(

128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))

Giac [A]
time = 6.14, size = 161, normalized size = 1.24

$$\frac{(b^2+2\,ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}+\frac{2\,b^2cx^6+4\,ac^2x^6+3\,b^3x^4+6\,abcx^4+10\,ab^2x^2-4\,a^2cx^2+6\,a^2b}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (b^2 + 2*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*b^2*c*x^6 + 4*a*c^2*x^6 + 3*b^3*x^
4 + 6*a*b*c*x^4 + 10*a*b^2*x^2 - 4*a^2*c*x^2 + 6*a^2*b)/((c*x^4 + b*x^2 + a
)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

Mupad [B]
time = 4.46, size = 460, normalized size = 3.54

$$\frac{\frac{3\,a^2\,b}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,(5\,a\,b^2-2\,a^2\,c)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,b\,x^4\,(b^2+2\,a\,c)}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{c\,x^6\,(b^2+2\,a\,c)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^4\,(b^2+2\,a\,c)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}+\frac{\mathrm{atan}\left(\frac{\left(x^2\,\left(\frac{(b^2+2\,a\,c)\,(b^2\,c^2+2\,a\,c^3)}{a\,(4\,a\,c-b^2)^{9/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{b\,(b^2+2\,a\,c)^2\,(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2)}{2\,a\,(4\,a\,c-b^2)^{15/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)+\frac{2\,b\,c^2\,(b^2+2\,a\,c)^2}{(4\,a\,c-b^2)^{15/2}}\right)\,\left(b^4\,(4\,a\,c-b^2)^5+16\,a^2\,c^2\,(4\,a\,c-b^2)^5-8\,a\,b^2\,c\,(4\,a\,c-b^2)^5\right)}{8\,a^2\,c^4+8\,a\,b^2\,c^3+2\,b^4\,c^2}\right)\,(b^2+2\,a\,c)}{(4\,a\,c-b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*a^2*b)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(5*a*b^2 - 2*a^2*c))/(
2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*x^4*(2*a*c + b^2))/(4*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) + (c*x^6*(2*a*c + b^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + (atan(((
x^2*(((2*a*c + b^2)*(2*a*c^3 + b^2*c^2))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a
^2*c^2 - 8*a*b^2*c)) + (b*(2*a*c + b^2)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^
2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b
*c^2*(2*a*c + b^2)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c
^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(8*a^2*c^4 + 2*b^4*c^2 + 8
*a*b^2*c^3))*(2*a*c + b^2))/(4*a*c - b^2)^(5/2)
```

3.877 $\int \frac{x^3}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=113

$$\frac{2a+bx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3b(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] 1/4*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/4*b*(2*c*x^2+b)/(-4*a*c+b^
2)^2/(c*x^4+b*x^2+a)+3*b*c*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(5/2)
```

Rubi [A]

time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1128, 652, 628, 632, 212}

$$\frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{2a+bx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3b(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a + b*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*b*(b + 2*c*x^2))
/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*b*c*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= \frac{2a+bx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{(3b)\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4(b^2-4ac)}$$

$$= \frac{2a+bx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3b(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(3bc)\text{Subst}\left(\int \frac{}{a+b\ldots}\right.}{2(b^2-\ldots}$$

$$= \frac{2a+bx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3b(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(3bc)\text{Subst}\left(\int \frac{}{b^2-\ldots}\right.}{(b^2\ldots}$$

$$= \frac{2a+bx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3b(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3bc\tanh^{-1}\left(\frac{b\ldots}{\sqrt{b^2\ldots}}\right.}{(b^2-4ac)^{\ldots}}$$

Mathematica [A]
time = 0.07, size = 114, normalized size = 1.01

$$\frac{\frac{(b^2-4ac)(2a+bx^2)}{(a+bx^2+cx^4)^2} - \frac{3b(b+2cx^2)}{a+bx^2+cx^4} - \frac{12bc\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(2*a + b*x^2))/(a + b*x^2 + c*x^4)^2 - (3*b*(b + 2*c*x^2))/
(a + b*x^2 + c*x^4) - (12*b*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqr
t[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2)
```

Maple [A]
time = 0.06, size = 128, normalized size = 1.13

method	result
default	$\frac{-b\,x^{2}-2a}{4(4ac-b^{2})(c\,x^{4}+b\,x^{2}+a)^{2}}-\frac{3b\left(\frac{2c\,x^{2}+b}{\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{2}+a\right)}+\frac{4c\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}\right)}{4(4ac-b^{2})}$
risch	$\frac{-\frac{3b\,c^{2}x^{6}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{9b^{2}c\,x^{4}}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5ac+b^{2}\right)b\,x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(8ac+b^{2}\right)}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{(c\,x^{4}+b\,x^{2}+a)^{2}}-\frac{3cb\ln\left(\left(-\left(-4ac+b^{2}\right)^{\frac{5}{2}}-16a^{2}b\,c^{2}+8\right.\right.}{2(-4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2-3/4*b/(4*a*c-b^2)*((2*c*x^2+
b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(105) = 210.
time = 0.36, size = 808, normalized size = 7.15

$$\left[-\frac{6\,(b^{3}c^{2}-4\,abc^{3})x^{6}+ab^{4}+4\,a^{2}b^{2}c-32\,a^{3}c^{2}+9\,(b^{4}c-4\,ab^{2}c^{2})x^{4}+2\,(b^{5}+ab^{3}c-20\,a^{2}bc^{2})x^{2}-6\,(bc^{3}x^{8}+2\,b^{2}c^{2}x^{6}+2\,ab^{2}cx^{2}+(b^{3}c+2\,abc^{2})x^{4}+a^{2}bc)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)}{4\,((b^{6}c^{2}-12\,ab^{4}c^{3}+48\,a^{2}b^{2}c^{4}-64\,a^{3}c^{5})x^{8}+a^{2}b^{6}-12\,a^{3}b^{4}c+48\,a^{4}b^{2}c^{2}-64\,a^{5}c^{3}+2\,(b^{7}c-12\,ab^{5}c^{2}+48\,a^{2}b^{3}c^{3}-64\,a^{3}bc^{4})x^{6}+(b^{8}-10\,ab^{6}c+24\,a^{2}b^{4}c^{2}+32\,a^{3}b^{2}c^{3}-128\,a^{4}c^{4})x^{4}+2\,(ab^{7}-12\,a^{2}b^{5}c+48\,a^{3}b^{3}c^{2}-64\,a^{4}bc^{3})x^{2})},\ -\frac{6\,(b^{3}c^{2}-4\,abc^{3})x^{6}+ab^{4}+4\,a^{2}b^{2}c-32\,a^{3}c^{2}+9\,(b^{4}c-4\,ab^{2}c^{2})x^{4}+2\,(b^{5}+ab^{3}c-20\,a^{2}bc^{2})x^{2}-12\,(bc^{3}x^{8}+2\,b^{2}c^{2}x^{6}+2\,ab^{2}cx^{2}+(b^{3}c+2\,abc^{2})x^{4}+a^{2}bc)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{4\,((b^{6}c^{2}-12\,ab^{4}c^{3}+48\,a^{2}b^{2}c^{4}-64\,a^{3}c^{5})x^{8}+a^{2}b^{6}-12\,a^{3}b^{4}c+48\,a^{4}b^{2}c^{2}-64\,a^{5}c^{3}+2\,(b^{7}c-12\,ab^{5}c^{2}+48\,a^{2}b^{3}c^{3}-64\,a^{3}bc^{4})x^{6}+(b^{8}-10\,ab^{6}c+24\,a^{2}b^{4}c^{2}+32\,a^{3}b^{2}c^{3}-128\,a^{4}c^{4})x^{4}+2\,(ab^{7}-12\,a^{2}b^{5}c+48\,a^{3}b^{3}c^{2}-64\,a^{4}bc^{3})x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*x^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(
b^4*c - 4*a*b^2*c^2)*x^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x^2 - 6*(b*c^3*
x^8 + 2*b^2*c^2*x^6 + 2*a*b^2*c*x^2 + (b^3*c + 2*a*b*c^2)*x^4 + a^2*b*c)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sq
rt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 -
 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7
 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), -1/4*(6*(b^3*c^2 - 4
*a*b*c^3)*x^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*
x^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x^2 - 12*(b*c^3*x^8 + 2*b^2*c^2*x^6
+ 2*a*b^2*c*x^2 + (b^3*c + 2*a*b*c^2)*x^4 + a^2*b*c)*sqrt(-b^2 + 4*a*c)*arc
tan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)
*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(105) = 210$.
time = 1.40, size = 491, normalized size = 4.35

$$\frac{3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^2c}{6bc^2}\right)}{2}-\frac{3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^2c}{6bc^2}\right)}{2}+\frac{-8a^2c-ab^2-9b^2cx^4-6bc^2x^6+x^2(-10abc-2b^3)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\cdot(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6\cdot(128a^2bc^3-64ab^3c^2+8b^5c)+x^4\cdot(128a^3c^3-24ab^4c+4b^6)+x^2\cdot(128a^3bc^2-64a^2b^3c+8ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*b*c**4*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c
**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b*
*2*c)/(6*b*c**2))/2 - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**3
*b*c**4*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b*
*2)**5) + 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a
*c - b**2)**5) + 3*b**2*c)/(6*b*c**2))/2 + (-8*a**2*c - a*b**2 - 9*b**2*c*x
**4 - 6*b*c**2*x**6 + x**2*(-10*a*b*c - 2*b**3))/(64*a**4*c**2 - 32*a**3*b*
*2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x
**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 2
4*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

Giac [A]
time = 6.00, size = 143, normalized size = 1.27

$$-\frac{3\,bc\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2x^6+9\,b^2cx^4+2\,b^3x^2+10\,abcx^2+ab^2+8\,a^2c}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*
c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*b*c^2*x^6 + 9*b^2*c*x^4 + 2*b^3*x^2 + 10*
a*b*c*x^2 + a*b^2 + 8*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a
^2*c^2))
```

Mupad [B]
time = 4.39, size = 400, normalized size = 3.54

$$-\frac{\frac{8\,c\,a^2+a\,b^2}{4\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\,\left(b^3+5\,a\,c\,b\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b^2\,c\,x^4}{4\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,b\,c^2\,x^6}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}-\frac{3\,b\,c\,\mathrm{atan}\left(\frac{\left(x^2\,\left(\frac{9\,b^2\,c^4}{a\,{\left(4\,a\,c-b^2\right)}^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b^3\,c^2\,\left(144\,a^2\,b\,c^4-72\,a\,b^3\,c^3+9\,b^5\,c^2\right)}{a\,{\left(4\,a\,c-b^2\right)}^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{18\,b^3\,c^4}{{\left(4\,a\,c-b^2\right)}^{15/2}}\right)\,\left(b^4\,{\left(4\,a\,c-b^2\right)}^5+16\,a^2\,c^2\,{\left(4\,a\,c-b^2\right)}^5-8\,a\,b^2\,c\,{\left(4\,a\,c-b^2\right)}^5\right)}{18\,b^2\,c^4}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((a*b^2 + 8*a^2*c)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3 + 5*a*b
*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^4)/(4*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) + (3*b*c^2*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2
*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (3*b*c*atan(((x^2*((
9*b^2*c^4)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^3*c^
2*(9*b^5*c^2 - 72*a*b^3*c^3 + 144*a^2*b*c^4))/(a*(4*a*c - b^2)^(15/2)*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))) + (18*b^3*c^4)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*
c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*b
^2*c^4)))/(4*a*c - b^2)^(5/2)
```

3.878 $\int \frac{x}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=113

$$-\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3c(b+2cx^2)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{6c^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/4*(-2*c*x^2-b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/2*c*(2*c*x^2+b)/(-4*a*c+b
^2)^2/(c*x^4+b*x^2+a)-6*c^2*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1121, 628, 632, 212}

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c(b+2cx^2)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*c*(b + 2*c*x^
2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*c^2*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$
$$= -\frac{b+2cx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{(3c)\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{2(b^2-4ac)}$$
$$= -\frac{b+2cx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3c(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(3c^2)\text{Subst}\left(\int \frac{}{a+}\right.}{(b^2-}$$
$$= -\frac{b+2cx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3c(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(6c^2)\text{Subst}\left(\int \frac{}{b^2-}\right.}{(b^2}$$
$$= -\frac{b+2cx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3c(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{6c^2\tanh^{-1}\left(\frac{b}{\sqrt{b^2}}\right.}{(b^2-4ac)}$$

Mathematica [A]
time = 0.07, size = 106, normalized size = 0.94

$$\frac{-\frac{(b+2cx^2)(b^2-6bcx^2-2c(5a+3cx^4))}{(a+bx^2+cx^4)^2}+\frac{24c^2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-(((b + 2*c*x^2)*(b^2 - 6*b*c*x^2 - 2*c*(5*a + 3*c*x^4)))/(a + b*x^2 + c*x
^4)^2) + (24*c^2*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c])/(4*(b^2 - 4*a*c)^2)
```

Maple [A]
time = 0.05, size = 126, normalized size = 1.12

method	result
default	$\frac{2c\,x^{2}+b}{4(4ac-b^{2})(c\,x^{4}+b\,x^{2}+a)^{2}}+\frac{3c\left(\frac{2c\,x^{2}+b}{\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{2}+a\right)}+\frac{4c\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}\right)}{2(4ac-b^{2})}$
risch	$\frac{\frac{3c^{3}x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{9b\,c^{2}x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(5ac+b^{2}\right)c\,x^{2}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{b\left(10ac-b^{2}\right)}{64a^{2}c^{2}-32a\,b^{2}c+4b^{4}}}{(c\,x^{4}+b\,x^{2}+a)^{2}}-\frac{3c^{2}\ln\left(\left(\left(-4ac+b^{2}\right)^{\frac{5}{2}}-16a^{2}b\,c^{2}+8a\,b^{3}c-b^{5}\right.\right.}{\left(-4ac+b^{2}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3/2*c/(4*a*c-b^2)*((2*c*x^2+b
)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a
*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(105) = 210$.
time = 0.40, size = 809, normalized size = 7.16

$$\left[\frac{12\,(b^{2}c^{3}-4\,ac^{4})x^{6}-b^{5}+14\,ab^{3}c-40\,a^{2}bc^{2}+18\,(b^{3}c^{2}-4\,abc^{3})x^{4}+4\,(b^{4}c+ab^{2}c^{2}-20\,a^{2}c^{3})x^{2}+12\,(c^{4}x^{8}+2\,bc^{3}x^{6}+2\,abc^{2}x^{2}+(b^{2}c^{2}+2\,ac^{3})x^{4}+a^{2}c^{2})\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac-(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)}{4\,((b^{6}c^{2}-12\,ab^{4}c^{3}+48\,a^{2}b^{2}c^{4}-64\,a^{3}c^{5})x^{8}+a^{2}b^{6}-12\,a^{3}b^{4}c+48\,a^{4}b^{2}c^{2}-64\,a^{5}c^{3}+2\,(b^{7}c-12\,ab^{5}c^{2}+48\,a^{2}b^{3}c^{3}-64\,a^{3}bc^{4})x^{6}+(b^{8}-10\,ab^{6}c+24\,a^{2}b^{4}c^{2}+32\,a^{3}b^{2}c^{3}-128\,a^{4}c^{4})x^{4}+2\,(ab^{7}-12\,a^{2}b^{5}c+48\,a^{3}b^{3}c^{2}-64\,a^{4}bc^{3})x^{2})},\ \frac{12\,(b^{2}c^{3}-4\,ac^{4})x^{6}-b^{5}+14\,ab^{3}c-40\,a^{2}bc^{2}+18\,(b^{3}c^{2}-4\,abc^{3})x^{4}+4\,(b^{4}c+ab^{2}c^{2}-20\,a^{2}c^{3})x^{2}-24\,(c^{4}x^{8}+2\,bc^{3}x^{6}+2\,abc^{2}x^{2}+(b^{2}c^{2}+2\,ac^{3})x^{4}+a^{2}c^{2})\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{4\,((b^{6}c^{2}-12\,ab^{4}c^{3}+48\,a^{2}b^{2}c^{4}-64\,a^{3}c^{5})x^{8}+a^{2}b^{6}-12\,a^{3}b^{4}c+48\,a^{4}b^{2}c^{2}-64\,a^{5}c^{3}+2\,(b^{7}c-12\,ab^{5}c^{2}+48\,a^{2}b^{3}c^{3}-64\,a^{3}bc^{4})x^{6}+(b^{8}-10\,ab^{6}c+24\,a^{2}b^{4}c^{2}+32\,a^{3}b^{2}c^{3}-128\,a^{4}c^{4})x^{4}+2\,(ab^{7}-12\,a^{2}b^{5}c+48\,a^{3}b^{3}c^{2}-64\,a^{4}bc^{3})x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*x^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^
3*c^2 - 4*a*b*c^3)*x^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x^2 + 12*(c^4*x
^8 + 2*b*c^3*x^6 + 2*a*b*c^2*x^2 + (b^2*c^2 + 2*a*c^3)*x^4 + a^2*c^2)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
```

```
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), 1/4*(12*(b^2*c^3 - 4*a*
c^4)*x^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*x^4 +
 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x^2 - 24*(c^4*x^8 + 2*b*c^3*x^6 + 2*a*b
*c^2*x^2 + (b^2*c^2 + 2*a*c^3)*x^4 + a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6
+ (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 +
2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(107) = 214$.
time = 1.37, size = 481, normalized size = 4.26

$$-3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3bc^2}{6c^3}\right)+3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-3b^6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3bc^2}{6c^3}\right)+\frac{10abc-b^3+18bc^2x^4+12c^3x^6+x^2\cdot(20ac^2+4b^2c)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\cdot(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6\cdot(128a^2bc^3-64ab^3c^2+8b^5c)+x^4\cdot(128a^3c^3-24ab^4c+4b^6)+x^2\cdot(128a^3bc^2-64a^2b^3c+8ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*c**5*sqrt(-1/(4*a*
c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**4*c
**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3
*b*c**2)/(6*c**3)) + 3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**3
*c**5*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2
)**5) + 36*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*sqrt(-1/(4*
a*c - b**2)**5) + 3*b*c**2)/(6*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**4 +
 12*c**3*x**6 + x**2*(20*a*c**2 + 4*b**2*c))/(64*a**4*c**2 - 32*a**3*b**2*c
 + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6*
(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 24*a*
b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

Giac [A]
time = 5.28, size = 144, normalized size = 1.27

$$\frac{6\,c^2\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2+20\,ac^2x^2-b^3+10\,abc}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c
^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(12*c^3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 + 20
```

```
*a*c^2*x^2 - b^3 + 10*a*b*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a
^2*c^2))
```

Mupad [B]
time = 4.34, size = 386, normalized size = 3.42

$$\frac{\frac{3\,c^3\,x^6}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{b^3-10\,a\,b\,c}{4\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\,\left(b^2\,c+5\,a\,c^2\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{9\,b\,c^2\,x^4}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}+\frac{6\,c^2\,\mathrm{atan}\left(\frac{\left(x^2\,\left(\frac{36\,c^6}{a\,{\left(4\,a\,c-b^2\right)}^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{36\,b\,c^4\,\left(16\,a^2\,b\,c^4-8\,a\,b^3\,c^3+b^5\,c^2\right)}{a\,{\left(4\,a\,c-b^2\right)}^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{72\,b\,c^6}{{\left(4\,a\,c-b^2\right)}^{15/2}}\right)\,\left(b^4\,{\left(4\,a\,c-b^2\right)}^5+16\,a^2\,c^2\,{\left(4\,a\,c-b^2\right)}^5-8\,a\,b^2\,c\,{\left(4\,a\,c-b^2\right)}^5\right)}{72\,c^6}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*c^3*x^6)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(4*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (x^2*(5*a*c^2 + b^2*c))/(b^4 + 16*a^2*c^2 - 8*a*b^
2*c) + (9*b*c^2*x^4)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2)
 + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + (6*c^2*atan(((x^2*((36*c^6)/(a*
(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*(b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4))/(a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c))) + (72*b*c^6)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2
*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(72*c^6)))/(4*a*c - b^2)
^(5/2)
```

3.879 $\int \frac{1}{x(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=200

$$\frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{b(b^4-10ab^2c+30a^2c^2)\tanh^{-1}\big(}{2a^3\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/4*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*(2*b^4-15*a*b^
2*c+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1
/2*b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^
3/(-4*a*c+b^2)^(5/2)+ln(x)/a^3-1/4*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A]
time = 0.20, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1128, 754, 836, 814, 648, 632, 212, 642}

$$-\frac{\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)}{a^3}+\frac{16a^2c^2+2bcx^2(b^2-7ac)-15ab^2c+2b^4}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{b(30a^2c^2-10ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*b^4
- 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x^2)/(4*a^2*(b^2 - 4*a*c)^2
*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a
+ b*x^2 + c*x^4]/(4*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
```

```
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-4ac\right)-3bcx}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Sub}}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Sub}}{}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(}{a}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(}{a}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\log(}{a}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{b(b^4}{}$$

<u>Mathematica</u> [A]
time = 0.33, size = 342, normalized size = 1.71

$$\frac{\frac{a^2(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{a(2b^4-15ab^2c+16a^2c^2+2b^3cx^2-14abc^2x^2)}{(b^2-4ac)^2(a+bx^2+cx^4)}+4\log(x)-\frac{\left(b^5-10ab^3c+30a^2bc^2+b^4\sqrt{b^2-4ac}-8ab^2c\sqrt{b^2-4ac}+16a^2c^2\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}}+\frac{\left(b^5-10ab^3c+30a^2bc^2-b^4\sqrt{b^2-4ac}+8ab^2c\sqrt{b^2-4ac}-16a^2c^2\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((a^2*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(
2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*c^2*x^2))/((b^2 - 4*
a*c)^2*(a + b*x^2 + c*x^4)) + 4*Log[x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2
+ b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2
 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + ((b^
5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2
```

```
- 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

Maple [A]
time = 0.08, size = 360, normalized size = 1.80

method	result
default	$-\frac{\frac{\frac{ab\,c^{2}\left(7ac-b^{2}\right)x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{ca\left(16a^{2}c^{2}-29a\,b^{2}c+4b^{4}\right)x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{ab\left(a^{2}c^{2}+6a\,b^{2}c-b^{4}\right)x^{2}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{3a^{2}\left(8a^{2}c^{2}-7a\,b^{2}c+b^{4}\right)}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{\frac{\left(16a^{2}c^{3}-8a\,b^{2}c^{2}+b^{4}c\right)\ln\left(c\,x^{4}+b\,x\right.}{2c}}{}}{2a^{3}}$
risch	$\frac{-\frac{b\,c^{2}\left(7ac-b^{2}\right)x^{6}}{2a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c\left(16a^{2}c^{2}-29a\,b^{2}c+4b^{4}\right)x^{4}}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)a^{2}}-\frac{b\left(a^{2}c^{2}+6a\,b^{2}c-b^{4}\right)x^{2}}{2a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{6a^{2}c^{2}-\frac{21}{4}a\,b^{2}c+\frac{3}{4}b^{4}}{a\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{\ln(x)}{a^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(1\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3*((a*b*c^2*(7*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-1/2*c*a*(16*a
^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+a*b*(a^2*c^2+6*a*b^
2*c-b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/(
16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*(
1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)/c*ln(c*x^4+b*x^2+a)+2*(23*a^2*b*c^2-9*a*
b^3*c+b^5-1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))))+ln(x)/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(188) = 376$.
time = 0.65, size = 2017, normalized size = 10.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^6 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*
a^3*b^2*c^3 - 64*a^4*c^4)*x^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 +
4*a^4*b*c^3)*x^2 + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8 + a^2*b^5 -
 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^
6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^4 + 2*(a*b^6 - 10*a
^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a
^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^
4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b
^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(x))/(a^5*b^6 -
12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3
+ 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5
*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 3
2*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c
^2 - 64*a^7*b*c^3)*x^2), 1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 -
96*a^5*c^3 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^6 + (4*a*b^6*c
 - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*x^4 + 2*(a*b^7 - 10*a^2*b
^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x^2 + 2*((b^5*c^2 - 10*a*b^3*c^3 + 30*
a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^
4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*
c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(-b^2 + 4*a*c
)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^
4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*
log(c*x^4 + b*x^2 + a) + 4*((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(x))/(a^5*b^6 - 12*a^6*b^4*c +
 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c
^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*
a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3
```

```
- 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*
c^3)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

Giac [**A**]
time = 6.43, size = 323, normalized size = 1.62

$$-\frac{(b^5 - 10\,ab^3c + 30\,a^2bc^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^3b^4 - 8\,a^4b^2c + 16\,a^5c^2)\sqrt{-b^2+4\,ac}} + \frac{3\,b^4c^2x^8 - 24\,ab^2c^3x^8 + 48\,a^2c^4x^8 + 6\,b^5cx^6 - 44\,ab^3c^2x^6 + 68\,a^2bc^3x^6 + 3\,b^6x^4 - 10\,ab^4cx^4 - 58\,a^2b^2c^2x^4 + 128\,a^3c^3x^4 + 10\,ab^5x^2 - 72\,a^2b^3cx^2 + 92\,a^3bc^2x^2 + 9\,a^2b^4 - 66\,a^3b^2c + 96\,a^4c^2}{8\,(a^3b^4 - 8\,a^4b^2c + 16\,a^5c^2)(cx^4+bx^2+a)^2} - \frac{\log(cx^4+bx^2+a)}{4\,a^3} + \frac{\log(x^2)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
[Out] -1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a
*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) + 1/8*(3*b^4
*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8 + 6*b^5*c*x^6 - 44*a*b^3*c^2*x
^6 + 68*a^2*b*c^3*x^6 + 3*b^6*x^4 - 10*a*b^4*c*x^4 - 58*a^2*b^2*c^2*x^4 + 1
28*a^3*c^3*x^4 + 10*a*b^5*x^2 - 72*a^2*b^3*c*x^2 + 92*a^3*b*c^2*x^2 + 9*a^2
*b^4 - 66*a^3*b^2*c + 96*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*
x^4 + b*x^2 + a)^2) - 1/4*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*log(x^2)/a^3
```

Mupad [**B**]
time = 10.95, size = 2500, normalized size = 12.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^3),x)
[Out] log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(4*a*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (x^4*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(4*a^2*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (b*x^2*(a^2*c^2 - b^4 + 6*a*b^2*c))/(2*a^2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^6*(7*a*c - b^2))/(2*a^2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
- (log((((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^
5))^(1/2) + 1)*((b^2*c^3*(4*b^6 - 497*a^3*c^3 + 302*a^2*b^2*c^2 - 61*a*b^4*
c))/(a^4*(4*a*c - b^2)^4) - ((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)
/(a^6*(4*a*c - b^2)^5))^(1/2) + 1)*((4*b^2*c^2*(b^4 + 23*a^2*c^2 - 9*a*b^2*
```

c))/(a^2*(4*a*c - b^2)^2) + (b*c^2*(a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) + 1)*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (2*b*c^3*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3) + (b*c^4*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) - (b^2*c^4*(7*a*c - b^2)^2)/(a^6*(4*a*c - b^2)^4) + (b^3*c^5*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6))*(((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) - 1)*((4*b^2*c^2*(b^4 + 23*a^2*c^2 - 9*a*b^2*c))/(a^2*(4*a*c - b^2)^2) - (b*c^2*(a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) - 1)*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (2*b*c^3*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3) + (b^2*c^3*(4*b^6 - 497*a^3*c^3 + 302*a^2*b^2*c^2 - 61*a*b^4*c))/(a^4*(4*a*c - b^2)^4) + (b*c^4*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (b^2*c^4*(7*a*c - b^2)^2)/(a^6*(4*a*c - b^2)^4) - (b^3*c^5*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) + (b*atan((((b*((4*a^2*b^8*c^3 - 61*a^3*b^6*c^4 + 302*a^4*b^4*c^5 - 497*a^5*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) - (((4*a^4*b^10*c^2 - 68*a^5*b^8*c^3 + 444*a^6*b^6*c^4 - 1312*a^7*b^4*c^5 + 1472*a^8*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) + ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) - (((b*((4*a^4*b^10*c^2 - 68*a^5*b^8*c^3 + 444*a^6*b^6*c^4 - 1312*a^7*b^4*c^5 + 1472*a^8*b^2*c^6)/(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3) + ((4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(4*a^3*(4*a*c - b^2)^(5/2)) + (b*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)*(4*a^7*b^10*c^2 - 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6)*(2*b^10 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(8*a^3*(4*a*c - b^2)^(5/2)*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)))*(2*b^10 - 2048*a^5*c^5 + 320*a

```
^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 - 40*a*b^8*c))/(2*(4*a^3*b
^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 51
20*a^7*b^2*c^4)) + (b^3*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^3*(4*a^7*b^10*c^2 -
 64*a^8*b^8*c^3 + 384*a^9*b^6*c^4 - 1024*a^10*b^4*c^5 + 1024*a^11*b^2*c^6))
/(64*a^9*(4*a*c - b^2)^(15/2)*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a
^8*b^4*c^2 - 256*a^9*b^2*c^3)))*(3*b^8 + 160*a^4*c^4 + 180*a^2*b^4*c^2 - 32
5*a^3*b^2*c^3 - 39*a*b^6*c)*(16*a^9*b^12*(4*a*c - b^2)^(15/2) + 65536*a^15*
c^6*(4*a*c - b^2)^(15/2) - 384*a^10*b^10*c*(4*a*c - b^2)^(15/2) + 3840*a^11
*b^8*c^2*(4*a*c - b^2)^(15/2) - 20480*a^12*b^6*c^3*(4*a*c - b^2)^(15/2) + 6
1440*a^13*b^4*c^4*(4*a*c - b^2)^(15/2) - 98304*a^14*b^2*c^5*(4*a*c - b^2)^(
15/2)))/(8*a^3*c^2*(4*a*c - b^2)^(13/2)*(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*
b^6*c^4 - 600*a^3*b^4*c^5 + 900*a^4*b^2*c^6)*(6*b^10 - 6400*a^5*c^5 + 960*a
^2*b^6*c^2 - 3850*a^3*b^4*c^3 + 7775*a^4*b^2*c^...
```

3.880 $\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=255

$$-\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2x^2}+\frac{b^2-2ac+bcx^2}{4a(b^2-4ac)x^2(a+bx^2+cx^4)^2}+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)x^2}{4a^2(b^2-4ac)^2x^2(a+bx^2+cx^4)}-\frac{3(b^6$$

```
[Out] -3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x^2+1/4*(b*c*x^2-2*a*c+b^
2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2+a)^2+1/4*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*
b*c*(-6*a*c+b^2)*x^2)/a^2/(-4*a*c+b^2)^2/x^2/(c*x^4+b*x^2+a)-3/2*(-20*a^3*c
^3+30*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a
^4/(-4*a*c+b^2)^(5/2)-3*b*ln(x)/a^4+3/4*b*ln(c*x^4+b*x^2+a)/a^4
```

Rubi [A]
time = 0.26, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1128, 754, 836, 814, 648, 632, 212, 642}

$$\frac{3b\log(a+bx^2+cx^4)}{4a^4}-\frac{3b\log(x)}{a^4}-\frac{3(b^2-5ac)(b^2-2ac)}{2a^3x^2(b^2-4ac)^2}+\frac{20a^2c^2+3bcx^2(b^2-6ac)-20ab^2c+3b^4}{4a^2x^2(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{3(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{5/2}}+\frac{-2ac+b^2+bcx^2}{4ax^2(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*x^2) + (b^2 - 2*a*c
 + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)^2) + (3*b^4 - 20*a*b
^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x^2)/(4*a^2*(b^2 - 4*a*c)^2*x^2*(a
+ b*x^2 + c*x^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*Arc
Tanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*L
og[x])/a^4 + (3*b*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2 (a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{4a (b^2-4ac) x^2 (a+bx^2+cx^4)^2} - \frac{\text{Subst}\left(\int \frac{-3b^2+10ac-4bcx}{x^2(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4a (b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx^2}{4a (b^2-4ac) x^2 (a+bx^2+cx^4)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x^2}{4a^2 (b^2-4ac)^2 x^2 (a+bx^2+cx^4)} +$$

$$= \frac{b^2-2ac+bcx^2}{4a (b^2-4ac) x^2 (a+bx^2+cx^4)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x^2}{4a^2 (b^2-4ac)^2 x^2 (a+bx^2+cx^4)} +$$

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3 (b^2-4ac)^2 x^2} + \frac{b^2-2ac+bcx^2}{4a (b^2-4ac) x^2 (a+bx^2+cx^4)^2} + \frac{3b^4-20ab^2c+2}{4a^2 (b^2-4ac}$$

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3 (b^2-4ac)^2 x^2} + \frac{b^2-2ac+bcx^2}{4a (b^2-4ac) x^2 (a+bx^2+cx^4)^2} + \frac{3b^4-20ab^2c+2}{4a^2 (b^2-4ac}$$

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3 (b^2-4ac)^2 x^2} + \frac{b^2-2ac+bcx^2}{4a (b^2-4ac) x^2 (a+bx^2+cx^4)^2} + \frac{3b^4-20ab^2c+2}{4a^2 (b^2-4ac}$$

$$= -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3 (b^2-4ac)^2 x^2} + \frac{b^2-2ac+bcx^2}{4a (b^2-4ac) x^2 (a+bx^2+cx^4)^2} + \frac{3b^4-20ab^2c+2}{4a^2 (b^2-4ac}$$

Mathematica [A]
time = 0.39, size = 402, normalized size = 1.58

$$\frac{-\frac{2a}{x^2} + \frac{a^2(b^3-3abc+b^2cx^2-2ac^2x^2)}{(-b^2+4ac)(a+bx^2+cx^4)^2} - \frac{a(4b^5-29ab^3c+46a^2bc^2+4b^4cx^2-26ab^2c^2x^2+28a^2c^3x^2)}{(b^2-4ac)^2(a+bx^2+cx^4)} - 12b\log(x) + \frac{3\left(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}} + \frac{3\left(-b^6+10ab^4c-30a^2b^2c^2+20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}}}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*a)/x^2 + (a^2*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((-b^2 + 4*a*
c)*(a + b*x^2 + c*x^4)^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c
*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) - 12*b*Log[x] + (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 +
```

```
b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2
 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + (3*(
-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8
*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

Maple [A]
time = 0.10, size = 412, normalized size = 1.62

method	result
default	$-\frac{\frac{\frac{c^2 a \left(14a^2c^2-13a\,b^2c+2b^4\right)x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{abc\left(74a^2c^2-55a\,b^2c+8b^4\right)x^4}{32a^2c^2-16a\,b^2c+2b^4}+\frac{a\left(18a^3c^3+7a^2b^2c^2-12a\,b^4c+2b^6\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{a^2b\left(58a^2c^2-36a\,b^2c+5b^4\right)}{32a^2c^2-16a\,b^2c+2b^4}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{3(-16a}{}}{2a^4}$
risch	$\frac{-\frac{3c^2\left(10a^2c^2-7a\,b^2c+b^4\right)x^8}{2a^3\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3bc\left(46a^2c^2-29a\,b^2c+4b^4\right)x^6}{4\left(16a^2c^2-8a\,b^2c+b^4\right)a^3}-\frac{\left(50a^3c^3+7a^2b^2c^2-18a\,b^4c+3b^6\right)x^4}{2a^3\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{b\left(122a^2c^2-68a\,b^2c+9b^4\right)x^2}{4a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{1}{2a}}{x^2\left(c\,x^4+b\,x^2+a\right)^2}-\frac{3b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^4*((c^2*a*(14*a^2*c^2-13*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^6+1/2*a*b*c*(74*a^2*c^2-55*a*b^2*c+8*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+a
*(18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2
+1/2*a^2*b*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4
+b*x^2+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(-16*a^2*b*c^3+8*a*b^3*c^2-b^
5*c)/c*ln(c*x^4+b*x^2+a)+2*(10*a^3*c^3-23*a^2*b^2*c^2+9*a*b^4*c-b^6-1/2*(-1
6*a^2*b*c^3+8*a*b^3*c^2-b^5*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))))-1/2/a^3/x^2-3*b*ln(x)/a^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1144 vs. $2(241) = 482$.

time = 0.81, size = 2312, normalized size = 9.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c
^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^8 + 3*(4*a*b^7*c - 45*
a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^6 + 2*(3*a*b^8 - 30*a^2*b^
6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^4 + (9*a^2*b^7 - 104
*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x^2 + 3*((b^6*c^2 - 10*a*b^4*
c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^10 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*
b^3*c^3 - 20*a^3*b*c^4)*x^8 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^
2*c^3 - 40*a^4*c^4)*x^6 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4
*b*c^3)*x^4 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x^2)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^
2*b^3*c^4 - 64*a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 -
 64*a^3*b^2*c^4)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3
- 128*a^4*b*c^4)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^
2*c^3)*x^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*
log(c*x^4 + b*x^2 + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*
a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4
)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4
)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^4 + (a
^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(x))/((a^4*b
^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c
- 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b
^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12
*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)*x^2), -1/4*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5
*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 4
0*a^4*c^5)*x^8 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*
b*c^4)*x^6 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 -
200*a^5*c^4)*x^4 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b
*c^3)*x^2 + 6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^10
+ 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^8 + (b^8 - 8*a
*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^6 + 2*(a*b^7 - 10*
a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x^4 + (a^2*b^6 - 10*a^3*b^4*c +
30*a^4*b^2*c^2 - 20*a^5*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3
*c^4 - 64*a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a
^3*b^2*c^4)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128
*a^4*b*c^4)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3
```

```
)*x^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(c
*x^4 + b*x^2 + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b
*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^8
 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^6
 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^4 + (a^2*b^
7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2)*log(x))/((a^4*b^6*c^
2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*
a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c
+ 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*
b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.18, size = 382, normalized size = 1.50

$$\frac{3\,(b^6 - 10\,ab^4c + 30\,a^2b^2c^2 - 20\,a^3c^3)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^4b^4 - 8\,a^5b^2c + 16\,a^6c^2)\sqrt{-b^2+4\,ac}} - \frac{9\,b^5c^2x^8 - 72\,ab^3c^3x^8 + 144\,a^2bc^4x^8 + 18\,b^6cx^6 - 136\,ab^4c^2x^6 + 236\,a^2b^2c^3x^6 + 56\,a^3c^4x^6 + 9\,b^7x^4 - 38\,ab^5cx^4 - 110\,a^2b^3c^2x^4 + 436\,a^3bc^3x^4 + 26\,ab^6x^2 - 192\,a^2b^4cx^2 + 316\,a^3b^2c^2x^2 + 72\,a^4c^3x^2 + 19\,a^2b^5 - 144\,a^3b^3c + 260\,a^4bc^2}{8\,(a^4b^4 - 8\,a^5b^2c + 16\,a^6c^2)(cx^4+bx^2+a)^2} + \frac{3\,b\log\left(cx^4+bx^2+a\right)}{4\,a^4} - \frac{3\,b\log\left(x^2\right)}{2\,a^4} + \frac{3\,bx^2-a}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x^2 + b)/s
qrt(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)
) - 1/8*(9*b^5*c^2*x^8 - 72*a*b^3*c^3*x^8 + 144*a^2*b*c^4*x^8 + 18*b^6*c*x^
6 - 136*a*b^4*c^2*x^6 + 236*a^2*b^2*c^3*x^6 + 56*a^3*c^4*x^6 + 9*b^7*x^4 -
38*a*b^5*c*x^4 - 110*a^2*b^3*c^2*x^4 + 436*a^3*b*c^3*x^4 + 26*a*b^6*x^2 - 1
92*a^2*b^4*c*x^2 + 316*a^3*b^2*c^2*x^2 + 72*a^4*c^3*x^2 + 19*a^2*b^5 - 144*
a^3*b^3*c + 260*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(c*x^4 + b
*x^2 + a)^2) + 3/4*b*log(c*x^4 + b*x^2 + a)/a^4 - 3/2*b*log(x^2)/a^4 + 1/2*
(3*b*x^2 - a)/(a^4*x^2)
```

Mupad [**B**]
time = 11.76, size = 2500, normalized size = 9.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*x^2 + c*x^4)^3),x)

[Out] (log(((27*c^5*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) - ((3*b - 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((9*c^3*(4*b^10 - 100*a^5*c^5 + 342*a^2*b^6*c^2 - 837*a^3*b^4*c^3 + 780*a^4*b^2*c^4 - 61*a*b^8*c))/(a^6*(4*a*c - b^2)^4) - ((3*b - 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((6*c^3*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*(3*b - 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^4 + (12*b*c^2*(b^6 - 10*a^3*c^3 + 23*a^2*b^2*c^2 - 9*a*b^4*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4) + (9*b*c^4*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (27*b*c^4*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2)/(a^9*(4*a*c - b^2)^4))*((27*c^5*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) - ((3*b + 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((9*c^3*(4*b^10 - 100*a^5*c^5 + 342*a^2*b^6*c^2 - 837*a^3*b^4*c^3 + 780*a^4*b^2*c^4 - 61*a*b^8*c))/(a^6*(4*a*c - b^2)^4) - ((3*b + 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((6*c^3*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*(3*b + 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^4 + (12*b*c^2*(b^6 - 10*a^3*c^3 + 23*a^2*b^2*c^2 - 9*a*b^4*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4) + (9*b*c^4*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (27*b*c^4*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2)/(a^9*(4*a*c - b^2)^4)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (3*b*log(x))/a^4 - (1/(2*a) + (x^4*(3*b^6 + 50*a^3*c^3 + 7*a^2*b^2*c^2 - 18*a*b^4*c))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^6*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3))/(4*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(9*b^5 + 122*a^2*b*c^2 - 68*a*b^3*c))/(4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^2*x^8*(b^4 + 10*a^2*c^2 - 7*a*b^2*c))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^6*(2*a*c + b^2) + a^2*x^2 + c^2*x^10 + 2*a*b*x^4 + 2*b*c*x^8) - (3*atan((x^2*((((27000*a^6*c^11 + 27*b^12*c^5 - 567*a*b^10*c^6 + 4779*a^2*b^8*c^7 - 20601*a^3*b^6*c^8 + 47790*a^4*b^4*c^9 - 56700*a^5*b^2*c^10)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((129600*a^9*b*c^10 + 54*a^3*b^13*c^4 - 1233*a^4*b^11*c^5 + 11583*a^5*b^9*c^6 - 57204*a^6*b^7*c^7 + 156276*a^7*b^5*c^8 - 223200*a^8*b^3*c^9)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - (((153600*a^13*c^10 + 6*a^6*b^14*c^3 - 108*a^7*b^12*c^4 + 588*a^8*b^10*c^5 + 792*a^9*b^8*c^6 - 22272*a^10*b^6*c^7 + 100608*a^11*b^4*c^8 - 199680*a^12*b^2*c^9)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3

+ 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)))*(6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (3*((3*((153600*a^13*c^10 + 6*a^6*b^14*c^3 - 108*a^7*b^12*c^4 + 588*a^8*b^10*c^5 + 792*a^9*b^8*c^6 - 22272*a^10*b^6*c^7 + 100608*a^11*b^4*c^8 - 199680*a^12*b^2*c^9)/(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5) - ((6*b^11 - 6144*a^5*b*c^5 + 960*a^2*b^7*c^2 - 3840*a^3*b^5*c^3 + 7680*a^4*b^3*c^4 - 120*a*b^9*c)*(163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^10*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 + 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)*(a^9*b^12 + 4096*a^15*c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5)))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(4*a^4...

3.881 $\int \frac{x^{10}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=400

$$-\frac{3b(b^2-8ac)\,x}{8c^2\left(b^2-4ac\right)^2}+\frac{(b^2-28ac)\,x^3}{8c\left(b^2-4ac\right)^2}+\frac{x^7(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^5(12ab-(b^2-28ac)\,x^2)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\Big(b^4-9ab^2 \cdots}{\cdots}$$

```
[Out] -3/8*b*(-8*a*c+b^2)*x/c^2/(-4*a*c+b^2)^2+1/8*(-28*a*c+b^2)*x^3/c/(-4*a*c+b^
2)^2+1/4*x^7*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/8*x^5*(12*a*b-(-2
8*a*c+b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/16*arctan(x*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^4-9*a*b^2*c+28*a^2*c^2+(-44*a^2*b*c^2+11
*a*b^3*c-b^5)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2))*(b^4-9*a*b^2*c+28*a^2*c^2+(44*a^2*b*c^2-11*a*b^3*c+b^5)/(-4*a*c+b^2)^(1
/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.20, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1134, 1289, 1293, 1180, 211}

$$\frac{3\left(-\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{3bx(b^2-8ac)}{8c^2\left(b^2-4ac\right)^2}+\frac{x^3(b^2-28ac)}{8c\left(b^2-4ac\right)^2}+\frac{x^7(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^5(12ab-x^2(b^2-28ac))}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3*b*(b^2 - 8*a*c)*x)/(8*c^2*(b^2 - 4*a*c)^2) + ((b^2 - 28*a*c)*x^3)/(8*c*
(b^2 - 4*a*c)^2) + (x^7*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
^2) + (x^5*(12*a*b - (b^2 - 28*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*(b^4 - 9*a*b^2*c + 28*a^2*c^2 - (b^5 - 11*a*b^3*c + 44*a^2*b*c^
2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b
^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^{10}}{(a+bx^2+cx^4)^3}\,dx = \frac{x^7(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\int \frac{x^6(14a+bx^2)}{(a+bx^2+cx^4)^2}\,dx}{4(b^2-4ac)}$$

$$= \frac{x^7(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^5(12ab-(b^2-28ac)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{\int \frac{x^4(60ab-3(b^2-28ac)x}{a+bx^2+cx^4}}{8(b^2-4ac)^2}$$

$$= \frac{(b^2-28ac)x^3}{8c(b^2-4ac)^2} + \frac{x^7(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^5(12ab-(b^2-28ac)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} + \int$$

$$= -\frac{3b(b^2-8ac)x}{8c^2(b^2-4ac)^2} + \frac{(b^2-28ac)x^3}{8c(b^2-4ac)^2} + \frac{x^7(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^5(12ab-(}{8(b^2-4ac)^2}$$

$$= -\frac{3b(b^2-8ac)x}{8c^2(b^2-4ac)^2} + \frac{(b^2-28ac)x^3}{8c(b^2-4ac)^2} + \frac{x^7(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^5(12ab-(}{8(b^2-4ac)^2}$$

$$= -\frac{3b(b^2-8ac)x}{8c^2(b^2-4ac)^2} + \frac{(b^2-28ac)x^3}{8c(b^2-4ac)^2} + \frac{x^7(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^5(12ab-(}{8(b^2-4ac)^2}$$

Mathematica [A]
time = 0.74, size = 455, normalized size = 1.14

$$\frac{\frac{2x(2b^5-17ab^3c+48a^2bc^2-5b^4cx^2+37ab^2c^2x^2-44a^2c^3x^2)}{(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{4(b^4x^3+ab^2x(b-4cx^2)+a^2cx(-3b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3\sqrt{2}\sqrt{c}\left(-b^5+11ab^3c-44a^2bc^2+b^4\sqrt{b^2-4ac}-9ab^2c\sqrt{b^2-4ac}+28a^2c^2\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(b^5-11ab^3c+44a^2bc^2+b^4\sqrt{b^2-4ac}-9ab^2c\sqrt{b^2-4ac}+28a^2c^2\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}}{16c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(2*b^5 - 17*a*b^3*c + 48*a^2*b*c^2 - 5*b^4*c*x^2 + 37*a*b^2*c^2*x^2 -
 44*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*(b^4*x^3 + a*b
^2*x*(b - 4*c*x^2) + a^2*c*x*(-3*b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b^5 + 11*a*b^3*c - 44*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^5 - 11*a*b^3*c +
44*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2
*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^3)
```

Maple [A]
time = 0.07, size = 496, normalized size = 1.24

method	result
risch	$\frac{-\frac{\left(44a^2c^2-37a\,b^2c+5b^4\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)c}+\frac{b\left(4a^2c^2+20a\,b^2c-3b^4\right)x^5}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(28a^2c^2-49a\,b^2c+6b^4\right)x^3}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{3a^2b\left(8ac-b^2\right)x}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c+\right.}\right.}{}$
default	$\frac{-\frac{\left(44a^2c^2-37a\,b^2c+5b^4\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)c}+\frac{b\left(4a^2c^2+20a\,b^2c-3b^4\right)x^5}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(28a^2c^2-49a\,b^2c+6b^4\right)x^3}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{3a^2b\left(8ac-b^2\right)x}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{-\frac{3\left(28a^2c^2\sqrt{-4ac\cdots}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*(44*a^2*c^2-37*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^7+1/8*b*
(4*a^2*c^2+20*a*b^2*c-3*b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*a/c^2*(
28*a^2*c^2-49*a*b^2*c+6*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/8*a^2*b*(8*a*
c-b^2)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/2/c/(16*a^2*c^
2-8*a*b^2*c+b^4)*(-1/8*(28*a^2*c^2*(-4*a*c+b^2)^(1/2)-9*a*b^2*c*(-4*a*c+b^2
)^(1/2)+b^4*(-4*a*c+b^2)^(1/2)-44*a^2*b*c^2+11*a*b^3*c-b^5)/c/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(28*a^2*c^2*(-4*a*c+b^2)^(1/2)-9*a*b^2*c*(-
4*a*c+b^2)^(1/2)+b^4*(-4*a*c+b^2)^(1/2)+44*a^2*b*c^2-11*a*b^3*c+b^5)/c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^7 + (3*b^5 - 20*a*b^3*c - 4*a
^2*b*c^2)*x^5 + (6*a*b^4 - 49*a^2*b^2*c + 28*a^3*c^2)*x^3 + 3*(a^2*b^3 - 8*
a^3*b*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3
*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6
*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^
3*b*c^4)*x^2) + 3/8*integrate((a*b^3 - 8*a^2*b*c + (b^4 - 9*a*b^2*c + 28*a^
2*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4279 vs. $2(351) = 702$.
time = 0.83, size = 4279, normalized size = 10.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^7 + 2*(3*b^5 - 20*a*b^3*c
- 4*a^2*b*c^2)*x^5 + 2*(6*a*b^4 - 49*a^2*b^2*c + 28*a^3*c^2)*x^3 + 3*sqrt(1
/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3
 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6
*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)
*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^
4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 12
80*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 -
1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c
^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 2
0*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a
^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5
*b^2*c^3 + 38416*a^6*c^4)*x + 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*
b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*
a^6*b*c^6 - (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8
 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*
c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^
4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1
280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c
^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*
b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 2
2*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 -
 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 -
1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c
^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) - 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*
c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c
^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)
*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^9 - 21*a*
b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20
*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^
5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*
a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 +
 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^
6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27*(21*a^2
*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4
)*x - 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c
```

```
^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 - (b^14*c^5 -
30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 4
9664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b
^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a
*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*
a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1
680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^
8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*
c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2
*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c
^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 -
1024*a^5*c^10))) + 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 +
a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^
2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*
a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2
- 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6
*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a
*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20
*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 102
4*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8
+ 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4
189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4)*x + 27/2*sqrt(1/2)*(b^
13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 -
 27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 + (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*
b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 8192
0*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 -
 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*
c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 -
 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^
5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1
024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2432 vs. 2(351) = 702.
time = 9.22, size = 2432, normalized size = 6.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6
*c - 2*b^7*c + 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 24*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^5*c^2 + 32*a*b^5*c^2 + 2*b^6*c^2 - 128*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 160
*a^2*b^3*c^3 - 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^4 + 256*a^3*b*c^4 + 192*a^2*b^2*c^4 - 448*a^3*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 + 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*c^3 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^3 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^4 + 2*(b^2 - 4*a*c)*b^5*c - 24*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^
2 - 4*a*c)*b^4*c^2 + 64*(b^2 - 4*a*c)*a^2*b*c^3 + 20*(b^2 - 4*a*c)*a*b^2*c^
3 - 112*(b^2 - 4*a*c)*a^2*c^4)*arctan(2*sqrt(1/2)*x/sqrt((b^5*c^2 - 8*a*b^3
*c^3 + 16*a^2*b*c^4 + sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)^2 - 4*(a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))
)/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^8*c^2 - 16*a*b^6*c^3 - 2*b^7*c
^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4 - 256*a^3*b^2*c^5 - 96*a^2*b^3
*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b*c^6 + 48*a^2*b^2*c^6 - 64*a^3
*c^7)*abs(c)) + 3/32*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^6*c + 2*b^7*c + 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^3*c^2 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 32*a*b^5*c^2 + 2*b^6*c^2 - 128*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^3 + 160*a^2*b^3*c^3 - 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^4 - 256*a^3*b*c^4 + 192*a^2*b^2*c^4 - 448*a^3*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 14*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 96*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
```

```
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^5*c + 24*(b^2 - 4*a*c)*a
*b^3*c^2 - 2*(b^2 - 4*a*c)*b^4*c^2 - 64*(b^2 - 4*a*c)*a^2*b*c^3 + 20*(b^2 -
 4*a*c)*a*b^2*c^3 - 112*(b^2 - 4*a*c)*a^2*c^4)*arctan(2*sqrt(1/2)*x/sqrt((b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 - sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*
b*c^4)^2 - 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^3 - 8*a*b^2*c^
4 + 16*a^2*c^5)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^8*c^2 - 16*a*b
^6*c^3 - 2*b^7*c^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4 - 256*a^3*b^2*
c^5 - 96*a^2*b^3*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b*c^6 + 48*a^2*
b^2*c^6 - 64*a^3*c^7)*abs(c)) - 1/8*(5*b^4*c*x^7 - 37*a*b^2*c^2*x^7 + 44*a^
2*c^3*x^7 + 3*b^5*x^5 - 20*a*b^3*c*x^5 - 4*a^2*b*c^2*x^5 + 6*a*b^4*x^3 - 49
*a^2*b^2*c*x^3 + 28*a^3*c^2*x^3 + 3*a^2*b^3*x - 24*a^3*b*c*x)/((b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B]

time = 9.04, size = 2500, normalized size = 6.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((x^3*(6*a*b^4 + 28*a^3*c^2 - 49*a^2*b^2*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (x^7*(5*b^4 + 44*a^2*c^2 - 37*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)) - (b*x^5*(4*a^2*c^2 - 3*b^4 + 20*a*b^2*c))/(8*c^2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) - (3*a^2*b*x*(8*a*c - b^2))/(8*c^2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
- atan(((((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 8192
0*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a^6*b^3*
c^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 128
0*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*((9*(b^4*(-(4*a*
c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3
*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 +
 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576
*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^
8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 19660
80*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256
*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840
*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c
^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2)
 - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*
a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c
```

^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17
*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*
c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12
*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 +
2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) - (x*(9*b^10 - 14112*a
^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^4*b^2*c^4 - 198*a*b^
8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*
b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 -
769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*
c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*
a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2
)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*
b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 +
860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440
*a^9*b^2*c^14)))^(1/2)*1i - (((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168
*a^2*b^11*c^4 + 81920*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^
7 - 2883584*a^6*b^3*c^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 2
40*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) +
(x*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2
*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1
069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2
*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(
1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^
7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*
a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^
2*c^14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*
a^2*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 +
b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*
a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a
^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6
+ 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^1
5)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(10485
76*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*
c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 196
6080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2) +
(x*(9*b^10 - 14112*a^5*c^5 + 1881*a^2*b^6*c^2 - 9090*a^3*b^4*c^3 + 21312*a^
4*b^2*c^4 - 198*a*b^8*c))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^
2*b^4*c^5 - 256*a^3*b^2*c^6)))*((9*(b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 +
1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c
^4 + 316864*a^5*b^9*c^5 - 1069824*a^6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 30105
60*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*
b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a
*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258
048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^
8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*1i)/((((3*(256*a*b^13*c^3 + 2097

```
152*a^7*b*c^9 - 7168*a^2*b^11*c^4 + 81920*a^3*b...
```

3.882 $\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=348

$$-\frac{(b^2+20ac)\,x}{8c\,(b^2-4ac)^2}+\frac{x^5(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{x^3(12ab+(b^2+20ac)\,x^2)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{\left(b^3-16abc-\frac{b^4-18ab^2c-40a^2c}{\sqrt{b^2-4ac}}\right.}{8\sqrt{2}\,c^{3/2}\,(b^2-4a}$$

```
[Out] -1/8*(20*a*c+b^2)*x/c/(-4*a*c+b^2)^2+1/4*x^5*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^
4+b*x^2+a)^2+1/8*x^3*(12*a*b+(20*a*c+b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+
a)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^3-16*a*b*
c+(40*a^2*c^2+18*a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2))*(b^3-16*a*b*c+(-40*a^2*c^2-18*a*b^2*c+b^4)/(-4*a*c+b^2)
^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.59, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1134, 1289, 1293, 1180, 211}

$$\frac{\left(-\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{3/2}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,c^{3/2}(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x^3(x^2(20ac+b^2)+12ab)}{8(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x(20ac+b^2)}{8c(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/8*((b^2 + 20*a*c)*x)/(c*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x^2))/(4*(b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^3*(12*a*b + (b^2 + 20*a*c)*x^2))/(8*(b^
2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3 - 16*a*b*c - (b^4 - 18*a*b^2*c -
40*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) + ((b^3 - 16*a*b*c + (b^4 - 18*a*b^2*c - 40*a^2*c^2)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*
(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
```

```
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx = \frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\int \frac{x^4(10a-bx^2)}{(a+bx^2+cx^4)^2}\,dx}{4(b^2-4ac)}$$

$$= \frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^3(12ab+(b^2+20ac)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{\int \frac{x^2(36ab+(b^2+20ac)x^2}{a+bx^2+cx^4}}{8(b^2-4ac)^2}$$

$$= -\frac{(b^2+20ac)x}{8c(b^2-4ac)^2} + \frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^3(12ab+(b^2+20ac)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} +$$

$$= -\frac{(b^2+20ac)x}{8c(b^2-4ac)^2} + \frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^3(12ab+(b^2+20ac)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} +$$

$$= -\frac{(b^2+20ac)x}{8c(b^2-4ac)^2} + \frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^3(12ab+(b^2+20ac)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} +$$

Mathematica [A]
time = 0.61, size = 381, normalized size = 1.09

$$\frac{\frac{2x(-2b^4+11ab^2c-36a^2c^2+b^3cx^2-16abc^2x^2)}{(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{4(-2a^2cx+b^3x^3+abx(b-3cx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{\sqrt{2}\sqrt{c}\left(-b^4+18ab^2c+40a^2c^2+b^3\sqrt{b^2-4ac}-16abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b^4-18ab^2c-40a^2c^2+b^3\sqrt{b^2-4ac}-16abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(-2*b^4 + 11*a*b^2*c - 36*a^2*c^2 + b^3*c*x^2 - 16*a*b*c^2*x^2))/((b^
2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (4*(-2*a^2*c*x + b^3*x^3 + a*b*x*(b - 3
*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-b^4 +
18*a*b^2*c + 40*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b^4 - 18*a*b^2*c - 4
0*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

Maple [A]
time = 0.06, size = 420, normalized size = 1.21

method	result

risch	$\frac{-\frac{b\left(16ac-b^{2}\right)x^{7}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(36a^{2}c^{2}+5a\,b^{2}c+b^{4}\right)x^{5}}{8c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{ab\left(14ac+b^{2}\right)x^{3}}{4c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a^{2}\left(20ac+b^{2}\right)x}{8c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}c+\_Z^{2}b+a\right)}\left(-\right.}{}$
default	$\frac{-\frac{b\left(16ac-b^{2}\right)x^{7}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(36a^{2}c^{2}+5a\,b^{2}c+b^{4}\right)x^{5}}{8c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{ab\left(14ac+b^{2}\right)x^{3}}{4c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a^{2}\left(20ac+b^{2}\right)x}{8c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{-\frac{\left(-16\sqrt{-4ac+b^{2}}\,abc+\sqrt{\right.}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*b*(16*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-1/8*(36*a^2*c^2+5*a*b^2
*c+b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*a/c*b*(14*a*c+b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^3-1/8*a^2*(20*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c
*x^4+b*x^2+a)^2+1/2/(16*a^2*c^2-8*a*b^2*c+b^4)*(-1/8*(-16*(-4*a*c+b^2)^(1/2
)*a*b*c+(-4*a*c+b^2)^(1/2)*b^3+40*a^2*c^2+18*a*b^2*c-b^4)/c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-16*(-4*a*c+b^2)^(1/2)*a*b*c+(-4*a*c+b^2)^(1
/2)*b^3-40*a^2*c^2-18*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^3*c - 16*a*b*c^2)*x^7 - (b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^5 - 2*(a*b
^3 + 14*a^2*b*c)*x^3 - (a^2*b^2 + 20*a^3*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b
^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2) - 1/8*integrate(-(a*b^2 + 20*a^2
*c + (b^3 - 16*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3725 vs. $2(302) = 604$.
time = 0.59, size = 3725, normalized size = 10.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/16*(2*(b^3*c - 16*a*b*c^2)*x^7 - 2*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^5 - 4*(a*b^3 + 14*a^2*b*c)*x^3 + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x + 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 - (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x - 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 - (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 +

```
a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*
b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^
3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680
*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 +
 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^
10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^
10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*
b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c
+ 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x + 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c -
 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 + (
b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*
b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625
*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 12
80*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^
2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b
^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*
c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^
4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 -
640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - sqrt(1/2)*((b^4*c^3
- 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 +
2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^
3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 3
5*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 1
60*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b
^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 -
 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4558 vs. 2(302) = 604.
time = 6.82, size = 4558, normalized size = 13.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

[Out] -1/64*(2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*b^9*c^6 - 2688*a^3*b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a^6*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^13*c^2 + 34*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^11*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 - 344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 - 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c^5 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 5632*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 - 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 10240*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 + 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 + 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 - 2*(b^2 - 4*a*c)*b^11*c^4 + 60*(b^2 - 4*a*c)*a*b^9*c^5 - 448*(b^2 - 4*a*c)*a^2*b^7*c^6 + 896*(b^2 - 4*a*c)*a^3*b^5*c^7 + 1536*(b^2 - 4*a*c)*a^4*b^3*c^8 - 5120*(b^2 - 4*a*c)*a^5*b*c^9 - (2*b^5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 2*a*b^8*c^3 - 192*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 16*a^2*b^6*c^4 + 896*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^5 + 288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + 384*a^3*b^4*c^5 - 1280*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^6 - 640*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^6 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 1792*a^4*b^2*c^6 + 320*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^7 + 2560*a^5*c^7 + 2*(b^2 - 4*a*c)*a*b^6*c^3 + 24*(b^2 - 4*a*c)*a^2*b^4*c^4 - 288*(b^2 - 4*a*c)*a^3*b^2*c^5 + 640*(b^2 - 4*a*c)*a^4*c^6)*abs(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))*arctan(2*sqrt(1/2)*x/sqrt((b^5*c - 8*a*b^3*c^2

```
+ 16*a^2*b*c^3 + sqrt((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)^2 - 4*(a*b^4*c -
 8*a^2*b^2*c^2 + 16*a^3*c^3)*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/(b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)))/((a*b^10*c^3 - 20*a^2*b^8*c^4 - 2*a*b^9*c^4
 + 160*a^3*b^6*c^5 + 32*a^2*b^7*c^5 + a*b^8*c^5 - 640*a^4*b^4*c^6 - 192*a^3
*b^5*c^6 - 16*a^2*b^6*c^6 + 1280*a^5*b^2*c^7 + 512*a^4*b^3*c^7 + 96*a^3*b^4
*c^7 - 1024*a^6*c^8 - 512*a^5*b*c^8 - 256*a^4*b^2*c^8 + 256*a^5*c^9)*abs(b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*abs(c)) + 1/64*(2*b^13*c^4 - 68*a*b^11*c^5
+ 688*a^2*b^9*c^6 - 2688*a^3*b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9
 - 20480*a^6*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^13*c^2 + 34*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^11*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^12*c^3 - 344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^9*c^4 - 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^10*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^11*c
^4 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7
*c^5 + 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^
8*c^5 + 30*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^9*
c^5 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
5*c^6 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
^6*c^6 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^7*c^6 - 5632*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^3*c^7 - 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt...
```

Mupad [B]
time = 8.54, size = 2500, normalized size = 7.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*
b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^
6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c
^5 - 6144*a^5*b^2*c^6)) - (x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4
+ 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*
c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 -
40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 -
 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*
a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4
 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b
^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3
*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1
140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c
```

^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a
c - b^2)^15)^(1/2))/(512(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 7
20*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*
c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 26
21440*a^9*b^2*c^12))))^(1/2) - (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208
*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 17
20320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^
4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^1
5*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3
 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7
 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 294912
0*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*1i - (((5242880*a^7*c^8 - 25
6*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6
 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a
^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*
(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13
*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*
a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15
)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*
c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160
*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^
2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a
^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4
*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(
4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*
b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 186
3680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(10
48576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^
14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 19
66080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) +
 (x*(b^8 + 800*a^4*c^4 + 314*a^2*b^4*c^2 + 208*a^3*b^2*c^3 - 36*a*b^6*c))/(
32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4))
)*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^
13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 68096
0*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^
15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^1
6*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 8601
60*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*
b^2*c^12)))^(1/2)*1i)/((((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*
c^4 - 655360*a^4*b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(b^17 + b^2*(-(4*a*c - b^2)
^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 3
4880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3

```
*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c
^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 537
60*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^
6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3
 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c
^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*
b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720
320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b...
```

3.883 $\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=298

$$\frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3x(4ab+(b^2+4ac)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3\left(b^2+4ac-\frac{b(b^2+12ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b-\sqrt{b^2}}}\right.}{8\sqrt{2}\sqrt{c}\,(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] 1/4*x^3*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/8*x*(4*a*b+(4*a*c+b^2)
*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c-b*(12*a*c+b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^2*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*arctan(x*2^(1/2)*
c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(12*a*c+b^2)/(-4*a*c+b^2
)^(1/2))/(-4*a*c+b^2)^2*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1134, 1289, 1180, 211}

$$\frac{3\left(-\frac{b(12ac+b^2)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\,(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{b(12ac+b^2)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\sqrt{c}\,(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{3x(x^2(4ac+b^2)+4ab)}{8(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^3*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x*(4*a*b +
 (b^2 + 4*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2 + 4*
a*c - (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (3*(b^2 + 4*a*c + (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(
b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
```

```
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx = \frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\int \frac{x^2(6a-3bx^2)}{(a+bx^2+cx^4)^2}\,dx}{4(b^2-4ac)}$$

$$= \frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3x(4ab+(b^2+4ac)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{\int \frac{12ab-3(b^2+4ac)x^2}{a+bx^2+cx^4}}{8(b^2-4ac)^2}$$

$$= \frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3x(4ab+(b^2+4ac)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\left(3\left(b^2+4ac-\right.\right.}{}$$

$$= \frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3x(4ab+(b^2+4ac)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3\left(b^2+4ac-\frac{b(}{\sqrt{b}}\right.}{8\sqrt{2}\sqrt{c}}$$

Mathematica [A]

time = 0.55, size = 343, normalized size = 1.15

$$\frac{\frac{4b^3x+8abcx+6b^2cx^3+24ac^2x^3}{(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{4(b^2x^3+ax(b-2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3\sqrt{2}\sqrt{c}\left(-b^3-12abc+b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b^3+12abc+b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}}{16c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^3*x + 8*a*b*c*x + 6*b^2*c*x^3 + 24*a*c^2*x^3)/((b^2 - 4*a*c)^2*(a + b
*x^2 + c*x^4)) - (4*(b^2*x^3 + a*x*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b^3 - 12*a*b*c + b^2*Sqrt[b^2 - 4*a*c]
 + 4*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sq
rt[c]*(b^3 + 12*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)
```

Maple [A]

time = 0.06, size = 374, normalized size = 1.26

method	result
risch	$\frac{\frac{3c(4ac+b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)}+\frac{b(16ac+5b^2)x^5}{128a^2c^2-64a\,b^2c+8b^4}-\frac{(4ac-19b^2)a\,x^3}{8(16a^2c^2-8a\,b^2c+b^4)}+\frac{3a^2bx}{2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\left(\sum_{\_R=\text{RootOf}(\_Z^4c+\_Z^2b+a)}\frac{\left(4ac \ldots\right.}{16a^2c \ldots}\right.}{\ldots}$
default	$\frac{\frac{3c(4ac+b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)}+\frac{b(16ac+5b^2)x^5}{128a^2c^2-64a\,b^2c+8b^4}-\frac{(4ac-19b^2)a\,x^3}{8(16a^2c^2-8a\,b^2c+b^4)}+\frac{3a^2bx}{2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\frac{3c\left(-\frac{\left(4ac\sqrt{-4ac+b^2}+b^2\sqrt{-4 \ldots\right.}{8c\sqrt{- \ldots}}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (3/8*c*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*b*(16*a*c+5*b^2)/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(4*a*c-19*b^2)*a/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^3+3/2*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/2/(16*a^2*c
^2-8*a*b^2*c+b^4)*c*(-1/8*(4*a*c*(-4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^2)^(1/2)-
12*a*b*c-b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(4*a*c*(-4*a*c
+b^2)^(1/2)+b^2*(-4*a*c+b^2)^(1/2)+12*a*b*c+b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/
```

```
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^2*c + 4*a*c^2)*x^7 + (5*b^3 + 16*a*b*c)*x^5 + 12*a^2*b*x + (19*a*
b^2 - 4*a^2*c)*x^3)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6
 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2
) + 3/8*integrate(((b^2 + 4*a*c)*x^2 - 4*a*b)/(c*x^4 + b*x^2 + a), x)/(b^4
- 8*a*b^2*c + 16*a^2*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3128 vs. $2(254) = 508$.
time = 0.43, size = 3128, normalized size = 10.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^2*c + 4*a*c^2)*x^7 + 2*(5*b^3 + 16*a*b*c)*x^5 + 24*a^2*b*x + 2*(
19*a*b^2 - 4*a^2*c)*x^3 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c +
 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c +
 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a
*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*
c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 128
0*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 6
40*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*
c + 16*a^2*c^2)*x + 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*
a^3*b*c^3 + (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*
b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c
^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b^8*c^2 + 160*
a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*
c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 -
 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4
+ 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 -
```

8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x - 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 + (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x + 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 - (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x - 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 - (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4

```
 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c
^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)))
)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1750 vs. 2(254) = 508.
time = 6.54, size = 1750, normalized size = 5.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/16*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 16*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^4*c - 4*b^5*c + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 16
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^3*c^2 + 32*a*b^3*c^2 + 6*b^4*c^2 - 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 64*a^2*b*c^3 - 16*a*b^2*c^3 - 32*a^2*c^4
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 4*(b^2 - 4*a*c)*b^3*c - 16*(b^2 - 4*a*c)
*a*b*c^2 - 6*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*c^3)*arctan(2*sqrt(1
/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sqrt((b^5 - 8*a*b^3*c + 16*a^2
*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((b^8 - 16*a*b^6*c - 2*b^7*
c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2*c^3 - 96*a^2*b^3*
c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*abs(c)) - 3/16*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 16*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 4*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^4*c + 4*b^5*c + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2
```

```
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 32*a*b^3*c^2 - 6*b^4*c^2 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 64*a^2*b*c^3 + 16*a*b^2*c^3
 + 32*a^2*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
- 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 3*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 4*(b^2 - 4*a*c)*b^3*c + 16
*(b^2 - 4*a*c)*a*b*c^2 + 6*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a
rctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - sqrt((b^5 - 8*a*
b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((b^8 - 16*a*
b^6*c - 2*b^7*c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2*c^3
 - 96*a^2*b^3*c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*abs(c)) + 1/8*(3*b^2*c*x^7 + 12*a*c^2*x^7 + 5*b^3*x^5 +
16*a*b*c*x^5 + 19*a*b^2*x^3 - 4*a^2*c*x^3 + 12*a^2*b*x)/((c*x^4 + b*x^2 + a
)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

Mupad [B]
time = 8.18, size = 2500, normalized size = 8.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(1024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 16384
0*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 409
6*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^
5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 819
20*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 10
24*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a
^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4
*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8
+ 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*
a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327
680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3
 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 -
 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5
 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*
a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 25
8048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*
b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) - (x*(9*b^6*c - 288*a^3*c^4 + 126*a
*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*
```

a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*
a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*
a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10
*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^
12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2
949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*1i - (((3*(1024*a*b^11*c
^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4 - 655360*a^
4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c
^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c))
 + (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^
11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^
6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2
 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^
10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2
621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*
c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^
8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^
15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*
a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20
*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16
*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 86016
0*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2
c^10)))^(1/2) + (x(9*b^6*c - 288*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^
3))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)
))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*
c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*b
^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 + 7
20*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*
c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621
440*a^9*b^2*c^10)))^(1/2)*1i)/((3*(576*a^4*c^4 + 540*a^2*b^4*c^2 + 1584*a^3
*b^2*c^3 + 45*a*b^6*c))/(256*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*
a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (((3*(1
024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 163840*a^3*b^7*c^4
 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 2
40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 2
4*a*b^10*c)) - (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7
 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^
5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40
*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 2
58048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8
*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 2
62144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c
^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c
)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11
*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 1024*a^5*b^5*c^5 + 61440*a^6*

```
b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a^10*c^11 - 40*a*b^18*c^2 +
720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10
*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 262
1440*a^9*b^2*c^10)))^(1/2) - (x*(9*b^6*c - 288*...
```

3.884 $\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=289

$$\frac{x(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x(7b^2-4ac+12bcx^2)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\tan^{-1}\Bigg(\frac{\;}{\sqrt{\;}}}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-}}}$$

```
[Out] 1/4*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x*(12*b*c*x^2-4*a*c+7*
b^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/8*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1134, 1192, 1180, 211}

$$\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x(-4ac+7b^2+12bcx^2)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x*(7*b^2 - 4*a
*c + 12*b*c*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(3*b
^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
```

```
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{2a-5bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= \frac{x(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\int \frac{3a\left(b^2+4ac\right)-12abcx^2}{a+bx^2+cx^4}}{8a\left(b^2-4ac\right)^2}$$

$$= \frac{x(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\Big(3c\Big(3b^2+4ac-2 \cdots}{\cdots}$$

$$= \frac{x(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x\left(7b^2-4ac+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{c}\,\Big(3b^2+4ac- \cdots}{4\sqrt{2} \cdots}$$

Mathematica [A]
time = 0.47, size = 285, normalized size = 0.99

$$\frac{1}{8}\left(\frac{2(2ax+bx^3)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{-7b^2x+4acx-12bcx^3}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (-7*b^2*x + 4*
a*c*x - 12*b*c*x^3)/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt
[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/8
```

Maple [A]
time = 0.06, size = 338, normalized size = 1.17

method	result
risch	$\frac{-\frac{3bc^2x^7}{2(16a^2c^2-8ab^2c+b^4)}+\frac{c(4ac-19b^2)x^5}{128a^2c^2-64ab^2c+8b^4}-\frac{b(16ac+5b^2)x^3}{8(16a^2c^2-8ab^2c+b^4)}-\frac{3a(4ac+b^2)x}{8(16a^2c^2-8ab^2c+b^4)}}{(cx^4+bx^2+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4c+\_Z^2b+a)} \right.}{}$
default	$\frac{-\frac{3bc^2x^7}{2(16a^2c^2-8ab^2c+b^4)}+\frac{c(4ac-19b^2)x^5}{128a^2c^2-64ab^2c+8b^4}-\frac{b(16ac+5b^2)x^3}{8(16a^2c^2-8ab^2c+b^4)}-\frac{3a(4ac+b^2)x}{8(16a^2c^2-8ab^2c+b^4)}}{(cx^4+bx^2+a)^2}+\frac{3c\left(-\frac{\left(3b^2+4ac-2b\sqrt{-4ac+b^2}\right.}{4\sqrt{-4ac+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/2*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*c*(4*a*c-19*b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^5-1/8*b*(16*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3/8
*a*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/2/(16*a^2*
c^2-8*a*b^2*c+b^4)*c*(-1/4*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/4*(-2*b*(-4*a*c+b^2)^(1/2)-4*a*c-3*b^2)/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(12*b*c^2*x^7 + (19*b^2*c - 4*a*c^2)*x^5 + (5*b^3 + 16*a*b*c)*x^3 + 3*
(a*b^2 + 4*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^
6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^
2) - 3/8*integrate((4*b*c*x^2 - b^2 - 4*a*c)/(c*x^4 + b*x^2 + a), x)/(b^4 -
 8*a*b^2*c + 16*a^2*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3128 vs. $2(241) = 482$.
time = 0.45, size = 3128, normalized size = 10.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*b*c^2*x^7 + 2*(19*b^2*c - 4*a*c^2)*x^5 + 2*(5*b^3 + 16*a*b*c)*x^3
 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*
a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqr
t(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*
c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20
*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^
7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a
^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x +
 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4 - (a*b^13 -
 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 1433
6*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6
*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*
a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4
*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c +
160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^
10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 -
1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*
b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c
 + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqr
t(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^
2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b
^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 +
16*a^2*c^3)*x - 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*
c^4 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5
*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*
```

```
c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*s
qrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^
6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 -
20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*
a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280
*a^5*b^2*c^4 - 1024*a^6*c^5))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*
b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*
b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10
 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 10
24*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^
3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*
c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c +
40*a*b^2*c^2 + 16*a^2*c^3)*x + 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2
*c^3 - 256*a^4*c^4 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7
*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^1
0 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1
024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8
*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/s
qrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*
b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4
*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a
^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*
b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a
^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 -
 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c
 + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*lo
g(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x - 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*
c + 128*a^3*b^2*c^3 - 256*a^4*c^4 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2
 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^
6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*
a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^
10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 -
1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*
c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^
6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) + 6*(a*b^2 + 4
*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1863 vs. $2(241) = 482$.
time = 8.36, size = 1863, normalized size = 6.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 2*b^6*c - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*a*b^4*c^2 + 2*b^5*c^2 + 64*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 128*a^3*c^4 - 96*a^2*b*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 2*(b^2 - 4*a*c)*b^3*
c^2 - 32*(b^2 - 4*a*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*arctan(2*sqrt(1/
2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sqrt((b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((a*b^8 - 16*a^2*b^6*c - 2*a
*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*c^2 - 256*a^4*b^2*c^3 - 96
*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + 128*a^4*b*c^4 + 48*a^3*b^2*c^
4 - 64*a^4*c^5)*abs(c)) + 3/32*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6
 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*a
*b^4*c^2 + 2*b^5*c^2 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 +
 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^2*b^2*c^3 + 16
*a*b^3*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 128*a^3*c
^4 - 96*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 48*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 24*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 12*sqrt(2)*sqr
```

```
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^
4*c - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a^2*c^3 - 24*(b^2 - 4*a*c)
*a*b*c^3)*arctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - sqrt(
(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((
a*b^8 - 16*a^2*b^6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*
c^2 - 256*a^4*b^2*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + 128
*a^4*b*c^4 + 48*a^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) - 1/8*(12*b*c^2*x^7 + 19*
b^2*c*x^5 - 4*a*c^2*x^5 + 5*b^3*x^3 + 16*a*b*c*x^3 + 3*a*b^2*x + 12*a^2*c*x
)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

Mupad [B]
time = 7.59, size = 2500, normalized size = 8.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c
^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 + 2
40*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 2
4*a*b^10*c)) - (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 +
 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5
 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*
a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 25
8048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*
b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9*c^3 - 13
1072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6
))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c))
)*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^
2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3
*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720
*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^
5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 262144
0*a^10*b^2*c^9)))^(1/2) - (x*(144*a^2*c^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(1
6*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9
*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4
160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6
- 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*
b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 8
60160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^1
0*b^2*c^9)))^(1/2)*1i - (((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^
3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12
+ 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 61
```

44*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 +
81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4
+ 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 10485
76*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760
*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*
c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2
560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 +
163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*
c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7
+ 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c
^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 4
0*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 -
258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^
9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) + (x*(144*a^2*c^5 + 117*b^4*c^3 +
72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 56
0*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 -
61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^2
*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 25804
8*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4
*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*1i)/((((3*(262144*a^6*c^8 - 64*b^12*c^
2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2
c^7))/(128(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 384
0*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*((9*((-(4*a*c - b^2)^
15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 +
11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(5
12*(a*b^20 + 1048576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^
4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6
- 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)
*(128*b^11*c^2 - 2560*a*b^9*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81
920*a^3*b^5*c^5 + 163840*a^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*
c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15
+ 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^
4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 104
8576*a^11*c^10 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 537
60*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^
6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) - (x*(144*a^2*c
^5 + 117*b^4*c^3 + 72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 -
256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81
920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1
024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13...

3.885 $\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=311

$$-\frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x(b(b^2+8ac)+c(b^2+20ac)x^2)}{8a(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{\sqrt{c}\left(b^2+20ac+\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\cdots\right.}{8\sqrt{2}\,a(b^2-4ac)^2\sqrt{b-\sqrt{b\cdots}}}$$

```
[Out] -1/4*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/8*x*(b*(8*a*c+b^2)+c*(2
0*a*c+b^2)*x^2)/a/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arctan(x*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c+b*(-52*a*c+b^2)/(-4*a
*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*a
rctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c-b
*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A]

time = 0.49, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1133, 1192, 1180, 211}

$$\frac{\sqrt{c}\left(\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x(cx^2(20ac+b^2)+b(8ac+b^2))}{8a(b^2-4ac)^2(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(b*(b^2 +
 8*a*c) + c*(b^2 + 20*a*c)*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
+ (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c))
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
```

```
1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx = -\frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{\int \frac{b-10cx^2}{(a+bx^2+cx^4)^2}\,dx}{4(b^2-4ac)}$$

$$= -\frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b(b^2+8ac)+c(b^2+20ac)x^2)}{8a(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{\int \frac{-b(b^2-16ac)-}{a+bx^2}}{8a(b^2}$$

$$= -\frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b(b^2+8ac)+c(b^2+20ac)x^2)}{8a(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\Big(c\Big(b^2+20a}{}$$

$$= -\frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b(b^2+8ac)+c(b^2+20ac)x^2)}{8a(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\sqrt{c}\Big(b^2+20}{8}$$

Mathematica [A]

time = 0.55, size = 334, normalized size = 1.07

$$\frac{1}{16}\left(-\frac{4x(b+2cx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x(b^3+8abc+b^2cx^2+20ac^2x^2)}{a(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^3+52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(b^3 + 8
*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
 + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b
^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^3 + 5
2*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/16
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(267) = 534$.
time = 0.16, size = 836, normalized size = 2.69

method	result
risch	$\frac{\frac{c^2(20ac+b^2)x^7}{8a(16a^2c^2-8ab^2c+b^4)}+\frac{cb(14ac+b^2)x^5}{4a(16a^2c^2-8ab^2c+b^4)}+\frac{(36a^2c^2+5ab^2c+b^4)x^3}{8a(16a^2c^2-8ab^2c+b^4)}+\frac{b(16ac-b^2)x}{128a^2c^2-64ab^2c+8b^4}}{(cx^4+bx^2+a)^2}+\frac{\sum_{\_R=\text{RootOf}(\_Z^4c+\_Z^2b+a)} \left(\frac{c(2\ldots}{16\ldots}\right.}{\ldots}$
default	$64c^3\left(-\frac{-\frac{\left(320\sqrt{-4ac+b^2}\,a^3c^3-144\sqrt{-4ac+b^2}\,a^2b^2c^2+12\sqrt{-4ac+b^2}\,ab^4c+\sqrt{-4ac+b^2}\,b^6+64a^3bc^3-48\ldots\right.}{16ac^2}\ldots}{\left(x^2+\ldots\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 64*c^3*(-1/64/(-4*a*c+b^2)^(5/2)/c/(4*a*c-b^2)^2*((-1/16/a/c^2*(320*(-4*a*c
+b^2)^(1/2)*a^3*c^3-144*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2+12*(-4*a*c+b^2)^(1/2
)*a*b^4*c+(-4*a*c+b^2)^(1/2)*b^6+64*a^3*b*c^3-48*a^2*b^3*c^2+12*a*b^5*c-b^7
)*x^3+1/8/c^2*(-96*(-4*a*c+b^2)^(1/2)*a^2*b*c^2+48*(-4*a*c+b^2)^(1/2)*a*b^3
```

```
*c-6*(-4*a*c+b^2)^(1/2)*b^5+448*a^3*c^3-336*a^2*b^2*c^2+84*a*b^4*c-7*b^6)*x
)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2-1/16*(320*(-4*a*c+b^2)^(1/2)*a^3
*c^3-144*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2+12*(-4*a*c+b^2)^(1/2)*a*b^4*c+(-4*a
*c+b^2)^(1/2)*b^6+832*a^3*b*c^3-432*a^2*b^3*c^2+60*a*b^5*c-b^7)/a/c*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)))+1/64/(-4*a*c+b^2)^(5/2)/c/(4*a*c-b^2)^2*((-1/16/a/c^2*(-320*(-
4*a*c+b^2)^(1/2)*a^3*c^3+144*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2-12*(-4*a*c+b^2)
^(1/2)*a*b^4*c-(-4*a*c+b^2)^(1/2)*b^6+64*a^3*b*c^3-48*a^2*b^3*c^2+12*a*b^5*
c-b^7)*x^3+1/8/c^2*(96*(-4*a*c+b^2)^(1/2)*a^2*b*c^2-48*(-4*a*c+b^2)^(1/2)*a
*b^3*c+6*(-4*a*c+b^2)^(1/2)*b^5+448*a^3*c^3-336*a^2*b^2*c^2+84*a*b^4*c-7*b^
6)*x)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2-1/16*(320*(-4*a*c+b^2)^(1/2)
*a^3*c^3-144*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2+12*(-4*a*c+b^2)^(1/2)*a*b^4*c+(
-4*a*c+b^2)^(1/2)*b^6-832*a^3*b*c^3+432*a^2*b^3*c^2-60*a*b^5*c+b^7)/a/c*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^2*c^2 + 20*a*c^3)*x^7 + 2*(b^3*c + 14*a*b*c^2)*x^5 + (b^4 + 5*a*b^2
*c + 36*a^2*c^2)*x^3 - (a*b^3 - 16*a^2*b*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3
+ 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2
*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(
a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) + 1/8*integrate((b^3 - 16*a*b*c
+ (b^2*c + 20*a*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3777 vs. $2(267) = 534$.
time = 0.57, size = 3777, normalized size = 12.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*x^7 + 4*(b^3*c + 14*a*b*c^2)*x^5 + 2*(b^4 + 5*
a*b^2*c + 36*a^2*c^2)*x^3 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2
+ 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 -
 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2
```

+ 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x + 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x - 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3

```
*c^5)*x + 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5
*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 + (a^3*b^14 - 38*a^4*b^12*c + 48
0*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 3
2768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a
^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*
c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3
*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 128
0*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^
10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 -
 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*
c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^
2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c -
 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4
 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c +
280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c
^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^
2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4270 vs. $2(267) = 534$.
time = 5.26, size = 4270, normalized size = 13.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/64*(2*a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6
*c^5 + 27136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
```

(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^
2*b^10*c^2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4
 + 5120*(b^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6 + (2*b^4
*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*
c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 2
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2
- 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2)^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9 - 28*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^8*c - 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b^5*c^2 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 56*a^2*b^7*c^2 - 832*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
^5*c^3 - 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5
*b*c^4 + 512*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 144*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 1664*a^4*b^3*c^4 - 256*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2048*a^5*b*c^5 + 2*(b^2 -
 4*a*c)*a*b^7*c - 48*(b^2 - 4*a*c)*a^2*b^5*c^2 + 288*(b^2 - 4*a*c)*a^3*b^3*
c^3 - 512*(b^2 - 4*a*c)*a^4*b*c^4)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))*a
rctan(2*sqrt(1/2)*x/sqrt((a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + sqrt((a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)^2 - 4*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(a
*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^
3)))/((a^3*b^10 - 20*a^4*b^8*c - 2*a^3*b^9*c + 160*a^5*b^6*c^2 + 32*a^4*b^7
*c^2 + a^3*b^8*c^2 - 640*a^6*b^4*c^3 - 192*a^5*b^5*c^3 - 16*a^4*b^6*c^3 + 1
280*a^7*b^2*c^4 + 512*a^6*b^3*c^4 + 96*a^5*b^4*c^4 - 1024*a^8*c^5 - 512*a^7
*b*c^5 - 256*a^6*b^2*c^5 + 256*a^7*c^6)*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2)*abs(c)) - 1/64*(2*a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 1
0496*a^5*b^6*c^5 + 27136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4

```
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2
 - 4*a*c)*a^2*b^10*c^2 + 128*(b^2 - 4*a*c)*a^3*...
```

Mupad [B]
time = 8.37, size = 2500, normalized size = 8.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((b*x*(16*a*c - b^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(b^4 + 36*a
^2*c^2 + 5*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^5*(14*a*c^
2 + b^2*c))/(4*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*a*c^2 + b^2*c
))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8
 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9
216*a^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^
5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c
+ 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)
) - (x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a
^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 -
680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^
5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 +
 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440
*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*
c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^
2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-
(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c
^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^
6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^
(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^
2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a
^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2
```

c^9)))^(1/2) - (x(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5
))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2
c^3)))(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*
a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 -
680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c -
b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a
^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5
+ 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 262144
0*a^12*b^2*c^9)))^(1/2)*1i - (((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a
^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6
- 5505024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240
*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (
x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^
13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 68096
0*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^
15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^1
6*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 8601
60*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12
*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2 + 5120*a^3*b^9*c^3 -
40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b^3*c^6))/(32*(a^2*b^8
+ 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17
+ b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 -
10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5
*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2)
)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7
680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^
8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)
))^(1/2) + (x*(800*a^3*c^6 - b^6*c^3 + 34*a*b^4*c^4 - 1472*a^2*b^2*c^5))/(3
2*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)
))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b
^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 6809
60*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)
^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^
16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860
160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^1
2*b^2*c^9)))^(1/2)*1i)/((((256*a*b^13*c^2 + 4194304*a^7*b*c^8 - 9216*a^2*b^
11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*a^5*b^5*c^6 - 55
05024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*
b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(
b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^
2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6
*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(
1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2
- 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258...

3.886 $\int \frac{1}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=355

$$\frac{x(b^2-2ac+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x((b^2-7ac)(3b^2-4ac)+3bc(b^2-8ac)x^2)}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+\right.}{8\sqrt{2}\,a^2}$$

[Out] 1/4*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/8*x*((-7*a*c+b^2)*(-4*a*c+3*b^2)+3*b*c*(-8*a*c+b^2)*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^4-10*a*b^2*c+56*a^2*c^2+b*(-8*a*c+b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^3-8*a*b*c+(-56*a^2*c^2+10*a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)

Rubi [A]
time = 1.20, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1106, 1192, 1180, 211}

$$\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b(b^2-8ac)\sqrt{b^2-4ac}+b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a^2(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(3bcx^2(b^2-8ac)+(b^2-7ac)(3b^2-4ac))}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{x(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-3),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[c]*(b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx = \frac{x(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{\int \frac{b^2-2ac-4(b^2-4ac)-5bcx^2}{(a+bx^2+cx^4)^2}\,dx}{4a\,(b^2-4ac)}$$

$$= \frac{x(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{x((b^2-7ac)\,(3b^2-4ac)+3bc(b^2-8ac)\,x^2)}{8a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} +$$

$$= \frac{x(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{x((b^2-7ac)\,(3b^2-4ac)+3bc(b^2-8ac)\,x^2)}{8a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} -$$

$$= \frac{x(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{x((b^2-7ac)\,(3b^2-4ac)+3bc(b^2-8ac)\,x^2)}{8a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} +$$

Mathematica [A]

time = 0.65, size = 372, normalized size = 1.05

$$\frac{\frac{4ax(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x(3b^4-25ab^2c+28a^2c^2+3b^3cx^2-24abc^2x^2)}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b^3\sqrt{b^2-4ac}-8abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(b^4-10ab^2c+56a^2c^2-b^3\sqrt{b^2-4ac}+8abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-3),x]
```

```
[Out] ((4*a*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2
*x*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2))/((b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56
*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b^
3*Sqrt[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]))/(16*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(310) = 620$.
time = 0.14, size = 920, normalized size = 2.59

method	result
risch	$\frac{-\frac{3bc^2(8ac-b^2)x^7}{8a^2(16a^2c^2-8ab^2c+b^4)}+\frac{c(28a^2c^2-49ab^2c+6b^4)x^5}{8a^2(16a^2c^2-8ab^2c+b^4)}-\frac{b(4a^2c^2+20ab^2c-3b^4)x^3}{8a^2(16a^2c^2-8ab^2c+b^4)}+\frac{(44a^2c^2-37ab^2c+5b^4)x}{8(16a^2c^2-8ab^2c+b^4)a}}{(cx^4+bx^2+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4c+\_Z\right.}$
default	$64c^3\left(-\frac{\frac{3\left(128\sqrt{-4ac+b^2}\,a^3bc^3-80\sqrt{-4ac+b^2}\,a^2b^3c^2+16\sqrt{-4ac+b^2}\,ab^5c-\sqrt{-4ac+b^2}\,b^7+384a^4c^4-352a^3\right.}{64a^2c^3}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 64*c^3*(-1/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*((3/64/a^2/c^3*(128*(-4*a*c+
b^2)^(1/2)*a^3*b*c^3-80*(-4*a*c+b^2)^(1/2)*a^2*b^3*c^2+16*(-4*a*c+b^2)^(1/2
```

```
)*a*b^5*c-(-4*a*c+b^2)^(1/2)*b^7+384*a^4*c^4-352*a^3*b^2*c^3+120*a^2*b^4*c^
2-18*a*b^6*c+b^8)*x^3-1/64/a/c^3*(-704*(-4*a*c+b^2)^(1/2)*a^3*c^3+432*(-4*a
*c+b^2)^(1/2)*a^2*b^2*c^2-84*(-4*a*c+b^2)^(1/2)*a*b^4*c+5*(-4*a*c+b^2)^(1/2
)*b^6+320*a^3*b*c^3-240*a^2*b^3*c^2+60*a*b^5*c-5*b^7)*x)/(x^2+1/2/c*(-4*a*c
+b^2)^(1/2)+1/2*b/c)^2+3/64*(128*(-4*a*c+b^2)^(1/2)*a^3*b*c^3-80*(-4*a*c+b^
2)^(1/2)*a^2*b^3*c^2+16*(-4*a*c+b^2)^(1/2)*a*b^5*c-(-4*a*c+b^2)^(1/2)*b^7+8
96*a^4*c^4-608*a^3*b^2*c^3+152*a^2*b^4*c^2-18*a*b^6*c+b^8)/a^2/c^2*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)))+1/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*((3/64/a^2/c^3*(-128*(-4*
a*c+b^2)^(1/2)*a^3*b*c^3+80*(-4*a*c+b^2)^(1/2)*a^2*b^3*c^2-16*(-4*a*c+b^2)^
(1/2)*a*b^5*c+(-4*a*c+b^2)^(1/2)*b^7+384*a^4*c^4-352*a^3*b^2*c^3+120*a^2*b^
4*c^2-18*a*b^6*c+b^8)*x^3-1/64/a/c^3*(704*(-4*a*c+b^2)^(1/2)*a^3*c^3-432*(-
4*a*c+b^2)^(1/2)*a^2*b^2*c^2+84*(-4*a*c+b^2)^(1/2)*a*b^4*c-5*(-4*a*c+b^2)^(
1/2)*b^6+320*a^3*b*c^3-240*a^2*b^3*c^2+60*a*b^5*c-5*b^7)*x)/(x^2+1/2*b/c-1/
2/c*(-4*a*c+b^2)^(1/2))^2+3/64*(128*(-4*a*c+b^2)^(1/2)*a^3*b*c^3-80*(-4*a*c
+b^2)^(1/2)*a^2*b^3*c^2+16*(-4*a*c+b^2)^(1/2)*a*b^5*c-(-4*a*c+b^2)^(1/2)*b^
7-896*a^4*c^4+608*a^3*b^2*c^3-152*a^2*b^4*c^2+18*a*b^6*c-b^8)/a^2/c^2*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*x^7 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*x^
5 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 + (5*a*b^4 - 37*a^2*b^2*c + 44*a
^3*c^2)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^
5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (
a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5
*b*c^2)*x^2) - 3/8*integrate(-(b^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^3*c - 8*a*
b*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4323 vs. $2(309) = 618$.
time = 0.80, size = 4323, normalized size = 12.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^3*c^2 - 8*a*b*c^3)*x^7 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)
*x^5 + 2*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 - 3*sqrt(1/2)*((a^2*b^4*c^2
 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2
*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c +
32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9
- 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^
10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 -
1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3
+ 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^
4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160
*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*
(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*
a^4*c^7)*x + 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a
^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 8
7808*a^7*c^7 - (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*
c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a
^12*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 24
01*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^
3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*
b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160
*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^
8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b
^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c
^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*
b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 3*sqrt(1/2)*((a^2*b^4*c^2 -
 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(
a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32
*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 -
21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10
 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 10
24*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 +
2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*
c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a
^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(2
1*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^
4*c^7)*x - 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3
*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 878
08*a^7*c^7 - (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^
3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^1
2*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401
*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
+ 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^
5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a
^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8
- 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^1
```

```
0 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4
 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^
4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) - 3*sqrt(1/2)*((a^2*b^4*c^2 - 8
*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^
2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a
^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 - 21
*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 -
 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024
*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 24
01*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^
3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7
*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*
b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*
c^7)*x + 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b
^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808
*a^7*c^7 + (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3
+ 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*
b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a
^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 +
1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*
c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7
*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2707 vs. $2(309) = 618$.
time = 6.24, size = 2707, normalized size = 7.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7
*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c + sqr
```

t(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 + 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^4*c^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^6*c - 26*(b^2 - 4*a*c)*a*b^4*c^2 - 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2 - 4*a*c)*a^2*b^2*c^3 + 22*(b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*c^4 - 88*(b^2 - 4*a*c)*a^2*b*c^4)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) + 3/32*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c + 2*b^8*c + 116*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 736*a^3*b^2*c^4 + 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 896*a^4*c^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c

```
)*c)*a*b^3*c^3 - 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^6*c + 26*(b^2 - 4*a*c)*a*b^4*c^2 - 2*(b^2
- 4*a*c)*b^5*c^2 - 128*(b^2 - 4*a*c)*a^2*b^2*c^3 + 22*(b^2 - 4*a*c)*a*b^3*c
^3 + 224*(b^2 - 4*a*c)*a^3*c^4 - 88*(b^2 - 4*a*c)*a^2*b*c^4)*arctan(2*sqrt(
1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt((a^2*b^5 - 8*a^3*b
^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c
- 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/
((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 +
a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c
^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/8*(3*b^3*c^2*
x^7 - 24*a*b*c^3*x^7 + 6*b^4*c*x^5 - 49*a*b^2*c^2*x^5 + 28*a^2*c^3*x^5 + 3*
b^5*x^3 - 20*a*b^3*c*x^3 - 4*a^2*b*c^2*x^3 + 5*a*b^4*x - 37*a^2*b^2*c*x + 4
4*a^3*c^2*x)/((a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [**B**]
time = 9.00, size = 2500, normalized size = 7.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x*(5*b^4 + 44*a^2*c^2 - 37*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
 + (x^5*(6*b^4*c + 28*a^2*c^3 - 49*a*b^2*c^2))/(8*a^2*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) - (x^3*(4*a^2*b*c^2 - 3*b^5 + 20*a*b^3*c))/(8*a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) + (3*c*x^7*(b^3*c - 8*a*b*c^2))/(8*a^2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
- atan(((((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 9420
8*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4
*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*
c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^
5)) - (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 7
69*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c
^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a
^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)
^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b
^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 8
60160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*
a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c
^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4
*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(
9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*
c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824
*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*
a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))
```

/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) + (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i - (((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3 - 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2) - (x*(14112*a^4*c^7 + 9*b^8*c^3 - 180*a*b^6*c^4 + 1530*a^2*b^4*c^5 - 6192*a^3*b^2*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*1i)/((((3*(7340032*a^9*c^9 - 256*a^2*b^14*c...

3.887 $\int \frac{1}{x^2(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=425

$$-\frac{3(5b^2-12ac)\,(b^2-5ac)}{8a^3\,(b^2-4ac)^2\,x}+\frac{b^2-2ac+bcx^2}{4a\,(b^2-4ac)\,x\,(a+bx^2+cx^4)^2}+\frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)\,x^2}{8a^2\,(b^2-4ac)^2\,x\,(a+bx^2+cx^4)}-$$

[Out] -3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x+1/4*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)^2+1/8*(5*b^4-35*a*b^2*c+36*a^2*c^2+b*c*(-32*a*c+5*b^2)*x^2)/a^2/(-4*a*c+b^2)^2/x/(c*x^4+b*x^2+a)-3/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b^2)+b*(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b^2)+(-124*a^2*b*c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)

Rubi [A]

time = 0.61, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1135, 1291, 1295, 1180, 211}

$$-\frac{3(5b^2-12ac)\,(b^2-5ac)}{8a^3x\,(b^2-4ac)^2}+\frac{36a^2c^2+bcx^2(5b^2-32ac)-35ab^2c+5b^4}{8a^2x\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{3\sqrt{c}\left(\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}+(5b^2-12ac)\,(b^2-5ac)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^3\,(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left((5b^2-12ac)\,(b^2-5ac)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a^3\,(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bcx^2}{4ax\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x^2 + c*x^4)^3),x]

[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*x^2)/(8*a^2*(b^2 - 4*a*c)^2*x*(a + b*x^2 + c*x^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx = \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{-5b^2+18ac-7bcx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)\,x^2}{8a^2\left(b^2-4ac\right)^2x\left(a+bx^2+cx^4\right)}$$

$$= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c}{8a^2\left(b^2-\right.}$$

$$= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c}{8a^2\left(b^2-\right.}$$

$$= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} + \frac{5b^4-35ab^2c}{8a^2\left(b^2-\right.}$$

Mathematica [A]
time = 1.12, size = 454, normalized size = 1.07

$$-\frac{\frac{16}{x}+\frac{4ax\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2x\left(7b^5-52ab^3c+84a^2bc^2+7b^4cx^2-47ab^2c^2x^2+52a^2c^3x^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-5b^5+47ab^3c-124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/16*(16/x + (4*a*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(7*b^5 - 52*a*b^3*c + 84*a^2*b*c^2 + 7*b^
4*c*x^2 - 47*a*b^2*c^2*x^2 + 52*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 +
 c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*S
qrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^5 + 47*a*b^
3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c
] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a^3
```

Maple [A]
time = 0.09, size = 517, normalized size = 1.22

method	result

default	$-\frac{\frac{\frac{c^{2}\left(52a^{2}c^{2}-47a\,b^{2}c+7b^{4}\right)x^{7}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}+\frac{cb\left(136a^{2}c^{2}-99a\,b^{2}c+14b^{4}\right)x^{5}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}+\frac{\left(68a^{3}c^{3}+25a^{2}b^{2}c^{2}-43a\,b^{4}c+7b^{6}\right)x^{3}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}+\frac{3ab\left(36a^{2}c^{2}-22a\,b^{2}c+3b^{4}\right)x}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{3c\left(-\frac{\left(6\ldots\right.}{\ldots}\right.}{\ldots}}{\ldots}$
risch	$\frac{-\frac{3c^{2}\left(60a^{2}c^{2}-37a\,b^{2}c+5b^{4}\right)x^{8}}{8a^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{cb\left(392a^{2}c^{2}-227a\,b^{2}c+30b^{4}\right)x^{6}}{8a^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(324a^{3}c^{3}+25a^{2}b^{2}c^{2}-91a\,b^{4}c+15b^{6}\right)x^{4}}{8a^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{b\left(364a^{2}c^{2}-194a\,b^{2}c+25b^{4}\right)x^{2}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)a^{2}}-\frac{1}{a}}{x\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((1/8*c^2*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^7+1/8*c*b*(136*a^2*c^2-99*a*b^2*c+14*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+1
/8*(68*a^3*c^3+25*a^2*b^2*c^2-43*a*b^4*c+7*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^3+3/8*a*b*(36*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*
x^4+b*x^2+a)^2+3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*(-1/8*(60*a^2*c^2*(-4*a*c+b
^2)^(1/2)-37*a*b^2*c*(-4*a*c+b^2)^(1/2)+5*b^4*(-4*a*c+b^2)^(1/2)+124*a^2*b*
c^2-47*a*b^3*c+5*b^5)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(60*a^2*
c^2*(-4*a*c+b^2)^(1/2)-37*a*b^2*c*(-4*a*c+b^2)^(1/2)+5*b^4*(-4*a*c+b^2)^(1/
2)-124*a^2*b*c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-
1/a^3/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8 + (30*b^5*c - 227*a*b^3
*c^2 + 392*a^2*b*c^3)*x^6 + 8*a^2*b^4 - 64*a^3*b^2*c + 128*a^4*c^2 + (15*b^
6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*x^4 + (25*a*b^5 - 194*a^2*b^
3*c + 364*a^3*b*c^2)*x^2)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 +
 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c
+ 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)*x) - 3/8*integrate((5*b^5 - 42*a*b^3*c + 92*a^
```

```
2*b*c^2 + (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*x^2)/(c*x^4 + b*x^2 + a), x
)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4924 vs. $2(379) = 758$.
time = 1.38, size = 4924, normalized size = 11.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8 + 2*(30*b^5*c - 227*a*
b^3*c^2 + 392*a^2*b*c^3)*x^6 + 16*a^2*b^4 - 128*a^3*b^2*c + 256*a^4*c^2 + 2
*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*x^4 + 2*(25*a*b^5 - 1
94*a^2*b^3*c + 364*a^3*b*c^2)*x^2 + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (
a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6
*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*
a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*
a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 +
 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725
*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5
 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*
b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 1
60*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log
(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^
4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x + 27/2*sqrt(1/2)*(125*b^17
- 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^
9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1
324800*a^8*b*c^8 - (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960
*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4
*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10
*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a
^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 -
 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 4
95*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 184
80*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^
3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94
725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*
c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^
17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c
+ 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)))
 - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5
*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^
```

```
3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^
2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 1501
5*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^
8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^
5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3
 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20
*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 102
4*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3
 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*c^4 - 77825*a*b^8
*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810
000*a^5*c^9)*x - 27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*
c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 714
6736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 - (5*a^7*b^16 -
152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c
^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 1228
80*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a
^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14
*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2
*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 1
5015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8
*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12
*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*
c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 -
 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 -
1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*
c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))) + 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*
a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3
)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c
 + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^
11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4
 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*
b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*
c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^
5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5273 vs. $2(379) = 758$.

time = 7.08, size = 5273, normalized size = 12.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/64*(10*a^6*b^14*c^2 - 254*a^7*b^12*c^3 + 2712*a^8*b^10*c^4 - 15552*a^9*b^
8*c^5 + 50432*a^10*b^6*c^6 - 87552*a^11*b^4*c^7 + 63488*a^12*b^2*c^8 - 5*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^14 + 127*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^12*c + 10*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^13*c - 1356*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^2 - 214*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^11*c^2 - 5*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^12*c^2 + 7776
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^8*c^3 + 18
56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^9*c^3 +
107*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^10*c^3
- 25216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^6*
c^4 - 8128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^
7*c^4 - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b
^8*c^4 + 43776*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
11*b^4*c^5 + 17920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^10*b^5*c^5 + 4064*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^9*b^6*c^5 - 31744*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^12*b^2*c^6 - 15872*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^11*b^3*c^6 - 8960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^10*b^4*c^6 + 7936*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^11*b^2*c^7 - 10*(b^2 - 4*a*c)*a^6*b^12*c^2 + 214*(b^2 -
 4*a*c)*a^7*b^10*c^3 - 1856*(b^2 - 4*a*c)*a^8*b^8*c^4 + 8128*(b^2 - 4*a*c)*
a^9*b^6*c^5 - 17920*(b^2 - 4*a*c)*a^10*b^4*c^6 + 15872*(b^2 - 4*a*c)*a^11*b
^2*c^7 + (10*b^6*c^2 - 114*a*b^4*c^3 + 416*a^2*b^2*c^4 - 480*a^3*c^5 - 5*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 57*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 208*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 74*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 240*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 120*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 37*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 74*(b^2 - 4*
a*c)*a*b^2*c^3 - 120*(b^2 - 4*a*c)*a^2*c^4)*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5
*c^2)^2 - 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^11 - 102*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c - 10*sqrt(2)*sqrt(b*c + sqrt(b
```

```
^2 - 4*a*c)*c)*a^3*b^10*c - 10*a^3*b^11*c + 836*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b^7*c^2 + 164*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b^8*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 + 204*a^4*b
^9*c^2 - 3440*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^3 - 1016*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 - 82*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 - 1672*a^5*b^7*c^3 + 7104*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 + 2816*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^6*b^4*c^4 + 508*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^
4 + 6880*a^6*b^5*c^4 - 5888*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c
^5 - 2944*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 - 1408*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 - 14208*a^7*b^3*c^5 + 1472*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^6 + 11776*a^8*b*c^6 + 10*(b^2
 - 4*a*c)*a^3*b^9*c - 164*(b^2 - 4*a*c)*a^4*b^7*c^2 + 1016*(b^2 - 4*a*c)*a^
5*b^5*c^3 - 2816*(b^2 - 4*a*c)*a^6*b^3*c^4 + 2944*(b^2 - 4*a*c)*a^7*b*c^5)*
abs(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^5
 - 8*a^4*b^3*c + 16*a^5*b*c^2 + sqrt((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)
^2 - 4*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16
*a^5*c^3)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/((a^7*b^10 - 20*a^8*
b^8*c - 2*a^7*b^9*c + 160*a^9*b^6*c^2 + 32*a^8*b^7*c^2 + a^7*b^8*c^2 - 640*
a^10*b^4*c^3 - 192*a^9*b^5*c^3 - 16*a^8*b^6*c^3 + 1280*a^11*b^2*c^4 + 512*a
^10*b^3*c^4 + 96*a^9*b^4*c^4 - 1024*a^12*c^5 - 512*a^11*b*c^5 - 256*a^10*b^
2*c^5 + 256*a^11*c^6)*abs(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*abs(c)) + 3/6
4*(10*a^6*b^14*c^2 - 254*a^7*b^12*c^3 + 2712*a^8*b^10*c^4 - 15552*a^9*b^8*c
^5 + 50432*a^10*b^6*c^6 - 87552*a^11*b^4*c^7 + 63488*a^12*b^2*c^8 - 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^14 + 127*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^12*c + 10*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^13*c - 1356*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^2 - 214*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*...
```

Mupad [B]
time = 9.37, size = 2500, normalized size = 5.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] - atan(((x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^2
0*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^
13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 8698
15812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b
^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 - 25*b^6*(-(4*a*c -
b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^1
5*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
```

- 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(245760*a^12*b^23*c^2 - 1185410973696*a^23*b*c^13 - 10911744*a^13*b^21*c^3 + 220397568*a^14*b^19*c^4 - 2673082368*a^15*b^17*c^5 + 21630025728*a^16*b^15*c^6 - 122607894528*a^17*b^13*c^7 + 496773365760*a^18*b^11*c^8 - 1438679826432*a^19*b^9*c^9 + 2918430277632*a^20*b^7*c^10 - 3949222428672*a^21*b^5*c^11 + 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 2768240640*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + 124017180672*a^20*b^13*c^7 - 496068722688*a^21*b^11*c^8 + 1417339207680*a^22*b^9*c^9 - 2834678415360*a^23*b^7*c^10 + 3779571220480*a^24*b^5*c^11 - 3023656976384*a^25*b^3*c^12)))*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*1i + (x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 869815812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1185410973696*a^23*b*c^13 -

245760*a^12*b^23*c^2 + 10911744*a^13*b^21*c^3 - 220397568*a^14*b^19*c^4 +
2673082368*a^15*b^17*c^5 - 21630025728*a^16*b^15*c^6 + 122607894528*a^17*b^
13*c^7 - 496773365760*a^18*b^11*c^8 + 1438679826432*a^19*b^9*c^9 - 29184302
77632*a^20*b^7*c^10 + 3949222428672*a^21*b^5*c^11 - 3208340570112*a^22*b^3*
c^12 + x*(-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b
*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6
126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 6268416
0*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2
) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*
(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^1
8*c + 720*a^9*b^16*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*
a^12*b^10*c^5 + 860160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b
^4*c^8 - 2621440*a^16*b^2*c^9)))^(1/2)*(1099511627776*a^26*b*c^13 - 262144*
a^15*b^23*c^2 + 11534336*a^16*b^21*c^3 - 230686720*a^17*b^19*c^4 + 27682406
40*a^18*b^17*c^5 - 22145925120*a^19*b^15*c^6 + ...

3.888 $\int \frac{x^5}{a-bx^2+cx^4} \, dx$

Optimal. Leaf size=82

$$\frac{x^2}{2c}+\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log(a-bx^2+cx^4)}{4c^2}$$

```
[Out] 1/2*x^2/c+1/4*b*ln(c*x^4-b*x^2+a)/c^2+1/2*(-2*a*c+b^2)*arctanh((-2*c*x^2+b)
/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1128, 717, 648, 632, 212, 642}

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log(a-bx^2+cx^4)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a - b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) + (b*Log[a - b*x^2 + c*x^4])/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{a-bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a-bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2c}+\frac{\text{Subst}\left(\int \frac{-a+bx}{a-bx+cx^2}\,dx,x,x^2\right)}{2c}\\
&= \frac{x^2}{2c}+\frac{b\text{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx,x,x^2\right)}{4c^2}+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx,x,x^2\right)}{4c^2}\\
&= \frac{x^2}{2c}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,-b+2cx^2\right)}{2c^2}\\
&= \frac{x^2}{2c}+\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.98

$$
\frac{2cx^2+\frac{2(b^2-2ac)\tan^{-1}\left(\frac{-b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+b\log\left(a-bx^2+cx^4\right)}{4c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a - b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + b*Log[a - b*x^2 + c*x^4])/(4*c^2)
```

Maple [A]
time = 0.04, size = 86, normalized size = 1.05

method	result
default	$\frac{x^2}{2c}+\frac{\frac{b\ln\left(c\,x^4-b\,x^2+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$
risch	$\frac{x^2}{2c}+\frac{\ln\left(\left(\left(8a^2c^2-6a\,b^2c+b^4+\sqrt{-\left(4ac-b^2\right)\left(2ac-b^2\right)^2}\,b\right)x^2-2\sqrt{-\left(4ac-b^2\right)\left(2ac-b^2\right)^2}\,a\right)ab}{c\left(4ac-b^2\right)}-\frac{\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c+1/2/c*(1/2*b/c*ln(c*x^4-b*x^2+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^2-b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.36, size = 259, normalized size = 3.16

$$\left[\frac{2\,(b^2c-4\,ac^2)x^2-(b^2-2\,ac)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac+(2\,cx^2-b)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+(b^3-4\,abc)\log\left(cx^4-bx^2+a\right)}{4\,(b^2c^2-4\,ac^3)},\ \frac{2\,(b^2c-4\,ac^2)x^2-2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2-b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(b^3-4\,abc)\log\left(cx^4-bx^2+a\right)}{4\,(b^2c^2-4\,ac^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 - 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b
```

```
*x^2 + a)) + (b^3 - 4*a*b*c)*log(c*x^4 - b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*log(c*x^4 -
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(71) = 142$.

time = 1.15, size = 311, normalized size = 3.79

$$\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2 + \frac{ab - 8ac^2\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right) + 2b^2c\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \left(\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2 + \frac{ab - 8ac^2\left(\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right) + 2b^2c\left(\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4-b*x**2+a),x)
```

```
[Out] (b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*l
og(x**2 + (a*b - 8*a*c**2*(b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*
c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (b/(4*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (a*b - 8*a*
c**2*(b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2
))) + 2*b**2*c*(b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*
a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

Giac [A]

time = 5.64, size = 78, normalized size = 0.95

$$\frac{x^2}{2\,c} + \frac{b\log\left(cx^4 - bx^2 + a\right)}{4\,c^2} + \frac{(b^2 - 2\,ac)\arctan\left(\frac{2\,cx^2 - b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c + 1/4*b*log(c*x^4 - b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]

time = 4.74, size = 656, normalized size = 8.00

$$\frac{x^2}{2\,c} - \frac{\ln\left(c\,x^4 - b\,x^2 + a\right)\left(2\,b^3 - 8\,a\,b\,c\right)}{2\left(16\,a\,c^3 - 4\,b^2\,c^2\right)} - \frac{\mathrm{atan}\left(\frac{2\,c^2\left(4\,a\,c - b^2\right)\left(\frac{\frac{\left(8\,a\,b + \frac{8\,a\,c^2\left(2\,b^3 - 8\,a\,b\,c\right)}{16\,a\,c^3 - 4\,b^2\,c^2}\right)\left(2\,a\,c - b^2\right)}{8\,c^2\sqrt{4\,a\,c - b^2}} + \frac{a\left(2\,b^3 - 8\,a\,b\,c\right)\left(2\,a\,c - b^2\right)}{\sqrt{4\,a\,c - b^2}\left(16\,a\,c^3 - 4\,b^2\,c^2\right)}}{a} + x^2\left(\frac{\frac{\left(2\,a\,c - b^2\right)\left(\frac{4\,a\,c^3 - 6\,b^2\,c^2}{c^2} - \frac{4\,b\,c^2\left(2\,b^3 - 8\,a\,b\,c\right)}{16\,a\,c^3 - 4\,b^2\,c^2}\right)}{8\,c^2\sqrt{4\,a\,c - b^2}} - \frac{b\left(2\,b^3 - 8\,a\,b\,c\right)\left(2\,a\,c - b^2\right)}{2\sqrt{4\,a\,c - b^2}\left(16\,a\,c^3 - 4\,b^2\,c^2\right)}}{a} + \frac{b\left(\frac{\left(2\,b^3 - 8\,a\,b\,c\right)\left(\frac{4\,a\,c^3 - 6\,b^2\,c^2}{c^2} - \frac{4\,b\,c^2\left(2\,b^3 - 8\,a\,b\,c\right)}{16\,a\,c^3 - 4\,b^2\,c^2}\right)}{2\left(16\,a\,c^3 - 4\,b^2\,c^2\right)} - \frac{b^3 - a\,b\,c}{c^2} + \frac{b\left(2\,a\,c - b^2\right)^2}{2\,c^2\left(4\,a\,c - b^2\right)}\right)}{2\,a\sqrt{4\,a\,c - b^2}}\right) + \frac{b\left(\frac{a\,b^2}{c^2} + \frac{\left(2\,b^3 - 8\,a\,b\,c\right)\left(8\,a\,b + \frac{8\,a\,c^2\left(2\,b^3 - 8\,a\,b\,c\right)}{16\,a\,c^3 - 4\,b^2\,c^2}\right)}{2\left(16\,a\,c^3 - 4\,b^2\,c^2\right)} - \frac{a\left(2\,a\,c - b^2\right)^2}{c^2\left(4\,a\,c - b^2\right)}\right)}{2\,a\sqrt{4\,a\,c - b^2}}\right)}{4\,a^2\,c^2 - 4\,a\,b^2\,c + b^4}\right)\left(2\,a\,c - b^2\right)}{2\,c^2\sqrt{4\,a\,c - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a - b*x^2 + c*x^4),x)
```

```
[Out] x^2/(2*c) - (log(a - b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^2
*c^2)) - (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c))
/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*(2
*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2))
)/a + x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8
*a*b*c))/(16*a*c^3 - 4*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2*b^3 -
 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a
+ (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8*a
*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3 - a*b*c)/
c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))
) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c
))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*a*c - b^2)^2
)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a^2*c^2 - 4*a*
b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))
```

3.889 $\int \frac{x^3}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=64

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

```
[Out] 1/4*ln(c*x^4-b*x^2+a)/c+1/2*b*arctanh((-2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(-
4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1128, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b*x^2 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a - b*x^2 + c*x^4]/(4*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a-bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a-bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx,x,x^2\right)}{4c}+\frac{b\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx,x,x^2\right)}{4c}\\ &= \frac{\log\left(a-bx^2+cx^4\right)}{4c}-\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,-b+2cx^2\right)}{2c}\\ &= \frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 1.02

$$\frac{\frac{2b\tan^{-1}\left(\frac{-b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a-bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b*x^2 + c*x^4),x]
```

```
[Out] ((2*b*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 - b*x^2 + c*x^4])/(4*c)
```

Maple [A]
time = 0.03, size = 63, normalized size = 0.98

method	result

default	$\frac{\ln(c\,x^4-b\,x^2+a)}{4c}+\frac{b\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{2c\sqrt{4ac-b^2}}$
risch	$\frac{\ln\left(\left(4abc-b^3-\sqrt{-b^2\,(4ac-b^2)}\;b\right)x^2+2\sqrt{-b^2\,(4ac-b^2)}\;a\right)a}{4ac-b^2}-\frac{\ln\left(\left(4abc-b^3-\sqrt{-b^2\,(4ac-b^2)}\;b\right)x^2+2\sqrt{}\right.}{4c(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(c*x^4-b*x^2+a)/c+1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 206, normalized size = 3.22

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-(2\,cx^2-b)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+(b^2-4\,ac)\log\left(cx^4-bx^2+a\right)}{4\,(b^2c-4\,ac^2)},-\frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{(2\,cx^2-b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^2-4\,ac)\log\left(cx^4-bx^2+a\right)}{4\,(b^2c-4\,ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x
^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 -
 b*x^2 + a))/(b^2*c - 4*a*c^2), -1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x
^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^4 - b*x^2
 + a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54)=108$.
time = 0.51, size = 223, normalized size = 3.48

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)-2a-2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)-2a-2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (8*a*c*(
-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) - 2*a - 2*b**2*(-b*s
qrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (8*a*c*(b*sqrt(-4*a*c + b**
2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) - 2*a - 2*b**2*(b*sqrt(-4*a*c + b**2)/(4
*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

Giac [A]
time = 3.13, size = 62, normalized size = 0.97

$$\frac{b\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c}+\frac{\log\left(cx^4-bx^2+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/4
*log(c*x^4 - b*x^2 + a)/c
```

Mupad [B]
time = 4.40, size = 120, normalized size = 1.88

$$\frac{4\,a\,c\,\ln\left(c\,x^4-b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b^2\,\ln\left(c\,x^4-b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}-\frac{2\,c\,x^2}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b*x^2 + c*x^4),x)
```

```
[Out] (4*a*c*log(a - b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a - b*x^2 +
c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) - (2*c*x^2)/(4
*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

3.890 $\int \frac{x}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] arctanh((-2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1121, 632, 212}

$$\frac{\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b*x^2 + c*x^4),x]
```

```
[Out] ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{a-bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx, x, x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, -b+2cx^2\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{-b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.17

$$\frac{\tan^{-1}\left(\frac{-b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Maple [A]
time = 0.01, size = 38, normalized size = 1.09

method	result	size
default	$\frac{\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	38
risch	$-\frac{\ln\left(\left(b+\sqrt{-4ac+b^2}\right)x^2-2a\right)}{2\sqrt{-4ac+b^2}}+\frac{\ln\left(\left(-b+\sqrt{-4ac+b^2}\right)x^2+2a\right)}{2\sqrt{-4ac+b^2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.33, size = 134, normalized size = 3.83

$$\left[\frac{\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-(2\,cx^2-b)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}},-\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2-b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 - b)*sqrt(b^2 - 4*
a*c))/(c*x^4 - b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(32) = 64$.
time = 0.26, size = 131, normalized size = 3.74

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}-b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}-b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4-b*x**2+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) - b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) - b)/(2*c))/2
```

Giac [A]
time = 3.86, size = 37, normalized size = 1.06

$$\frac{\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B]
time = 4.30, size = 42, normalized size = 1.20

$$-\frac{\operatorname{atan}\left(\frac{a\,b-2\,a\,c\,x^2}{a\,\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b*x^2 + c*x^4),x)
[Out] -atan((a*b - 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

3.891 $\int \frac{1}{x(a-bx^2+cx^4)} dx$

Optimal. Leaf size=70

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}$$

```
[Out] ln(x)/a-1/4*ln(c*x^4-b*x^2+a)/a+1/2*b*arctanh((-2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1128, 719, 29, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b*x^2 + c*x^4)),x]
```

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a - b*x^2 + c*x^4]/(4*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a-bx^2+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a-bx+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a}+\frac{\text{Subst}\left(\int \frac{b-cx}{a-bx+cx^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{\log(x)}{a}-\frac{\text{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx,x,x^2\right)}{4a}+\frac{b\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\,dx,x,x^2\right)}{4a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}-\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,-b+2cx^2\right)}{2a}\\
&= \frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 117, normalized size = 1.67

$$\frac{4\sqrt{b^2-4ac}\,\log(x)+\left(b-\sqrt{b^2-4ac}\right)\log\left(-b-\sqrt{b^2-4ac}+2cx^2\right)-\left(b+\sqrt{b^2-4ac}\right)\log\left(-b+\sqrt{b^2-4ac}+2cx^2\right)}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] + (b - Sqrt[b^2 - 4*a*c])*Log[-b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2] - (b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.03, size = 68, normalized size = 0.97

method	result	size
default	$\frac{-\frac{\ln\left(c\,x^{4}-b\,x^{2}+a\right)}{2}+\frac{b\arctan\left(\frac{2c\,x^{2}-b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a}+\frac{\ln\left(x\right)}{a}$	68
risch	$\frac{\ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{2}c-a\,b^{2}\right)\_Z^{2}+\left(4ac-b^{2}\right)\_Z+c\right)}\_R\ln\left(\left(\left(-10ac+3b^{2}\right)\_R-5c\right)x^{2}-ab\_R+2b\right)\right)}{2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(-1/2*ln(c*x^4-b*x^2+a)+b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*c
-b^2)^(1/2)))+ln(x)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 230, normalized size = 3.29

$$\left[\frac{\sqrt{b^{2}-4\,ac}\,b\log\left(\frac{2\,c^{2}x^{4}-2\,bcx^{2}+b^{2}-2\,ac-\left(2\,cx^{2}-b\right)\sqrt{b^{2}-4\,ac}}{cx^{4}-bx^{2}+a}\right)-\left(b^{2}-4\,ac\right)\log\left(cx^{4}-bx^{2}+a\right)+4\,\left(b^{2}-4\,ac\right)\log\left(x\right)}{4\,\left(ab^{2}-4\,a^{2}c\right)},\,-\frac{2\,\sqrt{-b^{2}+4\,ac}\,b\arctan\left(-\frac{\left(2\,cx^{2}-b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)+\left(b^{2}-4\,ac\right)\log\left(cx^{4}-bx^{2}+a\right)-4\,\left(b^{2}-4\,ac\right)\log\left(x\right)}{4\,\left(ab^{2}-4\,a^{2}c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x
^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 -
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), -1/4*(2*sqrt(-b^2
+ 4*a*c)*b*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 -
 4*a*c)*log(c*x^4 - b*x^2 + a) - 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.
time = 8.31, size = 253, normalized size = 3.61

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ac-b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ac-b^2}{bc}\right)+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (8*a**2*
c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*b**2*(-b*sq
rt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*c - b**2)/(b*c)) +
(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (8*a**2*c
*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*b**2*(b*sqrt(
-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*c - b**2)/(b*c)) + log
(x)/a
```

Giac [A]
time = 3.63, size = 71, normalized size = 1.01

$$\frac{b\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\;a}-\frac{\log\left(cx^4-bx^2+a\right)}{4\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/4
*log(c*x^4 - b*x^2 + a)/a + 1/2*log(x^2)/a
```

Mupad [B]
time = 4.89, size = 1015, normalized size = 14.50

$$\frac{\ln(x)}{a}+\frac{\ln\left(c\,x^4-b\,x^2+a\right)\,\left(8\,a\,c-2\,b^2\right)}{2\,\left(4\,a\,b^2-16\,a^2\,c\right)}-\frac{b\,\mathrm{atan}\left(\frac{16\,a^3\,x^2\left(\frac{\left(3\,b^3-8\,a\,b\,c\right)\left(\frac{{\left(8\,a\,c-2\,b^2\right)}^2\left(10\,b\,c^3-\frac{\left(12\,b^3\,c^2-40\,a\,b\,c^3\right)\left(8\,a\,c-2\,b^2\right)}{2\,\left(4\,a\,b^2-16\,a^2\,c\right)}\right)}{4\,{\left(4\,a\,b^2-16\,a^2\,c\right)}^2}-\frac{b^2\left(10\,b\,c^3-\frac{\left(12\,b^3\,c^2-40\,a\,b\,c^3\right)\left(8\,a\,c-2\,b^2\right)}{2\,\left(4\,a\,b^2-16\,a^2\,c\right)}\right)}{16\,a^2\,\left(4\,a\,c-b^2\right)}+\frac{b^2\left(12\,b^3\,c^2-40\,a\,b\,c^3\right)\left(8\,a\,c-2\,b^2\right)}{16\,a^2\left(4\,a\,b^2-16\,a^2\,c\right)\left(4\,a\,c-b^2\right)}\right)}{8\,a^3\,c^2\,\left(25\,a\,c-6\,b^2\right)}-\frac{\left(10\,a^2\,c^2-14\,a\,b^2\,c+3\,b^4\right)\left(\frac{b^3\left(12\,b^3\,c^2-40\,a\,b\,c^3\right)}{64\,a^3\,{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{b\left(12\,b^3\,c^2-40\,a\,b\,c^3\right){\left(8\,a\,c-2\,b^2\right)}^2}{16\,a\,{\left(4\,a\,b^2-16\,a^2\,c\right)}^2\sqrt{4\,a\,c-b^2}}+\frac{b\left(8\,a\,c-2\,b^2\right)\left(10\,b\,c^3-\frac{\left(12\,b^3\,c^2-40\,a\,b\,c^3\right)\left(8\,a\,c-2\,b^2\right)}{2\,\left(4\,a\,b^2-16\,a^2\,c\right)}\right)}{4\,a\left(4\,a\,b^2-16\,a^2\,c\right)\sqrt{4\,a\,c-b^2}}\right)}{8\,a^3\,c^2\sqrt{4\,a\,c-b^2}\left(25\,a\,c-6\,b^2\right)}\right){\left(4\,a\,c-b^2\right)}^{3/2}}{b^2\,c^2}-\frac{2\left(3\,b^3-8\,a\,b\,c\right){\left(4\,a\,c-b^2\right)}^{3/2}\left(\frac{{\left(8\,a\,c-2\,b^2\right)}^2\left(4\,b^2\,c^2-\frac{2\,a\,b^2\,c^2\left(8\,a\,c-2\,b^2\right)}{4\,a\,b^2-16\,a^2\,c}\right)}{4\,{\left(4\,a\,b^2-16\,a^2\,c\right)}^2}-\frac{b^2\left(4\,b^2\,c^2-\frac{2\,a\,b^2\,c^2\left(8\,a\,c-2\,b^2\right)}{4\,a\,b^2-16\,a^2\,c}\right)}{16\,a^2\left(4\,a\,c-b^2\right)}+\frac{b^4\,c^2\left(8\,a\,c-2\,b^2\right)}{4\,a\left(4\,a\,b^2-16\,a^2\,c\right)\left(4\,a\,c-b^2\right)}\right)}{b^2\,c^4\left(25\,a\,c-6\,b^2\right)}+\frac{2\left(4\,a\,c-b^2\right)\left(10\,a^2\,c^2-14\,a\,b^2\,c+3\,b^4\right)\left(\frac{b^5\,c^2}{16\,a^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{b^3\,c^2\,{\left(8\,a\,c-2\,b^2\right)}^2}{4\,{\left(4\,a\,b^2-16\,a^2\,c\right)}^2\sqrt{4\,a\,c-b^2}}+\frac{b\left(8\,a\,c-2\,b^2\right)\left(4\,b^2\,c^2-\frac{2\,a\,b^2\,c^2\left(8\,a\,c-2\,b^2\right)}{4\,a\,b^2-16\,a^2\,c}\right)}{4\,a\left(4\,a\,b^2-16\,a^2\,c\right)\sqrt{4\,a\,c-b^2}}\right)}{b^2\,c^4\left(25\,a\,c-6\,b^2\right)}\right)}{2\,a\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b*x^2 + c*x^4)),x)
```

```
[Out] log(x)/a + (log(a - b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c)
) - (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3 -
 ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*
(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*
c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(12*b^
3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*(
(b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(12*b^3*c
^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(4*a*c - b
^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a
*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^
2)^(3/2))/(b^2*c^2) - (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*(((8*a*c - 2
*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(
4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))
/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(8*a*c - 2*b^2))/
(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*c - 6*b^2)) + (2*
(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2)/(16*a^2*(4*a*c -
 b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 - 16*a^2*c)^2*(4*a*c
 - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^
2))/(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))
/(b^2*c^4*(25*a*c - 6*b^2))))/(2*a*(4*a*c - b^2)^(1/2))
```

3.892 $\int \frac{1}{x^3(a-bx^2+cx^4)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2ax^2}+\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log(x)}{a^2}-\frac{b\log(a-bx^2+cx^4)}{4a^2}$$

```
[Out] -1/2/a/x^2+b*ln(x)/a^2-1/4*b*ln(c*x^4-b*x^2+a)/a^2+1/2*(-2*a*c+b^2)*arctanh
((-2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1128, 723, 814, 648, 632, 212, 642}

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log(a-bx^2+cx^4)}{4a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*1/(a*x^2) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
2*a^2*Sqrt[b^2 - 4*a*c]) + (b*Log[x])/a^2 - (b*Log[a - b*x^2 + c*x^4])/(4*a
^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a-bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a-bx+cx^2\right)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \frac{b-cx}{x(a-bx+cx^2)}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \left(\frac{b}{ax}-\frac{-b^2+ac+bcx}{a(a-bx+cx^2)}\right)\,dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{\text{Subst}\left(\int \frac{-b^2+ac+bcx}{a-bx+cx^2}\,dx, x, x^2\right)}{2a^2}$$
$$= -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\text{Subst}\left(\int \frac{-b+2cx}{a-bx+cx^2}\,dx, x, x^2\right)}{4a^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a-bx+cx^2}\right.}{4a^2}$$
$$= -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x,\right.}{2a^2}$$
$$= -\frac{1}{2ax^2}+\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}$$

Mathematica [A]
time = 0.10, size = 139, normalized size = 1.56

$$\frac{-\frac{2a}{x^2}+4b\log(x)+\frac{\left(b^2-2ac-b\sqrt{b^2-4ac}\right)\log\left(-b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(-b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^2 + 4*b*Log[x] + ((b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*Log[-b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((b^2 - 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A]
time = 0.04, size = 87, normalized size = 0.98

method	result
default	$-\frac{\frac{b\ln\left(c\,x^4-b\,x^2+a\right)}{2}+\frac{2\left(ac-\frac{b^2}{2}\right)\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a^2}-\frac{1}{2a\,x^2}+\frac{b\ln(x)}{a^2}$

risch	$-\frac{1}{2a\,x^2}+\frac{b\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(4abc-b^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(-10a^3c+3a^2b^2\right)\_R^2-4\_Rabc-2c^2\right)x^2-a^3b\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*(1/2*b*ln(c*x^4-b*x^2+a)+2*(a*c-1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^2-b)/(4*a*c-b^2)^(1/2)))-1/2/a/x^2+b*ln(x)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.36, size = 298, normalized size = 3.35

$$\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac+(2\,cx^2-b)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+(b^3-4\,abc)x^2\log\left(cx^4-bx^2+a\right)-4\,(b^3-4\,abc)x^2\log\left(x\right)+2\,ab^2-8\,a^2c}{4\,(a^2b^2-4\,a^3c)x^2},-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\,x^2\arctan\left(-\frac{(2\,cx^2-b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(b^3-4\,abc)x^2\log\left(cx^4-bx^2+a\right)-4\,(b^3-4\,abc)x^2\log\left(x\right)+2\,ab^2-8\,a^2c}{4\,(a^2b^2-4\,a^3c)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 - 2*b*c*x^2 + b^2
 - 2*a*c + (2*c*x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) + (b^3 - 4
*a*b*c)*x^2*log(c*x^4 - b*x^2 + a) - 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^2*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4
*a*b*c)*x^2*log(c*x^4 - b*x^2 + a) - 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.72, size = 95, normalized size = 1.07

$$-\frac{b\log\left(cx^4-bx^2+a\right)}{4\,a^2}+\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^2}-\frac{bx^2+a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*b*log(c*x^4 - b*x^2 + a)/a^2 + 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*
arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(b*
x^2 + a)/(a^2*x^2)
```

Mupad [B]
time = 5.84, size = 2032, normalized size = 22.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b*x^2 + c*x^4)),x)
```

```
[Out] (b*log(x))/a^2 - 1/(2*a*x^2) + (log(a - b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(
2*(16*a^3*c - 4*a^2*b^2)) + (atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*
c)*(c^5/a^3 + ((2*b^3 - 8*a*b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a
^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3
*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*(16
*a^3*c - 4*a^2*b^2)) - ((((2*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 +
 ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a
^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 -
 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^
2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8*a*b*c)*(4
0*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^
3*c - 4*a^2*b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((2*b^3
- 8*a*b*c)*(((2*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*
a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4
*a^2*(4*a*c - b^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c
^2)*(2*a*c - b^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*
(16*a^3*c - 4*a^2*b^2)) - ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)
/(64*a^9*(4*a*c - b^2)^(3/2)) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a
^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3
```

```
*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c
- b^2))/(4*a^2*(4*a*c - b^2)^(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(
8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2
)^(3/2))/(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3) + (2*a^3*(4*a*c - b^2)*(3*b^5
+ 13*a^2*b*c^2 - 15*a*b^3*c)*(((2*b^3 - 8*a*b*c)*((((4*a^3*b*c^3 - 4*a^2*b^
3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c
 - b^2))/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c -
b^2))/(2*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a
^2*b^2)) + ((2*a*c - b^2)*((a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)
*((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a
^3*c - 4*a^2*b^2)))/(2*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)
) - (b^2*c^2*(2*a*c - b^2)^3)/(16*a^5*(4*a*c - b^2)^(3/2))))/(c^2*(a^2*c^2
- 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3)) - (2*a^3*(4*a*c
- b^2)^(3/2)*(3*b^4 + a^2*c^2 - 9*a*b^2*c)*((b*c^4)/a^3 - ((2*b^3 - 8*a*b*c
)*((a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b
^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(
16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((((4*
a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c
- 4*a^2*b^2))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3
- 8*a*b*c)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))
)/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2)^2)
/(8*a^3*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*
a*b^2*c)*(4*a^2*c^4 + b^4*c^2 - 4*a*b^2*c^3)))*(2*a*c - b^2))/(2*a^2*(4*a*c
 - b^2)^(1/2))
```

3.893 $\int \frac{x^4}{a-bx^2+cx^4} \, dx$

Optimal. Leaf size=179

$$\frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] x/c-1/2*arctanh(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b
^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*ar
ctanh(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a
*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1136, 1180, 214}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{a-bx^2+cx^4}\,dx = \frac{x}{c} - \frac{\int \frac{a-bx^2}{a-bx^2+cx^4}\,dx}{c}$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-}}}{2c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}}{\sqrt{2}\,c^{3/2}\sqrt{b+}}$$

Mathematica [A]
time = 0.08, size = 208, normalized size = 1.16

$$\frac{x}{c} + \frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{-b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b*x^2 + c*x^4),x]
```

```
[Out] x/c + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - Sqrt
[b^2 - 4*a*c]]) + ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*S
qrt[-b + Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.04, size = 167, normalized size = 0.93

method	result

risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}-\_Z^{2}b+a\right)}\frac{\left(\_R^{2}b-a\right)\ln\left(x-\_R\right)}{2\_R^{3}c-\_Rb}}{2c}$
default	$\frac{x}{c}-\frac{\left(b\sqrt{-4ac+b^{2}}-2ac+b^{2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2c\sqrt{-4ac+b^{2}}\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\left(b\sqrt{-4ac+b^{2}}+2ac-b^{2}\right)\sqrt{2}}{2c\sqrt{-4ac+b^{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] x/c-1/2*(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))+1/2*(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] x/c + integrate((b*x^2 - a)/(c*x^4 - b*x^2 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. 2(143) = 286.
time = 0.37, size = 1051, normalized size = 5.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
- a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c
 + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)
))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a
```

```
*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 -
(b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))
)*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt((b^3 - 3*a
*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*
c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a
*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)
/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1
/2)*c*sqrt((b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x -
 sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c - (b^2*c^3
- 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
- 4*a*c^4))) - 2*x)/c
```

Sympy [**A**]
time = 1.22, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{2}c^{5}-128ab^{2}c^{4}+16b^{4}c^{3}\right)+t^{2}\left(-48a^{2}bc^{2}+28ab^{3}c-4b^{5}\right)+a^{3},\left(t\mapsto t\log\left(x+\frac{-32t^{3}abc^{4}+8t^{3}b^{3}c^{3}-4ta^{2}c^{2}+8tab^{2}c-2tb^{4}}{a^{2}c-ab^{2}}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(-48
*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + a**3, Lambda(_t, _t*log(x + (-32*_t*
*3*a*b*c**4 + 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**
4)/(a**2*c - a*b**2)))) + x/c
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2153 vs. $2(143) = 286$.
time = 4.78, size = 2153, normalized size = 12.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c +
sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^
```

```
5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(s
qrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(-b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 2*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*c^4 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(
-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(-b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 -
 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt(-(b*c + sqrt(b^
2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 + 2*a*b^3*c^4 + 16*a^3*
c^5 - 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*b^5*c^4 - 12*a*b^3
*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*
c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*
b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b
^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 +
 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*
b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c
- 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b
^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 + 2*(sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3
 + 2*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*
sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 8*sqrt(2)*sqrt(-b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^
5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c
))*arctan(2*sqrt(1/2)*x/sqrt(-(b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4
*c^3 - 8*a^2*b^2*c^4 + 2*a*b^3*c^4 + 16*a^3*c^5 - 8*a^2*b*c^5 + a*b^2*c^5 -
 4*a^2*c^6)*c^2)
```

Mupad [B]

time = 0.67, size = 3000, normalized size = 16.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b*x^2 + c*x^4),x)
```

```
[Out] x/c + atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((
b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(
b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*
c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^
3*c^3 - 16*a*b*c^4)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2
*c^4)))^(1/2))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a
*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c
^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 + b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2
*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5
 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((
b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*
a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 + b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 -
4*a*b^2*c))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2) - (2*a^2*b)/c))*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*
a*b^2*c^4)))^(1/2)*2i + atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*
c^3 - 16*a*b*c^4)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a
*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c
^4)))^(1/2))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b
^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 - b^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
```

```
(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c
^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^
5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
- b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16
*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 - b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 -
 4*a*b^2*c))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b
^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((
b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(
b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*
c^3 - 8*a*b^2*c^4)))^(1/2) - (2*a^2*b)/c))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

3.894 $\int \frac{x^2}{a-bx^2+cx^4} \, dx$

Optimal. Leaf size=150

$$\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)-1/2*arctanh(x*2^(1/2)*c^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1
/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1144, 214}

$$\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - (Sqrt[b + Sqrt[b^2 - 4*a*
c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt
[c]*Sqrt[b^2 - 4*a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx = -\left(\frac{1}{2}\left(-1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right)+\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)$$

$$= \frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tanh^{-1}\left(\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-}}$$

Mathematica [**A**]
time = 0.07, size = 137, normalized size = 0.91

$$\frac{-\sqrt{-b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)+\sqrt{-b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b*x^2 + c*x^4),x]
```

```
[Out] (-(Sqrt[-b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[b
^2 - 4*a*c]]]) + Sqrt[-b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [**A**]
time = 0.03, size = 149, normalized size = 0.99

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4-\_Z^2b+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^3c-\_Rb}\right)}{2}$
default	$4c\left(-\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\mathrm{arctan}\left(\right.}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(-b\right.}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/8*(b+(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1
```

```
/8/(-4*a*c+b^2)^(1/2)/c*(-b+(-4*a*c+b^2)^(1/2))*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 - b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(115) = 230$.
time = 0.38, size = 551, normalized size = 3.67

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{\frac{b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{\frac{b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}}{b^2c-4ac^2}}}{\sqrt{b^2c^2-4ac^3}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
 - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b + (b^2*c - 4*a*c^2)/sqr
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2
*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*
a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*sq
rt(1/2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c
^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c
^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2*sqrt(1
/2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))
*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2
 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

Sympy [A]
time = 0.43, size = 75, normalized size = 0.50

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{2}c^{3}-128ab^{2}c^{2}+16b^{4}c\right)+t^{2}\cdot\left(16abc-4b^{3}\right)+a,\left(t\mapsto t\log\left(64t^{3}ac^{2}-16t^{3}b^{2}c+2tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(16*a*b
*c - 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c + 2*_
t*b + x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(115) = 230.
time = 4.14, size = 513, normalized size = 3.42

$$\frac{\left(2 b^{2} c^{2}-8 a c^{3}-\sqrt{2} \sqrt{b^{2}-4 a c} \sqrt{-b c-\sqrt{b^{2}-4 a c}\, c}\, b^{2}+4 \sqrt{2} \sqrt{b^{2}-4 a c} \sqrt{-b c-\sqrt{b^{2}-4 a c}\, c}\, a c-2 \sqrt{2} \sqrt{b^{2}-4 a c} \sqrt{-b c-\sqrt{b^{2}-4 a c}\, c}\, b c-\sqrt{2} \sqrt{b^{2}-4 a c} \sqrt{-b c-\sqrt{b^{2}-4 a c}\, c}\, c^{2}-2 (b^{2}-4 a c) c^{2}\right) \arctan\left(\frac{2 \sqrt{\frac{1}{2}}\, x}{\sqrt{-\frac{b+\sqrt{b^{2}-4 a c}}{c}}}\right)}{2 (b^{4}-8 a b^{2} c+2 b^{3} c+16 a^{2} c^{2}-8 a b c^{2}+b^{2} c^{2}-4 a c^{3})|c|}-\frac{\left(2 b^{2} c^{2}-8 a c^{3}-\sqrt{2} \sqrt{b^{2}-4 a c} \sqrt{-b c+\sqrt{b^{2}-4 a c}\, c}\, b^{2}+4 \sqrt{2} \sqrt{b^{2}-4 a c} \sqrt{-b c+\sqrt{b^{2}-4 a c}\, c}\, a c-2 \sqrt{2} \sqrt{b^{2}-4 a c} \sqrt{-b c+\sqrt{b^{2}-4 a c}\, c}\, b c-\sqrt{2} \sqrt{b^{2}-4 a c} \sqrt{-b c+\sqrt{b^{2}-4 a c}\, c}\, c^{2}-2 (b^{2}-4 a c) c^{2}\right) \arctan\left(\frac{2 \sqrt{\frac{1}{2}}\, x}{\sqrt{-\frac{b-\sqrt{b^{2}-4 a c}}{c}}}\right)}{2 (b^{4}-8 a b^{2} c+2 b^{3} c+16 a^{2} c^{2}-8 a b c^{2}+b^{2} c^{2}-4 a c^{3})|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 -
 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*
c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b*c -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 -
4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b + sqrt(b^2 - 4*a*c))/c))/((b^4 -
8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c)) -
 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2
- 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)
*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b*c
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 -
 4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b - sqrt(b^2 - 4*a*c))/c))/((b^4 -
 8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c))
```

Mupad [B]
time = 4.54, size = 416, normalized size = 2.77

$$-2\,\mathrm{atanh}\left(\frac{\left(x\,(4\,a\,c^{2}-2\,b^{2}\,c)+\frac{x\,(8\,b^{3}\,c^{2}-32\,a\,b\,c^{3})\left(b^{3}+\sqrt{-(4\,a\,c-b^{2})^{3}}-4\,a\,b\,c\right)}{8\,(16\,a^{2}\,c^{3}-8\,a\,b^{2}\,c^{2}+b^{4}\,c)}\right)\sqrt{\frac{b^{3}+\sqrt{-(4\,a\,c-b^{2})^{3}}-4\,a\,b\,c}{8\,(16\,a^{2}\,c^{3}-8\,a\,b^{2}\,c^{2}+b^{4}\,c)}}}{a\,c}\right)\sqrt{\frac{b^{3}+\sqrt{-(4\,a\,c-b^{2})^{3}}-4\,a\,b\,c}{8\,(16\,a^{2}\,c^{3}-8\,a\,b^{2}\,c^{2}+b^{4}\,c)}}-2\,\mathrm{atanh}\left(\frac{\left(x\,(4\,a\,c^{2}-2\,b^{2}\,c)-\frac{x\,(8\,b^{3}\,c^{2}-32\,a\,b\,c^{3})\left(\sqrt{-(4\,a\,c-b^{2})^{3}}-b^{3}+4\,a\,b\,c\right)}{8\,(16\,a^{2}\,c^{3}-8\,a\,b^{2}\,c^{2}+b^{4}\,c)}\right)\sqrt{-\frac{\sqrt{-(4\,a\,c-b^{2})^{3}}-b^{3}+4\,a\,b\,c}{8\,(16\,a^{2}\,c^{3}-8\,a\,b^{2}\,c^{2}+b^{4}\,c)}}}{a\,c}\right)\sqrt{-\frac{\sqrt{-(4\,a\,c-b^{2})^{3}}-b^{3}+4\,a\,b\,c}{8\,(16\,a^{2}\,c^{3}-8\,a\,b^{2}\,c^{2}+b^{4}\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b*x^2 + c*x^4),x)
```

```
[Out] - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((
b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*
c^2)))^(1/2))/(a*c))*((b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c
+ 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) - (x*
(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(b^
4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b
*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(-((-(4*a*c - b^2
)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

3.895 $\int \frac{1}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] arctanh(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4
*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctanh(x*2^(1/2)*c^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1107, 214}

$$\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTanh[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
 + Sqrt[b^2 - 4*a*c]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a-bx^2+cx^4}\,dx = \frac{c\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{-\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.05, size = 137, normalized size = 0.91

$$\frac{\sqrt{2}\,\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{-b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]]/
Sqrt[-b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^
2 - 4*a*c]]]/Sqrt[-b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [A]
time = 0.02, size = 117, normalized size = 0.78

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4-\_Z^2 b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3 c-\_R b}\right)}{2}$	40
default	$4c\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/4/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 - b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(114) = 228$.
time = 0.34, size = 605, normalized size = 4.03

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
- 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(a
^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*
b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 -
 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b
^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt((b - (a*b^2 - 4*a^2
*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2
- 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b - (a*b^2 - 4
*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt((
b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x
 - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sq
rt((b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)))
```

Sympy [A]
time = 0.63, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\cdot\left(16abc-4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{-32t^3a^2bc+8t^3ab^3+4tac-2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(16*a*b
*c - 4*b**3) + c, Lambda(_t, _t*log(x + (-32*_t**3*a**2*b*c + 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(114) = 228$.
time = 6.16, size = 1050, normalized size = 7.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(-b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^
3*c + 2*b^4*c + 16*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqr
t(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(-b*c - sqrt(b^
2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
 sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^
2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^
2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b + sqrt(b^2 - 4*a*c))/c))/((
a*b^4 - 8*a^2*b^2*c + 2*a*b^3*c + 16*a^3*c^2 - 8*a^2*b*c^2 + a*b^2*c^2 - 4*
a^2*c^3)*abs(c)) + 1/4*(sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sq
rt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(-b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*c^2 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*
sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqr
t(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4
*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b - sqrt(b
^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c + 2*a*b^3*c + 16*a^3*c^2 - 8*a^2*b*c
^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

Mupad [B]
time = 0.49, size = 763, normalized size = 5.09

$$-\mathrm{atan}\left(\frac{b^4\,x\,1{}\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1{}\mathrm{i}+a^2\,c^2\,x\,16{}\mathrm{i}-a\,b^2\,c\,x\,8{}\mathrm{i}}{4\,a\,b^4\,\sqrt{\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\,\sqrt{\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2{}\mathrm{i}-\mathrm{atan}\left(\frac{b^4\,x\,1{}\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1{}\mathrm{i}+a^2\,c^2\,x\,16{}\mathrm{i}-a\,b^2\,c\,x\,8{}\mathrm{i}}{4\,a\,b^4\,\sqrt{-\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{-\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\,\sqrt{-\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*x^2 + c*x^4),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*((b^3 + (b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a
^2*b^2*c))^(1/2) + 64*a^3*c^2*((b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)
- 32*a^2*b^2*c*((b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*((b^3 + (b^6
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128
*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4
*a*b^4*(-((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*
a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-((b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4
+ 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(-((b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
- 64*a^2*b^2*c))^(1/2)))*(-((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

3.896 $\int \frac{1}{x^2(a-bx^2+cx^4)}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{ax}+\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/a/x+1/2*arctanh(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*
(1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctanh
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*a*c+b^2)^
(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1137, 1180, 214}

$$\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a-bx^2+cx^4\right)}\,dx = -\frac{1}{ax}+\frac{\int \frac{b-cx^2}{a-bx^2+cx^4}\,dx}{a}$$

$$= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{-\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$= -\frac{1}{ax}+\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{}{\sqrt{b^2\cdots}}\right.}{}$$

Mathematica [A]
time = 0.27, size = 199, normalized size = 1.16

$$-\frac{\frac{2}{x}+\frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(2/x + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - Sqrt[b^2
- 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b + Sqrt[b^2
- 4*a*c]]))/a
```

Maple [A]
time = 0.04, size = 159, normalized size = 0.92

method	result
default	$-\frac{1}{ax}+\frac{4c\left(-\frac{\left(b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\right.}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b\right.}}\right.}{a}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^5c^2-8a^4b^2c+a^3b^4\right)\_Z^4+\left(-12a^2b\,c^2+7a\,b^3c-b^5\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(40a^5c^2-22a^4b^2c+3a^3b^4\right)\_R^4+\left(-\right.\right.\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x+4/a*c*(-1/8*(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))+1/8*(-b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 - b)/(c*x^4 - b*x^2 + a), x)/a - 1/(a*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1108 vs. 2(137) = 274.
time = 0.38, size = 1108, normalized size = 6.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c
^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4
*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*
sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
```

```
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt((b^3 - 3*a
*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^
7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5 - 5
*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4
*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4
*a^4*c))) + sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2
*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4
- 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^
7*c)))*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt((b^
3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(
b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c - (a^3*
b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*
b^2 - 4*a^4*c))) - 2)/(a*x)
```

Sympy [A]
time = 1.88, size = 148, normalized size = 0.86

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{5}c^{2}-128a^{4}b^{2}c+16a^{3}b^{4}\right)+t^{2}\left(-48a^{2}bc^{2}+28ab^{3}c-4b^{5}\right)+c^{3},\left(t\mapsto t\log\left(x+\frac{-64t^{3}a^{5}c^{2}+48t^{3}a^{4}b^{2}c-8t^{3}a^{3}b^{4}+10ta^{2}bc^{2}-10tab^{3}c+2tb^{5}}{ac^{3}-b^{2}c^{2}}\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(-48
*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t*
*3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 + 10*_t*a**2*b*c**2
 - 10*_t*a*b**3*c + 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1877 vs. 2(137) = 274.
time = 4.72, size = 1877, normalized size = 10.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*b^
```

```
2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c
)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c -
 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^
2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt
(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(
-b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*
a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*ar
ctan(2*sqrt(1/2)*x/sqrt(-(a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4
- 8*a^4*b^2*c + 2*a^3*b^3*c + 16*a^5*c^2 - 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^
4*c^3)*abs(a)*abs(c)) + 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^
2 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(-b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*
a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c
)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*
c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqr
t(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*
b^5*c + 16*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*s
qrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(-b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*
c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*x/sqrt(-(a*b - sqrt(a^2*b^2 - 4*a^
3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c + 2*a^3*b^3*c + 16*a^5*c^2 - 8*a^4*b*
c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(c)) - 1/(a*x)
```

Mupad [B]
time = 4.93, size = 2979, normalized size = 17.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b*x^2 + c*x^4)),x)

[Out] - atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^
6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*
a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 + b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c^
2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*
a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 -
4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b
^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - (
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a
*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*
b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*((b^5 + b^2*(
-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - atan(((x*(4*a^
4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*
a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*
b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c +
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*
(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1
i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*
a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*
c^3 - 8*a^5*b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
```

```
b^2*c)))^(1/2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*
b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*
c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c^2 +
x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5
*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16
*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*((b^5 - b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(a*x)
```

3.897 $\int \frac{x^5}{a-b+2ax^2+ax^4} dx$

Optimal. Leaf size=69

$$\frac{x^2}{2a} - \frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}} - \frac{\log\left(a-b+2ax^2+ax^4\right)}{2a}$$

```
[Out] 1/2*x^2/a-1/2*ln(a*x^4+2*a*x^2+a-b)/a-1/2*(a+b)*arctanh((x^2+1)*a^(1/2)/b^(
1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1128, 717, 648, 632, 212, 642}

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}} - \frac{\log\left(ax^4+2ax^2+a-b\right)}{2a} + \frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x^2/(2*a) - ((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[
b]) - Log[a - b + 2*a*x^2 + a*x^4]/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a-b+2ax+ax^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2a}+\frac{\text{Subst}\left(\int \frac{-a+b-2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\text{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2a}+\frac{(a+b)\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{2a}-\frac{(a+b)\text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a(1+x^2)\right)}{a}\\
&= \frac{x^2}{2a}-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{2a}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.90

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x^2-\log\left(-b+a(1+x^2)^2\right)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a - b + 2*a*x^2 + a*x^4),x]

[Out] -1/2*((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(a^(3/2)*Sqrt[b]) + (x^2 - Log[-b + a*(1 + x^2)^2])/(2*a)

Maple [A]
time = 0.03, size = 63, normalized size = 0.91

method	result
default	$\frac{x^2}{2a}+\frac{-\ln\left(a\,x^4+2a\,x^2+a-b\right)-\frac{(a+b)\,\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}}{2a}$
risch	$\frac{x^2}{2a}-\frac{\ln\left(\left(a^2b+a\,b^2+\sqrt{ab\,(a+b)^2}\,a\right)x^2+\sqrt{ab\,(a+b)^2}\,a-\sqrt{ab\,(a+b)^2}\,b\right)}{2a}+\frac{\ln\left(\left(a^2b+a\,b^2+\sqrt{ab\,(a+b\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)

[Out] 1/2*x^2/a+1/2/a*(-ln(a*x^4+2*a*x^2+a-b)-(a+b)/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2)))

Maxima [A]
time = 0.50, size = 74, normalized size = 1.07

$$\frac{x^2}{2\,a}+\frac{(a+b)\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a}-\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")

[Out] 1/2*x^2/a + 1/4*(a + b)*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/(sqrt(a*b)*a) - 1/2*log(a*x^4 + 2*a*x^2 + a - b)/a

Fricas [A]
time = 0.36, size = 156, normalized size = 2.26

$$\left[\frac{2\,abx^2-2\,ab\log\left(ax^4+2\,ax^2+a-b\right)+\sqrt{ab}\,(a+b)\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{ab}\,(x^2+1)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,a^2b},\ \frac{abx^2-ab\log\left(ax^4+2\,ax^2+a-b\right)+\sqrt{-ab}\,(a+b)\arctan\left(\frac{\sqrt{-ab}}{ax^2+a}\right)}{2\,a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")

[Out] [1/4*(2*a*b*x^2 - 2*a*b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(a*b)*(a + b)*log((a*x^4 + 2*a*x^2 - 2*sqrt(a*b)*(x^2 + 1) + a + b)/(a*x^4 + 2*a*x^2 + a -

```
b)))/(a^2*b), 1/2*(a*b*x^2 - a*b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(-a*b)*
(a + b)*arctan(sqrt(-a*b)/(a*x^2 + a)))/(a^2*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(60) = 120$.
time = 0.64, size = 138, normalized size = 2.00

$$\left(-\frac{1}{2a} - \frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)\log\left(x^2 + \frac{-4ab\left(-\frac{1}{2a} - \frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right) + a - b}{a+b}\right) + \left(-\frac{1}{2a} + \frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)\log\left(x^2 + \frac{-4ab\left(-\frac{1}{2a} + \frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right) + a - b}{a+b}\right) + \frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (-1/(2*a) - sqrt(a**3*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(-1/(2*a) -
 sqrt(a**3*b)*(a + b)/(4*a**3*b)) + a - b)/(a + b)) + (-1/(2*a) + sqrt(a**3
*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(-1/(2*a) + sqrt(a**3*b)*(a + b)
/(4*a**3*b)) + a - b)/(a + b)) + x**2/(2*a)
```

Giac [A]
time = 3.30, size = 60, normalized size = 0.87

$$\frac{x^2}{2\,a} + \frac{(a+b)\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}\,a} - \frac{\log\left(ax^4 + 2\,ax^2 + a - b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a + 1/2*(a + b)*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/2
*log(a*x^4 + 2*a*x^2 + a - b)/a
```

Mupad [B]
time = 0.39, size = 166, normalized size = 2.41

$$\frac{x^2}{2\,a} - \ln\left(a\,\sqrt{a^3\,b} - b\,\sqrt{a^3\,b} - a^2\,b\,x^2 + a\,x^2\,\sqrt{a^3\,b}\right)\left(\frac{\frac{a^2}{2} + \frac{\sqrt{a^3\,b}}{4}}{a^3} + \frac{\sqrt{a^3\,b}}{4\,a^2\,b}\right) - \ln\left(a\,\sqrt{a^3\,b} - b\,\sqrt{a^3\,b} + a^2\,b\,x^2 + a\,x^2\,\sqrt{a^3\,b}\right)\left(\frac{\frac{a^2}{2} - \frac{\sqrt{a^3\,b}}{4}}{a^3} - \frac{\sqrt{a^3\,b}}{4\,a^2\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x^2/(2*a) - log(a*(a^3*b)^(1/2) - b*(a^3*b)^(1/2) - a^2*b*x^2 + a*x^2*(a^3*
b)^(1/2))*((a^2/2 + (a^3*b)^(1/2)/4)/a^3 + (a^3*b)^(1/2)/(4*a^2*b)) - log(a
*(a^3*b)^(1/2) - b*(a^3*b)^(1/2) + a^2*b*x^2 + a*x^2*(a^3*b)^(1/2))*((a^2/2
 - (a^3*b)^(1/2)/4)/a^3 - (a^3*b)^(1/2)/(4*a^2*b))
```

3.898 $\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}+\frac{\log\left(a-b+2ax^2+ax^4\right)}{4a}$$

```
[Out] 1/4*ln(a*x^4+2*a*x^2+a-b)/a+1/2*arctanh((x^2+1)*a^(1/2)/b^(1/2))/a^(1/2)/b^
(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1128, 648, 632, 212, 642}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}+\frac{\log\left(ax^4+2ax^2+a-b\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a - b + 2*a*
x^2 + a*x^4]/(4*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a-b+2ax+ax^2}\,dx, x, x^2\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)\right) + \frac{\text{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right.}{4a}\\ &= \frac{\log\left(a-b+2ax^2+ax^4\right)}{4a} + \text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a(1+x^2)\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}} + \frac{\log\left(a-b+2ax^2+ax^4\right)}{4a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 0.91

$$\frac{\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{\sqrt{b}} + \log\left(-b+a(1+x^2)^2\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ((2*Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[-b + a*(1 +
 x^2)^2])/(4*a)
```

Maple [A]
time = 0.02, size = 49, normalized size = 0.88

method	result

default	$\frac{\ln(a\,x^4+2a\,x^2+a-b)}{4a}+\frac{\operatorname{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$
risch	$\frac{\ln\left(\left(a\sqrt{ab}-ab\right)x^2+a\sqrt{ab}-\sqrt{ab}\,b\right)}{4a}+\frac{\ln\left(\left(a\sqrt{ab}-ab\right)x^2+a\sqrt{ab}-\sqrt{ab}\,b\right)\sqrt{ab}}{4ba}+\frac{\ln\left(\left(-a\sqrt{ab}-ab\right)x^2-a\sqrt{\right.}}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(a*x^4+2*a*x^2+a-b)/a+1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)
^(1/2))
```

Maxima [A]
time = 0.51, size = 60, normalized size = 1.07

$$-\frac{\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\sqrt{ab}}+\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
[Out] -1/4*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/sqrt(a*b) + 1/4*l
og(a*x^4 + 2*a*x^2 + a - b)/a
```

Fricas [A]
time = 0.35, size = 134, normalized size = 2.39

$$\left[\frac{b\log\left(ax^4+2\,ax^2+a-b\right)+\sqrt{ab}\,\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{ab}\,(x^2+1)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,ab},\frac{b\log\left(ax^4+2\,ax^2+a-b\right)-2\,\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{ax^2+a}\right)}{4\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
[Out] [1/4*(b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(a*b)*log((a*x^4 + 2*a*x^2 + 2*s
qrt(a*b)*(x^2 + 1) + a + b)/(a*x^4 + 2*a*x^2 + a - b)))/(a*b), 1/4*(b*log(a
*x^4 + 2*a*x^2 + a - b) - 2*sqrt(-a*b)*arctan(sqrt(-a*b)/(a*x^2 + a)))/(a*b
)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(48) = 96$.
time = 0.32, size = 110, normalized size = 1.96

$$\left(\frac{1}{4a}-\frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2+\frac{4ab\left(\frac{1}{4a}-\frac{\sqrt{a^3b}}{4a^2b}\right)+a-b}{a}\right)+\left(\frac{1}{4a}+\frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2+\frac{4ab\left(\frac{1}{4a}+\frac{\sqrt{a^3b}}{4a^2b}\right)+a-b}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (1/(4*a) - sqrt(a**3*b)/(4*a**2*b))*log(x**2 + (4*a*b*(1/(4*a) - sqrt(a**3*
b)/(4*a**2*b)) + a - b)/a) + (1/(4*a) + sqrt(a**3*b)/(4*a**2*b))*log(x**2 +
 (4*a*b*(1/(4*a) + sqrt(a**3*b)/(4*a**2*b)) + a - b)/a)
```

Giac [A]
time = 3.48, size = 46, normalized size = 0.82

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}}+\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b) + 1/4*log(a*x^4 + 2*a*x^2 +
a - b)/a
```

Mupad [B]
time = 0.17, size = 153, normalized size = 2.73

$$\frac{\ln\left(x^2\,\sqrt{a^3\,b}+a\,b-a^2-a^2\,x^2\right)}{4\,a}+\frac{\ln\left(x^2\,\sqrt{a^3\,b}-a\,b+a^2+a^2\,x^2\right)}{4\,a}-\frac{\ln\left(x^2\,\sqrt{a^3\,b}-a\,b+a^2+a^2\,x^2\right)\,\sqrt{a^3\,b}}{4\,a^2\,b}+\frac{\ln\left(x^2\,\sqrt{a^3\,b}+a\,b-a^2-a^2\,x^2\right)\,\sqrt{a^3\,b}}{4\,a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] log(x^2*(a^3*b)^(1/2) + a*b - a^2 - a^2*x^2)/(4*a) + log(x^2*(a^3*b)^(1/2)
- a*b + a^2 + a^2*x^2)/(4*a) - (log(x^2*(a^3*b)^(1/2) - a*b + a^2 + a^2*x^2
)*(a^3*b)^(1/2))/(4*a^2*b) + (log(x^2*(a^3*b)^(1/2) + a*b - a^2 - a^2*x^2)*
(a^3*b)^(1/2))/(4*a^2*b)
```

3.899 $\int \frac{x}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

```
[Out] -1/2*arctanh((x^2+1)*a^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1121, 632, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(Sqrt[a]*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx, x, x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx, x, 2a(1+x^2)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.84

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$	26
risch	$\frac{\ln\left(\left(\sqrt{ab}+a\right)x^2+a-b\right)}{4\sqrt{ab}}-\frac{\ln\left(\left(\sqrt{ab}-a\right)x^2-a+b\right)}{4\sqrt{ab}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 37, normalized size = 1.19

$$\frac{\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\,\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] 1/4*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/sqrt(a*b)
```

Fricas [A]
time = 0.36, size = 91, normalized size = 2.94

$$\left[\frac{\sqrt{ab}\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{ab}\,(x^2+1)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,ab}, \frac{\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{ax^2+a}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*b)*log((a*x^4 + 2*a*x^2 - 2*sqrt(a*b)*(x^2 + 1) + a + b)/(a*x^4
 + 2*a*x^2 + a - b))/(a*b), 1/2*sqrt(-a*b)*arctan(sqrt(-a*b)/(a*x^2 + a))/(
a*b)]
```

Sympy [A]
time = 0.15, size = 53, normalized size = 1.71

$$\frac{\sqrt{\frac{1}{ab}}\log\left(-b\sqrt{\frac{1}{ab}}+x^2+1\right)}{4}-\frac{\sqrt{\frac{1}{ab}}\log\left(b\sqrt{\frac{1}{ab}}+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] sqrt(1/(a*b))*log(-b*sqrt(1/(a*b)) + x**2 + 1)/4 - sqrt(1/(a*b))*log(b*sqrt
(1/(a*b)) + x**2 + 1)/4
```

Giac [A]
time = 3.42, size = 23, normalized size = 0.74

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b)
```

Mupad [B]
time = 4.34, size = 31, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sqrt{b}\,x^2}{a\,x^2+a-b}\right)}{2\,\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a - b + 2*a*x^2 + a*x^4),x)
[Out] atanh((a^(1/2)*b^(1/2)*x^2)/(a - b + a*x^2))/(2*a^(1/2)*b^(1/2))
```

3.900 $\int \frac{1}{x(a-b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2(a-b)\sqrt{b}}+\frac{\log(x)}{a-b}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{4(a-b)}$$

```
[Out] ln(x)/(a-b)-1/4*ln(a*x^4+2*a*x^2+a-b)/(a-b)+1/2*arctanh((x^2+1)*a^(1/2)/b^(
1/2))*a^(1/2)/(a-b)/b^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1128, 719, 29, 648, 632, 212, 642}

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4(a-b)}+\frac{\log(x)}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*(a - b)*Sqrt[b]) + Log[x]
/(a - b) - Log[a - b + 2*a*x^2 + a*x^4]/(4*(a - b))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a-b+2ax^2+ax^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a-b+2ax+ax^2\right)}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2(a-b)}+\frac{\text{Subst}\left(\int \frac{-2a-ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2(a-b)}\\
&= \frac{\log(x)}{a-b}-\frac{\text{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{4(a-b)}-\frac{a\text{Subst}\left(\int \frac{1}{a-b+2ax+ax^2}\,dx,x,x^2\right)}{2(a-b)}\\
&= \frac{\log(x)}{a-b}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{4(a-b)}+\frac{a\text{Subst}\left(\int \frac{1}{4ab-x^2}\,dx,x,2a(1+x^2)\right)}{a-b}\\
&= \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2(a-b)\sqrt{b}}+\frac{\log(x)}{a-b}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{4(a-b)}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 90, normalized size = 1.17

$$\frac{-4\sqrt{b}\,\log(x)+\left(\sqrt{a}+\sqrt{b}\right)\log\left(-\sqrt{b}+\sqrt{a}\,(1+x^2)\right)+\left(-\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{b}+\sqrt{a}\,(1+x^2)\right)}{4\sqrt{b}\,(-a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (-4*Sqrt[b]*Log[x] + (Sqrt[a] + Sqrt[b])*Log[-Sqrt[b] + Sqrt[a]*(1 + x^2)]
+ (-Sqrt[a] + Sqrt[b])*Log[Sqrt[b] + Sqrt[a]*(1 + x^2)])/(4*Sqrt[b]*(-a + b
))
```

Maple [A]
time = 0.02, size = 70, normalized size = 0.91

method	result	size
risch	$\frac{\ln(x)}{a-b}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(-1+\left(ab-b^2\right)\_Z^2+2b\_Z\right)}\_R\ln\left(\left((-a-5b)\_R+5\right)x^2+(-a+b)\_R+4\right)\right)}{4}$	64
default	$-\frac{a\left(\frac{\ln\left(a\,x^4+2a\,x^2+a-b\right)}{2a}-\frac{\operatorname{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{2(a-b)}+\frac{\ln(x)}{a-b}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/(a-b)*(1/2*ln(a*x^4+2*a*x^2+a-b)/a-1/(a*b)^(1/2)*arctanh(1/2*(2*a*x^
2+2*a)/(a*b)^(1/2)))+ln(x)/(a-b)
```

Maxima [A]
time = 0.51, size = 85, normalized size = 1.10

$$-\frac{a\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\sqrt{ab}\,(a-b)}-\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{4\,(a-b)}+\frac{\log\left(x^2\right)}{2\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] -1/4*a*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/(sqrt(a*b)*(a -
 b)) - 1/4*log(a*x^4 + 2*a*x^2 + a - b)/(a - b) + 1/2*log(x^2)/(a - b)
```

Fricas [A]
time = 0.35, size = 151, normalized size = 1.96

$$\left[-\frac{\sqrt{\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\,(bx^2+b)\sqrt{\frac{a}{b}}+a+b}{ax^4+2\,ax^2+a-b}\right)+\log\left(ax^4+2\,ax^2+a-b\right)-4\,\log\left(x\right)}{4\,(a-b)},\ -\frac{2\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{-\frac{a}{b}}}{ax^2+a}\right)+\log\left(ax^4+2\,ax^2+a-b\right)-4\,\log\left(x\right)}{4\,(a-b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(a/b) + a + b)/(a
*x^4 + 2*a*x^2 + a - b)) + log(a*x^4 + 2*a*x^2 + a - b) - 4*log(x))/(a - b)
, -1/4*(2*sqrt(-a/b)*arctan(b*sqrt(-a/b)/(a*x^2 + a)) + log(a*x^4 + 2*a*x^2
 + a - b) - 4*log(x))/(a - b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(61) = 122$.
time = 2.59, size = 184, normalized size = 2.39

$$\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+a-4b^2\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+b}{a}\right)+\left(-\frac{1}{4(a-b)}+\frac{\sqrt{ab}}{4b(a-b)}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{4(a-b)}+\frac{\sqrt{ab}}{4b(a-b)}\right)+a-4b^2\left(-\frac{1}{4(a-b)}+\frac{\sqrt{ab}}{4b(a-b)}\right)+b}{a}\right)+\frac{\log(x)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (-1/(4*(a - b)) - sqrt(a*b)/(4*b*(a - b)))*log(x**2 + (4*a*b*(-1/(4*(a - b)
) - sqrt(a*b)/(4*b*(a - b))) + a - 4*b**2*(-1/(4*(a - b)) - sqrt(a*b)/(4*b*
(a - b))) + b)/a) + (-1/(4*(a - b)) + sqrt(a*b)/(4*b*(a - b)))*log(x**2 + (
4*a*b*(-1/(4*(a - b)) + sqrt(a*b)/(4*b*(a - b))) + a - 4*b**2*(-1/(4*(a - b
)) + sqrt(a*b)/(4*b*(a - b))) + b)/a) + log(x)/(a - b)
```

Giac [A]
time = 4.75, size = 71, normalized size = 0.92

$$-\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}(a-b)}-\frac{\log(ax^4+2ax^2+a-b)}{4(a-b)}+\frac{\log(x^2)}{2(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b)*(a - b)) - 1/4*log(a*x^4
+ 2*a*x^2 + a - b)/(a - b) + 1/2*log(x^2)/(a - b)
```

Mupad [B]
time = 4.56, size = 183, normalized size = 2.38

$$\frac{\ln(x)}{a-b}-\frac{\ln\left(16\,a^4+20\,a^4\,x^2+\frac{\left(b-\sqrt{a\,b}\right)\,\left(x^2\,\left(16\,a^5+80\,b\,a^4\right)-16\,a^4\,b+16\,a^5\right)}{4\,\left(a\,b-b^2\right)}\right)\,\left(b-\sqrt{a\,b}\right)}{4\,\left(a\,b-b^2\right)}-\frac{\ln\left(16\,a^4+20\,a^4\,x^2+\frac{\left(b+\sqrt{a\,b}\right)\,\left(x^2\,\left(16\,a^5+80\,b\,a^4\right)-16\,a^4\,b+16\,a^5\right)}{4\,\left(a\,b-b^2\right)}\right)\,\left(b+\sqrt{a\,b}\right)}{4\,\left(a\,b-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a - b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] log(x)/(a - b) - (log(16*a^4 + 20*a^4*x^2 + ((b - (a*b)^(1/2))*(x^2*(80*a^4
*b + 16*a^5) - 16*a^4*b + 16*a^5))/(4*(a*b - b^2)))*(b - (a*b)^(1/2)))/(4*(
a*b - b^2)) - (log(16*a^4 + 20*a^4*x^2 + ((b + (a*b)^(1/2))*(x^2*(80*a^4*b
+ 16*a^5) - 16*a^4*b + 16*a^5))/(4*(a*b - b^2)))*(b + (a*b)^(1/2)))/(4*(a*b
 - b^2))
```

3.901 $\int \frac{1}{x^3(a-b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{2(a-b)x^2}-\frac{\sqrt{a}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2(a-b)^2\sqrt{b}}-\frac{2a\log(x)}{(a-b)^2}+\frac{a\log\left(a-b+2ax^2+ax^4\right)}{2(a-b)^2}$$

```
[Out] -1/2/(a-b)/x^2-2*a*ln(x)/(a-b)^2+1/2*a*ln(a*x^4+2*a*x^2+a-b)/(a-b)^2-1/2*(a
+b)*arctanh((x^2+1)*a^(1/2)/b^(1/2))*a^(1/2)/(a-b)^2/b^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1128, 723, 814, 648, 632, 212, 642}

$$-\frac{1}{2x^2(a-b)}-\frac{\sqrt{a}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)^2}+\frac{a\log\left(ax^4+2ax^2+a-b\right)}{2(a-b)^2}-\frac{2a\log(x)}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/2*1/((a - b)*x^2) - (Sqrt[a]*(a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]
])/(2*(a - b)^2*Sqrt[b]) - (2*a*Log[x])/(a - b)^2 + (a*Log[a - b + 2*a*x^2
+ a*x^4])/(2*(a - b)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3(a-b+2ax^2+ax^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a-b+2ax+ax^2)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2(a-b)x^2} + \frac{\text{Subst}\left(\int \frac{-2a-ax}{x(a-b+2ax+ax^2)}\,dx, x, x^2\right)}{2(a-b)}$$
$$= -\frac{1}{2(a-b)x^2} + \frac{\text{Subst}\left(\int \left(-\frac{2a}{(a-b)x} + \frac{a(3a+b+2ax)}{(a-b)(a-b+2ax+ax^2)}\right)\,dx, x, x^2\right)}{2(a-b)}$$
$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\text{Subst}\left(\int \frac{3a+b+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2(a-b)^2}$$
$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\text{Subst}\left(\int \frac{2a+2ax}{a-b+2ax+ax^2}\,dx, x, x^2\right)}{2(a-b)^2} + \frac{(a(a+b))\text{S}}{}$$
$$= -\frac{1}{2(a-b)x^2} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\log(a-b+2ax^2+ax^4)}{2(a-b)^2} - \frac{(a(a+b))\text{Subst}}{}$$
$$= -\frac{1}{2(a-b)x^2} - \frac{\sqrt{a}(a+b)\tanh^{-1}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2(a-b)^2\sqrt{b}} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\log(a-}{2}$$

Mathematica [A]

time = 0.06, size = 146, normalized size = 1.51

$$\frac{-8a\sqrt{b}\,x^2\log(x) + \sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^2 x^2\log\left(-\sqrt{b}+\sqrt{a}(1+x^2)\right) - \left(\sqrt{a}-\sqrt{b}\right)\left(2\left(\sqrt{a}\sqrt{b}+b\right) + \left(ax^2-\sqrt{a}\sqrt{b}\,x^2\right)\log\left(\sqrt{b}+\sqrt{a}(1+x^2)\right)\right)}{4(a-b)^2\sqrt{b}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (-8*a*Sqrt[b]*x^2*Log[x] + Sqrt[a]*(Sqrt[a] + Sqrt[b])^2*x^2*Log[-Sqrt[b] +
 Sqrt[a]*(1 + x^2)] - (Sqrt[a] - Sqrt[b])*(2*(Sqrt[a]*Sqrt[b] + b) + (a*x^2
 - Sqrt[a]*Sqrt[b]*x^2)*Log[Sqrt[b] + Sqrt[a]*(1 + x^2)]))/(4*(a - b)^2*Sqr
t[b]*x^2)
```

Maple [A]

time = 0.04, size = 82, normalized size = 0.85

method	result
default	$\frac{a\left(\ln\left(a\,x^4+2a\,x^2+a-b\right)-\frac{(a+b)\,\text{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{2(a-b)^2}-\frac{1}{2(a-b)x^2}-\frac{2a\ln(x)}{(a-b)^2}$

risch	$-\frac{1}{2(a-b)x^2}-\frac{2a\ln(x)}{a^2-2ab+b^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2b-2a\,b^2+b^3\right)\_Z^2-4ab\_Z-a\right)}\_R\ln\left(\left(\left(-a^3-3a^2b+9a\,b^2-5b^3\right)\_R^2+\left(-8a^2+8ab\right.\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a-b)^2*a*(ln(a*x^4+2*a*x^2+a-b)-(a+b)/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2
+2*a)/(a*b)^(1/2)))-1/2/(a-b)/x^2-2*a*ln(x)/(a-b)^2
```

Maxima [A]
time = 0.52, size = 123, normalized size = 1.27

$$\frac{a\log\left(ax^4+2\,ax^2+a-b\right)}{2\left(a^2-2\,ab+b^2\right)}-\frac{a\log\left(x^2\right)}{a^2-2\,ab+b^2}+\frac{\left(a^2+ab\right)\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\left(a^2-2\,ab+b^2\right)\sqrt{ab}}-\frac{1}{2\,(a-b)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a - b)/(a^2 - 2*a*b + b^2) - a*log(x^2)/(a^2 -
2*a*b + b^2) + 1/4*(a^2 + a*b)*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqr
t(a*b)))/((a^2 - 2*a*b + b^2)*sqrt(a*b)) - 1/2/((a - b)*x^2)
```

Fricas [A]
time = 0.37, size = 209, normalized size = 2.15

$$\left[\frac{(a+b)x^2\sqrt{\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\,(bx^2+b)\sqrt{\frac{a}{b}}+a+b}{ax^4+2\,ax^2+a-b}\right)+2\,ax^2\log\left(ax^4+2\,ax^2+a-b\right)-8\,ax^2\log\left(x\right)-2\,a+2\,b}{4\left(a^2-2\,ab+b^2\right)x^2},\ \frac{(a+b)x^2\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{-\frac{a}{b}}}{ax^2+a}\right)+ax^2\log\left(ax^4+2\,ax^2+a-b\right)-4\,ax^2\log\left(x\right)-a+b}{2\left(a^2-2\,ab+b^2\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*x^2*sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(a/b)
+ a + b)/(a*x^4 + 2*a*x^2 + a - b)) + 2*a*x^2*log(a*x^4 + 2*a*x^2 + a - b)
- 8*a*x^2*log(x) - 2*a + 2*b)/((a^2 - 2*a*b + b^2)*x^2), 1/2*((a + b)*x^2*s
qrt(-a/b)*arctan(b*sqrt(-a/b)/(a*x^2 + a)) + a*x^2*log(a*x^4 + 2*a*x^2 + a
- b) - 4*a*x^2*log(x) - a + b)/((a^2 - 2*a*b + b^2)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(85) = 170$.
time = 20.62, size = 372, normalized size = 3.84

$$-\frac{2a\log\left(x\right)}{\left(a-b\right)^2}+\left(\frac{a}{2\left(a-b\right)^2}-\frac{\sqrt{ab}\left(a+b\right)}{4b\left(a^2-2ab+b^2\right)}\right)\log\left(x^2+\frac{-4a^2b\left(\frac{a}{2(a-b)^2}-\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a-b)^2}-\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)+3ab-4b^3\left(\frac{a}{2(a-b)^2}-\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)}{a^2+ab}\right)+\left(\frac{a}{2\left(a-b\right)^2}+\frac{\sqrt{ab}\left(a+b\right)}{4b\left(a^2-2ab+b^2\right)}\right)\log\left(x^2+\frac{-4a^2b\left(\frac{a}{2(a-b)^2}+\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a-b)^2}+\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)+3ab-4b^3\left(\frac{a}{2(a-b)^2}+\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)}{a^2+ab}\right)-\frac{1}{x^2\cdot\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] -2*a*log(x)/(a - b)**2 + (a/(2*(a - b)**2) - sqrt(a*b)*(a + b)/(4*b*(a**2 -
 2*a*b + b**2)))*log(x**2 + (-4*a**2*b*(a/(2*(a - b)**2) - sqrt(a*b)*(a + b
)/(4*b*(a**2 - 2*a*b + b**2))) + a**2 + 8*a*b**2*(a/(2*(a - b)**2) - sqrt(a
*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))) + 3*a*b - 4*b**3*(a/(2*(a - b)**2)
 - sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))))/(a**2 + a*b)) + (a/(2*(a
 - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2)))*log(x**2 + (-4*a
**2*b*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))) +
a**2 + 8*a*b**2*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b +
b**2))) + 3*a*b - 4*b**3*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 -
 2*a*b + b**2))))/(a**2 + a*b)) - 1/(x**2*(2*a - 2*b))
```

Giac [A]
time = 4.37, size = 126, normalized size = 1.30

$$\frac{a\log\left(ax^4+2\,ax^2+a-b\right)}{2\,(a^2-2\,ab+b^2)}-\frac{a\log\left(x^2\right)}{a^2-2\,ab+b^2}+\frac{(a^2+ab)\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,(a^2-2\,ab+b^2)\sqrt{-ab}}+\frac{2\,ax^2-a+b}{2\,(a^2-2\,ab+b^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a - b)/(a^2 - 2*a*b + b^2) - a*log(x^2)/(a^2 -
2*a*b + b^2) + 1/2*(a^2 + a*b)*arctan((a*x^2 + a)/sqrt(-a*b))/((a^2 - 2*a*b
 + b^2)*sqrt(-a*b)) + 1/2*(2*a*x^2 - a + b)/((a^2 - 2*a*b + b^2)*x^2)
```

Mupad [B]
time = 4.87, size = 389, normalized size = 4.01

$$\frac{\ln\left(100\,a\,(a\,b)^{7/2}-198\,b\,(a\,b)^{7/2}-a^3\,(a\,b)^{5/2}+100\,b^3\,(a\,b)^{5/2}-b^5\,(a\,b)^{3/2}+a^2\,b^6-100\,a^3\,b^5+198\,a^4\,b^4-100\,a^5\,b^3+a^6\,b^2+a^2\,b^6\,x^2-100\,a^3\,b^5\,x^2+198\,a^4\,b^4\,x^2-100\,a^5\,b^3\,x^2+a^6\,b^2\,x^2\right)\left(\frac{a\,\sqrt{a\,b}}{4}+b\left(\frac{a}{2}+\frac{\sqrt{a\,b}}{4}\right)\right)}{a^2\,b-2\,a\,b^2+b^3}-\frac{2\,a\,\ln\left(x\right)}{a^2-2\,a\,b+b^2}-\frac{\ln\left(198\,b\,(a\,b)^{7/2}-100\,a\,(a\,b)^{7/2}+a^3\,(a\,b)^{5/2}-100\,b^3\,(a\,b)^{5/2}+b^5\,(a\,b)^{3/2}+a^2\,b^6-100\,a^3\,b^5+198\,a^4\,b^4-100\,a^5\,b^3+a^6\,b^2+a^2\,b^6\,x^2-100\,a^3\,b^5\,x^2+198\,a^4\,b^4\,x^2-100\,a^5\,b^3\,x^2+a^6\,b^2\,x^2\right)\left(\frac{a\,\sqrt{a\,b}}{4}-b\left(\frac{a}{2}-\frac{\sqrt{a\,b}}{4}\right)\right)}{a^2\,b-2\,a\,b^2+b^3}-\frac{1}{2\,x^2\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] (log(100*a*(a*b)^(7/2) - 198*b*(a*b)^(7/2) - a^3*(a*b)^(5/2) + 100*b^3*(a*b
)^(5/2) - b^5*(a*b)^(3/2) + a^2*b^6 - 100*a^3*b^5 + 198*a^4*b^4 - 100*a^5*b
^3 + a^6*b^2 + a^2*b^6*x^2 - 100*a^3*b^5*x^2 + 198*a^4*b^4*x^2 - 100*a^5*b^
3*x^2 + a^6*b^2*x^2)*((a*(a*b)^(1/2))/4 + b*(a/2 + (a*b)^(1/2)/4)))/(a^2*b
- 2*a*b^2 + b^3) - (2*a*log(x))/(a^2 - 2*a*b + b^2) - (log(198*b*(a*b)^(7/2
) - 100*a*(a*b)^(7/2) + a^3*(a*b)^(5/2) - 100*b^3*(a*b)^(5/2) + b^5*(a*b)^(
3/2) + a^2*b^6 - 100*a^3*b^5 + 198*a^4*b^4 - 100*a^5*b^3 + a^6*b^2 + a^2*b^
6*x^2 - 100*a^3*b^5*x^2 + 198*a^4*b^4*x^2 - 100*a^5*b^3*x^2 + a^6*b^2*x^2)*
((a*(a*b)^(1/2))/4 - b*(a/2 - (a*b)^(1/2)/4)))/(a^2*b - 2*a*b^2 + b^3) - 1/
(2*x^2*(a - b))
```

3.902 $\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=114

$$\frac{x}{a}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}$$

```
[Out] x/a+1/2*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))*(a^(1/2)-b^(1/2))^(3/2)/a
^(5/4)/b^(1/2)-1/2*arctan(a^(1/4)*x/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+b^(1/
2))^(3/2)/a^(5/4)/b^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1136, 1180, 211}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]
])/(2*a^(5/4)*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*a^(5/4)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx = \frac{x}{a} - \frac{\int \frac{a-b+2ax^2}{a-b+2ax^2+ax^4}\,dx}{a}$$
$$= \frac{x}{a} - \frac{1}{2}\left(2-\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx - \frac{1}{2}\left(2+\frac{a+b}{\sqrt{a}\sqrt{b}}\right)\int \frac{}{a+\sqrt{}}$$
$$= \frac{x}{a} + \frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{}{\sqrt{}}\right)}{2a^{5/4}\sqrt{b}}$$

Mathematica [A]
time = 0.06, size = 144, normalized size = 1.26

$$\frac{x}{a} + \frac{\left(\sqrt{a}-\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{a-\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((Sqrt[a] - Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]])
/(2*a*Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^2*ArcTan[(S
qrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(2*a*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b
])
```

Maple [A]
time = 0.04, size = 101, normalized size = 0.89

method	result	size
risch	$\frac{x}{a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4+2a\_Z^2+a-b\right)}\frac{\left(-2\_R^2a-a+b\right)\ln\left(x-\_R\right)}{\_R^3+\_R}}{4a^2}$	57

default	$\frac{x}{a}+\frac{\left(-2\sqrt{ab}-a-b\right)\arctan\left(\frac{ax}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)a}}-\frac{\left(-2\sqrt{ab}+a+b\right)\operatorname{arctanh}\left(\frac{ax}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)a}}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

```
[Out] x/a+1/2*(-2*(a*b)^(1/2)-a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a
*x/(((a*b)^(1/2)+a)*a)^(1/2))-1/2*(-2*(a*b)^(1/2)+a+b)/(a*b)^(1/2)/(((a*b)^
(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] x/a - integrate((2*a*x^2 + a - b)/(a*x^4 + 2*a*x^2 + a - b), x)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(74) = 148.
time = 0.37, size = 603, normalized size = 5.29

$$\frac{a\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\log\left(-(3a^2-2ab-b^2)x+\left(a^4b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-3a^2b-ab^2\right)\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\right)-a\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\log\left(-(3a^2-2ab-b^2)x-\left(a^4b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-3a^2b-ab^2\right)\sqrt{-\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+a+3b}{a^2b}}\right)-a\sqrt{\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-a-3b}{a^2b}}\log\left(-(3a^2-2ab-b^2)x+\left(a^4b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+3a^2b+ab^2\right)\sqrt{\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-a-3b}{a^2b}}\right)+a\sqrt{\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-a-3b}{a^2b}}\log\left(-(3a^2-2ab-b^2)x-\left(a^4b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}+3a^2b+ab^2\right)\sqrt{\frac{a^2b\sqrt{\frac{9a^2+6ab+b^2}{a^5b}}-a-3b}{a^2b}}\right)+4x}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] 1/4*(a*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3*b)/(a^2*b))
*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b))
- 3*a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3
*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3*
b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x - (a^4*b*sqrt((9*a^2 + 6*a*b + b^2
)/(a^5*b)) - 3*a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*
b)) + a + 3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)
) - a - 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*sqrt((9*a^2 + 6
*a*b + b^2)/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b
^2)/(a^5*b)) - a - 3*b)/(a^2*b))) + a*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b^2
)/(a^5*b)) - a - 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x - (a^4*b*sqrt((
```

```
9*a^2 + 6*a*b + b^2)/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*b*sqrt((9*a^2 +
6*a*b + b^2)/(a^5*b)) - a - 3*b)/(a^2*b))) + 4*x)/a
```

Sympy [A]
time = 0.78, size = 105, normalized size = 0.92

$$\operatorname{RootSum}\left(256t^4a^5b^2 + t^2\cdot\left(32a^4b + 96a^3b^2\right) + a^3 - 3a^2b + 3ab^2 - b^3, \left(t \mapsto t\log\left(x + \frac{64t^3a^4b + 4ta^3 + 24ta^2b + 4tab^2}{3a^2 - 2ab - b^2}\right)\right)\right) + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(256*_t**4*a**5*b**2 + _t**2*(32*a**4*b + 96*a**3*b**2) + a**3 - 3*a
**2*b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (64*_t**3*a**4*b + 4*_t*a**3
 + 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 - 2*a*b - b**2)))) + x/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(74) = 148.
time = 6.13, size = 512, normalized size = 4.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^4 - sqrt(a^2 + sqrt(a*b)*a)*sqrt
(a*b)*a^3*b - 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 2*(3*sqrt(a^2 +
 sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*b^2)*a^2
- (3*sqrt(a^2 + sqrt(a*b)*a)*a^3*b - 7*sqrt(a^2 + sqrt(a*b)*a)*a^2*b^2 + 4*
sqrt(a^2 + sqrt(a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 + sqrt(a^4 - (a^2
 - a*b)*a^2))/a^2))/(3*a^6*b - 7*a^5*b^2 + 4*a^4*b^3) + 1/2*(3*sqrt(a^2 - s
qrt(a*b)*a)*sqrt(a*b)*a^4 - sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^3*b - 4*sqr
t(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 2*(3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(
a*b)*a*b - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b^2)*a^2 - (3*sqrt(a^2 - sqr
t(a*b)*a)*a^3*b - 7*sqrt(a^2 - sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(a*b
)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - sqrt(a^4 - (a^2 - a*b)*a^2))/a^2))
/(3*a^6*b - 7*a^5*b^2 + 4*a^4*b^3) + x/a
```

Mupad [B]
time = 4.79, size = 1097, normalized size = 9.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x/a - 2*atanh((24*x*(a^5*b^3)^(1/2)*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a^5*b^
3)^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b^2 - (6*(
a^5*b^3)^(1/2))/a - 6*a^2*b + 2*b^3 + (2*b^2*(a^5*b^3)^(1/2))/a^3 + (4*b*(a
^5*b^3)^(1/2))/a^2) + (8*x*(a^5*b^3)^(1/2)*(- 3/(16*a^2) - 1/(16*a*b) - (3*
(a^5*b^3)^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((4*(a^5
*b^3)^(1/2))/a - (6*(a^5*b^3)^(1/2))/b + 2*a*b^2 + 4*a^2*b - 6*a^3 + (2*b*(
a^5*b^3)^(1/2))/a^2) - (8*a*b^2*x*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a^5*b^3)
^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b + (4*(a^5*
b^3)^(1/2))/a^2 - 6*a^2 + 2*b^2 - (6*(a^5*b^3)^(1/2))/(a*b) + (2*b*(a^5*b^3
)^(1/2))/a^3) - (24*a^2*b*x*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a^5*b^3)^(1/2)
)/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b + (4*(a^5*b^3)^(
1/2))/a^2 - 6*a^2 + 2*b^2 - (6*(a^5*b^3)^(1/2))/(a*b) + (2*b*(a^5*b^3)^(1/2
))/a^3))*(-(3*a*(a^5*b^3)^(1/2) + b*(a^5*b^3)^(1/2) + a^4*b + 3*a^3*b^2)/(1
6*a^5*b^2))^(1/2) + 2*atanh((24*x*(a^5*b^3)^(1/2)*((3*(a^5*b^3)^(1/2))/(16*
a^4*b^2) - 1/(16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((6
*(a^5*b^3)^(1/2))/a + 4*a*b^2 - 6*a^2*b + 2*b^3 - (2*b^2*(a^5*b^3)^(1/2))/a
^3 - (4*b*(a^5*b^3)^(1/2))/a^2) - (8*x*(a^5*b^3)^(1/2)*((3*(a^5*b^3)^(1/2))
/(16*a^4*b^2) - 1/(16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2)
)/((4*(a^5*b^3)^(1/2))/a - (6*(a^5*b^3)^(1/2))/b - 2*a*b^2 - 4*a^2*b + 6*a^
3 + (2*b*(a^5*b^3)^(1/2))/a^2) + (8*a*b^2*x*((3*(a^5*b^3)^(1/2))/(16*a^4*b^
2) - 1/(16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b -
(4*(a^5*b^3)^(1/2))/a^2 - 6*a^2 + 2*b^2 + (6*(a^5*b^3)^(1/2))/(a*b) - (2*b*
(a^5*b^3)^(1/2))/a^3) + (24*a^2*b*x*((3*(a^5*b^3)^(1/2))/(16*a^4*b^2) - 1/(
16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b - (4*(a^5*
b^3)^(1/2))/a^2 - 6*a^2 + 2*b^2 + (6*(a^5*b^3)^(1/2))/(a*b) - (2*b*(a^5*b^3
)^(1/2))/a^3))*((3*a*(a^5*b^3)^(1/2) + b*(a^5*b^3)^(1/2) - a^4*b - 3*a^3*b^
2)/(16*a^5*b^2))^(1/2)
```

3.903 $\int \frac{x^2}{a-b+2ax^2+ax^4} \, dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] -1/2*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))*(a^(1/2)-b^(1/2))^(1/2)/a^(3
/4)/b^(1/2)+1/2*arctan(a^(1/4)*x/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+b^(1/2))
^(1/2)/a^(3/4)/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1144, 211}

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -1/2*(Sqrt[Sqrt[a] - Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]])/
(a^(3/4)*Sqrt[b]) + (Sqrt[Sqrt[a] + Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a
] + Sqrt[b]]])/(2*a^(3/4)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx = -\left(\frac{1}{2}\left(-1+\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx\right)+\frac{1}{2}\left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{a+\sqrt{a}\sqrt{b}$$

$$= -\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{}}\right.}{2a^{3/4}\sqrt{b}}$$

Mathematica [A]
time = 0.07, size = 128, normalized size = 1.17

$$\frac{-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-\sqrt{a}\sqrt{b}}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (-(((Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]])/Sqrt
[a - Sqrt[a]*Sqrt[b]]) + ((Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a + S
qrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.04, size = 96, normalized size = 0.88

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4+2a\_Z^2+a-b\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3+\_R}}{4a}$	43
default	$a\left(\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{ax}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{2a\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)a}}-\frac{\left(\sqrt{ab}-a\right)\mathrm{arctanh}\left(\frac{ax}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{2a\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)a}}\right)$	96

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)

[Out] a*(1/2*((a*b)^(1/2)+a)/a/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)+a)*a)^(1/2))-1/2*((a*b)^(1/2)-a)/a/(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")

[Out] integrate(x^2/(a*x^4 + 2*a*x^2 + a - b), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(69) = 138.
time = 0.36, size = 267, normalized size = 2.45

$$\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\log\left(a^2b\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)+\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\log\left(-a^2b\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")

[Out] 1/4*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*sqrt(1/(a^3*b)) + x) - 1/4*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*log(-a^2*b*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*sqrt(1/(a^3*b)) + x) - 1/4*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*log(a^2*b*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*sqrt(1/(a^3*b)) + x) + 1/4*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*log(-a^2*b*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*sqrt(1/(a^3*b)) + x)

Sympy [A]
time = 0.27, size = 44, normalized size = 0.40

$$\operatorname{RootSum}\left(256t^4a^3b^2+32t^2a^2b+a-b,\left(t\mapsto t\log\left(-64t^3a^2b-4ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a*x**4+2*a*x**2+a-b),x)

[Out] RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b + a - b, Lambda(_t, _t*log(-64*_t**3*a**2*b - 4*_t*a + x)))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(69) = 138.

time = 4.29, size = 199, normalized size = 1.83

$$-\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a-4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^4b-4a^3b^2\right)}-\frac{\left(3\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a-4\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a-\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^4b-4a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] -1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a - 4*sqrt(a^2 + sqrt(a*b)*a)*sqr
t(a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a - b)*a + 4*a^2
))/a))/(3*a^4*b - 4*a^3*b^2) - 1/2*(3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a -
 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2
*a - sqrt(-4*(a - b)*a + 4*a^2))/a))/(3*a^4*b - 4*a^3*b^2)
```

Mupad [B]
time = 0.30, size = 216, normalized size = 1.98

$$-2\,\mathrm{atanh}\left(\frac{2\left(x\left(4a^3+4ba^2\right)-\frac{4ax\left(\sqrt{a^3b^3}+a^2b\right)}{b}\right)\sqrt{-\frac{\sqrt{a^3b^3}+a^2b}{16a^3b^2}}}{2ab-2a^2}\right)\sqrt{-\frac{\sqrt{a^3b^3}+a^2b}{16a^3b^2}}-2\,\mathrm{atanh}\left(\frac{2\left(x\left(4a^3+4ba^2\right)+\frac{4ax\left(\sqrt{a^3b^3}-a^2b\right)}{b}\right)\sqrt{\frac{\sqrt{a^3b^3}-a^2b}{16a^3b^2}}}{2ab-2a^2}\right)\sqrt{\frac{\sqrt{a^3b^3}-a^2b}{16a^3b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] - 2*atanh((2*(x*(4*a^2*b + 4*a^3) - (4*a*x*((a^3*b^3)^(1/2) + a^2*b))/b)*(-
((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2))/(2*a*b - 2*a^2))*(-((a^3*b^3
)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2) - 2*atanh((2*(x*(4*a^2*b + 4*a^3) + (4
*a*x*((a^3*b^3)^(1/2) - a^2*b))/b)*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))
^(1/2))/(2*a*b - 2*a^2))*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)
```

3.904 $\int \frac{1}{a-b+2ax^2+ax^4} dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{\sqrt{a}-\sqrt{b}}\,\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{\sqrt{a}+\sqrt{b}}\,\sqrt{b}}$$

```
[Out] 1/2*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))/a^(1/4)/b^(1/2)/(a^(1/2)-b^(1
/2))^(1/2)-1/2*arctan(a^(1/4)*x/(a^(1/2)+b^(1/2))^(1/2))/a^(1/4)/b^(1/2)/(a
^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1107, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[
b]]*Sqrt[b]) - ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]]/(2*a^(1/4)*Sqrt[
Sqrt[a] + Sqrt[b]]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx = \frac{\sqrt{a}\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\sqrt{b}} - \frac{\sqrt{a}\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\sqrt{b}}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}}$$

Mathematica [A]
time = 0.04, size = 105, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a-\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a - Sqrt[a]*Sqrt[b]]*
Sqrt[b]) - ArcTan[(Sqrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a + Sqrt[a
]*Sqrt[b]]*Sqrt[b])
```

Maple [A]
time = 0.02, size = 74, normalized size = 0.68

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4+2a\_Z^2+a-b\right)} \frac{\ln\left(x-\_R\right)}{\_R^3+\_R}}{4a}$	40
default	$a\left(-\frac{\arctan\left(\frac{ax}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)a}} - \frac{\operatorname{arctanh}\left(\frac{ax}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{2\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)a}}\right)$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)+a)*a
)^(1/2))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2
)-a)*a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] integrate(1/(a*x^4 + 2*a*x^2 + a - b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(69) = 138$.
time = 0.35, size = 553, normalized size = 5.07

$$-\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}\log\left(\left(b-\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}+x\right)+\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}\log\left(-\left(b-\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}+x\right)-\frac{1}{4}\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}\log\left(\left(b+\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}+x\right)+\frac{1}{4}\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}\log\left(-\left(b+\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2))*l
og((b - (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(-((a*b - b^2)
/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2)) + x) + 1/4*sqrt(-((a*b -
 b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2))*log(-(b - (a^2*b -
a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a
^2*b^2 + a*b^3) + 1)/(a*b - b^2)) + x) - 1/4*sqrt(((a*b - b^2)/sqrt(a^3*b -
 2*a^2*b^2 + a*b^3) - 1)/(a*b - b^2))*log((b + (a^2*b - a*b^2)/sqrt(a^3*b -
 2*a^2*b^2 + a*b^3))*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) - 1)
/(a*b - b^2)) + x) + 1/4*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3)
- 1)/(a*b - b^2))*log(-(b + (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3)
)*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) - 1)/(a*b - b^2)) + x)
```

Sympy [A]
time = 0.47, size = 63, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^2b^2-256ab^3\right)+32t^2ab+1,\left(t\mapsto t\log\left(-64t^3a^2b+64t^3ab^2-4ta-4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 - 256*a*b**3) + 32*_t**2*a*b + 1, Lambda(_t, _
t*log(-64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a - 4*_t*b + x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(69) = 138$.
time = 5.74, size = 299, normalized size = 2.74

$$\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,a^2b-4\sqrt{a^2+\sqrt{ab}\,a}\,ab^2-3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^2+4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,ab\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^5b-7a^4b^2+4a^3b^3\right)}+\frac{\left(3\sqrt{a^2-\sqrt{ab}\,a}\,a^2b-4\sqrt{a^2-\sqrt{ab}\,a}\,ab^2+3\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a^2-4\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,ab\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a-\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^5b-7a^4b^2+4a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^2*b - 4*sqrt(a^2 + sqrt(a*b)*a)*a*b^2 - 3*
sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2 + 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)
*a*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a - b)*a + 4*a^2))/a
))/(3*a^5*b - 7*a^4*b^2 + 4*a^3*b^3) + 1/2*(3*sqrt(a^2 - sqrt(a*b)*a)*a^2*b
 - 4*sqrt(a^2 - sqrt(a*b)*a)*a*b^2 + 3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^
2 - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a*b)*abs(a)*arctan(2*sqrt(1/2)*x/sq
rt((2*a - sqrt(-4*(a - b)*a + 4*a^2))/a))/(3*a^5*b - 7*a^4*b^2 + 4*a^3*b^3)
```

Mupad [B]
time = 5.78, size = 322, normalized size = 2.95

$$\frac{\ln\left(4a^3b\sqrt{-\frac{1}{ab+\sqrt{ab^3}}}-4a^3x+\frac{4a^4bx}{ab+\sqrt{ab^3}}\right)\sqrt{-\frac{1}{ab+\sqrt{ab^3}}}}{4}+\frac{\ln\left(4a^3x-4a^3b\sqrt{-\frac{1}{ab-\sqrt{ab^3}}}-\frac{4a^4bx}{ab-\sqrt{ab^3}}\right)\sqrt{-\frac{1}{ab-\sqrt{ab^3}}}}{4}-\ln\left(4a^3x+4a^3b\sqrt{-\frac{1}{ab+\sqrt{ab^3}}}-\frac{4a^4bx}{ab+\sqrt{ab^3}}\right)\sqrt{\frac{ab-\sqrt{ab^3}}{16\left(ab^3-a^2b^2\right)}}-\ln\left(4a^3x+16a^3b\sqrt{-\frac{1}{16ab-16\sqrt{ab^3}}}-\frac{4a^4bx}{ab-\sqrt{ab^3}}\right)\sqrt{\frac{ab+\sqrt{ab^3}}{16\left(ab^3-a^2b^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b + 2*a*x^2 + a*x^4),x)
```

```
[Out] (log(4*a^3*b*(-1/(a*b + (a*b^3)^(1/2)))^(1/2) - 4*a^3*x + (4*a^4*b*x)/(a*b
+ (a*b^3)^(1/2)))*(-1/(a*b + (a*b^3)^(1/2)))^(1/2))/4 + (log(4*a^3*x - 4*a^
3*b*(-1/(a*b - (a*b^3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b - (a*b^3)^(1/2)))*(
-1/(a*b - (a*b^3)^(1/2)))^(1/2))/4 - log(4*a^3*x + 4*a^3*b*(-1/(a*b + (a*b^
3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b + (a*b^3)^(1/2)))*((a*b - (a*b^3)^(1/2)
)/(16*(a*b^3 - a^2*b^2)))^(1/2) - log(4*a^3*x + 16*a^3*b*(-1/(16*a*b - 16*(
a*b^3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b - (a*b^3)^(1/2)))*((a*b + (a*b^3)^(
1/2))/(16*(a*b^3 - a^2*b^2)))^(1/2)
```

3.905 $\int \frac{1}{x^2(a-b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=121

$$-\frac{1}{(a-b)x}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}}$$

```
[Out] -1/(a-b)/x-1/2*a^(1/4)*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))/(a^(1/2)-b
^(1/2))^(3/2)/b^(1/2)+1/2*a^(1/4)*arctan(a^(1/4)*x/(a^(1/2)+b^(1/2))^(1/2))
/b^(1/2)/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1137, 1180, 211}

$$-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{1}{x(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(1/((a - b)*x)) - (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]])/(2
*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]) + (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt
[a] + Sqrt[b]]])/(2*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a-b+2ax^2+ax^4\right)}\,dx &= -\frac{1}{(a-b)x}-\frac{\int \frac{-2a-ax^2}{a-b+2ax^2+ax^4}\,dx}{-a+b}\\ &= -\frac{1}{(a-b)x}-\frac{a\int \frac{1}{a-\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\left(\sqrt{a}-\sqrt{b}\right)\sqrt{b}}+\frac{a\int \frac{1}{a+\sqrt{a}\sqrt{b}+ax^2}\,dx}{2\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}}\\ &= -\frac{1}{(a-b)x}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 143, normalized size = 1.18

$$\frac{\frac{2}{x}+\frac{\left(a+\sqrt{a}\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(a-\sqrt{a}\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}}{2(-a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (2/x + ((a + Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]]
)/(Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt
[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]))/(2*
(-a + b))
```

Maple [A]
time = 0.03, size = 119, normalized size = 0.98

method	result

default	$-\frac{a^{2}\left(\frac{\left(\sqrt{ab}-a\right)\arctan\left(\frac{ax}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{2a\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)a}}-\frac{\left(\sqrt{ab}+a\right)\operatorname{arctanh}\left(\frac{ax}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{2a\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{a-b}-\frac{1}{(a-b)x}$
risch	$-\frac{1}{(a-b)x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(b^{2}a^{3}-3a^{2}b^{3}+3b^{4}a-b^{5}\right)\_Z^{4}+\left(2a^{2}b+6a\,b^{2}\right)\_Z^{2}+a\right)}\_R\ln\left(\left(\left(b\,a^{4}+2b^{2}a^{3}-12a^{2}b^{3}+14b^{4}a-5b^{5}\right)\_R^{4}+\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a-b)*a^2*(1/2*((a*b)^(1/2)-a)/a/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*a
rctan(a*x/(((a*b)^(1/2)+a)*a)^(1/2))-1/2*((a*b)^(1/2)+a)/a/(a*b)^(1/2)/(((a
*b)^(1/2)-a)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2)))-1/(a-b)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

```
[Out] -a*integrate((x^2 + 2)/(a*x^4 + 2*a*x^2 + a - b), x)/(a - b) - 1/((a - b)*x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1612 vs. 2(81) = 162.
time = 0.36, size = 1612, normalized size = 13.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

```
[Out] 1/4*((a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*sqr
t((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 +
15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*log((3*a
^2 + a*b)*x + (6*a^2*b + 2*a*b^2 - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*sqrt
```

```
((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 1
5*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*
b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 -
 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 -
b^4))) - (a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)
*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^
4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*log(
(3*a^2 + a*b)*x - (6*a^2*b + 2*a*b^2 - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*
sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4
 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b
^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^
3 - b^4))) + (a - b)*x*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^3 -
b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^
3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*
log((3*a^2 + a*b)*x + (6*a^2*b + 2*a*b^2 + (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b
^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3
*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^
2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a
^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*
a*b^3 - b^4))) - (a - b)*x*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^
3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 2
0*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^
4))*log((3*a^2 + a*b)*x - (6*a^2*b + 2*a*b^2 + (a^4*b - 2*a^3*b^2 + 2*a*b^4
 - b^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20
*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^
2*b^2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 +
15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2
+ 3*a*b^3 - b^4))) - 4)/((a - b)*x)
```

Sympy [**A**]
time = 2.17, size = 134, normalized size = 1.11

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{3}b^{2}-768a^{2}b^{3}+768ab^{4}-256b^{5}\right)+t^{2}\cdot\left(32a^{2}b+96ab^{2}\right)+a,\left(t\mapsto t\log\left(x+\frac{64t^{3}a^{4}b-128t^{3}a^{3}b^{2}+128t^{3}ab^{4}-64t^{3}b^{5}+4ta^{3}+40ta^{2}b+20tab^{2}}{3a^{2}+ab}\right)\right)\right)-\frac{1}{x\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(_t**4*(256*a**3*b**2 - 768*a**2*b**3 + 768*a*b**4 - 256*b**5) + _t*
*2*(32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x + (64*_t**3*a**4*b - 12
8*_t**3*a**3*b**2 + 128*_t**3*a*b**4 - 64*_t**3*b**5 + 4*_t*a**3 + 40*_t*a*
*2*b + 20*_t*a*b**2)/(3*a**2 + a*b)))) - 1/(x*(a - b))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(81) = 162$.

time = 7.88, size = 698, normalized size = 5.77

$$\frac{\left(\left(3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,ab-4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,b^2\right)(a-b)^2|a|-2\left(3\sqrt{a^2+\sqrt{ab}\,a}\,a^3b-7\sqrt{a^2+\sqrt{ab}\,a}\,a^2b^2+4\sqrt{a^2+\sqrt{ab}\,a}\,ab^3\right)|a-b||a|+\left(3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^4-10\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^3b+11\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a^2b^2-4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,ab^3\right)|a|\right)\arctan\left(\frac{x}{\sqrt{\frac{a^2-ab+\sqrt{(a^2-ab)^2-(a^2-ab)(a^2-2ab+b^2)}}{a^2-ab}}}\right)}{2\left(3a^6b-13a^5b^2+21a^4b^3-15a^3b^4+4a^2b^5\right)|a-b|}-\frac{\left(\left(3\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,ab-4\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,b^2\right)(a-b)^2|a|+2\left(3\sqrt{a^2-\sqrt{ab}\,a}\,a^3b-7\sqrt{a^2-\sqrt{ab}\,a}\,a^2b^2+4\sqrt{a^2-\sqrt{ab}\,a}\,ab^3\right)|a-b||a|+\left(3\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a^4-10\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a^3b+11\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a^2b^2-4\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,ab^3\right)|a|\right)\arctan\left(\frac{x}{\sqrt{\frac{a^2-ab-\sqrt{(a^2-ab)^2-(a^2-ab)(a^2-2ab+b^2)}}{a^2-ab}}}\right)}{2\left(3a^6b-13a^5b^2+21a^4b^3-15a^3b^4+4a^2b^5\right)|a-b|}-\frac{1}{(a-b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

```
[Out] 1/2*((3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 + sqrt(a*b)*a)*s
qrt(a*b)*b^2)*(a - b)^2*abs(a) - 2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^3*b - 7*sqr
t(a^2 + sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2 + sqrt(a*b)*a)*a*b^3)*abs(a - b)*
abs(a) + (3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^4 - 10*sqrt(a^2 + sqrt(a*b)
*a)*sqrt(a*b)*a^3*b + 11*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 4*sqrt
(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - a*b + sqr
t((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - a*b)))/((3*a^6*b
 - 13*a^5*b^2 + 21*a^4*b^3 - 15*a^3*b^4 + 4*a^2*b^5)*abs(a - b)) - 1/2*((3*
sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)
*b^2)*(a - b)^2*abs(a) + 2*(3*sqrt(a^2 - sqrt(a*b)*a)*a^3*b - 7*sqrt(a^2 -
sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(a*b)*a)*a*b^3)*abs(a - b)*abs(a) +
 (3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^4 - 10*sqrt(a^2 - sqrt(a*b)*a)*sqrt
(a*b)*a^3*b + 11*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 4*sqrt(a^2 - s
qrt(a*b)*a)*sqrt(a*b)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - a*b - sqrt((a^2 -
 a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - a*b)))/((3*a^6*b - 13*a^
5*b^2 + 21*a^4*b^3 - 15*a^3*b^4 + 4*a^2*b^5)*abs(a - b)) - 1/((a - b)*x)
```

Mupad [B]
time = 5.12, size = 2774, normalized size = 22.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] atan(((x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) + (-(3*a*b^2 + a^2*b + 3
*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^
2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2
 - x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4
 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5
- 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b + 3*a*(a*b^
3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/
2)*1i + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) - (-(3*a*b^2 + a^2*b +
 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*
b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b
^2 + x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b
^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^
5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b + 3*a*(a*
```

b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i)/(6*a^6*b - 2*a^7 + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) + (-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2 - x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2) - (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) - (-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2 + x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b^3 - 6*a^5*b^2))*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*2i - 1/(x*(a - b)) + atan(((x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) + (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2 - x*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) - (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2 + x*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i)/(6*a^6*b - 2*a^7 + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) + (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2 - x*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2) - (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) - (-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2 + x*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2

```
))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b^3 - 6*a^5*b^
2))*(-(3*a*b^2 + a^2*b - 3*a*(a*b^3)^(1/2) - b*(a*b^3)^(1/2))/(16*(3*a*b^4
- b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*2i
```

3.906 $\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=69

$$\frac{x^2}{2a}+\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{2a}$$

```
[Out] 1/2*x^2/a-1/2*ln(a*x^4+2*a*x^2+a+b)/a+1/2*(a-b)*arctan((x^2+1)*a^(1/2)/b^(1
/2))/a^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1128, 717, 648, 632, 210, 642}

$$\frac{(a-b)\text{ArcTan}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{2a}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x^2/(2*a) + ((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[b
]) - Log[a + b + 2*a*x^2 + a*x^4]/(2*a)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+b+2ax+ax^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2a}+\frac{\text{Subst}\left(\int \frac{-a-b-2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\text{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2a}+\frac{(a-b)\text{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{x^2}{2a}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{2a}-\frac{(a-b)\text{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx,x,2a(1+x^2)\right)}{a}\\
&= \frac{x^2}{2a}+\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{2a}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.90

$$
\frac{\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{\sqrt{b}}+\sqrt{a}\left(x^2-\log\left(b+a(1+x^2)^2\right)\right)}{2a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Sqrt[a]*(x^2 - Log
[b + a*(1 + x^2)^2]))/(2*a^(3/2))
```

Maple [A]
time = 0.03, size = 62, normalized size = 0.90

method	result
default	$\frac{x^2}{2a}+\frac{-\ln\left(a\,x^4+2a\,x^2+a+b\right)+\frac{(a-b)\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}}{2a}$
risch	$\frac{x^2}{2a}-\frac{\ln\left(\left(a^2b-a\,b^2+\sqrt{-ab\,(a-b)^2}\,a\right)x^2+\sqrt{-ab\,(a-b)^2}\,a+\sqrt{-ab\,(a-b)^2}\,b\right)}{2a}+\frac{\ln\left(\left(a^2b-a\,b^2+\sqrt{-a\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/a+1/2/a*(-ln(a*x^4+2*a*x^2+a+b)+(a-b)/(a*b)^(1/2)*arctan(1/2*(2*a*x
^2+2*a)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 58, normalized size = 0.84

$$\frac{x^2}{2\,a}+\frac{(a-b)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/a + 1/2*(a - b)*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*l
og(a*x^4 + 2*a*x^2 + a + b)/a
```

Fricas [A]
time = 0.36, size = 157, normalized size = 2.28

$$\left[\frac{2\,abx^2-2\,ab\log\left(ax^4+2\,ax^2+a+b\right)+\sqrt{-ab}\,(a-b)\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{-ab}\,(x^2+1)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,a^2b},\ \frac{abx^2-ab\log\left(ax^4+2\,ax^2+a+b\right)-\sqrt{ab}\,(a-b)\arctan\left(\frac{\sqrt{ab}}{ax^2+a}\right)}{2\,a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x^2 - 2*a*b*log(a*x^4 + 2*a*x^2 + a + b) + sqrt(-a*b)*(a - b)*l
og((a*x^4 + 2*a*x^2 + 2*sqrt(-a*b)*(x^2 + 1) + a - b)/(a*x^4 + 2*a*x^2 + a
```

```
+ b)))/(a^2*b), 1/2*(a*b*x^2 - a*b*log(a*x^4 + 2*a*x^2 + a + b) - sqrt(a*b)
*(a - b)*arctan(sqrt(a*b)/(a*x^2 + a)))/(a^2*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(60) = 120$.

time = 0.64, size = 144, normalized size = 2.09

$$\left(-\frac{1}{2a} - \frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right)\log\left(x^2 + \frac{4ab\left(-\frac{1}{2a} - \frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right) + a + b}{a-b}\right) + \left(-\frac{1}{2a} + \frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right)\log\left(x^2 + \frac{4ab\left(-\frac{1}{2a} + \frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right) + a + b}{a-b}\right) + \frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (-1/(2*a) - sqrt(-a**3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(-1/(2*a) -
 sqrt(-a**3*b)*(a - b)/(4*a**3*b)) + a + b)/(a - b)) + (-1/(2*a) + sqrt(-a*
*3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(-1/(2*a) + sqrt(-a**3*b)*(a -
b)/(4*a**3*b)) + a + b)/(a - b)) + x**2/(2*a)
```

Giac [A]

time = 6.55, size = 58, normalized size = 0.84

$$\frac{x^2}{2\,a} + \frac{(a-b)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a} - \frac{\log\left(ax^4 + 2\,ax^2 + a + b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a + 1/2*(a - b)*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*l
og(a*x^4 + 2*a*x^2 + a + b)/a
```

Mupad [B]

time = 0.18, size = 302, normalized size = 4.38

$$\frac{x^2}{2\,a} - \frac{\ln\left(a\,x^4+2\,a\,x^2+a+b\right)}{2\,a} - \frac{\mathrm{atan}\left(\frac{a\,b\,\left(x^2\,\left(\frac{\frac{\sqrt{a}\,\left(2\,a-2\,b\right)}{\sqrt{b}}+\frac{\left(a-b\right)\,\left(4\,a\,b-12\,a^2\right)}{4\,a^{3/2}\,\sqrt{b}}}{a+b}+\frac{\sqrt{a}\,\left(6\,a-2\,b-\frac{(a-b)^2}{b}+\frac{2\,a\,b-6\,a^2}{a}\right)}{\sqrt{b}\,\left(a+b\right)}\right)-\frac{\frac{\left(a-b\right)\,\left(16\,a\,b-\frac{8\,a^3+8\,b\,a^2}{a}+16\,a^2\right)}{4\,a^{3/2}\,\sqrt{b}}-\frac{\left(16\,a^3+16\,b\,a^2\right)\,\left(a-b\right)}{8\,a^{5/2}\,\sqrt{b}}}{a+b}+\frac{\sqrt{a}\,\left(4\,a+4\,b-\frac{8\,a\,b-\frac{8\,a^3+8\,b\,a^2}{2\,a}+8\,a^2}{a}-\frac{(a-b)^2\,\left(a^3+b\,a^2\right)}{a^3\,b}\right)}{\sqrt{b}\,\left(a+b\right)}\right)}{a^2-2\,a\,b+b^2}\right)\,\left(a-b\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x^2/(2*a) - log(a + b + 2*a*x^2 + a*x^4)/(2*a) - (atan((a*b*(x^2*(((a^(1/2)
*(2*a - 2*b))/b^(1/2) + ((a - b)*(4*a*b - 12*a^2))/(4*a^(3/2)*b^(1/2)))/(a
+ b) + (a^(1/2)*(6*a - 2*b - (a - b)^2/b + (2*a*b - 6*a^2)/a))/(b^(1/2)*(a
+ b))) - (((a - b)*(16*a*b - (8*a^2*b + 8*a^3)/a + 16*a^2))/(4*a^(3/2)*b^(1
/2)) - ((16*a^2*b + 16*a^3)*(a - b))/(8*a^(5/2)*b^(1/2)))/(a + b) + (a^(1/2
)*(4*a + 4*b - (8*a*b - (8*a^2*b + 8*a^3)/(2*a) + 8*a^2)/a - ((a - b)^2*(a^
2*b + a^3))/(a^3*b)))/(b^(1/2)*(a + b))))/(a^2 - 2*a*b + b^2))*(a - b))/(2*
a^(3/2)*b^(1/2))
```

3.907 $\int \frac{x^3}{a+b+2ax^2+ax^4} \, dx$

Optimal. Leaf size=54

$$-\frac{\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}+\frac{\log\left(a+b+2ax^2+ax^4\right)}{4a}$$

```
[Out] 1/4*ln(a*x^4+2*a*x^2+a+b)/a-1/2*arctan((x^2+1)*a^(1/2)/b^(1/2))/a^(1/2)/b^(
1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1128, 648, 632, 210, 642}

$$\frac{\log\left(ax^4+2ax^2+a+b\right)}{4a}-\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]) + Log[a + b + 2*
a*x^2 + a*x^4]/(4*a)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+b+2ax+ax^2}\,dx, x, x^2\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx, x, x^2\right)\right) + \frac{\text{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx, x, x^2\right.}{4a} \\
&= \frac{\log\left(a+b+2ax^2+ax^4\right)}{4a} + \text{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx, x, 2a\left(1+x^2\right)\right) \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}} + \frac{\log\left(a+b+2ax^2+ax^4\right)}{4a}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.91

$$\frac{-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{\sqrt{b}} + \log\left(b+a(1+x^2)^2\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ((-2*Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[b + a*(1 +
x^2)^2])/(4*a)
```

Maple [A]
time = 0.02, size = 47, normalized size = 0.87

method	result

default	$\frac{\ln(a\,x^4+2a\,x^2+a+b)}{4a}-\frac{\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$
risch	$\frac{\ln\left(\left(a\sqrt{-ab}-ab\right)x^2+a\sqrt{-ab}+\sqrt{-ab}\,b\right)}{4a}+\frac{\ln\left(\left(a\sqrt{-ab}-ab\right)x^2+a\sqrt{-ab}+\sqrt{-ab}\,b\right)\sqrt{-ab}}{4ba}+\frac{\ln\left(\left(-a\sqrt{\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(a*x^4+2*a*x^2+a+b)/a-1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^
(1/2))
```

Maxima [A]
time = 0.49, size = 42, normalized size = 0.78

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}}+\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b) + 1/4*log(a*x^4 + 2*a*x^2 + a
+ b)/a
```

Fricas [A]
time = 0.38, size = 131, normalized size = 2.43

$$\left[\frac{b\log\left(ax^4+2\,ax^2+a+b\right)-\sqrt{-ab}\,\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{-ab}\,(x^2+1)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,ab},\ \frac{b\log\left(ax^4+2\,ax^2+a+b\right)+2\,\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{ax^2+a}\right)}{4\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log(a*x^4 + 2*a*x^2 + a + b) - sqrt(-a*b)*log((a*x^4 + 2*a*x^2 + 2*
sqrt(-a*b)*(x^2 + 1) + a - b)/(a*x^4 + 2*a*x^2 + a + b)))/(a*b), 1/4*(b*log
(a*x^4 + 2*a*x^2 + a + b) + 2*sqrt(a*b)*arctan(sqrt(a*b)/(a*x^2 + a)))/(a*b
)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(48) = 96.
time = 0.31, size = 117, normalized size = 2.17

$$\left(\frac{1}{4a}-\frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2+\frac{-4ab\left(\frac{1}{4a}-\frac{\sqrt{-a^3b}}{4a^2b}\right)+a+b}{a}\right)+\left(\frac{1}{4a}+\frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2+\frac{-4ab\left(\frac{1}{4a}+\frac{\sqrt{-a^3b}}{4a^2b}\right)+a+b}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (1/(4*a) - sqrt(-a**3*b)/(4*a**2*b))*log(x**2 + (-4*a*b*(1/(4*a) - sqrt(-a*
*3*b)/(4*a**2*b)) + a + b)/a) + (1/(4*a) + sqrt(-a**3*b)/(4*a**2*b))*log(x*
*2 + (-4*a*b*(1/(4*a) + sqrt(-a**3*b)/(4*a**2*b)) + a + b)/a)
```

Giac [A]
time = 5.19, size = 42, normalized size = 0.78

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}}+\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b) + 1/4*log(a*x^4 + 2*a*x^2 + a
+ b)/a
```

Mupad [B]
time = 0.09, size = 85, normalized size = 1.57

$$\frac{\ln\left(a\,x^4+2\,a\,x^2+a+b\right)}{4\,a}-\frac{\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}}{a+b}+\frac{a^{3/2}}{\sqrt{b}\,\left(a+b\right)}+\frac{\sqrt{a}\,\sqrt{b}\,x^2}{a+b}+\frac{a^{3/2}\,x^2}{\sqrt{b}\,\left(a+b\right)}\right)}{2\,\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] log(a + b + 2*a*x^2 + a*x^4)/(4*a) - atan((a^(1/2)*b^(1/2))/(a + b) + a^(3/
2)/(b^(1/2)*(a + b)) + (a^(1/2)*b^(1/2)*x^2)/(a + b) + (a^(3/2)*x^2)/(b^(1/
2)*(a + b)))/(2*a^(1/2)*b^(1/2))
```

3.908 $\int \frac{x}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

```
[Out] 1/2*arctan((x^2+1)*a^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1121, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{a+b+2ax^2+ax^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx, x, x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx, x, 2a(1+x^2)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b + 2*a*x^2 + a*x^4),x]
[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.84

method	result	size
default	$\frac{\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$	26
risch	$-\frac{\ln\left(\left(\sqrt{-ab}-a\right)x^2-a-b\right)}{4\sqrt{-ab}}+\frac{\ln\left(\left(\sqrt{-ab}+a\right)x^2+a+b\right)}{4\sqrt{-ab}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
[Out] 1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 21, normalized size = 0.68

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 0.35, size = 91, normalized size = 2.94

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{-ab}\,(x^2+1)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,ab}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{ax^2+a}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b)*log((a*x^4 + 2*a*x^2 - 2*sqrt(-a*b)*(x^2 + 1) + a - b)/(a*
x^4 + 2*a*x^2 + a + b))/(a*b), -1/2*sqrt(a*b)*arctan(sqrt(a*b)/(a*x^2 + a))
/(a*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.
time = 0.15, size = 60, normalized size = 1.94

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4}+\frac{\sqrt{-\frac{1}{ab}}\log\left(b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-b*sqrt(-1/(a*b)) + x**2 + 1)/4 + sqrt(-1/(a*b))*log(b*
sqrt(-1/(a*b)) + x**2 + 1)/4
```

Giac [A]
time = 2.56, size = 21, normalized size = 0.68

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b)
```

Mupad [B]
time = 0.05, size = 24, normalized size = 0.77

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a}+\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\,\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b + 2*a*x^2 + a*x^4),x)
[Out] atan((a^(1/2) + a^(1/2)*x^2)/b^(1/2))/(2*a^(1/2)*b^(1/2))
```

3.909 $\int \frac{1}{x(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)}+\frac{\log(x)}{a+b}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{4(a+b)}$$

```
[Out] ln(x)/(a+b)-1/4*ln(a*x^4+2*a*x^2+a+b)/(a+b)-1/2*arctan((x^2+1)*a^(1/2)/b^(1
/2))*a^(1/2)/(a+b)/b^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1128, 719, 29, 648, 632, 210, 642}

$$-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{4(a+b)}+\frac{\log(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(Sqrt[b]*(a + b)) + Log[
x]/(a + b) - Log[a + b + 2*a*x^2 + a*x^4]/(4*(a + b))
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+b+2ax^2+ax^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+b+2ax+ax^2\right)}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2(a+b)}+\frac{\text{Subst}\left(\int \frac{-2a-ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2(a+b)}\\
&= \frac{\log(x)}{a+b}-\frac{\text{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{4(a+b)}-\frac{a\text{Subst}\left(\int \frac{1}{a+b+2ax+ax^2}\,dx,x,x^2\right)}{2(a+b)}\\
&= \frac{\log(x)}{a+b}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{4(a+b)}+\frac{a\text{Subst}\left(\int \frac{1}{-4ab-x^2}\,dx,x,2a(1+x^2)\right)}{a+b}\\
&= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)}+\frac{\log(x)}{a+b}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{4(a+b)}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 105, normalized size = 1.52

$$
\frac{4\sqrt{b}\,\log(x)+i\left(\sqrt{a}+i\sqrt{b}\right)\log\left(-i\sqrt{b}+\sqrt{a}\,(1+x^2)\right)+\left(-i\sqrt{a}-\sqrt{b}\right)\log\left(i\sqrt{b}+\sqrt{a}\,(1+x^2)\right)}{4\sqrt{b}\,(a+b)}
$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]

[Out] (4*Sqrt[b]*Log[x] + I*(Sqrt[a] + I*Sqrt[b])*Log[(-I)*Sqrt[b] + Sqrt[a]*(1 + x^2)] + ((-I)*Sqrt[a] - Sqrt[b])*Log[I*Sqrt[b] + Sqrt[a]*(1 + x^2)])/(4*Sqrt[b]*(a + b))

Maple [A]
time = 0.02, size = 63, normalized size = 0.91

method	result	size
risch	$\frac{\ln(x)}{a+b}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(ab+b^2\right)\_Z^2+2b\_Z\right)}\_R\ln\left(\left((-a+5b)\_R+5\right)x^2+(-a-b)\_R+4\right)\right)}{4}$	62
default	$-\frac{a\left(\frac{\ln\left(a\,x^4+2a\,x^2+a+b\right)}{2a}+\frac{\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{2(a+b)}+\frac{\ln(x)}{a+b}$	63

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)

[Out] -1/2*a/(a+b)*(1/2*ln(a*x^4+2*a*x^2+a+b)/a+1/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2)))+ln(x)/(a+b)

Maxima [A]
time = 0.48, size = 61, normalized size = 0.88

$$-\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(a+b)}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,(a+b)}+\frac{\log\left(x^2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")

[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/4*log(a*x^4 + 2*a*x^2 + a + b)/(a + b) + 1/2*log(x^2)/(a + b)

Fricas [A]
time = 0.36, size = 147, normalized size = 2.13

$$\left[\frac{\sqrt{-\frac{a}{b}}\,\log\left(\frac{ax^4+2\,ax^2-2\,(bx^2+b)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-\log\left(ax^4+2\,ax^2+a+b\right)+4\,\log\left(x\right)}{4\,(a+b)},\ \frac{2\,\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{ax^2+a}\right)-\log\left(ax^4+2\,ax^2+a+b\right)+4\,\log\left(x\right)}{4\,(a+b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(-a/b) + a - b)/(
a*x^4 + 2*a*x^2 + a + b)) - log(a*x^4 + 2*a*x^2 + a + b) + 4*log(x))/(a + b
), 1/4*(2*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*x^2 + a)) - log(a*x^4 + 2*a*x^2 +
 a + b) + 4*log(x))/(a + b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(61) = 122$.

time = 2.52, size = 194, normalized size = 2.81

$$\left(-\frac{1}{4(a+b)} - \frac{\sqrt{-ab}}{4b(a+b)}\right)\log\left(x^2 + \frac{-4ab\left(-\frac{1}{4(a+b)} - \frac{\sqrt{-ab}}{4b(a+b)}\right) + a - 4b^2\left(-\frac{1}{4(a+b)} - \frac{\sqrt{-ab}}{4b(a+b)}\right) - b}{a}\right) + \left(-\frac{1}{4(a+b)} + \frac{\sqrt{-ab}}{4b(a+b)}\right)\log\left(x^2 + \frac{-4ab\left(-\frac{1}{4(a+b)} + \frac{\sqrt{-ab}}{4b(a+b)}\right) + a - 4b^2\left(-\frac{1}{4(a+b)} + \frac{\sqrt{-ab}}{4b(a+b)}\right) - b}{a}\right) + \frac{\log(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (-1/(4*(a + b)) - sqrt(-a*b)/(4*b*(a + b)))*log(x**2 + (-4*a*b*(-1/(4*(a +
b)) - sqrt(-a*b)/(4*b*(a + b))) + a - 4*b**2*(-1/(4*(a + b)) - sqrt(-a*b)/(
4*b*(a + b))) - b)/a) + (-1/(4*(a + b)) + sqrt(-a*b)/(4*b*(a + b)))*log(x**
2 + (-4*a*b*(-1/(4*(a + b)) + sqrt(-a*b)/(4*b*(a + b))) + a - 4*b**2*(-1/(4
*(a + b)) + sqrt(-a*b)/(4*b*(a + b))) - b)/a) + log(x)/(a + b)
```

Giac [A]

time = 2.55, size = 61, normalized size = 0.88

$$-\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(a+b)} - \frac{\log\left(ax^4 + 2\,ax^2 + a + b\right)}{4\,(a+b)} + \frac{\log\left(x^2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/4*log(a*x^4 +
2*a*x^2 + a + b)/(a + b) + 1/2*log(x^2)/(a + b)
```

Mupad [B]

time = 4.64, size = 71, normalized size = 1.03

$$\frac{\ln(x)}{a+b} - \frac{4\,b\,\ln\left(a\,x^4 + 2\,a\,x^2 + a + b\right)}{16\,b^2 + 16\,a\,b} - \frac{\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\,\sqrt{b}\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] log(x)/(a + b) - (4*b*log(a + b + 2*a*x^2 + a*x^4))/(16*a*b + 16*b^2) - (a^
(1/2)*atan(a^(1/2)/b^(1/2) + (a^(1/2)*x^2)/b^(1/2)))/(2*b^(1/2)*(a + b))
```

3.910 $\int \frac{1}{x^3(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2(a+b)x^2}+\frac{\sqrt{a}\,(a-b)\tan^{-1}\left(\frac{\sqrt{a}\,(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)^2}-\frac{2a\log(x)}{(a+b)^2}+\frac{a\log\left(a+b+2ax^2+ax^4\right)}{2(a+b)^2}$$

```
[Out] -1/2/(a+b)/x^2-2*a*ln(x)/(a+b)^2+1/2*a*ln(a*x^4+2*a*x^2+a+b)/(a+b)^2+1/2*(a
-b)*arctan((x^2+1)*a^(1/2)/b^(1/2))*a^(1/2)/(a+b)^2/b^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1128, 723, 814, 648, 632, 210, 642}

$$\frac{\sqrt{a}\,(a-b)\text{ArcTan}\left(\frac{\sqrt{a}\,(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)^2}-\frac{1}{2x^2(a+b)}+\frac{a\log\left(ax^4+2ax^2+a+b\right)}{2(a+b)^2}-\frac{2a\log(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/2*1/((a + b)*x^2) + (Sqrt[a]*(a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]
)/(2*Sqrt[b]*(a + b)^2) - (2*a*Log[x])/(a + b)^2 + (a*Log[a + b + 2*a*x^2 +
 a*x^4])/(2*(a + b)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3(a+b+2ax^2+ax^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+b+2ax+ax^2)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2(a+b)x^2} + \frac{\text{Subst}\left(\int \frac{-2a-ax}{x(a+b+2ax+ax^2)}\,dx, x, x^2\right)}{2(a+b)}$$
$$= -\frac{1}{2(a+b)x^2} + \frac{\text{Subst}\left(\int \left(-\frac{2a}{(a+b)x} + \frac{a(3a-b+2ax)}{(a+b)(a+b+2ax+ax^2)}\right)\,dx, x, x^2\right)}{2(a+b)}$$
$$= -\frac{1}{2(a+b)x^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\text{Subst}\left(\int \frac{3a-b+2ax}{a+b+2ax+ax^2}\,dx, x, x^2\right)}{2(a+b)^2}$$
$$= -\frac{1}{2(a+b)x^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\text{Subst}\left(\int \frac{2a+2ax}{a+b+2ax+ax^2}\,dx, x, x^2\right)}{2(a+b)^2} + \frac{(a(a-b))\text{Su}}{}$$
$$= -\frac{1}{2(a+b)x^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\log(a+b+2ax^2+ax^4)}{2(a+b)^2} - \frac{(a(a-b))\text{Subst}(\int}{}$$
$$= -\frac{1}{2(a+b)x^2} + \frac{\sqrt{a}(a-b)\tan^{-1}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\log(a+b}{2(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 163, normalized size = 1.83

$$-\frac{1}{2(a+b)x^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{\left(-ia^2+2a^{3/2}\sqrt{b}+iab\right)\log\left(\sqrt{a}-i\sqrt{b}+\sqrt{a}\,x^2\right)}{4\sqrt{a}\sqrt{b}(a+b)^2} + \frac{\left(ia^2+2a^{3/2}\sqrt{b}-iab\right)\log\left(\sqrt{a}+i\sqrt{b}+\sqrt{a}\,x^2\right)}{4\sqrt{a}\sqrt{b}(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
[Out] -1/2*1/((a + b)*x^2) - (2*a*Log[x])/(a + b)^2 + (((-I)*a^2 + 2*a^(3/2)*Sqrt
[b] + I*a*b)*Log[Sqrt[a] - I*Sqrt[b] + Sqrt[a]*x^2])/(4*Sqrt[a]*Sqrt[b]*(a
+ b)^2) + ((I*a^2 + 2*a^(3/2)*Sqrt[b] - I*a*b)*Log[Sqrt[a] + I*Sqrt[b] + Sq
rt[a]*x^2])/(4*Sqrt[a]*Sqrt[b]*(a + b)^2)
```

Maple [A]
time = 0.03, size = 75, normalized size = 0.84

method	result
default	$\frac{a\left(\ln(a\,x^4+2a\,x^2+a+b)+\frac{(a-b)\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{2(a+b)^2} - \frac{1}{2(a+b)x^2} - \frac{2a\ln(x)}{(a+b)^2}$

risch	$-\frac{1}{2(a+b)x^2}-\frac{2a\ln(x)}{a^2+2ab+b^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2b+2a\,b^2+b^3\right)\_Z^2-4ab\_Z+a\right)}\_R\ln\left(\left(\left(-a^3+3a^2b+9a\,b^2+5b^3\right)\_R^2+\left(-8a^2-8\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b)^2*a*(ln(a*x^4+2*a*x^2+a+b)+(a-b)/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+
2*a)/(a*b)^(1/2)))-1/2/(a+b)/x^2-2*a*ln(x)/(a+b)^2
```

Maxima [A]
time = 0.50, size = 104, normalized size = 1.17

$$\frac{a\log\left(ax^4+2\,ax^2+a+b\right)}{2\left(a^2+2\,ab+b^2\right)}-\frac{a\log\left(x^2\right)}{a^2+2\,ab+b^2}+\frac{\left(a^2-ab\right)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{ab}}-\frac{1}{2\,(a+b)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a + b)/(a^2 + 2*a*b + b^2) - a*log(x^2)/(a^2 +
2*a*b + b^2) + 1/2*(a^2 - a*b)*arctan((a*x^2 + a)/sqrt(a*b))/((a^2 + 2*a*b
+ b^2)*sqrt(a*b)) - 1/2/((a + b)*x^2)
```

Fricas [A]
time = 0.37, size = 208, normalized size = 2.34

$$\left[-\frac{(a-b)x^2\sqrt{-\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\,(bx^2+b)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-2\,ax^2\log\left(ax^4+2\,ax^2+a+b\right)+8\,ax^2\log\left(x\right)+2\,a+2\,b}{4\left(a^2+2\,ab+b^2\right)x^2},-\frac{(a-b)x^2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{ax^2+a}\right)-ax^2\log\left(ax^4+2\,ax^2+a+b\right)+4\,ax^2\log\left(x\right)+a+b}{2\left(a^2+2\,ab+b^2\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - b)*x^2*sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(-a/
b) + a - b)/(a*x^4 + 2*a*x^2 + a + b)) - 2*a*x^2*log(a*x^4 + 2*a*x^2 + a +
b) + 8*a*x^2*log(x) + 2*a + 2*b)/((a^2 + 2*a*b + b^2)*x^2), -1/2*((a - b)*x
^2*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*x^2 + a)) - a*x^2*log(a*x^4 + 2*a*x^2 +
a + b) + 4*a*x^2*log(x) + a + b)/((a^2 + 2*a*b + b^2)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(85) = 170.
time = 20.47, size = 386, normalized size = 4.34

$$-\frac{2a\log\left(x\right)}{\left(a+b\right)^2}+\left(\frac{a}{2\left(a+b\right)^2}-\frac{\sqrt{-ab}\left(a-b\right)}{4b\left(a^2+2ab+b^2\right)}\right)\log\left(x^2+\frac{4a^2b\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)-3ab+4b^3\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)}{a^2-ab}\right)+\left(\frac{a}{2\left(a+b\right)^2}+\frac{\sqrt{-ab}\left(a-b\right)}{4b\left(a^2+2ab+b^2\right)}\right)\log\left(x^2+\frac{4a^2b\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)-3ab+4b^3\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)}{a^2-ab}\right)-\frac{1}{x^2\cdot\left(2a+2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -2*a*log(x)/(a + b)**2 + (a/(2*(a + b)**2) - sqrt(-a*b)*(a - b)/(4*b*(a**2
+ 2*a*b + b**2)))*log(x**2 + (4*a**2*b*(a/(2*(a + b)**2) - sqrt(-a*b)*(a -
b)/(4*b*(a**2 + 2*a*b + b**2))) + a**2 + 8*a*b**2*(a/(2*(a + b)**2) - sqrt(
-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))) - 3*a*b + 4*b**3*(a/(2*(a + b)**
2) - sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))))/(a**2 - a*b)) + (a/(2
*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2)))*log(x**2 + (
4*a**2*b*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))
) + a**2 + 8*a*b**2*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a
*b + b**2))) - 3*a*b + 4*b**3*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(
a**2 + 2*a*b + b**2))))/(a**2 - a*b)) - 1/(x**2*(2*a + 2*b))
```

Giac [A]
time = 3.33, size = 125, normalized size = 1.40

$$\frac{a\log\left(ax^4+2\,ax^2+a+b\right)}{2\,(a^2+2\,ab+b^2)}-\frac{a\log\left(x^2\right)}{a^2+2\,ab+b^2}+\frac{(a^2-ab)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,(a^2+2\,ab+b^2)\sqrt{ab}}+\frac{2\,ax^2-a-b}{2\,(a^2+2\,ab+b^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*a*log(a*x^4 + 2*a*x^2 + a + b)/(a^2 + 2*a*b + b^2) - a*log(x^2)/(a^2 +
2*a*b + b^2) + 1/2*(a^2 - a*b)*arctan((a*x^2 + a)/sqrt(a*b))/((a^2 + 2*a*b
+ b^2)*sqrt(a*b)) + 1/2*(2*a*x^2 - a - b)/((a^2 + 2*a*b + b^2)*x^2)
```

Mupad [B]
time = 7.39, size = 2500, normalized size = 28.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] (8*a*b*log(((2*a^5)/(a + b)^3 - (a/(2*(a + b)^2) - (-(a*(a - b)^2)/(b*(a +
b)^4))^(1/2)/4)*((12*a^5*x^2)/(a + b)^2 - (a/(2*(a + b)^2) - (-(a*(a - b)^2
)/(b*(a + b)^4))^(1/2)/4)*((8*a^4*(3*a - b))/(a + b) + 16*a^4*(a/(2*(a + b)
^2) - (-(a*(a - b)^2)/(b*(a + b)^4))^(1/2)/4)*(a + b + a*x^2 - 5*b*x^2) + (
4*a^4*x^2*(7*a + 5*b))/(a + b)) + (a^4*(15*a - b))/(a + b)^2) + (a^5*x^2)/(
a + b)^3)*((2*a^5)/(a + b)^3 - (a/(2*(a + b)^2) + (-(a*(a - b)^2)/(b*(a + b
)^4))^(1/2)/4)*((12*a^5*x^2)/(a + b)^2 - (a/(2*(a + b)^2) + (-(a*(a - b)^2)
/(b*(a + b)^4))^(1/2)/4)*((8*a^4*(3*a - b))/(a + b) + 16*a^4*(a/(2*(a + b)^
2) + (-(a*(a - b)^2)/(b*(a + b)^4))^(1/2)/4)*(a + b + a*x^2 - 5*b*x^2) + (4
```

```
*a^4*x^2*(7*a + 5*b))/(a + b)) + (a^4*(15*a - b))/(a + b)^2) + (a^5*x^2)/(a
 + b)^3)))/(32*a*b^2 + 16*a^2*b + 16*b^3) - (2*a*log(x))/(2*a*b + a^2 + b^2
) - 1/(2*x^2*(a + b)) + (a^(1/2)*atan(((13*a^2 - 34*a*b + b^2)*((8*a*b*((14
*a^5*b + 15*a^6 - a^4*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*((40*a^
6*b + 24*a^7 - 8*a^4*b^3 + 8*a^5*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) + (8*
a*b*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/((32*a*b^2
+ 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(32*a*b^2 + 16*a^2*
b + 16*b^3)))/(32*a*b^2 + 16*a^2*b + 16*b^3) - (2*a^5)/(3*a*b^2 + 3*a^2*b +
 a^3 + b^3) + (a^(1/2)*((a^(1/2)*(a - b)*((40*a^6*b + 24*a^7 - 8*a^4*b^3 +
8*a^5*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) + (8*a*b*(64*a^7*b + 16*a^8 + 16
*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b
^2 + 3*a^2*b + a^3 + b^3))))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) + (2*a^(3/2)*b
^(1/2)*(a - b)*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/
((2*a*b + a^2 + b^2)*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^
3 + b^3)))*(a - b))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) + (a^2*(a - b)^2*(64*a^
7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/(2*(2*a*b + a^2 + b^2
)^2*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))*(24*a*
b^(13/2) + 4*b^(15/2) + 4*a^6*b^(3/2) + 24*a^5*b^(5/2) + 60*a^4*b^(7/2) + 8
0*a^3*b^(9/2) + 60*a^2*b^(11/2)))/((a + b)^3*(98*a*b + a^2 + b^2)*(a^(13/2)
 - 2*a^(11/2)*b + a^(9/2)*b^2)) - (x^2*(((13*a^2 - 34*a*b + b^2)*(a^5/(3*a*
b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*((12*a^5*b + 12*a^6)/(3*a*b^2 + 3*a^2*b
 + a^3 + b^3) - (8*a*b*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*b^2)/(3*a*
b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a
^5*b^3 + 192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b +
 a^3 + b^3))))/(32*a*b^2 + 16*a^2*b + 16*b^3)))/(32*a*b^2 + 16*a^2*b + 16*b
^3) - (a^(1/2)*((a^(1/2)*(a - b)*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*
b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b
^4 + 224*a^5*b^3 + 192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 +
 3*a^2*b + a^3 + b^3))))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) - (2*a^(3/2)*b^(1/
2)*(a - b)*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 + 192*a^6*b^2))/((
2*a*b + a^2 + b^2)*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3
+ b^3)))*(a - b))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) + (a^2*(a - b)^2*(32*a^7*
b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 + 192*a^6*b^2))/(2*(2*a*b + a^2 + b^2
)^2*(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/((a +
 b)^3*(98*a*b + a^2 + b^2)) + (a^(1/2)*(a^2 - 34*a*b + 13*b^2)*((8*a*b*((a^
(1/2)*(a - b)*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*b^2)/(3*a*b^2 + 3*a
^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 +
192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^
3))))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) - (2*a^(3/2)*b^(1/2)*(a - b)*(32*a^7*
b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^3 + 192*a^6*b^2))/((2*a*b + a^2 + b^2)*
(32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(32*a*b^2
 + 16*a^2*b + 16*b^3) - (a^(1/2)*(a - b)*((12*a^5*b + 12*a^6)/(3*a*b^2 + 3*
a^2*b + a^3 + b^3) - (8*a*b*((76*a^6*b + 28*a^7 + 20*a^4*b^3 + 68*a^5*b^2)/
(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (8*a*b*(32*a^7*b - 16*a^8 + 80*a^4*b^4 +
224*a^5*b^3 + 192*a^6*b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^
```

```
2*b + a^3 + b^3))))/(32*a*b^2 + 16*a^2*b + 16*b^3)))/(4*b^(1/2)*(2*a*b + a^
2 + b^2)) + (a^(3/2)*(a - b)^3*(32*a^7*b - 16*a^8 + 80*a^4*b^4 + 224*a^5*b^
3 + 192*a^6*b^2))/(64*b^(3/2)*(2*a*b + a^2 + b^2)^3*(3*a*b^2 + 3*a^2*b + a^
3 + b^3))))/(b^(1/2)*(a + b)^3*(98*a*b + a^2 + b^2)))*(24*a*b^(13/2) + 4*b^
(15/2) + 4*a^6*b^(3/2) + 24*a^5*b^(5/2) + 60*a^4*b^(7/2) + 80*a^3*b^(9/2) +
 60*a^2*b^(11/2)))/(a^(13/2) - 2*a^(11/2)*b + a^(9/2)*b^2) + (a^(1/2)*((a^(
1/2)*(a - b)*((14*a^5*b + 15*a^6 - a^4*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3)
 - (8*a*b*((40*a^6*b + 24*a^7 - 8*a^4*b^3 + 8*a^5*b^2)/(3*a*b^2 + 3*a^2*b +
 a^3 + b^3) + (8*a*b*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*
b^2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(3
2*a*b^2 + 16*a^2*b + 16*b^3)))/(4*b^(1/2)*(2*a*b + a^2 + b^2)) - (8*a*b*((a
^(1/2)*(a - b)*((40*a^6*b + 24*a^7 - 8*a^4*b^3 + 8*a^5*b^2)/(3*a*b^2 + 3*a^
2*b + a^3 + b^3) + (8*a*b*(64*a^7*b + 16*a^8 + ...
```

3.911 $\int \frac{x^4}{a+b+2ax^2+ax^4} \, dx$

Optimal. Leaf size=432

$$\frac{x}{a}+\frac{\left(a+b+2\sqrt{a}\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}}-\frac{\left(a+b+2\sqrt{a}\sqrt{a+b}\right)\tan^{-1}\left(\right.}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\,\sqrt{}}$$

```
[Out] x/a+1/8*ln(x^2*a^(1/2)+(a+b)^(1/2)-a^(1/4)*x*2^(1/2)*(-a^(1/2)+(a+b)^(1/2))
^(1/2))*(a+b-2*a^(1/2)*(a+b)^(1/2))/a^(5/4)*2^(1/2)/(a+b)^(1/2)/(-a^(1/2)+(
a+b)^(1/2))^(1/2)-1/8*ln(x^2*a^(1/2)+(a+b)^(1/2)+a^(1/4)*x*2^(1/2)*(-a^(1/2
)+(a+b)^(1/2))^(1/2))*(a+b-2*a^(1/2)*(a+b)^(1/2))/a^(5/4)*2^(1/2)/(a+b)^(1/
2)/(-a^(1/2)+(a+b)^(1/2))^(1/2)+1/4*arctan((-a^(1/4)*x*2^(1/2)+(-a^(1/2)+(a
+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))*(a+b+2*a^(1/2)*(a+b)^(1/2))/
a^(5/4)*2^(1/2)/(a+b)^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)-1/4*arctan((a^(1/4)
*x*2^(1/2)+(-a^(1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))*(a+b+
2*a^(1/2)*(a+b)^(1/2))/a^(5/4)*2^(1/2)/(a+b)^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1
/2)
```

Rubi [A]

time = 0.59, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1136, 1183, 648, 632, 210, 642}

$$\frac{\left(2\sqrt{a}\sqrt{a+b}+a+b\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}-\frac{\left(2\sqrt{a}\sqrt{a+b}+a+b\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(-\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((a + b + 2*Sqrt[a]*Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]]
 - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[2]*a^(5/4)*Sqrt
[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - ((a + b + 2*Sqrt[a]*Sqrt[a + b])*Arc
Tan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[
a + b]]])/(2*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ((a
 + b - 2*Sqrt[a]*Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[
a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[-Sq
rt[a] + Sqrt[a + b]]) - ((a + b - 2*Sqrt[a]*Sqrt[a + b])*Log[Sqrt[a + b] +
Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a
^(5/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx = \frac{x}{a} - \frac{\int \frac{a+b+2ax^2}{a+b+2ax^2+ax^4}\,dx}{a}$$

$$= \frac{x}{a} - \frac{\int \frac{\frac{\sqrt{2}\,(a+b)\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}} - \left(a+b-2\sqrt{a}\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}} - \frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}} + x^2}\,dx}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\int \frac{\frac{\sqrt{2}\,(a+b)\sqrt{-\cdots}}{\cdots}}{\frac{\sqrt{a+\cdots}}{\sqrt{a}}\cdots}}{2\sqrt{2}\cdots}$$

$$= \frac{x}{a} + \frac{\left(a+b-2\sqrt{a}\sqrt{a+b}\right)\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}+2x}{\frac{\sqrt{a+b}}{\sqrt{a}} - \frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}} + x^2}\,dx}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \cdots$$

$$= \frac{x}{a} + \frac{\left(a+b-2\sqrt{a}\sqrt{a+b}\right)\log\left(\sqrt{a+b} - \sqrt{2}\,\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x + \cdots\right.}{4\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

$$= \frac{x}{a} + \frac{\left(a+b+2\sqrt{a}\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} - \frac{(a\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 164, normalized size = 0.38

$$\frac{x}{a} - \frac{i\left(\sqrt{a}-i\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{a-i\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{i\left(\sqrt{a}+i\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{a+i\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a - ((I/2)*(Sqrt[a] - I*Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*
Sqrt[b]]])/(a*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((I/2)*(Sqrt[a] + I*Sq
```

```
rt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/(a*Sqrt[a + I*Sqr
t[a]*Sqrt[b]]*Sqrt[b])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(300) = 600$.

time = 0.10, size = 777, normalized size = 1.80

method	result
risch	$\frac{x}{a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4+2a\_Z^2+a+b\right)}\frac{\left(-2\_R^2a-a-b\right)\ln\left(x-\_R\right)}{\_R^3+\_R}}{4a^2}$
default	$\frac{x}{a}+\frac{-\frac{\left(-a^{\frac{3}{2}}\sqrt{a+b}\,\sqrt{2\sqrt{a^2+ab}-2a}-\sqrt{a}\,\sqrt{a+b}\,\sqrt{a^2+ab}\,\sqrt{2\sqrt{a^2+ab}-2a}+2\sqrt{a^2+ab}\,\sqrt{\cdots}\right.}{2\sqrt{\cdots}}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

```
[Out] x/a+1/a*(1/4/b/a*(-1/2*(-a^(3/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-
a^(1/2)*(a+b)^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+2*(a^2+a*
b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^
2)/a^(1/2)*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))+2*(
2*(a+b)^(1/2)*a*b+1/2*(-a^(3/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-a
^(1/2)*(a+b)^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+2*(a^2+a*b
)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^2
)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)/a^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))
^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^
(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)))+1/4/b/a*(1/2*(-a^(3/2)*(a+
b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-a^(1/2)*(a+b)^(1/2)*(a^2+a*b)^(1/2)*
(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+2*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/
2)*a+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^2)/a^(1/2)*ln(x^2*a^(1/2)+x*(2*(a*(a
+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))+2*(-2*(a+b)^(1/2)*a*b-1/2*(-a^(3/2)*(a+b
)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-a^(1/2)*(a+b)^(1/2)*(a^2+a*b)^(1/2)*(
2*(a^2+a*b)^(1/2)-2*a)^(1/2)+2*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2
)*a+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^2)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)/a^(1
/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2
)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2
*a)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] x/a - integrate((2*a*x^2 + a + b)/(a*x^4 + 2*a*x^2 + a + b), x)/a
```

Fricas [A]
time = 0.35, size = 615, normalized size = 1.42

$$\frac{a\sqrt{\frac{a^2b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}+a-3b}{a^2b}}\log\left(-(3a^2+2ab-b^2)x+\left(a^4b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}+3a^2b-ab^2\right)\sqrt{\frac{a^2b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}+a-3b}{a^2b}}\right)-a\sqrt{\frac{a^2b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}+a-3b}{a^2b}}\log\left(-(3a^2+2ab-b^2)x-\left(a^4b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}+3a^2b-ab^2\right)\sqrt{\frac{a^2b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}+a-3b}{a^2b}}\right)-a\sqrt{-\frac{a^2b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}-a+3b}{a^2b}}\log\left(-(3a^2+2ab-b^2)x+\left(a^4b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}-3a^2b+ab^2\right)\sqrt{-\frac{a^2b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}-a+3b}{a^2b}}\right)+a\sqrt{-\frac{a^2b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}-a+3b}{a^2b}}\log\left(-(3a^2+2ab-b^2)x-\left(a^4b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}-3a^2b+ab^2\right)\sqrt{-\frac{a^2b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}-a+3b}{a^2b}}\right)+4x}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*(a*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a - 3*b)/(a^2*b))
*log(-(3*a^2 + 2*a*b - b^2)*x + (a^4*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b))
 + 3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a -
3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a - 3
*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x - (a^4*b*sqrt(-(9*a^2 - 6*a*b + b
^2)/(a^5*b)) + 3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^
5*b)) + a - 3*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^
5*b)) - a + 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x + (a^4*b*sqrt(-(9*a^
2 - 6*a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*
a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))) + a*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*
a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x - (a^4
*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt(-(a^2*b*sqr
t(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))) + 4*x)/a
```

Sympy [A]
time = 0.78, size = 105, normalized size = 0.24

$$\operatorname{RootSum}\left(256t^4a^5b^2+t^2\left(-32a^4b+96a^3b^2\right)+a^3+3a^2b+3ab^2+b^3,\ \left(t\mapsto t\log\left(x+\frac{-64t^3a^4b+4ta^3-24ta^2b+4tab^2}{3a^2+2ab-b^2}\right)\right)\right)+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(256*_t**4*a**5*b**2 + _t**2*(-32*a**4*b + 96*a**3*b**2) + a**3 + 3*
a**2*b + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b + 4*_t*a*
*3 - 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 + 2*a*b - b**2)))) + x/a
```

Giac [A]
time = 3.70, size = 533, normalized size = 1.23

$$\frac{\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^4+\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^3b-4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2b^2+2\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab+4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,b^2\right)a^2-\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,a^3b+7\sqrt{a^2+\sqrt{-ab}\,a}\,a^2b^2+4\sqrt{a^2+\sqrt{-ab}\,a}\,ab^3\right)|a|\right)\arctan\left(\frac{x}{\sqrt{\frac{a^2+\sqrt{a^4-(a^2+ab)a^2}}{a^2}}}\right)}{2\left(3a^6b+7a^5b^2+4a^4b^3\right)}-\frac{\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^4+\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^3b-4\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^2b^2+2\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab+4\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,b^2\right)a^2+\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,a^3b+7\sqrt{a^2-\sqrt{-ab}\,a}\,a^2b^2+4\sqrt{a^2-\sqrt{-ab}\,a}\,ab^3\right)|a|\right)\arctan\left(\frac{x}{\sqrt{\frac{a^2-\sqrt{a^4-(a^2+ab)a^2}}{a^2}}}\right)}{2\left(3a^6b+7a^5b^2+4a^4b^3\right)}+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + sqrt(a^2 + sqrt(-a*b)*a)*s
qrt(-a*b)*a^3*b - 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 2*(3*sqrt
(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)
*b^2)*a^2 - (3*sqrt(a^2 + sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 + sqrt(-a*b)*a)*
a^2*b^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 + sq
rt(a^4 - (a^2 + a*b)*a^2))/a^2))/(3*a^6*b + 7*a^5*b^2 + 4*a^4*b^3) - 1/2*(3
*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a
*b)*a^3*b - 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 2*(3*sqrt(a^2 -
 sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*b^2)*
a^2 + (3*sqrt(a^2 - sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 - sqrt(-a*b)*a)*a^2*b^
2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - sqrt(a^4
 - (a^2 + a*b)*a^2))/a^2))/(3*a^6*b + 7*a^5*b^2 + 4*a^4*b^3) + x/a
```

Mupad [B]

time = 4.65, size = 1147, normalized size = 2.66

$$\frac{x}{a}+2\,\mathrm{atanh}\left(\frac{24\,x\,\sqrt{-a^5\,b^3}\,\sqrt{\frac{1}{16\,a\,b}-\frac{3}{16\,a^2}+\frac{3\,\sqrt{-a^5\,b^3}}{16\,a^4\,b^2}-\frac{\sqrt{-a^5\,b^3}}{16\,a^5\,b}}}{\frac{6\,\sqrt{-a^5\,b^3}}{a}+4\,a\,b^2+6\,a^2\,b-2\,b^3-\frac{2\,b^2\,\sqrt{-a^5\,b^3}}{a^3}+\frac{4\,b\,\sqrt{-a^5\,b^3}}{a^2}}-\frac{8\,x\,\sqrt{-a^5\,b^3}\,\sqrt{\frac{1}{16\,a\,b}-\frac{3}{16\,a^2}+\frac{3\,\sqrt{-a^5\,b^3}}{16\,a^4\,b^2}-\frac{\sqrt{-a^5\,b^3}}{16\,a^5\,b}}}{\frac{4\,\sqrt{-a^5\,b^3}}{a}+\frac{6\,\sqrt{-a^5\,b^3}}{b}-2\,a\,b^2+4\,a^2\,b+6\,a^3-\frac{2\,b\,\sqrt{-a^5\,b^3}}{a^2}}-\frac{8\,a\,b^2\,x\,\sqrt{\frac{1}{16\,a\,b}-\frac{3}{16\,a^2}+\frac{3\,\sqrt{-a^5\,b^3}}{16\,a^4\,b^2}-\frac{\sqrt{-a^5\,b^3}}{16\,a^5\,b}}}{4\,a\,b+\frac{4\,\sqrt{-a^5\,b^3}}{a^2}+6\,a^2-2\,b^2+\frac{6\,\sqrt{-a^5\,b^3}}{a\,b}-\frac{2\,b\,\sqrt{-a^5\,b^3}}{a^3}}+\frac{24\,a^2\,b\,x\,\sqrt{\frac{1}{16\,a\,b}-\frac{3}{16\,a^2}+\frac{3\,\sqrt{-a^5\,b^3}}{16\,a^4\,b^2}-\frac{\sqrt{-a^5\,b^3}}{16\,a^5\,b}}}{4\,a\,b+\frac{4\,\sqrt{-a^5\,b^3}}{a^2}+6\,a^2-2\,b^2+\frac{6\,\sqrt{-a^5\,b^3}}{a\,b}-\frac{2\,b\,\sqrt{-a^5\,b^3}}{a^3}}\right)\sqrt{\frac{3\,a\,\sqrt{-a^5\,b^3}-b\,\sqrt{-a^5\,b^3}+a^4\,b-3\,a^3\,b^2}{16\,a^5\,b^2}}+2\,\mathrm{atanh}\left(\frac{24\,x\,\sqrt{-a^5\,b^3}\,\sqrt{\frac{1}{16\,a\,b}-\frac{3}{16\,a^2}-\frac{3\,\sqrt{-a^5\,b^3}}{16\,a^4\,b^2}+\frac{\sqrt{-a^5\,b^3}}{16\,a^5\,b}}}{\frac{6\,\sqrt{-a^5\,b^3}}{a}-4\,a\,b^2-6\,a^2\,b+2\,b^3-\frac{2\,b^2\,\sqrt{-a^5\,b^3}}{a^3}+\frac{4\,b\,\sqrt{-a^5\,b^3}}{a^2}}-\frac{8\,x\,\sqrt{-a^5\,b^3}\,\sqrt{\frac{1}{16\,a\,b}-\frac{3}{16\,a^2}-\frac{3\,\sqrt{-a^5\,b^3}}{16\,a^4\,b^2}+\frac{\sqrt{-a^5\,b^3}}{16\,a^5\,b}}}{\frac{4\,\sqrt{-a^5\,b^3}}{a}+\frac{6\,\sqrt{-a^5\,b^3}}{b}+2\,a\,b^2-4\,a^2\,b-6\,a^3-\frac{2\,b\,\sqrt{-a^5\,b^3}}{a^2}}-\frac{8\,a\,b^2\,x\,\sqrt{\frac{1}{16\,a\,b}-\frac{3}{16\,a^2}-\frac{3\,\sqrt{-a^5\,b^3}}{16\,a^4\,b^2}+\frac{\sqrt{-a^5\,b^3}}{16\,a^5\,b}}}{4\,a\,b-\frac{4\,\sqrt{-a^5\,b^3}}{a^2}+6\,a^2-2\,b^2-\frac{6\,\sqrt{-a^5\,b^3}}{a\,b}+\frac{2\,b\,\sqrt{-a^5\,b^3}}{a^3}}+\frac{24\,a^2\,b\,x\,\sqrt{\frac{1}{16\,a\,b}-\frac{3}{16\,a^2}-\frac{3\,\sqrt{-a^5\,b^3}}{16\,a^4\,b^2}+\frac{\sqrt{-a^5\,b^3}}{16\,a^5\,b}}}{4\,a\,b-\frac{4\,\sqrt{-a^5\,b^3}}{a^2}+6\,a^2-2\,b^2-\frac{6\,\sqrt{-a^5\,b^3}}{a\,b}+\frac{2\,b\,\sqrt{-a^5\,b^3}}{a^3}}\right)\sqrt{-\frac{3\,a\,\sqrt{-a^5\,b^3}-b\,\sqrt{-a^5\,b^3}-a^4\,b+3\,a^3\,b^2}{16\,a^5\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] x/a + 2*atanh((24*x*(-a^5*b^3)^(1/2)*(1/(16*a*b) - 3/(16*a^2) + (3*(-a^5*b^
3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((6*(-a^5*b^3)
^(1/2))/a + 4*a*b^2 + 6*a^2*b - 2*b^3 - (2*b^2*(-a^5*b^3)^(1/2))/a^3 + (4*b
*(-a^5*b^3)^(1/2))/a^2) - (8*x*(-a^5*b^3)^(1/2)*(1/(16*a*b) - 3/(16*a^2) +
(3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((4
*(-a^5*b^3)^(1/2))/a + (6*(-a^5*b^3)^(1/2))/b - 2*a*b^2 + 4*a^2*b + 6*a^3 -
 (2*b*(-a^5*b^3)^(1/2))/a^2) - (8*a*b^2*x*(1/(16*a*b) - 3/(16*a^2) + (3*(-a
^5*b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b +
(4*(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 + (6*(-a^5*b^3)^(1/2))/(a*b) - (2*
b*(-a^5*b^3)^(1/2))/a^3) + (24*a^2*b*x*(1/(16*a*b) - 3/(16*a^2) + (3*(-a^5*
b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b + (4*
(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 + (6*(-a^5*b^3)^(1/2))/(a*b) - (2*b*(
-a^5*b^3)^(1/2))/a^3))*((3*a*(-a^5*b^3)^(1/2) - b*(-a^5*b^3)^(1/2) + a^4*b
- 3*a^3*b^2)/(16*a^5*b^2))^(1/2) + 2*atanh((24*x*(-a^5*b^3)^(1/2)*(1/(16*a*
b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)^(1/2)/(16*
a^5*b))^(1/2))/((6*(-a^5*b^3)^(1/2))/a - 4*a*b^2 - 6*a^2*b + 2*b^3 - (2*b^2
*(-a^5*b^3)^(1/2))/a^3 + (4*b*(-a^5*b^3)^(1/2))/a^2) - (8*x*(-a^5*b^3)^(1/2
)*(1/(16*a*b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)
^(1/2)/(16*a^5*b))^(1/2))/((4*(-a^5*b^3)^(1/2))/a + (6*(-a^5*b^3)^(1/2))/b
+ 2*a*b^2 - 4*a^2*b - 6*a^3 - (2*b*(-a^5*b^3)^(1/2))/a^2) - (8*a*b^2*x*(1/(
16*a*b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)^(1/2)
/(16*a^5*b))^(1/2))/(4*a*b - (4*(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 - (6*
(-a^5*b^3)^(1/2))/(a*b) + (2*b*(-a^5*b^3)^(1/2))/a^3) + (24*a^2*b*x*(1/(16*
```

```
a*b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*b^3)^(1/2)/(1
6*a^5*b))^(1/2))/(4*a*b - (4*(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 - (6*(-a
^5*b^3)^(1/2))/(a*b) + (2*b*(-a^5*b^3)^(1/2))/a^3))*(-(3*a*(-a^5*b^3)^(1/2)
 - b*(-a^5*b^3)^(1/2) - a^4*b + 3*a^3*b^2)/(16*a^5*b^2))^(1/2)
```

3.912 $\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=331

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}}+\frac{\log\left(\sqrt{a+b}-\sqrt{\right.}{4\sqrt{2}$$

```
[Out] 1/8*ln(x^2*a^(1/2)+(a+b)^(1/2)-a^(1/4)*x*2^(1/2)*(-a^(1/2)+(a+b)^(1/2))^(1/
2))/a^(3/4)*2^(1/2)/(-a^(1/2)+(a+b)^(1/2))^(1/2)-1/8*ln(x^2*a^(1/2)+(a+b)^(
1/2)+a^(1/4)*x*2^(1/2)*(-a^(1/2)+(a+b)^(1/2))^(1/2))/a^(3/4)*2^(1/2)/(-a^(1
/2)+(a+b)^(1/2))^(1/2)-1/4*arctan((-a^(1/4)*x*2^(1/2)+(-a^(1/2)+(a+b)^(1/2)
)^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))/a^(3/4)*2^(1/2)/(a^(1/2)+(a+b)^(1/2))
^(1/2)+1/4*arctan((a^(1/4)*x*2^(1/2)+(-a^(1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)
+(a+b)^(1/2))^(1/2))/a^(3/4)*2^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1143, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a]
 + Sqrt[a + b]]]/(Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ArcTan[(Sq
rt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]
]/(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt[a + b] - Sqrt[
2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)
*Sqrt[-Sqrt[a] + Sqrt[a + b]]) - Log[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sq
rt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*Sqrt[-Sqrt[a] + Sq
rt[a + b]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1143

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Dist[1/(2*c*r), Int[x^(m - 1)/(q + r*x + x^2), x],
x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m, 3
] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = \frac{\int \frac{x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\int \frac{x}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a-\cdots}}}{\sqrt[4]{a}}}}{2\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a-\cdots}}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{4a} + \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a-\cdots}}}{\sqrt[4]{a}}}}{4a}$$

$$= \frac{\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}} - \frac{\log\left(\sqrt{a+b}+\sqrt{\cdots}\right.}{4\sqrt{2}\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{\cdots}}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right.}{2\sqrt{2}\,a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 143, normalized size = 0.43

$$\frac{\frac{\left(i\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{a}\sqrt{b}}}+\frac{\left(-i\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (((I*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqrt[b]]])/Sq
rt[a - I*Sqrt[a]*Sqrt[b]] + (((-I)*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sq
rt[a + I*Sqrt[a]*Sqrt[b]]])/Sqrt[a + I*Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*Sqrt[b]
)
```

Maple [A]
time = 0.05, size = 339, normalized size = 1.02

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4+2a\_Z^2+a+b\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3+\_R}}{4a}$
default	$\frac{\sqrt{2\sqrt{a^2+ab}-2a}\,\left(\sqrt{a^2+ab}+a\right)\left(\frac{\ln\left(-x^2\sqrt{a}+x\sqrt{2\sqrt{a(a+b)}-2a}-\sqrt{a+b}\right)}{2\sqrt{a}}-\frac{\sqrt{2\sqrt{a(a+}}}{}\right.}{4ab}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*((a^2+a*b)^(1/2)+a)/a/b*(1/2/a^(1/2)*ln(-
x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))-1/a^(1/2)*(2*(a*(a
+b))^(1/2)-2*a)^(1/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*a
rctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2
*(a*(a+b))^(1/2)+2*a)^(1/2)))-1/4*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*((a^2+a*b)^
(1/2)+a)/a/b*(1/2/a^(1/2)*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a
+b)^(1/2))-1/a^(1/2)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)/(4*a^(1/2)*(a+b)^(1/2)-2
*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/
2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x^4 + 2*a*x^2 + a + b), x)
```

Fricas [A]
time = 0.35, size = 279, normalized size = 0.84

$$\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\log\left(a^2b\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)+\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\log\left(-a^2b\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((a*b*sqrt(-1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt((a*b*sqrt(-1/(a^3
*b)) + 1)/(a*b))*sqrt(-1/(a^3*b)) + x) - 1/4*sqrt((a*b*sqrt(-1/(a^3*b)) + 1
)/(a*b))*log(-a^2*b*sqrt((a*b*sqrt(-1/(a^3*b)) + 1)/(a*b))*sqrt(-1/(a^3*b))
 + x) - 1/4*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b))*log(a^2*b*sqrt(-(a*b*sq
rt(-1/(a^3*b)) - 1)/(a*b))*sqrt(-1/(a^3*b)) + x) + 1/4*sqrt(-(a*b*sqrt(-1/(
a^3*b)) - 1)/(a*b))*log(-a^2*b*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b))*sqrt
(-1/(a^3*b)) + x)
```

Sympy [A]
time = 0.26, size = 44, normalized size = 0.13

$$\operatorname{RootSum}\left(256t^4a^3b^2 - 32t^2a^2b + a + b, \left(t \mapsto t\log\left(64t^3a^2b - 4ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b + a + b, Lambda(_t, _t*log(64
*_t**3*a**2*b - 4*_t*a + x)))
```

Giac [A]
time = 3.44, size = 203, normalized size = 0.61

$$-\frac{\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a + 4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2\left(3a^4b+4a^3b^2\right)} + \frac{\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,a + 4\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a-\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2\left(3a^4b+4a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] -1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a + 4*sqrt(a^2 + sqrt(-a*b)*a)*
sqrt(-a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a + b)*a + 4
*a^2))/a))/(3*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*
b)*a + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x
/sqrt((2*a - sqrt(-4*(a + b)*a + 4*a^2))/a))/(3*a^4*b + 4*a^3*b^2)
```

Mupad [B]
time = 0.28, size = 222, normalized size = 0.67

$$-2\,\mathrm{atanh}\left(\frac{2\left(x\left(4a^2b-4a^3\right)+\frac{4ax\left(\sqrt{-a^3b^3}+a^2b\right)}{b}\right)\sqrt{\frac{\sqrt{-a^3b^3}+a^2b}{16a^3b^2}}}{2a^2+2ba}\right)\sqrt{\frac{\sqrt{-a^3b^3}+a^2b}{16a^3b^2}} - 2\,\mathrm{atanh}\left(\frac{2\left(x\left(4a^2b-4a^3\right)-\frac{4ax\left(\sqrt{-a^3b^3}-a^2b\right)}{b}\right)\sqrt{-\frac{\sqrt{-a^3b^3}-a^2b}{16a^3b^2}}}{2a^2+2ba}\right)\sqrt{-\frac{\sqrt{-a^3b^3}-a^2b}{16a^3b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] - 2*atanh((2*(x*(4*a^2*b - 4*a^3) + (4*a*x*((-a^3*b^3)^(1/2) + a^2*b))/b)*(
((-a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2))/(2*a*b + 2*a^2))*(((-a^3*b^
3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2) - 2*atanh((2*(x*(4*a^2*b - 4*a^3) - (
4*a*x*((-a^3*b^3)^(1/2) - a^2*b))/b)*(-((-a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b
^2))^(1/2))/(2*a*b + 2*a^2))*(-((-a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/
2)
```

3.913 $\int \frac{1}{a+b+2ax^2+ax^4} \, dx$

Optimal. Leaf size=359

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}}-\frac{\log\left(\sqrt{a+b}-\right.}{4\sqrt{2}\sqrt[4]{}}$$

```
[Out] -1/8*ln(x^2*a^(1/2)+(a+b)^(1/2)-a^(1/4)*x*2^(1/2)*(-a^(1/2)+(a+b)^(1/2))^(1
/2))/a^(1/4)*2^(1/2)/(a+b)^(1/2)/(-a^(1/2)+(a+b)^(1/2))^(1/2)+1/8*ln(x^2*a^
(1/2)+(a+b)^(1/2)+a^(1/4)*x*2^(1/2)*(-a^(1/2)+(a+b)^(1/2))^(1/2))/a^(1/4)*2
^(1/2)/(a+b)^(1/2)/(-a^(1/2)+(a+b)^(1/2))^(1/2)-1/4*arctan((-a^(1/4)*x*2^(1
/2)+(-a^(1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))/a^(1/4)*2^(1
/2)/(a+b)^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)+1/4*arctan((a^(1/4)*x*2^(1/2)+(
-a^(1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))/a^(1/4)*2^(1/2)/(
a+b)^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a]
 + Sqrt[a + b]]]/(Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]])
+ ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] +
Sqrt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) -
 Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]
*x^2]/(4*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]]) + Log[Sq
rt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(
4*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+b+2ax^2+ax^4}\,dx = \frac{\int \frac{\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}-x}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \frac{\int \frac{\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}} \cdots}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a} \cdots}}{\sqrt[4]{a}} \cdots}}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+ \cdots}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}}+x^2}\,dx}{4\sqrt{a}\sqrt{a+b}} + \frac{\int \frac{1}{\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a} \cdots}}{\sqrt[4]{a}} \cdots}}{4\sqrt{a}\sqrt{a+b}}$$

$$= -\frac{\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x+\sqrt{a}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \frac{\log\left(\sqrt{a+b}+ \cdots\right.}{4\sqrt{2} \cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+ \cdots}{\sqrt{\sqrt{a}+\sqrt{a+b}} \cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+ \cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 119, normalized size = 0.33

$$-\frac{i\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a-i\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{i\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a+i\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ((-1/2*I)*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqrt[b]]])/(Sqrt[a - I*Sqrt
[a]*Sqrt[b]]*Sqrt[b]) + ((I/2)*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b
]]])/(Sqrt[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(243) = 486.
time = 0.06, size = 499, normalized size = 1.39

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4+2a\_Z^2+a+b\right)} \frac{\ln\left(x-\_R\right)}{\_R^3+\_R}}{4a}$
default	$-\frac{\left(\sqrt{2\sqrt{a^2+ab}-2a}\,\sqrt{a^2+ab}+\sqrt{2\sqrt{a^2+ab}-2a}\,a\right)\ln\left(-x^2\sqrt{a}+x\sqrt{2\sqrt{a\left(a+b\right)}-2a}-\sqrt{a+\right.}{2\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(a+b)^(1/2)/a^(1/2)/b*(-1/2*((2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1
/2)+(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a)/a^(1/2)*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))
^(1/2)-2*a)^(1/2)-(a+b)^(1/2))+2*(-2*a^(1/2)*b+1/2*((2*(a^2+a*b)^(1/2)-2*a)
^(1/2)*(a^2+a*b)^(1/2)+(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a)*(2*(a*(a+b))^(1/2)-
2*a)^(1/2)/a^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arc
tan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(
a*(a+b))^(1/2)+2*a)^(1/2)))+1/4/(a+b)^(1/2)/a^(1/2)/b*(1/2*((2*(a^2+a*b)^(1
/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a)/a^(1/2)*ln(
x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a+b)^(1/2))+2*(2*a^(1/2)*b-1/2
*((2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+(2*(a^2+a*b)^(1/2)-2*a)^(1/
2)*a)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)/a^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a
+b))^(1/2)+2*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4
*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] integrate(1/(a*x^4 + 2*a*x^2 + a + b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(245) = 490$.
time = 0.34, size = 567, normalized size = 1.58

$$\frac{1}{4}\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}\log\left(\left((a^2b+ab^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+b\right)\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}+x\right)-\frac{1}{4}\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}\log\left(-\left((a^2b+ab^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+b\right)\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}+x\right)-\frac{1}{4}\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}\log\left(\left((a^2b+ab^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-b\right)\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}+x\right)+\frac{1}{4}\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}\log\left(-\left((a^2b+ab^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-b\right)\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)
)*log(((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + b)*sqrt(((a*b
 + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)) + x) - 1/4*s
qrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2))*log
(-((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + b)*sqrt(((a*b + b
^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)) + x) - 1/4*sqrt(
-((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2))*log(((
a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - b)*sqrt(-((a*b + b^2)
*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2)) + x) + 1/4*sqrt(-((
a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2))*log(-((a^
2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - b)*sqrt(-((a*b + b^2)*s
qrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2)) + x)
```

Sympy [A]
time = 0.45, size = 63, normalized size = 0.18

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{2}b^{2}+256ab^{3}\right)-32t^{2}ab+1,\left(t\mapsto t\log\left(64t^{3}a^{2}b+64t^{3}ab^{2}-4ta+4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 + 256*a*b**3) - 32*_t**2*a*b + 1, Lambda(_t, _
t*log(64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a + 4*_t*b + x)))
```

Giac [A]
time = 3.86, size = 307, normalized size = 0.86

$$\frac{\left(3\sqrt{a^{2}+\sqrt{-ab}\,a}\,a^{2}b+4\sqrt{a^{2}+\sqrt{-ab}\,a}\,ab^{2}+3\sqrt{a^{2}+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^{2}+4\sqrt{a^{2}+\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a+\sqrt{-4(a+b)a+4a^{2}}}{a}}}\right)}{2\left(3a^{5}b+7a^{4}b^{2}+4a^{3}b^{3}\right)}+\frac{\left(3\sqrt{a^{2}-\sqrt{-ab}\,a}\,a^{2}b+4\sqrt{a^{2}-\sqrt{-ab}\,a}\,ab^{2}+3\sqrt{a^{2}-\sqrt{-ab}\,a}\,\sqrt{-ab}\,a^{2}+4\sqrt{a^{2}-\sqrt{-ab}\,a}\,\sqrt{-ab}\,ab\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a-\sqrt{-4(a+b)a+4a^{2}}}{a}}}\right)}{2\left(3a^{5}b+7a^{4}b^{2}+4a^{3}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*a^2*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b^2 +
3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt
(-a*b)*a*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a + b)*a + 4*a
^2))/a))/(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3) + 1/2*(3*sqrt(a^2 - sqrt(-a*b)*a
)*a^2*b + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^2 + 3*sqrt(a^2 - sqrt(-a*b)*a)*sqr
t(-a*b)*a^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b)*abs(a)*arctan(2*sq
rt(1/2)*x/sqrt((2*a - sqrt(-4*(a + b)*a + 4*a^2))/a))/(3*a^5*b + 7*a^4*b^2
+ 4*a^3*b^3)
```

Mupad [B]
time = 5.16, size = 986, normalized size = 2.75

$$2\operatorname{atanh}\left(\frac{8a^3x\sqrt{\frac{ab}{16(a^2b^2+ab^3)}-\frac{\sqrt{-ab^3}}{16(a^2b^2+ab^3)}}}{\frac{2a^4b^2}{a^2b^2+ab^3}-\frac{2a^3b\sqrt{-ab^3}}{a^2b^2+ab^3}}-\frac{8a^5b^2x\sqrt{\frac{ab}{16(a^2b^2+ab^3)}-\frac{\sqrt{-ab^3}}{16(a^2b^2+ab^3)}}}{\frac{2a^5b^5}{a^2b^2+ab^3}+\frac{2a^6b^4}{a^2b^2+ab^3}-\frac{2a^4b^4\sqrt{-ab^3}}{a^2b^2+ab^3}-\frac{2a^5b^3\sqrt{-ab^3}}{a^2b^2+ab^3}}+\frac{8a^4bx\sqrt{\frac{ab}{16(a^2b^2+ab^3)}-\frac{\sqrt{-ab^3}}{16(a^2b^2+ab^3)}}\sqrt{-ab^3}}{\frac{2a^5b^5}{a^2b^2+ab^3}+\frac{2a^6b^4}{a^2b^2+ab^3}-\frac{2a^4b^4\sqrt{-ab^3}}{a^2b^2+ab^3}-\frac{2a^5b^3\sqrt{-ab^3}}{a^2b^2+ab^3}}\right)\sqrt{\frac{ab-\sqrt{-ab^3}}{16(a^2b^2+ab^3)}}-2\operatorname{atanh}\left(\frac{8a^5b^2x\sqrt{\frac{\sqrt{-ab^3}}{16(a^2b^2+ab^3)}+\frac{ab}{16(a^2b^2+ab^3)}}}{\frac{2a^5b^5}{a^2b^2+ab^3}+\frac{2a^6b^4}{a^2b^2+ab^3}+\frac{2a^4b^4\sqrt{-ab^3}}{a^2b^2+ab^3}+\frac{2a^5b^3\sqrt{-ab^3}}{a^2b^2+ab^3}}-\frac{8a^3x\sqrt{\frac{\sqrt{-ab^3}}{16(a^2b^2+ab^3)}+\frac{ab}{16(a^2b^2+ab^3)}}}{\frac{2a^4b^2}{a^2b^2+ab^3}+\frac{2a^3b\sqrt{-ab^3}}{a^2b^2+ab^3}}+\frac{8a^4bx\sqrt{\frac{\sqrt{-ab^3}}{16(a^2b^2+ab^3)}+\frac{ab}{16(a^2b^2+ab^3)}}\sqrt{-ab^3}}{\frac{2a^5b^5}{a^2b^2+ab^3}+\frac{2a^6b^4}{a^2b^2+ab^3}+\frac{2a^4b^4\sqrt{-ab^3}}{a^2b^2+ab^3}+\frac{2a^5b^3\sqrt{-ab^3}}{a^2b^2+ab^3}}\right)\sqrt{\frac{ab+\sqrt{-ab^3}}{16(a^2b^2+ab^3)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b + 2*a*x^2 + a*x^4),x)
```

```
[Out] 2*atanh((8*a^3*x*((a*b)/(16*(a*b^3 + a^2*b^2)) - (-a*b^3)^(1/2)/(16*(a*b^3
+ a^2*b^2)))^(1/2))/((2*a^4*b^2)/(a*b^3 + a^2*b^2) - (2*a^3*b*(-a*b^3)^(1/2
))/(a*b^3 + a^2*b^2)) - (8*a^5*b^2*x*((a*b)/(16*(a*b^3 + a^2*b^2)) - (-a*b^
3)^(1/2)/(16*(a*b^3 + a^2*b^2)))^(1/2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2
*a^6*b^4)/(a*b^3 + a^2*b^2) - (2*a^4*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)
- (2*a^5*b^3*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)) + (8*a^4*b*x*((a*b)/(16*(a*
b^3 + a^2*b^2)) - (-a*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2)))^(1/2)*(-a*b^3)^(1/
2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/(a*b^3 + a^2*b^2) - (2*a^4
*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) - (2*a^5*b^3*(-a*b^3)^(1/2))/(a*b^3
+ a^2*b^2)))*((a*b - (-a*b^3)^(1/2))/(16*(a*b^3 + a^2*b^2)))^(1/2) - 2*atan
h((8*a^5*b^2*x*((-a*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2)) + (a*b)/(16*(a*b^3 +
a^2*b^2)))^(1/2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/(a*b^3 + a^2
*b^2) + (2*a^4*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) + (2*a^5*b^3*(-a*b^3)^
(1/2))/(a*b^3 + a^2*b^2)) - (8*a^3*x*((-a*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2))
 + (a*b)/(16*(a*b^3 + a^2*b^2)))^(1/2))/((2*a^4*b^2)/(a*b^3 + a^2*b^2) + (2
*a^3*b*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)) + (8*a^4*b*x*((-a*b^3)^(1/2)/(16*
(a*b^3 + a^2*b^2)) + (a*b)/(16*(a*b^3 + a^2*b^2)))^(1/2)*(-a*b^3)^(1/2))/((
2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/(a*b^3 + a^2*b^2) + (2*a^4*b^4*(
-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) + (2*a^5*b^3*(-a*b^3)^(1/2))/(a*b^3 + a^2*
b^2)))*((a*b + (-a*b^3)^(1/2))/(16*(a*b^3 + a^2*b^2)))^(1/2)
```

3.914 $\int \frac{1}{x^2(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=433

$$-\frac{1}{(a+b)x}+\frac{\sqrt[4]{a}\left(2\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a}+\sqrt{a+b}}}-\frac{\sqrt[4]{a}\left(2\sqrt{a}+\sqrt{a+b}\right)\tan}{2\sqrt{2}(a+}$$

```
[Out] -1/(a+b)/x+1/8*a^(1/4)*ln(x^2*a^(1/2)+(a+b)^(1/2)-a^(1/4)*x*2^(1/2)*(-a^(1/
2)+(a+b)^(1/2))^(1/2))*(2*a^(1/2)-(a+b)^(1/2))/(a+b)^(3/2)*2^(1/2)/(-a^(1/2
)+(a+b)^(1/2))^(1/2)-1/8*a^(1/4)*ln(x^2*a^(1/2)+(a+b)^(1/2)+a^(1/4)*x*2^(1/
2)*(-a^(1/2)+(a+b)^(1/2))^(1/2))*(2*a^(1/2)-(a+b)^(1/2))/(a+b)^(3/2)*2^(1/2
)/(-a^(1/2)+(a+b)^(1/2))^(1/2)+1/4*a^(1/4)*arctan((-a^(1/4)*x*2^(1/2)+(-a^(
1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))*(2*a^(1/2)+(a+b)^(1/2
))/(a+b)^(3/2)*2^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)-1/4*a^(1/4)*arctan((a^(1
/4)*x*2^(1/2)+(-a^(1/2)+(a+b)^(1/2))^(1/2))/(a^(1/2)+(a+b)^(1/2))^(1/2))*(2
*a^(1/2)+(a+b)^(1/2))/(a+b)^(3/2)*2^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1137, 1183, 648, 632, 210, 642}

$$\frac{\sqrt[4]{a}\left(\sqrt{a+b}+2\sqrt{a}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}+\sqrt{a}}}-\frac{\sqrt[4]{a}\left(\sqrt{a+b}+2\sqrt{a}\right)\text{ArcTan}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}\,x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\,x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}\,x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{1}{x(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(1/((a + b)*x)) + (a^(1/4)*(2*Sqrt[a] + Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a]
 + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[
2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] + Sqrt[
a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt
[a] + Sqrt[a + b]]])/(2*Sqrt[2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]])
+ (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt
[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*(a + b)^(3/2)*Sqrt[-S
qrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*Log[Sqrt[a + b]
 + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2
]*(a + b)^(3/2)*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2(a+b+2ax^2+ax^4)}\,dx = -\frac{1}{(a+b)x} + \frac{\int \frac{-2a-ax^2}{a+b+2ax^2+ax^4}\,dx}{a+b}$$

$$= -\frac{1}{(a+b)x} + \frac{\int \frac{-2\sqrt{2}\,a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}} - \left(-2a+\sqrt{a}\sqrt{a+b}\right)x}{\frac{\sqrt{a+b}}{\sqrt{a}} - \frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}\,x}{\sqrt[4]{a}} + x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}\,(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}} + \cdots$$

$$= -\frac{1}{(a+b)x} + \frac{\left(\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\right)\int \frac{-\frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}} \cdots}{\frac{\sqrt{a+b}}{\sqrt{a}} - \frac{\sqrt{2}\sqrt{-\sqrt{a}+\sqrt{a+\cdots}}}{\sqrt[4]{a}} \cdots}}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}} \cdots$$

$$= -\frac{1}{(a+b)x} + \frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\cdots}\right.}{4\sqrt{2}\,(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}} \cdots$$

$$= -\frac{1}{(a+b)x} + \frac{\sqrt[4]{a}\left(2\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{\cdots}}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right.}{2\sqrt{2}\,(a+b)^{3/2}\sqrt{\sqrt{a}+\sqrt{a+b}}} \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 174, normalized size = 0.40

$$\frac{1}{(-a-b)x} + \frac{\left(ia-\sqrt{a}\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a-i\sqrt{a}\sqrt{b}}\,\sqrt{b}\,(a+b)} + \frac{\left(-ia-\sqrt{a}\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a+i\sqrt{a}\sqrt{b}}\,\sqrt{b}\,(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] 1/((-a - b)*x) + ((I*a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqr
t[a]*Sqrt[b]]])/(2*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*(a + b)) + (((-I)*a
- Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/(2*Sqrt
[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*(a + b))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1261 vs. $2(301) = 602$.

time = 0.07, size = 1262, normalized size = 2.91

method	result
risch	$-\frac{1}{(a+b)x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(b^2a^3+3a^2b^3+3b^4a+b^5\right)\_Z^4+\left(-2a^2b+6a\,b^2\right)\_Z^2+a\right)}\_R\ln\left(\left(\left(-b\,a^4+2b^2a^3+12a^2b^3+14b^4a+5b^5\right)\_R^4\right.\right.\right.}{4}$
default	$-\frac{1}{(a+b)x}-\frac{a\left(-\frac{\left(2\sqrt{2\sqrt{a^2+ab}-2a}\,a^{\frac{5}{2}}+2\sqrt{2\sqrt{a^2+ab}-2a}\,\sqrt{a^2+ab}\,a^{\frac{3}{2}}+2\sqrt{2\sqrt{a^2+ab}-2a}\,a\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)

[Out] -1/(a+b)/x-1/(a+b)*a*(1/4/a/b/(a+b)^(3/2)*(-1/2*(2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^(5/2)+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a^(3/2)+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^(3/2)*b+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a^(1/2)*b-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+b)^(1/2)*a-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+b)^(1/2)*b-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*a^2-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*a*b)/a^(1/2)*ln(-x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-(a+b)^(1/2))+2*(-4*a^2*b-4*a*b^2+1/2*(2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^(5/2)+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a^(3/2)+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^(3/2)*b+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a^(1/2)*b-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+b)^(1/2)*a-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+b)^(1/2)*b-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*a^2-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*a*b)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)/a^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)))+1/4/a/b/(a+b)^(3/2)*(1/2*(2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^(5/2)+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a^(3/2)+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^(3/2)*b+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a^(1/2)*b-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+b)^(1/2)*a-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+b)^(1/2)*b-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*a^2-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)

```
*(a+b)^(1/2)*a*b)/a^(1/2)*ln(x^2*a^(1/2)+x*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+(a
+b)^(1/2))+2*(4*a^2*b+4*a*b^2-1/2*(2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^(5/2)+
2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a^(3/2)+2*(2*(a^2+a*b)^(1/2
)-2*a)^(1/2)*a^(3/2)*b+2*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a^(1
/2)*b-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+b)^(1/2)*a-(2*(a^2+a
*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(a+b)^(1/2)*b-(2*(a^2+a*b)^(1/2)-2*a)^
(1/2)*(a+b)^(1/2)*a^2-(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*a*b)*(2*(a*
(a+b))^(1/2)-2*a)^(1/2)/a^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2
*a)^(1/2)*arctan((2*x*a^(1/2)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+
b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

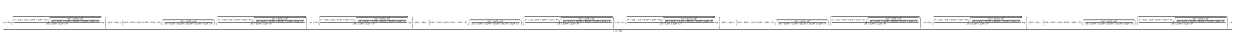
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

```
[Out] -a*integrate((x^2 + 2)/(a*x^4 + 2*a*x^2 + a + b), x)/(a + b) - 1/((a + b)*x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1582 vs. $2(299) = 598$.
time = 0.37, size = 1582, normalized size = 3.65



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

```
[Out] 1/4*((a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt
(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 +
15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))*log(-(3*
a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*sqr
t(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 +
 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3
 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4))) - (a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4
)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*
b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))*lo
g(-(3*a^2 - a*b)*x - (6*a^2*b - 2*a*b^2 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^
5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3
*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2
 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a
```

```
^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4))) + (a + b)*x*sqrt((a^2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 2
0*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^
4))*log(-(3*a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 - (a^4*b + 2*a^3*b^2 - 2*a*b^
4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 +
20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b - (a^3*b + 3*a
^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2
+ 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^
2 + 3*a*b^3 + b^4))) - (a + b)*x*sqrt((a^2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3
*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b
^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^
3 + b^4))*log(-(3*a^2 - a*b)*x - (6*a^2*b - 2*a*b^2 - (a^4*b + 2*a^3*b^2 -
2*a*b^4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*
b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b - (a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^
5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*
a^2*b^2 + 3*a*b^3 + b^4))) - 4)/((a + b)*x)
```

Sympy [**A**]
time = 2.11, size = 134, normalized size = 0.31

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{3}b^{2}+768a^{2}b^{3}+768ab^{4}+256b^{5}\right)+t^{2}\left(-32a^{2}b+96ab^{2}\right)+a,\ \left(t\mapsto t\log\left(x+\frac{-64t^{3}a^{4}b-128t^{3}a^{3}b^{2}+128t^{3}ab^{4}+64t^{3}b^{5}+4ta^{3}-40ta^{2}b+20tab^{2}}{3a^{2}-ab}\right)\right)\right)-\frac{1}{x\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(_t**4*(256*a**3*b**2 + 768*a**2*b**3 + 768*a*b**4 + 256*b**5) + _t*
*2*(-32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b -
128*_t**3*a**3*b**2 + 128*_t**3*a*b**4 + 64*_t**3*b**5 + 4*_t*a**3 - 40*_t*
a**2*b + 20*_t*a*b**2)/(3*a**2 - a*b)))) - 1/(x*(a + b))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 742 vs. 2(299) = 598.
time = 3.39, size = 742, normalized size = 1.71

$$\frac{\left(\left(3\sqrt{a^{2}-\sqrt{-ab}\,a}\sqrt{-ab}\,ab+4\sqrt{a^{2}-\sqrt{-ab}\,a}\sqrt{-ab}\,b^{2}\right)(a+b)^{2}|a|-2\left(3\sqrt{a^{2}-\sqrt{-ab}\,a}\,a^{3}b+7\sqrt{a^{2}-\sqrt{-ab}\,a}\,a^{2}b^{2}+4\sqrt{a^{2}-\sqrt{-ab}\,a}\,ab^{3}\right)|a||-a-b|-\left(3\sqrt{a^{2}-\sqrt{-ab}\,a}\sqrt{-ab}\,a^{4}+10\sqrt{a^{2}-\sqrt{-ab}\,a}\sqrt{-ab}\,a^{3}b+11\sqrt{a^{2}-\sqrt{-ab}\,a}\sqrt{-ab}\,a^{2}b^{2}+4\sqrt{a^{2}-\sqrt{-ab}\,a}\sqrt{-ab}\,ab^{3}\right)|a|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a^{2}+2ab+\sqrt{-4(a^{2}+2ab+b^{2})(a^{2}+ab)+4(a^{2}+ab)^{2}}}{a^{2}+ab}}}\right)}{2\left(3a^{6}b+13a^{5}b^{2}+21a^{4}b^{3}+15a^{3}b^{4}+4a^{2}b^{5}\right)|-a-b|}-\frac{\left(\left(3\sqrt{a^{2}+\sqrt{-ab}\,a}\sqrt{-ab}\,ab+4\sqrt{a^{2}+\sqrt{-ab}\,a}\sqrt{-ab}\,b^{2}\right)(a+b)^{2}|a|+2\left(3\sqrt{a^{2}+\sqrt{-ab}\,a}\,a^{3}b+7\sqrt{a^{2}+\sqrt{-ab}\,a}\,a^{2}b^{2}+4\sqrt{a^{2}+\sqrt{-ab}\,a}\,ab^{3}\right)|a||-a-b|-\left(3\sqrt{a^{2}+\sqrt{-ab}\,a}\sqrt{-ab}\,a^{4}+10\sqrt{a^{2}+\sqrt{-ab}\,a}\sqrt{-ab}\,a^{3}b+11\sqrt{a^{2}+\sqrt{-ab}\,a}\sqrt{-ab}\,a^{2}b^{2}+4\sqrt{a^{2}+\sqrt{-ab}\,a}\sqrt{-ab}\,ab^{3}\right)|a|\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{2a^{2}+2ab-\sqrt{-4(a^{2}+2ab+b^{2})(a^{2}+ab)+4(a^{2}+ab)^{2}}}{a^{2}+ab}}}\right)}{2\left(3a^{6}b+13a^{5}b^{2}+21a^{4}b^{3}+15a^{3}b^{4}+4a^{2}b^{5}\right)|-a-b|}-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

```
[Out] 1/2*((3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 - sqrt(-a*b)*a
)*sqrt(-a*b)*b^2)*(a + b)^2*abs(a) - 2*(3*sqrt(a^2 - sqrt(-a*b)*a)*a^3*b +
7*sqrt(a^2 - sqrt(-a*b)*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^3)*abs(
a)*abs(-a - b) - (3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + 10*sqrt(a^2 -
 sqrt(-a*b)*a)*sqrt(-a*b)*a^3*b + 11*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^
```

```
2*b^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b^3)*abs(a))*arctan(2*sqrt(
1/2)*x/sqrt((2*a^2 + 2*a*b + sqrt(-4*(a^2 + 2*a*b + b^2)*(a^2 + a*b) + 4*(a
^2 + a*b)^2))/(a^2 + a*b)))/((3*a^6*b + 13*a^5*b^2 + 21*a^4*b^3 + 15*a^3*b^
4 + 4*a^2*b^5)*abs(-a - b)) - 1/2*((3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a
*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*b^2)*(a + b)^2*abs(a) + 2*(3*sqr
t(a^2 + sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 + sqrt(-a*b)*a)*a^2*b^2 + 4*sqrt(a
^2 + sqrt(-a*b)*a)*a*b^3)*abs(a)*abs(-a - b) - (3*sqrt(a^2 + sqrt(-a*b)*a)*
sqrt(-a*b)*a^4 + 10*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^3*b + 11*sqrt(a^2
 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)
*a*b^3)*abs(a))*arctan(2*sqrt(1/2)*x/sqrt((2*a^2 + 2*a*b - sqrt(-4*(a^2 + 2
*a*b + b^2)*(a^2 + a*b) + 4*(a^2 + a*b)^2))/(a^2 + a*b)))/((3*a^6*b + 13*a^
5*b^2 + 21*a^4*b^3 + 15*a^3*b^4 + 4*a^2*b^5)*abs(-a - b)) - 1/((a + b)*x)
```

Mupad [B]
time = 5.27, size = 2848, normalized size = 6.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x)
```

```
[Out] - 1/(x*(a + b)) - atan((((-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^
3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*
a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 + x*(-(3*a*b^2 - a^2*b -
3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3
*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*
b^3 + 320*a^8*b^2)) - x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b
^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*
a^2*b^3 + a^3*b^2)))^(1/2)*1i - ((-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) +
b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8
*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 - x*(-(3*a*b^2 -
a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b
^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 +
640*a^7*b^3 + 320*a^8*b^2)) + x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*
(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 +
b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i)/(6*a^6*b + 2*a^7 + ((-(3*a*b^2 - a^2
*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3
+ a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128
*a^7*b^2 + x*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(1
6*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 32
0*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) - x*(8*a^7*b + 4*a^8
- 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b
^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2) + ((-(3*a*b^2
- a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2
*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3
```

+ 128*a^7*b^2 - x*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) + x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b^3 + 6*a^5*b^2))*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*2i - atan((((-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 + x*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) - x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i - ((-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 - x*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) + x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i)/(6*a^6*b + 2*a^7 + ((-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 + x*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) - x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2) + ((-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 - x*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) + x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2) + 2*a^4*b^3 + 6*a^5*b^2))*(-(3*a*b^2 - a^2*b + 3*a*(-a*b^3)^(1/2) - b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*2i

3.915 $\int \frac{x}{1+x^2+x^4} dx$

Optimal. Leaf size=20

$$\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1121, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{1+x^2+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) \\
&= -\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right) \\
&= \frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.95

method	result	size
default	$\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19
risch	$\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.49, size = 18, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Fricas [A]
time = 0.35, size = 18, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{2}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Sympy [A]
time = 0.03, size = 26, normalized size = 1.30

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^{2}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+x**2+1),x)
[Out] sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/3
```

Giac [A]
time = 2.77, size = 18, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{2}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Mupad [B]
time = 0.06, size = 20, normalized size = 1.00

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^{2}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + x^4 + 1),x)
[Out] (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/3
```

3.916 $\int \frac{x}{10+2x^2+x^4} dx$

Optimal. Leaf size=14

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}(1+x^2)\right)$$

```
[Out] 1/6*arctan(1/3*x^2+1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1121, 632, 210}

$$\frac{1}{6}\text{ArcTan}\left(\frac{1}{3}(x^2+1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{10+2x^2+x^4} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{10+2x+x^2} dx, x, x^2\right)\\ &= -\text{Subst}\left(\int \frac{1}{-36-x^2} dx, x, 2(1+x^2)\right)\\ &= \frac{1}{6}\tan^{-1}\left(\frac{1}{3}(1+x^2)\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}(1+x^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Maple [A]
time = 0.01, size = 11, normalized size = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$	11
risch	$\frac{\arctan\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^2+10),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arctan(1/3*x^2+1/3)
```

Maxima [A]
time = 0.50, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="maxima")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Fricas [A]
time = 0.37, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

<u>**Sympy**</u> [**A**]
time = 0.03, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**2+10),x)
```

```
[Out] atan(x**2/3 + 1/3)/6
```

<u>**Giac**</u> [**A**]
time = 4.16, size = 10, normalized size = 0.71

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="giac")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

<u>**Mupad**</u> [**B**]
time = 0.06, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2 + x^4 + 10),x)
```

```
[Out] atan(x^2/3 + 1/3)/6
```

3.917 $\int \frac{x^2}{20+9x^2+x^4} \, dx$

Optimal. Leaf size=23

$$-2\tan^{-1}\left(\frac{x}{2}\right)+\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] -2*arctan(1/2*x)+arctan(1/5*x*5^(1/2))*5^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1144, 209}

$$\sqrt{5}\,\text{ArcTan}\left(\frac{x}{\sqrt{5}}\right)-2\text{ArcTan}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(20 + 9*x^2 + x^4),x]
```

```
[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{20+9x^2+x^4}\,dx &= -\left(4\int \frac{1}{4+x^2}\,dx\right)+5\int \frac{1}{5+x^2}\,dx\\ &= -2\tan^{-1}\left(\frac{x}{2}\right)+\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$-2\tan^{-1}\left(\frac{x}{2}\right)+\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(20 + 9*x^2 + x^4),x]
[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.83

method	result	size
default	$-2\arctan\left(\frac{x}{2}\right)+\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$	19
risch	$-2\arctan\left(\frac{x}{2}\right)+\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+9*x^2+20),x,method=_RETURNVERBOSE)
[Out] -2*arctan(1/2*x)+arctan(1/5*x*5^(1/2))*5^(1/2)
```

Maxima [A]
time = 0.49, size = 18, normalized size = 0.78

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-2\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="maxima")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

Fricas [A]
time = 0.37, size = 18, normalized size = 0.78

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-2\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="fricas")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.87

$$-2\operatorname{atan}\left(\frac{x}{2}\right)+\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\,x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+9*x**2+20),x)
[Out] -2*atan(x/2) + sqrt(5)*atan(sqrt(5)*x/5)
```

Giac [A]
time = 4.09, size = 18, normalized size = 0.78

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,x\right)-2\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+9*x^2+20),x, algorithm="giac")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

Mupad [B]
time = 4.37, size = 18, normalized size = 0.78

$$\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\,x}{5}\right)-2\operatorname{atan}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(9*x^2 + x^4 + 20),x)
[Out] 5^(1/2)*atan((5^(1/2)*x)/5) - 2*atan(x/2)
```

3.918 $\int \frac{x^2}{1-x^2+x^4} dx$

Optimal. Leaf size=74

$$-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}$$

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))+1/12*ln(1+x^2-x*3^(1/2))*3^
(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1141, 1175, 632, 210, 1178, 642}

$$-\frac{1}{2}\text{ArcTan}\left(\sqrt{3}-2x\right)+\frac{1}{2}\text{ArcTan}\left(2x+\sqrt{3}\right)+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^2 + x^4),x]
```

```
[Out] -1/2*ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]/2 + Log[1 - Sqrt[3]*x +
x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
```

```
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{1-x^2+x^4}\,dx = -\left(\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1-x^2+x^4}\,dx$$
$$=\frac{1}{4}\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}\,x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}\,x-x^2}}{4\sqrt{3}}$$
$$=\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2\right.$$
$$=-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\right.}{4\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 94, normalized size = 1.27

$$\frac{\sqrt{-1-i\sqrt{3}}\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+\sqrt{-1+i\sqrt{3}}\left(-i+\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^2 + x^4),x]
```

[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2] + Sqrt[-1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/(2*Sqrt[6])

Maple [A]
time = 0.03, size = 69, normalized size = 0.93

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2+1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^3-\_R}\right)}{2}$	38
default	$-\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^2-x\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x-\sqrt{3}\right)\right)}{6}-\frac{\sqrt{3}\left(\frac{\ln\left(1+x^2+x\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x+\sqrt{3}\right)\right)}{6}$	69

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^4-x^2+1),x,method=_RETURNVERBOSE)

[Out] -1/6*3^(1/2)*(-1/2*ln(1+x^2-x*3^(1/2))-3^(1/2)*arctan(2*x-3^(1/2)))-1/6*3^(1/2)*(1/2*ln(1+x^2+x*3^(1/2))-3^(1/2)*arctan(2*x+3^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^4-x^2+1),x, algorithm="maxima")

[Out] integrate(x^2/(x^4 - x^2 + 1), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(56) = 112.
time = 0.39, size = 163, normalized size = 2.20

$$-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{18}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-18\sqrt{6}\sqrt{2}x+36x^2+36}+\sqrt{3}\right)-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{24}\sqrt{6}\sqrt{2}\log\left(18\sqrt{6}\sqrt{2}x+36x^2+36\right)+\frac{1}{24}\sqrt{6}\sqrt{2}\log\left(-18\sqrt{6}\sqrt{2}x+36x^2+36\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^4-x^2+1),x, algorithm="fricas")

[Out] -1/6*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/18*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36) + sqrt(3)) - 1/6*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/24*sqrt(6)*sqrt(2)*log(18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36) + 1/24*sqrt(6)*sqrt(2)*log(-18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36)

Sympy [A]
time = 0.07, size = 63, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-x**2+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/12
 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

Giac [A]
time = 3.70, size = 56, normalized size = 0.76

$$-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)+\frac{1}{2}\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{2}\arctan\left(2\,x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + 1/2*arctan(2*x + sqrt(3)) + 1/2*arctan(2*x - sqrt(3))
```

Mupad [B]
time = 0.08, size = 44, normalized size = 0.59

$$-\operatorname{atan}\left(\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\operatorname{atan}\left(\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 - x^2 + 1),x)
```

```
[Out] atan(x/2 + (3^(1/2)*x*1i)/2)*((3^(1/2)*1i)/6 + 1/2) - atan(x/2 - (3^(1/2)*x
*1i)/2)*((3^(1/2)*1i)/6 - 1/2)
```

3.919 $\int \frac{x^2}{2-2x^2+x^4} \, dx$

Optimal. Leaf size=188

$$-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)+\text{-}$$

```
[Out] -1/4*arctan((-2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^
(1/2)+1/4*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/
2))^(1/2)+1/4*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)-1/4
*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1141, 1175, 632, 210, 1178, 642}

$$-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - 2*x^2 + x^4),x]
```

```
[Out] -1/2*(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt[2*(-1
 + Sqrt[2])]]) + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x
)/Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]
/(4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(
4*Sqrt[2*(1 + Sqrt[2])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{2-2x^2+x^4}\,dx = -\left(\frac{1}{2}\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx+$$

$$= \frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\text{Sub}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2}-\sqrt{2\left(\right.}\right.}{4\sqrt{2\left(1\right.}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - 2*x^2 + x^4),x]
```

```
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)^
(3/2)
```

Maple [A]
time = 0.05, size = 168, normalized size = 0.89

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-2\_Z^2+2\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3-\_R}\right)}{4}$

default	$\frac{\sqrt{2+2\sqrt{2}}\,\left(\sqrt{2}-1\right)\left(\frac{\ln\left(x^{2}+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{\sqrt{2+2\sqrt{2}}\,\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{4}-\ldots\sqrt{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4-2*x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2+2*2^(1/2))^(1/2)*(2^(1/2)-1)*(1/2*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/
2))+(2+2*2^(1/2))^(1/2)/(-2+2*2^(1/2))^(1/2)*arctan((2*x-(2+2*2^(1/2))^(1/2
))/(-2+2*2^(1/2))^(1/2)))-1/4*(2+2*2^(1/2))^(1/2)*(2^(1/2)-1)*(1/2*ln(x^2+2
^(1/2)+x*(2+2*2^(1/2))^(1/2))-(2+2*2^(1/2))^(1/2)/(-2+2*2^(1/2))^(1/2)*arct
an((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^4 - 2*x^2 + 2), x)
```

Fricas [A]
time = 0.39, size = 240, normalized size = 1.28

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+x^{2}+\sqrt{2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+x^{2}+\sqrt{2}\right)-\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{2\sqrt{2}+4}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^{2}+2\sqrt{2}}\sqrt{2\sqrt{2}+4}-\sqrt{2}-1\right)-\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{2\sqrt{2}+4}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^{2}+2\sqrt{2}}\sqrt{2\sqrt{2}+4}+\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(1/2*2^(3/4)*x*sqrt(2*sqr
t(2) + 4) + x^2 + sqrt(2)) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)
*log(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + x^2 + sqrt(2)) - 1/4*2^(3/4)*sqrt
(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 1/2*2^(1/4)*sqr
t(2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2
*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2
*sqrt(2))*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1)
```

Sympy [A]
time = 0.28, size = 24, normalized size = 0.13

$$\operatorname{RootSum}\left(128t^{4}+16t^{2}+1,\left(t\mapsto t\log\left(64t^{3}+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4-2*x**2+2),x)
```

```
[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))
```

Giac [A]
time = 5.02, size = 147, normalized size = 0.78

$$\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x+2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2+2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)+\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2-2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4-2*x^2+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(sqrt(2) + 2)
)/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x - 2
^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 + 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2)) + 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 - 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2))
```

Mupad [B]
time = 4.37, size = 101, normalized size = 0.54

$$\operatorname{atanh}\left(32x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)+\operatorname{atanh}\left(32x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 - 2*x^2 + 2),x)
```

```
[Out] atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*
(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2)) + atanh(32*x*((-
 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32
- 1/32)^(1/2) - 2*(2^(1/2)/32 - 1/32)^(1/2))
```

3.920 $\int x^7 \sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=171

$$-\frac{b(7b^2-12ac)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}+\frac{\left(35b^2-32ac-42bcx^2\right)\left(a+bx^2+cx^4\right)}{480c^3}$$

```
[Out] 1/10*x^4*(c*x^4+b*x^2+a)^(3/2)/c+1/480*(-42*b*c*x^2-32*a*c+35*b^2)*(c*x^4+b
*x^2+a)^(3/2)/c^3+1/512*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2
+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(9/2)-1/256*b*(-12*a*c+7*b^2)*(2*c*x^2
+b)*(c*x^4+b*x^2+a)^(1/2)/c^4
```

Rubi [A]
time = 0.11, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1128, 756, 793, 626, 635, 212}

$$\frac{b(7b^2-12ac)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{9/2}}-\frac{b(7b^2-12ac)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{\left(-32ac+35b^2-42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{480c^3}+\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -1/256*(b*(7*b^2 - 12*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c^4 + (x^
4*(a + b*x^2 + c*x^4)^(3/2))/(10*c) + ((35*b^2 - 32*a*c - 42*b*c*x^2)*(a +
b*x^2 + c*x^4)^(3/2))/(480*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x^3\sqrt{a+bx+cx^2}\,dx,x,x^2\right)$$
$$= \frac{x^4(a+bx^2+cx^4)^{3/2}}{10c} + \frac{\text{Subst}\left(\int x\left(-2a-\frac{7bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{10c}$$
$$= \frac{x^4(a+bx^2+cx^4)^{3/2}}{10c} + \frac{(35b^2-32ac-42bcx^2)(a+bx^2+cx^4)^{3/2}}{480c^3} - \frac{(b(7b^2-1}{}$$
$$= -\frac{b(7b^2-12ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{256c^4} + \frac{x^4(a+bx^2+cx^4)^{3/2}}{10c} + \frac{(35b^2-3}{}$$
$$= -\frac{b(7b^2-12ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{256c^4} + \frac{x^4(a+bx^2+cx^4)^{3/2}}{10c} + \frac{(35b^2-3}{}$$
$$= -\frac{b(7b^2-12ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{256c^4} + \frac{x^4(a+bx^2+cx^4)^{3/2}}{10c} + \frac{(35b^2-3}{}$$

Mathematica [A]
time = 0.30, size = 166, normalized size = 0.97

$$\frac{\sqrt{a+bx^2+cx^4}\,(-105b^4+70b^3cx^2+4b^2c(115a-14cx^4)+8bc^2x^2(-29a+6cx^4)+128c^2(-2a^2+acx^4+3c^2x^8))}{3840c^4} - \frac{(7b^5-40ab^3c+48a^2bc^2)\log\left(c^4\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)\right)}{512c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-105*b^4 + 70*b^3*c*x^2 + 4*b^2*c*(115*a - 14*c*x
^4) + 8*b*c^2*x^2*(-29*a + 6*c*x^4) + 128*c^2*(-2*a^2 + a*c*x^4 + 3*c^2*x^8
)))/(3840*c^4) - ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*Log[c^4*(b + 2*c*x^2
- 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])])/(512*c^(9/2))
```

Maple [A]
time = 0.07, size = 296, normalized size = 1.73

method	result
risch	$-\frac{(-384c^4x^8-48bc^3x^6-128ac^3x^4+56b^2c^2x^4+232abc^2x^2-70b^3cx^2+256a^2c^2-460ab^2c+105b^4)\sqrt{cx^4+bx^2+a}}{3840c^4}+\frac{3ba^2}{}$
default	$\frac{x^4(cx^4+bx^2+a)^{\frac{3}{2}}}{10c}-\frac{7bx^2(cx^4+bx^2+a)^{\frac{3}{2}}}{80c^2}+\frac{7b^2(cx^4+bx^2+a)^{\frac{3}{2}}}{96c^3}-\frac{7b^3\sqrt{cx^4+bx^2+a}\,x^2}{128c^3}-\frac{7b^4\sqrt{cx^4+bx^2-}}{256c^4}$

elliptic	$\frac{x^{4} \left(c\,x^{4}+b\,x^{2}+a \right)^{\frac{3}{2}}}{10 c}-\frac{7 b\,x^{2} \left(c\,x^{4}+b\,x^{2}+a \right)^{\frac{3}{2}}}{80 c^{2}}+\frac{7 b^{2} \left(c\,x^{4}+b\,x^{2}+a \right)^{\frac{3}{2}}}{96 c^{3}}-\frac{7 b^{3} \sqrt{c\,x^{4}+b\,x^{2}+a}\, x^{2}}{128 c^{3}}-\frac{7 b^{4} \sqrt{c\,x^{4}+b\,x^{2}+}}{256 c^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x^4*(c*x^4+b*x^2+a)^(3/2)/c-7/80*b/c^2*x^2*(c*x^4+b*x^2+a)^(3/2)+7/96*
b^2/c^3*(c*x^4+b*x^2+a)^(3/2)-7/128*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)*x^2-7/256
*b^4/c^4*(c*x^4+b*x^2+a)^(1/2)-5/64*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c
*x^4+b*x^2+a)^(1/2))*a+7/512*b^5/c^(9/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*
x^2+a)^(1/2))+3/32*b/c^2*a*(c*x^4+b*x^2+a)^(1/2)*x^2+3/64*b^2/c^3*a*(c*x^4+
b*x^2+a)^(1/2)+3/32*b/c^(5/2)*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^
(1/2))-1/15*a/c^2*(c*x^4+b*x^2+a)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 367, normalized size = 2.15

$$\left[\frac{15\,(7\,b^{5}-40\,ab^{3}c+48\,a^{2}bc^{2})\sqrt{c}\,\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}-4\,\sqrt{cx^{4}+bx^{2}+a}\,(2\,cx^{2}+b)\sqrt{c}-4\,ac\right)+4\,(384\,c^{5}x^{8}+48\,bc^{4}x^{6}-105\,b^{4}c+460\,ab^{2}c^{2}-256\,a^{2}c^{3}-8\,(7\,b^{2}c^{3}-16\,ac^{4})x^{4}+2\,(35\,b^{3}c^{2}-116\,abc^{3})x^{2})\sqrt{cx^{4}+bx^{2}+a}}{15360\,c^{5}},\,-\frac{15\,(7\,b^{5}-40\,ab^{3}c+48\,a^{2}bc^{2})\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(2\,cx^{2}+b)\sqrt{-c}}{2\,(c^{2}x^{4}+bcx^{2}+ac)}\right)-2\,(384\,c^{5}x^{8}+48\,bc^{4}x^{6}-105\,b^{4}c+460\,ab^{2}c^{2}-256\,a^{2}c^{3}-8\,(7\,b^{2}c^{3}-16\,ac^{4})x^{4}+2\,(35\,b^{3}c^{2}-116\,abc^{3})x^{2})\sqrt{cx^{4}+bx^{2}+a}}{7680\,c^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/15360*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8
*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) +
 4*(384*c^5*x^8 + 48*b*c^4*x^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 -
8*(7*b^2*c^3 - 16*a*c^4)*x^4 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^2)*sqrt(c*x^4
 + b*x^2 + a))/c^5, -1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(-c
)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*
x^2 + a*c)) - 2*(384*c^5*x^8 + 48*b*c^4*x^6 - 105*b^4*c + 460*a*b^2*c^2 - 2
56*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^4 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^
2)*sqrt(c*x^4 + b*x^2 + a))/c^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 3.91, size = 172, normalized size = 1.01

$$\frac{1}{3840}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-460ab^2c+256a^2c^2}{c^4}\right)-\frac{(7b^5-40ab^3c+48a^2bc^2)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{512c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16
*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*b^2
*c + 256*a^2*c^2)/c^4) - 1/512*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(abs(
-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2)
```

Mupad [B]
time = 5.31, size = 315, normalized size = 1.84

$$\frac{x^4\left(cx^4+bx^2+a\right)^{3/2}}{10c}+\frac{7b\left(\frac{a\left(\left(\frac{b}{4c}+\frac{x^2}{2}\right)\sqrt{cx^4+bx^2+a}+\frac{\ln\left(\sqrt{cx^4+bx^2+a}+\frac{cx^2+\frac{b}{2}}{\sqrt{c}}\right)\left(ac-\frac{b^2}{4}\right)}{2c^{3/2}}\right)}{4c}-\frac{x^2\left(cx^4+bx^2+a\right)^{3/2}}{4c}+\frac{5b\left(\frac{\left(8c\left(cx^4+a\right)-3b^2+2bcx^2\right)\sqrt{cx^4+bx^2+a}}{24c^2}+\frac{\ln\left(2\sqrt{cx^4+bx^2+a}+\frac{2cx^2+b}{\sqrt{c}}\right)\left(b^3-4abc\right)}{16c^{5/2}}\right)}{8c}\right)}{20c}-\frac{a\left(\frac{\left(8c\left(cx^4+a\right)-3b^2+2bcx^2\right)\sqrt{cx^4+bx^2+a}}{24c^2}+\frac{\ln\left(2\sqrt{cx^4+bx^2+a}+\frac{2cx^2+b}{\sqrt{c}}\right)\left(b^3-4abc\right)}{16c^{5/2}}\right)}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(3/2))/(10*c) + (7*b*((a*((b/(4*c) + x^2/2)*(a + b
*x^2 + c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2
))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^2*(a + b*x^2 + c*x^4)^(3/2))/(4*
c) + (5*b*(((8*c*(a + c*x^4) - 3*b^2 + 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(1/2)
)/(24*c^2) + (log(2*(a + b*x^2 + c*x^4)^(1/2) + (b + 2*c*x^2)/c^(1/2))*(b^3
 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(20*c) - (a*(((8*c*(a + c*x^4) - 3*b^2
+ 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(24*c^2) + (log(2*(a + b*x^2 + c*x^
4)^(1/2) + (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(5*c)
```

3.921 $\int x^5 \sqrt{a+bx^2+cx^4}\, dx$

Optimal. Leaf size=153

$$\frac{(5b^2-4ac)\,(b+2cx^2)\,\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b(a+bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(a+bx^2+cx^4)^{3/2}}{8c}-\frac{(b^2-4ac)\,(5b^2-4ac)}{}$$

```
[Out] -5/48*b*(c*x^4+b*x^2+a)^(3/2)/c^2+1/8*x^2*(c*x^4+b*x^2+a)^(3/2)/c-1/256*(-4
*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1
/2))/c^(7/2)+1/128*(-4*a*c+5*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^3
```

Rubi [A]
time = 0.09, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$,
Rules used = {1128, 756, 654, 626, 635, 212}

$$-\frac{(b^2-4ac)\,(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{(5b^2-4ac)\,(b+2cx^2)\,\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b(a+bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(a+bx^2+cx^4)^{3/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^3) - (5*b*(a
 + b*x^2 + c*x^4)^(3/2))/(48*c^2) + (x^2*(a + b*x^2 + c*x^4)^(3/2))/(8*c) -
 ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(256*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int x^5\sqrt{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{x^2(a+bx^2+cx^4)^{3/2}}{8c}+\frac{\text{Subst}\left(\int\left(-a-\frac{5bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{8c}\\
&= -\frac{5b(a+bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(a+bx^2+cx^4)^{3/2}}{8c}+\frac{(5b^2-4ac)\,\text{Subst}\left(\int\sqrt{a+bx\right.}{32c^2}\\
&= \frac{(5b^2-4ac)\,(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b(a+bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(a+bx^2+}{8c}\\
&= \frac{(5b^2-4ac)\,(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b(a+bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(a+bx^2+}{8c}\\
&= \frac{(5b^2-4ac)\,(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b(a+bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(a+bx^2+}{8c}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 132, normalized size = 0.86

$$\frac{\sqrt{a+bx^2+cx^4}\,(15b^3-52abc-10b^2cx^2+24ac^2x^2+8bc^2x^4+48c^3x^6)}{384c^3}+\frac{(5b^4-24ab^2c+16a^2c^2)\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)}{256c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(15*b^3 - 52*a*b*c - 10*b^2*c*x^2 + 24*a*c^2*x^2 +
 8*b*c^2*x^4 + 48*c^3*x^6))/(384*c^3) + ((5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(256*c^(7/2))
```

Maple [A]
time = 0.04, size = 247, normalized size = 1.61

method	result
risch	$-\frac{(-48c^3x^6-8bc^2x^4-24c^2ax^2+10b^2cx^2+52abc-15b^3)\sqrt{cx^4+bx^2+a}}{384c^3}-\frac{a^2\ln\left(\frac{\frac{b}{2}+cx^2}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{16c^{\frac{3}{2}}}+$
default	$\frac{x^2(cx^4+bx^2+a)^{\frac{3}{2}}}{8c}-\frac{5b(cx^4+bx^2+a)^{\frac{3}{2}}}{48c^2}+\frac{5b^2\sqrt{cx^4+bx^2+a}\,x^2}{64c^2}+\frac{5b^3\sqrt{cx^4+bx^2+a}}{128c^3}+\frac{3b^2\ln\left(\frac{\frac{b}{2}+cx^2}{\sqrt{c}}+\sqrt{\right.}}{}$
elliptic	$\frac{x^2(cx^4+bx^2+a)^{\frac{3}{2}}}{8c}-\frac{5b(cx^4+bx^2+a)^{\frac{3}{2}}}{48c^2}+\frac{5b^2\sqrt{cx^4+bx^2+a}\,x^2}{64c^2}+\frac{5b^3\sqrt{cx^4+bx^2+a}}{128c^3}+\frac{3b^2\ln\left(\frac{\frac{b}{2}+cx^2}{\sqrt{c}}+\sqrt{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^2*(c*x^4+b*x^2+a)^(3/2)/c-5/48*b*(c*x^4+b*x^2+a)^(3/2)/c^2+5/64*b^2/c
^2*(c*x^4+b*x^2+a)^(1/2)*x^2+5/128*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)+3/32*b^2/c
^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-5/256*b^4/c^(7/2)*
ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/16*a/c*(c*x^4+b*x^2+a)^(1
/2)*x^2-1/32*a/c^2*(c*x^4+b*x^2+a)^(1/2)*b-1/16*a^2/c^(3/2)*ln((1/2*b+c*x^2
)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 303, normalized size = 1.98

$$\left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}-4\,ac\right)+4\,(48\,c^4x^6+8\,bc^3x^4+15\,b^3c-52\,abc^2-2\,(5\,b^2c^2-12\,ac^3)x^2)\sqrt{cx^4+bx^2+a}}{1536\,c^4},\ \frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)+2\,(48\,c^4x^6+8\,bc^3x^4+15\,b^3c-52\,abc^2-2\,(5\,b^2c^2-12\,ac^3)x^2)\sqrt{cx^4+bx^2+a}}{768\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c
*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(
48*c^4*x^6 + 8*b*c^3*x^4 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)
*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c
^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2
*x^4 + b*c*x^2 + a*c)) + 2*(48*c^4*x^6 + 8*b*c^3*x^4 + 15*b^3*c - 52*a*b*c^
2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 3.91, size = 134, normalized size = 0.88

$$\frac{1}{384}\,\sqrt{cx^4+bx^2+a}\,\left(2\,\left(4\,\left(6\,x^2+\frac{b}{c}\right)x^2-\frac{5\,b^2c-12\,ac^2}{c^3}\right)x^2+\frac{15\,b^3-52\,abc}{c^3}\right)+\frac{(5\,b^4-24\,ab^2c+16\,a^2c^2)\log\left(\left|-2\,\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*c^2
)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 1/256*(5*b^4 - 24*a*b^2*c + 16*a^2*
c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(7/
2)
```

Mupad [B]
time = 4.64, size = 193, normalized size = 1.26

$$\frac{x^2\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{8\,c}-\frac{a\,\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2+a}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{8\,c}-\frac{5\,b\,\left(\frac{\left(8\,c\,\left(c\,x^4+a\right)-3\,b^2+2\,b\,c\,x^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^4+b\,x^2+a}+\frac{2\,c\,x^2+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (x^2*(a + b*x^2 + c*x^4)^(3/2))/(8*c) - (a*((b/(4*c) + x^2/2)*(a + b*x^2 +
c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))*(a*c
 - b^2/4))/(2*c^(3/2))))/(8*c) - (5*b*(((8*c*(a + c*x^4) - 3*b^2 + 2*b*c*x^
2)*(a + b*x^2 + c*x^4)^(1/2))/(24*c^2) + (log(2*(a + b*x^2 + c*x^4)^(1/2) +
 (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(16*c)
```

3.922 $\int x^3\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=108

$$-\frac{b(b+2cx^2)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{(a+bx^2+cx^4)^{3/2}}{6c}+\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}$$

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c+1/32*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(
1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)-1/16*b*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2
)/c^2
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1128, 654, 626, 635, 212}

$$\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}-\frac{b(b+2cx^2)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{(a+bx^2+cx^4)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -1/16*(b*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c^2 + (a + b*x^2 + c*x^4)^(
3/2)/(6*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4])])/(32*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx=\frac{1}{2}\text{Subst}\left(\int x\sqrt{a+bx+cx^2}\,dx,x,x^2\right)$$

$$=\frac{(a+bx^2+cx^4)^{3/2}}{6c}-\frac{b\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^2\right)}{4c}$$

$$=-\frac{b(b+2cx^2)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{(a+bx^2+cx^4)^{3/2}}{6c}+\frac{(b(b^2-4ac))\,\text{Subst}\left(\int\frac{}{\sqrt{}}\right.}{32c}$$

$$=-\frac{b(b+2cx^2)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{(a+bx^2+cx^4)^{3/2}}{6c}+\frac{(b(b^2-4ac))\,\text{Subst}\left(\int\frac{1}{4c-}\right.}{1}$$

$$=-\frac{b(b+2cx^2)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{(a+bx^2+cx^4)^{3/2}}{6c}+\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{32c^{5/2}}$$

Mathematica [A]
time = 0.20, size = 103, normalized size = 0.95

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(-3b^2+2bcx^2+8c(a+cx^4)\right)-3(b^3-4abc)\log\left(c^2\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{96c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^2 + c*x^4],x]
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 2*b*c*x^2 + 8*c*(a + c*x^4)) -
 3*(b^3 - 4*a*b*c)*Log[c^2*(b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]
)])/(96*c^(5/2))
```

Maple [A]

time = 0.04, size = 139, normalized size = 1.29

method	result
risch	$\frac{(8c^2x^4+2bc\,x^2+8ac-3b^2)\sqrt{c\,x^4+b\,x^2+a}}{48c^2}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)a}{8c^{\frac{3}{2}}}+\frac{b^3\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{32c^{\frac{5}{2}}}$
default	$\frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{6c}-\frac{b\,x^2\sqrt{c\,x^4+b\,x^2+a}}{8c}-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{16c^2}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)a}{8c^{\frac{3}{2}}}+$
elliptic	$\frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{6c}-\frac{b\,x^2\sqrt{c\,x^4+b\,x^2+a}}{8c}-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{16c^2}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)a}{8c^{\frac{3}{2}}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c-1/8*b/c*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16*b^2/c^2*
(c*x^4+b*x^2+a)^(1/2)-1/8*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))*a+1/32*b^3/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]

time = 0.37, size = 237, normalized size = 2.19

$$\left[-\frac{3\,(b^3-4\,abc)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}-4\,ac\right)-4\,(8\,c^3x^4+2\,bc^2x^2-3\,b^2c+8\,ac^2)\sqrt{cx^4+bx^2+a}}{192\,c^3},-\frac{3\,(b^3-4\,abc)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-2\,(8\,c^3x^4+2\,bc^2x^2-3\,b^2c+8\,ac^2)\sqrt{cx^4+bx^2+a}}{96\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqr
t(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^4 + 2*b*c^
2*x^2 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/96*(3*(b^3 - 4*
```

```
a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(
c^2*x^4 + b*c*x^2 + a*c)) - 2*(8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c + 8*a*c^2)
*sqrt(c*x^4 + b*x^2 + a))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 3.95, size = 98, normalized size = 0.91

$$\frac{1}{48}\sqrt{cx^4 + bx^2 + a}\left(2\left(4\,x^2 + \frac{b}{c}\right)x^2 - \frac{3\,b^2 - 8\,ac}{c^2}\right) - \frac{(b^3 - 4\,abc)\log\left(\left|-2\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{32\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) -
1/32*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqr
t(c) - b))/c^(5/2)
```

Mupad [B]
time = 4.52, size = 87, normalized size = 0.81

$$\frac{\left(8\,c\,\left(c\,x^4 + a\right) - 3\,b^2 + 2\,b\,c\,x^2\right)\,\sqrt{c\,x^4 + b\,x^2 + a}}{48\,c^2} + \frac{\ln\left(2\,\sqrt{c\,x^4 + b\,x^2 + a} + \frac{2\,c\,x^2 + b}{\sqrt{c}}\right)\,\left(b^3 - 4\,a\,b\,c\right)}{32\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((8*c*(a + c*x^4) - 3*b^2 + 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(48*c^2)
+ (log(2*(a + b*x^2 + c*x^4)^(1/2) + (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*c)
)/(32*c^(5/2))
```

3.923 $\int x\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=83

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

```
[Out] -1/16*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c
^(3/2)+1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1121, 626, 635, 212}

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16c}\\ &= \frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8c}\\ &= \frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c} - \frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 85, normalized size = 1.02

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c} + \frac{(b^2-4ac)\log\left(bc+2c^2x^2-2c^{3/2}\sqrt{a+bx^2+cx^4}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) + ((b^2 - 4*a*c)*Log[b*c + 2*
c^2*x^2 - 2*c^(3/2)*Sqrt[a + b*x^2 + c*x^4]])/(16*c^(3/2))
```

Maple [A]
time = 0.04, size = 101, normalized size = 1.22

method	result	size
default	$\frac{(2c\,x^2+b)\sqrt{c\,x^4+b\,x^2+a}}{8c}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)a}{4\sqrt{c}}-\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)b^2}{16c^{\frac{3}{2}}}$	101
risch	$\frac{(2c\,x^2+b)\sqrt{c\,x^4+b\,x^2+a}}{8c}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)a}{4\sqrt{c}}-\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)b^2}{16c^{\frac{3}{2}}}$	101

elliptic	$\frac{(2c x^2+b)\sqrt{c x^4+b x^2+a}}{8c}+\frac{\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)a}{4\sqrt{c}}-\frac{\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)b^2}{16c^{\frac{3}{2}}}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c+1/4/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2
)+(c*x^4+b*x^2+a)^(1/2))*a-1/16/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))*b^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 197, normalized size = 2.37

$$\left[-\frac{(b^2-4ac)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\,(2cx^2+b)\sqrt{c}-4ac\right)-4\sqrt{cx^4+bx^2+a}\,(2c^2x^2+bc)}{32c^2}, \frac{(b^2-4ac)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2cx^2+b)\sqrt{-c}}{2(c^2x^4+bcx^2+ac)}\right)+2\sqrt{cx^4+bx^2+a}\,(2c^2x^2+bc)}{16c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x
^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a)*
(2*c^2*x^2 + b*c))/c^2, 1/16*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x
^4 + b*x^2 + a)*(2*c^2*x^2 + b*c))/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 3.71, size = 76, normalized size = 0.92

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{(b^2-4ac)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(-2*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2)
```

Mupad [B]
time = 4.62, size = 72, normalized size = 0.87

$$\frac{\left(\frac{b}{4c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2+a}}{2}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b/(4*c) + x^2/2)*(a + b*x^2 + c*x^4)^(1/2))/2 + (log((a + b*x^2 + c*x^4)^
(1/2) + (b/2 + c*x^2)/c^(1/2))*(a*c - b^2/4))/(4*c^(3/2))
```

3.924 $\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx$

Optimal. Leaf size=109

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)+1/4*b*a
rctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/2*(c*x^4+b*
x^2+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1128, 748, 857, 635, 212, 738}

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/2 - (Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[
a + b*x^2 + c*x^4])])/2 + (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(4*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right) \\
&= \frac{1}{2}\sqrt{a+bx^2+cx^4} - \frac{1}{4}\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\sqrt{a+bx^2+cx^4} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) + \frac{1}{4}b\text{Subst}\Bigg(\int \frac{}{\sqrt{a}} \\
&= \frac{1}{2}\sqrt{a+bx^2+cx^4} - a\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right) + \frac{1}{2}b\text{Subst}\Bigg(\int \\
&= \frac{1}{2}\sqrt{a+bx^2+cx^4} - \frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) + \frac{b\tanh^{-1}\Bigg(\frac{}{2\sqrt{c}\sqrt{a}}}{4\sqrt{c}}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 104, normalized size = 0.95

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}+\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)-\frac{b\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/2 + Sqrt[a]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 +
 c*x^4])/Sqrt[a]] - (b*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]
)/(4*Sqrt[c])
```

Maple [A]
time = 0.03, size = 91, normalized size = 0.83

method	result	s
default	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2}+\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4\sqrt{c}}-\frac{\sqrt{a}\,\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2}$	9
elliptic	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2}+\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4\sqrt{c}}-\frac{\sqrt{a}\,\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)+1/4*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1
/2))/c^(1/2)-1/2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2
)
```

Maple is followed by **Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 566, normalized size = 5.19

$$\left[\frac{b\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\,(2cx^2+b)\sqrt{c}-4ac\right)+2\sqrt{a}\,c\log\left(-\frac{(b^2+4ac)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\,(bx^2+2a)\sqrt{a}+8a^2}{x^4}\right)+4\sqrt{cx^4+bx^2+a}\,c}{8c},\ -\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2cx^2+b)\sqrt{-c}}{2(c^2x^4+bcx^2+ac)}\right)-\sqrt{a}\,c\log\left(-\frac{(b^2+4ac)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}\,(bx^2+2a)\sqrt{a}+8a^2}{x^4}\right)-2\sqrt{cx^4+bx^2+a}\,c}{4c},\ \frac{4\sqrt{-a}\,c\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2a)\sqrt{-a}}{2(acx^4+abx^2+a^2)}\right)+b\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\,(2cx^2+b)\sqrt{c}-4ac\right)+4\sqrt{cx^4+bx^2+a}\,c}{8c},\ \frac{2\sqrt{-a}\,c\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2a)\sqrt{-a}}{2(acx^4+abx^2+a^2)}\right)-b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2cx^2+b)\sqrt{-c}}{2(c^2x^4+bcx^2+ac)}\right)+2\sqrt{cx^4+bx^2+a}\,c}{4c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4
*sqrt(c*x^4 + b*x^2 + a)*c)/c, -1/4*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x
^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(a)*c*log(-
((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sq
rt(a) + 8*a^2)/x^4) - 2*sqrt(c*x^4 + b*x^2 + a)*c)/c, 1/8*(4*sqrt(-a)*c*arc
tan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 +
 a^2)) + b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*c)/c, 1/4*(
2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c
*x^4 + a*b*x^2 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c
*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*c
)/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [B]
time = 4.42, size = 88, normalized size = 0.81

$$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2}-\frac{\sqrt{a}\,\ln\left(\frac{b}{2}+\frac{a}{x^2}+\frac{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2}+\frac{b\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x,x)
```

```
[Out] (a + b*x^2 + c*x^4)^(1/2)/2 - (a^(1/2)*log(b/2 + a/x^2 + (a^(1/2)*(a + b*x^
2 + c*x^4)^(1/2))/x^2))/2 + (b*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2
)/c^(1/2)))/(4*c^(1/2))
```

3.925 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3} \, dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

```
[Out] -1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)+1/2*a
rctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*c^(1/2)-1/2*(c*x^4+b*
x^2+a)^(1/2)/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1128, 746, 857, 635, 212, 738}

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/x^2 - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt
[a + b*x^2 + c*x^4])])/(4*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}+\frac{1}{4}\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}+\frac{1}{4}b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)+\frac{1}{2}c\text{Subst}\Bigg(\int \frac{}{\sqrt{}}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{1}{2}b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)+c\text{Subst}\Bigg(\int\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\Bigg(\frac{}{2\sqrt{c}}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 107, normalized size = 0.96

$$\frac{1}{2}\left(-\frac{\sqrt{a+bx^2+cx^4}}{x^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}-\sqrt{c}\,\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

```
[Out] (-(Sqrt[a + b*x^2 + c*x^4]/x^2) + (b*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2
+ c*x^4])/Sqrt[a]])/Sqrt[a] - Sqrt[c]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4]])/2
```

Maple [A]
time = 0.05, size = 140, normalized size = 1.25

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2x^2}+\frac{\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2}-\frac{b\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4\sqrt{a}}$
default	$-\frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\sqrt{c\,x^4+b\,x^2+a}}{2a}-\frac{b\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4\sqrt{a}}+\frac{c\sqrt{c\,x^4+b\,x^2+a}\,x^2}{2a}$
elliptic	$-\frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\sqrt{c\,x^4+b\,x^2+a}}{2a}-\frac{b\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4\sqrt{a}}+\frac{c\sqrt{c\,x^4+b\,x^2+a}\,x^2}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2*(c*x^4+b*x^2+a)^(3/2)+1/2*b/a*(c*x^4+b*x^2+a)^(1/2)-1/4*b/a^(1/2
)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/2*c/a*(c*x^4+b*x^2+
a)^(1/2)*x^2+1/2*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 601, normalized size = 5.37

$$\left[\frac{2\,a\sqrt{c}\,x^2\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}-4\,ac\right)+\sqrt{a}\,bx^2\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}\,a}{8\,ax^2},\ -\frac{4\,a\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-\sqrt{a}\,bx^2\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,\sqrt{cx^4+bx^2+a}\,a}{8\,ax^2},\ \frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+a\sqrt{c}\,x^2\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}-4\,ac\right)-2\,\sqrt{cx^4+bx^2+a}\,a}{4\,ax^2},\ \frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)-2\,a\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-2\,\sqrt{cx^4+bx^2+a}\,a}{4\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x
^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*
x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/
x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2), -1/8*(4*a*sqrt(-c)*x^2*arctan(
1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c
)) - sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x
^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 + b*x^2 + a)*a)/
(a*x^2), 1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*
a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + a*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b
*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 2
*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2), 1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*
x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*a*sq
rt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x
^4 + b*c*x^2 + a*c)) - 2*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**3, x)
```

Giac [A]
time = 4.00, size = 148, normalized size = 1.32

$$\frac{b\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}}-\frac{1}{2}\,\sqrt{c}\,\log\left(\left|-2\,\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)+\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\,\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) -
1/2*sqrt(c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b)
) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/((sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)
```

Mupad [**B**]
time = 4.55, size = 91, normalized size = 0.81

$$\frac{\sqrt{c}\ \ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2}-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2\,x^2}-\frac{b\,\ln\left(\frac{b}{2}+\frac{a}{x^2}+\frac{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^3,x)
```

```
[Out] (c^(1/2)*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/2 - (a + b
*x^2 + c*x^4)^(1/2)/(2*x^2) - (b*log(b/2 + a/x^2 + (a^(1/2)*(a + b*x^2 + c*
x^4)^(1/2))/x^2))/(4*a^(1/2))
```

3.926 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx$

Optimal. Leaf size=88

$$-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}+\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}$$

```
[Out] 1/16*(-4*a*c+b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^
(3/2)-1/8*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a/x^4
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1128, 734, 738, 212}

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -1/8*((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) + ((b^2 - 4*a*c)*ArcTa
nh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^2\right)\\ &= -\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16a}\\ &= -\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4} + \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8a}\\ &= -\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4} + \frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 88, normalized size = 1.00

$$-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4} - \frac{(b^2-4ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -1/8*((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) - ((b^2 - 4*a*c)*ArcTa
nh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(74) = 148.
time = 0.04, size = 193, normalized size = 2.19

method	result

risch	$-\frac{(b\,x^2+2a)\sqrt{c\,x^4+b\,x^2+a}}{8a\,x^4}-\frac{c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4\sqrt{a}}+\frac{b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x}}{x^2}\right.}{16a^{\frac{3}{2}}}$
default	$-\frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}+\frac{b(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{8a^2x^2}-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{8a^2}+\frac{b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16a^{\frac{3}{2}}}-b$
elliptic	$-\frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}+\frac{b(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{8a^2x^2}-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{8a^2}+\frac{b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16a^{\frac{3}{2}}}-b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/x^4*(c*x^4+b*x^2+a)^(3/2)+1/8*b/a^2/x^2*(c*x^4+b*x^2+a)^(3/2)-1/8*b^
2/a^2*(c*x^4+b*x^2+a)^(1/2)+1/16*b^2/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4
+b*x^2+a)^(1/2))/x^2)-1/8*b/a^2*c*(c*x^4+b*x^2+a)^(1/2)*x^2+1/4*c/a*(c*x^4+
b*x^2+a)^(1/2)-1/4*c/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))
/x^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 215, normalized size = 2.44

$$\left[-\frac{(b^2-4\,ac)\sqrt{a}\,x^4\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,\sqrt{cx^4+bx^2+a}\,(abx^2+2\,a^2)}{32\,a^2x^4},-\frac{(b^2-4\,ac)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\,\sqrt{cx^4+bx^2+a}\,(abx^2+2\,a^2)}{16\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*s
qrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 +
```

```
 b*x^2 + a)*(a*b*x^2 + 2*a^2))/(a^2*x^4), -1/16*((b^2 - 4*a*c)*sqrt(-a)*x^4
*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x
^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2))/(a^2*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(74) = 148.
time = 4.52, size = 241, normalized size = 2.74

$$-\frac{(b^2 - 4\,ac)\arctan\left(-\frac{\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a} + \frac{\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^3 b^2 + 4\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^3 ac + 8\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 ab\sqrt{c} + \left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)ab^2 + 4\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)a^2c}{8\left(\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 - a\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*(b^2 - 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a)
)/(sqrt(-a)*a) + 1/8*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2 + 4*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))^2*a*b*sqrt(c) + (sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b^2 +
 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*c)/(((sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))^2 - a)^2*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^5,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^5, x)
```

3.927 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx$

Optimal. Leaf size=116

$$\frac{b(2a+bx^2)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{(a+bx^2+cx^4)^{3/2}}{6ax^6}-\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}$$

```
[Out] -1/6*(c*x^4+b*x^2+a)^(3/2)/a/x^6-1/32*b*(-4*a*c+b^2)*arctanh(1/2*(b*x^2+2*a
)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)+1/16*b*(b*x^2+2*a)*(c*x^4+b*x^2+a)
^(1/2)/a^2/x^4
```

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1128, 744, 734, 738, 212}

$$-\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}+\frac{b(2a+bx^2)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{(a+bx^2+cx^4)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

```
[Out] (b*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a^2*x^4) - (a + b*x^2 + c*x^4
)^(3/2)/(6*a*x^6) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[
a + b*x^2 + c*x^4])])/(32*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{b\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx,x,x^2\right)}{4a}\\
&= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}+\frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{}{x\sqrt{a+}}\right.}{32a^2}\\
&= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{\left(b\left(b^2-4ac\right)\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\right.}{16a^2}\\
&= \frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{}{2\sqrt{a}\sqrt{a}}\right.}{32a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.41, size = 108, normalized size = 0.93

$$
\frac{\sqrt{a+bx^2+cx^4}\left(-8a^2-2abx^2+3b^2x^4-8acx^4\right)}{48a^2x^6}+\frac{\left(b^3-4abc\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 - 2*a*b*x^2 + 3*b^2*x^4 - 8*a*c*x^4))/(48*
a^2*x^6) + ((b^3 - 4*a*b*c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])
/Sqrt[a]])/(16*a^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(98) = 196$.
time = 0.05, size = 222, normalized size = 1.91

method	result
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(8c\,x^{4}a-3b^{2}x^{4}+2ab\,x^{2}+8a^{2}\right)}{48x^{6}a^{2}}+\frac{bc\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{8a^{\frac{3}{2}}}-\frac{b^{3}\ln\left(\frac{2a+b\,x^{2}+2}{}\right.}{}$
default	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}+\frac{b\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{8a^{2}x^{4}}-\frac{b^{2}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{16a^{3}x^{2}}+\frac{b^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16a^{3}}-\frac{b^{3}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}}}{x^{2}}\right.}{32a^{\frac{5}{2}}}$
elliptic	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}+\frac{b\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{8a^{2}x^{4}}-\frac{b^{2}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{16a^{3}x^{2}}+\frac{b^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16a^{3}}-\frac{b^{3}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}}}{x^{2}}\right.}{32a^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*x^4+b*x^2+a)^(3/2)/a/x^6+1/8*b/a^2/x^4*(c*x^4+b*x^2+a)^(3/2)-1/16*b
^2/a^3/x^2*(c*x^4+b*x^2+a)^(3/2)+1/16*b^3/a^3*(c*x^4+b*x^2+a)^(1/2)-1/32*b^
3/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/16*b^2/a^3*
c*(c*x^4+b*x^2+a)^(1/2)*x^2-1/8*b/a^2*c*(c*x^4+b*x^2+a)^(1/2)+1/8*b/a^(3/2)
*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 261, normalized size = 2.25

$$\left[-\frac{3\,(b^3-4\,abc)\sqrt{a}\,x^6\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,(2\,a^2bx^2-(3\,ab^2-8\,a^2c)x^4+8\,a^3)\sqrt{cx^4+bx^2+a}}{192\,a^3x^6},\ \frac{3\,(b^3-4\,abc)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)-2\,(2\,a^2bx^2-(3\,ab^2-8\,a^2c)x^4+8\,a^3)\sqrt{cx^4+bx^2+a}}{96\,a^3x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2
+ 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(2*a^2*
b*x^2 - (3*a*b^2 - 8*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^6)
, 1/96*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(2*a^2*b*x^2 - (3*a*b^
2 - 8*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**7, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(98) = 196.
time = 4.36, size = 359, normalized size = 3.09

$$\frac{(b^3-4\,abc)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}\,a^2}-\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5b^3-12\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5abc-48\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^4a^2c^{\frac{3}{2}}-8\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3ab^3-48\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3a^2bc-48\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2a^2b^2\sqrt{c}-3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^2b^3-36\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^3bc-16\,a^4c^{\frac{3}{2}}}{48\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/16*(b^3 - 4*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-
a))/(sqrt(-a)*a^2) - 1/48*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*b^3
- 12*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c - 48*(sqrt(c)*x^2 - sq
rt(c*x^4 + b*x^2 + a))^4*a^2*c^(3/2) - 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^3*a*b^3 - 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^2*b*c - 48*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^2*b^2*sqrt(c) - 3*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))*a^2*b^3 - 36*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
a))*a^3*b*c - 16*a^4*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 -
 a)^3*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^7,x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^7, x)
```

3.928 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=161

$$-\frac{(5b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{128a^3x^4}-\frac{(a+bx^2+cx^4)^{3/2}}{8ax^8}+\frac{5b(a+bx^2+cx^4)^{3/2}}{48a^2x^6}+\frac{(b^2-4ac)(5b^2-4ac)}{}$$

```
[Out] -1/8*(c*x^4+b*x^2+a)^(3/2)/a/x^8+5/48*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^6+1/256
*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a
)^(1/2))/a^(7/2)-1/128*(-4*a*c+5*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^3
/x^4
```

Rubi [A]
time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1128, 758, 820, 734, 738, 212}

$$\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{7/2}}-\frac{(5b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{128a^3x^4}+\frac{5b(a+bx^2+cx^4)^{3/2}}{48a^2x^6}-\frac{(a+bx^2+cx^4)^{3/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -1/128*((5*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(a^3*x^4) -
(a + b*x^2 + c*x^4)^(3/2)/(8*a*x^8) + (5*b*(a + b*x^2 + c*x^4)^(3/2))/(48*a
^2*x^6) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*S
qrt[a + b*x^2 + c*x^4])])/(256*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^5}\,dx,x,x^2\right)$$

$$= -\frac{(a+bx^2+cx^4)^{3/2}}{8ax^8} - \frac{\text{Subst}\left(\int \frac{\left(\frac{5b}{2}+cx\right)\sqrt{a+bx+cx^2}}{x^4}\,dx,x,x^2\right)}{8a}$$

$$= -\frac{(a+bx^2+cx^4)^{3/2}}{8ax^8} + \frac{5b(a+bx^2+cx^4)^{3/2}}{48a^2x^6} + \frac{(5b^2-4ac)\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx}}{x^3}\right.}{32a^2}$$

$$= -\frac{(5b^2-4ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{(a+bx^2+cx^4)^{3/2}}{8ax^8} + \frac{5b(a+bx^2+cx^4}{48a^2x^6}$$

$$= -\frac{(5b^2-4ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{(a+bx^2+cx^4)^{3/2}}{8ax^8} + \frac{5b(a+bx^2+cx^4}{48a^2x^6}$$

$$= -\frac{(5b^2-4ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{(a+bx^2+cx^4)^{3/2}}{8ax^8} + \frac{5b(a+bx^2+cx^4}{48a^2x^6}$$

Mathematica [A]
time = 0.50, size = 141, normalized size = 0.88

$$\frac{\sqrt{a+bx^2+cx^4}\,(-48a^3-8a^2bx^2+10ab^2x^4-24a^2cx^4-15b^3x^6+52abcx^6)}{384a^3x^8} + \frac{(-5b^4+24ab^2c-16a^2c^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{128a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-48*a^3 - 8*a^2*b*x^2 + 10*a*b^2*x^4 - 24*a^2*c*x
^4 - 15*b^3*x^6 + 52*a*b*c*x^6))/(384*a^3*x^8) + ((-5*b^4 + 24*a*b^2*c - 16
*a^2*c^2)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(128*a^
(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(139) = 278.
time = 0.06, size = 387, normalized size = 2.40

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(-52abc\,x^6+15b^3x^6+24a^2c\,x^4-10a\,b^2x^4+8a^2b\,x^2+48a^3\right)}{384x^8a^3} + \frac{c^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+}}{x^2}\right.}{16a^{\frac{3}{2}}}$

default	$-\frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{8a\,x^8}+\frac{5b(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{48a^2x^6}-\frac{5b^2(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{64a^3x^4}+\frac{5b^3(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{128a^4x^2}-\frac{5b^4\sqrt{c\,x^4+b\,x^2+a}}{128a^4}+\frac{5b^4}{}$
elliptic	$-\frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{8a\,x^8}+\frac{5b(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{48a^2x^6}-\frac{5b^2(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{64a^3x^4}+\frac{5b^3(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{128a^4x^2}-\frac{5b^4\sqrt{c\,x^4+b\,x^2+a}}{128a^4}+\frac{5b^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^4+b*x^2+a)^(3/2)/a/x^8+5/48*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^6-5/64*
b^2/a^3/x^4*(c*x^4+b*x^2+a)^(3/2)+5/128*b^3/a^4/x^2*(c*x^4+b*x^2+a)^(3/2)-5
/128*b^4/a^4*(c*x^4+b*x^2+a)^(1/2)+5/256*b^4/a^(7/2)*ln((2*a+b*x^2+2*a^(1/2
)*(c*x^4+b*x^2+a)^(1/2))/x^2)-5/128*b^3/a^4*c*(c*x^4+b*x^2+a)^(1/2)*x^2+7/6
4*b^2/a^3*c*(c*x^4+b*x^2+a)^(1/2)-3/32*b^2/a^(5/2)*c*ln((2*a+b*x^2+2*a^(1/2
)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/16*c/a^2/x^4*(c*x^4+b*x^2+a)^(3/2)-1/32*c/a
^3*b/x^2*(c*x^4+b*x^2+a)^(3/2)+1/32*c^2/a^3*b*(c*x^4+b*x^2+a)^(1/2)*x^2-1/1
6*c^2/a^2*(c*x^4+b*x^2+a)^(1/2)+1/16*c^2/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c
*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.42, size = 325, normalized size = 2.02

$$\left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{a}\,x^8\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)-4\,((15\,ab^3-52\,a^2bc)x^6+8\,a^3bx^2-2\,(5\,a^2b^2-12\,a^3c)x^4+48\,a^4)\sqrt{cx^4+bx^2+a}}{1536\,a^4x^8},\ -\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{-a}\,x^8\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\,((15\,ab^3-52\,a^2bc)x^6+8\,a^3bx^2-2\,(5\,a^2b^2-12\,a^3c)x^4+48\,a^4)\sqrt{cx^4+bx^2+a}}{768\,a^4x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^8*log(-((b^2 + 4*a*c
)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2
)/x^4) - 4*((15*a*b^3 - 52*a^2*b*c)*x^6 + 8*a^3*b*x^2 - 2*(5*a^2*b^2 - 12*a
^3*c)*x^4 + 48*a^4)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^8), -1/768*(3*(5*b^4 -
```

```
24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^8*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^3 - 52*a^2*b*c
)*x^6 + 8*a^3*b*x^2 - 2*(5*a^2*b^2 - 12*a^3*c)*x^4 + 48*a^4)*sqrt(c*x^4 + b
*x^2 + a))/(a^4*x^8)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**9, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(139) = 278.
time = 3.62, size = 617, normalized size = 3.83

$$-\frac{\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{128\,\sqrt{-a}\,a^3}+\frac{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^7b^4-72\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^7ab^2c+48\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^7a^2c^2-55\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5ab^4+264\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5a^2b^2c+336\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5a^3c^2+1152\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^4a^3bc^{\frac{3}{2}}+73\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3a^2b^4+648\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3a^3b^2c+336\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3a^4c^2+384\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2a^3b^3\sqrt{c}+256\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2a^4bc^{\frac{3}{2}}+15\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^3b^4+312\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^4b^2c+48\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^5c^2+128\,a^5bc^{\frac{3}{2}}}{384\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/128*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/384*(15*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))^7*b^4 - 72*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b^2*
c + 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*c^2 - 55*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^5*a*b^4 + 264*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^5*a^2*b^2*c + 336*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*c^2 +
 1152*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^3*b*c^(3/2) + 73*(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^2*b^4 + 648*(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))^3*a^3*b^2*c + 336*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^
4*c^2 + 384*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*b^3*sqrt(c) + 256
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^4*b*c^(3/2) + 15*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))*a^3*b^4 + 312*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*a^4*b^2*c + 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^5*c^2 + 128*
a^5*b*c^(3/2))/((((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^4*a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^9,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^9, x)
```

3.929 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=199

$$\frac{b(7b^2-12ac)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}}+\frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8}-\frac{(35b^2-32ac)\left(a+b\right.}{480a^3x^6}$$

```
[Out] -1/10*(c*x^4+b*x^2+a)^(3/2)/a/x^10+7/80*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^8-1/4
80*(-32*a*c+35*b^2)*(c*x^4+b*x^2+a)^(3/2)/a^3/x^6-1/512*b*(-12*a*c+7*b^2)*(
-4*a*c+b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(9/2)+
1/256*b*(-12*a*c+7*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^4/x^4
```

Rubi [A]
time = 0.16, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1128, 758, 848, 820, 734, 738, 212}

$$-\frac{b(7b^2-12ac)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{9/2}}+\frac{b(7b^2-12ac)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4}-\frac{(35b^2-32ac)\left(a+bx^2+cx^4\right)^{3/2}}{480a^3x^6}+\frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a^4*x^4) -
(a + b*x^2 + c*x^4)^(3/2)/(10*a*x^10) + (7*b*(a + b*x^2 + c*x^4)^(3/2))/(80
*a^2*x^8) - ((35*b^2 - 32*a*c)*(a + b*x^2 + c*x^4)^(3/2))/(480*a^3*x^6) - (
b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
b*x^2 + c*x^4])])/(512*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2+cx^4)^{3/2}}{10ax^{10}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+2cx\right)\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^2\right)}{10a}$$

$$= -\frac{(a+bx^2+cx^4)^{3/2}}{10ax^{10}} + \frac{7b(a+bx^2+cx^4)^{3/2}}{80a^2x^8} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{4}(35b^2-32ac)+\frac{7bcx}{2}\right)\sqrt{a+}}{x^4}\right.}{40a^2}$$

$$= -\frac{(a+bx^2+cx^4)^{3/2}}{10ax^{10}} + \frac{7b(a+bx^2+cx^4)^{3/2}}{80a^2x^8} - \frac{(35b^2-32ac)\,(a+bx^2+cx^4)^{3/2}}{480a^3x^6}$$

$$= \frac{b(7b^2-12ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{(a+bx^2+cx^4)^{3/2}}{10ax^{10}} + \frac{7b(a+bx^2+c}{80a^2x^8}$$

$$= \frac{b(7b^2-12ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{(a+bx^2+cx^4)^{3/2}}{10ax^{10}} + \frac{7b(a+bx^2+c}{80a^2x^8}$$

$$= \frac{b(7b^2-12ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{(a+bx^2+cx^4)^{3/2}}{10ax^{10}} + \frac{7b(a+bx^2+c}{80a^2x^8}$$

Mathematica [A]
time = 0.68, size = 176, normalized size = 0.88

$$\frac{\sqrt{a+bx^2+cx^4}\,(-384a^4-48a^3bx^2+56a^2b^2x^4-128a^3cx^4-70ab^3x^6+232a^2bcx^6+105b^4x^8-460ab^2cx^8+256a^2c^2x^8)}{3840a^4x^{10}} + \frac{(7b^5-40ab^3c+48a^2bc^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{256a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-384*a^4 - 48*a^3*b*x^2 + 56*a^2*b^2*x^4 - 128*a^
3*c*x^4 - 70*a*b^3*x^6 + 232*a^2*b*c*x^6 + 105*b^4*x^8 - 460*a*b^2*c*x^8 +
256*a^2*c^2*x^8))/(3840*a^4*x^10) + ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*Ar
cTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(256*a^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(173) = 346$.
time = 0.08, size = 442, normalized size = 2.22

method	result

risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(-256 a^{2} c^{2} x^{8}+460 a\,b^{2} c\,x^{8}-105 b^{4} x^{8}-232 a^{2} b c\,x^{6}+70 a\,b^{3} x^{6}+128 a^{3} c\,x^{4}-56 a^{2} b^{2} x^{4}+48 a^{3} b\,x^{2}+384 a^{4}\right)}{3840 x^{10} a^{4}}-$
default	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{10 a\,x^{10}}+\frac{7 b \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{80 a^{2} x^{8}}-\frac{7 b^{2} \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{96 a^{3} x^{6}}+\frac{7 b^{3} \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{128 a^{4} x^{4}}-\frac{7 b^{4} \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{256 a^{5} x^{2}}+\frac{7 b^{5} \sqrt{c\,x^{4}}}{25}$
elliptic	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{10 a\,x^{10}}+\frac{7 b \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{80 a^{2} x^{8}}-\frac{7 b^{2} \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{96 a^{3} x^{6}}+\frac{7 b^{3} \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{128 a^{4} x^{4}}-\frac{7 b^{4} \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{256 a^{5} x^{2}}+\frac{7 b^{5} \sqrt{c\,x^{4}}}{25}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(c*x^4+b*x^2+a)^(3/2)/a/x^10+7/80*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^8-7/9
6*b^2/a^3/x^6*(c*x^4+b*x^2+a)^(3/2)+7/128*b^3/a^4/x^4*(c*x^4+b*x^2+a)^(3/2)
-7/256*b^4/a^5/x^2*(c*x^4+b*x^2+a)^(3/2)+7/256*b^5/a^5*(c*x^4+b*x^2+a)^(1/2
)-7/512*b^5/a^(9/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+7/2
56*b^4/a^5*c*(c*x^4+b*x^2+a)^(1/2)*x^2-13/128*b^3/a^4*c*(c*x^4+b*x^2+a)^(1/
2)+5/64*b^3/a^(7/2)*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-3
/32*b/a^3*c/x^4*(c*x^4+b*x^2+a)^(3/2)+3/64*b^2/a^4*c/x^2*(c*x^4+b*x^2+a)^(3
/2)-3/64*b^2/a^4*c^2*(c*x^4+b*x^2+a)^(1/2)*x^2+3/32*b/a^3*c^2*(c*x^4+b*x^2+
a)^(1/2)-3/32*b/a^(5/2)*c^2*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/
x^2)+1/15*c/a^2/x^6*(c*x^4+b*x^2+a)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.45, size = 389, normalized size = 1.95

$$\left[\frac{15\,(7\,b^{5}-40\,ab^{3}c+48\,a^{2}bc^{2})\sqrt{a}\,x^{10}\log\left(-\frac{(b^{2}+4\,ac)x^{4}+8\,abx^{2}-4\,\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2\,a)\sqrt{a}+8\,a^{2}}{x^{4}}\right)+4\,((105\,ab^{4}-460\,a^{2}b^{2}c+256\,a^{3}c^{2})x^{8}-48\,a^{4}bx^{2}-2\,(35\,a^{2}b^{3}-116\,a^{3}bc)x^{6}-384\,a^{5}+8\,(7\,a^{3}b^{2}-16\,a^{4}c)x^{4})\sqrt{cx^{4}+bx^{2}+a}}{15360\,a^{5}x^{10}}, \frac{15\,(7\,b^{5}-40\,ab^{3}c+48\,a^{2}bc^{2})\sqrt{-a}\,x^{10}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2\,a)\sqrt{-a}}{2\,(acx^{4}+abx^{2}+a^{2})}\right)+2\,((105\,ab^{4}-460\,a^{2}b^{2}c+256\,a^{3}c^{2})x^{8}-48\,a^{4}bx^{2}-2\,(35\,a^{2}b^{3}-116\,a^{3}bc)x^{6}-384\,a^{5}+8\,(7\,a^{3}b^{2}-16\,a^{4}c)x^{4})\sqrt{cx^{4}+bx^{2}+a}}{7680\,a^{5}x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/15360*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^10*log(-((b^2 +
4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) +
8*a^2)/x^4) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^8 - 48*a^4*b*x
^2 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^6 - 384*a^5 + 8*(7*a^3*b^2 - 16*a^4*c)*
x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^10), 1/7680*(15*(7*b^5 - 40*a*b^3*c +
48*a^2*b*c^2)*sqrt(-a)*x^10*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^2*c + 256
*a^3*c^2)*x^8 - 48*a^4*b*x^2 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^6 - 384*a^5 +
 8*(7*a^3*b^2 - 16*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^10)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**11, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 842 vs. 2(173) = 346.
time = 5.50, size = 842, normalized size = 4.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/256*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/3840*(105*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^9*b^5 - 600*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a*
b^3*c + 720*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a^2*b*c^2 - 490*(sqrt
(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b^5 + 2800*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^7*a^2*b^3*c - 3360*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7
*a^3*b*c^2 - 7680*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^4*c^(5/2) + 8
96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*b^5 - 5120*(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))^5*a^3*b^3*c - 15360*(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))^5*a^4*b*c^2 - 24320*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^4
*b^2*c^(3/2) - 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^5*c^(5/2) -
 790*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^3*b^5 - 9200*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^3*a^4*b^3*c - 12000*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))^3*a^5*b*c^2 - 3840*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^
4*b^4*sqrt(c) - 5120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^5*b^2*c^(3
/2) - 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^6*c^(5/2) - 105*(sqr
```

```
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^4*b^5 - 3240*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))*a^5*b^3*c - 720*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^6
*b*c^2 - 1280*a^6*b^2*c^(3/2) + 512*a^7*c^(5/2))/(((sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))^2 - a)^5*a^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^11,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^11, x)
```

3.930 $\int x^4 \sqrt{a+bx^2+cx^4}\, dx$

Optimal. Leaf size=395

$$-\frac{2(2b^2-5ac)\,x\sqrt{a+bx^2+cx^4}}{105c^2}+\frac{b(8b^2-29ac)\,x\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x^3(b+5cx^2)\sqrt{a+bx^2+cx^4}}{35c}-\frac{\sqrt[4]{a}\,b(}{}$$

```
[Out] -2/105*(-5*a*c+2*b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/35*x^3*(5*c*x^2+b)*(c*x
^4+b*x^2+a)^(1/2)/c+1/105*b*(-29*a*c+8*b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)
/(a^(1/2)+x^2*c^(1/2))-1/105*a^(1/4)*b*(-29*a*c+8*b^2)*(cos(2*arctan(c^(1/4
)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/
2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a
)^(1/2)+1/210*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(8*b^3-29*a*b*c+2*(-5*a*c+2*b
^2)*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/
4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1130, 1293, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{a}\,b(8b^2-29ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{105c^{11/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}\,(2b^2-5ac)-29abc+8b^3\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{210c^{11/4}\sqrt{a+bx^2+cx^4}}+\frac{bx(8b^2-29ac)\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2x(2b^2-5ac)\sqrt{a+bx^2+cx^4}}{105c^2}+\frac{x^3(b+5cx^2)\sqrt{a+bx^2+cx^4}}{35c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-2*(2*b^2 - 5*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^2) + (b*(8*b^2 - 29*a
*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (x^3
*(b + 5*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c) - (a^(1/4)*b*(8*b^2 - 29*a*c
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
105*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^3 - 29*a*b*c + 2*Sqrt
[a]*Sqrt[c]*(2*b^2 - 5*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(210*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*((2*b*p + c*(m + 4*p - 1)*x^
2)/(c*(m + 4*p + 1)*(m + 4*p - 1))), x] - Dist[2*p*(d^2/(c*(m + 4*p + 1)*(m
 + 4*p - 1))), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx = \frac{x^3(b+5cx^2)\sqrt{a+bx^2+cx^4}}{35c} - \frac{\int \frac{x^2(3ab+2(2b^2-5ac)x^2)}{\sqrt{a+bx^2+cx^4}}\,dx}{35c}$$

$$= -\frac{2(2b^2-5ac)\,x\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{x^3(b+5cx^2)\sqrt{a+bx^2+cx^4}}{35c} + \frac{\int \frac{2a(2b^2-5a}{\sqrt{a+}}}{}$$

$$= -\frac{2(2b^2-5ac)\,x\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{x^3(b+5cx^2)\sqrt{a+bx^2+cx^4}}{35c} - \frac{(\sqrt{a}\,b(8b^2}{}$$

$$= -\frac{2(2b^2-5ac)\,x\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{b(8b^2-29ac)\,x\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{x^3(b+5}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.17, size = 538, normalized size = 1.36

$$\frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(10a^2c-4b^3x^2-b^2cx^4+18bc^2x^6+15c^3x^8+a(-4b^2+13bcx^2+25c^2x^4))+ib(8b^2-29ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-8b^4+37ab^2c-20a^2c^2+8b^3\sqrt{b^2-4ac}-29abc\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{420c^3\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(10*a^2*c - 4*b^3*x^2 - b^2*c*x^4 +
18*b*c^2*x^6 + 15*c^3*x^8 + a*(-4*b^2 + 13*b*c*x^2 + 25*c^2*x^4)) + I*b*(8*
b^2 - 29*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^4 + 37
*a*b^2*c - 20*a^2*c^2 + 8*b^3*Sqrt[b^2 - 4*a*c] - 29*a*b*c*Sqrt[b^2 - 4*a*c
])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*
b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Arc
Sinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
 - Sqrt[b^2 - 4*a*c])])/(420*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b
*x^2 + c*x^4])
```

Maple [A]
time = 0.05, size = 476, normalized size = 1.21

method	result

default	$\frac{x^5\sqrt{c\,x^4+b\,x^2+a}}{7}+\frac{b\,x^3\sqrt{c\,x^4+b\,x^2+a}}{35c}+\frac{\left(\frac{2a}{7}-\frac{4b^2}{35c}\right)x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{\left(\frac{2a}{7}-\frac{4b^2}{35c}\right)a\sqrt{2}\,\sqrt{4-\frac{2}{}}}{}$
elliptic	$\frac{x^5\sqrt{c\,x^4+b\,x^2+a}}{7}+\frac{b\,x^3\sqrt{c\,x^4+b\,x^2+a}}{35c}+\frac{\left(\frac{2a}{7}-\frac{4b^2}{35c}\right)x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{\left(\frac{2a}{7}-\frac{4b^2}{35c}\right)a\sqrt{2}\,\sqrt{4-\frac{2}{}}}{}$
risch	$\frac{x(15c^2x^4+3bc\,x^2+10ac-4b^2)\sqrt{c\,x^4+b\,x^2+a}}{105c^2}-\frac{\left(29abc-8b^3\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2(b}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x^5*(c*x^4+b*x^2+a)^(1/2)+1/35*b/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(2/7*a
-4/35/c*b^2)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(2/7*a-4/35/c*b^2)/c*a*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))-1/2*(-3/35*b/c*a-2/3*(2/7*a-4/35/c*b^2)/c*b)*a*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*
(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b
^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^4*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.931 $\int x^2\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=342

$$-\frac{2(b^2-3ac)\,x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x(b+3cx^2)\,\sqrt{a+bx^2+cx^4}}{15c}+\frac{2\sqrt[4]{a}\,(b^2-3ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2}{(\sqrt{a}+}}}{15c^{7/4}\sqrt{}}$$

```
[Out] 1/15*x*(3*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c-2/15*(-3*a*c+b^2)*x*(c*x^4+b*x^2
+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))+2/15*a^(1/4)*(-3*a*c+b^2)*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x
^4+b*x^2+a)^(1/2)-1/30*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),
1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/
2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+
b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1130, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}\,(b^2-3ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{2x(b^2-3ac)\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x(b+3cx^2)\sqrt{a+bx^2+cx^4}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-2*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(3/2)*(Sqrt[a] + Sqrt[c]
*x^2)) + (x*(b + 3*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) + (2*a^(1/4)*(b^2
 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]
))/4])/(15*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(2*b^2 + Sqrt[a]*b*S
qrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(30*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*((2*b*p + c*(m + 4*p - 1)*x^
2)/(c*(m + 4*p + 1)*(m + 4*p - 1))), x] - Dist[2*p*(d^2/(c*(m + 4*p + 1)*(m
 + 4*p - 1))), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx = \frac{x(b+3cx^2)\sqrt{a+bx^2+cx^4}}{15c} - \frac{\int \frac{ab+2(b^2-3ac)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{15c}$$

$$= \frac{x(b+3cx^2)\sqrt{a+bx^2+cx^4}}{15c} + \frac{\left(2\sqrt{a}\,(b^2-3ac)\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^{3/2}} - \frac{(\sqrt{}}{}$$

$$= -\frac{2(b^2-3ac)\,x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{x(b+3cx^2)\sqrt{a+bx^2+cx^4}}{15c} + \frac{2\sqrt[4]{a}\,(b^2-3}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.02, size = 479, normalized size = 1.40

$$\frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(b+3cx^2)(a+bx^2+cx^4) - i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right) + i\left(-b^3+4abc+b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{30c^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 3*c*x^2)*(a + b*x^2 + c*x^4) -
I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*
a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*
c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
])/(30*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.04, size = 417, normalized size = 1.22

method	result
risch	$\frac{x(3cx^2+b)\sqrt{cx^4+bx^2+a}}{15c} - \frac{(6ac-2b^2)a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)\cdots}{a}}\cdots}{\cdots}$
default	$\frac{x^3\sqrt{cx^4+bx^2+a}}{5} + \frac{bx\sqrt{cx^4+bx^2+a}}{15c} - \frac{ba\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4\cdots}\right)\cdots}{\cdots}}\cdots}{60c\sqrt{\frac{-b\cdots}{\cdots}}\cdots}$
elliptic	$\frac{x^3\sqrt{cx^4+bx^2+a}}{5} + \frac{bx\sqrt{cx^4+bx^2+a}}{15c} - \frac{ba\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4\cdots}\right)\cdots}{\cdots}}\cdots}{60c\sqrt{\frac{-b\cdots}{\cdots}}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^3*(c*x^4+b*x^2+a)^(1/2)+1/15*b/c*x*(c*x^4+b*x^2+a)^(1/2)-1/60*b/c*a*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Elli
```

```
pticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2))-1/2*(2/5*a-2/15/c*b^2)*a*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x**2*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^2*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.932 $\int \sqrt{a+bx^2+cx^4}\, dx$

Optimal. Leaf size=309

$$\frac{1}{3}x\sqrt{a+bx^2+cx^4}+\frac{bx\sqrt{a+bx^2+cx^4}}{3\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\right.}{3c^{3/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)+1/3*b*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+
x^2*c^(1/2))-1/3*a^(1/4)*b*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*
(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2
)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/6*a^(1/4)*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
F(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1105, 1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}\,\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{bx\sqrt{a+bx^2+cx^4}}{3\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/3 + (b*x*Sqrt[a + b*x^2 + c*x^4])/(3*Sqrt[c]*(S
qrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*b*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(
1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \sqrt{a+bx^2+cx^4}\,dx = \frac{1}{3}x\sqrt{a+bx^2+cx^4}+\frac{1}{3}\int \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{1}{3}x\sqrt{a+bx^2+cx^4}+\frac{1}{3}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx-\frac{\left(\sqrt{a}\,b\right)}{}$$

$$= \frac{1}{3}x\sqrt{a+bx^2+cx^4}+\frac{bx\sqrt{a+bx^2+cx^4}}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+}{\left(\sqrt{a}+\sqrt{c}\right.}}}{3c^{3/4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.38, size = 445, normalized size = 1.44

$$\frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(a+bx^2+cx^4)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-b^2+4ac+b\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{12c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4],x]
```

[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) + I*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(12*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])

Maple [A]
time = 0.05, size = 379, normalized size = 1.23

method	result
default	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{\ }}{}\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+}}$
risch	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{\ }}{}\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+}}$
elliptic	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{\ }}{}\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)+1/6*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/6*b*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/

```
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x^4+b\,x^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2), x)
```

3.933 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx$

Optimal. Leaf size=303

$$-\frac{\sqrt{a+bx^2+cx^4}}{x}+\frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2}-\frac{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{c}}\right.\right.}{\sqrt{a+bx^2+cx^4}}$$

```
[Out] -(c*x^4+b*x^2+a)^(1/2)/x+2*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/(a^(1/2)+x^2*c^(
1/2))-2*a^(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-
b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x
^2*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2
*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c
^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1131, 1211, 1117, 1209}

$$\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2}-\frac{\sqrt{a+bx^2+cx^4}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/x) + (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[
a] + Sqrt[c]*x^2) - (2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[
a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1131

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Dist[2*(p/
(d^2*(m + 1))), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{x} + \int \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{x} + \left(b+2\sqrt{a}\sqrt{c}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \left(2\sqrt{a}\sqrt{c}\right)\int \frac{}{\sqrt{}}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{x} + \frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2} - \frac{2\sqrt[4]{a}\,\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+}{(\sqrt{a}}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.54, size = 435, normalized size = 1.44

$$\frac{-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(a+bx^2+cx^4\right)+i\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\sqrt{2}\sqrt{b^2-4ac}\,x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^2,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*(-b + Sqrt[b^2
 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellipt
icE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*x*Sqrt[(b - S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]

time = 0.04, size = 381, normalized size = 1.26

method	result
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{x}+\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{\cdots}}{\cdots}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+\cdots}}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{x}+\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{\cdots}}{\cdots}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+\cdots}}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{x}+\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{\cdots}}{\cdots}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^4+b*x^2+a)^(1/2)/x+1/4*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4
-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-c*a*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2
*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**2,x)
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^2,x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^2, x)
```

3.934 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx$

Optimal. Leaf size=341

$$-\frac{\sqrt{a+bx^2+cx^4}}{3x^3}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax}+\frac{b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E}{3a^{3/4}\sqrt{a+b}}$$

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/x^3-1/3*b*(c*x^4+b*x^2+a)^(1/2)/a/x+1/3*b*x*c^(1
/2)*(c*x^4+b*x^2+a)^(1/2)/a/(a^(1/2)+x^2*c^(1/2))-1/3*b*c^(1/4)*(cos(2*arct
an(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+
x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4
+b*x^2+a)^(1/2)+1/6*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/2))*(
(c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)^(1/2
)
```

Rubi [A]

time = 0.10, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1131, 1295, 1211, 1117, 1209}

$$\frac{\sqrt[4]{c}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax}+\frac{b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt{a+bx^2+cx^4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^4,x]
```

```
[Out] -1/3*Sqrt[a + b*x^2 + c*x^4]/x^3 - (b*Sqrt[a + b*x^2 + c*x^4])/(3*a*x) + (b
*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a*(Sqrt[a] + Sqrt[c]*x^2)) - (b*c^(1
/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])
/(3*a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + ((b + 2*Sqrt[a]*Sqrt[c])*c^(1/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(
3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1131

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Dist[2*(p/
(d^2*(m + 1))), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{3x^3} + \frac{1}{3}\int \frac{b+2cx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3x^3} - \frac{b\sqrt{a+bx^2+cx^4}}{3ax} - \frac{\int \frac{-2ac-bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3x^3} - \frac{b\sqrt{a+bx^2+cx^4}}{3ax} - \frac{\left(b\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{a}} + \frac{1}{3}\Bigg(\Bigg(\cdots$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3x^3} - \frac{b\sqrt{a+bx^2+cx^4}}{3ax} + \frac{b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3a\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{b\sqrt[4]{c}\left(\sqrt{a}+\cdots\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.59, size = 459, normalized size = 1.35

$$\frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2)(a+bx^2+cx^4)+ib\left(-b+\sqrt{b^2-4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-b^2+4ac+b\sqrt{b^2-4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{12a\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^4,x]
```

```
[Out] (-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2)*(a + b*x^2 + c*x^4) + I*b*(
-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(12*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.04, size = 404, normalized size = 1.18

method	result

risch	$-\frac{\sqrt{c x^4+b x^2+a}\,(b x^2+a)}{3x^3 a}+\frac{c\left(-\frac{ba\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\cdots}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{\sqrt{c x^4+b x^2+a}}{3x^3}-\frac{b\sqrt{c x^4+b x^2+a}}{3ax}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4\cdots}\right)\cdots}{\cdots}}\cdots}{6\sqrt{\frac{-b+\cdots}{\cdots}}\cdots}$
elliptic	$-\frac{\sqrt{c x^4+b x^2+a}}{3x^3}-\frac{b\sqrt{c x^4+b x^2+a}}{3ax}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4\cdots}\right)\cdots}{\cdots}}\cdots}{6\sqrt{\frac{-b+\cdots}{\cdots}}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/x^3-1/3*b*(c*x^4+b*x^2+a)^(1/2)/a/x+1/6*c*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF
(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2))-1/6*b*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2
*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^4, x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**4,x)

[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**4, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^4, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2 + c*x^4)^(1/2)/x^4,x)

[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^4, x)

3.935 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=397

$$-\frac{\sqrt{a+bx^2+cx^4}}{5x^5}-\frac{b\sqrt{a+bx^2+cx^4}}{15ax^3}+\frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{15a^2x}-\frac{2\sqrt{c}\,(b^2-3ac)\,x\sqrt{a+bx^2+cx^4}}{15a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+$$

```
[Out] -1/5*(c*x^4+b*x^2+a)^(1/2)/x^5-1/15*b*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/15*(-3*
a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/a^2/x-2/15*(-3*a*c+b^2)*x*c^(1/2)*(c*x^4+b*x
^2+a)^(1/2)/a^2/(a^(1/2)+x^2*c^(1/2))+2/15*c^(1/4)*(-3*a*c+b^2)*(cos(2*arct
an(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+
x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4
+b*x^2+a)^(1/2)-1/30*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/2)
*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*
x^2+a)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1131, 1295, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{c}\,(b^2-3ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{15a^2x}-\frac{2\sqrt{c}\,x(b^2-3ac)\sqrt{a+bx^2+cx^4}}{15a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt{a+bx^2+cx^4}}{5x^5}-\frac{b\sqrt{a+bx^2+cx^4}}{15ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -1/5*Sqrt[a + b*x^2 + c*x^4]/x^5 - (b*Sqrt[a + b*x^2 + c*x^4])/(15*a*x^3) +
 (2*(b^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*x) - (2*Sqrt[c]*(b^2 - 3
*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*(Sqrt[a] + Sqrt[c]*x^2)) + (2*c^(1
/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(15*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(2*b^2 + Sqr
t[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(30*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1131

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Dist[2*(p/
(d^2*(m + 1))), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} + \frac{1}{5}\int \frac{b+2cx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} - \frac{\int \frac{2(b^2-3ac)+bcx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{15a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} + \frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{15a^2x} + \frac{\int \frac{-abc}{\sqrt{a}}}{}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} + \frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{15a^2x} + \frac{(2\sqrt{c}}{}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} + \frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{15a^2x} - \frac{2\sqrt{c}\,(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.88, size = 530, normalized size = 1.34

$$\frac{-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(3a^3-2b^2x^6(b+cx^2)+a^2(4bx^2+9cx^4)+a(-b^2x^4+7bcx^6+6c^2x^8)\right)-i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)x^5\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+i\left(-b^3+4abc+b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}\right)x^5\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{30a^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x^5\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/x^6,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(3*a^3 - 2*b^2*x^6*(b + c*x^2) + a^2*(4
*b*x^2 + 9*c*x^4) + a*(-(b^2*x^4) + 7*b*c*x^6 + 6*c^2*x^8)) - I*(b^2 - 3*a*
c)*(-b + Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b
^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] +
4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(3
0*a^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^5*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.05, size = 452, normalized size = 1.14

method	result

risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(6c\,x^4a-2b^2x^4+ab\,x^2+3a^2)}{15x^5a^2}-\frac{c\left(\frac{(6ac-2b^2)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2(}{}}}{}\right.}{}$
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{5x^5}-\frac{b\sqrt{c\,x^4+b\,x^2+a}}{15a\,x^3}-\frac{2(3ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}{15a^2x}-\frac{bc\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4a}\right.}{a}}}{}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{5x^5}-\frac{b\sqrt{c\,x^4+b\,x^2+a}}{15a\,x^3}-\frac{2(3ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}{15a^2x}-\frac{bc\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4a}\right.}{a}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(c*x^4+b*x^2+a)^(1/2)/x^5-1/15*b*(c*x^4+b*x^2+a)^(1/2)/a/x^3-2/15*(3*a
*c-b^2)/a^2*(c*x^4+b*x^2+a)^(1/2)/x-1/60*b*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1
/15*c*(3*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elli
pticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/x^6,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)/x^6, x)
```

3.936 $\int x^7(a+bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=223

$$\frac{3b(b^2-4ac)(3b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b(3b^2-4ac)(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{256c^4}+\frac{x^4(a+bx^2}{14}$$

```
[Out] -1/256*b*(-4*a*c+3*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c^4+1/14*x^4*(c*x
^4+b*x^2+a)^(5/2)/c+1/560*(-30*b*c*x^2-16*a*c+21*b^2)*(c*x^4+b*x^2+a)^(5/2)
/c^3-3/4096*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)
/(c*x^4+b*x^2+a)^(1/2))/c^(11/2)+3/2048*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(2*c*
x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^5
```

Rubi [A]

time = 0.14, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1128, 756, 793, 626, 635, 212}

$$-\frac{3b(b^2-4ac)^2(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4096c^{11/2}}+\frac{3b(b^2-4ac)(3b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b(3b^2-4ac)(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{256c^4}+\frac{(-16ac+21b^2-30bcx^2)(a+bx^2+cx^4)^{5/2}}{560c^3}+\frac{x^4(a+bx^2+cx^4)^{5/2}}{14c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(
2048*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(25
6*c^4) + (x^4*(a + b*x^2 + c*x^4)^(5/2))/(14*c) + ((21*b^2 - 16*a*c - 30*b*
c*x^2)*(a + b*x^2 + c*x^4)^(5/2))/(560*c^3) - (3*b*(b^2 - 4*a*c)^2*(3*b^2 -
 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4096*c
^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int x^7\left(a+bx^2+cx^4\right)^{3/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{x^4(a+bx^2+cx^4)^{5/2}}{14c}+\frac{\text{Subst}\left(\int x\left(-2a-\frac{9bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)}{14c}\\
&= \frac{x^4(a+bx^2+cx^4)^{5/2}}{14c}+\frac{(21b^2-16ac-30bcx^2)\left(a+bx^2+cx^4\right)^{5/2}}{560c^3}-\frac{(b(3b^2-\cdots}{\cdots}\\
&= -\frac{b(3b^2-4ac)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{256c^4}+\frac{x^4(a+bx^2+cx^4)^{5/2}}{14c}+\frac{(21b^2-\cdots}{\cdots}\\
&= \frac{3b(b^2-4ac)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b(3b^2-4ac)\left(b+2cx^2\right)\cdots}{256c^4\cdots}\\
&= \frac{3b(b^2-4ac)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b(3b^2-4ac)\left(b+2cx^2\right)\cdots}{256c^4\cdots}\\
&= \frac{3b(b^2-4ac)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b(3b^2-4ac)\left(b+2cx^2\right)\cdots}{256c^4\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.66, size = 220, normalized size = 0.99

$$
\frac{\sqrt{a+bx^2+cx^4}\left(315b^6-210b^5cx^2+16b^3c^2x^2(91a-9cx^4)+168b^4c(-15a+cx^4)+1024c^3(a+cx^4)^2(-2a+5cx^4)+16b^2c^2(343a^2-62acx^4+8c^2x^8)+32bc^3x^2(-73a^2+22acx^4+200c^2x^8)\right)}{71680c^5}+\frac{3b(b^2-4ac)^2(3b^2-4ac)\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{4096c^{11/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(315*b^6 - 210*b^5*c*x^2 + 16*b^3*c^2*x^2*(91*a -
9*c*x^4) + 168*b^4*c*(-15*a + c*x^4) + 1024*c^3*(a + c*x^4)^2*(-2*a + 5*c*x
^4) + 16*b^2*c^2*(343*a^2 - 62*a*c*x^4 + 8*c^2*x^8) + 32*b*c^3*x^2*(-73*a^2
 + 22*a*c*x^4 + 200*c^2*x^8)))/(71680*c^5) + (3*b*(b^2 - 4*a*c)^2*(3*b^2 -
4*a*c)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(4096*c^(11/2)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(197) = 394$.
time = 0.06, size = 534, normalized size = 2.39

method	result

risch	$-\frac{(-5120c^6x^{12}-6400b\,c^5x^{10}-8192a\,c^5x^8-128b^2c^4x^8-704ab\,c^4x^6+144b^3c^3x^6-1024a^2c^4x^4+992a\,b^2c^3x^4-168b^4c^2x^4+2336a^2b\,c^3}{71680c^5}$
default	$\frac{3b^4x^4\sqrt{c\,x^4+b\,x^2+a}}{1280c^3}-\frac{3b^5x^2\sqrt{c\,x^4+b\,x^2+a}}{1024c^4}+\frac{b^2x^8\sqrt{c\,x^4+b\,x^2+a}}{560c}-\frac{9b^3x^6\sqrt{c\,x^4+b\,x^2+a}}{4480c^2}$
elliptic	$\frac{3b^4x^4\sqrt{c\,x^4+b\,x^2+a}}{1280c^3}-\frac{3b^5x^2\sqrt{c\,x^4+b\,x^2+a}}{1024c^4}+\frac{b^2x^8\sqrt{c\,x^4+b\,x^2+a}}{560c}-\frac{9b^3x^6\sqrt{c\,x^4+b\,x^2+a}}{4480c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/1280*b^4/c^3*x^4*(c*x^4+b*x^2+a)^(1/2)-3/1024*b^5/c^4*x^2*(c*x^4+b*x^2+a)
^(1/2)+1/560*b^2*x^8/c*(c*x^4+b*x^2+a)^(1/2)-9/4480*b^3/c^2*x^6*(c*x^4+b*x^
2+a)^(1/2)+1/70*a^2*x^4/c*(c*x^4+b*x^2+a)^(1/2)+21/1024*a*b^5/c^(9/2)*ln((1
/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-15/256*a^2*b^3/c^(7/2)*ln((1/2*b
+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+49/640*a^2*b^2/c^3*(c*x^4+b*x^2+a)^(
1/2)+3/64*a^3*b/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-9/2
56*a*b^4/c^4*(c*x^4+b*x^2+a)^(1/2)+5/56*b*x^10*(c*x^4+b*x^2+a)^(1/2)-1/35*a
^3/c^2*(c*x^4+b*x^2+a)^(1/2)+11/1120*a*b*x^6/c*(c*x^4+b*x^2+a)^(1/2)-31/224
0*a*b^2/c^2*x^4*(c*x^4+b*x^2+a)^(1/2)+13/640*a*b^3/c^3*x^2*(c*x^4+b*x^2+a)^
(1/2)-73/2240*a^2*b/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+4/35*a*x^8*(c*x^4+b*x^2+a
)^(1/2)+9/2048*b^6/c^5*(c*x^4+b*x^2+a)^(1/2)-9/4096*b^7/c^(11/2)*ln((1/2*b+
c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/14*c*x^12*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 535, normalized size = 2.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/286720*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^12 + 6400*b*c^6*x^10 + 128*(b^2*c^5 + 64
*a*c^6)*x^8 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4
- 16*(9*b^3*c^4 - 44*a*b*c^5)*x^6 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2
*c^5)*x^4 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^2)*sqrt(c*x^
4 + b*x^2 + a))/c^6, 1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 6
4*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt
(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(5120*c^7*x^12 + 6400*b*c^6*x^10 + 128*
(b^2*c^5 + 64*a*c^6)*x^8 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 -
2048*a^3*c^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^6 + 8*(21*b^4*c^3 - 124*a*b^2*
c^4 + 128*a^2*c^5)*x^4 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x
^2)*sqrt(c*x^4 + b*x^2 + a))/c^6]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. 2(197) = 394.
time = 2.82, size = 669, normalized size = 3.00

$$\frac{1}{7680}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-460ab^2c+256a^2c^2}{c^4}\right)-\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{9}{2}}}\right)a+\frac{1}{30720}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(2\left(8\left(10x^2+\frac{b}{c}\right)x^2-\frac{9b^2c^3-20ac^4}{c^5}\right)x^2+\frac{21b^3c^2-68abc^3}{c^5}\right)x^2-\frac{105b^4c-448ab^2c^2+240a^2c^3}{c^5}\right)x^2+\frac{315b^5-1680ab^3c+1808a^2bc^2}{c^5}\right)+\frac{15\left(21b^6-140ab^4c+240a^2b^2c^2-64a^3c^3\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{11}{2}}}\right)b+\frac{1}{430080}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(2\left(8\left(10\left(12x^2+\frac{b}{c}\right)x^2-\frac{11b^2c^4-24ac^5}{c^6}\right)x^2+\frac{99b^3c^3-316abc^4}{c^6}\right)x^2-\frac{231b^4c^2-972ab^2c^3+512a^2c^4}{c^6}\right)x^2+\frac{1155b^5c-6048ab^3c^2+6352a^2bc^3}{c^6}\right)x^2-\frac{3465b^6-21840ab^4c+34608a^2b^2c^2-8192a^3c^3}{c^6}\right)-\frac{105\left(33b^7-252ab^5c+560a^2b^3c^2-320a^3bc^3\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{13}{2}}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 -
 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*
b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(abs(
-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2))*a + 1/307
20*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*x^2 + b/c)*x^2 - (9*b^2*c^3 -
 20*a*c^4)/c^5)*x^2 + (21*b^3*c^2 - 68*a*b*c^3)/c^5)*x^2 - (105*b^4*c - 448
*a*b^2*c^2 + 240*a^2*c^3)/c^5)*x^2 + (315*b^5 - 1680*a*b^3*c + 1808*a^2*b*c
^2)/c^5) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*log(abs
(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(11/2))*b + 1/4
30080*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*(12*x^2 + b/c)*x^2 - (11*b
```

```
^2*c^4 - 24*a*c^5)/c^6)*x^2 + (99*b^3*c^3 - 316*a*b*c^4)/c^6)*x^2 - (231*b^
4*c^2 - 972*a*b^2*c^3 + 512*a^2*c^4)/c^6)*x^2 + (1155*b^5*c - 6048*a*b^3*c^
2 + 6352*a^2*b*c^3)/c^6)*x^2 - (3465*b^6 - 21840*a*b^4*c + 34608*a^2*b^2*c^
2 - 8192*a^3*c^3)/c^6) - 105*(33*b^7 - 252*a*b^5*c + 560*a^2*b^3*c^2 - 320*
a^3*b*c^3)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))
/c^(13/2))*c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \left(c x^4 + b x^2 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^7*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.937 $\int x^5(a+bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=204

$$-\frac{(b^2-4ac)\,(7b^2-4ac)\,(b+2cx^2)\,\sqrt{a+bx^2+cx^4}}{1024c^4}+\frac{(7b^2-4ac)\,(b+2cx^2)\,(a+bx^2+cx^4)^{3/2}}{384c^3}-\frac{7b(a+bx^2+\dots}{120c\dots}$$

```
[Out] 1/384*(-4*a*c+7*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c^3-7/120*b*(c*x^4+b
*x^2+a)^(5/2)/c^2+1/12*x^2*(c*x^4+b*x^2+a)^(5/2)/c+1/2048*(-4*a*c+b^2)^2*(-
4*a*c+7*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(9/2)
-1/1024*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^4
```

Rubi [A]
time = 0.12, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1128, 756, 654, 626, 635, 212}

$$\frac{(b^2-4ac)^2\,(7b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2048c^{9/2}}-\frac{(b^2-4ac)\,(7b^2-4ac)\,(b+2cx^2)\,\sqrt{a+bx^2+cx^4}}{1024c^4}+\frac{(7b^2-4ac)\,(b+2cx^2)\,(a+bx^2+cx^4)^{3/2}}{384c^3}-\frac{7b(a+bx^2+cx^4)^{5/2}}{120c^2}+\frac{x^2(a+bx^2+cx^4)^{5/2}}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/1024*((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4
])/c^4 + ((7*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(384*c^3
) - (7*b*(a + b*x^2 + c*x^4)^(5/2))/(120*c^2) + (x^2*(a + b*x^2 + c*x^4)^(5
/2))/(12*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(2048*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^5 \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{1}{2}\text{Subst}\left(\int x^2 \left(a+bx+cx^2\right)^{3/2} dx, x, x^2\right)$$

$$= \frac{x^2(a+bx^2+cx^4)^{5/2}}{12c} + \frac{\text{Subst}\left(\int \left(-a-\frac{7bx}{2}\right)\left(a+bx+cx^2\right)^{3/2} dx, x, x^2\right)}{12c}$$

$$= -\frac{7b(a+bx^2+cx^4)^{5/2}}{120c^2} + \frac{x^2(a+bx^2+cx^4)^{5/2}}{12c} + \frac{(7b^2-4ac)\,\text{Subst}\left(\int (a+bx\right.}{48c^2}$$

$$= \frac{(7b^2-4ac)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384c^3} - \frac{7b(a+bx^2+cx^4)^{5/2}}{120c^2} + \frac{x^2(a+bx^2}{12}$$

$$= -\frac{(b^2-4ac)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4} + \frac{(7b^2-4ac)\left(b+2cx^2\right)(}{384c^3}$$

$$= -\frac{(b^2-4ac)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4} + \frac{(7b^2-4ac)\left(b+2cx^2\right)(}{384c^3}$$

$$= -\frac{(b^2-4ac)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4} + \frac{(7b^2-4ac)\left(b+2cx^2\right)(}{384c^3}$$

Mathematica [A]
time = 0.53, size = 194, normalized size = 0.95

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(-105b^5+70b^4cx^2+8b^3c(95a-7cx^4)+48b^2c^2x^2(-9a+cx^4)+160c^3x^2(3a^2+14acx^4+8c^2x^8)+16bc^2(-81a^2+18acx^4+104c^2x^8)\right)-15(b^2-4ac)^2(7b^2-4ac)\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{30720c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(-105*b^5 + 70*b^4*c*x^2 + 8*b^3*c*(95*a
 - 7*c*x^4) + 48*b^2*c^2*x^2*(-9*a + c*x^4) + 160*c^3*x^2*(3*a^2 + 14*a*c*x
^4 + 8*c^2*x^8) + 16*b*c^2*(-81*a^2 + 18*a*c*x^4 + 104*c^2*x^8)) - 15*(b^2
- 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x
^4]])/(30720*c^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(178) = 356$.
time = 0.06, size = 432, normalized size = 2.12

method	result
risch	$-\frac{(-1280c^5x^{10}-1664b\,c^4x^8-2240a\,c^4x^6-48b^2c^3x^6-288ab\,c^3x^4+56b^3c^2x^4-480a^2c^3x^2+432a\,b^2c^2x^2-70b^4c\,x^2+1296a^2b\,c^2-760a\,b^3}{15360c^4}$

default	$\frac{b^2x^6\sqrt{c\,x^4+b\,x^2+a}}{320c}-\frac{7b^3x^4\sqrt{c\,x^4+b\,x^2+a}}{1920c^2}+\frac{7b^4x^2\sqrt{c\,x^4+b\,x^2+a}}{1536c^3}+\frac{a^2x^2\sqrt{c\,x^4+b\,x^2+a}}{32c}$
elliptic	$\frac{b^2x^6\sqrt{c\,x^4+b\,x^2+a}}{320c}-\frac{7b^3x^4\sqrt{c\,x^4+b\,x^2+a}}{1920c^2}+\frac{7b^4x^2\sqrt{c\,x^4+b\,x^2+a}}{1536c^3}+\frac{a^2x^2\sqrt{c\,x^4+b\,x^2+a}}{32c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/320*b^2*x^6/c*(c*x^4+b*x^2+a)^(1/2)-7/1920*b^3/c^2*x^4*(c*x^4+b*x^2+a)^(1
/2)+7/1536*b^4/c^3*x^2*(c*x^4+b*x^2+a)^(1/2)+1/32*a^2*x^2/c*(c*x^4+b*x^2+a)
^(1/2)+19/384*a*b^3/c^3*(c*x^4+b*x^2+a)^(1/2)-15/512*a*b^4/c^(7/2)*ln((1/2*
b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+9/128*a^2*b^2/c^(5/2)*ln((1/2*b+c*x
^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-27/320*a^2*b/c^2*(c*x^4+b*x^2+a)^(1/2)+7
/48*a*x^6*(c*x^4+b*x^2+a)^(1/2)-7/1024*b^5/c^4*(c*x^4+b*x^2+a)^(1/2)+7/2048
*b^6/c^(9/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/12*c*x^10*(c
*x^4+b*x^2+a)^(1/2)-1/32*a^3/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+
a)^(1/2))+13/120*b*x^8*(c*x^4+b*x^2+a)^(1/2)+3/160*a*b*x^4/c*(c*x^4+b*x^2+a
)^(1/2)-9/320*a*b^2/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.36, size = 451, normalized size = 2.21

$$\left[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}-4\,ac\right)-4\,(1280\,c^6x^{10}+1664\,bc^5x^8+16\,(3\,b^2c^4+140\,ac^5)x^6-105\,b^5c+760\,ab^3c^2-1296\,a^2bc^3-8\,(7\,b^3c^3-36\,abc^4)x^4+2\,(35\,b^4c^2-216\,ab^2c^3+240\,a^2c^4)x^2)\sqrt{cx^4+bx^2+a}}{61440\,c^5},\ -\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-2\,(1280\,c^6x^{10}+1664\,bc^5x^8+16\,(3\,b^2c^4+140\,ac^5)x^6-105\,b^5c+760\,ab^3c^2-1296\,a^2bc^3-8\,(7\,b^3c^3-36\,abc^4)x^4+2\,(35\,b^4c^2-216\,ab^2c^3+240\,a^2c^4)x^2)\sqrt{cx^4+bx^2+a}}{30720\,c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/61440*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*l
og(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*s
qrt(c) - 4*a*c) - 4*(1280*c^6*x^10 + 1664*b*c^5*x^8 + 16*(3*b^2*c^4 + 140*a
```

```
*c^5)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 - 8*(7*b^3*c^3 - 36*
a*b*c^4)*x^4 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^2)*sqrt(c*x^4
 + b*x^2 + a))/c^5, -1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64
*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c
)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(1280*c^6*x^10 + 1664*b*c^5*x^8 + 16*(3*b^
2*c^4 + 140*a*c^5)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 - 8*(7*
b^3*c^3 - 36*a*b*c^4)*x^4 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^
2)*sqrt(c*x^4 + b*x^2 + a))/c^5]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5}\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(178) = 356.
time = 6.51, size = 535, normalized size = 2.62

$$\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{7}{2}}}\right)a+\frac{1}{7680}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-460ab^2c+256a^2c^2}{c^4}\right)-\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{9}{2}}}\right)b+\frac{1}{30720}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(2\left(8\left(10x^2+\frac{b}{c}\right)x^2-\frac{9b^2c^3-20ac^4}{c^5}\right)x^2+\frac{21b^3c^2-68abc^3}{c^5}\right)x^2-\frac{105b^4c-448ab^2c^2+240a^2c^3}{c^5}\right)x^2+\frac{315b^5-1680ab^3c+1808a^2bc^2}{c^5}\right)+\frac{15\left(21b^6-140ab^4c+240a^2b^2c^2-64a^3c^3\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{11}{2}}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*
c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c
^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(7/2
))*a + 1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^
2*c^2 - 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 -
 460*a*b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*l
og(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2))*b
+ 1/30720*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*x^2 + b/c)*x^2 - (9*b^
2*c^3 - 20*a*c^4)/c^5)*x^2 + (21*b^3*c^2 - 68*a*b*c^3)/c^5)*x^2 - (105*b^4*
c - 448*a*b^2*c^2 + 240*a^2*c^3)/c^5)*x^2 + (315*b^5 - 1680*a*b^3*c + 1808*
a^2*b*c^2)/c^5) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*
log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(11/2))*
c
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^5*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.938 $\int x^3(a+bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=150

$$\frac{3b(b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{32c^2}+\frac{(a+bx^2+cx^4)^{5/2}}{10c}-\frac{3b(b^2-4ac)^2}{}$$

```
[Out] -1/32*b*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c^2+1/10*(c*x^4+b*x^2+a)^(5/2)/c-
3/512*b*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2
))/c^(7/2)+3/256*b*(-4*a*c+b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^3
```

Rubi [A]
time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1128, 654, 626, 635, 212}

$$-\frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}+\frac{3b(b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{32c^2}+\frac{(a+bx^2+cx^4)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^3) - (b*(b
 + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*c^2) + (a + b*x^2 + c*x^4)^(5/2)
/(10*c) - (3*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*
x^2 + c*x^4])])/(512*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$=\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}-\frac{b\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^2\right)}{4c}$$

$$=-\frac{b(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}+\frac{(3b(b^2-4ac))\,\text{Subst}}{}$$

$$=\frac{3b(b^2-4ac)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+$$

$$=\frac{3b(b^2-4ac)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+$$

$$=\frac{3b(b^2-4ac)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+$$

Mathematica [A]
time = 0.37, size = 140, normalized size = 0.93

$$\frac{\sqrt{a+bx^2+cx^4}\left(15b^4-10b^3cx^2+128c^2\left(a+cx^4\right)^2+4b^2c\left(-25a+2cx^4\right)+8bc^2x^2\left(7a+22cx^4\right)\right)}{1280c^3}+\frac{3b\left(b^2-4ac\right)^2\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)}{512c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(15*b^4 - 10*b^3*c*x^2 + 128*c^2*(a + c*x^4)^2 + 4
*b^2*c*(-25*a + 2*c*x^4) + 8*b*c^2*x^2*(7*a + 22*c*x^4)))/(1280*c^3) + (3*b
```

```
*(b^2 - 4*a*c)^2*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(512
*c^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(128) = 256$.
time = 0.05, size = 316, normalized size = 2.11

method	result
risch	$\frac{\left(128c^4x^8+176b\,c^3x^6+256a\,c^3x^4+8b^2c^2x^4+56ab\,c^2x^2-10b^3c\,x^2+128a^2c^2-100a\,b^2c+15b^4\right)\sqrt{c\,x^4+b\,x^2+a}}{1280c^3}-\frac{3a^2b\ln\left(\frac{\frac{b}{2}+c}{\sqrt{c}}\right.}{}$
default	$-\frac{b^3x^2\sqrt{c\,x^4+b\,x^2+a}}{128c^2}+\frac{b^2x^4\sqrt{c\,x^4+b\,x^2+a}}{160c}-\frac{5a\,b^2\sqrt{c\,x^4+b\,x^2+a}}{64c^2}+\frac{3a\,b^3\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\right.}{64c^{\frac{5}{2}}}$
elliptic	$-\frac{b^3x^2\sqrt{c\,x^4+b\,x^2+a}}{128c^2}+\frac{b^2x^4\sqrt{c\,x^4+b\,x^2+a}}{160c}-\frac{5a\,b^2\sqrt{c\,x^4+b\,x^2+a}}{64c^2}+\frac{3a\,b^3\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\right.}{64c^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*b^3/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+1/160*b^2*x^4/c*(c*x^4+b*x^2+a)^(1
/2)-5/64*a*b^2/c^2*(c*x^4+b*x^2+a)^(1/2)+3/64*a*b^3/c^(5/2)*ln((1/2*b+c*x^2
)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/32*a^2*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2
)+(c*x^4+b*x^2+a)^(1/2))+11/80*b*x^6*(c*x^4+b*x^2+a)^(1/2)+1/5*a*x^4*(c*x^4
+b*x^2+a)^(1/2)+3/256*b^4/c^3*(c*x^4+b*x^2+a)^(1/2)-3/512*b^5/c^(7/2)*ln((1
/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/10*a^2/c*(c*x^4+b*x^2+a)^(1/2)
+7/160*a*b*x^2/c*(c*x^4+b*x^2+a)^(1/2)+1/10*c*x^8*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 361, normalized size = 2.41

$$\left[\frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2+4\sqrt{cx^4+bx^2+a}\,(2cx^2+b)\sqrt{c}-4ac\right)+4\left(128c^5x^8+176bc^4x^6+15b^4c-100ab^2c^2+128a^2c^3+8(b^2c^3+32ac^4)x^4-2(5b^3c^2-28abc^3)x^2\right)\sqrt{cx^4+bx^2+a}}{5120c^4},\ \frac{15\left(b^5-8ab^3c+16a^2bc^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2cx^2+b)\sqrt{-c}}{2(c^2x^4+bcx^2+ac)}\right)+2\left(128c^5x^8+176bc^4x^6+15b^4c-100ab^2c^2+128a^2c^3+8(b^2c^3+32ac^4)x^4-2(5b^3c^2-28abc^3)x^2\right)\sqrt{cx^4+bx^2+a}}{2560c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c
*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(
128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b
^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2
 + a))/c^4, 1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arctan(1/2
*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c))
+ 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 +
 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^4 +
b*x^2 + a))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(128) = 256.
time = 5.47, size = 414, normalized size = 2.76

$$\frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{3\left(b^3-4abc\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}\right)a+\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{7}{2}}}\right)b+\frac{1}{7680}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-460ab^2c+256a^2c^2}{c^4}\right)-\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{9}{2}}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2)
 - 3*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqr
t(c) - b))/c^(5/2))*a + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c
)*x^2 - (5*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4
 - 24*a*b^2*c + 16*a^2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
a))*sqrt(c) - b))/c^(7/2))*b + 1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(
8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^
2)/c^4)*x^2 - (105*b^4 - 460*a*b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a
*b^3*c + 48*a^2*b*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*s
qrt(c) - b))/c^(9/2))*c
```

Mupad [B]
time = 4.88, size = 223, normalized size = 1.49

$$\frac{\left(c\,x^4+b\,x^2+a\right)^{5/2}}{10\,c}-\frac{b\,\left(\frac{3\,a\,\left(\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\,\left(\frac{a}{2\,\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{\left(2\,c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2+a}}{4\,c}\right)}{4}+\frac{x^2\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}{4}-\frac{3\,b^2\,\left(\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\,\left(\frac{a}{2\,\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{\left(2\,c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2+a}}{4\,c}\right)}{16\,c}+\frac{b\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}{8\,c}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (a + b*x^2 + c*x^4)^(5/2)/(10*c) - (b*((3*a*(log((a + b*x^2 + c*x^4)^(1/2)
+ (b/2 + c*x^2)/c^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^2)
*(a + b*x^2 + c*x^4)^(1/2))/(4*c)))/4 + (x^2*(a + b*x^2 + c*x^4)^(3/2))/4 -
 (3*b^2*(log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))*(a/(2*c^(1/
2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(4*c)))/
(16*c) + (b*(a + b*x^2 + c*x^4)^(3/2))/(8*c)))/(4*c)
```

3.939 $\int x(a+bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{3(b^2-4ac)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{3(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}$$

[Out] 1/16*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c+3/256*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)-3/128*(-4*a*c+b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^2

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1121, 626, 635, 212}

$$\frac{3(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}-\frac{3(b^2-4ac)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^2) + ((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*c) + (3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int x(a+bx^2+cx^4)^{3/2}\,dx = \frac{1}{2}\text{Subst}\left(\int (a+bx+cx^2)^{3/2}\,dx, x, x^2\right)$$

$$= \frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{16c} - \frac{(3(b^2-4ac))\,\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^2\right.}{32c}$$

$$= -\frac{3(b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^2} + \frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{16c} + \frac{(3\cdots}{\cdots}$$

$$= -\frac{3(b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^2} + \frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{16c} + \frac{(3\cdots}{\cdots}$$

$$= -\frac{3(b^2-4ac)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^2} + \frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{16c} + \frac{3(b\cdots}{\cdots}$$

Mathematica [A]
time = 0.31, size = 110, normalized size = 0.89

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}\,(-3b^2+20ac+8bcx^2+8c^2x^4)}{128c^2} - \frac{3(-b^2+4ac)^2\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{256c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^(3/2),x]
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 20*a*c + 8*b*c*x^2 + 8*c^2
*x^4))/(128*c^2) - (3*(-b^2 + 4*a*c)^2*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4]])/(256*c^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(106) = 212$.
time = 0.04, size = 242, normalized size = 1.95

method	result
risch	$\frac{(16c^3x^6+24bc^2x^4+40c^2ax^2+2b^2cx^2+20abc-3b^3)\sqrt{cx^4+bx^2+a}}{128c^2} + \frac{3a^2\ln\left(\frac{\frac{b}{2}+cx^2}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{16\sqrt{c}} - \frac{3ab^2\cdots}{\cdots}$

default	$\frac{b^{2}x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{64c}+\frac{5ab\sqrt{c\,x^{4}+b\,x^{2}+a}}{32c}-\frac{3a\,b^{2}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{3}{2}}}+\frac{c\,x^{6}\sqrt{c\,x^{4}+b}}{8}$
elliptic	$\frac{b^{2}x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{64c}+\frac{5ab\sqrt{c\,x^{4}+b\,x^{2}+a}}{32c}-\frac{3a\,b^{2}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{3}{2}}}+\frac{c\,x^{6}\sqrt{c\,x^{4}+b}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)+5/32*a*b/c*(c*x^4+b*x^2+a)^(1/2)-3/32*
a*b^2/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/8*c*x^6*(c*
x^4+b*x^2+a)^(1/2)+3/16*b*x^4*(c*x^4+b*x^2+a)^(1/2)+5/16*a*x^2*(c*x^4+b*x^2
+a)^(1/2)-3/128*b^3/c^2*(c*x^4+b*x^2+a)^(1/2)+3/256*b^4/c^(5/2)*ln((1/2*b+c
*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.35, size = 297, normalized size = 2.40

$$\left[\frac{3\,(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2})\sqrt{c}\,\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}-4\,\sqrt{cx^{4}+bx^{2}+a}\,(2\,cx^{2}+b)\sqrt{c}-4\,ac\right)+4\,(16\,c^{4}x^{6}+24\,bc^{3}x^{4}-3\,b^{3}c+20\,abc^{2}+2\,(b^{2}c^{2}+20\,ac^{3})x^{2})\sqrt{cx^{4}+bx^{2}+a}}{512\,c^{3}},\,-\frac{3\,(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2})\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(2\,cx^{2}+b)\sqrt{-c}}{2\,(c^{2}x^{4}+bcx^{2}+ac)}\right)-2\,(16\,c^{4}x^{6}+24\,bc^{3}x^{4}-3\,b^{3}c+20\,abc^{2}+2\,(b^{2}c^{2}+20\,ac^{3})x^{2})\sqrt{cx^{4}+bx^{2}+a}}{256\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2
 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(16*c
^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^2)*
sqrt(c*x^4 + b*x^2 + a))/c^3, -1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b
*c*x^2 + a*c)) - 2*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b
^2*c^2 + 20*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(106) = 212.
time = 4.69, size = 317, normalized size = 2.56

$$\frac{1}{16}\left(2\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{(b^2-4ac)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}\right)a+\frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{3(b^3-4abc)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}\right)b+\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3(5b^4-24ab^2c+16a^2c^2)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{7}{2}}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + (b^2 - 4*a*c)*log(abs(-2*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2))*a + 1/96*(2*sq
rt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^3
- 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))
/c^(5/2))*b + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5
*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^
2*c + 16*a^2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c
) - b))/c^(7/2))*c
```

Mupad [B]
time = 4.96, size = 115, normalized size = 0.93

$$\frac{\left(c\,x^2+\frac{b}{2}\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{8\,c}+\frac{\left(3\,a\,c-\frac{3\,b^2}{4}\right)\,\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2+a}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b/2 + c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(8*c) + ((3*a*c - (3*b^2)/4)*((b/
(4*c) + x^2/2)*(a + b*x^2 + c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2) +
 (b/2 + c*x^2)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(8*c)
```

3.940 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x} dx$

Optimal. Leaf size=155

$$\frac{(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16c}+\frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2}-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-\frac{b(b^2-}{}$$

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)-1/2*a^(3/2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^
4+b*x^2+a)^(1/2))-1/32*b*(-12*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x
^4+b*x^2+a)^(1/2))/c^(3/2)+1/16*(2*b*c*x^2+8*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)
/c
```

Rubi [A]

time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1128, 748, 828, 857, 635, 212, 738}

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}}+\frac{(8ac+b^2+2bcx^2)\sqrt{a+bx^2+cx^4}}{16c}+\frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c) + (a + b*x^2 + c
*x^4)^(3/2)/6 - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
c*x^4])])/2 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4])])/(32*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - \frac{1}{4}\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} + \frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b(\cdots}{x\sqrt{a+b\cdots}}\right.}{1\cdots}$$

$$= \frac{(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{\cdots}{x\sqrt{\cdots}}\right.$$

$$= \frac{(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - a^2\text{Subst}\left(\int \frac{1}{4a-\cdots}\right.$$

$$= \frac{(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16c} + \frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2} - \frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\cdots}{2\sqrt{\cdots}}\right.$$

Mathematica [A]
time = 0.46, size = 142, normalized size = 0.92

$$\frac{\sqrt{a+bx^2+cx^4}\,(3b^2+14bcx^2+8c(4a+cx^4))}{48c} + a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right) + \frac{(b^3-12abc)\log\left(c\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)\right)}{32c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(3*b^2 + 14*b*c*x^2 + 8*c*(4*a + c*x^4)))/(48*c) +
 a^(3/2)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] + ((b^3 -
 12*a*b*c)*Log[c*(b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^
(3/2))
```

Maple [A]
time = 0.04, size = 192, normalized size = 1.24

method	result
default	$\frac{c x^4\sqrt{c x^4+b x^2+a}}{6}+\frac{7b x^2\sqrt{c x^4+b x^2+a}}{24}+\frac{b^2\sqrt{c x^4+b x^2+a}}{16c}-\frac{b^3\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+\cdots}\right.}{32c^{\frac{3}{2}}}$

elliptic	$\frac{c\,x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{6}+\frac{7 b\,x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{24}+\frac{b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16 c}-\frac{b^{3}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+}\right.}{32 c^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/c*b^2
*(c*x^4+b*x^2+a)^(1/2)-1/32/c^(3/2)*b^3*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))+3/8*a*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2
)+2/3*a*(c*x^4+b*x^2+a)^(1/2)-1/2*a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*
x^2+a)^(1/2))/x^2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.46, size = 727, normalized size = 4.69

$$\left[\frac{48\,a^{\frac{3}{2}}c^{2}\log\left(-\frac{(b^{2}+4ac)x^{4}+8abx^{2}-4\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{a}+8a^{2}}{x^{4}}\right)-3\,(b^{3}-12\,abc)\sqrt{c}\,\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}-4\sqrt{cx^{4}+bx^{2}+a}\,(2\,cx^{2}+b)\sqrt{c}-4\,ac\right)+4\,(8\,c^{3}x^{4}+14\,bc^{2}x^{2}+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{4}+bx^{2}+a}}{192\,c^{2}},\ \frac{24\,a^{\frac{3}{2}}c^{2}\log\left(-\frac{(b^{2}+4ac)x^{4}+8abx^{2}-4\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{a}+8a^{2}}{x^{4}}\right)+3\,(b^{3}-12\,abc)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(2cx^{2}+b)\sqrt{-c}}{2\,(c^{2}x^{4}+bcx^{2}+ac)}\right)+2\,(8\,c^{3}x^{4}+14\,bc^{2}x^{2}+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{4}+bx^{2}+a}}{96\,c^{2}},\ \frac{96\sqrt{-a}\,ac^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{-a}}{2\,(acx^{4}+abx^{2}+a^{2})}\right)-3\,(b^{3}-12\,abc)\sqrt{c}\,\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}-4\sqrt{cx^{4}+bx^{2}+a}\,(2\,cx^{2}+b)\sqrt{c}-4\,ac\right)+4\,(8\,c^{3}x^{4}+14\,bc^{2}x^{2}+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{4}+bx^{2}+a}}{192\,c^{2}},\ \frac{48\sqrt{-a}\,ac^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{-a}}{2\,(acx^{4}+abx^{2}+a^{2})}\right)+3\,(b^{3}-12\,abc)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(2cx^{2}+b)\sqrt{-c}}{2\,(c^{2}x^{4}+bcx^{2}+ac)}\right)+2\,(8\,c^{3}x^{4}+14\,bc^{2}x^{2}+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{4}+bx^{2}+a}}{96\,c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/192*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 3*(b^3 - 12*a*b*c)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c + 32*a*c^2)*sqrt
(c*x^4 + b*x^2 + a))/c^2, 1/96*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 3
*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)
*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*
c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2, 1/192*(96*sqrt(-a)*a*c^2*arctan
(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^
2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(
```

```
c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^4 + 14*b*c^2
*x^2 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2, 1/96*(48*sqrt(-a)*
a*c^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 + 14
*b*c^2*x^2 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x, x)
```

3.941 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=150

$$\frac{3}{8}(3b+2cx^2)\sqrt{a+bx^2+cx^4}-\frac{(a+bx^2+cx^4)^{3/2}}{2x^2}-\frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)+\frac{3(b^2+4ac)\,\mathrm{ta}}{}$$

```
[Out] -1/2*(c*x^4+b*x^2+a)^(3/2)/x^2-3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4
+b*x^2+a)^(1/2))*a^(1/2)+3/16*(4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(
c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/8*(2*c*x^2+3*b)*(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1128, 746, 828, 857, 635, 212, 738}

$$\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}}-\frac{(a+bx^2+cx^4)^{3/2}}{2x^2}+\frac{3}{8}(3b+2cx^2)\sqrt{a+bx^2+cx^4}-\frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/8 - (a + b*x^2 + c*x^4)^(3/2)/(
2*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x
^4])])/4 + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2
 + c*x^4])])/(16*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2+cx^4)^{3/2}}{2x^2} + \frac{3}{4}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{2x^2} - \frac{3\text{Subst}\left(\int \frac{-4abc-c(b^2+4ac)x}{x\sqrt{a+bx+cx^2}}\right.}{16c}$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{2x^2} + \frac{1}{4}(3ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+b}}\right.$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{2x^2} - \frac{1}{2}(3ab)\text{Subst}\left(\int \frac{1}{4a-x^2}\right.$$

$$= \frac{3}{8}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{2x^2} - \frac{3}{4}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{a}}\right.$$

Mathematica [A]
time = 0.49, size = 132, normalized size = 0.88

$$\frac{1}{16}\left(\frac{2\sqrt{a+bx^2+cx^4}\,(-4a+5bx^2+2cx^4)}{x^2} + 24\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right) - \frac{3(b^2+4ac)\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] ((2*Sqrt[a + b*x^2 + c*x^4]*(-4*a + 5*b*x^2 + 2*c*x^4))/x^2 + 24*Sqrt[a]*b*
ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] - (3*(b^2 + 4*a*c)
*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/Sqrt[c])/16
```

Maple [A]
time = 0.09, size = 170, normalized size = 1.13

method	result
default	$\frac{c\,x^2\sqrt{c\,x^4+b\,x^2+a}}{4} + \frac{5b\sqrt{c\,x^4+b\,x^2+a}}{8} + \frac{3b^2\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16\sqrt{c}} + \frac{3a\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\right.}{}$
risch	$\frac{c\,x^2\sqrt{c\,x^4+b\,x^2+a}}{4} + \frac{5b\sqrt{c\,x^4+b\,x^2+a}}{8} + \frac{3b^2\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16\sqrt{c}} + \frac{3a\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\right.}{}$

elliptic	$\frac{c x^2 \sqrt{c x^4+b x^2+a}}{4}+\frac{5 b \sqrt{c x^4+b x^2+a}}{8}+\frac{3 b^2 \ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{16 \sqrt{c}}+\frac{3 a \sqrt{c}\, \ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*x^2*(c*x^4+b*x^2+a)^(1/2)+5/8*b*(c*x^4+b*x^2+a)^(1/2)+3/16*b^2*ln((1/
2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/4*a*c^(1/2)*ln((1/2*b+c
*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2*a/x^2*(c*x^4+b*x^2+a)^(1/2)-3/4*a^
(1/2)*b*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.42, size = 713, normalized size = 4.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*sqrt(a)*b*c*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^
4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 3*(b^2 + 4*a*c)*sqrt(c
)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c*x^4 + b*x
^2 + a))/(c*x^2), 1/16*(6*sqrt(a)*b*c*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c
*x^4 + b*x^2 + a))/(c*x^2), 1/32*(24*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4
 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 3*(b^2 +
4*a*c)*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqr
```

```
t(c*x^4 + b*x^2 + a))/(c*x^2), 1/16*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*
x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c
*x^4 + b*x^2 + a))/(c*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0]%%%}+%%%{%%{[-2,0]:[1
,0,%%%
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^3,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^3, x)
```

3.942 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^5} dx$

Optimal. Leaf size=151

$$-\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2}-\frac{(a+bx^2+cx^4)^{3/2}}{4x^4}-\frac{3(b^2+4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}+\frac{3}{4}b\sqrt{c}$$

```
[Out] -1/4*(c*x^4+b*x^2+a)^(3/2)/x^4-3/16*(4*a*c+b^2)*arctanh(1/2*(b*x^2+2*a)/a^(
1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)+3/4*b*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(
c*x^4+b*x^2+a)^(1/2))*c^(1/2)-3/8*(-2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/x^2
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1128, 746, 826, 857, 635, 212, 738}

$$-\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}-\frac{(a+bx^2+cx^4)^{3/2}}{4x^4}-\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2}+\frac{3}{4}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (-3*(b - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*x^2) - (a + b*x^2 + c*x^4)^(3
/2)/(4*x^4) - (3*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*
x^2 + c*x^4])])/(16*Sqrt[a]) + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c
]*Sqrt[a + b*x^2 + c*x^4])])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^3}\,dx,x,x^2\right)$$
$$= -\frac{(a+bx^2+cx^4)^{3/2}}{4x^4} + \frac{3}{8}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}\,dx,x,x^2\right)$$
$$= -\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{(a+bx^2+cx^4)^{3/2}}{4x^4} - \frac{3}{16}\text{Subst}\left(\int \frac{-b^2-4ac}{x\sqrt{a+bx}}\right.$$
$$= -\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{(a+bx^2+cx^4)^{3/2}}{4x^4} + \frac{1}{4}(3bc)\text{Subst}\left(\int \frac{}{\sqrt{a+b}}\right.$$
$$= -\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{(a+bx^2+cx^4)^{3/2}}{4x^4} + \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{1}{4c-x^2}\right.$$
$$= -\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2} - \frac{(a+bx^2+cx^4)^{3/2}}{4x^4} - \frac{3(b^2+4ac)\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{16\sqrt{a}}$$

Mathematica [A]

time = 0.51, size = 131, normalized size = 0.87

$$\frac{1}{8}\left(\frac{\sqrt{a+bx^2+cx^4}\,(-2a-5bx^2+4cx^4)}{x^4} + \frac{3(b^2+4ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}} - 6b\sqrt{c}\,\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] ((Sqrt[a + b*x^2 + c*x^4]*(-2*a - 5*b*x^2 + 4*c*x^4))/x^4 + (3*(b^2 + 4*a*c
)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/Sqrt[a] - 6*b*S
qrt[c]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/8
```

Maple [A]

time = 0.07, size = 174, normalized size = 1.15

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(5b\,x^2+2a)}{8x^4}+\frac{c\sqrt{c\,x^4+b\,x^2+a}}{2}+\frac{3b\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4}-\frac{3\sqrt{a}}{}$
default	$\frac{c\sqrt{c\,x^4+b\,x^2+a}}{2}+\frac{3b\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4}-\frac{a\sqrt{c\,x^4+b\,x^2+a}}{4x^4}-\frac{5b\sqrt{c\,x^4+b\,x^2}}{8x^2}$

elliptic	$\frac{c\sqrt{c\,x^{4}+b\,x^{2}+a}}{2}+\frac{3b\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4}-\frac{a\sqrt{c\,x^{4}+b\,x^{2}+a}}{4x^{4}}-\frac{5b\sqrt{c\,x^{4}+b\,x^{2}+}}{8x^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c*(c*x^4+b*x^2+a)^(1/2)+3/4*b*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))-1/4*a/x^4*(c*x^4+b*x^2+a)^(1/2)-5/8*b/x^2*(c*x^4+b*x^2+a)^(1
/2)-3/16/a^(1/2)*b^2*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-3/
4*a^(1/2)*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.43, size = 713, normalized size = 4.72

$$\left[\frac{12\,ab\sqrt{c}\,x^{4}\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}-4\,\sqrt{cx^{4}+bx^{2}+a}\,(2\,cx^{2}+b)\sqrt{c}-4\,ac\right)+3\,(b^{2}+4\,ac)\sqrt{a}\,x^{4}\log\left(-\frac{(b^{2}+4ac)x^{4}+8abx^{2}-4\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{a}+8a^{2}}{x^{4}}\right)+4\,(4\,acx^{4}-5\,abx^{2}-2\,a^{2})\sqrt{cx^{4}+bx^{2}+a}}{32\,ax^{4}},\ -\frac{24\,ab\sqrt{-c}\,x^{4}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(2cx^{2}+b)\sqrt{-c}}{2\,(c^{2}x^{4}+bcx^{2}+ac)}\right)-3\,(b^{2}+4\,ac)\sqrt{a}\,x^{4}\log\left(-\frac{(b^{2}+4ac)x^{4}+8abx^{2}-4\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{a}+8a^{2}}{x^{4}}\right)-4\,(4\,acx^{4}-5\,abx^{2}-2\,a^{2})\sqrt{cx^{4}+bx^{2}+a}}{32\,ax^{4}},\ \frac{6\,ab\sqrt{c}\,x^{4}\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}-4\,\sqrt{cx^{4}+bx^{2}+a}\,(2\,cx^{2}+b)\sqrt{c}-4\,ac\right)+3\,(b^{2}+4\,ac)\sqrt{-a}\,x^{4}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{-a}}{2\,(acx^{4}+abx^{2}+a^{2})}\right)+2\,(4\,acx^{4}-5\,abx^{2}-2\,a^{2})\sqrt{cx^{4}+bx^{2}+a}}{16\,ax^{4}},\ -\frac{12\,ab\sqrt{-c}\,x^{4}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(2cx^{2}+b)\sqrt{-c}}{2\,(c^{2}x^{4}+bcx^{2}+ac)}\right)-3\,(b^{2}+4\,ac)\sqrt{-a}\,x^{4}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{-a}}{2\,(acx^{4}+abx^{2}+a^{2})}\right)-2\,(4\,acx^{4}-5\,abx^{2}-2\,a^{2})\sqrt{cx^{4}+bx^{2}+a}}{16\,ax^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(12*a*b*sqrt(c)*x^4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 +
 b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^4*lo
g(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(a) + 8*a^2)/x^4) + 4*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt(c*x^4 + b*x
^2 + a))/(a*x^4), -1/32*(24*a*b*sqrt(-c)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 3*(b^2 + 4*a*c)*sq
rt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt
(c*x^4 + b*x^2 + a))/(a*x^4), 1/16*(6*a*b*sqrt(c)*x^4*log(-8*c^2*x^4 - 8*b*
c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 3*
(b^2 + 4*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sq
```

```
rt(c*x^4 + b*x^2 + a))/(a*x^4), -1/16*(12*a*b*sqrt(-c)*x^4*arctan(1/2*sqrt(
c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 3*(b
^2 + 4*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*s
qrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt
(c*x^4 + b*x^2 + a))/(a*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**5,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(125) = 250.
time = 4.87, size = 302, normalized size = 2.00

$$-\frac{3}{4} b\sqrt{c} \log\left(\left|2\left(\sqrt{c}x^{2} - \sqrt{cx^{4} + bx^{2} + a}\right)\sqrt{c} + b\right|\right) + \frac{1}{2}\sqrt{cx^{4} + bx^{2} + a}\,c + \frac{3\left(b^{2} + 4\,ac\right)\arctan\left(-\frac{\sqrt{c}x^{2} - \sqrt{cx^{4} + bx^{2} + a}}{\sqrt{-a}}\right)}{8\sqrt{-a}} + \frac{5\left(\sqrt{c}x^{2} - \sqrt{cx^{4} + bx^{2} + a}\right)^{3} b^{2} + 4\left(\sqrt{c}x^{2} - \sqrt{cx^{4} + bx^{2} + a}\right)^{3} ac + 16\left(\sqrt{c}x^{2} - \sqrt{cx^{4} + bx^{2} + a}\right)^{2} ab\sqrt{c} - 3\left(\sqrt{c}x^{2} - \sqrt{cx^{4} + bx^{2} + a}\right)ab^{2} + 4\left(\sqrt{c}x^{2} - \sqrt{cx^{4} + bx^{2} + a}\right)a^{2}c - 8\,a^{2}b\sqrt{c}}{8\left(\left(\sqrt{c}x^{2} - \sqrt{cx^{4} + bx^{2} + a}\right)^{2} - a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] -3/4*b*sqrt(c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) +
b)) + 1/2*sqrt(c*x^4 + b*x^2 + a)*c + 3/8*(b^2 + 4*a*c)*arctan(-(sqrt(c)*x^
2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) + 1/8*(5*(sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))^3*b^2 + 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a
*c + 16*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a*b*sqrt(c) - 3*(sqrt(c)*
x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b^2 + 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*a^2*c - 8*a^2*b*sqrt(c))/((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 -
 a)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^5,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^5, x)
```

3.943 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=163

$$-\frac{\left(2ab+\left(b^2+8ac\right)x^2\right)\sqrt{a+bx^2+cx^4}}{16ax^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6}+\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{3/2}}$$

```
[Out] -1/6*(c*x^4+b*x^2+a)^(3/2)/x^6+1/32*b*(-12*a*c+b^2)*arctanh(1/2*(b*x^2+2*a)
/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)+1/2*c^(3/2)*arctanh(1/2*(2*c*x^2+b)
/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))-1/16*(2*a*b+(8*a*c+b^2)*x^2)*(c*x^4+b*x^2+a
)^(1/2)/a/x^4
```

Rubi [A]
time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1128, 746, 824, 857, 635, 212, 738}

$$\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{3/2}}-\frac{\left(x^2\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^2+cx^4}}{16ax^4}+\frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -1/16*((2*a*b + (b^2 + 8*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) - (a +
b*x^2 + c*x^4)^(3/2)/(6*x^6) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*S
qrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2+cx^4)^{3/2}}{6x^6} + \frac{1}{4}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{(2ab+(b^2+8ac)\,x^2)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{(a+bx^2+cx^4)^{3/2}}{6x^6} - \frac{\text{Subst}\Big(\int \frac{\frac{1}{2}b(b^2-}{x\sqrt{a+}}}{}$$

$$= -\frac{(2ab+(b^2+8ac)\,x^2)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{(a+bx^2+cx^4)^{3/2}}{6x^6} + \frac{1}{2}c^2\text{Subst}\Big(\int \frac{}{\sqrt{}}$$

$$= -\frac{(2ab+(b^2+8ac)\,x^2)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{(a+bx^2+cx^4)^{3/2}}{6x^6} + c^2\text{Subst}\Big(\int \frac{1}{4c-}$$

$$= -\frac{(2ab+(b^2+8ac)\,x^2)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{(a+bx^2+cx^4)^{3/2}}{6x^6} + \frac{b(b^2-12ac)\tan}{}$$

Mathematica [A]
time = 0.68, size = 148, normalized size = 0.91

$$\frac{\sqrt{a+bx^2+cx^4}\,(-8a^2-14abx^2-3b^2x^4-32acx^4)}{48ax^6} + \frac{(b^3-12abc)\tanh^{-1}\left(\frac{-\sqrt{c}\,x^2+\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{1}{2}c^{3/2}\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 - 14*a*b*x^2 - 3*b^2*x^4 - 32*a*c*x^4))/(4
8*a*x^6) + ((b^3 - 12*a*b*c)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x
^4])/Sqrt[a]])/(16*a^(3/2)) - (c^(3/2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4]])/2
```

Maple [A]
time = 0.07, size = 202, normalized size = 1.24

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(32c\,x^4a+3b^2x^4+14ab\,x^2+8a^2)}{48x^6a} + \frac{c^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2} - \frac{3bc\ln\Big(\frac{2a+b\,x^2+2\sqrt{a}}{}}{8}$

default	$\frac{c^{\frac{3}{2}} \ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2} - \frac{7b\sqrt{c\,x^4+b\,x^2+a}}{24x^4} - \frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{16a\,x^2} + \frac{b^3 \ln\left(\frac{2a+b\,x^2+2\sqrt{a}\ \cdots}{\cdots}\right.}{32\cdots}$
elliptic	$\frac{c^{\frac{3}{2}} \ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2} - \frac{7b\sqrt{c\,x^4+b\,x^2+a}}{24x^4} - \frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{16a\,x^2} + \frac{b^3 \ln\left(\frac{2a+b\,x^2+2\sqrt{a}\ \cdots}{\cdots}\right.}{32\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-7/24*b/x^4*(c*x
^4+b*x^2+a)^(1/2)-1/16/a*b^2/x^2*(c*x^4+b*x^2+a)^(1/2)+1/32/a^(3/2)*b^3*ln(
(2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-3/8*b*c/a^(1/2)*ln((2*a+b*
x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-2/3*c/x^2*(c*x^4+b*x^2+a)^(1/2)-1
/6*a/x^6*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.41, size = 771, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(48*a^2*c^(3/2)*x^6*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^
6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 +
2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8
*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x^6), -1/192*(96*a^2*sqrt(-c)*c*x^6*arc
tan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 +
 a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2
```

```
 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(14*a^
2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x
^6), 1/96*(24*a^2*c^(3/2)*x^6*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x
^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(-a
)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) - 2*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8*a^3)*sqrt(
c*x^4 + b*x^2 + a))/(a^2*x^6), -1/96*(48*a^2*sqrt(-c)*c*x^6*arctan(1/2*sqrt
(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 3*(
b^3 - 12*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*
a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^
2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**7,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**7, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(135) = 270.
time = 4.87, size = 412, normalized size = 2.53

$$-\frac{1}{2}\, c^{\frac{3}{2}} \log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right) - \frac{\left(b^3 - 12\,abc\right)\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{16\sqrt{-a}\,a} + \frac{3\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^5 b^3\sqrt{c} + 60\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^5 abc^{\frac{3}{2}} + 48\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^4 ab^2c + 96\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^4 a^2c^2 + 8\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^3 ab^3\sqrt{c} - 96\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 a^3c^2 - 3\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)a^2b^3\sqrt{c} + 36\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)a^3bc^{\frac{3}{2}} + 64\,a^4c^2}{48\left(\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 - a\right)^3 a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*c^(3/2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b
)) - 1/16*(b^3 - 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/
sqrt(-a))/(sqrt(-a)*a) + 1/48*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*
b^3*sqrt(c) + 60*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c^(3/2) + 48
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a*b^2*c + 96*(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))^4*a^2*c^2 + 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^
3*a*b^3*sqrt(c) - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*c^2 - 3*
(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*b^3*sqrt(c) + 36*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))*a^3*b*c^(3/2) + 64*a^4*c^2)/(((sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))^2 - a)^3*a*sqrt(c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^7,x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^7, x)
```

3.944 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=133

$$\frac{3(b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{128a^2x^4}-\frac{(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{16ax^8}-\frac{3(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx}}\right)}{256a^{5/2}}$$

```
[Out] -1/16*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(3/2)/a/x^8-3/256*(-4*a*c+b^2)^2*arctanh(
1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)+3/128*(-4*a*c+b^2)*(
b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4
```

Rubi [A]
time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1128, 734, 738, 212}

$$-\frac{3(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{5/2}}+\frac{3(b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{128a^2x^4}-\frac{(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{16ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (3*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^2*x^4) - ((2
*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*a*x^8) - (3*(b^2 - 4*a*c)^2*ArcT
anh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(256*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{(2a+bx^2)\,(a+bx^2+cx^4)^{3/2}}{16ax^8} - \frac{(3(b^2-4ac))\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x,\right.}{32a}$$

$$= \frac{3(b^2-4ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{(2a+bx^2)\,(a+bx^2+cx^4)^{3/2}}{16ax^8} + \frac{(3(b}{}$$

$$= \frac{3(b^2-4ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{(2a+bx^2)\,(a+bx^2+cx^4)^{3/2}}{16ax^8} - \frac{(3(b}{}$$

$$= \frac{3(b^2-4ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{(2a+bx^2)\,(a+bx^2+cx^4)^{3/2}}{16ax^8} - \frac{3(b^2}{}$$

Mathematica [**A**]
time = 0.68, size = 120, normalized size = 0.90

$$\frac{-\frac{\sqrt{a}\,(2a+bx^2)\sqrt{a+bx^2+cx^4}\,(8a^2+8abx^2-3b^2x^4+20acx^4)}{x^8}+3(b^2-4ac)^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{128a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (-((Sqrt[a]*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4]*(8*a^2 + 8*a*b*x^2 - 3*b^
2*x^4 + 20*a*c*x^4))/x^8) + 3*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a
 + b*x^2 + c*x^4])/Sqrt[a]])/(128*a^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(115) = 230$.
time = 0.06, size = 260, normalized size = 1.95

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(20abc\,x^6-3b^3x^6+40a^2c\,x^4+2a\,b^2x^4+24a^2b\,x^2+16a^3\right)}{128x^8a^2}-\frac{3c^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right.}{16\sqrt{a}}$
default	$-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{64a\,x^4}+\frac{3b^3\sqrt{c\,x^4+b\,x^2+a}}{128a^2x^2}-\frac{3c^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16\sqrt{a}}-\frac{5c\sqrt{c\,x^4+}}{16}$
elliptic	$-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{64a\,x^4}+\frac{3b^3\sqrt{c\,x^4+b\,x^2+a}}{128a^2x^2}-\frac{3c^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16\sqrt{a}}-\frac{5c\sqrt{c\,x^4+}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*b^2/a/x^4*(c*x^4+b*x^2+a)^(1/2)+3/128*b^3/a^2/x^2*(c*x^4+b*x^2+a)^(1/
2)-3/16*c^2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-5/1
6*c/x^4*(c*x^4+b*x^2+a)^(1/2)-3/256*b^4/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*
x^4+b*x^2+a)^(1/2))/x^2)-5/32/a*c*b/x^2*(c*x^4+b*x^2+a)^(1/2)+3/32/a^(3/2)*
c*b^2*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/8*a/x^8*(c*x^4+
b*x^2+a)^(1/2)-3/16*b/x^6*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.42, size = 319, normalized size = 2.40

$$\left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{a}\,x^8\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,((3\,ab^3-20\,a^2bc)x^6-24\,a^3bx^2-2\,(a^2b^2+20\,a^3c)x^4-16\,a^4)\sqrt{cx^4+bx^2+a}}{512\,a^3x^8},\ \frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-a}\,x^8\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\,((3\,ab^3-20\,a^2bc)x^6-24\,a^3bx^2-2\,(a^2b^2+20\,a^3c)x^4-16\,a^4)\sqrt{cx^4+bx^2+a}}{256\,a^3x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^8*log(-((b^2 + 4*a*c)*x^
4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^
```

```
4) + 4*((3*a*b^3 - 20*a^2*b*c)*x^6 - 24*a^3*b*x^2 - 2*(a^2*b^2 + 20*a^3*c)*
x^4 - 16*a^4)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^8), 1/256*(3*(b^4 - 8*a*b^2*c
 + 16*a^2*c^2)*sqrt(-a)*x^8*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*x^6 - 24*
a^3*b*x^2 - 2*(a^2*b^2 + 20*a^3*c)*x^4 - 16*a^4)*sqrt(c*x^4 + b*x^2 + a))/(
a^3*x^8)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}{x^{9}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**9,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**9, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 606 vs. 2(115) = 230.
time = 4.25, size = 606, normalized size = 4.56

$$\frac{3\,(b^4 - 8\,ab^2c + 16\,a^2c^2)\arctan\left(-\frac{\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{128\,\sqrt{-a}\,a^2} - \frac{3\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^7 b^4 - 24\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^7 ab^2c - 80\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^7 a^2c^2 - 256\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^6 a^2bc^{\frac{3}{2}} - 11\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^5 ab^4 - 168\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^5 a^2b^2c - 48\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^5 a^3c^2 - 128\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^4 a^2b^3\sqrt{c} - 11\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^3 a^2b^4 - 168\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^3 a^3b^2c - 48\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^3 a^4c^2 - 256\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 a^4bc^{\frac{3}{2}} + 3\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right) a^3b^4 - 24\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right) a^4b^2c - 80\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right) a^5c^2}{128\left(\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 - a\right)^4 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 3/128*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/128*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b
*x^2 + a))^7*b^4 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b^2*c - 8
0*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*c^2 - 256*(sqrt(c)*x^2 - sq
rt(c*x^4 + b*x^2 + a))^6*a^2*b*c^(3/2) - 11*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))^5*a*b^4 - 168*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*b^2*c
- 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*c^2 - 128*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^4*a^2*b^3*sqrt(c) - 11*(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 + a))^3*a^2*b^4 - 168*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^3
*b^2*c - 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^4*c^2 - 256*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^4*b*c^(3/2) + 3*(sqrt(c)*x^2 - sqrt(c*
x^4 + b*x^2 + a))*a^3*b^4 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^4*
b^2*c - 80*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^5*c^2)/(((sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^2 - a)^4*a^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^9} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^9,x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^9, x)
```

3.945 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{11}} dx$

Optimal. Leaf size=162

$$-\frac{3b(b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{256a^3x^4}+\frac{b(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{32a^2x^8}-\frac{(a+bx^2+cx^4)^{5/2}}{10ax^{10}}+\frac{3b(b^2-4a$$

```
[Out] 1/32*b*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(3/2)/a^2/x^8-1/10*(c*x^4+b*x^2+a)^(5/2)
/a/x^10+3/512*b*(-4*a*c+b^2)^2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2
+a)^(1/2))/a^(7/2)-3/256*b*(-4*a*c+b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a
^3/x^4
```

Rubi [A]
time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1128, 744, 734, 738, 212}

$$\frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{7/2}}-\frac{3b(b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{256a^3x^4}+\frac{b(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{32a^2x^8}-\frac{(a+bx^2+cx^4)^{5/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] (-3*b*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a^3*x^4) +
(b*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*a^2*x^8) - (a + b*x^2 + c*x
^4)^(5/2)/(10*a*x^10) + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[
a]*Sqrt[a + b*x^2 + c*x^4])])/(512*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{11}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^6}\,dx,x,x^2\right)\\
&= -\frac{(a+bx^2+cx^4)^{5/2}}{10ax^{10}} - \frac{b\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^5}\,dx,x,x^2\right)}{4a}\\
&= \frac{b(2a+bx^2)\,(a+bx^2+cx^4)^{3/2}}{32a^2x^8} - \frac{(a+bx^2+cx^4)^{5/2}}{10ax^{10}} + \frac{(3b(b^2-4ac))\,\text{Subst}\left(\int \frac{\sqrt{}}{}\right.}{64a}\\
&= -\frac{3b(b^2-4ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b(2a+bx^2)\,(a+bx^2+cx^4)^{3/2}}{32a^2x^8} - \frac{(a}{}\\
&= -\frac{3b(b^2-4ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b(2a+bx^2)\,(a+bx^2+cx^4)^{3/2}}{32a^2x^8} - \frac{(a}{}\\
&= -\frac{3b(b^2-4ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b(2a+bx^2)\,(a+bx^2+cx^4)^{3/2}}{32a^2x^8} - \frac{(a}{}
\end{aligned}$$

Mathematica [A]

time = 0.97, size = 160, normalized size = 0.99

$$\frac{-\frac{\sqrt{a}\sqrt{a+bx^2+cx^4}\left(128a^4+15b^4x^8-10ab^2x^6(b+10cx^2)+16a^3(11bx^2+16cx^4)+8a^2x^4(b^2+7bcx^2+16c^2x^4)\right)}{x^{10}}-15b(b^2-4ac)^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{1280a^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]

[Out] (-((Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(128*a^4 + 15*b^4*x^8 - 10*a*b^2*x^6*(b + 10*c*x^2) + 16*a^3*(11*b*x^2 + 16*c*x^4) + 8*a^2*x^4*(b^2 + 7*b*c*x^2 + 16*c^2*x^4)))/x^10) - 15*b*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(1280*a^(7/2))

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(140) = 280.
time = 0.08, size = 337, normalized size = 2.08

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(128a^2c^2x^8-100a\,b^2c\,x^8+15b^4x^8+56a^2bc\,x^6-10a\,b^3x^6+256a^3c\,x^4+8a^2b^2x^4+176a^3b\,x^2+128a^4\right)}{1280x^{10}a^3}+-\frac{3b}{}$
default	$-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{160a\,x^6}+\frac{b^3\sqrt{c\,x^4+b\,x^2+a}}{128a^2x^4}-\frac{3b^4\sqrt{c\,x^4+b\,x^2+a}}{256a^3x^2}+\frac{3b^5\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+}}{x^2}\right.}{512a^{\frac{7}{2}}}$
elliptic	$-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{160a\,x^6}+\frac{b^3\sqrt{c\,x^4+b\,x^2+a}}{128a^2x^4}-\frac{3b^4\sqrt{c\,x^4+b\,x^2+a}}{256a^3x^2}+\frac{3b^5\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+}}{x^2}\right.}{512a^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^(3/2)/x^11,x,method=_RETURNVERBOSE)

[Out] -1/160/a*b^2/x^6*(c*x^4+b*x^2+a)^(1/2)+1/128/a^2*b^3/x^4*(c*x^4+b*x^2+a)^(1/2)-3/256/a^3*b^4/x^2*(c*x^4+b*x^2+a)^(1/2)+3/512/a^(7/2)*b^5*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/10*c^2/a/x^2*(c*x^4+b*x^2+a)^(1/2)-3/64*b^3*c/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+3/32*b*c^2/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-7/160*b*c/a/x^4*(c*x^4+b*x^2+a)^(1/2)+5/64*b^2*c/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)-1/5*c/x^6*(c*x^4+b*x^2+a)^(1/2)-1/10*a/x^10*(c*x^4+b*x^2+a)^(1/2)-11/80*b/x^8*(c*x^4+b*x^2+a)^(1/2)

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.47, size = 383, normalized size = 2.36

$$\left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{a}\,x^{10}\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)-4\,((15\,ab^4-100\,a^2b^2c+128\,a^3c^2)x^8+176\,a^4bx^2-2\,(5\,a^2b^3-28\,a^3bc)x^6+128\,a^5+8\,(a^3b^2+32\,a^4c)x^4)\sqrt{cx^4+bx^2+a}}{5120\,a^4x^{10}},-\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-a}\,x^{10}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\,((15\,ab^4-100\,a^2b^2c+128\,a^3c^2)x^8+176\,a^4bx^2-2\,(5\,a^2b^3-28\,a^3bc)x^6+128\,a^5+8\,(a^3b^2+32\,a^4c)x^4)\sqrt{cx^4+bx^2+a}}{2560\,a^4x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^10*log(-((b^2 + 4*a*
c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^
2)/x^4) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^8 + 176*a^4*b*x^2 -
 2*(5*a^2*b^3 - 28*a^3*b*c)*x^6 + 128*a^5 + 8*(a^3*b^2 + 32*a^4*c)*x^4)*sqr
t(c*x^4 + b*x^2 + a))/(a^4*x^10), -1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*sqrt(-a)*x^10*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)
/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x
^8 + 176*a^4*b*x^2 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^6 + 128*a^5 + 8*(a^3*b^2
+ 32*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^10)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**11,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**11, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 832 vs. 2(140) = 280.
time = 3.18, size = 832, normalized size = 5.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -3/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/1280*(15*(sqrt(c)*x^2 - sqrt(c*x^
```

```
4 + b*x^2 + a))^9*b^5 - 120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a*b^3
*c + 240*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a^2*b*c^2 + 1280*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))^8*a^3*c^(5/2) - 70*(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))^7*a*b^5 + 560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^
2*b^3*c + 2720*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^3*b*c^2 + 5120*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^3*b^2*c^(3/2) + 128*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*b^5 + 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))^5*a^3*b^3*c + 3840*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^4*
b*c^2 + 1280*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^3*b^4*sqrt(c) + 25
60*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^4*b^2*c^(3/2) + 2560*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^5*c^(5/2) + 70*(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))^3*a^3*b^5 + 2000*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3
*a^4*b^3*c + 2400*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^5*b*c^2 + 256
0*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^5*b^2*c^(3/2) - 15*(sqrt(c)*x
^2 - sqrt(c*x^4 + b*x^2 + a))*a^4*b^5 + 120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))*a^5*b^3*c + 1040*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^6*b*c^2
 + 256*a^7*c^(5/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^5*a^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^11,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^11, x)
```

3.946 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=216

$$\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4}-\frac{(7b^2-4ac)(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{384a^3x^8}-\frac{(a+bx^2+cx}{12ax^{12}}$$

```
[Out] -1/384*(-4*a*c+7*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(3/2)/a^3/x^8-1/12*(c*x^4
+b*x^2+a)^(5/2)/a/x^12+7/120*b*(c*x^4+b*x^2+a)^(5/2)/a^2/x^10-1/2048*(-4*a*
c+b^2)^2*(-4*a*c+7*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/
2))/a^(9/2)+1/1024*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^
(1/2)/a^4/x^4
```

Rubi [A]
time = 0.14, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1128, 758, 820, 734, 738, 212}

$$-\frac{(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2048a^{9/2}}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4}-\frac{(7b^2-4ac)(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{384a^3x^8}+\frac{7b(a+bx^2+cx^4)^{5/2}}{120a^2x^{10}}-\frac{(a+bx^2+cx^4)^{5/2}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(1024
*a^4*x^4) - ((7*b^2 - 4*a*c)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(384*
a^3*x^8) - (a + b*x^2 + c*x^4)^(5/2)/(12*a*x^12) + (7*b*(a + b*x^2 + c*x^4)
^(5/2))/(120*a^2*x^10) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2048*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{(a+bx^2+cx^4)^{5/2}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+cx\right)(a+bx+cx^2)^{3/2}}{x^6}\,dx, x, x^2\right)}{12a}$$

$$= -\frac{(a+bx^2+cx^4)^{5/2}}{12ax^{12}} + \frac{7b(a+bx^2+cx^4)^{5/2}}{120a^2x^{10}} + \frac{(7b^2-4ac)\,\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^5}\right.}{48a^2}$$

$$= -\frac{(7b^2-4ac)(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{384a^3x^8} - \frac{(a+bx^2+cx^4)^{5/2}}{12ax^{12}} + \frac{7b(a+bx^2+}{120a^2x^1}$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{(7b^2-4ac)(2a+bx^2)(a+}{384a^3x^8}$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{(7b^2-4ac)(2a+bx^2)(a+}{384a^3x^8}$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{(7b^2-4ac)(2a+bx^2)(a+}{384a^3x^8}$$

Mathematica [A]
time = 1.35, size = 201, normalized size = 0.93

$$\frac{-\frac{\sqrt{a}\sqrt{a+bx^2+cx^4}\,(1280a^5-105b^5x^{10}+10ab^3x^8(7b+76cx^2)+64a^4(26bx^2+35cx^4)+48a^3x^4(b^2+6bcx^2+10c^2x^4)-8a^2bx^6(7b^2+54bcx^2+162c^2x^4))}{x^{12}}+15(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{15360a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] (-((Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(1280*a^5 - 105*b^5*x^10 + 10*a*b^3*x^8
*(7*b + 76*c*x^2) + 64*a^4*(26*b*x^2 + 35*c*x^4) + 48*a^3*x^4*(b^2 + 6*b*c*
x^2 + 10*c^2*x^4) - 8*a^2*b*x^6*(7*b^2 + 54*b*c*x^2 + 162*c^2*x^4)))/x^12)
+ 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2
+ c*x^4])/Sqrt[a]])/(15360*a^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(190) = 380$.
time = 0.09, size = 457, normalized size = 2.12

method	result

risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(-1296a^{2}b\,c^{2}x^{10}+760a\,b^{3}c\,x^{10}-105b^{5}x^{10}+480a^{3}c^{2}x^{8}-432a^{2}b^{2}c\,x^{8}+70b^{4}a\,x^{8}+288a^{3}bc\,x^{6}-56a^{2}b^{3}x^{6}+\right.}{15360x^{12}a^{4}}$
default	$-\frac{7b^{6}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2048a^{\frac{9}{2}}}-\frac{b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{320a\,x^{8}}+\frac{7b^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{1920a^{2}x^{6}}-\frac{7b^{4}\sqrt{c\,x^{4}}}{153}$
elliptic	$-\frac{7b^{6}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2048a^{\frac{9}{2}}}-\frac{b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{320a\,x^{8}}+\frac{7b^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{1920a^{2}x^{6}}-\frac{7b^{4}\sqrt{c\,x^{4}}}{153}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -7/2048/a^(9/2)*b^6*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/3
20/a*b^2/x^8*(c*x^4+b*x^2+a)^(1/2)+7/1920/a^2*b^3/x^6*(c*x^4+b*x^2+a)^(1/2)
-7/1536/a^3*b^4/x^4*(c*x^4+b*x^2+a)^(1/2)+7/1024/a^4*b^5/x^2*(c*x^4+b*x^2+a
)^(1/2)-9/128*c^2*b^2/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2)
)/x^2)-1/32*c^2/a/x^4*(c*x^4+b*x^2+a)^(1/2)-1/12*a/x^12*(c*x^4+b*x^2+a)^(1/
2)-7/48*c/x^8*(c*x^4+b*x^2+a)^(1/2)+27/320*c^2*b/a^2/x^2*(c*x^4+b*x^2+a)^(1
/2)+15/512/a^(7/2)*b^4*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2
)-19/384/a^3*b^3*c/x^2*(c*x^4+b*x^2+a)^(1/2)+9/320/a^2*b^2*c/x^4*(c*x^4+b*x
^2+a)^(1/2)-3/160/a*b*c/x^6*(c*x^4+b*x^2+a)^(1/2)-13/120*b/x^10*(c*x^4+b*x^
2+a)^(1/2)+1/32*c^3/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/
x^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.54, size = 473, normalized size = 2.19

$$\left[-\frac{15\,(7\,b^{6}-60\,ab^{4}c+144\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3})\sqrt{a}\,x^{12}\log\left(-\frac{(b^{2}+4\,ac)x^{4}+8\,abx^{2}+4\,\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2\,a)\sqrt{a}+8\,a^{2}}{x^{4}}\right)-4\,((105\,ab^{5}-760\,a^{2}b^{3}c+1296\,a^{3}bc^{2})x^{10}-2\,(35\,a^{2}b^{4}-216\,a^{3}b^{2}c+240\,a^{4}c^{2})x^{8}-1664\,a^{5}bx^{2}+8\,(7\,a^{3}b^{3}-36\,a^{4}bc)x^{6}-1280\,a^{6}-16\,(3\,a^{4}b^{2}+140\,a^{5}c)x^{4})\sqrt{cx^{4}+bx^{2}+a}}{61440\,a^{5}x^{12}},\ \frac{15\,(7\,b^{6}-60\,ab^{4}c+144\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3})\sqrt{-a}\,x^{12}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2\,a)\sqrt{-a}}{2\,(acx^{4}+abx^{2}+a^{2})}\right)+2\,((105\,ab^{5}-760\,a^{2}b^{3}c+1296\,a^{3}bc^{2})x^{10}-2\,(35\,a^{2}b^{4}-216\,a^{3}b^{2}c+240\,a^{4}c^{2})x^{8}-1664\,a^{5}bx^{2}+8\,(7\,a^{3}b^{3}-36\,a^{4}bc)x^{6}-1280\,a^{6}-16\,(3\,a^{4}b^{2}+140\,a^{5}c)x^{4})\sqrt{cx^{4}+bx^{2}+a}}{30720\,a^{5}x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] [-1/61440*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*x
^12*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c
^2)*x^10 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^8 - 1664*a^5*b*x^
2 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^6 - 1280*a^6 - 16*(3*a^4*b^2 + 140*a^5*c)*
x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^12), 1/30720*(15*(7*b^6 - 60*a*b^4*c +
 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((105*a*b^5 - 76
0*a^2*b^3*c + 1296*a^3*b*c^2)*x^10 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^
4*c^2)*x^8 - 1664*a^5*b*x^2 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^6 - 1280*a^6 - 1
6*(3*a^4*b^2 + 140*a^5*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^12)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**13,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**13, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. 2(190) = 380.
time = 3.41, size = 1235, normalized size = 5.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] 1/1024*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*arctan(-(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/15360*(105*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^11*b^6 - 900*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^11*a*b^4*c + 2160*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^11
*a^2*b^2*c^2 - 960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^11*a^3*c^3 - 595
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a*b^6 + 5100*(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))^9*a^2*b^4*c - 12240*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^9*a^3*b^2*c^2 - 15040*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a^4*c
^3 - 76800*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^8*a^4*b*c^(5/2) + 1386*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*b^6 - 11880*(sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))^7*a^3*b^4*c - 97440*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
 + a))^7*a^4*b^2*c^2 - 24960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^5*
```

```
c^3 - 112640*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^4*b^3*c^(3/2) - 61
440*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^5*b*c^(5/2) - 1686*(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*b^6 - 42600*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^5*a^4*b^4*c - 128160*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
^5*a^5*b^2*c^2 - 24960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^6*c^3 -
15360*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^4*b^5*sqrt(c) - 61440*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^5*b^3*c^(3/2) - 92160*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^4*a^6*b*c^(5/2) - 595*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^3*a^4*b^6 - 25620*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*
a^5*b^4*c - 58320*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^6*b^2*c^2 - 1
5040*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^7*c^3 - 30720*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^2*a^6*b^3*c^(3/2) - 12288*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^2*a^7*b*c^(5/2) + 105*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*a^5*b^6 - 900*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^6*b^4*c - 132
00*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^7*b^2*c^2 - 960*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))*a^8*c^3 - 3072*a^8*b*c^(5/2))/(((sqrt(c)*x^2 - sq
rt(c*x^4 + b*x^2 + a))^2 - a)^6*a^4)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^13,x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^13, x)
```

3.947 $\int x^4 (a+bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=495

$$\frac{(8b^4-51ab^2c+60a^2c^2)\,x\sqrt{a+bx^2+cx^4}}{1155c^3}-\frac{8b(2b^2-9ac)\,(b^2-3ac)\,x\sqrt{a+bx^2+cx^4}}{1155c^{7/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{x^3(b(2b^2+ac)+10$$

```
[Out] 1/33*x^3*(3*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c+1/1155*(60*a^2*c^2-51*a*b^2*c+
8*b^4)*x*(c*x^4+b*x^2+a)^(1/2)/c^3-1/385*x^3*(b*(a*c+2*b^2)+10*c*(-3*a*c+b^
2)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2-8/1155*b*(-9*a*c+2*b^2)*(-3*a*c+b^2)*x*(c
*x^4+b*x^2+a)^(1/2)/c^(7/2)/(a^(1/2)+x^2*c^(1/2))+8/1155*a^(1/4)*b*(-9*a*c+
2*b^2)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^
(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c
^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2+a)^(1/2)-1/2310*a^(1/4)*(cos(2*arcta
n(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x
^2*c^(1/2))*(8*b*(-9*a*c+2*b^2)*(-3*a*c+b^2)+(60*a^2*c^2-51*a*b^2*c+8*b^4)*
a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(15/4)/(
c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1130, 1287, 1293, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\,(60a^2c^2-51ab^2c+8b^4)+8b(2b^2-9ac)\,(b^2-3ac)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2310c^{15/4}\sqrt{a+bx^2+cx^4}}+\frac{x(60a^2c^2-51ab^2c+8b^4)\sqrt{a+bx^2+cx^4}}{1155c^3}+\frac{8\sqrt[4]{a}\,b(2b^2-9ac)\,(b^2-3ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{1155c^{15/4}\sqrt{a+bx^2+cx^4}}-\frac{8bx(2b^2-9ac)\,(b^2-3ac)\sqrt{a+bx^2+cx^4}}{1155c^{7/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{x^3(10cx^2(b^2-3ac)+b(ac+2b^2))\sqrt{a+bx^2+cx^4}}{385c^2}+\frac{x^3(b+3cx^2)\,(a+bx^2+cx^4)^{3/2}}{33c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((8*b^4 - 51*a*b^2*c + 60*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(1155*c^3) -
(8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(1155*c^(7/2)
*(Sqrt[a] + Sqrt[c]*x^2)) - (x^3*(b*(2*b^2 + a*c) + 10*c*(b^2 - 3*a*c)*x^2)
*Sqrt[a + b*x^2 + c*x^4])/(385*c^2) + (x^3*(b + 3*c*x^2)*(a + b*x^2 + c*x^4
)^(3/2))/(33*c) + (8*a^(1/4)*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(1155*c^(15/4)*Sq
rt[a + b*x^2 + c*x^4]) - (a^(1/4)*(8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c) + Sqrt
[a]*Sqrt[c]*(8*b^4 - 51*a*b^2*c + 60*a^2*c^2))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2310*c^(15/4)*Sqrt[a + b*x^2 +
 c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*((2*b*p + c*(m + 4*p - 1)*x^
2)/(c*(m + 4*p + 1)*(m + 4*p - 1))), x] - Dist[2*p*(d^2/(c*(m + 4*p + 1)*(m
 + 4*p - 1))), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1287

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Dist[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^4\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{x^3(b+3cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{33c}-\frac{\int x^2(3ab+6(b^2-3ac)\,x^2)\sqrt{a+bx^2+cx^4}}{33c}$$

$$=-\frac{x^3(b(2b^2+ac)+10c(b^2-3ac)\,x^2)\sqrt{a+bx^2+cx^4}}{385c^2}+\frac{x^3(b+3cx^2)\left(a+bx^2\right.}{33c}$$

$$=\frac{(8b^4-51ab^2c+60a^2c^2)\,x\sqrt{a+bx^2+cx^4}}{1155c^3}-\frac{x^3(b(2b^2+ac)+10c(b^2-3ac)\,x}{385c^2}$$

$$=\frac{(8b^4-51ab^2c+60a^2c^2)\,x\sqrt{a+bx^2+cx^4}}{1155c^3}-\frac{x^3(b(2b^2+ac)+10c(b^2-3ac)\,x}{385c^2}$$

$$=\frac{(8b^4-51ab^2c+60a^2c^2)\,x\sqrt{a+bx^2+cx^4}}{1155c^3}-\frac{8b(2b^2-9ac)\,(b^2-3ac)\,x\sqrt{a+b}}{1155c^{7/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.46, size = 657, normalized size = 1.33

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(60*a^3*c^2 + a^2*c*(-51*b^2 + 92*b*
c*x^2 + 255*c^2*x^4) + a*(8*b^4 - 57*b^3*c*x^2 - 14*b^2*c^2*x^4 + 367*b*c^3
*x^6 + 300*c^4*x^8) + x^2*(8*b^5 + 2*b^4*c*x^2 - b^3*c^2*x^4 + 145*b^2*c^3*
x^6 + 245*b*c^4*x^8 + 105*c^5*x^10)) - (4*I)*b*(2*b^4 - 15*a*b^2*c + 27*a^2
*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^
```

```
2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x]
, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-8*b^6 + 68*a*b^4*c
 - 159*a^2*b^2*c^2 + 60*a^3*c^3 + 8*b^5*Sqrt[b^2 - 4*a*c] - 60*a*b^3*c*Sqrt
[b^2 - 4*a*c] + 108*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]])/(2310
*c^4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.05, size = 674, normalized size = 1.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c*x^9*(c*x^4+b*x^2+a)^(1/2)+4/33*b*x^7*(c*x^4+b*x^2+a)^(1/2)+1/7*(13/1
1*a*c+1/33*b^2)/c*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(38/33*a*b-6/7*(13/11*a*c+1
/33*b^2)/c*b)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(a^2-5/7*(13/11*a*c+1/33*b^2)
/c*a-4/5*(38/33*a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/c*b)/c*x*(c*x^4+b*x^2+a)^
(1/2)-1/12*(a^2-5/7*(13/11*a*c+1/33*b^2)/c*a-4/5*(38/33*a*b-6/7*(13/11*a*c+
1/33*b^2)/c*b)/c*b)/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5*(38/33*a*b-6/7
*(13/11*a*c+1/33*b^2)/c*b)/c*a-2/3*(a^2-5/7*(13/11*a*c+1/33*b^2)/c*a-4/5*(3
8/33*a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/c*b)/c*b)*a*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^4*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.948 $\int x^2(a+bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=443

$$\frac{(8b^4-57ab^2c+84a^2c^2)\,x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{x(b(4b^2-9ac)+6c(2b^2-7ac)\,x^2)\sqrt{a+bx^2+cx^4}}{315c^2}+\frac{x(3b+7$$

```
[Out] 1/63*x*(7*c*x^2+3*b)*(c*x^4+b*x^2+a)^(3/2)/c-1/315*x*(b*(-9*a*c+4*b^2)+6*c*
(-7*a*c+2*b^2)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/315*(84*a^2*c^2-57*a*b^2*c+
8*b^4)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+x^2*c^(1/2))-1/315*a^(1/4)*
(84*a^2*c^2-57*a*b^2*c+8*b^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)+1/630*a^(1/4)*(co
s(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(
a^(1/2)+x^2*c^(1/2))*(8*b^4-57*a*b^2*c+84*a^2*c^2+4*b*(-6*a*c+b^2)*a^(1/2)*
c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*
x^2+a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1130, 1190, 1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}\,b\sqrt{c}\,(b^2-6ac)+8b^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{630c^{11/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+8b^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{a+bx^2+cx^4}}+\frac{x(84a^2c^2-57ab^2c+8b^4)\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{x(6cx^2(2b^2-7ac)+b(4b^2-9ac))\sqrt{a+bx^2+cx^4}}{315c^2}+\frac{x(3b+7cx^2)(a+bx^2+cx^4)^{3/2}}{63c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(315*c^(5/2)*
(Sqrt[a] + Sqrt[c]*x^2)) - (x*(b*(4*b^2 - 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)
*Sqrt[a + b*x^2 + c*x^4])/(315*c^2) + (x*(3*b + 7*c*x^2)*(a + b*x^2 + c*x^4
)^(3/2))/(63*c) - (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqr
t[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[
a]*b*Sqrt[c]*(b^2 - 6*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(630*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*((2*b*p + c*(m + 4*p - 1)*x^
2)/(c*(m + 4*p + 1)*(m + 4*p - 1))), x] - Dist[2*p*(d^2/(c*(m + 4*p + 1)*(m
 + 4*p - 1))), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int x^2\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{x(3b+7cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-\frac{\int\left(ab+2(2b^2-7ac)\,x^2\right)\sqrt{a+bx^2+cx^4}\,d}{21c}$$

$$=-\frac{x(b(4b^2-9ac)+6c(2b^2-7ac)\,x^2)\sqrt{a+bx^2+cx^4}}{315c^2}+\frac{x(3b+7cx^2)\,(a+bx}{63c}$$

$$=-\frac{x(b(4b^2-9ac)+6c(2b^2-7ac)\,x^2)\sqrt{a+bx^2+cx^4}}{315c^2}+\frac{x(3b+7cx^2)\,(a+bx}{63c}$$

$$=\frac{(8b^4-57ab^2c+84a^2c^2)\,x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{x(b(4b^2-9ac)+6c(2b^2-7ac)}{315c^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.23, size = 602, normalized size = 1.36

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b^4*x^2 - b^3*c*x^4 + 53*b^2*c^2
*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*b + 77*c*x^2) + a*(-4*b^3 + 2
7*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^3*x^6)) + I*(8*b^4 - 57*a*b^2*c + 84*a^
2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^5 + 65*a*b^3*
c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*b^2*c*Sqrt[b^2 - 4*a*c]
+ 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(1260*c^3*Sqrt[c/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.04, size = 545, normalized size = 1.23 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c*x^7*(c*x^4+b*x^2+a)^(1/2)+10/63*b*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(11/9
*a*c+1/21*b^2)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(76/63*a*b-4/5*(11/9*a*c+1/2
```

```
1*b^2)/c*b)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(76/63*a*b-4/5*(11/9*a*c+1/21*b^
2)/c*b)/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(a^2-3/5*(11/9*a*c+1/21*b^2)/c*a
-2/3*(76/63*a*b-4/5*(11/9*a*c+1/21*b^2)/c*b)/c*b)*a*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(El
lipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^2*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.949 $\int (a+bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=381

$$-\frac{2b(b^2-8ac)\,x\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x(b^2+10ac+3bcx^2)\sqrt{a+bx^2+cx^4}}{35c}+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}+\frac{2\sqrt[4]{a}\,b(b^2}{}$$

[Out] 1/7*x*(c*x^4+b*x^2+a)^(3/2)+1/35*x*(3*b*c*x^2+10*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/c-2/35*b*(-8*a*c+b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))+2/35*a^(1/4)*b*(-8*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/70*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b*(-8*a*c+b^2)+(-20*a*c+b^2)*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)

Rubi [A]
time = 0.14, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1105, 1190, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\,(b^2-20ac)+2b(b^2-8ac)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}\,b(b^2-8ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{2bx(b^2-8ac)\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x(10ac+b^2+3bcx^2)\sqrt{a+bx^2+cx^4}}{35c}+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2 + c*x^4)^(3/2),x]

[Out] (-2*b*(b^2 - 8*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(35*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (x*(b^2 + 10*a*c + 3*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c) + (x*(a + b*x^2 + c*x^4)^(3/2))/7 + (2*a^(1/4)*b*(b^2 - 8*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(70*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeqQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2} + \frac{3}{7}\int \left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}\, dx$$

$$= \frac{x(b^2+10ac+3bcx^2)\sqrt{a+bx^2+cx^4}}{35c} + \frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2} + \frac{\int \frac{-a(b^2-20ac)-2b}{\sqrt{a+bx^2}}}{35c}$$

$$= \frac{x(b^2+10ac+3bcx^2)\sqrt{a+bx^2+cx^4}}{35c} + \frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2} + \frac{\left(2\sqrt{a}\,b(b^2-8\right.}{}$$

$$= -\frac{2b(b^2-8ac)\,x\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{x(b^2+10ac+3bcx^2)\sqrt{a+bx^2+cx^4}}{35c} + \frac{1}{7}x$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 533, normalized size = 1.40

$$\frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(15a^2c+a(b^2+23bcx^2+20c^2x^4)+x^2(b^3+9b^2cx^2+13bc^2x^4+5c^3x^6))-ib(b^2-8ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+i\left(-b^4+9ab^2c-20a^2c^2+b^3\sqrt{b^2-4ac}-8abc\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{70c^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^2 + 23*b*c*x^2 + 20
*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c^3*x^6)) - I*b*(b^2
- 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a*b^2*c
 - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[
b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])])/(70*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4
])
```

Maple [A]
time = 0.04, size = 471, normalized size = 1.24

method	result

default	$\frac{c\,x^5\sqrt{c\,x^4+b\,x^2+a}}{7}+\frac{8b\,x^3\sqrt{c\,x^4+b\,x^2+a}}{35}+\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)x\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\left(a^2-\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)a}{3c}\right)\sqrt{\ }}{}$
elliptic	$\frac{c\,x^5\sqrt{c\,x^4+b\,x^2+a}}{7}+\frac{8b\,x^3\sqrt{c\,x^4+b\,x^2+a}}{35}+\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)x\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\left(a^2-\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)a}{3c}\right)\sqrt{\ }}{}$
risch	$\frac{x\left(5c^2x^4+8bc\,x^2+15ac+b^2\right)\sqrt{c\,x^4+b\,x^2+a}}{35c}+\frac{-\frac{\left(16abc-2b^3\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2(b}{}}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*c*x^5*(c*x^4+b*x^2+a)^(1/2)+8/35*b*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(9/7*a
*c+3/35*b^2)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(a^2-1/3*(9/7*a*c+3/35*b^2)/c*a)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(46/35*a*b-2/3*(9/7*a*c+3/35*b^2)/c*b)*a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,x^4+b\,x^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2), x)
```

3.950 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^2} dx$

Optimal. Leaf size=361

$$\frac{(b^2+12ac)\,x\sqrt{a+bx^2+cx^4}}{5\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{1}{5}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}-\frac{\sqrt[4]{a}\,(b^2+12ac)\left(\sqrt{a}\right.}{}$$

```
[Out] -(c*x^4+b*x^2+a)^(3/2)/x+1/5*x*(6*c*x^2+7*b)*(c*x^4+b*x^2+a)^(1/2)+1/5*(12*
a*c+b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-1/5*a^(1/4)*
(12*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/10*a^(1/4)*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arcta
n(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2)
)*(b^2+12*a*c+8*b*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2
)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1131, 1190, 1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\left(8\sqrt{a}\,b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{5c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{x(12ac+b^2)\sqrt{a+bx^2+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}+\frac{1}{5}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] ((b^2 + 12*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(5*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^
2)) + (x*(7*b + 6*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/5 - (a + b*x^2 + c*x^4)^(
3/2)/x - (a^(1/4)*(b^2 + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(5*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4
)*(b^2 + 8*Sqrt[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(10*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1131

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Dist[2*(p/
(d^2*(m + 1))), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx = -\frac{(a+bx^2+cx^4)^{3/2}}{x} + 3\int (b+2cx^2)\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{5}x(7b+6cx^2)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{x} + \frac{\int \frac{8abc+c(b^2+12ac)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{1}{5}x(7b+6cx^2)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{x} - \frac{\left(\sqrt{a}\,(b^2+12ac)\right)\int \frac{}{\sqrt{a}}}{5\sqrt{c}}$$

$$= \frac{(b^2+12ac)\,x\sqrt{a+bx^2+cx^4}}{5\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{1}{5}x(7b+6cx^2)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+}{x}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.82, size = 505, normalized size = 1.40

$$\frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(-5a^2-3abx^2+2b^2x^4-4acx^4+3bcx^6+c^2x^8\right)+i(b^2+12ac)\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-b^3+4abc+b^2\sqrt{b^2-4ac}+12ac\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{20c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-5*a^2 - 3*a*b*x^2 + 2*b^2*x^4 - 4*a*
c*x^4 + 3*b*c*x^6 + c^2*x^8) + I*(b^2 + 12*a*c)*(-b + Sqrt[b^2 - 4*a*c])*x*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSin
h[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
Sqrt[b^2 - 4*a*c])] - I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 12*a*c*Sq
rt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(20*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.04, size = 430, normalized size = 1.19

method	result

risch	$-\frac{\sqrt{c x^4+b x^2+a}\,(-c x^4-2 b x^2+5 a)}{5 x}-\frac{(12ac+b^2)a\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\sqrt{4+\frac{2(b+\sqrt{-4a}{a}}\cdots}{\cdots}$
default	$-\frac{a\sqrt{c x^4+b x^2+a}}{x}+\frac{c x^3\sqrt{c x^4+b x^2+a}}{5}+\frac{2bx\sqrt{c x^4+b x^2+a}}{5}+\frac{2ab\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac}}{a}}\cdots}{\cdots}$
elliptic	$-\frac{a\sqrt{c x^4+b x^2+a}}{x}+\frac{c x^3\sqrt{c x^4+b x^2+a}}{5}+\frac{2bx\sqrt{c x^4+b x^2+a}}{5}+\frac{2ab\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac}}{a}}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*(c*x^4+b*x^2+a)^(1/2)/x+1/5*c*x^3*(c*x^4+b*x^2+a)^(1/2)+2/5*b*x*(c*x^4+b
*x^2+a)^(1/2)+2/5*a*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(12/5*a*c+1/5*b^2)*a*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**2,x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^2,x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^2, x)
```

3.951 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=353

$$\frac{8b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{(3b-2cx^2)\sqrt{a+bx^2+cx^4}}{3x}-\frac{(a+bx^2+cx^4)^{3/2}}{3x^3}-\frac{8\sqrt[4]{a}\,b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\ldots}}{\ldots}$$

```
[Out] -1/3*(c*x^4+b*x^2+a)^(3/2)/x^3-1/3*(-2*c*x^2+3*b)*(c*x^4+b*x^2+a)^(1/2)/x+8
/3*b*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/(a^(1/2)+x^2*c^(1/2))-8/3*a^(1/4)*b*c^
(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(
1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))
^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)+1/6*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(3*b^2+4*a*c+8*b*a^(1/
2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)
/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1131, 1285, 1211, 1117, 1209}

$$\frac{\left(8\sqrt{a}\,b\sqrt{c}+4ac+3b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+bx^2+cx^4}}-\frac{8\sqrt[4]{a}\,b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3\sqrt{a+bx^2+cx^4}}-\frac{(3b-2cx^2)\sqrt{a+bx^2+cx^4}}{3x}+\frac{8b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{(a+bx^2+cx^4)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (8*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*(Sqrt[a] + Sqrt[c]*x^2)) - ((3*b
 - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(3*x) - (a + b*x^2 + c*x^4)^(3/2)/(3*x
^3) - (8*a^(1/4)*b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(3*Sqrt[a + b*x^2 + c*x^4]) + ((3*b^2 + 8*Sqrt[a]*b
*Sqrt[c] + 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(6*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1131

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Dist[2*(p/
(d^2*(m + 1))), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4}\,dx = -\frac{(a+bx^2+cx^4)^{3/2}}{3x^3} + \int \frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$

$$= -\frac{(3b-2cx^2)\sqrt{a+bx^2+cx^4}}{3x} - \frac{(a+bx^2+cx^4)^{3/2}}{3x^3} - \frac{1}{3}\int \frac{-3b^2-4ac-8bcx^2}{\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{(3b-2cx^2)\sqrt{a+bx^2+cx^4}}{3x} - \frac{(a+bx^2+cx^4)^{3/2}}{3x^3} - \frac{1}{3}\left(8\sqrt{a}\,b\sqrt{c}\right)\int \frac{1-}{\sqrt{a+bx}}$$

$$= \frac{8b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{(3b-2cx^2)\sqrt{a+bx^2+cx^4}}{3x} - \frac{(a+bx^2+cx^4)^{3/2}}{3x^3} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.59, size = 473, normalized size = 1.34

$$\frac{2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(-a^2-5abx^2-4b^2x^4-3bcx^6+c^2x^8\right)+4ib\left(-b+\sqrt{b^2-4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-b^2+4ac+4b\sqrt{b^2-4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{6\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-a^2 - 5*a*b*x^2 - 4*b^2*x^4 - 3*b*c*x^
6 + c^2*x^8) + (4*I)*b*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] +
4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I
*(-b^2 + 4*a*c + 4*b*Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(6*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.05, size = 428, normalized size = 1.21

method	result
default	$-\frac{a\sqrt{c\,x^4+b\,x^2+a}}{3x^3} - \frac{4b\sqrt{c\,x^4+b\,x^2+a}}{3x} + \frac{cx\sqrt{c\,x^4+b\,x^2+a}}{3} + \frac{\left(\frac{4ac}{3}+b^2\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4}\right.}{}}}{}$

elliptic	$-\frac{a\sqrt{c\,x^4+b\,x^2+a}}{3x^3}-\frac{4b\sqrt{c\,x^4+b\,x^2+a}}{3x}+\frac{cx\sqrt{c\,x^4+b\,x^2+a}}{3}+\frac{\left(\frac{4ac}{3}+b^2\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-\ldots}\right)\ldots}{\ldots}}\ldots}{\ldots}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-c\,x^4+4b\,x^2+a)}{3x^3}-\frac{4bca\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b\ldots}\right)\ldots}{a}}\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a*(c*x^4+b*x^2+a)^(1/2)/x^3-4/3*b*(c*x^4+b*x^2+a)^(1/2)/x+1/3*c*x*(c*x
^4+b*x^2+a)^(1/2)+1/4*(4/3*a*c+b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-4/3*b*c*a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**4,x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x^4 + b\,x^2 + a)^{3/2}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^4,x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^4, x)
```

3.952 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=400

$$-\frac{(b^2+12ac)\sqrt{a+bx^2+cx^4}}{5ax}+\frac{\sqrt{c}\,(b^2+12ac)\,x\sqrt{a+bx^2+cx^4}}{5a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{(b-6cx^2)\sqrt{a+bx^2+cx^4}}{5x^3}-\frac{(a+bx^2}{5\ldots}$$

```
[Out] -1/5*(c*x^4+b*x^2+a)^(3/2)/x^5-1/5*(12*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/a/x-1
/5*(-6*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/x^3+1/5*(12*a*c+b^2)*x*c^(1/2)*(c*x^4
+b*x^2+a)^(1/2)/a/(a^(1/2)+x^2*c^(1/2))-1/5*c^(1/4)*(12*a*c+b^2)*(cos(2*arc
tan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE
(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)
+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^
4+b*x^2+a)^(1/2)+1/10*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b^2+12*a*c+8*b*a^(1/
2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+
b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1131, 1285, 1295, 1211, 1117, 1209}

$$\frac{\sqrt[4]{c}\left(8\sqrt{a}\,b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\,(12ac+b^2)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{5a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(12ac+b^2)\sqrt{a+bx^2+cx^4}}{5ax}+\frac{\sqrt{c}\,x(12ac+b^2)\sqrt{a+bx^2+cx^4}}{5a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{(a+bx^2+cx^4)^{3/2}}{5x^5}-\frac{(b-6cx^2)\sqrt{a+bx^2+cx^4}}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] -1/5*((b^2 + 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a*x) + (Sqrt[c]*(b^2 + 12*a*
c)*x*Sqrt[a + b*x^2 + c*x^4])/(5*a*(Sqrt[a] + Sqrt[c]*x^2)) - ((b - 6*c*x^2
)*Sqrt[a + b*x^2 + c*x^4])/(5*x^3) - (a + b*x^2 + c*x^4)^(3/2)/(5*x^5) - (c
^(1/4)*(b^2 + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqr
t[a]*Sqrt[c]))/4])/(5*a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(b^2 + 8*
Sqrt[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(10*a^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1131

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Dist[2*(p/
(d^2*(m + 1))), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
```

```
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx = -\frac{(a+bx^2+cx^4)^{3/2}}{5x^5}+\frac{3}{5}\int \frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

$$= -\frac{(b-6cx^2)\sqrt{a+bx^2+cx^4}}{5x^3}-\frac{(a+bx^2+cx^4)^{3/2}}{5x^5}-\frac{1}{5}\int \frac{-b^2-12ac-8bcx^2}{x^2\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{(b^2+12ac)\sqrt{a+bx^2+cx^4}}{5ax}-\frac{(b-6cx^2)\sqrt{a+bx^2+cx^4}}{5x^3}-\frac{(a+bx^2+cx^4)}{5x^5}$$

$$= -\frac{(b^2+12ac)\sqrt{a+bx^2+cx^4}}{5ax}-\frac{(b-6cx^2)\sqrt{a+bx^2+cx^4}}{5x^3}-\frac{(a+bx^2+cx^4)}{5x^5}$$

$$= -\frac{(b^2+12ac)\sqrt{a+bx^2+cx^4}}{5ax}+\frac{\sqrt{c}\,(b^2+12ac)\,x\sqrt{a+bx^2+cx^4}}{5a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{(b-6cx^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.88, size = 527, normalized size = 1.32

$$\frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(a^3+b^2x^6(b+cx^2)+a^2(3bx^2+8cx^4)+a(3b^2x^4+9bcx^6+7c^2x^8)\right)+i(b^2+12ac)\left(-b+\sqrt{b^2-4ac}\right)x^5\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-b^3+4abc+b^2\sqrt{b^2-4ac}+12ac\sqrt{b^2-4ac}\right)x^5\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{20a\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x^5\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a^3 + b^2*x^6*(b + c*x^2) + a^2*(3*b*x
^2 + 8*c*x^4) + a*(3*b^2*x^4 + 9*b*c*x^6 + 7*c^2*x^8)) + I*(b^2 + 12*a*c)*(
-b + Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^3 + 4*a*b*c + b^2*S
qrt[b^2 - 4*a*c] + 12*a*c*Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(20*a
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^5*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.05, size = 450, normalized size = 1.12

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(7c\,x^4a+b^2x^4+2ab\,x^2+a^2)}{5x^5a}+\frac{c\left(-\frac{(12ac+b^2)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2(}{}}}{}\right.}{}$
default	$-\frac{a\sqrt{c\,x^4+b\,x^2+a}}{5x^5}-\frac{2b\sqrt{c\,x^4+b\,x^2+a}}{5x^3}-\frac{(7ac+b^2)\sqrt{c\,x^4+b\,x^2+a}}{5ax}+\frac{2bc\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4}\right.}{}}}{}$
elliptic	$-\frac{a\sqrt{c\,x^4+b\,x^2+a}}{5x^5}-\frac{2b\sqrt{c\,x^4+b\,x^2+a}}{5x^3}-\frac{(7ac+b^2)\sqrt{c\,x^4+b\,x^2+a}}{5ax}+\frac{2bc\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4}\right.}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a*(c*x^4+b*x^2+a)^(1/2)/x^5-2/5*b*(c*x^4+b*x^2+a)^(1/2)/x^3-1/5*(7*a*c
+b^2)/a*(c*x^4+b*x^2+a)^(1/2)/x+2/5*b*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(c^2
+1/5*c*(7*a*c+b^2)/a)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**6,x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^6,x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^6, x)
```

3.953 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=447

$$-\frac{(b^2-20ac)\sqrt{a+bx^2+cx^4}}{35ax^3}+\frac{2b(b^2-8ac)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{2b\sqrt{c}\,(b^2-8ac)\,x\sqrt{a+bx^2+cx^4}}{35a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{3(b+1}{}$$

```
[Out] -1/7*(c*x^4+b*x^2+a)^(3/2)/x^7-1/35*(-20*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/a/x
^3+2/35*b*(-8*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/a^2/x-3/35*(10*c*x^2+b)*(c*x^4
+b*x^2+a)^(1/2)/x^5-2/35*b*(-8*a*c+b^2)*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2
/(a^(1/2)+x^2*c^(1/2))+2/35*b*c^(1/4)*(-8*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*
((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a)^(1/
2)-1/70*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2
)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b*(-8*a*c+b^2)+(-20*a*c+b^2)*a^(
1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^
4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1131, 1285, 1295, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}\,(b^2-20ac)+2b(b^2-8ac)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt[4]{c}\,(b^2-8ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b(b^2-8ac)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{2b\sqrt{c}\,x(b^2-8ac)\sqrt{a+bx^2+cx^4}}{35a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{(b^2-20ac)\sqrt{a+bx^2+cx^4}}{35ax^3}-\frac{(a+bx^2+cx^4)^{3/2}}{7x^7}-\frac{3(b+10cx^2)\sqrt{a+bx^2+cx^4}}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/35*((b^2 - 20*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a*x^3) + (2*b*(b^2 - 8*a*c)
*Sqrt[a + b*x^2 + c*x^4])/(35*a^2*x) - (2*b*Sqrt[c]*(b^2 - 8*a*c)*x*Sqrt[a
+ b*x^2 + c*x^4])/(35*a^2*(Sqrt[a] + Sqrt[c]*x^2)) - (3*(b + 10*c*x^2)*Sqrt
[a + b*x^2 + c*x^4])/(35*x^5) - (a + b*x^2 + c*x^4)^(3/2)/(7*x^7) + (2*b*c^
(1/4)*(b^2 - 8*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[
a]*Sqrt[c]))/4])/(35*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqrt[a]*S
qrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(70*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]
)
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1131

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Dist[2*(p/
(d^2*(m + 1))), Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
```

```
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^8}\,dx = -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}+\frac{3}{7}\int \frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

$$= -\frac{3\left(b+10cx^2\right)\sqrt{a+bx^2+cx^4}}{35x^5}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}+\frac{3}{35}\int \frac{b^2-20ac-8bcx^2}{x^4\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3}-\frac{3\left(b+10cx^2\right)\sqrt{a+bx^2+cx^4}}{35x^5}-\frac{\left(a+bx^2+cx^4\right.}{7x^7}$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3}+\frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{3\left(b+10cx^2\right)}{35}$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3}+\frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{3\left(b+10cx^2\right)}{35}$$

$$= -\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3}+\frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{2b\sqrt{c}\left(b^2-8a\right.}{35a^2\left(\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.04, size = 572, normalized size = 1.28

$$\frac{-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(5a^4-2b^3x^8\left(b+cx^2\right)+a^3\left(13bx^2+20cx^4\right)+abx^6\left(-b^2+17bcx^2+16c^2x^4\right)+3a^2\left(3b^2x^4+13bcx^6+5c^2x^8\right)\right)-ib\left(b^2-8ac\right)\left(-b+\sqrt{b^2-4ac}\right)x^7\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+i\left(-b^4+9ab^2c-20a^2c^2+b^3\sqrt{b^2-4ac}-8abc\sqrt{b^2-4ac}\right)x^7\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{70a^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x^7\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(5*a^4 - 2*b^3*x^8*(b + c*x^2) + a^3*(1
3*b*x^2 + 20*c*x^4) + a*b*x^6*(-b^2 + 17*b*c*x^2 + 16*c^2*x^4) + 3*a^2*(3*b
^2*x^4 + 13*b*c*x^6 + 5*c^2*x^8)) - I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*
c])*x^7*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b
^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*x^7*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2
```

```
)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(70*a^2*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^7*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.06, size = 495, normalized size = 1.11

method	result
default	$-\frac{a\sqrt{c\,x^{4}+b\,x^{2}+a}}{7x^{7}}-\frac{8b\sqrt{c\,x^{4}+b\,x^{2}+a}}{35x^{5}}-\frac{\left(15ac+b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{35a\,x^{3}}-\frac{2b\left(8ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{35a^{2}x}$
elliptic	$-\frac{a\sqrt{c\,x^{4}+b\,x^{2}+a}}{7x^{7}}-\frac{8b\sqrt{c\,x^{4}+b\,x^{2}+a}}{35x^{5}}-\frac{\left(15ac+b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{35a\,x^{3}}-\frac{2b\left(8ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{35a^{2}x}$
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(16abc\,x^{6}-2b^{3}x^{6}+15a^{2}c\,x^{4}+a\,b^{2}x^{4}+8a^{2}b\,x^{2}+5a^{3}\right)}{35x^{7}a^{2}}+\frac{c\left(-\frac{\left(16abc-2b^{3}\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4}\right.}{}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a*(c*x^4+b*x^2+a)^(1/2)/x^7-8/35*b*(c*x^4+b*x^2+a)^(1/2)/x^5-1/35*(15*
a*c+b^2)/a*(c*x^4+b*x^2+a)^(1/2)/x^3-2/35*b*(8*a*c-b^2)/a^2*(c*x^4+b*x^2+a)
^(1/2)/x+1/4*(c^2-1/35*c*(15*a*c+b^2)/a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/35*b*
c*(8*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*
x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elliptic
E(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^
2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**8,x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**8, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/x^8,x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/x^8, x)
```

3.954 $\int \sqrt{3-2x^2-x^4}\,dx$

Optimal. Leaf size=48

$$\frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -2/3*EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(x,1/3*I*3^(1/2))*3^(1
/2)+1/3*x*(-x^4-2*x^2+3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.312$, Rules used = {1105, 1194, 538, 435, 430}

$$\frac{4F\left(\text{ArcSin}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\text{ArcSin}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{-x^4-2x^2+3}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt[3] + (4*E
llipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{3-2x^2-x^4}\,dx &= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{1}{3}\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx\\
&= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{2}{3}\int \frac{6-2x^2}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx\\
&= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2}{3}\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx+8\int \frac{1}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx\\
&= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.04, size = 59, normalized size = 1.23

$$\frac{1}{3}\left(x\sqrt{3-2x^2-x^4}-2iE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-4iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (4*I
)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(44) = 88.
time = 0.04, size = 114, normalized size = 2.38

method	result

default	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\mathrm{EllipticF}\right.}{3\sqrt{-x^4-2}}$
elliptic	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\mathrm{EllipticF}\right.}{3\sqrt{-x^4-2}}$
risch	$-\frac{x(x^4+2x^2-3)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\mathrm{Elliptic}\right.}{3\sqrt{-x^4-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3
)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4
-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 - 2*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 24, normalized size = 0.50

$$\frac{\sqrt{-x^4-2\,x^2+3}\,(x^2+2)}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-x^4 - 2*x^2 + 3)*(x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-x^4-2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 - 2*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 - 2*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-x^4-2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - x^4 - 2*x^2)^(1/2),x)
```

```
[Out] int((3 - x^4 - 2*x^2)^(1/2), x)
```

3.955 $\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{x^4\sqrt{a+bx^2+cx^4}}{6c}+\frac{(15b^2-16ac-10bcx^2)\sqrt{a+bx^2+cx^4}}{48c^3}-\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}$$

```
[Out] -1/32*b*(-12*a*c+5*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/
2))/c^(7/2)+1/6*x^4*(c*x^4+b*x^2+a)^(1/2)/c+1/48*(-10*b*c*x^2-16*a*c+15*b^2
)*(c*x^4+b*x^2+a)^(1/2)/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1128, 756, 793, 635, 212}

$$-\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{(-16ac+15b^2-10bcx^2)\sqrt{a+bx^2+cx^4}}{48c^3}+\frac{x^4\sqrt{a+bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2 - 16*a*c - 10*b*c*x^2)*Sqrt[
a + b*x^2 + c*x^4])/(48*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
```

```
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6c}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{(15b^2-16ac-10bcx^2)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{(b(5b^2-12ac))\,\text{S}}{}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{(15b^2-16ac-10bcx^2)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{(b(5b^2-12ac))\,\text{S}}{}$$

$$= \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{(15b^2-16ac-10bcx^2)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{b(5b^2-12ac)\tan}{}$$

Mathematica [A]
time = 0.22, size = 101, normalized size = 0.83

$$\frac{\sqrt{a+bx^2+cx^4}\,(15b^2-16ac-10bcx^2+8c^2x^4)}{48c^3} + \frac{(5b^3-12abc)\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)}{32c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(15*b^2 - 16*a*c - 10*b*c*x^2 + 8*c^2*x^4))/(48*c^
3) + ((5*b^3 - 12*a*b*c)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4
]])/(32*c^(7/2))
```

Maple [A]
time = 0.04, size = 162, normalized size = 1.34

method	result
risch	$-\frac{\left(-8 c^{2} x^{4}+10 b c \,x^{2}+16 a c -15 b^{2}\right) \sqrt{c \,x^{4}+b \,x^{2}+a}}{48 c^{3}}+\frac{3 b a \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{8 c^{\frac{5}{2}}}-\frac{5 b^{3} \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c}\right.}{32 c}$
default	$\frac{x^{4} \sqrt{c \,x^{4}+b \,x^{2}+a}}{6 c}-\frac{5 b \,x^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{24 c^{2}}+\frac{5 b^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{16 c^{3}}-\frac{5 b^{3} \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}}\right.}{32 c^{\frac{7}{2}}}$
elliptic	$\frac{x^{4} \sqrt{c \,x^{4}+b \,x^{2}+a}}{6 c}-\frac{5 b \,x^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{24 c^{2}}+\frac{5 b^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{16 c^{3}}-\frac{5 b^{3} \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}}\right.}{32 c^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^4*(c*x^4+b*x^2+a)^(1/2)/c-5/24*b/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+5/16*b
^2/c^3*(c*x^4+b*x^2+a)^(1/2)-5/32*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))+3/8*b/c^(5/2)*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)
^(1/2))-1/3*a/c^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.36, size = 241, normalized size = 1.99

$$\left[-\frac{3 \, {\left(5 \, b^{3} - 12 \, a b c\right)} \sqrt{c} \log\left(-8 \, c^{2} x^{4} - 8 \, b c x^{2} - b^{2} - 4 \, \sqrt{c x^{4} + b x^{2} + a} {\left(2 \, c x^{2} + b\right)} \sqrt{c} - 4 \, a c\right) - 4 \, {\left(8 \, c^{3} x^{4} - 10 \, b c^{2} x^{2} + 15 \, b^{2} c - 16 \, a c^{2}\right)} \sqrt{c x^{4} + b x^{2} + a}}{192 \, c^{4}}, \frac{3 \, {\left(5 \, b^{3} - 12 \, a b c\right)} \sqrt{-c} \arctan\left(\frac{\sqrt{c x^{4} + b x^{2} + a} {\left(2 \, c x^{2} + b\right)} \sqrt{-c}}{2 \, {\left(c^{2} x^{4} + b c x^{2} + a c\right)}}\right) + 2 \, {\left(8 \, c^{3} x^{4} - 10 \, b c^{2} x^{2} + 15 \, b^{2} c - 16 \, a c^{2}\right)} \sqrt{c x^{4} + b x^{2} + a}}{96 \, c^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*
sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^4 - 10*
b*c^2*x^2 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/96*(3*(5*b
^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sq
rt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 - 10*b*c^2*x^2 + 15*b^2*c
- 16*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^4]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [**A**]
time = 3.17, size = 103, normalized size = 0.85

$$\frac{1}{48}\sqrt{cx^4 + bx^2 + a}\left(2\,x^2\left(\frac{4\,x^2}{c} - \frac{5\,b}{c^2}\right) + \frac{15\,b^2 - 16\,ac}{c^3}\right) + \frac{(5\,b^3 - 12\,abc)\log\left(\left|-2\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{32\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c - 5*b/c^2) + (15*b^2 - 16*a*c)
/c^3) + 1/32*(5*b^3 - 12*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2 + a))*sqrt(c) - b))/c^(7/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\sqrt{c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^7/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.956 $\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=104

$$-\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}+\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}$$

```
[Out] 1/16*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/
c^(5/2)-3/8*b*(c*x^4+b*x^2+a)^(1/2)/c^2+1/4*x^2*(c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A]

time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1128, 756, 654, 635, 212}

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[a + b*x^2 + c*x^4])/(4*c
) + ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{\text{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}} \right)}{16c^2}$$

$$= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \right)}{8c^2}$$

$$= -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{a+bx^2+cx^4}}{4c} + \frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{5/2}}$$

Mathematica [A]
time = 0.18, size = 91, normalized size = 0.88

$$\frac{(-3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{(-3b^2+4ac)\log\left(bc^2+2c^3x^2-2c^{5/2}\sqrt{a+bx^2+cx^4}\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((-3*b^2 + 4*a*c)*Log[
b*c^2 + 2*c^3*x^2 - 2*c^(5/2)*Sqrt[a + b*x^2 + c*x^4]])/(16*c^(5/2))
```

Maple [A]
time = 0.04, size = 116, normalized size = 1.12

method	result
risch	$-\frac{\left(-2 c \,x^{2}+3 b \right) \sqrt{c \,x^{4}+b \,x^{2}+a}}{8 c^{2}}-\frac{a \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{4 c^{\frac{3}{2}}}+\frac{3 b^{2} \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{16 c^{\frac{5}{2}}}$
default	$\frac{x^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{4 c}-\frac{3 b \sqrt{c \,x^{4}+b \,x^{2}+a}}{8 c^{2}}+\frac{3 b^{2} \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{16 c^{\frac{5}{2}}}-\frac{a \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c x^{4}} \ldots\right.}{4 c^{\frac{3}{2}}}$
elliptic	$\frac{x^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{4 c}-\frac{3 b \sqrt{c \,x^{4}+b \,x^{2}+a}}{8 c^{2}}+\frac{3 b^{2} \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{16 c^{\frac{5}{2}}}-\frac{a \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c x^{4}} \ldots\right.}{4 c^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^2*(c*x^4+b*x^2+a)^(1/2)/c-3/8*b*(c*x^4+b*x^2+a)^(1/2)/c^2+3/16*b^2/c^
(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*a/c^(3/2)*ln((1/2
*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 203, normalized size = 1.95

$$\left[-\frac{(3\,b^2-4\,ac)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^4+bx^2+a}\,(2\,c^2x^2-3\,bc)}{32\,c^3},\ -\frac{(3\,b^2-4\,ac)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-2\,\sqrt{cx^4+bx^2+a}\,(2\,c^2x^2-3\,bc)}{16\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c
*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c^2*x^2 - 3*b*c))/c^3, -1/16*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt
(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*s
qrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 - 3*b*c))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 4.13, size = 82, normalized size = 0.79

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(\frac{2\,x^2}{c}-\frac{3\,b}{c^2}\right)-\frac{(3\,b^2-4\,ac)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2/c - 3*b/c^2) - 1/16*(3*b^2 - 4*a*c)*log(
abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^5/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.957 $\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

```
[Out] -1/4*b*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)+1/2*(
c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1128, 654, 635, 212}

$$\frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4c}\\ &= \frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2c}\\ &= \frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 70, normalized size = 1.03

$$\frac{\sqrt{a+bx^2+cx^4}}{2c}+\frac{b\log\left(bc+2c^2x^2-2c^{3/2}\sqrt{a+bx^2+cx^4}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 + c*x^4],x]
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) + (b*Log[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a +
 b*x^2 + c*x^4]])/(4*c^(3/2))
```

Maple [A]
time = 0.04, size = 56, normalized size = 0.82

method	result	size
default	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{3}{2}}}$	56
risch	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{3}{2}}}$	56

elliptic	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{3}{2}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)/c-1/4*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 161, normalized size = 2.37

$$\left[\frac{b\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}-4\,ac\right)+4\,\sqrt{cx^4+bx^2+a}\,c}{8\,c^2},\frac{b\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)+2\,\sqrt{cx^4+bx^2+a}\,c}{4\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*c)/c^2, 1/4*(b
*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^
4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*c)/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 4.79, size = 61, normalized size = 0.90

$$\frac{b\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3
/2) + 1/2*sqrt(c*x^4 + b*x^2 + a)/c
```

Mupad [B]
time = 4.43, size = 55, normalized size = 0.81

$$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2\,c}-\frac{b\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (a + b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 +
 c*x^2)/c^(1/2)))/(4*c^(3/2))
```

3.958 $\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1121, 635, 212}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [**A**]
time = 0.07, size = 41, normalized size = 0.95

$$-\frac{\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 + c*x^4],x]
[Out] -1/2*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]/Sqrt[c]
```

Maple [**A**]
time = 0.04, size = 35, normalized size = 0.81

method	result	size
default	$\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}$	35
elliptic	$\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.35, size = 118, normalized size = 2.74

$$\left[\frac{\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}\,-4\,ac\right)}{4\,\sqrt{c}},-\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
+ b)*sqrt(c) - 4*a*c)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c))/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 4.96, size = 40, normalized size = 0.93

$$-\frac{\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}\,-b\right|\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/sqrt(
c)
```

Mupad [B]
time = 4.69, size = 34, normalized size = 0.79

$$\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}\,+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))/(2*c^(1/2))
```

3.959 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1128, 738, 212}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Mathematica [**A**]
time = 0.07, size = 41, normalized size = 0.93

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/Sqrt[a]
```

Maple [**A**]
time = 0.02, size = 39, normalized size = 0.89

method	result	size
default	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	39
elliptic	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.36, size = 124, normalized size = 2.82

$$\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [**A**]
time = 5.65, size = 38, normalized size = 0.86

$$\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a)
```

Mupad [B]
time = 4.44, size = 44, normalized size = 1.00

$$-\frac{\ln\left(\frac{1}{x^2}\right)}{2\sqrt{a}}-\frac{\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] - log(1/x^2)/(2*a^(1/2)) - log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) +
b*x^2)/(2*a^(1/2))
```

3.960 $\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

```
[Out] 1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)-1/2*(c
*x^4+b*x^2+a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1128, 744, 738, 212}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(a*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
```

```
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}-\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4a}\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2a}\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 72, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(a*x^2) - (b*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b
*x^2 + c*x^4])/Sqrt[a]])/(2*a^(3/2))
```

Maple [A]
time = 0.04, size = 63, normalized size = 0.88

method	result	size
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4a^{\frac{3}{2}}}$	63

risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}$	63
elliptic	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*b/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x
^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.35, size = 179, normalized size = 2.49

$$\left[\frac{\sqrt{a}\,bx^{2}\log\left(-\frac{(b^{2}+4ac)x^{4}+8abx^{2}+4\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{a}+8a^{2}}{x^{4}}\right)-4\sqrt{cx^{4}+bx^{2}+a}\,a}{8a^{2}x^{2}},-\frac{\sqrt{-a}\,bx^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{-a}}{2\,(acx^{4}+abx^{2}+a^{2})}\right)+2\sqrt{cx^{4}+bx^{2}+a}\,a}{4a^{2}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a)
/(a^2*x^2), -1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*a)/(
a^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 5.53, size = 114, normalized size = 1.58

$$-\frac{b\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}\,a}+\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*
a) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a)
```

Mupad [B]
time = 4.48, size = 56, normalized size = 0.78

$$\frac{b\,\mathrm{atanh}\left(\frac{\frac{b\,x^2}{2}+a}{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}\right)}{4\,a^{3/2}}-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (b*atanh((a + (b*x^2)/2)/(a^(1/2)*(a + b*x^2 + c*x^4)^(1/2))))/(4*a^(3/2))
- (a + b*x^2 + c*x^4)^(1/2)/(2*a*x^2)
```

3.961 $\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}$$

```
[Out] -1/16*(-4*a*c+3*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))
/a^(5/2)-1/4*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^
2
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1128, 758, 820, 738, 212}

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/4*Sqrt[a + b*x^2 + c*x^4]/(a*x^4) + (3*b*Sqrt[a + b*x^2 + c*x^4])/(8*a^2
*x^2) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
c*x^4])])/(16*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
```

```
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}-\frac{\text{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4a}\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}+\frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{16a^2}\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8a^2}\\ &= -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}\end{aligned}$$

Mathematica [A]

time = 0.22, size = 91, normalized size = 0.84

$$\frac{(-2a+3bx^2)\sqrt{a+bx^2+cx^4}}{8a^2x^4}+\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-2*a + 3*b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((3*b^2 - 4*a*c)*A
rcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(5/2))
```

Maple [A]
time = 0.05, size = 127, normalized size = 1.18

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-3b\,x^2+2a)}{8a^2x^4}+\frac{c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4a^{\frac{3}{2}}}-\frac{3b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+}}{x^2}\right.}{16a^{\frac{5}{2}}}$
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{4a\,x^4}+\frac{3b\sqrt{c\,x^4+b\,x^2+a}}{8a^2x^2}-\frac{3b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16a^{\frac{5}{2}}}+\frac{c\ln\left(\frac{2a+b\,x^2+}{}\right.}{}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{4a\,x^4}+\frac{3b\sqrt{c\,x^4+b\,x^2+a}}{8a^2x^2}-\frac{3b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16a^{\frac{5}{2}}}+\frac{c\ln\left(\frac{2a+b\,x^2+}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^2-3/16*b
^2/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/4*c/a^(3/2
)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 221, normalized size = 2.05

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{a}\,x^4\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\,\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}\,\left(3\,abx^2-2\,a^2\right)}{32\,a^3x^4},\ \frac{\left(3\,b^2-4\,ac\right)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,\left(bx^2+2\,a\right)\sqrt{-a}}{2\,\left(acx^4+abx^2+a^2\right)}\right)+2\,\sqrt{cx^4+bx^2+a}\,\left(3\,abx^2-2\,a^2\right)}{16\,a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 - 4*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4
*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4
 + b*x^2 + a)*(3*a*b*x^2 - 2*a^2))/(a^3*x^4), 1/16*((3*b^2 - 4*a*c)*sqrt(-a
)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 +
a*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*(3*a*b*x^2 - 2*a^2))/(a^3*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(90) = 180.
time = 5.58, size = 221, normalized size = 2.05

$$\frac{\left(3\,b^2-4\,ac\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^2}-\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3b^2-4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3ac-5\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)ab^2-4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^2c-8\,a^2b\sqrt{c}}{8\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2 - 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a
))/(sqrt(-a)*a^2) - 1/8*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2 -
4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a*c - 5*(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))*a*b^2 - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*c -
8*a^2*b*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.962 $\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$-\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}$$

```
[Out] 1/32*b*(-12*a*c+5*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2
))/a^(7/2)-1/6*(c*x^4+b*x^2+a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2+a)^(1/2)/a^2
/x^4-1/48*(-16*a*c+15*b^2)*(c*x^4+b*x^2+a)^(1/2)/a^3/x^2
```

Rubi [A]
time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1128, 758, 848, 820, 738, 212}

$$\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}-\frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/6*Sqrt[a + b*x^2 + c*x^4]/(a*x^6) + (5*b*Sqrt[a + b*x^2 + c*x^4])/(24*a^
2*x^4) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^2 + c*x^4])/(48*a^3*x^2) + (b*(5*b
^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(3
2*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
```

```
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(15b^2-16ac)+\frac{5bcx}{2}}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x\right.}{12a^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{48a^3x^2} -$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{48a^3x^2} +$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{48a^3x^2} +$$

Mathematica [A]
time = 0.36, size = 110, normalized size = 0.76

$$\frac{\sqrt{a+bx^2+cx^4}\,(-8a^2+10abx^2-15b^2x^4+16acx^4)}{48a^3x^6} + \frac{(-5b^3+12abc)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 + 10*a*b*x^2 - 15*b^2*x^4 + 16*a*c*x^4))/(
48*a^3*x^6) + ((-5*b^3 + 12*a*b*c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 +
c*x^4])/Sqrt[a]])/(16*a^(7/2))
```

Maple [A]
time = 0.05, size = 176, normalized size = 1.21

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-16c\,x^4a+15b^2x^4-10ab\,x^2+8a^2)}{48a^3x^6} - \frac{3bc\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{8a^{\frac{5}{2}}} + \frac{5b^3\ln\left(\frac{2a\ldots}{}\right.}{}$

default	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{6a\,x^{6}}+\frac{5b\sqrt{c\,x^{4}+b\,x^{2}+a}}{24a^{2}x^{4}}-\frac{5b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16a^{3}x^{2}}+\frac{5b^{3}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x}}{x^{2}}\right.}{32a^{\frac{7}{2}}}$
elliptic	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{6a\,x^{6}}+\frac{5b\sqrt{c\,x^{4}+b\,x^{2}+a}}{24a^{2}x^{4}}-\frac{5b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16a^{3}x^{2}}+\frac{5b^{3}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x}}{x^{2}}\right.}{32a^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*x^4+b*x^2+a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-5/16*
b^2/a^3/x^2*(c*x^4+b*x^2+a)^(1/2)+5/32*b^3/a^(7/2)*ln((2*a+b*x^2+2*a^(1/2)*
(c*x^4+b*x^2+a)^(1/2))/x^2)-3/8*b/a^(5/2)*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+
b*x^2+a)^(1/2))/x^2)+1/3*c/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.42, size = 265, normalized size = 1.83

$$\left[-\frac{3\,(5\,b^{3}-12\,abc)\sqrt{a}\,x^{6}\log\left(-\frac{(b^{2}+4\,ac)x^{4}+8\,abx^{2}-4\,\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2\,a)\sqrt{a}+8\,a^{2}}{x^{4}}\right)-4\,(10\,a^{2}bx^{2}-(15\,ab^{2}-16\,a^{2}c)x^{4}-8\,a^{3})\sqrt{cx^{4}+bx^{2}+a}}{192\,a^{4}x^{6}},\ -\frac{3\,(5\,b^{3}-12\,abc)\sqrt{-a}\,x^{6}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2\,a)\sqrt{-a}}{2\,(acx^{4}+abx^{2}+a^{2})}\right)-2\,(10\,a^{2}bx^{2}-(15\,ab^{2}-16\,a^{2}c)x^{4}-8\,a^{3})\sqrt{cx^{4}+bx^{2}+a}}{96\,a^{4}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(10*
a^2*b*x^2 - (15*a*b^2 - 16*a^2*c)*x^4 - 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^
4*x^6), -1/96*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(10*a^2*b*x^
2 - (15*a*b^2 - 16*a^2*c)*x^4 - 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7}\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(123) = 246.
time = 4.32, size = 335, normalized size = 2.31

$$-\frac{(5\,b^3 - 12\,abc)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}\,a^3} + \frac{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5 b^3 - 36\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^5 abc - 40\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3 ab^3 + 96\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3 a^2bc + 96\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2 a^3c^{\frac{3}{2}} + 33\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^2b^3 + 36\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^3bc + 48\,a^3b^2\sqrt{c} - 32\,a^4c^{\frac{3}{2}}}{48\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^3 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*(5*b^3 - 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sq
rt(-a))/(sqrt(-a)*a^3) + 1/48*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5
*b^3 - 36*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c - 40*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^3*a*b^3 + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^3*a^2*b*c + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*c^(3/2)
+ 33*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*b^3 + 36*(sqrt(c)*x^2 - sq
rt(c*x^4 + b*x^2 + a))*a^3*b*c + 48*a^3*b^2*sqrt(c) - 32*a^4*c^(3/2))/(((sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^3*a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.963 $\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=313

$$\frac{x\sqrt{a+bx^2+cx^4}}{3c}-\frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{2\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\right.}{3c^{7/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-2/3*b*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(1/2
)+x^2*c^(1/2))+2/3*a^(1/4)*b*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1
/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/6*a^(1/4)*(cos(2*
arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipt
icF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1
/2)+x^2*c^(1/2))*(2*b+a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2
))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1136, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x\sqrt{a+bx^2+cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (2*b*x*Sqrt[a + b*x^2 + c*x^4])/(3*c^(3
/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (2*a^(1/4)*b*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]
) - (a^(1/4)*(2*b + Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{x\sqrt{a+bx^2+cx^4}}{3c} - \frac{\int \frac{a+2bx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c}$$

$$= \frac{x\sqrt{a+bx^2+cx^4}}{3c} + \frac{\left(2\sqrt{a}\,b\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}} - \frac{\left(\sqrt{a}\left(2b+\sqrt{a}\,\sqrt{c}\right)\right)\int \cdots}{3c^{3/2}}$$

$$= \frac{x\sqrt{a+bx^2+cx^4}}{3c} - \frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{2\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2 \cdots}{\left(\sqrt{a}+\sqrt{\cdots}\right.}}}{3c^{7/4}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.61, size = 444, normalized size = 1.42

$$\frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(a+bx^2+cx^4) - ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right) + i\left(-b^2+ac+b\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{6c^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) - I*b*(-b + Sqrt
[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ell
ipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*c])*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])])/(6*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 +
 c*x^4])
```

Maple [A]
time = 0.03, size = 388, normalized size = 1.24

method	result
default	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\cdots}{\cdots}\right.}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+\cdots}}$
risch	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{-\frac{ba\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x\cdots}{\cdots}\right.\right.}{\sqrt{\cdots}}}{\cdots}$
elliptic	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\cdots}{\cdots}\right.}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-1/12*a/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
```

```
2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c*a
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b
+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x**4/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.964 $\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=267

$$\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}+\cdots$$

```
[Out] x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^
2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b
*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(
2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)
+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1153, 1117, 1209}

$$\frac{\sqrt[4]{a}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3
/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}} - \frac{\sqrt{a}\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}}$$

$$= \frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.08, size = 278, normalized size = 1.04

$$\frac{i\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{2\sqrt{2}\,c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqr
t[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.02, size = 216, normalized size = 0.81

method	result
default	$-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+}}{a}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{}}$
elliptic	$-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+}}{a}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int(x^2/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.965 $\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=114

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)
/a^(1/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1117}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx=\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 186, normalized size = 1.63

$$-\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.02, size = 144, normalized size = 1.26

method	result
default	$\frac{\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}$
elliptic	$\frac{\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(
-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [A]
time = 0.08, size = 120, normalized size = 1.05

$$-\frac{\sqrt{\frac{1}{2}}\left(a\sqrt{\frac{b^2-4\,ac}{a^2}}+b\right)\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}\,\text{ellipticF}\left(\sqrt{\frac{1}{2}}\,x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}},\frac{ab\sqrt{\frac{b^2-4\,ac}{a^2}}+b^2-2\,ac}{2\,ac}\right)}{2\,\sqrt{a}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(1/2)*(a*sqrt((b^2 - 4*a*c)/a^2) + b)*sqrt((a*sqrt((b^2 - 4*a*c)/a
^2) - b)/a)*ellipticF(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a),
1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c))/(sqrt(a)*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int(1/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.966 $\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=294

$$-\frac{\sqrt{a+bx^2+cx^4}}{ax}+\frac{\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\right.}{a^{3/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -(c*x^4+b*x^2+a)^(1/2)/a/x+x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(a^(1/2)+x^2*c
^(1/2))-c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2
)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4
)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/
2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)
^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1137, 12, 1153, 1117, 1209}

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt{a+bx^2+cx^4}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(
Sqrt[a] + Sqrt[c]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
2*a^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx &= -\frac{\sqrt{a+bx^2+cx^4}}{ax}+\frac{\int \frac{cx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{ax}+\frac{c\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{ax}+\frac{\sqrt{c}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}}-\frac{\sqrt{c}\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{ax}+\frac{\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\right.}}}{a}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.34, size = 298, normalized size = 1.01

$$-\frac{\frac{4(a+bx^2+cx^4)}{x}+\frac{i\sqrt{2}\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}}}{4a\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-4*(a + b*x^2 + c*x^4))/x + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a
+ b*x^2 + c*x^4])
```

Maple [A]
time = 0.03, size = 239, normalized size = 0.81

method	result
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{ax}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x}{}\right.\right.}{2\sqrt{}}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{ax}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x}{}\right.\right.}{2\sqrt{}}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{ax}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x}{}\right.\right.}{2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^4+b*x^2+a)^(1/2)/a/x-1/2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.967 $\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=345

$$-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{2b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{2b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{3a^{7/4}\sqrt{a}}$$

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x-2/3*b*x*
c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(a^(1/2)+x^2*c^(1/2))+2/3*b*c^(1/4)*(cos(
2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elli
pticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^
(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/
(c*x^4+b*x^2+a)^(1/2)-1/6*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b+a^(1/2)*c^(1
/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a
)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1137, 1295, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{2b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/3*Sqrt[a + b*x^2 + c*x^4]/(a*x^3) + (2*b*Sqrt[a + b*x^2 + c*x^4])/(3*a^2
*x) - (2*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*(Sqrt[a] + Sqrt[c]*x^2
)) + (2*b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(3*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - ((2*b + Sqrt[a]*Sqrt[c
])*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(6*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{\int \frac{-2b-cx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{\int \frac{ac+2bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x} + \frac{\left(2b\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^{3/2}} - \cdots$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{2b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{2b\sqrt[4]{c}\,(\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.59, size = 459, normalized size = 1.33

$$\frac{-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a-2bx^2)(a+bx^2+cx^4) - ib\left(-b+\sqrt{b^2-4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right) + i\left(-b^2+ac+b\sqrt{b^2-4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{6a^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a - 2*b*x^2)*(a + b*x^2 + c*x^4) - I*b
*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2
 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
 (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + a*c + b*Sqrt[
b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(6*a^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.04, size = 413, normalized size = 1.20

method	result

risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-2b\,x^2+a)}{3a^2x^3}-\frac{c\left(-\frac{ba\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x}{a}}}{}\right.}{}$
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{c\,x^4+b\,x^2+a}}{3a^2x}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4a}\right.}{a}}}{12a\sqrt{\frac{-b-}{}}}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{c\,x^4+b\,x^2+a}}{3a^2x}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4a}\right.}{a}}}{12a\sqrt{\frac{-b-}{}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x-1/12*c/a
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*c*b/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.968 $\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=124

$$-\frac{x^4\sqrt{a+bx^2-cx^4}}{6c}-\frac{(15b^2+16ac+10bcx^2)\sqrt{a+bx^2-cx^4}}{48c^3}-\frac{b(5b^2+12ac)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{7/2}}$$

```
[Out] -1/32*b*(12*a*c+5*b^2)*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/
2))/c^(7/2)-1/6*x^4*(-c*x^4+b*x^2+a)^(1/2)/c-1/48*(10*b*c*x^2+16*a*c+15*b^2
)*(-c*x^4+b*x^2+a)^(1/2)/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1128, 756, 793, 635, 210}

$$-\frac{b(12ac+5b^2)\,\text{ArcTan}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{7/2}}-\frac{(16ac+15b^2+10bcx^2)\sqrt{a+bx^2-cx^4}}{48c^3}-\frac{x^4\sqrt{a+bx^2-cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/6*(x^4*Sqrt[a + b*x^2 - c*x^4])/c - ((15*b^2 + 16*a*c + 10*b*c*x^2)*Sqrt
[a + b*x^2 - c*x^4])/(48*c^3) - (b*(5*b^2 + 12*a*c)*ArcTan[(b - 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(32*c^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
```

```
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{(15b^2+16ac+10bcx^2)\sqrt{a+bx^2-cx^4}}{48c^3} + \frac{(b(5b^2+12ac$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{(15b^2+16ac+10bcx^2)\sqrt{a+bx^2-cx^4}}{48c^3} + \frac{(b(5b^2+12ac$$

$$= -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{(15b^2+16ac+10bcx^2)\sqrt{a+bx^2-cx^4}}{48c^3} - \frac{b(5b^2+12ac)$$

Mathematica [A]
time = 10.07, size = 107, normalized size = 0.86

$$\frac{-2\sqrt{c}\sqrt{a+bx^2-cx^4}\,(15b^2+10bcx^2+8c(2a+cx^4))-3b(5b^2+12ac)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]*(15*b^2 + 10*b*c*x^2 + 8*c*(2*a + c*x^4
)) - 3*b*(5*b^2 + 12*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 -
c*x^4])])/(96*c^(7/2))
```

Maple [A]
time = 0.07, size = 168, normalized size = 1.35

method	result
risch	$-\frac{(8c^2x^4+10bcx^2+16ac+15b^2)\sqrt{-c\,x^4+b\,x^2+a}}{48c^3}+\frac{3ba\arctan\left(\frac{\sqrt{c}\,\left(x^2-\frac{b}{2c}\right)}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{8c^{\frac{5}{2}}}+\frac{5b^3\arctan\left(\frac{\sqrt{c}}{\sqrt{-c\,x^4} \ldots}\right.}{32c^{\frac{7}{2}}}$
default	$-\frac{x^4\sqrt{-c\,x^4+b\,x^2+a}}{6c}-\frac{5b\,x^2\sqrt{-c\,x^4+b\,x^2+a}}{24c^2}-\frac{5b^2\sqrt{-c\,x^4+b\,x^2+a}}{16c^3}+\frac{5b^3\arctan\left(\frac{\sqrt{c}\,(x \ldots}{\sqrt{-c\,x^4+} \ldots}\right.}{32c^{\frac{7}{2}}}$
elliptic	$-\frac{x^4\sqrt{-c\,x^4+b\,x^2+a}}{6c}-\frac{5b\,x^2\sqrt{-c\,x^4+b\,x^2+a}}{24c^2}-\frac{5b^2\sqrt{-c\,x^4+b\,x^2+a}}{16c^3}+\frac{5b^3\arctan\left(\frac{\sqrt{c}\,(x \ldots}{\sqrt{-c\,x^4+} \ldots}\right.}{32c^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*x^4*(-c*x^4+b*x^2+a)^(1/2)/c-5/24*b/c^2*x^2*(-c*x^4+b*x^2+a)^(1/2)-5/1
6*b^2/c^3*(-c*x^4+b*x^2+a)^(1/2)+5/32*b^3/c^(7/2)*arctan(c^(1/2)*(x^2-1/2*b
/c)/(-c*x^4+b*x^2+a)^(1/2))+3/8*b/c^(5/2)*a*arctan(c^(1/2)*(x^2-1/2*b/c)/(-
c*x^4+b*x^2+a)^(1/2))-1/3*a/c^2*(-c*x^4+b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.50, size = 153, normalized size = 1.23

$$-\frac{\sqrt{-cx^4+bx^2+a}\,x^4}{6\,c}-\frac{5\,\sqrt{-cx^4+bx^2+a}\,bx^2}{24\,c^2}-\frac{5\,b^3\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{32\,c^{\frac{7}{2}}}-\frac{3\,ab\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{8\,c^{\frac{5}{2}}}-\frac{5\,\sqrt{-cx^4+bx^2+a}\,b^2}{16\,c^3}-\frac{\sqrt{-cx^4+bx^2+a}\,a}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-c*x^4 + b*x^2 + a)*x^4/c - 5/24*sqrt(-c*x^4 + b*x^2 + a)*b*x^2/c
^2 - 5/32*b^3*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(7/2) - 3/8*a*b*ar
csin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(5/2) - 5/16*sqrt(-c*x^4 + b*x^2 +
 a)*b^2/c^3 - 1/3*sqrt(-c*x^4 + b*x^2 + a)*a/c^2
```

Fricas [A]
time = 0.37, size = 249, normalized size = 2.01

$$\left[-\frac{3\,(5\,b^3+12\,abc)\sqrt{-c}\,\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}\,(2\,cx^2-b)\sqrt{-c}-4\,ac\right)+4\,(8\,c^3x^4+10\,bc^2x^2+15\,b^2c+16\,ac^2)\sqrt{-cx^4+bx^2+a}}{192\,c^4},\,-\frac{3\,(5\,b^3+12\,abc)\sqrt{c}\,\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}\,(2\,cx^2-b)\sqrt{c}}{2\,(c^2x^4-bcx^2-ac)}\right)+2\,(8\,c^3x^4+10\,bc^2x^2+15\,b^2c+16\,ac^2)\sqrt{-cx^4+bx^2+a}}{96\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 + 12*a*b*c)*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*
sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*(8*c^3*x^4 + 1
0*b*c^2*x^2 + 15*b^2*c + 16*a*c^2)*sqrt(-c*x^4 + b*x^2 + a))/c^4, -1/96*(3*
(5*b^3 + 12*a*b*c)*sqrt(c)*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b
)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c)) + 2*(8*c^3*x^4 + 10*b*c^2*x^2 + 15*b^2
*c + 16*a*c^2)*sqrt(-c*x^4 + b*x^2 + a))/c^4]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [**A**]
time = 4.31, size = 112, normalized size = 0.90

$$-\frac{1}{48}\sqrt{-cx^4 + bx^2 + a}\left(2\,x^2\left(\frac{4\,x^2}{c} + \frac{5\,b}{c^2}\right) + \frac{15\,b^2 + 16\,ac}{c^3}\right) - \frac{(5\,b^3 + 12\,abc)\log\left(\left|2\left(\sqrt{-c}\,x^2 - \sqrt{-cx^4 + bx^2 + a}\right)\sqrt{-c} + b\right|\right)}{32\,\sqrt{-c}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c + 5*b/c^2) + (15*b^2 + 16*a*
c)/c^3) - 1/32*(5*b^3 + 12*a*b*c)*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b
*x^2 + a))*sqrt(-c) + b))/(sqrt(-c)*c^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\sqrt{-c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int(x^7/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.969 $\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=107

$$-\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}-\frac{(3b^2+4ac)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}$$

```
[Out] -1/16*(4*a*c+3*b^2)*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2))
/c^(5/2)-3/8*b*(-c*x^4+b*x^2+a)^(1/2)/c^2-1/4*x^2*(-c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1128, 756, 654, 635, 210}

$$-\frac{(4ac+3b^2)\,\text{ArcTan}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2 - c*x^4])/(8*c^2) - (x^2*Sqrt[a + b*x^2 - c*x^4])/(4*c
) - ((3*b^2 + 4*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4
])])/(16*c^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{x^2\sqrt{a+bx^2-cx^4}}{4c} - \frac{\text{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2} - \frac{x^2\sqrt{a+bx^2-cx^4}}{4c} + \frac{(3b^2+4ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx-c}}\right.}{16c^2}$$

$$= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2} - \frac{x^2\sqrt{a+bx^2-cx^4}}{4c} + \frac{(3b^2+4ac)\,\text{Subst}\left(\int \frac{1}{-4c-x^2}\,dx, x,\right.}{8c^2}$$

$$= -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2} - \frac{x^2\sqrt{a+bx^2-cx^4}}{4c} - \frac{(3b^2+4ac)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2}}\right.}{16c^{5/2}}$$

Mathematica [A]
time = 10.04, size = 89, normalized size = 0.83

$$-\frac{(3b+2cx^2)\sqrt{a+bx^2-cx^4}}{8c^2} - \frac{(3b^2+4ac)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/8*((3*b + 2*c*x^2)*Sqrt[a + b*x^2 - c*x^4])/c^2 - ((3*b^2 + 4*a*c)*ArcTa
n[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(16*c^(5/2))
```

Maple [A]
time = 0.04, size = 120, normalized size = 1.12

method	result
risch	$-\frac{(2c\,x^2+3b)\sqrt{-c\,x^4+b\,x^2+a}}{8c^2}+\frac{a\arctan\left(\frac{\sqrt{c}\,\left(x^2-\frac{b}{2c}\right)}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{4c^{\frac{3}{2}}}+\frac{3b^2\arctan\left(\frac{\sqrt{c}\,\left(x^2-\frac{b}{2c}\right)}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{16c^{\frac{5}{2}}}$
default	$-\frac{x^2\sqrt{-c\,x^4+b\,x^2+a}}{4c}-\frac{3b\sqrt{-c\,x^4+b\,x^2+a}}{8c^2}+\frac{3b^2\arctan\left(\frac{\sqrt{c}\,\left(x^2-\frac{b}{2c}\right)}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{16c^{\frac{5}{2}}}+$ a arctan $\left(\frac{\sqrt{}}{\sqrt{-c}}\right.$
elliptic	$-\frac{x^2\sqrt{-c\,x^4+b\,x^2+a}}{4c}-\frac{3b\sqrt{-c\,x^4+b\,x^2+a}}{8c^2}+\frac{3b^2\arctan\left(\frac{\sqrt{c}\,\left(x^2-\frac{b}{2c}\right)}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{16c^{\frac{5}{2}}}+$ a arctan $\left(\frac{\sqrt{}}{\sqrt{-c}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^2*(-c*x^4+b*x^2+a)^(1/2)/c-3/8*b*(-c*x^4+b*x^2+a)^(1/2)/c^2+3/16*b^2
/c^(5/2)*arctan(c^(1/2)*(x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2))+1/4*a/c^(3/2)
*arctan(c^(1/2)*(x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.50, size = 105, normalized size = 0.98

$$-\frac{\sqrt{-cx^4+bx^2+a}\,x^2}{4\,c}-\frac{3\,b^2\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{16\,c^{\frac{5}{2}}}-\frac{a\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{4\,c^{\frac{3}{2}}}-\frac{3\,\sqrt{-cx^4+bx^2+a}\,b}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-c*x^4 + b*x^2 + a)*x^2/c - 3/16*b^2*arcsin(-(2*c*x^2 - b)/sqrt(b
^2 + 4*a*c))/c^(5/2) - 1/4*a*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(3/
2) - 3/8*sqrt(-c*x^4 + b*x^2 + a)*b/c^2
```

Fricas [A]
time = 0.38, size = 211, normalized size = 1.97

$$\left[-\frac{(3\,b^2+4\,ac)\sqrt{-c}\,\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}\,(2\,cx^2-b)\sqrt{-c}\,-4\,ac\right)+4\,\sqrt{-cx^4+bx^2+a}\,(2\,c^2x^2+3\,bc)}{32\,c^3},-\frac{(3\,b^2+4\,ac)\sqrt{c}\,\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}\,(2\,cx^2-b)\sqrt{c}}{2\,(c^2x^4-bcx^2-ac)}\right)+2\,\sqrt{-cx^4+bx^2+a}\,(2\,c^2x^2+3\,bc)}{16\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 + 4*a*c)*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-
c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*sqrt(-c*x^4 + b*x^2
+ a)*(2*c^2*x^2 + 3*b*c))/c^3, -1/16*((3*b^2 + 4*a*c)*sqrt(c)*arctan(1/2*sq
rt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c)) + 2
*sqrt(-c*x^4 + b*x^2 + a)*(2*c^2*x^2 + 3*b*c))/c^3]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [**A**]
time = 3.83, size = 91, normalized size = 0.85

$$-\frac{1}{8}\sqrt{-cx^4 + bx^2 + a}\left(\frac{2\,x^2}{c} + \frac{3\,b}{c^2}\right) - \frac{(3\,b^2 + 4\,ac)\log\left(\left|2\left(\sqrt{-c}\,x^2 - \sqrt{-cx^4 + bx^2 + a}\right)\sqrt{-c} + b\right|\right)}{16\,\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2/c + 3*b/c^2) - 1/16*(3*b^2 + 4*a*c)*lo
g(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/(sqrt(-c)*
c^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{-c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int(x^5/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.970 $\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{a+bx^2-cx^4}}{2c}-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}$$

```
[Out] -1/4*b*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2))/c^(3/2)-1/2*
(-c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1128, 654, 635, 210}

$$-\frac{b\text{ArcTan}\left(\frac{b-2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt{a+bx^2-cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 - c*x^4]/c - (b*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 - c*x^4])])/(4*c^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{a+bx^2-cx^4}}{2c}+\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)}{4c}\\ &= -\frac{\sqrt{a+bx^2-cx^4}}{2c}+\frac{b\text{Subst}\left(\int \frac{1}{-4c-x^2}\,dx, x, \frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}\right)}{2c}\\ &= -\frac{\sqrt{a+bx^2-cx^4}}{2c}-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 394 vs. 2(70) = 140.
time = 2.12, size = 394, normalized size = 5.63

$$\frac{1}{8}\left(\frac{16a^2\left(b^4\sqrt{-c^2}+8ab^2c\sqrt{-c^2}-16a^2(-c^2)^{3/2}+8b^3\sqrt{-c}\,c^{3/2}x^2-32ab(-c^2)^{3/2}x^2\right)}{b^3\sqrt{c}\,(b^2+4ac)\,(b^2+4ac+8bcx^2)}-\frac{4\sqrt{a+bx^2-cx^4}\left(16a^2c^{7/2}+4abc\left(b\left(c^{3/2}-\sqrt{-c}\sqrt{-c^2}\right)+8c^{5/2}x^2\right)-b^3\left(b\sqrt{-c}\sqrt{-c^2}-4c\left(c^{3/2}-\sqrt{-c}\sqrt{-c^2}\right)x^2\right)\right)}{c^{5/2}\,(b^2+4ac)\,(b^2+4ac+8bcx^2)}+\frac{2b\tan^{-1}\left(\frac{2\sqrt{c}\left(-\sqrt{-c}\,x^2+\sqrt{a+bx^2-cx^4}\right)}{b}\right)}{c^{3/2}}+\frac{b\log\left(b^2+4bcx^2+4c\left(a-2\left(cx^4+\sqrt{-c}\,x^2\sqrt{a+bx^2-cx^4}\right)\right)\right)}{(-c)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((16*a^2*(b^4*Sqrt[-c^2] + 8*a*b^2*c*Sqrt[-c^2] - 16*a^2*(-c^2)^(3/2) + 8*b
^3*Sqrt[-c]*c^(3/2)*x^2 - 32*a*b*(-c^2)^(3/2)*x^2))/(b^3*Sqrt[c]*(b^2 + 4*a
*c)*(b^2 + 4*a*c + 8*b*c*x^2)) - (4*Sqrt[a + b*x^2 - c*x^4]*(16*a^2*c^(7/2)
 + 4*a*b*c*(b*(c^(3/2) - Sqrt[-c]*Sqrt[-c^2]) + 8*c^(5/2)*x^2) - b^3*(b*Sqr
t[-c]*Sqrt[-c^2] - 4*c*(c^(3/2) - Sqrt[-c]*Sqrt[-c^2])*x^2)))/(c^(5/2)*(b^2
 + 4*a*c)*(b^2 + 4*a*c + 8*b*c*x^2)) + (2*b*ArcTan[(2*Sqrt[c]*(-(Sqrt[-c]*x
^2) + Sqrt[a + b*x^2 - c*x^4]))/b])/c^(3/2) + (b*Log[b^2 + 4*b*c*x^2 + 4*c*
(a - 2*(c*x^4 + Sqrt[-c]*x^2*Sqrt[a + b*x^2 - c*x^4]))])/(-c)^(3/2))/8
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.83

method	result	size

default	$-\frac{\sqrt{-c\,x^{4}+b\,x^{2}+a}}{2c}+\frac{b\arctan\left(\frac{\sqrt{c}\,\left(x^{2}-\frac{b}{2c}\right)}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{4c^{\frac{3}{2}}}$	58
risch	$-\frac{\sqrt{-c\,x^{4}+b\,x^{2}+a}}{2c}+\frac{b\arctan\left(\frac{\sqrt{c}\,\left(x^{2}-\frac{b}{2c}\right)}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{4c^{\frac{3}{2}}}$	58
elliptic	$-\frac{\sqrt{-c\,x^{4}+b\,x^{2}+a}}{2c}+\frac{b\arctan\left(\frac{\sqrt{c}\,\left(x^{2}-\frac{b}{2c}\right)}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{4c^{\frac{3}{2}}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-c*x^4+b*x^2+a)^(1/2)/c+1/4*b/c^(3/2)*arctan(c^(1/2)*(x^2-1/2*b/c)/(-
c*x^4+b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.50, size = 50, normalized size = 0.71

$$-\frac{b\arcsin\left(-\frac{2\,cx^{2}-b}{\sqrt{b^{2}+4\,ac}}\right)}{4\,c^{\frac{3}{2}}}-\frac{\sqrt{-cx^{4}+bx^{2}+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(3/2) - 1/2*sqrt(-c*x^4 +
 b*x^2 + a)/c
```

Fricas [A]
time = 0.35, size = 169, normalized size = 2.41

$$\left[-\frac{b\sqrt{-c}\,\log\left(8\,c^{2}x^{4}-8\,bcx^{2}+b^{2}-4\,\sqrt{-cx^{4}+bx^{2}+a}\,(2\,cx^{2}-b)\sqrt{-c}-4\,ac\right)+4\,\sqrt{-cx^{4}+bx^{2}+a}\,c}{8\,c^{2}},-\frac{b\sqrt{c}\,\arctan\left(\frac{\sqrt{-cx^{4}+bx^{2}+a}\,(2\,cx^{2}-b)\sqrt{c}}{2\,(c^{2}x^{4}-bcx^{2}-ac)}\right)+2\,\sqrt{-cx^{4}+bx^{2}+a}\,c}{4\,c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(b*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-c*x^4 + b*x^2 +
 a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*sqrt(-c*x^4 + b*x^2 + a)*c)/c^2, -1
/4*(b*sqrt(c)*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(c)/(c^
2*x^4 - b*c*x^2 - a*c)) + 2*sqrt(-c*x^4 + b*x^2 + a)*c)/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [A]
time = 3.74, size = 70, normalized size = 1.00

$$-\frac{b\log\left(\left|2\left(\sqrt{-c}\,x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{4\sqrt{-c}\,c}-\frac{\sqrt{-cx^4+bx^2+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/(
sqrt(-c)*c) - 1/2*sqrt(-c*x^4 + b*x^2 + a)/c
```

Mupad [B]
time = 4.59, size = 62, normalized size = 0.89

$$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{2\,c}-\frac{b\,\ln\left(\frac{\frac{b}{2}-c\,x^2}{\sqrt{-c}}+\sqrt{-c\,x^4+b\,x^2+a}\right)}{4\,{\left(-c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] - (a + b*x^2 - c*x^4)^(1/2)/(2*c) - (b*log((b/2 - c*x^2)/(-c)^(1/2) + (a +
b*x^2 - c*x^4)^(1/2)))/(4*(-c)^(3/2))
```

3.971 $\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] -1/2*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1121, 635, 210}

$$-\frac{\text{ArcTan}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/2*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])]/Sqrt[c]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx-cx^2}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{-4c-x^2}\,dx, x, \frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(44) = 88.
time = 0.15, size = 124, normalized size = 2.82

$$-\frac{\tan^{-1}\left(\frac{2\sqrt{-c}\sqrt{c}\,x^2}{b}-\frac{2\sqrt{c}\sqrt{a+bx^2-cx^4}}{b}\right)}{2\sqrt{c}}-\frac{\log\left(b^2+4ac+4bcx^2-8c^2x^4-8\sqrt{-c}\,cx^2\sqrt{a+bx^2-cx^4}\right)}{4\sqrt{-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 - c*x^4],x]
[Out] -1/2*ArcTan[(2*Sqrt[-c]*Sqrt[c]*x^2)/b - (2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]
)/b]/Sqrt[c] - Log[b^2 + 4*a*c + 4*b*c*x^2 - 8*c^2*x^4 - 8*Sqrt[-c]*c*x^2*S
qrt[a + b*x^2 - c*x^4]]/(4*Sqrt[-c])
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.82

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{c}\left(x^2-\frac{b}{2c}\right)}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{2\sqrt{c}}$	36
elliptic	$\frac{\arctan\left(\frac{\sqrt{c}\left(x^2-\frac{b}{2c}\right)}{\sqrt{-c\,x^4+b\,x^2+a}}\right)}{2\sqrt{c}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/c^(1/2)*arctan(c^(1/2)*(x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.49, size = 28, normalized size = 0.64

$$-\frac{\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/sqrt(c)
```

Fricas [A]
time = 0.37, size = 124, normalized size = 2.82

$$\left[-\frac{\sqrt{-c}\,\log\left(8\,c^{2}x^{4}-8\,bcx^{2}+b^{2}-4\,\sqrt{-cx^{4}+bx^{2}+a}\,(2\,cx^{2}-b)\sqrt{-c}-4\,ac\right)}{4\,c},\,-\frac{\arctan\left(\frac{\sqrt{-cx^{4}+bx^{2}+a}\,(2\,cx^{2}-b)\sqrt{c}}{2\,(c^{2}x^{4}-bcx^{2}-ac)}\right)}{2\,\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-c*x^4 + b*x^2 + a)
*(2*c*x^2 - b)*sqrt(-c) - 4*a*c)/c, -1/2*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a
)*(2*c*x^2 - b)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c))/sqrt(c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^{2}-cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [A]
time = 3.22, size = 45, normalized size = 1.02

$$-\frac{\log\left(\left|2\left(\sqrt{-c}\,x^{2}-\sqrt{-cx^{4}+bx^{2}+a}\right)\sqrt{-c}+b\right|\right)}{2\,\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/sqr
t(-c)
```

Mupad [B]
time = 4.79, size = 40, normalized size = 0.91

$$\frac{\ln\left(\frac{\frac{b}{2}-c\,x^{2}}{\sqrt{-c}}+\sqrt{-c\,x^{4}+b\,x^{2}+a}\right)}{2\,\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 - c*x^4)^(1/2),x)
[Out] log((b/2 - c*x^2)/(-c)^(1/2) + (a + b*x^2 - c*x^4)^(1/2))/(2*(-c)^(1/2))
```

3.972 $\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctan(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2-a)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1128, 738, 210}

$$-\frac{\text{ArcTan}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])]/Sqrt[a]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx, x, \frac{-2a+bx^2}{\sqrt{-a+bx^2+cx^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{-2a+bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Mathematica [A]
time = 0.08, size = 44, normalized size = 0.94

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -(ArcTan[(Sqrt[c]*x^2 - Sqrt[-a + b*x^2 + c*x^4])/Sqrt[a]]/Sqrt[a])
```

Maple [A]
time = 0.03, size = 45, normalized size = 0.96

method	result	size
default	$-\frac{\ln\left(\frac{-2a+b\,x^2+2\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}{x^2}\right)}{2\sqrt{-a}}$	45
elliptic	$-\frac{\ln\left(\frac{-2a+b\,x^2+2\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}{x^2}\right)}{2\sqrt{-a}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

Maxima [A]
time = 0.49, size = 36, normalized size = 0.77

$$-\frac{\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}\,x^2}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] -1/2*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/sqrt(a)
```

Fricas [A]
time = 0.35, size = 129, normalized size = 2.74

$$\left[-\frac{\sqrt{-a}\log\left(\frac{(b^2-4\,ac)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}\,(bx^2-2\,a)\sqrt{-a}+8\,a^2}{x^4}\right)}{4\,a},\ \frac{\arctan\left(\frac{\sqrt{cx^4+bx^2-a}\,(bx^2-2\,a)\sqrt{a}}{2\,(acx^4+abx^2-a^2)}\right)}{2\,\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
[Out] [-1/4*sqrt(-a)*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 -
a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4)/a, 1/2*arctan(1/2*sqrt(c*x^4 + b*x^
2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2))/sqrt(a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2-a)**(1/2),x)
[Out] Integral(1/(x*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 3.54, size = 36, normalized size = 0.77

$$\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a))/sqrt(a)
```

Mupad [B]
time = 4.52, size = 52, normalized size = 1.11

$$-\frac{\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{-a}}-\frac{\ln\left(2\,\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}-2\,a+b\,x^2\right)}{2\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] - log(1/x^2)/(2*(-a)^(1/2)) - log(2*(-a)^(1/2)*(b*x^2 - a + c*x^4)^(1/2) -
2*a + b*x^2)/(2*(-a)^(1/2))
```

3.973 $\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

```
[Out] -1/4*b*arctan(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2-a)^(1/2))/a^(3/2)+1/2*(
c*x^4+b*x^2-a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1128, 744, 738, 210}

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\text{ArcTan}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) - (b*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqr
t[-a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
```

```
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}+\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)}{4a}\\ &= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx,x,\frac{-2a+bx^2}{\sqrt{-a+bx^2+cx^4}}\right)}{2a}\\ &= \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 76, normalized size = 0.99

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) - (b*ArcTan[(Sqrt[c]*x^2 - Sqrt[-a + b*x
^2 + c*x^4])/Sqrt[a]])/(2*a^(3/2))
```

Maple [A]
time = 0.04, size = 74, normalized size = 0.96

method	result	size
default	$\frac{\sqrt{c\,x^4+b\,x^2-a}}{2a\,x^2}-\frac{b\ln\left(\frac{-2a+b\,x^2+2\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}{x^2}\right)}{4a\sqrt{-a}}$	74

elliptic	$\frac{\sqrt{c\,x^{4}+b\,x^{2}-a}}{2 a\,x^{2}}-\frac{b \ln\left(\frac{-2 a+b\,x^{2}+2 \sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{4 a \sqrt{-a}}$	74
risch	$-\frac{-c\,x^{4}-b\,x^{2}+a}{2 a\,x^{2} \sqrt{c\,x^{4}+b\,x^{2}-a}}-\frac{b \ln\left(\frac{-2 a+b\,x^{2}+2 \sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{4 a \sqrt{-a}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^4+b*x^2-a)^(1/2)/a/x^2-1/4*b/a/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1
/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

Maxima [A]
time = 0.51, size = 62, normalized size = 0.81

$$-\frac{b \arcsin\left(-\frac{b}{\sqrt{b^{2}+4 \, ac}}+\frac{2 \, a}{\sqrt{b^{2}+4 \, ac}\, x^{2}}\right)}{4 \, a^{\frac{3}{2}}}+\frac{\sqrt{cx^{4}+bx^{2}-a}}{2 \, ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(3/2) +
 1/2*sqrt(c*x^4 + b*x^2 - a)/(a*x^2)
```

Fricas [A]
time = 0.36, size = 188, normalized size = 2.44

$$\left[-\frac{\sqrt{-a}\, bx^{2} \log\left(\frac{(b^{2}-4\, ac)x^{4}-8\, abx^{2}-4\, \sqrt{cx^{4}+bx^{2}-a}\,(bx^{2}-2\, a)\sqrt{-a}+8\, a^{2}}{x^{4}}\right)-4\, \sqrt{cx^{4}+bx^{2}-a}\, a}{8\, a^{2}x^{2}}, \frac{\sqrt{a}\, bx^{2} \arctan\left(\frac{\sqrt{cx^{4}+bx^{2}-a}\,(bx^{2}-2\, a)\sqrt{a}}{2\,(acx^{4}+abx^{2}-a^{2})}\right)+2\, \sqrt{cx^{4}+bx^{2}-a}\, a}{4\, a^{2}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b*x^2*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4*sqrt(c*x^4 + b
*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 - a)*
a)/(a^2*x^2), 1/4*(sqrt(a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 - a)*(b*x^2
- 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2)) + 2*sqrt(c*x^4 + b*x^2 - a)*a)/(a
^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \sqrt{-a+bx^{2}+cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 7.40, size = 111, normalized size = 1.44

$$\frac{b\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}-\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)b-2\,a\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a))/a^(3/2) - 1/
2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*b - 2*a*sqrt(c))/(((sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 - a))^2 + a)*a)
```

Mupad [B]
time = 4.55, size = 64, normalized size = 0.83

$$\frac{\sqrt{c\,x^4+b\,x^2-a}}{2\,a\,x^2}-\frac{b\,\mathrm{atanh}\left(\frac{a-\frac{b\,x^2}{2}}{\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}\right)}{4\,{\left(-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] (b*x^2 - a + c*x^4)^(1/2)/(2*a*x^2) - (b*atanh((a - (b*x^2)/2)/((-a)^(1/2)*
(b*x^2 - a + c*x^4)^(1/2))))/(4*(-a)^(3/2))
```

3.974 $\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}-\frac{(3b^2+4ac)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}$$

```
[Out] -1/16*(4*a*c+3*b^2)*arctan(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2-a)^(1/2))/
a^(5/2)+1/4*(c*x^4+b*x^2-a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1128, 758, 820, 738, 210}

$$-\frac{(4ac+3b^2)\,\text{ArcTan}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[-a + b*x^2 + c*x^4])/(8*a^2*
x^2) - ((3*b^2 + 4*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c
*x^4])])/(16*a^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
```

```
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}+\frac{\text{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{-a+bx+cx^2}}\,dx,x,x^2\right)}{4a}\\ &= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{(3b^2+4ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{-a+\cdots}}\cdots\right)}{16a^2}\\ &= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}-\frac{(3b^2+4ac)\,\text{Subst}\left(\int \frac{1}{-4a-x^2}\,dx\cdots\right)}{8a^2}\\ &= \frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}-\frac{(3b^2+4ac)\tan^{-1}\left(\frac{2a\cdots}{2\sqrt{a}\sqrt{-a\cdots}}\right)}{16a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 95, normalized size = 0.83

$$\frac{(2a+3bx^2)\sqrt{-a+bx^2+cx^4}}{8a^2x^4}+\frac{(-3b^2-4ac)\tan^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((2*a + 3*b*x^2)*Sqrt[-a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((-3*b^2 - 4*a*c)*
ArcTan[(Sqrt[c]*x^2 - Sqrt[-a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(5/2))
```

Maple [A]
time = 0.05, size = 149, normalized size = 1.30

method	result
risch	$-\frac{\left(-c\,x^{4}-b\,x^{2}+a \right) \left(3 b\,x^{2}+2 a \right)}{8 a^{2} x^{4} \sqrt{c\,x^{4}+b\,x^{2}-a}}-\frac{c \ln \left(\frac{-2 a +b\,x^{2}+2 \sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{4 a \sqrt{-a}}-\frac{3 b^{2} \ln \left(\frac{-2 a +b\,x^{2}+2 \sqrt{-a}\,\sqrt{c\,x^{4}+ \cdots}}{x^{2}}\right.}{16 a^{2} \sqrt{-a}}$
default	$\frac{\sqrt{c\,x^{4}+b\,x^{2}-a}}{4 a\,x^{4}}+\frac{3 b \sqrt{c\,x^{4}+b\,x^{2}-a}}{8 a^{2} x^{2}}-\frac{3 b^{2} \ln \left(\frac{-2 a +b\,x^{2}+2 \sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{16 a^{2} \sqrt{-a}}-\frac{c \ln \left(\frac{-2 a +b\,x^{2} \cdots}{}\right.}{}$
elliptic	$\frac{\sqrt{c\,x^{4}+b\,x^{2}-a}}{4 a\,x^{4}}+\frac{3 b \sqrt{c\,x^{4}+b\,x^{2}-a}}{8 a^{2} x^{2}}-\frac{3 b^{2} \ln \left(\frac{-2 a +b\,x^{2}+2 \sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{16 a^{2} \sqrt{-a}}-\frac{c \ln \left(\frac{-2 a +b\,x^{2} \cdots}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(c*x^4+b*x^2-a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^2-3/16*b^
2/a^2/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)-1/
4*c/a/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

Maxima [A]
time = 0.52, size = 126, normalized size = 1.10

$$-\frac{3\,b^{2} \arcsin\left(-\frac{b}{\sqrt{b^{2}+4\,ac}}+\frac{2\,a}{\sqrt{b^{2}+4\,ac}\,x^{2}}\right)}{16\,a^{\frac{5}{2}}}-\frac{c \arcsin\left(-\frac{b}{\sqrt{b^{2}+4\,ac}}+\frac{2\,a}{\sqrt{b^{2}+4\,ac}\,x^{2}}\right)}{4\,a^{\frac{3}{2}}}+\frac{3\,\sqrt{cx^{4}+bx^{2}-a}\,b}{8\,a^{2}x^{2}}+\frac{\sqrt{cx^{4}+bx^{2}-a}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/16*b^2*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(5/2
) - 1/4*c*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(3/2
) + 3/8*sqrt(c*x^4 + b*x^2 - a)*b/(a^2*x^2) + 1/4*sqrt(c*x^4 + b*x^2 - a)/(
a*x^4)
```

Fricas [A]
time = 0.40, size = 230, normalized size = 2.00

$$\left[-\frac{(3\,b^{2}+4\,ac)\sqrt{-a}\,x^{4}\log\left(\frac{(b^{2}-4\,ac)x^{4}-8\,abx^{2}-4\,\sqrt{cx^{4}+bx^{2}-a}\,(bx^{2}-2\,a)\sqrt{-a}+8\,a^{2}}{x^{4}}\right)-4\,\sqrt{cx^{4}+bx^{2}-a}\,(3\,abx^{2}+2\,a^{2})}{32\,a^{3}x^{4}},\ \frac{(3\,b^{2}+4\,ac)\sqrt{a}\,x^{4}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}-a}\,(bx^{2}-2\,a)\sqrt{a}}{2\,(acx^{4}+abx^{2}-a^{2})}\right)+2\,\sqrt{cx^{4}+bx^{2}-a}\,(3\,abx^{2}+2\,a^{2})}{16\,a^{3}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 + 4*a*c)*sqrt(-a)*x^4*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4
*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*sqrt(c*x^
4 + b*x^2 - a)*(3*a*b*x^2 + 2*a^2))/(a^3*x^4), 1/16*((3*b^2 + 4*a*c)*sqrt(a
)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a
*b*x^2 - a^2)) + 2*sqrt(c*x^4 + b*x^2 - a)*(3*a*b*x^2 + 2*a^2))/(a^3*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(96) = 192.
time = 9.12, size = 224, normalized size = 1.95

$$\frac{(3\,b^2+4\,ac)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}-\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^3b^2+4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^3ac+5\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)ab^2-4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)a^2c-8\,a^2b\sqrt{c}}{8\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2-a}\right)^2+a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2 + 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a)
)/a^(5/2) - 1/8*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^3*b^2 + 4*(sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^3*a*c + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 - a))*a*b^2 - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*a^2*c - 8*a^2*b*
sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^2 + a)^2*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^5*(b*x^2 - a + c*x^4)^(1/2)), x)
```

3.975 $\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=154

$$\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{(15b^2+16ac)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}-\frac{b(5b^2+12ac)\tan^{-1}\left(\frac{\cdots}{2\sqrt{a}\sqrt{-\cdots}}\right.}{32a^{7/2}}$$

```
[Out] -1/32*b*(12*a*c+5*b^2)*arctan(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2-a)^(1/2
))/a^(7/2)+1/6*(c*x^4+b*x^2-a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2-a)^(1/2)/a^2
/x^4+1/48*(16*a*c+15*b^2)*(c*x^4+b*x^2-a)^(1/2)/a^3/x^2
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1128, 758, 848, 820, 738, 210}

$$-\frac{b(12ac+5b^2)\,\text{ArcTan}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{(16ac+15b^2)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[-a + b*x^2 + c*x^4])/(24*a^2
*x^4) + ((15*b^2 + 16*a*c)*Sqrt[-a + b*x^2 + c*x^4])/(48*a^3*x^2) - (b*(5*b
^2 + 12*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])])/(3
2*a^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
```

```
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{\text{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right)}{6a}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(15b^2+16ac)+\frac{5bcx}{2}}{x^2\sqrt{-a+bx+cx^2}}\,dx\right.}{12a^2}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{(15b^2+16ac)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{(15b^2+16ac)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}$$

$$= \frac{\sqrt{-a+bx^2+cx^4}}{6ax^6} + \frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4} + \frac{(15b^2+16ac)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}$$

Mathematica [A]
time = 0.32, size = 114, normalized size = 0.74

$$\frac{\sqrt{-a+bx^2+cx^4}\,(8a^2+10abx^2+15b^2x^4+16acx^4)}{48a^3x^6} + \frac{(-5b^3-12abc)\tan^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[-a + b*x^2 + c*x^4]*(8*a^2 + 10*a*b*x^2 + 15*b^2*x^4 + 16*a*c*x^4))/(
48*a^3*x^6) + ((-5*b^3 - 12*a*b*c)*ArcTan[(Sqrt[c]*x^2 - Sqrt[-a + b*x^2 +
c*x^4])/Sqrt[a]])/(16*a^(7/2))
```

Maple [A]
time = 0.05, size = 202, normalized size = 1.31

method	result
risch	$-\frac{(-c\,x^4-b\,x^2+a)(16c\,x^4a+15b^2x^4+10ab\,x^2+8a^2)}{48a^3x^6\sqrt{c\,x^4+b\,x^2-a}} - \frac{3bc\ln\left(\frac{-2a+b\,x^2+2\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}{x^2}\right)}{8a^2\sqrt{-a}} - \frac{5b^3\ln\left(\frac{-2a+b\,x^2-\right.}{}$

default	$\frac{\sqrt{c\,x^{4}+b\,x^{2}-a}}{6a\,x^{6}}+\frac{5b\sqrt{c\,x^{4}+b\,x^{2}-a}}{24a^{2}x^{4}}+\frac{5b^{2}\sqrt{c\,x^{4}+b\,x^{2}-a}}{16a^{3}x^{2}}-\frac{5b^{3}\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+}}{x^{2}}\right.}{32a^{3}\sqrt{-a}}$
elliptic	$\frac{\sqrt{c\,x^{4}+b\,x^{2}-a}}{6a\,x^{6}}+\frac{5b\sqrt{c\,x^{4}+b\,x^{2}-a}}{24a^{2}x^{4}}+\frac{5b^{2}\sqrt{c\,x^{4}+b\,x^{2}-a}}{16a^{3}x^{2}}-\frac{5b^{3}\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+}}{x^{2}}\right.}{32a^{3}\sqrt{-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(c*x^4+b*x^2-a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^4+5/16*b
^2/a^3/x^2*(c*x^4+b*x^2-a)^(1/2)-5/32*b^3/a^3/(-a)^(1/2)*ln((-2*a+b*x^2+2*(
-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)-3/8*b/a^2*c/(-a)^(1/2)*ln((-2*a+b*x^2
+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)+1/3*c/a^2/x^2*(c*x^4+b*x^2-a)^(1/
2)
```

Maxima [A]
time = 0.48, size = 179, normalized size = 1.16

$$-\frac{5\,b^{3}\arcsin\left(-\frac{b}{\sqrt{b^{2}+4\,ac}}+\frac{2\,a}{\sqrt{b^{2}+4\,ac}\,x^{2}}\right)}{32\,a^{\frac{7}{2}}}-\frac{3\,bc\arcsin\left(-\frac{b}{\sqrt{b^{2}+4\,ac}}+\frac{2\,a}{\sqrt{b^{2}+4\,ac}\,x^{2}}\right)}{8\,a^{\frac{5}{2}}}+\frac{5\,\sqrt{cx^{4}+bx^{2}-a}\,b^{2}}{16\,a^{3}x^{2}}+\frac{\sqrt{cx^{4}+bx^{2}-a}\,c}{3\,a^{2}x^{2}}+\frac{5\,\sqrt{cx^{4}+bx^{2}-a}\,b}{24\,a^{2}x^{4}}+\frac{\sqrt{cx^{4}+bx^{2}-a}}{6\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/32*b^3*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(7/2
) - 3/8*b*c*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(5
/2) + 5/16*sqrt(c*x^4 + b*x^2 - a)*b^2/(a^3*x^2) + 1/3*sqrt(c*x^4 + b*x^2 -
 a)*c/(a^2*x^2) + 5/24*sqrt(c*x^4 + b*x^2 - a)*b/(a^2*x^4) + 1/6*sqrt(c*x^4
 + b*x^2 - a)/(a*x^6)
```

Fricas [A]
time = 0.41, size = 272, normalized size = 1.77

$$\left[-\frac{3\,(5\,b^{3}+12\,abc)\sqrt{-a}\,x^{6}\log\left(\frac{(b^{2}-4\,ac)x^{4}-8\,abx^{2}-4\,\sqrt{cx^{4}+bx^{2}-a}\,(bx^{2}-2\,a)\sqrt{-a}+8\,a^{2}}{x^{4}}\right)-4\,(10\,a^{2}bx^{2}+(15\,ab^{2}+16\,a^{2}c)x^{4}+8\,a^{3})\sqrt{cx^{4}+bx^{2}-a}}{192\,a^{4}x^{6}},\ \frac{3\,(5\,b^{3}+12\,abc)\sqrt{a}\,x^{6}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}-a}\,(bx^{2}-2\,a)\sqrt{a}}{2\,(acx^{4}+abx^{2}-a^{2})}\right)+2\,(10\,a^{2}bx^{2}+(15\,ab^{2}+16\,a^{2}c)x^{4}+8\,a^{3})\sqrt{cx^{4}+bx^{2}-a}}{96\,a^{4}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 + 12*a*b*c)*sqrt(-a)*x^6*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*(10
*a^2*b*x^2 + (15*a*b^2 + 16*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 - a))/(a
^4*x^6), 1/96*(3*(5*b^3 + 12*a*b*c)*sqrt(a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x
^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2)) + 2*(10*a^2*b*x^2
+ (15*a*b^2 + 16*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 - a))/(a^4*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\sqrt{-a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(131) = 262.
time = 7.70, size = 344, normalized size = 2.23

$$\frac{(5b^3 + 12abc)\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 - a}}{\sqrt{a}}\right)}{16a^{\frac{7}{2}}} - \frac{15\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 - a}\right)^5 b^3 + 36\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 - a}\right)^5 abc + 40\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 - a}\right)^3 ab^3 + 96\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 - a}\right)^3 a^2bc - 96\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 - a}\right)^2 a^3c^{\frac{3}{2}} + 33\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 - a}\right)a^2b^3 - 36\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 - a}\right)a^3bc - 48a^3b^2\sqrt{c} - 32a^4c^{\frac{3}{2}}}{48\left(\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 - a}\right)^2 + a\right)^3 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(5*b^3 + 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqr
t(a))/a^(7/2) - 1/48*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^5*b^3 + 36
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^5*a*b*c + 40*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 - a))^3*a*b^3 + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^3*a
^2*b*c - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^2*a^3*c^(3/2) + 33*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*a^2*b^3 - 36*(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 - a))*a^3*b*c - 48*a^3*b^2*sqrt(c) - 32*a^4*c^(3/2))/(((sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 - a))^2 + a)^3*a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,\sqrt{c\,x^4 + b\,x^2 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(b*x^2 - a + c*x^4)^(1/2)), x)
```

3.976 $\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=409

$$-\frac{x\sqrt{a+bx^2-cx^4}}{3c}-\frac{b\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}{3\sqrt{2}\,c^{5/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c-1/6*b*EllipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c
+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-
(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(
1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(5/2)*2^(1/2)/(-c*x^4
+b*x^2+a)^(1/2)+1/6*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2)
,((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b^2+a*c-b*(4*a*c+b^2
)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2
)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(5/2)*2^(1/2)/(-c*x^4+b*x^2+a)^
(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1136, 1216, 538, 435, 430}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}\,c^{5/2}\sqrt{a+bx^2-cx^4}}-\frac{b\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}\,c^{5/2}\sqrt{a+bx^2-cx^4}}-\frac{x\sqrt{a+bx^2-cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/3*(x*Sqrt[a + b*x^2 - c*x^4])/c - (b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sq
rt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*
x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*
Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^
2 + a*c - b*Sqrt[b^2 + 4*a*c])*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*
Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 +
 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{x\sqrt{a+bx^2-cx^4}}{3c} + \frac{\int \frac{a+2bx^2}{\sqrt{a+bx^2-cx^4}}\,dx}{3c}$$

$$= -\frac{x\sqrt{a+bx^2-cx^4}}{3c} + \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\ldots}{\sqrt{1-\ldots}}}{3c\sqrt{a+bx^2-c\ldots}}$$

$$= -\frac{x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(b\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2\ldots}{b+\sqrt{\ldots}}}\ldots\right.}{3c^2\sqrt{a+bx^2-cx\ldots}}$$

$$= -\frac{x\sqrt{a+bx^2-cx^4}}{3c} - \frac{b\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4a\ldots}}}\ldots}{3\sqrt{\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.52, size = 459, normalized size = 1.12

$$\frac{2c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,x(-a-bx^2+cx^4)-i\sqrt{2}\,b\left(-b+\sqrt{b^2+4ac}\right)\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2cx^2}{-b+\sqrt{b^2+4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+i\sqrt{2}\left(-b^2-ac+b\sqrt{b^2+4ac}\right)\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2cx^2}{-b+\sqrt{b^2+4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{6c^2\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x*(-a - b*x^2 + c*x^4) - I*Sqrt[2]*
b*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt
[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*
a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (
b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-b^2 - a*c + b
*Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 +
4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqr
t[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*c^2*Sqrt[-(c/(b + Sqrt[b^2 + 4
*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A]
time = 0.04, size = 391, normalized size = 0.96

method	result
default	$-\frac{x\sqrt{-c\,x^4+b\,x^2+a}}{3c}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{}\right.}{12c\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+}}$
risch	$-\frac{x\sqrt{-c\,x^4+b\,x^2+a}}{3c}+\frac{-\frac{ba\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x}{}\right.\right.}{\sqrt{-b}}}{}$
elliptic	$-\frac{x\sqrt{-c\,x^4+b\,x^2+a}}{3c}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}}{}\right.}{12c\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c+1/12*a/c*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b/c*a*2
^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*
a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(x**4/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{-c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 - c*x^4)^(1/2),x)
[Out] int(x^4/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.977 $\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=377

$$-\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}}{\sqrt{b+\sqrt{b^2}}}\right.\right.}{2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -1/4*EllipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2
)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-
(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+
b^2)^(1/2)))^(1/2)/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/4*EllipticF(x*2
^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c
+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2))
)^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)
/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1154, 507, 435, 430}

$$\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}}-\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/2*((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^
2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x
^4]) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x
^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*El
lipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[
b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 -
 c*x^4])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 1154

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(
b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[x^2/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqr
t[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\cdots}}}{\sqrt{a+bx^2-cx^4}}$$

$$= \frac{\left(\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\cdots}{\sqrt{1-\frac{\cdots}{b-\cdots}}}}{2c\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4\cdots}}}\cdots}{2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.09, size = 271, normalized size = 0.72

$$-\frac{i\left(-b+\sqrt{b^2+4ac}\right)\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|-\frac{b+\sqrt{b^2+4ac}}{-b+\sqrt{b^2+4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|-\frac{b+\sqrt{b^2+4ac}}{-b+\sqrt{b^2+4ac}}\right)\right)}{2\sqrt{2}\,c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-1/2*I)*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*
c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + S
qrt[b^2 + 4*a*c]))] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 +
4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))]))/(Sqrt[
2]*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A]
time = 0.02, size = 217, normalized size = 0.58

method	result
default	$-\frac{a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\right.\right.}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\sqrt{-c\,x^4+}}$
elliptic	$-\frac{a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\right.\right.}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\sqrt{-c\,x^4+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2
)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)
^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2
)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(
1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int(x^2/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.978 $\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=169

$$\frac{\sqrt{b+\sqrt{b^2+4ac}}\,\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\,\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\,\sqrt{c}\,\sqrt{a+bx^2-cx^4}}$$

```
[Out] 1/2*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)
^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/
2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)*2^(1
/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1118, 430}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\,\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\,\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\,\sqrt{c}\,\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sq
rt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4
*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1118

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2
*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\cdots}}}{\sqrt{a+bx^2-cx^4}}$$

$$= \frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,F\left(\sin^{-1}\left(\frac{\cdots}{\sqrt{b\cdots}}\right.\right.}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 177, normalized size = 1.05

$$-\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right)|-\frac{b+\sqrt{b^2+4ac}}{-b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*
a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[2]*
Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A]
time = 0.02, size = 145, normalized size = 0.86

method	result
default	$\frac{\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\sqrt{-\cdots}\right.}{4\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\sqrt{-c\,x^4+b\,x^2+a}}$

elliptic	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}}{2},\sqrt{-4 \ldots}\right.}{4\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*E
llipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a
*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [A]
time = 0.08, size = 126, normalized size = 0.75

$$\frac{\sqrt{\frac{1}{2}}\left(a^{\frac{3}{2}}\sqrt{\frac{b^{2}+4\,ac}{a^{2}}}+\sqrt{a}\,b\right)\sqrt{\frac{a\sqrt{\frac{b^{2}+4\,ac}{a^{2}}}-b}{a}}\,\mathrm{ellipticF}\left(\sqrt{\frac{1}{2}}\,x\sqrt{\frac{a\sqrt{\frac{b^{2}+4\,ac}{a^{2}}}-b}{a}},-\frac{ab\sqrt{\frac{b^{2}+4\,ac}{a^{2}}}+b^{2}+2\,ac}{2\,ac}\right)}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*(a^(3/2)*sqrt((b^2 + 4*a*c)/a^2) + sqrt(a)*b)*sqrt((a*sqrt((b
^2 + 4*a*c)/a^2) - b)/a)*ellipticF(sqrt(1/2)*x*sqrt((a*sqrt((b^2 + 4*a*c)/a
^2) - b)/a), -1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) + b^2 + 2*a*c)/(a*c))/(a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^{2}-cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 - c*x^4)^(1/2),x)
```

```
[Out] int(1/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.979 $\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=408

$$-\frac{\sqrt{a+bx^2-cx^4}}{ax}+\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\Big(}{2\sqrt{2}\,a\sqrt{c}\,\sqrt{a+bx^2-cx^4}}$$

```
[Out] -(-c*x^4+b*x^2+a)^(1/2)/a/x+1/4*EllipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^
(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c
+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^
(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/a*2^(1/2)/c^(1/2)/(-c*x^4+b*x
^2+a)^(1/2)-1/4*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b
+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-
2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2
/(b+(4*a*c+b^2)^(1/2)))^(1/2)/a*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1137, 12, 1154, 507, 435, 430}

$$-\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}a\sqrt{c}\sqrt{a+bx^2-cx^4}}+\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}a\sqrt{c}\sqrt{a+bx^2-cx^4}}-\frac{\sqrt{a+bx^2-cx^4}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2 - c*x^4]/(a*x)) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b
^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)
/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt
[2]*a*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b +
Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*
c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(
2*Sqrt[2]*a*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1154

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(
b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[x^2/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqr
t[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{\int \frac{cx^2}{\sqrt{a+bx^2-cx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{c\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{\left(c\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\quad}{\sqrt{1 \cdots}}}{a\sqrt{a+bx^2-c \cdots}}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} - \frac{\left(\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2c \cdots}{b+\sqrt{b^2 \cdots}}}\right.}{2a\sqrt{a+ \cdots}}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{ax} + \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}} \cdots}{2\sqrt{2} \cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.28, size = 283, normalized size = 0.69

$$\frac{-\frac{4a}{x}-4bx+4cx^3+\frac{i\left(-b+\sqrt{b^2+4ac}\right)\sqrt{2+\frac{4cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)\right)}{\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}}}{4a\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-4*a)/x - 4*b*x + 4*c*x^3 + (I*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[2 + (4*c*x^2
)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(El
lipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[
b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-
(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*
a*c])]))/Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))])/(4*a*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A]
time = 0.04, size = 241, normalized size = 0.59

method	result
default	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{ax}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\Big(\mathrm{EllipticF}\Big(\ldots}{2\sqrt{\ldots}}$
risch	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{ax}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\Big(\mathrm{EllipticF}\Big(\ldots}{2\sqrt{\ldots}}$
elliptic	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{ax}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\Big(\mathrm{EllipticF}\Big(\ldots}{2\sqrt{\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-c*x^4+b*x^2+a)^(1/2)/a/x+1/2*c*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*
(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)
*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1
/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(
b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**2 - c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 - c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2 - c*x^4)^(1/2)), x)
```

3.980 $\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=445

$$-\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{b\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1 \cdots}}{3\sqrt{2}\,a^2\sqrt{c}\cdots}$$

```
[Out] -1/3*(-c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(-c*x^4+b*x^2+a)^(1/2)/a^2/x-1/6*b*
EllipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/
2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*
c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^
(1/2)))^(1/2)/a^2*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/6*EllipticF(x*2^
(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+
b^2)^(1/2)))^(1/2))*(b^2+a*c-b*(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)
^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2))
)^(1/2)/a^2*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1137, 1295, 1216, 538, 435, 430}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}\,a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}-\frac{b\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}\,a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -1/3*Sqrt[a + b*x^2 - c*x^4]/(a*x^3) + (2*b*Sqrt[a + b*x^2 - c*x^4])/(3*a^2
*x) - (b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*
x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*E
llipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt
[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*a^2*Sqrt[c]*Sqrt[a + b*
x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^2 + a*c - b*Sqrt[b^2 + 4*a*
c])*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqr
t[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4
*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*a^2*S
qrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3} + \frac{\int \frac{-2b+cx^2}{x^2\sqrt{a+bx^2-cx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x} - \frac{\int \frac{-ac-2bcx^2}{\sqrt{a+bx^2-cx^4}}\,dx}{3a^2}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x} - \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{\ldots}{b-\ldots}}\right.\ldots}{\ldots}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x} - \frac{\left(b\left(b-\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2\ldots}{b-\sqrt{\ldots}}}\right.\ldots}{\ldots}$$

$$= -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x} - \frac{b\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4\ldots}}\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.49, size = 472, normalized size = 1.06

$$\frac{-2\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}(a-2bx^2)(a+bx^2-cx^4)-i\sqrt{2}b\left(-b+\sqrt{b^2+4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2cx^2}{-b+\sqrt{b^2+4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+i\sqrt{2}\left(-b^2-ac+b\sqrt{b^2+4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2cx^2}{-b+\sqrt{b^2+4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{6a^2\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x^3\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] (-2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*(a - 2*b*x^2)*(a + b*x^2 - c*x^4) -
I*Sqrt[2]*b*(-b + Sqrt[b^2 + 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*
x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b +
 Sqrt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4
*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(
-b^2 - a*c + b*Sqrt[b^2 + 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2
)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sq
rt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*
c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*a^2*Sqrt[-(c
/(b + Sqrt[b^2 + 4*a*c]))]*x^3*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A]
time = 0.04, size = 417, normalized size = 0.94

method	result
risch	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}\,(-2b\,x^2+a)}{3a^2x^3}+\frac{c\left(-\frac{ba\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}}{}\right.}{}$
default	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{-c\,x^4+b\,x^2+a}}{3a^2x}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4a}\right.}{}}}{12a\sqrt{\frac{-b-}{}}}$
elliptic	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{-c\,x^4+b\,x^2+a}}{3a^2x}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4a}\right.}{}}}{12a\sqrt{\frac{-b-}{}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(-c*x^4+b*x^2+a)^(1/2)/a^2/x+1/12*c
/a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*
c+b^2)^(1/2))/a/c)^(1/2))-1/3*c*b/a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2
)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/
2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^
(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b
*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(a + b*x**2 - c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2 - c*x^4)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^2 - c*x^4)^(1/2)), x)
```

3.981 $\int \frac{x^9}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{x^6(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{bx^4\sqrt{a+bx^2+cx^4}}{c\,(b^2-4ac)}-\frac{(b(15b^2-52ac)-2c(5b^2-12ac)\,x^2)\sqrt{a+bx^2+cx^4}}{8c^3\,(b^2-4ac)}+$$

```
[Out] 3/16*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/
c^(7/2)+x^6*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*x^4*(c*x^4+b*x
^2+a)^(1/2)/c/(-4*a*c+b^2)-1/8*(b*(-52*a*c+15*b^2)-2*c*(-12*a*c+5*b^2)*x^2)
*(c*x^4+b*x^2+a)^(1/2)/c^3/(-4*a*c+b^2)
```

Rubi [A]
time = 0.16, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$,
Rules used = {1128, 752, 846, 793, 635, 212}

$$\frac{3(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{(b(15b^2-52ac)-2cx^2(5b^2-12ac))\sqrt{a+bx^2+cx^4}}{8c^3\,(b^2-4ac)}-\frac{bx^4\sqrt{a+bx^2+cx^4}}{c\,(b^2-4ac)}+\frac{x^6(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^6*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x^4*Sqrt[a
 + b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*(5*b^2 -
 12*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^3*(b^2 - 4*a*c)) + (3*(5*b^2 -
4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(7
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
```

```
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{x^6(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{x^2(6a+3bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}$$

$$= \frac{x^6(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)} - \frac{\text{Subst}\left(\int \frac{x\left(-6ab-\frac{3}{2}(5b^2-12ac)x\right.}{\sqrt{a+bx+cx^2}}\right.}{3c(b^2-4ac)}$$

$$= \frac{x^6(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-}{8c^3(b^2}$$

$$= \frac{x^6(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-}{8c^3(b^2}$$

$$= \frac{x^6(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-}{8c^3(b^2}$$

Mathematica [A]
time = 0.62, size = 166, normalized size = 0.87

$$\frac{4a^2c(-13b+6cx^2)+b^2x^2(15b^2+5bcx^2-2c^2x^4)+a(15b^3-62b^2cx^2-20bc^2x^4+8c^3x^6)}{8c^3(-b^2+4ac)\sqrt{a+bx^2+cx^4}} + \frac{3(-5b^2+4ac)\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{16c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (4*a^2*c*(-13*b + 6*c*x^2) + b^2*x^2*(15*b^2 + 5*b*c*x^2 - 2*c^2*x^4) + a*(
15*b^3 - 62*b^2*c*x^2 - 20*b*c^2*x^4 + 8*c^3*x^6))/(8*c^3*(-b^2 + 4*a*c)*Sq
rt[a + b*x^2 + c*x^4]) + (3*(-5*b^2 + 4*a*c)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sq
rt[a + b*x^2 + c*x^4]])/(16*c^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(172) = 344.
time = 0.07, size = 354, normalized size = 1.86

method	result

default	$\frac{x^6}{4c\sqrt{c\,x^4+b\,x^2+a}}-\frac{5b\,x^4}{8c^2\sqrt{c\,x^4+b\,x^2+a}}-\frac{15b^2x^2}{16c^3\sqrt{c\,x^4+b\,x^2+a}}+\frac{15b^3}{32c^4\sqrt{c\,x^4+b\,x^2+a}}+\frac{}{16c}$
elliptic	$\frac{x^6}{4c\sqrt{c\,x^4+b\,x^2+a}}-\frac{5b\,x^4}{8c^2\sqrt{c\,x^4+b\,x^2+a}}-\frac{15b^2x^2}{16c^3\sqrt{c\,x^4+b\,x^2+a}}+\frac{15b^3}{32c^4\sqrt{c\,x^4+b\,x^2+a}}+\frac{}{16c}$
risch	$-\frac{(-2c\,x^2+7b)\sqrt{c\,x^4+b\,x^2+a}}{8c^3}+\frac{3a\,x^2}{4c^2\sqrt{c\,x^4+b\,x^2+a}}-\frac{15b^2x^2}{16c^3\sqrt{c\,x^4+b\,x^2+a}}-\frac{3ba}{8c^3\sqrt{c\,x^4+b\,x^2-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^6/c/(c*x^4+b*x^2+a)^(1/2)-5/8*b/c^2*x^4/(c*x^4+b*x^2+a)^(1/2)-15/16*b
^2/c^3*x^2/(c*x^4+b*x^2+a)^(1/2)+15/32*b^3/c^4/(c*x^4+b*x^2+a)^(1/2)+15/16*
b^4/c^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+15/32*b^5/c^4/(4*a*c-b^2)/(c*
x^4+b*x^2+a)^(1/2)+15/16*b^2/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+
a)^(1/2))-13/8*b/c^3*a/(c*x^4+b*x^2+a)^(1/2)-13/4*b^2/c^2*a/(4*a*c-b^2)/(c*
x^4+b*x^2+a)^(1/2)*x^2-13/8*b^3/c^3*a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+3/4
*a/c^2*x^2/(c*x^4+b*x^2+a)^(1/2)-3/4*a/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*
x^4+b*x^2+a)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.44, size = 591, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^4*c - 24*a*b^2*c^2 +
16*a^2*c^3)*x^4 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2)*sqrt(c)*log(-8*c
```

```
^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c)
- 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - 5*(b^
3*c^2 - 4*a*b*c^3)*x^4 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^2)*sqrt(c
*x^4 + b*x^2 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^4 + (b^3*
c^4 - 4*a*b*c^5)*x^2), -1/16*(3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b
^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)
*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c
^2*x^4 + b*c*x^2 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^6 - 15*a*b^3*c + 52*a
^2*b*c^2 - 5*(b^3*c^2 - 4*a*b*c^3)*x^4 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*
c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*
c^6)*x^4 + (b^3*c^4 - 4*a*b*c^5)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x**9/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A]
time = 5.02, size = 215, normalized size = 1.13

$$\frac{\left(\left(\frac{2\,(b^2c^2-4\,ac^3)x^2}{b^2c^3-4\,ac^4}-\frac{5\,(b^3c-4\,abc^2)}{b^2c^3-4\,ac^4}\right)x^2-\frac{15\,b^4-62\,ab^2c+24\,a^2c^2}{b^2c^3-4\,ac^4}\right)x^2-\frac{15\,ab^3-52\,a^2bc}{b^2c^3-4\,ac^4}}{8\,\sqrt{cx^4+bx^2+a}}-\frac{3\,(5\,b^2-4\,ac)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] 1/8*(((2*(b^2*c^2 - 4*a*c^3)*x^2/(b^2*c^3 - 4*a*c^4) - 5*(b^3*c - 4*a*b*c^2
)/(b^2*c^3 - 4*a*c^4))*x^2 - (15*b^4 - 62*a*b^2*c + 24*a^2*c^2)/(b^2*c^3 -
4*a*c^4))*x^2 - (15*a*b^3 - 52*a^2*b*c)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^4 + b
*x^2 + a) - 3/16*(5*b^2 - 4*a*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))*sqrt(c) - b))/c^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^9/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.982 $\int \frac{x^7}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{(3b^2-8ac-2bcx^2)\sqrt{a+bx^2+cx^4}}{2c^2(b^2-4ac)}-\frac{3b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

```
[Out] -3/4*b*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)+x^4*(
b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(-2*b*c*x^2-8*a*c+3*b^2)*
(c*x^4+b*x^2+a)^(1/2)/c^2/(-4*a*c+b^2)
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1128, 752, 793, 635, 212}

$$\frac{(-8ac+3b^2-2bcx^2)\sqrt{a+bx^2+cx^4}}{2c^2(b^2-4ac)}+\frac{x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{3b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^4*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((3*b^2 - 8*a
*c - 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(2*c^2*(b^2 - 4*a*c)) - (3*b*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
```

```
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{x(4a+2bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}$$

$$= \frac{x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{(3b^2-8ac-2bcx^2)\sqrt{a+bx^2+cx^4}}{2c^2(b^2-4ac)} - \frac{(3b)\text{Subst}}{}$$

$$= \frac{x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{(3b^2-8ac-2bcx^2)\sqrt{a+bx^2+cx^4}}{2c^2(b^2-4ac)} - \frac{(3b)\text{Subst}}{}$$

$$= \frac{x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{(3b^2-8ac-2bcx^2)\sqrt{a+bx^2+cx^4}}{2c^2(b^2-4ac)} - \frac{3b\tanh^{-1}}{}$$

Mathematica [A]

time = 0.45, size = 131, normalized size = 0.98

$$\frac{-3ab^2+8a^2c-3b^3x^2+10abcx^2-b^2cx^4+4ac^2x^4}{2c^2(-b^2+4ac)\sqrt{a+bx^2+cx^4}}+\frac{3b\log\left(bc^2+2c^3x^2-2c^{5/2}\sqrt{a+bx^2+cx^4}\right)}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*a*b^2 + 8*a^2*c - 3*b^3*x^2 + 10*a*b*c*x^2 - b^2*c*x^4 + 4*a*c^2*x^4)/(
2*c^2*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (3*b*Log[b*c^2 + 2*c^3*x^2
- 2*c^(5/2)*Sqrt[a + b*x^2 + c*x^4]])/(4*c^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(118) = 236$.
time = 0.06, size = 264, normalized size = 1.97

method	result
risch	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2c^2}+\frac{3b\,x^2}{4c^2\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b^2}{8c^3\sqrt{c\,x^4+b\,x^2+a}}-\frac{b^3x^2}{4c^2(4ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}-$
default	$\frac{x^4}{2c\sqrt{c\,x^4+b\,x^2+a}}+\frac{3b\,x^2}{4c^2\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b^2}{8c^3\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b^3x^2}{4c^2(4ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}$
elliptic	$\frac{x^4}{2c\sqrt{c\,x^4+b\,x^2+a}}+\frac{3b\,x^2}{4c^2\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b^2}{8c^3\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b^3x^2}{4c^2(4ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^4/c/(c*x^4+b*x^2+a)^(1/2)+3/4*b/c^2*x^2/(c*x^4+b*x^2+a)^(1/2)-3/8*b^2
/c^3/(c*x^4+b*x^2+a)^(1/2)-3/4*b^3/c^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^
2-3/8*b^4/c^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-3/4*b/c^(5/2)*ln((1/2*b+c*x
^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+a/c^2/(c*x^4+b*x^2+a)^(1/2)+2*a/c*b/(4*a
*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+a/c^2*b^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/
2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.41, size = 459, normalized size = 3.43

$$\left[\frac{3\left((b^3c-4\,abc^2)x^4+ab^3-4\,a^2bc+(b^4-4\,ab^2c)x^2\right)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}-4\,ac\right)+4\left((b^2c^2-4\,ac^3)x^4+3\,ab^2c-8\,a^2c^2+(3\,b^3c-10\,abc^2)x^2\right)\sqrt{cx^4+bx^2+a}}{8\,(ab^2c^3-4\,a^2c^4+(b^2c^4-4\,ac^5)x^4+(b^3c^3-4\,abc^4)x^2)}, \frac{3\left((b^3c-4\,abc^2)x^4+ab^3-4\,a^2bc+(b^4-4\,ab^2c)x^2\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)+2\left((b^2c^2-4\,ac^3)x^4+3\,ab^2c-8\,a^2c^2+(3\,b^3c-10\,abc^2)x^2\right)\sqrt{cx^4+bx^2+a}}{4\,(ab^2c^3-4\,a^2c^4+(b^2c^4-4\,ac^5)x^4+(b^3c^3-4\,abc^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^
2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^4 + 3*a*b^2*c - 8*a^
2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^3 - 4
*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2), 1/4*(3*((b
^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^2)*sqrt(-c)
*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x
^2 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^4 + 3*a*b^2*c - 8*a^2*c^2 + (3*b^3*c
- 10*a*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c
^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A]
time = 8.48, size = 154, normalized size = 1.15

$$\frac{\left(\frac{(b^2c-4\,ac^2)x^2}{b^2c^2-4\,ac^3}+\frac{3\,b^3-10\,abc}{b^2c^2-4\,ac^3}\right)x^2+\frac{3\,ab^2-8\,a^2c}{b^2c^2-4\,ac^3}}{2\,\sqrt{cx^4+bx^2+a}}+\frac{3\,b\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((b^2*c - 4*a*c^2)*x^2/(b^2*c^2 - 4*a*c^3) + (3*b^3 - 10*a*b*c)/(b^2*c
^2 - 4*a*c^3))*x^2 + (3*a*b^2 - 8*a^2*c)/(b^2*c^2 - 4*a*c^3))/sqrt(c*x^4 +
```

```
b*x^2 + a) + 3/4*b*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(
c) - b))/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^7/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.983 $\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=115

$$\frac{x^2(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\,(b^2-4ac)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)+x^2*(b*x
^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*(c*x^4+b*x^2+a)^(1/2)/c/(-4*a*
c+b^2)
```

Rubi [A]

time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1128, 752, 654, 635, 212}

$$\frac{x^2(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\,(b^2-4ac)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*Sqrt[a + b
*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])]/(2*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{2a+bx}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{b^2-4ac}\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx\right.}{2c}\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b}{\sqrt{a+}}\right.}{c}\\
&= \frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+c}}\right.}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.37, size = 96, normalized size = 0.83

$$
\frac{ab+b^2x^2-2acx^2}{c\left(-b^2+4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\log\left(bc+2c^2x^2-2c^{3/2}\sqrt{a+bx^2+cx^4}\right)}{2c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*b + b^2*x^2 - 2*a*c*x^2)/(c*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - Lo
g[b*c + 2*c^2*x^2 - 2*c^(3/2)*Sqrt[a + b*x^2 + c*x^4]]/(2*c^(3/2))
```

Maple [A]
time = 0.03, size = 149, normalized size = 1.30

method	result
default	$-\frac{x^{2}}{2c\sqrt{c\,x^{4}+b\,x^{2}+a}}+\frac{b}{4c^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}+\frac{b^{2}x^{2}}{2c(4ac-b^{2})\sqrt{c\,x^{4}+b\,x^{2}+a}}+\frac{b^{3}}{4c^{2}(4ac-b^{2})\sqrt{c\,x^{4}+b\,x}}$
elliptic	$-\frac{x^{2}}{2c\sqrt{c\,x^{4}+b\,x^{2}+a}}+\frac{b}{4c^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}+\frac{b^{2}x^{2}}{2c(4ac-b^{2})\sqrt{c\,x^{4}+b\,x^{2}+a}}+\frac{b^{3}}{4c^{2}(4ac-b^{2})\sqrt{c\,x^{4}+b\,x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2/c/(c*x^4+b*x^2+a)^(1/2)+1/4*b/c^2/(c*x^4+b*x^2+a)^(1/2)+1/2*b^2/c/
(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/4*b^3/c^2/(4*a*c-b^2)/(c*x^4+b*x^2+
a)^(1/2)+1/2/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 387, normalized size = 3.37

$$\left[\frac{((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^4+bx^2+a}\,(abc+(b^2c-2\,ac^2)x^2)}{4\,(ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^4+(b^3c^2-4\,abc^3)x^2)},-\frac{((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)+2\,\sqrt{cx^4+bx^2+a}\,(abc+(b^2c-2\,ac^2)x^2)}{2\,(ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^4+(b^3c^2-4\,abc^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(
c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a)*(a*b*c + (b^2*c - 2*a*c^2)*
x^2))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c
^3)*x^2), -1/2*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*
x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^
2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*(a*b*c + (b^2*c - 2*a*c
^2)*x^2))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a
*b*c^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A]
time = 10.24, size = 101, normalized size = 0.88

$$-\frac{\frac{(b^2-2\,ac)x^2}{b^2c-4\,ac^2} + \frac{ab}{b^2c-4\,ac^2}}{\sqrt{cx^4+bx^2+a}} - \frac{\log\left(\left|-2\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)\sqrt{c} - b\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((b^2 - 2*a*c)*x^2/(b^2*c - 4*a*c^2) + a*b/(b^2*c - 4*a*c^2))/sqrt(c*x^4 +
 b*x^2 + a) - 1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c
) - b))/c^(3/2)
```

Mupad [B]
time = 4.76, size = 84, normalized size = 0.73

$$\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a} + \frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,c^{3/2}} + \frac{\frac{a\,b}{2} - x^2\left(a\,c - \frac{b^2}{2}\right)}{2\,c\,\left(a\,c - \frac{b^2}{4}\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))/(2*c^(3/2)) + ((a*b)
/2 - x^2*(a*c - b^2/2))/(2*c*(a*c - b^2/4)*(a + b*x^2 + c*x^4)^(1/2))
```

3.984 $\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}} dx$

Optimal. Leaf size=36

$$\frac{2a + bx^2}{(b^2 - 4ac)\sqrt{a + bx^2 + cx^4}}$$

```
[Out] (b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1128, 650}

$$\frac{2a + bx^2}{(b^2 - 4ac)\sqrt{a + bx^2 + cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a + bx^2 + cx^4)^{3/2}} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a + bx + cx^2)^{3/2}} dx, x, x^2\right) \\
&= \frac{2a + bx^2}{(b^2 - 4ac)\sqrt{a + bx^2 + cx^4}}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 36, normalized size = 1.00

$$\frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.03, size = 38, normalized size = 1.06

method	result	size
gosper	$-\frac{b\,x^2+2a}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$	38
default	$-\frac{b\,x^2+2a}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$	38
trager	$-\frac{b\,x^2+2a}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$	38
elliptic	$-\frac{b\,x^2+2a}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -(b*x^2+2*a)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 67, normalized size = 1.86

$$\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^
2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A]
time = 7.04, size = 44, normalized size = 1.22

$$\frac{\frac{bx^2}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (b*x^2/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [B]
time = 4.47, size = 37, normalized size = 1.03

$$-\frac{b\,x^2+2\,a}{(4\,a\,c-b^2)\,\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -(2*a + b*x^2)/((4*a*c - b^2)*(a + b*x^2 + c*x^4)^(1/2))
```

3.985 $\int \frac{x}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{b+2cx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (-2*c*x^2-b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1121, 627}

$$-\frac{b+2cx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b + 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]))
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^2+cx^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{b+2cx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 37, normalized size = 1.03

$$\frac{-b-2cx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + c*x^4)^(3/2),x]
[Out] (-b - 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.03, size = 36, normalized size = 1.00

method	result	size
gosper	$\frac{2 c \,x^{2}+b}{\sqrt{c \,x^{4}+b \,x^{2}+a}\, \left(4 a c -b^{2}\right)}$	36
default	$\frac{2 c \,x^{2}+b}{\sqrt{c \,x^{4}+b \,x^{2}+a}\, \left(4 a c -b^{2}\right)}$	36
trager	$\frac{2 c \,x^{2}+b}{\sqrt{c \,x^{4}+b \,x^{2}+a}\, \left(4 a c -b^{2}\right)}$	36
elliptic	$\frac{2 c \,x^{2}+b}{\sqrt{c \,x^{4}+b \,x^{2}+a}\, \left(4 a c -b^{2}\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] (2*c*x^2+b)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 67, normalized size = 1.86

$$-\frac{\sqrt{cx^{4} + bx^{2} + a}\,(2\,cx^{2} + b)}{(b^{2}c - 4\,ac^{2})x^{4} + ab^{2} - 4\,a^{2}c + (b^{3} - 4\,abc)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a
^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [A]
time = 3.08, size = 45, normalized size = 1.25

$$-\frac{\frac{2\,cx^2}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(2*c*x^2/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [B]
time = 4.36, size = 35, normalized size = 0.97

$$\frac{2\,c\,x^2+b}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (b + 2*c*x^2)/((4*a*c - b^2)*(a + b*x^2 + c*x^4)^(1/2))
```

3.986 $\int \frac{1}{x(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{b^2-2ac+bcx^2}{a\,(b^2-4ac)\,\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)+(b*c*x^
2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1128, 754, 12, 738, 212}

$$\frac{-2ac+b^2+bcx^2}{a\,(b^2-4ac)\,\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh
[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2a}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{a}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 124, normalized size = 1.39

$$-\frac{\left(b^2-2ac+bcx^2\right)\sqrt{a+bx^2+cx^4}}{a\left(4a^2c-b^2x^2\left(b+cx^2\right)+a\left(-b^2+4bcx^2+4c^2x^4\right)\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -(((b^2 - 2*a*c + b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(a*(4*a^2*c - b^2*x^2*(
b + c*x^2) + a*(-b^2 + 4*b*c*x^2 + 4*c^2*x^4)))) + ArcTanh[(Sqrt[c]*x^2 - S
qrt[a + b*x^2 + c*x^4])/Sqrt[a]]/a^(3/2)
```

Maple [A]
time = 0.04, size = 99, normalized size = 1.11

method	result	size
default	$\frac{1}{2a\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{b\left(2c\,x^{2}+b\right)}{2a\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}$	99
elliptic	$\frac{1}{2a\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{b\left(2c\,x^{2}+b\right)}{2a\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/(c*x^4+b*x^2+a)^(1/2)-1/2*b/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)
^(1/2)-1/2/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(77) = 154$.
time = 0.40, size = 389, normalized size = 4.37

$$\left[\frac{\left((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2\right)\sqrt{a}\,\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,(abcx^2+ab^2-2\,a^2c)\sqrt{cx^4+bx^2+a}}{4\,(a^3b^2-4\,a^4c+(a^2b^2c-4\,a^3c^2)x^4+(a^2b^3-4\,a^3bc)x^2)},\ \frac{\left((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\,(abcx^2+ab^2-2\,a^2c)\sqrt{cx^4+bx^2+a}}{2\,(a^3b^2-4\,a^4c+(a^2b^2c-4\,a^3c^2)x^4+(a^2b^3-4\,a^3bc)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(
a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 +
 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(c*x^4 +
b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4
*a^3*b*c)*x^2), 1/2*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a
)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(c*x^4 +
 b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 -
4*a^3*b*c)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [A]
time = 3.24, size = 110, normalized size = 1.24

$$\frac{\frac{abcx^2}{a^2b^2-4\,a^3c}+\frac{ab^2-2\,a^2c}{a^2b^2-4\,a^3c}}{\sqrt{cx^4+bx^2+a}}+\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (a*b*c*x^2/(a^2*b^2 - 4*a^3*c) + (a*b^2 - 2*a^2*c)/(a^2*b^2 - 4*a^3*c))/sqr
t(c*x^4 + b*x^2 + a) + arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt
(-a))/(sqrt(-a)*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.987 $\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}$$

```
[Out] 3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)+(b*c*x
^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2+a)^(1/2)-1/2*(-8*a*c+3*b^2)*(
c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1128, 754, 820, 738, 212}

$$\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2x^2\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]) - ((3
*b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^2) + (3*b*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
```

```
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}}-\frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{a\left(b^2-4ac\right)}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2}-\frac{(3b)\text{Subst}\left(\right.}{}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2}+\frac{(3b)\text{Subst}\left(\right.}{}\\
&= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2}+\frac{3b\tanh^{-1}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.51, size = 127, normalized size = 0.91

$$\frac{-4a^2c+3b^2x^2(b+cx^2)+a(b^2-10bcx^2-8c^2x^4)}{2a^2\left(-b^2+4ac\right)x^2\sqrt{a+bx^2+cx^4}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2*x^4))/(2*a^
2*(-b^2 + 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]) - (3*b*ArcTanh[(Sqrt[c]*x^2 -
 Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(5/2))
```

Maple [A]
time = 0.08, size = 195, normalized size = 1.40

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2a^2x^2}+\frac{b^2x^2c}{a^2(4ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}-\frac{2c^2x^2}{a(4ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}+\frac{b^3}{4a^2(4ac-b^2)\sqrt{c\,x^4+}}$
default	$-\frac{1}{2a\,x^2\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b}{4a^2\sqrt{c\,x^4+b\,x^2+a}}+\frac{3b^2x^2c}{2a^2(4ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}+\frac{3b^3}{4a^2(4ac-b^2)\sqrt{c\,x^4+}}$
elliptic	$-\frac{1}{2a\,x^2\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b}{4a^2\sqrt{c\,x^4+b\,x^2+a}}+\frac{3b^2x^2c}{2a^2(4ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}+\frac{3b^3}{4a^2(4ac-b^2)\sqrt{c\,x^4+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2/(c*x^4+b*x^2+a)^(1/2)-3/4*b/a^2/(c*x^4+b*x^2+a)^(1/2)+3/2*b^2/a^
2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2*c+3/4*b^3/a^2/(4*a*c-b^2)/(c*x^4+b*
x^2+a)^(1/2)+3/4*b/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x
^2)-2*c/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 485, normalized size = 3.49

$$\left[\frac{3\,\left((b^{3}c-4\,abc^{2})x^{6}+(b^{4}-4\,ab^{2}c)x^{4}+(ab^{3}-4\,a^{2}bc)x^{2}\right)\sqrt{a}\,\log\left(-\frac{(b^{2}+4\,ac)x^{4}+8\,abx^{2}+4\,\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2\,a)\sqrt{a}+8\,a^{2}}{x^{4}}\right)-4\,\left((3\,ab^{2}c-8\,a^{2}c^{2})x^{4}+a^{2}b^{2}-4\,a^{3}c+(3\,ab^{3}-10\,a^{2}bc)x^{2}\right)\sqrt{cx^{4}+bx^{2}+a}}{8\,\left((a^{3}b^{2}c-4\,a^{4}c^{2})x^{6}+(a^{3}b^{3}-4\,a^{4}bc)x^{4}+(a^{4}b^{2}-4\,a^{5}c)x^{2}\right)},\,-\frac{3\,\left((b^{3}c-4\,abc^{2})x^{6}+(b^{4}-4\,ab^{2}c)x^{4}+(ab^{3}-4\,a^{2}bc)x^{2}\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2\,a)\sqrt{-a}}{2\,(acx^{4}+abx^{2}+a^{2})}\right)+2\,\left((3\,ab^{2}c-8\,a^{2}c^{2})x^{4}+a^{2}b^{2}-4\,a^{3}c+(3\,ab^{3}-10\,a^{2}bc)x^{2}\right)\sqrt{cx^{4}+bx^{2}+a}}{4\,\left((a^{3}b^{2}c-4\,a^{4}c^{2})x^{6}+(a^{3}b^{3}-4\,a^{4}bc)x^{4}+(a^{4}b^{2}-4\,a^{5}c)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2
 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((3*a*b^2*c - 8*a^2*c^2)*x^4
+ a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5
*c)*x^2), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3
 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a
)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^
2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^
3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5*c)*
x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [A]
time = 5.18, size = 200, normalized size = 1.44

$$-\frac{\frac{(a^{2}b^{2}c-2\,a^{3}c^{2})x^{2}}{a^{4}b^{2}-4\,a^{5}c}+\frac{a^{2}b^{3}-3\,a^{3}bc}{a^{4}b^{2}-4\,a^{5}c}}{\sqrt{cx^{4}+bx^{2}+a}}-\frac{3\,b\arctan\left(-\frac{\sqrt{c}\,x^{2}-\sqrt{cx^{4}+bx^{2}+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^{2}}+\frac{\left(\sqrt{c}\,x^{2}-\sqrt{cx^{4}+bx^{2}+a}\right)b+2\,a\sqrt{c}}{2\,\left(\left(\sqrt{c}\,x^{2}-\sqrt{cx^{4}+bx^{2}+a}\right)^{2}-a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b^2*c - 2*a^3*c^2)*x^2/(a^4*b^2 - 4*a^5*c) + (a^2*b^3 - 3*a^3*b*c)/(
a^4*b^2 - 4*a^5*c))/sqrt(c*x^4 + b*x^2 + a) - 3/2*b*arctan(-(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/2*((sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^2 - a)*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.988 $\int \frac{1}{x^5(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{b^2-2ac+bcx^2}{a(b^2-4ac)x^4\sqrt{a+bx^2+cx^4}}-\frac{(5b^2-12ac)\sqrt{a+bx^2+cx^4}}{4a^2(b^2-4ac)x^4}+\frac{b(15b^2-52ac)\sqrt{a+bx^2+cx^4}}{8a^3(b^2-4ac)x^2}-\frac{3(5b^2-}{}$$

```
[Out] -3/16*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))
/a^(7/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^4/(c*x^4+b*x^2+a)^(1/2)-1/4*(
-12*a*c+5*b^2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/x^4+1/8*b*(-52*a*c+15
*b^2)*(c*x^4+b*x^2+a)^(1/2)/a^3/(-4*a*c+b^2)/x^2
```

Rubi [A]
time = 0.15, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1128, 754, 848, 820, 738, 212}

$$-\frac{3(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{7/2}}+\frac{b(15b^2-52ac)\sqrt{a+bx^2+cx^4}}{8a^3x^2(b^2-4ac)}-\frac{(5b^2-12ac)\sqrt{a+bx^2+cx^4}}{4a^2x^4(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{ax^4(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^4*Sqrt[a + b*x^2 + c*x^4]) - ((5
*b^2 - 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(15*
b^2 - 52*a*c)*Sqrt[a + b*x^2 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x^2) - (3*(5*b^
2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*
a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
```

```
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5 (a+bx^2+cx^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3 (a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)x^4\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-5b^2+12ac)-2bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{a(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)x^4\sqrt{a+bx^2+cx^4}} - \frac{(5b^2-12ac)\sqrt{a+bx^2+cx^4}}{4a^2(b^2-4ac)x^4} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)x^4\sqrt{a+bx^2+cx^4}} - \frac{(5b^2-12ac)\sqrt{a+bx^2+cx^4}}{4a^2(b^2-4ac)x^4} + \frac{b(15b^2-}{8a}$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)x^4\sqrt{a+bx^2+cx^4}} - \frac{(5b^2-12ac)\sqrt{a+bx^2+cx^4}}{4a^2(b^2-4ac)x^4} + \frac{b(15b^2-}{8a}$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)x^4\sqrt{a+bx^2+cx^4}} - \frac{(5b^2-12ac)\sqrt{a+bx^2+cx^4}}{4a^2(b^2-4ac)x^4} + \frac{b(15b^2-}{8a}$$

Mathematica [A]
time = 0.69, size = 166, normalized size = 0.85

$$\frac{-8a^3c-15b^3x^4(b+cx^2)+2a^2(b^2+10bcx^2-12c^2x^4)+abx^2(-5b^2+62bcx^2+52c^2x^4)}{8a^3(-b^2+4ac)x^4\sqrt{a+bx^2+cx^4}} + \frac{3(5b^2-4ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-8*a^3*c - 15*b^3*x^4*(b + c*x^2) + 2*a^2*(b^2 + 10*b*c*x^2 - 12*c^2*x^4)
+ a*b*x^2*(-5*b^2 + 62*b*c*x^2 + 52*c^2*x^4))/(8*a^3*(-b^2 + 4*a*c)*x^4*Sqr
t[a + b*x^2 + c*x^4]) + (3*(5*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a +
b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(7/2))
```

Maple [A]
time = 0.09, size = 314, normalized size = 1.61

method	result
default	$-\frac{1}{4a\,x^4\sqrt{c\,x^4+b\,x^2+a}}+\frac{5b}{8a^2x^2\sqrt{c\,x^4+b\,x^2+a}}+\frac{15b^2}{16a^3\sqrt{c\,x^4+b\,x^2+a}}-\frac{15b^3x^2c}{8a^3(4ac-b^2)\sqrt{c\,x^4+b}}$

elliptic	$-\frac{1}{4a\,x^4\sqrt{c\,x^4+b\,x^2+a}}+\frac{5b}{8a^2x^2\sqrt{c\,x^4+b\,x^2+a}}+\frac{15b^2}{16a^3\sqrt{c\,x^4+b\,x^2+a}}-\frac{15b^3x^2c}{8a^3(4ac-b^2)\sqrt{c\,x^4+b\,x^2}}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-7b\,x^2+2a)}{8a^3x^4}+\frac{3b\,c^2x^2}{a^2(4ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}+\frac{c^2}{a\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}-\frac{}{a^3(4ac-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/x^4/(c*x^4+b*x^2+a)^(1/2)+5/8*b/a^2/x^2/(c*x^4+b*x^2+a)^(1/2)+15/16*
b^2/a^3/(c*x^4+b*x^2+a)^(1/2)-15/8*b^3/a^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2
)*x^2*c-15/16*b^4/a^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-15/16*b^2/a^(7/2)*l
n((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+13/2*b/a^2*c^2/(4*a*c-b^
2)/(c*x^4+b*x^2+a)^(1/2)*x^2+13/4*b^2/a^2*c/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/
2)-3/4*c/a^2/(c*x^4+b*x^2+a)^(1/2)+3/4*c/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c
*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.46, size = 615, normalized size = 3.15

$$\left[-\frac{3\left((5b^4c-24ab^2c^2+16a^2c^3)x^8+(5b^5-24ab^3c+16a^2bc^2)x^6+(5ab^4-24a^2b^2c+16a^3c^2)x^4\right)\sqrt{a}\log\left(-\frac{(b^2+4ac)x^4+8abx^2+4\sqrt{cx^4+bx^2+a}\,(bx^2+2a)\sqrt{a}+8a^2}{x^4}\right)-4\left((15ab^3c-52a^2bc^2)x^6-2a^3b^2+8a^4c+(15ab^4-62a^2b^2c+24a^3c^2)x^4+5(a^2b^3-4a^3bc)x^2\right)\sqrt{cx^4+bx^2+a}}{32\left((a^4b^2c-4a^5c^2)x^8+(a^4b^3-4a^5bc)x^6+(a^5b^2-4a^6c)x^4\right)},\ \frac{3\left((5b^4c-24ab^2c^2+16a^2c^3)x^8+(5b^5-24ab^3c+16a^2bc^2)x^6+(5ab^4-24a^2b^2c+16a^3c^2)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2a)\sqrt{-a}}{2(acx^4+abx^2+a^2)}\right)+2\left((15ab^3c-52a^2bc^2)x^6-2a^3b^2+8a^4c+(15ab^4-62a^2b^2c+24a^3c^2)x^4+5(a^2b^3-4a^3bc)x^2\right)\sqrt{cx^4+bx^2+a}}{16\left((a^4b^2c-4a^5c^2)x^8+(a^4b^3-4a^5bc)x^6+(a^5b^2-4a^6c)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^8 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^6 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^4)*sqrt(a)*lo
g(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(a) + 8*a^2)/x^4) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^6 - 2*a^3*b^2 + 8
*a^4*c + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^4 + 5*(a^2*b^3 - 4*a^3*b*
```

```
c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^8 + (a^4*b^3 -
4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c)*x^4), 1/16*(3*((5*b^4*c - 24*a*b^2*c^2
 + 16*a^2*c^3)*x^8 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^6 + (5*a*b^4 - 2
4*a^2*b^2*c + 16*a^3*c^2)*x^4)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*
(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^3*c - 52*a^2
*b*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*
x^4 + 5*(a^2*b^3 - 4*a^3*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^4*b^2*c - 4
*a^5*c^2)*x^8 + (a^4*b^3 - 4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c)*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [A]
time = 5.01, size = 350, normalized size = 1.79

$$\frac{\frac{\left(a^3b^3c-3\,a^4bc^2\right)x^2}{a^6b^2-4\,a^7c}+\frac{a^3b^4-4\,a^4b^2c+2\,a^5c^2}{a^6b^2-4\,a^7c}}{\sqrt{cx^4+bx^2+a}}+\frac{3\left(5\,b^2-4\,ac\right)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^3}-\frac{7\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3b^2-4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3ac+8\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2ab\sqrt{c}-9\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)ab^2-4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)a^2c-16\,a^2b\sqrt{c}}{8\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((a^3*b^3*c - 3*a^4*b*c^2)*x^2/(a^6*b^2 - 4*a^7*c) + (a^3*b^4 - 4*a^4*b^2*c
 + 2*a^5*c^2)/(a^6*b^2 - 4*a^7*c))/sqrt(c*x^4 + b*x^2 + a) + 3/8*(5*b^2 - 4
*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a
^3) - 1/8*(7*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2 - 4*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))^3*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a
))^2*a*b*sqrt(c) - 9*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b^2 - 4*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*c - 16*a^2*b*sqrt(c))/(((sqrt(c)*x^
2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.989 $\int \frac{x^6}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=408

$$\frac{x^3(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{c\,(b^2-4ac)}+\frac{2(b^2-3ac)\,x\sqrt{a+bx^2+cx^4}}{c^{3/2}\,(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2\sqrt[4]{a}\,(b^2-3ac)\left(\sqrt{a}+\right.}{}$$

```
[Out] x^3*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*x*(c*x^4+b*x^2+a)^(1/2
)/c/(-4*a*c+b^2)+2*(-3*a*c+b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(-4*a*c+b^2
)/(a^(1/2)+x^2*c^(1/2))-2*a^(1/4)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*
x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(-4*a*c+b^2)/(c*x^4+b*x
^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-
b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/2)*c^(1
/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1134, 1293, 1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{2x(b^2-3ac)\sqrt{a+bx^2+cx^4}}{c^{3/2}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{bx\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{x^3(2a+bx^2)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^3*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x*Sqrt[a +
 b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + (2*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*
x^4])/(c^(3/2)*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*(b^2 - 3
*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4
])/(c^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*b^2 + Sqrt
[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(2*c^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x^3(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{\int \frac{x^2(6a+3bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx}{-b^2+4ac}$$

$$= \frac{x^3(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)} + \frac{\int \frac{3ab+6(b^2-3ac)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c(b^2-4ac)}$$

$$= \frac{x^3(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)} + \frac{\left(\sqrt{a}\left(2b-3\sqrt{a}\sqrt{c}\right)\right)\int \frac{}{\sqrt{}}}{\left(b-2\sqrt{a}\sqrt{c}\right)}$$

$$= \frac{x^3(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)} + \frac{2(b^2-3ac)x\sqrt{a+bx^2+cx}}{c^{3/2}(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.90, size = 489, normalized size = 1.20

$$\frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(b^2x^2+a(b-2cx^2)) - i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right) + i\left(-b^3+4abc+b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{2c^2(-b^2+4ac)\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2*x^2 + a*(b - 2*c*x^2)) - I*(b^2
 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sq
rt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2*
c^2*(-b^2 + 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.04, size = 482, normalized size = 1.18

method	result

default	$-\frac{2c\left(\frac{\left(2ac-b^{2}\right)x^{3}}{2c^{2}\left(4ac-b^{2}\right)}-\frac{abx}{2c^{2}\left(4ac-b^{2}\right)}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}-\frac{ab\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\ \mathrm{El}}{4c(4ac-b^{2})\sqrt{\frac{-b+\sqrt{-4ac+}}{a}}}$
elliptic	$-\frac{2c\left(\frac{\left(2ac-b^{2}\right)x^{3}}{2c^{2}\left(4ac-b^{2}\right)}-\frac{abx}{2c^{2}\left(4ac-b^{2}\right)}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}-\frac{ab\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\ \mathrm{El}}{4c(4ac-b^{2})\sqrt{\frac{-b+\sqrt{-4ac+}}{a}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(1/2/c^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3-1/2*a*b/c^2/(4*a*c-b^2)*x)/((x^4+
b/c*x^2+a/c)*c)^(1/2)-1/4*a*b/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*
(1/c+(2*a*c-b^2)/c/(4*a*c-b^2))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**6/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^6/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.990 $\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=342

$$\frac{x(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\Big(}{c^{3/4}\,(b^2-4ac)\sqrt{a}\ldots}$$

```
[Out] x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*x*(c*x^4+b*x^2+a)^(1/2)/
(-4*a*c+b^2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))+a^(1/4)*b*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arcta
n(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2)
)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-4*a*c+b^2)/(c*x
^4+b*x^2+a)^(1/2)-1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^(1
/2)
```

Rubi [A]
time = 0.09, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1134, 1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\,(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x*Sqrt[a + b
*x^2 + c*x^4])/(Sqrt[c]*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*b
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c
^(3/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*(b - 2*Sqrt[a]*Sqr
t[c])*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{\int \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{-b^2+4ac}$$

$$= \frac{x(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{a}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{c}} + \frac{\left(\sqrt{a}\,b\right)\int \frac{1-\frac{\sqrt{c}}{\sqrt{a}}}{\sqrt{a+bx^2}}}{\sqrt{c}\,(b^2-4a}$$

$$= \frac{x(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{\sqrt[4]{a}\,b\left(\sqrt{a}+\sqrt{c}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.54, size = 452, normalized size = 1.32

$$\frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(2a+bx^2)-ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+i\left(-b^2+4ac+b\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{4c\,(b^2-4ac)\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(2*a + b*x^2) - I*b*(-b + Sqrt[b^2 -
 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sq
rt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])])/(4*c*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a +
 b*x^2 + c*x^4])
```

Maple [A]
time = 0.03, size = 450, normalized size = 1.32

method	result
default	$-\frac{2c\left(\frac{b\,x^{3}}{2c\left(4ac-b^{2}\right)}+\frac{ax}{c\left(4ac-b^{2}\right)}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\ \mathrm{Ellipti}}{2\left(4ac-b^{2}\right)\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}$
elliptic	$-\frac{2c\left(\frac{b\,x^{3}}{2c\left(4ac-b^{2}\right)}+\frac{ax}{c\left(4ac-b^{2}\right)}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\ \mathrm{Ellipti}}{2\left(4ac-b^{2}\right)\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(1/2*b/c/(4*a*c-b^2)*x^3+a/c/(4*a*c-b^2)*x)/((x^4+b/c*x^2+a/c)*c)^(1/2
)+1/2*a/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*a*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+
b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x**4/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{(c\,x^4 + b\,x^2 + a)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^4/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.991 $\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=341

$$-\frac{x(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E}{(b^2-4ac)\sqrt{a+b}}$$

```
[Out] -x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+2*x*c^(1/2)*(c*x^4+b*x^2+
a)^(1/2)/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2))-2*a^(1/4)*c^(1/4)*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^
2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(-4*a*c+b^2)/(c*
x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b
/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^
2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^(
1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1133, 1211, 1117, 1209}

$$-\frac{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(b-2\sqrt{a}\,\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{x(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) + (2*Sqrt[c]*x
*Sqrt[a + b*x^2 + c*x^4])/((b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1
/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[
c]))/4])/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(b - 2*Sqrt[a]*
Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{\int \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{-b^2+4ac}$$

$$= -\frac{x(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{\left(2\sqrt{a}\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{b^2-4ac} - \frac{\left(b+2\sqrt{a}\right.\cdots}{\cdots}$$

$$= -\frac{x(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{2\sqrt[4]{a}\,\sqrt[4]{c}\left(\sqrt{a}+\sqrt{\cdots}\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.50, size = 437, normalized size = 1.28

$$\frac{-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(b+2cx^2)+i\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{2\left(b^2-4ac\right)\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 2*c*x^2) + I*(-b + Sqrt[b^2 - 4*
a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2
*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.05, size = 446, normalized size = 1.31

method	result
default	$-\frac{2c\left(-\frac{x^3}{4ac-b^2}-\frac{bx}{2\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}-\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \text{EllipticF}}{4\left(4ac-b^2\right)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}$
elliptic	$-\frac{2c\left(-\frac{x^3}{4ac-b^2}-\frac{bx}{2\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}-\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \text{EllipticF}}{4\left(4ac-b^2\right)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(-1/(4*a*c-b^2)*x^3-1/2*b/(4*a*c-b^2)/c*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)
-1/4*b/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+c/(4*a*c-b^2)*a*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2
*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x**2/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^2/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.992 $\int \frac{1}{(a+bx^2+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=353

$$\frac{x(b^2-2ac+bcx^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\Big(2}{a^{3/4}(b^2-4ac)\sqrt{a+}}$$

```
[Out] x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*x*c^(1/2)*(c*x
^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2))+b*c^(1/4)*(cos(2*arc
tan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE
(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)
+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(-4*a
*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b
*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^4
+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1106, 1211, 1117, 1209}

$$\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*
Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)
) + (b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3
/4)*(b - 2*Sqrt[a]*Sqrt[c])*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x(b^2-2ac+bcx^2)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\int \frac{2ac+bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{x(b^2-2ac+bcx^2)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{c}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\left(b-2\sqrt{a}\sqrt{c}\right)} + \frac{\left(b\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}\cdots}{\sqrt{\cdots}}}{\sqrt{a+bx^2\cdots}}\cdots}{\sqrt{a}\left(b^2-4\cdots\right)}$$

$$= \frac{x(b^2-2ac+bcx^2)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\cdots\right)}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.55, size = 456, normalized size = 1.29

$$-\frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(b^2-2ac+bcx^2)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-b^2+4ac+b\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{4a\left(b^2-4ac\right)\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] -1/4*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2) + I*b*(-
b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.03, size = 481, normalized size = 1.36

method	result
default	$-\frac{2c\left(\frac{b\,x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)x}{2a\left(4ac-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^{2}}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac\cdots}\right.}{a}}}{4\sqrt{\frac{-b+\sqrt{-4ac\cdots}}{a}}}$
elliptic	$-\frac{2c\left(\frac{b\,x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)x}{2a\left(4ac-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^{2}}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac\cdots}\right.}{a}}}{4\sqrt{\frac{-b+\sqrt{-4ac\cdots}}{a}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(1/2/a*b/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+b/c*
x^2+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))-1/2*b*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((a + b*x**2 + c*x**4)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(1/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.993 $\int \frac{1}{x^2(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=428

$$\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x\sqrt{a+bx^2+cx^4}}-\frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)x}+\frac{2\sqrt{c}\left(b^2-3ac\right)x\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2\sqrt[4]{c}\,(b^2 \cdots}{\cdots}$$

```
[Out] (b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)^(1/2)-2*(-3*a*c+b^2)*(
c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/x+2*(-3*a*c+b^2)*x*c^(1/2)*(c*x^4+b*x
^2+a)^(1/2)/a^2/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2))-2*c^(1/4)*(-3*a*c+b^2)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7
/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*
(2*b^2-6*a*c+b*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
1/2)/a^(7/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 428, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1135, 1295, 1211, 1117, 1209}

$$\frac{\sqrt[4]{c}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{c}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{a^2x\left(b^2-4ac\right)}+\frac{2\sqrt{c}\,x(b^2-3ac)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x*Sqrt[a + b*x^2 + c*x^4]) - (2*(b
^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b^2 - 4*a*c)*x) + (2*Sqrt[c]*(b^
2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]
*x^2)) - (2*c^(1/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^
4]) + (c^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(7/4)*(b^2 - 4*a*c)*Sq
rt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^2+cx^4)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a(b^2-4ac)x\sqrt{a+bx^2+cx^4}} - \frac{\int \frac{-2(b^2-3ac)-bcx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{a(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)x\sqrt{a+bx^2+cx^4}} - \frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{a^2(b^2-4ac)x} + \frac{\int \frac{abc+2c(b^2\cdots}{\sqrt{a+bx^2\cdots}}}{a^2(b^2\cdots}$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)x\sqrt{a+bx^2+cx^4}} - \frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{a^2(b^2-4ac)x} - \frac{(2\sqrt{c}(b^2-\cdots}{\cdots}$$

$$= \frac{b^2-2ac+bcx^2}{a(b^2-4ac)x\sqrt{a+bx^2+cx^4}} - \frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{a^2(b^2-4ac)x} + \frac{2\sqrt{c}(b^2-\cdots}{a^2(b^2-\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.86, size = 515, normalized size = 1.20

$$-\frac{2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(-4a^2c+2b^2x^2(b+cx^2)+a(b^2-7bcx^2-6c^2x^4)\right)-i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+i\left(-b^3+4abc+b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/2*(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-4*a^2*c + 2*b^2*x^2*(b + c*x^2) +
 a*(b^2 - 7*b*c*x^2 - 6*c^2*x^4)) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c]
)*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2
*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c
*Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sq
rt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a^2*(b^2 - 4*a*c)*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.08, size = 536, normalized size = 1.25

method	result

default	$-\frac{2c\left(\frac{(2ac-b^2)x^3}{2(4ac-b^2)a^2}+\frac{b(3ac-b^2)x}{2a^2(4ac-b^2)c}\right)}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}-\frac{\sqrt{c\,x^4+b\,x^2+a}}{a^2x}+\frac{\left(-\frac{b}{a^2}+\frac{b(3ac-b^2)}{a^2(4ac-b^2)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right.}{a}}}{}$
elliptic	$-\frac{2c\left(\frac{(2ac-b^2)x^3}{2(4ac-b^2)a^2}+\frac{b(3ac-b^2)x}{2a^2(4ac-b^2)c}\right)}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}-\frac{\sqrt{c\,x^4+b\,x^2+a}}{a^2x}+\frac{\left(-\frac{b}{a^2}+\frac{b(3ac-b^2)}{a^2(4ac-b^2)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right.}{a}}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(1/2*(2*a*c-b^2)/(4*a*c-b^2)/a^2*x^3+1/2*b*(3*a*c-b^2)/a^2/(4*a*c-b^2)
/c*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)-1/a^2*(c*x^4+b*x^2+a)^(1/2)/x+1/4*(-b/a^2
+b*(3*a*c-b^2)/a^2/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(
4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(c*(2*a*c-b^
2)/(4*a*c-b^2)/a^2+c/a^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*
(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.994 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=50

$$-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}+\frac{x\sqrt{bx^2+cx^4}}{3c}$$

```
[Out] -2/3*b*(c*x^4+b*x^2)^(1/2)/c^2/x+1/3*x*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3, 2041, 1602}

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{x\sqrt{bx^2+cx^4}}{3c} - \frac{(2b)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c}$$
$$= -\frac{2b\sqrt{bx^2+cx^4}}{3c^2x} + \frac{x\sqrt{bx^2+cx^4}}{3c}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.68

$$\frac{(-2b+cx^2)\sqrt{x^2(b+cx^2)}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
[Out] ((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

Maple [A]
time = 0.12, size = 37, normalized size = 0.74

method	result	size
trager	$-\frac{(-cx^2+2b)\sqrt{cx^4+bx^2}}{3c^2x}$	32
gosper	$-\frac{(cx^2+b)(-cx^2+2b)x}{3c^2\sqrt{cx^4+bx^2}}$	37
default	$-\frac{(cx^2+b)(-cx^2+2b)x}{3c^2\sqrt{cx^4+bx^2}}$	37
risch	$-\frac{x(cx^2+b)(-cx^2+2b)}{3\sqrt{x^2(cx^2+b)}\,c^2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(c*x^2+b)*(-c*x^2+2*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.30, size = 34, normalized size = 0.68

$$\frac{c^2x^4-bcx^2-2\,b^2}{3\,\sqrt{cx^2+b}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

Fricas [A]
time = 0.32, size = 30, normalized size = 0.60

$$\frac{\sqrt{cx^4+bx^2}\,(cx^2-2\,b)}{3\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 3.76, size = 48, normalized size = 0.96

$$\frac{2\,b^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,c^2}+\frac{(cx^2+b)^{\frac{3}{2}}}{3\,c^2\mathrm{sgn}\,(x)}-\frac{\sqrt{cx^2+b}\,b}{c^2\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*b^(3/2)*sgn(x)/c^2 + 1/3*(c*x^2 + b)^(3/2)/(c^2*sgn(x)) - sqrt(c*x^2 +
b)*b/(c^2*sgn(x))
```

Mupad [B]
time = 4.60, size = 33, normalized size = 0.66

$$-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{2\,b}{3\,c^2}-\frac{x^2}{3\,c}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*((2*b)/(3*c^2) - x^2/(3*c)))/x
```

3.995 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] -1/2*b*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/2*(c*x^4+b*x^2)^(
1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3, 2043, 654, 634, 212}

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/
(2*c^(3/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right) \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{4c} \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c} \\
&= \frac{\sqrt{bx^2+cx^4}}{2c} - \frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 1.28

$$\frac{x\left(\sqrt{c}\,x(b+cx^2)+b\sqrt{b+cx^2}\,\log\left(-\sqrt{c}\,x+\sqrt{b+cx^2}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2) + b*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c
*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.13, size = 64, normalized size = 1.10

method	result	size
default	$\frac{x\sqrt{c\,x^{2}+b}\,\left(x\sqrt{c\,x^{2}+b}\,c^{\frac{3}{2}}-b\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)c\right)}{2\sqrt{c\,x^{4}+b\,x^{2}}\,c^{\frac{5}{2}}}$	64
risch	$\frac{x^{2}\left(c\,x^{2}+b\right)}{2c\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+b}\right)x\sqrt{c\,x^{2}+b}}{2c^{\frac{3}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(c*x^2+b)^(1/2)*(x*(c*x^2+b)^(1/2)*c^(3/2)-b*ln(x*c^(1/2)+(c*x^2+b)^(
1/2))*c)/(c*x^4+b*x^2)^(1/2)/c^(5/2)
```

Maxima [A]
time = 0.28, size = 52, normalized size = 0.90

$$-\frac{b\log\left(2\,cx^{2}+b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^{4}+bx^{2}}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 1/2*sqrt(
c*x^4 + b*x^2)/c
```

Fricas [A]
time = 0.39, size = 114, normalized size = 1.97

$$\left[\frac{b\sqrt{c}\,\log\left(-2\,cx^{2}-b+2\,\sqrt{cx^{4}+bx^{2}}\,\sqrt{c}\right)+2\,\sqrt{cx^{4}+bx^{2}}\,c}{4\,c^{2}},\ \frac{b\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-c}}{cx^{2}+b}\right)+\sqrt{cx^{4}+bx^{2}}\,c}{2\,c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt(
c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/
(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}}{\sqrt{x^{2}\left(b+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 4.11, size = 59, normalized size = 1.02

$$-\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^2+b}\,x}{2\,c\operatorname{sgn}\left(x\right)}+\frac{b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\,\right|\right)}{2\,c^{\frac{3}{2}}\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b*log(abs(b))*sgn(x)/c^(3/2) + 1/2*sqrt(c*x^2 + b)*x/(c*sgn(x)) + 1/2*
b*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(3/2)*sgn(x))
```

Mupad [B]
time = 4.61, size = 53, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{2\,c}-\frac{b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4
)^(1/2)))/(4*c^(3/2))
```

3.996 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

```
[Out] (c*x^4+b*x^2)^(1/2)/c/x
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3, 1602}

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx\\ &= \frac{\sqrt{bx^2+cx^4}}{cx}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\sqrt{x^2 (b + cx^2)}}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[x^2*(b + c*x^2)]/(c*x)
```

Maple [A]
time = 0.14, size = 26, normalized size = 1.18

method	result	size
trager	$\frac{\sqrt{c\,x^4+b\,x^2}}{cx}$	21
gosper	$\frac{(c\,x^2+b)x}{c\sqrt{c\,x^4+b\,x^2}}$	26
default	$\frac{(c\,x^2+b)x}{c\sqrt{c\,x^4+b\,x^2}}$	26
risch	$\frac{x(c\,x^2+b)}{\sqrt{x^2\,(c\,x^2+b)}\,c}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b)/c*x/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.30, size = 13, normalized size = 0.59

$$\frac{\sqrt{cx^2 + b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2 + b)/c
```

Fricas [A]
time = 0.39, size = 20, normalized size = 0.91

$$\frac{\sqrt{cx^4 + bx^2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 4.48, size = 28, normalized size = 1.27

$$-\frac{\sqrt{b}\,\mathrm{sgn}(x)}{c}+\frac{\sqrt{cx^2+b}}{c\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b)*sgn(x)/c + sqrt(c*x^2 + b)/(c*sgn(x))
```

Mupad [B]
time = 4.37, size = 20, normalized size = 0.91

$$\frac{\sqrt{c\,x^4+b\,x^2}}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (b*x^2 + c*x^4)^(1/2)/(c*x)
```

3.997 $\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

[Out] arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3, 2038, 634, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]

[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
```

```
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{x}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.68

$$\frac{x\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\,\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]*Sqrt[x^2*
(b + c*x^2)])
```

Maple [A]
time = 0.13, size = 44, normalized size = 1.42

method	result	size
default	$\frac{x\sqrt{c\,x^2+b}\,\ln\left(x\sqrt{c}+\sqrt{c\,x^2+b}\right)}{\sqrt{c\,x^4+b\,x^2}\,\sqrt{c}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)*ln(x*c^(1/2)+(c*x^2+b)^(1/2))/c^(1/
2)
```

Maxima [A]
time = 0.31, size = 32, normalized size = 1.03

$$\frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c)
```

Fricas [A]
time = 0.35, size = 74, normalized size = 2.39

$$\left[\frac{\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\,\sqrt{c}\right)}{2\,\sqrt{c}},-\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-c}}{cx^2+b}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c), -sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

Giac [A]
time = 4.78, size = 38, normalized size = 1.23

$$\frac{\log\left(|b|\right)\mathrm{sgn}(x)}{2\,\sqrt{c}}-\frac{\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+b}\,\right|\right)}{\sqrt{c}\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b))*sgn(x)/sqrt(c) - log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(sq
rt(c)*sgn(x))
```

Mupad [B]
time = 4.56, size = 33, normalized size = 1.06

$$\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(1/2))
```

3.998 $\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] -arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3, 2033, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeqQ[n, 2]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Mathematica [**A**]
time = 0.01, size = 52, normalized size = 1.73

$$-\frac{x\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] -((x*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[x^2*(b
 + c*x^2)]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.
time = 0.13, size = 50, normalized size = 1.67

method	result	size
default	$-\frac{x\sqrt{c\,x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)}{\sqrt{c\,x^4+b\,x^2}\,\sqrt{b}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1
/2)+b)/x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(c*x^4 + b*x^2), x)
```

Fricas [A]
time = 0.34, size = 80, normalized size = 2.67

$$\left[\frac{\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)/sqrt(b), sqr
t(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/sqrt(b*x**2 + c*x**4), x)
```

Giac [A]
time = 4.19, size = 46, normalized size = 1.53

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] -arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(c*x^2 + b)/sqrt(-b)
)/(sqrt(-b)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2 + c*x^4)^(1/2),x)
[Out] int(1/(b*x^2 + c*x^4)^(1/2), x)
```

3.999 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

```
[Out] -(c*x^4+b*x^2)^(1/2)/b/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3, 2039}

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{bx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

Maple [A]
time = 0.13, size = 26, normalized size = 1.13

method	result	size
trager	$-\frac{\sqrt{c\,x^4+b\,x^2}}{b\,x^2}$	22
gosper	$-\frac{c\,x^2+b}{b\sqrt{c\,x^4+b\,x^2}}$	26
default	$-\frac{c\,x^2+b}{b\sqrt{c\,x^4+b\,x^2}}$	26
risch	$-\frac{c\,x^2+b}{\sqrt{x^2\,(c\,x^2+b)}\;b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+b)/b/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Fricas [A]
time = 0.35, size = 21, normalized size = 0.91

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 4.14, size = 34, normalized size = 1.48

$$\frac{2\sqrt{c}}{\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*sgn(x))
```

Mupad [B]
time = 4.31, size = 21, normalized size = 0.91

$$-\frac{\sqrt{c\,x^4+b\,x^2}}{b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -(b*x^2 + c*x^4)^(1/2)/(b*x^2)
```

3.1000 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}$$

```
[Out] 1/2*c*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/2*(c*x^4+b*x^2)^(1/2
)/b/x^3
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3, 2050, 2033, 212}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^
4]])/(2*b^(3/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}-\frac{c\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b}\\ &= -\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 76, normalized size = 1.29

$$\frac{-\sqrt{b}\,(b+cx^2)+cx^2\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{3/2}x\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
[Out] (-(Sqrt[b]*(b + c*x^2)) + c*x^2*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqr
t[b]])/(2*b^(3/2)*x*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.13, size = 73, normalized size = 1.24

method	result	size
default	$-\frac{\sqrt{c\,x^2+b}\,\left(-c\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b\,x^2+\sqrt{c\,x^2+b}\,b^{\frac{3}{2}}\right)}{2x\sqrt{c\,x^4+b\,x^2}\,b^{\frac{5}{2}}}$	73

risch	$-\frac{c\,x^2+b}{2bx\sqrt{x^2\,(c\,x^2+b)}}+\frac{c\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)x\sqrt{c\,x^2+b}}{2b^{\frac{3}{2}}\sqrt{x^2\,(c\,x^2+b)}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x*(c*x^2+b)^(1/2)*(-c*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*x^2+(c*x^2
+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

Fricas [A]
time = 0.37, size = 133, normalized size = 2.25

$$\left[\frac{\sqrt{b}\,cx^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\,\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\,b}{4\,b^2x^3},-\frac{\sqrt{-b}\,cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\,\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}\,b}{2\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 4.89, size = 55, normalized size = 0.93

$$-\frac{\frac{c^{2}\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{\sqrt{cx^{2}+b}\,c}{bx^{2}}}{2\,c\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(c^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(c*x^2 + b)*c
/(b*x^2))/(c*sgn(x))
```

Mupad [B]
time = 4.64, size = 76, normalized size = 1.29

$$-\frac{\left(\frac{\sqrt{c}\,x^{2}\,\sqrt{c+\frac{b}{x^{2}}}}{2\,b}+\frac{c^{3/2}\,x^{3}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,1\mathrm{i}}{2\,b^{3/2}}\right)\,\sqrt{\frac{b}{c\,x^{2}}+1}}{x\,\sqrt{c\,x^{4}+b\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((((c^(1/2)*x^2*(c + b/x^2)^(1/2))/(2*b) + (c^(3/2)*x^3*asin((b^(1/2)*1i)/(
c^(1/2)*x))*1i)/(2*b^(3/2)))*(b/(c*x^2) + 1)^(1/2))/(x*(b*x^2 + c*x^4)^(1/2
))
```

3.1001 $\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{bx^2+cx^4}}{3bx^4}+\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

```
[Out] -1/3*(c*x^4+b*x^2)^(1/2)/b/x^4+2/3*c*(c*x^4+b*x^2)^(1/2)/b^2/x^2
```

Rubi [A]

time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3, 2041, 2039}

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -1/3*Sqrt[b*x^2 + c*x^4]/(b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{\sqrt{bx^2+cx^4}}{3bx^4} - \frac{(2c)\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx}{3b}$$
$$= -\frac{\sqrt{bx^2+cx^4}}{3bx^4} + \frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-b+2cx^2\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

Maple [A]
time = 0.14, size = 37, normalized size = 0.71

method	result	size
trager	$-\frac{\left(-2c\,x^2+b\right)\sqrt{c\,x^4+b\,x^2}}{3b^2x^4}$	30
gosper	$-\frac{\left(c\,x^2+b\right)\left(-2c\,x^2+b\right)}{3x^2b^2\sqrt{c\,x^4+b\,x^2}}$	37
default	$-\frac{\left(c\,x^2+b\right)\left(-2c\,x^2+b\right)}{3x^2b^2\sqrt{c\,x^4+b\,x^2}}$	37
risch	$-\frac{\left(c\,x^2+b\right)\left(-2c\,x^2+b\right)}{3x^2\sqrt{x^2\left(c\,x^2+b\right)}\,b^2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(c*x^2+b)*(-2*c*x^2+b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [A]
time = 0.31, size = 44, normalized size = 0.85

$$\frac{2\sqrt{cx^4+bx^2}\,c}{3\,b^2x^2} - \frac{\sqrt{cx^4+bx^2}}{3\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/(b*x^4)
```

Fricas [A]
time = 0.36, size = 31, normalized size = 0.60

$$\frac{\sqrt{cx^4+bx^2}\,(2\,cx^2-b)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 4.55, size = 59, normalized size = 1.13

$$\frac{4\left(3\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)c^{\frac{3}{2}}}{3\left(\left(\sqrt{c}\,x-\sqrt{cx^2+b}\right)^2-b\right)^3\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 4/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*c^(3/2)/(((sqrt(c)*x - sqrt(c*x
^2 + b))^2 - b)^3*sgn(x))
```

Mupad [B]
time = 4.47, size = 29, normalized size = 0.56

$$-\frac{(b-2\,c\,x^2)\,\sqrt{c\,x^4+b\,x^2}}{3\,b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
[Out] -((b - 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b^2*x^4)
```

3.1002 $\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}$$

```
[Out] -3/8*c^2*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/4*(c*x^4+b*x^2)^(
1/2)/b/x^5+3/8*c*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3, 2050, 2033, 212}

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -1/4*Sqrt[b*x^2 + c*x^4]/(b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2*x^3) -
(3*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/2))
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx &= \int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx \\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(3c)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b} \\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5} + \frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3} + \frac{(3c^2)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2} \\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5} + \frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3} - \frac{(3c^2)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \right.}{8b^2} \\
&= -\frac{\sqrt{bx^2+cx^4}}{4bx^5} + \frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3} - \frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 91, normalized size = 1.05

$$
\frac{\sqrt{b}\left(-2b^2+bcx^2+3c^2x^4\right)-3c^2x^4\sqrt{b+cx^2}\,\tanh^{-1}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{8b^{5/2}x^3\sqrt{x^2\left(b+cx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[b]*(-2*b^2 + b*c*x^2 + 3*c^2*x^4) - 3*c^2*x^4*Sqrt[b + c*x^2]*ArcTanh
[Sqrt[b + c*x^2]/Sqrt[b]])/(8*b^(5/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A]
time = 0.14, size = 94, normalized size = 1.08

method	result	size

default	$-\frac{\sqrt{c\,x^{2}+b}\,\left(3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)b\,c^{2}x^{4}-3b^{\frac{3}{2}}\sqrt{c\,x^{2}+b}\,c\,x^{2}+2\sqrt{c\,x^{2}+b}\,b^{\frac{5}{2}}\right)}{8x^{3}\sqrt{c\,x^{4}+b\,x^{2}}\,b^{\frac{7}{2}}}$	94
risch	$-\frac{(c\,x^{2}+b)\,(-3c\,x^{2}+2b)}{8b^{2}x^{3}\sqrt{x^{2}\,(c\,x^{2}+b)}}-\frac{3c^{2}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)x\sqrt{c\,x^{2}+b}}{8b^{\frac{5}{2}}\sqrt{x^{2}\,(c\,x^{2}+b)}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/8/x^3*(c*x^2+b)^(1/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*c^2*x^4-3
*b^(3/2)*(c*x^2+b)^(1/2)*c*x^2+2*(c*x^2+b)^(1/2)*b^(5/2))/(c*x^4+b*x^2)^(1/
2)/b^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

Fricas [A]
time = 0.37, size = 163, normalized size = 1.87

$$\left[\frac{3\sqrt{b}\,c^{2}x^{5}\log\left(-\frac{cx^{3}+2\,bx-2\sqrt{cx^{4}+bx^{2}}\,\sqrt{b}}{x^{3}}\right)+2\sqrt{cx^{4}+bx^{2}}\,(3\,bcx^{2}-2\,b^{2})}{16\,b^{3}x^{5}},\ \frac{3\sqrt{-b}\,c^{2}x^{5}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\,\sqrt{-b}}{cx^{3}+bx}\right)+\sqrt{cx^{4}+bx^{2}}\,(3\,bcx^{2}-2\,b^{2})}{8\,b^{3}x^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b
))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5), 1/8*(3*sqrt
(-b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^
4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{4}\sqrt{x^{2}\,(b+cx^{2})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

Giac [A]
time = 4.51, size = 79, normalized size = 0.91

$$\frac{\frac{3\,c^3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^2}+\frac{3\,(cx^2+b)^{\frac{3}{2}}c^3-5\,\sqrt{cx^2+b}\,bc^3}{b^2c^2x^4}}{8\,c\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(c*x^2 + b)
^(3/2)*c^3 - 5*sqrt(c*x^2 + b)*b*c^3)/(b^2*c^2*x^4))/(c*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

3.1003 $\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{x\sqrt{a+cx^4}}{3c}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

```
[Out] 1/3*x*(c*x^4+a)^(1/2)/c-1/6*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)
^(1/2)/c^(5/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4, 327, 226}

$$\frac{x\sqrt{a+cx^4}}{3c}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/(3*c) - (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(6*c^(5/4)*Sqrt[a + c*x^4])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{x\sqrt{a+cx^4}}{3c} - \frac{a\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3c}$$
$$= \frac{x\sqrt{a+cx^4}}{3c} - \frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan\right.}{6c^{5/4}\sqrt{a+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 62, normalized size = 0.57

$$\frac{x\left(a+cx^4-a\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)\right)}{3c\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
[Out] (x*(a + c*x^4 - a*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c
*x^4)/a)]))/(3*c*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 91, normalized size = 0.84

method	result	size
default	$\frac{x\sqrt{c\,x^4+a}}{3c}-\frac{a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	91

risch	$\frac{x\sqrt{c\,x^4+a}}{3c}-\frac{a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	91
elliptic	$\frac{x\sqrt{c\,x^4+a}}{3c}-\frac{a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(c*x^4+a)^(1/2)/c-1/3*a/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + a), x)
```

Fricas [A]
time = 0.09, size = 44, normalized size = 0.41

$$-\frac{\sqrt{c}\,\left(-\frac{a}{c}\right)^{\frac{3}{4}}\,\mathrm{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x},-1\right)-\sqrt{cx^4+a}\,x}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(sqrt(c)*(-a/c)^(3/4)*ellipticF((-a/c)^(1/4)/x, -1) - sqrt(c*x^4 + a)*
x)/c
```

Sympy [C] Result contains complex when optimal does not.
time = 0.39, size = 37, normalized size = 0.34

$$\frac{x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+a)**(1/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + c*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a + c*x^4)^(1/2), x)
```

3.1004 $\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{a+cx^4}}{2c}$$

```
[Out] 1/2*(c*x^4+a)^(1/2)/c
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {4, 267}

$$\frac{\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
[Out] Sqrt[a + c*x^4]/(2*c)
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{x^3}{\sqrt{a+cx^4}}\,dx\\ &= \frac{\sqrt{a+cx^4}}{2c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + c*x^4]/(2*c)
```

Maple [A]
time = 0.14, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
derivativedivides	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
default	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
trager	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
risch	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
elliptic	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^4+a)^(1/2)/c
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.78

$$\frac{\sqrt{cx^4 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^4 + a)/c
```

Fricas [A]
time = 0.35, size = 14, normalized size = 0.78

$$\frac{\sqrt{cx^4 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^4 + a)/c
```

Sympy [A]
time = 0.20, size = 22, normalized size = 1.22

$$\begin{cases} \frac{\sqrt{a+cx^4}}{2c} & \text{for } c \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+a)**(1/2),x)
[Out] Piecewise((sqrt(a + c*x**4)/(2*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True))
```

Giac [A]
time = 3.95, size = 14, normalized size = 0.78

$$\frac{\sqrt{cx^4+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(c*x^4 + a)/c
```

Mupad [B]
time = 4.66, size = 14, normalized size = 0.78

$$\frac{\sqrt{c\,x^4+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + c*x^4)^(1/2),x)
[Out] (a + c*x^4)^(1/2)/(2*c)
```

3.1005 $\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=210

$$\frac{x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right.}{}$$

```
[Out] x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*(cos(2*arctan(c^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*ar
ctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*arcta
n(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+
a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4, 311, 226, 1210}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{x^2}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{\sqrt{a}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}} - \frac{\sqrt{a}\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}$$

$$= \frac{x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{c^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 51, normalized size = 0.24

$$\frac{x^3\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(3
*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 97, normalized size = 0.46

method	result	size
default	$\frac{i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$	97
elliptic	$\frac{i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^
(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.36, size = 37, normalized size = 0.18

$$\frac{x^{3}\Gamma\left(\frac{3}{4}\right)\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^{4}e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+a)**(1/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + c*x^4)^(1/2),x)
```

```
[Out] int(x^2/(a + c*x^4)^(1/2), x)
```

3.1006 $\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] 1/2*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4, 281, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{x}{\sqrt{a+cx^4}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{a+cx^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}\end{aligned}$$

Mathematica [**A**]
time = 0.10, size = 30, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{c}\,x^2}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
[Out] ArcTanh[Sqrt[a + c*x^4]/(Sqrt[c]*x^2)]/(2*Sqrt[c])
```

Maple [**A**]
time = 0.13, size = 24, normalized size = 0.80

method	result	size
default	$\frac{\ln\left(x^2\sqrt{c}+\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}$	24
elliptic	$\frac{\ln\left(x^2\sqrt{c}+\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

time = 0.49, size = 45, normalized size = 1.50

$$-\frac{\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x^2))/
sqrt(c)
```

Fricas [A]
time = 0.36, size = 63, normalized size = 2.10

$$\left[\frac{\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\,\sqrt{c}\,x^2-a\right)}{4\,\sqrt{c}},-\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2}{\sqrt{cx^4+a}}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a)/sqrt(c), -1/2*sqrt(-
c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + a))/c]
```

Sympy [A]
time = 0.44, size = 20, normalized size = 0.67

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+a)**(1/2),x)
```

```
[Out] asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))
```

Giac [A]
time = 3.95, size = 25, normalized size = 0.83

$$-\frac{\log\left(\left|-\sqrt{c}\,x^2+\sqrt{cx^4+a}\,\right|\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + c*x^4)^(1/2),x)
```

```
[Out] int(x/(a + c*x^4)^(1/2), x)
```

3.1007 $\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

```
[Out] 1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^4+a)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4, 226}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]
)
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\int \frac{1}{\sqrt{a+cx^4}}\,dx$$
$$=\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 74, normalized size = 0.84

$$-\frac{i\sqrt{1+\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
 -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 70, normalized size = 0.80

method	result	size
default	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	70
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

Fricas [A]
time = 0.08, size = 28, normalized size = 0.32

$$-\frac{\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}},-1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(a)*(-c/a)^(3/4)*ellipticF(x*(-c/a)^(1/4), -1)/c
```

Sympy [C] Result contains complex when optimal does not.
time = 0.35, size = 36, normalized size = 0.41

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^{4}e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**(1/2),x)
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

Mupad [B]
time = 4.35, size = 37, normalized size = 0.42

$$\frac{x\sqrt{\frac{c\,x^4}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{c\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^(1/2),x)
```

```
[Out] (x*((c*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(c*x^4)/a))/(a + c*x^4
)^(1/2)
```

3.1008 $\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=27

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh((c*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4, 272, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/Sqrt[a]
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{1}{x\sqrt{a+cx^4}}\,dx\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^4\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^4}\right)}{2c}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
[Out] -1/2*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/Sqrt[a]
```

Maple [A]
time = 0.12, size = 29, normalized size = 1.07

method	result	size
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{2\sqrt{a}}$	29
elliptic	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{2\sqrt{a}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^4+a)^(1/2))/x^2)

Maxima [A]
time = 0.49, size = 37, normalized size = 1.37

$$\frac{\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] 1/4*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))/sqrt(a)

Fricas [A]
time = 0.36, size = 63, normalized size = 2.33

$$\left[\frac{\log\left(\frac{cx^4-2\sqrt{cx^4+a}\sqrt{a}+2a}{x^4}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-a}}{a}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="fricas")

[Out] [1/4*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(sqrt(c*x^4 + a)*sqrt(-a)/a)/a]

Sympy [A]
time = 0.45, size = 22, normalized size = 0.81

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}\,x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**4+a)**(1/2),x)

[Out] -asinh(sqrt(a)/(sqrt(c)*x**2))/(2*sqrt(a))

Giac [A]
time = 4.69, size = 23, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] 1/2*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B]
time = 4.55, size = 19, normalized size = 0.70

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)^(1/2)),x)
[Out] -atanh((a + c*x^4)^(1/2)/a^(1/2))/(2*a^(1/2))
```

3.1009 $\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=232

$$-\frac{\sqrt{a+cx^4}}{ax}+\frac{\sqrt{c}\,x\sqrt{a+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+cx^4}}+\frac{\sqrt[4]{c}\,(\sqrt{}$$

```
[Out] -(c*x^4+a)^(1/2)/a/x+x*c^(1/2)*(c*x^4+a)^(1/2)/a/(a^(1/2)+x^2*c^(1/2))-c^(1
/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+a)^(1/2)+1
/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+a)^
(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4, 331, 311, 226, 1210}

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{ax}+\frac{\sqrt{c}\,x\sqrt{a+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[a + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + c*x^4])/(a*(Sqrt[a] + Sqrt[c
]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a
+ c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*Sqr
t[a + c*x^4])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{1}{x^2\sqrt{a+cx^4}}\,dx \\
&= -\frac{\sqrt{a+cx^4}}{ax}+\frac{c\int \frac{x^2}{\sqrt{a+cx^4}}\,dx}{a} \\
&= -\frac{\sqrt{a+cx^4}}{ax}+\frac{\sqrt{c}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{a}}-\frac{\sqrt{c}\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{a}} \\
&= -\frac{\sqrt{a+cx^4}}{ax}+\frac{\sqrt{c}\,x\sqrt{a+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\cdots}}{\cdots}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 49, normalized size = 0.21

$$-\frac{\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{cx^4}{a}\right)}{x\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -((Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((c*x^4)/a)])/(x*
Sqrt[a + c*x^4]))
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 115, normalized size = 0.50

method	result
default	$-\frac{\sqrt{c\,x^4+a}}{ax}+\frac{i\sqrt{c}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$
risch	$-\frac{\sqrt{c\,x^4+a}}{ax}+\frac{i\sqrt{c}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$
elliptic	$-\frac{\sqrt{c\,x^4+a}}{ax}+\frac{i\sqrt{c}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^4+a)^(1/2)/a/x+I*c^(1/2)/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(Elli
pticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),
I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 0.40, size = 39, normalized size = 0.17

$$\frac{\Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| {\frac{c x^{4} e^{i \pi}}{a}} \right)}{4 \sqrt{a} x \Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+a)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*x*gamma(3/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^2), x)
```

Mupad [**B**]
time = 4.56, size = 40, normalized size = 0.17

$$-\frac{\sqrt{\frac{a}{c\,x^4}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{a}{c\,x^4}\right)}{3\,x\,\sqrt{c\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^4)^(1/2)),x)
```

```
[Out] -((a/(c*x^4) + 1)^(1/2)*hypergeom([1/2, 3/4], 7/4, -a/(c*x^4)))/(3*x*(a + c
*x^4)^(1/2))
```

3.1010 $\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

```
[Out] -1/2*(c*x^4+a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {4, 270}

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + c*x^4]/(a*x^2)
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx &= \int \frac{1}{x^3\sqrt{a+cx^4}}\,dx\\ &= -\frac{\sqrt{a+cx^4}}{2ax^2}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 21, normalized size = 1.00

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + c*x^4]/(a*x^2)
```

Maple [A]
time = 0.13, size = 18, normalized size = 0.86

method	result	size
gosper	$-\frac{\sqrt{c\,x^{4}+a}}{2 a\,x^{2}}$	18
default	$-\frac{\sqrt{c\,x^{4}+a}}{2 a\,x^{2}}$	18
trager	$-\frac{\sqrt{c\,x^{4}+a}}{2 a\,x^{2}}$	18
risch	$-\frac{\sqrt{c\,x^{4}+a}}{2 a\,x^{2}}$	18
elliptic	$-\frac{\sqrt{c\,x^{4}+a}}{2 a\,x^{2}}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^4+a)^(1/2)/a/x^2
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.81

$$-\frac{\sqrt{cx^4+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^4 + a)/(a*x^2)
```

Fricas [A]
time = 0.34, size = 17, normalized size = 0.81

$$-\frac{\sqrt{cx^4+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^4 + a)/(a*x^2)
```

Sympy [A]
time = 0.33, size = 20, normalized size = 0.95

$$-\frac{\sqrt{c}\,\sqrt{\frac{a}{cx^4}+1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4+a)**(1/2),x)
[Out] -sqrt(c)*sqrt(a/(c*x**4) + 1)/(2*a)
```

Giac [A]
time = 4.54, size = 31, normalized size = 1.48

$$\frac{\sqrt{c}}{\left(\sqrt{c}\,x^2-\sqrt{cx^4+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] sqrt(c)/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)
```

Mupad [B]
time = 4.51, size = 17, normalized size = 0.81

$$-\frac{\sqrt{c\,x^4+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^4)^(1/2)),x)
[Out] -(a + c*x^4)^(1/2)/(2*a*x^2)
```

3.1011 $\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{a+cx^4}}{3ax^3}-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}$$

```
[Out] -1/3*(c*x^4+a)^(1/2)/a/x^3-1/6*c^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/a^(5/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4, 331, 226}

$$-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -1/3*Sqrt[a + c*x^4]/(a*x^3) - (c^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(6*a^(5/4)*Sqrt[a + c*x^4])
```

Rule 4

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{1}{x^4\sqrt{a+cx^4}}\,dx$$
$$= -\frac{\sqrt{a+cx^4}}{3ax^3} - \frac{c\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3a}$$
$$= -\frac{\sqrt{a+cx^4}}{3ax^3} - \frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\Big(2\mathrm{t}}{6a^{5/4}\sqrt{a+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 51, normalized size = 0.46

$$-\frac{\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{cx^4}{a}\right)}{3x^3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -1/3*(Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((c*x^4)/a)])/
(x^3*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 93, normalized size = 0.85

method	result	size
default	$-\frac{\sqrt{c\,x^4+a}}{3a\,x^3}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	93

risch	$-\frac{\sqrt{c\,x^4+a}}{3a\,x^3}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	93
elliptic	$-\frac{\sqrt{c\,x^4+a}}{3a\,x^3}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^4+a)^(1/2)/a/x^3-1/3*c/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^4), x)
```

Fricas [A]
time = 0.08, size = 47, normalized size = 0.43

$$\frac{\sqrt{a}\,x^3\left(-\frac{c}{a}\right)^{\frac{3}{4}}\,\mathrm{ellipticF}\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}},-1\right)-\sqrt{cx^4+a}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(a)*x^3*(-c/a)^(3/4)*ellipticF(x*(-c/a)^(1/4), -1) - sqrt(c*x^4 +
a))/(a*x^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 41, normalized size = 0.37

$$\frac{\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+a)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*x**3*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + c*x^4)^(1/2)), x)
```

3.1012 $\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=73

$$-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

```
[Out] 3/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-3/8*a*x*(b*x^2+a)^(1/2)/
b^2+1/4*x^3*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 327, 223, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) + (3*a^2*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x^4}{\sqrt{a+bx^2}}\,dx \\
&= \frac{x^3\sqrt{a+bx^2}}{4b} - \frac{(3a)\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b} \\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{(3a^2)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2} \\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{(3a^2)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, -\right.}{8b^2} \\
&= -\frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 63, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\,(-3ax+2bx^3)}{8b^2} - \frac{3a^2\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a*x + 2*b*x^3))/(8*b^2) - (3*a^2*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/(8*b^(5/2))
```

Maple [A]
time = 0.14, size = 63, normalized size = 0.86

method	result	size
risch	$-\frac{x(-2b\,x^2+3a)\sqrt{b\,x^2+a}}{8b^2}+\frac{3a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}$	51

default	$\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^3*(b*x^2+a)^(1/2)/b-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.29, size = 51, normalized size = 0.70

$$\frac{\sqrt{bx^2+a}\,x^3}{4\,b}-\frac{3\,\sqrt{bx^2+a}\,ax}{8\,b^2}+\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*x^3/b - 3/8*sqrt(b*x^2 + a)*a*x/b^2 + 3/8*a^2*arcsinh(b
*x/sqrt(a*b))/b^(5/2)
```

Fricas [A]
time = 0.39, size = 124, normalized size = 1.70

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(2\,b^2x^3-3\,abx)\sqrt{bx^2+a}}{16\,b^3},-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(2\,b^2x^3-3\,abx)\sqrt{bx^2+a}}{8\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A]
time = 2.46, size = 95, normalized size = 1.30

$$-\frac{3a^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*x**3/(8*b*sqrt(1 + b*x*
*2/a)) + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + x**5/(4*sqrt(a)*sqr
t(1 + b*x**2/a))
```

Giac [**A**]
time = 3.17, size = 54, normalized size = 0.74

$$\frac{1}{8}\sqrt{bx^2+a}\,x\left(\frac{2\,x^2}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(5/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^4/(a + b*x^2)^(1/2), x)
```

3.1013 $\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=36

$$-\frac{a\sqrt{a+bx^2}}{b^2}+\frac{\left(a+bx^2\right)^{3/2}}{3b^2}$$

```
[Out] 1/3*(b*x^2+a)^(3/2)/b^2-a*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {5, 272, 45}

$$\frac{\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x^3}{\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, x^2\right) \\
&= -\frac{a\sqrt{a+bx^2}}{b^2}+\frac{(a+bx^2)^{3/2}}{3b^2}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 27, normalized size = 0.75

$$
\frac{(-2a+bx^2)\sqrt{a+bx^2}}{3b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

Maple [**A**]
time = 0.14, size = 34, normalized size = 0.94

method	result	size
gosper	$-\frac{\sqrt{b\,x^2+a}\,(-b\,x^2+2a)}{3b^2}$	25
trager	$-\frac{\sqrt{b\,x^2+a}\,(-b\,x^2+2a)}{3b^2}$	25
risch	$-\frac{\sqrt{b\,x^2+a}\,(-b\,x^2+2a)}{3b^2}$	25
default	$\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a*(b*x^2+a)^(1/2)/b^2
```

Maxima [**A**]
time = 0.28, size = 33, normalized size = 0.92

$$
\frac{\sqrt{bx^2+a}\,x^2}{3\,b}-\frac{2\,\sqrt{bx^2+a}\,a}{3\,b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*x^2/b - 2/3*sqrt(b*x^2 + a)*a/b^2
```

Fricas [A]
time = 0.35, size = 23, normalized size = 0.64

$$\frac{\sqrt{bx^2+a}\,(bx^2-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*(b*x^2 - 2*a)/b^2
```

Sympy [A]
time = 0.21, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne
(b, 0)), (x**4/(4*sqrt(a)), True))
```

Giac [A]
time = 4.61, size = 30, normalized size = 0.83

$$\frac{(bx^2+a)^{\frac{3}{2}}}{3\,b^2}-\frac{\sqrt{bx^2+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x^2 + a)^(3/2)/b^2 - sqrt(b*x^2 + a)*a/b^2
```

Mupad [B]
time = 4.60, size = 24, normalized size = 0.67

$$-\frac{\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^2)^(1/2),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

3.1014 $\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=49

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*x*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 327, 223, 212}

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(
3/2))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x^2}{\sqrt{a+bx^2}}\,dx\\ &= \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\ &= \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\ &= \frac{x\sqrt{a+bx^2}}{2b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.04

$$\frac{x\sqrt{a+bx^2}}{2b} + \frac{a\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] (x*Sqrt[a + b*x^2])/(2*b) + (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3
/2))
```

Maple [A]
time = 0.14, size = 39, normalized size = 0.80

method	result	size
default	$\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$	39
risch	$\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.63

$$\frac{\sqrt{bx^2+a}\,x}{2\,b}-\frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b - 1/2*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Fricas [A]
time = 0.36, size = 93, normalized size = 1.90

$$\left[\frac{2\,\sqrt{bx^2+a}\,bx+a\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b^2},\frac{\sqrt{bx^2+a}\,bx+a\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*x + a*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/b^2]
```

Sympy [A]
time = 1.12, size = 42, normalized size = 0.86

$$\frac{\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)
)
```

Giac [A]
time = 4.66, size = 40, normalized size = 0.82

$$\frac{\sqrt{bx^2+a}\,x}{2\,b}+\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3
/2)
```

Mupad [B]
time = 4.64, size = 56, normalized size = 1.14

$$\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{if } b=0 \\ \frac{x\sqrt{b\,x^2+a}}{2\,b}-\frac{a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, x^3/(3*a^(1/2)), b ~= 0, (x*(a + b*x^2)^(1/2))/(2*b) - (a
*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)))
```

3.1015 $\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt{a+bx^2}}{b}$$

```
[Out] (b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {5, 267}

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] Sqrt[a + b*x^2]/b
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x}{\sqrt{a+bx^2}}\,dx\\ &= \frac{\sqrt{a+bx^2}}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] Sqrt[a + b*x^2]/b
```

Maple [A]
time = 0.13, size = 14, normalized size = 0.93

method	result	size
gosper	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
derivativedivides	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
default	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
trager	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
risch	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)/b
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)/b
```

Fricas [A]
time = 0.36, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2 + a)/b
```

Sympy [A]
time = 0.17, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True))
```

Giac [A]
time = 5.70, size = 13, normalized size = 0.87

$$\frac{\sqrt{bx^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b
```

Mupad [B]
time = 4.33, size = 13, normalized size = 0.87

$$\frac{\sqrt{b\,x^2+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)/b
```

3.1016 $\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Maple [A]
time = 0.13, size = 21, normalized size = 0.84

method	result	size
default	$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.52

$$\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Fricas [A]
time = 0.35, size = 59, normalized size = 2.36

$$\left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 0.43, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Giac [A]
time = 6.60, size = 37, normalized size = 1.48

$$\frac{1}{2}\,\sqrt{bx^2+a}\,x-\frac{a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

Mupad [B]
time = 0.12, size = 20, normalized size = 0.80

$$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2)^(1/2),x)
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

3.1017 $\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] -arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {5, 272, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Maple [A]
time = 0.13, size = 29, normalized size = 1.16

method	result	size
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.68

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

Fricas [A]
time = 0.37, size = 60, normalized size = 2.40

$$\left[\frac{\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)/sqrt(a), sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a))/a]
```

Sympy [A]
time = 0.45, size = 19, normalized size = 0.76

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/2),x)
[Out] -asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

Giac [A]
time = 4.89, size = 22, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B]
time = 4.57, size = 19, normalized size = 0.76

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] -atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(1/2)
```

3.1018 $\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a+bx^2}}{ax}$$

[Out] -(b*x^2+a)^(1/2)/a/x

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {5, 270}

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -(Sqrt[a + b*x^2]/(a*x))

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x^2\sqrt{a+bx^2}}\,dx\\ &= -\frac{\sqrt{a+bx^2}}{ax}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.00

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Maple [A]
time = 0.14, size = 18, normalized size = 0.95

method	result	size
gosper	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
default	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
trager	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
risch	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(b*x^2+a)^(1/2)/a/x
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -sqrt(b*x^2 + a)/(a*x)
```

Fricas [A]
time = 0.35, size = 17, normalized size = 0.89

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -sqrt(b*x^2 + a)/(a*x)
```

Sympy [A]
time = 0.31, size = 19, normalized size = 1.00

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**(1/2),x)
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

Giac [A]
time = 7.80, size = 30, normalized size = 1.58

$$\frac{2\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B]
time = 0.04, size = 17, normalized size = 0.89

$$-\frac{\sqrt{b\,x^2+a}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^(1/2)),x)
[Out] -(a + b*x^2)^(1/2)/(a*x)
```

3.1019 $\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{a+bx^2}}{2ax^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

```
[Out] 1/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5, 272, 44, 65, 214}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2]/(a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3
/2))
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{x^3\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a} \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a} \\
&= -\frac{\sqrt{a+bx^2}}{2ax^2} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 1.00

$$
-\frac{\sqrt{a+bx^2}}{2ax^2} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -1/2*Sqrt[a + b*x^2]/(a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3
/2))
```

Maple [A]
time = 0.15, size = 48, normalized size = 0.96

method	result	size
default	$-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}$	48
risch	$-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.72

$$\frac{b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/2*sqrt(b*x^2 + a)/(a*x^2)
```

Fricas [A]
time = 0.37, size = 105, normalized size = 2.10

$$\left[\frac{\sqrt{a}\,bx^2\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,\sqrt{bx^2+a}\,a}{4\,a^2x^2},\,-\frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\,a}{2\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2
*sqrt(b*x^2 + a)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + sqrt(b*x^2 + a)*a)/(a^2*x^2)]
```

Sympy [A]
time = 1.14, size = 42, normalized size = 0.84

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**
(3/2))
```

Giac [A]
time = 6.50, size = 51, normalized size = 1.02

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{bx^2+a}\,b}{ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^2 + a)*b
/(a*x^2))/b
```

Mupad [B]
time = 4.54, size = 38, normalized size = 0.76

$$\frac{b\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (a + b*x^2)^(1/2)/(2*a*x
^2)
```

3.1020 $\int \frac{1}{x^4 \sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\sqrt{a+bx^2}}{3ax^3}+\frac{2b\sqrt{a+bx^2}}{3a^2x}$$

```
[Out] -1/3*(b*x^2+a)^(1/2)/a/x^3+2/3*b*(b*x^2+a)^(1/2)/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {5, 277, 270}

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/3*Sqrt[a + b*x^2]/(a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rule 5

```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^2}}\,dx$$
$$= -\frac{\sqrt{a+bx^2}}{3ax^3} - \frac{(2b)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{3a}$$
$$= -\frac{\sqrt{a+bx^2}}{3ax^3} + \frac{2b\sqrt{a+bx^2}}{3a^2x}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\,(-a+2bx^2)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] (Sqrt[a + b*x^2]*(-a + 2*b*x^2))/(3*a^2*x^3)
```

Maple [A]
time = 0.14, size = 37, normalized size = 0.84

method	result	size
gosper	$-\frac{\sqrt{b\,x^2+a}\,(-2b\,x^2+a)}{3a^2x^3}$	26
trager	$-\frac{\sqrt{b\,x^2+a}\,(-2b\,x^2+a)}{3a^2x^3}$	26
risch	$-\frac{\sqrt{b\,x^2+a}\,(-2b\,x^2+a)}{3a^2x^3}$	26
default	$-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(b*x^2+a)^(1/2)/a/x^3+2/3*b*(b*x^2+a)^(1/2)/a^2/x
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.82

$$\frac{2\sqrt{bx^2+a}\,b}{3\,a^2x} - \frac{\sqrt{bx^2+a}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(b*x^2 + a)*b/(a^2*x) - 1/3*sqrt(b*x^2 + a)/(a*x^3)
```

Fricas [A]
time = 0.34, size = 27, normalized size = 0.61

$$\frac{(2\,bx^2 - a)\sqrt{bx^2 + a}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*b*x^2 - a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

Sympy [A]
time = 0.40, size = 46, normalized size = 1.05

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{3ax^2} + \frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2} + 1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**(1/2),x)
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(3*a**2)
```

Giac [A]
time = 7.68, size = 55, normalized size = 1.25

$$\frac{4\left(3\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2 - a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2 - a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 4/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*b^(3/2)/((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^3
```

Mupad [B]
time = 4.55, size = 25, normalized size = 0.57

$$-\frac{\sqrt{b\,x^2 + a}\,\left(a - 2\,b\,x^2\right)}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^(1/2)),x)
[Out] -((a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3)
```

3.1021 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}} \, dx$

Optimal. Leaf size=16

$$\frac{x^5}{3\sqrt{cx^4}}$$

```
[Out] 1/3*x^5/(c*x^4)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1, 15, 30}

$$\frac{x^5}{3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] x^5/(3*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}} \, dx &= \int \frac{x^4}{\sqrt{cx^4}} \, dx\\ &= \frac{x^2 \int x^2 \, dx}{\sqrt{cx^4}}\\ &= \frac{x^5}{3\sqrt{cx^4}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^5}{3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^5/(3*Sqrt[c*x^4])
```

Maple [A]
time = 0.02, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{x^5}{3\sqrt{c\,x^4}}$	13
default	$\frac{x^5}{3\sqrt{c\,x^4}}$	13
risch	$\frac{x^5}{3\sqrt{c\,x^4}}$	13
trager	$\frac{(x^2+x+1)(-1+x)\sqrt{c\,x^4}}{3c\,x^2}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^5/(c*x^4)^(1/2)
```

Maxima [A]
time = 0.29, size = 12, normalized size = 0.75

$$\frac{x^5}{3\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^5/sqrt(c*x^4)
```

Fricas [A]
time = 0.36, size = 13, normalized size = 0.81

$$\frac{\sqrt{cx^4}\,x}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4)*x/c
```

Sympy [A]
time = 0.20, size = 12, normalized size = 0.75

$$\frac{x^5}{3\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4)**(1/2),x)
```

```
[Out] x**5/(3*sqrt(c*x**4))
```

Giac [A]
time = 5.11, size = 8, normalized size = 0.50

$$\frac{x^3}{3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*x^3/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x^4}{\sqrt{c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4)^(1/2),x)
```

```
[Out] int(x^4/(c*x^4)^(1/2), x)
```

3.1022 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$\frac{x^4}{2\sqrt{cx^4}}$$

```
[Out] 1/2*x^4/(c*x^4)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 15, 30}

$$\frac{x^4}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^4/(2*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx &= \int \frac{x^3}{\sqrt{cx^4}}\,dx\\ &= \frac{x^2\int x\,dx}{\sqrt{cx^4}}\\ &= \frac{x^4}{2\sqrt{cx^4}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^4}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^4/(2*Sqrt[c*x^4])
```

Maple [A]
time = 0.02, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{x^{4}}{2\sqrt{c\,x^{4}}}$	13
derivativedivides	$\frac{\sqrt{c\,x^{4}}}{2c}$	13
default	$\frac{x^{4}}{2\sqrt{c\,x^{4}}}$	13
risch	$\frac{x^{4}}{2\sqrt{c\,x^{4}}}$	13
trager	$\frac{(1+x)(-1+x)\sqrt{c\,x^{4}}}{2c\,x^{2}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^4/(c*x^4)^(1/2)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.75

$$\frac{\sqrt{cx^4}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^4)/c
```

Fricas [A]
time = 0.33, size = 12, normalized size = 0.75

$$\frac{\sqrt{cx^4}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^4)/c
```

Sympy [A]
time = 0.19, size = 12, normalized size = 0.75

$$\frac{x^4}{2\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4)**(1/2),x)
```

```
[Out] x**4/(2*sqrt(c*x**4))
```

Giac [A]
time = 7.79, size = 8, normalized size = 0.50

$$\frac{x^2}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/sqrt(c)
```

Mupad [B]
time = 4.50, size = 10, normalized size = 0.62

$$\frac{\sqrt{x^4}}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4)^(1/2),x)
```

```
[Out] (x^4)^(1/2)/(2*c^(1/2))
```

3.1023 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=13

$$\frac{x^3}{\sqrt{cx^4}}$$

```
[Out] x^3/(c*x^4)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1, 15, 8}

$$\frac{x^3}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^3/Sqrt[c*x^4]
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx &= \int \frac{x^2}{\sqrt{cx^4}}\,dx\\
&= \frac{x^2\int 1\,dx}{\sqrt{cx^4}}\\
&= \frac{x^3}{\sqrt{cx^4}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{x^3}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^3/Sqrt[c*x^4]
```

Maple [A]
time = 0.03, size = 12, normalized size = 0.92

method	result	size
default	$\frac{x^3}{\sqrt{c\,x^4}}$	12
risch	$\frac{x^3}{\sqrt{c\,x^4}}$	12
trager	$\frac{(-1+x)\sqrt{c\,x^4}}{c\,x^2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^3/(c*x^4)^(1/2)
```

Maxima [A]
time = 0.29, size = 11, normalized size = 0.85

$$\frac{x^3}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] x^3/sqrt(c*x^4)
```

Fricas [A]
time = 0.33, size = 14, normalized size = 1.08

$$\frac{\sqrt{cx^4}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4)/(c*x)
```

Sympy [A]
time = 0.17, size = 10, normalized size = 0.77

$$\frac{x^3}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4)**(1/2),x)
[Out] x**3/sqrt(c*x**4)
```

Giac [A]
time = 5.20, size = 5, normalized size = 0.38

$$\frac{x}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4)^(1/2),x, algorithm="giac")
[Out] x/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x^2}{\sqrt{c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4)^(1/2),x)
[Out] int(x^2/(c*x^4)^(1/2), x)
```

3.1024 $\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=15

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

[Out] x^2*ln(x)/(c*x^4)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1, 15, 29}

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]

[Out] (x^2*Log[x])/Sqrt[c*x^4]

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx &= \int \frac{x}{\sqrt{cx^4}}\,dx\\ &= \frac{x^2\int \frac{1}{x}\,dx}{\sqrt{cx^4}}\\ &= \frac{x^2\log(x)}{\sqrt{cx^4}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] (x^2*Log[x])/Sqrt[c*x^4]
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.93

method	result	size
default	$\frac{x^{2} \ln \left(x \right)}{\sqrt{c \,x^{4}}}$	14
risch	$\frac{x^{2} \ln \left(x \right)}{\sqrt{c \,x^{4}}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] x^2*ln(x)/(c*x^4)^(1/2)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.87

$$\frac{x^2\log\left(x\right)}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="maxima")
[Out] x^2*log(x)/sqrt(c*x^4)
```

Fricas [A]
time = 0.33, size = 16, normalized size = 1.07

$$\frac{\sqrt{cx^4}\,\log\left(x\right)}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^4)*log(x)/(c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4)**(1/2),x)
```

```
[Out] Integral(x/sqrt(c*x**4), x)
```

Giac [A]
time = 4.58, size = 14, normalized size = 0.93

$$\frac{\log\left(x^2\sqrt{|c|}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2*sqrt(abs(c)))/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{\sqrt{c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4)^(1/2),x)
```

```
[Out] int(x/(c*x^4)^(1/2), x)
```

3.1025 $\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{x}{\sqrt{cx^4}}$$

[Out] -x/(c*x^4)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1, 15, 30}

$$-\frac{x}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]

[Out] -(x/Sqrt[c*x^4])

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx &= \int \frac{1}{\sqrt{cx^4}}\,dx\\ &= \frac{x^2\int \frac{1}{x^2}\,dx}{\sqrt{cx^4}}\\ &= -\frac{x}{\sqrt{cx^4}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{x}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
[Out] -(x/Sqrt[c*x^4])
```

Maple [A]
time = 0.02, size = 11, normalized size = 0.92

method	result	size
gosper	$-\frac{x}{\sqrt{c\,x^{4}}}$	11
default	$-\frac{x}{\sqrt{c\,x^{4}}}$	11
risch	$-\frac{x}{\sqrt{c\,x^{4}}}$	11
trager	$\frac{(-1+x)\sqrt{c\,x^{4}}}{c\,x^{3}}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -x/(c*x^4)^(1/2)
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="maxima")
[Out] -x/sqrt(c*x^4)
```

Fricas [A]
time = 0.35, size = 15, normalized size = 1.25

$$-\frac{\sqrt{cx^4}}{cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4)/(c*x^3)
```

Sympy [A]
time = 0.16, size = 10, normalized size = 0.83

$$-\frac{x}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4)**(1/2),x)
```

```
[Out] -x/sqrt(c*x**4)
```

Giac [A]
time = 3.72, size = 8, normalized size = 0.67

$$-\frac{1}{\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/(sqrt(c)*x)
```

Mupad [B]
time = 4.30, size = 13, normalized size = 1.08

$$-\frac{\sqrt{x^4}}{\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4)^(1/2),x)
```

```
[Out] -(x^4)^(1/2)/(c^(1/2)*x^3)
```

3.1026 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2\sqrt{cx^4}}$$

```
[Out] -1/2/(c*x^4)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1, 15, 30}

$$-\frac{1}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/2*1/Sqrt[c*x^4]
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx &= \int \frac{1}{x\sqrt{cx^4}}\,dx\\ &= \frac{x^2\int \frac{1}{x^3}\,dx}{\sqrt{cx^4}}\\ &= -\frac{1}{2\sqrt{cx^4}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/2*1/Sqrt[c*x^4]
```

Maple [A]
time = 0.02, size = 10, normalized size = 0.77

method	result	size
gosper	$-\frac{1}{2\sqrt{c\,x^4}}$	10
derivativedivides	$-\frac{1}{2\sqrt{c\,x^4}}$	10
default	$-\frac{1}{2\sqrt{c\,x^4}}$	10
risch	$-\frac{1}{2\sqrt{c\,x^4}}$	10
trager	$\frac{(1+x)(-1+x)\sqrt{c\,x^4}}{2c\,x^4}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(c*x^4)^(1/2)
```

Maxima [A]
time = 0.28, size = 9, normalized size = 0.69

$$-\frac{1}{2\,\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2/sqrt(c*x^4)
```

Fricas [A]
time = 0.34, size = 15, normalized size = 1.15

$$-\frac{\sqrt{cx^4}}{2\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^4)/(c*x^4)
```

Sympy [A]
time = 0.18, size = 12, normalized size = 0.92

$$-\frac{1}{2\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4)**(1/2),x)
```

```
[Out] -1/(2*sqrt(c*x**4))
```

Giac [A]
time = 3.88, size = 8, normalized size = 0.62

$$-\frac{1}{2\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2/(sqrt(c)*x^2)
```

Mupad [B]
time = 4.34, size = 10, normalized size = 0.77

$$-\frac{1}{2\sqrt{c}\,\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^4)^(1/2)),x)
```

```
[Out] -1/(2*c^(1/2)*(x^4)^(1/2))
```

3.1027 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3x\sqrt{cx^4}}$$

```
[Out] -1/3/x/(c*x^4)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1, 15, 30}

$$-\frac{1}{3x\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/3*1/(x*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx &= \int \frac{1}{x^2\sqrt{cx^4}}\,dx\\
&= \frac{x^2\int \frac{1}{x^4}\,dx}{\sqrt{cx^4}}\\
&= -\frac{1}{3x\sqrt{cx^4}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\frac{1}{3x\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/3*1/(x*Sqrt[c*x^4])
```

Maple [A]
time = 0.02, size = 13, normalized size = 0.81

method	result	size
gosper	$-\frac{1}{3x\sqrt{c\,x^4}}$	13
default	$-\frac{1}{3x\sqrt{c\,x^4}}$	13
risch	$-\frac{1}{3x\sqrt{c\,x^4}}$	13
trager	$\frac{(-1+x)(x^2+x+1)\sqrt{c\,x^4}}{3c\,x^5}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x/(c*x^4)^(1/2)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$-\frac{1}{3\sqrt{cx^4}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3/(sqrt(c*x^4)*x)
```

Fricas [A]
time = 0.34, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{3\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(c*x^4)/(c*x^5)
```

Sympy [A]
time = 0.19, size = 14, normalized size = 0.88

$$-\frac{1}{3x\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4)**(1/2),x)
```

```
[Out] -1/(3*x*sqrt(c*x**4))
```

Giac [A]
time = 3.96, size = 8, normalized size = 0.50

$$-\frac{1}{3\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3/(sqrt(c)*x^3)
```

Mupad [B]
time = 4.31, size = 13, normalized size = 0.81

$$-\frac{1}{3\sqrt{c}\,x\,\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^4)^(1/2)),x)
```

```
[Out] -1/(3*c^(1/2)*x*(x^4)^(1/2))
```

3.1028 $\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{4x^2\sqrt{cx^4}}$$

```
[Out] -1/4/x^2/(c*x^4)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1, 15, 30}

$$-\frac{1}{4x^2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/4*1/(x^2*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx &= \int \frac{1}{x^3\sqrt{cx^4}}\,dx\\ &= \frac{x^2\int \frac{1}{x^5}\,dx}{\sqrt{cx^4}}\\ &= -\frac{1}{4x^2\sqrt{cx^4}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.06

$$-\frac{cx^2}{4\left(cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/4*(c*x^2)/(c*x^4)^(3/2)
```

Maple [A]
time = 0.02, size = 13, normalized size = 0.81

method	result	size
gosper	$-\frac{1}{4x^2\sqrt{c\,x^4}}$	13
default	$-\frac{1}{4x^2\sqrt{c\,x^4}}$	13
risch	$-\frac{1}{4x^2\sqrt{c\,x^4}}$	13
trager	$\frac{(-1+x)(x^3+x^2+x+1)\sqrt{c\,x^4}}{4c\,x^6}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^2/(c*x^4)^(1/2)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$-\frac{1}{4\sqrt{cx^4}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4/(sqrt(c*x^4)*x^2)
```

Fricas [A]
time = 0.34, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{4\,cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(c*x^4)/(c*x^6)
```

Sympy [A]
time = 0.21, size = 15, normalized size = 0.94

$$-\frac{1}{4x^2\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**4)**(1/2),x)
```

```
[Out] -1/(4*x**2*sqrt(c*x**4))
```

Giac [A]
time = 4.04, size = 8, normalized size = 0.50

$$-\frac{1}{4\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4/(sqrt(c)*x^4)
```

Mupad [B]
time = 4.27, size = 13, normalized size = 0.81

$$-\frac{1}{4\sqrt{c}\,x^2\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c*x^4)^(1/2)),x)
```

```
[Out] -1/(4*c^(1/2)*x^2*(x^4)^(1/2))
```

3.1029 $\int \frac{1}{x^4 \sqrt{2+2a-2(1+a)+cx^4}} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{5x^3\sqrt{cx^4}}$$

```
[Out] -1/5/x^3/(c*x^4)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1, 15, 30}

$$-\frac{1}{5x^3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/5*1/(x^3*Sqrt[c*x^4])
```

Rule 1

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}} \, dx &= \int \frac{1}{x^4\sqrt{cx^4}} \, dx \\
&= \frac{x^2 \int \frac{1}{x^6} \, dx}{\sqrt{cx^4}} \\
&= -\frac{1}{5x^3\sqrt{cx^4}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 0.94

$$-\frac{cx}{5\left(cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/5*(c*x)/(c*x^4)^(3/2)
```

Maple [A]
time = 0.02, size = 13, normalized size = 0.81

method	result	size
gosper	$-\frac{1}{5x^3\sqrt{c\,x^4}}$	13
default	$-\frac{1}{5x^3\sqrt{c\,x^4}}$	13
risch	$-\frac{1}{5x^3\sqrt{c\,x^4}}$	13
trager	$\frac{(-1+x)(x^4+x^3+x^2+x+1)\sqrt{c\,x^4}}{5c\,x^7}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^3/(c*x^4)^(1/2)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$-\frac{1}{5\sqrt{cx^4}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5/(sqrt(c*x^4)*x^3)
```

Fricas [A]
time = 0.32, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{5\,cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(c*x^4)/(c*x^7)
```

Sympy [A]
time = 0.23, size = 15, normalized size = 0.94

$$-\frac{1}{5x^3\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4)**(1/2),x)
```

```
[Out] -1/(5*x**3*sqrt(c*x**4))
```

Giac [A]
time = 3.58, size = 8, normalized size = 0.50

$$-\frac{1}{5\sqrt{c}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/5/(sqrt(c)*x^5)
```

Mupad [B]
time = 4.33, size = 13, normalized size = 0.81

$$-\frac{1}{5\sqrt{c}\,x^3\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c*x^4)^(1/2)),x)
```

```
[Out] -1/(5*c^(1/2)*x^3*(x^4)^(1/2))
```

3.1030 $\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^5}{5\sqrt{a}}$$

```
[Out] 1/5*x^5/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2, 12, 30}

$$\frac{x^5}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^5/(5*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x^4}{\sqrt{a}}\,dx\\ &= \frac{\int x^4\,dx}{\sqrt{a}}\\ &= \frac{x^5}{5\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^5}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^5/(5*Sqrt[a])
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.75

method	result	size
gosper	$\frac{x^5}{5\sqrt{a}}$	9
default	$\frac{x^5}{5\sqrt{a}}$	9
norman	$\frac{x^5}{5\sqrt{a}}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/a^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5/a^(1/2)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^5/sqrt(a)
```

Fricas [A]
time = 0.33, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*x^5/sqrt(a)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/a**(1/2),x)
[Out] x**5/(5*sqrt(a))
```

Giac [A]
time = 3.47, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/a^(1/2),x, algorithm="giac")
[Out] 1/5*x^5/sqrt(a)
```

Mupad [B]
time = 0.02, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/a^(1/2),x)
[Out] x^5/(5*a^(1/2))
```

3.1031 $\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^4}{4\sqrt{a}}$$

```
[Out] 1/4*x^4/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2, 12, 30}

$$\frac{x^4}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^4/(4*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x^3}{\sqrt{a}}\,dx\\ &= \frac{\int x^3\,dx}{\sqrt{a}}\\ &= \frac{x^4}{4\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^4}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^4/(4*Sqrt[a])
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.75

method	result	size
gosper	$\frac{x^4}{4\sqrt{a}}$	9
default	$\frac{x^4}{4\sqrt{a}}$	9
norman	$\frac{x^4}{4\sqrt{a}}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/a^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4/a^(1/2)
```

Maxima [A]
time = 0.28, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/sqrt(a)
```

Fricas [A]
time = 0.33, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4/sqrt(a)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/a**(1/2),x)
[Out] x**4/(4*sqrt(a))
```

Giac [A]
time = 4.11, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/a^(1/2),x, algorithm="giac")
[Out] 1/4*x^4/sqrt(a)
```

Mupad [B]
time = 0.03, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/a^(1/2),x)
[Out] x^4/(4*a^(1/2))
```

3.1032 $\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^3}{3\sqrt{a}}$$

[Out] 1/3*x^3/a^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2, 12, 30}

$$\frac{x^3}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x^3/(3*Sqrt[a])

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x^2}{\sqrt{a}}\,dx\\ &= \frac{\int x^2\,dx}{\sqrt{a}}\\ &= \frac{x^3}{3\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^3}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] x^3/(3*Sqrt[a])
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.75

method	result	size
gosper	$\frac{x^3}{3\sqrt{a}}$	9
default	$\frac{x^3}{3\sqrt{a}}$	9
norman	$\frac{x^3}{3\sqrt{a}}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/a^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/a^(1/2)
```

Maxima [A]
time = 0.28, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="maxima")
[Out] 1/3*x^3/sqrt(a)
```

Fricas [A]
time = 0.34, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*x^3/sqrt(a)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/a**(1/2),x)
```

```
[Out] x**3/(3*sqrt(a))
```

Giac [A]
time = 4.22, size = 8, normalized size = 0.67

$$\frac{x^3}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/a^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*x^3/sqrt(a)
```

Mupad [B]
time = 0.01, size = 8, normalized size = 0.67

$$\frac{x^3}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/a^(1/2),x)
```

```
[Out] x^3/(3*a^(1/2))
```

3.1033 $\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^2}{2\sqrt{a}}$$

```
[Out] 1/2*x^2/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2, 12, 30}

$$\frac{x^2}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^2/(2*Sqrt[a])
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{x}{\sqrt{a}}\,dx\\ &= \frac{\int x\,dx}{\sqrt{a}}\\ &= \frac{x^2}{2\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{x^2}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
[Out] x^2/(2*Sqrt[a])
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.75

method	result	size
gosper	$\frac{x^2}{2\sqrt{a}}$	9
default	$\frac{x^2}{2\sqrt{a}}$	9
norman	$\frac{x^2}{2\sqrt{a}}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/a^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2/a^(1/2)
```

Maxima [A]
time = 0.30, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="maxima")
[Out] 1/2*x^2/sqrt(a)
```

Fricas [A]
time = 0.34, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2/sqrt(a)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a**(1/2),x)
```

```
[Out] x**2/(2*sqrt(a))
```

Giac [A]
time = 4.21, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/a^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/sqrt(a)
```

Mupad [B]
time = 0.02, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/a^(1/2),x)
```

```
[Out] x^2/(2*a^(1/2))
```

3.1034 $\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=7

$$\frac{x}{\sqrt{a}}$$

```
[Out] x/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2, 8}

$$\frac{x}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x/Sqrt[a]
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{\sqrt{a}}\,dx \\ &= \frac{x}{\sqrt{a}} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\frac{x}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x/Sqrt[a]
```

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
default	$\frac{x}{\sqrt{a}}$	6
norman	$\frac{x}{\sqrt{a}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/a^(1/2)
```

Maxima [A]
time = 0.28, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(a)
```

Fricas [A]
time = 0.33, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="fricas")
```

```
[Out] x/sqrt(a)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a**(1/2),x)
```

```
[Out] x/sqrt(a)
```

Giac [**A**]
time = 4.46, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/a^(1/2),x, algorithm="giac")
```

```
[Out] x/sqrt(a)
```

Mupad [**B**]
time = 0.00, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/a^(1/2),x)
```

```
[Out] x/a^(1/2)
```

3.1035 $\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=8

$$\frac{\log(x)}{\sqrt{a}}$$

```
[Out] ln(x)/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2, 12, 29}

$$\frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] Log[x]/Sqrt[a]
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{\sqrt{a}\,x}\,dx\\
&= \frac{\int \frac{1}{x}\,dx}{\sqrt{a}}\\
&= \frac{\log(x)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] Log[x]/Sqrt[a]
```

Maple [A]
time = 0.02, size = 7, normalized size = 0.88

method	result	size
default	$\frac{\ln(x)}{\sqrt{a}}$	7
norman	$\frac{\ln(x)}{\sqrt{a}}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/a^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x)/a^(1/2)
```

Maxima [A]
time = 0.28, size = 6, normalized size = 0.75

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="maxima")
[Out] log(x)/sqrt(a)
```

Fricas [A]
time = 0.35, size = 6, normalized size = 0.75

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="fricas")
[Out] log(x)/sqrt(a)
```

Sympy [**A**]
time = 0.01, size = 7, normalized size = 0.88

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a**(1/2),x)
```

```
[Out] log(x)/sqrt(a)
```

Giac [**A**]
time = 4.73, size = 7, normalized size = 0.88

$$\frac{\log(|x|)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/a^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(x))/sqrt(a)
```

Mupad [**B**]
time = 4.24, size = 6, normalized size = 0.75

$$\frac{\ln(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x),x)
```

```
[Out] log(x)/a^(1/2)
```

3.1036 $\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{\sqrt{a}\,x}$$

```
[Out] -1/x/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2, 12, 30}

$$-\frac{1}{\sqrt{a}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(1/(Sqrt[a]*x))
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{\sqrt{a}\,x^2}\,dx\\ &= \frac{\int \frac{1}{x^2}\,dx}{\sqrt{a}}\\ &= -\frac{1}{\sqrt{a}\,x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{\sqrt{a}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -(1/(Sqrt[a]*x))
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.90

method	result	size
gosper	$-\frac{1}{x\sqrt{a}}$	9
default	$-\frac{1}{x\sqrt{a}}$	9
norman	$-\frac{1}{x\sqrt{a}}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/a^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x/a^(1/2)
```

Maxima [A]
time = 0.28, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(a)*x)
```

Fricas [A]
time = 0.36, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="fricas")
```

```
[Out] -1/(sqrt(a)*x)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/a**(1/2),x)
[Out] -1/(sqrt(a)*x)
```

Giac [A]
time = 3.73, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/a^(1/2),x, algorithm="giac")
[Out] -1/(sqrt(a)*x)
```

Mupad [B]
time = 0.03, size = 8, normalized size = 0.80

$$-\frac{1}{\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x^2),x)
[Out] -1/(a^(1/2)*x)
```

3.1037 $\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{2\sqrt{a}\,x^2}$$

```
[Out] -1/2/x^2/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2, 12, 30}

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/2*1/(Sqrt[a]*x^2)
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{\sqrt{a}\,x^3}\,dx\\
&= \frac{\int \frac{1}{x^3}\,dx}{\sqrt{a}}\\
&= -\frac{1}{2\sqrt{a}\,x^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
[Out] -1/2*1/(Sqrt[a]*x^2)
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.75

method	result	size
gosper	$-\frac{1}{2x^2\sqrt{a}}$	9
default	$-\frac{1}{2x^2\sqrt{a}}$	9
norman	$-\frac{1}{2x^2\sqrt{a}}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/a^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/x^2/a^(1/2)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.67

$$-\frac{1}{2\,\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="maxima")
[Out] -1/2/(sqrt(a)*x^2)
```

Fricas [A]
time = 0.33, size = 8, normalized size = 0.67

$$-\frac{1}{2\,\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2/(sqrt(a)*x^2)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{1}{2\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/a**(1/2),x)
[Out] -1/(2*sqrt(a)*x**2)
```

Giac [A]
time = 5.09, size = 8, normalized size = 0.67

$$-\frac{1}{2\,\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/a^(1/2),x, algorithm="giac")
[Out] -1/2/(sqrt(a)*x^2)
```

Mupad [B]
time = 4.40, size = 8, normalized size = 0.67

$$-\frac{1}{2\,\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x^3),x)
[Out] -1/(2*a^(1/2)*x^2)
```

3.1038 $\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{3\sqrt{a}\,x^3}$$

```
[Out] -1/3/x^3/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2, 12, 30\}$

$$-\frac{1}{3\sqrt{a}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/3*1/(Sqrt[a]*x^3)
```

Rule 2

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx &= \int \frac{1}{\sqrt{a}\,x^4}\,dx\\ &= \frac{\int \frac{1}{x^4}\,dx}{\sqrt{a}}\\ &= -\frac{1}{3\sqrt{a}\,x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{3\sqrt{a}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -1/3*1/(Sqrt[a]*x^3)
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.75

method	result	size
gosper	$-\frac{1}{3x^3\sqrt{a}}$	9
default	$-\frac{1}{3x^3\sqrt{a}}$	9
norman	$-\frac{1}{3x^3\sqrt{a}}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/a^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3/a^(1/2)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.67

$$-\frac{1}{3\,\sqrt{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3/(sqrt(a)*x^3)
```

Fricas [A]
time = 0.34, size = 8, normalized size = 0.67

$$-\frac{1}{3\,\sqrt{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3/(sqrt(a)*x^3)
```

Sympy [**A**]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{1}{3\sqrt{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/a**(1/2),x)
[Out] -1/(3*sqrt(a)*x**3)
```

Giac [**A**]
time = 4.50, size = 8, normalized size = 0.67

$$-\frac{1}{3\,\sqrt{a}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/a^(1/2),x, algorithm="giac")
[Out] -1/3/(sqrt(a)*x^3)
```

Mupad [**B**]
time = 4.33, size = 8, normalized size = 0.67

$$-\frac{1}{3\,\sqrt{a}\,\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^(1/2)*x^4),x)
[Out] -1/(3*a^(1/2)*x^3)
```

3.1039 $\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 430}

$$\frac{F\left(\text{ArcSin}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx &= 2\int \frac{1}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx\\ &= \frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.02, size = 18, normalized size = 1.50

$$-iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(13) = 26.
time = 0.02, size = 43, normalized size = 3.58

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

Fricas [A]
time = 0.07, size = 8, normalized size = 0.67

$$\frac{1}{3}\sqrt{3}\,\mathrm{ellipticF}\left(x,-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*ellipticF(x, -1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4 - 2x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 - 2*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{-x^4 - 2\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3 - x^4 - 2*x^2)^(1/2),x)
```

```
[Out] int(1/(3 - x^4 - 2*x^2)^(1/2), x)
```

3.1040 $\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx$

Optimal. Leaf size=39

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)|\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

```
[Out] -1/21*(x^2/(5+21^(1/2)))^(1/2)/x*(5+21^(1/2))^(1/2)*EllipticF((1-2*x^2/(5+2
1^(1/2)))^(1/2),1/42*(882+210*21^(1/2))^(1/2))*21^(3/4)
```

Rubi [A]
time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1109, 431}

$$-\frac{F\left(\text{ArcCos}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)|\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + 5*x^2 - x^4],x]
```

```
[Out] -(EllipticF[ArcCos[Sqrt[2/(5 + Sqrt[21])]*x], (21 + 5*Sqrt[21])/42]/21^(1/4
))
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx = 2\int \frac{1}{\sqrt{5+\sqrt{21}-2x^2}\,\sqrt{-5+\sqrt{21}+2x^2}}\,dx$$
$$= -\frac{F\left(\cos^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)|\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.
time = 10.09, size = 87, normalized size = 2.23

$$\frac{\sqrt{5-\sqrt{21}-2x^2}\,\sqrt{2+\left(-5+\sqrt{21}\right)x^2}\,F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)|\frac{23}{2}-\frac{5\sqrt{21}}{2}\right)}{2\sqrt{-1+5x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + 5*x^2 - x^4],x]
```

```
[Out] (Sqrt[5 - Sqrt[21] - 2*x^2]*Sqrt[2 + (-5 + Sqrt[21])*x^2]*EllipticF[ArcSin[
Sqrt[(5 + Sqrt[21])/2]*x], 23/2 - (5*Sqrt[21])/2])/(2*Sqrt[-1 + 5*x^2 - x^4
])
```

Maple [**A**]
time = 0.06, size = 82, normalized size = 2.10

method	result	size
default	$\frac{\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\,\sqrt{1-\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)x^2}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right),\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{-x^4+5x^2-1}}$	82
elliptic	$\frac{\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\,\sqrt{1-\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)x^2}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right),\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{-x^4+5x^2-1}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+5*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1/2*7^(1/2)-1/2*3^(1/2))*(1-(5/2-1/2*21^(1/2))*x^2)^(1/2)*(1-(5/2+1/2*21
^(1/2))*x^2)^(1/2)/(-x^4+5*x^2-1)^(1/2)*EllipticF(x*(1/2*7^(1/2)-1/2*3^(1/2
)),5/2+1/2*21^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-x^4 + 5*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4+5x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+5*x**2-1)**(1/2),x)
[Out] Integral(1/sqrt(-x**4 + 5*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+5*x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-x^4 + 5*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-x^4+5\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2 - x^4 - 1)^(1/2),x)
[Out] int(1/(5*x^2 - x^4 - 1)^(1/2), x)
```

3.1041 $\int x^{5/2}(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=31

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

```
[Out] 2/7*a*x^(7/2)+2/11*b*x^(11/2)+2/15*c*x^(15/2)
```

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx&=\int\left(ax^{5/2}+bx^{9/2}+cx^{13/2}\right)\,dx\\&=\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.81

$$\frac{2x^{7/2}\left(165a+105bx^2+77cx^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(7/2)*(165*a + 105*b*x^2 + 77*c*x^4))/1155
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.65

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{11}{2}}}{11}+\frac{2c\,x^{\frac{15}{2}}}{15}$	20
default	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{11}{2}}}{11}+\frac{2c\,x^{\frac{15}{2}}}{15}$	20
gosper	$\frac{2x^{\frac{7}{2}}\left(77c\,x^{4}+105b\,x^{2}+165a\right)}{1155}$	22
trager	$\frac{2x^{\frac{7}{2}}\left(77c\,x^{4}+105b\,x^{2}+165a\right)}{1155}$	22
risch	$\frac{2x^{\frac{7}{2}}\left(77c\,x^{4}+105b\,x^{2}+165a\right)}{1155}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 2/7*a*x^(7/2)+2/11*b*x^(11/2)+2/15*c*x^(15/2)
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.61

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

Fricas [A]
time = 0.33, size = 24, normalized size = 0.77

$$\frac{2}{1155}\left(77\,cx^{7}+105\,bx^{5}+165\,ax^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 2/1155*(77*c*x^7 + 105*b*x^5 + 165*a*x^3)*sqrt(x)
```

Sympy [A]
time = 0.48, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{11}{2}}}{11}+\frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(7/2)/7 + 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

Giac [A]
time = 3.88, size = 19, normalized size = 0.61

$$\frac{2}{15}\,cx^{\frac{15}{2}} + \frac{2}{11}\,bx^{\frac{11}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

Mupad [B]
time = 4.29, size = 21, normalized size = 0.68

$$\frac{2\,x^{7/2}\,\left(77\,c\,x^4+105\,b\,x^2+165\,a\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(7/2)*(165*a + 105*b*x^2 + 77*c*x^4))/1155
```

3.1042 $\int x^{3/2}(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=31

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

```
[Out] 2/5*a*x^(5/2)+2/9*b*x^(9/2)+2/13*c*x^(13/2)
```

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx&=\int\left(ax^{3/2}+bx^{7/2}+cx^{11/2}\right)\,dx\\&=\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.81

$$\frac{2}{585}x^{5/2}\left(117a+65bx^2+45cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(5/2)*(117*a + 65*b*x^2 + 45*c*x^4))/585
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.65

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{9}{2}}}{9}+\frac{2c\,x^{\frac{13}{2}}}{13}$	20
default	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{9}{2}}}{9}+\frac{2c\,x^{\frac{13}{2}}}{13}$	20
gosper	$\frac{2x^{\frac{5}{2}}\left(45c\,x^{4}+65b\,x^{2}+117a\right)}{585}$	22
trager	$\frac{2x^{\frac{5}{2}}\left(45c\,x^{4}+65b\,x^{2}+117a\right)}{585}$	22
risch	$\frac{2x^{\frac{5}{2}}\left(45c\,x^{4}+65b\,x^{2}+117a\right)}{585}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*a*x^(5/2)+2/9*b*x^(9/2)+2/13*c*x^(13/2)
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.61

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

Fricas [A]
time = 0.33, size = 24, normalized size = 0.77

$$\frac{2}{585}\left(45\,cx^{6}+65\,bx^{4}+117\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/585*(45*c*x^6 + 65*b*x^4 + 117*a*x^2)*sqrt(x)
```

Sympy [A]
time = 0.29, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{5}{2}}}{5}+\frac{2bx^{\frac{9}{2}}}{9}+\frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

Giac [A]
time = 3.23, size = 19, normalized size = 0.61

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

Mupad [B]
time = 0.04, size = 21, normalized size = 0.68

$$\frac{2\,x^{5/2}\,\left(45\,c\,x^4+65\,b\,x^2+117\,a\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (2*x^(5/2)*(117*a + 65*b*x^2 + 45*c*x^4))/585
```

3.1043 $\int \sqrt{x} \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=31

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

```
[Out] 2/3*a*x^(3/2)+2/7*b*x^(7/2)+2/11*c*x^(11/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{14\}$

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4),x]
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x} \left(a+bx^2+cx^4\right) \, dx &= \int \left(a\sqrt{x}+bx^{5/2}+cx^{9/2}\right) \, dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.81

$$\frac{2}{231}x^{3/2}\left(77a+33bx^2+21cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4),x]
[Out] (2*x^(3/2)*(77*a + 33*b*x^2 + 21*c*x^4))/231
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.65

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{7}{2}}}{7}+\frac{2c\,x^{\frac{11}{2}}}{11}$	20
default	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{7}{2}}}{7}+\frac{2c\,x^{\frac{11}{2}}}{11}$	20
gosper	$\frac{2x^{\frac{3}{2}}\left(21c\,x^{4}+33b\,x^{2}+77a\right)}{231}$	22
trager	$\frac{2x^{\frac{3}{2}}\left(21c\,x^{4}+33b\,x^{2}+77a\right)}{231}$	22
risch	$\frac{2x^{\frac{3}{2}}\left(21c\,x^{4}+33b\,x^{2}+77a\right)}{231}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a*x^(3/2)+2/7*b*x^(7/2)+2/11*c*x^(11/2)
```

Maxima [A]
time = 0.31, size = 19, normalized size = 0.61

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

Fricas [A]
time = 0.32, size = 22, normalized size = 0.71

$$\frac{2}{231}\left(21\,cx^{5}+33\,bx^{3}+77\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 2/231*(21*c*x^5 + 33*b*x^3 + 77*a*x)*sqrt(x)
```

Sympy [A]
time = 0.89, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{7}{2}}}{7}+\frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a),x)
[Out] 2*a*x**(3/2)/3 + 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

Giac [A]
time = 3.25, size = 19, normalized size = 0.61

$$\frac{2}{11}cx^{\frac{11}{2}} + \frac{2}{7}bx^{\frac{7}{2}} + \frac{2}{3}ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.68

$$\frac{2\,x^{3/2}\,\left(21\,c\,x^4+33\,b\,x^2+77\,a\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2 + c*x^4),x)
[Out] (2*x^(3/2)*(77*a + 33*b*x^2 + 21*c*x^4))/231
```

3.1044 $\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx$

Optimal. Leaf size=29

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

```
[Out] 2/5*b*x^(5/2)+2/9*c*x^(9/2)+2*a*x^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{14\}$

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx &= \int \left(\frac{a}{\sqrt{x}}+bx^{3/2}+cx^{7/2}\right)dx\\ &= 2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 25, normalized size = 0.86

$$\frac{2}{45}\sqrt{x}\left(45a+9bx^2+5cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(45*a + 9*b*x^2 + 5*c*x^4))/45
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{5}{2}}}{5}+\frac{2c\,x^{\frac{9}{2}}}{9}+2a\sqrt{x}$	20
default	$\frac{2b\,x^{\frac{5}{2}}}{5}+\frac{2c\,x^{\frac{9}{2}}}{9}+2a\sqrt{x}$	20
trager	$\left(\frac{2}{9}c\,x^{4}+\frac{2}{5}b\,x^{2}+2a\right)\sqrt{x}$	21
gosper	$\frac{2\sqrt{x}\,\left(5c\,x^{4}+9b\,x^{2}+45a\right)}{45}$	22
risch	$\frac{2\sqrt{x}\,\left(5c\,x^{4}+9b\,x^{2}+45a\right)}{45}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*b*x^(5/2)+2/9*c*x^(9/2)+2*a*x^(1/2)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.66

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

Fricas [A]
time = 0.33, size = 21, normalized size = 0.72

$$\frac{2}{45}\left(5\,cx^{4}+9\,bx^{2}+45\,a\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*c*x^4 + 9*b*x^2 + 45*a)*sqrt(x)
```

Sympy [A]
time = 0.14, size = 27, normalized size = 0.93

$$2a\sqrt{x}+\frac{2bx^{\frac{5}{2}}}{5}+\frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

Giac [A]
time = 3.73, size = 19, normalized size = 0.66

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.72

$$\frac{2\,\sqrt{x}\,\left(5\,c\,x^4+9\,b\,x^2+45\,a\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(45*a + 9*b*x^2 + 5*c*x^4))/45
```

3.1045 $\int \frac{a+bx^2+cx^4}{x^{3/2}} \, dx$

Optimal. Leaf size=29

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

```
[Out] 2/3*b*x^(3/2)+2/7*c*x^(7/2)-2*a/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{14\}$

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{3/2}} \, dx &= \int \left(\frac{a}{x^{3/2}}+b\sqrt{x}+cx^{5/2}\right) \, dx\\ &= -\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.86

$$-\frac{2(21a-7bx^2-3cx^4)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (-2*(21*a - 7*b*x^2 - 3*c*x^4))/(21*Sqrt[x])
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.69

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{2c\,x^{\frac{7}{2}}}{7}-\frac{2a}{\sqrt{x}}$	20
default	$\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{2c\,x^{\frac{7}{2}}}{7}-\frac{2a}{\sqrt{x}}$	20
gosper	$-\frac{2(-3c\,x^{4}-7b\,x^{2}+21a)}{21\sqrt{x}}$	22
trager	$-\frac{2(-3c\,x^{4}-7b\,x^{2}+21a)}{21\sqrt{x}}$	22
risch	$-\frac{2(-3c\,x^{4}-7b\,x^{2}+21a)}{21\sqrt{x}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*b*x^(3/2)+2/7*c*x^(7/2)-2*a/x^(1/2)
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.66

$$\frac{2}{7}\,cx^{\frac{7}{2}}+\frac{2}{3}\,bx^{\frac{3}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="maxima")
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

Fricas [A]
time = 0.37, size = 21, normalized size = 0.72

$$\frac{2\,(3\,cx^{4}+7\,bx^{2}-21\,a)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="fricas")
[Out] 2/21*(3*c*x^4 + 7*b*x^2 - 21*a)/sqrt(x)
```

Sympy [A]
time = 0.24, size = 27, normalized size = 0.93

$$-\frac{2a}{\sqrt{x}}+\frac{2bx^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(3/2),x)
[Out] -2*a/sqrt(x) + 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

Giac [A]
time = 3.67, size = 19, normalized size = 0.66

$$\frac{2}{7} c x^{\frac{7}{2}} + \frac{2}{3} b x^{\frac{3}{2}} - \frac{2 a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="giac")
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

Mupad [B]
time = 0.04, size = 21, normalized size = 0.72

$$\frac{6\,c\,x^4+14\,b\,x^2-42\,a}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(3/2),x)
[Out] (14*b*x^2 - 42*a + 6*c*x^4)/(21*x^(1/2))
```

3.1046 $\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

```
[Out] -2/3*a/x^(3/2)+2/5*c*x^(5/2)+2*b*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = $\{14\}$

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{\sqrt{x}}+cx^{3/2}\right)dx\\ &= -\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.86

$$-\frac{2(5a-15bx^2-3cx^4)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] (-2*(5*a - 15*b*x^2 - 3*c*x^4))/(15*x^(3/2))
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.69

method	result	size
derivativedivides	$-\frac{2a}{3x^{\frac{3}{2}}}+\frac{2c\,x^{\frac{5}{2}}}{5}+2b\sqrt{x}$	20
default	$-\frac{2a}{3x^{\frac{3}{2}}}+\frac{2c\,x^{\frac{5}{2}}}{5}+2b\sqrt{x}$	20
gosper	$-\frac{2\left(-3c\,x^{4}-15b\,x^{2}+5a\right)}{15x^{\frac{3}{2}}}$	22
trager	$-\frac{2\left(-3c\,x^{4}-15b\,x^{2}+5a\right)}{15x^{\frac{3}{2}}}$	22
risch	$-\frac{2\left(-3c\,x^{4}-15b\,x^{2}+5a\right)}{15x^{\frac{3}{2}}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3*a/x^(3/2)+2/5*c*x^(5/2)+2*b*x^(1/2)
```

Maxima [A]
time = 0.33, size = 19, normalized size = 0.66

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="maxima")
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

Fricas [A]
time = 0.33, size = 21, normalized size = 0.72

$$\frac{2\left(3\,cx^{4}+15\,bx^{2}-5\,a\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="fricas")
[Out] 2/15*(3*c*x^4 + 15*b*x^2 - 5*a)/x^(3/2)
```

Sympy [A]
time = 0.31, size = 27, normalized size = 0.93

$$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}+\frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) + 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

Giac [A]
time = 4.11, size = 19, normalized size = 0.66

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.72

$$\frac{6\,c\,x^4+30\,b\,x^2-10\,a}{15\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(5/2),x)
```

```
[Out] (30*b*x^2 - 10*a + 6*c*x^4)/(15*x^(3/2))
```

3.1047 $\int \frac{a+bx^2+cx^4}{x^{7/2}} \, dx$

Optimal. Leaf size=29

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}$$

```
[Out] -2/5*a/x^(5/2)+2/3*c*x^(3/2)-2*b/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{7/2}} \, dx &= \int \left(\frac{a}{x^{7/2}}+\frac{b}{x^{3/2}}+c\sqrt{x}\right) \, dx\\ &= -\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.86

$$\frac{2(-3a-15bx^2+5cx^4)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^(7/2),x]
```

```
[Out] (2*(-3*a - 15*b*x^2 + 5*c*x^4))/(15*x^(5/2))
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.69

method	result	size
derivativedivides	$-\frac{2a}{5x^{\frac{5}{2}}}+\frac{2c\,x^{\frac{3}{2}}}{3}-\frac{2b}{\sqrt{x}}$	20
default	$-\frac{2a}{5x^{\frac{5}{2}}}+\frac{2c\,x^{\frac{3}{2}}}{3}-\frac{2b}{\sqrt{x}}$	20
gosper	$-\frac{2(-5c\,x^{4}+15b\,x^{2}+3a)}{15x^{\frac{5}{2}}}$	22
trager	$-\frac{2(-5c\,x^{4}+15b\,x^{2}+3a)}{15x^{\frac{5}{2}}}$	22
risch	$-\frac{2(-5c\,x^{4}+15b\,x^{2}+3a)}{15x^{\frac{5}{2}}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5*a/x^(5/2)+2/3*c*x^(3/2)-2*b/x^(1/2)
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.69

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\,(5\,bx^{2}+a)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="maxima")
[Out] 2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

Fricas [A]
time = 0.32, size = 21, normalized size = 0.72

$$\frac{2\,(5\,cx^{4}-15\,bx^{2}-3\,a)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="fricas")
[Out] 2/15*(5*c*x^4 - 15*b*x^2 - 3*a)/x^(5/2)
```

Sympy [A]
time = 0.34, size = 27, normalized size = 0.93

$$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}+\frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**(7/2),x)
```

```
[Out] -2*a/(5*x**(5/2)) - 2*b/sqrt(x) + 2*c*x**(3/2)/3
```

Giac [A]
time = 3.55, size = 20, normalized size = 0.69

$$\frac{2}{3}cx^{\frac{3}{2}}-\frac{2\,(5\,bx^2+a)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

Mupad [B]
time = 4.33, size = 21, normalized size = 0.72

$$-\frac{-10\,c\,x^4+30\,b\,x^2+6\,a}{15\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/x^(7/2),x)
```

```
[Out] -(6*a + 30*b*x^2 - 10*c*x^4)/(15*x^(5/2))
```

3.1048 $\int x^{5/2}(a+bx^2+cx^4)^2\,dx$

Optimal. Leaf size=64

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}\left(b^2+2ac\right)x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

```
[Out] 2/7*a^2*x^(7/2)+4/11*a*b*x^(11/2)+2/15*(2*a*c+b^2)*x^(15/2)+4/19*b*c*x^(19/
2)+2/23*c^2*x^(23/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1122}

$$\frac{2}{7}a^2x^{7/2}+\frac{2}{15}x^{15/2}\left(2ac+b^2\right)+\frac{4}{11}abx^{11/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/2))/15 + (
4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx&=\int\left(a^2x^{5/2}+2abx^{9/2}+\left(b^2+2ac\right)x^{13/2}+2bcx^{17/2}+c^2x^{21/2}\right)\,dx\\&=\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}\left(b^2+2ac\right)x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.97

$$\frac{2\left(72105a^2x^{7/2}+91770abx^{11/2}+33649b^2x^{15/2}+67298acx^{15/2}+53130bcx^{19/2}+21945c^2x^{23/2}\right)}{504735}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*(72105*a^2*x^(7/2) + 91770*a*b*x^(11/2) + 33649*b^2*x^(15/2) + 67298*a*c
*x^(15/2) + 53130*b*c*x^(19/2) + 21945*c^2*x^(23/2)))/504735
```

Maple [A]
time = 0.05, size = 45, normalized size = 0.70

method	result	size
derivativedivides	$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{11}{2}}}{11}+\frac{2(2ac+b^2)x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$	45
default	$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{11}{2}}}{11}+\frac{2(2ac+b^2)x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$	45
gosper	$\frac{2x^{\frac{7}{2}}(21945c^2x^8+53130bc\,x^6+67298c\,x^4a+33649b^2x^4+91770ab\,x^2+72105a^2)}{504735}$	49
trager	$\frac{2x^{\frac{7}{2}}(21945c^2x^8+53130bc\,x^6+67298c\,x^4a+33649b^2x^4+91770ab\,x^2+72105a^2)}{504735}$	49
risch	$\frac{2x^{\frac{7}{2}}(21945c^2x^8+53130bc\,x^6+67298c\,x^4a+33649b^2x^4+91770ab\,x^2+72105a^2)}{504735}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*a^2*x^(7/2)+4/11*a*b*x^(11/2)+2/15*(2*a*c+b^2)*x^(15/2)+4/19*b*c*x^(19/
2)+2/23*c^2*x^(23/2)
```

Maxima [A]
time = 0.30, size = 44, normalized size = 0.69

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,(b^2+2\,ac)x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*(b^2 + 2*a*c)*x^(15/2) + 4/11*
a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

Fricas [A]
time = 0.34, size = 49, normalized size = 0.77

$$\frac{2}{504735}\left(21945\,c^2x^{11}+53130\,bcx^9+33649\,(b^2+2\,ac)x^7+91770\,abx^5+72105\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*c^2*x^11 + 53130*b*c*x^9 + 33649*(b^2 + 2*a*c)*x^7 + 91770*
a*b*x^5 + 72105*a^2*x^3)*sqrt(x)
```

Sympy [A]
time = 1.15, size = 70, normalized size = 1.09

$$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{4acx^{\frac{15}{2}}}{15}+\frac{2b^2x^{\frac{15}{2}}}{15}+\frac{4bcx^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 4*a*c*x**(15/2)/15 + 2*b**2*x**(15
/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

Giac [A]
time = 3.45, size = 46, normalized size = 0.72

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}+\frac{4}{15}\,acx^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2) + 4/15*a*c*x^(15/
2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

Mupad [B]
time = 4.41, size = 45, normalized size = 0.70

$$x^{15/2}\left(\frac{2\,b^2}{15}+\frac{4\,a\,c}{15}\right)+\frac{2\,a^2\,x^{7/2}}{7}+\frac{2\,c^2\,x^{23/2}}{23}+\frac{4\,a\,b\,x^{11/2}}{11}+\frac{4\,b\,c\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(15/2)*((4*a*c)/15 + (2*b^2)/15) + (2*a^2*x^(7/2))/7 + (2*c^2*x^(23/2))/2
3 + (4*a*b*x^(11/2))/11 + (4*b*c*x^(19/2))/19
```

3.1049 $\int x^{3/2}(a+bx^2+cx^4)^2\,dx$

Optimal. Leaf size=64

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}\left(b^2+2ac\right)x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

```
[Out] 2/5*a^2*x^(5/2)+4/9*a*b*x^(9/2)+2/13*(2*a*c+b^2)*x^(13/2)+4/17*b*c*x^(17/2)
+2/21*c^2*x^(21/2)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {1122}

$$\frac{2}{5}a^2x^{5/2}+\frac{2}{13}x^{13/2}\left(2ac+b^2\right)+\frac{4}{9}abx^{9/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*(b^2 + 2*a*c)*x^(13/2))/13 + (4*
b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx&=\int\left(a^2x^{3/2}+2abx^{7/2}+\left(b^2+2ac\right)x^{11/2}+2bcx^{15/2}+c^2x^{19/2}\right)dx\\&=\frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}\left(b^2+2ac\right)x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.89

$$\frac{2x^{5/2}(13923a^2+1190a(13bx^2+9cx^4)+15x^4(357b^2+546bcx^2+221c^2x^4))}{69615}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(5/2)*(13923*a^2 + 1190*a*(13*b*x^2 + 9*c*x^4) + 15*x^4*(357*b^2 + 546
*b*c*x^2 + 221*c^2*x^4)))/69615
```

Maple [A]
time = 0.05, size = 45, normalized size = 0.70

method	result	size
derivativedivides	$\frac{2a^{2}x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2(2ac+b^{2})x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{17}{2}}}{17}+\frac{2c^{2}x^{\frac{21}{2}}}{21}$	45
default	$\frac{2a^{2}x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2(2ac+b^{2})x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{17}{2}}}{17}+\frac{2c^{2}x^{\frac{21}{2}}}{21}$	45
gosper	$\frac{2x^{\frac{5}{2}}\left(3315c^{2}x^{8}+8190bc\,x^{6}+10710c\,x^{4}a+5355b^{2}x^{4}+15470ab\,x^{2}+13923a^{2}\right)}{69615}$	49
trager	$\frac{2x^{\frac{5}{2}}\left(3315c^{2}x^{8}+8190bc\,x^{6}+10710c\,x^{4}a+5355b^{2}x^{4}+15470ab\,x^{2}+13923a^{2}\right)}{69615}$	49
risch	$\frac{2x^{\frac{5}{2}}\left(3315c^{2}x^{8}+8190bc\,x^{6}+10710c\,x^{4}a+5355b^{2}x^{4}+15470ab\,x^{2}+13923a^{2}\right)}{69615}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*a^2*x^(5/2)+4/9*a*b*x^(9/2)+2/13*(2*a*c+b^2)*x^(13/2)+4/17*b*c*x^(17/2)
+2/21*c^2*x^(21/2)
```

Maxima [A]
time = 0.30, size = 44, normalized size = 0.69

$$\frac{2}{21}\,c^{2}x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,\left(b^{2}+2\,ac\right)x^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^{2}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*(b^2 + 2*a*c)*x^(13/2) + 4/9*a
*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

Fricas [A]
time = 0.32, size = 49, normalized size = 0.77

$$\frac{2}{69615}\,\left(3315\,c^{2}x^{10}+8190\,bcx^{8}+5355\,\left(b^{2}+2\,ac\right)x^{6}+15470\,abx^{4}+13923\,a^{2}x^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*c^2*x^10 + 8190*b*c*x^8 + 5355*(b^2 + 2*a*c)*x^6 + 15470*a*b*
x^4 + 13923*a^2*x^2)*sqrt(x)
```

Sympy [A]
time = 0.74, size = 70, normalized size = 1.09

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{4acx^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 4*a*c*x**(13/2)/13 + 2*b**2*x**(13/2
)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

Giac [A]
time = 3.92, size = 46, normalized size = 0.72

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{13}\,acx^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2) + 4/13*a*c*x^(13/
2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

Mupad [B]
time = 0.03, size = 45, normalized size = 0.70

$$x^{13/2}\left(\frac{2\,b^2}{13}+\frac{4\,a\,c}{13}\right)+\frac{2\,a^2\,x^{5/2}}{5}+\frac{2\,c^2\,x^{21/2}}{21}+\frac{4\,a\,b\,x^{9/2}}{9}+\frac{4\,b\,c\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(13/2)*((4*a*c)/13 + (2*b^2)/13) + (2*a^2*x^(5/2))/5 + (2*c^2*x^(21/2))/2
1 + (4*a*b*x^(9/2))/9 + (4*b*c*x^(17/2))/17
```

3.1050 $\int \sqrt{x}\,(a+bx^2+cx^4)^2\,dx$

Optimal. Leaf size=64

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}\left(b^2+2ac\right)x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

```
[Out] 2/3*a^2*x^(3/2)+4/7*a*b*x^(7/2)+2/11*(2*a*c+b^2)*x^(11/2)+4/15*b*c*x^(15/2)
+2/19*c^2*x^(19/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1122}

$$\frac{2}{3}a^2x^{3/2}+\frac{2}{11}x^{11/2}\left(2ac+b^2\right)+\frac{4}{7}abx^{7/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*(b^2 + 2*a*c)*x^(11/2))/11 + (4*
b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2\sqrt{x}+2abx^{5/2}+\left(b^2+2ac\right)x^{9/2}+2bcx^{13/2}+c^2x^{17/2}\right)dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}\left(b^2+2ac\right)x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.89

$$\frac{2x^{3/2}(7315a^2+570a(11bx^2+7cx^4)+7(285b^2x^4+418bcx^6+165c^2x^8))}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*(7315*a^2 + 570*a*(11*b*x^2 + 7*c*x^4) + 7*(285*b^2*x^4 + 418*b*
c*x^6 + 165*c^2*x^8)))/21945
```

Maple [A]
time = 0.06, size = 45, normalized size = 0.70

method	result	size
derivativedivides	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2(2ac+b^2)x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}$	45
default	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2(2ac+b^2)x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}$	45
gosper	$\frac{2x^{\frac{3}{2}}(1155c^2x^8+2926bc\,x^6+3990c\,x^4a+1995b^2x^4+6270ab\,x^2+7315a^2)}{21945}$	49
trager	$\frac{2x^{\frac{3}{2}}(1155c^2x^8+2926bc\,x^6+3990c\,x^4a+1995b^2x^4+6270ab\,x^2+7315a^2)}{21945}$	49
risch	$\frac{2x^{\frac{3}{2}}(1155c^2x^8+2926bc\,x^6+3990c\,x^4a+1995b^2x^4+6270ab\,x^2+7315a^2)}{21945}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a^2*x^(3/2)+4/7*a*b*x^(7/2)+2/11*(2*a*c+b^2)*x^(11/2)+4/15*b*c*x^(15/2)
+2/19*c^2*x^(19/2)
```

Maxima [A]
time = 0.30, size = 44, normalized size = 0.69

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\left(b^2+2\,ac\right)x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*(b^2 + 2*a*c)*x^(11/2) + 4/7*a
*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

Fricas [A]
time = 0.32, size = 47, normalized size = 0.73

$$\frac{2}{21945}\left(1155\,c^2x^9+2926\,bcx^7+1995\left(b^2+2\,ac\right)x^5+6270\,abx^3+7315\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*c^2*x^9 + 2926*b*c*x^7 + 1995*(b^2 + 2*a*c)*x^5 + 6270*a*b*x^
3 + 7315*a^2*x)*sqrt(x)
```

Sympy [A]
time = 1.56, size = 63, normalized size = 0.98

$$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}+\frac{2x^{\frac{11}{2}}\cdot(2ac+b^2)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/2
)/19 + 2*x**(11/2)*(2*a*c + b**2)/11
```

Giac [A]
time = 3.28, size = 46, normalized size = 0.72

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{11}\,acx^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2) + 4/11*a*c*x^(11/
2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

Mupad [B]
time = 0.03, size = 45, normalized size = 0.70

$$x^{11/2}\left(\frac{2\,b^2}{11}+\frac{4\,a\,c}{11}\right)+\frac{2\,a^2\,x^{3/2}}{3}+\frac{2\,c^2\,x^{19/2}}{19}+\frac{4\,a\,b\,x^{7/2}}{7}+\frac{4\,b\,c\,x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(11/2)*((4*a*c)/11 + (2*b^2)/11) + (2*a^2*x^(3/2))/3 + (2*c^2*x^(19/2))/1
9 + (4*a*b*x^(7/2))/7 + (4*b*c*x^(15/2))/15
```

3.1051 $\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}\left(b^2+2ac\right)x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

```
[Out] 4/5*a*b*x^(5/2)+2/9*(2*a*c+b^2)*x^(9/2)+4/13*b*c*x^(13/2)+2/17*c^2*x^(17/2)
+2*a^2*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1122}

$$2a^2\sqrt{x}+\frac{2}{9}x^{9/2}\left(2ac+b^2\right)+\frac{4}{5}abx^{5/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*(b^2 + 2*a*c)*x^(9/2))/9 + (4*b*c*x^
(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2}{\sqrt{x}}+2abx^{3/2}+\left(b^2+2ac\right)x^{7/2}+2bcx^{11/2}+c^2x^{15/2}\right)dx\\ &= 2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}\left(b^2+2ac\right)x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.92

$$\frac{2\sqrt{x}\,(9945a^2+442a(9bx^2+5cx^4)+5x^4(221b^2+306bcx^2+117c^2x^4))}{9945}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(9945*a^2 + 442*a*(9*b*x^2 + 5*c*x^4) + 5*x^4*(221*b^2 + 306*b*c
*x^2 + 117*c^2*x^4)))/9945
```

Maple [A]
time = 0.03, size = 45, normalized size = 0.73

method	result	size
derivativedivides	$\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2(2ac+b^{2})x^{\frac{9}{2}}}{9}+\frac{4bc\,x^{\frac{13}{2}}}{13}+\frac{2c^{2}x^{\frac{17}{2}}}{17}+2a^{2}\sqrt{x}$	45
default	$\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2(2ac+b^{2})x^{\frac{9}{2}}}{9}+\frac{4bc\,x^{\frac{13}{2}}}{13}+\frac{2c^{2}x^{\frac{17}{2}}}{17}+2a^{2}\sqrt{x}$	45
trager	$\left(\frac{2}{17}c^{2}x^{8}+\frac{4}{13}bc\,x^{6}+\frac{4}{9}c\,x^{4}a+\frac{2}{9}b^{2}x^{4}+\frac{4}{5}ab\,x^{2}+2a^{2}\right)\sqrt{x}$	48
gosper	$\frac{2\sqrt{x}\,\left(585c^{2}x^{8}+1530bc\,x^{6}+2210c\,x^{4}a+1105b^{2}x^{4}+3978ab\,x^{2}+9945a^{2}\right)}{9945}$	49
risch	$\frac{2\sqrt{x}\,\left(585c^{2}x^{8}+1530bc\,x^{6}+2210c\,x^{4}a+1105b^{2}x^{4}+3978ab\,x^{2}+9945a^{2}\right)}{9945}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/5*a*b*x^(5/2)+2/9*(2*a*c+b^2)*x^(9/2)+4/13*b*c*x^(13/2)+2/17*c^2*x^(17/2)
+2*a^2*x^(1/2)
```

Maxima [A]
time = 0.33, size = 48, normalized size = 0.77

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^2x^{\frac{9}{2}}+2\,a^2\sqrt{x}+\frac{4}{45}\left(5\,cx^{\frac{9}{2}}+9\,bx^{\frac{5}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 2*a^2*sqrt(x) + 4
/45*(5*c*x^(9/2) + 9*b*x^(5/2))*a
```

Fricas [A]
time = 0.35, size = 46, normalized size = 0.74

$$\frac{2}{9945}\left(585\,c^2x^8+1530\,bcx^6+1105\,\left(b^2+2\,ac\right)x^4+3978\,abx^2+9945\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*c^2*x^8 + 1530*b*c*x^6 + 1105*(b^2 + 2*a*c)*x^4 + 3978*a*b*x^2
+ 9945*a^2)*sqrt(x)
```

Sympy [A]
time = 0.47, size = 68, normalized size = 1.10

$$2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{4acx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{9}{2}}}{9} + \frac{4bcx^{\frac{13}{2}}}{13} + \frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 4*a*c*x**(9/2)/9 + 2*b**2*x**(9/2)/9 +
4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

Giac [A]
time = 3.48, size = 46, normalized size = 0.74

$$\frac{2}{17}\,c^2x^{\frac{17}{2}} + \frac{4}{13}\,bcx^{\frac{13}{2}} + \frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{9}\,acx^{\frac{9}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 4/9*a*c*x^(9/2) +
 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

Mupad [B]
time = 0.03, size = 45, normalized size = 0.73

$$x^{9/2}\left(\frac{2\,b^2}{9} + \frac{4\,a\,c}{9}\right) + 2\,a^2\,\sqrt{x} + \frac{2\,c^2\,x^{17/2}}{17} + \frac{4\,a\,b\,x^{5/2}}{5} + \frac{4\,b\,c\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(1/2),x)
```

```
[Out] x^(9/2)*((4*a*c)/9 + (2*b^2)/9) + 2*a^2*x^(1/2) + (2*c^2*x^(17/2))/17 + (4*
a*b*x^(5/2))/5 + (4*b*c*x^(13/2))/13
```

3.1052 $\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}} \, dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}\left(b^2+2ac\right)x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

```
[Out] 4/3*a*b*x^(3/2)+2/7*(2*a*c+b^2)*x^(7/2)+4/11*b*c*x^(11/2)+2/15*c^2*x^(15/2)
-2*a^2/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1122}

$$-\frac{2a^2}{\sqrt{x}}+\frac{2}{7}x^{7/2}\left(2ac+b^2\right)+\frac{4}{3}abx^{3/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*(b^2 + 2*a*c)*x^(7/2))/7 + (4*b*c
*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2}{x^{3/2}}+2ab\sqrt{x}+\left(b^2+2ac\right)x^{5/2}+2bcx^{9/2}+c^2x^{13/2}\right)dx\\ &= -\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}\left(b^2+2ac\right)x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.84

$$-\frac{2(1155a^2-770abx^2-165b^2x^4-330acx^4-210bcx^6-77c^2x^8)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (-2*(1155*a^2 - 770*a*b*x^2 - 165*b^2*x^4 - 330*a*c*x^4 - 210*b*c*x^6 - 77*
c^2*x^8))/(1155*Sqrt[x])
```

Maple [A]
time = 0.04, size = 47, normalized size = 0.76

method	result	size
derivativedivides	$\frac{2c^2x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{11}{2}}}{11}+\frac{4ac\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{3}{2}}}{3}-\frac{2a^2}{\sqrt{x}}$	47
default	$\frac{2c^2x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{11}{2}}}{11}+\frac{4ac\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{3}{2}}}{3}-\frac{2a^2}{\sqrt{x}}$	47
gosper	$-\frac{2\left(-77c^2x^8-210bc\,x^6-330c\,x^4a-165b^2x^4-770ab\,x^2+1155a^2\right)}{1155\sqrt{x}}$	49
trager	$-\frac{2\left(-77c^2x^8-210bc\,x^6-330c\,x^4a-165b^2x^4-770ab\,x^2+1155a^2\right)}{1155\sqrt{x}}$	49
risch	$-\frac{2\left(-77c^2x^8-210bc\,x^6-330c\,x^4a-165b^2x^4-770ab\,x^2+1155a^2\right)}{1155\sqrt{x}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*c^2*x^(15/2)+4/11*b*c*x^(11/2)+4/7*a*c*x^(7/2)+2/7*b^2*x^(7/2)+4/3*a*b
*x^(3/2)-2*a^2/x^(1/2)
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.71

$$\frac{2}{15}\,c^2x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\left(b^2+2\,ac\right)x^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*(b^2 + 2*a*c)*x^(7/2) + 4/3*a*b
*x^(3/2) - 2*a^2/sqrt(x)
```

Fricas [A]
time = 0.33, size = 46, normalized size = 0.74

$$\frac{2\left(77\,c^2x^8+210\,bcx^6+165\,(b^2+2\,ac)x^4+770\,abx^2-1155\,a^2\right)}{1155\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*c^2*x^8 + 210*b*c*x^6 + 165*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2 - 11
55*a^2)/sqrt(x)
```

Sympy [A]
time = 0.63, size = 68, normalized size = 1.10

$$-\frac{2a^2}{\sqrt{x}} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{4acx^{\frac{7}{2}}}{7} + \frac{2b^2x^{\frac{7}{2}}}{7} + \frac{4bcx^{\frac{11}{2}}}{11} + \frac{2c^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(3/2),x)
```

```
[Out] -2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 4*a*c*x**(7/2)/7 + 2*b**2*x**(7/2)/7 +
 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

Giac [A]
time = 4.51, size = 46, normalized size = 0.74

$$\frac{2}{15}\,c^2x^{\frac{15}{2}} + \frac{4}{11}\,bcx^{\frac{11}{2}} + \frac{2}{7}\,b^2x^{\frac{7}{2}} + \frac{4}{7}\,acx^{\frac{7}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} - \frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2) + 4/7*a*c*x^(7/2) +
 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

Mupad [B]
time = 0.03, size = 45, normalized size = 0.73

$$x^{7/2}\left(\frac{2\,b^2}{7}+\frac{4\,a\,c}{7}\right)-\frac{2\,a^2}{\sqrt{x}}+\frac{2\,c^2\,x^{15/2}}{15}+\frac{4\,a\,b\,x^{3/2}}{3}+\frac{4\,b\,c\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(3/2),x)
```

```
[Out] x^(7/2)*((4*a*c)/7 + (2*b^2)/7) - (2*a^2)/x^(1/2) + (2*c^2*x^(15/2))/15 + (
4*a*b*x^(3/2))/3 + (4*b*c*x^(11/2))/11
```

3.1053 $\int \frac{(a+bx^2+cx^4)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}\left(b^2+2ac\right)x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

```
[Out] -2/3*a^2/x^(3/2)+2/5*(2*a*c+b^2)*x^(5/2)+4/9*b*c*x^(9/2)+2/13*c^2*x^(13/2)+
4*a*b*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {1122}

$$-\frac{2a^2}{3x^{3/2}}+\frac{2}{5}x^{5/2}\left(2ac+b^2\right)+4ab\sqrt{x}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*(b^2 + 2*a*c)*x^(5/2))/5 + (4*b*c
*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{\sqrt{x}}+\left(b^2+2ac\right)x^{3/2}+2bcx^{7/2}+c^2x^{11/2}\right)dx\\ &= -\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}\left(b^2+2ac\right)x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.84

$$-\frac{2(195a^2-1170abx^2-117b^2x^4-234acx^4-130bcx^6-45c^2x^8)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (-2*(195*a^2 - 1170*a*b*x^2 - 117*b^2*x^4 - 234*a*c*x^4 - 130*b*c*x^6 - 45*
c^2*x^8))/(585*x^(3/2))
```

Maple [A]
time = 0.04, size = 47, normalized size = 0.76

method	result	size
derivativedivides	$\frac{2c^{2}x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{9}{2}}}{9}+\frac{4ac\,x^{\frac{5}{2}}}{5}+\frac{2b^{2}x^{\frac{5}{2}}}{5}+4ab\sqrt{x}-\frac{2a^{2}}{3x^{\frac{3}{2}}}$	47
default	$\frac{2c^{2}x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{9}{2}}}{9}+\frac{4ac\,x^{\frac{5}{2}}}{5}+\frac{2b^{2}x^{\frac{5}{2}}}{5}+4ab\sqrt{x}-\frac{2a^{2}}{3x^{\frac{3}{2}}}$	47
gosper	$-\frac{2\left(-45c^{2}x^{8}-130bc\,x^{6}-234c\,x^{4}a-117b^{2}x^{4}-1170ab\,x^{2}+195a^{2}\right)}{585x^{\frac{3}{2}}}$	49
trager	$-\frac{2\left(-45c^{2}x^{8}-130bc\,x^{6}-234c\,x^{4}a-117b^{2}x^{4}-1170ab\,x^{2}+195a^{2}\right)}{585x^{\frac{3}{2}}}$	49
risch	$-\frac{2\left(-45c^{2}x^{8}-130bc\,x^{6}-234c\,x^{4}a-117b^{2}x^{4}-1170ab\,x^{2}+195a^{2}\right)}{585x^{\frac{3}{2}}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13*c^2*x^(13/2)+4/9*b*c*x^(9/2)+4/5*a*c*x^(5/2)+2/5*b^2*x^(5/2)+4*a*b*x^(
1/2)-2/3*a^2/x^(3/2)
```

Maxima [A]
time = 0.31, size = 44, normalized size = 0.71

$$\frac{2}{13}\,c^{2}x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\,\left(b^{2}+2\,ac\right)x^{\frac{5}{2}}+4\,ab\sqrt{x}-\frac{2\,a^{2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*(b^2 + 2*a*c)*x^(5/2) + 4*a*b*sqr
t(x) - 2/3*a^2/x^(3/2)
```

Fricas [A]
time = 0.34, size = 46, normalized size = 0.74

$$\frac{2\,\left(45\,c^{2}x^{8}+130\,bcx^{6}+117\,\left(b^{2}+2\,ac\right)x^{4}+1170\,abx^{2}-195\,a^{2}\right)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*c^2*x^8 + 130*b*c*x^6 + 117*(b^2 + 2*a*c)*x^4 + 1170*a*b*x^2 - 19
5*a^2)/x^(3/2)
```

Sympy [A]
time = 0.70, size = 68, normalized size = 1.10

$$-\frac{2a^2}{3x^{\frac{3}{2}}} + 4ab\sqrt{x} + \frac{4acx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{5}{2}}}{5} + \frac{4bcx^{\frac{9}{2}}}{9} + \frac{2c^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 4*a*c*x**(5/2)/5 + 2*b**2*x**(5/2)/5
 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

Giac [A]
time = 3.66, size = 46, normalized size = 0.74

$$\frac{2}{13}\,c^2x^{\frac{13}{2}} + \frac{4}{9}\,bcx^{\frac{9}{2}} + \frac{2}{5}\,b^2x^{\frac{5}{2}} + \frac{4}{5}\,acx^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2) + 4/5*a*c*x^(5/2) + 4
*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

Mupad [B]
time = 0.03, size = 45, normalized size = 0.73

$$x^{5/2}\left(\frac{2\,b^2}{5} + \frac{4\,a\,c}{5}\right) - \frac{2\,a^2}{3\,x^{3/2}} + \frac{2\,c^2\,x^{13/2}}{13} + 4\,a\,b\,\sqrt{x} + \frac{4\,b\,c\,x^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(5/2),x)
```

```
[Out] x^(5/2)*((4*a*c)/5 + (2*b^2)/5) - (2*a^2)/(3*x^(3/2)) + (2*c^2*x^(13/2))/13
 + 4*a*b*x^(1/2) + (4*b*c*x^(9/2))/9
```

3.1054 $\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}\left(b^2+2ac\right)x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

```
[Out] -2/5*a^2/x^(5/2)+2/3*(2*a*c+b^2)*x^(3/2)+4/7*b*c*x^(7/2)+2/11*c^2*x^(11/2)-
4*a*b/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {1122}

$$-\frac{2a^2}{5x^{5/2}}+\frac{2}{3}x^{3/2}\left(2ac+b^2\right)-\frac{4ab}{\sqrt{x}}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*(b^2 + 2*a*c)*x^(3/2))/3 + (4*b
*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{7/2}}\,dx &= \int \left(\frac{a^2}{x^{7/2}}+\frac{2ab}{x^{3/2}}+\left(b^2+2ac\right)\sqrt{x}+2bcx^{5/2}+c^2x^{9/2}\right)dx\\ &= -\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}\left(b^2+2ac\right)x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 0.84

$$-\frac{2(231a^2+2310abx^2-385b^2x^4-770acx^4-330bcx^6-105c^2x^8)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (-2*(231*a^2 + 2310*a*b*x^2 - 385*b^2*x^4 - 770*a*c*x^4 - 330*b*c*x^6 - 105
*c^2*x^8))/(1155*x^(5/2))
```

Maple [A]
time = 0.04, size = 47, normalized size = 0.76

method	result	size
derivativedivides	$\frac{2c^2x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{7}{2}}}{7}+\frac{4ac\,x^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{3}{2}}}{3}-\frac{4ab}{\sqrt{x}}-\frac{2a^2}{5x^{\frac{5}{2}}}$	47
default	$\frac{2c^2x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{7}{2}}}{7}+\frac{4ac\,x^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{3}{2}}}{3}-\frac{4ab}{\sqrt{x}}-\frac{2a^2}{5x^{\frac{5}{2}}}$	47
gosper	$-\frac{2\left(-105c^2x^8-330bc\,x^6-770c\,x^4a-385b^2x^4+2310ab\,x^2+231a^2\right)}{1155x^{\frac{5}{2}}}$	49
trager	$-\frac{2\left(-105c^2x^8-330bc\,x^6-770c\,x^4a-385b^2x^4+2310ab\,x^2+231a^2\right)}{1155x^{\frac{5}{2}}}$	49
risch	$-\frac{2\left(-105c^2x^8-330bc\,x^6-770c\,x^4a-385b^2x^4+2310ab\,x^2+231a^2\right)}{1155x^{\frac{5}{2}}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*c^2*x^(11/2)+4/7*b*c*x^(7/2)+4/3*a*c*x^(3/2)+2/3*b^2*x^(3/2)-4*a*b/x^(
1/2)-2/5*a^2/x^(5/2)
```

Maxima [A]
time = 0.30, size = 45, normalized size = 0.73

$$\frac{2}{11}\,c^2x^{\frac{11}{2}}+\frac{4}{7}\,bcx^{\frac{7}{2}}+\frac{2}{3}\,\left(b^2+2\,ac\right)x^{\frac{3}{2}}-\frac{2\,\left(10\,abx^2+a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*(b^2 + 2*a*c)*x^(3/2) - 2/5*(10*a
*b*x^2 + a^2)/x^(5/2)
```

Fricas [A]
time = 0.34, size = 46, normalized size = 0.74

$$\frac{2\,\left(105\,c^2x^8+330\,bcx^6+385\,\left(b^2+2\,ac\right)x^4-2310\,abx^2-231\,a^2\right)}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^2*x^8 + 330*b*c*x^6 + 385*(b^2 + 2*a*c)*x^4 - 2310*a*b*x^2 -
231*a^2)/x^(5/2)
```

Sympy [A]
time = 0.87, size = 68, normalized size = 1.10

$$-\frac{2a^2}{5x^{\frac{5}{2}}} - \frac{4ab}{\sqrt{x}} + \frac{4acx^{\frac{3}{2}}}{3} + \frac{2b^2x^{\frac{3}{2}}}{3} + \frac{4bcx^{\frac{7}{2}}}{7} + \frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/x**(7/2),x)
```

```
[Out] -2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 4*a*c*x**(3/2)/3 + 2*b**2*x**(3/2)/3
 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

Giac [A]
time = 3.70, size = 47, normalized size = 0.76

$$\frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{7}\,bcx^{\frac{7}{2}} + \frac{2}{3}\,b^2x^{\frac{3}{2}} + \frac{4}{3}\,acx^{\frac{3}{2}} - \frac{2\,(10\,abx^2 + a^2)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2) + 4/3*a*c*x^(3/2) - 2
/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.77

$$x^{3/2}\left(\frac{2\,b^2}{3} + \frac{4\,a\,c}{3}\right) - \frac{\frac{2\,a^2}{5} + 4\,b\,a\,x^2}{x^{5/2}} + \frac{2\,c^2\,x^{11/2}}{11} + \frac{4\,b\,c\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/x^(7/2),x)
```

```
[Out] x^(3/2)*((4*a*c)/3 + (2*b^2)/3) - ((2*a^2)/5 + 4*a*b*x^2)/x^(5/2) + (2*c^2*
x^(11/2))/11 + (4*b*c*x^(7/2))/7
```

3.1055 $\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}a\left(b^2+ac\right)x^{15/2}+\frac{2}{19}b\left(b^2+6ac\right)x^{19/2}+\frac{6}{23}c\left(b^2+ac\right)x^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

```
[Out] 2/7*a^3*x^(7/2)+6/11*a^2*b*x^(11/2)+2/5*a*(a*c+b^2)*x^(15/2)+2/19*b*(6*a*c+
b^2)*x^(19/2)+6/23*c*(a*c+b^2)*x^(23/2)+2/9*b*c^2*x^(27/2)+2/31*c^3*x^(31/2
)
```

Rubi [A]
time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1122}

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(15/2))/5 +
(2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(23/2))/23 + (2*b*c^2*
x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3x^{5/2}+3a^2bx^{9/2}+3a\left(b^2+ac\right)x^{13/2}+b\left(b^2+6ac\right)x^{17/2}+3c\left(b^2+ac\right)x^{21}\right.\\ &= \frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}a\left(b^2+ac\right)x^{15/2}+\frac{2}{19}b\left(b^2+6ac\right)x^{19/2}+\frac{6}{23}c\left(b^2+a\right.\end{aligned}$$

Mathematica [A]
time = 0.05, size = 111, normalized size = 1.08

$$\frac{2\left(6705765a^3x^{7/2}+12801915a^2bx^{11/2}+9388071ab^2x^{15/2}+9388071a^2cx^{15/2}+2470545b^3x^{19/2}+14823270abcx^{19/2}+6122655b^2cx^{23/2}+6122655ac^2x^{23/2}+5215595bc^2x^{27/2}+1514205c^3x^{31/2}\right)}{46940355}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(6705765*a^3*x^(7/2) + 12801915*a^2*b*x^(11/2) + 9388071*a*b^2*x^(15/2)
+ 9388071*a^2*c*x^(15/2) + 2470545*b^3*x^(19/2) + 14823270*a*b*c*x^(19/2) +
 6122655*b^2*c*x^(23/2) + 6122655*a*c^2*x^(23/2) + 5215595*b*c^2*x^(27/2) +
 1514205*c^3*x^(31/2)))/46940355
```

Maple [A]
time = 0.06, size = 111, normalized size = 1.08

method	result
gosper	$\frac{2x^{\frac{7}{2}}\left(1514205c^3x^{12}+5215595b\,c^2x^{10}+6122655x^8c^2a+6122655x^8b^2c+14823270abc\,x^6+2470545b^3x^6+9388071x^4a^2c+9\right.}{46940355}$
trager	$\frac{2x^{\frac{7}{2}}\left(1514205c^3x^{12}+5215595b\,c^2x^{10}+6122655x^8c^2a+6122655x^8b^2c+14823270abc\,x^6+2470545b^3x^6+9388071x^4a^2c+9\right.}{46940355}$
risch	$\frac{2x^{\frac{7}{2}}\left(1514205c^3x^{12}+5215595b\,c^2x^{10}+6122655x^8c^2a+6122655x^8b^2c+14823270abc\,x^6+2470545b^3x^6+9388071x^4a^2c+9\right.}{46940355}$
derivativedivides	$\frac{2c^3x^{\frac{31}{2}}}{31}+\frac{2b\,c^2x^{\frac{27}{2}}}{9}+\frac{2\left(c^2a+2b^2c+c\left(2ac+b^2\right)\right)x^{\frac{23}{2}}}{23}+\frac{2\left(4abc+b\left(2ac+b^2\right)\right)x^{\frac{19}{2}}}{19}+\frac{2\left(a\left(2ac+b^2\right)+2a\,b^2+a^2c\right)x^{\frac{15}{2}}}{15}$
default	$\frac{2c^3x^{\frac{31}{2}}}{31}+\frac{2b\,c^2x^{\frac{27}{2}}}{9}+\frac{2\left(c^2a+2b^2c+c\left(2ac+b^2\right)\right)x^{\frac{23}{2}}}{23}+\frac{2\left(4abc+b\left(2ac+b^2\right)\right)x^{\frac{19}{2}}}{19}+\frac{2\left(a\left(2ac+b^2\right)+2a\,b^2+a^2c\right)x^{\frac{15}{2}}}{15}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/31*c^3*x^(31/2)+2/9*b*c^2*x^(27/2)+2/23*(c^2*a+2*b^2*c+c*(2*a*c+b^2))*x^(
23/2)+2/19*(4*a*b*c+b*(2*a*c+b^2))*x^(19/2)+2/15*(a*(2*a*c+b^2)+2*a*b^2+a^2
*c)*x^(15/2)+6/11*a^2*b*x^(11/2)+2/7*a^3*x^(7/2)
```

Maxima [A]
time = 0.30, size = 81, normalized size = 0.79

$$\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,\left(b^2c+ac^2\right)x^{\frac{23}{2}}+\frac{2}{19}\,\left(b^3+6\,abc\right)x^{\frac{19}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{5}\,\left(ab^2+a^2c\right)x^{\frac{15}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*(b^2*c + a*c^2)*x^(23/2) + 2/
19*(b^3 + 6*a*b*c)*x^(19/2) + 6/11*a^2*b*x^(11/2) + 2/5*(a*b^2 + a^2*c)*x^(
15/2) + 2/7*a^3*x^(7/2)
```

Fricas [A]
time = 0.37, size = 86, normalized size = 0.83

$$\frac{2}{46940355}\,\left(1514205\,c^3x^{15}+5215595\,bc^2x^{13}+6122655\,\left(b^2c+ac^2\right)x^{11}+2470545\,\left(b^3+6\,abc\right)x^9+12801915\,a^2bx^5+9388071\,\left(ab^2+a^2c\right)x^7+6705765\,a^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/46940355*(1514205*c^3*x^15 + 5215595*b*c^2*x^13 + 6122655*(b^2*c + a*c^2)
*x^11 + 2470545*(b^3 + 6*a*b*c)*x^9 + 12801915*a^2*b*x^5 + 9388071*(a*b^2 +
 a^2*c)*x^7 + 6705765*a^3*x^3)*sqrt(x)
```

Sympy [A]
time = 2.15, size = 129, normalized size = 1.25

$$\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{6a^2bx^{\frac{11}{2}}}{11}+\frac{2a^2cx^{\frac{15}{2}}}{5}+\frac{2ab^2x^{\frac{15}{2}}}{5}+\frac{12abcx^{\frac{19}{2}}}{19}+\frac{6ac^2x^{\frac{23}{2}}}{23}+\frac{2b^3x^{\frac{19}{2}}}{19}+\frac{6b^2cx^{\frac{23}{2}}}{23}+\frac{2bc^2x^{\frac{27}{2}}}{9}+\frac{2c^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a**2*c*x**(15/2)/5 + 2*a*b**2
*x**(15/2)/5 + 12*a*b*c*x**(19/2)/19 + 6*a*c**2*x**(23/2)/23 + 2*b**3*x**(1
9/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/9 + 2*c**3*x**(31/2)/3
1
```

Giac [A]
time = 3.28, size = 87, normalized size = 0.84

$$\frac{2}{31}c^3x^{\frac{31}{2}}+\frac{2}{9}bc^2x^{\frac{27}{2}}+\frac{6}{23}b^2cx^{\frac{23}{2}}+\frac{6}{23}ac^2x^{\frac{23}{2}}+\frac{2}{19}b^3x^{\frac{19}{2}}+\frac{12}{19}abcx^{\frac{19}{2}}+\frac{2}{5}ab^2x^{\frac{15}{2}}+\frac{2}{5}a^2cx^{\frac{15}{2}}+\frac{6}{11}a^2bx^{\frac{11}{2}}+\frac{2}{7}a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 6/23*a*c^2*x
^(23/2) + 2/19*b^3*x^(19/2) + 12/19*a*b*c*x^(19/2) + 2/5*a*b^2*x^(15/2) + 2
/5*a^2*c*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7/2)
```

Mupad [B]
time = 0.04, size = 76, normalized size = 0.74

$$x^{19/2}\left(\frac{2\,b^3}{19}+\frac{12\,a\,c\,b}{19}\right)+\frac{2\,a^3\,x^{7/2}}{7}+\frac{2\,c^3\,x^{31/2}}{31}+\frac{6\,a^2\,b\,x^{11/2}}{11}+\frac{2\,b\,c^2\,x^{27/2}}{9}+\frac{2\,a\,x^{15/2}\,\left(b^2+a\,c\right)}{5}+\frac{6\,c\,x^{23/2}\,\left(b^2+a\,c\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(19/2)*((2*b^3)/19 + (12*a*b*c)/19) + (2*a^3*x^(7/2))/7 + (2*c^3*x^(31/2)
)/31 + (6*a^2*b*x^(11/2))/11 + (2*b*c^2*x^(27/2))/9 + (2*a*x^(15/2)*(a*c +
b^2))/5 + (6*c*x^(23/2)*(a*c + b^2))/23
```

3.1056 $\int x^{3/2}(a+bx^2+cx^4)^3\,dx$

Optimal. Leaf size=103

$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}a\left(b^2+ac\right)x^{13/2}+\frac{2}{17}b\left(b^2+6ac\right)x^{17/2}+\frac{2}{7}c\left(b^2+ac\right)x^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

```
[Out] 2/5*a^3*x^(5/2)+2/3*a^2*b*x^(9/2)+6/13*a*(a*c+b^2)*x^(13/2)+2/17*b*(6*a*c+b
^2)*x^(17/2)+2/7*c*(a*c+b^2)*x^(21/2)+6/25*b*c^2*x^(25/2)+2/29*c^3*x^(29/2)
```

Rubi [A]
time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {1122}

$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{2}{7}cx^{21/2}\left(ac+b^2\right)+\frac{2}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{6}{13}ax^{13/2}\left(ac+b^2\right)+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*(b^2 + a*c)*x^(13/2))/13 + (
2*b*(b^2 + 6*a*c)*x^(17/2))/17 + (2*c*(b^2 + a*c)*x^(21/2))/7 + (6*b*c^2*x^
(25/2))/25 + (2*c^3*x^(29/2))/29
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3x^{3/2}+3a^2bx^{7/2}+3a\left(b^2+ac\right)x^{11/2}+b\left(b^2+6ac\right)x^{15/2}+3c\left(b^2+ac\right)x\right.\\ &= \frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}a\left(b^2+ac\right)x^{13/2}+\frac{2}{17}b\left(b^2+6ac\right)x^{17/2}+\frac{2}{7}c\left(b^2+ac\right.\end{aligned}$$

Mathematica [A]
time = 0.05, size = 111, normalized size = 1.08

$$\frac{2\left(672945a^3x^{5/2}+1121575a^2bx^{9/2}+776475ab^2x^{13/2}+776475a^2cx^{13/2}+197925b^3x^{17/2}+1187550abcx^{17/2}+480675b^2cx^{21/2}+480675ac^2x^{21/2}+403767bc^2x^{25/2}+116025c^3x^{29/2}\right)}{3364725}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(672945*a^3*x^(5/2) + 1121575*a^2*b*x^(9/2) + 776475*a*b^2*x^(13/2) + 77
6475*a^2*c*x^(13/2) + 197925*b^3*x^(17/2) + 1187550*a*b*c*x^(17/2) + 480675
*b^2*c*x^(21/2) + 480675*a*c^2*x^(21/2) + 403767*b*c^2*x^(25/2) + 116025*c^
3*x^(29/2)))/3364725
```

Maple [A]
time = 0.07, size = 111, normalized size = 1.08

method	result
gosper	$\frac{2x^{\frac{5}{2}}\left(116025c^{3}x^{12}+403767b\,c^{2}x^{10}+480675x^{8}c^{2}a+480675x^{8}b^{2}c+1187550abc\,x^{6}+197925b^{3}x^{6}+776475x^{4}a^{2}c+776475a\,b^{2}\right.}{3364725}$
trager	$\frac{2x^{\frac{5}{2}}\left(116025c^{3}x^{12}+403767b\,c^{2}x^{10}+480675x^{8}c^{2}a+480675x^{8}b^{2}c+1187550abc\,x^{6}+197925b^{3}x^{6}+776475x^{4}a^{2}c+776475a\,b^{2}\right.}{3364725}$
risch	$\frac{2x^{\frac{5}{2}}\left(116025c^{3}x^{12}+403767b\,c^{2}x^{10}+480675x^{8}c^{2}a+480675x^{8}b^{2}c+1187550abc\,x^{6}+197925b^{3}x^{6}+776475x^{4}a^{2}c+776475a\,b^{2}\right.}{3364725}$
derivativedivides	$\frac{2c^{3}x^{\frac{29}{2}}}{29}+\frac{6b\,c^{2}x^{\frac{25}{2}}}{25}+\frac{2\left(c^{2}a+2b^{2}c+c\left(2ac+b^{2}\right)\right)x^{\frac{21}{2}}}{21}+\frac{2\left(4abc+b\left(2ac+b^{2}\right)\right)x^{\frac{17}{2}}}{17}+\frac{2\left(a\left(2ac+b^{2}\right)+2a\,b^{2}+a^{2}c\right)x^{\frac{13}{2}}}{13}$
default	$\frac{2c^{3}x^{\frac{29}{2}}}{29}+\frac{6b\,c^{2}x^{\frac{25}{2}}}{25}+\frac{2\left(c^{2}a+2b^{2}c+c\left(2ac+b^{2}\right)\right)x^{\frac{21}{2}}}{21}+\frac{2\left(4abc+b\left(2ac+b^{2}\right)\right)x^{\frac{17}{2}}}{17}+\frac{2\left(a\left(2ac+b^{2}\right)+2a\,b^{2}+a^{2}c\right)x^{\frac{13}{2}}}{13}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/29*c^3*x^(29/2)+6/25*b*c^2*x^(25/2)+2/21*(c^2*a+2*b^2*c+c*(2*a*c+b^2))*x^
(21/2)+2/17*(4*a*b*c+b*(2*a*c+b^2))*x^(17/2)+2/13*(a*(2*a*c+b^2)+2*a*b^2+a^
2*c)*x^(13/2)+2/3*a^2*b*x^(9/2)+2/5*a^3*x^(5/2)
```

Maxima [A]
time = 0.29, size = 81, normalized size = 0.79

$$\frac{2}{29}\,c^{3}x^{\frac{29}{2}}+\frac{6}{25}\,bc^{2}x^{\frac{25}{2}}+\frac{2}{7}\,\left(b^{2}c+ac^{2}\right)x^{\frac{21}{2}}+\frac{2}{17}\,\left(b^{3}+6\,abc\right)x^{\frac{17}{2}}+\frac{2}{3}\,a^{2}bx^{\frac{9}{2}}+\frac{6}{13}\,\left(ab^{2}+a^{2}c\right)x^{\frac{13}{2}}+\frac{2}{5}\,a^{3}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*(b^2*c + a*c^2)*x^(21/2) + 2/
17*(b^3 + 6*a*b*c)*x^(17/2) + 2/3*a^2*b*x^(9/2) + 6/13*(a*b^2 + a^2*c)*x^(1
3/2) + 2/5*a^3*x^(5/2)
```

Fricas [A]
time = 0.40, size = 86, normalized size = 0.83

$$\frac{2}{3364725}\,\left(116025\,c^{3}x^{14}+403767\,bc^{2}x^{12}+480675\,\left(b^{2}c+ac^{2}\right)x^{10}+197925\,\left(b^{3}+6\,abc\right)x^{8}+1121575\,a^{2}bx^{4}+776475\,\left(ab^{2}+a^{2}c\right)x^{6}+672945\,a^{3}x^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/3364725*(116025*c^3*x^14 + 403767*b*c^2*x^12 + 480675*(b^2*c + a*c^2)*x^1
0 + 197925*(b^3 + 6*a*b*c)*x^8 + 1121575*a^2*b*x^4 + 776475*(a*b^2 + a^2*c)
*x^6 + 672945*a^3*x^2)*sqrt(x)
```

Sympy [A]

time = 1.65, size = 129, normalized size = 1.25

$$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2bx^{\frac{9}{2}}}{3}+\frac{6a^2cx^{\frac{13}{2}}}{13}+\frac{6ab^2x^{\frac{13}{2}}}{13}+\frac{12abcx^{\frac{17}{2}}}{17}+\frac{2ac^2x^{\frac{21}{2}}}{7}+\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{21}{2}}}{7}+\frac{6bc^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a**2*c*x**(13/2)/13 + 6*a*b**2*
x**(13/2)/13 + 12*a*b*c*x**(17/2)/17 + 2*a*c**2*x**(21/2)/7 + 2*b**3*x**(17
/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/25 + 2*c**3*x**(29/2)/29
```

Giac [A]

time = 3.85, size = 87, normalized size = 0.84

$$\frac{2}{29}c^3x^{\frac{29}{2}}+\frac{6}{25}bc^2x^{\frac{25}{2}}+\frac{2}{7}b^2cx^{\frac{21}{2}}+\frac{2}{7}ac^2x^{\frac{21}{2}}+\frac{2}{17}b^3x^{\frac{17}{2}}+\frac{12}{17}abcx^{\frac{17}{2}}+\frac{6}{13}ab^2x^{\frac{13}{2}}+\frac{6}{13}a^2cx^{\frac{13}{2}}+\frac{2}{3}a^2bx^{\frac{9}{2}}+\frac{2}{5}a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/7*a*c^2*x^
(21/2) + 2/17*b^3*x^(17/2) + 12/17*a*b*c*x^(17/2) + 6/13*a*b^2*x^(13/2) + 6
/13*a^2*c*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/2)
```

Mupad [B]

time = 0.04, size = 76, normalized size = 0.74

$$x^{17/2}\left(\frac{2\,b^3}{17}+\frac{12\,a\,c\,b}{17}\right)+\frac{2\,a^3\,x^{5/2}}{5}+\frac{2\,c^3\,x^{29/2}}{29}+\frac{2\,a^2\,b\,x^{9/2}}{3}+\frac{6\,b\,c^2\,x^{25/2}}{25}+\frac{6\,a\,x^{13/2}\,\left(b^2+a\,c\right)}{13}+\frac{2\,c\,x^{21/2}\,\left(b^2+a\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(17/2)*((2*b^3)/17 + (12*a*b*c)/17) + (2*a^3*x^(5/2))/5 + (2*c^3*x^(29/2)
)/29 + (2*a^2*b*x^(9/2))/3 + (6*b*c^2*x^(25/2))/25 + (6*a*x^(13/2)*(a*c + b
^2))/13 + (2*c*x^(21/2)*(a*c + b^2))/7
```

3.1057 $\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}a\left(b^2+ac\right)x^{11/2}+\frac{2}{15}b\left(b^2+6ac\right)x^{15/2}+\frac{6}{19}c\left(b^2+ac\right)x^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

```
[Out] 2/3*a^3*x^(3/2)+6/7*a^2*b*x^(7/2)+6/11*a*(a*c+b^2)*x^(11/2)+2/15*b*(6*a*c+b
^2)*x^(15/2)+6/19*c*(a*c+b^2)*x^(19/2)+6/23*b*c^2*x^(23/2)+2/27*c^3*x^(27/2
)
```

Rubi [A]
time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{1122\}$

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/2))/11 + (
2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19/2))/19 + (6*b*c^2*x
^(23/2))/23 + (2*c^3*x^(27/2))/27
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3\sqrt{x}+3a^2bx^{5/2}+3a\left(b^2+ac\right)x^{9/2}+b\left(b^2+6ac\right)x^{13/2}+3c\left(b^2+ac\right)x^{17}\right.\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}a\left(b^2+ac\right)x^{11/2}+\frac{2}{15}b\left(b^2+6ac\right)x^{15/2}+\frac{6}{19}c\left(b^2+ac\right.\end{aligned}$$

Mathematica [A]
time = 0.05, size = 107, normalized size = 1.04

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{77}a^2x^{7/2}\left(11b+7cx^2\right)+\frac{2ax^{11/2}\left(285b^2+418bcx^2+165c^2x^4\right)}{1045}+\frac{2x^{15/2}\left(3933b^3+9315b^2cx^2+7695bc^2x^4+2185c^3x^6\right)}{58995}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*x^(7/2)*(11*b + 7*c*x^2))/77 + (2*a*x^(11/2)*(28
5*b^2 + 418*b*c*x^2 + 165*c^2*x^4))/1045 + (2*x^(15/2)*(3933*b^3 + 9315*b^2
*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^6))/58995
```

Maple [A]
time = 0.06, size = 111, normalized size = 1.08

method	result
gosper	$\frac{2x^{\frac{3}{2}}\left(168245c^{3}x^{12}+592515b\,c^{2}x^{10}+717255x^{8}c^{2}a+717255x^{8}b^{2}c+1817046abc\,x^{6}+302841b^{3}x^{6}+1238895x^{4}a^{2}c+1238895\right.}{4542615}$
trager	$\frac{2x^{\frac{3}{2}}\left(168245c^{3}x^{12}+592515b\,c^{2}x^{10}+717255x^{8}c^{2}a+717255x^{8}b^{2}c+1817046abc\,x^{6}+302841b^{3}x^{6}+1238895x^{4}a^{2}c+1238895\right.}{4542615}$
risch	$\frac{2x^{\frac{3}{2}}\left(168245c^{3}x^{12}+592515b\,c^{2}x^{10}+717255x^{8}c^{2}a+717255x^{8}b^{2}c+1817046abc\,x^{6}+302841b^{3}x^{6}+1238895x^{4}a^{2}c+1238895\right.}{4542615}$
derivativedivides	$\frac{2c^{3}x^{\frac{27}{2}}}{27}+\frac{6b\,c^{2}x^{\frac{23}{2}}}{23}+\frac{2\left(c^{2}a+2b^{2}c+c\left(2ac+b^{2}\right)\right)x^{\frac{19}{2}}}{19}+\frac{2\left(4abc+b\left(2ac+b^{2}\right)\right)x^{\frac{15}{2}}}{15}+\frac{2\left(a\left(2ac+b^{2}\right)+2a\,b^{2}+a^{2}c\right)x^{\frac{11}{2}}}{11}$
default	$\frac{2c^{3}x^{\frac{27}{2}}}{27}+\frac{6b\,c^{2}x^{\frac{23}{2}}}{23}+\frac{2\left(c^{2}a+2b^{2}c+c\left(2ac+b^{2}\right)\right)x^{\frac{19}{2}}}{19}+\frac{2\left(4abc+b\left(2ac+b^{2}\right)\right)x^{\frac{15}{2}}}{15}+\frac{2\left(a\left(2ac+b^{2}\right)+2a\,b^{2}+a^{2}c\right)x^{\frac{11}{2}}}{11}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*c^3*x^(27/2)+6/23*b*c^2*x^(23/2)+2/19*(c^2*a+2*b^2*c+c*(2*a*c+b^2))*x^
(19/2)+2/15*(4*a*b*c+b*(2*a*c+b^2))*x^(15/2)+2/11*(a*(2*a*c+b^2)+2*a*b^2+a^
2*c)*x^(11/2)+6/7*a^2*b*x^(7/2)+2/3*a^3*x^(3/2)
```

Maxima [A]
time = 0.29, size = 81, normalized size = 0.79

$$\frac{2}{27}\,c^{3}x^{\frac{27}{2}}+\frac{6}{23}\,bc^{2}x^{\frac{23}{2}}+\frac{6}{19}\,\left(b^{2}c+ac^{2}\right)x^{\frac{19}{2}}+\frac{2}{15}\,\left(b^{3}+6\,abc\right)x^{\frac{15}{2}}+\frac{6}{7}\,a^{2}bx^{\frac{7}{2}}+\frac{6}{11}\,\left(ab^{2}+a^{2}c\right)x^{\frac{11}{2}}+\frac{2}{3}\,a^{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*(b^2*c + a*c^2)*x^(19/2) + 2
/15*(b^3 + 6*a*b*c)*x^(15/2) + 6/7*a^2*b*x^(7/2) + 6/11*(a*b^2 + a^2*c)*x^(
11/2) + 2/3*a^3*x^(3/2)
```

Fricas [A]
time = 0.32, size = 84, normalized size = 0.82

$$\frac{2}{4542615}\,\left(168245\,c^{3}x^{13}+592515\,bc^{2}x^{11}+717255\,\left(b^{2}c+ac^{2}\right)x^{9}+302841\,\left(b^{3}+6\,abc\right)x^{7}+1946835\,a^{2}bx^{3}+1238895\,\left(ab^{2}+a^{2}c\right)x^{5}+1514205\,a^{3}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 2/4542615*(168245*c^3*x^13 + 592515*b*c^2*x^11 + 717255*(b^2*c + a*c^2)*x^9
 + 302841*(b^3 + 6*a*b*c)*x^7 + 1946835*a^2*b*x^3 + 1238895*(a*b^2 + a^2*c)
*x^5 + 1514205*a^3*x)*sqrt(x)
```

Sympy [A]
time = 2.45, size = 112, normalized size = 1.09

$$\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{6bc^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{27}{2}}}{27}+\frac{2x^{\frac{19}{2}}\cdot(3ac^2+3b^2c)}{19}+\frac{2x^{\frac{15}{2}}\cdot(6abc+b^3)}{15}+\frac{2x^{\frac{11}{2}}\cdot(3a^2c+3ab^2)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*b*c**2*x**(23/2)/23 + 2*c**3*x*
*(27/2)/27 + 2*x**(19/2)*(3*a*c**2 + 3*b**2*c)/19 + 2*x**(15/2)*(6*a*b*c +
b**3)/15 + 2*x**(11/2)*(3*a**2*c + 3*a*b**2)/11
```

Giac [A]
time = 4.00, size = 87, normalized size = 0.84

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\,b^2cx^{\frac{19}{2}}+\frac{6}{19}\,ac^2x^{\frac{19}{2}}+\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{4}{5}\,abcx^{\frac{15}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{6}{11}\,a^2cx^{\frac{11}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 6/19*a*c^2*
x^(19/2) + 2/15*b^3*x^(15/2) + 4/5*a*b*c*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6
/11*a^2*c*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/2)
```

Mupad [B]
time = 0.04, size = 76, normalized size = 0.74

$$x^{15/2}\left(\frac{2\,b^3}{15}+\frac{4\,a\,c\,b}{5}\right)+\frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,c^3\,x^{27/2}}{27}+\frac{6\,a^2\,b\,x^{7/2}}{7}+\frac{6\,b\,c^2\,x^{23/2}}{23}+\frac{6\,a\,x^{11/2}\,(b^2+a\,c)}{11}+\frac{6\,c\,x^{19/2}\,(b^2+a\,c)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(15/2)*((2*b^3)/15 + (4*a*b*c)/5) + (2*a^3*x^(3/2))/3 + (2*c^3*x^(27/2))/
27 + (6*a^2*b*x^(7/2))/7 + (6*b*c^2*x^(23/2))/23 + (6*a*x^(11/2)*(a*c + b^2
))/11 + (6*c*x^(19/2)*(a*c + b^2))/19
```

3.1058 $\int \frac{\left(a+bx^2+cx^4\right)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=101

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}a\left(b^2+ac\right)x^{9/2}+\frac{2}{13}b\left(b^2+6ac\right)x^{13/2}+\frac{6}{17}c\left(b^2+ac\right)x^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

```
[Out] 6/5*a^2*b*x^(5/2)+2/3*a*(a*c+b^2)*x^(9/2)+2/13*b*(6*a*c+b^2)*x^(13/2)+6/17*
c*(a*c+b^2)*x^(17/2)+2/7*b*c^2*x^(21/2)+2/25*c^3*x^(25/2)+2*a^3*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1122}

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{17}cx^{17/2}\left(ac+b^2\right)+\frac{2}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{2}{3}ax^{9/2}\left(ac+b^2\right)+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*(b^2 + a*c)*x^(9/2))/3 + (2*b*(b
^2 + 6*a*c)*x^(13/2))/13 + (6*c*(b^2 + a*c)*x^(17/2))/17 + (2*b*c^2*x^(21/2
))/7 + (2*c^3*x^(25/2))/25
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{\sqrt{x}}\,dx=\int \left(\frac{a^3}{\sqrt{x}}+3a^2bx^{3/2}+3a\left(b^2+ac\right)x^{7/2}+b\left(b^2+6ac\right)x^{11/2}+3c\left(b^2+ac\right)x^{15/2}+$$

$$=2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}a\left(b^2+ac\right)x^{9/2}+\frac{2}{13}b\left(b^2+6ac\right)x^{13/2}+\frac{6}{17}c\left(b^2+ac\right)x^{17/}$$

Mathematica [A]
time = 0.06, size = 97, normalized size = 0.96

$$\frac{2\sqrt{x}\left(116025a^3+7735a^2(9bx^2+5cx^4)+3x^6(2975b^3+6825b^2cx^2+5525bc^2x^4+1547c^3x^6)+175a(221b^2x^4+306bcx^6+117c^2x^8)\right)}{116025}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]

[Out] (2*Sqrt[x]*(116025*a^3 + 7735*a^2*(9*b*x^2 + 5*c*x^4) + 3*x^6*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^6) + 175*a*(221*b^2*x^4 + 306*b*c*x^6 + 117*c^2*x^8)))/116025

Maple [A]
time = 0.04, size = 111, normalized size = 1.10

method	result
trager	$\left(\frac{2}{25}c^3x^{12}+\frac{2}{7}b\,c^2x^{10}+\frac{6}{17}x^8c^2a+\frac{6}{17}x^8b^2c+\frac{12}{13}abc\,x^6+\frac{2}{13}b^3x^6+\frac{2}{3}x^4a^2c+\frac{2}{3}a\,b^2x^4+\frac{6}{5}a^2b\right.$
gosper	$\frac{2\sqrt{x}\,\left(4641c^3x^{12}+16575b\,c^2x^{10}+20475x^8c^2a+20475x^8b^2c+53550abc\,x^6+8925b^3x^6+38675x^4a^2c+38675a\,b^2x^4+69615\right.}{116025}$
risch	$\frac{2\sqrt{x}\,\left(4641c^3x^{12}+16575b\,c^2x^{10}+20475x^8c^2a+20475x^8b^2c+53550abc\,x^6+8925b^3x^6+38675x^4a^2c+38675a\,b^2x^4+69615\right.}{116025}$
derivativedivides	$\frac{2c^3x^{\frac{25}{2}}}{25}+\frac{2b\,c^2x^{\frac{21}{2}}}{7}+\frac{2\left(c^2a+2b^2c+c\left(2ac+b^2\right)\right)x^{\frac{17}{2}}}{17}+\frac{2\left(4abc+b\left(2ac+b^2\right)\right)x^{\frac{13}{2}}}{13}+\frac{2\left(a\left(2ac+b^2\right)+2a\,b^2+a^2c\right)x^{\frac{9}{2}}}{9}+$
default	$\frac{2c^3x^{\frac{25}{2}}}{25}+\frac{2b\,c^2x^{\frac{21}{2}}}{7}+\frac{2\left(c^2a+2b^2c+c\left(2ac+b^2\right)\right)x^{\frac{17}{2}}}{17}+\frac{2\left(4abc+b\left(2ac+b^2\right)\right)x^{\frac{13}{2}}}{13}+\frac{2\left(a\left(2ac+b^2\right)+2a\,b^2+a^2c\right)x^{\frac{9}{2}}}{9}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^3/x^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/25*c^3*x^(25/2)+2/7*b*c^2*x^(21/2)+2/17*(c^2*a+2*b^2*c+c*(2*a*c+b^2))*x^(17/2)+2/13*(4*a*b*c+b*(2*a*c+b^2))*x^(13/2)+2/9*(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*x^(9/2)+6/5*a^2*b*x^(5/2)+2*a^3*x^(1/2)

Maxima [A]
time = 0.28, size = 88, normalized size = 0.87

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}+2\,a^3\sqrt{x}+\frac{2}{15}\left(5\,cx^{\frac{9}{2}}+9\,bx^{\frac{5}{2}}\right)a^2+\frac{2}{663}\left(117\,c^2x^{\frac{17}{2}}+306\,bcx^{\frac{13}{2}}+221\,b^2x^{\frac{9}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="maxima")

[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^(13/2) + 2*a^3*sqrt(x) + 2/15*(5*c*x^(9/2) + 9*b*x^(5/2))*a^2 + 2/663*(117*c^2*x^(17/2) + 306*b*c*x^(13/2) + 221*b^2*x^(9/2))*a

Fricas [A]
time = 0.39, size = 83, normalized size = 0.82

$$\frac{2}{116025}\left(4641\,c^3x^{12}+16575\,bc^2x^{10}+20475\,\left(b^2c+ac^2\right)x^8+8925\,\left(b^3+6\,abc\right)x^6+69615\,a^2bx^2+38675\,\left(ab^2+a^2c\right)x^4+116025\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*c^3*x^12 + 16575*b*c^2*x^10 + 20475*(b^2*c + a*c^2)*x^8 + 89
25*(b^3 + 6*a*b*c)*x^6 + 69615*a^2*b*x^2 + 38675*(a*b^2 + a^2*c)*x^4 + 1160
25*a^3)*sqrt(x)
```

Sympy [A]
time = 1.15, size = 128, normalized size = 1.27

$$2a^3\sqrt{x} + \frac{6a^2bx^{\frac{5}{2}}}{5} + \frac{2a^2cx^{\frac{9}{2}}}{3} + \frac{2ab^2x^{\frac{9}{2}}}{3} + \frac{12abcx^{\frac{13}{2}}}{13} + \frac{6ac^2x^{\frac{17}{2}}}{17} + \frac{2b^3x^{\frac{13}{2}}}{13} + \frac{6b^2cx^{\frac{17}{2}}}{17} + \frac{2bc^2x^{\frac{21}{2}}}{7} + \frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a**2*c*x**(9/2)/3 + 2*a*b**2*x**(9
/2)/3 + 12*a*b*c*x**(13/2)/13 + 6*a*c**2*x**(17/2)/17 + 2*b**3*x**(13/2)/13
 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/7 + 2*c**3*x**(25/2)/25
```

Giac [A]
time = 3.90, size = 87, normalized size = 0.86

$$\frac{2}{25}\,c^3x^{\frac{25}{2}} + \frac{2}{7}\,bc^2x^{\frac{21}{2}} + \frac{6}{17}\,b^2cx^{\frac{17}{2}} + \frac{6}{17}\,ac^2x^{\frac{17}{2}} + \frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{12}{13}\,abcx^{\frac{13}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{2}{3}\,a^2cx^{\frac{9}{2}} + \frac{6}{5}\,a^2bx^{\frac{5}{2}} + 2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 6/17*a*c^2*x
^(17/2) + 2/13*b^3*x^(13/2) + 12/13*a*b*c*x^(13/2) + 2/3*a*b^2*x^(9/2) + 2/
3*a^2*c*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

Mupad [B]
time = 0.03, size = 76, normalized size = 0.75

$$x^{13/2}\left(\frac{2\,b^3}{13} + \frac{12\,a\,c\,b}{13}\right) + 2\,a^3\,\sqrt{x} + \frac{2\,c^3\,x^{25/2}}{25} + \frac{6\,a^2\,b\,x^{5/2}}{5} + \frac{2\,b\,c^2\,x^{21/2}}{7} + \frac{2\,a\,x^{9/2}\,(b^2+a\,c)}{3} + \frac{6\,c\,x^{17/2}\,(b^2+a\,c)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(1/2),x)
```

```
[Out] x^(13/2)*((2*b^3)/13 + (12*a*b*c)/13) + 2*a^3*x^(1/2) + (2*c^3*x^(25/2))/25
 + (6*a^2*b*x^(5/2))/5 + (2*b*c^2*x^(21/2))/7 + (2*a*x^(9/2)*(a*c + b^2))/3
 + (6*c*x^(17/2)*(a*c + b^2))/17
```

3.1059 $\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}a\left(b^2+ac\right)x^{7/2}+\frac{2}{11}b\left(b^2+6ac\right)x^{11/2}+\frac{2}{5}c\left(b^2+ac\right)x^{15/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

```
[Out] 2*a^2*b*x^(3/2)+6/7*a*(a*c+b^2)*x^(7/2)+2/11*b*(6*a*c+b^2)*x^(11/2)+2/5*c*(
a*c+b^2)*x^(15/2)+6/19*b*c^2*x^(19/2)+2/23*c^3*x^(23/2)-2*a^3/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{1122\}$

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{2}{5}cx^{15/2}\left(ac+b^2\right)+\frac{2}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{6}{7}ax^{7/2}\left(ac+b^2\right)+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*(b^2 + a*c)*x^(7/2))/7 + (2*b*(b^
2 + 6*a*c)*x^(11/2))/11 + (2*c*(b^2 + a*c)*x^(15/2))/5 + (6*b*c^2*x^(19/2))
/19 + (2*c^3*x^(23/2))/23
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x^{3/2}}\,dx=\int \left(\frac{a^3}{x^{3/2}}+3a^2b\sqrt{x}+3a\left(b^2+ac\right)x^{5/2}+b\left(b^2+6ac\right)x^{9/2}+3c\left(b^2+ac\right)x^{13/2}+3b$$
$$=-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}a\left(b^2+ac\right)x^{7/2}+\frac{2}{11}b\left(b^2+6ac\right)x^{11/2}+\frac{2}{5}c\left(b^2+ac\right)x^{15/2}+$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.94

$$-\frac{2(168245a^3-168245a^2bx^2-72105ab^2x^4-72105a^2cx^4-15295b^3x^6-91770abcx^6-33649b^2cx^8-33649ac^2x^8-26565bc^2x^{10}-7315c^3x^{12})}{168245\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (-2*(168245*a^3 - 168245*a^2*b*x^2 - 72105*a*b^2*x^4 - 72105*a^2*c*x^4 - 15
295*b^3*x^6 - 91770*a*b*c*x^6 - 33649*b^2*c*x^8 - 33649*a*c^2*x^8 - 26565*b
*c^2*x^10 - 7315*c^3*x^12))/(168245*Sqrt[x])
```

Maple [A]
time = 0.04, size = 88, normalized size = 0.89

method	result
derivativedivides	$\frac{2c^3x^{\frac{23}{2}}}{23}+\frac{6b\,c^2x^{\frac{19}{2}}}{19}+\frac{2a\,c^2x^{\frac{15}{2}}}{5}+\frac{2b^2c\,x^{\frac{15}{2}}}{5}+\frac{12abc\,x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{6a^2c\,x^{\frac{7}{2}}}{7}+\frac{6a\,b^2x^{\frac{7}{2}}}{7}+2a^2b\,x^{\frac{3}{2}}-$
default	$\frac{2c^3x^{\frac{23}{2}}}{23}+\frac{6b\,c^2x^{\frac{19}{2}}}{19}+\frac{2a\,c^2x^{\frac{15}{2}}}{5}+\frac{2b^2c\,x^{\frac{15}{2}}}{5}+\frac{12abc\,x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{6a^2c\,x^{\frac{7}{2}}}{7}+\frac{6a\,b^2x^{\frac{7}{2}}}{7}+2a^2b\,x^{\frac{3}{2}}-$
gosper	$-\frac{2(-7315c^3x^{12}-26565b\,c^2x^{10}-33649x^8c^2a-33649x^8b^2c-91770abc\,x^6-15295b^3x^6-72105x^4a^2c-72105a\,b^2x^4-1682}{168245\sqrt{x}}$
trager	$-\frac{2(-7315c^3x^{12}-26565b\,c^2x^{10}-33649x^8c^2a-33649x^8b^2c-91770abc\,x^6-15295b^3x^6-72105x^4a^2c-72105a\,b^2x^4-1682}{168245\sqrt{x}}$
risch	$-\frac{2(-7315c^3x^{12}-26565b\,c^2x^{10}-33649x^8c^2a-33649x^8b^2c-91770abc\,x^6-15295b^3x^6-72105x^4a^2c-72105a\,b^2x^4-1682}{168245\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*c^3*x^(23/2)+6/19*b*c^2*x^(19/2)+2/5*a*c^2*x^(15/2)+2/5*b^2*c*x^(15/2)
+12/11*a*b*c*x^(11/2)+2/11*b^3*x^(11/2)+6/7*a^2*c*x^(7/2)+6/7*a*b^2*x^(7/2)
+2*a^2*b*x^(3/2)-2*a^3/x^(1/2)
```

Maxima [A]
time = 0.36, size = 81, normalized size = 0.82

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\left(b^2c+ac^2\right)x^{\frac{15}{2}}+\frac{2}{11}\left(b^3+6\,abc\right)x^{\frac{11}{2}}+2\,a^2bx^{\frac{3}{2}}+\frac{6}{7}\left(ab^2+a^2c\right)x^{\frac{7}{2}}-\frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*(b^2*c + a*c^2)*x^(15/2) + 2/
11*(b^3 + 6*a*b*c)*x^(11/2) + 2*a^2*b*x^(3/2) + 6/7*(a*b^2 + a^2*c)*x^(7/2)
 - 2*a^3/sqrt(x)
```

Fricas [A]
time = 0.32, size = 83, normalized size = 0.84

$$\frac{2\,(7315\,c^3x^{12}+26565\,bc^2x^{10}+33649\,(b^2c+ac^2)x^8+15295\,(b^3+6\,abc)x^6+168245\,a^2bx^2+72105\,(ab^2+a^2c)x^4-168245\,a^3)}{168245\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*c^3*x^12 + 26565*b*c^2*x^10 + 33649*(b^2*c + a*c^2)*x^8 + 15
295*(b^3 + 6*a*b*c)*x^6 + 168245*a^2*b*x^2 + 72105*(a*b^2 + a^2*c)*x^4 - 16
8245*a^3)/sqrt(x)
```

Sympy [A]
time = 1.39, size = 126, normalized size = 1.27

$$-\frac{2a^3}{\sqrt{x}} + 2a^2bx^{\frac{3}{2}} + \frac{6a^2cx^{\frac{7}{2}}}{7} + \frac{6ab^2x^{\frac{7}{2}}}{7} + \frac{12abcx^{\frac{11}{2}}}{11} + \frac{2ac^2x^{\frac{15}{2}}}{5} + \frac{2b^3x^{\frac{11}{2}}}{11} + \frac{2b^2cx^{\frac{15}{2}}}{5} + \frac{6bc^2x^{\frac{19}{2}}}{19} + \frac{2c^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a**2*c*x**(7/2)/7 + 6*a*b**2*x**(7/
2)/7 + 12*a*b*c*x**(11/2)/11 + 2*a*c**2*x**(15/2)/5 + 2*b**3*x**(11/2)/11 +
 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/19 + 2*c**3*x**(23/2)/23
```

Giac [A]
time = 4.03, size = 87, normalized size = 0.88

$$\frac{2}{23}\,c^3x^{\frac{23}{2}} + \frac{6}{19}\,bc^2x^{\frac{19}{2}} + \frac{2}{5}\,b^2cx^{\frac{15}{2}} + \frac{2}{5}\,ac^2x^{\frac{15}{2}} + \frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{12}{11}\,abcx^{\frac{11}{2}} + \frac{6}{7}\,ab^2x^{\frac{7}{2}} + \frac{6}{7}\,a^2cx^{\frac{7}{2}} + 2\,a^2bx^{\frac{3}{2}} - \frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/5*a*c^2*x^
(15/2) + 2/11*b^3*x^(11/2) + 12/11*a*b*c*x^(11/2) + 6/7*a*b^2*x^(7/2) + 6/7
*a^2*c*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

Mupad [B]
time = 0.04, size = 76, normalized size = 0.77

$$x^{11/2}\left(\frac{2\,b^3}{11}+\frac{12\,a\,c\,b}{11}\right)-\frac{2\,a^3}{\sqrt{x}}+\frac{2\,c^3\,x^{23/2}}{23}+2\,a^2\,b\,x^{3/2}+\frac{6\,b\,c^2\,x^{19/2}}{19}+\frac{6\,a\,x^{7/2}\,\left(b^2+a\,c\right)}{7}+\frac{2\,c\,x^{15/2}\,\left(b^2+a\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(3/2),x)
```

```
[Out] x^(11/2)*((2*b^3)/11 + (12*a*b*c)/11) - (2*a^3)/x^(1/2) + (2*c^3*x^(23/2))/
23 + 2*a^2*b*x^(3/2) + (6*b*c^2*x^(19/2))/19 + (6*a*x^(7/2)*(a*c + b^2))/7
+ (2*c*x^(15/2)*(a*c + b^2))/5
```

3.1060 $\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}a\left(b^2+ac\right)x^{5/2}+\frac{2}{9}b\left(b^2+6ac\right)x^{9/2}+\frac{6}{13}c\left(b^2+ac\right)x^{13/2}+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

```
[Out] -2/3*a^3/x^(3/2)+6/5*a*(a*c+b^2)*x^(5/2)+2/9*b*(6*a*c+b^2)*x^(9/2)+6/13*c*(
a*c+b^2)*x^(13/2)+6/17*b*c^2*x^(17/2)+2/21*c^3*x^(21/2)+6*a^2*b*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1122}

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{13}cx^{13/2}\left(ac+b^2\right)+\frac{2}{9}bx^{9/2}\left(6ac+b^2\right)+\frac{6}{5}ax^{5/2}\left(ac+b^2\right)+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*(b^2 + a*c)*x^(5/2))/5 + (2*b
*(b^2 + 6*a*c)*x^(9/2))/9 + (6*c*(b^2 + a*c)*x^(13/2))/13 + (6*b*c^2*x^(17/
2))/17 + (2*c^3*x^(21/2))/21
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx &= \int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{\sqrt{x}}+3a\left(b^2+ac\right)x^{3/2}+b\left(b^2+6ac\right)x^{7/2}+3c\left(b^2+ac\right)x^{11/2}+3bc^2\right.\\ &= -\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}a\left(b^2+ac\right)x^{5/2}+\frac{2}{9}b\left(b^2+6ac\right)x^{9/2}+\frac{6}{13}c\left(b^2+ac\right)x^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.92

$$-\frac{2\left(23205a^3-208845a^2bx^2-41769ab^2x^4-41769a^2cx^4-7735b^3x^6-46410abcx^6-16065b^2cx^8-16065ac^2x^8-12285bc^2x^{10}-3315c^3x^{12}\right)}{69615x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (-2*(23205*a^3 - 208845*a^2*b*x^2 - 41769*a*b^2*x^4 - 41769*a^2*c*x^4 - 773
5*b^3*x^6 - 46410*a*b*c*x^6 - 16065*b^2*c*x^8 - 16065*a*c^2*x^8 - 12285*b*c
^2*x^10 - 3315*c^3*x^12))/(69615*x^(3/2))
```

Maple [A]
time = 0.04, size = 88, normalized size = 0.87

method	result
derivativedivides	$\frac{2c^3x^{\frac{21}{2}}}{21}+\frac{6bc^2x^{\frac{17}{2}}}{17}+\frac{6ac^2x^{\frac{13}{2}}}{13}+\frac{6b^2cx^{\frac{13}{2}}}{13}+\frac{4abcx^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6a^2cx^{\frac{5}{2}}}{5}+\frac{6ab^2x^{\frac{5}{2}}}{5}+6a^2b\sqrt{x}-\frac{2a}{3x}$
default	$\frac{2c^3x^{\frac{21}{2}}}{21}+\frac{6bc^2x^{\frac{17}{2}}}{17}+\frac{6ac^2x^{\frac{13}{2}}}{13}+\frac{6b^2cx^{\frac{13}{2}}}{13}+\frac{4abcx^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6a^2cx^{\frac{5}{2}}}{5}+\frac{6ab^2x^{\frac{5}{2}}}{5}+6a^2b\sqrt{x}-\frac{2a}{3x}$
gosper	$-\frac{2(-3315c^3x^{12}-12285bc^2x^{10}-16065x^8c^2a-16065x^8b^2c-46410abcx^6-7735b^3x^6-41769x^4a^2c-41769ab^2x^4-208845a}{69615x^{\frac{3}{2}}}$
trager	$-\frac{2(-3315c^3x^{12}-12285bc^2x^{10}-16065x^8c^2a-16065x^8b^2c-46410abcx^6-7735b^3x^6-41769x^4a^2c-41769ab^2x^4-208845a}{69615x^{\frac{3}{2}}}$
risch	$-\frac{2(-3315c^3x^{12}-12285bc^2x^{10}-16065x^8c^2a-16065x^8b^2c-46410abcx^6-7735b^3x^6-41769x^4a^2c-41769ab^2x^4-208845a}{69615x^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*c^3*x^(21/2)+6/17*b*c^2*x^(17/2)+6/13*a*c^2*x^(13/2)+6/13*b^2*c*x^(13/
2)+4/3*a*b*c*x^(9/2)+2/9*b^3*x^(9/2)+6/5*a^2*c*x^(5/2)+6/5*a*b^2*x^(5/2)+6*
a^2*b*x^(1/2)-2/3*a^3/x^(3/2)
```

Maxima [A]
time = 0.28, size = 81, normalized size = 0.80

$$\frac{2}{21}c^3x^{\frac{21}{2}}+\frac{6}{17}bc^2x^{\frac{17}{2}}+\frac{6}{13}\left(b^2c+ac^2\right)x^{\frac{13}{2}}+\frac{2}{9}\left(b^3+6\,abc\right)x^{\frac{9}{2}}+6\,a^2b\sqrt{x}+\frac{6}{5}\left(ab^2+a^2c\right)x^{\frac{5}{2}}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*(b^2*c + a*c^2)*x^(13/2) + 2
/9*(b^3 + 6*a*b*c)*x^(9/2) + 6*a^2*b*sqrt(x) + 6/5*(a*b^2 + a^2*c)*x^(5/2)
- 2/3*a^3/x^(3/2)
```

Fricas [A]
time = 0.33, size = 83, normalized size = 0.82

$$\frac{2\left(3315\,c^3x^{12}+12285\,bc^2x^{10}+16065\left(b^2c+ac^2\right)x^8+7735\left(b^3+6\,abc\right)x^6+208845\,a^2bx^2+41769\left(ab^2+a^2c\right)x^4-23205\,a^3\right)}{69615\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*c^3*x^12 + 12285*b*c^2*x^10 + 16065*(b^2*c + a*c^2)*x^8 + 773
5*(b^3 + 6*a*b*c)*x^6 + 208845*a^2*b*x^2 + 41769*(a*b^2 + a^2*c)*x^4 - 2320
5*a^3)/x^(3/2)
```

Sympy [A]
time = 1.52, size = 128, normalized size = 1.27

$$-\frac{2a^3}{3x^{\frac{3}{2}}}+6a^2b\sqrt{x}+\frac{6a^2cx^{\frac{5}{2}}}{5}+\frac{6ab^2x^{\frac{5}{2}}}{5}+\frac{4abcx^{\frac{9}{2}}}{3}+\frac{6ac^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2cx^{\frac{13}{2}}}{13}+\frac{6bc^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a**2*c*x**(5/2)/5 + 6*a*b**2*x*
*(5/2)/5 + 4*a*b*c*x**(9/2)/3 + 6*a*c**2*x**(13/2)/13 + 2*b**3*x**(9/2)/9 +
 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17 + 2*c**3*x**(21/2)/21
```

Giac [A]
time = 3.98, size = 87, normalized size = 0.86

$$\frac{2}{21}c^3x^{\frac{21}{2}}+\frac{6}{17}bc^2x^{\frac{17}{2}}+\frac{6}{13}b^2cx^{\frac{13}{2}}+\frac{6}{13}ac^2x^{\frac{13}{2}}+\frac{2}{9}b^3x^{\frac{9}{2}}+\frac{4}{3}abcx^{\frac{9}{2}}+\frac{6}{5}ab^2x^{\frac{5}{2}}+\frac{6}{5}a^2cx^{\frac{5}{2}}+6\,a^2b\sqrt{x}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 6/13*a*c^2*
x^(13/2) + 2/9*b^3*x^(9/2) + 4/3*a*b*c*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6/5*a^
2*c*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

Mupad [B]
time = 0.04, size = 76, normalized size = 0.75

$$x^{9/2}\left(\frac{2\,b^3}{9}+\frac{4\,a\,c\,b}{3}\right)-\frac{2\,a^3}{3\,x^{3/2}}+\frac{2\,c^3\,x^{21/2}}{21}+6\,a^2\,b\,\sqrt{x}+\frac{6\,b\,c^2\,x^{17/2}}{17}+\frac{6\,a\,x^{5/2}\,\left(b^2+a\,c\right)}{5}+\frac{6\,c\,x^{13/2}\,\left(b^2+a\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(5/2),x)
```

```
[Out] x^(9/2)*((2*b^3)/9 + (4*a*b*c)/3) - (2*a^3)/(3*x^(3/2)) + (2*c^3*x^(21/2))/
21 + 6*a^2*b*x^(1/2) + (6*b*c^2*x^(17/2))/17 + (6*a*x^(5/2)*(a*c + b^2))/5
+ (6*c*x^(13/2)*(a*c + b^2))/13
```

3.1061 $\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}} dx$

Optimal. Leaf size=99

$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2a(b^2+ac)\,x^{3/2}+\frac{2}{7}b(b^2+6ac)\,x^{7/2}+\frac{6}{11}c(b^2+ac)\,x^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

```
[Out] -2/5*a^3/x^(5/2)+2*a*(a*c+b^2)*x^(3/2)+2/7*b*(6*a*c+b^2)*x^(7/2)+6/11*c*(a*
c+b^2)*x^(11/2)+2/5*b*c^2*x^(15/2)+2/19*c^3*x^(19/2)-6*a^2*b/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{1122\}$

$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+\frac{6}{11}cx^{11/2}(ac+b^2)+\frac{2}{7}bx^{7/2}(6ac+b^2)+2ax^{3/2}(ac+b^2)+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*(b^2 + a*c)*x^(3/2) + (2*b*(
b^2 + 6*a*c)*x^(7/2))/7 + (6*c*(b^2 + a*c)*x^(11/2))/11 + (2*b*c^2*x^(15/2)
)/5 + (2*c^3*x^(19/2))/19
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}}\,dx &= \int \left(\frac{a^3}{x^{7/2}}+\frac{3a^2b}{x^{3/2}}+3a(b^2+ac)\sqrt{x}+b(b^2+6ac)\,x^{5/2}+3c(b^2+ac)\,x^{9/2}+3bc^2x^{1}\right.\\ &= -\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2a(b^2+ac)\,x^{3/2}+\frac{2}{7}b(b^2+6ac)\,x^{7/2}+\frac{6}{11}c(b^2+ac)\,x^{11/2}+\frac{2}{5}bc\end{aligned}$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.94

$$-\frac{2(1463a^3+21945a^2bx^2-7315ab^2x^4-7315a^2cx^4-1045b^3x^6-6270abcx^6-1995b^2cx^8-1995ac^2x^8-1463bc^2x^{10}-385c^3x^{12})}{7315x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (-2*(1463*a^3 + 21945*a^2*b*x^2 - 7315*a*b^2*x^4 - 7315*a^2*c*x^4 - 1045*b^
3*x^6 - 6270*a*b*c*x^6 - 1995*b^2*c*x^8 - 1995*a*c^2*x^8 - 1463*b*c^2*x^10
- 385*c^3*x^12))/(7315*x^(5/2))
```

Maple [A]
time = 0.05, size = 88, normalized size = 0.89

method	result
derivativedivides	$\frac{2c^3x^{\frac{19}{2}}}{19}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{6ac^2x^{\frac{11}{2}}}{11}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{12abcx^{\frac{7}{2}}}{7}+\frac{2b^3x^{\frac{7}{2}}}{7}+2a^2cx^{\frac{3}{2}}+2ab^2x^{\frac{3}{2}}-\frac{6a^2b}{\sqrt{x}}-\frac{2\cdots}{5\cdots}$
default	$\frac{2c^3x^{\frac{19}{2}}}{19}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{6ac^2x^{\frac{11}{2}}}{11}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{12abcx^{\frac{7}{2}}}{7}+\frac{2b^3x^{\frac{7}{2}}}{7}+2a^2cx^{\frac{3}{2}}+2ab^2x^{\frac{3}{2}}-\frac{6a^2b}{\sqrt{x}}-\frac{2\cdots}{5\cdots}$
gosper	$-\frac{2(-385c^3x^{12}-1463bc^2x^{10}-1995x^8c^2a-1995x^8b^2c-6270abcx^6-1045b^3x^6-7315x^4a^2c-7315ab^2x^4+21945a^2bx^2+\cdots}{7315x^{\frac{5}{2}}}$
trager	$-\frac{2(-385c^3x^{12}-1463bc^2x^{10}-1995x^8c^2a-1995x^8b^2c-6270abcx^6-1045b^3x^6-7315x^4a^2c-7315ab^2x^4+21945a^2bx^2+\cdots}{7315x^{\frac{5}{2}}}$
risch	$-\frac{2(-385c^3x^{12}-1463bc^2x^{10}-1995x^8c^2a-1995x^8b^2c-6270abcx^6-1045b^3x^6-7315x^4a^2c-7315ab^2x^4+21945a^2bx^2+\cdots}{7315x^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*c^3*x^(19/2)+2/5*b*c^2*x^(15/2)+6/11*a*c^2*x^(11/2)+6/11*b^2*c*x^(11/2
)+12/7*a*b*c*x^(7/2)+2/7*b^3*x^(7/2)+2*a^2*c*x^(3/2)+2*a*b^2*x^(3/2)-6*a^2*
b/x^(1/2)-2/5*a^3/x^(5/2)
```

Maxima [A]
time = 0.29, size = 82, normalized size = 0.83

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,(b^2c+ac^2)x^{\frac{11}{2}}+\frac{2}{7}\,(b^3+6\,abc)x^{\frac{7}{2}}+2\,(ab^2+a^2c)x^{\frac{3}{2}}-\frac{2\,(15\,a^2bx^2+a^3)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*(b^2*c + a*c^2)*x^(11/2) + 2/
7*(b^3 + 6*a*b*c)*x^(7/2) + 2*(a*b^2 + a^2*c)*x^(3/2) - 2/5*(15*a^2*b*x^2 +
 a^3)/x^(5/2)
```

Fricas [A]
time = 0.35, size = 83, normalized size = 0.84

$$\frac{2\,(385\,c^3x^{12}+1463\,bc^2x^{10}+1995\,(b^2c+ac^2)x^8+1045\,(b^3+6\,abc)x^6-21945\,a^2bx^2+7315\,(ab^2+a^2c)x^4-1463\,a^3)}{7315\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*c^3*x^12 + 1463*b*c^2*x^10 + 1995*(b^2*c + a*c^2)*x^8 + 1045*(b
^3 + 6*a*b*c)*x^6 - 21945*a^2*b*x^2 + 7315*(a*b^2 + a^2*c)*x^4 - 1463*a^3)/
x^(5/2)
```

Sympy [A]
time = 1.85, size = 124, normalized size = 1.25

$$-\frac{2a^3}{5x^{\frac{5}{2}}}-\frac{6a^2b}{\sqrt{x}}+2a^2cx^{\frac{3}{2}}+2ab^2x^{\frac{3}{2}}+\frac{12abcx^{\frac{7}{2}}}{7}+\frac{6ac^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3/x**(7/2),x)
```

```
[Out] -2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a**2*c*x**(3/2) + 2*a*b**2*x**(
3/2) + 12*a*b*c*x**(7/2)/7 + 6*a*c**2*x**(11/2)/11 + 2*b**3*x**(7/2)/7 + 6*
b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5 + 2*c**3*x**(19/2)/19
```

Giac [A]
time = 4.24, size = 88, normalized size = 0.89

$$\frac{2}{19}c^3x^{\frac{19}{2}}+\frac{2}{5}bc^2x^{\frac{15}{2}}+\frac{6}{11}b^2cx^{\frac{11}{2}}+\frac{6}{11}ac^2x^{\frac{11}{2}}+\frac{2}{7}b^3x^{\frac{7}{2}}+\frac{12}{7}abcx^{\frac{7}{2}}+2\,ab^2x^{\frac{3}{2}}+2\,a^2cx^{\frac{3}{2}}-\frac{2\,(15\,a^2bx^2+a^3)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 6/11*a*c^2*x
^(11/2) + 2/7*b^3*x^(7/2) + 12/7*a*b*c*x^(7/2) + 2*a*b^2*x^(3/2) + 2*a^2*c*
x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

Mupad [B]
time = 0.04, size = 79, normalized size = 0.80

$$x^{7/2}\left(\frac{2\,b^3}{7}+\frac{12\,a\,c\,b}{7}\right)-\frac{\frac{2\,a^3}{5}+6\,b\,a^2\,x^2}{x^{5/2}}+\frac{2\,c^3\,x^{19/2}}{19}+\frac{2\,b\,c^2\,x^{15/2}}{5}+2\,a\,x^{3/2}\,\left(b^2+a\,c\right)+\frac{6\,c\,x^{11/2}\,\left(b^2+a\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3/x^(7/2),x)
```

```
[Out] x^(7/2)*((2*b^3)/7 + (12*a*b*c)/7) - ((2*a^3)/5 + 6*a^2*b*x^2)/x^(5/2) + (2
*c^3*x^(19/2))/19 + (2*b*c^2*x^(15/2))/5 + 2*a*x^(3/2)*(a*c + b^2) + (6*c*x
^(11/2)*(a*c + b^2))/11
```

3.1062 $\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=389

$$\frac{2x^{3/2}}{3c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

```
[Out] 2/3*x^(3/2)/c-1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1
/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(
1/2))^(1/4)+1/2*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/
4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1
/2))^(1/4)-1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2)
)^(1/4)+1/2*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*
(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^
(1/4)
```

Rubi [A]

time = 0.60, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1129, 1381, 1524, 304, 211, 214}

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{2x^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-
b + Sqrt[b^2 - 4*a*c])^(1/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcT
anh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(
7/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c
])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3
/4)*c^(7/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx = 2\text{Subst}\left(\int \frac{x^{10}}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2x^{3/2}}{3c} - \frac{2\text{Subst}\left(\int \frac{x^2(3a+3bx^4)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{3c}$$
$$= \frac{2x^{3/2}}{3c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\left(b+\frac{b^2-}{\sqrt{b^2-}}\right.}{}$$
$$= \frac{2x^{3/2}}{3c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}c^{3/2}}$$
$$= \frac{2x^{3/2}}{3c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ta}}{2^{3/4}c^{7/4}\sqrt[4]{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.09, size = 80, normalized size = 0.21

$$\frac{4x^{3/2} - 3\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{a\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4),x]
[Out] (4*x^(3/2) - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] + b*Log
[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(6*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 65, normalized size = 0.17

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3c} - \frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)} \frac{\left(\_R^6b+\_R^2a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2c}$	65

default	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{6}b+\_R^{2}a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2c}$	65
risch	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{6}b+\_R^{2}a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2c}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)/c-1/2/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R
=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/c - integrate((b*x^(5/2) + a*sqrt(x))/(c^2*x^4 + b*c*x^2 + a*c)
, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6649 vs. $2(307) = 614$.
time = 3.69, size = 6649, normalized size = 17.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*
c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^
6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^
2*c^8 + 16*a^2*c^9)))*arctan(1/2*((b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^
3*b^3*c^3 + 4*a^4*b*c^4 - (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3
*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*
b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*
c^16 - 64*a^3*c^17)))*sqrt((a^10*b^12 - 10*a^11*b^10*c + 37*a^12*b^8*c^2 -
62*a^13*b^6*c^3 + 46*a^14*b^4*c^4 - 12*a^15*b^2*c^5 + a^16*c^6)*x - 1/2*sqr
t(1/2)*(a^7*b^17 - 17*a^8*b^15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 9
```

24*a^11*b^9*c^4 - 1078*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 + 12*a^15*b*c^8 - (a^7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491*a^10*b^8*c^10 + 997*a^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^13*b^2*c^13 - 64*a^14*c^14)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))) + (a^5*b^15 - 14*a^6*b^13*c + 77*a^7*b^11*c^2 - 210*a^8*b^9*c^3 + 294*a^9*b^7*c^4 - 196*a^10*b^5*c^5 + 49*a^11*b^3*c^6 - 4*a^12*b*c^7 - (a^5*b^12*c^7 - 15*a^6*b^10*c^8 + 88*a^7*b^8*c^9 - 253*a^8*b^6*c^10 + 362*a^9*b^4*c^11 - 224*a^10*b^2*c^12 + 32*a^11*c^13)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^7*b^12 - 10*a^8*b^10*c + 37*a^9*b^8*c^2 - 62*a^10*b^6*c^3 + 46*a^11*b^4*c^4 - 12*a^12*b^2*c^5 + a^13*c^6)) - 12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*arctan(-1/2*((b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt((a^10*b^12 - 10*a^11*b^10*c + 37*a^12*b^8*c^2 - 62*a^13*b^6*c^3 + 46*a^14*b^4*c^4 - 12*a^15*b^2*c^5 + a^16*c^6)*x - 1/2*sqrt(1/2)*(a^7*b^17 - 17*a^8*b^15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 924*a^11*b^9*c^4 - 1078*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 + 12*a^15*b*c^8 + (a^7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491*a^10*b^8*c^10 + 997*a^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^13*b^2*c^13 - 64*a^14*c^14)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c

```
^8 + 16*a^2*c^9))) + (a^5*b^15 - 14*a^6*b^13*c + 77*a^7*b^11*c^2 - 210*a^8*
b^9*c^3 + 294*a^9*b^7*c^4 - 196*a^10*b^5*c^5 + 49*a^11*b^3*c^6 - 4*a^12*b*c
^7 + (a^5*b^12*c^7 - 15*a^6*b^10*c^8 + 88*a^7*b^8*c^9 - 253*a^8*b^6*c^10 +
362*a^9*b^4*c^11 - 224*a^10*b^2*c^12 + 32*a^11*c^13)*sqrt((b^12 - 10*a*b^10
*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^
6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(x)
*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^
4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2
 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 -
12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 1
6*a^2*c^9))))/(a^7*b^12 - 10*a^8*b^10*c + 37*a^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2 + a), x)
```

Mupad [**B**]
time = 5.82, size = 2500, normalized size = 6.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^
3*c^5))/c^3 - (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5
*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*
a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6
*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6
```

*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i - (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 + (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i)/((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 - (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b^3*c^5))/c^3 + (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-

```
(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2
) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*
b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a^3*
b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c
^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) -
 (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(
4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 +
280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(25
6*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^
(1/4) - (256*(a^8*c - a^7*b^2))/c^3))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^
3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6...
```

3.1063 $\int \frac{x^{7/2}}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=385

$$\frac{2\sqrt{x}}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right.}{\sqrt[4]{2}\,c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] 1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*
c+b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/
2*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c
+b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/2
*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/2*ar
ctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)
/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+2*x^(1/2
)/c
```

Rubi [A]

time = 0.57, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1129, 1381, 1436, 218, 214, 211}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\sqrt{x}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b - Sqrt[b
^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4
)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b +
Sqrt[b^2 - 4*a*c])^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[
(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*
c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = 2\text{Subst}\left(\int \frac{x^8}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2\sqrt{x}}{c} - \frac{2\text{Subst}\left(\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{c}$$

$$= \frac{2\sqrt{x}}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{c} - \frac{\left(b+\frac{b^2\ldots}{\sqrt{b^2\ldots}}\right.\ldots}{\ldots}$$

$$= \frac{2\sqrt{x}}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{c\sqrt{-b+\sqrt{b^2-4ac}}}$$

$$= \frac{2\sqrt{x}}{c} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ta}\ldots}{\sqrt[4]{2}\,c^{5/4}\left(-\ldots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.05, size = 80, normalized size = 0.21

$$-\frac{-4\sqrt{x}+\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*(-4*Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] + b
*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 64, normalized size = 0.17

method	result	size
risch	$\frac{2\sqrt{x}}{c}-\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^4b+a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2c}$	61

derivativedivides	$\frac{2\sqrt{x}}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^4b-a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2c}$	64
default	$\frac{2\sqrt{x}}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^4b-a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2c}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/c+1/2/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootO
f(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5319 vs. $2(307) = 614$.
time = 1.17, size = 5319, normalized size = 13.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(-1/4*(sqrt(1/2)*(b^11 - 13
*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^
5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b
^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*
sqrt(4*(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*x
 + 2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 125
*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 + (b^11*c^5 - 15*a*b^9*c^6 + 85*a
^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqrt((b^8 -
```

6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^
4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) -
 2*sqrt(1/2)*(a*b^15 - 16*a^2*b^13*c + 103*a^3*b^11*c^2 - 340*a^4*b^9*c^3 +
 605*a^5*b^7*c^4 - 554*a^6*b^5*c^5 + 224*a^7*b^3*c^6 - 32*a^8*b*c^7 + (a*b^
14*c^5 - 19*a^2*b^12*c^6 + 147*a^3*b^10*c^7 - 590*a^4*b^8*c^8 + 1290*a^5*b^
6*c^9 - 1464*a^6*b^4*c^10 + 736*a^7*b^2*c^11 - 128*a^8*c^12)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt(1/
2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c
^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*
c^6 + 16*a^2*c^7)))/(a^5*b^8 - 6*a^6*b^6*c + 11*a^7*b^4*c^2 - 6*a^8*b^2*c^3
 + a^9*c^4)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/4*(sqrt(1/2)*(b
^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*
a^5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 3
52*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))*sqrt(4*(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^
6*c^4)*x + 2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c
^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 - (b^11*c^5 - 15*a*b^9*c^
6 + 85*a^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt
(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^
2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b
^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*
a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))
/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) - 2*sqrt(1/2)*(a*b^15 - 16*a^2*b^13*

```
c + 103*a^3*b^11*c^2 - 340*a^4*b^9*c^3 + 605*a^5*b^7*c^4 - 554*a^6*b^5*c^5
+ 224*a^7*b^3*c^6 - 32*a^8*b*c^7 - (a*b^14*c^5 - 19*a^2*b^12*c^6 + 147*a^3*
b^10*c^7 - 590*a^4*b^8*c^8 + 1290*a^5*b^6*c^9 - 1464*a^6*b^4*c^10 + 736*a^7
*b^2*c^11 - 128*a^8*c^12)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 -
 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13
)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b
*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqr...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a), x)
```

Mupad [B]
time = 6.86, size = 2500, normalized size = 6.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (256
*x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*
c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a
*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4
 - 128*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1
```

3*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (256*x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (256*x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (256*x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)))*(-(b^9 +

```
b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3
*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c -
 b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*2i + atan(((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b
^4*c + 13*a^5*b^2*c^2))/c - (256*x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a
^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)
*(256*a^5*b*c^6 + 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a
*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1
/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4) - (256*x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b
^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3
*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 1...
```

3.1064 $\int \frac{x^{5/2}}{a+bx^2+cx^4} dx$

Optimal. Leaf size=331

$$-\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*
a*c+b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/2*arctanh(2^(1/4
)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(3
/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(
-b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)
/(-4*a*c+b^2)^(1/2)-1/2*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1
/2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2
)
```

Rubi [A]

time = 0.31, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1129, 1388, 304, 211, 214}

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c])) + ((-b + Sqrt
[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*
c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])
^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan
h[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/
4)*Sqrt[b^2 - 4*a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx = 2\text{Subst}\left(\int \frac{x^6}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right) + \left(1+\frac{b}{\sqrt{b^2-4ac}}\right.$$

$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{2}\sqrt{c}} + \frac{\left(1-\right.}{}$$

$$= -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}} + \frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\tan}{2^{3/4}c^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.06, size = 48, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{\log\left(\sqrt{x}-\#1\right)\#1^3}{b+2c\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1^3)/(b + 2*c*#1^4) & ]
/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 45, normalized size = 0.14

method	result	size
derivativedivides	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^6\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}\right)}{2}$	45
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^6\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}\right)}{2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4058 vs. 2(253) = 506.
time = 0.64, size = 4058, normalized size = 12.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^
8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(1/2*((b^4 -
 5*a*b^2*c + 4*a^2*c^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*sqrt((b^4 -
```

2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9
)))*sqrt((a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*x - 1/2*sqrt(1/2)*(a^3*b^7 - 6*a
^4*b^5*c + 9*a^5*b^3*c^2 - 4*a^6*b*c^3 + (a^3*b^8*c^3 - 13*a^4*b^6*c^4 + 60
*a^5*b^4*c^5 - 112*a^6*b^2*c^6 + 64*a^7*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*
a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*
b^2*c^4 + 16*a^2*c^5))) + (a^2*b^6 - 6*a^3*b^4*c + 9*a^4*b^2*c^2 - 4*a^5*c^
3 + (a^2*b^7*c^3 - 9*a^3*b^5*c^4 + 24*a^4*b^3*c^5 - 16*a^5*b*c^6)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c
^9)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4
 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 4
8*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a^3*b
^4 - 2*a^4*b^2*c + a^5*c^2)) + 2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4
*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)))*arctan(-1/2*((b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^5*c^3 - 8*a*b^3*
c^4 + 16*a^2*b*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^
7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt((a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*x
 - 1/2*sqrt(1/2)*(a^3*b^7 - 6*a^4*b^5*c + 9*a^5*b^3*c^2 - 4*a^6*b*c^3 - (a^
3*b^8*c^3 - 13*a^4*b^6*c^4 + 60*a^5*b^4*c^5 - 112*a^6*b^2*c^6 + 64*a^7*c^7)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(sqrt(1/2)*sqrt(
-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))) + (a^2*b^6 - 6*a^3*b^4*c + 9*a^4*b^2*c^2
- 4*a^5*c^3 - (a^2*b^7*c^3 - 9*a^3*b^5*c^4 + 24*a^4*b^3*c^5 - 16*a^5*b*c^6)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*
a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^
4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
)))/(a^3*b^4 - 2*a^4*b^2*c + a^5*c^2)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*
a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*
b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2
 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^
6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 +
 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 -
 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x

```
)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*
b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*sqr
t(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b
^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqr
t(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (
b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3
- 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3
*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^
2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c
^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

Mupad [**B**]
time = 6.51, size = 2500, normalized size = 7.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2 + c*x^4),x)
```

[Out] - atan(((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + (x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) - (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 - x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i)/((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) - (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 - x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) - (x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4) + 256*a^4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*2i - atan(((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 - b^2

```
*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*
a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 - b^
2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a
*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96
*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4
- 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 - b^2*
(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a
^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + (x^(1/2)*(256*a^3*b^3*c - 768*a^
4*b*c^2) - (-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c
^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^
5 - x^(1/2)*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b
^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*
c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*
b^2*c^4))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3
*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^
3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i)/((x^(1/2)*
(256*a^3*b^3*c - 768*a^4*b*c^2) - (-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 4
8*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c...
```

3.1065 $\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$\frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}-\frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}+$$

```
[Out] 1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a
*c+b^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)+1/2*arctanh(2^(1/4)
*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(1/
4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)-1/2*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-
b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/
(-4*a*c+b^2)^(1/2)-1/2*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/
2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1129, 1388, 218, 214, 211}

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt
[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^
2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])
^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1
/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(
1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(
2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*
Sqrt[b^2 - 4*a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx = 2\text{Subst}\left(\int \frac{x^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right) + \left(1+\frac{b}{\sqrt{b^2-4a}}\right.$$

$$= \frac{\sqrt{-b-\sqrt{b^2-4ac}}\,\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} + \frac{\sqrt{-b}}{}$$

$$= \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\right.}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.05, size = 46, normalized size = 0.14

$$\frac{1}{2}\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{\log\left(\sqrt{x}-\#1\right)\#1}{b+2c\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1)/(b + 2*c*#1^4) & ]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 45, normalized size = 0.14

method	result	size
derivativedivides	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^4\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}\right)}{2}$	45
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^4\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}\right)}{2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2482 vs. 2(253) = 506.
time = 0.44, size = 2482, normalized size = 7.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)))*arctan(1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (b^7*c - 1
2*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b + (b^4*
```

c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) + x)*sqrt(-(b + (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*(b^4 - 8*a*b^2*
c + 16*a^2*c^2 - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqr
t(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x)*sqrt(-(b +
(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sq
rt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 2*sq
rt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)))*arctan(-1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5))*sqrt(sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b - (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64
*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) + x)*sqrt(-(b - (b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^
6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x)*sqrt(-(b - (b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt(-
(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 1/2*sqrt
(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3
)))*log((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(-(b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5) + sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log((b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) +
sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)
/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b
^2*c^2 + 16*a^2*c^3)))*log(-(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/

```
2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

Mupad [**B**]
time = 6.02, size = 2500, normalized size = 7.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^5
)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2
 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^
(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c
^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6) - x^(1/2)*(65536*a^4*b*c^6 + 4
096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 1
6*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b
^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i + (x^
(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) - (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) +
 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2
*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 1
```

6*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b
^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 9830
4*a^3*b^4*c^5 - 393216*a^4*b^2*c^6) + x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b
^5*c^4 - 32768*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 -
256*a^3*b^2*c^4)))^(3/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b^5 + (-(4
*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 -
16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i)/((x^(1/2)*(51
2*a^3*c^4 - 256*a^2*b^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
- 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 -
256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4
*c^5 - 393216*a^4*b^2*c^6) - x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 -
32768*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*
b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b
^2*c^4)))^(3/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b^5 + (-(4*a*c - b^
2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6
*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - (x^(1/2)*(512*a^3*c^4 -
256*a^2*b^2*c^3) - (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b
^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^
2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3
c)/(32(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*
c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 39321
6*a^4*b^2*c^6) + x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^
3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3
/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^
2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*
b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*2i - 2*atan(((x^(1/2)*(512*a^3*c^4 - 256
*a^2*b^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c
^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)
/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4
)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a
^4*b^2*c^6)*1i + x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^
3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3
/4)*1i - 2048*a^3*b*c^4 + 512*a^2*b^3*c^3)*1i)*(-(b^5 + (-(4*a*c - b^2)^5)^
(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) + (x^(1/2)*(512*a^3*c^4 - 256*a^
2*b^2*c^3) - (-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/
(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)

```
))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(3
2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))
^(1/4)*(524288*a^5*c^7 - 8192*a^2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*
b^2*c^6)*1i - x^(1/2)*(65536*a^4*b*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c
^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8
*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)
*1i - 2048*a^3*b*c^4 + 512*a^2*b^3*c^3)*1i)*(-(b^5 + (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256...
```

3.1066 $\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$-\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\ldots\right.}{\sqrt{b^2-4ac}\sqrt[4]{-b\ldots}}$$

```
[Out] -2^(1/4)*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/
4))/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(-4*a*c+b^2)^(1/2)+2^(1/4)*c^(1/4)*arctan
h(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))/(-b-(-4*a*c+b^2)^(
1/2))^(1/4)/(-4*a*c+b^2)^(1/2)+2^(1/4)*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/
2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2)
)^(1/4)-2^(1/4)*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1
/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A]
time = 0.25, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1129, 1389, 304, 211, 214}

$$-\frac{\sqrt[4]{2}\sqrt[4]{c}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\,\mathrm{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((2^(1/4)*c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (2^(1/4)*c^
(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sq
rt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (2^(1/4)*c^(1/4)*ArcTanh[
(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*
c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (2^(1/4)*c^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx &= 2\text{Subst}\left(\int \frac{x^2}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)\\
&= \frac{(2c)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, \right.}{\sqrt{b^2-4ac}}\\
&= \frac{\left(\sqrt{2}\sqrt{c}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{\left(\sqrt{2}\sqrt{c}\right)\text{Subs}}{}\\
&= -\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.05, size = 47, normalized size = 0.14

$$\frac{1}{2}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log\left(\sqrt{x}-\#1\right)}{b\#1+2c\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1 + 2*c*#1^5) & ]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 45, normalized size = 0.14

method	result	size
derivativedivides	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^2\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}\right)}{2}$	45
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^2\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}\right)}{2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2769 vs. 2(251) = 502.
time = 0.45, size = 2769, normalized size = 8.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)))*arctan(((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(c^2*x - 1/2*sqrt
(1/2)*(b^3*c - 4*a*b*c^2 - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*
a^4*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-
(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - (a*b^4*c - 8*a^2*b^2*c^2 +
16*a^3*c^3)*sqrt(x)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)))/c) + 2*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2)))*arctan(((a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt
(c^2*x - 1/2*sqrt(1/2)*(b^3*c - 4*a*b*c^2 + (a*b^6*c - 12*a^2*b^4*c^2 + 48*
a^3*b^2*c^3 - 64*a^4*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)))/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - (a
*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b - (a*b
^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/c) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b + (a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b
^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a
^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a
^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*s
qrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/
sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a
*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*
a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqr
t(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sq
rt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*
```

```
c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c
^2)) + c*sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16
*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^
2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 -
 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*sqrt(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

Mupad [**B**]
time = 5.31, size = 2500, normalized size = 7.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2 + c*x^4),x)
[Out] 2*atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*
(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(
3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 - x^(1/2)*(-(b^5
```

- (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i)*1i - 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) - ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 + x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i)*1i + 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4))/(((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 - x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i)*1i - 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - 256*a*c^5 + ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 + x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i)*1i + 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) - atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 + x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)) - 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 - x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(131072*a^4*c^7

```
- 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)) + 256*a*b*c^5*x
^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(
a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1
/4)*1i)/(256*a*c^5 + ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*
a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4
*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 + x
^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a
*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/
4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^
6)) - 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 25
6*a^4*b^2*c^3)))^(1/4) + ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
- 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256
*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5
 - x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(3
2*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))
^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^
2*c^6)) + 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16...
```

3.1067 $\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=331

$$\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{-b-}}\right.}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2}\right.}$$

```
[Out] 2^(3/4)*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4
))/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)+2^(3/4)*c^(3/4)*arctanh
(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))/(-b-(-4*a*c+b^2)^(1
/2))^(3/4)/(-4*a*c+b^2)^(1/2)-2^(3/4)*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2
)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))
^(3/4)-2^(3/4)*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/
2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A]
time = 0.28, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1129, 1361, 218, 214, 211}

$$\frac{2^{3/4}c^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (2^(3/4)*c^(3/
4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[
b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (2^(3/4)*c^(3/4)*ArcTanh[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^
2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)}\,dx = 2\text{Subst}\left(\int \frac{1}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, \sqrt{x}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{(2c)\text{Subst}\left(\int -\right.}{\sqrt{}}$$

$$= \frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 49, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log\left(\sqrt{x}-\#1\right)}{b\#1^3+2c\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1^3 + 2*c*#1^7) & ]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 42, normalized size = 0.13

method	result	size
derivativedivides	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}\right)}{2}$	42
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}\right)}{2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum(1/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)/a - integrate((c*x^(7/2) + b*x^(3/2))/(a*c*x^4 + a*b*x^2 + a^2),
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4045 vs. 2(251) = 502.
time = 0.60, size = 4045, normalized size = 12.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(-1/4*(sqrt(
1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (a^3*b^8 - 14*a^4*b
^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(
4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x + 2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 21*a
^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4 - (a^3*b^9 - 13*a^4*b^7*c + 60*a^5*
b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5
*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + 2*sqrt(1/2)*(b
^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5 - (a^3
*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c
^5 - 128*a^8*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(x)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 -
8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^
7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*
c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^4*c^3 - 2*a*b
^2*c^4 + a^2*c^5)) + 2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))
)*arctan(1/4*(sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 +
(a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)))*sqrt(4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x + 2*sqrt(1/2)*(b^8
 - 8*a*b^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4 + (a^3*b^9 - 13*
a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
 - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c
```

```
^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + 2*sqrt(1/2)*(b^9
*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5 + (a^3*b
^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5
 - 128*a^8*c^6)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
- (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^4*c^3 - 2*a*b^2
*c^4 + a^2*c^5)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))
)*log(-2*(b^2*c - a*c^2)*sqrt(x) + (b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*
a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^
4*b^2*c + 16*a^5*c^2)))) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 -
12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a
^5*c^2)))*log(-2*(b^2*c - a*c^2)*sqrt(x) - (b^4 - 5*a*b^2*c + 4*a^2*c^2 - (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(
b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x)), x)
```

Mupad [B]
time = 6.26, size = 2500, normalized size = 7.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4
 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 5
12*b^2*c^6 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^
7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^
7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) + x^(1/2
)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))
*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11
*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^
4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) + 512*c^7*x^(1/2))*(-(
b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b
^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^
6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c
^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b^7 + b^2*(-(
4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-
(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*
b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 983
04*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) - x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*
c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^
5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)
^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256
*a^6*b^2*c^3)))^(3/4)) - 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(
1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^
(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6
*b^2*c^3)))^(1/4)*1i)/(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^
3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^
8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*
(2048*a*c^7 - 512*b^2*c^6 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3
*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a
^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(
1/4)*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^
3*c^6) + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 16384
0*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a
^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*
```

a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) + 512*c
^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b
^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4) + ((-(b^7 +
b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c -
 a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c +
96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 + ((-(b
^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^
5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6
*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 - 524288*a^4
*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) - x^(1/2)*(4096*b^7*c^4 -
45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4
*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(
4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b
^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) - 512*c^7*x^(1/2))*(-(b^7 + b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*
c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c
^2 - 256*a^6*b^2*c^3)))^(1/4)))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*
a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32
*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))
)^(1/4)*2i - atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 +
256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(204
8*a*c^7 - 512*b^2*c^6 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c
^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b
^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)
*(8192*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^
6) + x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^
2*b^3*c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b
^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1...

3.1068 $\int \frac{1}{x^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=371

$$-\frac{2}{a\sqrt{x}}-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}}{\sqrt[4]{-b+}}\right.}{2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(1-b/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/2*c^(1/4
)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(1-b/(-4*a
*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/2*c^(1/4)*arctan(2
^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/
2))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/2*c^(1/4)*arctanh(2^(1/4)*c^(
1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/4
)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-2/a/x^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1129, 1382, 1524, 304, 211, 214}

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*
c])^(1/4)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^
(1/4)) + (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) + (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx^2+cx^4)}\,dx = 2\text{Subst}\left(\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{a\sqrt{x}} + \frac{2\text{Subst}\left(\int \frac{x^2(-b-cx^4)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{a}$$
$$= -\frac{2}{a\sqrt{x}} + \frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}}\right.}{\sqrt{2}\,a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(1+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.08, size = 78, normalized size = 0.21

$$-\frac{\frac{4}{\sqrt{x}} + \text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b\log\left(\sqrt{x}-\#1\right)+c\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(4/Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] + c*
Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/a
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 65, normalized size = 0.18

method	result	size
derivativedivides	$-\frac{2}{a\sqrt{x}} - \frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)} \frac{\left(\_R^6c+\_R^2b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2a}$	65

default	$-\frac{2}{a\sqrt{x}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6c+\_R^2b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2a}$	65
risch	$-\frac{2}{a\sqrt{x}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6c+\_R^2b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2a}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/x^(1/2)-1/2/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=
RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2/(a*sqrt(x)) - integrate((c*x^(5/2) + b*sqrt(x))/(a*c*x^4 + a*b*x^2 + a^2
), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5384 vs. $2(289) = 578$.
time = 1.88, size = 5384, normalized size = 14.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(1/2*((b^6 - 7*a*b^4*c +
13*a^2*b^2*c^2 - 4*a^3*c^3 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((b^8*c^8 - 6*a*b^6*c^9 +
11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x - 1/2*sqrt(1/2)*(b^13*c^5 -
13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 + 175*a^4*b^5*c^9 - 79*a^5
*b^3*c^10 + 12*a^6*b*c^11 + (a^5*b^12*c^5 - 16*a^6*b^10*c^6 + 100*a^7*b^8*c
```

^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*b^2*c^10 + 64*a^11*c^11)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c
+ 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))) - (b^1
0*c^4 - 10*a*b^8*c^5 + 35*a^2*b^6*c^6 - 50*a^3*b^4*c^7 + 25*a^4*b^2*c^8 - 4
*a^5*c^9 + (a^5*b^9*c^4 - 11*a^6*b^7*c^5 + 41*a^7*b^5*c^6 - 56*a^8*b^3*c^7
+ 16*a^9*b*c^8)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^
4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(x)
)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^
2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 -
 6*a^3*b^2*c^8 + a^4*c^9)) - 4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-
1/2*((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (a^5*b^5 - 8*a^6*b^3*c
 + 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((
b^8*c^8 - 6*a*b^6*c^9 + 11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x - 1/
2*sqrt(1/2)*(b^13*c^5 - 13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 +
175*a^4*b^5*c^9 - 79*a^5*b^3*c^10 + 12*a^6*b*c^11 - (a^5*b^12*c^5 - 16*a^6*
b^10*c^6 + 100*a^7*b^8*c^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*b
^2*c^10 + 64*a^11*c^11)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*
c + 16*a^7*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^
5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a
^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))) - (b^10*c^4 - 10*a*b^8*c^
5 + 35*a^2*b^6*c^6 - 50*a^3*b^4*c^7 + 25*a^4*b^2*c^8 - 4*a^5*c^9 - (a^5*b^9
*c^4 - 11*a^6*b^7*c^5 + 41*a^7*b^5*c^6 - 56*a^8*b^3*c^7 + 16*a^9*b*c^8)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(x)*sqrt(sqrt(1/2)*sqr
t(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c +
16*a^7*c^2))))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 - 6*a^3*b^2*c^8 + a^
4*c^9)) - a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*

```
c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*
a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5
 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^
2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*s
qrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c
 + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*x^(3/2)), x)
```

Mupad [**B**]
time = 5.74, size = 2500, normalized size = 6.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] 2*atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*
c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a
*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
+ 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^
9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*
```

b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i + 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i - 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4))/(((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i + 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i

+ 2048*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2*c^7)*1i - 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - atan((((-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4 - 49152*a^13*b^...

3.1069 $\int \frac{1}{x^{5/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=371

$$-\frac{2}{3ax^{3/2}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\cdots}{\sqrt[4]{-b+\cdots}}\right)}{\sqrt[4]{2}\,a\left(-b+\sqrt{b^2-4ac}\right)^{3\cdots}}$$

```
[Out] -2/3/a/x^(3/2)+1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^
(1/2))^(1/4))*(1-b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3
/4)+1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/
4))*(1-b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/2*c^
(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-
4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/2*c^(3/4)*arcta
nh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)
^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A]

time = 0.36, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1129, 1382, 1436, 218, 214, 211}

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*
a*c])^(3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c]
)^(3/4)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x]
)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx^2+cx^4)}\,dx = 2\text{Subst}\left(\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{3ax^{3/2}} + \frac{2\text{Subst}\left(\int \frac{-3b-3cx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{3a}$$
$$= -\frac{2}{3ax^{3/2}} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, \sqrt{x}\right)}{a}$$
$$= -\frac{2}{3ax^{3/2}} + \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\right.}{a\sqrt{-b-\sqrt{b^2-4ac}}}$$
$$= -\frac{2}{3ax^{3/2}} + \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\,a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.06, size = 82, normalized size = 0.22

$$-\frac{\frac{4}{x^{3/2}} + 3\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b\log\left(\sqrt{x}-\#1\right)+c\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
[Out] -1/6*(4/x^(3/2) + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] +
c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/a
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 64, normalized size = 0.17

method	result	size
risch	$-\frac{2}{3a\,x^{\frac{3}{2}}} - \frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)} \frac{\left(\_R^4c+b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2a}$	61

derivativedivides	$-\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}c-b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2a}$	64
default	$-\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}c-b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2a}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)+1/2/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=Ro
otOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2/3*(3*b*sqrt(x) + a/x^(3/2))/a^2 + integrate((b*c*x^(7/2) + (b^2 - a*c)*x
^(3/2))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6671 vs. $2(289) = 578$.
time = 2.54, size = 6671, normalized size = 17.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(12*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 3
7*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*
a^8*b^2*c + 16*a^9*c^2)))*arctan(-1/4*(sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*
a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^
6*b^2*c^6 - 16*a^7*c^7 + (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a
^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(4*(b^1
2*c^4 - 10*a*b^10*c^5 + 37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 -
```

12*a^5*b^2*c^9 + a^6*c^10)*x + 2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2*b^14*c^2 - 546*a^3*b^12*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*a^6*b^6*c^6 - 592*a^7*b^4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 + (a^7*b^15 - 19*a^8*b^13*c + 148*a^9*b^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 1672*a^12*b^5*c^5 + 928*a^13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) + 2*sqrt(1/2)*(b^20*c^2 - 21*a*b^18*c^3 + 188*a^2*b^16*c^4 - 935*a^3*b^14*c^5 + 2821*a^4*b^12*c^6 - 5292*a^5*b^10*c^7 + 6083*a^6*b^8*c^8 - 4071*a^7*b^6*c^9 + 1449*a^8*b^4*c^10 - 248*a^9*b^2*c^11 + 16*a^10*c^12 + (a^7*b^17*c^2 - 22*a^8*b^15*c^3 + 204*a^9*b^13*c^4 - 1032*a^10*b^11*c^5 + 3075*a^11*b^9*c^6 - 5417*a^12*b^7*c^7 + 5324*a^13*b^5*c^8 - 2480*a^14*b^3*c^9 + 320*a^15*b*c^10)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(x)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(b^12*c^7 - 10*a*b^10*c^8 + 37*a^2*b^8*c^9 - 62*a^3*b^6*c^10 + 46*a^4*b^4*c^11 - 12*a^5*b^2*c^12 + a^6*c^13)) - 12*a*x^2*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*arctan(1/4*(sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(4*(b^12*c^4 - 10*a*b^10*c^5 + 37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 - 12*a^5*b^2*c^9 + a^6*c^10)*x + 2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2*b^14*c^2 - 546*a^3*b^12*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*a^6*b^6*c^6 - 592*a^7*b^

```
4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 - (a^7*b^15 - 19*a^8*b^13*c + 148*a^9*b
^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 1672*a^12*b^5*c^5 + 928*a^
13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^
15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2
*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 -
10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2
*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3))
)/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^
5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*s
qrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*x^(5/2)), x)
```

Mupad [**B**]
time = 8.64, size = 2500, normalized size = 6.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 +
 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(
(x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 2498
```

56*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) + 4096*a^11*b*c^9 + 512*a^9*b^5*c^7 - 3072*a^10*b^3*c^8))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5 + 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3

+ 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 4096*a^11*b*c^9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - (x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b...

3.1070 $\int \frac{1}{x^{7/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=412

$$-\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a^2\sqrt[4]{-b-\sqrt{b^2-4ac}}}+\frac{\sqrt[4]{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{2^{3/4}a^2\sqrt[4]{-b+\sqrt{}}}$$

[Out] -2/5/a/x^(5/2)+1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/2*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/2*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+2*b/a^2/x^(1/2)

Rubi [A]

time = 0.61, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1129, 1382, 1518, 1524, 304, 211, 214}

$$\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]

[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) + (c^(1/4)*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b + Sqrt[b^2 - 4*a*c])^(1/4))

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}(a+bx^2+cx^4)}\,dx = 2\text{Subst}\left(\int \frac{1}{x^6(a+bx^4+cx^8)}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2\text{Subst}\left(\int \frac{-5b-5cx^4}{x^2(a+bx^4+cx^8)}\,dx, x, \sqrt{x}\right)}{5a}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{2\text{Subst}\left(\int \frac{x^2(-5(b^2-ac)-5bcx^4)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{5a^2}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right.}{a^2}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} - \frac{\left(\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}}\right.}{\sqrt{2}\,a^2}$$
$$= -\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a^2\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.11, size = 106, normalized size = 0.26

$$\frac{-\frac{4(a-5bx^2)}{x^{5/2}} + 5\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b^2\log\left(\sqrt{x}-\#1\right)-ac\log\left(\sqrt{x}-\#1\right)+bc\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{10a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*(a - 5*b*x^2))/x^(5/2) + 5*RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log[Sq
rt[x] - #1] - a*c*Log[Sqrt[x] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2
*c*#1^5) & ])/(10*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 84, normalized size = 0.20

method	result	size

risch	$-\frac{2(-5b\,x^2+a)}{5a^2x^{\frac{5}{2}}}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4b+a)}\frac{(bc\_R^6+(-ac+b^2)\_R^2)\ln(\sqrt{x}-\_R)}{2\_R^7c+\_R^3b}}{2a^2}$	81
derivativedivides	$-\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4b+a)}\frac{(-bc\_R^6+(ac-b^2)\_R^2)\ln(\sqrt{x}-\_R)}{2\_R^7c+\_R^3b}}{2a^2}-\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{a^2\sqrt{x}}$	84
default	$-\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4b+a)}\frac{(-bc\_R^6+(ac-b^2)\_R^2)\ln(\sqrt{x}-\_R)}{2\_R^7c+\_R^3b}}{2a^2}-\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{a^2\sqrt{x}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*sum((-b*c*_R^6+(a*c-b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R
=RootOf(_Z^8*c+_Z^4*b+a))-2/5/a/x^(5/2)+2*b/a^2/x^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/5*(5*b/sqrt(x) - a/x^(5/2))/a^2 + integrate((b*c*x^(5/2) + (b^2 - a*c)*sq
rt(x))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7995 vs. $2(328) = 656$.
time = 11.60, size = 7995, normalized size = 19.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/10*(20*a^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 3
0*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((
b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 -
 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 -
12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c +
16*a^11*c^2)))*arctan(1/2*((b^11 - 11*a*b^9*c + 43*a^2*b^7*c^2 - 70*a^3*b^5
```

*c^3 + 41*a^4*b^3*c^4 - 4*a^5*b*c^5 - (a^9*b^6 - 10*a^10*b^4*c + 32*a^11*b^
2*c^2 - 32*a^12*c^3)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b
^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*
c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))
*sqrt((b^16*c^14 - 14*a*b^14*c^15 + 79*a^2*b^12*c^16 - 230*a^3*b^10*c^17 +
367*a^4*b^8*c^18 - 314*a^5*b^6*c^19 + 130*a^6*b^4*c^20 - 20*a^7*b^2*c^21 +
a^8*c^22)*x - 1/2*sqrt(1/2)*(b^23*c^9 - 23*a*b^21*c^10 + 230*a^2*b^19*c^11
- 1311*a^3*b^17*c^12 + 4692*a^4*b^15*c^13 - 10947*a^5*b^13*c^14 + 16731*a^6
*b^11*c^15 - 16380*a^7*b^9*c^16 + 9711*a^8*b^7*c^17 - 3109*a^9*b^5*c^18 + 4
25*a^10*b^3*c^19 - 20*a^11*b*c^20 - (a^9*b^18*c^9 - 22*a^10*b^16*c^10 + 205
*a^11*b^14*c^11 - 1050*a^12*b^12*c^12 + 3206*a^13*b^10*c^13 - 5909*a^14*b^8
*c^14 + 6333*a^15*b^6*c^15 - 3580*a^16*b^4*c^16 + 880*a^17*b^2*c^17 - 64*a^
18*c^18)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 36
7*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^
8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(-(b^9
- 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*
a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*
a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7
*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c
^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2))) - (b^19*c^7 - 18*a*b^17*c^8
+ 135*a^2*b^15*c^9 - 546*a^3*b^13*c^10 + 1287*a^4*b^11*c^11 - 1782*a^5*b^9*
c^12 + 1386*a^6*b^7*c^13 - 540*a^7*b^5*c^14 + 81*a^8*b^3*c^15 - 4*a^9*b*c^1
6 - (a^9*b^14*c^7 - 17*a^10*b^12*c^8 + 117*a^11*b^10*c^9 - 416*a^12*b^8*c^1
0 + 805*a^13*b^6*c^11 - 810*a^14*b^4*c^12 + 352*a^15*b^2*c^13 - 32*a^16*c^1
4)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*
b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^
18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(x))*sqrt(sqr
t(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c
^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a
^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^
6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b
^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))/(b^16*c^9
- 14*a*b^14*c^10 + 79*a^2*b^12*c^11 - 230*a^3*b^10*c^12 + 367*a^4*b^8*c^13
- 314*a^5*b^6*c^14 + 130*a^6*b^4*c^15 - 20*a^7*b^2*c^16 + a^8*c^17)) - 20*a
^2*x^3*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*
c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*
a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b
^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^
4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^
2)))*arctan(-1/2*((b^11 - 11*a*b^9*c + 43*a^2*b^7*c^2 - 70*a^3*b^5*c^3 + 41
*a^4*b^3*c^4 - 4*a^5*b*c^5 + (a^9*b^6 - 10*a^10*b^4*c + 32*a^11*b^2*c^2 - 3
2*a^12*c^3)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 +
367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8
*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt((b^
16*c^14 - 14*a*b^14*c^15 + 79*a^2*b^12*c^16 - 230*a^3*b^10*c^17 + 367*a^4*b

```
^8*c^18 - 314*a^5*b^6*c^19 + 130*a^6*b^4*c^20 - 20*a^7*b^2*c^21 + a^8*c^22)
*x - 1/2*sqrt(1/2)*(b^23*c^9 - 23*a*b^21*c^10 + 230*a^2*b^19*c^11 - 1311*a^
3*b^17*c^12 + 4692*a^4*b^15*c^13 - 10947*a^5*b^13*c^14 + 16731*a^6*b^11*c^1
5 - 16380*a^7*b^9*c^16 + 9711*a^8*b^7*c^17 - 3109*a^9*b^5*c^18 + 425*a^10*b
^3*c^19 - 20*a^11*b*c^20 + (a^9*b^18*c^9 - 22*a^10*b^16*c^10 + 205*a^11*b^1
4*c^11 - 1050*a^12*b^12*c^12 + 3206*a^13*b^10*c^13 - 5909*a^14*b^8*c^14 + 6
333*a^15*b^6*c^15 - 3580*a^16*b^4*c^16 + 880*a^17*b^2*c^17 - 64*a^18*c^18)*
sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8
*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*
b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(-(b^9 - 9*a*b^7
*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*
c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*
c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7
+ a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^
9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c
 + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate(1/((c*x^4 + b*x^2 + a)*x^(7/2)), x)
```

Mupad [B]
time = 6.48, size = 2500, normalized size = 6.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2 + c*x^4)),x)
[Out] atan((((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9
*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a
```

*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2)
- 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1
/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^
12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a
^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^
3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96
*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10
*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655
360*a^27*b^2*c^8) - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9
*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8) + x
^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)
^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5
*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 1
5*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 1
6*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i + ((-(b^13 +
b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^
7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2
) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*
(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8
 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/
4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 14
4*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5
*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*
(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^
6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c +
 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b
^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 -
655360*a^27*b^2*c^8) - 2048*a^21*b^11*c^4 + 28672*a^22*b^9*c^5 - 151552*a^2
3*b^7*c^6 + 368640*a^24*b^5*c^7 - 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*
b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a
^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^
3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c
*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96
*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*1i)/(256*a^20*c^12 - ((-(b^13 + b
^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7
*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2)
 - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(
-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8
+ 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4
)*(x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2
*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-

(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 4096*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b^4*c^7 - 655360*a^27*b^2*c^8) - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 28672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^25*b^3*c^8) + x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4) + ((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(3/4)*(131072*a^26*b*c^9 + x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - ...

3.1071 $\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=544

$$-\frac{bx^{3/2}}{2c(b^2-4ac)}+\frac{x^{7/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(3b^3-20abc+(3b^2-14ac)\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{}}{\sqrt[4]{-b-\sqrt{}}}\right.}{4\,2^{3/4}c^{7/4}(b^2-4ac)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*b*x^(3/2)/c/(-4*a*c+b^2)+1/2*x^(7/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b
*x^2+a)-1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(
3*b^3-20*a*b*c-(-14*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-4*a*c+
b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/8*arctanh(2^(1/4)*c^(1/4)*x^(1/2
)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^3-20*a*b*c-(-14*a*c+3*b^2)*(-4*a*c+b^
2)^(1/2))*2^(1/4)/c^(7/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+
1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^3-20
*a*b*c+(-14*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-4*a*c+b^2)^(3/
2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/8*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-
4*a*c+b^2)^(1/2))^(1/4))*(3*b^3-20*a*b*c+(-14*a*c+3*b^2)*(-4*a*c+b^2)^(1/2)
)*2^(1/4)/c^(7/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A]
time = 1.77, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1129, 1379, 1516, 1524, 304, 211, 214}

$$\frac{\left((3b^2-14ac)\sqrt{b^2-4ac}-20abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(-(3b^2-14ac)\sqrt{b^2-4ac}-20abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{bx^{3/2}}{2c(b^2-4ac)}+\frac{x^{7/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\left((3b^2-14ac)\sqrt{b^2-4ac}-20abc+3b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(-(3b^2-14ac)\sqrt{b^2-4ac}-20abc+3b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(b*x^(3/2))/(c*(b^2 - 4*a*c)) + (x^(7/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) + ((3*b^3 - 20*a*b*c + (3*b^2 - 14*a*c)*Sqrt[b^2 -
4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((3
*b^3 - 20*a*b*c - (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*
c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b^3 - 20*a*b*c + (3*b^2 - 14
*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*
a*c])^(1/4)) + ((3*b^3 - 20*a*b*c - (3*b^2 - 14*a*c)*Sqrt[b^2 - 4*a*c])*Arc
Tanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*
c^(7/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1379

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*
p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c,
 d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1]
 && GtQ[m, 2*n - 1]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx = 2\text{Subst}\left(\int \frac{x^{14}}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{7/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x^6(14a+3bx^4)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2(b^2-4ac)}$$

$$= -\frac{bx^{3/2}}{2c(b^2-4ac)} + \frac{x^{7/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{x^2(9ab+3(3b^2-14ac)x^4)}{a+bx^4+cx^8}\,dx,\right.}{6c(b^2-4ac)}$$

$$= -\frac{bx^{3/2}}{2c(b^2-4ac)} + \frac{x^{7/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(3b^2-14ac+\frac{3b^3}{\sqrt{b^2-4ac}} - \right.}{}$$

$$= -\frac{bx^{3/2}}{2c(b^2-4ac)} + \frac{x^{7/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^2-14ac+\frac{3b^3}{\sqrt{b^2-4ac}} - \right.}{}$$

$$= -\frac{bx^{3/2}}{2c(b^2-4ac)} + \frac{x^{7/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(3b^3-20abc+(3b^2-14ac)\sqrt{b\right.}{4\,2^{3/4}c^{7/4}(b^2-4a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.48, size = 232, normalized size = 0.43

$$-\frac{4\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b\log\left(\sqrt{x}-\#1\right)-c\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right] + \frac{\frac{4cx^{3/2}(ab+b^2x^2-2acx^2)}{a+bx^2+cx^4}+\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{-4b^3\log\left(\sqrt{x}-\#1\right)+13abc\log\left(\sqrt{x}-\#1\right)+b^2c\log\left(\sqrt{x}-\#1\right)\#1^4-2ac^2\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{b^2-4ac}}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/8*(4*RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] - c*Log[Sqrt[x
] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ] + ((4*c*x^(3/2)*(a*b + b^2*x^2 - 2*a*c*
```

```
x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#1^8 & , (-4*b^3*Log[Sqr
t[x] - #1] + 13*a*b*c*Log[Sqrt[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 - 2*
a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(b^2 - 4*a*c))/c^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 149, normalized size = 0.27

method	result	size
derivativedivides	$\frac{-\frac{\left(2ac-b^{2}\right)x^{\frac{7}{2}}}{2c\left(4ac-b^{2}\right)}+\frac{ab\,x^{\frac{3}{2}}}{2c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\left(-14ac+3b^{2}\right)\_R^{6}+3b\_R^{2}a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{8c\left(4ac-b^{2}\right)}$	149
default	$\frac{-\frac{\left(2ac-b^{2}\right)x^{\frac{7}{2}}}{2c\left(4ac-b^{2}\right)}+\frac{ab\,x^{\frac{3}{2}}}{2c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\left(-14ac+3b^{2}\right)\_R^{6}+3b\_R^{2}a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{8c\left(4ac-b^{2}\right)}$	149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/4*(2*a*c-b^2)/c/(4*a*c-b^2)*x^(7/2)+1/4*a*b/c/(4*a*c-b^2)*x^(3/2))/(c
*x^4+b*x^2+a)-1/8/c/(4*a*c-b^2)*sum(((-14*a*c+3*b^2)*_R^6+3*b*_R^2*a)/(2*_R
^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^(7/2) + a*b*x^(3/2))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2
*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + integrate(1/4*((3*b^2 - 14*a*c)
*x^(5/2) + 3*a*b*sqrt(x))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 +
(b^3*c - 4*a*b*c^2)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 13791 vs. 2(440) = 880.
time = 101.30, size = 13791, normalized size = 25.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

[Out] -1/8*(4*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2
)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^7*c^2 - 1
00023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*c^5 - (b^12*c^7 - 24*
a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144
*a^5*b^2*c^12 + 4096*a^6*c^13)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*
a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^
5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^
8)/(b^18*c^14 - 36*a*b^16*c^15 + 576*a^2*b^14*c^16 - 5376*a^3*b^12*c^17 + 3
2256*a^4*b^10*c^18 - 129024*a^5*b^8*c^19 + 344064*a^6*b^6*c^20 - 589824*a^7
*b^4*c^21 + 589824*a^8*b^2*c^22 - 262144*a^9*c^23)))/(b^12*c^7 - 24*a*b^10*
c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^
2*c^12 + 4096*a^6*c^13)))*arctan(-1/2*((243*b^14 - 7857*a*b^12*c + 105732*a
^2*b^10*c^2 - 760311*a^3*b^8*c^3 + 3104898*a^4*b^6*c^4 - 6982136*a^5*b^4*c^
5 + 7430752*a^6*b^2*c^6 - 2151296*a^7*c^7 + (3*b^15*c^7 - 92*a*b^13*c^8 + 1
200*a^2*b^11*c^9 - 8640*a^3*b^9*c^10 + 37120*a^4*b^7*c^11 - 95232*a^5*b^5*c
^12 + 135168*a^6*b^3*c^13 - 81920*a^7*b*c^14)*sqrt((6561*b^16 - 258066*a*b^
14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4
 - 596117340*a^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 +
92236816*a^8*c^8)/(b^18*c^14 - 36*a*b^16*c^15 + 576*a^2*b^14*c^16 - 5376*a^
3*b^12*c^17 + 32256*a^4*b^10*c^18 - 129024*a^5*b^8*c^19 + 344064*a^6*b^6*c^
20 - 589824*a^7*b^4*c^21 + 589824*a^8*b^2*c^22 - 262144*a^9*c^23)))*sqrt((8
473321344609*a^10*b^28 - 635242333532202*a^11*b^26*c + 21886199991740997*a^
12*b^24*c^2 - 458493787100930526*a^13*b^22*c^3 + 6509864083385855214*a^14*b
^20*c^4 - 66075187799368237644*a^15*b^18*c^5 + 492507017632546570017*a^16*b
^16*c^6 - 2724207428940903583776*a^17*b^14*c^7 + 11153433166801409792448*a^
18*b^12*c^8 - 33271004636573820028416*a^19*b^10*c^9 + 699941713492700510469
12*a^20*b^8*c^10 - 97955045940955934416896*a^21*b^6*c^11 + 8212571997378131
4453504*a^22*b^4*c^12 - 33936868909745621041152*a^23*b^2*c^13 + 52292659640
80300097536*a^24*c^14)*x - 1/2*sqrt(1/2)*(70027449129*a^7*b^35 - 6122096689
005*a^8*b^33*c + 249990341171571*a^9*b^31*c^2 - 6326257121127405*a^10*b^29*
c^3 + 110988161834764032*a^11*b^27*c^4 - 1430544999765797622*a^12*b^25*c^5
+ 14002838155817044317*a^13*b^23*c^6 - 106093205101688563539*a^14*b^21*c^7
+ 628167444650802051336*a^15*b^19*c^8 - 2912813018384776691184*a^16*b^17*c^
9 + 10528165589869901546496*a^17*b^15*c^10 - 29303647903181578139136*a^18*b
^13*c^11 + 61475256193651087011840*a^19*b^11*c^12 - 93889042597843790094336
*a^20*b^9*c^13 + 98726339583924763820032*a^21*b^7*c^14 - 651242399980388409
01632*a^22*b^5*c^15 + 22857309144118234447872*a^23*b^3*c^16 - 3171100059850
444374016*a^24*b*c^17 + (864536409*a^7*b^36*c^7 - 74140546590*a^8*b^34*c^8
+ 2982905446851*a^9*b^32*c^9 - 74784329591895*a^10*b^30*c^10 + 130869752244
5889*a^11*b^28*c^11 - 16967005712892648*a^12*b^26*c^12 + 168795289141715376
*a^13*b^24*c^13 - 1316597783295499776*a^14*b^22*c^14 + 8155166523802290432*
a^15*b^20*c^15 - 40370095751433271296*a^16*b^18*c^16 + 1598659134024418385 9
2*a^17*b^16*c^17 - 504303068561780178944*a^18*b^14*c^18 + 12547267724208679
81312*a^19*b^12*c^19 - 2420355022911817908224*a^20*b^10*c^20 + 352197657620
4010946560*a^21*b^8*c^21 - 3700574176420976132096*a^22*b^6*c^22 + 260802437

```
8623876136960*a^23*b^4*c^23 - 1071919511651804512256*a^24*b^2*c^24 + 182059
119829942534144*a^25*c^25)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*
b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^
6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(
b^18*c^14 - 36*a*b^16*c^15 + 576*a^2*b^14*c^16 - 5376*a^3*b^12*c^17 + 32256
*a^4*b^10*c^18 - 129024*a^5*b^8*c^19 + 344064*a^6*b^6*c^20 - 589824*a^7*b^4
*c^21 + 589824*a^8*b^2*c^22 - 262144*a^9*c^23)))*sqrt(-(81*b^11 - 2079*a*b^
9*c + 20790*a^2*b^7*c^2 - 100023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 181104*
a^5*b*c^5 - (b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10
 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12 + 4096*a^6*c^13)*sqrt((6561*b^16 -
 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582
*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^
7*b^2*c^7 + 92236816*a^8*c^8)/(b^18*c^14 - 36*a*b^16*c^15 + 576*a^2*b^14*c^
16 - 5376*a^3*b^12*c^17 + 32256*a^4*b^10*c^18 - 129024*a^5*b^8*c^19 + 34406
4*a^6*b^6*c^20 - 589824*a^7*b^4*c^21 + 589824*a^8*b^2*c^22 - 262144*a^9*c^2
3)))/(b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840
*a^4*b^4*c^11 - 6144*a^5*b^2*c^12 + 4096*a^6*c^13))) + (707347971*a^5*b^28
- 49385749248*a^6*b^26*c + 1581659292411*a^7*b^24*c^2 - 30741508944180*a^8*
b^22*c^3 + 404164995546078*a^9*b^20*c^4 - 3791005310369520*a^10*b^18*c^5 +
26061549261652395*a^11*b^16*c^6 - 132704982981551106*a^12*b^14*c^7 + 499360
586530184568*a^13*b^12*c^8 - 1367638655348841120*a^14*b^10*c^9 + 2641915738
391610240*a^15*b^8*c^10 - 3404159129874875904*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [**B**]
time = 7.01, size = 2500, normalized size = 4.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(13/2)/(a + b*x^2 + c*x^4)^2,x)

[Out] - ((x^(7/2)*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x^(3/2))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) - (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(3/4) - (x^(1/2)*(9801*a^5*b^11 - 256905*a^6*b^9*c - 29042496*a^10*b*c^5 + 2642841*a^7*b^7*c^2 - 13243020*a^8*b^5*c^3 + 31945648*a^9*b^3*c^4))/(16*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26

313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15
)^(1/2))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^
20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12
+ 3784704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 -
57671680*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^
(1/4)*1i - (((46036680704*a^12*c^12 - 110592*a^3*b^18*c^3 + 4423680*a^4*b^1
6*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*b^12*c^6 - 5065015296*a^7*b^1
0*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400*a^9*b^6*c^9 + 104312340480*a^
10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(128*(16384*a^7*c^10 - b^14*c^3 +
 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^3*b^8*c^6 - 8960*a^4*b^6*c^7 + 2
1504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) + (x^(1/2)*((81*b^8*(-(4*a*c - b^2)^
15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*
a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^
6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^
9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/
2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^
3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(16777216*a^12*c^19 + b^24*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9
 - 14080*a^3*b^18*c^10 + 126720*a^4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784
704*a^6*b^12*c^13 - 12976128*a^7*b^10*c^14 + 32440320*a^8*b^8*c^15 - 576716
80*a^9*b^6*c^16 + 69206016*a^10*b^4*c^17 - 50331648*a^11*b^2*c^18)))^(1/4)*
(6576668672*a^11*c^13 + 36864*a^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 2048000
0*a^5*b^12*c^7 - 185991168*a^6*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 38860226
56*a^8*b^6*c^10 + 8883535872*a^9*b^4*c^11 - 11576279040*a^10*b^2*c^12))/(16
*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))*((...

3.1072 $\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=520

$$-\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)}+\frac{x^{5/2}(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(b^2-10ac+\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] 1/2*x^(5/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/8*arctan(2^(1/4)*c^(
1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2-10*a*c+b*(-12*a*c+b^2)/(-4
*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)
-1/8*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2-10
*a*c+b*(-12*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)/(-b-(
-4*a*c+b^2)^(1/2))^(3/4)-1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2
)^(1/2))^(1/4))*(b^2-10*a*c-b*(-12*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(
5/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/8*arctanh(2^(1/4)*c^(1/4)
*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b^2-10*a*c-b*(-12*a*c+b^2)/(-4*a*c
+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/2
*b*x^(1/2)/c/(-4*a*c+b^2)
```

Rubi [A]
time = 1.07, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1129, 1379, 1516, 1436, 218, 214, 211}

$$-\frac{\left(\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x^{5/2}(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\sqrt{x}}{2c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(b*Sqrt[x])/(c*(b^2 - 4*a*c)) + (x^(5/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*
a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*
2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*
a*c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x]
)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b + S
qrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4
*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 1
0*a*c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b
+ Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1379

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*
p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c,
 d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1]
 && GtQ[m, 2*n - 1]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
```

```
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx = 2\text{Subst}\left(\int \frac{x^{12}}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)$$
$$= \frac{x^{5/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x^4(10a+bx^4)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2(b^2-4ac)}$$
$$= -\frac{b\sqrt{x}}{2c(b^2-4ac)} + \frac{x^{5/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{ab+(b^2-10ac)x^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right.}{2c(b^2-4ac)}$$
$$= -\frac{b\sqrt{x}}{2c(b^2-4ac)} + \frac{x^{5/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b^2-10ac-\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}\right)\text{Subs}}{4c(}$$
$$= -\frac{b\sqrt{x}}{2c(b^2-4ac)} + \frac{x^{5/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(b^2-10ac+\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}\right)\text{Subs}}{4c(b^2-}$$
$$= -\frac{b\sqrt{x}}{2c(b^2-4ac)} + \frac{x^{5/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(b^2-10ac+\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}\right)\tan^{-}}{4\sqrt[4]{2}\,c^{5/4}(b^2-4ac)\left(-b\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.39, size = 237, normalized size = 0.46

$$-\frac{4\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log\left(\sqrt{x}-\#1\right)-c\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]+\frac{\frac{4c\sqrt{x}\,(ab+b^2x^2-2acx^2)}{a+bx^2+cx^4}+\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{-4b^3\log\left(\sqrt{x}-\#1\right)+15abc\log\left(\sqrt{x}-\#1\right)+3b^2c\log\left(\sqrt{x}-\#1\right)\#1^4-6ac^2\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{b^2-4ac}}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/8*(4*RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] - c*Log[Sqrt[x
] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ] + ((4*c*Sqrt[x]*(a*b + b^2*x^2 - 2*a*
c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#1^8 & , (-4*b^3*Log[S
qrt[x] - #1] + 15*a*b*c*Log[Sqrt[x] - #1] + 3*b^2*c*Log[Sqrt[x] - #1]*#1^4
- 6*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(b^2 - 4*a*c))/c
^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 146, normalized size = 0.28

method	result	size
derivativedivides	$\frac{-\frac{(2ac-b^2)x^{\frac{5}{2}}}{2c(4ac-b^2)}+\frac{ab\sqrt{x}}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4b+a)}\frac{((10ac-b^2)\_R^4-ab)\ln(\sqrt{x}-\_R)}{2\_R^7c+\_R^3b}}{8c(4ac-b^2)}$	146
default	$\frac{-\frac{(2ac-b^2)x^{\frac{5}{2}}}{2c(4ac-b^2)}+\frac{ab\sqrt{x}}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4b+a)}\frac{((10ac-b^2)\_R^4-ab)\ln(\sqrt{x}-\_R)}{2\_R^7c+\_R^3b}}{8c(4ac-b^2)}$	146

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/4*(2*a*c-b^2)/c/(4*a*c-b^2)*x^(5/2)+1/4*a*b/c/(4*a*c-b^2)*x^(1/2))/(c
*x^4+b*x^2+a)+1/8/c/(4*a*c-b^2)*sum(((10*a*c-b^2)*_R^4-a*b)/(2*_R^7*c+_R^3*
b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^(9/2) + 2*a*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*x^2) + integrate(-1/4*(b*x^(7/2) + 10*a*x^(3/2))/((b^2*c - 4*
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11906 vs. 2(424) = 848.
time = 17.99, size = 11906, normalized size = 22.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/8*(4*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2
)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*
b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1
280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqr
t((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b
^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11
 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^
5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^1
8 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a
^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))*arctan
(-1/2*(sqrt(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c
^3 + 1338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6
- 497651072*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 +
 4582400000*a^10*b^2*c^10 - 2560000000*a^11*c^11 - (b^25*c^5 - 70*a*b^23*c^
6 + 2192*a^2*b^21*c^7 - 40672*a^3*b^19*c^8 + 498432*a^4*b^17*c^9 - 4254720*
a^5*b^15*c^10 + 25976832*a^6*b^13*c^11 - 114475008*a^7*b^11*c^12 + 36195532
8*a^8*b^9*c^13 - 802029568*a^9*b^7*c^14 + 1183842304*a^10*b^5*c^15 - 104647
8848*a^11*b^3*c^16 + 419430400*a^12*b*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571
*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5
 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*
a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*
c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(
(81*a^2*b^16 - 8118*a^3*b^14*c + 358651*a^4*b^12*c^2 - 9129750*a^5*b^10*c^3
 + 146540625*a^6*b^8*c^4 - 1519250000*a^7*b^6*c^5 + 9937500000*a^8*b^4*c^6
- 37500000000*a^9*b^2*c^7 + 62500000000*a^10*c^8)*x + 1/2*sqrt(1/2)*(b^22 -
 112*a*b^20*c + 5735*a^2*b^18*c^2 - 176820*a^3*b^16*c^3 + 3634845*a^4*b^14*
c^4 - 52073994*a^5*b^12*c^5 + 527503968*a^6*b^10*c^6 - 3751826400*a^7*b^8*c
^7 + 18208800000*a^8*b^6*c^8 - 56920000000*a^9*b^4*c^9 + 102400000000*a^10*
b^2*c^10 - 80000000000*a^11*c^11 - (b^25*c^5 - 91*a*b^23*c^6 + 3641*a^2*b^2
1*c^7 - 84776*a^3*b^19*c^8 + 1280016*a^4*b^17*c^9 - 13215744*a^5*b^15*c^10
+ 95875584*a^6*b^13*c^11 - 493891584*a^7*b^11*c^12 + 1798938624*a^8*b^9*c^1
3 - 4533059584*a^9*b^7*c^14 + 7523860480*a^10*b^5*c^15 - 7405568000*a^11*b^
3*c^16 + 3276800000*a^12*b*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^
2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*
a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^
13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 5898
24*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(-(b^9 - 45*
a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5
- 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6
144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c
^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000
*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c
^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589
```

```
824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*
a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a
^5*b^2*c^10 + 4096*a^6*c^11)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 -
5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*
c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470
625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*
b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 -
129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^
8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7
 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)
) - sqrt(1/2)*(9*a*b^30 - 1270*a^2*b^28*c + 82813*a^3*b^26*c^2 - 3305978*a^
4*b^24*c^3 + 90231255*a^5*b^22*c^4 - 1780615316*a^6*b^20*c^5 + 26199812170*
a^7*b^18*c^6 - 292147074792*a^8*b^16*c^7 + 2484388440192*a^9*b^14*c^8 - 160
82985454080*a^10*b^12*c^9 + 78485701504000*a^11*b^10*c^10 - 283191078400000
*a^12*b^8*c^11 + 730734080000000*a^13*b^6*c^12 - 1272576000000000*a^14*b^4*
c^13 + 1337600000000000*a^15*b^2*c^14 - 640000000000000*a^16*c^15 - (9*a*b^
33*c^5 - 1081*a^2*b^31*c^6 + 59923*a^3*b^29*c^7 - 2033390*a^4*b^27*c^8 + 47
234960*a^5*b^25*c^9 - 795781312*a^6*b^23*c^10 + 10050046208*a^7*b^21*c^11 -
 96993186304*a^8*b^19*c^12 + 722648002560*a^9*b^17*c^13 - 4169749463040*a^1
0*b^15*c^14 + 18574068219904*a^11*b^13*c^15 - 63226237812736*a^12*b^11*c^16
 + 161327426306048*a^13*b^9*c^17 - 298510607974400*a^14*b^7*c^18 + 37806407
6800000*a^15*b^5*c^19 - 293076992000000*a^16*b^3*c^20 + 104857600000000*a^1
7*b*c^21)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [B]
time = 11.85, size = 2500, normalized size = 4.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] 2*atan(((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2
- 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^
3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^15*c
^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7 - 29
8844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10))/(16
*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 + b^6*(-(4*a*c - b^2)^
15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 +
 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 505036
80*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c
^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2
)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^
17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 12
6720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128
*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a
^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*c^11 + 40960*
a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 52428800*a^6*b^6
*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1i)/(2*(b^8*c + 256*
a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(b^21 + b^6*
(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 3632
0*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*
b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*
c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(
16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a
^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12
*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c
^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(3/4)*1i)*(-(b^21
+ b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2
- 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 1506457
6*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^
9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*
b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(
8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 1
4080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^
6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9
*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*1i - (
```

```
x^(1/2)*(81*a^4*b^10 - 2000000*a^9*c^5 - 3744*a^5*b^8*c + 66322*a^6*b^6*c^2
 - 547800*a^7*b^4*c^3 + 1980000*a^8*b^2*c^4))/(16*(b^12*c + 4096*a^6*c^7 -
24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 614
4*a^5*b^2*c^6)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*
c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 30013
44*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a
^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(
4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^
6 + 1056*a^2*b^20*c^7 - 14080*a^3*b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a
^5*b^14*c^10 + 3784704*a^6*b^12*c^11 - 12976128*a^7*b^10*c^12 + 32440320*a^
8*b^8*c^13 - 57671680*a^9*b^6*c^14 + 69206016*a^10*b^4*c^15 - 50331648*a^11
*b^2*c^16)))^(1/4) - (((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549
*a^5*b^5*c^2 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)) - ((x^(1/2)*(1006632960*a^10*b*c^11 + 40
96*a^3*b^15*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6
*b^9*c^7 - 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b
^3*c^10))/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 12
80*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + ((-(b^21 + b^6*(-(
4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a
^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9
*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9
 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(
-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(167
77216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*
b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c^
11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^14
 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a^9*
c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 - 524
28800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251...
```

3.1073 $\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=471

$$\frac{x^{3/2}(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2+12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}}+\frac{\left(b-\frac{b^2+12ac}{\sqrt{b^2-4a\ldots}}\right.}{4\,2^{3/4}c^{3/4}\,(\ldots}$$

```
[Out] 1/2*x^(3/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/8*arctan(2^(1/4)*c^(
1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-12*a*c-b^2)/(-4*a*c+b^2)^(
1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/8*arctan
h(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-12*a*c-b^2)/(
-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/
4)+1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2+1
2*a*c+b*(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+
b^2)^(1/2))^(1/4)-1/8*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2
))^(1/4))*(b^2+12*a*c+b*(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(3
/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A]
time = 0.62, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1129, 1379, 1524, 304, 211, 214}

$$\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{3/2}(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(3/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2 + 12*
a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4
*a*c])^(1/4)) + ((b - (b^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*
a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]
)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(
3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((b - (b
^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1379

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*
p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c,
 d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1]
 && GtQ[m, 2*n - 1]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^2}\,dx = 2\text{Subst}\left(\int \frac{x^{10}}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x^2(6a-bx^4)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2(b^2-4ac)}$$

$$= \frac{x^{3/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(b^2+12ac-b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4}}\right.}{4(b^2-4ac)^{3/2}}$$

$$= \frac{x^{3/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b^2+12ac-b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{}{\sqrt{-b+\sqrt{b^2}}}\right.}{4\sqrt{2}\sqrt{c}\,(b^2-4ac)}$$

$$= \frac{x^{3/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b^2+12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,\sqrt{}}{\sqrt[4]{-b-\sqrt{b^2-}}}\right.}{4\,2^{3/4}c^{3/4}(b^2-4ac)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.32, size = 189, normalized size = 0.40

$$\frac{1}{8}\left(\frac{4x^{3/2}(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{4\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{\log\left(\sqrt{x}-\#1\right)}{b\#1+2c\#1^5}\&\right]}{c} + \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{-4b^2\log\left(\sqrt{x}-\#1\right)+10ac\log\left(\sqrt{x}-\#1\right)+bc\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{c(b^2-4ac)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*x^(3/2)*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (4*RootSum
[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1 + 2*c*#1^5) & ])/c + RootS
um[a + b*#1^4 + c*#1^8 & , (-4*b^2*Log[Sqrt[x] - #1] + 10*a*c*Log[Sqrt[x] -
 #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(c*(b^2 - 4*a*c)))
/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 121, normalized size = 0.26

method	result	size

derivativedivides	$\frac{-\frac{b\,x^{\frac{7}{2}}}{2\left(4ac-b^2\right)}-\frac{a\,x^{\frac{3}{2}}}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^6b+6\_R^2a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{32ac-8b^2}$	121
default	$\frac{-\frac{b\,x^{\frac{7}{2}}}{2\left(4ac-b^2\right)}-\frac{a\,x^{\frac{3}{2}}}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^6b+6\_R^2a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{32ac-8b^2}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/4*b/(4*a*c-b^2)*x^(7/2)-1/2*a/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*x^2+a)+1/
8/(4*a*c-b^2)*sum((-_R^6*b+6*_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=Ro
otOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^(7/2) + 2*a*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*x^2) - integrate(-1/4*(b*x^(5/2) - 6*a*sqrt(x))/((b^2*c - 4*a
*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11032 vs. 2(375) = 750.
time = 22.26, size = 11032, normalized size = 23.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b
^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4
*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b
^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 57
6*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c
^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262
144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
```

c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*arctan(-1/2*((b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 - (b^14*c^3 - 12*a*b^12*c^4 - 48*a^2*b^10*c^5 + 1600*a^3*b^8*c^6 - 11520*a^4*b^6*c^7 + 39936*a^5*b^4*c^8 - 69632*a^6*b^2*c^9 + 49152*a^7*c^10)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt((117649*a^4*b^20 + 9983358*a^5*b^18*c + 404714961*a^6*b^16*c^2 + 9897860448*a^7*b^14*c^3 + 158656107456*a^8*b^12*c^4 + 1707655509504*a^9*b^10*c^5 + 12338818573824*a^10*b^8*c^6 + 58812305154048*a^11*b^6*c^7 + 177024646692864*a^12*b^4*c^8 + 304679870005248*a^13*b^2*c^9 + 228509902503936*a^14*c^10)*x - 1/2*sqrt(1/2)*(2401*a^3*b^25 + 294294*a^4*b^23*c + 13335105*a^5*b^21*c^2 + 323354360*a^6*b^19*c^3 + 4269253584*a^7*b^17*c^4 + 24537890304*a^8*b^15*c^5 - 79436754432*a^9*b^13*c^6 - 1621756588032*a^10*b^11*c^7 - 3506876964864*a^11*b^9*c^8 + 27305557622784*a^12*b^7*c^9 + 100201644490752*a^13*b^5*c^10 - 142936235311104*a^14*b^3*c^11 - 677066377789440*a^15*b*c^12 - (2401*a^3*b^30*c^3 - 49049*a^4*b^28*c^4 - 1432760*a^5*b^26*c^5 - 6473264*a^6*b^24*c^6 + 373184512*a^7*b^22*c^7 - 319185152*a^8*b^20*c^8 - 27408852992*a^9*b^18*c^9 + 93871525888*a^10*b^16*c^10 + 774145638400*a^11*b^14*c^11 - 4486009651200*a^12*b^12*c^12 - 5590781263872*a^13*b^10*c^13 + 81717925773312*a^14*b^8*c^14 - 108093958520832*a^15*b^6*c^15 - 454721122861056*a^16*b^4*c^16 + 1497904875307008*a^17*b^2*c^17 - 1283918464548864*a^18*c^18)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9))) - (343*a^2*b^19 + 21070*a^3*b^17*c + 600271*a^4*b^15*c^2 + 8903196*a^5*b^13*c^3 + 62719920*a^6*b^11*c^4 - 15909696*a^7*b^9*c^5 - 2396812032*a^8*b^7*c^6 - 6953610240*a^9*b^5*c^7 + 19591041024*a^10*b^3*c^8 + 78364164096*a^11*b*c^9 - (343*a^2*b^24*c^3 + 10437*a^3*b^22*c^4 + 90132*a^4*b^20*c^5 - 1028432*a^5*b^18*c^6 - 14041152*a^6*b^16*c^7 + 70390272*a^7*b^14*c^8 + 646137856*a^8*b^12*c^9 - 3121520640*a^9*b^10*c^10 - 11091935232*a^10*b^8*c^11 + 68335239168*a^11*b^6*c^12 + 24652283904*a^12*b^4*c^13 - 557256278016*a^13*b^2*c^14 + 743008370688*a^14*c^15)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(x))*sqrt(sqrt(1/2

```
)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 1
7496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14
*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 3440
64*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^
15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840
*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))/(2401*a^3*b^16 + 179046*a
^4*b^14*c + 6354369*a^5*b^12*c^2 + 131902344*a^6*b^10*c^3 + 1713103344*a^7*
b^8*c^4 + 13740938496*a^8*b^6*c^5 + 65167421184*a^9*b^4*c^6 + 166523848704*
a^10*b^2*c^7 + 176319369216*a^11*c^8)) - 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 -
 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 16
8*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^
5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [B]
time = 6.44, size = 2500, normalized size = 5.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((a*x^(3/2))/(4*a*c - b^2) + (b*x^(7/2))/(2*(4*a*c - b^2)))/(a + b*x^2 +
c*x^4) - atan(((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^4*
b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^7*b
```

^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 - 1638
4*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*
a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*((b^4*(-(4*a*c -
 b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^
13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10
665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216
*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*
c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12
976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 6920
6016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959552*a^10*c^11
- 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*b^10*c^6 + 3329
22880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8*b^4*c^9 - 26508
00128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*
b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a
*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^
3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6
- 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677
7216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b
^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9
- 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 +
69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(3/4) + (x^(1/2)*(49*a^3
*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b
^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 38
40*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15
)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 +
 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a
^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^
15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 12
6720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a
^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^1
0*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*1i - (((5435817984*a^10*b*c^10
 - 4096*a^3*b^15*c^3 + 1425408*a^4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 32374
7840*a^6*b^9*c^6 - 1714421760*a^7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 817050
4192*a^9*b^3*c^9)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*
b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b
^12*c)) + (x^(1/2)*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*
c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5
*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^
8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*
c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 +
1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b
^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*

```
c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c
^14)))^(1/4)*(1207959552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12
*c^5 - 56229888*a^5*b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c
^8 + 2390753280*a^8*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^
6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^
2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^
9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216
*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^
3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(
4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^
4 + 1056*a^2*b^20*c^5 - 14080*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a
^5*b^14*c^8 + 3784704*a^6*b^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*
b^8*c^11 - 57671680*a^9*b^6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b
^2*c^14)))^(3/4) - (x^(1/2)*(49*a^3*b^9*c + 15552*a^7*b*c^5 + 945*a^4*b^7*c
^2 + 6420*a^5*b^5*c^3 + 17712*a^6*b^3*c^4))/(16*(b^12 + 4096*a^6*c^6 + 240*
a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a
*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96
*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5
 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 +...
```

3.1074 $\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=483

$$\frac{\sqrt{x}\,(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{\left(3b^2+4ac+3b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\,\sqrt[4]{c}\,(b^2-4ac)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(3b^2+4ac-3\right.}{4\sqrt[4]{2}\,\sqrt[4]{}}$$

```
[Out] 1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^2+4*
a*c-3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+
b^2)^(1/2))^(3/4)+1/8*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2
))^(1/4))*(3*b^2+4*a*c-3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)
^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/8*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b
-(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^2+4*a*c+3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/c^
(1/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/8*arctanh(2^(1/4)*
c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^2+4*a*c+3*b*(-4*a*c+b^2
)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1
/2*(b*x^2+2*a)*x^(1/2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
```

Rubi [A]
time = 0.71, antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1129, 1379, 1436, 218, 214, 211}

$$-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,\sqrt[4]{c}\,(b^2-4ac)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,\sqrt[4]{c}\,(b^2-4ac)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\,(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,\sqrt[4]{c}\,(b^2-4ac)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,\sqrt[4]{c}\,(b^2-4ac)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2 + 4
*a*c + 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b
^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2
- 4*a*c])^(3/4)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2
- 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((3*b^2 + 4*a*c + 3*b*Sqrt
[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(
1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4
)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2
)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1379

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*
p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c,
 d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1]
 && GtQ[m, 2*n - 1]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx = 2\text{Subst}\left(\int \frac{x^8}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\,(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{2a-3bx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2\,(b^2-4ac)}$$

$$= \frac{\sqrt{x}\,(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(3b^2+4ac-3b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-}\right.}{4\,(b^2-4ac)^{3/2}}$$

$$= \frac{\sqrt{x}\,(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\left(3b^2+4ac-3b\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{}{\sqrt{-b+\sqrt{}}}\right.}{4\,(b^2-4ac)^{3/2}\sqrt{-b+\sqrt{}}}$$

$$= \frac{\sqrt{x}\,(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(3b^2+4ac+3b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}}{\sqrt[4]{-b-\sqrt{b^2}}}\right.}{4\sqrt[4]{2}\,\sqrt[4]{c}\,(b^2-4ac)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.27, size = 194, normalized size = 0.40

$$\frac{1}{8}\left(\frac{4\sqrt{x}\,(2a+bx^2)}{(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{4\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{\log\left(\sqrt{x}-\#1\right)}{b\#1^3+2c\#1^7}\&\right]}{c} + \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{-4b^2\log\left(\sqrt{x}-\#1\right)+14ac\log\left(\sqrt{x}-\#1\right)+3bc\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{c\,(b^2-4ac)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*Sqrt[x]*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (4*RootSum
[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1^3 + 2*c*#1^7) & ])/c + Roo
tSum[a + b*#1^4 + c*#1^8 & , (-4*b^2*Log[Sqrt[x] - #1] + 14*a*c*Log[Sqrt[x]
 - #1] + 3*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(c*(b^2 - 4*
a*c)))/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 118, normalized size = 0.24

method	result	size

derivativedivides	$\frac{-\frac{b\,x^{\frac{5}{2}}}{2\left(4ac-b^{2}\right)}-\frac{a\sqrt{x}}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-3\_R^{4}b+2a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{32ac-8b^{2}}$	118
default	$\frac{-\frac{b\,x^{\frac{5}{2}}}{2\left(4ac-b^{2}\right)}-\frac{a\sqrt{x}}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-3\_R^{4}b+2a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{32ac-8b^{2}}$	118

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] 2*(-1/4*b/(4*a*c-b^2)*x^(5/2)-1/2*a/(4*a*c-b^2)*x^(1/2))/(c*x^4+b*x^2+a)+1/8/(4*a*c-b^2)*sum((-3*_R^4*b+2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")

[Out] -1/2*(2*c*x^(9/2) + b*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - integrate(-1/4*(2*c*x^(7/2) + 5*b*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9245 vs. 2(383) = 766.
time = 3.00, size = 9245, normalized size = 19.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7))* arctan(-1/2*(sqrt(1/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^7*b*c^7 - (27*b^22*c - 820*a*b^20*c^2 + 10064*a^2*b^18*c^3 - 57024*a^3*b^16*c^4 + 44544*a^4*b^14*c^5 + 1505280*a^5*b^12*c^6 - 10838016*a^6*b^10*c^7 + 38436864*a^7*b^8*c^8 - 79233024*a^8*b^6*c^9 + 92012544*a^9*b^4*c^10 - 49283072*a^10*b^2*c^11 + 4194304*a^11*c^12)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt((1476225*b^8 + 641520*a*b^6*c + 30816*a^2*b^4*c^2 - 8448*a^3*b^2*c^3 + 256*a^4*c^4)*x + sqrt(1/2)*(111537*b^12 - 1375704*a*b^10*c + 5803760*a^2*b^8*c^2 - 8961280*a^3*b^6*c^3 + 2522880*a^4*b^4*c^4 - 186368*a^5*b^2*c^5 + 4096*a^6*c^6 + 8*(81*b^19*c - 2596*a*b^17*c^2 + 36416*a^2*b^15*c^3 - 292096*a^3*b^13*c^4 + 1465856*a^4*b^11*c^5 - 4716544*a^5*b^9*c^6 + 9519104*a^6*b^7*c^7 - 11075584*a^7*b^5*c^8 + 5832704*a^8*b^3*c^9 - 262144*a^9*b*c^10)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)) + sqrt(1/2)*(2657205*b^19 - 57028212*a*b^17*c + 502044480*a^2*b^15*c^2 - 2306152704*a^3*b^13*c^3 + 5758457344*a^4*b^11*c^4 - 7169792000*a^5*b^9*c^5 + 2897625088*a^6*b^7*c^6 + 946012160*a^7*b^5*c^7 - 111345664*a^8*b^3*c^8 + 2883584*a^9*b*c^9 - (32805*b^26*c - 989172*a*b^24*c^2 + 12010848*a^2*b^22*c^3 - 66614144*a^3*b^20*c^4 + 38905600*a^4*b^18*c^5 + 1841587200*a^5*b^16*c^6 - 12771508224*a^6*b^14*c^7 + 43815469056*a^7*b^12*c^8 - 85947383808*a^8*b^10*c^9 + 90262732800*a^9*b^8*c^10 - 34319892480*a^10*b^6*c^11 - 9386852352*a^11*b^4*c^12 + 1895825408*a^12*b^2*c^13 - 67108864*a^13*c^14)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(x)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6

```
561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^1
4*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 34406
4*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)
))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*
b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 +
 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 -
1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt
((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*
b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 34
4064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^
11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a
^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))/(332150625*a*b^12 + 3214890
00*a^2*b^10*c + 107535600*a^3*b^8*c^2 + 12061440*a^4*b^6*c^3 - 463104*a^5*b
^4*c^4 - 104448*a^6*b^2*c^5 + 4096*a^7*c^6)) - ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [B]
time = 10.88, size = 2500, normalized size = 5.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan((((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3
*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b
^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 409
```

6*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4) - (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4) - (x^(1/2)*(128*a^6*c^7 + 2025*a^2*b^8*c^3 - 270*a^3*b^6*c^4 + 1224*a^4*b^4*c^5 + 864*a^5*b^2*c^6))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*1i + ((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c -

b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 20480*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(3/4) + (405*a^2*b^6*c^3 - 32*a^5*c^6 + 918*a^3*b^4*c^4 + 96*a^4*b^2*c^5)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3...

3.1075 $\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=450

$$-\frac{x^{3/2}(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(4b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}}+\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4\right.}{2\,2^{3/4}\left(b^2-\right.}$$

```
[Out] -1/2*x^(3/2)*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/4)*arctan(2^
(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(4*b-(-4*a*c+b^2)^(1/2
))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^(1/4)*arc
tanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(4*b-(-4*a*c+b^
2)^(1/2))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^(1
/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(4*b+(-4*
a*c+b^2)^(1/2))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/
4*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(4
*b+(-4*a*c+b^2)^(1/2))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(
1/4)
```

Rubi [A]
time = 0.47, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1129, 1378, 1524, 304, 211, 214}

$$-\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{x^{3/2}(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(x^(3/2)*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (c^(1/4)
*(4*b + Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(
1/4)) + (c^(1/4)*(4*b - Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt
[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(4*b + Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)
^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(4*b - Sqrt[b^2 - 4*a*c])
*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3
/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1378

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*((a + b*x^n + c*
x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^2}\,dx = 2\text{Subst}\left(\int \frac{x^6}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{x^2(3b-2cx^4)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2(b^2-4ac)}$$

$$= -\frac{x^{3/2}(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(c\left(4b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}}\right.}{2(b^2-4ac)^{3/2}}$$

$$= -\frac{x^{3/2}(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(\sqrt{c}\left(4b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{}{\sqrt{-b+\sqrt{b}}}\right.}{2\sqrt{2}\,(b^2-4ac)^{3}}$$

$$= -\frac{x^{3/2}(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt[4]{c}\left(4b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4}}}\right.}{2\,2^{3/4}(b^2-4ac)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.21, size = 109, normalized size = 0.24

$$\frac{-\frac{4x^{3/2}(b+2cx^2)}{a+bx^2+cx^4} + \text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{3b\log\left(\sqrt{x}-\#1\right)-2c\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{8(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*x^(3/2)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#1
^8 & , (3*b*Log[Sqrt[x] - #1] - 2*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^
5) & ])/(8*(b^2 - 4*a*c))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 121, normalized size = 0.27

method	result	size

derivativedivides	$\frac{\frac{2c\,x^{\frac{7}{2}}}{8ac-2b^{2}}+\frac{2b\,x^{\frac{3}{2}}}{16ac-4b^{2}}}{c\,x^{4}+b\,x^{2}+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-2\_R^{6}c+3\_R^{2}b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{8(4ac-b^{2})}$	121
default	$\frac{\frac{2c\,x^{\frac{7}{2}}}{8ac-2b^{2}}+\frac{2b\,x^{\frac{3}{2}}}{16ac-4b^{2}}}{c\,x^{4}+b\,x^{2}+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-2\_R^{6}c+3\_R^{2}b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{8(4ac-b^{2})}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/2*c/(4*a*c-b^2)*x^(7/2)+1/4*b/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*x^2+a)-1/8
/(4*a*c-b^2)*sum((-2*_R^6*c+3*_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=R
ootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^(7/2) + b*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*x^2) + integrate(-1/4*(2*c*x^(5/2) - 3*b*sqrt(x))/((b^2*c -
4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9757 vs. $2(350) = 700$.
time = 8.38, size = 9757, normalized size = 21.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*
b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b
^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18
 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^
4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a
^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2
- 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))*
```

arctan(-((81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4*(a*b^13 - 24*a^2*b^11*c + 240*a^3*b^9*c^2 - 1280*a^4*b^7*c^3 + 3840*a^5*b^5*c^4 - 6144*a^6*b^3*c^5 + 4096*a^7*b*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt((74733890625*b^16*c^2 + 112193100000*a*b^14*c^3 + 68088600000*a^2*b^12*c^4 + 20761920000*a^3*b^10*c^5 + 3063744000*a^4*b^8*c^6 + 113909760*a^5*b^6*c^7 - 19021824*a^6*b^4*c^8 - 1179648*a^7*b^2*c^9 + 65536*a^8*c^10)*x - 1/2*sqrt(1/2)*(2989355625*b^21*c - 23678649000*a*b^19*c^2 + 7135160400*a^2*b^17*c^3 + 277460328960*a^3*b^15*c^4 - 338956033536*a^4*b^13*c^5 - 492326940672*a^5*b^11*c^6 - 183476674560*a^6*b^9*c^7 - 21980119040*a^7*b^7*c^8 + 750059520*a^8*b^5*c^9 + 190316544*a^9*b^3*c^10 - 7340032*a^10*b*c^11 + (36905625*a*b^28*c - 1159839000*a^2*b^26*c^2 + 15854324400*a^3*b^24*c^3 - 122710429440*a^4*b^22*c^4 + 584418357504*a^5*b^20*c^5 - 1728949905408*a^6*b^18*c^6 + 2983008514048*a^7*b^16*c^7 - 2317983285248*a^8*b^14*c^8 - 462348419072*a^9*b^12*c^9 + 1339972648960*a^10*b^10*c^10 + 254402363392*a^11*b^8*c^11 - 161849802752*a^12*b^6*c^12 - 51220840448*a^13*b^4*c^13 - 2550136832*a^14*b^2*c^14 + 268435456*a^15*c^15)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))) + (22143375*b^14*c - 161619300*a*b^12*c^2 + 233100720*a^2*b^10*c^3 + 224213184*a^3*b^8*c^4 + 48450816*a^4*b^6*c^5 + 185344*a^5*b^4*c^6 - 487424*a^6*b^2*c^7 + 16384*a^7*c^8 - 4*(273375*a*b^21*c - 6355800*a^2*b^19*c^2 + 60732720*a^3*b^17*c^3 - 301810176*a^4*b^15*c^4 + 798453248*a^5*b^13*c^5 - 951914496*a^6*b^11*c^6 + 38461440*a^7*b^9*c^7 + 557711360*a^8*b^7*c^8 + 179503104*a^9*b^5*c^9 + 11010048*a^10*b^3*c^10 - 1048576*a^11*b*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))/(332150625*b^12*c + 321489000*a*b^10*c^2 + 107535600*a^2*b^8*c^3 + 12061440

```
*a^3*b^6*c^4 - 463104*a^4*b^4*c^5 - 104448*a^5*b^2*c^6 + 4096*a^6*c^7)) - 4
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(
1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c
+ 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5
+ 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a
^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129
024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2
*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*
a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*arctan(
((81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(a*b^13 - 24*a^2
*b^11*c + 240*a^3*b^9*c^2 - 1280*a^4*b^7*c^3 + 3840*a^5*b^5*c^4 - 6144*a^6*
b^3*c^5 + 4096*a^7*b*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b
^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10
*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [B]
time = 6.06, size = 2500, normalized size = 5.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((b*x^(3/2))/(2*(4*a*c - b^2)) + (c*x^(7/2))/(4*a*c - b^2))/(a + b*x^2 + c*
x^4) - atan(((((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14*c
^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c^8
```

+ 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a ^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/ 2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a ^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a *c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1 056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^ 14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^ 8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2*b^14*c^5 + 94 69952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b^8*c^8 - 362807 296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^ 12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^ (1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a ^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^ 7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 167772 16*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126 720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^ 8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b ^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) + (x^(1/2)*(576*a^4*b*c^8 - 5625* a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8 *b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 27279 36*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48* a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440 320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648 *a^12*b^2*c^11)))^(1/4)*1i - (((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 24 33024*a^2*b^14*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693 440*a^5*b^8*c^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296 256*a^8*b^2*c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3* b^8*c^3 + 8960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b ^12*c)) + (x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a ^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 272 7936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15* c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 4 8*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 324 40320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 503316 48*a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a ^2*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5* b^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*a^12*b^2*c^11)))^(3/4) - (x^(1/2)*(576*a^4*b*c^8 - 5625*a*b^7*c^5 + 5100*a^2*b^5*c^6 + 3920*a^3*b^3*c^7))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 811008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 32440320*a^9*b^8*c^8 - 57671680*a^10...

3.1076 $\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=442

$$-\frac{\sqrt{x}\,(b+2cx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{c^{3/4}\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4a}}\right.}{2\sqrt[4]{2}\,(b^2-}$$

```
[Out] 1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(
3+4*b/(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4
)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
)*(3+4*b/(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(
3/4)+1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/
4))*(3-4*b/(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))
^(3/4)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^
(1/4))*(3-4*b/(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/
2))^(3/4)-1/2*(2*c*x^2+b)*x^(1/2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
```

Rubi [A]
time = 0.47, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1129, 1378, 1436, 218, 214, 211}

$$\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,(b^2-4ac)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,(b^2-4ac)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,(b^2-4ac)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,(b^2-4ac)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\sqrt{x}\,(b+2cx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(Sqrt[x]*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (c^(3/4)
*(3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[
b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/
4)) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2
 - 4*a*c])^(3/4)) + (c^(3/4)*(3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4
)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1378

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*((a + b*x^n + c*
x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2}\,dx = 2\text{Subst}\left(\int \frac{x^4}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}\,(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{b-6cx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2(b^2-4ac)}$$

$$= -\frac{\sqrt{x}\,(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(c\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}\right.}{2(b^2-4ac)}$$

$$= -\frac{\sqrt{x}\,(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(c\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{}{\sqrt{-b-\sqrt{b^2-}}}\right.}{2(b^2-4ac)\sqrt{-b-\sqrt{b^2}}}$$

$$= -\frac{\sqrt{x}\,(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{c^{3/4}\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4}}}\right.}{2\sqrt[4]{2}\,(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.18, size = 110, normalized size = 0.25

$$\frac{-\frac{4\sqrt{x}\,(b+2cx^2)}{a+bx^2+cx^4} + \text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b\log\left(\sqrt{x}-\#1\right)-6c\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{8(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*Sqrt[x]*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#1
^8 & , (b*Log[Sqrt[x] - #1] - 6*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^
7) & ])/(8*(b^2 - 4*a*c))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 118, normalized size = 0.27

method	result	size

derivativedivides	$\frac{\frac{2c\,x^{\frac{5}{2}}}{8ac-2b^2}+\frac{2b\sqrt{x}}{16ac-4b^2}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(6\_R^4c-b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{32ac-8b^2}$	118
default	$\frac{\frac{2c\,x^{\frac{5}{2}}}{8ac-2b^2}+\frac{2b\sqrt{x}}{16ac-4b^2}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(6\_R^4c-b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{32ac-8b^2}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/2*c/(4*a*c-b^2)*x^(5/2)+1/4*b/(4*a*c-b^2)*x^(1/2))/(c*x^4+b*x^2+a)+1/8
/(4*a*c-b^2)*sum((6*_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z
^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^(9/2) + (b^2 - 2*a*c)*x^(5/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-1/4*(b*c*x^(7/2) + (b
^2 + 6*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 10570 vs. $2(350) = 700$.
time = 6.56, size = 10570, normalized size = 23.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a
^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4
*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b
^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 57
6*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*
c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262
```

144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*arctan(1/2*(sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 - (a^3*b^23 - 20*a^4*b^21*c + 432*a^5*b^19*c^2 - 11712*a^6*b^17*c^3 + 195072*a^7*b^15*c^4 - 1935360*a^8*b^13*c^5 + 12214272*a^9*b^11*c^6 - 50823168*a^10*b^9*c^7 + 139788288*a^11*b^7*c^8 - 245628928*a^12*b^5*c^9 + 250609664*a^13*b^3*c^10 - 113246208*a^14*b*c^11)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt((49*b^12*c^2 + 3150*a*b^10*c^3 + 95985*a^2*b^8*c^4 + 1621296*a^3*b^6*c^5 + 15746400*a^4*b^4*c^6 + 75582720*a^5*b^2*c^7 + 136048896*a^6*c^8)*x + 1/2*sqrt(1/2)*(b^18 + 52*a*b^16*c + 1269*a^2*b^14*c^2 + 14294*a^3*b^12*c^3 + 48608*a^4*b^10*c^4 - 679392*a^5*b^8*c^5 - 4209408*a^6*b^6*c^6 - 4105728*a^7*b^4*c^7 + 214990848*a^8*b^2*c^8 - 483729408*a^9*c^9 - (a^3*b^23 + 7*a^4*b^21*c - 152*a^5*b^19*c^2 - 2960*a^6*b^17*c^3 + 44032*a^7*b^15*c^4 + 60928*a^8*b^13*c^5 - 4444160*a^9*b^11*c^6 + 36855808*a^10*b^9*c^7 - 153681920*a^11*b^7*c^8 + 363528192*a^12*b^5*c^9 - 467140608*a^13*b^3*c^10 + 254803968*a^14*b*c^11)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)) - sqrt(1/2)*(7*b^24*c + 400*a*b^22*c^2 + 7843*a^2*b^20*c^3 + 22574*a^3*b^18*c^4 - 1395688*a^4*b^16*c^5 - 11961472*a^5*b^14*c^6 + 98703360*a^6*b^12*c^7 + 1408361472*a^7*b^10*c^8 - 12100202496*a^8*b^8*c^9 + 1218281472*a^9*b^6*c^10 + 241219731456*a^10*b^4*c^11 - 812665405440*a^11*b^2*c^12 + 835884417024*a^12*c^13 - (7*a^3*b^29*c + 85*a^4*b^27*c^2 + 1764*a^5*b^25*c^3 - 37920*a^6*b^23*c^4 - 103296*a^7*b^21*c^5 - 2564352*a^8*b^19*c^6 + 145468416*a^9*b^17*c^7 - 1

```
602797568*a^10*b^15*c^8 + 6543507456*a^11*b^13*c^9 + 7533166592*a^12*b^11*c
^10 - 193399619584*a^13*b^9*c^11 + 890247315456*a^14*b^7*c^12 - 20785209016
32*a^15*b^5*c^13 + 2556193406976*a^16*b^3*c^14 - 1320903770112*a^17*b*c^15)
*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32
256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(x)*sqrt(-(b^7 + 21
*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4
096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3
+ 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b
^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 -
 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 -
 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 61
44*a^8*b^2*c^5 + 4096*a^9*c^6)))*sqrt(sqrt(1/2)...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [**B**]
time = 10.63, size = 2500, normalized size = 5.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*
a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5
```

+ 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(3/4) + (2232*a*b^3*c^7 - 7*b^5*c^6 + 11664*a^2*b*c^8)/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4) - (x^(1/2)*(1225*b^6*c^7 - 46656*a^3*c^10 + 10836*a*b^4*c^8 + 14256*a^2*b^2*c^9))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*1i - (((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15

)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 393216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) + (x^(1/2)*(2048*b^17*c^4 - 30720*a*b^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c^8 - 57671680*a^12*b^6*c^9 + 69206016*a^...

3.1077 $\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=489

$$\frac{x^{3/2}(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a(b^2-4ac)\sqrt[4]{-b-\sqrt{b^2-4ac}}}+\frac{\sqrt[4]{c}\left(b+\frac{b^2-20ac}{\sqrt{b^2-4c}}\right.}{4\,2^{3/4}a(b^2}$$

```
[Out] 1/2*x^(3/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/8*c^(1/4)*
arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(20*a*c-b^
2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
-1/8*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))
*(b+(20*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)/(-b-(-4*a*c+b^2
)^(1/2))^(1/4)+1/8*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^
(1/2))^(1/4))*(b+(-20*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)/(
-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/8*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-
b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-20*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a
/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A]
time = 0.65, antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1129, 1380, 1524, 304, 211, 214}

$$\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a(b^2-4ac)\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a(b^2-4ac)\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{3/2}(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a(b^2-4ac)\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a(b^2-4ac)\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (c^(1/4)*(b - (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*A
rcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)
*a*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b - (b^2 - 20*
a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
- (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b +
Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1380

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b
^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(
p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}\,dx = 2\text{Subst}\left(\int \frac{x^2}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x^2(-b^2+10ac-bcx^4)}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2a(b^2-4ac)}$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(c\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}}\right.}{4a(b^2-4ac)}$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(\sqrt{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{}{\sqrt{-b-\sqrt{b}}}\right.}{4\sqrt{2}\,a(b^2-4ac}$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4a}}}\right.}{4\,2^{3/4}a(b^2-4ac)\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.26, size = 136, normalized size = 0.28

$$-\frac{\frac{4x^{3/2}(b^2-2ac+bcx^2)}{a+bx^2+cx^4} + \text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b^2\log(\sqrt{x}-\#1)-10ac\log(\sqrt{x}-\#1)+bc\log(\sqrt{x}-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{8a(-b^2+4ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/8*((4*x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a +
 b*#1^4 + c*#1^8 & , (b^2*Log[Sqrt[x] - #1] - 10*a*c*Log[Sqrt[x] - #1] + b*
c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(a*(-b^2 + 4*a*c))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 149, normalized size = 0.30

method	result	size
derivativedivides	$\frac{-\frac{bc\,x^{\frac{7}{2}}}{2a(4ac-b^2)}+\frac{(2ac-b^2)x^{\frac{3}{2}}}{2a(4ac-b^2)}}{c\,x^4+b\,x^2+a} + \frac{\sum_{\_R=\text{RootOf}(c\_Z^8+\_Z^4 b+a)} \frac{(-bc\_R^6+(10ac-b^2)\_R^2)\ln(\sqrt{x}-\_R)}{2\_R^7 c+\_R^3 b}}{8a(4ac-b^2)}$	149

default	$\frac{-\frac{b c x^{\frac{7}{2}}}{2a\left(4ac-b^{2}\right)}+\frac{\left(2ac-b^{2}\right)x^{\frac{3}{2}}}{2a\left(4ac-b^{2}\right)}}{c x^{4}+b x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-bc\_R^{6}+\left(10ac-b^{2}\right)\_R^{2}\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{8a(4ac-b^{2})}$	149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/4/a*b*c/(4*a*c-b^2)*x^(7/2)+1/4*(2*a*c-b^2)/a/(4*a*c-b^2)*x^(3/2))/(c
*x^4+b*x^2+a)+1/8/a/(4*a*c-b^2)*sum((-b*c*_R^6+(10*a*c-b^2)*_R^2)/(2*_R^7*c
+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^(7/2) + (b^2 - 2*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - integrate(-1/4*(b*c*x^(5/2) + (b
^2 - 10*a*c)*sqrt(x))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b
^3 - 4*a^2*b*c)*x^2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 12411 vs. 2(397) = 794.
time = 42.36, size = 12411, normalized size = 25.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c
)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*
b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1
280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqr
t((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b
^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c
 + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^
15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^
8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a
^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*arctan
(1/2*((b^11 - 47*a*b^9*c + 853*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b
```

^3*c^4 - 40000*a^5*b*c^5 - (a^5*b^14 - 44*a^6*b^12*c + 720*a^7*b^10*c^2 - 6
080*a^8*b^8*c^3 + 29440*a^9*b^6*c^4 - 82944*a^10*b^4*c^5 + 126976*a^11*b^2*
c^6 - 81920*a^12*c^7)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a
^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a
^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*
a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4
*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt((531441*b^24*c^8 - 768
81798*a*b^22*c^9 + 5113978011*a^2*b^20*c^10 - 206852401350*a^3*b^18*c^11 +
5667080000625*a^4*b^16*c^12 - 110792866500000*a^5*b^14*c^13 + 1584936775000
000*a^6*b^12*c^14 - 16715805000000000*a^7*b^10*c^15 + 128988375000000000*a^
8*b^8*c^16 - 710150000000000000*a^9*b^6*c^17 + 2647500000000000000*a^10*b^4
*c^18 - 6000000000000000000*a^11*b^2*c^19 + 6250000000000000000*a^12*c^20)*
x - 1/2*sqrt(1/2)*(6561*b^31*c^5 - 1032993*a*b^29*c^6 + 75634965*a^2*b^27*c
^7 - 3414264975*a^3*b^25*c^8 + 106186248955*a^4*b^23*c^9 - 2407919378459*a^
5*b^21*c^10 + 41083864936232*a^6*b^19*c^11 - 536376931701360*a^7*b^17*c^12
+ 5394460343808000*a^8*b^15*c^13 - 41720627697600000*a^9*b^13*c^14 + 245614
092480000000*a^10*b^11*c^15 - 1078472304000000000*a^11*b^9*c^16 + 341052480
0000000000*a^12*b^7*c^17 - 7314160000000000000*a^13*b^5*c^18 + 948800000000
0000000*a^14*b^3*c^19 - 5600000000000000000*a^15*b*c^20 - (6561*a^5*b^34*c^
5 - 895212*a^6*b^32*c^6 + 56697732*a^7*b^30*c^7 - 2212069617*a^8*b^28*c^8 +
 59497163992*a^9*b^26*c^9 - 1169816993840*a^10*b^24*c^10 + 17397456159488*a
^11*b^22*c^11 - 199763116583168*a^12*b^20*c^12 + 1791922585643008*a^13*b^18
*c^13 - 12624164431147008*a^14*b^16*c^14 + 69835076189159424*a^15*b^14*c^15
 - 301610411758387200*a^16*b^12*c^16 + 1004700278784000000*a^17*b^10*c^17 -
 2527971917824000000*a^18*b^8*c^18 + 4641908326400000000*a^19*b^6*c^19 - 58
64652800000000000*a^20*b^4*c^20 + 4554752000000000000*a^21*b^2*c^21 - 16384
00000000000000*a^22*c^22)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 459
50*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6
)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32
256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17
*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(-(b^9 - 45*a*b^7*c
 + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^
6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^1
0*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45
950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^
6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 3
2256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^1
7*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^1
0*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2
*c^5 + 4096*a^11*c^6))) - (729*b^23*c^4 - 86994*a*b^21*c^5 + 4700619*a^2*b^
19*c^6 - 151648714*a^3*b^17*c^7 + 3240737969*a^4*b^15*c^8 - 48070563100*a^5
*b^13*c^9 + 503690450000*a^6*b^11*c^10 - 3715387000000*a^7*b^9*c^11 + 18824
300000000*a^8*b^7*c^12 - 62050000000000*a^9*b^5*c^13 + 119000000000000*a^10
*b^3*c^14 - 100000000000000*a^11*b*c^15 - (729*a^5*b^26*c^4 - 84807*a^6*b^2
4*c^5 + 4445469*a^7*b^22*c^6 - 138927340*a^8*b^20*c^7 + 2884712240*a^9*b^18

```
*c^8 - 41968650816*a^10*b^16*c^9 + 439511597568*a^11*b^14*c^10 - 3350499342
336*a^12*b^12*c^11 + 18578963128320*a^13*b^10*c^12 - 74005426176000*a^14*b^
8*c^13 + 205936435200000*a^15*b^6*c^14 - 379514880000000*a^16*b^4*c^15 + 41
5744000000000*a^17*b^2*c^16 - 204800000000000*a^18*c^17)*sqrt((b^12 - 78*a*
b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 262500
0*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^1
4*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 34
4064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19
*c^9)))*sqrt(x))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 -
 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b
^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [**B**]
time = 6.56, size = 2500, normalized size = 5.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 28528
64*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 18574213
12*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386
073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^
4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 2867
2*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 737280
```

00*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c
^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 1
08380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)
^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a
*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48
*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4
- 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 3
2440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 503
31648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 1966
08*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*
a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 762524467
2*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 2
4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c
^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300134
4*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^
8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
- 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c
+ 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^
10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^1
3*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*
b^2*c^11)))^(3/4) + (x^(1/2)*(81*b^7*c^8 + 3060*a*b^5*c^9 + 600000*a^3*b*c^
11 - 98000*a^2*b^3*c^10))/(16*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 24
0*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(
-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^
17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 +
15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 13467
6480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 5
25*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(
1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*
c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 378
4704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671
680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)
*1i - (((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 28528
64*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 18574213
12*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19386
073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 336*a^
4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 + 2867
2*a^8*b^2*c^6)) + (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 737280
00*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c
^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 1
08380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)
^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a
*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48

```
*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4
- 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12*b^10*c^7 + 3
2440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*b^4*c^10 - 503
31648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*b^18*c^4 + 1966
08*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*c^7 - 320471040*
a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6*c^10 + 762524467
2*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12 + 4096*a^8*c^6 - 2
4*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144
*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c
^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4*b^13*c^4 - 300134
4*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7*c^7 + 108380160*a^
8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
- 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4
*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^17*c^12 - 48*a^6*b^22*c
 + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a^9*b^16*c^4 - 811008*a^
10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*...
```

3.1078 $\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=503

$$\frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{c^{3/4}\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\,a\,(b^2-4ac)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{c^{3/4}\left(3b^2\right.}{}$$

```
[Out] 1/8*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(
3*b^2-28*a*c-3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4*a*c+b^2)^(3/2)/(-b-(-4*a
*c+b^2)^(1/2))^(3/4)+1/8*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*
c+b^2)^(1/2))^(1/4))*(3*b^2-28*a*c-3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4*a*
c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/8*c^(3/4)*arctan(2^(1/4)*c^(1/
4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^2-28*a*c+3*b*(-4*a*c+b^2)^(1
/2))*2^(3/4)/a/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/8*c^(3/4)
*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(3*b^2-28*a
*c+3*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1
/2))^(3/4)+1/2*(b*c*x^2-2*a*c+b^2)*x^(1/2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
```

Rubi [A]
time = 0.89, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1129, 1359, 1436, 218, 214, 211}

$$\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,a\,(b^2-4ac)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,a\,(b^2-4ac)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,a\,(b^2-4ac)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\,a\,(b^2-4ac)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\,(-2ac+b^2+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (c^(3/4)*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 -
 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b - Sq
rt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 - 4*a*c])
*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1
/4)*a*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1359

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-
x)*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c +
n*(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n)
)^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p, -1]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^2}\,dx = 2\text{Subst}\left(\int \frac{1}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{b^2-2ac-4(b^2-4ac)-3bcx^4}{a+bx^4+cx^8}\,dx, x, \sqrt{x}\right)}{2a\,(b^2-4ac)}$$

$$= \frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(c\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \right.}{4a\,(b^2-4ac)^{3/}}$$

$$= \frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\left(c\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \right.}{4a\,(b^2-4ac)^{3/2}\sqrt{}}$$

$$= \frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{c^{3/4}\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\right.}{4\sqrt[4]{2}\,a\,(b^2-4ac)^{3/2}\left(-b-\sqrt{b^2-}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.21, size = 140, normalized size = 0.28

$$-\frac{\frac{4\sqrt{x}\,(b^2-2ac+bcx^2)}{a+bx^2+cx^4} + \text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{3b^2\log\left(\sqrt{x}-\#1\right)-14ac\log\left(\sqrt{x}-\#1\right)+3bc\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{8a\,(-b^2+4ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/8*((4*Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a +
 b*#1^4 + c*#1^8 & , (3*b^2*Log[Sqrt[x] - #1] - 14*a*c*Log[Sqrt[x] - #1] +
3*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(a*(-b^2 + 4*a*c))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 144, normalized size = 0.29

method	result	size
derivativedivides	$\frac{-\frac{bc\,x^{\frac{5}{2}}}{2a\left(4ac-b^2\right)}+\frac{\left(2ac-b^2\right)\sqrt{x}}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a} + \frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-3\_R^4bc+14ac-3b^2\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{8a(4ac-b^2)}$	144

default	$\frac{-\frac{b c x^{\frac{5}{2}}}{2a(4ac-b^2)}+\frac{(2ac-b^2)\sqrt{x}}{2a(4ac-b^2)}}{c x^4+b x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4 b+a)}\frac{(-3\_R^4 bc+14ac-3b^2)\ln(\sqrt{x}-\_R)}{2\_R^7 c+\_R^3 b}}{8a(4ac-b^2)}$	144

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/4/a*b*c/(4*a*c-b^2)*x^(5/2)+1/4*(2*a*c-b^2)/a/(4*a*c-b^2)*x^(1/2))/(c
*x^4+b*x^2+a)+1/8/a/(4*a*c-b^2)*sum((-3*_R^4*b*c+14*a*c-3*b^2)/(2*_R^7*c+_R
^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((3*b^2*c - 14*a*c^2)*x^(9/2) + (3*b^3 - 13*a*b*c)*x^(5/2) + 4*(a*b^2 -
 4*a^2*c)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*
b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((3*b^2*c - 14*a*c^2)*x^(7/2) + (3*b^
3 - 17*a*b*c)*x^(3/2))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (
a^2*b^3 - 4*a^3*b*c)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 13351 vs. $2(403) = 806$.
time = 32.05, size = 13351, normalized size = 26.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c
)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^7*c^2 - 1
00023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*c^5 + (a^7*b^12 - 24*
a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144
*a^12*b^2*c^5 + 4096*a^13*c^6)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*
a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^
5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^
8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 3
2256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 589824*a^2
1*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))/(a^7*b^12 - 24*a^8*b^1
```

0*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)))*arctan(1/2*(sqrt(1/2)*(2187*b^25 - 123930*a*b^23*c + 3181356*a^2*b^21*c^2 - 48848184*a^3*b^19*c^3 + 498738681*a^4*b^17*c^4 - 3558667203*a^5*b^15*c^5 + 18137948052*a^6*b^13*c^6 - 66233459696*a^7*b^11*c^7 + 170799777856*a^8*b^9*c^8 - 299946354944*a^9*b^7*c^9 + 333598899200*a^10*b^5*c^10 - 201931706368*a^11*b^3*c^11 + 44333907968*a^12*b*c^12 - (27*a^7*b^26 - 1485*a^8*b^24*c + 37431*a^9*b^22*c^2 - 572228*a^10*b^20*c^3 + 5915536*a^11*b^18*c^4 - 43630272*a^12*b^16*c^5 + 235931136*a^13*b^14*c^6 - 945629184*a^14*b^12*c^7 + 2803507200*a^15*b^10*c^8 - 6054641664*a^16*b^8*c^9 + 9220325376*a^17*b^6*c^10 - 9316859904*a^18*b^4*c^11 + 5541724160*a^19*b^2*c^12 - 1438646272*a^20*c^13)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))*sqrt((7144929*b^20*c^4 - 365906970*a*b^18*c^5 + 8249676741*a^2*b^16*c^6 - 107186466510*a^3*b^14*c^7 + 881134553646*a^4*b^12*c^8 - 4726564284204*a^5*b^10*c^9 + 16406600944545*a^6*b^8*c^10 - 35077504886928*a^7*b^6*c^11 + 41166213348960*a^8*b^4*c^12 - 20609394167040*a^9*b^2*c^13 + 3543369523456*a^10*c^14)*x + 1/2*sqrt(1/2)*(59049*b^28 - 3818502*a*b^26*c + 112685175*a^2*b^24*c^2 - 2006183214*a^3*b^22*c^3 + 23996183904*a^4*b^20*c^4 - 203172719904*a^5*b^18*c^5 + 1249344303669*a^6*b^16*c^6 - 5630371599204*a^7*b^14*c^7 + 18522020088594*a^8*b^12*c^8 - 43727956655856*a^9*b^10*c^9 + 71711400072928*a^10*b^8*c^10 - 77372920636928*a^11*b^6*c^11 + 50337719029248*a^12*b^4*c^12 - 17166378299392*a^13*b^2*c^13 + 2314037239808*a^14*c^14 - (729*a^7*b^29 - 45927*a^8*b^27*c + 1331640*a^9*b^25*c^2 - 23536197*a^10*b^23*c^3 + 283046346*a^11*b^21*c^4 - 2447287920*a^12*b^19*c^5 + 15665468896*a^13*b^17*c^6 - 75268700672*a^14*b^15*c^7 + 272035456000*a^15*b^13*c^8 - 733077069824*a^16*b^11*c^9 + 1441307017216*a^17*b^9*c^10 - 1985616084992*a^18*b^7*c^11 + 1781248688128*a^19*b^5*c^12 - 899513581568*a^20*b^3*c^13 + 176234168320*a^21*b*c^14)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))*sqrt(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^7*c^2 - 100023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*c^5 + (a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))/(a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 12

```
80*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6))*
sqrt(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^7*c^2 - 100023*a^3*b^5*c^3 + 22
6072*a^4*b^3*c^4 - 181104*a^5*b*c^5 + (a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b
^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a
^13*c^6)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 3849946
2*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137*
a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^18 - 36*a^1
5*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32256*a^18*b^10*c^4 - 1
29024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^2
2*b^2*c^8 - 262144*a^23*c^9)))/(a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2
- 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6
)) + sqrt(1/2)*(5845851*b^35*c^2 - 480954105*a*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^2*sqrt(x)), x)
```

Mupad [B]
time = 7.29, size = 2500, normalized size = 4.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^(1/2)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^(5/2))/(2*a*(4*a*c -
b^2)))/(a + b*x^2 + c*x^4) + atan(((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) -
 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^
3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6
+ 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 -
```

2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a
*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(
-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a
^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^
10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^
12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6
*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)*(285212672*
a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 4620288*a^6*b^11*c
^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352*a^9*b^5*c^9 -
478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^
6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*b*c^13 - 36864*a
^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6 + 233816064*a^5*
b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c^9 - 16436428800*a
^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^10*b^3*c^12))/(16*(a
^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^
3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/
2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^1
7*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*
c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c
^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 40
23*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c
^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(819
2*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 1408
0*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^1
3*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16
*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(3/4) - (5378
24*a^4*c^11 + 891*b^8*c^7 - 19548*a*b^6*c^8 + 155358*a^2*b^4*c^9 - 510384*a
^3*b^2*c^10)/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 25
6*a^7*b^2*c^3)))*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a
^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*
c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^
7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c
^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^
4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/
2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^1
9*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a
^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14
*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b^6*c^9 + 69206016*a^17*b
^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4) + (x^(1/2)*(15059072*a^4*c^13 + 9
801*b^8*c^9 - 227502*a*b^6*c^10 + 2092104*a^2*b^4*c^11 - 8989344*a^3*b^2*c^
12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280
*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*((81*b^8*(-(4*a*c - b
^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201
623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 27957196
8*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 249411993

```
6*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)
^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 2631
3*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^
(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20
*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 +
3784704*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 57
671680*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1
/4)*1i - ((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*
b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4
+ 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 -
1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10
+ 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2) -
 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^7*b^24 + 16777216*a^19*c^
12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 - 14080*a^10*b^18*c^3 + 126720*a^11*
b^16*c^4 - 811008*a^12*b^14*c^5 + 3784704*a^13*b^12*c^6 - 12976128*a^14*b^1
0*c^7 + 32440320*a^15*b^8*c^8 - 57671680*a^16*b...
```

3.1079 $\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=573

$$-\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\,\left(a+bx^2+cx^4\right)}+\frac{\sqrt[4]{c}\left(5b^3-28abc-\left(5b^2-18ac\right)\sqrt{b^2-4ac}\right)\text{ta}}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{-b-}}$$

```
[Out] 1/8*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(
5*b^3-28*a*b*c-(-18*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)
^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/8*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^
(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^3-28*a*b*c-(-18*a*c+5*b^2)*(-4*a*
c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-
1/8*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(
5*b^3-28*a*b*c+(-18*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)
^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/8*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^
(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^3-28*a*b*c+(-18*a*c+5*b^2)*(-4*a*
c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+
1/2*(18*a*c-5*b^2)/a^2/(-4*a*c+b^2)/x^(1/2)+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a
*c+b^2)/(c*x^4+b*x^2+a)/x^(1/2)
```

Rubi [A]

time = 1.67, antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1129, 1380, 1518, 1524, 304, 211, 214}

$$\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{5b^2-18ac}{2a^2\sqrt{x}\left(b^2-4ac\right)}-\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{-2ac+b^2+bcx^2}{2a\sqrt{x}\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(5*b^2 - 18*a*c)/(a^2*(b^2 - 4*a*c)*Sqrt[x]) + (b^2 - 2*a*c + b*c*x^2)
/(2*a*(b^2 - 4*a*c)*Sqrt[x]*(a + b*x^2 + c*x^4)) + (c^(1/4)*(5*b^3 - 28*a*b
*c - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(5*b^3 - 28*a*b*c + (5*b^2 - 18*a*c)*Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) -
 (c^(1/4)*(5*b^3 - 28*a*b*c - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(
2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b
^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^3 - 28*a*
b*c + (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
```

```
/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b +
Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1380

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b
^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(
p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx = 2\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-5b^2+18ac-5bcx^4}{x^2\left(a+bx^4+cx^8\right)}\,dx, x, \sqrt{x}\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2(-b(5\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(5b^2-18ac\cdots\right.\right.}{\cdots}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} + \frac{\left(\sqrt{c}\left(5b^2-18\cdots\right.\right.}{\cdots}$$

$$= -\frac{5b^2-18ac}{2a^2\left(b^2-4ac\right)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)} - \frac{\sqrt[4]{c}\left(5b^2-18a\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.44, size = 265, normalized size = 0.46

$$-\frac{-\frac{4\left(16a^2c-5b^2x^2\left(b+cx^2\right)+a\left(-4b^2+19bcx^2+18c^2x^4\right)\right)}{\left(b^2-4ac\right)\sqrt{x}\left(a+bx^2+cx^4\right)}+4\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log\left(\sqrt{x}-\#1\right)+c\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]+\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^3\log\left(\sqrt{x}-\#1\right)-7abc\log\left(\sqrt{x}-\#1\right)+b^2c\log\left(\sqrt{x}-\#1\right)\#1^4-2ac^2\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{b^2-4ac}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/8*((-4*(16*a^2*c - 5*b^2*x^2*(b + c*x^2) + a*(-4*b^2 + 19*b*c*x^2 + 18*c
^2*x^4)))/((b^2 - 4*a*c)*Sqrt[x]*(a + b*x^2 + c*x^4)) + 4*RootSum[a + b*#1^
4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] + c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c
*#1^5) & ] + RootSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 7*a*b
*c*Log[Sqrt[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 - 2*a*c^2*Log[Sqrt[x] -
 #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(b^2 - 4*a*c))/a^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 172, normalized size = 0.30

method	result
derivativedivides	$-\frac{2\left(\frac{\frac{c\left(2ac-b^{2}\right)x^{\frac{7}{2}}}{16ac-4b^{2}}+\frac{b\left(3ac-b^{2}\right)x^{\frac{3}{2}}}{16ac-4b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(c\left(18ac-5b^{2}\right)\_R^{6}+b\left(23ac-5b^{2}\right)\_R^{2}\right)\ln\left(\sqrt{x}-\_\right.}{2\_R^{7}c+\_R^{3}b}}{64ac-16b^{2}}\right.}{a^{2}}$
default	$-\frac{2\left(\frac{\frac{c\left(2ac-b^{2}\right)x^{\frac{7}{2}}}{16ac-4b^{2}}+\frac{b\left(3ac-b^{2}\right)x^{\frac{3}{2}}}{16ac-4b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(c\left(18ac-5b^{2}\right)\_R^{6}+b\left(23ac-5b^{2}\right)\_R^{2}\right)\ln\left(\sqrt{x}-\_\right.}{2\_R^{7}c+\_R^{3}b}}{64ac-16b^{2}}\right.}{a^{2}}$
risch	$-\frac{2}{a^{2}\sqrt{x}}-\frac{c^{2}x^{\frac{7}{2}}}{a(c\,x^{4}+b\,x^{2}+a)(4ac-b^{2})}+\frac{c\,x^{\frac{7}{2}}b^{2}}{2a^{2}(c\,x^{4}+b\,x^{2}+a)(4ac-b^{2})}-\frac{3b\,x^{\frac{3}{2}}c}{2a(c\,x^{4}+b\,x^{2}+a)(4ac-b^{2})}+\frac{b^{3}}{2a^{2}(c\,x^{4}+b\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^2*((1/4*c*(2*a*c-b^2)/(4*a*c-b^2)*x^(7/2)+1/4*b*(3*a*c-b^2)/(4*a*c-b^2
)*x^(3/2))/(c*x^4+b*x^2+a)+1/16/(4*a*c-b^2)*sum((c*(18*a*c-5*b^2)*_R^6+b*(2
3*a*c-5*b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b
+a)))-2/a^2/x^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((5*b^2*c - 18*a*c^2)*x^(7/2) + (5*b^3 - 19*a*b*c)*x^(3/2) + 4*(a*b^2
- 4*a^2*c)/sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2
```

```
*b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((5*b^2*c - 18*a*c^2)*x^(5/2) + (5*b
^3 - 23*a*b*c)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 +
(a^2*b^3 - 4*a^3*b*c)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 15175 vs. $2(469) = 938$.
time = 203.77, size = 15175, normalized size = 26.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2
 - 4*a^4*c)*x)*sqrt(sqrt(1/2)*sqrt(-(625*b^13 - 14625*a*b^11*c + 137475*a^2
*b^9*c^2 - 655590*a^3*b^7*c^3 + 1634841*a^4*b^5*c^4 - 1932840*a^5*b^3*c^5 +
 758160*a^6*b*c^6 - (a^9*b^12 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^
12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5 + 4096*a^15*c^6)*sqrt((3
90625*b^20 - 13593750*a*b^18*c + 203859375*a^2*b^16*c^2 - 1716818750*a^3*b^
14*c^3 + 8861859375*a^4*b^12*c^4 - 28693007250*a^5*b^10*c^5 + 57219314050*a
^6*b^8*c^6 - 66126243780*a^7*b^6*c^7 + 39067874721*a^8*b^4*c^8 - 9017490888
*a^9*b^2*c^9 + 688747536*a^10*c^10)/(a^18*b^18 - 36*a^19*b^16*c + 576*a^20*
b^14*c^2 - 5376*a^21*b^12*c^3 + 32256*a^22*b^10*c^4 - 129024*a^23*b^8*c^5 +
 344064*a^24*b^6*c^6 - 589824*a^25*b^4*c^7 + 589824*a^26*b^2*c^8 - 262144*a
^27*c^9)))/(a^9*b^12 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^
3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^5 + 4096*a^15*c^6)))*arctan(-1/2*((
3125*b^16 - 90625*a*b^14*c + 1113125*a^2*b^12*c^2 - 7497500*a^3*b^10*c^3 +
29884825*a^4*b^8*c^4 - 70582238*a^5*b^6*c^5 + 92418696*a^6*b^4*c^6 - 553573
44*a^7*b^2*c^7 + 7558272*a^8*c^8 + (5*a^9*b^15 - 148*a^10*b^13*c + 1872*a^1
1*b^11*c^2 - 13120*a^12*b^9*c^3 + 55040*a^13*b^7*c^4 - 138240*a^14*b^5*c^5
+ 192512*a^15*b^3*c^6 - 114688*a^16*b*c^7)*sqrt((390625*b^20 - 13593750*a*b
^18*c + 203859375*a^2*b^16*c^2 - 1716818750*a^3*b^14*c^3 + 8861859375*a^4*b
^12*c^4 - 28693007250*a^5*b^10*c^5 + 57219314050*a^6*b^8*c^6 - 66126243780*
a^7*b^6*c^7 + 39067874721*a^8*b^4*c^8 - 9017490888*a^9*b^2*c^9 + 688747536*
a^10*c^10)/(a^18*b^18 - 36*a^19*b^16*c + 576*a^20*b^14*c^2 - 5376*a^21*b^12
*c^3 + 32256*a^22*b^10*c^4 - 129024*a^23*b^8*c^5 + 344064*a^24*b^6*c^6 - 58
9824*a^25*b^4*c^7 + 589824*a^26*b^2*c^8 - 262144*a^27*c^9)))*sqrt((29460504
150390625*b^32*c^14 - 1906321237792968750*a*b^30*c^15 + 570167859060058593
75*a^2*b^28*c^16 - 1044377592206152343750*a^3*b^26*c^17 + 130832447751894433
59375*a^4*b^24*c^18 - 118530802949736785156250*a^5*b^22*c^19 + 800604258039
631850781250*a^6*b^20*c^20 - 4094671424700247652812500*a^7*b^18*c^21 + 1593
6145834452127418390625*a^8*b^16*c^22 - 47001721928239386407700000*a^9*b^14*
c^23 + 103639285070964365022360000*a^10*b^12*c^24 - 16660512931934708766662
4000*a^11*b^10*c^25 + 187261266135093054493017600*a^12*b^8*c^26 - 137324626
675735662427299840*a^13*b^6*c^27 + 58232653591556101463064576*a^14*b^4*c^28
```

```
 - 1139078513779834762808524 8*a^15*b^2*c^29 + 796764763524754885902336*a^16
*c^30)*x - 1/2*sqrt(1/2)*(174322509765625*b^41*c^9 - 13760750732421875*a*b^
39*c^10 + 510877840332031250*a^2*b^37*c^11 - 11849949052880859375*a^3*b^35*
c^12 + 192395929770468750000*a^4*b^33*c^13 - 2321414560345529296875*a^5*b^3
1*c^14 + 21567618386883643359375*a^6*b^29*c^15 - 157732451339060345625000*a
^7*b^27*c^16 + 920408933613565924734375*a^8*b^25*c^17 - 4317209296832579116
815625*a^9*b^23*c^18 + 16318538811415112602143125*a^10*b^21*c^19 - 49611749
311377582465071000*a^11*b^19*c^20 + 120537558422009905693280400*a^12*b^17*c
^21 - 231348963491962085612943360*a^13*b^15*c^22 + 344476360343672339627579
904*a^14*b^13*c^23 - 387248525625301291868147712*a^15*b^11*c^24 + 315446330
908747793410007040*a^16*b^9*c^25 - 174596571921689117767434240*a^17*b^7*c^2
6 + 58984300529125601196441600*a^18*b^5*c^27 - 10043642926062594638217216*a
^19*b^3*c^28 + 647029821682654173462528*a^20*b*c^29 + (278916015625*a^9*b^4
0*c^9 - 22184550781250*a^10*b^38*c^10 + 832288361328125*a^11*b^36*c^11 - 19
570939608593750*a^12*b^34*c^12 + 323271617376718750*a^13*b^32*c^13 - 398389
3429538503125*a^14*b^30*c^14 + 37970074015626890625*a^15*b^28*c^15 - 286266
780074640405000*a^16*b^26*c^16 + 1731540717669235278000*a^17*b^24*c^17 - 84
71573265172367334400*a^18*b^22*c^18 + 33638878345847517227264*a^19*b^20*c^1
9 - 108320215826758770219008*a^20*b^18*c^20 + 281439039542942343016448*a^21
*b^16*c^21 - 584308534716933471797248*a^22*b^14*c^22 + 95441552680292910386
3808*a^23*b^12*c^23 - 1198125396055113290219520*a^24*b^10*c^24 + 1116075874
846199793057792*a^25*b^8*c^25 - 730808896711525612388352*a^26*b^6*c^26 + 30
7792791549408498941952*a^27*b^4*c^27 - 70848277770584815828992*a^28*b^2*c^2
8 + 6140942214464815497216*a^29*c^29)*sqrt((390625*b^20 - 13593750*a*b^18*c
 + 203859375*a^2*b^16*c^2 - 1716818750*a^3*b^14*c^3 + 8861859375*a^4*b^12*c
^4 - 28693007250*a^5*b^10*c^5 + 57219314050*a^6*b^8*c^6 - 66126243780*a^7*b
^6*c^7 + 39067874721*a^8*b^4*c^8 - 9017490888*a^9*b^2*c^9 + 688747536*a^10*
c^10)/(a^18*b^18 - 36*a^19*b^16*c + 576*a^20*b^14*c^2 - 5376*a^21*b^12*c^3
+ 32256*a^22*b^10*c^4 - 129024*a^23*b^8*c^5 + 344064*a^24*b^6*c^6 - 589824*
a^25*b^4*c^7 + 589824*a^26*b^2*c^8 - 262144*a^27*c^9)))*sqrt(-(625*b^13 - 1
4625*a*b^11*c + 137475*a^2*b^9*c^2 - 655590*a^3*b^7*c^3 + 1634841*a^4*b^5*c
^4 - 1932840*a^5*b^3*c^5 + 758160*a^6*b*c^6 - (a^9*b^12 - 24*a^10*b^10*c +
240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4*c^4 - 6144*a^14*b^2*c^
5 + 4096*a^15*c^6)*sqrt((390625*b^20 - 13593750*a*b^18*c + 203859375*a^2*b^
16*c^2 - 1716818750*a^3*b^14*c^3 + 8861859375*a...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^2*x^(3/2)), x)
```

Mupad [B]
time = 11.42, size = 2500, normalized size = 4.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan(((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 260
4992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^1
7*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175
670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143
360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*
b^3*c^20) + (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a
^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17
*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^
11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^
10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)
^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181
990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b
^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 +
16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*
c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 -
 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 6
9206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(32768000*a^21*b^34
*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32*c^5 + 64398295040
*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 15898363035648*a^25*b^26*c^8
 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*a^27*b^22*c^10 - 649291
7279490048*a^28*b^20*c^11 + 29298398985191424*a^29*b^18*c^12 - 104398826088
955904*a^30*b^16*c^13 + 293000581579014144*a^31*b^14*c^14 - 641705669216436
224*a^32*b^12*c^15 + 1077743462209552384*a^33*b^10*c^16 - 13483557107143802
88*a^34*b^8*c^17 + 1198053158392168448*a^35*b^6*c^18 - 695801744382230528*a
^36*b^4*c^19 + 223957324438437888*a^37*b^2*c^20 + x^(1/2)*(-(625*b^25 - 625
*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*
c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5
```

+ 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*(91197892454252544*a^40*c^21 - 52428800*a^23*b^34*c^4 + 3418357760*a^24*b^32*c^5 - 104457043968*a^25*b^30*c^6 + 1986074247168*a^26*b^28*c^7 - 26302715265024*a^27*b^26*c^8 + 257340683059200*a^28*b^24*c^9 - 1924694567550976*a^29*b^22*c^10 + 11230133666971648*a^30*b^20*c^11 - 51694329453871104*a^31*b^18*c^12 + 188531248770056192*a^32*b^16*c^13 - 543721556635811840*a^33*b^14*c^14 + 1229750704231415808*a^34*b^12*c^15 - 2146620531372195840*a^35*b^10*c^16 + 2815880065059913728*a^36*b^8*c^17 - 2657721914474102784*a^37*b^6*c^18 + 1675831642591068160*a^38*b^4*c^19 - 612489549322387456*a^39*b^2*c^20)))*(-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(1/4)*1i + (x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 2604992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 - 175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 1197821248143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*a^30*b^3*c^20) + (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 3105423360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 5996689920*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c - b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c ...

3.1080 $\int \frac{x^{15/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=621

$$-\frac{3(b^2+12ac)\sqrt{x}}{16c\left(b^2-4ac\right)^2}+\frac{x^{9/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\Big(b^3-28abc+\frac{b^4-30ab}{\sqrt{b^2}}}{32\sqrt[4]{2}\,c^{5/4}\,(b}$$

```
[Out] 1/4*x^(9/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/16*x^(5/2)*(8*a*b+
(12*a*c+b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-3/64*arctan(2^(1/4)*c^(1/4
)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^3-28*a*b*c+(-24*a^2*c^2-30*a*b^
2*c+b^4)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)^2/(-b-(-4*a*c+b^2
)^(1/2))^(3/4)-3/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))
^(1/4))*(b^3-28*a*b*c+(-24*a^2*c^2-30*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2))*2^(3
/4)/c^(5/4)/(-4*a*c+b^2)^2/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3/64*arctan(2^(1/4
)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b^3-28*a*b*c+(24*a^2*c^2+
30*a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)^2/(-b+(-4*
a*c+b^2)^(1/2))^(3/4)-3/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*(b^3-28*a*b*c+(24*a^2*c^2+30*a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2)
)*2^(3/4)/c^(5/4)/(-4*a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-3/16*(12*a*c
+b^2)*x^(1/2)/c/(-4*a*c+b^2)^2
```

Rubi [A]
time = 1.30, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1129, 1379, 1512, 1516, 1436, 218, 214, 211}

$$-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,c^{5/4}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x^{9/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x^{5/2}\left(x^2(12ac+b^2)+8ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\sqrt{x}\,(12ac+b^2)}{16c\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3*(b^2 + 12*a*c)*Sqrt[x])/(16*c*(b^2 - 4*a*c)^2) + (x^(9/2)*(2*a + b*x^2)
)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(5/2)*(8*a*b + (b^2 + 12*a
*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (3*(b^3 - 28*a*b*c + (
b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^
2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c
- 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2
- 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c + (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqr
t[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
```

```
(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1
/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1379

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*
p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c,
 d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1]
 && GtQ[m, 2*n - 1]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1512

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[f^(n - 1)*(f*x)^(m - n + 1)*(a +
b*x^n + c*x^(2*n))^(p + 1)*((b*d - 2*a*e - (b*e - 2*c*d)*x^n)/(n*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^{15/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\text{Subst}\left(\int \frac{x^{16}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{9/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^8\left(18a-3bx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{9/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^4(-120ab}{a+}\right.}{16(}$$

$$= -\frac{3(b^2+12ac)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx\right.}$$

$$= -\frac{3(b^2+12ac)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx\right.}$$

$$= -\frac{3(b^2+12ac)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx\right.}$$

$$= -\frac{3(b^2+12ac)\sqrt{x}}{16c\left(b^2-4ac\right)^2} + \frac{x^{9/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{5/2}\left(8ab+\left(b^2+12ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.84, size = 506, normalized size = 0.81

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*c^2*Sqrt[x]*(36*a^3*c + b^3*x^4*(3*b - c*x^2) + a*b*x^2*(6*b^2 + 7*b*c
*x^2 + 28*c^2*x^4) + a^2*(3*b^2 + 48*b*c*x^2 + 68*c^2*x^4)))/((b^2 - 4*a*c)
^2*(a + b*x^2 + c*x^4)^2) + 32*c*RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x
] - #1]/(b*#1^3 + 2*c*#1^7) & ] + (8*RootSum[a + b*#1^4 + c*#1^8 & , (3*b^4
*Log[Sqrt[x] - #1] - 22*a*b^2*c*Log[Sqrt[x] - #1] + 28*a^2*c^2*Log[Sqrt[x]
- #1] + 3*b^3*c*Log[Sqrt[x] - #1]*#1^4 + 6*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ])/(a*(b^2 - 4*a*c)) - (3*RootSum[a + b*#1^4 + c*#1^8
```

```
& , (8*b^6*Log[Sqrt[x] - #1] - 80*a*b^4*c*Log[Sqrt[x] - #1] + 223*a^2*b^2*
c^2*Log[Sqrt[x] - #1] - 140*a^3*c^3*Log[Sqrt[x] - #1] + 8*b^5*c*Log[Sqrt[x]
 - #1]*#1^4 - 17*a*b^3*c^2*Log[Sqrt[x] - #1]*#1^4 - 36*a^2*b*c^3*Log[Sqrt[x
] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(a*(b^2 - 4*a*c)^2))/(64*c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 275, normalized size = 0.44

method	result
derivativedivides	$\frac{-\frac{3a^2(12ac+b^2)\sqrt{x}}{16c(16a^2c^2-8a\,b^2c+b^4)}-\frac{3ab(8ac+b^2)x^{\frac{5}{2}}}{8c(16a^2c^2-8a\,b^2c+b^4)}-\frac{(68a^2c^2+7a\,b^2c+3b^4)x^{\frac{9}{2}}}{16c(16a^2c^2-8a\,b^2c+b^4)}-\frac{b(28ac-b^2)x^{\frac{13}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\Big(\underset{\_R=\mathrm{RootOf}(}{}}{}$
default	$\frac{-\frac{3a^2(12ac+b^2)\sqrt{x}}{16c(16a^2c^2-8a\,b^2c+b^4)}-\frac{3ab(8ac+b^2)x^{\frac{5}{2}}}{8c(16a^2c^2-8a\,b^2c+b^4)}-\frac{(68a^2c^2+7a\,b^2c+3b^4)x^{\frac{9}{2}}}{16c(16a^2c^2-8a\,b^2c+b^4)}-\frac{b(28ac-b^2)x^{\frac{13}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\Big(\underset{\_R=\mathrm{RootOf}(}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-3/32*a^2*(12*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-3/16*a/c*b*(
8*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)-1/32*(68*a^2*c^2+7*a*b^2*c+3*
b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)-1/32*b*(28*a*c-b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64/c/(16*a^2*c^2-8*a*b^2*c+b^4)
*sum((b*(-28*a*c+b^2)*_R^4+12*a^2*c+a*b^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R)
,_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^2*c + 12*a*c^2)*x^(17/2) + (7*b^3 + 44*a*b*c)*x^(13/2) + 24*a^2*
b*x^(5/2) + (35*a*b^2 + 4*a^2*c)*x^(9/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2
*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2)*x^2) - integrate(3/32*((b^2 + 12*a*c)*x^(7/2) + 40*a*b*x
^(3/2))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18827 vs. 2(521) = 1042.
time = 115.55, size = 18827, normalized size = 30.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2
*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c -
 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)
*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^13 - 117*a*b^11*c + 5109*a^2*b^9*c^2 - 97968*
a^3*b^7*c^3 + 670176*a^4*b^5*c^4 + 2895360*a^5*b^3*c^5 - 449280*a^6*b*c^6 +
 (b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a
^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*
c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14 + 1048576*a^10*c^15)*sqr
t((b^16 - 214*a*b^14*c + 19467*a^2*b^12*c^2 - 967222*a^3*b^10*c^3 + 2776934
5*a^4*b^8*c^4 - 438445008*a^5*b^6*c^5 + 2996795232*a^6*b^4*c^6 - 141647616*
a^7*b^2*c^7 + 1679616*a^8*c^8)/(b^30*c^10 - 60*a*b^28*c^11 + 1680*a^2*b^26*
c^12 - 29120*a^3*b^24*c^13 + 349440*a^4*b^22*c^14 - 3075072*a^5*b^20*c^15 +
 20500480*a^6*b^18*c^16 - 105431040*a^7*b^16*c^17 + 421724160*a^8*b^14*c^18
 - 1312030720*a^9*b^12*c^19 + 3148873728*a^10*b^10*c^20 - 5725224960*a^11*b
^8*c^21 + 7633633280*a^12*b^6*c^22 - 7046430720*a^13*b^4*c^23 + 4026531840*
a^14*b^2*c^24 - 1073741824*a^15*c^25)))/(b^20*c^5 - 40*a*b^18*c^6 + 720*a^2
*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 +
 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 262144
0*a^9*b^2*c^14 + 1048576*a^10*c^15)))*arctan(1/4*(2*sqrt(1/2)*(39*a*b^45 -
14092*a^2*b^43*c + 2342105*a^3*b^41*c^2 - 236978592*a^4*b^39*c^3 + 16281606
995*a^5*b^37*c^4 - 802176194594*a^6*b^35*c^5 + 29187154410144*a^7*b^33*c^6
- 796345615508224*a^8*b^31*c^7 + 16398991931243520*a^9*b^29*c^8 - 255116875
223562240*a^10*b^27*c^9 + 2991680147781025792*a^11*b^25*c^10 - 263516522130
68988416*a^12*b^23*c^11 + 173550335869159342080*a^13*b^21*c^12 - 8484882575
40540071936*a^14*b^19*c^13 + 3040685322832896327680*a^15*b^17*c^14 - 781064
7267905472823296*a^16*b^15*c^15 + 13826407321898900783104*a^17*b^13*c^16 -
15695440087743077548032*a^18*b^11*c^17 + 9831375748320473382912*a^19*b^9*c^
18 - 2132270903021870776320*a^20*b^7*c^19 - 302763289379535323136*a^21*b^5*
c^20 + 174927391948233768896*a^22*b^3*c^21 - 21912208461633945 6*a^23*b*c^22
- (39*a*b^52*c^5 - 11089*a^2*b^50*c^6 + 1437201*a^3*b^48*c^7 - 112481582*a^
4*b^46*c^8 + 5937253574*a^5*b^44*c^9 - 223744431280*a^6*b^42*c^10 + 6224393
237408*a^7*b^40*c^11 - 130617052733440*a^8*b^38*c^12 + 2099519233610240*a^9
*b^36*c^13 - 26151440607784960*a^10*b^34*c^14 + 254528845453582336*a^11*b^3
2*c^15 - 1944123359024644096*a^12*b^30*c^16 + 11639486870997172224*a^13*b^2
8*c^17 - 54083721102022934528*a^14*b^26*c^18 + 189937300181956427776*a^15*b
^24*c^19 - 470443868125682204672*a^16*b^22*c^20 + 634824840388053827584*a^1
7*b^20*c^21 + 533578961559771676672*a^18*b^18*c^22 - 5318575782259016597504
*a^19*b^16*c^23 + 14710673859192740118528*a^20*b^14*c^24 - 2424005214877221
3358592*a^21*b^12*c^25 + 25158757634824950775808*a^22*b^10*c^26 - 150818559
06511551725568*a^23*b^8*c^27 + 3658254071754544644096*a^24*b^6*c^28 + 38659
1606652134227968*a^25*b^4*c^29 - 402054458686989926 40*a^26*b^2*c^30 + 69253
```

```
3995824480256*a^27*c^31)*sqrt((b^16 - 214*a*b^14*c + 19467*a^2*b^12*c^2 - 9
67222*a^3*b^10*c^3 + 27769345*a^4*b^8*c^4 - 438445008*a^5*b^6*c^5 + 2996795
232*a^6*b^4*c^6 - 141647616*a^7*b^2*c^7 + 1679616*a^8*c^8)/(b^30*c^10 - 60*
a*b^28*c^11 + 1680*a^2*b^26*c^12 - 29120*a^3*b^24*c^13 + 349440*a^4*b^22*c^
14 - 3075072*a^5*b^20*c^15 + 20500480*a^6*b^18*c^16 - 105431040*a^7*b^16*c^
17 + 421724160*a^8*b^14*c^18 - 1312030720*a^9*b^12*c^19 + 3148873728*a^10*b
^10*c^20 - 5725224960*a^11*b^8*c^21 + 7633633280*a^12*b^6*c^22 - 7046430720
*a^13*b^4*c^23 + 4026531840*a^14*b^2*c^24 - 1073741824*a^15*c^25)))*sqrt(x)
*sqrt(-(b^13 - 117*a*b^11*c + 5109*a^2*b^9*c^2 - 97968*a^3*b^7*c^3 + 670176
*a^4*b^5*c^4 + 2895360*a^5*b^3*c^5 - 449280*a^6*b*c^6 + (b^20*c^5 - 40*a*b^
18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048
*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b
^4*c^13 - 2621440*a^9*b^2*c^14 + 1048576*a^10*c^15)*sqrt((b^16 - 214*a*b^14
*c + 19467*a^2*b^12*c^2 - 967222*a^3*b^10*c^3 + 27769345*a^4*b^8*c^4 - 4384
45008*a^5*b^6*c^5 + 2996795232*a^6*b^4*c^6 - 141647616*a^7*b^2*c^7 + 167961
6*a^8*c^8)/(b^30*c^10 - 60*a*b^28*c^11 + 1680*a^2*b^26*c^12 - 29120*a^3*b^2
4*c^13 + 349440*a^4*b^22*c^14 - 3075072*a^5*b^20*c^15 + 20500480*a^6*b^18*c
^16 - 105431040*a^7*b^16*c^17 + 421724160*a^8*b^14*c^18 - 1312030720*a^9*b^
12*c^19 + 3148873728*a^10*b^10*c^20 - 5725224960*a^11*b^8*c^21 + 7633633280
*a^12*b^6*c^22 - 7046430720*a^13*b^4*c^23 + 4026531840*a^14*b^2*c^24 - 1073
741824*a^15*c^25)))/(b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3
*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11
 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14 + 104
8576*a^10*c^15)) + (b^33 - 225*a*b^31*c + 22235*a^2*b^29*c^2 - 1265758*a^3*
b^27*c^3 + 45810016*a^4*b^25*c^4 - 1102080960*a^5*b^23*c^5 + 18046071552*a^
6*b^21*c^6 - 204854161920*a^7*b^19*c^7 + 163725...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(15/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B]
time = 9.35, size = 2500, normalized size = 4.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 1590021
9*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 284032396
8*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10 -
36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c^5 -
 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824*a^8*
b^2*c^9)) + ((3*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*
a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c
^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19
*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a
^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 +
 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*
a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 400
9*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)
^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(10995116277
76*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^3
4*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*
c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^
9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2
113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560
*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c
^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558
138880*a^19*b^2*c^24)))^(1/4)*(703687441776640*a^13*b*c^15 + 671088640*a^3*
b^21*c^5 - 26843545600*a^4*b^19*c^6 + 483183820800*a^5*b^17*c^7 - 515396075
5200*a^6*b^15*c^8 + 36077725286400*a^7*b^13*c^9 - 173173081374720*a^8*b^11*
c^10 + 577243604582400*a^9*b^9*c^11 - 1319413953331200*a^10*b^7*c^12 + 1979
120929996800*a^11*b^5*c^13 - 1759218604441600*a^12*b^3*c^14))/(65536*(b^18*
c - 262144*a^9*c^10 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4
+ 32256*a^4*b^10*c^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7
*b^4*c^8 + 589824*a^8*b^2*c^9)) - (9*x^(1/2)*(16777216*a^3*b^25*c^4 - 31243
722414882816*a^15*b*c^16 + 23890755584*a^4*b^23*c^5 - 1000190509056*a^5*b^2
1*c^6 + 18747532247040*a^6*b^19*c^7 - 209186382151680*a^7*b^17*c^8 + 154495
1275978752*a^8*b^15*c^9 - 7925554690916352*a^9*b^13*c^10 + 2878301539192012
8*a^10*b^11*c^11 - 73870688712130560*a^11*b^9*c^12 + 130973825100677120*a^1
2*b^7*c^13 - 152242778028376064*a^13*b^5*c^14 + 103864266406232064*a^14*b^3
*c^15))/(4194304*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^
20*c^3 - 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3
784704*a^6*b^12*c^7 - 12976128*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 576716
```

80*a^9*b^6*c^10 + 69206016*a^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20809116549120*a^16*b^8*c^21 - 19585050869760*a^17*b^6*c^22 + 13056700579840*a^18*b^4*c^23 - 5497558138880*a^19*b^2*c^24)))^(3/4))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7 - 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 7044752998...

3.1081 $\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=569

$$\frac{x^{7/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x^{3/2}(24ab+(5b^2+28ac)x^2)}{16(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{\left(5b^3+172abc+\sqrt{b^2-4ac}\,(5b^2+28ac)\right)\tan}{32\,2^{3/4}c^{3/4}(b^2-4ac)^{5/2}\sqrt[4]{-b-}}$$

```
[Out] 1/4*x^(7/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*x^(3/2)*(24*a*b
+(28*a*c+5*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/64*arctan(2^(1/4)*c^(
1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^2+28*a*c+(-172*a*b*c-5*b^3
)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2)
)^(1/4)-1/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))
*(5*b^2+28*a*c+(-172*a*b*c-5*b^3)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a
*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/64*arctan(2^(1/4)*c^(1/4)*x^(1/2)
/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^3+172*a*b*c+(28*a*c+5*b^2)*(-4*a*c+b^2
)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1
/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^3+1
72*a*b*c+(28*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(5
/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A]

time = 1.27, antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1129, 1379, 1512, 1524, 304, 211, 214}

$$\frac{\left(\sqrt{b^2-4ac}\,(28ac+5b^2)+172abc+5b^3\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}(b^2-4ac)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}(b^2-4ac)^2\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{7/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x^{3/2}(x^2(28ac+5b^2)+24ab)}{16(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{\left(\sqrt{b^2-4ac}\,(28ac+5b^2)+172abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}(b^2-4ac)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}(b^2-4ac)^2\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(7/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^(3/2)*
(24*a*b + (5*b^2 + 28*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b^2 + 28*a*c))*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2
- 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((5*b^2 + 28*a*c - (5*b^3
+ 172*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt
[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 -
 4*a*c])^(1/4)) - ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b^2 + 28*a*c))
*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(
3/4)*c^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((5*b^2
+ 28*a*c - (5*b^3 + 172*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*
```

```
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)
^2*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1379

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*
p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c,
 d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1]
 && GtQ[m, 2*n - 1]
```

Rule 1512

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[f^(n - 1)*(f*x)^(m - n + 1)*(a +
b*x^n + c*x^(2*n))^(p + 1)*((b*d - 2*a*e - (b*e - 2*c*d)*x^n)/(n*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = 2\text{Subst}\left(\int \frac{x^{14}}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{7/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^6(14a-5bx^4)}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}$$

$$= \frac{x^{7/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}(24ab+(5b^2+28ac)\,x^2)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{x^2(-72ab+}{a+}\right.}{16(}$$

$$= \frac{x^{7/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}(24ab+(5b^2+28ac)\,x^2)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(5b^3+172abc+\right.}{}$$

$$= \frac{x^{7/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}(24ab+(5b^2+28ac)\,x^2)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(5b^3+172abc+\right.}{}$$

$$= \frac{x^{7/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x^{3/2}(24ab+(5b^2+28ac)\,x^2)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(5b^3+172abc+\right.}{32}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.76, size = 392, normalized size = 0.69

$$\frac{1}{64}\left(\frac{4x^{3/2}(4a^2(6b-cx^2)+b^2x^4(9b+5cx^2)+a(37b^2x^2+36bcx^4+28c^2x^6))}{(b^2-4ac)^2\,(a+bx^2+cx^4)^2} + \frac{8\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b^3\log\left(\sqrt{x}-\#1\right)-13abc\log\left(\sqrt{x}-\#1\right)+b^2c\log\left(\sqrt{x}-\#1\right)\#1^4+2ac^2\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{ac^2\,(-b^2+4ac)} + \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{8b^5\log\left(\sqrt{x}-\#1\right)-136ab^3c\log\left(\sqrt{x}-\#1\right)+344a^2bc^2\log\left(\sqrt{x}-\#1\right)+8b^4c\log\left(\sqrt{x}-\#1\right)\#1^4-11ab^2c^2\log\left(\sqrt{x}-\#1\right)\#1^4-36a^2c^3\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{ac^2\,(b^2-4ac)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*x^(3/2)*(4*a^2*(6*b - c*x^2) + b^2*x^4*(9*b + 5*c*x^2) + a*(37*b^2*x^2
+ 36*b*c*x^4 + 28*c^2*x^6)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) + (8*R
```

```
ootSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 13*a*b*c*Log[Sqrt[x
] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 2*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b
*#1 + 2*c*#1^5) & ])/(a*c^2*(-b^2 + 4*a*c)) + RootSum[a + b*#1^4 + c*#1^8 &
 , (8*b^5*Log[Sqrt[x] - #1] - 136*a*b^3*c*Log[Sqrt[x] - #1] + 344*a^2*b*c^2
*Log[Sqrt[x] - #1] + 8*b^4*c*Log[Sqrt[x] - #1]*#1^4 - 11*a*b^2*c^2*Log[Sqrt
[x] - #1]*#1^4 - 36*a^2*c^3*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(
a*c^2*(b^2 - 4*a*c)^2))/64
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 242, normalized size = 0.43

method	result
derivativedivides	$\frac{\frac{3a^2bx^{\frac{3}{2}}}{2(16a^2c^2-8ab^2c+b^4)}-\frac{a(4ac-37b^2)x^{\frac{7}{2}}}{16(16a^2c^2-8ab^2c+b^4)}+\frac{9b(4ac+b^2)x^{\frac{11}{2}}}{16(16a^2c^2-8ab^2c+b^4)}+\frac{2c(28ac+5b^2)x^{\frac{15}{2}}}{512a^2c^2-256ab^2c+32b^4}}{(cx^4+bx^2+a)^2} - \frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+}}{}$
default	$\frac{\frac{3a^2bx^{\frac{3}{2}}}{2(16a^2c^2-8ab^2c+b^4)}-\frac{a(4ac-37b^2)x^{\frac{7}{2}}}{16(16a^2c^2-8ab^2c+b^4)}+\frac{9b(4ac+b^2)x^{\frac{11}{2}}}{16(16a^2c^2-8ab^2c+b^4)}+\frac{2c(28ac+5b^2)x^{\frac{15}{2}}}{512a^2c^2-256ab^2c+32b^4}}{(cx^4+bx^2+a)^2} - \frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/4*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)-1/32*a*(4*a*c-37*b^2)/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+9/32*b*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^(11/2)+1/32*c*(28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(c*x^4
+b*x^2+a)^2-1/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum(((-28*a*c-5*b^2)*_R^6+72*b*
_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^2*c + 28*a*c^2)*x^(15/2) + 9*(b^3 + 4*a*b*c)*x^(11/2) + 24*a^2*b
*x^(3/2) + (37*a*b^2 - 4*a^2*c)*x^(7/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*
c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*
c + 16*a^3*b*c^2)*x^2) + integrate(1/32*((5*b^2 + 28*a*c)*x^(5/2) - 72*a*b*
sqrt(x))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [**B**]
time = 8.01, size = 2500, normalized size = 4.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((9*x^(11/2)*(b^3 + 4*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(7/2
)*(37*a*b^2 - 4*a^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^(15/2)*(
28*a*c + 5*b^2))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a^2*b*x^(3/2))/(2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*
b*x^2 + 2*b*c*x^6) - atan(((((386183668047020032*a^16*c^16 + 2097152000*a^3
*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5 - 515
4027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 3505726682667
41760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 4628236966960300032
```

*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 7924026369753743360*
a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 1182321565924274995 2*a^
14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^28 + 2684354
56*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4
 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 1
96804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 -
 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 939524096*a^13*b^2*c
^13 - 56*a*b^26*c)) - (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25)^(1
/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^25*c
^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*a^6*
b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 144629
70429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a^11*
b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^13 +
150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) + 2
3125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4
c(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40*c^3
 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4*b^3
2*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7*b^2
6*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 19373070745
6*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^16*c^
15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 1664729
3239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*a^17
*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)))^(
1/4)*(27584547717644288*a^15*c^16 + 99891544064*a^3*b^24*c^4 - 409256696217
6*a^4*b^22*c^5 + 75824426385408*a^5*b^20*c^6 - 837991069122560*a^6*b^18*c^7
 + 6133342147706880*a^7*b^16*c^8 - 31188471955587072*a^8*b^14*c^9 + 1123431
50323826688*a^9*b^12*c^10 - 286537128244936704*a^10*b^10*c^11 + 50774347459
0679040*a^11*b^8*c^12 - 599365778533253120*a^12*b^6*c^13 + 4363565826456944
64*a^13*b^4*c^14 - 170573835886657536*a^14*b^2*c^15))/(4194304*(b^24 + 1677
7216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c
^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 3
2440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331
648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^6*(-(4*a*c - b^2)^25
)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27186416*a^3*b^
25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 - 265188833280*
a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^8*b^15*c^8 + 14
462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10 - 70455242260480*a
^11*b^9*c^11 + 206669464207360*a^12*b^7*c^12 - 267459844112384*a^13*b^5*c^1
3 + 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2)
 + 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a
*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(1099511627776*a^20*c^23 + b^40
*c^3 - 80*a*b^38*c^4 + 3040*a^2*b^36*c^5 - 72960*a^3*b^34*c^6 + 1240320*a^4
*b^32*c^7 - 15876096*a^5*b^30*c^8 + 158760960*a^6*b^28*c^9 - 1270087680*a^7
*b^26*c^10 + 8255569920*a^8*b^24*c^11 - 44029706240*a^9*b^22*c^12 + 1937307
07456*a^10*b^20*c^13 - 704475299840*a^11*b^18*c^14 + 2113425899520*a^12*b^1

```
6*c^15 - 5202279137280*a^13*b^14*c^16 + 10404558274560*a^14*b^12*c^17 - 166
47293239296*a^15*b^10*c^18 + 20809116549120*a^16*b^8*c^19 - 19585050869760*
a^17*b^6*c^20 + 13056700579840*a^18*b^4*c^21 - 5497558138880*a^19*b^2*c^22)
))^(3/4) - (x^(1/2)*(3705625*a^3*b^15*c - 6402256896*a^10*b*c^8 + 281098125
*a^4*b^13*c^2 + 7885779000*a^5*b^11*c^3 + 95525940400*a^6*b^9*c^4 + 3874698
62400*a^7*b^7*c^5 - 497953639680*a^8*b^5*c^6 - 117420369920*a^9*b^3*c^7))/(
4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3
 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976
128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a
^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*(-(625*b^31 + 625*b^
6*(-(4*a*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c
^2 + 27186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21
*c^5 - 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 66645041479
68*a^8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 -...
```

3.1082 $\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=569

$$\frac{x^{5/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(7b^3+36abc+\sqrt{b^2-4ac}\left(7b^2+20ac\right)\right)\text{ta}}{32\sqrt[4]{2}\,\sqrt[4]{c}\,\left(b^2-4ac\right)^{5/2}\left(-b-\right.}$$

```
[Out] 1/4*x^(5/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/64*arctan(2^(1/4)*
c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(7*b^2+20*a*c+(-36*a*b*c-7*b
^3)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/
2))^(3/4)-3/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4
))*(7*b^2+20*a*c+(-36*a*b*c-7*b^3)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-4*
a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-3/64*arctan(2^(1/4)*c^(1/4)*x^(1/2
)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(7*b^3+36*a*b*c+(20*a*c+7*b^2)*(-4*a*c+b^2
)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3
/64*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(7*b^3+3
6*a*b*c+(20*a*c+7*b^2)*(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(5/
2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/16*(24*a*b+(20*a*c+7*b^2)*x^2)*x^(1/2)/(
-4*a*c+b^2)^2/(c*x^4+b*x^2+a)
```

Rubi [A]
time = 1.44, antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1129, 1379, 1512, 1436, 218, 214, 211}

$$-\frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,\sqrt[4]{c}\,\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,\sqrt[4]{c}\,\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\left(x^2\left(20ac+7b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,\sqrt[4]{c}\,\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,\sqrt[4]{c}\,\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(5/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[x]*
(24*a*b + (7*b^2 + 20*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) -
 (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*b^2 + 20*a*c))*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2
 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7*b^2 + 20*a*c - (7*b
^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sq
rt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2
 - 4*a*c])^(3/4)) - (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*b^2 + 20*a*
c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*
2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7
*b^2 + 20*a*c - (7*b^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1
```

```
/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*
a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1379

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*
p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c,
 d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1]
 && GtQ[m, 2*n - 1]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1512

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[f^(n - 1)*(f*x)^(m - n + 1)*(a +
b*x^n + c*x^(2*n))^(p + 1)*((b*d - 2*a*e - (b*e - 2*c*d)*x^n)/(n*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[f^n/(n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - n)*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2*
n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
n - 1] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3}\,dx &= 2\text{Subst}\left(\int \frac{x^{12}}{(a+bx^4+cx^8)^3}\,dx, x, \sqrt{x}\right)\\
&= \frac{x^{5/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x^4(10a-7bx^4)}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}\\
&= \frac{x^{5/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\Big(\int \frac{-24ab+3}{a+}\cdots}{16(\cdots}\\
&= \frac{x^{5/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\Big(3\Big(7b^3+36abc\cdots}{\cdots}\\
&= \frac{x^{5/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\Big(3\Big(7b^3+36abc\cdots}{\cdots}\\
&= \frac{x^{5/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\sqrt{x}\left(24ab+\left(7b^2+20ac\right)x^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{3\Big(7b^3+36abc+\cdots}{32\cdots}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.65, size = 397, normalized size = 0.70

$$\frac{1}{64}\left(\frac{4\sqrt{x}\left(12a^2(2b-cx^2)+b^2x^4(11b+7cx^2)+a(39b^2x^2+28bcx^4+20c^2x^6)\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}+\frac{24\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^3\log\left(\sqrt{x}-\#1\right)-5abc\log\left(\sqrt{x}-\#1\right)+b^2c\log\left(\sqrt{x}-\#1\right)\#1^4+2ac^2\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{ac^2\left(-b^2+4ac\right)}+\frac{3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{8b^5\log\left(\sqrt{x}-\#1\right)-72ab^3c\log\left(\sqrt{x}-\#1\right)+152a^2bc^2\log\left(\sqrt{x}-\#1\right)+8b^4c\log\left(\sqrt{x}-\#1\right)\#1^4-9ab^2c^2\log\left(\sqrt{x}-\#1\right)\#1^4-44a^2c^3\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{ac^2\left(b^2-4ac\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*Sqrt[x]*(12*a^2*(2*b - c*x^2) + b^2*x^4*(11*b + 7*c*x^2) + a*(39*b^2*x^
2 + 28*b*c*x^4 + 20*c^2*x^6)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) + (2
4*RootSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 5*a*b*c*Log[Sqrt
[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 2*a*c^2*Log[Sqrt[x] - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ])/(a*c^2*(-b^2 + 4*a*c)) + (3*RootSum[a + b*#1^4 + c
*#1^8 & , (8*b^5*Log[Sqrt[x] - #1] - 72*a*b^3*c*Log[Sqrt[x] - #1] + 152*a^2
*b*c^2*Log[Sqrt[x] - #1] + 8*b^4*c*Log[Sqrt[x] - #1]*#1^4 - 9*a*b^2*c^2*Log
[Sqrt[x] - #1]*#1^4 - 44*a^2*c^3*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7
) & ])/(a*c^2*(b^2 - 4*a*c)^2))/64
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.09, size = 241, normalized size = 0.42

method	result
derivativedivides	$\frac{\frac{3a^2b\sqrt{x}}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{3(4ac-13b^2)a\,x^{\frac{5}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}+\frac{2b(28ac+11b^2)x^{\frac{9}{2}}}{512a^2c^2-256a\,b^2c+32b^4}+\frac{2c(20ac+7b^2)x^{\frac{13}{2}}}{512a^2c^2-256a\,b^2c+32b^4}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^8}\right.}{}$
default	$\frac{\frac{3a^2b\sqrt{x}}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{3(4ac-13b^2)a\,x^{\frac{5}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}+\frac{2b(28ac+11b^2)x^{\frac{9}{2}}}{512a^2c^2-256a\,b^2c+32b^4}+\frac{2c(20ac+7b^2)x^{\frac{13}{2}}}{512a^2c^2-256a\,b^2c+32b^4}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^8}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/4*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-3/32*(4*a*c-13*b^2)*a/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/32*b*(28*a*c+11*b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^(9/2)+1/32*c*(20*a*c+7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*
x^4+b*x^2+a)^2+3/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum(((20*a*c+7*b^2)*_R^4-8*a
*b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(24*b*c^2*x^(17/2) + (41*b^2*c - 20*a*c^2)*x^(13/2) + (13*b^3 + 20*a*
b*c)*x^(9/2) + 3*(3*a*b^2 + 4*a^2*c)*x^(5/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3
*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*x^2) + integrate(3/32*(8*b*c*x^(7/2) + 5*(3*b^2 + 4*
a*c)*x^(3/2))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16
*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16141 vs. $2(469) = 938$.
time = 37.23, size = 16141, normalized size = 28.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4
*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(sqr
t(1/2)*sqrt(-(2401*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c^2 + 833280*a^3*b^
3*c^3 + 672000*a^4*b*c^4 + (b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 768
0*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*
c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 10
48576*a^10*c^11)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c
^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 +
 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5
*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*
b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 57252249
60*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 40
26531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 +
720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10
*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 262
1440*a^9*b^2*c^10 + 1048576*a^10*c^11)))*arctan(1/2*(sqrt(1/2)*(823543*b^26
 - 20526492*a*b^24*c + 207055136*a^2*b^22*c^2 - 1054403200*a^3*b^20*c^3 + 2
668489984*a^4*b^18*c^4 - 1980797952*a^5*b^16*c^5 - 9183117312*a^6*b^14*c^6
+ 60969517056*a^7*b^12*c^7 - 195146219520*a^8*b^10*c^8 + 30249058304*a^9*b^
8*c^9 + 1320167669760*a^10*b^6*c^10 - 1880096768000*a^11*b^4*c^11 - 2569011
200000*a^12*b^2*c^12 + 5242880000000*a^13*c^13 - (343*b^37*c - 13432*a*b^35
*c^2 + 172016*a^2*b^33*c^3 + 568320*a^3*b^31*c^4 - 48501760*a^4*b^29*c^5 +
770760704*a^5*b^27*c^6 - 7125221376*a^6*b^25*c^7 + 44060377088*a^7*b^23*c^8
 - 187151810560*a^8*b^21*c^9 + 522563092480*a^9*b^19*c^10 - 768025296896*a^
10*b^17*c^11 - 334135033856*a^11*b^15*c^12 + 3645823254528*a^12*b^13*c^13 -
 3363496263680*a^13*b^11*c^14 - 16498043125760*a^14*b^9*c^15 + 623113855303
68*a^15*b^7*c^16 - 100059853094912*a^16*b^5*c^17 + 83047487635456*a^17*b^3*
c^18 - 29205777612800*a^18*b*c^19)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 1
36380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2
 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22
*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^
9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^
10*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*
a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))*sqrt((63
556931025*b^16 + 1107801807840*a*b^14*c + 8229675066816*a^2*b^12*c^2 + 3408
```

```
9163584000*a^3*b^10*c^3 + 86221374400000*a^4*b^8*c^4 + 136341632000000*a^5*
b^6*c^5 + 131424000000000*a^6*b^4*c^6 + 70400000000000*a^7*b^2*c^7 + 160000
00000000*a^8*c^8)*x + sqrt(1/2)*(789777737*b^22 - 7443973964*a*b^20*c - 270
5008400*a^2*b^18*c^2 + 166642188480*a^3*b^16*c^3 - 23017121280*a^4*b^14*c^4
 - 1866033297408*a^5*b^12*c^5 - 803898138624*a^6*b^10*c^6 + 11168850739200*
a^7*b^8*c^7 + 14678630400000*a^8*b^6*c^8 - 23490560000000*a^9*b^4*c^9 - 643
07200000000*a^10*b^2*c^10 - 40960000000000*a^11*c^11 + 8*(26411*b^33*c - 12
21952*a*b^31*c^2 + 25385088*a^2*b^29*c^3 - 309750784*a^3*b^27*c^4 + 2424181
760*a^4*b^25*c^5 - 12295815168*a^5*b^23*c^6 + 36966465536*a^6*b^21*c^7 - 34
375204864*a^7*b^19*c^8 - 198547734528*a^8*b^17*c^9 + 848696442880*a^9*b^15*
c^10 - 948860616704*a^10*b^13*c^11 - 2216807104512*a^11*b^11*c^12 + 8103865
090048*a^12*b^9*c^13 - 6260988575744*a^13*b^7*c^14 - 10597831802880*a^14*b^
5*c^15 + 23622320128000*a^15*b^3*c^16 - 13421772800000*a^16*b*c^17)*sqrt((5
764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c
^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 291
20*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6
*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a
^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 76336
33280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 -
 1073741824*a^15*c^17)))*sqrt(-(2401*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c
^2 + 833280*a^3*b^3*c^3 + 672000*a^4*b*c^4 + (b^20*c - 40*a*b^18*c^2 + 720*
a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6
 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440
*a^9*b^2*c^10 + 1048576*a^10*c^11)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 1
36380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2
 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22
*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^
9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^
10*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*
a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))/(b^20*c
- 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B]
time = 8.52, size = 2500, normalized size = 4.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^(9/2)*(11*b^3 + 28*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^(
5/2)*(13*a*b^2 - 4*a^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^(13/2
)*(20*a*c + 7*b^2))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a^2*b*x^(1/2))
/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2
*a*b*x^2 + 2*b*c*x^6) - atan((((((3*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2
) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*
a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 398985216
0*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977
994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^1
1 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2
*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^2
5)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 30
40*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*
b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^
8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 7044
75299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13
*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16
+ 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579
840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)*(351843720888320*a
^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b^20*c^5 + 167772160000*
a^4*b^18*c^6 - 1691143372800*a^5*b^16*c^7 + 10952166604800*a^6*b^14*c^8 - 4
6901042872320*a^7*b^12*c^9 + 129879811031040*a^8*b^10*c^10 - 20615843020800
0*a^9*b^8*c^11 + 82463372083200*a^10*b^6*c^12 + 329853488332800*a^11*b^4*c^
13 - 615726511554560*a^12*b^2*c^14))/(65536*(b^18 - 262144*a^9*c^9 + 576*a^
2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b^8*c^5 +
344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*a*b^16*c)
) - (9*x^(1/2)*(3774873600*a^2*b^25*c^4 - 4222124650659840*a^14*b*c^16 - 14
7907936256*a^3*b^23*c^5 + 2590402150400*a^4*b^21*c^6 - 26607322398720*a^5*b
^19*c^7 + 176329882337280*a^6*b^17*c^8 - 777217281884160*a^7*b^15*c^9 + 223
3932749733888*a^8*b^13*c^10 - 3727344418160640*a^9*b^11*c^11 + 159978941841
4080*a^10*b^9*c^12 + 7124835347988480*a^11*b^7*c^13 - 16008889300418560*a^1
2*b^5*c^14 + 13792273858822144*a^13*b^3*c^15))/(4194304*(b^24 + 16777216*a^
```

12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 81
1008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320
*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^1
1*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401
*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23
*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^
17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a
^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 2307
70606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*
c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)
))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b
^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6
 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c
^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840
*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^
14 + 10404558274560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c^16 + 208091
16549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*c^18 + 13056700579840*a^18
*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(3/4) + (3*(570240000*a^7*b*c^8
+ 2917215*a^2*b^11*c^3 + 49009212*a^3*b^9*c^4 + 303385824*a^4*b^7*c^5 + 879
403392*a^5*b^5*c^6 + 1191801600*a^6*b^3*c^7))/(65536*(b^18 - 262144*a^9*c^9
 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*c^4 - 129024*a^5*b
^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824*a^8*b^2*c^8 - 36*
a*b^16*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 70464307
2000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040
*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799
680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66
059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^
5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/
2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(b^
40*c + 1099511627776*a^20*c^21 - 80*a*b^38*c^2 + 3040*a^2*b^36*c^3 - 72960*
a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 - 15876096*a^5*b^30*c^6 + 158760960*a^6
*b^28*c^7 - 1270087680*a^7*b^26*c^8 + 8255569920*a^8*b^24*c^9 - 44029706240
*a^9*b^22*c^10 + 193730707456*a^10*b^20*c^11 - 704475299840*a^11*b^18*c^12
+ 2113425899520*a^12*b^16*c^13 - 5202279137280*a^13*b^14*c^14 + 10404558274
560*a^14*b^12*c^15 - 16647293239296*a^15*b^10*c...

3.1083 $\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=533

$$\frac{x^{3/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3x^{3/2}(5b^2-4ac+8bcx^2)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\sqrt[4]{c}\left(11b^2+20ac+4b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\cdots\right.}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-b-\sqrt{\cdots}}}$$

```
[Out] 1/4*x^(3/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/16*x^(3/2)*(8*b*c*
x^2-4*a*c+5*b^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/32*c^(1/4)*arctan(2^(1/4)
*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(11*b^2+20*a*c-4*b*(-4*a*c+
b^2)^(1/2))*2^(1/4)/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-3/32*c
^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(11*b
^2+20*a*c-4*b*(-4*a*c+b^2)^(1/2))*2^(1/4)/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^
2)^(1/2))^(1/4)-3/32*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2
)^(1/2))^(1/4))*(11*b^2+20*a*c+4*b*(-4*a*c+b^2)^(1/2))*2^(1/4)/(-4*a*c+b^2)
^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+3/32*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x
^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(11*b^2+20*a*c+4*b*(-4*a*c+b^2)^(1/2)
)*2^(1/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A]
time = 0.96, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1129, 1379, 1514, 1524, 304, 211, 214}

$$-\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{3\sqrt[4]{c}\left(-4b\sqrt{b^2-4ac}+20ac+11b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{3x^{3/2}(-4ac+5b^2+8bcx^2)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{3/2}(2a+bx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{3\sqrt[4]{c}\left(-4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*x^(3/2
)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (
3*c^(1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*
(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^
2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)
])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(
1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b
- Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^2 -
4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1379

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*
p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c,
 d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1]
 && GtQ[m, 2*n - 1]
```

Rule 1514

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m +
 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*
x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b
^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx = 2\text{Subst}\left(\int \frac{x^{10}}{(a+bx^4+cx^8)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\text{Subst}\left(\int \frac{x^2(6a-9bx^4)}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)}{4(b^2-4ac)}$$

$$= \frac{x^{3/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3x^{3/2}(5b^2-4ac+8bcx^2)}{16(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{x^2(3a(7b\ldots}{\ldots}\right.}{16\ldots}$$

$$= \frac{x^{3/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3x^{3/2}(5b^2-4ac+8bcx^2)}{16(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\left(3c\left(11b^2+20a\ldots\right.\right.}{\ldots}$$

$$= \frac{x^{3/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3x^{3/2}(5b^2-4ac+8bcx^2)}{16(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{\left(3\sqrt{c}\left(11b^2+2\ldots\right.\right.}{\ldots}$$

$$= \frac{x^{3/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3x^{3/2}(5b^2-4ac+8bcx^2)}{16(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{3\sqrt[4]{c}\left(11b^2+20\ldots\right.}{16\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.67, size = 339, normalized size = 0.64

$$\frac{1}{64}\left(-\frac{4x^{3/2}(20a^2c+a(7b^2+28bcx^2-12c^2x^4)+bx^2(11b^2+39bcx^2+24c^2x^4))}{(b^2-4ac)^2(a+bx^2+cx^4)^2} + \frac{8\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b^2\log(\sqrt{x}-\#1)-10ac\log(\sqrt{x}-\#1)+bc\log(\sqrt{x}-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{ab^2c-4a^2c^2} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{8b^4\log(\sqrt{x}-\#1)-133ab^2c\log(\sqrt{x}-\#1)+260a^2c^2\log(\sqrt{x}-\#1)+8b^3c\log(\sqrt{x}-\#1)\#1^4-8abc^2\log(\sqrt{x}-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{ac(b^2-4ac)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*x^(3/2)*(20*a^2*c + a*(7*b^2 + 28*b*c*x^2 - 12*c^2*x^4) + b*x^2*(11*b^
2 + 39*b*c*x^2 + 24*c^2*x^4)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) + (8
```

```
*RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log[Sqrt[x] - #1] - 10*a*c*Log[Sqrt[x
] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(a*b^2*c - 4*a
^2*c^2) - RootSum[a + b*#1^4 + c*#1^8 & , (8*b^4*Log[Sqrt[x] - #1] - 133*a*
b^2*c*Log[Sqrt[x] - #1] + 260*a^2*c^2*Log[Sqrt[x] - #1] + 8*b^3*c*Log[Sqrt[
x] - #1]*#1^4 - 8*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(a*
c*(b^2 - 4*a*c)^2))/64
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 244, normalized size = 0.46

method	result
derivativedivides	$\frac{-\frac{a(20ac+7b^2)x^{\frac{3}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}-\frac{b(28ac+11b^2)x^{\frac{7}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}+\frac{3(4ac-13b^2)c\,x^{\frac{11}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}-\frac{3b\,c^2x^{\frac{15}{2}}}{2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\Big(\sum_{\_R=\mathrm{RootOf}(c\_Z^8}$
default	$\frac{-\frac{a(20ac+7b^2)x^{\frac{3}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}-\frac{b(28ac+11b^2)x^{\frac{7}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}+\frac{3(4ac-13b^2)c\,x^{\frac{11}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}-\frac{3b\,c^2x^{\frac{15}{2}}}{2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\Big(\sum_{\_R=\mathrm{RootOf}(c\_Z^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/32*a*(20*a*c+7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)-1/32*b*(28*a*c
+11*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+3/32*(4*a*c-13*b^2)*c/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^(11/2)-3/4*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(
c*x^4+b*x^2+a)^2+3/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((-8*b*c*_R^6+(20*a*c+7
*b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(24*b*c^2*x^(15/2) + 3*(13*b^2*c - 4*a*c^2)*x^(11/2) + (11*b^3 + 28*a
*b*c)*x^(7/2) + (7*a*b^2 + 20*a^2*c)*x^(3/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*
a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3
*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*x^2) - integrate(3/32*(8*b*c*x^(5/2) - (7*b^2 + 20*a
*c)*sqrt(x))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17098 vs. $2(429) = 858$.

time = 140.91, size = 17098, normalized size = 32.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4
*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(sqr
t(1/2)*sqrt(-(2401*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c^2 + 833280*a^3*b^
3*c^3 + 672000*a^4*b*c^4 + (a*b^20 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 768
0*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*
c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9 + 10
48576*a^11*c^10)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c
^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(a^2*b^30 - 60*a^3*b^28*c +
 1680*a^4*b^26*c^2 - 29120*a^5*b^24*c^3 + 349440*a^6*b^22*c^4 - 3075072*a^7
*b^20*c^5 + 20500480*a^8*b^18*c^6 - 105431040*a^9*b^16*c^7 + 421724160*a^10
*b^14*c^8 - 1312030720*a^11*b^12*c^9 + 3148873728*a^12*b^10*c^10 - 57252249
60*a^13*b^8*c^11 + 7633633280*a^14*b^6*c^12 - 7046430720*a^15*b^4*c^13 + 40
26531840*a^16*b^2*c^14 - 1073741824*a^17*c^15)))/(a*b^20 - 40*a^2*b^18*c +
720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258048*a^6*b^10
*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*b^4*c^8 - 262
1440*a^10*b^2*c^9 + 1048576*a^11*c^10)))*arctan((sqrt(1/2)*(9604*b^11 - 776
48*a*b^9*c + 49792*a^2*b^7*c^2 + 710144*a^3*b^5*c^3 - 486400*a^4*b^3*c^4 -
2560000*a^5*b*c^5 + (11*a*b^22 - 420*a^2*b^20*c + 7120*a^3*b^18*c^2 - 70080
*a^4*b^16*c^3 + 437760*a^5*b^14*c^4 - 1763328*a^6*b^12*c^5 + 4300800*a^7*b^
10*c^6 - 4423680*a^8*b^8*c^7 - 6881280*a^9*b^6*c^8 + 30146560*a^10*b^4*c^9
- 40894464*a^11*b^2*c^10 + 20971520*a^12*c^11)*sqrt((5764801*b^8 + 45138800
*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^
4)/(a^2*b^30 - 60*a^3*b^28*c + 1680*a^4*b^26*c^2 - 29120*a^5*b^24*c^3 + 349
440*a^6*b^22*c^4 - 3075072*a^7*b^20*c^5 + 20500480*a^8*b^18*c^6 - 105431040
*a^9*b^16*c^7 + 421724160*a^10*b^14*c^8 - 1312030720*a^11*b^12*c^9 + 314887
3728*a^12*b^10*c^10 - 5725224960*a^13*b^8*c^11 + 7633633280*a^14*b^6*c^12 -
 7046430720*a^15*b^4*c^13 + 4026531840*a^16*b^2*c^14 - 1073741824*a^17*c^15
)))*sqrt(2*(7725384689170640625*b^32*c^2 + 282981388151158200000*a*b^30*c^3
 + 4771356315680228400000*a^2*b^28*c^4 + 49128817241078789760000*a^3*b^26*c
^5 + 345588150973149899136000*a^4*b^24*c^6 + 1760255679328066642821120*a^5*
b^22*c^7 + 6713248089010572194512896*a^6*b^20*c^8 + 19549244243462644098662
400*a^7*b^18*c^9 + 43918980013757300981760000*a^8*b^16*c^10 + 7635946154611
4482176000000*a^9*b^14*c^11 + 102391405638952550400000000*a^10*b^12*c^12 +
104771599798763520000000000*a^11*b^10*c^13 + 80203570069504000000000000*a^1
2*b^8*c^14 + 44410083737600000000000000*a^13*b^6*c^15 + 1677950976000000000
0000000*a^14*b^4*c^16 + 3866624000000000000000000*a^15*b^2*c^17 + 409600000
000000000000000*a^16*c^18)*x - sqrt(1/2)*(3433504306298O625*b^39*c + 639383
```

```
997250120500*a*b^37*c^2 + 1702582978320327600*a^2*b^35*c^3 - 33679909088480
047680*a^3*b^33*c^4 - 226356577432679586816*a^4*b^31*c^5 + 3070408568282459
87328*a^5*b^29*c^6 + 670867909094283436320*a^6*b^27*c^7 + 1318244029442002
7473920*a^7*b^25*c^8 - 692094766373645405388880*a^8*b^23*c^9 - 3378275967658
55207653376*a^9*b^21*c^10 - 1238682216715922094161192*a^10*b^19*c^11 + 24318
1666792442727235584 0*a^11*b^17*c^12 + 6364547618530696101888000*a^12*b^15*c
^13 + 1670048530810955366400000*a^13*b^13*c^14 - 22547190842413547520000000
*a^14*b^11*c^15 - 5515442024506982400000000*a^15*b^9*c^16 - 65861891339059
200000000000*a^16*b^7*c^17 - 4507498971136000000000000*a^17*b^5*c^18 - 169
470853120000000000000000*a^18*b^3*c^19 - 2726297600000000000000000*a^19*b*c^
20 - (14300309480625*a*b^50*c - 350025834154500*a^2*b^48*c^2 + 228323733089
7600*a^3*b^46*c^3 + 11058755739736320*a^4*b^44*c^4 - 187590866560022016*a^5
*b^42*c^5 + 269999460492023808*a^6*b^40*c^6 + 5419159153196974080*a^7*b^38*
c^7 - 20060103804412428288*a^8*b^36*c^8 - 81517089629835689984*a^9*b^34*c^9
 + 483114981234839912448*a^10*b^32*c^10 + 726270796025389121536*a^11*b^30*c
^11 - 6970397383273176104960*a^12*b^28*c^12 - 4001856093795548921856*a^13*b
^26*c^13 + 70422664348663810097152*a^14*b^24*c^14 + 113864965666319777136 64
*a^15*b^22*c^15 - 516502296636516332470272*a^16*b^20*c^16 - 516252615049635
43040*a^17*b^18*c^17 + 2676850158007685356716032*a^18*b^16*c^18 + 118294207
420914997395456*a^19*b^14*c^19 - 9367572054094396574924800*a^20*b^12*c^20 -
 2497949014239792332800000*a^21*b^10*c^21 + 19834918598023839744000000*a^22
*b^8*c^22 + 12864258557188505600000000*a^23*b^6*c^23 - 17798344474624000000
000000*a^24*b^4*c^24 - 2247126889267200000000000*a^25*b^2*c^25 - 687194767
360000000000000*a^26*c^26)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 13638000
0*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(a^2*b^30 - 60*a
^3*b^28*c + 1680*a^4*b^26*c^2 - 29120*a^5*b^24*c^3 + 349440*a^6*b^22*c^4 -
3075072*a^7*b^20*c^5 + 20500480*a^8*b^18*c^6 - 105431040*a^9*b^16*c^7 + 421
724160*a^10*b^14*c^8 - 1312030720*a^11*b^12*c^9 + 3148873728*a^12*b^10*c^10
 - 5725224960*a^13*b^8*c^11 + 7633633280*a^14*b...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B]
time = 7.66, size = 2500, normalized size = 4.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - atan(((((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 161
128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b^2
1*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 196180
3621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 1581647476555776
0*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*a^11*
b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13*b^3*c
^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^
3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16
*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*
c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12
*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (9*x^(1/2)*((81*(2401
*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 132
3600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 143728
4352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 133
27073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^1
0 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*
a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9
400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a
^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320
*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680
*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 19373
0707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b
^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 1
6647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 1958505086976
0*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^1
9)))^(1/4)*(822083584*a*b^26*c^4 - 140737488355328000*a^14*c^17 - 2795084185
6*a^2*b^24*c^5 + 399431958528*a^3*b^22*c^6 - 2968896143360*a^4*b^20*c^7 + 1
0329396346880*a^5*b^18*c^8 + 6262062317568*a^6*b^16*c^9 - 202859895324672*a
^7*b^14*c^10 + 658057709223936*a^8*b^12*c^11 + 346346162749440*a^9*b^10*c^1
2 - 8653156510597120*a^10*b^8*c^13 + 28569710136131584*a^11*b^6*c^14 - 4707
6689854857216*a^12*b^4*c^15 + 40250921669623808*a^13*b^2*c^16))/(4194304*(b
^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*
a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^
10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^
```

10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a^14*b^14*c^13 + 10404558274560*a^15*b^12*c^14 - 16647293239296*a^16*b^10*c^15 + 20809116549120*a^17*b^8*c^16 - 19585050869760*a^18*b^6*c^17 + 13056700579840*a^19*b^4*c^18 - 5497558138880*a^20*b^2*c^19)))^(3/4) - (9*x^(1/2)*(200930625*a*b^13*c^5 - 3110400000*a^7*b*c^11 + 2093250600*a^2*b^11*c^6 + 7523454960*a^3*b^9*c^7 + 10328580864*a^4*b^7*c^8 + 2354261760*a^5*b^5*c^9 - 5453568000*a^6*b^3*c^10))/(4194304*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c)))*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^7*b^28*c^6 - 1270087680*a^8*b^26*c^7 + 8255569920*a^9*b^24*c^8 - 44029706240*a^10*b^22*c^9 + 193730707456*a^11*b^20*c^10 - 704475299840*a^12*b^18*c^11 + 2113425899520*a^13*b^16*c^12 - 5202279137280*a...

3.1084 $\int \frac{x^{7/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=533

$$\frac{\sqrt{x}\,(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{\sqrt{x}\,(13b^2-4ac+24bcx^2)}{16\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{c^{3/4}\left(41b^2+28ac+36b\sqrt{b^2-4ac}\right)\tan^{-1}\Big(}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(-b-\sqrt{b^2\right.}}$$

```
[Out] -1/32*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))
*(41*b^2+28*a*c-36*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)^(5/2)/(-b+(-4
*a*c+b^2)^(1/2))^(3/4)-1/32*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4
*a*c+b^2)^(1/2))^(1/4))*(41*b^2+28*a*c-36*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4
*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/32*c^(3/4)*arctan(2^(1/4)*c
^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(41*b^2+28*a*c+36*b*(-4*a*c+b
^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/32*c^
(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(41*b^
2+28*a*c+36*b*(-4*a*c+b^2)^(1/2))*2^(3/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^
2)^(1/2))^(3/4)+1/4*(b*x^2+2*a)*x^(1/2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/16
*(24*b*c*x^2-4*a*c+13*b^2)*x^(1/2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)
```

Rubi [A]
time = 0.97, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$,
Rules used = {1129, 1379, 1444, 1436, 218, 214, 211}

$$\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\,(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{\sqrt{x}\,(-4ac+13b^2+24bcx^2)}{16\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (Sqrt[x]*
(13*b^2 - 4*a*c + 24*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
c^(3/4)*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(
-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2
- 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)
*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*
2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1379

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2*
p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c,
 d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1]
 && GtQ[m, 2*n - 1]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
```

```
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^3}\,dx = 2\text{Subst}\left(\int \frac{x^8}{(a+bx^4+cx^8)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\,(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{\text{Subst}\left(\int \frac{2a-11bx^4}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)}{4\,(b^2-4ac)}$$

$$= \frac{\sqrt{x}\,(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{\sqrt{x}\,(13b^2-4ac+24bcx^2)}{16\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{a(5b^2+28}{a+b}\right.}{16a\,(}$$

$$= \frac{\sqrt{x}\,(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{\sqrt{x}\,(13b^2-4ac+24bcx^2)}{16\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{\left(c\left(41b^2+28ac\right.\right.}{}$$

$$= \frac{\sqrt{x}\,(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{\sqrt{x}\,(13b^2-4ac+24bcx^2)}{16\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{\left(c\left(41b^2+28ac\right.\right.}{}$$

$$= \frac{\sqrt{x}\,(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{\sqrt{x}\,(13b^2-4ac+24bcx^2)}{16\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{c^{3/4}\left(41b^2+28a\right.}{16\sqrt[4]{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.57, size = 345, normalized size = 0.65

$$\frac{1}{64}\left(-\frac{4\sqrt{x}\,(28a^2c+a(5b^2+36bcx^2-4c^2x^4)+bx^2(9b^2+37bcx^2+24c^2x^4))}{(b^2-4ac)^2\,(a+bx^2+cx^4)^2}+\frac{8\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{3b^2\log(\sqrt{x}-\#1)-14ac\log(\sqrt{x}-\#1)+3bc\log(\sqrt{x}-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{ab^2c-4a^2c^2}-\frac{3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{8b^4\log(\sqrt{x}-\#1)-71ab^2c\log(\sqrt{x}-\#1)+140a^2c^2\log(\sqrt{x}-\#1)+8b^3c\log(\sqrt{x}-\#1)\#1^4-8abc^2\log(\sqrt{x}-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{ac\,(b^2-4ac)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*Sqrt[x]*(28*a^2*c + a*(5*b^2 + 36*b*c*x^2 - 4*c^2*x^4) + b*x^2*(9*b^2
+ 37*b*c*x^2 + 24*c^2*x^4)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) + (8*R
```

```
ootSum[a + b*#1^4 + c*#1^8 & , (3*b^2*Log[Sqrt[x] - #1] - 14*a*c*Log[Sqrt[x
] - #1] + 3*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(a*b^2*c -
 4*a^2*c^2) - (3*RootSum[a + b*#1^4 + c*#1^8 & , (8*b^4*Log[Sqrt[x] - #1] -
 71*a*b^2*c*Log[Sqrt[x] - #1] + 140*a^2*c^2*Log[Sqrt[x] - #1] + 8*b^3*c*Log
[Sqrt[x] - #1]*#1^4 - 8*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7)
 & ])/(a*c*(b^2 - 4*a*c)^2))/64
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.09, size = 237, normalized size = 0.44

method	result
derivativedivides	$\frac{-\frac{a\left(28ac+5b^2\right)\sqrt{x}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{9b\left(4ac+b^2\right)x^{\frac{5}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{2c\left(4ac-37b^2\right)x^{\frac{9}{2}}}{512a^2c^2-256a\,b^2c+32b^4}-\frac{3b\,c^2x^{\frac{13}{2}}}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_\right.}}{}$
default	$\frac{-\frac{a\left(28ac+5b^2\right)\sqrt{x}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{9b\left(4ac+b^2\right)x^{\frac{5}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{2c\left(4ac-37b^2\right)x^{\frac{9}{2}}}{512a^2c^2-256a\,b^2c+32b^4}-\frac{3b\,c^2x^{\frac{13}{2}}}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/32*a*(28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-9/32*b*(4*a*c+
b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/32*c*(4*a*c-37*b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(9/2)-3/4*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*x^4
+b*x^2+a)^2+1/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((-72*_R^4*b*c+28*a*c+5*b^2)
/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^2*c^2 + 28*a*c^3)*x^(17/2) + 2*(5*b^3*c + 16*a*b*c^2)*x^(13/2) +
 (5*b^4 + a*b^2*c + 60*a^2*c^2)*x^(9/2) + (a*b^3 + 20*a^2*b*c)*x^(5/2))/((a
*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5
*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*
c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) - integ
rate(1/32*((5*b^2*c + 28*a*c^2)*x^(7/2) + 5*(b^3 + 20*a*b*c)*x^(3/2))/(a^2*
b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x^4
 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17577 vs. 2(429) = 858.
time = 62.02, size = 17577, normalized size = 32.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*
c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(sqrt
(1/2)*sqrt(-(625*b^11 + 48125*a*b^9*c + 1386000*a^2*b^7*c^2 + 52776416*a^3*
b^5*c^3 + 106839040*a^4*b^3*c^4 - 14488320*a^5*b*c^5 + (a^3*b^20 - 40*a^4*b
^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*
a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4
*c^8 - 2621440*a^12*b^2*c^9 + 1048576*a^13*c^10)*sqrt((390625*b^12 + 679687
50*a*b^10*c + 5345390625*a^2*b^8*c^2 + 207773230000*a^3*b^6*c^3 + 364774326
0000*a^4*b^4*c^4 - 146825952000*a^5*b^2*c^5 + 1475789056*a^6*c^6)/(a^6*b^30
 - 60*a^7*b^28*c + 1680*a^8*b^26*c^2 - 29120*a^9*b^24*c^3 + 349440*a^10*b^2
2*c^4 - 3075072*a^11*b^20*c^5 + 20500480*a^12*b^18*c^6 - 105431040*a^13*b^1
6*c^7 + 421724160*a^14*b^14*c^8 - 1312030720*a^15*b^12*c^9 + 3148873728*a^1
6*b^10*c^10 - 5725224960*a^17*b^8*c^11 + 7633633280*a^18*b^6*c^12 - 7046430
720*a^19*b^4*c^13 + 4026531840*a^20*b^2*c^14 - 1073741824*a^21*c^15)))/(a^3
*b^20 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^
12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 +
2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9 + 1048576*a^13*c^10)))*arctan(-
1/2*(sqrt(1/2)*(78125*b^29 + 8984375*a*b^27*c + 126057500*a^2*b^25*c^2 - 11
728892000*a^3*b^23*c^3 - 337997264000*a^4*b^21*c^4 + 16158402819328*a^5*b^1
9*c^5 - 250796782021632*a^6*b^17*c^6 + 2135157383479296*a^7*b^15*c^7 - 1132
0551861780480*a^8*b^13*c^8 + 38910774658269184*a^9*b^11*c^9 - 8628777086694
1952*a^10*b^9*c^10 + 117618334863720448*a^11*b^7*c^11 - 87309254272221184*a
^12*b^5*c^12 + 25678828902809600*a^13*b^3*c^13 - 481612735184896*a^14*b*c^1
4 - (125*a^3*b^38 - 250*a^4*b^36*c + 371808*a^5*b^34*c^2 - 26229792*a^6*b^3
2*c^3 + 772832256*a^7*b^30*c^4 - 13482362880*a^8*b^28*c^5 + 158496522240*a^
9*b^26*c^6 - 1335540350976*a^10*b^24*c^7 + 8313808748544*a^11*b^22*c^8 - 38
599981268992*a^12*b^20*c^9 + 132159729827840*a^13*b^18*c^10 - 3182480798515
20*a^14*b^16*c^11 + 457413413044224*a^15*b^14*c^12 - 34096268967936*a^16*b^
12*c^13 - 1579740511076352*a^17*b^10*c^14 + 4004015473950720*a^18*b^8*c^15
 - 5258908281077760*a^19*b^6*c^16 + 3850859087659008*a^20*b^4*c^17 - 1292063
601590272*a^21*b^2*c^18 + 47141561040896*a^22*c^19)*sqrt((390625*b^12 + 679
68750*a*b^10*c + 5345390625*a^2*b^8*c^2 + 207773230000*a^3*b^6*c^3 + 364774
3260000*a^4*b^4*c^4 - 146825952000*a^5*b^2*c^5 + 1475789056*a^6*c^6)/(a^6*b
^30 - 60*a^7*b^28*c + 1680*a^8*b^26*c^2 - 29120*a^9*b^24*c^3 + 349440*a^10*
b^22*c^4 - 3075072*a^11*b^20*c^5 + 20500480*a^12*b^18*c^6 - 105431040*a^13*
```

```
b^16*c^7 + 421724160*a^14*b^14*c^8 - 1312030720*a^15*b^12*c^9 + 3148873728*
a^16*b^10*c^10 - 5725224960*a^17*b^8*c^11 + 7633633280*a^18*b^6*c^12 - 7046
430720*a^19*b^4*c^13 + 4026531840*a^20*b^2*c^14 - 1073741824*a^21*c^15)))*s
qrt((57900390625*b^20*c^2 + 12324511718750*a*b^18*c^3 + 1205884775390625*a^
2*b^16*c^4 + 65432990485500000*a^3*b^14*c^5 + 2040478510359000000*a^4*b^12*
c^6 + 32800938614539200000*a^5*b^10*c^7 + 207889507133921625600*a^6*b^8*c^8
 + 346156698905880576000*a^7*b^6*c^9 + 6035004398894284800*a^8*b^4*c^10 - 729
38453798748160*a^9*b^2*c^11 + 907102598004736*a^10*c^12)*x + 1/2*sqrt(1/2)*
(9765625*b^28 + 1875000000*a*b^26*c + 164073828125*a^2*b^24*c^2 + 751246278
1250*a^3*b^22*c^3 + 172640593125000*a^4*b^20*c^4 + 833591286220000*a^5*b^18
*c^5 - 25244646454473600*a^6*b^16*c^6 - 247207744997452800*a^7*b^14*c^7 + 4
921285802233810944*a^8*b^12*c^8 - 27172264471646322688*a^9*b^10*c^9 + 66112
218162983731200*a^10*b^8*c^10 - 64377573435729510440*a^11*b^6*c^11 + 837800
4190477680640*a^12*b^4*c^12 - 260810320323280896*a^13*b^2*c^13 + 2369574133
563392*a^14*c^14 - (15625*a^3*b^37 + 1171875*a^4*b^35*c + 21472500*a^5*b^33
*c^2 - 1105910400*a^6*b^31*c^3 - 5996659200*a^7*b^29*c^4 + 651559609344*a^8
*b^27*c^5 - 11564016242688*a^9*b^25*c^6 + 104479260966912*a^10*b^23*c^7 - 4
96723697074176*a^11*b^21*c^8 + 361399517839360*a^12*b^19*c^9 + 121012429683
42528*a^13*b^17*c^10 - 94903845745852416*a^14*b^15*c^11 + 40202208140486246
4*a^15*b^13*c^12 - 1116052372473249792*a^16*b^11*c^13 + 2095432170272194560
*a^17*b^9*c^14 - 2587208019729186816*a^18*b^7*c^15 + 1913685844816822272*a^
19*b^5*c^16 - 655401757282664448*a^20*b^3*c^17 + 12415908639145984*a^21*b*c
^18)*sqrt((390625*b^12 + 67968750*a*b^10*c + 5345390625*a^2*b^8*c^2 + 20777
3230000*a^3*b^6*c^3 + 3647743260000*a^4*b^4*c^4 - 146825952000*a^5*b^2*c^5
+ 1475789056*a^6*c^6)/(a^6*b^30 - 60*a^7*b^28*c + 1680*a^8*b^26*c^2 - 29120
*a^9*b^24*c^3 + 349440*a^10*b^22*c^4 - 3075072*a^11*b^20*c^5 + 20500480*a^1
2*b^18*c^6 - 105431040*a^13*b^16*c^7 + 421724160*a^14*b^14*c^8 - 1312030720
*a^15*b^12*c^9 + 3148873728*a^16*b^10*c^10 - 5725224960*a^17*b^8*c^11 + 763
3633280*a^18*b^6*c^12 - 7046430720*a^19*b^4*c^13 + 4026531840*a^20*b^2*c^14
 - 1073741824*a^21*c^15)))*sqrt(-(625*b^11 + 48125*a*b^9*c + 1386000*a^2*b^
7*c^2 + 52776416*a^3*b^5*c^3 + 106839040*a^4*b^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B]
time = 8.38, size = 2500, normalized size = 4.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 3520856
800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536*(
b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^
10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589
824*a^8*b^2*c^8 - 36*a*b^16*c)) + ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 6
25*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^
25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188833280*
a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14
462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 70455242260480*a
^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^13*b^5*c^1
3 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2)
 - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a
*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 1099511627776*a^23*
c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7
*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 - 1270087680*a^1
0*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^22*c^9 + 1937307
07456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 2113425899520*a^15*b^1
6*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^17*b^12*c^14 - 166
47293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^16 - 19585050869760*
a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)
))^(1/4)*(83886080*a*b^23*c^4 + 1759218604441600*a^12*b*c^15 - 1677721600*a
^2*b^21*c^5 - 6710886400*a^3*b^19*c^6 + 563714457600*a^4*b^17*c^7 - 8375186
227200*a^5*b^15*c^8 + 68547678044160*a^6*b^13*c^9 - 360777252864000*a^7*b^1
1*c^10 + 1278182267289600*a^8*b^9*c^11 - 3051144767078400*a^9*b^7*c^12 + 47
27899999436800*a^10*b^5*c^13 - 4310085580881920*a^11*b^3*c^14))/(65536*(b^1
8 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4*b^10*
c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 + 589824
*a^8*b^2*c^8 - 36*a*b^16*c)) - (x^(1/2)*(209715200*b^27*c^4 - 629145600*a*b
^25*c^5 - 91620104919318528*a^13*b*c^17 - 94623498240*a^2*b^23*c^6 + 129842
2300672*a^3*b^21*c^7 + 1803886264320*a^4*b^19*c^8 - 197235635650560*a^5*b^1
7*c^9 + 2330621053501440*a^6*b^15*c^10 - 15146459867381760*a^7*b^13*c^11 +
63613894492422144*a^8*b^11*c^12 - 180146733873889280*a^9*b^9*c^13 + 3426518
```

03680112640*a^10*b^7*c^14 - 419309754368655360*a^11*b^5*c^15 + 296956100429
742080*a^12*b^3*c^16))/(2097152*(b^24 + 16777216*a^12*c^12 + 1056*a^2*b^20*
c^2 - 14080*a^3*b^18*c^3 + 126720*a^4*b^16*c^4 - 811008*a^5*b^14*c^5 + 3784
704*a^6*b^12*c^6 - 12976128*a^7*b^10*c^7 + 32440320*a^8*b^8*c^8 - 57671680*
a^9*b^6*c^9 + 69206016*a^10*b^4*c^10 - 50331648*a^11*b^2*c^11 - 48*a*b^22*c
)))*((625*b^6*(-(4*a*c - b^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*
c^15 + 89000*a^2*b^27*c^2 - 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4
 - 25492409600*a^5*b^21*c^5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7
*b^17*c^7 + 6664504147968*a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163
326443520*a^10*b^11*c^10 + 70455242260480*a^11*b^9*c^11 - 206669464207360*a
^12*b^7*c^12 + 267459844112384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^1
4 - 38416*a^3*c^3*(-(4*a*c - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*
b^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2)
)/(33554432*(a^3*b^40 + 1099511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*
b^36*c^2 - 72960*a^6*b^34*c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^
5 + 158760960*a^9*b^28*c^6 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^2
4*c^8 - 44029706240*a^12*b^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299
840*a^14*b^18*c^11 + 2113425899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14
*c^13 + 10404558274560*a^17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 208
09116549120*a^19*b^8*c^16 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a
^21*b^4*c^18 - 5497558138880*a^22*b^2*c^19)))^(3/4))*((625*b^6*(-(4*a*c - b
^2)^25)^(1/2) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2
- 27186416*a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^
5 + 265188833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*
a^8*b^15*c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 +
 70455242260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 26745984411
2384*a^13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c
 - b^2)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25
)^(1/2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 10
99511627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*
c^3 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6
 - 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b
^22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 21134
25899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^1
7*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 2...

3.1085 $\int \frac{x^{5/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=594

$$-\frac{x^{3/2}(b+2cx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x^{3/2}(b(b^2+4ac)+c(b^2+12ac)\,x^2)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt[4]{c}\left(b^2+12ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{}{\sqrt{}}\right.}{32\,2^{3/4}a\left(b^2-4a\right.}$$

```
[Out] -1/4*x^(3/2)*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/16*x^(3/2)*(b*(4*
a*c+b^2)+c*(12*a*c+b^2)*x^2)/a/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/64*c^(1/4)*
arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2+12*a*c-b
^3/(-4*a*c+b^2)^(1/2)+68*a*b*c/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)^2
/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-3/64*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)
/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2+12*a*c-b^3/(-4*a*c+b^2)^(1/2)+68*a*b*c
/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)^2/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
+3/64*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))
*(b^3-68*a*b*c+(12*a*c+b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-4*a*c+b^2)^(5/2
)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-3/64*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2
)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b^3-68*a*b*c+(12*a*c+b^2)*(-4*a*c+b^2)^(1
/2))*2^(1/4)/a/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A]
time = 1.59, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1129, 1378, 1514, 1524, 304, 211, 214}

$$\frac{3\sqrt[4]{c}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{3\sqrt[4]{c}\left(\sqrt{b^2-4ac}\,(12ac+b^2)-68abc+b^3\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{3x^{3/2}(cx^2(12ac+b^2)+b(4ac+b^2))}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^{3/2}(b+2cx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\sqrt[4]{c}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{3\sqrt[4]{c}\left(\sqrt{b^2-4ac}\,(12ac+b^2)-68abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x^(3/2)*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(
3/2)*(b*(b^2 + 4*a*c) + c*(b^2 + 12*a*c)*x^2))/(16*a*(b^2 - 4*a*c)^2*(a + b
*x^2 + c*x^4)) + (3*c^(1/4)*(b^2 + 12*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*
a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
 + (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 12*a*c))*ArcTan[(2
^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2
 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(b^2 + 12*a*c
- b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^
(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)
^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2
```

```
- 4*a*c]*(b^2 + 12*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 -
 4*a*c])^(1/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])
^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1378

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*((a + b*x^n + c*
x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1514

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m +
 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*
x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b
```

```
^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^3}\,dx &= 2\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4+cx^8\right)^3}\,dx, x, \sqrt{x}\right)\\
&= -\frac{x^{3/2}(b+2cx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3b-18cx^4\right)}{\left(a+bx^4+cx^8\right)^2}\,dx, x, \sqrt{x}\right)}{4\left(b^2-4ac\right)}\\
&= -\frac{x^{3/2}(b+2cx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b(b^2+4ac)+c(b^2+12ac)\,x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \text{Subst}\Big(\int\\
&= -\frac{x^{3/2}(b+2cx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b(b^2+4ac)+c(b^2+12ac)\,x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \Big(3c\Big(b^2\\
&= -\frac{x^{3/2}(b+2cx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b(b^2+4ac)+c(b^2+12ac)\,x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \Big(3\sqrt{c}\,\Big(\\
&= -\frac{x^{3/2}(b+2cx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}\left(b(b^2+4ac)+c(b^2+12ac)\,x^2\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + 3\sqrt[4]{c}\,\Big(b
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.39, size = 214, normalized size = 0.36

$$
\frac{\frac{4x^{3/2}\left(3b^2x^2\left(b+cx^2\right)^2+4a^2c\left(7b+17cx^2\right)+a\left(-b^3+7b^2cx^2+48bc^2x^4+36c^3x^6\right)\right)}{\left(a+bx^2+cx^4\right)^2}+3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^3\log\left(\sqrt{x}-\#1\right)-28abc\log\left(\sqrt{x}-\#1\right)+b^2c\log\left(\sqrt{x}-\#1\right)\#1^4+12ac^2\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{64a\left(b^2-4ac\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*x^(3/2)*(3*b^2*x^2*(b + c*x^2)^2 + 4*a^2*c*(7*b + 17*c*x^2) + a*(-b^3 +
 7*b^2*c*x^2 + 48*b*c^2*x^4 + 36*c^3*x^6)))/(a + b*x^2 + c*x^4)^2 + 3*RootS
um[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 28*a*b*c*Log[Sqrt[x] -
#1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 12*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1
 + 2*c*#1^5) & ])/(64*a*(b^2 - 4*a*c)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 281, normalized size = 0.47

method	result
derivativedivides	$\frac{\frac{2b\left(28ac-b^2\right)x^{\frac{3}{2}}}{512a^2c^2-256a\,b^2c+32b^4}+\frac{\left(68a^2c^2+7a\,b^2c+3b^4\right)x^{\frac{7}{2}}}{16a\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{3cb\left(8ac+b^2\right)x^{\frac{11}{2}}}{8a\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{3c^2\left(12ac+b^2\right)x^{\frac{15}{2}}}{16a\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}-\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_\right.}\right.}{}$
default	$\frac{\frac{2b\left(28ac-b^2\right)x^{\frac{3}{2}}}{512a^2c^2-256a\,b^2c+32b^4}+\frac{\left(68a^2c^2+7a\,b^2c+3b^4\right)x^{\frac{7}{2}}}{16a\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{3cb\left(8ac+b^2\right)x^{\frac{11}{2}}}{8a\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{3c^2\left(12ac+b^2\right)x^{\frac{15}{2}}}{16a\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}-\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*b*(28*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)+1/32*(68*a^2*c^2+
7*a*b^2*c+3*b^4)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+3/16*c/a*b*(8*a*c+b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(11/2)+3/32*c^2*(12*a*c+b^2)/a/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(15/2))/(c*x^4+b*x^2+a)^2-3/64/a/(16*a^2*c^2-8*a*b^2*c+b^4)
*sum((c*(-12*a*c-b^2)*_R^6+b*(28*a*c-b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2
)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^2*c^2 + 12*a*c^3)*x^(15/2) + 6*(b^3*c + 8*a*b*c^2)*x^(11/2) + (3
*b^4 + 7*a*b^2*c + 68*a^2*c^2)*x^(7/2) - (a*b^3 - 28*a^2*b*c)*x^(3/2))/((a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c
 + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) + integr
ate(3/32*((b^2*c + 12*a*c^2)*x^(5/2) + (b^3 - 28*a*b*c)*sqrt(x))/(a^2*b^4 -
 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x^4 + (a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [**B**]
time = 8.02, size = 2500, normalized size = 4.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*x^(11/2)*(b^3*c + 8*a*b*c^2))/(8*(a*b^4 + 16*a^3*c^2 - 8*a^2*b^2*c)) -
(x^(3/2)*(b^3 - 28*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(7/2)*(
3*b^4 + 68*a^2*c^2 + 7*a*b^2*c))/(16*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3
*c^2*x^(15/2)*(12*a*c + b^2))/(16*(a*b^4 + 16*a^3*c^2 - 8*a^2*b^2*c)))/(x^4
*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - atan(((((27*(3799
912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14019461
120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^22*c^8 -
 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 356438262153
```

2160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362027008*a^9*b^12*c^13 + 56529603635707904*a^10*b^10*c^14 - 33767651356442624*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^13*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 268435456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 + 256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 1049624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^12 - 939524096*a^15*b^2*c^13)) - (9*x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 704475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(1/4)*(5066549580791808*a^15*c^18 + 16777216*a*b^28*c^4 - 1677721600*a^2*b^26*c^5 + 67947724800*a^3*b^24*c^6 - 1491964264448*a^4*b^22*c^7 + 20440823103488*a^5*b^20*c^8 - 188712273051648*a^6*b^18*c^9 + 1225740716605440*a^7*b^16*c^10 - 5727081191178240*a^8*b^14*c^11 + 19380541706993664*a^9*b^12*c^12 - 47173446878101504*a^10*b^10*c^13 + 80798711478747136*a^11*b^8*c^14 - 93414507895848960*a^12*b^6*c^15 + 67905838131445760*a^13*b^4*c^16 - 27584547717644288*a^14*b^2*c^17))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 811008*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 13151174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 4213765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^5*b^40 + 1099511627776*a^25*c^20 - 80*a^6*b^38*c + 3040*a^7*b^36*c^2 - 72960*a^8*b^34*c^3 + 1240320*a^9*b^32*c^4 - 15876096*a^10*

b^30*c^5 + 158760960*a^11*b^28*c^6 - 1270087680*a^12*b^26*c^7 + 8255569920*
a^13*b^24*c^8 - 44029706240*a^14*b^22*c^9 + 193730707456*a^15*b^20*c^10 - 7
04475299840*a^16*b^18*c^11 + 2113425899520*a^17*b^16*c^12 - 5202279137280*a
^18*b^14*c^13 + 10404558274560*a^19*b^12*c^14 - 16647293239296*a^20*b^10*c^
15 + 20809116549120*a^21*b^8*c^16 - 19585050869760*a^22*b^6*c^17 + 13056700
579840*a^23*b^4*c^18 - 5497558138880*a^24*b^2*c^19)))^(3/4) + (9*x^(1/2)*(2
982998016*a^6*b*c^14 - 173138472*a*b^11*c^9 - 123201*b^13*c^8 + 10695194640
*a^2*b^9*c^10 - 166726460160*a^3*b^7*c^11 + 147581948160*a^4*b^5*c^12 + 449
37566208*a^5*b^3*c^13))/(4194304*(a^2*b^24 + 16777216*a^14*c^12 - 48*a^3*b^
22*c + 1056*a^4*b^20*c^2 - 14080*a^5*b^18*c^3 + 126720*a^6*b^16*c^4 - 81100
8*a^7*b^14*c^5 + 3784704*a^8*b^12*c^6 - 12976128*a^9*b^10*c^7 + 32440320*a^
10*b^8*c^8 - 57671680*a^11*b^6*c^9 + 69206016*a^12*b^4*c^10 - 50331648*a^13
*b^2*c^11)))*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 471104225280*a^1
6*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27...

3.1086 $\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=594

$$-\frac{\sqrt{x}\,(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{\sqrt{x}\,(b(b^2+20ac)+c(b^2+44ac)\,x^2)}{16a\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{3c^{3/4}\left(b^2+44ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{\ldots}{\sqrt{b\ldots}}\right.}{32\sqrt[4]{2}\,a\,(b^2-4ac)^2\ldots}$$

```
[Out] -3/64*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))
*(b^2+44*a*c-b^3/(-4*a*c+b^2)^(1/2)+68*a*b*c/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/
(-4*a*c+b^2)^2/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3/64*c^(3/4)*arctanh(2^(1/4)*c
^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b^2+44*a*c-b^3/(-4*a*c+b^2)^
(1/2)+68*a*b*c/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4*a*c+b^2)^2/(-b-(-4*a*c+b^2
)^(1/2))^(3/4)-3/64*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*(b^3-68*a*b*c+(44*a*c+b^2)*(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4
*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-3/64*c^(3/4)*arctanh(2^(1/4)*
c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b^3-68*a*b*c+(44*a*c+b^2)*(
-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/
4)-1/4*(2*c*x^2+b)*x^(1/2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*(b*(20*a*c+b
^2)+c*(44*a*c+b^2)*x^2)*x^(1/2)/a/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)
```

Rubi [A]
time = 1.77, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1129, 1378, 1444, 1436, 218, 214, 211}

$$-\frac{3c^{3/4}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a\,(b^2-4ac)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(\sqrt{b^2-4ac}\,(44ac+b^2)-68abc+b^3\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a\,(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\sqrt{x}\,(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{\sqrt{x}\,(cx^2(44ac+b^2)+b(20ac+b^2))}{16a\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{3c^{3/4}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a\,(b^2-4ac)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(\sqrt{b^2-4ac}\,(44ac+b^2)-68abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a\,(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(Sqrt[x]*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[
x]*(b*(b^2 + 20*a*c) + c*(b^2 + 44*a*c)*x^2))/(16*a*(b^2 - 4*a*c)^2*(a + b*
x^2 + c*x^4)) - (3*c^(3/4)*(b^2 + 44*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a
*c])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
- (3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 44*a*c))*ArcTan[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*(b^2
- 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^2 + 44*a*c -
 b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^
2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 -
```

```
 4*a*c]*(b^2 + 44*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 -
4*a*c])^(1/4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^
(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1378

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*((a + b*x^n + c*
x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] - Dist[d^n/(n*(p + 1)*(b^2
- 4*a*c)), Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^n
)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
1] && LeQ[m, 2*n - 1]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx = 2\text{Subst}\left(\int \frac{x^4}{(a+bx^4+cx^8)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}\,(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\text{Subst}\left(\int \frac{b-22cx^4}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)}{4\,(b^2-4ac)}$$

$$= -\frac{\sqrt{x}\,(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\sqrt{x}\,(b(b^2+20ac)+c(b^2+44ac)\,x^2)}{16a\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{\text{Subst}\Big(\int}{}$$

$$= -\frac{\sqrt{x}\,(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\sqrt{x}\,(b(b^2+20ac)+c(b^2+44ac)\,x^2)}{16a\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{\Big(3c\Big(b^2-}{}$$

$$= -\frac{\sqrt{x}\,(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\sqrt{x}\,(b(b^2+20ac)+c(b^2+44ac)\,x^2)}{16a\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{\Big(3c\Big(b^2-}{}$$

$$= -\frac{\sqrt{x}\,(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\sqrt{x}\,(b(b^2+20ac)+c(b^2+44ac)\,x^2)}{16a\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{3c^{3/4}\Big(b^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.33, size = 215, normalized size = 0.36

$$\frac{\frac{4\sqrt{x}\,\left(b^2x^2(b+cx^2)^2+4a^2c(9b+19cx^2)+a(-3b^3+13b^2cx^2+64bc^2x^4+44c^3x^6)\right)}{(a+bx^2+cx^4)^2} + 3\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b^3\log\left(\sqrt{x}-\#1\right)-12abc\log\left(\sqrt{x}-\#1\right)+b^2c\log\left(\sqrt{x}-\#1\right)\#1^4+44ac^2\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{64a\,(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*Sqrt[x]*(b^2*x^2*(b + c*x^2)^2 + 4*a^2*c*(9*b + 19*c*x^2) + a*(-3*b^3 +
 13*b^2*c*x^2 + 64*b*c^2*x^4 + 44*c^3*x^6)))/(a + b*x^2 + c*x^4)^2 + 3*Root
Sum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 12*a*b*c*Log[Sqrt[x] -
 #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 44*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#
1^3 + 2*c*#1^7) & ])/(64*a*(b^2 - 4*a*c)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.09, size = 270, normalized size = 0.45

method	result
derivativedivides	$\frac{\frac{3b\left(12ac-b^{2}\right)\sqrt{x}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(76a^{2}c^{2}+13a\,b^{2}c+b^{4}\right)x^{\frac{5}{2}}}{16a\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{cb\left(32ac+b^{2}\right)x^{\frac{9}{2}}}{8a\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c^{2}\left(44ac+b^{2}\right)x^{\frac{13}{2}}}{16a\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{3\Bigg(\sum_{\_R=\mathrm{RootOf}(c}$
default	$\frac{\frac{3b\left(12ac-b^{2}\right)\sqrt{x}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(76a^{2}c^{2}+13a\,b^{2}c+b^{4}\right)x^{\frac{5}{2}}}{16a\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{cb\left(32ac+b^{2}\right)x^{\frac{9}{2}}}{8a\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c^{2}\left(44ac+b^{2}\right)x^{\frac{13}{2}}}{16a\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{3\Bigg(\sum_{\_R=\mathrm{RootOf}(c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/32*b*(12*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)+1/32*(76*a^2*c^2+
13*a*b^2*c+b^4)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/16*c/a*b*(32*a*c+b^2
)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)+1/32*c^2*(44*a*c+b^2)/a/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64/a/(16*a^2*c^2-8*a*b^2*c+b^4)*
sum((c*(44*a*c+b^2)*_R^4-12*a*b*c+b^3)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=
RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^3*c^2 - 12*a*b*c^3)*x^(17/2) + (6*b^4*c - 71*a*b^2*c^2 + 44*a^2*
c^3)*x^(13/2) + (3*b^5 - 28*a*b^3*c - 8*a^2*b*c^2)*x^(9/2) + (7*a*b^4 - 59*
a^2*b^2*c + 76*a^3*c^2)*x^(5/2))/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 +
 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + integrate(-3/32*((b^3*c - 12*a*b*c^2)*
x^(7/2) + (b^4 - 13*a*b^2*c - 44*a^2*c^2)*x^(3/2))/(a^3*b^4 - 8*a^4*b^2*c +
```

```
16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a
^3*b^3*c + 16*a^4*b*c^2)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20151 vs. $2(492) = 984$.
time = 157.95, size = 20151, normalized size = 33.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6
 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)
*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^15 - 55*a*b^13*c + 990*a^2*b^11*c^2 - 4455*a^
3*b^9*c^3 - 37840*a^4*b^7*c^4 + 155232*a^5*b^5*c^5 + 1281280*a^6*b^3*c^6 +
11925760*a^7*b*c^7 + (a^7*b^20 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^
10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8*
c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9 +
1048576*a^17*c^10)*sqrt((b^20 - 90*a*b^18*c + 3045*a^2*b^16*c^2 - 41870*a^3
*b^14*c^3 + 10990*a^4*b^12*c^4 + 4635988*a^5*b^10*c^5 - 9414335*a^6*b^8*c^6
 - 375477520*a^7*b^6*c^7 + 203802720*a^8*b^4*c^8 + 15873186560*a^9*b^2*c^9
+ 54875873536*a^10*c^10)/(a^14*b^30 - 60*a^15*b^28*c + 1680*a^16*b^26*c^2 -
 29120*a^17*b^24*c^3 + 349440*a^18*b^22*c^4 - 3075072*a^19*b^20*c^5 + 20500
480*a^20*b^18*c^6 - 105431040*a^21*b^16*c^7 + 421724160*a^22*b^14*c^8 - 131
2030720*a^23*b^12*c^9 + 3148873728*a^24*b^10*c^10 - 5725224960*a^25*b^8*c^1
1 + 7633633280*a^26*b^6*c^12 - 7046430720*a^27*b^4*c^13 + 4026531840*a^28*b
^2*c^14 - 1073741824*a^29*c^15)))/(a^7*b^20 - 40*a^8*b^18*c + 720*a^9*b^16*
c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860
160*a^13*b^8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^
16*b^2*c^9 + 1048576*a^17*c^10)))*arctan(1/4*(2*sqrt(1/2)*(55*b^50*c^2 - 96
15*a*b^48*c^3 + 753144*a^2*b^46*c^4 - 34618329*a^3*b^44*c^5 + 1022996351*a^
4*b^42*c^6 - 19829625848*a^5*b^40*c^7 + 239371021765*a^6*b^38*c^8 - 1349373
222101*a^7*b^36*c^9 - 6208608433093*a^8*b^34*c^10 + 162046752443092*a^9*b^3
2*c^11 - 812380117242496*a^10*b^30*c^12 - 4269511492139520*a^11*b^28*c^13 +
 64483194304611328*a^12*b^26*c^14 - 112921634620248064*a^13*b^24*c^15 - 220
1189026874097664*a^14*b^22*c^16 + 14616435842612723712*a^15*b^20*c^17 - 272
064120668160000*a^16*b^18*c^18 - 34144707089706673764*a^17*b^16*c^19 + 103
456428186131025108*a^18*b^14*c^20 + 41812952976409157959668*a^19*b^12*c^21
- 373354628924470397501444*a^20*b^10*c^22 + 966417047758501524275200*a^21*b^8
*c^23 - 23704136640124663365632*a^22*b^6*c^24 - 41093578558303672415027*a^
23*b^4*c^25 + 873713261936696337891328*a^24*b^2*c^26 - 593095678509622759522
2304*a^25*c^27 - (55*a^7*b^55*c^2 - 8790*a^8*b^53*c^3 + 639444*a^9*b^51*c^4
 - 27972744*a^10*b^49*c^5 + 818164461*a^11*b^47*c^6 - 16808585403*a^12*b^45
```

```
*c^7 + 247606057400*a^13*b^43*c^8 - 2618903955536*a^14*b^41*c^9 + 196792126
98112*a^15*b^39*c^10 - 106705071378688*a^16*b^37*c^11 + 532157820729344*a^1
7*b^35*c^12 - 4164077631959040*a^18*b^33*c^13 + 38697192507441152*a^19*b^31
*c^14 - 260154033577000960*a^20*b^29*c^15 + 1111838695929937920*a^21*b^27*c
^16 - 3408465526029025280*a^22*b^25*c^17 + 18861661252013260800*a^23*b^23*c
^18 - 181703629277052796928*a^24*b^21*c^19 + 1257805826981487443968*a^25*b^
19*c^20 - 5574358677366926999552*a^26*b^17*c^21 + 15471803807132282781696*a
^27*b^15*c^22 - 21577312571980702023680*a^28*b^13*c^23 - 166693308180935223
54176*a^29*b^11*c^24 + 154518440770998877814784*a^30*b^9*c^25 - 36483518232
6156934250496*a^31*b^7*c^26 + 474150347838602533142528*a^32*b^5*c^27 - 3435
23164402121399336960*a^33*b^3*c^28 + 109360198129920071696384*a^34*b*c^29)*
sqrt((b^20 - 90*a*b^18*c + 3045*a^2*b^16*c^2 - 41870*a^3*b^14*c^3 + 10990*a
^4*b^12*c^4 + 4635988*a^5*b^10*c^5 - 9414335*a^6*b^8*c^6 - 375477520*a^7*b^
6*c^7 + 203802720*a^8*b^4*c^8 + 15873186560*a^9*b^2*c^9 + 54875873536*a^10*
c^10)/(a^14*b^30 - 60*a^15*b^28*c + 1680*a^16*b^26*c^2 - 29120*a^17*b^24*c^
3 + 349440*a^18*b^22*c^4 - 3075072*a^19*b^20*c^5 + 20500480*a^20*b^18*c^6 -
 105431040*a^21*b^16*c^7 + 421724160*a^22*b^14*c^8 - 1312030720*a^23*b^12*c
^9 + 3148873728*a^24*b^10*c^10 - 5725224960*a^25*b^8*c^11 + 7633633280*a^26
*b^6*c^12 - 7046430720*a^27*b^4*c^13 + 4026531840*a^28*b^2*c^14 - 107374182
4*a^29*c^15)))*sqrt(x)*sqrt(-(b^15 - 55*a*b^13*c + 990*a^2*b^11*c^2 - 4455*
a^3*b^9*c^3 - 37840*a^4*b^7*c^4 + 155232*a^5*b^5*c^5 + 1281280*a^6*b^3*c^6
+ 11925760*a^7*b*c^7 + (a^7*b^20 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*
a^10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^
8*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9
+ 1048576*a^17*c^10)*sqrt((b^20 - 90*a*b^18*c + 3045*a^2*b^16*c^2 - 41870*a
^3*b^14*c^3 + 10990*a^4*b^12*c^4 + 4635988*a^5*b^10*c^5 - 9414335*a^6*b^8*c
^6 - 375477520*a^7*b^6*c^7 + 203802720*a^8*b^4*c^8 + 15873186560*a^9*b^2*c^
9 + 54875873536*a^10*c^10)/(a^14*b^30 - 60*a^15*b^28*c + 1680*a^16*b^26*c^2
 - 29120*a^17*b^24*c^3 + 349440*a^18*b^22*c^4 - 3075072*a^19*b^20*c^5 + 205
00480*a^20*b^18*c^6 - 105431040*a^21*b^16*c^7 + 421724160*a^22*b^14*c^8 - 1
312030720*a^23*b^12*c^9 + 3148873728*a^24*b^10*c^10 - 5725224960*a^25*b^8*c
^11 + 7633633280*a^26*b^6*c^12 - 7046430720*a^27*b^4*c^13 + 4026531840*a^28
*b^2*c^14 - 1073741824*a^29*c^15)))/(a^7*b^20 - 40*a^8*b^18*c + 720*a^9*b^1
6*c^2 - 7680*a^10*b^14*c^3 + 53760*a^11*b^12*c^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B]
time = 9.35, size = 2500, normalized size = 4.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 36
79344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 9760227
840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c + 576
*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8*c^5
 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) + ((3*
(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*c^17 +
 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 12356816*a
^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 668723200*a^
8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10 + 502626
713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418560*a^13*b
^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5*c^15 - 299
19144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^(1/2) - 95*a
*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3*b^4*c^3*(-(4
*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)^(1/2) - 45*a*
b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099511627776*a^27*
c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*c^3 + 1240320*a^
11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*c^6 - 1270087680
*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^16*b^22*c^9 + 193
730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 2113425899520*a^19
*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*a^21*b^12*c^14 -
 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^16 - 19585050869
760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 5497558138880*a^26*b^2*c
^19)))^(1/4)*(774056185954304*a^16*c^16 - 16777216*a^4*b^24*c^4 + 889192448
*a^5*b^22*c^5 - 20065550336*a^6*b^20*c^6 + 256355860480*a^7*b^18*c^7 - 2045
478174720*a^8*b^16*c^8 + 10385230921728*a^9*b^14*c^9 - 31026843746304*a^10*
b^12*c^10 + 30099130810368*a^11*b^10*c^11 + 156680406958080*a^12*b^8*c^12 -
 764160581304320*a^13*b^6*c^13 + 1587694790508544*a^14*b^4*c^14 - 170644204
6308352*a^15*b^2*c^15))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c
```

```
+ 576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^
8*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) -
 (9*x^(1/2)*(3096224743817216*a^16*b*c^18 - 16777216*a^2*b^29*c^4 + 1157627
904*a^3*b^27*c^5 - 34175188992*a^4*b^25*c^6 + 570425344000*a^5*b^23*c^7 - 5
968393928704*a^6*b^21*c^8 + 40450001993728*a^7*b^19*c^9 - 171227461189632*a
^8*b^17*c^10 + 350881648214016*a^9*b^15*c^11 + 523642412728320*a^10*b^13*c^
12 - 6226534348095488*a^11*b^11*c^13 + 21186489555615744*a^12*b^9*c^14 - 39
951854506868736*a^13*b^7*c^15 + 42889749576286208*a^14*b^5*c^16 - 225179981
36852480*a^15*b^3*c^17))/(4194304*(a^4*b^24 + 16777216*a^16*c^12 - 48*a^5*b
^22*c + 1056*a^6*b^20*c^2 - 14080*a^7*b^18*c^3 + 126720*a^8*b^16*c^4 - 8110
08*a^9*b^14*c^5 + 3784704*a^10*b^12*c^6 - 12976128*a^11*b^10*c^7 + 32440320
*a^12*b^8*c^8 - 57671680*a^13*b^6*c^9 + 69206016*a^14*b^4*c^10 - 50331648*a
^15*b^2*c^11)))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 125050657177
60*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*
c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^
7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^
15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291
284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^1
5*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^
25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*
a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^2
5)^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 109
9511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*
c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28*
c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a^1
6*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 + 21
13425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558274560*
a^21*b^12*c^14 - 16647293239296*a^22*b^10*c^15 + 20809116549120*a^23*b^8*c^
16 - 19585050869760*a^24*b^6*c^17 + 13056700579840*a^25*b^4*c^18 - 54975581
38880*a^26*b^2*c^19)))^(3/4))*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2)
+ 12505065717760*a^17*b*c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 132
9320*a^4*b^27*c^4 - 12356816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 1811904
00*a^7*b^21*c^7 - 668723200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490
242560*a^10*b^15*c^10 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^
11*c^12 + 8291284418560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 3197
4471237632*a^15*b^5*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-
(4*a*c - b^2)^25)^(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)
^(1/2) + 2015*a^3*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(
4*a*c - b^2)^25)^(1/2) - 45*a*b^8*c*(-(4*a*c - ...
```

3.1087 $\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=658

$$\frac{x^{3/2}(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{x^{3/2}(5b^4-45ab^2c+52a^2c^2+bc(5b^2-44ac)\,x^2)}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{\sqrt[4]{c}\,\Big(5b^4-54ab^2c+520a^2 \ldots}{32 \ldots}$$

[Out] 1/4*x^(3/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*x^(3/2)*(5*b^4-45*a*b^2*c+52*a^2*c^2+b*c*(-44*a*c+5*b^2)*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-1/64*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^4-54*a*b^2*c+520*a^2*c^2-b*(-44*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/64*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^4-54*a*b^2*c+520*a^2*c^2-b*(-44*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/64*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^4-54*a*b^2*c+520*a^2*c^2+b*(-44*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/64*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(5*b^4-54*a*b^2*c+520*a^2*c^2+b*(-44*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*2^(1/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)

Rubi [A]
time = 3.62, antiderivative size = 658, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = $\{1129, 1380, 1514, 1524, 304, 211, 214\}$

$$-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b(5b^2-44ac)\sqrt{b^2-4ac}+5b^4\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\;2^{3/4}a^2\,(b^2-4ac)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c+b(5b^2-44ac)\sqrt{b^2-4ac}+5b^4\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\;2^{3/4}a^2\,(b^2-4ac)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{3/2}(52a^2c^2+bcx^2(5b^2-44ac)-45ab^2c+5b^4)}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b(5b^2-44ac)\sqrt{b^2-4ac}+5b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\;2^{3/4}a^2\,(b^2-4ac)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c+b(5b^2-44ac)\sqrt{b^2-4ac}+5b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\;2^{3/4}a^2\,(b^2-4ac)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{3/2}(-2ac+b^2+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]

[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + b*c*(5*b^2 - 44*a*c)*x^2))/(16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 + b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^

```
2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 + b
*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt
[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1129

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1380

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b
^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(
p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1514

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(
```

```
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m +
 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*
x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b
^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx = 2\text{Subst}\left(\int \frac{x^2}{(a+bx^4+cx^8)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{\text{Subst}\left(\int \frac{x^2(-5b^2+26ac-9bcx^4)}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)}{4a\,(b^2-4ac)}$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{x^{3/2}(5b^4-45ab^2c+52a^2c^2+bc(5b^2-44ac)\,x^2)}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{x^{3/2}(5b^4-45ab^2c+52a^2c^2+bc(5b^2-44ac)\,x^2)}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{x^{3/2}(5b^4-45ab^2c+52a^2c^2+bc(5b^2-44ac)\,x^2)}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{x^{3/2}(5b^4-45ab^2c+52a^2c^2+bc(5b^2-44ac)\,x^2)}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.48, size = 255, normalized size = 0.39

$$\frac{\frac{4x^{3/2}\left(84a^3c^2+5b^3x^2(b+cx^2)^2+a^2c(-69b^2-8bcx^2+52c^2x^4)+ab(9b^3-36b^2cx^2-89bc^2x^4-44c^3x^6)\right)}{(a+bx^2+cx^4)^2}+\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{5b^4\log\left(\sqrt{x}-\#1\right)-49ab^2c\log\left(\sqrt{x}-\#1\right)+260a^2c^2\log\left(\sqrt{x}-\#1\right)+5b^3c\log\left(\sqrt{x}-\#1\right)\#1^4-44abc^2\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{64a^2\,(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*x^(3/2)*(84*a^3*c^2 + 5*b^3*x^2*(b + c*x^2)^2 + a^2*c*(-69*b^2 - 8*b*c*
x^2 + 52*c^2*x^4) + a*b*(9*b^3 - 36*b^2*c*x^2 - 89*b*c^2*x^4 - 44*c^3*x^6))
)/(a + b*x^2 + c*x^4)^2 + RootSum[a + b*#1^4 + c*#1^8 & , (5*b^4*Log[Sqrt[x
] - #1] - 49*a*b^2*c*Log[Sqrt[x] - #1] + 260*a^2*c^2*Log[Sqrt[x] - #1] + 5*
b^3*c*Log[Sqrt[x] - #1]*#1^4 - 44*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2
*c*#1^5) & ])/(64*a^2*(b^2 - 4*a*c)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 321, normalized size = 0.49

method	result
derivativedivides	$\frac{\frac{3\left(28a^{2}c^{2}-23a\,b^{2}c+3b^{4}\right)x^{\frac{3}{2}}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)a}-\frac{b\left(8a^{2}c^{2}+36a\,b^{2}c-5b^{4}\right)x^{\frac{7}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c\left(52a^{2}c^{2}-89a\,b^{2}c+10b^{4}\right)x^{\frac{11}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{b\,c^{2}\left(44ac-5b^{2}\right)x^{\frac{15}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{\_R=}{}$
default	$\frac{\frac{3\left(28a^{2}c^{2}-23a\,b^{2}c+3b^{4}\right)x^{\frac{3}{2}}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)a}-\frac{b\left(8a^{2}c^{2}+36a\,b^{2}c-5b^{4}\right)x^{\frac{7}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c\left(52a^{2}c^{2}-89a\,b^{2}c+10b^{4}\right)x^{\frac{11}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{b\,c^{2}\left(44ac-5b^{2}\right)x^{\frac{15}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{\_R=}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/32*(28*a^2*c^2-23*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(3/2)-
1/32*b*(8*a^2*c^2+36*a*b^2*c-5*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+
1/32/a^2*c*(52*a^2*c^2-89*a*b^2*c+10*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(11/
2)-1/32*b*c^2*(44*a*c-5*b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(c*x^
4+b*x^2+a)^2+1/64/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((b*c*(-44*a*c+5*b^2)*_
R^6+(260*a^2*c^2-49*a*b^2*c+5*b^4)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_
R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^3*c^2 - 44*a*b*c^3)*x^(15/2) + (10*b^4*c - 89*a*b^2*c^2 + 52*a^2
*c^3)*x^(11/2) + (5*b^5 - 36*a*b^3*c - 8*a^2*b*c^2)*x^(7/2) + 3*(3*a*b^4 -
23*a^2*b^2*c + 28*a^3*c^2)*x^(3/2))/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*
c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^
```

```
2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b
^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) - integrate(-1/32*((5*b^3*c - 44*a*b*
c^2)*x^(5/2) + (5*b^4 - 49*a*b^2*c + 260*a^2*c^2)*sqrt(x))/(a^3*b^4 - 8*a^4
*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B]
time = 8.75, size = 2500, normalized size = 3.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^(11/2)*(10*b^4*c + 52*a^2*c^3 - 89*a*b^2*c^2))/(16*(a^2*b^4 + 16*a^4*c^
2 - 8*a^3*b^2*c)) - (x^(7/2)*(8*a^2*b*c^2 - 5*b^5 + 36*a*b^3*c))/(16*a*(a*b
```

^4 + 16*a^3*c^2 - 8*a^2*b^2*c)) + (3*x^(3/2)*(3*b^4 + 28*a^2*c^2 - 23*a*b^2
*c))/(16*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^(15/2)*(44*a*c - 5*b^
2))/(16*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c
^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((2097152000*a*b^33*c^4 + 46617885
6428188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 5340020080640*a^3*b
^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*c^8 - 23
398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 - 163109930
0505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 452077026065
68226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 - 4878820944
58626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^16 - 1771946
621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^5*c^18 - 1418
770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 268435456*a^20*c^1
4 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 + 256256*a^10*b^
20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 56229888*a^13*b^1
4*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9 + 1049624576*a^16
*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^4*c^12 - 939524096
*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*a*c - b^2)^25)^(1/2
) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b^33*c^2 - 57758230*a^3*b^3
1*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5*b^27*c^5 + 188531780400*a
^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 13317068448000*a^8*b^21*c^8 - 82
629338933248*a^9*b^19*c^9 + 419701532733440*a^10*b^17*c^10 - 17375022953267
20*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^12 - 15422593991966720*a^1
3*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 48851227886223360*a^15*b^7*
c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189126635520*a^17*b^3*c^17 -
285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 52625*a*b^35*c - 380775*a^2*b
^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^6*c^3*(-(4*a*c - b^2)^25)^
(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^(1/2) + 121578600*a^5*b^2*
c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-(4*a*c - b^2)^25)^(1/2))/(
33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80*a^10*b^38*c + 3040*a^11*b
^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32*c^4 - 15876096*a^14*b^30*
c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b^26*c^7 + 8255569920*a^17*
b^24*c^8 - 44029706240*a^18*b^22*c^9 + 193730707456*a^19*b^20*c^10 - 704475
299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c^12 - 5202279137280*a^22*b
^14*c^13 + 10404558274560*a^23*b^12*c^14 - 16647293239296*a^24*b^10*c^15 +
20809116549120*a^25*b^8*c^16 - 19585050869760*a^26*b^6*c^17 + 1305670057984
0*a^27*b^4*c^18 - 5497558138880*a^28*b^2*c^19)))^(1/4)*(2378463553205043200
*a^18*c^19 - 419430400*a^3*b^30*c^4 + 26675773440*a^4*b^28*c^5 - 8147183861
76*a^5*b^26*c^6 + 15745652097024*a^6*b^24*c^7 - 214134184476672*a^7*b^22*c^
8 + 2159815572848640*a^8*b^20*c^9 - 16615360157450240*a^9*b^18*c^10 + 98862
579421544448*a^10*b^16*c^11 - 456983970538586112*a^11*b^14*c^12 + 163543943
3677275136*a^12*b^12*c^13 - 4480548366094172160*a^13*b^10*c^14 + 9201889778
671288320*a^14*b^8*c^15 - 13675039531022155776*a^15*b^6*c^16 + 138416023484
90686464*a^16*b^4*c^17 - 8502514621498785792*a^17*b^2*c^18))/(4194304*(a^6*
b^24 + 16777216*a^18*c^12 - 48*a^7*b^22*c + 1056*a^8*b^20*c^2 - 14080*a^9*b
^18*c^3 + 126720*a^10*b^16*c^4 - 811008*a^11*b^14*c^5 + 3784704*a^12*b^12*c

```
^6 - 12976128*a^13*b^10*c^7 + 32440320*a^14*b^8*c^8 - 57671680*a^15*b^6*c^9
 + 69206016*a^16*b^4*c^10 - 50331648*a^17*b^2*c^11)))*(-(625*b^37 - 625*b^1
2*(-(4*a*c - b^2)^25)^(1/2) + 11279020326912000*a^18*b*c^18 + 2168275*a^2*b
^33*c^2 - 57758230*a^3*b^31*c^3 + 1109954201*a^4*b^29*c^4 - 16285749400*a^5
*b^27*c^5 + 188531780400*a^6*b^25*c^6 - 1756313913600*a^7*b^23*c^7 + 133170
68448000*a^8*b^21*c^8 - 82629338933248*a^9*b^19*c^9 + 419701532733440*a^10*
b^17*c^10 - 1737502295326720*a^11*b^15*c^11 + 5807000541921280*a^12*b^13*c^
12 - 15422593991966720*a^13*b^11*c^13 + 31764369743282176*a^14*b^9*c^14 - 4
8851227886223360*a^15*b^7*c^15 + 52725360025927680*a^16*b^5*c^16 - 35577189
126635520*a^17*b^3*c^17 - 285610000*a^6*c^6*(-(4*a*c - b^2)^25)^(1/2) - 526
25*a*b^35*c - 380775*a^2*b^8*c^2*(-(4*a*c - b^2)^25)^(1/2) + 4075730*a^3*b^
6*c^3*(-(4*a*c - b^2)^25)^(1/2) - 28545201*a^4*b^4*c^4*(-(4*a*c - b^2)^25)^
(1/2) + 121578600*a^5*b^2*c^5*(-(4*a*c - b^2)^25)^(1/2) + 21375*a*b^10*c*(-
(4*a*c - b^2)^25)^(1/2))/(33554432*(a^9*b^40 + 1099511627776*a^29*c^20 - 80
*a^10*b^38*c + 3040*a^11*b^36*c^2 - 72960*a^12*b^34*c^3 + 1240320*a^13*b^32
*c^4 - 15876096*a^14*b^30*c^5 + 158760960*a^15*b^28*c^6 - 1270087680*a^16*b
^26*c^7 + 8255569920*a^17*b^24*c^8 - 44029706240*a^18*b^22*c^9 + 1937307074
56*a^19*b^20*c^10 - 704475299840*a^20*b^18*c^11 + 2113425899520*a^21*b^16*c
^12 - 5202279137280*a^22*b^14*c^13 + 10404558274560*a^23*b^12*c^14 - 166472
93239296*a^24*b^10*c^15 + 20809116549120*a^25*b...
```

3.1088 $\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=658

$$\frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{\sqrt{x}\,(7b^4-55ab^2c+60a^2c^2+bc(7b^2-52ac)\,x^2)}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{3c^{3/4}\Big(7b^4-66ab^2c+28\cdots}{3\cdots}$$

```
[Out] 3/64*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(7*b^4-66*a*b^2*c+280*a^2*c^2-b*(-52*a*c+7*b^2)*(-4*a*c+b^2)^(1/2))*2^(3/4)
/a^2/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+3/64*c^(3/4)*arctanh(
2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(7*b^4-66*a*b^2*c+28
0*a^2*c^2-b*(-52*a*c+7*b^2)*(-4*a*c+b^2)^(1/2))*2^(3/4)/a^2/(-4*a*c+b^2)^(5
/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3/64*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/
2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(7*b^4-66*a*b^2*c+280*a^2*c^2+b*(-52*a*c+
7*b^2)*(-4*a*c+b^2)^(1/2))*2^(3/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^
(1/2))^(3/4)-3/64*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^
(1/2))^(1/4))*(7*b^4-66*a*b^2*c+280*a^2*c^2+b*(-52*a*c+7*b^2)*(-4*a*c+b^2)^
(1/2))*2^(3/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*(b*
c*x^2-2*a*c+b^2)*x^(1/2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*(7*b^4-55*a*
b^2*c+60*a^2*c^2+b*c*(-52*a*c+7*b^2)*x^2)*x^(1/2)/a^2/(-4*a*c+b^2)^2/(c*x^4
+b*x^2+a)
```

Rubi [A]
time = 3.81, antiderivative size = 658, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1129, 1359, 1444, 1436, 218, 214, 211}

$$\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b(7b^2-52ac)\sqrt{b^2-4ac}+7b^4\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a^2\,(b^2-4ac)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(280a^2c^2-66ab^2c+b(7b^2-52ac)\sqrt{b^2-4ac}+7b^4\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a^2\,(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\,(60a^2c^2+bcx^2(7b^2-52ac)-55ab^2c+7b^4)}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b(7b^2-52ac)\sqrt{b^2-4ac}+7b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a^2\,(b^2-4ac)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(280a^2c^2-66ab^2c+b(7b^2-52ac)\sqrt{b^2-4ac}+7b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\,a^2\,(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\,(-2ac+b^2+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
 + (Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + b*c*(7*b^2 - 52*a*c)*x^2))/(
16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*c^(3/4)*(7*b^4 - 66*a*b^2*
c + 280*a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c
)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c +
280*a^2*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5
/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*
```

```
a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*a^2
*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[
x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-
b + Sqrt[b^2 - 4*a*c])^(3/4))
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

<u>Rule 218</u>

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

<u>Rule 1129</u>

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Dist[k/d, Subst[Int[x^(k*(m + 1) - 1)*(a + b*
(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a, b
, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

<u>Rule 1359</u>

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-
x)*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c +
n*(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n)
)^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*
a*c, 0] && ILtQ[p, -1]
```

<u>Rule 1436</u>

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
```

```
*c] ||  !IGtQ[n/2, 0])
```

Rule 1444

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/
(a*n*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*d
*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^3}\,dx = 2\text{Subst}\left(\int \frac{1}{(a+bx^4+cx^8)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{\text{Subst}\left(\int \frac{b^2-2ac-8(b^2-4ac)-11bcx^4}{(a+bx^4+cx^8)^2}\,dx, x, \sqrt{x}\right)}{4a\,(b^2-4ac)}$$

$$= \frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\sqrt{x}\,(7b^4-55ab^2c+60a^2c^2+bc(7b^2-52ac}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

$$= \frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\sqrt{x}\,(7b^4-55ab^2c+60a^2c^2+bc(7b^2-52ac}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

$$= \frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\sqrt{x}\,(7b^4-55ab^2c+60a^2c^2+bc(7b^2-52ac}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

$$= \frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\sqrt{x}\,(7b^4-55ab^2c+60a^2c^2+bc(7b^2-52ac}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.43, size = 259, normalized size = 0.39

$$\frac{\frac{4\sqrt{x}\left(92a^3c^2+7b^3x^2(b+cx^2)^2+a^2c(-79b^2-8bcx^2+60c^2x^4)+ab(11b^3-44b^2cx^2-107bc^2x^4-52c^3x^6)\right)}{(a+bx^2+cx^4)^2}+3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{7b^4\log\left(\sqrt{x}-\#1\right)-59ab^2c\log\left(\sqrt{x}-\#1\right)+140a^2c^2\log\left(\sqrt{x}-\#1\right)+7b^3c\log\left(\sqrt{x}-\#1\right)\#1^4-52abc^2\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{64a^2\,(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((4*Sqrt[x]*(92*a^3*c^2 + 7*b^3*x^2*(b + c*x^2)^2 + a^2*c*(-79*b^2 - 8*b*c*
x^2 + 60*c^2*x^4) + a*b*(11*b^3 - 44*b^2*c*x^2 - 107*b*c^2*x^4 - 52*c^3*x^6
)))/(a + b*x^2 + c*x^4)^2 + 3*RootSum[a + b*#1^4 + c*#1^8 & , (7*b^4*Log[Sq
rt[x] - #1] - 59*a*b^2*c*Log[Sqrt[x] - #1] + 140*a^2*c^2*Log[Sqrt[x] - #1]
+ 7*b^3*c*Log[Sqrt[x] - #1]*#1^4 - 52*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1
^3 + 2*c*#1^7) & ])/(64*a^2*(b^2 - 4*a*c)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 316, normalized size = 0.48

method	result
derivativedivides	$\frac{\frac{\left(92a^2c^2-79a\,b^2c+11b^4\right)\sqrt{x}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a}-\frac{b\left(8a^2c^2+44a\,b^2c-7b^4\right)x^{\frac{5}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(60a^2c^2-107a\,b^2c+14b^4\right)x^{\frac{9}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{b\,c^2\left(52ac-7b^2\right)x^{\frac{13}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{3\left(\ldots\right.}{-\ldots}$
default	$\frac{\frac{\left(92a^2c^2-79a\,b^2c+11b^4\right)\sqrt{x}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a}-\frac{b\left(8a^2c^2+44a\,b^2c-7b^4\right)x^{\frac{5}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(60a^2c^2-107a\,b^2c+14b^4\right)x^{\frac{9}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{b\,c^2\left(52ac-7b^2\right)x^{\frac{13}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{3\left(\ldots\right.}{-\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(92*a^2*c^2-79*a*b^2*c+11*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(1/2)
-1/32*b*(8*a^2*c^2+44*a*b^2*c-7*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)
+1/32/a^2*c*(60*a^2*c^2-107*a*b^2*c+14*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9
/2)-1/32*b*c^2*(52*a*c-7*b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*x
^4+b*x^2+a)^2+3/64/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((b*c*(-52*a*c+7*b^2)*
_R^4+140*a^2*c^2-59*a*b^2*c+7*b^4)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=Root
Of(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(7*b^4*c^2 - 59*a*b^2*c^3 + 140*a^2*c^4)*x^(17/2) + (42*b^5*c - 347
*a*b^3*c^2 + 788*a^2*b*c^3)*x^(13/2) + (21*b^6 - 121*a*b^4*c - 41*a^2*b^2*c
^2 + 900*a^3*c^3)*x^(9/2) + (49*a*b^5 - 398*a^2*b^3*c + 832*a^3*b*c^2)*x^(5
/2) + 32*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*sqrt(x))/(a^5*b^4 - 8*a^6*b^2
```

```
*c + 16*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^8 + 2*(a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*
c^3)*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2) - integrate(3/32*(
(7*b^4*c - 59*a*b^2*c^2 + 140*a^2*c^3)*x^(7/2) + (7*b^5 - 66*a*b^3*c + 192*
a^2*b*c^2)*x^(3/2))/(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + (a^3*b^4*c - 8*a^
4*b^2*c^2 + 16*a^5*c^3)*x^4 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2),
x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^3*sqrt(x)), x)
```

Mupad [B]
time = 9.85, size = 2500, normalized size = 3.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2 + c*x^4)^3),x)
```

[Out] ((x^(9/2)*(14*b^4*c + 60*a^2*c^3 - 107*a*b^2*c^2))/(16*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(1/2)*(11*b^4 + 92*a^2*c^2 - 79*a*b^2*c))/(16*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^(5/2)*(8*a^2*b*c^2 - 7*b^5 + 44*a*b^3*c))/(16*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^(13/2)*(52*a*c - 7*b^2))/(16*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - atan((((((9*x^(1/2)*(1546704997025054720*a^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 1473677099008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 + 113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 1545406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 7315227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 9650897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 - 14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 3784704*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 57671680*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - (3*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(1/4)*(3377699720527872*a^19*b*c^16 + 117440512*a^7*b^25*c^4 - 5804916736*a^8*b^23*c^5 + 132070244352*a^9*b^21*c^6 - 1828045455360*a^10*b^19*c^7 + 17136919511040*a^11*b^17*c^8 - 114572547588096*a^12*b^15*c^9 + 559926296444928*a^13*b^13*c^10 - 2014580179992576*a^14*b^11*c^11 + 5294148487741440*a^15*b^9*c^12 - 9906599766261760*a^16*b^7*c^13 + 12525636463624192*a^17*b^5*c^14 - 9605333580251136*a^18*b^3*c^15))/(65536*(a^8*b^18 - 262144*a^17*c^9 - 36*a^9*b^16*c + 576*a^10*b^14*c^2 - 5376*a^11*b^12*c^3 + 32256*a^12*b^10*c^4 - 129024*a^13*b^8*c^5 + 344064*a^14*b^6*c^6 - 589824*a^

15*b^4*c^7 + 589824*a^16*b^2*c^8)))*(-(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276813600400*a^7*b^25*c^7 + 21341140889600*a^8*b^23*c^8 - 113330748025600*a^9*b^21*c^9 + 492398189373440*a^10*b^19*c^10 - 1748923551027200*a^11*b^17*c^11 + 5052644161945600*a^12*b^15*c^12 - 11756581147443200*a^13*b^13*c^13 + 21683350423470080*a^14*b^11*c^14 - 30929025701511168*a^15*b^9*c^15 + 32836636093972480*a^16*b^7*c^16 - 24359874477424640*a^17*b^5*c^17 + 11224950044098560*a^18*b^3*c^18 + 24010000*a^7*c^7*(-(4*a*c - b^2)^25)^(1/2) - 193795*a*b^37*c - 996660*a^2*b^10*c^2*(-(4*a*c - b^2)^25)^(1/2) + 7556115*a^3*b^8*c^3*(-(4*a*c - b^2)^25)^(1/2) - 34052295*a^4*b^6*c^4*(-(4*a*c - b^2)^25)^(1/2) + 87808681*a^5*b^4*c^5*(-(4*a*c - b^2)^25)^(1/2) - 108025400*a^6*b^2*c^6*(-(4*a*c - b^2)^25)^(1/2) + 73745*a*b^12*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^11*b^40 + 1099511627776*a^31*c^20 - 80*a^12*b^38*c + 3040*a^13*b^36*c^2 - 72960*a^14*b^34*c^3 + 1240320*a^15*b^32*c^4 - 15876096*a^16*b^30*c^5 + 158760960*a^17*b^28*c^6 - 1270087680*a^18*b^26*c^7 + 8255569920*a^19*b^24*c^8 - 44029706240*a^20*b^22*c^9 + 193730707456*a^21*b^20*c^10 - 704475299840*a^22*b^18*c^11 + 2113425899520*a^23*b^16*c^12 - 5202279137280*a^24*b^14*c^13 + 10404558274560*a^25*b^12*c^14 - 16647293239296*a^26*b^10*c^15 + 20809116549120*a^27*b^8*c^16 - 19585050869760*a^28*b^6*c^17 + 13056700579840*a^29*b^4*c^18 - 5497558138880*a^30*b^2*c^19)))^(3/4) + (3*(4356374400000*a^8*c^16 + 18475695*...

3.1089 $\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/5*(d*x)^(5/2)*AppellF1(5/4,-1/2,-1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1155, 524}

$$\frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
```

```
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx &= \frac{\sqrt{a+bx^2+cx^4}\int (dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(147) = 294.
time = 10.38, size = 365, normalized size = 2.48

$$\frac{2d\sqrt{dx}\left(5(2b+5cx^2)(a+bx^2+cx^4)-10ab\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+2(-3b^2+10ac)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{225c\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*Sqrt[d*x]*(5*(2*b + 5*c*x^2)*(a + b*x^2 + c*x^4) - 10*a*b*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(-3*
b^2 + 10*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appe
llF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])]))/(225*c*Sqrt[a + b*x^2 + c*x^4])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)*d*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \sqrt{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.1090 $\int \sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/3*(d*x)^(3/2)*AppellF1(3/4,-1/2,-1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$\frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
```

```
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int \sqrt{dx}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$
$$= \frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(147) = 294.
time = 10.29, size = 342, normalized size = 2.33

$$\frac{2x\sqrt{dx}\left(21(a+bx^2+cx^4)+28a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+6bx^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{147\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*Sqrt[d*x]*(21*(a + b*x^2 + c*x^4) + 28*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 6*b*x^2*Sqrt[(b - Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(147*Sqr
t[a + b*x^2 + c*x^4])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(d*x)*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.1091 $\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2*AppellF1(1/4,-1/2,-1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-
4*a*c+b^2)^(1/2)))*(d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$\frac{2\sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/4, (-2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[1
 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
```

```
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{\sqrt{dx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 342 vs. $2(145) = 290$.
time = 10.26, size = 342, normalized size = 2.36

$$\frac{2x\left(5(a+bx^2+cx^4)+20a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+2bx^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{25\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*x*(5*(a + b*x^2 + c*x^4) + 20*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(25*Sqrt[d*x]*Sqrt[a
 + b*x^2 + c*x^4])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x)
[Out] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/sqrt(d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{\sqrt{d\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(1/2),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(1/2), x)
```

3.1092 $\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -2*AppellF1(-1/4,-1/2,-1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+
(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(d*x)^(1/2)/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$-\frac{2\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
```

```
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{(dx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{2\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 345 vs. 2(145) = 290.
time = 10.26, size = 345, normalized size = 2.38

$$\frac{x\left(-42(a+bx^2+cx^4)+28bx^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+24cx^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{21(dx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(d*x)^(3/2),x]
```

```
[Out] (x*(-42*(a + b*x^2 + c*x^4) + 28*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 24*c*x^4*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*(d*x)^(3/2
)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x)
[Out] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{(dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(3/2),x)
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{{(d\,x)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(3/2),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(3/2), x)
```

3.1093 $\int (dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=148

$$\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}\, F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/5*a*(d*x)^(5/2)*AppellF1(5/4,-3/2,-3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}\, F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
```

```
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int (dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2} \, dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right) \int (dx)^{3/2} \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2} \left(1+\frac{2cx^2}{b+\sqrt{b^2-4a}}\right.}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}\, F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4}}\right.}{5d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 459 vs. 2(148) = 296.
time = 10.58, size = 459, normalized size = 3.10

$$\frac{2d\sqrt{dx}\left(5(-28b^4x^2-8b^3cx^4+305b^2c^2x^6+480bc^3x^8+195c^4x^{10}+a^2c(176b+455cx^2)+a(-28b^3+196b^2cx^2+916bc^2x^4+650c^3x^6))+20ab(7b^2-44ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+4(21b^4-157ab^2c+260a^2c^2)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{16575c^2\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*Sqrt[d*x]*(5*(-28*b^4*x^2 - 8*b^3*c*x^4 + 305*b^2*c^2*x^6 + 480*b*c^3*
x^8 + 195*c^4*x^10 + a^2*c*(176*b + 455*c*x^2) + a*(-28*b^3 + 196*b^2*c*x^2
 + 916*b*c^2*x^4 + 650*c^3*x^6)) + 20*a*b*(7*b^2 - 44*a*c)*Sqrt[(b - Sqrt[b
^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 4*(21*b^4
- 157*a*b^2*c + 260*a^2*c^2)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(16575*c^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral((c*d*x^5 + b*d*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^{\frac{3}{2}} \left(a + bx^2 + cx^4\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((d*x)**(3/2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,x\right)}^{3/2}\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.1094 $\int \sqrt{dx}\,(a+bx^2+cx^4)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2/3*a*(d*x)^(3/2)*AppellF1(3/4,-3/2,-3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$\frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
```

```
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \sqrt{dx}\,\left(a+bx^2+cx^4\right)^{3/2}\,dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int \sqrt{dx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right.}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4a}\right.}{3d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 417 vs. 2(148) = 296.
time = 10.54, size = 417, normalized size = 2.82

$$\frac{2x\sqrt{dx}\left(7(12ab^2+209a^2c+12b^3x^2+328abcx^2+131b^2cx^4+286ac^2x^4+196bc^2x^6+77c^3x^8)-28a(3b^2-44ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+12b(-5b^2+36ac)\,x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{8085c\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[d*x]*(7*(12*a*b^2 + 209*a^2*c + 12*b^3*x^2 + 328*a*b*c*x^2 + 131*
b^2*c*x^4 + 286*a*c^2*x^4 + 196*b*c^2*x^6 + 77*c^3*x^8) - 28*a*(3*b^2 - 44*
a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2
, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])] + 12*b*(-5*b^2 + 36*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(8085*c*Sqrt[a + b*x^2 + c*x^4
])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{dx}\,\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx} \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(sqrt(d*x)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x}\,\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.1095 $\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=146

$$\frac{2a\sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 2*a*AppellF1(1/4,-3/2,-3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+
(-4*a*c+b^2)^(1/2)))*(d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/d/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$\frac{2a\sqrt{dx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2, 5/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
```

```
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{dx}}\,dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{\sqrt{dx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2a\sqrt{dx}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 415 vs. 2(146) = 292.
time = 10.51, size = 415, normalized size = 2.84

$$\frac{2x\left(5(4ab^2+51a^2c+4b^3x^2+76abcx^2+29b^2cx^4+66ac^2x^4+40bc^2x^6+15c^3x^8)-20a(b^2-36ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)-4b(3b^2-28ac)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{975c\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*x*(5*(4*a*b^2 + 51*a^2*c + 4*b^3*x^2 + 76*a*b*c*x^2 + 29*b^2*c*x^4 + 66*
a*c^2*x^4 + 40*b*c^2*x^6 + 15*c^3*x^8) - 20*a*(b^2 - 36*a*c)*Sqrt[(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 4*b*(3*b
^2 - 28*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])]))/(975*c*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x)
[Out] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)/(d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(1/2),x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/sqrt(d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{\sqrt{d\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(1/2),x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(1/2), x)
```

3.1096 $\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2a\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -2*a*AppellF1(-1/4,-3/2,-3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(d*x)^(1/2)/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$-\frac{2a\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
```

```
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{(dx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{2a\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 384 vs. 2(146) = 292.
time = 10.50, size = 384, normalized size = 2.63

$$\frac{x\left(14(-77a^2-64abx^2+13b^2x^4-70acx^4+20bcx^6+7c^2x^8)+896abx^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+24(b^2+28ac)x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{539(dx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (x*(14*(-77*a^2 - 64*a*b*x^2 + 13*b^2*x^4 - 70*a*c*x^4 + 20*b*c*x^6 + 7*c^2
*x^8) + 896*a*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])] + 24*(b^2 + 28*a*c)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(539*(d*x)^(3/2)*Sqr
t[a + b*x^2 + c*x^4])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x)
[Out] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)/(d^2*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(3/2),x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{\left(d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2), x)
```

3.1097 $\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/5*(d*x)^(5/2)*AppellF1(5/4,1/2,1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*Sqrt[a + b*x^2
+ c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^{3/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1-\cdots}}}{\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2(dx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4\cdots}}\right.}{5d\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 10.10, size = 173, normalized size = 1.18

$$\frac{2x(dx)^{3/2}\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{5\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])])/(5*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x)*d*x/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d*x)**(3/2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x\right)}^{3/2}}{\sqrt{c\,x^4+b\,x^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.1098 $\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/3*(d*x)^(3/2)*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*Sqrt[a + b*x^2
+ c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{dx}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+}}}{\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right.}{3d\sqrt{a+bx^2+cx^4}}$$

Mathematica [**A**]
time = 10.08, size = 173, normalized size = 1.18

$$\frac{2x\sqrt{dx}\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*Sqrt[d*x]*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF
1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])/(3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(d*x)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x}}{\sqrt{c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.1099 $\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2*AppellF1(1/4,1/2,1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*
a*c+b^2)^(1/2)))*(d*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[a + b*x^2 + c*
x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\,\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{}{\sqrt{dx}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-}}}}}{\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2\sqrt{dx}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2c}{b-\sqrt{b^2}}\right.}{d\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 10.07, size = 171, normalized size = 1.18

$$\frac{2x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{dx}\,\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2
, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
4*a*c])])/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c*d*x^5 + b*d*x^3 + a*d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(d*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d\,x}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.1100 $\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -2*AppellF1(-1/4,1/2,1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-
4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/d/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sqrt[a + b*x^2 +
c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{}{(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-}}}}}{\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-}}\right.}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 348 vs. 2(145) = 290.
time = 10.36, size = 348, normalized size = 2.40

$$\frac{x\left(-42(a+bx^2+cx^4)+14bx^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+18cx^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{21a(dx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (x*(-42*(a + b*x^2 + c*x^4) + 14*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 18*c*x^4*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*a*(d*x)^(3
/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c*d^2*x^6 + b*d^2*x^4 + a*d^2*x
^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/((d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{(d\,x)}^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.1101 $\int \frac{(dx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{2(dx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/5*(d*x)^(5/2)*AppellF1(5/4,3/2,3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*d*Sqrt[a + b*x^
2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(dx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^{3/2}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{}{b+}\right.}}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2(dx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-}}\right.}{5ad\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 348 vs. 2(150) = 300.
time = 10.28, size = 348, normalized size = 2.32

$$\frac{d\sqrt{dx}\left(-5(b+2cx^2)+5b\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+2cx^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{5\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (d*Sqrt[d*x]*(-5*(b + 2*c*x^2) + 5*b*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*c*x^2*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*(b^2 - 4*a*c)*S
qrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)*d*x/(c^2*x^8 + 2*b*c*x^6 + (b^2
+ 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^{\frac{3}{2}}}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((d*x)**(3/2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^{3/2}}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.1102 $\int \frac{\sqrt{dx}}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{2(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2/3*(d*x)^(3/2)*AppellF1(3/4,3/2,3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+
2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*d*Sqrt[a + b*x^
2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{dx}}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{dx}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{}{b+\sqrt{}}\right.}}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4}}\right.}{3ad\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 367 vs. 2(150) = 300.
time = 10.40, size = 367, normalized size = 2.45

$$\frac{x\sqrt{dx}\left(-21(b^2-2ac+bcx^2)+7(b^2+2ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+9bcx^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{21a(-b^2+4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[d*x]*(-21*(b^2 - 2*a*c + b*c*x^2) + 7*(b^2 + 2*a*c)*Sqrt[(b - Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*b*c*x^2
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/
2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])]))/(21*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*
a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx}}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(d*x)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x}}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.1103 $\int \frac{1}{\sqrt{dx}\,(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{2\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2*AppellF1(1/4,3/2,3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*
a*c+b^2)^(1/2)))*(d*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*
c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*Sqrt[a + b*x^2 +
c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{dx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{}{\sqrt{dx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4a}}\right.}}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2\sqrt{dx}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{}{b-\sqrt{}}\right.}{ad\sqrt{a+bx^2+cx^4}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 366 vs. 2(148) = 296.
time = 10.37, size = 366, normalized size = 2.47

$$\frac{x\left(-5(b^2-2ac+bcx^2)-5(b^2-6ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+bcx^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{5a\,(-b^2+4ac)\,\sqrt{dx}\,\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (x*(-5*(b^2 - 2*a*c + b*c*x^2) - 5*(b^2 - 6*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + b*c*x^2*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*a*(
-b^2 + 4*a*c)*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*d*x^9 + 2*b*c*d*x^7 + (b^2
+ 2*a*c)*d*x^5 + 2*a*b*d*x^3 + a^2*d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx}\,\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d\,x}\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.1104 $\int \frac{1}{(dx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -2*AppellF1(-1/4,3/2,3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-
4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1155, 524}

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*Sqrt[d*x]*Sqrt[a + b*x^2
+ c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{}{(dx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4a}}\right.}}{a\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2c}{b-\sqrt{b^2}}\right.}{ad\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 409 vs. 2(148) = 296.
time = 10.51, size = 409, normalized size = 2.76

$$-\frac{x\left(-7(8a^2c-3b^2x^2(b+cx^2)+a(-2b^2+11bcx^2+10c^2x^4))-7b(b^2-3ac)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+3c(-3b^2+10ac)x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{7a^2\left(b^2-4ac\right)(dx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/7*(x*(-7*(8*a^2*c - 3*b^2*x^2*(b + c*x^2) + a*(-2*b^2 + 11*b*c*x^2 + 10*
c^2*x^4)) - 7*b*(b^2 - 3*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 3*c*(-3*b^2 + 10*a*c)*x^4*Sqrt[(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(a
^2*(b^2 - 4*a*c)*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*d^2*x^10 + 2*b*c*d^2*x^8 +
(b^2 + 2*a*c)*d^2*x^6 + 2*a*b*d^2*x^4 + a^2*d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(d\,x\right)}^{3/2}\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.1105 $\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=156

$$\frac{a^3(dx)^{1+m}}{d(1+m)}+\frac{3a^2b(dx)^{3+m}}{d^3(3+m)}+\frac{3a(b^2+ac)\,(dx)^{5+m}}{d^5(5+m)}+\frac{b(b^2+6ac)\,(dx)^{7+m}}{d^7(7+m)}+\frac{3c(b^2+ac)\,(dx)^{9+m}}{d^9(9+m)}+\frac{3bc^2(dx)^{11+m}}{d^{11}(11+m)}+$$

```
[Out] a^3*(d*x)^(1+m)/d/(1+m)+3*a^2*b*(d*x)^(3+m)/d^3/(3+m)+3*a*(a*c+b^2)*(d*x)^(
5+m)/d^5/(5+m)+b*(6*a*c+b^2)*(d*x)^(7+m)/d^7/(7+m)+3*c*(a*c+b^2)*(d*x)^(9+m
)/d^9/(9+m)+3*b*c^2*(d*x)^(11+m)/d^11/(11+m)+c^3*(d*x)^(13+m)/d^13/(13+m)
```

Rubi [A]
time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = $\{1122\}$

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2b(dx)^{m+3}}{d^3(m+3)}+\frac{3c(ac+b^2)\,(dx)^{m+9}}{d^9(m+9)}+\frac{b(6ac+b^2)\,(dx)^{m+7}}{d^7(m+7)}+\frac{3a(ac+b^2)\,(dx)^{m+5}}{d^5(m+5)}+\frac{3bc^2(dx)^{m+11}}{d^{11}(m+11)}+\frac{c^3(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*(d*x)^(1 + m))/(d*(1 + m)) + (3*a^2*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (
3*a*(b^2 + a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (b*(b^2 + 6*a*c)*(d*x)^(7 +
m))/(d^7*(7 + m)) + (3*c*(b^2 + a*c)*(d*x)^(9 + m))/(d^9*(9 + m)) + (3*b*c^
2*(d*x)^(11 + m))/(d^11*(11 + m)) + (c^3*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx = \int \left(a^3(dx)^m+\frac{3a^2b(dx)^{2+m}}{d^2}+\frac{3a(b^2+ac)\,(dx)^{4+m}}{d^4}+\frac{b(b^2+6ac)\,(dx)^{6+m}}{d^6}+\right.$$

$$=\frac{a^3(dx)^{1+m}}{d(1+m)}+\frac{3a^2b(dx)^{3+m}}{d^3(3+m)}+\frac{3a(b^2+ac)\,(dx)^{5+m}}{d^5(5+m)}+\frac{b(b^2+6ac)\,(dx)^{7+m}}{d^7(7+m)}+$$

Mathematica [A]
time = 0.39, size = 111, normalized size = 0.71

$$(dx)^m \left(\frac{a^3x}{1+m}+\frac{3a^2bx^3}{3+m}+\frac{3a(b^2+ac)\,x^5}{5+m}+\frac{b(b^2+6ac)\,x^7}{7+m}+\frac{3c(b^2+ac)\,x^9}{9+m}+\frac{3bc^2x^{11}}{11+m}+\frac{c^3x^{13}}{13+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (d*x)^m*((a^3*x)/(1 + m) + (3*a^2*b*x^3)/(3 + m) + (3*a*(b^2 + a*c)*x^5)/(5
 + m) + (b*(b^2 + 6*a*c)*x^7)/(7 + m) + (3*c*(b^2 + a*c)*x^9)/(9 + m) + (3*
b*c^2*x^11)/(11 + m) + (c^3*x^13)/(13 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(156) = 312$.
time = 0.02, size = 782, normalized size = 5.01

method	result
gosper	$\frac{x(c^3m^6x^{12}+36c^3m^5x^{12}+3bc^2m^6x^{10}+505c^3m^4x^{12}+114bc^2m^5x^{10}+3480c^3m^3x^{12}+3ac^2m^6x^8+3b^2cm^6x^8+1665bc^2m^4x^{10}+12139}{}$
risch	$\frac{x(c^3m^6x^{12}+36c^3m^5x^{12}+3bc^2m^6x^{10}+505c^3m^4x^{12}+114bc^2m^5x^{10}+3480c^3m^3x^{12}+3ac^2m^6x^8+3b^2cm^6x^8+1665bc^2m^4x^{10}+12139}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x*(c^3*m^6*x^12+36*c^3*m^5*x^12+3*b*c^2*m^6*x^10+505*c^3*m^4*x^12+114*b*c^2
*m^5*x^10+3480*c^3*m^3*x^12+3*a*c^2*m^6*x^8+3*b^2*c*m^6*x^8+1665*b*c^2*m^4*
x^10+12139*c^3*m^2*x^12+120*a*c^2*m^5*x^8+120*b^2*c*m^5*x^8+11820*b*c^2*m^3
*x^10+19524*c^3*m*x^12+6*a*b*c*m^6*x^6+1839*a*c^2*m^4*x^8+b^3*m^6*x^6+1839*
b^2*c*m^4*x^8+42117*b*c^2*m^2*x^10+10395*c^3*x^12+252*a*b*c*m^5*x^6+13584*a
*c^2*m^3*x^8+42*b^3*m^5*x^6+13584*b^2*c*m^3*x^8+68706*b*c^2*m*x^10+3*a^2*c*
m^6*x^4+3*a*b^2*m^6*x^4+4074*a*b*c*m^4*x^6+49881*a*c^2*m^2*x^8+679*b^3*m^4*
x^6+49881*b^2*c*m^2*x^8+36855*b*c^2*x^10+132*a^2*c*m^5*x^4+132*a*b^2*m^5*x^
4+31752*a*b*c*m^3*x^6+83064*a*c^2*m*x^8+5292*b^3*m^3*x^6+83064*b^2*c*m*x^8+
3*a^2*b*m^6*x^2+2259*a^2*c*m^4*x^4+2259*a*b^2*m^4*x^4+122010*a*b*c*m^2*x^6+
45045*a*c^2*x^8+20335*b^3*m^2*x^6+45045*b^2*c*x^8+138*a^2*b*m^5*x^2+18840*a
^2*c*m^3*x^4+18840*a*b^2*m^3*x^4+209916*a*b*c*m*x^6+34986*b^3*m*x^6+a^3*m^6
+2505*a^2*b*m^4*x^2+77937*a^2*c*m^2*x^4+77937*a*b^2*m^2*x^4+115830*a*b*c*x^
6+19305*b^3*x^6+48*a^3*m^5+22620*a^2*b*m^3*x^2+142308*a^2*c*m*x^4+142308*a*
b^2*m*x^4+925*a^3*m^4+104277*a^2*b*m^2*x^2+81081*a^2*c*x^4+81081*a*b^2*x^4+
9120*a^3*m^3+219162*a^2*b*m*x^2+48259*a^3*m^2+135135*a^2*b*x^2+129072*a^3*m
+135135*a^3)*(d*x)^m/(13+m)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [A]
time = 0.32, size = 195, normalized size = 1.25

$$\frac{c^3d^mx^{13}x^m}{m+13}+\frac{3\,bc^2d^mx^{11}x^m}{m+11}+\frac{3\,b^2cd^mx^9x^m}{m+9}+\frac{3\,ac^2d^mx^9x^m}{m+9}+\frac{b^3d^mx^7x^m}{m+7}+\frac{6\,abcd^mx^7x^m}{m+7}+\frac{3\,ab^2d^mx^5x^m}{m+5}+\frac{3\,a^2cd^mx^5x^m}{m+5}+\frac{3\,a^2bd^mx^3x^m}{m+3}+\frac{(dx)^{m+1}\,a^3}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

[Out] c^3*d^m*x^13*x^m/(m + 13) + 3*b*c^2*d^m*x^11*x^m/(m + 11) + 3*b^2*c*d^m*x^9*x^m/(m + 9) + 3*a*c^2*d^m*x^9*x^m/(m + 9) + b^3*d^m*x^7*x^m/(m + 7) + 6*a*b*c*d^m*x^7*x^m/(m + 7) + 3*a*b^2*d^m*x^5*x^m/(m + 5) + 3*a^2*c*d^m*x^5*x^m/(m + 5) + 3*a^2*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^3/(d*(m + 1))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(156) = 312$.
time = 0.35, size = 594, normalized size = 3.81

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] ((c^3*m^6 + 36*c^3*m^5 + 505*c^3*m^4 + 3480*c^3*m^3 + 12139*c^3*m^2 + 19524*c^3*m + 10395*c^3)*x^13 + 3*(b*c^2*m^6 + 38*b*c^2*m^5 + 555*b*c^2*m^4 + 3940*b*c^2*m^3 + 14039*b*c^2*m^2 + 22902*b*c^2*m + 12285*b*c^2)*x^11 + 3*((b^2*c + a*c^2)*m^6 + 40*(b^2*c + a*c^2)*m^5 + 613*(b^2*c + a*c^2)*m^4 + 4528*(b^2*c + a*c^2)*m^3 + 15015*b^2*c + 15015*a*c^2 + 16627*(b^2*c + a*c^2)*m^2 + 27688*(b^2*c + a*c^2)*m)*x^9 + ((b^3 + 6*a*b*c)*m^6 + 42*(b^3 + 6*a*b*c)*m^5 + 679*(b^3 + 6*a*b*c)*m^4 + 5292*(b^3 + 6*a*b*c)*m^3 + 19305*b^3 + 115830*a*b*c + 20335*(b^3 + 6*a*b*c)*m^2 + 34986*(b^3 + 6*a*b*c)*m)*x^7 + 3*((a*b^2 + a^2*c)*m^6 + 44*(a*b^2 + a^2*c)*m^5 + 753*(a*b^2 + a^2*c)*m^4 + 6280*(a*b^2 + a^2*c)*m^3 + 27027*a*b^2 + 27027*a^2*c + 25979*(a*b^2 + a^2*c)*m^2 + 47436*(a*b^2 + a^2*c)*m)*x^5 + 3*(a^2*b*m^6 + 46*a^2*b*m^5 + 835*a^2*b*m^4 + 7540*a^2*b*m^3 + 34759*a^2*b*m^2 + 73054*a^2*b*m + 45045*a^2*b)*x^3 + (a^3*m^6 + 48*a^3*m^5 + 925*a^3*m^4 + 9120*a^3*m^3 + 48259*a^3*m^2 + 129072*a^3*m + 135135*a^3)*x)*(d*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4332 vs. $2(143) = 286$.
time = 1.05, size = 4332, normalized size = 27.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**3,x)

[Out] Piecewise(((-a**3/(12*x**12) - 3*a**2*b/(10*x**10) - 3*a**2*c/(8*x**8) - 3*a*b**2/(8*x**8) - a*b*c/x**6 - 3*a*c**2/(4*x**4) - b**3/(6*x**6) - 3*b**2*c/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*log(x))/d**13, Eq(m, -13)), ((-a**3/(10*x**10) - 3*a**2*b/(8*x**8) - a**2*c/(2*x**6) - a*b**2/(2*x**6) - 3*a*b*c/(2*x**4) - 3*a*c**2/(2*x**2) - b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b*c**2*log(x) + c**3*x**2/2)/d**11, Eq(m, -11)), ((-a**3/(8*x**8) - a**2*b/(2*x**6) - 3*a**2*c/(4*x**4) - 3*a*b**2/(4*x**4) - 3*a*b*c/x**2 + 3*a*c**2*log(x)

- b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4)/d**9, Eq(m, -9)), ((-a**3/(6*x**6) - 3*a**2*b/(4*x**4) - 3*a**2*c/(2*x**2) - 3*a*b**2/(2*x**2) + 6*a*b*c*log(x) + 3*a*c**2*x**2/2 + b**3*log(x) + 3*b**2*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6)/d**7, Eq(m, -7)), ((-a**3/(4*x**4) - 3*a**2*b/(2*x**2) + 3*a**2*c*log(x) + 3*a*b**2*log(x) + 3*a*b*c*x**2 + 3*a*c**2*x**4/4 + b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 + c**3*x**8/8)/d**5, Eq(m, -5)), ((-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a**2*c*x**2/2 + 3*a*b**2*x**2/2 + 3*a*b*c*x**4/2 + a*c**2*x**6/2 + b**3*x**4/4 + b**2*c*x**6/2 + 3*b*c**2*x**8/8 + c**3*x**10/10)/d**3, Eq(m, -3)), ((a**3*log(x) + 3*a**2*b*x**2/2 + 3*a**2*c*x**4/4 + 3*a*b**2*x**4/4 + a*b*c*x**6 + 3*a*c**2*x**8/8 + b**3*x**6/6 + 3*b**2*c*x**8/8 + 3*b*c**2*x**10/10 + c**3*x**12/12)/d, Eq(m, -1)), (a**3*m**6*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48*a**3*m**5*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 925*a**3*m**4*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9120*a**3*m**3*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48259*a**3*m**2*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 129072*a**3*m*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**3*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*a**2*b*m**6*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 138*a**2*b*m**5*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 2505*a**2*b*m**4*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 22620*a**2*b*m**3*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 104277*a**2*b*m**2*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 219162*a**2*b*m*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**2*b*x**3*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*a**2*c*m**6*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 132*a**2*c*m**5*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 2259*a**2*c*m**4*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 18840*a**2*c*m**3*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 77937*a**2*c*m**2*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 142308*a**2*c*m*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 81081*a**2*c*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3*a*b**2*m**6*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**

```
5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 132*a*b**2
*m**5*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 2259*a*b**2*m**4*x**5*(d*x)**m/(m**7 + 4
9*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 1351
35) + 18840*a*b**2*m**3*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m*
*4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 77937*a*b**2*m**2*x**5
*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**
2 + 264207*m + 135135) + 142308*a*b**2*m*x**5*(d*x)**m/(m**7 + 49*m**6 + 97
3*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 81081
*a*b**2*x**5*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3
+ 177331*m**2 + 264207*m + 135135) + 6*a*b*c*m**6*x**7*(d*x)**m/(m**7 + 49*
m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135
) + 252*a*b*c*m**5*x**7*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 +
57379*m**3 + 177331*m**2 + 264207*m + 135135) + 4074*a*b*c*m**4*x**7*(d*x)*
*m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. $2(156) = 312$.
time = 3.21, size = 1132, normalized size = 7.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^3*m^6*x^13 + 36*(d*x)^m*c^3*m^5*x^13 + 3*(d*x)^m*b*c^2*m^6*x^11
+ 505*(d*x)^m*c^3*m^4*x^13 + 114*(d*x)^m*b*c^2*m^5*x^11 + 3480*(d*x)^m*c^3*
m^3*x^13 + 3*(d*x)^m*b^2*c*m^6*x^9 + 3*(d*x)^m*a*c^2*m^6*x^9 + 1665*(d*x)^m
*b*c^2*m^4*x^11 + 12139*(d*x)^m*c^3*m^2*x^13 + 120*(d*x)^m*b^2*c*m^5*x^9 +
120*(d*x)^m*a*c^2*m^5*x^9 + 11820*(d*x)^m*b*c^2*m^3*x^11 + 19524*(d*x)^m*c^
3*m*x^13 + (d*x)^m*b^3*m^6*x^7 + 6*(d*x)^m*a*b*c*m^6*x^7 + 1839*(d*x)^m*b^2
*c*m^4*x^9 + 1839*(d*x)^m*a*c^2*m^4*x^9 + 42117*(d*x)^m*b*c^2*m^2*x^11 + 10
395*(d*x)^m*c^3*x^13 + 42*(d*x)^m*b^3*m^5*x^7 + 252*(d*x)^m*a*b*c*m^5*x^7 +
 13584*(d*x)^m*b^2*c*m^3*x^9 + 13584*(d*x)^m*a*c^2*m^3*x^9 + 68706*(d*x)^m*
b*c^2*m*x^11 + 3*(d*x)^m*a*b^2*m^6*x^5 + 3*(d*x)^m*a^2*c*m^6*x^5 + 679*(d*x
)^m*b^3*m^4*x^7 + 4074*(d*x)^m*a*b*c*m^4*x^7 + 49881*(d*x)^m*b^2*c*m^2*x^9
+ 49881*(d*x)^m*a*c^2*m^2*x^9 + 36855*(d*x)^m*b*c^2*x^11 + 132*(d*x)^m*a*b^
2*m^5*x^5 + 132*(d*x)^m*a^2*c*m^5*x^5 + 5292*(d*x)^m*b^3*m^3*x^7 + 31752*(d
*x)^m*a*b*c*m^3*x^7 + 83064*(d*x)^m*b^2*c*m*x^9 + 83064*(d*x)^m*a*c^2*m*x^9
 + 3*(d*x)^m*a^2*b*m^6*x^3 + 2259*(d*x)^m*a*b^2*m^4*x^5 + 2259*(d*x)^m*a^2*
c*m^4*x^5 + 20335*(d*x)^m*b^3*m^2*x^7 + 122010*(d*x)^m*a*b*c*m^2*x^7 + 4504
5*(d*x)^m*b^2*c*x^9 + 45045*(d*x)^m*a*c^2*x^9 + 138*(d*x)^m*a^2*b*m^5*x^3 +
 18840*(d*x)^m*a*b^2*m^3*x^5 + 18840*(d*x)^m*a^2*c*m^3*x^5 + 34986*(d*x)^m*
b^3*m*x^7 + 209916*(d*x)^m*a*b*c*m*x^7 + (d*x)^m*a^3*m^6*x + 2505*(d*x)^m*a
^2*b*m^4*x^3 + 77937*(d*x)^m*a*b^2*m^2*x^5 + 77937*(d*x)^m*a^2*c*m^2*x^5 +
```

19305*(d*x)^m*b^3*x^7 + 115830*(d*x)^m*a*b*c*x^7 + 48*(d*x)^m*a^3*m^5*x + 2
2620*(d*x)^m*a^2*b*m^3*x^3 + 142308*(d*x)^m*a*b^2*m*x^5 + 142308*(d*x)^m*a^
2*c*m*x^5 + 925*(d*x)^m*a^3*m^4*x + 104277*(d*x)^m*a^2*b*m^2*x^3 + 81081*(d
*x)^m*a*b^2*x^5 + 81081*(d*x)^m*a^2*c*x^5 + 9120*(d*x)^m*a^3*m^3*x + 219162
*(d*x)^m*a^2*b*m*x^3 + 48259*(d*x)^m*a^3*m^2*x + 135135*(d*x)^m*a^2*b*x^3 +
 129072*(d*x)^m*a^3*m*x + 135135*(d*x)^m*a^3*x)/(m^7 + 49*m^6 + 973*m^5 + 1
0045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)

Mupad [B]
time = 4.83, size = 546, normalized size = 3.50

$$\frac{a^3\,x\,{\left(d\,x\right)}^m\,\left(m^6+48\,m^5+925\,m^4+9120\,m^3+48259\,m^2+129072\,m+135135\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{c^3\,x^{13}\,{\left(d\,x\right)}^m\,\left(m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{3\,a^2\,b\,x^3\,{\left(d\,x\right)}^m\,\left(m^6+46\,m^5+835\,m^4+7540\,m^3+34759\,m^2+73054\,m+45045\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{3\,b\,c^2\,x^{11}\,{\left(d\,x\right)}^m\,\left(m^6+38\,m^5+555\,m^4+3940\,m^3+14039\,m^2+22902\,m+12285\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{3\,a\,x^5\,{\left(d\,x\right)}^m\,\left(b^2+a\,c\right)\,\left(m^6+44\,m^5+753\,m^4+6280\,m^3+25979\,m^2+47436\,m+27027\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{b\,x^7\,{\left(d\,x\right)}^m\,\left(b^2+6\,a\,c\right)\,\left(m^6+42\,m^5+679\,m^4+5292\,m^3+20335\,m^2+34986\,m+19305\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{3\,c\,x^9\,{\left(d\,x\right)}^m\,\left(b^2+a\,c\right)\,\left(m^6+40\,m^5+613\,m^4+4528\,m^3+16627\,m^2+27688\,m+15015\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (a^3*x*(d*x)^m*(129072*m + 48259*m^2 + 9120*m^3 + 925*m^4 + 48*m^5 + m^6 +
135135))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6
+ m^7 + 135135) + (c^3*x^13*(d*x)^m*(19524*m + 12139*m^2 + 3480*m^3 + 505*m
^4 + 36*m^5 + m^6 + 10395))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4
+ 973*m^5 + 49*m^6 + m^7 + 135135) + (3*a^2*b*x^3*(d*x)^m*(73054*m + 34759*
m^2 + 7540*m^3 + 835*m^4 + 46*m^5 + m^6 + 45045))/(264207*m + 177331*m^2 +
57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (3*b*c^2*x^11*(d
*x)^m*(22902*m + 14039*m^2 + 3940*m^3 + 555*m^4 + 38*m^5 + m^6 + 12285))/(2
64207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135
135) + (3*a*x^5*(d*x)^m*(a*c + b^2)*(47436*m + 25979*m^2 + 6280*m^3 + 753*m
^4 + 44*m^5 + m^6 + 27027))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4
+ 973*m^5 + 49*m^6 + m^7 + 135135) + (b*x^7*(d*x)^m*(6*a*c + b^2)*(34986*m
+ 20335*m^2 + 5292*m^3 + 679*m^4 + 42*m^5 + m^6 + 19305))/(264207*m + 17733
1*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (3*c*x^9
*(d*x)^m*(a*c + b^2)*(27688*m + 16627*m^2 + 4528*m^3 + 613*m^4 + 40*m^5 + m
^6 + 15015))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*
m^6 + m^7 + 135135)
```

3.1106 $\int (dx)^m (a+bx^2+cx^4)^2 \, dx$

Optimal. Leaf size=101

$$\frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{(b^2+2ac)\,(dx)^{5+m}}{d^5(5+m)}+\frac{2bc(dx)^{7+m}}{d^7(7+m)}+\frac{c^2(dx)^{9+m}}{d^9(9+m)}$$

```
[Out] a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*(d*x)^(3+m)/d^3/(3+m)+(2*a*c+b^2)*(d*x)^(5+m)
/d^5/(5+m)+2*b*c*(d*x)^(7+m)/d^7/(7+m)+c^2*(d*x)^(9+m)/d^9/(9+m)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{1122\}$

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{(2ac+b^2)\,(dx)^{m+5}}{d^5(m+5)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+7}}{d^7(m+7)}+\frac{c^2(dx)^{m+9}}{d^9(m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + ((b
^2 + 2*a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (2*b*c*(d*x)^(7 + m))/(d^7*(7 +
m)) + (c^2*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2(dx)^m+\frac{2ab(dx)^{2+m}}{d^2}+\frac{(b^2+2ac)\,(dx)^{4+m}}{d^4}+\frac{2bc(dx)^{6+m}}{d^6}+\frac{c^2(dx)^{8+m}}{d^8}\right) dx\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{(b^2+2ac)\,(dx)^{5+m}}{d^5(5+m)}+\frac{2bc(dx)^{7+m}}{d^7(7+m)}+\frac{c^2(dx)^{9+m}}{d^9(9+m)}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 70, normalized size = 0.69

$$(dx)^m \left(\frac{a^2x}{1+m}+\frac{2abx^3}{3+m}+\frac{(b^2+2ac)\,x^5}{5+m}+\frac{2bcx^7}{7+m}+\frac{c^2x^9}{9+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (d*x)^m*((a^2*x)/(1 + m) + (2*a*b*x^3)/(3 + m) + ((b^2 + 2*a*c)*x^5)/(5 + m
) + (2*b*c*x^7)/(7 + m) + (c^2*x^9)/(9 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(101) = 202$.
time = 0.01, size = 301, normalized size = 2.98

method	result
gosper	$\frac{x(c^2m^4x^8+16c^2m^3x^8+2bc\,m^4x^6+86c^2m^2x^8+36bc\,m^3x^6+176m\,x^8c^2+2ac\,m^4x^4+b^2m^4x^4+208bc\,m^2x^6+105c^2x^8+40ac\,m^3x^4+20}{}$
risch	$\frac{x(c^2m^4x^8+16c^2m^3x^8+2bc\,m^4x^6+86c^2m^2x^8+36bc\,m^3x^6+176m\,x^8c^2+2ac\,m^4x^4+b^2m^4x^4+208bc\,m^2x^6+105c^2x^8+40ac\,m^3x^4+20}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(c^2*m^4*x^8+16*c^2*m^3*x^8+2*b*c*m^4*x^6+86*c^2*m^2*x^8+36*b*c*m^3*x^6+1
76*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^4+208*b*c*m^2*x^6+105*c^2*x^8+40*a*c*m
^3*x^4+20*b^2*m^3*x^4+444*b*c*m*x^6+2*a*b*m^4*x^2+260*a*c*m^2*x^4+130*b^2*m
^2*x^4+270*b*c*x^6+44*a*b*m^3*x^2+600*a*c*m*x^4+300*b^2*m*x^4+a^2*m^4+328*a
*b*m^2*x^2+378*a*c*x^4+189*b^2*x^4+24*a^2*m^3+916*a*b*m*x^2+206*a^2*m^2+630
*a*b*x^2+744*a^2*m+945*a^2)*(d*x)^m/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [A]
time = 0.30, size = 110, normalized size = 1.09

$$\frac{c^2d^mx^9x^m}{m+9} + \frac{2\,bcd^mx^7x^m}{m+7} + \frac{b^2d^mx^5x^m}{m+5} + \frac{2\,acd^mx^5x^m}{m+5} + \frac{2\,abd^mx^3x^m}{m+3} + \frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x^9*x^m/(m + 9) + 2*b*c*d^m*x^7*x^m/(m + 7) + b^2*d^m*x^5*x^m/(m +
5) + 2*a*c*d^m*x^5*x^m/(m + 5) + 2*a*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*
a^2/(d*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(101) = 202$.
time = 0.33, size = 241, normalized size = 2.39

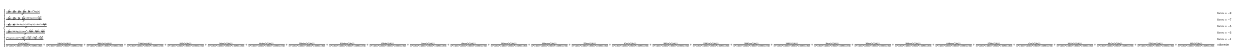
$$\frac{((c^2m^4+16\,c^2m^3+86\,c^2m^2+176\,c^2m+105\,c^2)x^9+2\,(bcm^4+18\,bcm^3+104\,bcm^2+222\,bcm+135\,bc)x^7+((b^2+2\,ac)m^4+20\,(b^2+2\,ac)m^3+130\,(b^2+2\,ac)m^2+189\,b^2+378\,ac+300\,(b^2+2\,ac)m)x^5+2\,(abm^4+22\,abm^3+164\,abm^2+458\,abm+315\,ab)x^3+(a^2m^4+24\,a^2m^3+206\,a^2m^2+744\,a^2m+945\,a^2)x)(dx)^m}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^9 + 2*(b*c*m^4
 + 18*b*c*m^3 + 104*b*c*m^2 + 222*b*c*m + 135*b*c)*x^7 + ((b^2 + 2*a*c)*m^4
 + 20*(b^2 + 2*a*c)*m^3 + 130*(b^2 + 2*a*c)*m^2 + 189*b^2 + 378*a*c + 300*(
b^2 + 2*a*c)*m)*x^5 + 2*(a*b*m^4 + 22*a*b*m^3 + 164*a*b*m^2 + 458*a*b*m + 3
15*a*b)*x^3 + (a^2*m^4 + 24*a^2*m^3 + 206*a^2*m^2 + 744*a^2*m + 945*a^2)*x)
*(d*x)^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. $2(90) = 180$.
time = 0.53, size = 1435, normalized size = 14.21



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4*x**4) -
b*c/x**2 + c**2*log(x))/d**9, Eq(m, -9)), ((-a**2/(6*x**6) - a*b/(2*x**4) -
 a*c/x**2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2)/d**7, Eq(m, -7)), (
(-a**2/(4*x**4) - a*b/x**2 + 2*a*c*log(x) + b**2*log(x) + b*c*x**2 + c**2*x
**4/4)/d**5, Eq(m, -5)), ((-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 + b**2*
x**2/2 + b*c*x**4/2 + c**2*x**6/6)/d**3, Eq(m, -3)), ((a**2*log(x) + a*b*x*
*2 + a*c*x**4/2 + b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8)/d, Eq(m, -1)), (a
**2*m**4*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
 24*a**2*m**3*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 206*a**2*m**2*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 744*a**2*m*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 945*a**2*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
1689*m + 945) + 2*a*b*m**4*x**3*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 44*a*b*m**3*x**3*(d*x)**m/(m**5 + 25*m**4 + 230*m**3
+ 950*m**2 + 1689*m + 945) + 328*a*b*m**2*x**3*(d*x)**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945) + 916*a*b*m*x**3*(d*x)**m/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a*b*x**3*(d*x)**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 2*a*c*m**4*x**5*(d*x)**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 40*a*c*m**3*x**5*(d*x)**
m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 260*a*c*m**2*x**5
*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 600*a*c*m
*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 378*
a*c*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b
**2*m**4*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945
) + 20*b**2*m**3*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 130*b**2*m**2*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 300*b**2*m*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 +
950*m**2 + 1689*m + 945) + 189*b**2*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**
```

```
3 + 950*m**2 + 1689*m + 945) + 2*b*c*m**4*x**7*(d*x)**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945) + 36*b*c*m**3*x**7*(d*x)**m/(m**5 + 25*m
**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 208*b*c*m**2*x**7*(d*x)**m/(m**
5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 444*b*c*m*x**7*(d*x)**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 270*b*c*x**7*(d*x)
**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + c**2*m**4*x**9*
(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*c**2*m*
*3*x**9*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86
*c**2*m**2*x**9*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 176*c**2*m*x**9*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 105*c**2*x**9*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
1689*m + 945), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(101) = 202.
time = 3.74, size = 449, normalized size = 4.45

$$\frac{\left(dx\right)^{m} c^{2} m^{4} x^{9} + 16 \left(dx\right)^{m} c^{2} m^{3} x^{9} + 2 \left(dx\right)^{m} b c m^{4} x^{7} + 86 \left(dx\right)^{m} c^{2} m^{2} x^{9} + 36 \left(dx\right)^{m} b c m^{3} x^{7} + 176 \left(dx\right)^{m} c^{2} m x^{9} + \left(dx\right)^{m} b^{2} m^{4} x^{5} + 2 \left(dx\right)^{m} a c m^{4} x^{5} + 208 \left(dx\right)^{m} b c m^{2} x^{7} + 105 \left(dx\right)^{m} c^{2} x^{9} + 20 \left(dx\right)^{m} b^{2} m^{3} x^{5} + 40 \left(dx\right)^{m} a c m^{3} x^{5} + 444 \left(dx\right)^{m} b c m x^{7} + 2 \left(dx\right)^{m} a b m^{4} x^{3} + 130 \left(dx\right)^{m} b^{2} m^{2} x^{5} + 260 \left(dx\right)^{m} a c m^{2} x^{5} + 270 \left(dx\right)^{m} b c x^{7} + 44 \left(dx\right)^{m} a b m^{3} x^{3} + 300 \left(dx\right)^{m} b^{2} m x^{5} + 600 \left(dx\right)^{m} a c m x^{5} + \left(dx\right)^{m} a^{2} m^{4} x + 328 \left(dx\right)^{m} a b m^{2} x^{3} + 189 \left(dx\right)^{m} b^{2} x^{5} + 378 \left(dx\right)^{m} a c x^{5} + 24 \left(dx\right)^{m} a^{2} m^{3} x + 916 \left(dx\right)^{m} a b m x^{3} + 206 \left(dx\right)^{m} a^{2} m^{2} x + 630 \left(dx\right)^{m} a b x^{3} + 744 \left(dx\right)^{m} a^{2} m x + 945 \left(dx\right)^{m} a^{2} x}{m^{5} + 25 m^{4} + 230 m^{3} + 950 m^{2} + 1689 m + 945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^2*m^4*x^9 + 16*(d*x)^m*c^2*m^3*x^9 + 2*(d*x)^m*b*c*m^4*x^7 + 86*
(d*x)^m*c^2*m^2*x^9 + 36*(d*x)^m*b*c*m^3*x^7 + 176*(d*x)^m*c^2*m*x^9 + (d*x
)^m*b^2*m^4*x^5 + 2*(d*x)^m*a*c*m^4*x^5 + 208*(d*x)^m*b*c*m^2*x^7 + 105*(d*
x)^m*c^2*x^9 + 20*(d*x)^m*b^2*m^3*x^5 + 40*(d*x)^m*a*c*m^3*x^5 + 444*(d*x)^
m*b*c*m*x^7 + 2*(d*x)^m*a*b*m^4*x^3 + 130*(d*x)^m*b^2*m^2*x^5 + 260*(d*x)^m
*a*c*m^2*x^5 + 270*(d*x)^m*b*c*x^7 + 44*(d*x)^m*a*b*m^3*x^3 + 300*(d*x)^m*b
^2*m*x^5 + 600*(d*x)^m*a*c*m*x^5 + (d*x)^m*a^2*m^4*x + 328*(d*x)^m*a*b*m^2*
x^3 + 189*(d*x)^m*b^2*x^5 + 378*(d*x)^m*a*c*x^5 + 24*(d*x)^m*a^2*m^3*x + 91
6*(d*x)^m*a*b*m*x^3 + 206*(d*x)^m*a^2*m^2*x + 630*(d*x)^m*a*b*x^3 + 744*(d*
x)^m*a^2*m*x + 945*(d*x)^m*a^2*x)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*
m + 945)
```

Mupad [B]
time = 4.58, size = 260, normalized size = 2.57

$$(d\,x)^m\,\left(\frac{c^2\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{x^5\,\left(b^2+2\,a\,c\right)\,\left(m^4+20\,m^3+130\,m^2+300\,m+189\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{a^2\,x\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{2\,a\,b\,x^3\,\left(m^4+22\,m^3+164\,m^2+458\,m+315\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{2\,b\,c\,x^7\,\left(m^4+18\,m^3+104\,m^2+222\,m+135\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (d*x)^m*((c^2*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2
+ 230*m^3 + 25*m^4 + m^5 + 945) + (x^5*(2*a*c + b^2)*(300*m + 130*m^2 + 20*
m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (a^2*
x*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^2 + 230*m^3 + 25*
```

```
m^4 + m^5 + 945) + (2*a*b*x^3*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689
*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (2*b*c*x^7*(222*m + 104*m^2
+ 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))
```

3.1107 $\int (dx)^m (a+bx^2+cx^4)\,dx$

Optimal. Leaf size=52

$$\frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{3+m}}{d^3(3+m)}+\frac{c(dx)^{5+m}}{d^5(5+m)}$$

```
[Out] a*(d*x)^(1+m)/d/(1+m)+b*(d*x)^(3+m)/d^3/(3+m)+c*(d*x)^(5+m)/d^5/(5+m)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+3}}{d^3(m+3)}+\frac{c(dx)^{m+5}}{d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(3 + m))/(d^3*(3 + m)) + (c*(d*x)^
(5 + m))/(d^5*(5 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+bx^2+cx^4\right)\,dx &= \int \left(a(dx)^m+\frac{b(dx)^{2+m}}{d^2}+\frac{c(dx)^{4+m}}{d^4}\right)\,dx\\ &= \frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{3+m}}{d^3(3+m)}+\frac{c(dx)^{5+m}}{d^5(5+m)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 35, normalized size = 0.67

$$x(dx)^m\left(\frac{a}{1+m}+\frac{bx^2}{3+m}+\frac{cx^4}{5+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4),x]
[Out] x*(d*x)^m*(a/(1 + m) + (b*x^2)/(3 + m) + (c*x^4)/(5 + m))
```

Maple [A]
time = 0.01, size = 51, normalized size = 0.98

method	result	size
norman	$\frac{a x\,\mathrm{e}^{m\ln(dx)}}{1+m}+\frac{b\,x^{3}\mathrm{e}^{m\ln(dx)}}{3+m}+\frac{c\,x^{5}\mathrm{e}^{m\ln(dx)}}{5+m}$	51
gosper	$\frac{x\left(c\,m^{2}x^{4}+4cm\,x^{4}+b\,m^{2}x^{2}+3c\,x^{4}+6bm\,x^{2}+a\,m^{2}+5b\,x^{2}+8am+15a\right)(dx)^{m}}{(5+m)(3+m)(1+m)}$	78
risch	$\frac{x\left(c\,m^{2}x^{4}+4cm\,x^{4}+b\,m^{2}x^{2}+3c\,x^{4}+6bm\,x^{2}+a\,m^{2}+5b\,x^{2}+8am+15a\right)(dx)^{m}}{(5+m)(3+m)(1+m)}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] a/(1+m)*x*exp(m*ln(d*x))+b/(3+m)*x^3*exp(m*ln(d*x))+c/(5+m)*x^5*exp(m*ln(d*
x))
```

Maxima [A]
time = 0.30, size = 50, normalized size = 0.96

$$\frac{cd^{m}x^{5}x^{m}}{m+5}+\frac{bd^{m}x^{3}x^{m}}{m+3}+\frac{(dx)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] c*d^m*x^5*x^m/(m + 5) + b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Fricas [A]
time = 0.36, size = 71, normalized size = 1.37

$$\frac{\left((cm^{2}+4\,cm+3\,c)x^{5}+(bm^{2}+6\,bm+5\,b)x^{3}+(am^{2}+8\,am+15\,a)x\right)(dx)^{m}}{m^{3}+9\,m^{2}+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] ((c*m^2 + 4*c*m + 3*c)*x^5 + (b*m^2 + 6*b*m + 5*b)*x^3 + (a*m^2 + 8*a*m + 1
5*a)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(42) = 84$.
time = 0.24, size = 299, normalized size = 5.75

$$\begin{cases} \frac{-\frac{a}{4x^{4}}-\frac{b}{2x^{2}}+c\log(x)}{d^{5}} & \text{for}\: m=-5 \\ \frac{-\frac{a}{2x^{2}}+b\log(x)+\frac{cx^{2}}{2}}{d^{3}} & \text{for}\: m=-3 \\ \frac{a\log(x)+\frac{bx^{2}}{2}+\frac{cx^{4}}{4}}{d} & \text{for}\: m=-1 \\ \frac{am^{2}x(dx)^{m}}{m^{3}+9m^{2}+23m+15}+\frac{8amx(dx)^{m}}{m^{3}+9m^{2}+23m+15}+\frac{15ax(dx)^{m}}{m^{3}+9m^{2}+23m+15}+\frac{bm^{2}x^{3}(dx)^{m}}{m^{3}+9m^{2}+23m+15}+\frac{6bmx^{3}(dx)^{m}}{m^{3}+9m^{2}+23m+15}+\frac{5bx^{3}(dx)^{m}}{m^{3}+9m^{2}+23m+15}+\frac{cm^{2}x^{5}(dx)^{m}}{m^{3}+9m^{2}+23m+15}+\frac{4cmx^{5}(dx)^{m}}{m^{3}+9m^{2}+23m+15}+\frac{3cx^{5}(dx)^{m}}{m^{3}+9m^{2}+23m+15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-a/(4*x**4) - b/(2*x**2) + c*log(x))/d**5, Eq(m, -5)), ((-a/(2*
x**2) + b*log(x) + c*x**2/2)/d**3, Eq(m, -3)), ((a*log(x) + b*x**2/2 + c*x*
*4/4)/d, Eq(m, -1)), (a*m**2*x*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 8*a*m
*x*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a*x*(d*x)**m/(m**3 + 9*m**2 +
23*m + 15) + b*m**2*x**3*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 6*b*m*x**3*
(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 5*b*x**3*(d*x)**m/(m**3 + 9*m**2 + 2
3*m + 15) + c*m**2*x**5*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 4*c*m*x**5*(
d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 3*c*x**5*(d*x)**m/(m**3 + 9*m**2 + 23
*m + 15), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(52) = 104$.
time = 3.96, size = 119, normalized size = 2.29

$$\frac{(dx)^m cm^2x^5 + 4\,(dx)^m cmx^5 + (dx)^m bm^2x^3 + 3\,(dx)^m cx^5 + 6\,(dx)^m bmx^3 + (dx)^m am^2x + 5\,(dx)^m bx^3 + 8\,(dx)^m amx + 15\,(dx)^m ax}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*c*m^2*x^5 + 4*(d*x)^m*c*m*x^5 + (d*x)^m*b*m^2*x^3 + 3*(d*x)^m*c*x^
5 + 6*(d*x)^m*b*m*x^3 + (d*x)^m*a*m^2*x + 5*(d*x)^m*b*x^3 + 8*(d*x)^m*a*m*x
 + 15*(d*x)^m*a*x)/(m^3 + 9*m^2 + 23*m + 15)
```

Mupad [B]
time = 4.40, size = 89, normalized size = 1.71

$$(d\,x)^m\,\left(\frac{b\,x^3\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}+\frac{c\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15}+\frac{a\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4),x)
```

```
[Out] (d*x)^m*((b*x^3*(6*m + m^2 + 5))/(23*m + 9*m^2 + m^3 + 15) + (c*x^5*(4*m +
m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (a*x*(8*m + m^2 + 15))/(23*m + 9*m^2
+ m^3 + 15))
```

3.1108 $\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=173

$$\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)}-\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

```
[Out] 2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^
2)^(1/2)))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*(d*x)^(1+m
)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(
1+m)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.17, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1145, 371}

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m
)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(
1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1145

```
Int[((d_.)*(x_))^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^2), x]
, x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx = \frac{c\int \frac{(dx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{}{b+\sqrt{}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.08, size = 82, normalized size = 0.47

$$\frac{(dx)^m\text{RootSum}\left[a+b\#1^2+c\#1^4\&,\frac{{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1+2c\#1^3}\&\right]}{2m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d*x)^m*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -
(#1/(x - #1))]/((x/(x - #1))^m*(b*#1 + 2*c*#1^3)) & ])/(2*m)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a),x)
```

```
[Out] int((d*x)^m/(c*x^4+b*x^2+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a),x)
[Out] Integral((d*x)**m/(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{c\,x^4 + b\,x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^2 + c*x^4),x)
[Out] int((d*x)^m/(a + b*x^2 + c*x^4), x)
```

3.1109 $\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=315

$$\frac{(dx)^{1+m}\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)d\left(a+bx^2+cx^4\right)}+\frac{c\left(b^2(1-m)+b\sqrt{b^2-4ac}\,(1-m)-4ac(3-m)\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};3\right.}{2a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)}$$

```
[Out] 1/2*(d*x)^(1+m)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^4+b*x^2+a)-1/2*c*
(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(
1/2)))*(b^2*(1-m)-4*a*c*(3-m)-b*(1-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3
/2)/d/(1+m)/(b+(-4*a*c+b^2)^(1/2))+1/2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*
m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(b^2*(1-m)-4*a*c*(3-m)+b*(1
-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))
```

Rubi [A]

time = 0.46, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1135, 1299, 371}

$$\frac{c(dx)^{m+1}\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^2\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c
*x^4)) + (c*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*(d*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
 - (c*(b^2*(1 - m) - b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
```

```
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^2)}{2a(b^2-4ac)d(a+bx^2+cx^4)} - \frac{\int \frac{(dx)^m(-b^2(1-m)+2ac(3-m)-bc(1-m)x^2)}{a+bx^2+cx^4}\,dx}{2a(b^2-4ac)}$$

$$= \frac{(dx)^{1+m}(b^2-2ac+bcx^2)}{2a(b^2-4ac)d(a+bx^2+cx^4)} - \frac{\left(c\left(b^2(1-m)-b\sqrt{b^2-4ac}\,(1-m)-4ac(3-\right.\right.}{4a(b^2-4ac)^{3/2}}$$

$$= \frac{(dx)^{1+m}(b^2-2ac+bcx^2)}{2a(b^2-4ac)d(a+bx^2+cx^4)} + \frac{c\left(b^2(1-m)+b\sqrt{b^2-4ac}\,(1-m)-4ac(3-m\right.}{2a(b^2-4ac)^{3/2}\left(b-\sqrt{\right.}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.10, size = 78, normalized size = 0.25

$$\frac{x(dx)^m F_1\left(\frac{1+m}{2};2,2;\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{a^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(d*x)^m*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(a^2*(1 + m))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(c\,x^4+b\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] int((d*x)^m/(c*x^4+b*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2 + c*x**4)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^m}{{\left(c\,x^4+b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^2 + c*x^4)^2,x)
[Out] int((d*x)^m/(a + b*x^2 + c*x^4)^2, x)
```

3.1110 $\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=158

$$\frac{a(dx)^{1+m}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1+m}{2};-\frac{3}{2},-\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] a*(d*x)^(1+m)*AppellF1(1/2+1/2*m,-3/2,-3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^
2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+m)/(1
+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1155, 524}

$$\frac{a(dx)^{m+1}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2, -3/2, (3
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
```

```
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2}\,dx = \frac{\left(a\sqrt{a+bx^2+cx^4}\right)\int (dx)^m \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{a(dx)^{1+m}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1+m}{2};-\frac{3}{2},-\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2c}{b+\sqrt{b^2}}\right.}{d(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(158) = 316.
time = 0.69, size = 357, normalized size = 2.26

$$\frac{x(dx)^m\sqrt{a+bx^2+cx^4}\left(a(15+8m+m^2)F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+(1+m)x^2\left(b(5+m)F_1\left(\frac{3+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{5+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+c(3+m)x^2F_1\left(\frac{5+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{7+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{(1+m)(3+m)(5+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^2 + c*x^4]*(a*(15 + 8*m + m^2)*AppellF1[(1 + m)/2,
-1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])] + (1 + m)*x^2*(b*(5 + m)*AppellF1[(3 + m)/2, -1/2, -1/2
, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
 4*a*c])] + c*(3 + m)*x^2*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 +
m)*(3 + m)*(5 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((d*x)**m*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,x\right)}^m\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((d*x)^m*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.1111 $\int (dx)^m \sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{1+m}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/2+1/2*m,-1/2,-1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/d/(1+m)/(1+2
*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/
2)
```

Rubi [A]
time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1155, 524}

$$\frac{(dx)^{m+1}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3 +
 m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
```

```
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int (dx)^m \sqrt{a+bx^2+cx^4}\,dx = \frac{\sqrt{a+bx^2+cx^4}\int (dx)^m \sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{(dx)^{1+m}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-}}\right.}{d(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**A**]
time = 0.50, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3 + m)/
2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,x\right)}^m\,\sqrt{c\,x^4+b\,x^2+a}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d*x)^m*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.1112 $\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{2};\frac{1}{2},\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/2+1/2*m,1/2,1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(1+m)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1155, 524}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 +
m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\cdots}}}{\sqrt{a+bx^2+cx^4}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{2};\frac{1}{2},\frac{1}{2};\frac{3+m}{2};-\frac{2c\cdots}{b-\sqrt{b^2\cdots}}\right.}{d(1+m)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 0.74, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{2};\frac{1}{2},\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1
 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((d*x)^m/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.1113 $\int \frac{(dx)^m}{(a+bx^2+cx^4)^{3/2}} dx$

Optimal. Leaf size=160

$$\frac{(dx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{2};\frac{3}{2},\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/2+1/2*m,3/2,3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(1+m)/(c*x^4+b*x^2+a)^(1/
2)
```

Rubi [A]
time = 0.10, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1155, 524}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*(1
+ m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
```

```
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{}{b+}\right.}}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{2};\frac{3}{2},\frac{3}{2};\frac{3+m}{2};-\frac{}{b-\sqrt{}}\right.}{ad(1+m)\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 8.68, size = 221, normalized size = 1.38

$$\frac{x(dx)^m\left(-b+\sqrt{b^2-4ac}-2cx^2\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}F_1\left(\frac{1+m}{2};\frac{3}{2},\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{\left(-b+\sqrt{b^2-4ac}\right)(1+m)\left(a+bx^2+cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(d*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqr
t[b^2 - 4*a*c])*(1 + m)*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*
c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((d*x)^m/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.1114 $\int (dx)^m \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=155

$$\frac{(dx)^{1+m}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1+m}{2};-p,-p;\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-}}\right.}{d(1+m)}$$

```
[Out] (d*x)^(1+m)*(c*x^4+b*x^2+a)^p*AppellF1(1/2+1/2*m,-p,-p,3/2+1/2*m,-2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x^2/
(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.07, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1155, 524}

$$\frac{(dx)^{m+1}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{m+1}{2};-p,-p;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2 + c*x^4)^p*AppellF1[(1 + m)/2, -p, -p, (3 + m)/2,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
d*(1 + m)*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int (\cdots$$

$$= \frac{(dx)^{1+m}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1 \cdots}{d(1+m)}$$

Mathematica [A]
time = 0.19, size = 179, normalized size = 1.15

$$\frac{x(dx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1+m}{2};-p,-p;\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x*(d*x)^m*(a + b*x^2 + c*x^4)^p*AppellF1[(1 + m)/2, -p, -p, (3 + m)/2, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((1 +
 m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m \left(c\,x^4+b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(c\,x^4+b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^2 + c*x^4)^p,x)
```

```
[Out] int((d*x)^m*(a + b*x^2 + c*x^4)^p, x)
```

3.1115 $\int x^7(a+bx^2+cx^4)^p \, dx$

Optimal. Leaf size=257

$$\frac{x^4(a+bx^2+cx^4)^{1+p}}{4c(2+p)}+\frac{(b^2(2+p)(3+p)-2ac(3+2p)-2bc(1+p)(3+p)x^2)\,(a+bx^2+cx^4)^{1+p}}{8c^3(1+p)(2+p)(3+2p)}-\frac{2^{-2+p}b(6}{}$$

```
[Out] 1/4*x^4*(c*x^4+b*x^2+a)^(1+p)/c/(2+p)+1/8*(b^2*(2+p)*(3+p)-2*a*c*(3+2*p)-2*
b*c*(1+p)*(3+p)*x^2)*(c*x^4+b*x^2+a)^(1+p)/c^3/(2+p)/(2*p^2+5*p+3)-2^(-2+p)
*b*(6*a*c-b^2*(3+p))*(c*x^4+b*x^2+a)^(1+p)*hypergeom([-p, 1+p],[2+p],1/2*(2
*c*x^2+(-4*a*c+b^2)^(1/2)+b)/(-4*a*c+b^2)^(1/2))*((-2*c*x^2+(-4*a*c+b^2)^(1
/2)-b)/(-4*a*c+b^2)^(1/2))^(-1-p)/c^3/(1+p)/(3+2*p)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1128, 756, 793, 638}

$$-\frac{b2^{p-2}(6ac-b^2(p+3))\,(a+bx^2+cx^4)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^3(p+1)(2p+3)\sqrt{b^2-4ac}}+\frac{(-2ac(2p+3)+b^2(p+2)(p+3)-2bc(p+1)(p+3)x^2)\,(a+bx^2+cx^4)^{p+1}}{8c^3(p+1)(p+2)(2p+3)}+\frac{x^4(a+bx^2+cx^4)^{p+1}}{4c(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(1 + p))/(4*c*(2 + p)) + ((b^2*(2 + p)*(3 + p) - 2
*a*c*(3 + 2*p) - 2*b*c*(1 + p)*(3 + p)*x^2)*(a + b*x^2 + c*x^4)^(1 + p))/(8
*c^3*(1 + p)*(2 + p)*(3 + 2*p)) - (2^(-2 + p)*b*(6*a*c - b^2*(3 + p))*(-((b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c
*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c^3*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
```

```
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^7\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{2}\text{Subst}\left(\int x^3\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)$$
$$=\frac{x^4\left(a+bx^2+cx^4\right)^{1+p}}{4c(2+p)}+\frac{\text{Subst}\left(\int x(-2a-b(3+p)x)\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)}{4c(2+p)}$$
$$=\frac{x^4\left(a+bx^2+cx^4\right)^{1+p}}{4c(2+p)}+\frac{\left(b^2(2+p)(3+p)-2ac(3+2p)-2bc(1+p)(3+p)x^2\right.}{8c^3(1+p)(2+p)(3+2p)}$$
$$=\frac{x^4\left(a+bx^2+cx^4\right)^{1+p}}{4c(2+p)}+\frac{\left(b^2(2+p)(3+p)-2ac(3+2p)-2bc(1+p)(3+p)x^2\right.}{8c^3(1+p)(2+p)(3+2p)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.38, size = 162, normalized size = 0.63

$$\frac{1}{8}x^8\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(4;-p,-p;5;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^8*(a + b*x^2 + c*x^4)^p*AppellF1[4, -p, -p, 5, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(8*((b - Sqrt[b^2 - 4*a*c]
```

```
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{7} \left(c\,x^{4}+b\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x^4+b*x^2+a)^p,x)
[Out] int(x^7*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^7, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^p*x^7, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{7} \left(a + bx^{2} + cx^{4}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*x**4+b*x**2+a)**p,x)
[Out] Integral(x**7*(a + b*x**2 + c*x**4)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^7 \left(c x^4 + b x^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x^2 + c*x^4)^p,x)
```

```
[Out] int(x^7*(a + b*x^2 + c*x^4)^p, x)
```

3.1116 $\int x^5(a+bx^2+cx^4)^p\,dx$

Optimal. Leaf size=223

$$-\frac{b(2+p)\left(a+bx^2+cx^4\right)^{1+p}}{4c^2(1+p)(3+2p)}+\frac{x^2\left(a+bx^2+cx^4\right)^{1+p}}{2c(3+2p)}+\frac{2^{-1+p}\left(2ac-b^2(2+p)\right)\left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p}(a}{c^2\sqrt{b^2-4a}}$$

```
[Out] -1/4*b*(2+p)*(c*x^4+b*x^2+a)^(1+p)/c^2/(2*p^2+5*p+3)+1/2*x^2*(c*x^4+b*x^2+a
)^(1+p)/c/(3+2*p)+2^(-1+p)*(2*a*c-b^2*(2+p))*(c*x^4+b*x^2+a)^(1+p)*hypergeo
m([-p, 1+p],[2+p],1/2*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/(-4*a*c+b^2)^(1/2))*((
-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(-1-p)/c^2/(1+p)/(3+2*p)
/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1128, 756, 654, 638}

$$\frac{2^{p-1}\left(2ac-b^2(p+2)\right)\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}}-\frac{b(p+2)\left(a+bx^2+cx^4\right)^{p+1}}{4c^2(p+1)(2p+3)}+\frac{x^2\left(a+bx^2+cx^4\right)^{p+1}}{2c(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] -1/4*(b*(2 + p)*(a + b*x^2 + c*x^4)^(1 + p))/(c^2*(1 + p)*(3 + 2*p)) + (x^2
*(a + b*x^2 + c*x^4)^(1 + p))/(2*c*(3 + 2*p)) + (2^(-1 + p)*(2*a*c - b^2*(2
 + p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a
 + b*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)
*(3 + 2*p))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^5\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{2}\text{Subst}\left(\int x^2\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)$$
$$=\frac{x^2\left(a+bx^2+cx^4\right)^{1+p}}{2c(3+2p)}+\frac{\text{Subst}\left(\int(-a-b(2+p)x)\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)}{2c(3+2p)}$$
$$=-\frac{b(2+p)\left(a+bx^2+cx^4\right)^{1+p}}{4c^2(1+p)(3+2p)}+\frac{x^2\left(a+bx^2+cx^4\right)^{1+p}}{2c(3+2p)}-\frac{\left(2ac-b^2(2+p)\right)\text{Subs}}{4c}$$
$$=-\frac{b(2+p)\left(a+bx^2+cx^4\right)^{1+p}}{4c^2(1+p)(3+2p)}+\frac{x^2\left(a+bx^2+cx^4\right)^{1+p}}{2c(3+2p)}+\frac{2^{-1+p}\left(2ac-b^2(2+p)\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.34, size = 162, normalized size = 0.73

$$\frac{1}{6}x^6\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(3;-p,-p;4;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^6*(a + b*x^2 + c*x^4)^p*AppellF1[3, -p, -p, 4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(6*((b - Sqrt[b^2 - 4*a*c]
```

```
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{5} \left(c\,x^{4}+b\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^p,x)
[Out] int(x^5*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^p*x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5} \left(a + bx^{2} + cx^{4}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**p,x)
[Out] Integral(x**5*(a + b*x**2 + c*x**4)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\left(c\,x^4+b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2 + c*x^4)^p,x)
```

```
[Out] int(x^5*(a + b*x^2 + c*x^4)^p, x)
```

3.1117 $\int x^3(a+bx^2+cx^4)^p\,dx$

Optimal. Leaf size=160

$$\frac{(a+bx^2+cx^4)^{1+p}}{4c(1+p)}+\frac{2^{-1+p}b\left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^2+cx^4)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4a}}{2\sqrt{b^2-}}\right.}{c\sqrt{b^2-4ac}\,(1+p)}$$

```
[Out] 1/4*(c*x^4+b*x^2+a)^(1+p)/c/(1+p)+2^(-1+p)*b*(c*x^4+b*x^2+a)^(1+p)*hypergeo
m([-p, 1+p],[2+p],1/2*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/(-4*a*c+b^2)^(1/2))*((
-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2))^(-1-p)/c/(1+p)/(-4*a*c+b
^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1128, 654, 638}

$$\frac{b2^{p-1}(a+bx^2+cx^4)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\frac{(a+bx^2+cx^4)^{p+1}}{4c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(4*c*(1 + p)) + (2^(-1 + p)*b*(-((b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c*x^4)^(1 + p
)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(2*
Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{2}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)$$

$$=\frac{\left(a+bx^2+cx^4\right)^{1+p}}{4c(1+p)}-\frac{b\text{Subst}\left(\int\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)}{4c}$$

$$=\frac{\left(a+bx^2+cx^4\right)^{1+p}}{4c(1+p)}+\frac{2^{-1+p}b\left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^2+cx^4\right)^{1+p}\ {}_2F}{c\sqrt{b^2-4ac}\,(1+p}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.25, size = 162, normalized size = 1.01

$$\frac{1}{4}x^4\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^pF_1\left(2;-p,-p;3;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^4*(a + b*x^2 + c*x^4)^p*AppellF1[2, -p, -p, 3, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^3\left(c\,x^4+b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^3*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^p*x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+bx^{2}+cx^{4}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**p,x)
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(c\,x^4+b\,x^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2 + c*x^4)^p,x)
[Out] int(x^3*(a + b*x^2 + c*x^4)^p, x)
```

3.1118 $\int x(a+bx^2+cx^4)^p\,dx$

Optimal. Leaf size=126

$$-\frac{2^p\left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^2+cx^4)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2cx^2}{2\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(1+p)}$$

```
[Out] -2^p*(c*x^4+b*x^2+a)^(1+p)*hypergeom([-p, 1+p],[2+p],1/2*(2*c*x^2+(-4*a*c+b
^2)^(1/2)+b)/(-4*a*c+b^2)^(1/2))*((-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b
^2)^(1/2))^(-1-p)/(1+p)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1121, 638}

$$-\frac{2^p\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx^2+cx^4)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] -((2^p*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a
 + b*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int x(a+bx^2+cx^4)^p\,dx = \frac{1}{2}\text{Subst}\left(\int (a+bx+cx^2)^p\,dx, x, x^2\right)$$
$$= -\frac{2^p\left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^2+cx^4)^{1+p}\ {}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2}}{2\sqrt{b}}\right.}{\sqrt{b^2-4ac}\,(1+p)}$$

Mathematica [A]
time = 0.10, size = 135, normalized size = 1.07

$$\frac{2^{-2+p}\left(b-\sqrt{b^2-4ac}+2cx^2\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\ {}_2F_1\left(-p,1+p;2+p;\frac{-b+\sqrt{b^2-4ac}-2cx^2}{2\sqrt{b^2-4ac}}\right)}{c(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2 + c*x^4)^p,x]
[Out] (2^(-2 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(a + b*x^2 + c*x^4)^p*Hyperge
ometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)/(2*Sqrt[b^2
 - 4*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c
])^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x\left(c\,x^4+b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^p,x)
[Out] int(x*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^p*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a + bx^{2} + cx^{4}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**p,x)
[Out] Integral(x*(a + b*x**2 + c*x**4)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(c\,x^4+b\,x^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2 + c*x^4)^p,x)
[Out] int(x*(a + b*x^2 + c*x^4)^p, x)
```

3.1119 $\int \frac{(a+bx^2+cx^4)^p}{x}\,dx$

Optimal. Leaf size=152

$$\frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2}\right.}{p}$$

```
[Out] 4^(-1+p)*(c*x^4+b*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,1/2*(-b-(-4*a*c+b^2)^(
1/2))/c/x^2,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^2)/p/(((2*c*x^2-(-4*a*c+b^2)^(1
/2)+b)/c/x^2)^p)/(((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/c/x^2)^p)
```

Rubi [A]
time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1128, 772, 138}

$$\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x,x]
```

```
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b -
 Sqrt[b^2 - 4*a*c])/(c*x^2), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^2)])/(p*((b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(c*x^2))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p)), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
- q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
+ e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m
, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^p}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^p}{x}\,dx, x, x^2\right)$$

$$= -\left(\left(2^{-1+2p}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(\right.\right.$$

$$= \frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-2p;\right.}{p}$$

Mathematica [A]
time = 0.22, size = 152, normalized size = 1.00

$$\frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{-b+\sqrt{b^2-4ac}}{2cx^2}\right)}{p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x,x]
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b +
 Sqrt[b^2 - 4*a*c])/(c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/(p*((b -
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
(c*x^2))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^4+b\,x^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x,x)
[Out] int((c*x^4+b*x^2+a)^p/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^p/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x,x)
[Out] Integral((a + b*x**2 + c*x**4)**p/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^p/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^4 + b\,x^2 + a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p/x,x)
[Out] int((a + b*x^2 + c*x^4)^p/x, x)
```

3.1120 $\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx$

Optimal. Leaf size=166

$$-\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b}{}\right.}{(1-2p)x^2}$$

```
[Out] -2^(-1+2*p)*(c*x^4+b*x^2+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,1/2*(-b-(-4*a*c+b^
2)^(1/2))/c/x^2,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^2)/(1-2*p)/x^2/(((2*c*x^2-(
-4*a*c+b^2)^(1/2)+b)/c/x^2)^p)/(((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/c/x^2)^p)
```

Rubi [A]
time = 0.10, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1128, 772, 138}

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-2p)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^3,x]
```

```
[Out] -((2^(-1 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p),
-1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^2), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^2)]
)/((1 - 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p)), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
- q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
+ e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m
, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^p}{x^2}\,dx, x, x^2\right)$$

$$= -\left(\left(2^{-1+2p}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a\right.\right.$$

$$= -\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(1-\right.}{(1-2p)x^2}$$

Mathematica [**A**]
time = 0.24, size = 163, normalized size = 0.98

$$\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(1-2p;-p,-p;2-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{-b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(-1+2p)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^3,x]
[Out] (2^(-1 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2
*(b + Sqrt[b^2 - 4*a*c])/(c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((-1
 + 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(c*x^2))^p)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^4+b\,x^2+a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^3,x)
[Out] int((c*x^4+b*x^2+a)^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^p/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^p}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p/x^3,x)
[Out] int((a + b*x^2 + c*x^4)^p/x^3, x)
```

3.1121 $\int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx$

Optimal. Leaf size=164

$$-\frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{}}{}\right.}{(1-p)x^4}$$

```
[Out] -4^(-1+p)*(c*x^4+b*x^2+a)^p*AppellF1(2-2*p,-p,-p,3-2*p,1/2*(-b-(-4*a*c+b^2)
^(1/2))/c/x^2,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^2)/(1-p)/x^4/(((2*c*x^2-(-4*a
*c+b^2)^(1/2)+b)/c/x^2)^p)/(((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/c/x^2)^p)
```

Rubi [A]

time = 0.10, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1128, 772, 138}

$$-\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-p)x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^5,x]
```

```
[Out] -((4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[2*(1 - p), -p, -p, 3 - 2*p, -1
/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^2), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^2)])/
((1 - p)*x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(c*x^2))^p))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p)), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
- q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
+ e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m
, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^p}{x^3}\,dx, x, x^2\right)$$

$$= -\left(\left(2^{-1+2p}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\right.\right.$$

$$= -\frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(2(1\right.}{(1-p)x^4}$$

Mathematica [A]
time = 0.27, size = 159, normalized size = 0.97

$$\frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(2-2p;-p,-p;3-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{-b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(-1+p)x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^5,x]
```

```
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -1/2*(
b + Sqrt[b^2 - 4*a*c])/(c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((-1 +
 p)*x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(c*x^2))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^4+b\,x^2+a)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^5,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^p/x^5, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**5,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p/x^5,x)
[Out] int((a + b*x^2 + c*x^4)^p/x^5, x)
```

3.1122 $\int x^4(a+bx^2+cx^4)^p\,dx$

Optimal. Leaf size=138

$$\frac{1}{5}x^5\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right.$$

```
[Out] 1/5*x^5*(c*x^4+b*x^2+a)^p*AppellF1(5/2,-p,-p,7/2,-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1155, 524}

$$\frac{1}{5}x^5\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^5*(a + b*x^2 + c*x^4)^p*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int x^4(a+bx^2+cx^4)^p\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\right)\int x^4\Big($$
$$= \frac{1}{5}x^5\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\Big(\frac{5}{2};$$

Mathematica [A]
time = 0.39, size = 166, normalized size = 1.20

$$\frac{1}{5}x^5\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^5*(a + b*x^2 + c*x^4)^p*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(5*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^4(c\,x^4+b\,x^2+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^4*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^p*x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a+bx^2+cx^4\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)**p,x)
[Out] Integral(x**4*(a + b*x**2 + c*x**4)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,{\left(c\,x^4+b\,x^2+a\right)}^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2 + c*x^4)^p,x)
[Out] int(x^4*(a + b*x^2 + c*x^4)^p, x)
```

3.1123 $\int x^2(a+bx^2+cx^4)^p\,dx$

Optimal. Leaf size=138

$$\frac{1}{3}x^3\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\,F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right.$$

```
[Out] 1/3*x^3*(c*x^4+b*x^2+a)^p*AppellF1(3/2,-p,-p,5/2,-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1155, 524}

$$\frac{1}{3}x^3\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p\,F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^3*(a + b*x^2 + c*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*(1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int x^2(a+bx^2+cx^4)^p\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\right)\int x^2($$
$$= \frac{1}{3}x^3\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(\frac{3}{2}\right.$$

Mathematica [A]
time = 0.39, size = 166, normalized size = 1.20

$$\frac{1}{3}x^3\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^3*(a + b*x^2 + c*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(3*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2(c\,x^4+b\,x^2+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^2*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx^2+cx^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Integral(x**2*(a + b*x**2 + c*x**4)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(c\,x^4+b\,x^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2 + c*x^4)^p,x)
```

```
[Out] int(x^2*(a + b*x^2 + c*x^4)^p, x)
```

3.1124 $\int (a+bx^2+cx^4)^p \, dx$

Optimal. Leaf size=133

$$x\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\right.$$

```
[Out] x*(c*x^4+b*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^p)/((1
+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]

time = 0.04, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1119, 440}

$$x\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x*(a + b*x^2 + c*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b - Sqrt[b^
2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1119

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p])), Int[(1
+ 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^p\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\right)\int \Bigg(1+$$

$$= x\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\Bigg(\frac{1}{2};-p,$$

Mathematica [A]
time = 0.12, size = 161, normalized size = 1.21

$$x\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p,x]
[Out] (x*(a + b*x^2 + c*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(c\,x^4+b\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p,x)
[Out] int((c*x^4+b*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2+cx^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,x^4+b\,x^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^p, x)
```

3.1125 $\int \frac{(a+bx^2+cx^4)^p}{x^2} dx$

Optimal. Leaf size=136

$$-\frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{x}$$

```
[Out] -(c*x^4+b*x^2+a)^p*AppellF1(-1/2,-p,-p,1/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/x/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^p)/(
(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1155, 524}

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^2,x]
```

```
[Out] -(((a + b*x^2 + c*x^4)^p*AppellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^p}{x^2}\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\right)\int \frac{\left(1+\right.}{}$$

$$= -\frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{1}{2};-p,-\right.}{x}$$

Mathematica [A]
time = 0.40, size = 164, normalized size = 1.21

$$-\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^2,x]
```

```
[Out] -(((a + b*x^2 + c*x^4)^p*AppellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(x*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^4+b\,x^2+a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^2,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**2,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**p/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^4 + b\,x^2 + a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p/x^2,x)
```

```
[Out] int((a + b*x^2 + c*x^4)^p/x^2, x)
```

3.1126 $\int \frac{(a+bx^2+cx^4)^p}{x^4} dx$

Optimal. Leaf size=138

$$-\frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3x^3}$$

```
[Out] -1/3*(c*x^4+b*x^2+a)^p*AppellF1(-3/2,-p,-p,-1/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/x^3/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
))^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]

time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1155, 524}

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^p/x^4,x]
```

```
[Out] -1/3*((a + b*x^2 + c*x^4)^p*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(x^3*(1 + (2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^p}{x^4}\,dx = \left(\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\right)\int \frac{\left(1+\right.}{b}$$

$$= -\frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{3}{2};-p,-p\right.}{3x^3}$$

Mathematica [A]
time = 0.39, size = 166, normalized size = 1.20

$$-\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2 + c*x^4)^p/x^4,x]
[Out] -1/3*((a + b*x^2 + c*x^4)^p*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(x^3*((b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^4+b\,x^2+a)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^p/x^4,x)
[Out] int((c*x^4+b*x^2+a)^p/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="fricas")
[Out] integral((c*x^4 + b*x^2 + a)^p/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**p/x**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^4+b\,x^2+a)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^p/x^4,x)
[Out] int((a + b*x^2 + c*x^4)^p/x^4, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```